Computer Algebra Independent Integration Tests

Summer 2023 edition

8-Special-functions/205-8.2-Fresnel-integral-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [218]. This is test number [205].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (218)	0.00 (0)
Mathematica	87.16 (190)	12.84 (28)
Fricas	87.16 (190)	12.84 (28)
Maple	70.64 (154)	29.36 (64)
Maxima	55.05 (120)	44.95 (98)
Sympy	54.13 (118)	45.87 (100)
Mupad	27.52 (60)	72.48 (158)
Giac	27.52 (60)	72.48 (158)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

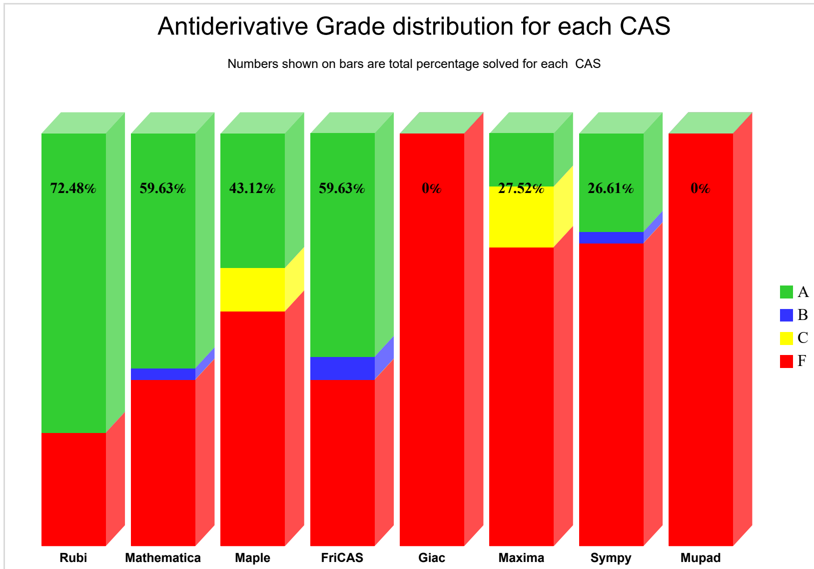
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

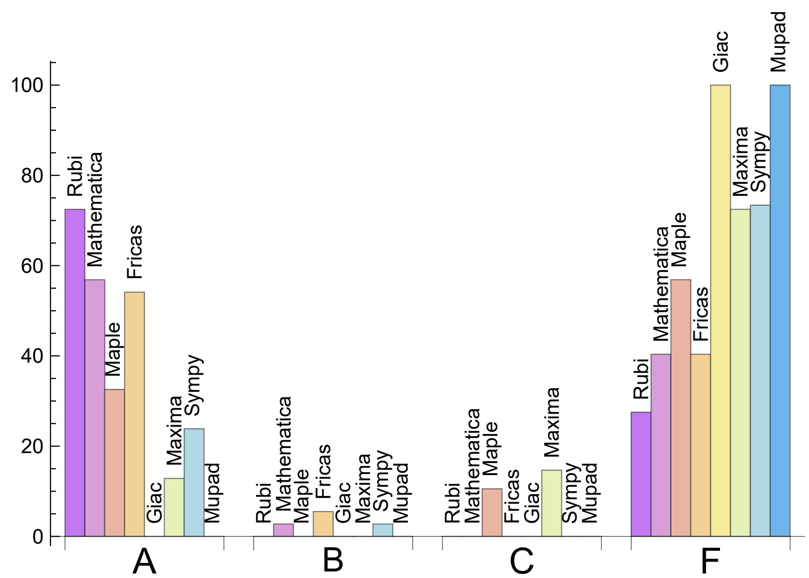
System	% A grade	% B grade	% C grade	% F grade
Rubi	72.477	0.000	0.000	27.523
Mathematica	56.881	2.752	0.000	40.367
Fricas	54.128	5.505	0.000	40.367
Maple	32.569	0.000	10.550	56.881
Sympy	23.853	2.752	0.000	73.394
Maxima	12.844	0.000	14.679	72.477
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	28	100.00	0.00	0.00
Fricas	28	100.00	0.00	0.00
Maple	64	100.00	0.00	0.00
Maxima	98	100.00	0.00	0.00
Sympy	100	100.00	0.00	0.00
Mupad	158	0.00	100.00	0.00
Giac	158	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.10
Fricas	0.26
Giac	0.27
Maxima	0.31
Mathematica	0.31
Maple	0.57
Sympy	4.35
Mupad	4.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	16.90	1.08	19.50	1.00
Giac	16.90	1.08	19.50	1.00
Sympy	49.28	1.06	20.00	1.00
Maxima	51.11	1.10	20.00	1.00
Maple	57.58	0.88	24.50	0.91
Mathematica	86.49	1.04	59.00	1.00
Fricas	97.03	1.06	53.50	1.00
Rubi	101.00	1.00	71.50	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

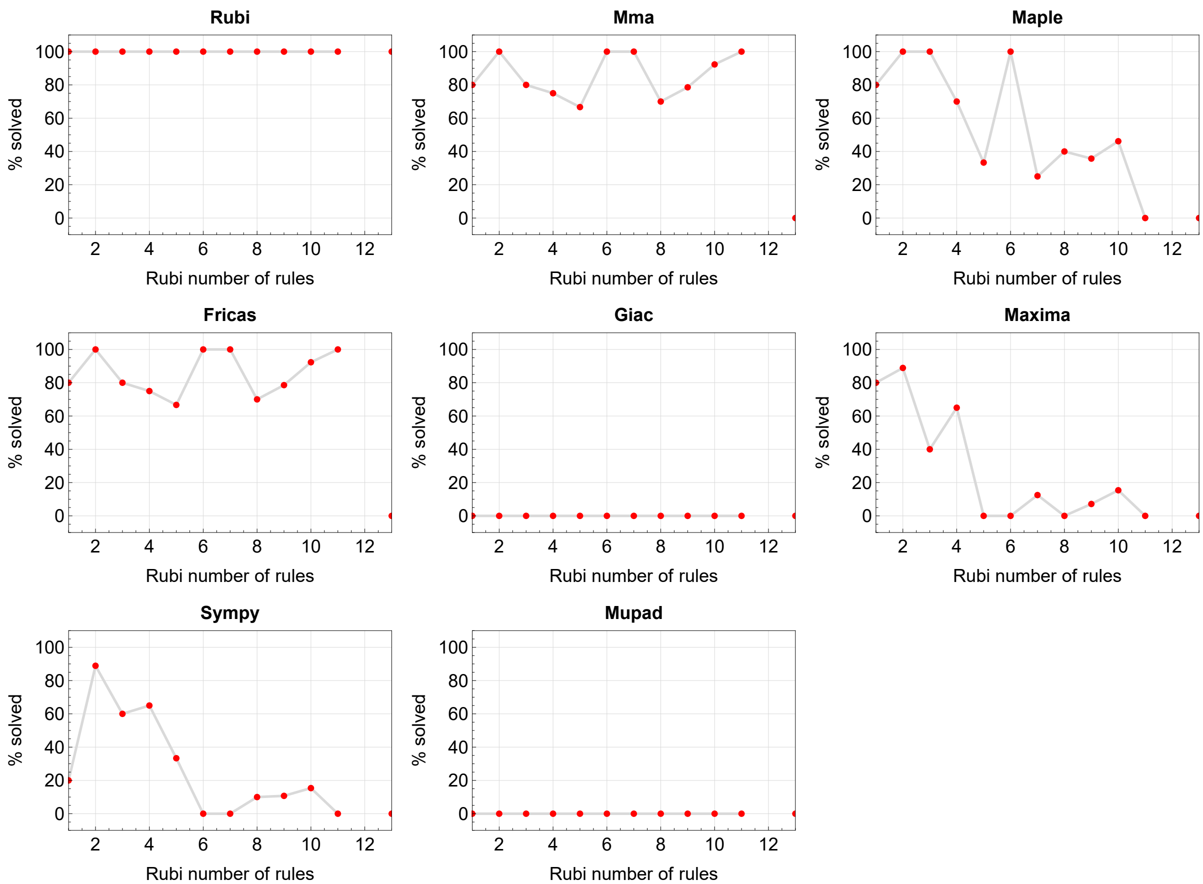


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

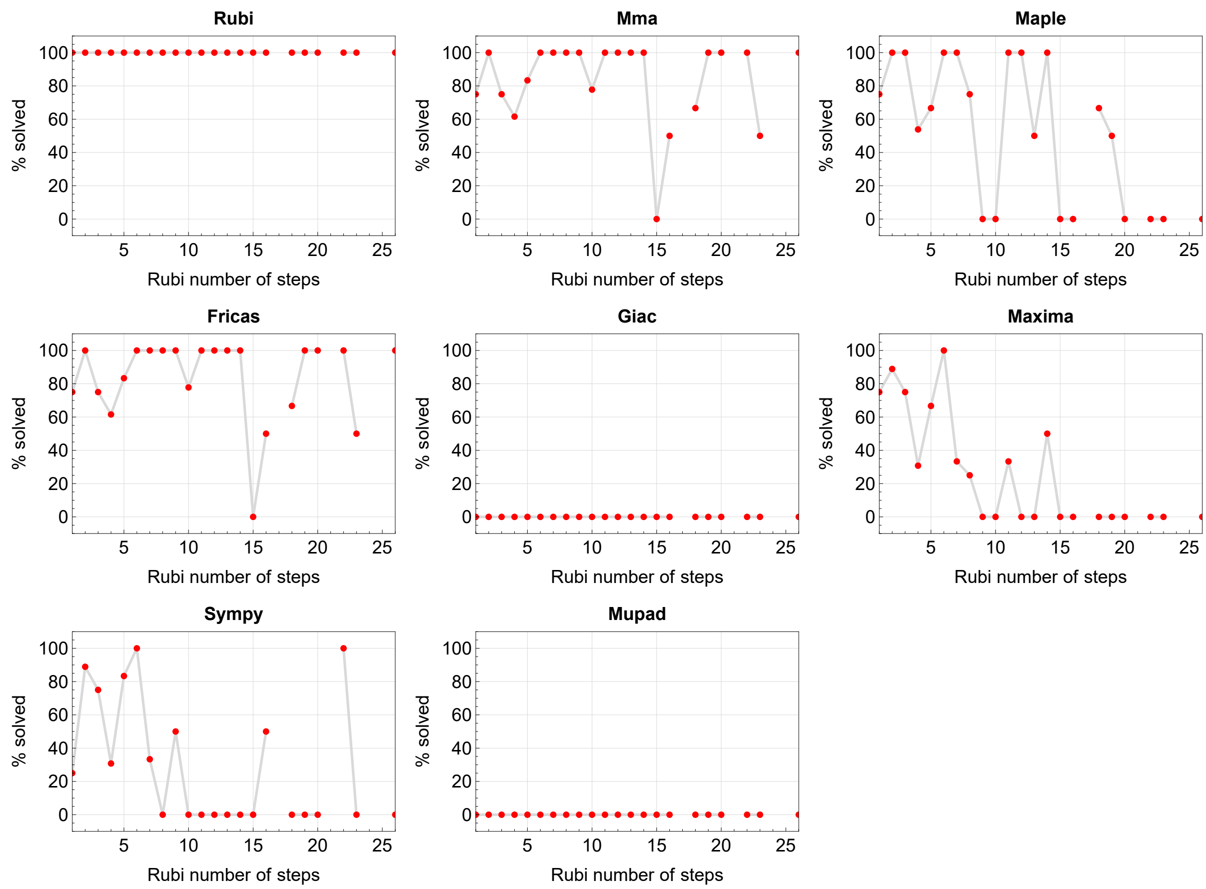


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

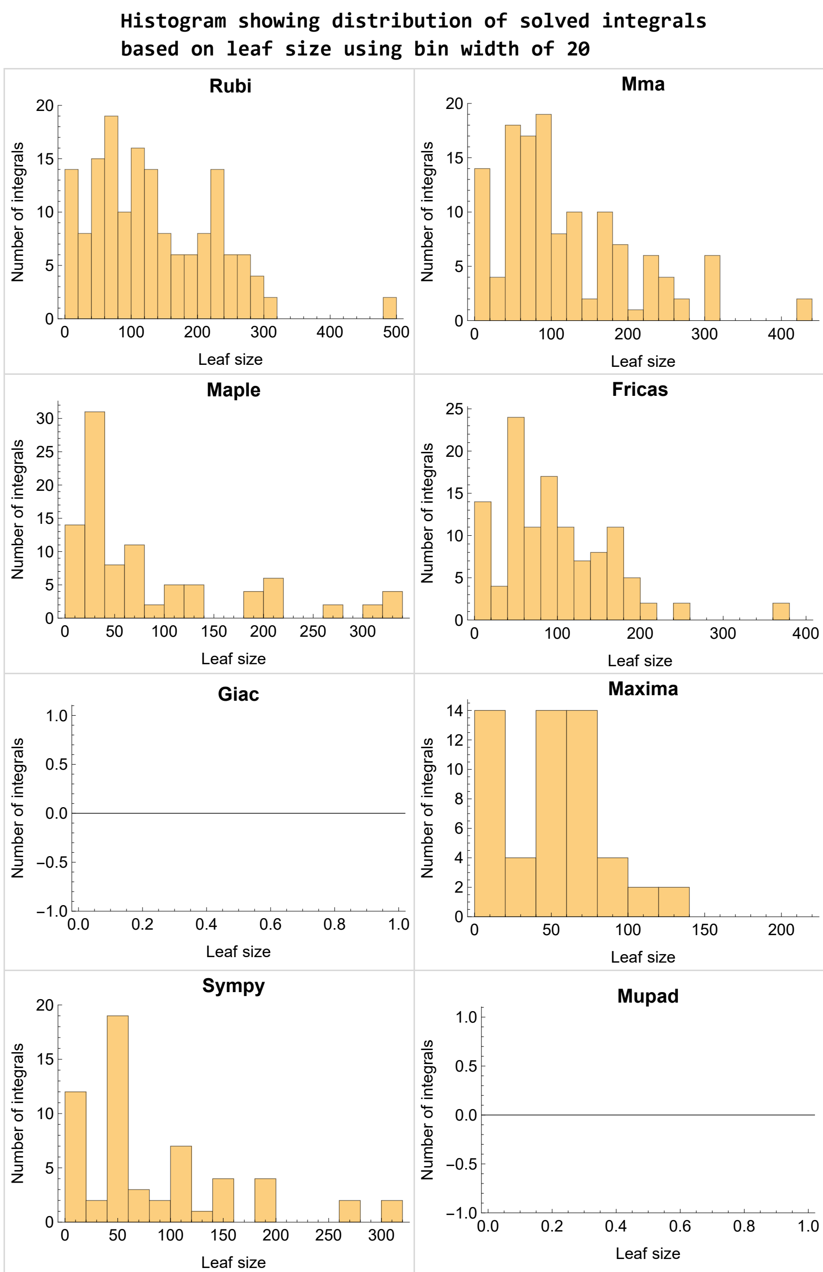


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

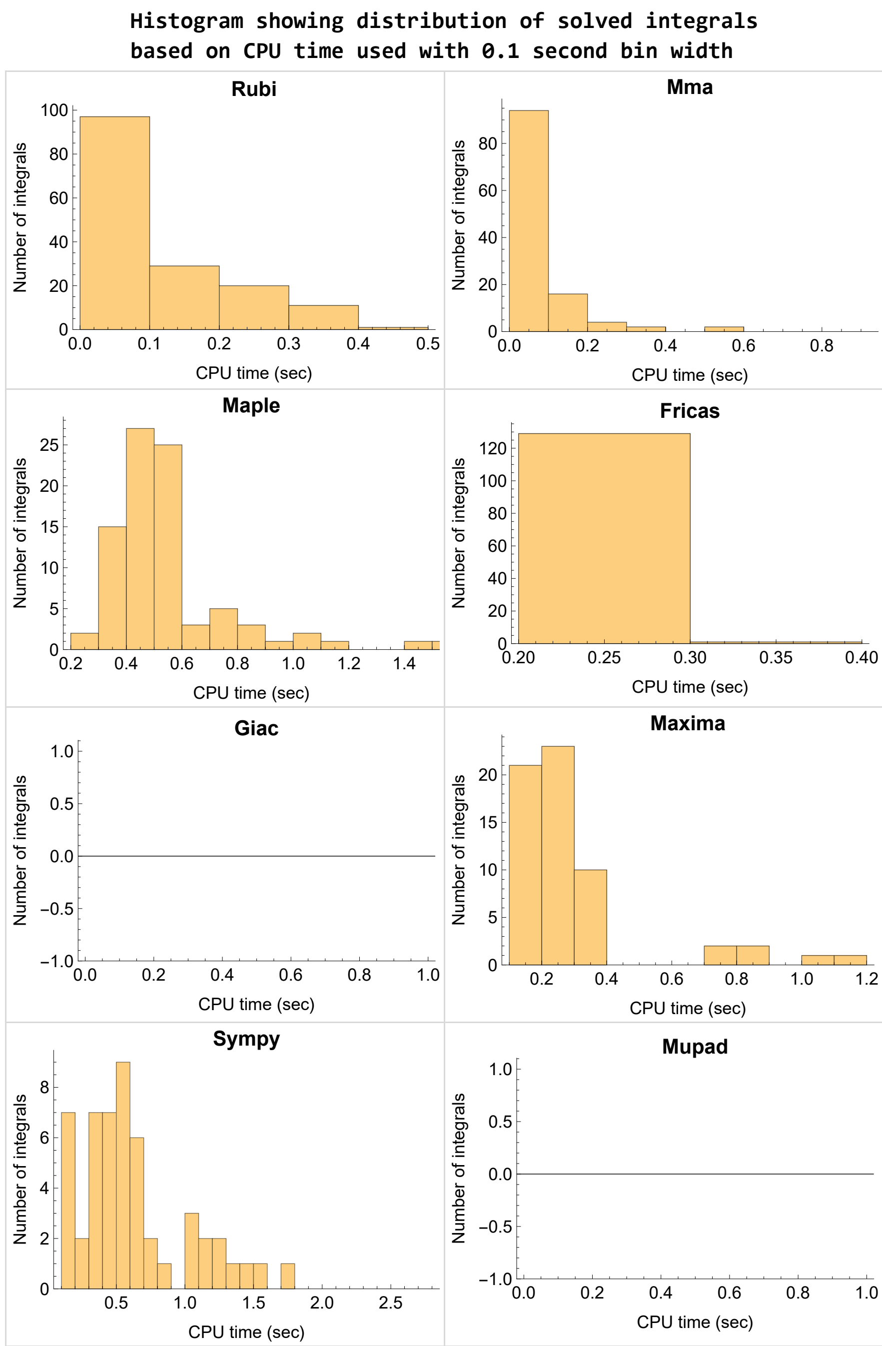


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

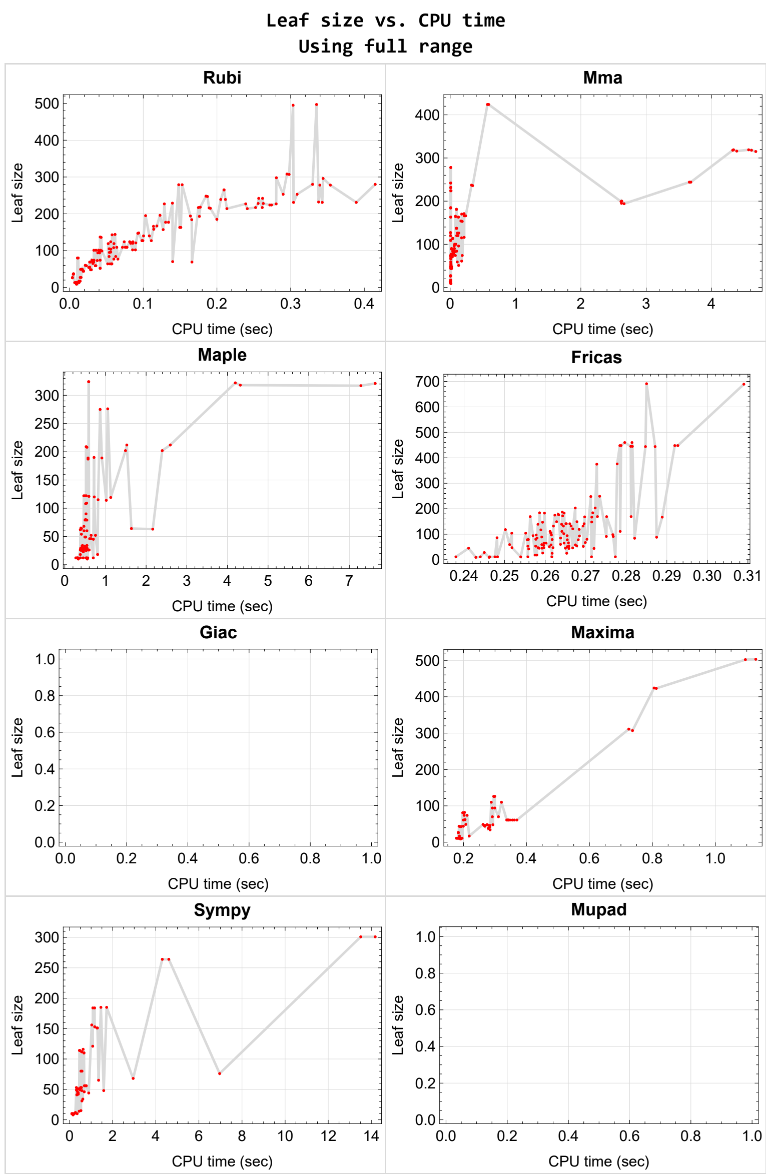


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

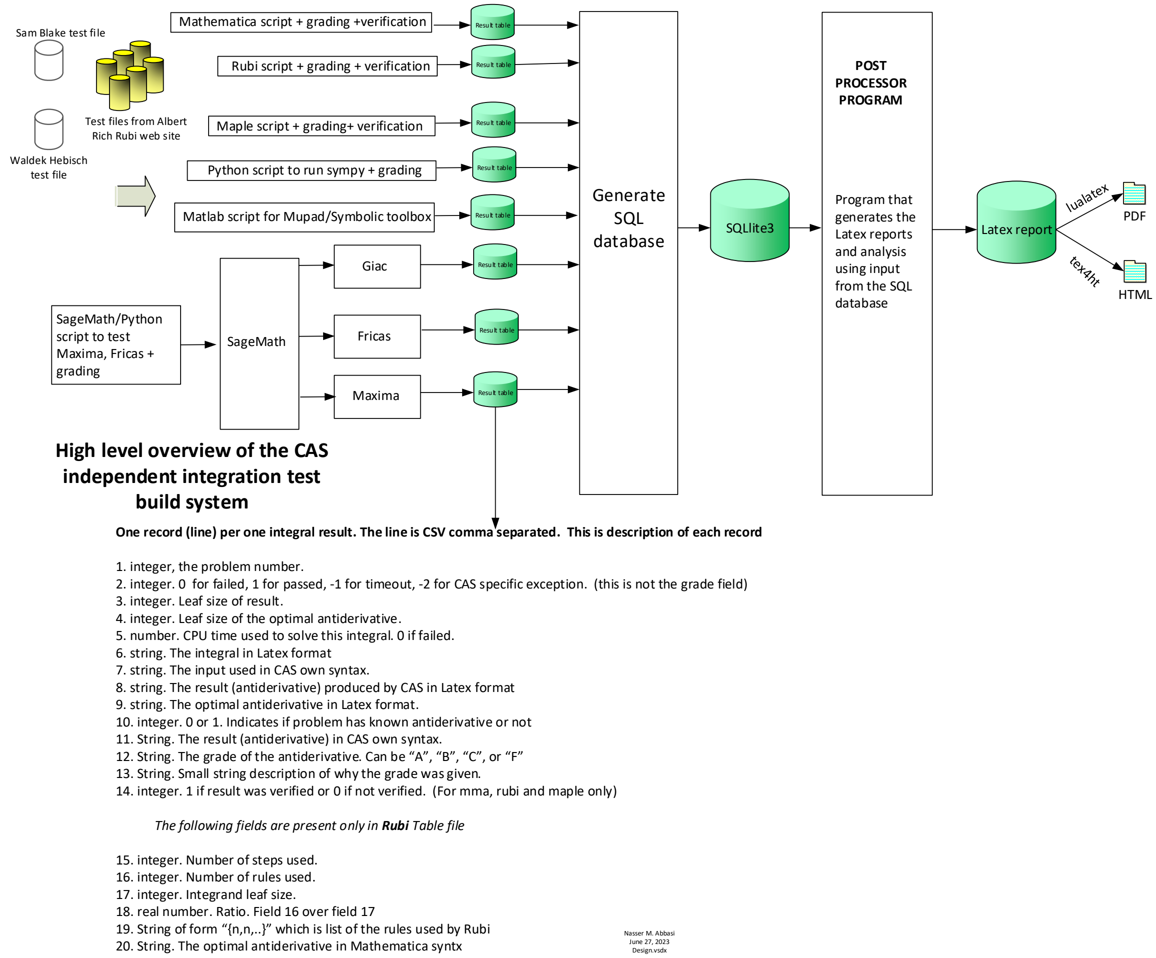
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 27, 31, 32, 34, 35, 36, 38, 43, 47, 51, 54, 55, 56, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 83, 87, 92, 93, 94, 96, 97, 98, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 134, 135, 136, 140, 141, 143, 144, 145, 147, 152, 156, 160, 163, 164, 165, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 192, 196, 201, 202, 203, 205, 206, 207, 210, 214, 218 }

B grade { 22, 28, 57, 131, 137, 166 }

C grade { }

F normal fail { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 6, 8, 9, 10, 12, 13, 19, 20, 21, 22, 25, 26, 27, 28, 32, 34, 36, 38, 51, 57, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 79, 92, 94, 96, 98, 114, 117, 118, 119, 122, 123, 128, 129, 130, 131, 134, 135, 136, 137, 141, 143, 145, 147, 160, 166, 174, 175, 176, 177, 178, 179, 181, 183, 185, 187, 188, 201, 203, 205, 207 }

B grade { }

C grade { 1, 2, 3, 4, 7, 11, 14, 15, 16, 17, 18, 110, 111, 112, 113, 115, 116, 120, 121, 124, 125, 126, 127 }

F normal fail { 31, 33, 35, 37, 43, 47, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 73, 75, 77, 83, 87, 91, 93, 95, 97, 99, 101, 105, 109, 140, 142, 144, 146, 152, 156, 158, 159, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 182, 184, 186, 192, 196, 200, 202, 204, 206, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 34, 35, 36, 38, 43, 47, 51, 57, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 83, 87, 92, 93, 94, 96, 97, 98, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 143, 144, 145, 147, 152, 156, 160, 166, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 192, 196, 201, 202, 203, 205, 206, 207, 210, 214, 218 }

B grade { 54, 55, 56, 58, 59, 60, 163, 164, 165, 167, 168, 169 }

C grade { }

F normal fail { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 2, 4, 6, 8, 22, 28, 57, 65, 66, 67, 68, 69, 70, 79, 111, 113, 115, 117, 131, 137, 166, 174, 175, 176, 177, 178, 179, 188 }

B grade { }

C grade { 1, 3, 5, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 26, 27, 110, 112, 114, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136 }

F normal fail { 9, 19, 20, 21, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 118, 128, 129, 130, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 181, 182, 183, 184, 185, 186, 187, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 65, 66, 67, 68, 69, 71, 75, 79, 93, 97, 110, 111, 112, 113, 114, 115, 116, 118, 120, 121, 122, 123, 124, 125, 126, 127, 174, 175, 176, 177, 178, 180, 184, 188, 202, 206 }

B grade { 8, 10, 70, 117, 119, 179 }

C grade { }

F normal fail { 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 72, 73, 74, 76, 77, 78, 83, 87, 91, 92, 94, 95, 96, 98, 99, 101, 105, 109, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 181, 182, 183, 185, 186, 187, 192, 196, 200, 201, 203, 204, 205, 207, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	88	29	126	84	184	0	0
N.S.	1	1.00	0.71	0.23	1.02	0.68	1.48	0.00	0.00
time (sec)	N/A	0.086	0.053	0.438	0.296	0.282	1.161	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	29	74	78	156	0	0
N.S.	1	1.00	0.76	0.27	0.68	0.72	1.43	0.00	0.00
time (sec)	N/A	0.078	0.043	0.399	0.211	0.256	1.028	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	79	29	110	85	53	0	0
N.S.	1	1.00	0.80	0.29	1.11	0.86	0.54	0.00	0.00
time (sec)	N/A	0.044	0.056	0.437	0.320	0.264	0.511	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	29	62	65	121	0	0
N.S.	1	1.00	0.85	0.35	0.74	0.77	1.44	0.00	0.00
time (sec)	N/A	0.063	0.032	0.455	0.205	0.260	1.073	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	94	58	112	0	0
N.S.	1	1.00	1.00	0.84	1.27	0.78	1.51	0.00	0.00
time (sec)	N/A	0.033	0.013	0.383	0.292	0.267	0.566	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	49	53	80	0	0
N.S.	1	1.00	1.00	0.92	0.83	0.90	1.36	0.00	0.00
time (sec)	N/A	0.036	0.010	0.408	0.262	0.258	0.528	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	29	70	51	53	0	0
N.S.	1	1.00	1.00	0.59	1.43	1.04	1.08	0.00	0.00
time (sec)	N/A	0.017	0.009	0.434	0.311	0.265	0.316	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	26	26	48	0	0
N.S.	1	1.00	1.00	0.96	1.00	1.00	1.85	0.00	0.00
time (sec)	N/A	0.004	0.005	0.525	0.184	0.259	0.572	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	0	29	0	0	46	0	0
N.S.	1	1.00	0.00	0.40	0.00	0.00	0.63	0.00	0.00
time (sec)	N/A	0.031	0.000	0.549	0.000	0.000	0.368	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	38	25	42	0	0
N.S.	1	1.00	1.00	0.89	1.41	0.93	1.56	0.00	0.00
time (sec)	N/A	0.014	0.010	0.444	0.278	0.265	0.360	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	27	61	45	51	0	0
N.S.	1	1.00	1.00	0.61	1.39	1.02	1.16	0.00	0.00
time (sec)	N/A	0.018	0.010	0.392	0.370	0.265	0.421	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	46	42	44	56	0	0
N.S.	1	1.00	1.00	0.88	0.81	0.85	1.08	0.00	0.00
time (sec)	N/A	0.042	0.012	0.611	0.283	0.266	0.775	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	65	61	54	110	0	0
N.S.	1	1.00	1.00	0.94	0.88	0.78	1.59	0.00	0.00
time (sec)	N/A	0.027	0.012	0.396	0.355	0.266	0.670	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	29	48	65	46	0	0
N.S.	1	1.00	1.00	0.38	0.62	0.84	0.60	0.00	0.00
time (sec)	N/A	0.060	0.016	0.467	0.274	0.265	0.682	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	76	29	61	80	56	0	0
N.S.	1	1.00	0.81	0.31	0.65	0.85	0.60	0.00	0.00
time (sec)	N/A	0.041	0.046	0.388	0.361	0.270	0.748	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	85	79	46	78	68	0	0
N.S.	1	1.00	0.83	0.77	0.45	0.76	0.67	0.00	0.00
time (sec)	N/A	0.086	0.048	0.498	0.268	0.268	2.950	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	84	29	61	80	185	0	0
N.S.	1	1.00	0.71	0.24	0.51	0.67	1.55	0.00	0.00
time (sec)	N/A	0.060	0.045	0.506	0.337	0.258	1.720	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	29	48	91	48	0	0
N.S.	1	1.00	0.76	0.23	0.38	0.72	0.38	0.00	0.00
time (sec)	N/A	0.111	0.129	0.454	0.276	0.275	1.584	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	424	276	0	376	0	0	0
N.S.	1	1.00	1.43	0.93	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.344	0.589	1.055	0.000	0.278	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	189	0	248	0	0	0
N.S.	1	1.00	1.22	0.98	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.176	0.344	0.911	0.000	0.271	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	61	109	0	132	0	0	0
N.S.	1	1.00	0.50	0.90	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.090	0.176	0.537	0.000	0.264	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	43	45	0	0	0
N.S.	1	1.00	2.47	0.92	1.19	1.25	0.00	0.00	0.00
time (sec)	N/A	0.005	0.026	0.493	0.189	0.241	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14
time (sec)	N/A	0.013	0.021	0.344	0.796	0.253	0.466	0.263	4.941

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14
time (sec)	N/A	0.011	2.304	0.310	0.458	0.271	0.582	0.276	4.817

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	166	189	503	175	0	0	0
N.S.	1	1.00	0.72	0.83	2.20	0.76	0.00	0.00	0.00
time (sec)	N/A	0.140	0.238	0.571	1.129	0.262	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	115	121	424	147	0	0	0
N.S.	1	1.00	0.78	0.82	2.88	1.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.174	0.593	0.805	0.259	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	51	80	307	104	0	0	0
N.S.	1	1.00	0.53	0.83	3.20	1.08	0.00	0.00	0.00
time (sec)	N/A	0.056	0.130	0.510	0.737	0.255	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	43	45	0	0	0
N.S.	1	1.00	2.47	0.92	1.19	1.25	0.00	0.00	0.00
time (sec)	N/A	0.005	0.019	0.423	0.192	0.259	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.012	0.015	0.171	0.780	0.253	0.378	0.256	4.650

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.010	1.498	0.157	0.445	0.268	0.283	0.262	4.675

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	181	0	0	183	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.309	0.096	0.000	0.000	0.260	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	171	324	0	184	0	0	0
N.S.	1	1.00	0.72	1.36	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.212	0.213	0.585	0.000	0.272	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	137	208	0	149	0	0	0
N.S.	1	1.00	0.77	1.18	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.130	0.101	0.544	0.000	0.263	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	113	0	0	117	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.100	0.038	0.000	0.000	0.259	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	111	0	0	0
N.S.	1	1.00	0.81	0.98	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.074	0.088	0.466	0.000	0.279	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	60	0	0	0
N.S.	1	1.00	1.00	0.89	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.027	0.012	0.513	0.000	0.264	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.012	0.014	0.025	0.227	0.247	1.082	0.260	4.655

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.025	0.019	0.070	0.221	0.265	1.185	0.262	4.673

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.026	0.017	0.066	0.226	0.256	1.066	0.257	4.833

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.058	0.021	0.091	0.225	0.250	1.123	0.253	4.805

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	111	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.098	0.005	0.000	0.000	0.261	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.101	0.021	0.075	0.214	0.257	1.401	0.259	4.802

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.154	0.016	0.068	0.214	0.259	1.327	0.261	4.848

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.168	0.025	0.066	0.221	0.261	1.618	0.257	4.746

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	187	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.262	0.008	0.000	0.000	0.264	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.240	0.024	0.072	0.213	0.247	2.499	0.255	4.848

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	497	497	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	60	0	89	0	0	0
N.S.	1	1.00	0.96	0.86	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.140	0.011	0.470	0.000	0.259	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.021	0.027	0.189	0.222	0.249	0.422	0.263	4.877

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	29	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.81	0.94	1.12	1.12
time (sec)	N/A	0.018	0.062	0.261	0.227	0.255	0.596	0.262	4.847

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	319	0	0	448	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.389	4.568	0.000	0.000	0.292	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	319	0	0	448	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.280	4.337	0.000	0.000	0.279	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	316	0	0	445	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.241	4.386	0.000	0.000	0.281	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	164	63	81	119	0	0	0
N.S.	1	1.00	2.52	0.97	1.25	1.83	0.00	0.00	0.00
time (sec)	N/A	0.032	0.084	2.163	0.197	0.268	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	195	0	0	444	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.261	2.622	0.000	0.000	0.285	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	228	200	0	0	460	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.263	2.626	0.000	0.000	0.281	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	691	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.415	3.683	0.000	0.000	0.285	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.009	0.005	0.296	0.188	0.238	0.220	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.008	0.003	0.270	0.180	0.256	0.155	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	8	0	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.89	0.00	0.00
time (sec)	N/A	0.009	0.012	0.332	0.191	0.243	0.162	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	0	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	0.00	0.00
time (sec)	N/A	0.010	0.004	0.384	0.184	0.248	0.351	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	14	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.08	0.00	0.00
time (sec)	N/A	0.010	0.005	0.535	0.177	0.244	0.443	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	18	31	0	0
N.S.	1	1.00	1.00	1.06	1.00	1.06	1.82	0.00	0.00
time (sec)	N/A	0.013	0.007	0.381	0.187	0.257	0.573	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	232	0	0	169	301	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.73	1.30	0.00	0.00
time (sec)	N/A	0.338	0.010	0.000	0.000	0.256	13.508	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	153	318	0	167	0	0	0
N.S.	1	1.00	0.71	1.47	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.189	0.185	4.317	0.000	0.289	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	120	202	0	132	0	0	0
N.S.	1	1.00	0.76	1.28	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.114	0.119	1.492	0.000	0.269	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	105	151	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.88	1.26	0.00	0.00
time (sec)	N/A	0.083	0.006	0.000	0.000	0.261	1.270	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	115	0	94	0	0	0
N.S.	1	1.00	0.79	1.10	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.054	0.076	0.818	0.000	0.265	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	46	0	47	0	0	0
N.S.	1	1.00	0.90	0.94	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.015	0.019	0.645	0.000	0.260	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.007	0.000	0.309	0.191	0.262	0.104	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.011	0.023	0.137	0.281	0.274	1.065	0.275	4.784

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.013	0.023	0.133	0.275	0.258	1.009	0.274	4.875

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.044	0.024	0.147	0.275	0.250	1.265	0.286	4.837

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	98	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.076	0.007	0.000	0.000	0.277	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.085	0.026	0.135	0.270	0.251	3.134	0.279	4.821

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.137	0.026	0.125	0.280	0.258	5.694	0.286	4.831

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.147	0.023	0.137	0.263	0.251	11.356	0.281	4.774

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	172	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.274	0.014	0.000	0.000	0.264	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.268	0.030	0.125	0.279	0.252	36.284	0.264	4.976

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.339	0.027	0.141	0.262	0.256	66.992	0.270	4.938

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.012	0.053	0.151	0.290	0.258	1.106	0.298	4.832

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	307	307	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	163	321	0	169	0	0	0
N.S.	1	1.00	0.75	1.48	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.175	0.127	7.636	0.000	0.281	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	184	0	0	141	264	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.77	1.43	0.00	0.00
time (sec)	N/A	0.166	0.008	0.000	0.000	0.265	4.604	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	126	212	0	139	0	0	0
N.S.	1	1.00	0.76	1.28	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.114	0.133	2.594	0.000	0.258	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	90	119	0	97	0	0	0
N.S.	1	1.00	0.83	1.10	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.060	0.064	1.129	0.000	0.258	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	58	114	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.79	1.56	0.00	0.00
time (sec)	N/A	0.038	0.005	0.000	0.000	0.261	0.474	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	52	0	53	0	0	0
N.S.	1	1.00	0.81	0.88	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.023	0.021	0.754	0.000	0.256	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.011	0.026	0.136	0.266	0.242	1.077	0.273	4.795

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	46	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.028	0.005	0.000	0.000	0.267	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.038	0.026	0.139	0.276	0.254	1.191	0.269	4.787

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.063	0.026	0.145	0.271	0.263	1.680	0.280	4.748

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.094	0.026	0.137	0.267	0.273	3.082	0.289	4.977

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	141	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.151	0.008	0.000	0.000	0.265	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.136	0.025	0.138	0.268	0.258	11.227	0.283	4.717

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.221	0.026	0.155	0.283	0.265	21.230	0.271	4.763

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.217	0.027	0.135	0.289	0.247	38.007	0.282	4.701

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	203	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.340	0.009	0.000	0.000	0.267	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	26	126	85	184	0	0
N.S.	1	1.00	0.72	0.21	1.02	0.69	1.48	0.00	0.00
time (sec)	N/A	0.056	0.053	0.598	0.299	0.258	1.080	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	26	74	79	153	0	0
N.S.	1	1.00	0.76	0.24	0.68	0.72	1.40	0.00	0.00
time (sec)	N/A	0.072	0.041	0.579	0.202	0.256	1.165	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	26	110	86	49	0	0
N.S.	1	1.00	0.81	0.26	1.11	0.87	0.49	0.00	0.00
time (sec)	N/A	0.040	0.048	0.394	0.288	0.248	0.485	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	26	61	66	116	0	0
N.S.	1	1.00	0.85	0.31	0.73	0.79	1.38	0.00	0.00
time (sec)	N/A	0.054	0.031	0.401	0.200	0.266	0.628	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	94	59	112	0	0
N.S.	1	1.00	1.00	0.84	1.27	0.80	1.51	0.00	0.00
time (sec)	N/A	0.030	0.013	0.382	0.300	0.251	0.555	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	26	49	54	80	0	0
N.S.	1	1.00	1.00	0.44	0.83	0.92	1.36	0.00	0.00
time (sec)	N/A	0.035	0.011	0.380	0.207	0.268	0.576	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	26	70	51	49	0	0
N.S.	1	1.00	1.00	0.53	1.43	1.04	1.00	0.00	0.00
time (sec)	N/A	0.017	0.010	0.389	0.292	0.261	0.319	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	27	28	44	0	0
N.S.	1	1.00	1.00	0.96	1.00	1.04	1.63	0.00	0.00
time (sec)	N/A	0.004	0.004	0.476	0.183	0.245	0.405	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	0	23	0	0	41	0	0
N.S.	1	1.00	0.00	0.33	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.033	0.000	0.457	0.000	0.000	0.336	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	34	25	53	0	0
N.S.	1	1.00	1.00	0.89	1.26	0.93	1.96	0.00	0.00
time (sec)	N/A	0.016	0.010	0.438	0.284	0.262	0.551	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	26	61	42	51	0	0
N.S.	1	1.00	1.00	0.59	1.39	0.95	1.16	0.00	0.00
time (sec)	N/A	0.019	0.010	0.443	0.339	0.256	0.358	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	26	44	44	42	0	0
N.S.	1	1.00	1.00	0.50	0.85	0.85	0.81	0.00	0.00
time (sec)	N/A	0.041	0.014	0.428	0.268	0.272	0.403	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	64	61	56	110	0	0
N.S.	1	1.00	1.00	0.93	0.88	0.81	1.59	0.00	0.00
time (sec)	N/A	0.028	0.012	0.410	0.342	0.260	0.663	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	46	65	65	0	0
N.S.	1	1.00	1.00	0.88	0.60	0.84	0.84	0.00	0.00
time (sec)	N/A	0.066	0.014	0.469	0.265	0.260	1.346	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	26	61	79	56	0	0
N.S.	1	1.00	0.79	0.28	0.65	0.84	0.60	0.00	0.00
time (sec)	N/A	0.038	0.079	0.394	0.342	0.262	0.688	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	84	26	48	78	44	0	0
N.S.	1	1.00	0.82	0.25	0.47	0.76	0.43	0.00	0.00
time (sec)	N/A	0.089	0.100	0.424	0.293	0.265	0.889	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	85	26	61	81	185	0	0
N.S.	1	1.00	0.71	0.22	0.51	0.68	1.55	0.00	0.00
time (sec)	N/A	0.052	0.045	0.428	0.348	0.258	1.453	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	90	46	91	76	0	0
N.S.	1	1.00	0.76	0.71	0.36	0.72	0.60	0.00	0.00
time (sec)	N/A	0.100	0.062	0.521	0.283	0.277	6.963	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	424	275	0	375	0	0	0
N.S.	1	1.00	1.42	0.92	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.281	0.571	0.868	0.000	0.273	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	237	190	0	249	0	0	0
N.S.	1	1.00	1.22	0.98	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.164	0.327	0.718	0.000	0.273	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	74	108	0	132	0	0	0
N.S.	1	1.00	0.61	0.89	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.083	0.185	0.512	0.000	0.266	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	44	47	0	0	0
N.S.	1	1.00	2.43	0.92	1.19	1.27	0.00	0.00	0.00
time (sec)	N/A	0.005	0.023	0.544	0.199	0.258	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14
time (sec)	N/A	0.012	0.020	0.314	0.748	0.261	0.381	0.270	4.652

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14
time (sec)	N/A	0.011	1.500	0.283	0.456	0.250	0.577	0.264	4.665

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	166	187	502	176	0	0	0
N.S.	1	1.00	0.73	0.82	2.21	0.78	0.00	0.00	0.00
time (sec)	N/A	0.129	0.215	0.571	1.096	0.263	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	116	122	423	148	0	0	0
N.S.	1	1.00	0.78	0.82	2.86	1.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.214	0.527	0.812	0.271	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	59	79	311	104	0	0	0
N.S.	1	1.00	0.62	0.83	3.27	1.09	0.00	0.00	0.00
time (sec)	N/A	0.054	0.109	0.542	0.725	0.252	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	44	47	0	0	0
N.S.	1	1.00	2.43	0.92	1.19	1.27	0.00	0.00	0.00
time (sec)	N/A	0.005	0.020	0.434	0.186	0.252	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.009	0.017	0.175	0.767	0.262	0.314	0.263	4.587

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.009	0.941	0.128	0.438	0.247	0.260	0.258	4.637

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	181	0	0	183	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.290	0.098	0.000	0.000	0.265	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	170	324	0	184	0	0	0
N.S.	1	1.00	0.71	1.36	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.205	0.177	0.589	0.000	0.259	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	137	209	0	149	0	0	0
N.S.	1	1.00	0.77	1.18	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.134	0.093	0.524	0.000	0.268	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	114	0	0	118	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.108	0.039	0.000	0.000	0.250	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	111	0	0	0
N.S.	1	1.00	0.81	0.98	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.082	0.068	0.495	0.000	0.264	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	0	59	0	0	0
N.S.	1	1.00	1.00	0.91	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.026	0.011	0.493	0.000	0.259	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.012	0.016	0.022	0.219	0.262	1.110	0.261	4.788

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.045	0.020	0.069	0.218	0.262	1.039	0.267	4.907

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.043	0.015	0.063	0.215	0.249	1.085	0.274	4.918

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.067	0.023	0.063	0.217	0.257	1.110	0.256	4.959

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	102	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.099	0.006	0.000	0.000	0.265	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.096	0.022	0.073	0.214	0.251	1.197	0.268	4.902

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.147	0.018	0.069	0.222	0.260	1.294	0.260	4.952

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.158	0.022	0.063	0.225	0.256	1.494	0.250	5.111

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	178	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.257	0.009	0.000	0.000	0.263	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.229	0.021	0.062	0.256	0.257	2.175	0.263	4.719

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	495	495	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	60	0	88	0	0	0
N.S.	1	1.00	0.96	0.87	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.166	0.010	0.543	0.000	0.287	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.018	0.032	0.171	0.224	0.278	0.436	0.250	4.751

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	29	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.81	0.94	1.12	1.12
time (sec)	N/A	0.019	0.057	0.239	0.230	0.263	0.571	0.273	4.641

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	318	0	0	448	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.304	4.612	0.000	0.000	0.293	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	318	0	0	448	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.239	4.324	0.000	0.000	0.278	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	315	0	0	445	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.213	4.677	0.000	0.000	0.282	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	165	64	82	121	0	0	0
N.S.	1	1.00	2.50	0.97	1.24	1.83	0.00	0.00	0.00
time (sec)	N/A	0.032	0.081	1.638	0.203	0.267	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	444	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.252	2.661	0.000	0.000	0.287	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	228	199	0	0	460	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.256	2.615	0.000	0.000	0.280	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	689	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.330	3.661	0.000	0.000	0.309	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.010	0.009	0.522	0.188	0.277	0.181	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.008	0.006	0.451	0.187	0.271	0.110	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	10	0	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	1.11	0.00	0.00
time (sec)	N/A	0.010	0.010	0.551	0.190	0.246	0.113	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	12	0	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	1.09	0.00	0.00
time (sec)	N/A	0.010	0.006	0.564	0.183	0.248	0.272	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.00	0.00
time (sec)	N/A	0.013	0.006	0.700	0.195	0.246	0.527	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	18	34	0	0
N.S.	1	1.00	1.00	1.06	1.00	1.06	2.00	0.00	0.00
time (sec)	N/A	0.014	0.010	0.806	0.218	0.258	0.615	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	231	0	0	169	301	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.73	1.30	0.00	0.00
time (sec)	N/A	0.343	0.008	0.000	0.000	0.275	14.179	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	154	317	0	167	0	0	0
N.S.	1	1.00	0.72	1.47	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.191	0.164	7.284	0.000	0.272	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	120	202	0	132	0	0	0
N.S.	1	1.00	0.76	1.29	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.127	0.100	2.396	0.000	0.262	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	105	151	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.88	1.26	0.00	0.00
time (sec)	N/A	0.086	0.008	0.000	0.000	0.261	1.294	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	83	114	0	94	0	0	0
N.S.	1	1.00	0.80	1.10	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.056	0.066	1.020	0.000	0.257	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	45	0	47	0	0	0
N.S.	1	1.00	0.92	0.94	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.017	0.023	0.702	0.000	0.262	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.008	0.000	0.324	0.191	0.254	0.113	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.011	0.023	0.134	0.278	0.261	0.963	0.279	4.781

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.012	0.023	0.126	0.283	0.253	0.931	0.277	4.667

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.040	0.023	0.129	0.290	0.258	1.169	0.289	4.649

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	93	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.077	0.008	0.000	0.000	0.268	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.085	0.024	0.132	0.292	0.253	2.956	0.280	4.641

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.138	0.024	0.131	0.289	0.260	5.583	0.286	4.703

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.149	0.025	0.126	0.282	0.267	11.051	0.277	4.610

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	168	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.272	0.008	0.000	0.000	0.270	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.228	0.028	0.126	0.294	0.257	36.966	0.281	4.620

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.334	0.023	0.125	0.296	0.257	65.464	0.281	4.715

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.052	0.129	0.284	0.257	1.129	0.306	4.700

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	163	322	0	169	0	0	0
N.S.	1	1.00	0.75	1.48	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.177	0.125	4.194	0.000	0.273	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	185	0	0	141	264	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.76	1.43	0.00	0.00
time (sec)	N/A	0.200	0.008	0.000	0.000	0.266	4.301	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	126	212	0	139	0	0	0
N.S.	1	1.00	0.75	1.27	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.118	0.111	1.525	0.000	0.264	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	90	120	0	97	0	0	0
N.S.	1	1.00	0.83	1.10	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.064	0.064	0.724	0.000	0.262	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	67	114	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.91	1.54	0.00	0.00
time (sec)	N/A	0.042	0.006	0.000	0.000	0.270	0.458	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	52	0	52	0	0	0
N.S.	1	1.00	0.80	0.87	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.021	0.021	0.647	0.000	0.269	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.012	0.023	0.121	0.309	0.268	1.178	0.274	4.715

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	45	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.030	0.007	0.000	0.000	0.259	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.037	0.028	0.128	0.277	0.264	1.277	0.275	4.821

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.058	0.024	0.119	0.267	0.252	1.801	0.272	4.814

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.080	0.026	0.126	0.276	0.264	3.027	0.286	4.805

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	141	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.149	0.009	0.000	0.000	0.270	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.134	0.024	0.138	0.274	0.248	11.018	0.297	4.775

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.219	0.026	0.135	0.262	0.262	20.384	0.292	4.781

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.218	0.024	0.130	0.271	0.250	36.669	0.285	4.769

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	203	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.354	0.010	0.000	0.000	0.272	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [140] had the largest ratio of [1.10000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	8	0.500
2	A	6	4	1.00	8	0.500
3	A	5	4	1.00	8	0.500
4	A	5	4	1.00	8	0.500
5	A	4	4	1.00	8	0.500
6	A	4	4	1.00	8	0.500
7	A	3	3	1.00	6	0.500
8	A	1	1	1.00	4	0.250
9	A	3	3	1.00	8	0.375
10	A	2	2	1.00	8	0.250
11	A	3	3	1.00	8	0.375
12	A	4	4	1.00	8	0.500
13	A	4	4	1.00	8	0.500
14	A	5	4	1.00	8	0.500
15	A	5	4	1.00	8	0.500
16	A	6	4	1.00	8	0.500
17	A	6	4	1.00	8	0.500
18	A	7	4	1.00	8	0.500
19	A	14	10	1.00	14	0.714
20	A	11	9	1.00	14	0.643
21	A	8	7	1.00	12	0.583
22	A	1	1	1.00	6	0.167
23	N/A	0	0	1.00	14	0.000
24	N/A	0	0	1.00	14	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
25	A	14	10	1.00	10	1.000
26	A	11	9	1.00	10	0.900
27	A	8	7	1.00	8	0.875
28	A	1	1	1.00	6	0.167
29	N/A	0	0	1.00	10	0.000
30	N/A	0	0	1.00	10	0.000
31	A	23	10	1.00	10	1.000
32	A	19	10	1.00	10	1.000
33	A	16	9	1.00	10	0.900
34	A	12	9	1.00	10	0.900
35	A	10	9	1.00	10	0.900
36	A	8	6	1.00	10	0.600
37	A	5	5	1.00	8	0.625
38	A	4	4	1.00	6	0.667
39	N/A	0	0	1.00	10	0.000
40	N/A	0	0	1.00	10	0.000
41	N/A	0	0	1.00	10	0.000
42	N/A	0	0	1.00	10	0.000
43	A	9	9	1.00	10	0.900
44	N/A	0	0	1.00	10	0.000
45	N/A	0	0	1.00	10	0.000
46	N/A	0	0	1.00	10	0.000
47	A	20	10	1.00	10	1.000
48	N/A	0	0	1.00	10	0.000
49	A	18	13	1.00	16	0.812
50	A	10	9	1.00	14	0.643
51	A	4	3	1.00	8	0.375
52	N/A	0	0	1.00	16	0.000
53	N/A	0	0	1.00	16	0.000
54	A	10	7	1.00	17	0.412
55	A	10	7	1.00	15	0.467
56	A	10	7	1.00	13	0.538
57	A	3	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	10	7	1.00	17	0.412
59	A	10	7	1.00	17	0.412
60	A	10	7	1.00	19	0.368
61	A	4	4	1.00	22	0.182
62	A	4	4	1.00	22	0.182
63	A	4	4	1.00	19	0.210
64	A	4	4	1.00	19	0.210
65	A	2	2	1.00	19	0.105
66	A	2	2	1.00	17	0.118
67	A	2	2	1.00	19	0.105
68	A	2	2	1.00	19	0.105
69	A	2	2	1.00	19	0.105
70	A	2	2	1.00	19	0.105
71	A	22	9	1.00	20	0.450
72	A	18	9	1.00	20	0.450
73	A	15	8	1.00	20	0.400
74	A	11	8	1.00	20	0.400
75	A	9	8	1.00	20	0.400
76	A	7	5	1.00	20	0.250
77	A	4	4	1.00	20	0.200
78	A	2	2	1.00	18	0.111
79	A	2	2	1.00	17	0.118
80	N/A	0	0	1.00	20	0.000
81	N/A	0	0	1.00	20	0.000
82	N/A	0	0	1.00	20	0.000
83	A	8	8	1.00	20	0.400
84	N/A	0	0	1.00	20	0.000
85	N/A	0	0	1.00	20	0.000
86	N/A	0	0	1.00	20	0.000
87	A	19	9	1.00	20	0.450
88	N/A	0	0	1.00	20	0.000
89	N/A	0	0	1.00	20	0.000
90	N/A	0	0	1.00	19	0.000
91	A	23	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
92	A	18	8	1.00	20	0.400
93	A	16	9	1.00	20	0.450
94	A	13	9	1.00	20	0.450
95	A	10	8	1.00	20	0.400
96	A	7	6	1.00	20	0.300
97	A	5	5	1.00	20	0.250
98	A	4	3	1.00	18	0.167
99	A	1	1	1.00	17	0.059
100	N/A	0	0	1.00	20	0.000
101	A	4	4	1.00	20	0.200
102	N/A	0	0	1.00	20	0.000
103	N/A	0	0	1.00	20	0.000
104	N/A	0	0	1.00	20	0.000
105	A	13	9	1.00	20	0.450
106	N/A	0	0	1.00	20	0.000
107	N/A	0	0	1.00	20	0.000
108	N/A	0	0	1.00	20	0.000
109	A	26	9	1.00	20	0.450
110	A	6	4	1.00	8	0.500
111	A	6	4	1.00	8	0.500
112	A	5	4	1.00	8	0.500
113	A	5	4	1.00	8	0.500
114	A	4	4	1.00	8	0.500
115	A	4	4	1.00	8	0.500
116	A	3	3	1.00	6	0.500
117	A	1	1	1.00	4	0.250
118	A	3	3	1.00	8	0.375
119	A	2	2	1.00	8	0.250
120	A	3	3	1.00	8	0.375
121	A	4	4	1.00	8	0.500
122	A	4	4	1.00	8	0.500
123	A	5	4	1.00	8	0.500
124	A	5	4	1.00	8	0.500
125	A	6	4	1.00	8	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
126	A	6	4	1.00	8	0.500
127	A	7	4	1.00	8	0.500
128	A	14	10	1.00	14	0.714
129	A	11	9	1.00	14	0.643
130	A	8	7	1.00	12	0.583
131	A	1	1	1.00	6	0.167
132	N/A	0	0	1.00	14	0.000
133	N/A	0	0	1.00	14	0.000
134	A	14	10	1.00	10	1.000
135	A	11	9	1.00	10	0.900
136	A	8	7	1.00	8	0.875
137	A	1	1	1.00	6	0.167
138	N/A	0	0	1.00	10	0.000
139	N/A	0	0	1.00	10	0.000
140	A	23	11	1.00	10	1.100
141	A	19	10	1.00	10	1.000
142	A	16	10	1.00	10	1.000
143	A	12	9	1.00	10	0.900
144	A	10	10	1.00	10	1.000
145	A	8	6	1.00	10	0.600
146	A	5	5	1.00	8	0.625
147	A	4	4	1.00	6	0.667
148	N/A	0	0	1.00	10	0.000
149	N/A	0	0	1.00	10	0.000
150	N/A	0	0	1.00	10	0.000
151	N/A	0	0	1.00	10	0.000
152	A	9	9	1.00	10	0.900
153	N/A	0	0	1.00	10	0.000
154	N/A	0	0	1.00	10	0.000
155	N/A	0	0	1.00	10	0.000
156	A	20	10	1.00	10	1.000
157	N/A	0	0	1.00	10	0.000
158	A	18	13	1.00	16	0.812

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	10	9	1.00	14	0.643
160	A	4	3	1.00	8	0.375
161	N/A	0	0	1.00	16	0.000
162	N/A	0	0	1.00	16	0.000
163	A	10	7	1.00	17	0.412
164	A	10	7	1.00	15	0.467
165	A	10	7	1.00	13	0.538
166	A	3	1	1.00	17	0.059
167	A	10	7	1.00	17	0.412
168	A	10	7	1.00	17	0.412
169	A	10	7	1.00	19	0.368
170	A	4	4	1.00	22	0.182
171	A	4	4	1.00	22	0.182
172	A	4	4	1.00	19	0.210
173	A	4	4	1.00	19	0.210
174	A	2	2	1.00	19	0.105
175	A	2	2	1.00	17	0.118
176	A	2	2	1.00	19	0.105
177	A	2	2	1.00	19	0.105
178	A	2	2	1.00	19	0.105
179	A	2	2	1.00	19	0.105
180	A	22	10	1.00	20	0.500
181	A	18	9	1.00	20	0.450
182	A	15	9	1.00	20	0.450
183	A	11	8	1.00	20	0.400
184	A	9	9	1.00	20	0.450
185	A	7	5	1.00	20	0.250
186	A	4	4	1.00	20	0.200
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	17	0.118
189	N/A	0	0	1.00	20	0.000
190	N/A	0	0	1.00	20	0.000
191	N/A	0	0	1.00	20	0.000
192	A	8	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	N/A	0	0	1.00	20	0.000
194	N/A	0	0	1.00	20	0.000
195	N/A	0	0	1.00	20	0.000
196	A	19	9	1.00	20	0.450
197	N/A	0	0	1.00	20	0.000
198	N/A	0	0	1.00	20	0.000
199	N/A	0	0	1.00	19	0.000
200	A	23	9	1.00	20	0.450
201	A	18	8	1.00	20	0.400
202	A	16	10	1.00	20	0.500
203	A	13	9	1.00	20	0.450
204	A	10	9	1.00	20	0.450
205	A	7	6	1.00	20	0.300
206	A	5	5	1.00	20	0.250
207	A	4	3	1.00	18	0.167
208	A	1	1	1.00	17	0.059
209	N/A	0	0	1.00	20	0.000
210	A	4	4	1.00	20	0.200
211	N/A	0	0	1.00	20	0.000
212	N/A	0	0	1.00	20	0.000
213	N/A	0	0	1.00	20	0.000
214	A	13	9	1.00	20	0.450
215	N/A	0	0	1.00	20	0.000
216	N/A	0	0	1.00	20	0.000
217	N/A	0	0	1.00	20	0.000
218	A	26	9	1.00	20	0.450

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^7 \text{FresnelS}(bx) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 124

$$\int x^7 \text{FresnelS}(bx) \, dx = -\frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105 \text{FresnelS}(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 \text{FresnelS}(bx) + \frac{105x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2}$$

```
[Out] -35/8*x^3*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/8*x^7*cos(1/2*b^2*Pi*x^2)/b/Pi-105
/8*FresnelS(b*x)/b^8/Pi^4+1/8*x^8*FresnelS(b*x)+105/8*x*sin(1/2*b^2*Pi*x^2)
/b^7/Pi^4-7/8*x^5*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3466, 3467, 3432}

$$\int x^7 \text{FresnelS}(bx) \, dx = -\frac{105 \text{FresnelS}(bx)}{8\pi^4 b^8} + \frac{x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi b} + \frac{105x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^4 b^7} - \frac{35x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^3 b^5} - \frac{7x^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^2 b^3} + \frac{1}{8}x^8 \text{FresnelS}(bx)$$

```
[In] Int[x^7*FresnelS[b*x],x]
```

```
[Out] (-35*x^3*Cos[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2])/(8*b
*Pi) - (105*FresnelS[b*x])/(8*b^8*Pi^4) + (x^8*FresnelS[b*x])/8 + (105*x*Si
n[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Sin[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^8\,\text{FresnelS}(bx) - \frac{1}{8}b\int x^8\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx) - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b\pi} \\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{35\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^3\pi^2} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx) \\
&\quad - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{105\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^5\pi^3} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx) \\
&\quad + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105\int\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^7\pi^4} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\,\text{FresnelS}(bx)}{8b^8\pi^4} \\
&\quad + \frac{1}{8}x^8\,\text{FresnelS}(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int x^7 \operatorname{FresnelS}(bx)\,dx$$
$$= \frac{b^3\pi x^3(-35+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+(-105+b^8\pi^4x^8)\operatorname{FresnelS}(bx)-7bx(-15+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^8\pi^4}$$

```
[In] Integrate[x^7*FresnelS[b*x],x]
```

```
[Out] (b^3*Pi*x^3*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8
)*FresnelS[b*x] - 7*b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

method	result
meijerg	$\frac{\pi b^{3} x^{11} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{11}{4}\right],\left[\frac{3}{2}, \frac{7}{4}, \frac{15}{4}\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{66}$
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx) b^{8} x^{8}}{8}+\frac{b^{7} x^{7} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{8 \pi}-\frac{7\left(\frac{b^{5} x^{5} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{5\left(-\frac{b^{3} x^{3} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}+\frac{\frac{3 b x \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{3 \operatorname{FresnelS}(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8 \pi}}{b^{8}}$
default	$\frac{\frac{\operatorname{FresnelS}(bx) b^{8} x^{8}}{8}+\frac{b^{7} x^{7} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{8 \pi}-\frac{7\left(\frac{b^{5} x^{5} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{5\left(-\frac{b^{3} x^{3} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}+\frac{\frac{3 b x \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{3 \operatorname{FresnelS}(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8 \pi}}{b^{8}}$
parts	$\frac{x^{8} \operatorname{FresnelS}(bx)}{8}-\frac{b\left(-\frac{x^{7} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{7 x^{5} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}-\frac{35\left(-\frac{x^{3} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{\frac{3 x \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}-\frac{3 \operatorname{FresnelS}\left(\frac{\sqrt{\pi}\, b^{2} x}{\sqrt{b^{2} \pi}}\right)}{b^{2} \sqrt{\pi}\, \sqrt{b^{2} \pi}}}{b^{2} \pi}\right.}{b^{2} \pi}\right.}{8}$

```
[In] int(x^7*FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] 1/66*Pi*b^3*x^11*hypergeom([3/4,11/4],[3/2,7/4,15/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int x^{7} \operatorname{FresnelS}(bx)\, dx = \frac{\left(\pi^{3} b^{7} x^{7}-35\, \pi b^{3} x^{3}\right) \cos\left(\frac{1}{2}\, \pi b^{2} x^{2}\right)+\left(\pi^{4} b^{8} x^{8}-105\right) \mathrm{S}\left(bx\right)-7\left(\pi^{2} b^{5} x^{5}-15\, bx\right) \sin\left(\frac{1}{2}\, \pi b^{2} x^{2}\right)}{8\, \pi^{4} b^{8}}$$

```
[In] integrate(x^7*fresnel_sin(b*x),x, algorithm="fricas")
[Out] 1/8*((pi^3*b^7*x^7 - 35*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 - 1
05)*fresnel_sin(b*x) - 7*(pi^2*b^5*x^5 - 15*b*x)*sin(1/2*pi*b^2*x^2))/(pi^4
*b^8)
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \frac{231x^8S(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\Gamma\left(\frac{15}{4}\right)} + \frac{231x^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi b\Gamma\left(\frac{15}{4}\right)} - \frac{1617x^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{15}{4}\right)} - \frac{8085x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{15}{4}\right)} + \frac{24255x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^4b^7\Gamma\left(\frac{15}{4}\right)} - \frac{24255S(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\pi^4b^8\Gamma\left(\frac{15}{4}\right)}$$

```
[In] integrate(x**7*fresnels(b*x),x)
```

```
[Out] 231*x**8*fresnels(b*x)*gamma(3/4)/(512*gamma(15/4)) + 231*x**7*cos(pi*b**2*
x**2/2)*gamma(3/4)/(512*pi*b*gamma(15/4)) - 1617*x**5*sin(pi*b**2*x**2/2)*g
amma(3/4)/(512*pi**2*b**3*gamma(15/4)) - 8085*x**3*cos(pi*b**2*x**2/2)*gamm
a(3/4)/(512*pi**3*b**5*gamma(15/4)) + 24255*x*sin(pi*b**2*x**2/2)*gamma(3/4
)/(512*pi**4*b**7*gamma(15/4)) - 24255*fresnels(b*x)*gamma(3/4)/(512*pi**4*
b**8*gamma(15/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \frac{1}{8}\,x^8\,\mathrm{S}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(\left(105i + 105\right)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\,\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - \left(105i - 105\right)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\,\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right) - 4\left(\sqrt{\frac{1}{2}}\pi^4b^7x^7 - 35\right.\right.}{16\,\pi^5b^8}$$

```
[In] integrate(x^7*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*x^8*fresnel_sin(b*x) - 1/16*sqrt(1/2)*((105*I + 105)*(1/4)^(1/4)*pi*erf
(sqrt(1/2*I*pi)*b*x) - (105*I - 105)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x
) - 4*(sqrt(1/2)*pi^4*b^7*x^7 - 35*sqrt(1/2)*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x
^2) + 28*(sqrt(1/2)*pi^3*b^5*x^5 - 15*sqrt(1/2)*pi*b*x)*sin(1/2*pi*b^2*x^2)
)/(pi^5*b^8)
```

Giac [F]

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \int x^7 \operatorname{S}(bx)\,dx$$

```
[In] integrate(x^7*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^7*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \int x^7 \operatorname{FresnelS}(b\,x)\,dx$$

```
[In] int(x^7*FresnelS(b*x),x)
[Out] int(x^7*FresnelS(b*x), x)
```

3.2 $\int x^6 \operatorname{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 109

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7\operatorname{FresnelS}(bx) + \frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}$$

```
[Out] -24/7*x^2*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/7*x^6*cos(1/2*b^2*Pi*x^2)/b/Pi+1/7
*x^7*FresnelS(b*x)+48/7*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*sin(1/2*b^2*Pi
*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3377, 2717}

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b} + \frac{48\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} - \frac{24x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5} - \frac{6x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3} + \frac{1}{7}x^7\operatorname{FresnelS}(bx)$$

```
[In] Int[x^6*FresnelS[b*x],x]
```

```
[Out] (-24*x^2*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*Cos[(b^2*Pi*x^2)/2])/(7*b
*Pi) + (x^7*FresnelS[b*x])/7 + (48*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x
^4*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}x^7 \operatorname{FresnelS}(bx) - \frac{1}{7}b\int x^7 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{7}x^7 \operatorname{FresnelS}(bx) - \frac{1}{14}b\text{Subst}\left(\int x^3 \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 \operatorname{FresnelS}(bx) - \frac{3\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b\pi} \\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 \operatorname{FresnelS}(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{12\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b^3\pi^2} \\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 \operatorname{FresnelS}(bx) \\
&\quad - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{24\text{Subst}\left(\int \cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b^5\pi^3} \\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} \\
&\quad + \frac{1}{7}x^7 \operatorname{FresnelS}(bx) + \frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{1}{7}x^7 \operatorname{FresnelS}(bx) - \frac{6(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}$$

```
[In] Integrate[x^6*FresnelS[b*x],x]
[Out] (x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^7*FresnelS
[b*x])/7 - (6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{\pi b^3 x^{10} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{3}{2},\frac{7}{4},\frac{7}{2}\right],-\frac{x^4\pi^2b^4}{16}\right)}{60}$	29
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx)b^7x^7}{7}+\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}}{b^7}$	107
default	$\frac{\frac{\operatorname{FresnelS}(bx)b^7x^7}{7}+\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}}{b^7}$	107
parts	$\frac{x^7\operatorname{FresnelS}(bx)}{7}-\frac{b\left(-\frac{x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{6x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{24\left(-\frac{x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}\right)}{b^2\pi}}{b^2\pi}\right)}{7}$	113

```
[In] int(x^6*FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] 1/60*Pi*b^3*x^10*hypergeom([3/4,5/2],[3/2,7/4,7/2],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{\pi^4 b^7 x^7 \operatorname{S}(bx) + (\pi^3 b^6 x^6 - 24\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 6\,(\pi^2 b^4 x^4 - 8)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{7\,\pi^4 b^7}$$

```
[In] integrate(x^6*fresnel_sin(b*x),x, algorithm="fricas")
[Out] 1/7*(pi^4*b^7*x^7*fresnel_sin(b*x) + (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*cos(1/2
*pi*b^2*x^2) - 6*(pi^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.43

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{3x^7 S(bx)\,\Gamma\left(\frac{3}{4}\right)}{28\Gamma\left(\frac{7}{4}\right)} + \frac{3x^6\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{28\pi b\Gamma\left(\frac{7}{4}\right)} - \frac{9x^4\sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{14\pi^2 b^3\Gamma\left(\frac{7}{4}\right)} - \frac{18x^2\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^3 b^5\Gamma\left(\frac{7}{4}\right)} + \frac{36\sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^4 b^7\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**6*fresnels(b*x),x)
[Out] 3*x**7*fresnels(b*x)*gamma(3/4)/(28*gamma(7/4)) + 3*x**6*cos(pi*b**2*x**2/2
)*gamma(3/4)/(28*pi*b*gamma(7/4)) - 9*x**4*sin(pi*b**2*x**2/2)*gamma(3/4)/(
14*pi**2*b**3*gamma(7/4)) - 18*x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**3
*b**5*gamma(7/4)) + 36*sin(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**4*b**7*gamma(7
/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{1}{7}\,x^7\operatorname{S}(bx) + \frac{(\pi^3 b^6 x^6 - 24\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 6\,(\pi^2 b^4 x^4 - 8)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{7\,\pi^4 b^7}$$

```
[In] integrate(x^6*fresnel_sin(b*x),x, algorithm="maxima")
[Out] 1/7*x^7*fresnel_sin(b*x) + 1/7*((pi^3*b^6*x^6 - 24*pi*b^2*x^2)*cos(1/2*pi*b
^2*x^2) - 6*(pi^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Giac [F]

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \int x^6 \operatorname{S}(bx)\;dx$$

```
[In] integrate(x^6*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^6*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \int x^6 \operatorname{FresnelS}(b\,x)\;dx$$

```
[In] int(x^6*FresnelS(b*x),x)
[Out] int(x^6*FresnelS(b*x), x)
```

3.3 $\int x^5 \operatorname{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 99

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\operatorname{FresnelC}(bx)}{2b^6\pi^3} + \frac{1}{6}x^6\operatorname{FresnelS}(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}$$

```
[Out] -5/2*x*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/6*x^5*cos(1/2*b^2*Pi*x^2)/b/Pi+5/2*Fr
esnelC(b*x)/b^6/Pi^3+1/6*x^6*FresnelS(b*x)-5/6*x^3*sin(1/2*b^2*Pi*x^2)/b^3/
Pi^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3466, 3467, 3433}

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{5\operatorname{FresnelC}(bx)}{2\pi^3b^6} + \frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b} - \frac{5x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5} - \frac{5x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3} + \frac{1}{6}x^6\operatorname{FresnelS}(bx)$$

```
[In] Int[x^5*FresnelS[b*x],x]
```

```
[Out] (-5*x*Cos[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2])/(6*b*Pi
) + (5*FresnelC[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x])/6 - (5*x^3*Sin[(b^
2*Pi*x^2)/2])/(6*b^3*Pi^2)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\,\text{FresnelS}(bx) - \frac{1}{6}b\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6\,\text{FresnelS}(bx) - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{6b\pi} \\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6\,\text{FresnelS}(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^3\pi^2} \\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6\,\text{FresnelS}(bx) \\
&\quad - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^5\pi^3} \\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\,\text{FresnelC}(bx)}{2b^6\pi^3} + \frac{1}{6}x^6\,\text{FresnelS}(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+15\operatorname{FresnelC}(bx)+b^6\pi^3x^6\operatorname{FresnelS}(bx)-5b^3\pi x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^6\pi^3}$$

```
[In] Integrate[x^5*FresnelS[b*x],x]
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + 15*FresnelC[b*x] + b^6*Pi^3
*x^6*FresnelS[b*x] - 5*b^3*Pi*x^3*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

method	result	size
meijerg	$\frac{\pi b^3 x^9 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{9}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{13}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{54}$	29
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx)b^6x^6}{6}+\frac{b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\operatorname{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$	96
default	$\frac{\frac{\operatorname{FresnelS}(bx)b^6x^6}{6}+\frac{b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\operatorname{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$	96
parts	$\frac{x^6\operatorname{FresnelS}(bx)}{6}-\frac{b\left(-\frac{x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{5x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{15\left(-\frac{x\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\operatorname{FresnelC}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{b^2\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b^2\pi}}{b^2\pi}\right)}{6}$	123

```
[In] int(x^5*FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] 1/54*Pi*b^3*x^9*hypergeom([3/4,9/4],[3/2,7/4,13/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{\pi^3 b^7 x^6 \operatorname{S}(bx) - 5\,\pi b^4 x^3 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + \left(\pi^2 b^6 x^5 - 15\,b^2 x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 15\,\sqrt{b^2}\,\operatorname{C}\left(\sqrt{b^2}x\right)}{6\,\pi^3 b^7}$$

```
[In] integrate(x^5*fresnel_sin(b*x),x, algorithm="fricas")
[Out] 1/6*(pi^3*b^7*x^6*fresnel_sin(b*x) - 5*pi*b^4*x^3*sin(1/2*pi*b^2*x^2) + (pi
^2*b^6*x^5 - 15*b^2*x)*cos(1/2*pi*b^2*x^2) + 15*sqrt(b^2)*fresnel_cos(sqrt(
b^2)*x))/(pi^3*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{\pi b^3 x^9 \Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right) {}_2F_3\left(\left.\begin{matrix}\frac{3}{4}, \frac{9}{4}\\ \frac{3}{2}, \frac{7}{4}, \frac{13}{4}\end{matrix}\right| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate(x**5*fresnels(b*x),x)
[Out] pi*b**3*x**9*gamma(3/4)*gamma(9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -pi*
*2*b**4*x**4/16)/(32*gamma(7/4)*gamma(13/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{1}{6}\,x^6\operatorname{S}(bx) - \frac{\sqrt{\frac{1}{2}}\left(20\,\sqrt{\frac{1}{2}}\pi^2 b^3 x^3 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + (15i - 15)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi \operatorname{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - (15i + 15)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi \operatorname{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}b\right.\right.}{12\,\pi^4 b^6}$$

```
[In] integrate(x^5*fresnel_sin(b*x),x, algorithm="maxima")
[Out] 1/6*x^6*fresnel_sin(b*x) - 1/12*sqrt(1/2)*(20*sqrt(1/2)*pi^2*b^3*x^3*sin(1/
2*pi*b^2*x^2) + (15*I - 15)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (15*I
+ 15)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x) - 4*(sqrt(1/2)*pi^3*b^5*x^5 -
 15*sqrt(1/2)*pi*b*x)*cos(1/2*pi*b^2*x^2))/(pi^4*b^6)
```

Giac [F]

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \int x^5 \operatorname{S}(bx)\,dx$$

```
[In] integrate(x^5*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^5*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \int x^5 \operatorname{FresnelS}(b\,x)\,dx$$

```
[In] int(x^5*FresnelS(b*x),x)
[Out] int(x^5*FresnelS(b*x), x)
```

3.4 $\int x^4 \operatorname{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 84

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = -\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5\operatorname{FresnelS}(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}$$

```
[Out] -8/5*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/5*x^4*cos(1/2*b^2*Pi*x^2)/b/Pi+1/5*x^5*
FresnelS(b*x)-4/5*x^2*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3377, 2718}

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{x^4\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b} - \frac{8\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5} - \frac{4x^2\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3} + \frac{1}{5}x^5\operatorname{FresnelS}(bx)$$

```
[In] Int[x^4*FresnelS[b*x],x]
```

```
[Out] (-8*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*Cos[(b^2*Pi*x^2)/2])/(5*b*Pi)
+ (x^5*FresnelS[b*x])/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\,\text{FresnelS}(bx) - \frac{1}{5}b\int x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{5}x^5\,\text{FresnelS}(bx) - \frac{1}{10}b\text{Subst}\left(\int x^2 \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx) - \frac{2\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b\pi} \\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{4\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b^3\pi^2} \\
&= -\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int x^4\,\text{FresnelS}(bx)\,dx = \frac{(-8 + b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{1}{5}x^5\,\text{FresnelS}(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}$$

```
[In] Integrate[x^4*FresnelS[b*x],x]
```

```
[Out] ((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^5*FresnelS[b*x]
)/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

method	result	size
meijerg	$\frac{\pi b^3 x^8 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},2\right],\left[\frac{3}{2},\frac{7}{4},3\right],-\frac{x^4\pi^2 b^4}{16}\right)}{48}$	29
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)b^5x^5}{5}+\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$	80
default	$\frac{\frac{\mathrm{FresnelS}(bx)b^5x^5}{5}+\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$	80
parts	$\frac{x^5\,\mathrm{FresnelS}(bx)}{5}-\frac{b\left(-\frac{x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{4x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}}{b^2\pi}\right)}{5}$	83

```
[In] int(x^4*FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] 1/48*Pi*b^3*x^8*hypergeom([3/4,2],[3/2,7/4,3],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int x^4\,\mathrm{FresnelS}(bx)\,dx=\frac{\pi^3b^5x^5\,\mathrm{S}\left(bx\right)-4\,\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{5\,\pi^3b^5}$$

```
[In] integrate(x^4*fresnel_sin(b*x),x, algorithm="fricas")
[Out] 1/5*(pi^3*b^5*x^5*fresnel_sin(b*x) - 4*pi*b^2*x^2*sin(1/2*pi*b^2*x^2) + (pi
^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{3x^5 S(bx)\,\Gamma\left(\frac{3}{4}\right)}{20\Gamma\left(\frac{7}{4}\right)} + \frac{3x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{20\pi b\Gamma\left(\frac{7}{4}\right)} - \frac{3x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^2 b^3\Gamma\left(\frac{7}{4}\right)} - \frac{6\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^3 b^5\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**4*fresnels(b*x),x)
```

```
[Out] 3*x**5*fresnels(b*x)*gamma(3/4)/(20*gamma(7/4)) + 3*x**4*cos(pi*b**2*x**2/2
)*gamma(3/4)/(20*pi*b*gamma(7/4)) - 3*x**2*sin(pi*b**2*x**2/2)*gamma(3/4)/(
5*pi**2*b**3*gamma(7/4)) - 6*cos(pi*b**2*x**2/2)*gamma(3/4)/(5*pi**3*b**5*g
amma(7/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{1}{5}\,x^5\,\mathrm{S}\left(bx\right) - \frac{4\,\pi b^2 x^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - \left(\pi^2 b^4 x^4 - 8\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{5\,\pi^3 b^5}$$

```
[In] integrate(x^4*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*fresnel_sin(b*x) - 1/5*(4*pi*b^2*x^2*sin(1/2*pi*b^2*x^2) - (pi^2*b^
4*x^4 - 8)*cos(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Giac [F]

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \int x^4\,\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(x^4*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \int x^4 \operatorname{FresnelS}(b\,x)\ dx$$

```
[In] int(x^4*FresnelS(b*x),x)
[Out] int(x^4*FresnelS(b*x), x)
```

3.5 $\int x^3 \operatorname{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 74

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\operatorname{FresnelS}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelS}(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] 1/4*x^3*cos(1/2*b^2*Pi*x^2)/b/Pi+3/4*FresnelS(b*x)/b^4/Pi^2+1/4*x^4*Fresnel
S(b*x)-3/4*x*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3466, 3467, 3432}

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{3\operatorname{FresnelS}(bx)}{4\pi^2 b^4} + \frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b} - \frac{3x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4\operatorname{FresnelS}(bx)$$

```
[In] Int[x^3*FresnelS[b*x],x]
```

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\,\text{FresnelS}(bx) - \frac{1}{4}b\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4\,\text{FresnelS}(bx) - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi}\\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4\,\text{FresnelS}(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\int\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^3\pi^2}\\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\,\text{FresnelS}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\,\text{FresnelS}(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^3\,\text{FresnelS}(bx)\,dx = \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\,\text{FresnelS}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\,\text{FresnelS}(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[In] Integrate[x^3*FresnelS[b*x],x]
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{\frac{\pi x^3 b^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4}-\frac{3 \sin\left(\frac{b^2 \pi x^2}{2}\right) b x}{4}+\frac{\left(21 x^4 \pi^2 b^4+63\right) \mathrm{FresnelS}(b x)}{84}}{\pi^2 b^4}$	62
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(b x) b^4 x^4}{4}+\frac{b^3 x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4 \pi}-\frac{3\left(\frac{b x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}-\frac{\mathrm{FresnelS}(b x)}{\pi}\right)}{4 \pi}}{b^4}$	70
default	$\frac{\frac{\mathrm{FresnelS}(b x) b^4 x^4}{4}+\frac{b^3 x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4 \pi}-\frac{3\left(\frac{b x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}-\frac{\mathrm{FresnelS}(b x)}{\pi}\right)}{4 \pi}}{b^4}$	70
parts	$\frac{x^4 \mathrm{FresnelS}(b x)}{4}-\frac{b\left(-\frac{x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}+\frac{\frac{3 x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}-\frac{3\ \mathrm{FresnelS}\left(\frac{\sqrt{\pi}\, b^2 x}{\sqrt{b^2 \pi}}\right)}{b^2 \sqrt{\pi}\, \sqrt{b^2 \pi}}}{b^2 \pi}\right)}{4}$	94

```
[In] int(x^3*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/Pi^2/b^4*(1/8*Pi*x^3*b^3*cos(1/2*b^2*Pi*x^2)-3/8*sin(1/2*b^2*Pi*x^2)*b*x+
1/168*(21*Pi^2*b^4*x^4+63)*FresnelS(b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int x^3 \operatorname{FresnelS}(bx)\, dx = \frac{\pi b^3 x^3 \cos\left(\frac{1}{2}\, \pi b^2 x^2\right) - 3\, b x \sin\left(\frac{1}{2}\, \pi b^2 x^2\right) + \left(\pi^2 b^4 x^4 + 3\right) \mathrm{S}\left(bx\right)}{4\, \pi^2 b^4}$$

```
[In] integrate(x^3*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - 3*b*x*sin(1/2*pi*b^2*x^2) + (pi^2*b^4
*x^4 + 3)*fresnel_sin(b*x))/(pi^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int x^3 \,\text{FresnelS}(bx)\, dx = \frac{21x^4 S(bx)\,\Gamma\left(\frac{3}{4}\right)}{64\Gamma\left(\frac{11}{4}\right)} + \frac{21x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi b\Gamma\left(\frac{11}{4}\right)} - \frac{63x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{11}{4}\right)} + \frac{63S(bx)\,\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**3*fresnels(b*x),x)
```

```
[Out] 21*x**4*fresnels(b*x)*gamma(3/4)/(64*gamma(11/4)) + 21*x**3*cos(pi*b**2*x**
2/2)*gamma(3/4)/(64*pi*b*gamma(11/4)) - 63*x*sin(pi*b**2*x**2/2)*gamma(3/4)
/(64*pi**2*b**3*gamma(11/4)) + 63*fresnels(b*x)*gamma(3/4)/(64*pi**2*b**4*g
amma(11/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int x^3 \,\text{FresnelS}(bx)\, dx = \frac{1}{4}\,x^4\,\mathrm{S}\left(bx\right) + \frac{\sqrt{\frac{1}{2}}\left(4\,\sqrt{\frac{1}{2}}\pi^2b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right) - 12\,\sqrt{\frac{1}{2}}\pi bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + (3i+3)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - (3i-3)}{8\,\pi^3b^4}$$

```
[In] integrate(x^3*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_sin(b*x) + 1/8*sqrt(1/2)*(4*sqrt(1/2)*pi^2*b^3*x^3*cos(1/2*
pi*b^2*x^2) - 12*sqrt(1/2)*pi*b*x*sin(1/2*pi*b^2*x^2) + (3*I + 3)*(1/4)^(1/
4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (3*I - 3)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi
)*b*x))/(pi^3*b^4)
```

Giac [F]

$$\int x^3 \,\text{FresnelS}(bx)\, dx = \int x^3\,\mathrm{S}\left(bx\right)\, dx$$

```
[In] integrate(x^3*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \int x^3 \operatorname{FresnelS}(b\,x)\;dx$$

```
[In] int(x^3*FresnelS(b*x),x)
[Out] int(x^3*FresnelS(b*x), x)
```

3.6 $\int x^2 \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x^2 \text{FresnelS}(bx)\,dx = \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

```
[Out] 1/3*x^2*cos(1/2*b^2*Pi*x^2)/b/Pi+1/3*x^3*FresnelS(b*x)-2/3*sin(1/2*b^2*Pi*x
^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3377, 2717}

$$\int x^2 \text{FresnelS}(bx)\,dx = \frac{x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} - \frac{2\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{1}{3}x^3 \text{FresnelS}(bx)$$

```
[In] Int[x^2*FresnelS[b*x],x]
```

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\,\text{FresnelS}(bx) - \frac{1}{3}b\int x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{3}x^3\,\text{FresnelS}(bx) - \frac{1}{6}b\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3\,\text{FresnelS}(bx) - \frac{\text{Subst}\left(\int\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{3b\pi} \\
&= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3\,\text{FresnelS}(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^2\,\text{FresnelS}(bx)\,dx = \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3\,\text{FresnelS}(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

```
[In] Integrate[x^2*FresnelS[b*x],x]
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx)b^3x^3}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
default	$\frac{\frac{\operatorname{FresnelS}(bx)b^3x^3}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
parts	$\frac{x^3\operatorname{FresnelS}(bx)}{3}-\frac{b\left(-\frac{x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}\right)}{3}$	54
meijerg	$\frac{\frac{\sqrt{\pi}\,x^2b^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\sqrt{\pi}}+\frac{\pi^{\frac{3}{2}}x^3b^3\operatorname{FresnelS}(bx)}{3}}{b^3\pi^{\frac{3}{2}}}$	60

```
[In] int(x^2*FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] 1/b^3*(1/3*FresnelS(b*x)*b^3*x^3+1/3/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)-2/3/Pi^
2*sin(1/2*b^2*Pi*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^2\operatorname{FresnelS}(bx)\,dx=\frac{\pi^2b^3x^3\operatorname{S}(bx)+\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)-2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}$$

```
[In] integrate(x^2*fresnel_sin(b*x),x, algorithm="fricas")
[Out] 1/3*(pi^2*b^3*x^3*fresnel_sin(b*x) + pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - 2*sin
(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int x^2\operatorname{FresnelS}(bx)\,dx=\frac{x^3S(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi^2b^3\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**2*fresnels(b*x),x)
[Out] x**3*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + x**2*cos(pi*b**2*x**2/2)*gam
ma(3/4)/(4*pi*b*gamma(7/4)) - sin(pi*b**2*x**2/2)*gamma(3/4)/(2*pi**2*b**3*
gamma(7/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \frac{1}{3}\,x^3\,\mathrm{S}\left(bx\right) + \frac{\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 2\,\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{3\,\pi^2 b^3}$$

```
[In] integrate(x^2*fresnel_sin(b*x),x, algorithm="maxima")
[Out] 1/3*x^3*fresnel_sin(b*x) + 1/3*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - 2*sin(1/2*
pi*b^2*x^2))/(pi^2*b^3)
```

Giac [F]

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \int x^2\,\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(x^2*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^2*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \int x^2 \operatorname{FresnelS}(b\,x)\,dx$$

```
[In] int(x^2*FresnelS(b*x),x)
[Out] int(x^2*FresnelS(b*x), x)
```

3.7 $\int x \operatorname{FresnelS}(bx) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 49

$$\int x \operatorname{FresnelS}(bx) \, dx = \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{\operatorname{FresnelC}(bx)}{2b^2\pi} + \frac{1}{2}x^2 \operatorname{FresnelS}(bx)$$

```
[Out] 1/2*x*cos(1/2*b^2*Pi*x^2)/b/Pi-1/2*FresnelC(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*
x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3466, 3433}

$$\int x \operatorname{FresnelS}(bx) \, dx = -\frac{\operatorname{FresnelC}(bx)}{2\pi b^2} + \frac{x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{1}{2}x^2 \operatorname{FresnelS}(bx)$$

```
[In] Int[x*FresnelS[b*x],x]
```

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\,\text{FresnelS}(bx) - \frac{1}{2}b\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{1}{2}x^2\,\text{FresnelS}(bx) - \frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi} \\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{\text{FresnelC}(bx)}{2b^2\pi} + \frac{1}{2}x^2\,\text{FresnelS}(bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x\,\text{FresnelS}(bx)\,dx = \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{\text{FresnelC}(bx)}{2b^2\pi} + \frac{1}{2}x^2\,\text{FresnelS}(bx)$$

```
[In] Integrate[x*FresnelS[b*x],x]
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

method	result	size
meijerg	$\frac{b^{3}\pi\,x^{5}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{9}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{30}$	29
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)b^{2}x^{2}}{2}+\frac{bx\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx)}{2\pi}}{b^{2}}$	44
default	$\frac{\frac{\mathrm{FresnelS}(bx)b^{2}x^{2}}{2}+\frac{bx\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx)}{2\pi}}{b^{2}}$	44
parts	$\frac{x^{2}\,\mathrm{FresnelS}(bx)}{2}-\frac{b\left(-\frac{x\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}\right)}{2}$	64

```
[In] int(x*FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] 1/30*b^3*Pi*x^5*hypergeom([3/4,5/4],[3/2,7/4,9/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int x\,\mathrm{FresnelS}(bx)\,dx=\frac{\pi b^{3}x^{2}\,\mathrm{S}\left(bx\right)+b^{2}x\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-\sqrt{b^{2}}\,\mathrm{C}\left(\sqrt{b^{2}}x\right)}{2\,\pi b^{3}}$$

```
[In] integrate(x*fresnel_sin(b*x),x, algorithm="fricas")
[Out] 1/2*(pi*b^3*x^2*fresnel_sin(b*x) + b^2*x*cos(1/2*pi*b^2*x^2) - sqrt(b^2)*fr
esnel_cos(sqrt(b^2)*x))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int x\,\mathrm{FresnelS}(bx)\,dx=\frac{\pi b^{3}x^{5}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_{2}F_{3}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x*fresnels(b*x),x)
[Out] pi*b**3*x**5*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -pi**
2*b**4*x**4/16)/(32*gamma(7/4)*gamma(9/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int x\,\mathrm{FresnelS}(bx)\,dx = \frac{1}{2}x^2\,\mathrm{S}\left(bx\right) + \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right) + (i-1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - (i+1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right)\right)}{4\,\pi^2b^2}$$

```
[In] integrate(x*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*fresnel_sin(b*x) + 1/4*sqrt(1/2)*(4*sqrt(1/2)*pi*b*x*cos(1/2*pi*b^2
*x^2) + (I - 1)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (I + 1)*(1/4)^(1/4
)*pi*erf(sqrt(-1/2*I*pi)*b*x))/(pi^2*b^2)
```

Giac [F]

$$\int x\,\mathrm{FresnelS}(bx)\,dx = \int x\,\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(x*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\,\mathrm{FresnelS}(bx)\,dx = \int x\,\mathrm{FresnelS}(b\,x)\,dx$$

```
[In] int(x*FresnelS(b*x),x)
```

```
[Out] int(x*FresnelS(b*x), x)
```

3.8 $\int \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \text{FresnelS}(bx)\,dx = \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + x\,\text{FresnelS}(bx)$$

```
[Out] cos(1/2*b^2*Pi*x^2)/b/Pi+x*FresnelS(b*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6553}

$$\int \text{FresnelS}(bx)\,dx = \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} + x\,\text{FresnelS}(bx)$$

```
[In] Int[FresnelS[b*x],x]
```

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + x\,\text{FresnelS}(bx)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\,dx = \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + x\,\text{FresnelS}(bx)$$

```
[In] Integrate[FresnelS[b*x],x]
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
parts	$\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{b\pi} + x\ \text{FresnelS}\left(bx\right)$	25
derivativedivides	$\frac{\text{FresnelS}(bx)bx+\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}}{b}$	27
default	$\frac{\text{FresnelS}(bx)bx+\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}}{b}$	27
meijerg	$\frac{b^3\pi x^4\ \text{hypergeom}\left(\left[\frac{3}{4},1\right],\left[\frac{3}{2},\frac{7}{4},2\right],-\frac{x^4\pi^2b^4}{16}\right)}{24}$	29

```
[In] int(FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] cos(1/2*b^2*Pi*x^2)/b/Pi+x*FresnelS(b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\,dx = \frac{\pi bx\,\text{S}\left(bx\right) + \cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi b}$$

```
[In] integrate(fresnel_sin(b*x),x, algorithm="fricas")
[Out] (pi*b*x*fresnel_sin(b*x) + cos(1/2*pi*b^2*x^2))/(pi*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \text{FresnelS}(bx)\,dx = \frac{3xS(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x),x)
[Out] 3*x*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + 3*cos(pi*b**2*x**2/2)*gamma(3
/4)/(4*pi*b*gamma(7/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\,dx = \frac{bx\,\mathrm{S}\,(bx) + \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi}}{b}$$

```
[In] integrate(fresnel_sin(b*x),x, algorithm="maxima")
[Out] (b*x*fresnel_sin(b*x) + cos(1/2*pi*b^2*x^2)/pi)/b
```

Giac [F]

$$\int \text{FresnelS}(bx)\,dx = \int \mathrm{S}\,(bx)\,dx$$

```
[In] integrate(fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)\,dx = \int \text{FresnelS}(b\,x)\,\,dx$$

```
[In] int(FresnelS(b*x),x)
[Out] int(FresnelS(b*x), x)
```

3.9 $\int \frac{\text{FresnelS}(bx)}{x} dx$

Optimal result

Integrand size = 8, antiderivative size = 73

$$\int \frac{\text{FresnelS}(bx)}{x} dx = \frac{1}{2} ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{2} ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*I*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)-1/2*I*b*x*hype
rgeom([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6559, 6493, 6495}

$$\int \frac{\text{FresnelS}(bx)}{x} dx = \frac{1}{2} ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{2} ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[In] Int[FresnelS[b*x]/x,x]
```

```
[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-1/2*I)*b^2*Pi*x^2] -
(I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]
```

Rule 6493

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

Rule 6495

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

Rule 6559

```
Int[FresnelS[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 + I)/4, Int[Erf[(Sqrt[P
i]/2)*(1 + I)*b*x]/x, x], x] + Dist[(1 - I)/4, Int[Erf[(Sqrt[Pi]/2)*(1 - I)
*b*x]/x, x], x] /; FreeQ[b, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(-\frac{1}{4}-\frac{i}{4}\right)\int\frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int\frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx\\ &= \frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F]

$$\int\frac{\text{FresnelS}(bx)}{x}\,dx=\int\frac{\text{FresnelS}(bx)}{x}\,dx$$

```
[In] Integrate[FresnelS[b*x]/x,x]
[Out] Integrate[FresnelS[b*x]/x, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

method	result	size
meijerg	$\frac{\pi x^3 b^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{18}$	29

```
[In] int(FresnelS(b*x)/x,x,method=_RETURNVERBOSE)
[Out] 1/18*Pi*x^3*b^3*hypergeom([3/4,3/4],[3/2,7/4,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [F]

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{S}\left(bx\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \frac{\pi b^3 x^3 \Gamma^2\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{3}{2}, \frac{7}{4}, \frac{7}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma^2\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x,x)
[Out] pi*b**3*x**3*gamma(3/4)**2*hyper((3/4, 3/4), (3/2, 7/4, 7/4), -pi**2*b**4*x
**4/16)/(32*gamma(7/4)**2)
```

Maxima [F]

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{S}\left(bx\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)/x, x)
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{S}\left(bx\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x}\,dx$$

```
[In] int(FresnelS(b*x)/x,x)
[Out] int(FresnelS(b*x)/x, x)
```

3.10 $\int \frac{\text{FresnelS}(bx)}{x^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\text{FresnelS}(bx)}{x^2} \, dx = -\frac{\text{FresnelS}(bx)}{x} + \frac{1}{2} b \text{Si}\left(\frac{1}{2} b^2 \pi x^2\right)$$

```
[Out] -FresnelS(b*x)/x+1/2*b*Si(1/2*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6561, 3456}

$$\int \frac{\text{FresnelS}(bx)}{x^2} \, dx = \frac{1}{2} b \text{Si}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\text{FresnelS}(bx)}{x}$$

```
[In] Int[FresnelS[b*x]/x^2,x]
```

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelS}(bx)}{x} + b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$
$$= -\frac{\text{FresnelS}(bx)}{x} + \frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\text{FresnelS}(bx)}{x} + \frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[In] Integrate[FresnelS[b*x]/x^2,x]
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\text{FresnelS}(bx)}{x}+\frac{b\,\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}$	24
derivativedivides	$b\left(-\frac{\text{FresnelS}(bx)}{bx}+\frac{\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}\right)$	28
default	$b\left(-\frac{\text{FresnelS}(bx)}{bx}+\frac{\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}\right)$	28
meijerg	$\frac{b^3\pi\,x^2\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{12}$	29

```
[In] int(FresnelS(b*x)/x^2,x,method=_RETURNVERBOSE)
[Out] -FresnelS(b*x)/x+1/2*b*Si(1/2*b^2*Pi*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\text{FresnelS}(bx)}{x^2} \, dx = \frac{bx \, \text{Si}\left(\frac{1}{2} \pi b^2 x^2\right) - 2 \, \text{S}\left(bx\right)}{2 \, x}$$

```
[In] integrate(fresnel_sin(b*x)/x^2,x, algorithm="fricas")
[Out] 1/2*(b*x*sin_integral(1/2*pi*b^2*x^2) - 2*fresnel_sin(b*x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\text{FresnelS}(bx)}{x^2} \, dx = \frac{\pi b^3 x^2 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16 \Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**2,x)
[Out] pi*b**3*x**2*gamma(3/4)*hyper((1/2, 3/4), (3/2, 3/2, 7/4), -pi**2*b**4*x**4
/16)/(16*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\text{FresnelS}(bx)}{x^2} \, dx = -\frac{1}{4} \, b \left(i \, \text{Ei}\left(\frac{1}{2} i \, \pi b^2 x^2\right) - i \, \text{Ei}\left(-\frac{1}{2} i \, \pi b^2 x^2\right)\right) - \frac{\text{S}\left(bx\right)}{x}$$

```
[In] integrate(fresnel_sin(b*x)/x^2,x, algorithm="maxima")
[Out] -1/4*b*(I*Ei(1/2*I*pi*b^2*x^2) - I*Ei(-1/2*I*pi*b^2*x^2)) - fresnel_sin(b*x
)/x
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\text{S}\,(bx)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^2}\,dx$$

```
[In] int(FresnelS(b*x)/x^2,x)
[Out] int(FresnelS(b*x)/x^2, x)
```

3.11 $\int \frac{\text{FresnelS}(bx)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \frac{\text{FresnelS}(bx)}{x^3} dx = \frac{1}{2}b^2\pi \,\text{FresnelC}(bx) - \frac{\text{FresnelS}(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

```
[Out] 1/2*b^2*Pi*FresnelC(b*x)-1/2*FresnelS(b*x)/x^2-1/2*b*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6561, 3468, 3433}

$$\int \frac{\text{FresnelS}(bx)}{x^3} dx = \frac{1}{2}\pi b^2 \,\text{FresnelC}(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{\text{FresnelS}(bx)}{2x^2}$$

```
[In] Int[FresnelS[b*x]/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{2x^2}+\frac{1}{2}b\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{\text{FresnelS}(bx)}{2x^2}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}+\frac{1}{2}\left(b^3\pi\right)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{1}{2}b^2\pi\,\text{FresnelC}(bx)-\frac{\text{FresnelS}(bx)}{2x^2}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelS}(bx)}{x^3}\,dx=\frac{1}{2}b^2\pi\,\text{FresnelC}(bx)-\frac{\text{FresnelS}(bx)}{2x^2}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

```
[In] Integrate[FresnelS[b*x]/x^3,x]
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
meijerg	$\frac{\pi b^{3} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{5}{4}, \frac{3}{2}, \frac{7}{4}\right], -\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{6}$	27
derivativedivides	$b^{2} \left(-\frac{\mathrm{FresnelS}(bx)}{2 b^{2} x^{2}}-\frac{\sin\left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{2 b x}+\frac{\pi \,\mathrm{FresnelC}(bx)}{2}\right)$	43
default	$b^{2} \left(-\frac{\mathrm{FresnelS}(bx)}{2 b^{2} x^{2}}-\frac{\sin\left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{2 b x}+\frac{\pi \,\mathrm{FresnelC}(bx)}{2}\right)$	43
parts	$-\frac{\mathrm{FresnelS}(bx)}{2 x^{2}}+\frac{b \left(-\frac{\sin\left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{x}+\frac{b^{2} \pi^{\frac{3}{2}} \mathrm{FresnelC}\left(\frac{\sqrt{\pi}\, b^{2} x}{\sqrt{b^{2} \pi}}\right)}{\sqrt{b^{2} \pi}}\right)}{2}$	60

```
[In] int(FresnelS(b*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*Pi*b^3*x*hypergeom([1/4,3/4],[5/4,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\mathrm{FresnelS}(bx)}{x^3}\,dx = \frac{\pi\sqrt{b^2}bx^2\,\mathrm{C}\left(\sqrt{b^2}x\right) - bx\sin\left(\frac{1}{2}\pi b^2x^2\right) - \mathrm{S}\left(bx\right)}{2\,x^2}$$

```
[In] integrate(fresnel_sin(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(pi*sqrt(b^2)*b*x^2*fresnel_cos(sqrt(b^2)*x) - b*x*sin(1/2*pi*b^2*x^2)
- fresnel_sin(b*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{\mathrm{FresnelS}(bx)}{x^3}\,dx = \frac{\pi b^3x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**3,x)
```

```
[Out] pi*b**3*x*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (5/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*gamma(5/4)*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = -\frac{\sqrt{\frac{1}{2}}\sqrt{\pi x^2}\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{1}{2}i\,\pi b^2x^2\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^2}{16\,x}-\frac{\mathrm{S}\left(bx\right)}{2\,x^2}$$

```
[In] integrate(fresnel_sin(b*x)/x^3,x, algorithm="maxima")
[Out] -1/16*sqrt(1/2)*sqrt(pi*x^2)*((I - 1)*sqrt(2)*gamma(-1/2, 1/2*I*pi*b^2*x^2)
 - (I + 1)*sqrt(2)*gamma(-1/2, -1/2*I*pi*b^2*x^2))*b^2/x - 1/2*fresnel_sin(
b*x)/x^2
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\mathrm{S}\left(bx\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^3,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^3}\,dx$$

```
[In] int(FresnelS(b*x)/x^3,x)
[Out] int(FresnelS(b*x)/x^3, x)
```

3.12 $\int \frac{\text{FresnelS}(bx)}{x^4} dx$

Optimal result

Integrand size = 8, antiderivative size = 52

$$\int \frac{\text{FresnelS}(bx)}{x^4} dx = \frac{1}{12}b^3\pi \text{ CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{3x^3} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}$$

```
[Out] 1/12*b^3*Pi*Ci(1/2*b^2*Pi*x^2)-1/3*FresnelS(b*x)/x^3-1/6*b*sin(1/2*b^2*Pi*x
^2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3378, 3383}

$$\int \frac{\text{FresnelS}(bx)}{x^4} dx = -\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2} + \frac{1}{12}\pi b^3 \text{ CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{3x^3}$$

```
[In] Int[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{3x^3} + \frac{1}{3}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{\text{FresnelS}(bx)}{3x^3} + \frac{1}{6}b\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{\text{FresnelS}(bx)}{3x^3} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} + \frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{3x^3} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{3x^3} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}$$

```
[In] Integrate[FresnelS[b*x]/x^4,x]
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

method	result	size
parts	$-\frac{\mathrm{FresnelS}(bx)}{3x^3}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}+\frac{b^2\pi\,\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{3}$	46
derivativedivides	$b^3\left(-\frac{\mathrm{FresnelS}(bx)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
default	$b^3\left(-\frac{\mathrm{FresnelS}(bx)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
meijerg	$\frac{\pi^{\frac{3}{2}}b^3\left(-\frac{\pi^{\frac{3}{2}}x^4b^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],\left[2,2,\frac{5}{2},\frac{11}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{21}+\frac{\frac{16\gamma}{3}-\frac{16\ln(2)}{3}-\frac{80}{9}+\frac{32\ln(x)}{3}+\frac{16\ln(\pi)}{3}+\frac{32\ln(b)}{3}}{\sqrt{\pi}}\right)}{64}$	68

```
[In] int(FresnelS(b*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*FresnelS(b*x)/x^3+1/3*b*(-1/2*sin(1/2*b^2*Pi*x^2)/x^2+1/4*b^2*Pi*Ci(1/
2*b^2*Pi*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\mathrm{FresnelS}(bx)}{x^4}\,dx=\frac{\pi b^3x^3\,\mathrm{Ci}\left(\frac{1}{2}\pi b^2x^2\right)-2\,bx\sin\left(\frac{1}{2}\pi b^2x^2\right)-4\,\mathrm{S}\left(bx\right)}{12\,x^3}$$

```
[In] integrate(fresnel_sin(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(pi*b^3*x^3*cos_integral(1/2*pi*b^2*x^2) - 2*b*x*sin(1/2*pi*b^2*x^2) -
 4*fresnel_sin(b*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{\mathrm{FresnelS}(bx)}{x^4}\,dx=-\frac{\pi^3b^7x^4\Gamma\left(\frac{7}{4}\right)\,{}_3F_4\left(\left.\begin{matrix}1,1,\frac{7}{4}\\2,2,\frac{5}{2},\frac{11}{4}\end{matrix}\right|-\frac{\pi^2b^4x^4}{16}\right)}{768\Gamma\left(\frac{11}{4}\right)}+\frac{\pi b^3\log\left(b^4x^4\right)}{24}$$

```
[In] integrate(fresnels(b*x)/x**4,x)
```

```
[Out] -pi**3*b**7*x**4*gamma(7/4)*hyper((1, 1, 7/4), (2, 2, 5/2, 11/4), -pi**2*b*
*4*x**4/16)/(768*gamma(11/4)) + pi*b**3*log(b**4*x**4)/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \frac{1}{24}\left(\pi\Gamma\left(-1, \frac{1}{2}i\,\pi b^2x^2\right) + \pi\Gamma\left(-1, -\frac{1}{2}i\,\pi b^2x^2\right)\right)b^3 - \frac{\text{S}\,(bx)}{3\,x^3}$$

```
[In] integrate(fresnel_sin(b*x)/x^4,x, algorithm="maxima")
[Out] 1/24*(pi*gamma(-1, 1/2*I*pi*b^2*x^2) + pi*gamma(-1, -1/2*I*pi*b^2*x^2))*b^3
 - 1/3*fresnel_sin(b*x)/x^3
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\text{S}\,(bx)}{x^4}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^4,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^4}\,dx$$

```
[In] int(FresnelS(b*x)/x^4,x)
[Out] int(FresnelS(b*x)/x^4, x)
```

3.13 $\int \frac{\text{FresnelS}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx) - \frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}$$

```
[Out] -1/12*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x-1/12*b^4*Pi^2*FresnelS(b*x)-1/4*FresnelS
(b*x)/x^4-1/12*b*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3468, 3469, 3432}

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = -\frac{1}{12}\pi^2b^4\,\text{FresnelS}(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3} - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x} - \frac{\text{FresnelS}(bx)}{4x^4}$$

```
[In] Int[FresnelS[b*x]/x^5,x]
```

```
[Out] -1/12*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{4x^4} + \frac{1}{4}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} + \frac{1}{12}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}\left(b^5\pi^2\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx) - \frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$
\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx) - \frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}
$$

```
[In] Integrate[FresnelS[b*x]/x^5,x]
```

```
[Out] -1/12*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$b^{4}\left(-\frac{\mathrm{FresnelS}(bx)}{4b^{4}x^{4}}-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{12b^{3}x^{3}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{bx}-\pi\ \mathrm{FresnelS}(bx)\right)}{12}\right)$	65
default	$b^{4}\left(-\frac{\mathrm{FresnelS}(bx)}{4b^{4}x^{4}}-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{12b^{3}x^{3}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{bx}-\pi\ \mathrm{FresnelS}(bx)\right)}{12}\right)$	65
meijerg	$\frac{\pi^{2}b^{4}\left(-\frac{32\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3\pi xb}-\frac{32\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3\pi^{2}x^{3}b^{3}}-\frac{32\left(x^{4}\pi^{2}b^{4}+3\right)\mathrm{FresnelS}(bx)}{3\pi^{2}x^{4}b^{4}}\right)}{128}$	79
parts	$-\frac{\mathrm{FresnelS}(bx)}{4x^{4}}+\frac{b\left(-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3x^{3}}+\frac{b^{2}\pi\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x}-\frac{b^{2}\pi^{\frac{3}{2}}\ \mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{\sqrt{b^{2}\pi}}\right)}{3}\right)}{4}$	83

```
[In] int(FresnelS(b*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-1/4*FresnelS(b*x)/b^4/x^4-1/12/b^3/x^3*sin(1/2*b^2*Pi*x^2)+1/12*Pi*(-
1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{\mathrm{FresnelS}(bx)}{x^5}\,dx = -\frac{\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^4x^4+3\right)\mathrm{S}\left(bx\right)}{12\,x^4}$$

```
[In] integrate(fresnel_sin(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(pi*b^3*x^3*cos(1/2*pi*b^2*x^2) + b*x*sin(1/2*pi*b^2*x^2) + (pi^2*b^4
*x^4 + 3)*fresnel_sin(b*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = \frac{\pi^2 b^4 S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64\Gamma\left(\frac{7}{4}\right)} + \frac{\pi b^3 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x\Gamma\left(\frac{7}{4}\right)} + \frac{b \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x^3\Gamma\left(\frac{7}{4}\right)} + \frac{3S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64x^4\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnels(b*x)*gamma(-1/4)/(64*gamma(7/4)) + pi*b**3*cos(pi*b**2*
x**2/2)*gamma(-1/4)/(64*x*gamma(7/4)) + b*sin(pi*b**2*x**2/2)*gamma(-1/4)/(
64*x**3*gamma(7/4)) + 3*fresnels(b*x)*gamma(-1/4)/(64*x**4*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{3}{2}}\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{1}{2}i\pi b^2x^2\right) + (i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^4}{64\,x^3} - \frac{\mathrm{S}\,(bx)}{4\,x^4}$$

```
[In] integrate(fresnel_sin(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(1/2)*(pi*x^2)^(3/2)*(-(I + 1)*sqrt(2)*gamma(-3/2, 1/2*I*pi*b^2*x
^2) + (I - 1)*sqrt(2)*gamma(-3/2, -1/2*I*pi*b^2*x^2))*b^4/x^3 - 1/4*fresnel
_sin(b*x)/x^4
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\mathrm{S}\,(bx)}{x^5}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^5} \, dx = \int \frac{\text{FresnelS}(b\,x)}{x^5} \, dx$$

```
[In] int(FresnelS(b*x)/x^5,x)
[Out] int(FresnelS(b*x)/x^5, x)
```

3.14 $\int \frac{\text{FresnelS}(bx)}{x^6} dx$

Optimal result

Integrand size = 8, antiderivative size = 77

$$\int \frac{\text{FresnelS}(bx)}{x^6} dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{\text{FresnelS}(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -1/40*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^2-1/5*FresnelS(b*x)/x^5-1/80*b^5*Pi^2*Si
(1/2*b^2*Pi*x^2)-1/20*b*sin(1/2*b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3378, 3380}

$$\int \frac{\text{FresnelS}(bx)}{x^6} dx = -\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4} - \frac{1}{80}\pi^2b^5\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2} - \frac{\text{FresnelS}(bx)}{5x^5}$$

```
[In] Int[FresnelS[b*x]/x^6,x]
```

```
[Out] -1/40*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{5x^5}+\frac{1}{5}b\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= -\frac{\text{FresnelS}(bx)}{5x^5}+\frac{1}{10}b\text{Subst}\left(\int\frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\text{FresnelS}(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}+\frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{\text{FresnelS}(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}\\
&\quad-\frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{\text{FresnelS}(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{\text{FresnelS}(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[In] Integrate[FresnelS[b*x]/x^6,x]
[Out] -1/40*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

method	result	size
meijerg	$-\frac{\pi\,b^3\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{12x^2}$	29
parts	$-\frac{\text{FresnelS}(bx)}{5x^5}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi\,\text{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{5}$	68
derivativedivides	$b^5\left(-\frac{\text{FresnelS}(bx)}{5b^5x^5}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\text{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71
default	$b^5\left(-\frac{\text{FresnelS}(bx)}{5b^5x^5}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\text{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71

```
[In] int(FresnelS(b*x)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/12*Pi*b^3/x^2*hypergeom([-1/2,3/4],[1/2,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{\pi^2 b^5 x^5\,\text{Si}\left(\frac{1}{2}\,\pi b^2 x^2\right) + 2\,\pi b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,bx \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 16\,\text{S}\,(bx)}{80\,x^5}$$

```
[In] integrate(fresnel_sin(b*x)/x^6,x, algorithm="fricas")
[Out] -1/80*(pi^2*b^5*x^5*sin_integral(1/2*pi*b^2*x^2) + 2*pi*b^3*x^3*cos(1/2*pi*
b^2*x^2) + 4*b*x*sin(1/2*pi*b^2*x^2) + 16*fresnel_sin(b*x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{\pi b^3 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16 x^2 \Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**6,x)
[Out] -pi*b**3*gamma(3/4)*hyper((-1/2, 3/4), (1/2, 3/2, 7/4), -pi**2*b**4*x**4/16
)/(16*x**2*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{1}{80}\left(-i\,\pi^2\Gamma\left(-2, \frac{1}{2}i\,\pi b^2 x^2\right) + i\,\pi^2\Gamma\left(-2, -\frac{1}{2}i\,\pi b^2 x^2\right)\right)b^5 - \frac{\text{S}\,(bx)}{5\,x^5}$$

```
[In] integrate(fresnel_sin(b*x)/x^6,x, algorithm="maxima")
[Out] -1/80*(-I*pi^2*gamma(-2, 1/2*I*pi*b^2*x^2) + I*pi^2*gamma(-2, -1/2*I*pi*b^2
*x^2))*b^5 - 1/5*fresnel_sin(b*x)/x^5
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\text{S}\,(bx)}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^6,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^6}\,dx$$

```
[In] int(FresnelS(b*x)/x^6,x)
[Out] int(FresnelS(b*x)/x^6, x)
```

3.15 $\int \frac{\text{FresnelS}(bx)}{x^7} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \frac{\text{FresnelS}(bx)}{x^7} \, dx = -\frac{b^3 \pi \cos\left(\frac{1}{2} b^2 \pi x^2\right)}{90 x^3} - \frac{1}{90} b^6 \pi^3 \text{FresnelC}(bx) - \frac{\text{FresnelS}(bx)}{6 x^6} - \frac{b \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{30 x^5} + \frac{b^5 \pi^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{90 x}$$

```
[Out] -1/90*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^3-1/90*b^6*Pi^3*FresnelC(b*x)-1/6*Fresne
lS(b*x)/x^6-1/30*b*sin(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*sin(1/2*b^2*Pi*x^2
)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3468, 3469, 3433}

$$\int \frac{\text{FresnelS}(bx)}{x^7} \, dx = -\frac{1}{90} \pi^3 b^6 \text{FresnelC}(bx) - \frac{b \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{30 x^5} + \frac{\pi^2 b^5 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{90 x} - \frac{\pi b^3 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{90 x^3} - \frac{\text{FresnelS}(bx)}{6 x^6}$$

```
[In] Int[FresnelS[b*x]/x^7,x]
```

```
[Out] -1/90*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^6*Pi^3*FresnelC[b*x])/90 - Fres
nelS[b*x]/(6*x^6) - (b*Sin[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Sin[(b^2*P
i*x^2)/2])/(90*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{6x^6}+\frac{1}{6}b\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{\text{FresnelS}(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{1}{30}\left(b^3\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{\text{FresnelS}(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{1}{90}\left(b^5\pi^2\right)\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{\text{FresnelS}(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}\\
&\quad+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{1}{90}\left(b^7\pi^3\right)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}b^6\pi^3\,\text{FresnelC}(bx)\\
&\quad-\frac{\text{FresnelS}(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int \frac{\text{FresnelS}(bx)}{x^7} dx = \frac{1}{90} \left(-\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} - b^6\pi^3 \,\text{FresnelC}(bx) - \frac{15\,\text{FresnelS}(bx)}{x^6} + \frac{b(-3+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} \right)$$

```
[In] Integrate[FresnelS[b*x]/x^7,x]
[Out] (-((b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3) - b^6*Pi^3*FresnelC[b*x] - (15*Fresnel
S[b*x])/x^6 + (b*(-3 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^5)/90
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.31

method	result	size
meijerg	$-\frac{\pi b^{3} \text{hypergeom}\left(\left[-\frac{3}{4}, \frac{3}{4}\right], \left[\frac{1}{4}, \frac{3}{2}, \frac{7}{4}\right], -\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{18 x^{3}}$	29
derivativedivides	$b^{6}\left(-\frac{\text{FresnelS}(b x)}{6 b^{6} x^{6}}-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{30 b^{5} x^{5}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{3 b^{3} x^{3}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b x}+\pi\ \text{FresnelC}(b x)\right)}{3}\right)}{30}\right)$	86
default	$b^{6}\left(-\frac{\text{FresnelS}(b x)}{6 b^{6} x^{6}}-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{30 b^{5} x^{5}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{3 b^{3} x^{3}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b x}+\pi\ \text{FresnelC}(b x)\right)}{3}\right)}{30}\right)$	86
parts	$-\frac{\text{FresnelS}(b x)}{6 x^{6}}+\frac{b\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 x^{5}}+\frac{b^{2} \pi\left(-\frac{\cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{3 x^{3}}-\frac{b^{2} \pi\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{x}+\frac{b^{2} \pi^{\frac{3}{2}} \text{FresnelC}\left(\frac{\sqrt{\pi}\, b^{2} x}{\sqrt{b^{2} \pi}}\right)}{\sqrt{b^{2} \pi}}\right)}{3}\right)}{5}\right)}{6}$	104

```
[In] int(FresnelS(b*x)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/18*Pi*b^3/x^3*hypergeom([-3/4,3/4],[1/4,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = -\frac{\pi^3\sqrt{b^2}b^5x^6\,\text{C}\left(\sqrt{b^2}x\right) + \pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right) - \left(\pi^2b^5x^5 - 3\,bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + 15\,\text{S}\left(bx\right)}{90\,x^6}$$

```
[In] integrate(fresnel_sin(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(pi^3*sqrt(b^2)*b^5*x^6*fresnel_cos(sqrt(b^2)*x) + pi*b^3*x^3*cos(1/2
*pi*b^2*x^2) - (pi^2*b^5*x^5 - 3*b*x)*sin(1/2*pi*b^2*x^2) + 15*fresnel_sin(
b*x))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = \frac{\pi b^3\Gamma\left(-\frac{3}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32x^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**7,x)
```

```
[Out] pi*b**3*gamma(-3/4)*gamma(3/4)*hyper((-3/4, 3/4), (1/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*x**3*gamma(1/4)*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(\pi x^2\right)^{\frac{5}{2}}\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{1}{2}i\,\pi b^2x^2\right) + (i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^6}{192\,x^5} - \frac{\text{S}\left(bx\right)}{6\,x^6}$$

```
[In] integrate(fresnel_sin(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] -1/192*sqrt(1/2)*(pi*x^2)^(5/2)*(-(I - 1)*sqrt(2)*gamma(-5/2, 1/2*I*pi*b^2*
x^2) + (I + 1)*sqrt(2)*gamma(-5/2, -1/2*I*pi*b^2*x^2))*b^6/x^5 - 1/6*fresne
l_sin(b*x)/x^6
```

Giac [F]

$$\int \frac{\mathrm{FresnelS}(bx)}{x^7}\,dx = \int \frac{\mathrm{S}\,(bx)}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^7,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{FresnelS}(bx)}{x^7}\,dx = \int \frac{\mathrm{FresnelS}(b\,x)}{x^7}\,dx$$

```
[In] int(FresnelS(b*x)/x^7,x)
[Out] int(FresnelS(b*x)/x^7, x)
```

3.16 $\int \frac{\text{FresnelS}(bx)}{x^8} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 102

$$\int \frac{\text{FresnelS}(bx)}{x^8} \, dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{672}b^7\pi^3\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}$$

```
[Out] -1/672*b^7*Pi^3*Ci(1/2*b^2*Pi*x^2)-1/168*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^4-1/7
*FresnelS(b*x)/x^7-1/42*b*sin(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*sin(1/2*b^
2*Pi*x^2)/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3378, 3383}

$$\int \frac{\text{FresnelS}(bx)}{x^8} \, dx = -\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6} - \frac{1}{672}\pi^3b^7\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2} - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4} - \frac{\text{FresnelS}(bx)}{7x^7}$$

```
[In] Int[FresnelS[b*x]/x^8,x]
```

```
[Out] -1/168*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^7*Pi^3*CosIntegral[(b^2*Pi*x^2
)/2])/672 - FresnelS[b*x]/(7*x^7) - (b*Sin[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5
*Pi^2*Sin[(b^2*Pi*x^2)/2])/(336*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{7x^7} + \frac{1}{7}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{\text{FresnelS}(bx)}{7x^7} + \frac{1}{14}b\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\text{FresnelS}(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{\text{FresnelS}(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} \\
&\quad - \frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{\text{FresnelS}(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} \\
&\quad + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right)
\end{aligned}
$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{672}b^7\pi^3\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \frac{1}{672}\left(-\frac{4b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} - b^7\pi^3\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{96\,\text{FresnelS}(bx)}{x^7} + \frac{2b(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\right)$$

```
[In] Integrate[FresnelS[b*x]/x^8,x]
```

```
[Out] ((-4*b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - b^7*Pi^3*CosIntegral[(b^2*Pi*x^2)/2]
 - (96*FresnelS[b*x])/x^7 + (2*b*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x
^6)/672
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result	siz
meijerg	$\frac{\pi^{\frac{7}{2}} b^{7} \left(\frac{\pi^{\frac{3}{2}} x^{4} b^{4} \operatorname{hypergeom}\left(\left[1,1,\frac{11}{4}\right],\left[2,3,\frac{7}{2},\frac{15}{4}\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{165}-\frac{16\left(-\frac{89}{21}+2 \gamma-2 \ln(2)+4 \ln(x)+2 \ln(\pi)+4 \ln(b)\right)}{21 \sqrt{\pi}}-\frac{128}{3 \pi^{\frac{5}{2}} x^{4} b^{4}}\right)}{1024}$	79
parts	$-\frac{\operatorname{FresnelS}(bx)}{7 x^{7}}+\frac{b\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{6 x^{6}}+\frac{b^{2} \pi\left(-\frac{\cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{4 x^{4}}-\frac{b^{2} \pi\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{2 x^{2}}+\frac{b^{2} \pi \operatorname{Ci}\left(\frac{b^{2} \pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{7}$	90
derivativedivides	$b^{7}\left(-\frac{\operatorname{FresnelS}(bx)}{7 b^{7} x^{7}}-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{42 b^{6} x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{4 b^{4} x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{2 b^{2} x^{2}}+\frac{\pi \operatorname{Ci}\left(\frac{b^{2} \pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93
default	$b^{7}\left(-\frac{\operatorname{FresnelS}(bx)}{7 b^{7} x^{7}}-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{42 b^{6} x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{4 b^{4} x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{2 b^{2} x^{2}}+\frac{\pi \operatorname{Ci}\left(\frac{b^{2} \pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93

```
[In] int(FresnelS(b*x)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1024*Pi^(7/2)*b^7*(1/165*Pi^(3/2)*x^4*b^4*hypergeom([1,1,11/4],[2,3,7/2,1
5/4],-1/16*x^4*Pi^2*b^4)-16/21*(-89/21+2*gamma-2*ln(2)+4*ln(x)+2*ln(Pi)+4*l
n(b))/Pi^(1/2)-128/3/Pi^(5/2)/x^4/b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = -\frac{\pi^3 b^7 x^7 \,\text{Ci}\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,\pi b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 2\,(\pi^2 b^5 x^5 - 8\,bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 96\,\text{S}\,(bx)}{672\,x^7}$$

```
[In] integrate(fresnel_sin(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] -1/672*(pi^3*b^7*x^7*cos_integral(1/2*pi*b^2*x^2) + 4*pi*b^3*x^3*cos(1/2*pi
*b^2*x^2) - 2*(pi^2*b^5*x^5 - 8*b*x)*sin(1/2*pi*b^2*x^2) + 96*fresnel_sin(b
*x))/x^7
```

Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \frac{\pi^5 b^{11} x^4 \Gamma\left(\frac{11}{4}\right) {}_3F_4\left(\left.\begin{matrix} 1, 1, \frac{11}{4} \\ 2, 3, \frac{7}{2}, \frac{15}{4} \end{matrix}\right| -\frac{\pi^2 b^4 x^4}{16}\right)}{61440\Gamma\left(\frac{15}{4}\right)} - \frac{\pi^3 b^7 \log\left(b^4 x^4\right)}{1344} - \frac{\pi b^3}{24x^4}$$

```
[In] integrate(fresnels(b*x)/x**8,x)
```

```
[Out] pi**5*b**11*x**4*gamma(11/4)*hyper((1, 1, 11/4), (2, 3, 7/2, 15/4), -pi**2*
b**4*x**4/16)/(61440*gamma(15/4)) - pi**3*b**7*log(b**4*x**4)/1344 - pi*b**
3/(24*x**4)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.45

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = -\frac{1}{224}\left(\pi^3 \Gamma\left(-3, \frac{1}{2} i\,\pi b^2 x^2\right) + \pi^3 \Gamma\left(-3, -\frac{1}{2} i\,\pi b^2 x^2\right)\right) b^7 - \frac{\text{S}\,(bx)}{7\,x^7}$$

```
[In] integrate(fresnel_sin(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] -1/224*(pi^3*gamma(-3, 1/2*I*pi*b^2*x^2) + pi^3*gamma(-3, -1/2*I*pi*b^2*x^2
))*b^7 - 1/7*fresnel_sin(b*x)/x^7
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{S(bx)}{x^8}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^8,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^8}\,dx$$

```
[In] int(FresnelS(b*x)/x^8,x)
[Out] int(FresnelS(b*x)/x^8, x)
```

3.17 $\int \frac{\text{FresnelS}(bx)}{x^9} dx$

Optimal result

Integrand size = 8, antiderivative size = 119

$$\int \frac{\text{FresnelS}(bx)}{x^9} dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx) - \frac{\text{FresnelS}(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}$$

```
[Out] -1/280*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^5+1/840*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x+
1/840*b^8*Pi^4*FresnelS(b*x)-1/8*FresnelS(b*x)/x^8-1/56*b*sin(1/2*b^2*Pi*x^
2)/x^7+1/840*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3468, 3469, 3432}

$$\int \frac{\text{FresnelS}(bx)}{x^9} dx = \frac{1}{840}\pi^4 b^8\,\text{FresnelS}(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7} + \frac{\pi^3 b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x} + \frac{\pi^2 b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3} - \frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5} - \frac{\text{FresnelS}(bx)}{8x^8}$$

```
[In] Int[FresnelS[b*x]/x^9,x]
```

```
[Out] -1/280*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^5 + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(8
40*x) + (b^8*Pi^4*FresnelS[b*x])/840 - FresnelS[b*x]/(8*x^8) - (b*Sin[(b^2*
Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Sin[(b^2*Pi*x^2)/2])/(840*x^3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{8x^8}+\frac{1}{8}b\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx\\
&= -\frac{\text{FresnelS}(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{1}{56}\left(b^3\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{\text{FresnelS}(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}-\frac{1}{280}\left(b^5\pi^2\right)\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{\text{FresnelS}(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}\\
&\quad+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}-\frac{1}{840}\left(b^7\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x}-\frac{\text{FresnelS}(bx)}{8x^8}\\
&\quad-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}+\frac{1}{840}\left(b^9\pi^4\right)\int\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x}+\frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx)\\
&\quad-\frac{\text{FresnelS}(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx$$
$$= \frac{b^3\pi x^3(-3+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+(-105+b^8\pi^4x^8)\,\text{FresnelS}(bx)+bx(-15+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^8}$$

```
[In] Integrate[FresnelS[b*x]/x^9,x]
```

```
[Out] (b^3*Pi*x^3*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)
*FresnelS[b*x] + b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

method	result
meijerg	$-\frac{\pi b^{3} \mathrm{hypergeom}\left(\left[-\frac{5}{4},\frac{3}{4}\right],\left[-\frac{1}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{30x^{5}}$
derivativedivides	$b^{8}\left(-\frac{\mathrm{FresnelS}(bx)}{8b^{8}x^{8}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{56b^{7}x^{7}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5b^{5}x^{5}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3b^{3}x^{3}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}-\pi\ \mathrm{FresnelS}(bx)\right)}{3}\right)}{5}\right)}{56}\right.$
default	$b^{8}\left(-\frac{\mathrm{FresnelS}(bx)}{8b^{8}x^{8}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{56b^{7}x^{7}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5b^{5}x^{5}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3b^{3}x^{3}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}-\pi\ \mathrm{FresnelS}(bx)\right)}{3}\right)}{5}\right)}{56}\right.$
	$b\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{7x^{7}}+\frac{b^{2}\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5x^{5}}-\frac{b^{2}\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3x^{3}}+\frac{b^{2}\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{x}-\frac{b^{2}\pi^{\frac{3}{2}}\ \mathrm{FresnelS}\left(\frac{\sqrt{\pi}\ldots}{\sqrt{\ldots}}\right)}{\sqrt{b^{2}\pi}}\ldots\right.}{3}\right.}{5}\right.}{7}\right.$

```
[In] int(FresnelS(b*x)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*Pi*b^3/x^5*hypergeom([-5/4,3/4],[-1/4,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{\mathrm{FresnelS}(bx)}{x^9}\,dx = \frac{\left(\pi^3 b^7 x^7 - 3\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + \left(\pi^4 b^8 x^8 - 105\right)\mathrm{S}\left(bx\right) + \left(\pi^2 b^5 x^5 - 15\,bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{840\,x^8}$$

```
[In] integrate(fresnel_sin(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] 1/840*((pi^3*b^7*x^7 - 3*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 -
105)*fresnel_sin(b*x) + (pi^2*b^5*x^5 - 15*b*x)*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \frac{\mathrm{FresnelS}(bx)}{x^9}\,dx = \frac{\pi^4 b^8 S(bx)\,\Gamma\left(-\frac{5}{4}\right)}{3584\Gamma\left(\frac{7}{4}\right)} + \frac{\pi^3 b^7 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x\Gamma\left(\frac{7}{4}\right)} + \frac{\pi^2 b^5 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^3\Gamma\left(\frac{7}{4}\right)} - \frac{3\pi b^3 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^5\Gamma\left(\frac{7}{4}\right)} - \frac{15b \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^7\Gamma\left(\frac{7}{4}\right)} - \frac{15S(bx)\,\Gamma\left(-\frac{5}{4}\right)}{512x^8\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**9,x)
```

```
[Out] pi**4*b**8*fresnels(b*x)*gamma(-5/4)/(3584*gamma(7/4)) + pi**3*b**7*cos(pi*
b**2*x**2/2)*gamma(-5/4)/(3584*x*gamma(7/4)) + pi**2*b**5*sin(pi*b**2*x**2/
2)*gamma(-5/4)/(3584*x**3*gamma(7/4)) - 3*pi*b**3*cos(pi*b**2*x**2/2)*gamma
(-5/4)/(3584*x**5*gamma(7/4)) - 15*b*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3584*
x**7*gamma(7/4)) - 15*fresnels(b*x)*gamma(-5/4)/(512*x**8*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{7}{2}}\left((i+1)\sqrt{2}\Gamma\left(-\frac{7}{2},\frac{1}{2}i\pi b^2x^2\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^8}{512\,x^7}-\frac{S\left(bx\right)}{8\,x^8}$$

```
[In] integrate(fresnel_sin(b*x)/x^9,x, algorithm="maxima")
[Out] -1/512*sqrt(1/2)*(pi*x^2)^(7/2)*((I + 1)*sqrt(2)*gamma(-7/2, 1/2*I*pi*b^2*x
^2) - (I - 1)*sqrt(2)*gamma(-7/2, -1/2*I*pi*b^2*x^2))*b^8/x^7 - 1/8*fresnel
_sin(b*x)/x^8
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{S\left(bx\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^9,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^9}\,dx$$

```
[In] int(FresnelS(b*x)/x^9,x)
[Out] int(FresnelS(b*x)/x^9, x)
```

3.18 $\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 127

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}-\frac{\text{FresnelS}(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

```
[Out] -1/432*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^6+1/3456*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x
^2-1/9*FresnelS(b*x)/x^9+1/6912*b^9*Pi^4*Si(1/2*b^2*Pi*x^2)-1/72*b*sin(1/2*
b^2*Pi*x^2)/x^8+1/1728*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3378, 3380}

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^4b^9\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{\text{FresnelS}(bx)}{9x^9}$$

```
[In] Int[FresnelS[b*x]/x^10,x]
```

```
[Out] -1/432*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^6 + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(3
456*x^2) - FresnelS[b*x]/(9*x^9) - (b*Sin[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*
Pi^2*Sin[(b^2*Pi*x^2)/2])/(1728*x^4) + (b^9*Pi^4*SinIntegral[(b^2*Pi*x^2)/2
])/6912
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)}{9x^9}+\frac{1}{9}b\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx\\
&= -\frac{\text{FresnelS}(bx)}{9x^9}+\frac{1}{18}b\text{Subst}\left(\int\frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx,x,x^2\right)\\
&= -\frac{\text{FresnelS}(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{\text{FresnelS}(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}\\
&\quad-\frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{\text{FresnelS}(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}\\
&\quad+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int\frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)}{3456}
\end{aligned}$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{\text{FresnelS}(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right)}{6912}$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{\text{FresnelS}(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int\frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \frac{\frac{2b^3\pi(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} - \frac{768\,\text{FresnelS}(bx)}{x^9} + \frac{4b(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} + b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

```
[In] Integrate[FresnelS[b*x]/x^10,x]
```

```
[Out] ((2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (768*FresnelS[b*x
])/x^9 + (4*b*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*SinI
ntegral[(b^2*Pi*x^2)/2])/6912
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

method	result
meijerg	$-\frac{\pi b^3 \operatorname{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{36x^6}$
parts	$-\frac{\operatorname{FresnelS}(bx)}{9x^9}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{8x^8}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6x^6}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi \operatorname{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right.}{4}\right.}{6}\right.}{8}\right.}{9}$
derivativedivides	$b^9\left(-\frac{\operatorname{FresnelS}(bx)}{9b^9x^9}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi \operatorname{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right.$
default	$b^9\left(-\frac{\operatorname{FresnelS}(bx)}{9b^9x^9}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi \operatorname{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right.$

```
[In] int(FresnelS(b*x)/x^10,x,method=_RETURNVERBOSE)
[Out] -1/36*Pi*b^3/x^6*hypergeom([-3/2,3/4],[-1/2,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \frac{\pi^4 b^9 x^9 \,\text{Si}\left(\frac{1}{2}\pi b^2 x^2\right) + 2\left(\pi^3 b^7 x^7 - 8\pi b^3 x^3\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) + 4\left(\pi^2 b^5 x^5 - 24\,bx\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right) - 768\,\text{S}\left(bx\right)}{6912\,x^9}$$

```
[In] integrate(fresnel_sin(b*x)/x^10,x, algorithm="fricas")
[Out] 1/6912*(pi^4*b^9*x^9*sin_integral(1/2*pi*b^2*x^2) + 2*(pi^3*b^7*x^7 - 8*pi*
b^3*x^3)*cos(1/2*pi*b^2*x^2) + 4*(pi^2*b^5*x^5 - 24*b*x)*sin(1/2*pi*b^2*x^2
) - 768*fresnel_sin(b*x))/x^9
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{\pi b^3 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{48 x^6 \Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(fresnels(b*x)/x**10,x)
[Out] -pi*b**3*gamma(3/4)*hyper((-3/2, 3/4), (-1/2, 3/2, 7/4), -pi**2*b**4*x**4/1
6)/(48*x**6*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{1}{576}\left(i\,\pi^4 \Gamma\left(-4, \frac{1}{2} i\,\pi b^2 x^2\right) - i\,\pi^4 \Gamma\left(-4, -\frac{1}{2} i\,\pi b^2 x^2\right)\right) b^9 - \frac{\text{S}\left(bx\right)}{9\,x^9}$$

```
[In] integrate(fresnel_sin(b*x)/x^10,x, algorithm="maxima")
[Out] -1/576*(I*pi^4*gamma(-4, 1/2*I*pi*b^2*x^2) - I*pi^4*gamma(-4, -1/2*I*pi*b^2
*x^2))*b^9 - 1/9*fresnel_sin(b*x)/x^9
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\text{S}\left(bx\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_sin(b*x)/x^10,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^{10}}\,dx$$

```
[In] int(FresnelS(b*x)/x^10,x)
[Out] int(FresnelS(b*x)/x^10, x)
```

3.19 $\int (c+dx)^3 \text{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 296

$$\begin{aligned}\int (c+dx)^3 \text{FresnelS}(a+bx)\,dx &= \frac{(bc-ad)^3 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} \\ &+ \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} \\ &+ \frac{d^2(bc-ad)(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} \\ &+ \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} \\ &- \frac{3d(bc-ad)^2 \text{FresnelC}(a+bx)}{2b^4\pi} \\ &- \frac{(bc-ad)^4 \text{FresnelS}(a+bx)}{4b^4d} \\ &+ \frac{3d^3 \text{FresnelS}(a+bx)}{4b^4\pi^2} + \frac{(c+dx)^4 \text{FresnelS}(a+bx)}{4d} \\ &- \frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} \\ &- \frac{3d^3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}\end{aligned}$$

```
[Out] (-a*d+b*c)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*d*(-a*d+b*c)^2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+d^2*(-a*d+b*c)*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+1/4*d^3*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)^2*FresnelC(b*x+a)/b^4/Pi-1/4*(-a*d+b*c)^4*FresnelS(b*x+a)/b^4/d+3/4*d^3*FresnelS(b*x+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelS(b*x+a)/d-2*d^2*(-a*d+b*c)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717, 3467}

$$\begin{aligned}\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = &-\frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4}\\ &+\frac{d^2(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}\\ &-\frac{3d(bc-ad)^2\operatorname{FresnelC}(a+bx)}{2\pi b^4}\\ &-\frac{(bc-ad)^4\operatorname{FresnelS}(a+bx)}{4b^4 d}\\ &+\frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}\\ &+\frac{3d(a+bx)(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}\\ &+\frac{3d^3\operatorname{FresnelS}(a+bx)}{4\pi^2 b^4}-\frac{3d^3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2 b^4}\\ &+\frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4}\\ &+\frac{(c+dx)^4\operatorname{FresnelS}(a+bx)}{4d}\end{aligned}$$

```
[In] Int[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (3*d*(b*c - a*d)^2*(a +
b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (d^2*(b*c - a*d)*(a + b*x)^2*Cos
[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (d^3*(a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/
(4*b^4*Pi) - (3*d*(b*c - a*d)^2*FresnelC[a + b*x])/(2*b^4*Pi) - ((b*c - a*d
)^4*FresnelS[a + b*x])/(4*b^4*d) + (3*d^3*FresnelS[a + b*x])/(4*b^4*Pi^2) +
 ((c + d*x)^4*FresnelS[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

<u>Rule 2717</u>

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

<u>Rule 2718</u>

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^4 \text{FresnelS}(a+bx)}{4d} - \frac{b\int (c+dx)^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{4d}$$

$$= \frac{(c+dx)^4 \text{FresnelS}(a+bx)}{4d} - \frac{\text{Subst}\left(\int \left(b^4c^4\left(1+\frac{ad(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{b^4c^4}\right)\sin\left(\frac{\pi x^2}{2}\right)+4b^3c^3d\left(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\right)x\right.}{}$$

$$= \frac{(c+dx)^4 \text{FresnelS}(a+bx)}{4d} - \frac{d^3\text{Subst}\left(\int x^4 \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} - \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^4} - \frac{\left(3d(bc-ad)^2\right)\text{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^4} - \frac{(bc-ad)^3\text{Subst}\left(\int x \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^4} - \frac{(bc-ad)^4\text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4d}$$

$$= \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{(bc-ad)^4 \text{FresnelS}(a+bx)}{4b^4d} + \frac{(c+dx)^4 \text{FresnelS}(a+bx)}{4d} - \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int x \sin\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{2b^4} - \frac{(bc-ad)^3\text{Subst}\left(\int \sin\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{2b^4} - \frac{\left(3d^3\right)\text{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4\pi} - \frac{\left(3d(bc-ad)^2\right)\text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^4\pi}$$

$$= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-ad)(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{3d(bc-ad)^2\,\text{FresnelC}(a+bx)}{2b^4\pi} - \frac{(bc-ad)^4\,\text{FresnelS}(a+bx)}{4b^4d} + \frac{(c+dx)^4\,\text{FresnelS}(a+bx)}{4d} - \frac{3d^3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} + \frac{(3d^3)\,\text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4\pi^2} - \frac{(d^2(bc-ad))\,\text{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{b^4\pi}$$

$$= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-ad)(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{3d(bc-ad)^2\,\text{FresnelC}(a+bx)}{2b^4\pi} - \frac{(bc-ad)^4\,\text{FresnelS}(a+bx)}{4b^4d} + \frac{3d^3\,\text{FresnelS}(a+bx)}{4b^4\pi^2} + \frac{(c+dx)^4\,\text{FresnelS}(a+bx)}{4d} - \frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3d^3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.43

$$\int (c+dx)^3\,\text{FresnelS}(a+bx)\,dx = \frac{4b^3c^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 6ab^2c^2d\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 4a^2bcd^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - a^3d^3\pi\cos\left(\frac{1}{2}\pi(a+\right.}{}$$

```
[In] Integrate[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] (4*b^3*c^3*Pi*Cos[(Pi*(a + b*x)^2)/2] - 6*a*b^2*c^2*d*Pi*Cos[(Pi*(a + b*x)^
2)/2] + 4*a^2*b*c*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a^3*d^3*Pi*Cos[(Pi*(a +
b*x)^2)/2] + 6*b^3*c^2*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - 4*a*b^2*c*d^2*Pi*x*
Cos[(Pi*(a + b*x)^2)/2] + a^2*b*d^3*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + 4*b^3*c*
d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] - a*b^2*d^3*Pi*x^2*Cos[(Pi*(a + b*x)^2)/
2] + b^3*d^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] - 6*d*(b*c - a*d)^2*Pi*FresnelC
[a + b*x] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi^2*(-a^2 + b^2*x^2) +
 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 + b^4*Pi^2*x^4))*Fresne
lS[a + b*x] - 8*b*c*d^2*Sin[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Sin[(Pi*(a + b*x)
^2)/2] - 3*b*d^3*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\text{FresnelS}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d} - \frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{\text{FresnelS}(bx+a)}{\pi}\right)}{\pi} + \frac{4(ad-bc)}{}}{}$
default	$\frac{\text{FresnelS}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d} - \frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{\text{FresnelS}(bx+a)}{\pi}\right)}{\pi} + \frac{4(ad-bc)}{}}{}$
parts	Expression too large to display

```
[In] int((d*x+c)^3*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*FresnelS(b*x+a)*(a*d-b*c-d*(b*x+a))^4/b^3/d-1/4/b^3/d*(-d^4/Pi*(b*
x+a)^3*cos(1/2*Pi*(b*x+a)^2)+3*d^4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1
/Pi*FresnelS(b*x+a))+4*(a*d-b*c)*d^3/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-8*(
a*d-b*c)*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-6*(a*d-b*c)^2*d^2/Pi*(b*x+a)*cos(1/
2*Pi*(b*x+a)^2)+6*(a*d-b*c)^2*d^2/Pi*FresnelC(b*x+a)+4*(a*d-b*c)^3*d/Pi*cos
(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^4*FresnelS(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.27

$$\int (c+dx)^3 \,\text{FresnelS}(a+bx)\,dx =$$

$$-\frac{6\,\pi(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{b^2}\,\text{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2(4\,ab^3c^3 - 6\,a^2b^2c^2d + 4\,a^3bcd^2 - a^4d^3) + 3\,d^3)\sqrt{b^2}\,\text{S}}{}$$

```
[In] integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(6*pi*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(b^2)*fresnel_cos(sqrt(b
^2)*(b*x + a)/b) - (pi^2*(4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 - a
^4*d^3) + 3*d^3)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) - (pi*b^4*d^3
*x^3 + pi*(4*b^4*c*d^2 - a*b^3*d^3)*x^2 + pi*(6*b^4*c^2*d - 4*a*b^3*c*d^2 +
 a^2*b^2*d^3)*x + pi*(4*b^4*c^3 - 6*a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - a^3*b*d
^3))*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (pi^2*b^5*d^3*x^4 + 4*pi
^2*b^5*c*d^2*x^3 + 6*pi^2*b^5*c^2*d*x^2 + 4*pi^2*b^5*c^3*x)*fresnel_sin(b*x
 + a) + (3*b^2*d^3*x + 8*b^2*c*d^2 - 5*a*b*d^3)*sin(1/2*pi*b^2*x^2 + pi*a*b
*x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [F]

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int (c+dx)^3 S(a+bx)\, dx$$

```
[In] integrate((d*x+c)**3*fresnels(b*x+a),x)
[Out] Integral((c + d*x)**3*fresnels(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^3 \operatorname{S}(bx+a)\, dx$$

```
[In] integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^3*fresnel_sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^3 \operatorname{S}(bx+a)\, dx$$

```
[In] integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int \operatorname{FresnelS}(a+b\,x)\,\left(c+d\,x\right)^3 dx$$

```
[In] int(FresnelS(a + b*x)*(c + d*x)^3,x)
[Out] int(FresnelS(a + b*x)*(c + d*x)^3, x)
```

3.20 $\int (c+dx)^2 \,\text{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 193

$$
\begin{aligned}
\int (c+dx)^2 \,\text{FresnelS}(a+bx)\,dx = {} & \frac{(bc-ad)^2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
& + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
& + \frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} \\
& - \frac{d(bc-ad)\,\text{FresnelC}(a+bx)}{b^3\pi} \\
& - \frac{(bc-ad)^3\,\text{FresnelS}(a+bx)}{3b^3d} \\
& + \frac{(c+dx)^3\,\text{FresnelS}(a+bx)}{3d} - \frac{2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}
\end{aligned}
$$

```
[Out] (-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(
b*x+a)^2)/b^3/Pi+1/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c
)*FresnelC(b*x+a)/b^3/Pi-1/3*(-a*d+b*c)^3*FresnelS(b*x+a)/b^3/d+1/3*(d*x+c)
^3*FresnelS(b*x+a)/d-2/3*d^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717}

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = -\frac{d(bc-ad)\operatorname{FresnelC}(a+bx)}{\pi b^3} - \frac{(bc-ad)^3\operatorname{FresnelS}(a+bx)}{3b^3d} + \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} + \frac{d(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} - \frac{2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3} + \frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3} + \frac{(c+dx)^3\operatorname{FresnelS}(a+bx)}{3d}$$

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{b \int (c+dx)^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right) dx}{3d}$$

$$= \frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{\operatorname{Subst}\left(\int \left(b^3c^3\left(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\right)\sin\left(\frac{\pi x^2}{2}\right) + 3b^2c^2d\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)x\sin\left(\frac{\pi x^2}{2}\right) + 3bcd\right.}{3b^3d}$$

$$= \frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{d^2 \operatorname{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right)\, dx, x, a+bx\right)}{3b^3} - \frac{(d(bc-ad)) \operatorname{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right)\, dx, x, a+bx\right)}{b^3} - \frac{(bc-ad)^2 \operatorname{Subst}\left(\int x \sin\left(\frac{\pi x^2}{2}\right)\, dx, x, a+bx\right)}{b^3} - \frac{(bc-ad)^3 \operatorname{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\, dx, x, a+bx\right)}{3b^3 d}$$

$$= \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3 \pi} - \frac{(bc-ad)^3 \operatorname{FresnelS}(a+bx)}{3b^3 d} + \frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{d^2 \operatorname{Subst}\left(\int x \sin\left(\frac{\pi x}{2}\right)\, dx, x, (a+bx)^2\right)}{6b^3} - \frac{(bc-ad)^2 \operatorname{Subst}\left(\int \sin\left(\frac{\pi x}{2}\right)\, dx, x, (a+bx)^2\right)}{2b^3} - \frac{(d(bc-ad)) \operatorname{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\, dx, x, a+bx\right)}{b^3 \pi}$$

$$= \frac{(bc-ad)^2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3 \pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3 \pi} + \frac{d^2(a+bx)^2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3 \pi} - \frac{d(bc-ad) \operatorname{FresnelC}(a+bx)}{b^3 \pi} - \frac{(bc-ad)^3 \operatorname{FresnelS}(a+bx)}{3b^3 d} + \frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{d^2 \operatorname{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right)\, dx, x, (a+bx)^2\right)}{3b^3 \pi}$$

$$= \frac{(bc-ad)^2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3 \pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3 \pi} + \frac{d^2(a+bx)^2 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3 \pi} - \frac{d(bc-ad) \operatorname{FresnelC}(a+bx)}{b^3 \pi} - \frac{(bc-ad)^3 \operatorname{FresnelS}(a+bx)}{3b^3 d} + \frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{2d^2 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3 \pi^2}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\int (c+dx)^2 \, \text{FresnelS}(a+bx) \, dx$$
$$= \frac{3b^2c^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-3abcd\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+a^2d^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+3b^2cd\pi x\cos\left(\frac{1}{2}\pi(a+b\right.}{}$$

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] (3*b^2*c^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - 3*a*b*c*d*Pi*Cos[(Pi*(a + b*x)^2)/2
] + a^2*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] + 3*b^2*c*d*Pi*x*Cos[(Pi*(a + b*x)^2
)/2] - a*b*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*d^2*Pi*x^2*Cos[(Pi*(a + b
*x)^2)/2] + 3*d*(-(b*c) + a*d)*Pi*FresnelC[a + b*x] + Pi^2*(3*a*b^2*c^2 - 3
*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelS[a + b*x]
 - 2*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\text{FresnelS}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}-\frac{3(ad-bc)d^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+}{3b^2d}}{b}$
default	$\frac{-\frac{\text{FresnelS}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}-\frac{3(ad-bc)d^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+}{3b^2d}}{b}$
parts	$\frac{\text{FresnelS}(bx+a)d^2x^3}{3}+\text{FresnelS}(bx+a)\,dc\,x^2+\text{FresnelS}(bx+a)\,c^2x+\frac{\text{FresnelS}(bx+a)c^3}{3d}-\frac{b\left(\right.}{}$

```
[In] int((d*x+c)^2*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3*FresnelS(b*x+a)*(a*d-b*c-d*(b*x+a))^3/b^2/d+1/3/b^2/d*(d^3/Pi*(b*
x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-3*(a*d-b*c)*d
```

```
^2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+3*(a*d-b*c)*d^2/Pi*FresnelC(b*x+a)+3*(a
*d-b*c)^2*d/Pi*cos(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^3*FresnelS(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.28

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{\pi^2(3ab^2c^2-3a^2bcd+a^3d^2)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-2bd^2\sin\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)-3\pi(bcd-ad^2)\sqrt{b^2}\,\mathrm{C}$$

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2)*sqrt(b^2)*fresnel_sin(sqrt(
b^2)*(b*x + a)/b) - 2*b*d^2*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - 3
*pi*(b*c*d - a*d^2)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + (pi*b^3*
d^2*x^2 + pi*(3*b^3*c*d - a*b^2*d^2)*x + pi*(3*b^3*c^2 - 3*a*b^2*c*d + a^2*
b*d^2))*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (pi^2*b^4*d^2*x^3 + 3
*pi^2*b^4*c*d*x^2 + 3*pi^2*b^4*c^2*x)*fresnel_sin(b*x + a))/(pi^2*b^4)
```

Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int (c+dx)^2\, S(a+bx)\,dx$$

```
[In] integrate((d*x+c)**2*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^2\,\mathrm{S}\,(bx+a)\,dx$$

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^2 \operatorname{S}(bx+a)\,dx$$

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int \operatorname{FresnelS}(a+b\,x)\,\left(c+d\,x\right)^2\,dx$$

```
[In] int(FresnelS(a + b*x)*(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)*(c + d*x)^2, x)
```

3.21 $\int (c+dx) \text{FresnelS}(a+bx) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int (c+dx) \text{FresnelS}(a+bx) \, dx = \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{d \,\text{FresnelC}(a+bx)}{2b^2\pi} - \frac{(bc-ad)^2 \text{FresnelS}(a+bx)}{2b^2 d} + \frac{(c+dx)^2 \text{FresnelS}(a+bx)}{2d}$$

```
[Out] (-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)
/b^2/Pi-1/2*d*FresnelC(b*x+a)/b^2/Pi-1/2*(-a*d+b*c)^2*FresnelS(b*x+a)/b^2/d
+1/2*(d*x+c)^2*FresnelS(b*x+a)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433}

$$\int (c+dx) \text{FresnelS}(a+bx) \, dx = -\frac{(bc-ad)^2 \text{FresnelS}(a+bx)}{2b^2 d} + \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2} - \frac{d \,\text{FresnelC}(a+bx)}{2\pi b^2} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2} + \frac{(c+dx)^2 \text{FresnelS}(a+bx)}{2d}$$

```
[In] Int[(c + d*x)*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a +
b*x)^2)/2])/(2*b^2*Pi) - (d*FresnelC[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)^2*
FresnelS[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelS[a + b*x])/(2*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d} - \frac{b\int (c+dx)^2 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{2d}$$

$$= \frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\sin\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)x\sin\left(\frac{\pi x^2}{2}\right)+d^2x^2\sin\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a}{2b^2d}$$

$$= \frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d} - \frac{d\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2} - \frac{(bc-ad)\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^2} - \frac{(bc-ad)^2\text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2d}$$

$$= \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{(bc-ad)^2\,\text{FresnelS}(a+bx)}{2b^2d} + \frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \sin\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^2} - \frac{d\text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2\pi}$$

$$= \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{d\,\text{FresnelC}(a+bx)}{2b^2\pi} - \frac{(bc-ad)^2\,\text{FresnelS}(a+bx)}{2b^2d} + \frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx = \frac{-d\,\text{FresnelC}(a+bx)+(2bc-ad+bdx)\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi(a+bx)\,\text{FresnelS}(a+bx)\right)}{2b^2\pi}$$

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x],x]
[Out] (-(d*FresnelC[a + b*x]) + (2*b*c - a*d + b*d*x)*(Cos[(Pi*(a + b*x)^2)/2] +
Pi*(a + b*x)*FresnelS[a + b*x]))/(2*b^2*Pi)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{\mathrm{FresnelS}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\ \mathrm{FresnelC}(bx+a)}{\pi}-\frac{(2ad-2bc)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
default	$\frac{-\frac{\mathrm{FresnelS}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\ \mathrm{FresnelC}(bx+a)}{\pi}-\frac{(2ad-2bc)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
parts	$\frac{\mathrm{FresnelS}(bx+a)d\,x^2}{2}+\mathrm{FresnelS}\,(bx+a)\,cx-\frac{b\left(-\frac{dx\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{da\left(-\frac{\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\right.}{b^2\pi}\right.}{}\right.}{}$

[In] int((d*x+c)*FresnelS(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b*(-FresnelS(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/2/b*(d/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-d/Pi*FresnelC(b*x+a)-(2*a*d-2*b*c)/Pi*cos(1/2*Pi*(b*x+a)^2)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int (c+dx)\,\mathrm{FresnelS}(a+bx)\,dx$$
$$=\frac{\pi(2\,abc-a^2d)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-\sqrt{b^2}d\,\mathrm{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(b^2dx+2\,b^2c-abd)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a\right.}{2\,\pi b^3}$$

[In] integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="fricas")

[Out] 1/2*(pi*(2*a*b*c - a^2*d)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) - sqrt(b^2)*d*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + (b^2*d*x + 2*b^2*c - a*b*d)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (pi*b^3*d*x^2 + 2*pi*b^3*c*x)*fresnel_sin(b*x + a))/(pi*b^3)

Sympy [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx=\int (c+dx)\,S(a+bx)\;dx$$

```
[In] integrate((d*x+c)*fresnels(b*x+a),x)
[Out] Integral((c + d*x)*fresnels(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx=\int (dx+c)\,\mathrm{S}\left(bx+a\right)\;dx$$

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)*fresnel_sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx=\int (dx+c)\,\mathrm{S}\left(bx+a\right)\;dx$$

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)*fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx=\int \text{FresnelS}(a+b\,x)\;\left(c+d\,x\right)\;dx$$

```
[In] int(FresnelS(a + b*x)*(c + d*x),x)
[Out] int(FresnelS(a + b*x)*(c + d*x), x)
```

3.22 $\int \text{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \text{FresnelS}(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}$$

```
[Out] cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6553}

$$\int \text{FresnelS}(a+bx)\,dx=\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral}=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

$$\int \text{FresnelS}(a+bx)\,dx = \frac{\cos\left(\frac{a^2\pi}{2}\right)\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}+\frac{a\,\text{FresnelS}(a+bx)}{b}+x\,\text{FresnelS}(a+bx)-\frac{\sin\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

[In] Integrate[FresnelS[a + b*x],x]

[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
default	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
parts	$x\ \text{FresnelS}(bx+a)-b\left(-\frac{\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelS}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	86

[In] int(FresnelS(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b*(FresnelS(b*x+a)*(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \text{FresnelS}(a+bx)\,dx = \frac{(\pi bx+\pi a)\,\text{S}\,(bx+a)+\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
[Out] ((pi*b*x + pi*a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]

$$\int \text{FresnelS}(a+bx)\,dx = \int S(a+bx)\ dx$$

```
[In] integrate(fresnels(b*x+a),x)
[Out] Integral(fresnels(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \text{FresnelS}(a+bx)\,dx = \frac{(bx+a)\,\text{S}\,(bx+a)+\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Giac [F]

$$\int \text{FresnelS}(a+bx)\,dx = \int S(bx+a)\,dx$$

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(a+bx)\,dx = \int \text{FresnelS}(a+b\,x)\,dx$$

```
[In] int(FresnelS(a + b*x),x)
[Out] int(FresnelS(a + b*x), x)
```

3.23 $\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx$$

```
[In] Int[FresnelS[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx=\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx$$

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x),x]
[Out] Integrate[FresnelS[a + b*x]/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)}{dx+c}dx$$

```
[In] int(FresnelS(b*x+a)/(d*x+c),x)
[Out] int(FresnelS(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx=\int \frac{\text{S}\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx=\int \frac{S(a+bx)}{c+dx}\,dx$$

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x)
[Out] Integral(fresnels(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{S}\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{S}\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{c+d\,x}\,dx$$

```
[In] int(FresnelS(a + b*x)/(c + d*x),x)
[Out] int(FresnelS(a + b*x)/(c + d*x), x)
```

3.24 $\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} dx = \int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} dx$$

```
[In] Int[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx$$

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x)^2,x]
[Out] Integrate[FresnelS[a + b*x]/(c + d*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)}{\left(dx+c\right)^{2}}dx$$

```
[In] int(FresnelS(b*x+a)/(d*x+c)^2,x)
[Out] int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{S(a+bx)}{\left(c+dx\right)^{2}}\,dx$$

```
[In] integrate(fresnels(b*x+a)/(d*x+c)**2,x)
[Out] Integral(fresnels(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(FresnelS(a + b*x)/(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)/(c + d*x)^2, x)
```

3.25 $\int x^3 \operatorname{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 229

$$\begin{aligned}\int x^3 \operatorname{FresnelS}(a+bx)\,dx = &-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi}\\ &-\frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}\\ &-\frac{3a^2\operatorname{FresnelC}(a+bx)}{2b^4\pi}-\frac{a^4\operatorname{FresnelS}(a+bx)}{4b^4}\\ &+\frac{3\operatorname{FresnelS}(a+bx)}{4b^4\pi^2}+\frac{1}{4}x^4\operatorname{FresnelS}(a+bx)\\ &+\frac{2a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2}-\frac{3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}\end{aligned}$$

```
[Out] -a^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*a^2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4
/Pi-a*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+1/4*(b*x+a)^3*cos(1/2*Pi*(b*x+
a)^2)/b^4/Pi-3/2*a^2*FresnelC(b*x+a)/b^4/Pi-1/4*a^4*FresnelS(b*x+a)/b^4+3/4
*FresnelS(b*x+a)/b^4/Pi^2+1/4*x^4*FresnelS(b*x+a)+2*a*sin(1/2*Pi*(b*x+a)^2)
/b^4/Pi^2-3/4*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules

used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717, 3467}

$$\int x^3\,\mathrm{FresnelS}(a+bx)\,dx = -\frac{a^4\,\mathrm{FresnelS}(a+bx)}{4b^4} - \frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4} - \frac{3a^2\,\mathrm{FresnelC}(a+bx)}{2\pi b^4} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4} + \frac{3\,\mathrm{FresnelS}(a+bx)}{4\pi^2 b^4} + \frac{2a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4} - \frac{3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2 b^4} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4} + \frac{1}{4}x^4\,\mathrm{FresnelS}(a+bx)$$

```
[In] Int[x^3*FresnelS[a + b*x],x]
```

```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b
*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) +
((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x]
)/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b
^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*P
i^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4 \,\text{FresnelS}(a+bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx$$

$$= \frac{1}{4}x^4 \,\text{FresnelS}(a+bx) - \frac{\text{Subst}\left(\int\left(a^4\sin\left(\frac{\pi x^2}{2}\right) - 4a^3x\sin\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\sin\left(\frac{\pi x^2}{2}\right) - 4ax^3\sin\left(\frac{\pi x^2}{2}\right) + x^4\sin\left(\frac{\pi x^2}{2}\right)\right)\,d\right.}{4b^4}$$

$$= \frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) - \frac{\operatorname{Subst}\left(\int x^4 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{4b^4} + \frac{a\operatorname{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^4} - \frac{(3a^2)\operatorname{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{2b^4} + \frac{a^3\operatorname{Subst}\left(\int x \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{b^4} - \frac{a^4\operatorname{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{4b^4}$$

$$= \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{a^4 \operatorname{FresnelS}(a+bx)}{4b^4} + \frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) + \frac{a\operatorname{Subst}\left(\int x \sin\left(\frac{\pi x}{2}\right) dx, x, (a+bx)^2\right)}{2b^4} + \frac{a^3\operatorname{Subst}\left(\int \sin\left(\frac{\pi x}{2}\right) dx, x, (a+bx)^2\right)}{2b^4} - \frac{3\operatorname{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{4b^4\pi} - \frac{(3a^2)\operatorname{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{2b^4\pi}$$

$$= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{3a^2 \operatorname{FresnelC}(a+bx)}{2b^4\pi} - \frac{a^4 \operatorname{FresnelS}(a+bx)}{4b^4} + \frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) - \frac{3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} + \frac{3\operatorname{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right) dx, x, a+bx\right)}{4b^4\pi^2} + \frac{a\operatorname{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right) dx, x, (a+bx)^2\right)}{b^4\pi}$$

$$= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{3a^2 \operatorname{FresnelC}(a+bx)}{2b^4\pi} - \frac{a^4 \operatorname{FresnelS}(a+bx)}{4b^4} + \frac{3 \operatorname{FresnelS}(a+bx)}{4b^4\pi^2} + \frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) + \frac{2a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx$$
$$= \frac{-a^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+a^2b\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-ab^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+b^3\pi x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{}$$

```
[In] Integrate[x^3*FresnelS[a + b*x],x]
```

```
[Out] (-(a^3*Pi*Cos[(Pi*(a + b*x)^2)/2]) + a^2*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - a
*b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] -
6*a^2*Pi*FresnelC[a + b*x] + (3 - a^4*Pi^2 + b^4*Pi^2*x^4)*FresnelS[a + b*x
] + 5*a*Sin[(Pi*(a + b*x)^2)/2] - 3*b*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx+a)b^4x^4}{4}-\frac{a^4\,\mathrm{FresnelS}(bx+a)}{4}-\frac{a^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{3a^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{3a^2\,\mathrm{FresnelC}(bx+a)}{2\pi}-\frac{a(bx+a)^2\cos(}{\pi}}{b^4}$
default	$\frac{\frac{\mathrm{FresnelS}(bx+a)b^4x^4}{4}-\frac{a^4\,\mathrm{FresnelS}(bx+a)}{4}-\frac{a^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{3a^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{3a^2\,\mathrm{FresnelC}(bx+a)}{2\pi}-\frac{a(bx+a)^2\cos(}{\pi}}{b^4}$
parts	$\frac{x^4\,\mathrm{FresnelS}(bx+a)}{4}-\frac{b\left(-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\right.}{b^2\pi}\right.}{}\right.}{}\right.}{}$

```
[In] int(x^3*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*FresnelS(b*x+a)*b^4*x^4-1/4*a^4*FresnelS(b*x+a)-a^3/Pi*cos(1/2*P
i*(b*x+a)^2)+3/2*a^2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*FresnelC(b
*x+a)-a/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*sin(1/2*Pi*(b*x+a)^2)+1
/4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)-3/4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+
a)^2)-1/Pi*FresnelS(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \frac{\pi^2 b^5 x^4 \operatorname{S}(bx+a) - 6\,\pi a^2 \sqrt{b^2} \operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2 a^4 - 3)\sqrt{b^2} \operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^4 x^3 - \pi a b^3 x^2 + \pi a^2 b^2 x -}{4\,\pi^2 b^5}$$

```
[In] integrate(x^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(pi^2*b^5*x^4*fresnel_sin(b*x + a) - 6*pi*a^2*sqrt(b^2)*fresnel_cos(sqr
t(b^2)*(b*x + a)/b) - (pi^2*a^4 - 3)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x +
 a)/b) + (pi*b^4*x^3 - pi*a*b^3*x^2 + pi*a^2*b^2*x - pi*a^3*b)*cos(1/2*pi*b
^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (3*b^2*x - 5*a*b)*sin(1/2*pi*b^2*x^2 + pi
*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [F]

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \int x^3 S(a+bx)\,dx$$

```
[In] integrate(x**3*fresnels(b*x+a),x)
```

```
[Out] Integral(x**3*fresnels(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.20

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \frac{1}{4}\,x^4 \operatorname{S}(bx+a) - \frac{\left(16\left(\pi^2 e^{\left(\frac{1}{2} i\,\pi b^2 x^2 + i\,\pi a b x + \frac{1}{2} i\,\pi a^2\right)} + \pi^2 e^{\left(-\frac{1}{2} i\,\pi b^2 x^2 - i\,\pi a b x - \frac{1}{2} i\,\pi a^2\right)}\right) a^4 + 32\left(-i\,\pi \Gamma\left(2, \frac{1}{2} i\,\pi b^2 x^2 + i\,\pi a b x + \frac{1}{2} i\,\pi a^2\right.\right.\right.}{}$$

```
[In] integrate(x^3*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_sin(b*x + a) - 1/32*(16*(pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*
b*x + 1/2*I*pi*a^2) + pi^2*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2
))*a^4 + 32*(-I*pi*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) +
```

```
 I*pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 16*((p
i^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi^2*e^(-1/2*I*pi*b^
2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^3 + 2*(-I*pi*gamma(2, 1/2*I*pi*b^2*x^
2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b
*x - 1/2*I*pi*a^2))*a)*b*x - ((-(I + 1)*sqrt(2)*pi^(5/2)*(erf(sqrt(1/2*I*pi
*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqrt(2)*pi^(5/2)*(erf
(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^4 - 12*((I -
1)*sqrt(2)*pi*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) - (I
 + 1)*sqrt(2)*pi*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))
*a^2 - (4*I + 4)*sqrt(2)*gamma(5/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*p
i*a^2) + (4*I - 4)*sqrt(2)*gamma(5/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*
I*pi*a^2))*sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2))*b/(pi^3*b^6*x + pi^3
*a*b^5)
```

Giac [F]

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \int x^3\,\mathrm{S}\left(bx+a\right)\,dx$$

```
[In] integrate(x^3*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \int x^3\,\operatorname{FresnelS}(a+b\,x)\,dx$$

```
[In] int(x^3*FresnelS(a + b*x),x)
[Out] int(x^3*FresnelS(a + b*x), x)
```

3.26 $\int x^2 \operatorname{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 147

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{a\operatorname{FresnelC}(a+bx)}{b^3\pi} + \frac{a^3\operatorname{FresnelS}(a+bx)}{3b^3} + \frac{1}{3}x^3\operatorname{FresnelS}(a+bx) - \frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

```
[Out] a^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-a*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+1/3
*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*FresnelC(b*x+a)/b^3/Pi+1/3*a^3*Fr
esnelS(b*x+a)/b^3+1/3*x^3*FresnelS(b*x+a)-2/3*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^
2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717}

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{a^3\operatorname{FresnelS}(a+bx)}{3b^3} + \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} + \frac{a\operatorname{FresnelC}(a+bx)}{\pi b^3} - \frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3} + \frac{1}{3}x^3\operatorname{FresnelS}(a+bx)$$

```
[In] Int[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (a*(a + b*x)*Cos[(Pi*(a + b*x)^2)/
2])/(b^3*Pi) + ((a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi) + (a*Fresne
lC[a + b*x])/(b^3*Pi) + (a^3*FresnelS[a + b*x])/(3*b^3) + (x^3*FresnelS[a +
 b*x])/3 - (2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\,\text{FresnelS}(a+bx)-\frac{1}{3}b\int x^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\
&= \frac{1}{3}x^3\,\text{FresnelS}(a+bx)\\
&\quad-\frac{\text{Subst}\left(\int\left(-a^3\sin\left(\frac{\pi x^2}{2}\right)+3a^2x\sin\left(\frac{\pi x^2}{2}\right)-3ax^2\sin\left(\frac{\pi x^2}{2}\right)+x^3\sin\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a+bx\right)}{3b^3}\\
&= \frac{1}{3}x^3\,\text{FresnelS}(a+bx)-\frac{\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3}\\
&\quad+\frac{a\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3}\\
&\quad-\frac{a^2\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3}+\frac{a^3\text{Subst}\left(\int\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3}\\
&= -\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}+\frac{a^3\,\text{FresnelS}(a+bx)}{3b^3}\\
&\quad+\frac{1}{3}x^3\,\text{FresnelS}(a+bx)-\frac{\text{Subst}\left(\int x\sin\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{6b^3}\\
&\quad-\frac{a^2\text{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^3}+\frac{a\text{Subst}\left(\int\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3\pi}\\
&= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}\\
&\quad+\frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}+\frac{a\,\text{FresnelC}(a+bx)}{b^3\pi}+\frac{a^3\,\text{FresnelS}(a+bx)}{3b^3}\\
&\quad+\frac{1}{3}x^3\,\text{FresnelS}(a+bx)-\frac{\text{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{3b^3\pi}
\end{aligned}$$

$$= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{a\,\text{FresnelC}(a+bx)}{b^3\pi} + \frac{a^3\,\text{FresnelS}(a+bx)}{3b^3} + \frac{1}{3}x^3\,\text{FresnelS}(a+bx) - \frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int x^2\,\text{FresnelS}(a+bx)\,dx = \frac{a^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - ab\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + b^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 3a\pi\,\text{FresnelC}(a+bx) + \pi^2(a}{3b^3\pi^2}$$

```
[In] Integrate[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*Pi
*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*a*Pi*FresnelC[a + b*x] + Pi^2*(a^3 + b^3*x
^3)*FresnelS[a + b*x] - 2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\text{FresnelS}(bx+a)b^3x^3}{3} + \frac{a^3\,\text{FresnelS}(bx+a)}{3} + \frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a\,\text{FresnelC}(bx+a)}{\pi} + \frac{(bx+a)^2\cos\left(\frac{\pi(bx\cdots}{2}\right)}{3\pi}}{b^3}$
default	$\frac{\frac{\text{FresnelS}(bx+a)b^3x^3}{3} + \frac{a^3\,\text{FresnelS}(bx+a)}{3} + \frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a\,\text{FresnelC}(bx+a)}{\pi} + \frac{(bx+a)^2\cos\left(\frac{\pi(bx\cdots}{2}\right)}{3\pi}}{b^3}$
parts	$\frac{x^3\,\text{FresnelS}(bx+a)}{3} - \frac{b\left(-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} - \frac{a\left(-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} - \frac{a\left(-\frac{\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi\cdots\right)}{b^2\pi}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{\cdots}$

```
[In] int(x^2*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelS(b*x+a)*b^3*x^3+1/3*a^3*FresnelS(b*x+a)+a^2/Pi*cos(1/2*P
i*(b*x+a)^2)-a/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+a/Pi*FresnelC(b*x+a)+1/3/Pi
*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2/3/Pi^2*sin(1/2*Pi*(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{\pi^2 b^4 x^3 \operatorname{S}(bx+a) + \pi^2 a^3 \sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + 3\pi a\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^3 x^2 - \pi a b^2 x + \pi a^2 b)\cos\left(\frac{1}{2}\pi b^2 x\right.}{3\pi^2 b^4}$$

```
[In] integrate(x^2*fresnel_sin(b*x+a),x, algorithm="fricas")
[Out] 1/3*(pi^2*b^4*x^3*fresnel_sin(b*x + a) + pi^2*a^3*sqrt(b^2)*fresnel_sin(sqr
t(b^2)*(b*x + a)/b) + 3*pi*a*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) +
 (pi*b^3*x^2 - pi*a*b^2*x + pi*a^2*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*p
i*a^2) - 2*b*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

Sympy [F]

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \int x^2 S(a+bx)\,dx$$

```
[In] integrate(x**2*fresnels(b*x+a),x)
[Out] Integral(x**2*fresnels(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.88

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{1}{3}x^3 \operatorname{S}(bx+a) + \frac{\left(12\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)} + \pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^3 + 4\left(3\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)} + \pi e^{\left(-\frac{1}{2}i\pi b^2\right.}\right.\right.}{}$$

```
[In] integrate(x^2*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*fresnel_sin(b*x + a) + 1/24*(12*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*
x + 1/2*I*pi*a^2) + pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a
^3 + 4*(3*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*e^(-1/2
*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - 2*I*gamma(2, 1/2*I*pi*b^2
*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + 2*I*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a
*b*x - 1/2*I*pi*a^2))*b*x + 8*a*(-I*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x
+ 1/2*I*pi*a^2) + I*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)
) - sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2)*((-(I + 1)*sqrt(2)*pi^(3/2)*
(erf(sqrt(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqr
t(2)*pi^(3/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1
))*a^3 - 6*((I - 1)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*
I*pi*a^2) - (I + 1)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2
*I*pi*a^2))*a))*b/(pi^2*b^5*x + pi^2*a*b^4)
```

Giac [F]

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \int x^2 \operatorname{S}(bx+a)\,dx$$

```
[In] integrate(x^2*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \int x^2 \operatorname{FresnelS}(a+b\,x)\,dx$$

```
[In] int(x^2*FresnelS(a + b*x),x)
[Out] int(x^2*FresnelS(a + b*x), x)
```

3.27 $\int x \operatorname{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int x \operatorname{FresnelS}(a+bx)\,dx = -\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} -\frac{\operatorname{FresnelC}(a+bx)}{2b^2\pi}-\frac{a^2\operatorname{FresnelS}(a+bx)}{2b^2}+\frac{1}{2}x^2\operatorname{FresnelS}(a+bx)$$

```
[Out] -a*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/
2*FresnelC(b*x+a)/b^2/Pi-1/2*a^2*FresnelS(b*x+a)/b^2+1/2*x^2*FresnelS(b*x+a
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433}

$$\int x \operatorname{FresnelS}(a+bx)\,dx = -\frac{a^2\operatorname{FresnelS}(a+bx)}{2b^2}-\frac{\operatorname{FresnelC}(a+bx)}{2\pi b^2}-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2} +\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2\operatorname{FresnelS}(a+bx)$$

```
[In] Int[x*FresnelS[a + b*x],x]
```

```
[Out] -((a*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi)) + ((a + b*x)*Cos[(Pi*(a + b*x)^2)/2
])/(2*b^2*Pi) - FresnelC[a + b*x]/(2*b^2*Pi) - (a^2*FresnelS[a + b*x])/(2*b
^2) + (x^2*FresnelS[a + b*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\,\text{FresnelS}(a+bx)-\frac{1}{2}b\int x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelS}(a+bx)\\
&\quad-\frac{\text{Subst}\left(\int\left(a^2\sin\left(\frac{\pi x^2}{2}\right)-2ax\sin\left(\frac{\pi x^2}{2}\right)+x^2\sin\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a+bx\right)}{2b^2}\\
&= \frac{1}{2}x^2\,\text{FresnelS}(a+bx)-\frac{\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2}\\
&\quad+\frac{a\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^2}-\frac{a^2\text{Subst}\left(\int\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2}\\
&= \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{a^2\,\text{FresnelS}(a+bx)}{2b^2}+\frac{1}{2}x^2\,\text{FresnelS}(a+bx)\\
&\quad+\frac{a\text{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^2}-\frac{\text{Subst}\left(\int\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2\pi}\\
&= -\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}\\
&\quad-\frac{\text{FresnelC}(a+bx)}{2b^2\pi}-\frac{a^2\,\text{FresnelS}(a+bx)}{2b^2}+\frac{1}{2}x^2\,\text{FresnelS}(a+bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$
\begin{aligned}
&\int x\,\text{FresnelS}(a+bx)\,dx\\
&= -\frac{\text{FresnelC}(a+bx)+(a-bx)\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi(a+bx)\,\text{FresnelS}(a+bx)\right)}{2b^2\pi}
\end{aligned}
$$

```
[In] Integrate[x*FresnelS[a + b*x],x]
[Out] -1/2*(FresnelC[a + b*x] + (a - b*x)*(Cos[(Pi*(a + b*x)^2)/2] + Pi*(a + b*x)
*FresnelS[a + b*x]))/(b^2*Pi)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\operatorname{FresnelS}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\operatorname{FresnelC}(bx+a)}{2\pi}}{b^2}$
default	$\frac{\operatorname{FresnelS}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\operatorname{FresnelC}(bx+a)}{2\pi}}{b^2}$
parts	$\frac{x^2\operatorname{FresnelS}(bx+a)}{2}-\frac{b\left(-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(-\frac{\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\operatorname{FresnelS}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)}{b}+\right.}{2}$

```
[In] int(x*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(FresnelS(b*x+a)*(-(b*x+a)*a+1/2*(b*x+a)^2)-a/Pi*cos(1/2*Pi*(b*x+a)^2
)+1/2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-1/2/Pi*FresnelC(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int x\operatorname{FresnelS}(a+bx)\,dx$$
$$=\frac{\pi b^3x^2\operatorname{S}(bx+a)-\pi a^2\sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(b^2x-ab)\cos\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)-\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{2\pi b^3}$$

```
[In] integrate(x*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^3*x^2*fresnel_sin(b*x + a) - pi*a^2*sqrt(b^2)*fresnel_sin(sqrt(b^
2)*(b*x + a)/b) + (b^2*x - a*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)
 - sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b))/(pi*b^3)
```

Sympy [F]

$$\int x\,\text{FresnelS}(a+bx)\,dx=\int xS(a+bx)\;dx$$

```
[In] integrate(x*fresnels(b*x+a),x)
[Out] Integral(x*fresnels(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.20

$$\int x\,\text{FresnelS}(a+bx)\,dx=\frac{1}{2}\,x^2\,\text{S}\left(bx+a\right)$$
$$-\frac{\left(8\left(\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)abx+8\left(\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-\right.}\right.\right.}{}$$

```
[In] integrate(x*fresnel_sin(b*x+a),x, algorithm="maxima")
[Out] 1/2*x^2*fresnel_sin(b*x + a) - 1/16*(8*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x
 + 1/2*I*pi*a^2) + pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a*
b*x + 8*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*e^(-1/2*I
*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - sqrt(2*pi*b^2*x^2 + 4*pi*a*
b*x + 2*pi*a^2)*((-(I + 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(1/2*I*pi*b^2*x^2 + I*
pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(-1/2*I*
pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^2 - (2*I - 2)*sqrt(2)*gamm
a(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + (2*I + 2)*sqrt(2)*ga
mma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)))*b/(pi^2*b^4*x + p
i^2*a*b^3)
```

Giac [F]

$$\int x\,\text{FresnelS}(a+bx)\,dx=\int x\,\text{S}\left(bx+a\right)\;dx$$

```
[In] integrate(x*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate(x*fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\,\mathrm{FresnelS}(a+bx)\,dx=\int x\,\mathrm{FresnelS}(a+b\,x)\;dx$$

```
[In] int(x*FresnelS(a + b*x),x)
[Out] int(x*FresnelS(a + b*x), x)
```

3.28 $\int \text{FresnelS}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \text{FresnelS}(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}$$

```
[Out] cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6553}

$$\int \text{FresnelS}(a+bx)\,dx=\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral}=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. $2(36) = 72$.

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

$$\int \text{FresnelS}(a+bx)\,dx = \frac{\cos\left(\frac{a^2\pi}{2}\right)\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{a\,\text{FresnelS}(a+bx)}{b} + x\,\text{FresnelS}(a+bx) - \frac{\sin\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

[In] Integrate[FresnelS[a + b*x],x]

[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi)

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
default	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
parts	$x\ \text{FresnelS}\left(bx+a\right)-b\left(-\frac{\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelS}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	86

[In] int(FresnelS(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b*(FresnelS(b*x+a)*(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \text{FresnelS}(a+bx)\,dx = \frac{(\pi bx+\pi a)\,\text{S}\,(bx+a)+\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
[Out] ((pi*b*x + pi*a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]

$$\int \text{FresnelS}(a+bx)\,dx = \int S(a+bx)\;dx$$

```
[In] integrate(fresnels(b*x+a),x)
[Out] Integral(fresnels(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \text{FresnelS}(a+bx)\,dx = \frac{(bx+a)\,\text{S}\,(bx+a)+\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Giac [F]

$$\int \text{FresnelS}(a+bx)\,dx = \int \text{S}\left(bx+a\right)\,dx$$

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(a+bx)\,dx = \int \text{FresnelS}(a+b\,x)\,dx$$

```
[In] int(FresnelS(a + b*x),x)
[Out] int(FresnelS(a + b*x), x)
```

3.29 $\int \frac{\text{FresnelS}(a+bx)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(a+bx)}{x} dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(a+bx)}{x} dx = \int \frac{\text{FresnelS}(a+bx)}{x} dx$$

```
[In] Int[FresnelS[a + b*x]/x,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{\text{FresnelS}(a+bx)}{x}\,dx$$

```
[In] Integrate[FresnelS[a + b*x]/x,x]
[Out] Integrate[FresnelS[a + b*x]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx+a)}{x}dx$$

```
[In] int(FresnelS(b*x+a)/x,x)
[Out] int(FresnelS(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{\text{S}\,(bx+a)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/x,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{S(a+bx)}{x}\,dx$$

```
[In] integrate(fresnels(b*x+a)/x,x)
[Out] Integral(fresnels(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x} \, dx = \int \frac{\text{S}\left(bx+a\right)}{x} \, dx$$

```
[In] integrate(fresnel_sin(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x} \, dx = \int \frac{\text{S}\left(bx+a\right)}{x} \, dx$$

```
[In] integrate(fresnel_sin(b*x+a)/x,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x} \, dx = \int \frac{\text{FresnelS}(a+b\,x)}{x} \, dx$$

```
[In] int(FresnelS(a + b*x)/x,x)
[Out] int(FresnelS(a + b*x)/x, x)
```

3.30 $\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx$$

```
[In] Int[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[FresnelS[a + b*x]/x^2,x]
[Out] Integrate[FresnelS[a + b*x]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx+a)}{x^2}dx$$

```
[In] int(FresnelS(b*x+a)/x^2,x)
[Out] int(FresnelS(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\text{S}\,(bx+a)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{S(a+bx)}{x^2}\,dx$$

```
[In] integrate(fresnels(b*x+a)/x**2,x)
[Out] Integral(fresnels(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{S\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{S\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{x^2}\,dx$$

```
[In] int(FresnelS(a + b*x)/x^2,x)
[Out] int(FresnelS(a + b*x)/x^2, x)
```

3.31 $\int x^7 \text{FresnelS}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 253

$$\begin{aligned}\int x^7 \text{FresnelS}(bx)^2 \, dx = & -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} \\ & + \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b^5\pi^3} \\ & + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} - \frac{105\,\text{FresnelS}(bx)^2}{8b^8\pi^4} \\ & + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 + \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} \\ & - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{10\sin(b^2\pi x^2)}{b^8\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3}\end{aligned}$$

```
[Out] -105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2-55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4+1
/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2-35/4*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/b^5/Pi^3+1/4*x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-105/8*FresnelS(b*x
)^2/b^8/Pi^4+1/8*x^8*FresnelS(b*x)^2+105/4*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/b^7/Pi^4-7/4*x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2+10*sin(b^2*
Pi*x^2)/b^8/Pi^5-5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6565, 6589, 6597, 3460, 3390, 30, 3377, 2717, 2714, 6575}

$$\int x^7 \operatorname{FresnelS}(bx)^2\,dx = -\frac{105 \operatorname{FresnelS}(bx)^2}{8\pi^4 b^8} - \frac{105x^2}{16\pi^4 b^6} + \frac{x^7 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{7x^6}{48\pi^2 b^2} + \frac{x^6\cos(\pi b^2 x^2)}{16\pi^2 b^2} + \frac{10\sin(\pi b^2 x^2)}{\pi^5 b^8} + \frac{105x \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^4 b^7} - \frac{55x^2\cos(\pi b^2 x^2)}{16\pi^4 b^6} - \frac{35x^3 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} - \frac{5x^4\sin(\pi b^2 x^2)}{8\pi^3 b^4} - \frac{7x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{8}x^8 \operatorname{FresnelS}(bx)^2$$

```
[In] Int[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\text{integral} = \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{8\pi} - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{4b\pi}$$

$$= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^3\pi^2} + \frac{7\int x^5\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^2\pi^2} - \frac{\text{Subst}\left(\int x^3\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{16\pi}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{105\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{4b^5\pi^3} + \frac{35\int x^3\sin\left(b^2\pi x^2\right)\,dx}{8b^4\pi^3} - \frac{3\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{16b^2\pi^2} + \frac{7\text{Subst}\left(\int x^2\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{8b^2\pi^2}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 + \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} - \frac{3x^4\sin\left(b^2\pi x^2\right)}{16b^4\pi^3} - \frac{105\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^7\pi^4} - \frac{105\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^6\pi^4} + \frac{3\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{8b^4\pi^3} + \frac{35\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{16b^4\pi^3} + \frac{7\text{Subst}\left(\int x^2\,dx, x, x^2\right)}{16b^2\pi^2} - \frac{7\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{16b^2\pi^2}$$

$$= \frac{7x^6}{48b^2\pi^2} - \frac{41x^2\cos(b^2\pi x^2)}{16b^6\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 + \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} - \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3} - \frac{105\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)}{4b^8\pi^4} + \frac{3\text{Subst}\left(\int\cos(b^2\pi x)\,dx, x, x^2\right)}{8b^6\pi^4} + \frac{35\text{Subst}\left(\int\cos(b^2\pi x)\,dx, x, x^2\right)}{16b^6\pi^4} - \frac{105\text{Subst}\left(\int\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{8b^6\pi^4} + \frac{7\text{Subst}\left(\int x\sin(b^2\pi x)\,dx, x, x^2\right)}{8b^4\pi^3}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} - \frac{105\,\text{FresnelS}(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 + \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{73\sin(b^2\pi x^2)}{8b^8\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3} + \frac{7\text{Subst}\left(\int\cos(b^2\pi x)\,dx, x, x^2\right)}{8b^6\pi^4}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4b\pi} - \frac{105\,\text{FresnelS}(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8\,\text{FresnelS}(bx)^2 + \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} - \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{10\sin(b^2\pi x^2)}{b^8\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.72

$$\int x^7\,\text{FresnelS}(bx)^2\,dx$$

$$= \frac{-315b^2\pi x^2 + 7b^6\pi^3x^6 + 3b^2\pi x^2(-55 + b^4\pi^2x^4)\cos(b^2\pi x^2) + 6\pi(-105 + b^8\pi^4x^8)\,\text{FresnelS}(bx)^2 + 12b\pi x\,\text{I}}{}$$

```
[In] Integrate[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-315*b^2*Pi*x^2 + 7*b^6*Pi^3*x^6 + 3*b^2*Pi*x^2*(-55 + b^4*Pi^2*x^4)*Cos[b
^2*Pi*x^2] + 6*Pi*(-105 + b^8*Pi^4*x^8)*FresnelS[b*x]^2 + 12*b*Pi*x*Fresnel
```

```
S[b*x]*(b^2*Pi*x^2*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 7*(-15 + b^4*
Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 480*Sin[b^2*Pi*x^2] - 30*b^4*Pi^2*x^4*Sin[
b^2*Pi*x^2])/(48*b^8*Pi^5)
```

Maple [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx$$

```
[In] int(x^7*FresnelS(b*x)^2,x)
[Out] int(x^7*FresnelS(b*x)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \frac{2\,\pi^3 b^6 x^6 - 75\,\pi b^2 x^2 + 3\,(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 6\,(\pi^4 b^7 x^7 - 35\,\pi^2 b^3 x^3)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) -}{24\,\pi^5}$$

```
[In] integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="fricas")
[Out] 1/24*(2*pi^3*b^6*x^6 - 75*pi*b^2*x^2 + 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*cos
(1/2*pi*b^2*x^2)^2 + 6*(pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x^2)
*fresnel_sin(b*x) - 3*(105*pi - pi^5*b^8*x^8)*fresnel_sin(b*x)^2 - 6*(5*(pi
^2*b^4*x^4 - 16)*cos(1/2*pi*b^2*x^2) + 7*(pi^3*b^5*x^5 - 15*pi*b*x)*fresnel
_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^5*b^8)
```

Sympy [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \int x^7 S^2(bx) \, dx$$

```
[In] integrate(x**7*fresnels(b*x)**2,x)
[Out] Integral(x**7*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \int x^7 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \int x^7 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \int x^7 \operatorname{FresnelS}(b\,x)^2 \, dx$$

```
[In] int(x^7*FresnelS(b*x)^2,x)
```

```
[Out] int(x^7*FresnelS(b*x)^2, x)
```

3.32 $\int x^6 \operatorname{FresnelS}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 239

$$\begin{aligned}\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = & -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4} + \frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2} \\ & + \frac{531\operatorname{FresnelC}(\sqrt{2}bx)}{56\sqrt{2}b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{7b^5\pi^3} \\ & + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{7b\pi} \\ & + \frac{1}{7}x^7\operatorname{FresnelS}(bx)^2 + \frac{96\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} \\ & - \frac{12x^4\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} - \frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3}\end{aligned}$$

```
[Out] -48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2-21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4+1/14*x^5
*cos(b^2*Pi*x^2)/b^2/Pi^2-48/7*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi
^3+2/7*x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/7*x^7*FresnelS(b*x)^2+9
6/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-12/7*x^4*FresnelS(b*x)*sin(1
/2*b^2*Pi*x^2)/b^3/Pi^2-17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3+531/112*FresnelC
(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6565, 6589, 6597, 3472, 30, 3467, 3466, 3433, 6595, 3438}

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \frac{531 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}\pi^4 b^7} - \frac{48x}{7\pi^4 b^6} + \frac{2x^6 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} + \frac{6x^5}{35\pi^2 b^2} + \frac{x^5\cos\left(\pi b^2 x^2\right)}{14\pi^2 b^2} + \frac{96 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} - \frac{21x\cos\left(\pi b^2 x^2\right)}{8\pi^4 b^6} - \frac{48x^2 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} - \frac{17x^3\sin\left(\pi b^2 x^2\right)}{28\pi^3 b^4} - \frac{12x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{1}{7}x^7 \operatorname{FresnelS}(bx)^2$$

```
[In] Int[x^6*FresnelS[b*x]^2,x]
```

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) - (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (531*FresnelC[Sqrt[2]*b*x])
/(56*Sqrt[2]*b^7*Pi^4) - (48*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b^5*
Pi^3) + (2*x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b*Pi) + (x^7*FresnelS[
b*x]^2)/7 + (96*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (12*x^4*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) - (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{1}{7}(2b)\int x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{7\pi} - \frac{12\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{7b\pi}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{12x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{48\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b^3\pi^2} - \frac{5\int x^4\cos\left(b^2\pi x^2\right)\,dx}{14b^2\pi^2} + \frac{12\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b^2\pi^2}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{12x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} - \frac{5x^3\sin\left(b^2\pi x^2\right)}{28b^4\pi^3} + \frac{96\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{7b^5\pi^3} + \frac{15\int x^2\sin\left(b^2\pi x^2\right)\,dx}{28b^4\pi^3} + \frac{24\int x^2\sin\left(b^2\pi x^2\right)\,dx}{7b^4\pi^3} + \frac{6\int x^4\,dx}{7b^2\pi^2} - \frac{6\int x^4\cos\left(b^2\pi x^2\right)\,dx}{7b^2\pi^2}$$

$$= \frac{6x^5}{35b^2\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 + \frac{96\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{12x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} - \frac{17x^3\sin\left(b^2\pi x^2\right)}{28b^4\pi^3} + \frac{15\int\cos\left(b^2\pi x^2\right)\,dx}{56b^6\pi^4} + \frac{12\int\cos\left(b^2\pi x^2\right)\,dx}{7b^6\pi^4} - \frac{96\int\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b^6\pi^4} + \frac{9\int x^2\sin\left(b^2\pi x^2\right)\,dx}{7b^4\pi^3}$$

$$= \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{15\,\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{6\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 + \frac{96\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{12x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} - \frac{17x^3\sin\left(b^2\pi x^2\right)}{28b^4\pi^3} + \frac{9\int\cos\left(b^2\pi x^2\right)\,dx}{14b^6\pi^4} - \frac{96\int\left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\,dx}{7b^6\pi^4}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4} + \frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{51\,\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{6\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 + \frac{96\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{12x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} - \frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3} + \frac{48\int\cos(b^2\pi x^2)\,dx}{7b^6\pi^4}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4} + \frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{51\,\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{30\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7b\pi} + \frac{1}{7}x^7\,\text{FresnelS}(bx)^2 + \frac{96\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{12x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} - \frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int x^6\,\text{FresnelS}(bx)^2\,dx$$

$$= \frac{2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 80b^7\pi^4x^7\,\text{FresnelS}(bx)^2 + 160\,\text{FresnelS}(bx)\left(b^2\pi x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)\right.}{}$$

```
[In] Integrate[x^6*FresnelS[b*x]^2,x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*b^7*Pi^4*x^7*FresnelS[b*x]^2 + 160
*FresnelS[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Co
s[b^2*Pi*x^2] - 2*(960 - 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))
/(560*b^7*Pi^4)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\text{FresnelS}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelS}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} + \frac{\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} - \frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}}{7\pi}\right.}{}$
default	$\frac{\frac{\text{FresnelS}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelS}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} + \frac{\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} - \frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}}{7\pi}\right.}{}$

```
[In] int(x^6*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/7*FresnelS(b*x)^2*b^7*x^7-2*FresnelS(b*x)*(-1/7/Pi*b^6*x^6*cos(1/2
*b^2*Pi*x^2)+6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*c
os(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*b^5*x^5*Pi^2
-8*b*x)-6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^
2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1
/2)))-1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(
b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x
*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int x^6\,\text{FresnelS}(bx)^2\,dx$$
$$= \frac{80\,\pi^4 b^8 x^7\,\mathrm{S}\,(bx)^2 + 56\,\pi^2 b^6 x^5 - 2370\,b^2 x + 20\,(4\,\pi^2 b^6 x^5 - 147\,b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 160\,(\pi^3 b^7 x^6 - 24\,\pi b^3 x}{}$$

```
[In] integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/560*(80*pi^4*b^8*x^7*fresnel_sin(b*x)^2 + 56*pi^2*b^6*x^5 - 2370*b^2*x +
20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2)^2 + 160*(pi^3*b^7*x^6 -
```

```
24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 2655*sqrt(2)*sqrt(b^
2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) - 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)
 + 24*(pi^2*b^5*x^4 - 8*b)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^8
)
```

Sympy [F]

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 S^2(bx) \, dx$$

```
[In] integrate(x**6*fresnels(b*x)**2,x)
[Out] Integral(x**6*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^6*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^6*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 \operatorname{FresnelS}(b\,x)^2 \, dx$$

```
[In] int(x^6*FresnelS(b*x)^2,x)
[Out] int(x^6*FresnelS(b*x)^2, x)
```

3.33 $\int x^5 \operatorname{FresnelS}(bx)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 265

$$\begin{aligned}\int x^5 \operatorname{FresnelS}(bx)^2\,dx =\ & \frac{5x^4}{24b^2\pi^2}-\frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4}+\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} \\ & -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^5\pi^3}+\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{3b\pi} \\ & +\frac{5\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^6\pi^3}+\frac{1}{6}x^6\operatorname{FresnelS}(bx)^2 \\ & -\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\ & -\frac{5x^3\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}-\frac{7x^2\sin\left(b^2\pi x^2\right)}{12b^4\pi^3}\end{aligned}$$

```
[Out] 5/24*x^4/b^2/Pi^2-11/6*cos(b^2*Pi*x^2)/b^6/Pi^4+1/12*x^4*cos(b^2*Pi*x^2)/b^
2/Pi^2-5*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi^3+1/3*x^5*cos(1/2*b^2*P
i*x^2)*FresnelS(b*x)/b/Pi+5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3+1/6*x^6*
FresnelS(b*x)^2-5/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^4/
Pi^3+5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^4/Pi^3-5/3*x^3
*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-7/12*x^2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6565, 6589, 6597, 3460, 3390, 30, 3377, 2718, 6581}

$$\int x^5 \text{FresnelS}(bx)^2 \, dx = -\frac{5ix^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^4} + \frac{5ix^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3 b^4} + \frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2\pi^3 b^6} + \frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b} + \frac{5x^4}{24\pi^2 b^2} + \frac{x^4\cos\left(\pi b^2 x^2\right)}{12\pi^2 b^2} - \frac{11\cos\left(\pi b^2 x^2\right)}{6\pi^4 b^6} - \frac{5x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^5} - \frac{7x^2\sin\left(\pi b^2 x^2\right)}{12\pi^3 b^4} - \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3} + \frac{1}{6}x^6\,\text{FresnelS}(bx)^2$$

```
[In] Int[x^5*FresnelS[b*x]^2,x]
```

```
[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) +
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*Fresne
lS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3
*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/
(12*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x^6\,\text{FresnelS}(bx)^2 - \frac{1}{3}b\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{\int x^5 \sin\left(b^2\pi x^2\right)\, dx}{6\pi} - \frac{5\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx}{3b\pi}$$

$$= \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{5x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{5\int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^3\pi^2} + \frac{5\int x^3 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{3b^2\pi^2} - \frac{\text{Subst}\left(\int x^2 \sin\left(b^2\pi x\right)\, dx, x, x^2\right)}{12\pi}$$

$$= \frac{x^4 \cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^5\pi^3} + \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{5x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{5\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx}{b^5\pi^3} + \frac{5\int x \sin\left(b^2\pi x^2\right)\, dx}{2b^4\pi^3} - \frac{\text{Subst}\left(\int x \cos\left(b^2\pi x\right)\, dx, x, x^2\right)}{6b^2\pi^2} + \frac{5\text{Subst}\left(\int x \sin^2\left(\frac{1}{2}b^2\pi x\right)\, dx, x, x^2\right)}{6b^2\pi^2}$$

$$= \frac{x^4 \cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^5\pi^3} + \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{5\,\text{FresnelC}(bx)\, \text{FresnelS}(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{5ix^2\, {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5ix^2\, {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} - \frac{5x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{x^2 \sin\left(b^2\pi x^2\right)}{6b^4\pi^3} + \frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\, dx, x, x^2\right)}{6b^4\pi^3} + \frac{5\text{Subst}\left(\int \sin\left(b^2\pi x\right)\, dx, x, x^2\right)}{4b^4\pi^3} + \frac{5\text{Subst}\left(\int x\, dx, x, x^2\right)}{12b^2\pi^2} - \frac{5\text{Subst}\left(\int x \cos\left(b^2\pi x\right)\, dx, x, x^2\right)}{12b^2\pi^2}$$

$$= \frac{5x^4}{24b^2\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} + \frac{x^4 \cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^5\pi^3} + \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{5\,\text{FresnelC}(bx)\, \text{FresnelS}(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{5ix^2\, {}_2F_2\left(1, 1; \frac{3}{2}, 2; -\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5ix^2\, {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} - \frac{5x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{7x^2 \sin\left(b^2\pi x^2\right)}{12b^4\pi^3} + \frac{5\text{Subst}\left(\int \sin\left(b^2\pi x\right)\, dx, x, x^2\right)}{12b^4\pi^3}$$

$$
\begin{aligned}
&= \frac{5x^4}{24b^2\pi^2} - \frac{11\cos(b^2\pi x^2)}{6b^6\pi^4} + \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^5\pi^3} \\
&\quad + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3b\pi} + \frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^6\pi^3} \\
&\quad + \frac{1}{6}x^6\,\text{FresnelS}(bx)^2 - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\
&\quad + \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} - \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3}
\end{aligned}
$$

Mathematica [F]

$$\int x^5\,\text{FresnelS}(bx)^2\,dx = \int x^5\,\text{FresnelS}(bx)^2\,dx$$

```
[In] Integrate[x^5*FresnelS[b*x]^2,x]
[Out] Integrate[x^5*FresnelS[b*x]^2, x]
```

Maple [F]

$$\int x^5\,\text{FresnelS}\,(bx)^2\,dx$$

```
[In] int(x^5*FresnelS(b*x)^2,x)
[Out] int(x^5*FresnelS(b*x)^2,x)
```

Fricas [F]

$$\int x^5\,\text{FresnelS}(bx)^2\,dx = \int x^5\,\text{S}\,(bx)^2\,dx$$

```
[In] integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="fricas")
[Out] integral(x^5*fresnel_sin(b*x)^2, x)
```

Sympy [F]

$$\int x^5 \operatorname{FresnelS}(bx)^2 \, dx = \int x^5 S^2(bx) \, dx$$

```
[In] integrate(x**5*fresnels(b*x)**2,x)
[Out] Integral(x**5*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x^5 \operatorname{FresnelS}(bx)^2 \, dx = \int x^5 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^5*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x^5 \operatorname{FresnelS}(bx)^2 \, dx = \int x^5 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^5*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelS}(bx)^2 \, dx = \int x^5 \operatorname{FresnelS}(b\,x)^2 \, dx$$

```
[In] int(x^5*FresnelS(b*x)^2,x)
[Out] int(x^5*FresnelS(b*x)^2, x)
```

3.34 $\int x^4 \operatorname{FresnelS}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 177

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \frac{4x^3}{15b^2\pi^2} + \frac{x^3 \cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{16 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{5b^5\pi^3} + \frac{2x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{5b\pi} + \frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 + \frac{43 \operatorname{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} - \frac{8x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} - \frac{11x \sin(b^2\pi x^2)}{20b^4\pi^3}$$

```
[Out] 4/15*x^3/b^2/Pi^2+1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-16/5*cos(1/2*b^2*Pi*x^2
)*FresnelS(b*x)/b^5/Pi^3+2/5*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/5
*x^5*FresnelS(b*x)^2-8/5*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-11/
20*x*sin(b^2*Pi*x^2)/b^4/Pi^3+43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6565, 6589, 6597, 3472, 30, 3467, 3432, 6587, 3466}

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \frac{43 \operatorname{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}\pi^3b^5} + \frac{2x^4 \operatorname{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b} + \frac{4x^3}{15\pi^2b^2} + \frac{x^3 \cos(\pi b^2x^2)}{10\pi^2b^2} - \frac{16 \operatorname{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5} - \frac{11x \sin(\pi b^2x^2)}{20\pi^3b^4} - \frac{8x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3} + \frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2$$

```
[In] Int[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (16*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(5*b^5*Pi^3) + (2*x^4*Cos[(b^2*Pi*x^2)/2]*Fresnel
S[b*x])/(5*b*Pi) + (x^5*FresnelS[b*x]^2)/5 + (43*FresnelS[Sqrt[2]*b*x])/(20
*Sqrt[2]*b^5*Pi^3) - (8*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
 - (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 3432</u>

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

<u>Rule 3466</u>

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

<u>Rule 3467</u>

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

<u>Rule 3472</u>

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

<u>Rule 6565</u>

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

<u>Rule 6587</u>

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
```

```
/; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\,\text{FresnelS}(bx)^2 - \frac{1}{5}(2b)\int x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx)^2 \\
&\quad - \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{5\pi} - \frac{8\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{5b\pi} \\
&= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx)^2 \\
&\quad - \frac{8x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{16\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{5b^3\pi^2} \\
&\quad - \frac{3\int x^2\cos\left(b^2\pi x^2\right)\,dx}{10b^2\pi^2} + \frac{8\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{5b^2\pi^2} \\
&= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b\pi} \\
&\quad + \frac{1}{5}x^5\,\text{FresnelS}(bx)^2 - \frac{8x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} - \frac{3x\sin\left(b^2\pi x^2\right)}{20b^4\pi^3} \\
&\quad + \frac{3\int \sin\left(b^2\pi x^2\right)\,dx}{20b^4\pi^3} + \frac{8\int \sin\left(b^2\pi x^2\right)\,dx}{5b^4\pi^3} + \frac{4\int x^2\,dx}{5b^2\pi^2} - \frac{4\int x^2\cos\left(b^2\pi x^2\right)\,dx}{5b^2\pi^2}
\end{aligned}$$

$$= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx)^2 + \frac{3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} + \frac{4\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{5b^5\pi^3} - \frac{8x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} - \frac{11x\sin(b^2\pi x^2)}{20b^4\pi^3} + \frac{2\int\sin(b^2\pi x^2)\,dx}{5b^4\pi^3}$$

$$= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5b\pi} + \frac{1}{5}x^5\,\text{FresnelS}(bx)^2 + \frac{3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} + \frac{\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^5\pi^3} - \frac{8x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} - \frac{11x\sin(b^2\pi x^2)}{20b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int x^4\,\text{FresnelS}(bx)^2\,dx = \frac{32b^3\pi x^3 + 12b^3\pi x^3\cos(b^2\pi x^2) + 24b^5\pi^3x^5\,\text{FresnelS}(bx)^2 + 129\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 48\,\text{FresnelS}(bx)\left((-8\right.}{120b^5\pi^3}$$

```
[In] Integrate[x^4*FresnelS[b*x]^2,x]
[Out] (32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelS[b
*x]^2 + 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^
2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right) + \frac{4b^3x^3}{15\pi^2} - \frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right.}{2\pi}\right.}{}}{b^5}$
default	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right) + \frac{4b^3x^3}{15\pi^2} - \frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right.}{2\pi}\right.}{}}{b^5}$

```
[In] int(x^4*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*FresnelS(b*x)^2*b^5*x^5-2*FresnelS(b*x)*(-1/5/Pi*b^4*x^4*cos(1/2
*b^2*Pi*x^2)+4/5/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3-4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))-1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int x^4\,\mathrm{FresnelS}(bx)^2\,dx$$
$$= \frac{24\,\pi^3 b^6 x^5\,\mathrm{S}\left(bx\right)^2 + 24\,\pi b^4 x^3\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 20\,\pi b^4 x^3 + 48\left(\pi^2 b^5 x^4 - 8\,b\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) + 129\,\sqrt{2}\sqrt{}}{120\,\pi^3 b^6}$$

```
[In] integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(24*pi^3*b^6*x^5*fresnel_sin(b*x)^2 + 24*pi*b^4*x^3*cos(1/2*pi*b^2*x^
2)^2 + 20*pi*b^4*x^3 + 48*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_
sin(b*x) + 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(16*
pi*b^3*x^2*fresnel_sin(b*x) + 11*b^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*
x^2))/(pi^3*b^6)
```

Sympy [F]

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 S^2(bx) \, dx$$

```
[In] integrate(x**4*fresnels(b*x)**2,x)
[Out] Integral(x**4*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^4*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^4*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 \operatorname{FresnelS}(b\,x)^2 \, dx$$

```
[In] int(x^4*FresnelS(b*x)^2,x)
[Out] int(x^4*FresnelS(b*x)^2, x)
```

3.35 $\int x^3 \operatorname{FresnelS}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 140

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \frac{3x^2}{8b^2\pi^2} + \frac{x^2\cos(b^2\pi x^2)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{2b\pi} + \frac{3\operatorname{FresnelS}(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelS}(bx)^2 - \frac{3x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} - \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

```
[Out] 3/8*x^2/b^2/Pi^2+1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^3*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b/Pi+3/4*FresnelS(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelS(b*x)^2-
3/2*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-1/2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6565, 6589, 6597, 3460, 2714, 6575, 30, 3377, 2717}

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \frac{3\operatorname{FresnelS}(bx)^2}{4\pi^2b^4} + \frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{3x^2}{8\pi^2b^2} + \frac{x^2\cos(\pi b^2x^2)}{8\pi^2b^2} - \frac{\sin(\pi b^2x^2)}{2\pi^3b^4} - \frac{3x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{1}{4}x^4\operatorname{FresnelS}(bx)^2$$

```
[In] Int[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\,\text{FresnelS}(bx)^2-\frac{1}{2}b\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2b\pi}+\frac{1}{4}x^4\,\text{FresnelS}(bx)^2\\
&\quad-\frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{4\pi}-\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{2b\pi}\\
&= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2b\pi}+\frac{1}{4}x^4\,\text{FresnelS}(bx)^2\\
&\quad-\frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2}+\frac{3\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^3\pi^2}\\
&\quad+\frac{3\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^2\pi^2}-\frac{\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{8\pi}\\
&= \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2}+\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2b\pi}+\frac{1}{4}x^4\,\text{FresnelS}(bx)^2\\
&\quad-\frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2}+\frac{3\text{Subst}\left(\int x\,dx,x,\text{FresnelS}(bx)\right)}{2b^4\pi^2}\\
&\quad-\frac{\text{Subst}\left(\int\cos\left(b^2\pi x\right)\,dx,x,x^2\right)}{8b^2\pi^2}+\frac{3\text{Subst}\left(\int\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)}{4b^2\pi^2}\\
&= \frac{3x^2}{8b^2\pi^2}+\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2}+\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2b\pi}+\frac{3\,\text{FresnelS}(bx)^2}{4b^4\pi^2}\\
&\quad+\frac{1}{4}x^4\,\text{FresnelS}(bx)^2-\frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2}-\frac{\sin\left(b^2\pi x^2\right)}{2b^4\pi^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int x^3 \operatorname{FresnelS}(bx)^2\,dx = \frac{3b^2\pi x^2 + b^2\pi x^2\cos(b^2\pi x^2) + 2\pi(3+b^4\pi^2x^4)\operatorname{FresnelS}(bx)^2 + 4b\pi x\operatorname{FresnelS}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 3\sin\right.}{8b^4\pi^3}$$

```
[In] Integrate[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*b^2*Pi*x^2 + b^2*Pi*x^2*Cos[b^2*Pi*x^2] + 2*Pi*(3 + b^4*Pi^2*x^4)*Fresne
lS[b*x]^2 + 4*b*Pi*x*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 3*Sin[
(b^2*Pi*x^2)/2]) - 4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Maple [F]

$$\int x^3 \operatorname{FresnelS}(bx)^2\,dx$$

```
[In] int(x^3*FresnelS(b*x)^2,x)
```

```
[Out] int(x^3*FresnelS(b*x)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{FresnelS}(bx)^2\,dx = \frac{2\,\pi^2b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\,(bx) + \pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + \pi b^2x^2 + (3\,\pi + \pi^3b^4x^4)\,\mathrm{S}\,(bx)^2 - 2\left(3\,\pi bx\,\mathrm{S}\,(bx) + 2\right.}{4\,\pi^3b^4}$$

```
[In] integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + pi*b^2*x^2*cos(1
/2*pi*b^2*x^2)^2 + pi*b^2*x^2 + (3*pi + pi^3*b^4*x^4)*fresnel_sin(b*x)^2 -
2*(3*pi*b*x*fresnel_sin(b*x) + 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/
(pi^3*b^4)
```

Sympy [F]

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \int x^3 S^2(bx) \, dx$$

```
[In] integrate(x**3*fresnels(b*x)**2,x)
[Out] Integral(x**3*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \int x^3 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^3*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \int x^3 \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^3*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \int x^3 \operatorname{FresnelS}(b\,x)^2 \, dx$$

```
[In] int(x^3*FresnelS(b*x)^2,x)
[Out] int(x^3*FresnelS(b*x)^2, x)
```

3.36 $\int x^2 \operatorname{FresnelS}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 124

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx = \frac{2x}{3b^2\pi^2} + \frac{x\cos(b^2\pi x^2)}{6b^2\pi^2} - \frac{5\operatorname{FresnelC}(\sqrt{2}bx)}{6\sqrt{2}b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{3b\pi} + \frac{1}{3}x^3\operatorname{FresnelS}(bx)^2 - \frac{4\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

```
[Out] 2/3*x/b^2/Pi^2+1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2+2/3*x^2*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/b/Pi+1/3*x^3*FresnelS(b*x)^2-4/3*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^3/Pi^2-5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6565, 6589, 6595, 3438, 3433, 3466}

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx = -\frac{5\operatorname{FresnelC}(\sqrt{2}bx)}{6\sqrt{2}\pi^2 b^3} + \frac{2x^2\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} + \frac{x\cos(\pi b^2x^2)}{6\pi^2b^2} + \frac{2x}{3\pi^2b^2} - \frac{4\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{1}{3}x^3\operatorname{FresnelS}(bx)^2$$

```
[In] Int[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]
*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*
```

```
b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*
b^3*Pi^2)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\,\text{FresnelS}(bx)^2 - \frac{1}{3}(2b)\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{2x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{\int x^2 \sin\left(b^2\pi x^2\right)\,dx}{3\pi} - \frac{4\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx}{3b\pi}$$

$$= \frac{x \cos\left(b^2\pi x^2\right)}{6b^2\pi^2} + \frac{2x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{4\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{\int \cos\left(b^2\pi x^2\right)\,dx}{6b^2\pi^2} + \frac{4\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b^2\pi^2}$$

$$= \frac{x \cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} + \frac{2x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{4\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{4\int \left(\frac{1}{2} - \frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{3b^2\pi^2}$$

$$= \frac{2x}{3b^2\pi^2} + \frac{x \cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} + \frac{2x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{4\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{2\int \cos\left(b^2\pi x^2\right)\,dx}{3b^2\pi^2}$$

$$= \frac{2x}{3b^2\pi^2} + \frac{x \cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} - \frac{\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{3b^3\pi^2} + \frac{2x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{4\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int x^2 \text{FresnelS}(bx)^2\,dx = \frac{2bx\left(4+\cos\left(b^2\pi x^2\right)\right) - 5\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 4b^3\pi^2x^3\,\text{FresnelS}(bx)^2 + 8\,\text{FresnelS}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right.\right.}{12b^3\pi^2}$$

```
[In] Integrate[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*b*x*(4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*b^3*Pi^2
*x^3*FresnelS[b*x]^2 + 8*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*
Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^3 x^3}{3} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi} +}{b^3}$
default	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^3 x^3}{3} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi} +}{b^3}$

```
[In] int(x^2*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelS(b*x)^2*b^3*x^3-2*FresnelS(b*x)*(-1/3/Pi*b^2*x^2*cos(1/2
*b^2*Pi*x^2)+2/3/Pi^2*sin(1/2*b^2*Pi*x^2))+2/3*b*x/Pi^2-1/3/Pi^2*2^(1/2)*Fr
esnelC(b*x*2^(1/2))-1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fres
nelC(b*x*2^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x^2\,\mathrm{FresnelS}(bx)^2\,dx$$
$$= \frac{4\pi^2 b^4 x^3\,\mathrm{S}(bx)^2 + 8\pi b^3 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)\mathrm{S}(bx) + 4b^2 x \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 6b^2 x - 16b\,\mathrm{S}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right) -}{12\pi^2 b^4}$$

```
[In] integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi^2*b^4*x^3*fresnel_sin(b*x)^2 + 8*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*
fresnel_sin(b*x) + 4*b^2*x*cos(1/2*pi*b^2*x^2)^2 + 6*b^2*x - 16*b*fresnel_s
in(b*x)*sin(1/2*pi*b^2*x^2) - 5*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(
b^2)*x))/(pi^2*b^4)
```

Sympy [F]

$$\int x^2 \operatorname{FresnelS}(bx)^2\,dx = \int x^2 S^2(bx)\,dx$$

```
[In] integrate(x**2*fresnels(b*x)**2,x)
[Out] Integral(x**2*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \operatorname{FresnelS}(bx)^2\,dx = \int x^2\,\mathrm{S}\left(bx\right)^2\,dx$$

```
[In] integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^2*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x^2 \operatorname{FresnelS}(bx)^2\,dx = \int x^2\,\mathrm{S}\left(bx\right)^2\,dx$$

```
[In] integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^2*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(bx)^2\,dx = \int x^2 \operatorname{FresnelS}(b\,x)^2\,dx$$

```
[In] int(x^2*FresnelS(b*x)^2,x)
[Out] int(x^2*FresnelS(b*x)^2, x)
```

3.37 $\int x \operatorname{FresnelS}(bx)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 143

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \frac{\cos(b^2\pi x^2)}{4b^2\pi^2} + \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b\pi} - \frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^2\pi} + \frac{1}{2}x^2\operatorname{FresnelS}(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}$$

```
[Out] 1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-1/2*F
resnelC(b*x)*FresnelS(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*x)^2+1/8*I*x^2*hyperge
om([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/Pi-1/8*I*x^2*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)/Pi
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6565, 6589, 6581, 3460, 2718}

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2\pi b^2} + \frac{x\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + \frac{\cos(\pi b^2x^2)}{4\pi^2b^2} + \frac{1}{2}x^2\operatorname{FresnelS}(bx)^2$$

```
[In] Int[x*FresnelS[b*x]^2,x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi)
 - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I
/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/Pi - ((I/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\,\text{FresnelS}(bx)^2 - b\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} + \frac{1}{2}x^2\,\text{FresnelS}(bx)^2\\ &\quad - \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2\pi} - \frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b\pi}\end{aligned}$$

$$= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^2\pi} + \frac{1}{2}x^2\,\text{FresnelS}(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{4\pi}$$

$$= \frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^2\pi} + \frac{1}{2}x^2\,\text{FresnelS}(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}$$

Mathematica [F]

$$\int x\,\text{FresnelS}(bx)^2\,dx = \int x\,\text{FresnelS}(bx)^2\,dx$$

```
[In] Integrate[x*FresnelS[b*x]^2,x]
[Out] Integrate[x*FresnelS[b*x]^2, x]
```

Maple [F]

$$\int x\,\text{FresnelS}\left(bx\right)^2\,dx$$

```
[In] int(x*FresnelS(b*x)^2,x)
[Out] int(x*FresnelS(b*x)^2,x)
```

Fricas [F]

$$\int x\,\text{FresnelS}(bx)^2\,dx = \int x\,\text{S}\left(bx\right)^2\,dx$$

```
[In] integrate(x*fresnel_sin(b*x)^2,x, algorithm="fricas")
[Out] integral(x*fresnel_sin(b*x)^2, x)
```

Sympy [F]

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \int x S^2(bx) \, dx$$

```
[In] integrate(x*fresnels(b*x)**2,x)
[Out] Integral(x*fresnels(b*x)**2, x)
```

Maxima [F]

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \int x \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x*fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \int x \operatorname{S}(bx)^2 \, dx$$

```
[In] integrate(x*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x*fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \int x \operatorname{FresnelS}(b\,x)^2 \, dx$$

```
[In] int(x*FresnelS(b*x)^2,x)
[Out] int(x*FresnelS(b*x)^2, x)
```

3.38 $\int \text{FresnelS}(bx)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 55

$$\int \text{FresnelS}(bx)^2\,dx = \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} + x\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi}$$

```
[Out] 2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+x*FresnelS(b*x)^2-1/2*FresnelS(b*x
*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6555, 12, 6587, 3432}

$$\int \text{FresnelS}(bx)^2\,dx = \frac{2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + x\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

```
[In] Int[FresnelS[b*x]^2,x]
```

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\,\text{FresnelS}(bx)^2 - 2\int bx\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= x\,\text{FresnelS}(bx)^2 - (2b)\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} + x\,\text{FresnelS}(bx)^2 - \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{\pi} \\
&= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} + x\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$
\int \text{FresnelS}(bx)^2\,dx = \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi} + x\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi}
$$

```
[In] Integrate[FresnelS[b*x]^2,x]
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\mathrm{FresnelS}(bx)^{2} bx+\frac{2\,\mathrm{FresnelS}(bx)\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49
default	$\frac{\mathrm{FresnelS}(bx)^{2} bx+\frac{2\,\mathrm{FresnelS}(bx)\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49

```
[In] int(FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x)^2*b*x+2*FresnelS(b*x)/Pi*cos(1/2*b^2*Pi*x^2)-1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \mathrm{FresnelS}(bx)^2\,dx = \frac{2\,\pi b^2 x\,\mathrm{S}\,(bx)^2 + 4\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\,(bx) - \sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{2\,\pi b^2}$$

```
[In] integrate(fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*pi*b^2*x*fresnel_sin(b*x)^2 + 4*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*
x) - sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F]

$$\int \mathrm{FresnelS}(bx)^2\,dx = \int S^2(bx)\,dx$$

```
[In] integrate(fresnels(b*x)**2,x)
```

```
[Out] Integral(fresnels(b*x)**2, x)
```

Maxima [F]

$$\int \text{FresnelS}(bx)^2\,dx = \int \text{S}\,(bx)^2\,\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2, x)
```

Giac [F]

$$\int \text{FresnelS}(bx)^2\,dx = \int \text{S}\,(bx)^2\,\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)^2\,dx = \int \text{FresnelS}(b\,x)^2\,dx$$

```
[In] int(FresnelS(b*x)^2,x)
[Out] int(FresnelS(b*x)^2, x)
```

3.39 $\int \frac{\text{FresnelS}(bx)^2}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \text{Int}\left(\frac{\text{FresnelS}(bx)^2}{x}, x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{FresnelS}(bx)^2}{x} \, dx$$

```
[In] Int[FresnelS[b*x]^2/x,x]
```

```
[Out] Defer[Int][FresnelS[b*x]^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(bx)^2}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x,x]
[Out] Integrate[FresnelS[b*x]^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx)^2}{x}dx$$

```
[In] int(FresnelS(b*x)^2/x,x)
[Out] int(FresnelS(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x}\,dx = \int \frac{\text{S}\,(bx)^2}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelS}(bx)^2}{x}\,dx = \int \frac{S^2(bx)}{x}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x,x)
[Out] Integral(fresnels(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x} \, dx$$

```
[In] int(FresnelS(b*x)^2/x,x)
[Out] int(FresnelS(b*x)^2/x, x)
```

3.40 $\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = -\frac{\text{FresnelS}(bx)^2}{x} + 2b\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -FresnelS(b*x)^2/x+2*b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^2,x]
```

```
[Out] -(FresnelS[b*x]^2/x) + 2*b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x
, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelS}(bx)^2}{x} + (2b)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^2,x]
[Out] Integrate[FresnelS[b*x]^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^2}{x^2}dx$$

```
[In] int(FresnelS(b*x)^2/x^2,x)
[Out] int(FresnelS(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{S^2(bx)}{x^2}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**2,x)
[Out] Integral(fresnels(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{FresnelS}\left(b\,x\right)^2}{x^2}\,dx$$

```
[In] int(FresnelS(b*x)^2/x^2,x)
[Out] int(FresnelS(b*x)^2/x^2, x)
```

3.41 $\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = -\frac{\text{FresnelS}(bx)^2}{2x^2} + b\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}, x\right)$$

```
[Out] -1/2*FresnelS(b*x)^2/x^2+b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x
^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^3,x]
```

```
[Out] -1/2*FresnelS[b*x]^2/x^2 + b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x^2, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelS}(bx)^2}{2x^2} + b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3} \, dx = \int \frac{\text{FresnelS}(bx)^2}{x^3} \, dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^3,x]
[Out] Integrate[FresnelS[b*x]^2/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^{2}}{x^{3}}dx$$

```
[In] int(FresnelS(b*x)^2/x^3,x)
[Out] int(FresnelS(b*x)^2/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3} \, dx = \int \frac{\text{S}\left(bx\right)^{2}}{x^{3}} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^3} \, dx = \int \frac{S^{2}(bx)}{x^{3}} \, dx$$

```
[In] integrate(fresnels(b*x)**2/x**3,x)
[Out] Integral(fresnels(b*x)**2/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{S}\,(bx)^2}{x^3}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{S}\,(bx)^2}{x^3}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^3}\,dx$$

```
[In] int(FresnelS(b*x)^2/x^3,x)
[Out] int(FresnelS(b*x)^2/x^3, x)
```

3.42 $\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx = -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{\text{FresnelS}(bx)^2}{3x^3} + \frac{b^3\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{3\sqrt{2}}-\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} + \frac{1}{3}b^3\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)$$

```
[Out] -1/6*b^2/x+1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*FresnelS(b*x)^2/x^3-1/3*b*FresnelS
(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/3*
b^3*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^4,x]
```

```
[Out] -1/6*b^2/x + (b^2*Cos[b^2*Pi*x^2])/(6*x) - FresnelS[b*x]^2/(3*x^3) + (b^3*P
i*FresnelS[Sqrt[2]*b*x])/(3*Sqrt[2]) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(3*x^2) + (b^3*Pi*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/3
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{3x^3} + \frac{1}{3}(2b)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{b^2}{6x} - \frac{\text{FresnelS}(bx)^2}{3x^3} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} \\
&\quad - \frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx \\
&= -\frac{b^2}{6x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{6x} - \frac{\text{FresnelS}(bx)^2}{3x^3} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} \\
&\quad + \frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx + \frac{1}{3}\left(b^4\pi\right)\int \sin\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b^2}{6x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{6x} - \frac{\text{FresnelS}(bx)^2}{3x^3} + \frac{b^3\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{3\sqrt{2}} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} + \frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^4,x]
[Out] Integrate[FresnelS[b*x]^2/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^2}{x^4}dx$$

```
[In] int(FresnelS(b*x)^2/x^4,x)
[Out] int(FresnelS(b*x)^2/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} \, dx = \int \frac{\text{S}\,(bx)^2}{x^4} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} \, dx = \int \frac{S^2(bx)}{x^4} \, dx$$

```
[In] integrate(fresnels(b*x)**2/x**4,x)
[Out] Integral(fresnels(b*x)**2/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} \, dx = \int \frac{\text{S}\,(bx)^2}{x^4} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} dx = \int \frac{S(bx)^2}{x^4} dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^4} dx$$

```
[In] int(FresnelS(b*x)^2/x^4,x)
[Out] int(FresnelS(b*x)^2/x^4, x)
```

3.43 $\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 127

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = -\frac{b^2}{24x^2} + \frac{b^2\cos(b^2\pi x^2)}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)^2}{4x^4} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{12}b^4\pi\text{Si}(b^2\pi x^2)$$

```
[Out] -1/24*b^2/x^2+1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b^3*Pi*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/x-1/12*b^4*Pi^2*FresnelS(b*x)^2-1/4*FresnelS(b*x)^2/x^4+1/12*b
^4*Pi*Si(b^2*Pi*x^2)-1/6*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6565, 6591, 6599, 6575, 30, 3456, 3461, 3378, 3380}

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = -\frac{1}{12}\pi^2b^4\,\text{FresnelS}(bx)^2 - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3} - \frac{b^2}{24x^2} + \frac{b^2\cos(\pi b^2x^2)}{24x^2} + \frac{1}{12}\pi b^4\text{Si}(b^2\pi x^2) - \frac{\pi b^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x} - \frac{\text{FresnelS}(bx)^2}{4x^4}$$

```
[In] Int[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{4x^4} + \frac{1}{2}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b^2}{24x^2} - \frac{\text{FresnelS}(bx)^2}{4x^4} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} \\
&\quad - \frac{1}{12}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx + \frac{1}{6}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx \\
&= -\frac{b^2}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} - \frac{\text{FresnelS}(bx)^2}{4x^4} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{24}b^2\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad + \frac{1}{12}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx - \frac{1}{6}\left(b^5\pi^2\right)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{b^2}{24x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} \\
&\quad - \frac{\text{FresnelS}(bx)^2}{4x^4} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} \\
&\quad + \frac{1}{24}b^4\pi\text{Si}\left(b^2\pi x^2\right) + \frac{1}{24}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&\quad - \frac{1}{6}\left(b^4\pi^2\right)\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right) \\
&= -\frac{b^2}{24x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx)^2 \\
&\quad - \frac{\text{FresnelS}(bx)^2}{4x^4} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = -\frac{b^2}{24x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} -\frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)^2}{4x^4} -\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}+\frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)$$

```
[In] Integrate[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^5}dx$$

```
[In] int(FresnelS(b*x)^2/x^5,x)
```

```
[Out] int(FresnelS(b*x)^2/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \frac{\pi b^4x^4\,\text{Si}\left(\pi b^2x^2\right)-2\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)+b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-b^2x^2-2\,bx\,\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-(}{12\,x^4}$$

```
[In] integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(pi*b^4*x^4*sin_integral(pi*b^2*x^2) - 2*pi*b^3*x^3*cos(1/2*pi*b^2*x^2
)*fresnel_sin(b*x) + b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - b^2*x^2 - 2*b*x*fresne
l_sin(b*x)*sin(1/2*pi*b^2*x^2) - (pi^2*b^4*x^4 + 3)*fresnel_sin(b*x)^2)/x^4
```

Sympy [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{S^2(bx)}{x^5}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**5,x)
[Out] Integral(fresnels(b*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^5}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^5, x)
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^5}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^5}\,dx$$

```
[In] int(FresnelS(b*x)^2/x^5,x)
[Out] int(FresnelS(b*x)^2/x^5, x)
```

3.44 $\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx =& -\frac{b^2}{60x^3}+\frac{b^2\cos(b^2\pi x^2)}{60x^3}+\frac{7b^5\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}}\\&-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{20x^2}-\frac{\text{FresnelS}(bx)^2}{5x^5}\\&-\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{7b^4\pi\sin(b^2\pi x^2)}{120x}\\&-\frac{1}{20}b^5\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

```
[Out] -1/60*b^2/x^3+1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/20*b^3*Pi*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x^2-1/5*FresnelS(b*x)^2/x^5-1/10*b*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/x^4-7/120*b^4*Pi*sin(b^2*Pi*x^2)/x+7/120*b^5*Pi^2*FresnelC(b*x*2^(1
/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x
,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx=\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^6,x]
```

```
[Out] -1/60*b^2/x^3 + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
 - FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
 - (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]
*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{5x^5} + \frac{1}{5}(2b)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{b^2}{60x^3} - \frac{\text{FresnelS}(bx)^2}{5x^5} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} \\
&\quad - \frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx + \frac{1}{10}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx \\
&= -\frac{b^2}{60x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{20x^2} - \frac{\text{FresnelS}(bx)^2}{5x^5} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} + \frac{1}{40}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad + \frac{1}{30}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{20}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&= -\frac{b^2}{60x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{20x^2} \\
&\quad - \frac{\text{FresnelS}(bx)^2}{5x^5} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} \\
&\quad - \frac{7b^4\pi\sin\left(b^2\pi x^2\right)}{120x} - \frac{1}{20}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&\quad + \frac{1}{20}\left(b^6\pi^2\right)\int \cos\left(b^2\pi x^2\right)\,dx + \frac{1}{15}\left(b^6\pi^2\right)\int \cos\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b^2}{60x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} + \frac{7b^5\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}} \\
&\quad - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{20x^2} - \frac{\text{FresnelS}(bx)^2}{5x^5} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} \\
&\quad - \frac{7b^4\pi\sin\left(b^2\pi x^2\right)}{120x} - \frac{1}{20}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^6,x]
[Out] Integrate[FresnelS[b*x]^2/x^6, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^2}{x^6}dx$$

```
[In] int(FresnelS(b*x)^2/x^6,x)
[Out] int(FresnelS(b*x)^2/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{S^2(bx)}{x^6}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**6,x)
[Out] Integral(fresnels(b*x)**2/x**6, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^6, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^6, x)
```

Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelS}\left(b\,x\right)^2}{x^6}\,dx$$

```
[In] int(FresnelS(b*x)^2/x^6,x)
[Out] int(FresnelS(b*x)^2/x^6, x)
```

3.45 $\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = -\frac{b^2}{120x^4}+\frac{b^2\cos(b^2\pi x^2)}{120x^4}+\frac{1}{72}b^6\pi^2\,\text{CosIntegral}(b^2\pi x^2) - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{45x^3}-\frac{\text{FresnelS}(bx)^2}{6x^6} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{b^4\pi\sin(b^2\pi x^2)}{72x^2} - \frac{1}{45}b^5\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] -1/120*b^2/x^4+1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)+1/120*b^2*cos(b^2*Pi*x^2)/x^4-1
/45*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/6*FresnelS(b*x)^2/x^6-1/
15*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/72*b^4*Pi*sin(b^2*Pi*x^2)/x^2-
1/45*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^7,x]
```

```
[Out] -1/120*b^2/x^4 + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - Fresn
elS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{6x^6} + \frac{1}{3}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx \\
&= -\frac{b^2}{120x^4} - \frac{\text{FresnelS}(bx)^2}{6x^6} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} \\
&\quad - \frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{15}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx \\
&= -\frac{b^2}{120x^4} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{45x^3} - \frac{\text{FresnelS}(bx)^2}{6x^6} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{1}{60}b^2\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&\quad + \frac{1}{90}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b^2}{120x^4} + \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{45x^3} - \frac{\text{FresnelS}(bx)^2}{6x^6} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} + \frac{1}{180}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad + \frac{1}{120}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b^2}{120x^4} + \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{45x^3} \\
&\quad - \frac{\text{FresnelS}(bx)^2}{6x^6} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{b^4\pi\sin\left(b^2\pi x^2\right)}{72x^2} \\
&\quad - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{180}\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&\quad + \frac{1}{120}\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= -\frac{b^2}{120x^4} + \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} + \frac{1}{72}b^6\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right) \\
&\quad - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{45x^3} - \frac{\text{FresnelS}(bx)^2}{6x^6} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} \\
&\quad - \frac{b^4\pi\sin\left(b^2\pi x^2\right)}{72x^2} - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^7,x]
[Out] Integrate[FresnelS[b*x]^2/x^7, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^2}{x^7}dx$$

```
[In] int(FresnelS(b*x)^2/x^7,x)
[Out] int(FresnelS(b*x)^2/x^7,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^7, x)
```

Sympy [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{S^2(bx)}{x^7}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**7,x)
[Out] Integral(fresnels(b*x)**2/x**7, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{S}\,(bx)^2}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^7, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{S}\,(bx)^2}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^7, x)
```

Mupad [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^7}\,dx$$

```
[In] int(FresnelS(b*x)^2/x^7,x)
[Out] int(FresnelS(b*x)^2/x^7, x)
```

3.46 $\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = & -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}+\frac{b^2\cos(b^2\pi x^2)}{210x^5}-\frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} \\ & -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{84x^4}-\frac{\text{FresnelS}(bx)^2}{7x^7} \\ & -\frac{b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{72\sqrt{2}}-\frac{2}{315}\sqrt{2}b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) \\ & -\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6}+\frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^2} \\ & -\frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3}-\frac{1}{168}b^7\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)\end{aligned}$$

```
[Out] -1/210*b^2/x^5+1/336*b^6*Pi^2/x+1/210*b^2*cos(b^2*Pi*x^2)/x^5-67/5040*b^6*P
i^2*cos(b^2*Pi*x^2)/x-1/84*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/7
*FresnelS(b*x)^2/x^7-1/21*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/168*b^5
*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-13/2520*b^4*Pi*sin(b^2*Pi*x^2)/
x^3-67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)-1/168*b^7*Pi^3*Unintegra
ble(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^8,x]
```

```
[Out] -1/210*b^2/x^5 + (b^6*Pi^2)/(336*x) + (b^2*Cos[b^2*Pi*x^2])/(210*x^5) - (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(84*x^4) - FresnelS[b*x]^2/(7*x^7) - (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])
/(72*Sqrt[2]) - (2*Sqrt[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 - (b*Fresnel
S[b*x]*Sin[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*
x^2)/2])/(168*x^2) - (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*Def
er[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/168
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{b^2}{210x^5} - \frac{\text{FresnelS}(bx)^2}{7x^7} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} \\
&\quad - \frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{21}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx \\
&= -\frac{b^2}{210x^5} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{84x^4} - \frac{\text{FresnelS}(bx)^2}{7x^7} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{1}{168}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad + \frac{1}{105}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{84}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{84x^4} \\
&\quad - \frac{\text{FresnelS}(bx)^2}{7x^7} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^2} \\
&\quad - \frac{13b^4\pi\sin\left(b^2\pi x^2\right)}{2520x^3} + \frac{1}{336}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{252}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad + \frac{1}{315}\left(2b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{168}\left(b^7\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos(b^2\pi x^2)}{210x^5} - \frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} \\
&\quad - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{84x^4} - \frac{\text{FresnelS}(bx)^2}{7x^7} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} \\
&\quad + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^2} - \frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3} \\
&\quad - \frac{1}{168}\left(b^7\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx - \frac{1}{168}\left(b^8\pi^3\right)\int\sin\left(b^2\pi x^2\right)\,dx \\
&\quad - \frac{1}{126}\left(b^8\pi^3\right)\int\sin\left(b^2\pi x^2\right)\,dx - \frac{1}{315}\left(4b^8\pi^3\right)\int\sin\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos(b^2\pi x^2)}{210x^5} - \frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} \\
&\quad - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{84x^4} - \frac{\text{FresnelS}(bx)^2}{7x^7} \\
&\quad - \frac{b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{72\sqrt{2}} - \frac{2}{315}\sqrt{2}b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^2} \\
&\quad - \frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3} - \frac{1}{168}\left(b^7\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int\frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int\frac{\text{FresnelS}(bx)^2}{x^8}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^8,x]
[Out] Integrate[FresnelS[b*x]^2/x^8, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelS}(bx)^2}{x^8}dx$$

```
[In] int(FresnelS(b*x)^2/x^8,x)
[Out] int(FresnelS(b*x)^2/x^8,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^8}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^8, x)
```

Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{S^2(bx)}{x^8}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**8,x)
[Out] Integral(fresnels(b*x)**2/x**8, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^8}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^8, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{S}\,(bx)^2}{x^8}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^8, x)
```

Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^8}\,dx$$

```
[In] int(FresnelS(b*x)^2/x^8,x)
[Out] int(FresnelS(b*x)^2/x^8, x)
```

3.47 $\int \frac{\text{FresnelS}(bx)^2}{x^9} dx$

Optimal result

Integrand size = 10, antiderivative size = 242

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^9} dx = &-\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} \\ &- \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x} \\ &+ \frac{1}{840}b^8\pi^4\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)^2}{8x^8} \\ &- \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} \\ &- \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} - \frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)\end{aligned}$$

```
[Out] -1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2+1/336*b^2*cos(b^2*Pi*x^2)/x^6-1/336*b^6*
Pi^2*cos(b^2*Pi*x^2)/x^2-1/140*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5
+1/420*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/840*b^8*Pi^4*FresnelS
(b*x)^2-1/8*FresnelS(b*x)^2/x^8-1/280*b^8*Pi^3*Si(b^2*Pi*x^2)-1/28*b*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/420*b^5*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^3-1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6565, 6591, 6599, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = \frac{1}{840}\pi^4 b^8\,\text{FresnelS}(bx)^2 + \frac{\pi^2 b^6}{1680x^2} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7} - \frac{b^2}{336x^6} + \frac{b^2\cos(\pi b^2x^2)}{336x^6} - \frac{1}{280}\pi^3 b^8\text{Si}(b^2\pi x^2) + \frac{\pi^3 b^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x} - \frac{\pi^2 b^6\cos(\pi b^2x^2)}{336x^2} + \frac{\pi^2 b^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3} - \frac{\pi b^4\sin(\pi b^2x^2)}{420x^4} - \frac{\pi b^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5} - \frac{\text{FresnelS}(bx)^2}{8x^8}$$

```
[In] Int[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{8x^8} + \frac{1}{4}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx \\
&= -\frac{b^2}{336x^6} - \frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} \\
&\quad - \frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{28}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx \\
&= -\frac{b^2}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} - \frac{\text{FresnelS}(bx)^2}{8x^8} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{112}b^2\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&\quad + \frac{1}{280}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{140}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} \\
&\quad - \frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} \\
&\quad + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} + \frac{1}{560}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&\quad + \frac{1}{336}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) + \frac{1}{840}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad - \frac{1}{420}\left(b^7\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx \\
&= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} \\
&\quad + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x} - \frac{\text{FresnelS}(bx)^2}{8x^8} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} \\
&\quad - \frac{b^4\pi\sin\left(b^2\pi x^2\right)}{420x^4} + \frac{\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right)}{1680} \\
&\quad + \frac{\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right)}{1120} + \frac{1}{672}\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad - \frac{1}{840}\left(b^8\pi^3\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx + \frac{1}{420}\left(b^9\pi^4\right)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx
\end{aligned}
$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x} - \frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} - \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} - \frac{b^8\pi^3\text{Si}(b^2\pi x^2)}{1680} - \frac{(b^8\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}dx, x, x^2\right)}{1680} - \frac{(b^8\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}dx, x, x^2\right)}{1120} - \frac{1}{672}(b^8\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}dx, x, x^2\right) + \frac{1}{420}(b^8\pi^4)\,\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x} + \frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} - \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} - \frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelS}(bx)^2}{x^9}dx = -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x} + \frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} - \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} - \frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)$$

```
[In] Integrate[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
```

```
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^9}dx$$

```
[In] int(FresnelS(b*x)^2/x^9,x)
[Out] int(FresnelS(b*x)^2/x^9,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = -\frac{3\,\pi^3 b^8 x^8\,\text{Si}\,(\pi b^2 x^2) - 3\,\pi^2 b^6 x^6 + 5\,b^2 x^2 + 5\,(\pi^2 b^6 x^6 - b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 2\,(\pi^3 b^7 x^7 - 3\,\pi b^3 x^3)\cos\left(\frac{1}{2}\right.}{84}$$

```
[In] integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="fricas")
[Out] -1/840*(3*pi^3*b^8*x^8*sin_integral(pi*b^2*x^2) - 3*pi^2*b^6*x^6 + 5*b^2*x^
2 + 5*(pi^2*b^6*x^6 - b^2*x^2)*cos(1/2*pi*b^2*x^2)^2 - 2*(pi^3*b^7*x^7 - 3*
pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - (pi^4*b^8*x^8 - 105)*fre
snel_sin(b*x)^2 + 2*(2*pi*b^4*x^4*cos(1/2*pi*b^2*x^2) - (pi^2*b^5*x^5 - 15*
b*x)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = \int \frac{S^2(bx)}{x^9}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**9,x)
[Out] Integral(fresnels(b*x)**2/x**9, x)
```

Maxima [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^9} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^9, x)
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^9} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^9} \, dx$$

```
[In] int(FresnelS(b*x)^2/x^9,x)
[Out] int(FresnelS(b*x)^2/x^9, x)
```

3.48 $\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx &= -\frac{b^2}{504x^7}+\frac{b^6\pi^2}{5184x^3}+\frac{b^2\cos(b^2\pi x^2)}{504x^7}\\ &-\frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}-\frac{853b^9\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{181440\sqrt{2}}\\ &-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{1728x^2}\\ &-\frac{\text{FresnelS}(bx)^2}{9x^9}-\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8}\\ &+\frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4}-\frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5}\\ &+\frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x}+\frac{b^9\pi^4\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)}{1728}\end{aligned}$$

```
[Out] -1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3+1/504*b^2*cos(b^2*Pi*x^2)/x^7-187/181440
*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/216*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x
)/x^6+1/1728*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/9*FresnelS(b*
x)^2/x^9-1/36*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8+1/864*b^5*Pi^2*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5+853/362
880*b^8*Pi^3*sin(b^2*Pi*x^2)/x-853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/2))*2^
(1/2)+1/1728*b^9*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$$

```
[In] Int[FresnelS[b*x]^2/x^10,x]
[Out] -1/504*b^2/x^7 + (b^6*Pi^2)/(5184*x^3) + (b^2*Cos[b^2*Pi*x^2])/(504*x^7) -
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (853*b^9*Pi^4*FresnelC[Sqrt[2
]*b*x])/(181440*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(216*
x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(1728*x^2) - FresnelS[b
*x]^2/(9*x^9) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(36*x^8) + (b^5*Pi^2*
FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(864*x^4) - (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) + (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/1728
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx \\
&= -\frac{b^2}{504x^7} - \frac{\text{FresnelS}(bx)^2}{9x^9} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} \\
&\quad - \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{36}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx \\
&= -\frac{b^2}{504x^7} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6} - \frac{\text{FresnelS}(bx)^2}{9x^9} \\
&\quad - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{1}{432}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad + \frac{1}{252}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{216}\left(b^5\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6} \\
&\quad - \frac{\text{FresnelS}(bx)^2}{9x^9} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4} \\
&\quad - \frac{19b^4\pi\sin\left(b^2\pi x^2\right)}{15120x^5} + \frac{\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx}{1728} + \frac{\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx}{1080} \\
&\quad + \frac{1}{630}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{864}\left(b^7\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx
\end{aligned}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos(b^2\pi x^2)}{504x^7} - \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}$$
$$- \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{1728x^2} - \frac{\text{FresnelS}(bx)^2}{9x^9}$$
$$- \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4} - \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5}$$
$$- \frac{(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx}{3456} - \frac{(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx}{2592} - \frac{(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx}{1620}$$
$$- \frac{1}{945}(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx + \frac{(b^9\pi^4)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx}{1728}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos(b^2\pi x^2)}{504x^7} - \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}$$
$$- \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{1728x^2}$$
$$- \frac{\text{FresnelS}(bx)^2}{9x^9} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4}$$
$$- \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5} + \frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x} + \frac{(b^9\pi^4)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx}{1728}$$
$$- \frac{(b^{10}\pi^4)\int\cos(b^2\pi x^2)\,dx}{1728} - \frac{(b^{10}\pi^4)\int\cos(b^2\pi x^2)\,dx}{1296}$$
$$- \frac{1}{810}(b^{10}\pi^4)\int\cos\left(b^2\pi x^2\right)\,dx - \frac{1}{945}(2b^{10}\pi^4)\int\cos\left(b^2\pi x^2\right)\,dx$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos(b^2\pi x^2)}{504x^7} - \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}$$
$$- \frac{67b^9\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{25920\sqrt{2}} - \frac{1}{945}\sqrt{2}b^9\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)$$
$$- \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{1728x^2}$$
$$- \frac{\text{FresnelS}(bx)^2}{9x^9} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4}$$
$$- \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5} + \frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x} + \frac{(b^9\pi^4)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx}{1728}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$$

```
[In] Integrate[FresnelS[b*x]^2/x^10,x]
[Out] Integrate[FresnelS[b*x]^2/x^10, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^{10}}dx$$

```
[In] int(FresnelS(b*x)^2/x^10,x)
[Out] int(FresnelS(b*x)^2/x^10,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx = \int \frac{\text{S}\,(bx)^2}{x^{10}}\,dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^10, x)
```

Sympy [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx = \int \frac{S^2(bx)}{x^{10}}\,dx$$

```
[In] integrate(fresnels(b*x)**2/x**10,x)
[Out] Integral(fresnels(b*x)**2/x**10, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^{10}} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^10, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^{10}} \, dx$$

```
[In] integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^10, x)
```

Mupad [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^{10}} \, dx$$

```
[In] int(FresnelS(b*x)^2/x^10,x)
[Out] int(FresnelS(b*x)^2/x^10, x)
```

3.49 $\int (c+dx)^2 \text{FresnelS}(a+bx)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 497

$$\begin{aligned}
\int (c+dx)^2 \text{FresnelS}(a+bx)^2 \, dx &= \frac{2d^2x}{3b^2\pi^2} + \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} + \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} \\
&- \frac{5d^2 \text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} + \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} \\
&+ \frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} \\
&+ \frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{3b^3\pi} \\
&- \frac{d(bc-ad)\text{FresnelC}(a+bx)\text{FresnelS}(a+bx)}{b^3\pi} \\
&+ \frac{(bc-ad)^2(a+bx)\text{FresnelS}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\text{FresnelS}(a+bx)^2}{b^3} \\
&+ \frac{d^2(a+bx)^3\text{FresnelS}(a+bx)^2}{3b^3} - \frac{(bc-ad)^2\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&+ \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&- \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&- \frac{4d^2\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}
\end{aligned}$$

```
[Out] 2/3*d^2*x/b^2/Pi^2+1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+1/6*d^2*(b*x
+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+2*(-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)*Fresnel
S(b*x+a)/b^3/Pi+2*d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a
```

```
)/b^3/Pi+2/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^3/Pi-d*(
-a*d+b*c)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+(-a*d+b*c)^2*(b*x+a)*Fresn
elS(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^2*FresnelS(b*x+a)^2/b^3+1/3*d^2*(b*x+
a)^3*FresnelS(b*x+a)^2/b^3+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hypergeom([1, 1],[3
/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hypergeom([
1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-4/3*d^2*FresnelS(b*x+a)*sin(1/2*P
i*(b*x+a)^2)/b^3/Pi^2-5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^3/Pi^2*2^(1/2)-1
/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6567, 6555, 6587, 3432, 6565, 6589, 6581, 3460, 2718, 6595, 3438, 3433, 3466}

$$\begin{aligned}
&\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2 \, dx \\
&= \frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3} \\
&\quad - \frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3} \\
&\quad - \frac{d(bc-ad)\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{\pi b^3} + \frac{d(a+bx)^2(bc-ad)\operatorname{FresnelS}(a+bx)^2}{b^3} \\
&\quad + \frac{(a+bx)(bc-ad)^2\operatorname{FresnelS}(a+bx)^2}{b^3} - \frac{(bc-ad)^2\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b^3} \\
&\quad + \frac{2d(a+bx)(bc-ad)\operatorname{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} \\
&\quad + \frac{2(bc-ad)^2\operatorname{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} \\
&\quad + \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2\pi^2 b^3} - \frac{5d^2\operatorname{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}\pi^2 b^3} \\
&\quad + \frac{d^2(a+bx)^3\operatorname{FresnelS}(a+bx)^2}{3b^3} - \frac{4d^2\operatorname{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3} \\
&\quad + \frac{2d^2(a+bx)^2\operatorname{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3} + \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6\pi^2 b^3} + \frac{2d^2x}{3\pi^2 b^2}
\end{aligned}$$

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2
]*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x
)^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^
2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
```

```
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
 + (d*(b*c - a*d)*(a + b*x)^2*FresnelS[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*F
resnelS[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(
Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a +
b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi)
- (4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
```

```
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6567

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\text{FresnelS}(x)^2+2bcd\left(1-\frac{ad}{bc}\right)x\,\text{FresnelS}(x)^2+d^2x^2\,\text{FresnelS}(x)^2\right)\,dx,x,a\right.}{b^3}$$

$$=\frac{d^2\text{Subst}\left(\int x^2\,\text{FresnelS}(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\text{Subst}\left(\int x\,\text{FresnelS}(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(bc-ad)^2\text{Subst}\left(\int \text{FresnelS}(x)^2\,dx,x,a+bx\right)}{b^3}$$

$$=\frac{(bc-ad)^2(a+bx)\,\text{FresnelS}(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2\,\text{FresnelS}(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3\,\text{FresnelS}(a+bx)^2}{3b^3}-\frac{(2d^2)\,\text{Subst}\left(\int x^3\,\text{FresnelS}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3}-\frac{(2d(bc-ad))\text{Subst}\left(\int x^2\,\text{FresnelS}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3}-\frac{(2(bc-ad)^2)\,\text{Subst}\left(\int x\,\text{FresnelS}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3}$$

$$=\frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi}+\frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi}+\frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{3b^3\pi}+\frac{(bc-ad)^2(a+bx)\,\text{FresnelS}(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2\,\text{FresnelS}(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3\,\text{FresnelS}(a+bx)^2}{3b^3}-\frac{d^2\text{Subst}\left(\int x^2\sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{3b^3\pi}-\frac{(4d^2)\,\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelS}(x)\,dx,x,a+bx\right)}{3b^3\pi}-\frac{(d(bc-ad))\text{Subst}\left(\int x\sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b^3\pi}-\frac{(2d(bc-ad))\text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\text{FresnelS}(x)\,dx,x,a+bx\right)}{b^3\pi}-\frac{(bc-ad)^2\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b^3\pi}$$

$$
\begin{aligned}
&= \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} + \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{3b^3\pi} \\
&\quad - \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{(bc-ad)^2(a+bx)\,\text{FresnelS}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelS}(a+bx)^2}{b^3} \\
&\quad + \frac{d^2(a+bx)^3\,\text{FresnelS}(a+bx)^2}{3b^3} - \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&\quad + \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{4d^2\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} - \frac{d^2\text{Subst}\left(\int\cos\left(\pi x^2\right)\,dx,x,a+bx\right)}{6b^3\pi^2} \\
&\quad + \frac{(4d^2)\,\text{Subst}\left(\int\sin^2\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3\pi^2} \\
&\quad - \frac{(d(bc-ad))\text{Subst}\left(\int\sin(\pi x)\,dx,x,(a+bx)^2\right)}{2b^3\pi}
\end{aligned}
$$

$$= \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} + \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} + \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} + \frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} + \frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{3b^3\pi} - \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi} + \frac{(bc-ad)^2(a+bx)\,\text{FresnelS}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelS}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\,\text{FresnelS}(a+bx)^2}{3b^3} - \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} + \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} - \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} - \frac{4d^2\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} + \frac{(4d^2)\,\text{Subst}\left(\int\left(\frac{1}{2}-\frac{1}{2}\cos\left(\pi x^2\right)\right)\,dx,x,a+bx\right)}{3b^3\pi^2}$$

$$= \frac{2d^2x}{3b^2\pi^2} + \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} + \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} + \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} + \frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} + \frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{3b^3\pi} - \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi} + \frac{(bc-ad)^2(a+bx)\,\text{FresnelS}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelS}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\,\text{FresnelS}(a+bx)^2}{3b^3} - \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} + \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} - \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} - \frac{4d^2\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} - \frac{(2d^2)\,\text{Subst}\left(\int\cos\left(\pi x^2\right)\,dx,x,a+bx\right)}{3b^3\pi^2}$$

$$
\begin{aligned}
&= \frac{2d^2x}{3b^2\pi^2} + \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} + \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} \\
&\quad - \frac{d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} - \frac{\sqrt{2}d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{3b^3\pi^2} \\
&\quad + \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{3b^3\pi} \\
&\quad - \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{(bc-ad)^2(a+bx)\,\text{FresnelS}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelS}(a+bx)^2}{b^3} \\
&\quad + \frac{d^2(a+bx)^3\,\text{FresnelS}(a+bx)^2}{3b^3} - \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&\quad + \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{4d^2\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}
\end{aligned}
$$

Mathematica [F]

$$
\int (c+dx)^2\,\text{FresnelS}(a+bx)^2\,dx = \int (c+dx)^2\,\text{FresnelS}(a+bx)^2\,dx
$$

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2, x]
```

Maple [F]

$$
\int (dx+c)^2\,\text{FresnelS}\,(bx+a)^2\,dx
$$

```
[In] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
[Out] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

Fricas [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int (dx+c)^2 \operatorname{S}(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnel_sin(b*x + a)^2, x)
```

Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int (c+dx)^2 S^2(a+bx)\,dx$$

```
[In] integrate((d*x+c)**2*fresnels(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*fresnels(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int (dx+c)^2 \operatorname{S}(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int (dx+c)^2 \operatorname{S}(bx+a)^2\,dx$$

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2 \, dx = \int \operatorname{FresnelS}(a+b\,x)^2 \, (c+d\,x)^2 \, dx$$

```
[In] int(FresnelS(a + b*x)^2*(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)^2*(c + d*x)^2, x)
```

3.50 $\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 279

$$\begin{aligned}\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx = & \frac{d \cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2} \\ & + \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{b^2\pi} \\ & + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{b^2\pi} \\ & - \frac{d \operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{2b^2\pi} \\ & + \frac{(bc-ad)(a+bx)\operatorname{FresnelS}(a+bx)^2}{b^2} \\ & + \frac{d(a+bx)^2\operatorname{FresnelS}(a+bx)^2}{2b^2} \\ & - \frac{(bc-ad)\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} \\ & + \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} \\ & - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi}\end{aligned}$$

```
[Out] 1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)*Fresnel
S(b*x+a)/b^2/Pi+d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^2/Pi-1/2*
d*FresnelC(b*x+a)*FresnelS(b*x+a)/b^2/Pi+(-a*d+b*c)*(b*x+a)*FresnelS(b*x+a)
^2/b^2+1/2*d*(b*x+a)^2*FresnelS(b*x+a)^2/b^2+1/8*I*d*(b*x+a)^2*hypergeom([1
, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/8*I*d*(b*x+a)^2*hypergeom([1, 1
],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/2*(-a*d+b*c)*FresnelS((b*x+a)*2^(1/
2))/b^2/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6567, 6555, 6587, 3432, 6565, 6589, 6581, 3460, 2718}

$$\begin{aligned}\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=&\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}\\&-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}\\&+\frac{(a+bx)(bc-ad)\,\text{FresnelS}(a+bx)^2}{b^2}\\&-\frac{(bc-ad)\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b^2}\\&+\frac{2(bc-ad)\,\text{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}\\&-\frac{d\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{2\pi b^2}\\&+\frac{d(a+bx)^2\,\text{FresnelS}(a+bx)^2}{2b^2}\\&+\frac{d(a+bx)\,\text{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}\\&+\frac{d\cos\left(\pi(a+bx)^2\right)}{4\pi^2b^2}\end{aligned}$$

```
[In] Int[(c + d*x)*FresnelS[a + b*x]^2,x]
```

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/
2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*Fresn
elS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi)
 + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Fresnel
S[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*
b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)
*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6567

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
```

[m, 1]

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)\text{FresnelS}(x)^2+dx\,\text{FresnelS}(x)^2\right)\,dx,x,a+bx\right)}{b^2}$$

$$= \frac{d\text{Subst}\left(\int x\,\text{FresnelS}(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\text{Subst}\left(\int \text{FresnelS}(x)^2\,dx,x,a+bx\right)}{b^2}$$

$$= \frac{(bc-ad)(a+bx)\,\text{FresnelS}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\,\text{FresnelS}(a+bx)^2}{2b^2} - \frac{d\text{Subst}\left(\int x^2\,\text{FresnelS}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^2} - \frac{(2(bc-ad))\text{Subst}\left(\int x\,\text{FresnelS}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^2}$$

$$= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^2\pi} + \frac{(bc-ad)(a+bx)\,\text{FresnelS}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelS}(a+bx)^2}{2b^2} - \frac{d\text{Subst}\left(\int x\sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{2b^2\pi} - \frac{d\text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\text{FresnelS}(x)\,dx,x,a+bx\right)}{b^2\pi} - \frac{(bc-ad)\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b^2\pi}$$

$$= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^2\pi} - \frac{d\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{2b^2\pi} + \frac{(bc-ad)(a+bx)\,\text{FresnelS}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelS}(a+bx)^2}{2b^2} - \frac{(bc-ad)\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} + \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{d\text{Subst}\left(\int \sin(\pi x)\,dx,x,(a+bx)^2\right)}{4b^2\pi}$$

$$= \frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2} + \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b^2\pi} - \frac{d\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{2b^2\pi} + \frac{(bc-ad)(a+bx)\,\text{FresnelS}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelS}(a+bx)^2}{2b^2} - \frac{(bc-ad)\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} + \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi}$$

Mathematica [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx = \int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx$$

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x]^2,x]
[Out] Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

Maple [F]

$$\int (dx+c)\,\text{FresnelS}\left(bx+a\right)^2\,dx$$

```
[In] int((d*x+c)*FresnelS(b*x+a)^2,x)
[Out] int((d*x+c)*FresnelS(b*x+a)^2,x)
```

Fricas [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx = \int (dx+c)\,\text{S}\left(bx+a\right)^2\,dx$$

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

Sympy [F]

$$\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx = \int (c+dx) \, S^2(a+bx) \, dx$$

```
[In] integrate((d*x+c)*fresnels(b*x+a)**2,x)
[Out] Integral((c + d*x)*fresnels(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx = \int (dx+c) \, \mathrm{S}\left(bx+a\right)^2 \, dx$$

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx = \int (dx+c) \, \mathrm{S}\left(bx+a\right)^2 \, dx$$

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx = \int \operatorname{FresnelS}(a+b\,x)^2 \, (c+d\,x) \, dx$$

```
[In] int(FresnelS(a + b*x)^2*(c + d*x),x)
[Out] int(FresnelS(a + b*x)^2*(c + d*x), x)
```

3.51 $\int \text{FresnelS}(a+bx)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 70

$$\int \text{FresnelS}(a+bx)^2 \, dx = \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b\pi} + \frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}$$

```
[Out] 2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b/Pi+(b*x+a)*FresnelS(b*x+a)^2/b-1/
2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6555, 6587, 3432}

$$\int \text{FresnelS}(a+bx)^2 \, dx = \frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b} + \frac{2\,\text{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[In] Int[FresnelS[a + b*x]^2,x]
```

```
[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[
a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - 2\int (a+bx)\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx$$

$$= \frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int x\,\text{FresnelS}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b}$$

$$= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b\pi} + \frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx, x, a+bx\right)}{b\pi}$$

$$= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b\pi} + \frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \text{FresnelS}(a+bx)^2\,dx$$
$$= \frac{4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx) + 2\pi(a+bx)\,\text{FresnelS}(a+bx)^2 - \sqrt{2}\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{2b\pi}$$

```
[In] Integrate[FresnelS[a + b*x]^2,x]
```

```
[Out] (4*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x] + 2*Pi*(a + b*x)*FresnelS[a +
b*x]^2 - Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)])/(2*b*Pi)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\mathrm{FresnelS}(bx+a)^2(bx+a)+\frac{2\,\mathrm{FresnelS}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60
default	$\frac{\mathrm{FresnelS}(bx+a)^2(bx+a)+\frac{2\,\mathrm{FresnelS}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60

```
[In] int(FresnelS(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x+a)^2*(b*x+a)+2*FresnelS(b*x+a)/Pi*cos(1/2*Pi*(b*x+a)^2)-1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int \mathrm{FresnelS}(a+bx)^2\,dx$$
$$=\frac{4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)\mathrm{S}\left(bx+a\right)+2\left(\pi b^2x+\pi ab\right)\mathrm{S}\left(bx+a\right)^2-\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{2\,\pi b^2}$$

```
[In] integrate(fresnel_sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)*fresnel_sin(b*x + a) +
 2*(pi*b^2*x + pi*a*b)*fresnel_sin(b*x + a)^2 - sqrt(2)*sqrt(b^2)*fresnel_s
in(sqrt(2)*sqrt(b^2)*(b*x + a)/b))/(pi*b^2)
```

Sympy [F]

$$\int \mathrm{FresnelS}(a+bx)^2\,dx=\int S^2(a+bx)\;dx$$

```
[In] integrate(fresnels(b*x+a)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)**2, x)
```

Maxima [F]

$$\int \text{FresnelS}(a+bx)^2\,dx = \int \text{S}\left(bx+a\right)^2\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)^2, x)
```

Giac [F]

$$\int \text{FresnelS}(a+bx)^2\,dx = \int \text{S}\left(bx+a\right)^2\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(a+bx)^2\,dx = \int \text{FresnelS}(a+b\,x)^2\,dx$$

```
[In] int(FresnelS(a + b*x)^2,x)
[Out] int(FresnelS(a + b*x)^2, x)
```

3.52 $\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)^2}{c+dx}, x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)^2/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx$$

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx$$

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x),x]
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)^{2}}{dx+c}dx$$

```
[In] int(FresnelS(b*x+a)^2/(d*x+c),x)
[Out] int(FresnelS(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{S}\left(bx+a\right)^{2}}{dx+c}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx=\int \frac{S^{2}(a+bx)}{c+dx}\,dx$$

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c),x)
[Out] Integral(fresnels(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{FresnelS}(a+b\,x)^2}{c+d\,x}\,dx$$

```
[In] int(FresnelS(a + b*x)^2/(c + d*x),x)
[Out] int(FresnelS(a + b*x)^2/(c + d*x), x)
```

3.53 $\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx = \int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx$$

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx$$

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

```
[In] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
[Out] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{S^{2}(a+bx)}{\left(c+dx\right)^{2}}\,dx$$

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(fresnels(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{FresnelS}\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(FresnelS(a + b*x)^2/(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)^2/(c + d*x)^2, x)
```

3.54 $\int x^2 \text{FresnelS}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 231

$$\begin{aligned}
&\int x^2 \text{FresnelS}(d(a+b\log(cx^n)))\,dx \\
&= \left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) \\
&\quad+\left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) \\
&\quad+\frac{1}{3}x^3\,\text{FresnelS}(d(a+b\log(cx^n)))
\end{aligned}$$

```
[Out] (1/12-1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+I
*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+(1/12-1/1
2*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*d^2
*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*FresnelS(
d*(a+b*ln(c*x^n)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{6606, 4713, 2314, 2308, 2266, 2235, 2236\}$

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$
$$= \left(\frac{1}{12}-\frac{i}{12}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\left(\frac{1}{12}-\frac{i}{12}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\frac{1}{3}x^3\operatorname{FresnelS}(d(a+b\log(cx^n)))$$

```
[In] Int[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 - I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) + ((1/12 - I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n])])/3
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{3}(bdn)\int x^2\sin\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)\,dx\\
&= \frac{1}{3}x^3\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{6}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx\\
&\quad+\frac{1}{6}(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx\\
&= \frac{1}{3}x^3\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{6}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{2-iabd^2n\pi}\,dx\\
&\quad+\frac{1}{6}\left(ibdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{2+iabd^2n\pi}\,dx
\end{aligned}
$$

$$= \frac{1}{3}x^3 \operatorname{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{6}\left(ibdx^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(3-iabd^2n\pi)x}{n} - \frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right) + \frac{1}{6}\left(ibdx^3(cx^n)^{iabd^2\pi-\frac{3+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(3+iabd^2n\pi)x}{n} + \frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right)$$

$$= \frac{1}{3}x^3 \operatorname{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{6}\left(ibde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{i\left(\frac{3-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \right. + \frac{1}{6}\left(ibde^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{iabd^2\pi-\frac{3+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{i\left(\frac{3+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \right.$$

$$= \left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \left(\frac{1}{12} - \frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \frac{1}{3}x^3\operatorname{FresnelS}(d(a+b\log(cx^n)))$$

Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.38

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \frac{1}{12}x^3\left(4\operatorname{FresnelS}(d(a+b\log(cx^n))) + \sqrt[4]{-1}\sqrt{2}e^{\frac{1}{2}\left(-\frac{6a}{bn}-\frac{9i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}(cx^n)^{-3/n}\left(e^{\frac{9i}{b^2d^2n^2\pi}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}\right.}{}\right.\right.\right.$$

```
[In] Integrate[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
```

```
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(E^((9*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

Maple [F]

$$\int x^2 \,\mathrm{FresnelS}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

```
[In] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(187) = 374$.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.94

$$\begin{aligned}\int x^2 \,\mathrm{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx &= \frac{1}{3}\,x^3\,\mathrm{S}\left(bd\log\left(cx^n\right)+ad\right)\\
&\quad -\frac{1}{6}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}-\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&\quad +\frac{1}{6}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}+\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&\quad -\frac{1}{6}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}-\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&\quad -\frac{1}{6}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}+\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\end{aligned}$$

```
[In] integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*fresnel_sin(b*d*log(c*x^n) + a*d) - 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-
3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^
2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^2*d^2*n^2)/(pi*
b^2*d^2*n^2)) + 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) + 9/2
*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(
c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*pi*sqrt(
b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_s
in((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(
```

```
b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n -
3*a/(b*n) + 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi
*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
```

Sympy [F]

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int x^2 S(ad+bd\log(cx^n))\,dx$$

```
[In] integrate(x**2*fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**2*fresnels(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int x^2\,\mathrm{S}((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x^2*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int x^2\,\mathrm{S}((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int x^2\,\mathrm{FresnelS}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

```
[In] int(x^2*FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*FresnelS(d*(a + b*log(c*x^n))), x)
```

3.55 $\int x \,\text{FresnelS}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 227

$$\begin{aligned}
&\int x \,\text{FresnelS}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx \\
&= \left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) \\
&\quad+\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) \\
&\quad+\frac{1}{2}x^2\,\text{FresnelS}\left(d\left(a+b\log\left(cx^n\right)\right)\right)
\end{aligned}$$

```
[Out] (1/8-1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(2
/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))+(1/8-
1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/P
i^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2*Fre
snelS(d*(a+b*ln(c*x^n)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used

= {6606, 4713, 2314, 2308, 2266, 2235, 2236}

$$\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$
$$= \left(\frac{1}{8}-\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\left(\frac{1}{8}-\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{-\frac{2(\pi abd^2n+i)}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\frac{1}{2}x^2\operatorname{FresnelS}(d(a+b\log(cx^n)))$$

```
[In] Int[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 - I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) + ((1/8 - I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelS[d*(a + b*Log[c*x^n])])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-\frac{1}{2}(bdn)\int x\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-\frac{1}{4}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx\\
&\quad+\frac{1}{4}(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-\frac{1}{4}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)x^{1-iabd^2n\pi}\,dx\\
&\quad+\frac{1}{4}\left(ibdnx^{-iabd^2n\pi}(cx^n)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)x^{1+iabd^2n\pi}\,dx
\end{aligned}
$$

$$= \frac{1}{2}x^2 \operatorname{FresnelS}(d(a + b\log(cx^n))) - \frac{1}{4}\left(ibdx^2(cx^n)^{-iabd^2\pi - \frac{2-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(2-iabd^2n\pi)x}{n} - \frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right) + \frac{1}{4}\left(ibdx^2(cx^n)^{iabd^2\pi - \frac{2+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{1}{2}ia^2d^2\pi + \frac{(2+iabd^2n\pi)x}{n} + \frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right)$$

$$= \frac{1}{2}x^2 \operatorname{FresnelS}(d(a + b\log(cx^n))) - \frac{1}{4}\left(ibde^{-\frac{2(i+abd^2n\pi)}{b^2d^2n^2\pi}}x^2(cx^n)^{-iabd^2\pi - \frac{2-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{i\left(\frac{2-iabd^2n\pi}{n} - ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \mathrm{lo}\right. + \frac{1}{4}\left(ibde^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{iabd^2\pi - \frac{2+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{i\left(\frac{2+iabd^2n\pi}{n} + ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \mathrm{log}\right.$$

$$= \left(\frac{1}{8} - \frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n} + iabd^2\pi + ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \left(\frac{1}{8} - \frac{i}{8}\right)e^{-\frac{2(i+abd^2n\pi)}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n} - iabd^2\pi - ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \frac{1}{2}x^2\operatorname{FresnelS}(d(a + b\log(cx^n)))$$

Mathematica [A] (verified)

Time = 4.34 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.41

$$\int x \operatorname{FresnelS}(d(a + b\log(cx^n)))\,dx = \frac{1}{8}x^2\left(4\operatorname{FresnelS}(d(a + b\log(cx^n))) + \sqrt[4]{-1}\sqrt{2}e^{-\frac{2a}{bn} - \frac{2i}{b^2d^2n^2\pi} - \frac{1}{2}ia^2d^2\pi + iabd^2\pi(n\log(x) - \log(cx^n)) - \frac{1}{2}ib^2d^2\pi(-n\log(x) + \log(cx^n))^2}(cx^n)^{-2/n}\left(e^{\frac{4i}{b^2d^2n^2\pi}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\right.}{}\right.\right.\right.$$

```
[In] Integrate[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(E^((4*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
```

```
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

Maple [F]

$$\int x \operatorname{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

```
[In] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
[Out] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.97

$$\begin{aligned}\int x \operatorname{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx =& -\frac{1}{4}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\,\log(c)}{n}-\frac{2\,a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\&+\frac{1}{4}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\,\log(c)}{n}-\frac{2\,a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\&-\frac{1}{4}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\,\log(c)}{n}-\frac{2\,a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\&-\frac{1}{4}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\,\log(c)}{n}-\frac{2\,a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\&+\frac{1}{2}\,x^2\,\mathrm{S}\left(bd\log\left(cx^n\right)+ad\right)\end{aligned}$$

```
[In] integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] -1/4*I*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^
2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
 + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/4*I*pi*sqrt(b^2*d^2*n^2)*e^
(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^
2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi*
b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(
pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
 pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*
```

```
d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi
*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*x^2*fresnel_sin(b*d*log(c*x^n) + a*d)
```

Sympy [F]

$$\int x\,\mathrm{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int xS(ad+bd\log\left(cx^n\right))\,dx$$

```
[In] integrate(x*fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*fresnels(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\,\mathrm{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\,\mathrm{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\,\mathrm{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\,\mathrm{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\,\mathrm{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\,\mathrm{FresnelS}(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

```
[In] int(x*FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(x*FresnelS(d*(a + b*log(c*x^n))), x)
```

3.56 $\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 214

$$\begin{aligned}
&\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx \\
&=\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) \\
&\quad+\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) \\
&\quad+x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)
\end{aligned}$$

```
[Out] (1/4-1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d
/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+(1/4-1/4*I)*
x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/
exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelS(d*(a+b*ln(c*
x^n)))
```

Rubi [A] (verified)

Time = 0.24 (sec), antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used

= {6603, 4711, 2312, 2308, 2266, 2235, 2236}

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}\,e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}\,e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)$$

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) + ((1/4 - I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelS[d*(a + b*Log[c*x^n])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2312

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Dist[(c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log[
```

```
F]), Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2
), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[F
]]
```

Rule 4711

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[I
/2, Int[E^((-I)*d*(a + b*Log[c*x^n])^2), x], x] - Dist[I/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 6603

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelS[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Sin[(Pi/2)*(d*(a +
 b*Log[c*x^n]))^2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-(bdn)\int\sin\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)\,dx\\
&= x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{2}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx\\
&\quad+\frac{1}{2}(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx\\
&= x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{2}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-iabd^2n\pi}\,dx\\
&\quad+\frac{1}{2}\left(ibdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{iabd^2n\pi}\,dx\\
&= x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\\
&\quad-\frac{1}{2}\left(ibdx\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(1-iabd^2n\pi)\,x}{n}\right.\right.\\
&\qquad\left.\left.-\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log\left(cx^n\right)\right)\\
&\quad+\frac{1}{2}\left(ibdx\left(cx^n\right)^{iabd^2\pi-\frac{1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(1+iabd^2n\pi)\,x}{n}\right.\right.\\
&\qquad\left.\left.+\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log\left(cx^n\right)\right)
\end{aligned}$$

$$= x\,\text{FresnelS}\,(d(a+b\log(cx^n)))$$
$$-\frac{1}{2}\left(ibde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{1-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log$$
$$+\frac{1}{2}\left(ibde^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{iabd^2\pi-\frac{1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{1+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log$$
$$=\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)$$
$$+\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)$$
$$+x\,\text{FresnelS}\,(d(a+b\log(cx^n)))$$

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int\text{FresnelS}\,(d(a+b\log(cx^n)))\,dx = x\,\text{FresnelS}\,(d(a+b\log(cx^n)))$$
$$+\frac{\sqrt[4]{-1}e^{\frac{1}{2}\left(-\frac{2a}{bn}-\frac{i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}x(cx^n)^{-1/n}\left(e^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\right.}{}\right.\right.}{}$$

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1
/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]
+ I*Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

Maple [F]

$$\int \operatorname{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

[In] int(FresnelS(d*(a+b*ln(c*x^n))),x)

[Out] int(FresnelS(d*(a+b*ln(c*x^n))),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(176) = 352$.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.08

$$\begin{aligned}\int \operatorname{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx = & -\frac{1}{2}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\ & +\frac{1}{2}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\ & -\frac{1}{2}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\ & -\frac{1}{2}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\ & +x\operatorname{S}\left(bd\log\left(cx^{n}\right)+ad\right)\end{aligned}$$

[In] integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] -1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + x*fresnel_sin(b*d*log(c*x^n) + a*d)

Sympy [F]

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int S(d(a+b\log\left(cx^n\right)))\,dx$$

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral(fresnels(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{FresnelS}(d\,(a+b\,\ln\left(c\,x^n\right)))\,dx$$

```
[In] int(FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(FresnelS(d*(a + b*log(c*x^n))), x)
```

3.57 $\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)}{bdn\pi}+\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] cos(1/2*d^2*Pi*(a+b*ln(c*x^n))^2)/b/d/n/Pi+FresnelS(d*(a+b*ln(c*x^n)))*(a+b
*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6553}

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\frac{\cos\left(\frac{1}{2}\pi d^2(a+b\log\left(cx^n\right))^2\right)}{\pi bdn}+\frac{\left(a+b\log\left(cx^n\right)\right)\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{bn}$$

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*
x^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \text{FresnelS}(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\text{Subst}\left(\int \text{FresnelS}(x)\,dx, x, ad+bd\log\left(cx^n\right)\right)}{bdn}\\
&= \frac{\cos\left(\frac{1}{2}\pi\left(ad+bd\log\left(cx^n\right)\right)^2\right)}{bdn\pi}+\frac{\text{FresnelS}\left(ad+bd\log\left(cx^n\right)\right)\left(a+b\log\left(cx^n\right)\right)}{bn}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 164 vs. $2(65) = 130$.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\begin{aligned}
\int \frac{\text{FresnelS}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\cos\left(\frac{1}{2}a^2d^2\pi\right)\cos\left(abd^2\pi\log\left(cx^n\right)+\frac{1}{2}b^2d^2\pi\log^2\left(cx^n\right)\right)}{bdn\pi}\\
&+\frac{a\,\text{FresnelS}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&+\frac{\text{FresnelS}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\log\left(cx^n\right)}{n}\\
&-\frac{\sin\left(\frac{1}{2}a^2d^2\pi\right)\sin\left(abd^2\pi\log\left(cx^n\right)+\frac{1}{2}b^2d^2\pi\log^2\left(cx^n\right)\right)}{bdn\pi}
\end{aligned}$$

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Cos[(a^2*d^2*Pi)/2]*Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/
2])/(b*d*n*Pi) + (a*FresnelS[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelS[d*(a
+ b*Log[c*x^n])]*Log[c*x^n])/n - (Sin[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\text{FresnelS}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))+\frac{\cos\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{ndb}$	63
default	$\frac{\text{FresnelS}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))+\frac{\cos\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{ndb}$	63

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/d/b*(FresnelS(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))+1/Pi*cos(1/2*Pi*(a
*d+b*d*ln(c*x^n))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.83

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(\pi bdn\log\left(x\right)+\pi bd\log\left(c\right)+\pi ad)\,\text{S}\left(bd\log\left(cx^n\right)+ad\right)+\cos\left(\frac{1}{2}\,\pi b^2d^2n^2\log\left(x\right)^2+\pi b^2d^2n\log\left(c\right)\log\left(x\right)+\right.}{\pi bdn}$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] ((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*fresnel_sin(b*d*log(c*x^n) + a*
d) + cos(1/2*pi*b^2*d^2*n^2*log(x)^2 + pi*b^2*d^2*n*log(c)*log(x) + 1/2*pi*
b^2*d^2*log(c)^2 + pi*a*b*d^2*n*log(x) + pi*a*b*d^2*log(c) + 1/2*pi*a^2*d^2
))/(pi*b*d*n)
```

Sympy [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \int \frac{S(ad+bd\log\left(cx^n\right))}{x}\,dx$$

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(b\log\left(cx^n\right)+a)d\,\text{S}\left((b\log\left(cx^n\right)+a)d\right)+\frac{\cos\left(\frac{1}{2}\,\pi b^2d^2\log(cx^n)^2+\pi abd^2\log(cx^n)+\frac{1}{2}\,\pi a^2d^2\right)}{\pi}}{bdn}$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*fresnel_sin((b*log(c*x^n) + a)*d) + cos(1/2*pi*b^2*d^
2*log(c*x^n)^2 + pi*a*b*d^2*log(c*x^n) + 1/2*pi*a^2*d^2)/pi)/(b*d*n)
```

Giac [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\int \frac{\text{S}\left((b\log\left(cx^n\right)+a)d\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\int \frac{\text{FresnelS}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x}\,dx$$

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x,x)
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x, x)
```

3.58 $\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 217

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}$$

```
[Out] (1/4-1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2*
I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x+(1/4-1/4*I)*ex
p(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*a*b
*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelS(d*(a+b*ln(c*x^n)))
/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{6606, 4713, 2314, 2308, 2266, 2235, 2236\}$

$$\int \frac{\text{FresnelS}\,(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}\,e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}$$
$$+ \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}\,e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x}$$
$$- \frac{\text{FresnelS}\,(d(a+b\log(cx^n)))}{x}$$

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 - I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x + ((1/4 - I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+(bdn)\int\frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)}{x^2}\,dx\\
&= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+\frac{1}{2}(ibdn)\int\frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx\\
&\quad-\frac{1}{2}(ibdn)\int\frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx\\
&= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+\frac{1}{2}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-2-iabd^2n\pi}\,dx\\
&\quad-\frac{1}{2}\left(ibdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-2+iabd^2n\pi}\,dx
\end{aligned}
$$

$$= -\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x} + \frac{\left(ibd(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(-1-iabd^2n\pi)x}{n}-\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(\right.}{2x} - \frac{\left(ibd(cx^n)^{iabd^2\pi-\frac{-1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(-1+iabd^2n\pi)x}{n}+\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(cx^n\right.}{2x}$$

$$= -\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x} + \frac{\left(ibde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{-1-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx^n)\right)}{2x} - \frac{\left(ibde^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{iabd^2\pi-\frac{-1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{-1+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx^n)\right)}{2x}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}$$

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\sqrt[4]{-1}\sqrt{2}e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\left(i\text{erfi}\left(\frac{(-1)^{3/4}(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+e^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+abd^2n\pi+b^2d^2n\pi\log(c\right.}{bdn\sqrt{\pi}}\right.}{4x}$$

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(
-1)*(I*Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d
*n*Sqrt[2*Pi])] + E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*P
i + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelS[d*(a + b*Log[
c*x^n])])/x
```

Maple [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)

[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(177) = 354$.

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.05

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx$$
$$= \frac{-i\,\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\,\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{}{2}\right.}}{}$$

[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")

```
[Out] 1/2*(-I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^
2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
 + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x*e^(log
(c)/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x
) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n
^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2
))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
+ I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)
/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) +
 pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)
) - 2*fresnel_sin(b*d*log(c*x^n) + a*d))/x
```

Sympy [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx = \int \frac{S(ad+bd\log\left(cx^{n}\right))}{x^{2}}\,dx$$

[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**2,x)

[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**2, x)

Maxima [F]

$$\int \frac{\text{FresnelS}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{S}\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\text{FresnelS}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{S}\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(d\left(a + b\,\ln\left(c\,x^n\right)\right))}{x^2}\,dx$$

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x^2,x)
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x^2, x)
```

3.59 $\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 228

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}$$

```
[Out] (1/8-1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/2
+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2+(1/8-1/
8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n
))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*FresnelS(d*
(a+b*ln(c*x^n)))/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{6606, 4713, 2314, 2308, 2266, 2235, 2236\}$

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(cx^n\right)^{2/n} e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(cx^n\right)^{2/n} e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2} - \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}$$

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 - I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 + ((1/8 - I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}(bdn)\int\frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)}{x^3}\,dx\\
&= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{4}(ibdn)\int\frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx\\
&\quad-\frac{1}{4}(ibdn)\int\frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx\\
&= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{4}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\quad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-3-iabd^2n\pi}\,dx\\
&\quad-\frac{1}{4}\left(ibdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi\right.\\
&\quad\left.+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-3+iabd^2n\pi}\,dx
\end{aligned}
$$

$$= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2} + \frac{\left(ibd(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(-2-iabd^2n\pi)x}{n}-\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(\right.}{4x^2} - \frac{\left(ibd(cx^n)^{iabd^2\pi-\frac{-2+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(-2+iabd^2n\pi)x}{n}+\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(cx^n\right.}{4x^2}$$

$$= -\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2} + \frac{\left(ibde^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{-2-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log\left(cx^n\right)\right.}{4x^2} - \frac{\left(ibde^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{iabd^2\pi-\frac{-2+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{-2+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log\left(cx^n\right)\right)}{4x^2}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}$$

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.88

$$\int\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = -\frac{\sqrt[4]{-1}e^{\frac{2\left(\frac{an}{b}-\frac{i}{b^2d^2\pi}+n(-n\log(x)+\log(cx^n))\right)}{n^2}}\left(i\,\text{erfi}\left(\frac{(-1)^{3/4}(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+e^{\frac{4i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\sqrt[4]{-1}(2i+abd^2n\right.\right.}{bdn}\right.}{4\sqrt{2}} - \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}$$

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x
^n])))/n^2)*(I*Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[2*Pi])] + E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*
I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt[2]
- FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Maple [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(183) = 366$.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.02

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \frac{-i\,\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\text{C}\left(\frac{\left(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\,\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\right.}}{}$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
[Out] 1/4*(-I*pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(pi*b^2*d^
2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d
^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x^
2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2
*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(
pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I
/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c)
 + pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^
2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((p
i*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d
^2*n^2)/(pi*b^2*d^2*n^2)) - 2*fresnel_sin(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{S(ad+bd\log\left(cx^{n}\right))}{x^{3}}\,dx$$

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**3,x)
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\mathrm{FresnelS}\left(d(a + b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{\mathrm{S}\left(\left(b\log\left(cx^{n}\right) + a\right)d\right)}{x^{3}}\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\mathrm{FresnelS}\left(d(a + b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{\mathrm{S}\left(\left(b\log\left(cx^{n}\right) + a\right)d\right)}{x^{3}}\,dx$$

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{FresnelS}\left(d(a + b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{\mathrm{FresnelS}(d\left(a + b\,\ln\left(c\,x^{n}\right)\right))}{x^{3}}\,dx$$

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x^3,x)
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x^3, x)
```

3.60 $\int (ex)^m \text{FresnelS}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 280

$$\int (ex)^m \text{FresnelS}(d(a+b\log(cx^n)))\,dx = \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{i(1+m)\left(1+m-2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m-iabd^2n\pi-ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{(ex)^{1+m}\text{FresnelS}(d(a+b\log(cx^n)))}{e(1+m)}$$

```
[Out] (1/4-1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)^
m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1
/2))/(1+m)/((c*x^n)^((1+m)/n))+(1/4-1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1+m-
I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m)*(1
+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*
FresnelS(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{6606, 4713, 2314, 2308, 2266, 2235, 2236\}$

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$
$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{m+1}$$
$$+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(-\frac{i(m+1)(-2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-i\pi abd^2n-i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{m+1}$$
$$+\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)}$$

```
[In] Int[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) + ((1/4 - I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelS[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

integral

$$= \frac{(ex)^{1+m}\,\text{FresnelS}\,(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int (ex)^m \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelS}\,(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)} + \frac{(ibdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelS}\,(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{-m+iabd^2n\pi}(ex)^m\,(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)x^{m-iabd^2n\pi}\,dx}{2(1+m)} + \frac{\left(ibdnx^{-m-iabd^2n\pi}(ex)^m\,(cx^n)^{iabd^2\pi}\right)\int \exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)x^{m+iabd^2n\pi}\,dx}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelS}\,(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdx(ex)^m\,(cx^n)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(1+m-iabd^2n\pi)x}{n}-\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\text{lo}}{2(1+m)} + \frac{\left(ibdx(ex)^m\,(cx^n)^{iabd^2\pi-\frac{1+m+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(1+m+iabd^2n\pi)x}{n}+\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log(c}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelS}\,(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibd\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\,(cx^n)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{1+m-iabd^2n\pi}{n}-ib^2d^2\pi x\right)}{2b^2d^2\pi}\right.\right.}{2(1+m)} + \frac{\left(ibd\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\,(cx^n)^{iabd^2\pi-\frac{1+m+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{1+m+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right.\right.}{2(1+m)}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\,(cx^n)^{-\frac{1+m}{n}}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\,(cx^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m-iabd^2n\pi-ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{(ex)^{1+m}\,\text{FresnelS}\,(d(a+b\log(cx^n)))}{e(1+m)}$$

Mathematica [A] (verified)

Time = 3.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int (ex)^m\,\text{FresnelS}\,(d(a+b\log(cx^n)))\,dx = \frac{(ex)^m\left(-\sqrt[4]{-1}\sqrt{2}e^{-\frac{(1+m)\left(i+im+2abd^2n\pi+2b^2d^2n\pi(-n\log(x)+\log(cx^n))\right)}{2b^2d^2n^2\pi}}x^{-m}\left(\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+im+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+\right.\right.}{4(1+m)}$$

```
[In] Integrate[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
[Out] ((e*x)^m*(-(((-1)^(1/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi +
 b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + E^((I*(1 + m)^2)/(b^2*d^2*n^
2*Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]
))/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2
*n*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m)) + 4*x*FresnelS
[d*(a + b*Log[c*x^n])]))/(4*(1 + m))
```

Maple [F]

$$\int (ex)^m \operatorname{FresnelS}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)dx$$

```
[In] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(310) = 620$.

Time = 0.29 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.47

$$\int (ex)^m \operatorname{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{-i\,\pi\sqrt{b^2d^2n^2}e^{\left(m\log(e)-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i\,m^2}{2\,\pi b^2d^2n^2}-\frac{i\,m}{\pi b^2d^2n^2}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\,m+\cdots}{\pi b^2d^2n^2}\right.\cdots}{\cdots}$$

```
[In] integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/
n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi
*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + p
i*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*
d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I*
m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fresn
el_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m -
I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) -
 m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) -
 I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2
*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(
pi*b^2*d^2*n^2)) - pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n
) - log(c)/n - a/(b*n) + 1/2*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2)
+ 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n
*log(c) + pi*a*b*d^2*n - I*m - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 2*x
*e^(m*log(e) + m*log(x))*fresnel_sin(b*d*log(c*x^n) + a*d))/(m + 1)
```

Sympy [F]

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int (ex)^m S(ad+bd\log(cx^n))\,dx$$

```
[In] integrate((e*x)**m*fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*fresnels(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{S}((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate((e*x)^m*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{S}((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate((e*x)^m*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int \operatorname{FresnelS}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m,x)
[Out] int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.61 $\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx = -\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b} + \frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*I*b*exp(c)*x^2*hypergeom([
1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6571, 6511, 6510, 30}

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx = \frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] -1/8*(E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (I/4)*b*E^c*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6510

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*
(Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 6511

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6571

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&\quad+\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&= \frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{Subst}\left(\int x\,dx,x,\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\
&= -\frac{e^c\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)
\end{aligned}
$$

Mathematica [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx$$

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [F]

$$\int e^{c+\frac{i b^2 \pi x^2}{2}} \operatorname{FresnelS}(bx)\, dx$$

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelS}(bx)\, dx = \int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)} \operatorname{S}(bx)\; dx$$

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Sympy [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelS}(bx)\, dx = e^c \int e^{\frac{i\pi b^2x^2}{2}} S(bx)\; dx$$

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelS}(bx)\, dx = \int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)} \operatorname{S}(bx)\; dx$$

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelS}(bx)\,dx = \int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\operatorname{S}(bx)\;dx$$

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelS}(bx)\,dx = \int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\operatorname{FresnelS}(b\,x)\;dx$$

```
[In] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
[Out] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

3.62 $\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\frac{e^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b-1/4*I*b*exp(c)*x^2*hypergeom([
1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6571, 6508, 30, 6513}

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\frac{e^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6508

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*(
Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
 x] && EqQ[d, -b^2]
```

Rule 6513

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6571

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&\quad+\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&= -\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{e^c\text{Subst}\left(\int x\,dx,x,\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\
&= \frac{e^c\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)
\end{aligned}
$$

Mathematica [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx$$

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [F]

$$\int \mathrm{e}^{c-\frac{ib^2\pi x^2}{2}} \operatorname{FresnelS}(bx)\,dx$$

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\mathrm{FresnelS}(bx)\,dx = \int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Sympy [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\mathrm{FresnelS}(bx)\,dx = e^c\int e^{-\frac{i\pi b^2x^2}{2}}S(bx)\,dx$$

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\mathrm{FresnelS}(bx)\,dx = \int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\mathrm{FresnelS}(bx)\,dx = \int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\mathrm{FresnelS}(bx)\,dx = \int \mathrm{e}^{c-\frac{\Pi\,b^2\,x^2\,1\mathrm{i}}{2}}\,\mathrm{FresnelS}(b\,x)\,dx$$

```
[In] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
[Out] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

3.63 $\int \text{FresnelS}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \text{FresnelS}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx = \frac{\cos(c)\,\text{FresnelS}(bx)^2}{2b} + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)$$

```
[Out] 1/2*cos(c)*FresnelS(b*x)^2/b+1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*I
*b*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*x^2*hype
rgeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6577, 6581, 6575, 30}

$$\int \text{FresnelS}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx = -\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\sin(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} + \frac{\cos(c)\,\text{FresnelS}(bx)^2}{2b}$$

```
[In] Int[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]*Sin
[c] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin
[c]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6577

```
Int[FresnelS[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cos(c)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx+\sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx\\
&= \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)\\
&\quad+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{\cos(c)\text{Subst}\left(\int x\,dx,x,\text{FresnelS}(bx)\right)}{b}\\
&= \frac{\cos(c)\,\text{FresnelS}(bx)^2}{2b}+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b}\\
&\quad-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)
\end{aligned}$$

Mathematica [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$$

```
[In] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int \text{FresnelS}\left(bx\right)\sin\left(c+\frac{b^2\pi\,x^2}{2}\right)dx$$

```
[In] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
[Out] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Sympy [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \sin\left(\frac{\pi b^2x^2}{2}+c\right)S(bx)\,dx$$

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

Maxima [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Giac [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \sin\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\,\text{FresnelS}(b\,x)\,dx$$

```
[In] int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
[Out] int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

3.64 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{\text{FresnelS}(bx)^2\sin(c)}{2b}$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*cos(c)*hypergeom([1, 1
],[3/2, 2],-1/2*I*b^2*Pi*x^2)+1/8*I*b*x^2*cos(c)*hypergeom([1, 1],[3/2, 2],
1/2*I*b^2*Pi*x^2)-1/2*FresnelS(b*x)^2*sin(c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6583, 6581, 6575, 30}

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = -\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{\sin(c)\,\text{FresnelS}(bx)^2}{2b}$$

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2
*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6583

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx - \sin(c)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) \\
&\quad + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{\sin(c)\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)}{b} \\
&= \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) \\
&\quad + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{\text{FresnelS}(bx)^2\sin(c)}{2b}
\end{aligned}$$

Mathematica [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]

$$\int \cos\left(c+\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\left(bx\right)dx$$

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Sympy [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(\frac{\pi b^2x^2}{2}+c\right)S(bx)\,dx$$

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

Maxima [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\text{FresnelS}(b\,x)\,dx$$

```
[In] int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
[Out] int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

3.65 $\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{FresnelS}(bx)^3}{3b}$$

```
[Out] 1/3*FresnelS(b*x)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6575, 30}

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{FresnelS}(bx)^3}{3b}$$

```
[In] Int[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^2\,dx, x, \text{FresnelS}(bx)\right)}{b}$$
$$= \frac{\text{FresnelS}(bx)^3}{3b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^3}{3b}$$

```
[In] Integrate[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
[Out] FresnelS[b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^3}{3b}$	12
default	$\frac{\text{FresnelS}(bx)^3}{3b}$	12

```
[In] int(FresnelS(b*x)^2*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
[Out] 1/3*FresnelS(b*x)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{S\left(bx\right)^3}{3\,b}$$

```
[In] integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] 1/3*fresnel_sin(b*x)^3/b
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \operatorname{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \begin{cases} \frac{S^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(fresnels(b*x)**2*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((fresnels(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \operatorname{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\mathrm{S}\,(bx)^3}{3\,b}$$

```
[In] integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] 1/3*fresnel_sin(b*x)^3/b
```

Giac [F]

$$\int \operatorname{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \mathrm{S}\,(bx)^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

```
[In] integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \operatorname{FresnelS}(b\,x)^2\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2), x)
```

3.66 $\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{FresnelS}(bx)^2}{2b}$$

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6575, 30}

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{FresnelS}(bx)^2}{2b}$$

```
[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, \text{FresnelS}(bx))}{b}$$
$$= \frac{\text{FresnelS}(bx)^2}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^2}{2b}$$

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] FresnelS[b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelS}(bx)^2}{2b}$	12

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*FresnelS(b*x)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{S}\,(bx)^2}{2\,b}$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{S}\,(bx)^2}{2\,b}$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Giac [F]

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{S}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.67 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log(\text{FresnelS}(bx))}{b}$$

```
[Out] ln(FresnelS(b*x))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 29}

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log(\text{FresnelS}(bx))}{b}$$

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
[Out] Log[FresnelS[b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \text{FresnelS}(bx)\right)}{b}\\ &= \frac{\log(\text{FresnelS}(bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log(\text{FresnelS}(bx))}{b}$$

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
[Out] Log[FresnelS[b*x]]/b
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\text{FresnelS}(bx))}{b}$	10
default	$\frac{\ln(\text{FresnelS}(bx))}{b}$	10

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x),x,method=_RETURNVERBOSE)
[Out] ln(FresnelS(b*x))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log\left(\text{S}\left(bx\right)\right)}{b}$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="fricas")
[Out] log(fresnel_sin(b*x))/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \begin{cases} \frac{\log\left(S(bx)\right)}{b} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x),x)
[Out] Piecewise((log(fresnels(b*x))/b, Ne(b, 0)), (nan, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log\left(\text{S}\left(bx\right)\right)}{b}$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="maxima")
[Out] log(fresnel_sin(b*x))/b
```

Giac [F]

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \int \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\text{S}\left(bx\right)}\,dx$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelS}\left(b\,x\right)}\,dx$$

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x),x)
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x), x)
```

3.68 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelS}(bx)}$$

```
[Out] -1/b/FresnelS(b*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6575, 30}

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelS}(bx)}$$

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelS[b*x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \text{FresnelS}(bx)\right)}{b}$$
$$= -\frac{1}{b\,\text{FresnelS}(bx)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelS}(bx)}$$

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
[Out] -(1/(b*FresnelS[b*x]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{1}{b\ \text{FresnelS}(bx)}$	12
default	$-\frac{1}{b\ \text{FresnelS}(bx)}$	12

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/b/FresnelS(b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{S}\,(bx)}$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="fricas")
[Out] -1/(b*fresnel_sin(b*x))
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = \begin{cases} -\frac{1}{bS(bx)} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**2,x)
[Out] Piecewise((-1/(b*fresnels(b*x)), Ne(b, 0)), (nan, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{S}\,(bx)}$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] -1/(b*fresnel_sin(b*x))
```

Giac [F]

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = \int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{S}\,(bx)^2}\,dx$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = \int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelS}\,(b\,x)^2}\,dx$$

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2,x)
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2, x)
```

3.69 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

```
[Out] -1/2/b/FresnelS(b*x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 30}

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelS[b*x]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, \text{FresnelS}(bx)\right)}{b}$$
$$= -\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
[Out] -1/2*1/(b*FresnelS[b*x]^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2b\,\text{FresnelS}(bx)^2}$	12
default	$-\frac{1}{2b\,\text{FresnelS}(bx)^2}$	12

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/FresnelS(b*x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2\,b\,\text{S}\,(bx)^2}$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="fricas")
[Out] -1/2/(b*fresnel_sin(b*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = \begin{cases} -\frac{1}{2bS^2(bx)} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**3,x)
[Out] Piecewise((-1/(2*b*fresnels(b*x)**2), Ne(b, 0)), (nan, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2\,b\,\text{S}\left(bx\right)^2}$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="maxima")
[Out] -1/2/(b*fresnel_sin(b*x)^2)
```

Giac [F]

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = \int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{S}\left(bx\right)^3}\,dx$$

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="giac")
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = \int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelS}\left(b\,x\right)^3}\,dx$$

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3,x)
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3, x)
```

3.70 $\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$$

```
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6575, 30}

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^{n+1}}{b(n+1)}$$

```
[In] Int[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^n\,dx, x, \text{FresnelS}(bx)\right)}{b}\\ &= \frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$$

```
[In] Integrate[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$	18
default	$\frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$	18

```
[In] int(FresnelS(b*x)^n*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{S}\left(bx\right)^n \text{S}\left(bx\right)}{bn+b}$$

```
[In] integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] fresnel_sin(b*x)^n*fresnel_sin(b*x)/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \begin{cases} 0 & \text{for } b = 0 \wedge (b = 0 \vee n = -1) \\ \frac{\log(S(bx))}{b} & \text{for } n = -1 \\ \frac{S(bx)S^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

```
[In] integrate(fresnels(b*x)**n*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (log(fresnels(b*x))/b, Eq
(n, -1)), (fresnels(b*x)*fresnels(b*x)**n/(b*n + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{S}(bx)^{n+1}}{b(n+1)}$$

```
[In] integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] fresnel_sin(b*x)^(n + 1)/(b*(n + 1))
```

Giac [F]

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{S}(bx)^n \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int \text{FresnelS}(b\,x)^n \, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) \, dx$$

```
[In] int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

3.71 $\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 232

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^2\pi} + \frac{105\operatorname{FresnelS}(bx)^2}{2b^9\pi^4} - \frac{105x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}$$

```
[Out] 105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2+55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*
x^6*cos(b^2*Pi*x^2)/b^3/Pi^2+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/P
i^3-x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+105/2*FresnelS(b*x)^2/b^9/
Pi^4-105*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelS(b*x)*s
in(1/2*b^2*Pi*x^2)/b^4/Pi^2-40*sin(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*sin(b^2*Pi*
x^2)/b^5/Pi^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6589, 6597, 3460, 3390, 30, 3377, 2717, 2714, 6575}

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{105\operatorname{FresnelS}(bx)^2}{2\pi^4 b^9} + \frac{105x^2}{4\pi^4 b^7} - \frac{7x^6}{12\pi^2 b^3} - \frac{x^7 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{40\sin(\pi b^2 x^2)}{\pi^5 b^9} - \frac{105x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} + \frac{55x^2\cos(\pi b^2 x^2)}{4\pi^4 b^7} + \frac{35x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{5x^4\sin(\pi b^2 x^2)}{2\pi^3 b^5} + \frac{7x^5\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^6\cos(\pi b^2 x^2)}{4\pi^2 b^3}$$

```
[In] Int[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\text{integral} = & -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^2\pi} \\ & + \frac{7\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx}{b^2\pi} + \frac{\int x^7 \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\end{aligned}$$

$$= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{35\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} - \frac{7\int x^5\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} + \frac{\text{Subst}\left(\int x^3\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{105\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^6\pi^3} - \frac{35\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b^5\pi^3} + \frac{3\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} - \frac{7\text{Subst}\left(\int x^2\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{3x^4\sin\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{105\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^8\pi^4} + \frac{105\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^7\pi^4} - \frac{3\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3} - \frac{35\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^5\pi^3} - \frac{7\text{Subst}\left(\int x^2\,dx, x, x^2\right)}{4b^3\pi^2} + \frac{7\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2}$$

$$= -\frac{7x^6}{12b^3\pi^2} + \frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{5x^4\sin\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{105\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)}{b^9\pi^4} - \frac{3\text{Subst}\left(\int \cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^7\pi^4} - \frac{35\text{Subst}\left(\int \cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^7\pi^4} + \frac{105\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^7\pi^4} - \frac{7\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{105\,\text{FresnelS}(bx)^2}{2b^9\pi^4} - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{73\sin(b^2\pi x^2)}{2b^9\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3} - \frac{7\text{Subst}\left(\int\cos(b^2\pi x)\,dx, x, x^2\right)}{2b^7\pi^4}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{105\,\text{FresnelS}(bx)^2}{2b^9\pi^4} - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00

$$\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{105\,\text{FresnelS}(bx)^2}{2b^9\pi^4} - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}$$

```
[In] Integrate[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Maple [F]

$$\int x^{8} \operatorname{FresnelS}\left(b x\right) \sin\left(\frac{b^{2} \pi\, x^{2}}{2}\right)dx$$

```
[In] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx =$$
$$-\frac{2\,\pi^{3} b^{6} x^{6} - 75\,\pi b^{2} x^{2} + 3\,\left(\pi^{3} b^{6} x^{6} - 55\,\pi b^{2} x^{2}\right) \cos\left(\frac{1}{2}\,\pi b^{2} x^{2}\right)^{2} + 6\,\left(\pi^{4} b^{7} x^{7} - 35\,\pi^{2} b^{3} x^{3}\right) \cos\left(\frac{1}{2}\,\pi b^{2} x^{2}\right) \mathrm{S}\left(bx\right)}{6\,\pi^{5} b^{9}}$$

```
[In] integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*pi^3*b^6*x^6 - 75*pi*b^2*x^2 + 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*cos
(1/2*pi*b^2*x^2)^2 + 6*(pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x^2)
*fresnel_sin(b*x) - 315*pi*fresnel_sin(b*x)^2 - 6*(5*(pi^2*b^4*x^4 - 16)*co
s(1/2*pi*b^2*x^2) + 7*(pi^3*b^5*x^5 - 15*pi*b*x)*fresnel_sin(b*x))*sin(1/2*
pi*b^2*x^2))/(pi^5*b^9)
```

Sympy [A] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.30

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \begin{cases} -\frac{x^{7} \cos\left(\frac{\pi b^{2} x^{2}}{2}\right) S\left(bx\right)}{\pi b^{2}} - \frac{x^{6} \sin^{2}\left(\frac{\pi b^{2} x^{2}}{2}\right)}{3\pi^{2} b^{3}} - \frac{5x^{6} \cos^{2}\left(\frac{\pi b^{2} x^{2}}{2}\right)}{6\pi^{2} b^{3}} + \frac{7x^{5} \sin\left(\frac{\pi b^{2} x^{2}}{2}\right) S\left(bx\right)}{\pi^{2} b^{4}} + \frac{5x^{4} \sin\left(\frac{\pi b^{2} x^{2}}{2}\right) \cos\left(\frac{\pi b^{2} x^{2}}{2}\right)}{\pi^{3} b^{5}} + \frac{35x^{3} \cos\left(\right.}{\pi} \\ 0 \end{cases}$$

```
[In] integrate(x**8*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((-x**7*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**6*sin(pi*
b**2*x**2/2)**2/(3*pi**2*b**3) - 5*x**6*cos(pi*b**2*x**2/2)**2/(6*pi**2*b**
3) + 7*x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 5*x**4*sin(pi*
b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) + 35*x**3*cos(pi*b**2*x**2/2)
*fresnels(b*x)/(pi**3*b**6) + 25*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**4*b**7)
 + 40*x**2*cos(pi*b**2*x**2/2)**2/(pi**4*b**7) - 105*x*sin(pi*b**2*x**2/2)*
fresnels(b*x)/(pi**4*b**8) - 80*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi
**5*b**9) + 105*fresnels(b*x)**2/(2*pi**4*b**9), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

```
[In] integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^8*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

```
[In] integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^8*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.72 $\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 216

$$\begin{aligned}\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = {} & \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x\cos(b^2\pi x^2)}{16b^7\pi^4} \\ & - \frac{x^5\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{531\,\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\ & + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\ & - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\ & - \frac{48\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\ & + \frac{6x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{17x^3\sin(b^2\pi x^2)}{8b^5\pi^3}\end{aligned}$$

```
[Out] 24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2+147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^5*co
s(b^2*Pi*x^2)/b^3/Pi^2+24*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^
6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi-48*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^8/Pi^4+6*x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+17/8*x^3*sin
(b^2*Pi*x^2)/b^5/Pi^3-531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6589, 6597, 3472, 30, 3467, 3466, 3433, 6595, 3438}

$$\int x^7 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{531 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4 b^8} + \frac{24x}{\pi^4 b^7} - \frac{3x^5}{5\pi^2 b^3} - \frac{x^6 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{48 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^4 b^8} + \frac{147x\cos\left(\pi b^2 x^2\right)}{16\pi^4 b^7} + \frac{24x^2 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{17x^3\sin\left(\pi b^2 x^2\right)}{8\pi^3 b^5} + \frac{6x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} - \frac{x^5\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

```
[In] Int[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) + (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (531*FresnelC[Sqrt[2]*b*x])/(1
6*Sqrt[2]*b^8*Pi^4) + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3)
 - (x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (48*FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^4*Pi^2) + (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad+\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^2\pi}+\frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}+\frac{6x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad-\frac{24\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2}+\frac{5\int x^4\cos\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2}-\frac{6\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2}
\end{aligned}$$

$$= -\frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^6\pi^3} - \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^2\pi} + \frac{6x^4 \operatorname{FresnelS}(b}{b^4}$$
$$+ \frac{5x^3 \sin(b^2\pi x^2)}{8b^5\pi^3} - \frac{48 \int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)\, dx}{b^6\pi^3} - \frac{15 \int x^2 \sin(b^2\pi x^2)\, dx}{8b^5\pi^3} - \frac{12 \int x^2 \sin(b^2\pi}{b^5\pi^3}$$
$$- \frac{3 \int x^4\, dx}{b^3\pi^2} + \frac{3 \int x^4 \cos(b^2\pi x^2)\, dx}{b^3\pi^2}$$

$$= -\frac{3x^5}{5b^3\pi^2} + \frac{111x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^6\pi^3}$$
$$- \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^2\pi} - \frac{48 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$
$$+ \frac{6x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{17x^3 \sin(b^2\pi x^2)}{8b^5\pi^3} - \frac{15 \int \cos(b^2\pi x^2)\, dx}{16b^7\pi^4}$$
$$- \frac{6 \int \cos(b^2\pi x^2)\, dx}{b^7\pi^4} + \frac{48 \int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^7\pi^4} - \frac{9 \int x^2 \sin(b^2\pi x^2)\, dx}{2b^5\pi^3}$$

$$= -\frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{15 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}$$
$$- \frac{3\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{b^8\pi^4} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^6\pi^3}$$
$$- \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^2\pi} - \frac{48 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$
$$+ \frac{6x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{17x^3 \sin(b^2\pi x^2)}{8b^5\pi^3}$$
$$- \frac{9 \int \cos(b^2\pi x^2)\, dx}{4b^7\pi^4} + \frac{48 \int \left(\frac{1}{2} - \frac{1}{2}\cos(b^2\pi x^2)\right)\, dx}{b^7\pi^4}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{51 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}$$
$$- \frac{3\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{b^8\pi^4} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^6\pi^3}$$
$$- \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^2\pi} - \frac{48 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$
$$+ \frac{6x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{17x^3 \sin(b^2\pi x^2)}{8b^5\pi^3} - \frac{24 \int \cos(b^2\pi x^2)\, dx}{b^7\pi^4}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x \cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5 \cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{51 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}$$
$$- \frac{15\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{b^8\pi^4} + \frac{24x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^6\pi^3}$$
$$- \frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^2\pi} - \frac{48 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$
$$+ \frac{6x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{17x^3 \sin(b^2\pi x^2)}{8b^5\pi^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int x^7 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \frac{-2655\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right) - 160\operatorname{FresnelS}(bx)\left(b^2\pi x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 6(-8+b^4\pi^2x^4)\sin\left(\right.\right.}{160b^8\pi^4}$$

```
[In] Integrate[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24 +
 b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)
/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(960 - 24*b^4*Pi^
2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.47

method	result
default	$\frac{\frac{\operatorname{FresnelS}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^7}-\frac{\frac{3}{5}b^5x^5\pi^2-24bx}{\pi^4}-\frac{3\left(\frac{\pi b^3x^3\sin\left(b^2\pi x\right.}{2}\right.}{}}{}$

```
[In] int(x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*b^5*x^5*Pi^2-8*b*x)-3/Pi^4*(1/2*Pi*b^3*x^3*
sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC
(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(-1/2*Pi*b^5*x^5*c
os(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*
x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int x^7 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx =$$
$$-\frac{56\,\pi^2 b^6 x^5 - 2370\,b^2 x + 20\,(4\,\pi^2 b^6 x^5 - 147\,b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 160\,(\pi^3 b^7 x^6 - 24\,\pi b^3 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) S}{160}$$

```
[In] integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/160*(56*pi^2*b^6*x^5 - 2370*b^2*x + 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(
1/2*pi*b^2*x^2)^2 + 160*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*
fresnel_sin(b*x) + 2655*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x)
- 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) + 24*(pi^2*b^5*x^4 - 8*b)*fresnel_s
in(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [F]

$$\int x^7 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

```
[In] integrate(x**7*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**7*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^7 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^7\,\mathrm{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

```
[In] integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \text{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

[In] integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(x^7*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \text{FresnelS}(b\,x) \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

[In] int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)

[Out] int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)

3.73 $\int x^6 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 248

$$\begin{aligned}\int x^6 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = &-\frac{5x^4}{8b^3\pi^2}+\frac{11\cos(b^2\pi x^2)}{2b^7\pi^4}-\frac{x^4\cos(b^2\pi x^2)}{4b^3\pi^2} \\ &+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\ &-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\ &-\frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} \\ &+\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} \\ &-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} \\ &+\frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{7x^2\sin(b^2\pi x^2)}{4b^5\pi^3}\end{aligned}$$

```
[Out] -5/8*x^4/b^3/Pi^2+11/2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^4*cos(b^2*Pi*x^2)/b^3
/Pi^2+15*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b^2/Pi-15/2*FresnelC(b*x)*FresnelS(b*x)/b^7/Pi^3+15/8*I*x^
2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^5/Pi^3-15/8*I*x^2*hypergeo
m([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/Pi^3+5*x^3*FresnelS(b*x)*sin(1/2*b^
2*Pi*x^2)/b^4/Pi^2+7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6589, 6597, 3460, 3390, 30, 3377, 2718, 6581}

$$\int x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5} - \frac{15\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2\pi^3b^7} - \frac{5x^4}{8\pi^2b^3} - \frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{11\cos\left(\pi b^2x^2\right)}{2\pi^4b^7} + \frac{15x\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{7x^2\sin\left(\pi b^2x^2\right)}{4\pi^3b^5} + \frac{5x^3\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^4\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[In] Int[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) + (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) -
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (15*FresnelC[b*x]*Fresn
elS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, (-1/2*I)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) + (5*x^3*FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^4*Pi^2) + (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\text{integral} = -\frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^2\pi} + \frac{5\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx}{b^2\pi} + \frac{\int x^5 \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$
\begin{aligned}
&= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad - \frac{15\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} \\
&\quad - \frac{5\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} + \frac{\text{Subst}\left(\int x^2\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi} \\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad - \frac{15\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^6\pi^3} - \frac{15\int x\sin\left(b^2\pi x^2\right)\,dx}{2b^5\pi^3} \\
&\quad + \frac{\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2} - \frac{5\text{Subst}\left(\int x\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2} \\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad - \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} \\
&\quad - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} + \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^2\sin\left(b^2\pi x^2\right)}{2b^5\pi^3} \\
&\quad - \frac{\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3} - \frac{15\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^5\pi^3} \\
&\quad - \frac{5\text{Subst}\left(\int x\,dx, x, x^2\right)}{4b^3\pi^2} + \frac{5\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} \\
&= -\frac{5x^4}{8b^3\pi^2} + \frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} \\
&\quad + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} \\
&\quad + \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{7x^2\sin\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{5\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^5\pi^3} \\
&= -\frac{5x^4}{8b^3\pi^2} + \frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} \\
&\quad + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} \\
&\quad + \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{7x^2\sin\left(b^2\pi x^2\right)}{4b^5\pi^3}
\end{aligned}
$$

Mathematica [F]

$$\int x^6 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

```
[In] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int x^6 \,\text{FresnelS}\,(bx) \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

```
[In] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int x^6 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \,\text{S}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

```
[In] integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]

$$\int x^6 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

```
[In] integrate(x**6*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**6*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{FresnelS}(b\,x) \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.74 $\int x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 158

$$\int x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{2x^3}{3b^3\pi^2}-\frac{x^3\cos(b^2\pi x^2)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^2\pi}-\frac{43\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{11x\sin(b^2\pi x^2)}{8b^5\pi^3}$$

```
[Out] -2/3*x^3/b^3/Pi^2-1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+8*cos(1/2*b^2*Pi*x^2)*Fr
esnelS(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+4*x^2*Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3-43/
16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6589, 6597, 3472, 30, 3467, 3432, 6587, 3466}

$$\int x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{43\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6}-\frac{2x^3}{3\pi^2b^3} - \frac{x^4\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{8\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\sin(\pi b^2x^2)}{8\pi^3b^5} + \frac{4x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^3\cos(\pi b^2x^2)}{4\pi^2b^3}$$

```
[In] Int[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]
)/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Fresn
elS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
```

```
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad+\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^2\pi}+\frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}+\frac{4x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad-\frac{8\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2}+\frac{3\int x^2\cos\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2}-\frac{4\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} \\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad+\frac{4x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{3x\sin\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{3\int\sin\left(b^2\pi x^2\right)\,dx}{8b^5\pi^3} \\
&\quad-\frac{4\int\sin\left(b^2\pi x^2\right)\,dx}{b^5\pi^3}-\frac{2\int x^2\,dx}{b^3\pi^2}+\frac{2\int x^2\cos\left(b^2\pi x^2\right)\,dx}{b^3\pi^2} \\
&= -\frac{2x^3}{3b^3\pi^2}-\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\
&\quad-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}-\frac{3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3}-\frac{2\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^6\pi^3} \\
&\quad+\frac{4x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{11x\sin\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{b^5\pi^3} \\
&= -\frac{2x^3}{3b^3\pi^2}-\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\
&\quad-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}-\frac{11\,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} \\
&\quad-\frac{2\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^6\pi^3}+\frac{4x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{11x\sin\left(b^2\pi x^2\right)}{8b^5\pi^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx =$$

$$-\frac{32b^3\pi x^3 + 12b^3\pi x^3 \cos(b^2\pi x^2) + 129\sqrt{2}\operatorname{FresnelS}\left(\sqrt{2}bx\right) + 48\operatorname{FresnelS}(bx)\left((-8 + b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)\right.}{48b^6\pi^3}$$

```
[In] Integrate[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -1/48*(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS
[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] -
 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(b^6*Pi^3)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{\operatorname{FresnelS}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}-\frac{2\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\operatorname{FresnelS}\left(bx\sqrt{2}\right)}{4\pi}\right)}{\pi^2}-\frac{-\frac{\pi b^3x^3\cos(}{2}}{}}{}}{b}$

```
[In] int(x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3-2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx =$$

$$-\frac{24\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 20\,\pi b^4x^3 + 48\,(\pi^2b^5x^4 - 8\,b)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\,(bx) + 129\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right) -}{48\,\pi^3b^7}$$

```
[In] integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/48*(24*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 + 20*pi*b^4*x^3 + 48*(pi^2*b^5*x
^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 129*sqrt(2)*sqrt(b^2)*fres
nel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(16*pi*b^3*x^2*fresnel_sin(b*x) + 11*b^2*
x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

```
[In] integrate(x**5*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^5*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^5*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{FresnelS}(b\,x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

```
[In] int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.75 $\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{3x^2}{4b^3\pi^2} - \frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^2\pi} - \frac{3\operatorname{FresnelS}(bx)^2}{2b^5\pi^2} + \frac{3x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

```
[Out] -3/4*x^2/b^3/Pi^2-1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2-x^3*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/b^2/Pi-3/2*FresnelS(b*x)^2/b^5/Pi^2+3*x*FresnelS(b*x)*sin(1/2
*b^2*Pi*x^2)/b^4/Pi^2+sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6589, 6597, 3460, 2714, 6575, 30, 3377, 2717}

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{3\operatorname{FresnelS}(bx)^2}{2\pi^2b^5} - \frac{3x^2}{4\pi^2b^3} - \frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\sin(\pi b^2x^2)}{\pi^3b^5} + \frac{3x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2\cos(\pi b^2x^2)}{4\pi^2b^3}$$

```
[In] Int[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
```

```
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^2\pi} + \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\
&= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} - \frac{3\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} + \frac{\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi}\\
&= -\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)}{b^5\pi^2} + \frac{\text{Subst}\left(\int \cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} - \frac{3\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2}\\
&= -\frac{3x^2}{4b^3\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{3\,\text{FresnelS}(bx)^2}{2b^5\pi^2} + \frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\sin\left(b^2\pi x^2\right)}{b^5\pi^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{3x^2}{4b^3\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{3\,\text{FresnelS}(bx)^2}{2b^5\pi^2} + \frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\sin\left(b^2\pi x^2\right)}{b^5\pi^3}$$

```
[In] Integrate[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Maple [F]

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

```
[In] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx =$$
$$-\frac{2\,\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) + \pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + \pi b^2 x^2 + 3\,\pi\,\mathrm{S}\left(bx\right)^2 - 2\left(3\,\pi bx\,\mathrm{S}\left(bx\right) + 2\,\cos\left(\frac{1}{2}\,\pi\right.\right.}{2\,\pi^3 b^5}$$

```
[In] integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] -1/2*(2*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + pi*b^2*x^2*cos(
1/2*pi*b^2*x^2)^2 + pi*b^2*x^2 + 3*pi*fresnel_sin(b*x)^2 - 2*(3*pi*b*x*fres
nel_sin(b*x) + 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \begin{cases} -\frac{x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi b^2} - \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^2 b^3} + \frac{3x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi^2 b^4} + \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{3S^2(bx)}{2\pi^2 b^5} & \text{fo} \\ 0 & \text{o} \end{cases}$$

```
[In] integrate(x**4*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((-x**3*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**2*sin(pi*
b**2*x**2/2)**2/(2*pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(pi**2*b**3) +
 3*x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*sin(pi*b**2*x**2/2)
*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnels(b*x)**2/(2*pi**2*b**5), Ne(b
, 0)), (0, True))
```

Maxima [F]

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^4*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^4*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.76 $\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{x}{b^3\pi^2} - \frac{x\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^2\pi} + \frac{2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

```
[Out] -x/b^3/Pi^2-1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2-x^2*cos(1/2*b^2*Pi*x^2)*FresnelS
(b*x)/b^2/Pi+2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+5/8*FresnelC(b*x*
2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6589, 6595, 3438, 3433, 3466}

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{5\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2b^4} - \frac{x}{\pi^2b^3} - \frac{x^2\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x\cos(\pi b^2x^2)}{4\pi^2b^3}$$

```
[In] Int[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -(x/(b^3*Pi^2)) - (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*FresnelC[Sqrt[2]*b*
x])/(4*Sqrt[2]*b^4*Pi^2) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)
 + (2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^2\pi}+\frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad+\frac{2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\int\cos\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2}-\frac{2\int\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad + \frac{2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{2\int\left(\frac{1}{2} - \frac{1}{2}\cos(b^2\pi x^2)\right)dx}{b^3\pi^2} \\
&= -\frac{x}{b^3\pi^2} - \frac{x\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} \\
&\quad + \frac{2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{\int\cos(b^2\pi x^2)\,dx}{b^3\pi^2} \\
&= -\frac{x}{b^3\pi^2} - \frac{x\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} \\
&\quad - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{-2bx(4+\cos(b^2\pi x^2)) + 5\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) - 8\,\text{FresnelS}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{8b^4\pi^2}
\end{aligned}
$$

```
[In] Integrate[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*b*x*(4 + Cos[b^2*Pi*x^2]) + 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 8*Fresnel
S[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\text{FresnelS}(bx)\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}-\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}-\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	115

```
[In] int(x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(b*x/Pi^2-1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))-1/2/Pi*(-1/2/
Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{8\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) + 4\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 6\,b^2 x - 16\,b\,\mathrm{S}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 5\,\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\sqrt{b^2}\,x\right)}{8\,\pi^2 b^5}$$

```
[In] integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] -1/8*(8*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 4*b^2*x*cos(1/2*p
i*b^2*x^2)^2 + 6*b^2*x - 16*b*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2) - 5*sqrt
(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi^2*b^5)
```

Sympy [F]

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

```
[In] integrate(x**3*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3\,\mathrm{S}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

```
[In] integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^3*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^3 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^3\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^3*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^3 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.77 $\int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}$$

```
[Out] -1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/
2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1
/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
/b/Pi
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6589, 6581, 3460, 2718}

$$\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b} + \frac{\text{FresnelC}(bx)\text{FresnelS}(bx)}{2\pi b^3} - \frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\cos(\pi b^2x^2)}{4\pi^2b^3}$$

```
[In] Int[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -1/4*Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - ((I/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b*Pi) + ((I/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rubi steps

integral

$$= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx}{b^2\pi} + \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{4b\pi}$$

$$= -\frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}$$

Mathematica [F]

$$\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

```
[In] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int x^2\,\text{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

```
[In] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^2\,\text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]

$$\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^2\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

```
[In] integrate(x**2*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**2*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

[In] integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)

Giac [F]

$$\int x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

[In] integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

[In] int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)

[Out] int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)

3.78 $\int x \operatorname{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int x \operatorname{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) dx = -\frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \operatorname{FresnelS}(bx)}{b^2 \pi} + \frac{\operatorname{FresnelS}\left(\sqrt{2} bx\right)}{2\sqrt{2} b^2 \pi}$$

```
[Out] -cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6587, 3432}

$$\int x \operatorname{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) dx = \frac{\operatorname{FresnelS}\left(\sqrt{2} bx\right)}{2\sqrt{2} \pi b^2} - \frac{\operatorname{FresnelS}(bx) \cos \left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}$$

```
[In] Int[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*
Sqrt[2]*b^2*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
```

```
/; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}+\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}+\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{-4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)+\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4b^2\pi}$$

```
[In] Integrate[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] (-4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x] + Sqrt[2]*FresnelS[Sqrt[2]*b*x])/(4*b
^2*Pi)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}(bx)}{b\pi}+\frac{\text{FresnelS}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$	46

```
[In] int(x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
[Out] (-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(
1/2))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int x \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{4 b \cos\left(\frac{1}{2} \pi b^2 x^2\right) \mathrm{S}(bx) - \sqrt{2}\sqrt{b^2}\, \mathrm{S}\left(\sqrt{2}\sqrt{b^2} x\right)}{4 \pi b^3}$$

```
[In] integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] -1/4*(4*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - sqrt(2)*sqrt(b^2)*fresnel_
sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]

$$\int x \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x\, \mathrm{S}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

```
[In] integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x\, \mathrm{S}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

```
[In] integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int x \,\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) \, dx$$

```
[In] int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.79 $\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \frac{\text{FresnelS}(bx)^2}{2b}$$

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6575, 30}

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \frac{\text{FresnelS}(bx)^2}{2b}$$

```
[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}(\int x\,dx, x, \text{FresnelS}(bx))}{b} \\ &= \frac{\text{FresnelS}(bx)^2}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^2}{2b}$$

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] FresnelS[b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelS}(bx)^2}{2b}$	12

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*FresnelS(b*x)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{S}\,(bx)^2}{2\,b}$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \text{FresnelS}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) \, dx = \begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) \, dx = \frac{\text{S}(bx)^2}{2b}$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Giac [F]

$$\int \text{FresnelS}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int \text{S}(bx) \sin \left(\frac{1}{2}\pi b^2x^2\right) \, dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int \text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) \,dx$$

```
[In] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.80 $\int \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x} \, dx = \text{Int}\left(\frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x}, x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x} \, dx = \int \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x} \, dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelS}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

3.81 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^2}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{S\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} \, dx = \int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^2} \, dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} \, dx = \int \frac{\text{S}\left(bx\right) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2} \, dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} \, dx = \int \frac{\text{S}\left(bx\right) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2} \, dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)
```

3.82 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = -\frac{b}{4x} + \frac{b\cos\left(b^2\pi x^2\right)}{4x} + \frac{b^2\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} + \frac{1}{2}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)$$

```
[Out] -1/4*b/x+1/4*b*cos(b^2*Pi*x^2)/x-1/2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+
1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^
2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] -1/4*b/x + (b*Cos[b^2*Pi*x^2])/(4*x) + (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sq
rt[2]) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) + (b^2*Pi*Defer[Int][(
Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/2
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b}{4x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} - \frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad + \frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx \\
&= -\frac{b}{4x} + \frac{b\cos\left(b^2\pi x^2\right)}{4x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} \\
&\quad + \frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx + \frac{1}{2}\left(b^3\pi\right)\int\sin\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b}{4x} + \frac{b\cos\left(b^2\pi x^2\right)}{4x} + \frac{b^2\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} + \frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$
\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx
$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$
\int\frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^3}dx
$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^3}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

3.83 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = -\frac{b}{12x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} - \frac{1}{6}b^3\pi^2\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b^3\pi\,\text{Si}\left(b^2\pi x^2\right)$$

```
[Out] -1/12*b/x^2+1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresn
elS(b*x)/x-1/6*b^3*Pi^2*FresnelS(b*x)^2+1/6*b^3*Pi*Si(b^2*Pi*x^2)-1/3*Fresn
elS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6591, 6599, 6575, 30, 3456, 3461, 3378, 3380}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = -\frac{1}{6}\pi^2 b^3\,\text{FresnelS}(bx)^2 - \frac{\pi b^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3} + \frac{b\cos\left(\pi b^2 x^2\right)}{12x^2} + \frac{1}{6}\pi b^3\,\text{Si}\left(b^2\pi x^2\right) - \frac{b}{12x^2}$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -1/12*b/x^2 + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{12x^2} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad + \frac{1}{3}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx \\
&= -\frac{b}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} \\
&\quad - \frac{1}{12}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) + \frac{1}{6}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx \\
&\quad - \frac{1}{3}\left(b^4\pi^2\right)\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{b}{12x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} \\
&\quad + \frac{1}{12}b^3\pi\text{Si}\left(b^2\pi x^2\right) + \frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&\quad - \frac{1}{3}\left(b^3\pi^2\right)\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right) \\
&= -\frac{b}{12x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} \\
&\quad - \frac{1}{6}b^3\pi^2\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx &= -\frac{b}{12x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} \\
&\quad - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} - \frac{1}{6}b^3\pi^2\,\text{FresnelS}(bx)^2 \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)
\end{aligned}$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -1/12*b/x^2 + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^4}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = -\frac{\pi^2b^3x^3\,\text{S}\,(bx)^2-\pi b^3x^3\,\text{Si}\,(\pi b^2x^2)+2\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)-bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+bx+2\,\text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{6\,x^3}$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(pi^2*b^3*x^3*fresnel_sin(b*x)^2 - pi*b^3*x^3*sin_integral(pi*b^2*x^2)
 + 2*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - b*x*cos(1/2*pi*b^2*x
^2)^2 + b*x + 2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2))/x^3
```

Sympy [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^4}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

Maxima [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

3.84 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}+\frac{7b^4\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^2} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}b^4\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -1/24*b/x^3+1/24*b*cos(b^2*Pi*x^2)/x^3-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresn
elS(b*x)/x^2-1/4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-7/48*b^3*Pi*sin(b^2*
Pi*x^2)/x+7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unintegr
able(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -1/24*b/x^3 + (b*Cos[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*FresnelC[Sqrt[2]*b
*x])/(24*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^2) - (F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{24x^3} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{1}{8}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad + \frac{1}{4}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx \\
&= -\frac{b}{24x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^2} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} + \frac{1}{16}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad + \frac{1}{12}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{8}\left(b^4\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&= -\frac{b}{24x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^2} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} \\
&\quad - \frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}\left(b^4\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&\quad + \frac{1}{8}\left(b^5\pi^2\right)\int \cos\left(b^2\pi x^2\right)\,dx + \frac{1}{6}\left(b^5\pi^2\right)\int \cos\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b}{24x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} + \frac{7b^4\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^2} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}\left(b^4\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right) \sin\left(\frac{b^{2}\pi\, x^{2}}{2}\right)}{x^{5}}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2} x^{2}\right)}{x^{5}}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\sin\left(\frac{\pi b^{2} x^{2}}{2}\right) S(bx)}{x^{5}}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

3.85 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} + \frac{1}{24}b^5\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x^3} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2} - \frac{1}{15}b^4\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] -1/40*b/x^4+1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)+1/40*b*cos(b^2*Pi*x^2)/x^4-1/15*b^
2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/5*FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^5-1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/40*b/x^4 + (b*Cos[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x^3) - (FresnelS[b
*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/15
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{40x^4} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx \\
&\quad + \frac{1}{5}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx \\
&= -\frac{b}{40x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x^3} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} \\
&\quad - \frac{1}{20}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) + \frac{1}{30}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad - \frac{1}{15}\left(b^4\pi^2\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x^3} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{60}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad + \frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad - \frac{1}{15}\left(b^4\pi^2\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x^3} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} \\
&\quad - \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2} - \frac{1}{15}\left(b^4\pi^2\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&\quad + \frac{1}{60}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&\quad + \frac{1}{40}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} + \frac{1}{24}b^5\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x^3} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2} - \frac{1}{15}\left(b^4\pi^2\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx=\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^6}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx=\int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 5.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^6}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

3.86 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = & -\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}\\ & -\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{24x^4}\\ & -\frac{7b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)\\ & -\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}\\ & +\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2}-\frac{13b^3\pi\sin\left(b^2\pi x^2\right)}{720x^3}\\ & -\frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)\end{aligned}$$

```
[Out] -1/60*b/x^5+1/96*b^5*Pi^2/x+1/60*b*cos(b^2*Pi*x^2)/x^5-67/1440*b^5*Pi^2*cos
(b^2*Pi*x^2)/x-1/24*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/6*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/x^2-13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3-67/1440*b^6*Pi^3*FresnelS(b*x*2^
(1/2))*2^(1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -1/60*b/x^5 + (b^5*Pi^2)/(96*x) + (b*Cos[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
24*x^4) - (7*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*P
i^3*FresnelS[Sqrt[2]*b*x])/45 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6)
 + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/x, x])/48
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{60x^5} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad + \frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx \\
&= -\frac{b}{60x^5} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{24x^4} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{48}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad + \frac{1}{30}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{24}\left(b^4\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{24x^4} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} \\
&\quad - \frac{13b^3\pi\sin\left(b^2\pi x^2\right)}{720x^3} + \frac{1}{96}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{72}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad + \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{48}\left(b^6\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx
\end{aligned}$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos(b^2\pi x^2)}{60x^5} - \frac{67b^5\pi^2\cos(b^2\pi x^2)}{1440x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{24x^4} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} - \frac{13b^3\pi\sin(b^2\pi x^2)}{720x^3} - \frac{1}{48}(b^6\pi^3)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx - \frac{1}{48}(b^7\pi^3)\int\sin\left(b^2\pi x^2\right)\,dx - \frac{1}{36}(b^7\pi^3)\int\sin\left(b^2\pi x^2\right)\,dx - \frac{1}{45}(2b^7\pi^3)\int\sin\left(b^2\pi x^2\right)\,dx$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos(b^2\pi x^2)}{60x^5} - \frac{67b^5\pi^2\cos(b^2\pi x^2)}{1440x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{24x^4} - \frac{7b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} - \frac{13b^3\pi\sin(b^2\pi x^2)}{720x^3} - \frac{1}{48}(b^6\pi^3)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\text{FresnelS}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^7}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Sympy [N/A]

Not integrable

Time = 11.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^7}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx=\int \frac{S\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx=\int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

3.87 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = &-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}\\ &-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5}\\ &+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x}\\ &+\frac{1}{210}b^7\pi^4\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}\\ &+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3}\\ &-\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{70}b^7\pi^3\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

```
[Out] -1/84*b/x^6+1/420*b^5*Pi^2/x^2+1/84*b*cos(b^2*Pi*x^2)/x^6-1/84*b^5*Pi^2*cos
(b^2*Pi*x^2)/x^2-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5+1/105*b^
6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/210*b^7*Pi^4*FresnelS(b*x)^2-1
/70*b^7*Pi^3*Si(b^2*Pi*x^2)-1/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/105
*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/105*b^3*Pi*sin(b^2*Pi*x^2
)/x^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6591, 6599, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = &\frac{1}{210}\pi^4 b^7\,\text{FresnelS}(bx)^2+\frac{\pi^2 b^5}{420x^2}\\&-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}\\&-\frac{\pi b^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}\\&+\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}-\frac{1}{70}\pi^3 b^7\text{Si}\left(b^2\pi x^2\right)\\&+\frac{\pi^3 b^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x}-\frac{\pi^2 b^5\cos\left(\pi b^2x^2\right)}{84x^2}\\&+\frac{\pi^2 b^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{\pi b^3\sin\left(\pi b^2x^2\right)}{105x^4}-\frac{b}{84x^6}\end{aligned}$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{b}{84x^6} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$$

$$= -\frac{b}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}$$
$$- \frac{1}{28}b\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) + \frac{1}{70}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx$$
$$- \frac{1}{35}\left(b^4\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5}$$
$$- \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3}$$
$$+ \frac{1}{140}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right)$$
$$+ \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) + \frac{1}{210}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx$$
$$- \frac{1}{105}\left(b^6\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5}$$
$$+ \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}$$
$$+ \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}$$
$$+ \frac{1}{420}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right)$$
$$+ \frac{1}{280}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right)$$
$$+ \frac{1}{168}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{210}\left(b^7\pi^3\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx$$
$$+ \frac{1}{105}\left(b^8\pi^4\right)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos(b^2\pi x^2)}{84x^6} - \frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b^3\pi\sin(b^2\pi x^2)}{105x^4} - \frac{1}{420}b^7\pi^3\text{Si}(b^2\pi x^2) - \frac{1}{420}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{280}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{168}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right) + \frac{1}{105}(b^7\pi^4)\,\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos(b^2\pi x^2)}{84x^6} - \frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x} + \frac{1}{210}b^7\pi^4\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b^3\pi\sin(b^2\pi x^2)}{105x^4} - \frac{1}{70}b^7\pi^3\text{Si}(b^2\pi x^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos(b^2\pi x^2)}{84x^6} - \frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x} + \frac{1}{210}b^7\pi^4\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b^3\pi\sin(b^2\pi x^2)}{105x^4} - \frac{1}{70}b^7\pi^3\text{Si}(b^2\pi x^2)$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
```

```
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^8}dx$$

[In] `int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)`

[Out] `int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)`

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$
$$= \frac{\pi^4 b^7 x^7\,\mathrm{S}\,(bx)^2 - 3\,\pi^3 b^7 x^7\,\mathrm{Si}\,(\pi b^2 x^2) + 3\,\pi^2 b^5 x^5 - 5\,(\pi^2 b^5 x^5 - bx)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\,(\pi^3 b^6 x^6 - 3\,\pi b^2 x^2)\,\mathrm{co}}{210\,x^7}$$

[In] `integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")`

```
[Out] 1/210*(pi^4*b^7*x^7*fresnel_sin(b*x)^2 - 3*pi^3*b^7*x^7*sin_integral(pi*b^2
*x^2) + 3*pi^2*b^5*x^5 - 5*(pi^2*b^5*x^5 - b*x)*cos(1/2*pi*b^2*x^2)^2 + 2*(
pi^3*b^6*x^6 - 3*pi*b^2*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 5*b*x -
 2*(2*pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - (pi^2*b^4*x^4 - 15)*fresnel_sin(b*x)
)*sin(1/2*pi*b^2*x^2))/x^7
```

Sympy [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^8}\,dx$$

[In] `integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)`

[Out] `Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)`

Maxima [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Giac [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

3.88 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx &= -\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}\\ &-\frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3}-\frac{853b^8\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}\\ &-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^6}\\ &+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{384x^2}\\ &-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}\\ &+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4}\\ &-\frac{19b^3\pi\sin\left(b^2\pi x^2\right)}{3360x^5}+\frac{853b^7\pi^3\sin\left(b^2\pi x^2\right)}{80640x}\\ &+\frac{1}{384}b^8\pi^4\,\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

```
[Out] -1/112*b/x^7+1/1152*b^5*Pi^2/x^3+1/112*b*cos(b^2*Pi*x^2)/x^7-187/40320*b^5*
Pi^2*cos(b^2*Pi*x^2)/x^3-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6+
1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/8*FresnelS(b*x)*sin(
1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19
/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5+853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x-853/8
0640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx=\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -1/112*b/x^7 + (b^5*Pi^2)/(1152*x^3) + (b*Cos[b^2*Pi*x^2])/(112*x^7) - (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (853*b^8*Pi^4*FresnelC[Sqrt[2]*b*x
])/(40320*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^6) +
(b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(384*x^2) - (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(1
92*x^4) - (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x, x])/384
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{112x^7}-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}-\frac{1}{16}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx\\
&\quad+\frac{1}{8}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx\\
&=-\frac{b}{112x^7}+\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^6}\\
&\quad-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}+\frac{1}{96}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx\\
&\quad+\frac{1}{56}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx-\frac{1}{48}\left(b^4\pi^2\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&=-\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^6}\\
&\quad-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4}\\
&\quad-\frac{19b^3\pi\sin\left(b^2\pi x^2\right)}{3360x^5}+\frac{1}{384}\left(b^5\pi^2\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{240}\left(b^5\pi^2\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx\\
&\quad+\frac{1}{140}\left(b^5\pi^2\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{192}\left(b^6\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos(b^2\pi x^2)}{112x^7} - \frac{187b^5\pi^2\cos(b^2\pi x^2)}{40320x^3} \\
&\quad - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{384x^2} \\
&\quad - \frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} \\
&\quad - \frac{19b^3\pi\sin(b^2\pi x^2)}{3360x^5} - \frac{1}{768}\left(b^7\pi^3\right)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx \\
&\quad - \frac{1}{576}\left(b^7\pi^3\right)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx - \frac{1}{360}\left(b^7\pi^3\right)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx \\
&\quad - \frac{1}{210}\left(b^7\pi^3\right)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos(b^2\pi x^2)}{112x^7} - \frac{187b^5\pi^2\cos(b^2\pi x^2)}{40320x^3} \\
&\quad - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{384x^2} \\
&\quad - \frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} - \frac{19b^3\pi\sin(b^2\pi x^2)}{3360x^5} \\
&\quad + \frac{853b^7\pi^3\sin(b^2\pi x^2)}{80640x} + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&\quad - \frac{1}{384}\left(b^9\pi^4\right)\int\cos\left(b^2\pi x^2\right)\,dx - \frac{1}{288}\left(b^9\pi^4\right)\int\cos\left(b^2\pi x^2\right)\,dx \\
&\quad - \frac{1}{180}\left(b^9\pi^4\right)\int\cos\left(b^2\pi x^2\right)\,dx - \frac{1}{105}\left(b^9\pi^4\right)\int\cos\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos(b^2\pi x^2)}{112x^7} - \frac{187b^5\pi^2\cos(b^2\pi x^2)}{40320x^3} - \frac{853b^8\pi^4\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}} \\
&\quad - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{384x^2} \\
&\quad - \frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} - \frac{19b^3\pi\sin(b^2\pi x^2)}{3360x^5} \\
&\quad + \frac{853b^7\pi^3\sin(b^2\pi x^2)}{80640x} + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^9}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Sympy [N/A]

Not integrable

Time = 36.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^9}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Mupad [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

3.89 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx &= -\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}\\ &\quad-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4}-\frac{5b^9\pi^4\,\text{CosIntegral}\left(b^2\pi x^2\right)}{2016}\\ &\quad-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7}\\ &\quad+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x^3}\\ &\quad-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}\\ &\quad+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}\\ &\quad-\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\sin\left(b^2\pi x^2\right)}{2016x^2}\\ &\quad+\frac{1}{945}b^8\pi^4\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)\end{aligned}$$

```
[Out] -1/144*b/x^8+1/2520*b^5*Pi^2/x^4-5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)+1/144*b*cos
(b^2*Pi*x^2)/x^8-67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/63*b^2*Pi*cos(1/2*
b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b
*x)/x^3-1/9*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelS(b
*x)*sin(1/2*b^2*Pi*x^2)/x^5-11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6+5/2016*b^7*P
i^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b
^2*Pi*x^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
[Out] -1/144*b/x^8 + (b^5*Pi^2)/(2520*x^4) + (b*Cos[b^2*Pi*x^2])/(144*x^8) - (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) - (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(63*x^7) + (b^6*Pi^3*Co
s[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x^3) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(9*x^9) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5) - (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/945
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{144x^8} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}\,dx \\
&\quad + \frac{1}{9}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx \\
&= -\frac{b}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} \\
&\quad - \frac{1}{36}b\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^5}\,dx, x, x^2\right) + \frac{1}{126}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx \\
&\quad - \frac{1}{63}\left(b^4\pi^2\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} \\
&\quad + \frac{1}{252}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&\quad + \frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) + \frac{1}{630}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx \\
&\quad - \frac{1}{315}\left(b^6\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7} \\
&\quad+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x^3}-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} \\
&\quad+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}-\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6} \\
&\quad+\frac{\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^3}\,dx,x,x^2\right)}{1260}+\frac{1}{756}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^3}\,dx,x,x^2\right) \\
&\quad+\frac{1}{432}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^3}\,dx,x,x^2\right)-\frac{\left(b^7\pi^3\right)\int\frac{\sin(b^2\pi x^2)}{x^3}\,dx}{1890} \\
&\quad+\frac{1}{945}\left(b^8\pi^4\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} \\
&\quad-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x^3} \\
&\quad-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}-\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6} \\
&\quad-\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx,x,x^2\right)}{3780}-\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx,x,x^2\right)}{2520} \\
&\quad-\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx,x,x^2\right)}{1512}-\frac{1}{864}\left(b^7\pi^3\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right) \\
&\quad+\frac{1}{945}\left(b^8\pi^4\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} \\
&\quad-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x^3} \\
&\quad-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}-\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6} \\
&\quad+\frac{5b^7\pi^3\sin\left(b^2\pi x^2\right)}{2016x^2}+\frac{1}{945}\left(b^8\pi^4\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&\quad-\frac{\left(b^9\pi^4\right)\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx,x,x^2\right)}{3780}-\frac{\left(b^9\pi^4\right)\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx,x,x^2\right)}{2520} \\
&\quad-\frac{\left(b^9\pi^4\right)\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx,x,x^2\right)}{1512}-\frac{1}{864}\left(b^9\pi^4\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x}\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{5b^9\pi^4\,\text{CosIntegral}\left(b^2\pi x^2\right)}{2016} \\
&\quad - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x^3} \\
&\quad - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} - \frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6} \\
&\quad + \frac{5b^7\pi^3\sin\left(b^2\pi x^2\right)}{2016x^2} + \frac{1}{945}\left(b^8\pi^4\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^{10}}dx$$

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int\frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Sympy [N/A]

Not integrable

Time = 66.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^{10}}\,dx$$

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

3.90 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \text{Int}\left(\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n, x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]
```

```
[Out] Defer[Int][Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]
```

Rubi steps

$$\text{integral} = \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]
[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\left(bx\right)^n dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \text{S}\left(bx\right)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \cos\left(\frac{\pi b^2x^2}{2}\right)S^n(bx)\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)**n,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)**n, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \mathrm{S}\left(bx\right)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \mathrm{S}\left(bx\right)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \text{FresnelS}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

3.91 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 307

$$\begin{aligned}\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = {} & \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40\cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} \\ & - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4} \\ & + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} \\ & + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} \\ & - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} \\ & + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} \\ & - \frac{35x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ & + \frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ & - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

```
[Out] 35/8*x^4/b^5/Pi^3-1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^2
*Pi*x^2)/b^5/Pi^3-105*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+7*x^5*co
s(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)*FresnelS(b*x)/
b^9/Pi^4-105/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7/Pi^4+
105/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/Pi^4-35*x^3*Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^
2)/b^2/Pi-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2*Pi*x^2)/b^3/Pi^
2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6597, 3460, 3390, 30, 3377, 2718, 6589, 6581}

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = -\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7} + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2\pi^4b^9} + \frac{35x^4}{8\pi^3b^5} + \frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{40\cos\left(\pi b^2x^2\right)}{\pi^5b^9} - \frac{105x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8} - \frac{55x^2\sin\left(\pi b^2x^2\right)}{4\pi^4b^7} - \frac{35x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{5x^4\cos\left(\pi b^2x^2\right)}{2\pi^3b^5} + \frac{7x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^6\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - \frac{x^8}{16\pi b}$$

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) + (
5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b
*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (1
05*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^
2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x
^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelS[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin
[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^7 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{7\int x^6 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^2\pi} - \frac{\int x^7 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b\pi} \\
&= \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} + \frac{x^7 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{35\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx}{b^4\pi^2} - \frac{7\int x^5 \sin\left(b^2\pi x^2\right)\, dx}{2b^3\pi^2} \\
&\quad - \frac{\text{Subst}\left(\int x^3 \sin^2\left(\frac{1}{2}b^2\pi x\right)\, dx, x, x^2\right)}{2b\pi} \\
&= \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{35x^3 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^7 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{105\int x^2 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^6\pi^3} \\
&\quad + \frac{35\int x^3 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^5\pi^3} - \frac{7\text{Subst}\left(\int x^2 \sin\left(b^2\pi x\right)\, dx, x, x^2\right)}{4b^3\pi^2} \\
&\quad - \frac{\text{Subst}\left(\int x^3\, dx, x, x^2\right)}{4b\pi} + \frac{\text{Subst}\left(\int x^3 \cos\left(b^2\pi x\right)\, dx, x, x^2\right)}{4b\pi} \\
&= -\frac{x^8}{16b\pi} + \frac{7x^4 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^8\pi^4} \\
&\quad + \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{35x^3 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^7 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^6 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \\
&\quad + \frac{105\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx}{b^8\pi^4} \\
&\quad + \frac{105\int x \sin\left(b^2\pi x^2\right)\, dx}{2b^7\pi^4} - \frac{7\text{Subst}\left(\int x \cos\left(b^2\pi x\right)\, dx, x, x^2\right)}{2b^5\pi^3} \\
&\quad + \frac{35\text{Subst}\left(\int x \sin^2\left(\frac{1}{2}b^2\pi x\right)\, dx, x, x^2\right)}{2b^5\pi^3} - \frac{3\text{Subst}\left(\int x^2 \sin\left(b^2\pi x\right)\, dx, x, x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

$$= -\frac{x^8}{16b\pi} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{35x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7x^2\sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2} + \frac{7\text{Subst}\left(\int\sin(b^2\pi x)\,dx,x,x^2\right)}{2b^7\pi^4} + \frac{105\text{Subst}\left(\int\sin(b^2\pi x)\,dx,x,x^2\right)}{4b^7\pi^4} - \frac{3\text{Subst}\left(\int x\cos(b^2\pi x)\,dx,x,x^2\right)}{2b^5\pi^3} + \frac{35\text{Subst}\left(\int x\,dx,x,x^2\right)}{4b^5\pi^3} - \frac{35\text{Subst}\left(\int x\cos(b^2\pi x)\,dx,x,x^2\right)}{4b^5\pi^3}$$

$$= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{119\cos(b^2\pi x^2)}{4b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{35x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2} + \frac{3\text{Subst}\left(\int\sin(b^2\pi x)\,dx,x,x^2\right)}{2b^7\pi^4} + \frac{35\text{Subst}\left(\int\sin(b^2\pi x)\,dx,x,x^2\right)}{4b^7\pi^4}$$

$$= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40\cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{35x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2}$$

Mathematica [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$$

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]

$$\int x^8 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \text{FresnelS}\,(bx)\,dx$$

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\,(bx)\;dx$$

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Sympy [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\;dx$$

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x**8*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\pi b^2x^2\right) S\left(bx\right)\,dx$$

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\pi b^2x^2\right) S\left(bx\right)\,dx$$

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.92 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 217

$$\begin{aligned}\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = {} & \frac{4x^3}{b^5\pi^3}-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} \\ & -\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4} \\ & +\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} \\ & +\frac{531\,\text{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\ & -\frac{24x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ & +\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ & -\frac{147x\sin\left(b^2\pi x^2\right)}{16b^7\pi^4}+\frac{x^5\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

```
[Out] 4*x^3/b^5/Pi^3-1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3-48*cos(1/2*b
^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+6*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b
^4/Pi^2-24*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^6*FresnelS(b*x)
*sin(1/2*b^2*Pi*x^2)/b^2/Pi-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*sin(b
^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6597, 3472, 30, 3467, 3466, 3432, 6589, 6587}

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{531\,\text{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8}+\frac{4x^3}{\pi^3b^5} + \frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{48\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\sin\left(\pi b^2x^2\right)}{16\pi^4b^7} - \frac{24x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{17x^3\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{6x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^5\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^7}{14\pi b}$$

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
- (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^
2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b
^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{6\int x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^6\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} + \frac{x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{24\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)\,dx}{b^4\pi^2} \\
&\quad - \frac{3\int x^4\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2} - \frac{\int x^6\,dx}{2b\pi} + \frac{\int x^6\cos\left(b^2\pi x^2\right)\,dx}{2b\pi}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x^7}{14b\pi} + \frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} \\
&\quad - \frac{24x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad + \frac{x^5\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{48\int x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^6\pi^3} \\
&\quad - \frac{9\int x^2\cos\left(b^2\pi x^2\right)\,dx}{2b^5\pi^3} + \frac{24\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^5\pi^3} - \frac{5\int x^4\sin\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2} \\
&= -\frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} - \frac{24x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^6\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{9x\sin\left(b^2\pi x^2\right)}{4b^7\pi^4} \\
&\quad + \frac{x^5\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{9\int\sin\left(b^2\pi x^2\right)\,dx}{4b^7\pi^4} + \frac{24\int\sin\left(b^2\pi x^2\right)\,dx}{b^7\pi^4} \\
&\quad - \frac{15\int x^2\cos\left(b^2\pi x^2\right)\,dx}{8b^5\pi^3} + \frac{12\int x^2\,dx}{b^5\pi^3} - \frac{12\int x^2\cos\left(b^2\pi x^2\right)\,dx}{b^5\pi^3} \\
&= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} + \frac{9\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^8\pi^4} + \frac{12\sqrt{2}\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{b^8\pi^4} \\
&\quad - \frac{24x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{147x\sin\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15\int\sin\left(b^2\pi x^2\right)\,dx}{16b^7\pi^4} + \frac{6\int\sin\left(b^2\pi x^2\right)\,dx}{b^7\pi^4} \\
&= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} + \frac{51\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\
&\quad + \frac{15\sqrt{2}\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{b^8\pi^4} - \frac{24x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^6\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{147x\sin\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$$
$$= \frac{896b^3\pi x^3 - 16b^7\pi^3x^7 + 476b^3\pi x^3\cos(b^2\pi x^2) + 3717\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 224\,\text{FresnelS}(bx)\left(6(-8 + b^4\pi\right.}{224b^8\pi}$$

[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]

[Out] (896*b^3*Pi*x^3 - 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*S
qrt[2]*FresnelS[Sqrt[2]*b*x] + 224*FresnelS[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 2
058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)

Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{\text{FresnelS}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7}-\frac{\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos\left(b^2\pi\right.}{2}}{}}{}}{}$

[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)

[Out] (FresnelS(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi*b^
3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*Fre
snelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5*x
^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi*b
*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b^2
*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx =$$
$$-\frac{16\,\pi^3b^8x^7-952\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-420\,\pi b^4x^3-1344\,(\pi^2b^5x^4-8\,b)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)-3717\,\sqrt{2}\sqrt{}}{224\,\pi}$$

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/224*(16*pi^3*b^8*x^7 - 952*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 420*pi*b^4
*x^3 - 1344*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 371
7*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 28*((4*pi^2*b^6*x^5
- 147*b^2*x)*cos(1/2*pi*b^2*x^2) + 8*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel
_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [F]

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^7\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**7*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^7\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^7\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.93 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} + \frac{15\,\text{FresnelS}(bx)^2}{2b^7\pi^3} - \frac{15x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11\sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] 15/4*x^2/b^5/Pi^3-1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+5*x^3*cos(
1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+15/2*FresnelS(b*x)^2/b^7/Pi^3-15*x*F
resnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^2/Pi-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

= {6597, 3460, 3390, 30, 3377, 2717, 6589, 2714, 6575}

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \frac{15\,\text{FresnelS}(bx)^2}{2\pi^3 b^7} + \frac{15x^2}{4\pi^3 b^5} + \frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{11\sin\left(\pi b^2 x^2\right)}{2\pi^4 b^7} - \frac{15x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{7x^2\cos\left(\pi b^2 x^2\right)}{4\pi^3 b^5} + \frac{5x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{x^4\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} - \frac{x^6}{12\pi b}$$

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
```

```
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{5\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{b^4\pi^2} + \frac{x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{15\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)\,dx}{b^4\pi^2} - \frac{5\int x^3 \sin\left(b^2\pi x^2\right)\,dx}{2b^3\pi^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int x^2 \sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b\pi}
\end{aligned}$$

$$= \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{15x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{15 \int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx}{b^6\pi^3} + \frac{15 \int x \sin^2\left(\frac{1}{2}b^2\pi x^2\right) \, dx}{b^5\pi^3} - \frac{5\text{Subst}\left(\int x \sin\left(b^2\pi x\right) \, dx, x, x^2\right)}{4b^3\pi^2} - \frac{\text{Subst}\left(\int x^2 \, dx, x, x^2\right)}{4b\pi} + \frac{\text{Subst}\left(\int x^2 \cos\left(b^2\pi x\right) \, dx, x, x^2\right)}{4b\pi}$$

$$= -\frac{x^6}{12b\pi} + \frac{5x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{15x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15\text{Subst}(\int x \, dx, x, \text{FresnelS}(bx))}{b^7\pi^3} - \frac{5\text{Subst}\left(\int \cos\left(b^2\pi x\right) \, dx, x, x^2\right)}{4b^5\pi^3} + \frac{15\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right) \, dx, x, x^2\right)}{2b^5\pi^3} - \frac{\text{Subst}\left(\int x \sin\left(b^2\pi x\right) \, dx, x, x^2\right)}{2b^3\pi^2}$$

$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} + \frac{15 \,\text{FresnelS}(bx)^2}{2b^7\pi^3} - \frac{15x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5 \sin\left(b^2\pi x^2\right)}{b^7\pi^4} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\text{Subst}\left(\int \cos\left(b^2\pi x\right) \, dx, x, x^2\right)}{2b^5\pi^3}$$

$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} + \frac{15 \,\text{FresnelS}(bx)^2}{2b^7\pi^3} - \frac{15x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11 \sin\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx) \, dx = \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} + \frac{15 \,\text{FresnelS}(bx)^2}{2b^7\pi^3} - \frac{15x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11 \sin\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Maple [F]

$$\int x^6 \cos\left(\frac{b^2\pi x^2}{2}\right) \mathrm{FresnelS}(bx)\,dx$$

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = -\frac{\pi^3 b^6 x^6 - 60\,\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) - 42\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 24\,\pi b^2 x^2 - 90\,\pi\,\mathrm{S}\left(bx\right)^2 - 6\,\left((\pi^2 b^4 x^4\right.}{12\,\pi^4 b^7}$$

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/12*(pi^3*b^6*x^6 - 60*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)
- 42*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 24*pi*b^2*x^2 - 90*pi*fresnel_sin(b
*x)^2 - 6*((pi^2*b^4*x^4 - 22)*cos(1/2*pi*b^2*x^2) + 2*(pi^3*b^5*x^5 - 15*p
i*b*x)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = \begin{cases} -\frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right)\cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{\pi^2 b^4} + \frac{2x^2 \sin^2\left(\frac{\pi b^2}{}\right.}{\pi^3 b^5} \\ 0 \end{cases}$$

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) - x**6*cos(pi*b**2*x**2/2
)**2/(12*pi*b) + x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + x**4*si
n(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*cos(pi*b**2*x
**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*x**2*sin(pi*b**2*x**2/2)**2/(pi**3*b*
*5) + 11*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 15*x*sin(pi*b**2*x**2
/2)*fresnels(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)
/(pi**4*b**7) + 15*fresnels(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\, dx = \int x^6 \cos\left(\frac{1}{2}\pi b^2x^2\right) \mathrm{S}\left(bx\right)\, dx$$

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\, dx = \int x^6 \cos\left(\frac{1}{2}\pi b^2x^2\right) \mathrm{S}\left(bx\right)\, dx$$

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\, dx = \int x^6\, \mathrm{FresnelS}(b\,x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

```
[In] int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.94 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 166

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{43\,\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^3\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] 4*x/b^5/Pi^3-1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+4*x^2*cos(1/2*b^
2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2-8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi
^3+x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x^3*sin(b^2*Pi*x^2)/b^3
/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6597, 3472, 30, 3467, 3466, 3433, 6589, 6595, 3438}

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{43\,\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6} + \frac{4x}{\pi^3b^5} + \frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{11x\cos\left(\pi b^2x^2\right)}{8\pi^3b^5} + \frac{4x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^3\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - \frac{x^5}{10\pi b}$$

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x)/(b^5*Pi^3) - x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (4
3*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(b^4*Pi^2) - (8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3
) + (x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^3*Sin[b^2*Pi*x^2]
)/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{4\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^4 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}
\end{aligned}
$$

$$= \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} + \frac{x^4 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx}{b^4\pi^2} - \frac{2\int x^2 \sin\left(b^2\pi x^2\right)\, dx}{b^3\pi^2} - \frac{\int x^4\, dx}{2b\pi} + \frac{\int x^4 \cos\left(b^2\pi x^2\right)\, dx}{2b\pi}$$

$$= -\frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{8\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^3 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\int \cos\left(b^2\pi x^2\right)\, dx}{b^5\pi^3} + \frac{8\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^5\pi^3} - \frac{3\int x^2 \sin\left(b^2\pi x^2\right)\, dx}{4b^3\pi^2}$$

$$= -\frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{\sqrt{2}b^6\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{8\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^3 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{3\int \cos\left(b^2\pi x^2\right)\, dx}{8b^5\pi^3} + \frac{8\int \left(\frac{1}{2} - \frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\, dx}{b^5\pi^3}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11\,\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{8\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^3 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{4\int \cos\left(b^2\pi x^2\right)\, dx}{b^5\pi^3}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11\,\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} - \frac{2\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{b^6\pi^3} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^4\pi^2} - \frac{8\,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^3 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.76

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \frac{-215\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 80\,\text{FresnelS}(bx)\left(4b^2\pi x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) + \left(-8 + b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx(}{80b^6\pi^3}$$

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*FresnelS[b*x]*(4*b^2*Pi*x^2*Cos[(b
^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(160 - 4*b
^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6*
Pi^3)
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{\text{FresnelS}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{\frac{1}{5}b^5x^5\pi^2-8bx}{2\pi^3}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{\pi}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}-\frac{\pi b^3}{}}{}}{b}$

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(1/2/Pi^3*(1/5*b^5*x^5
*Pi^2-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*
x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*
2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx =$$
$$-\frac{8\,\pi^2b^6x^5-320\,\pi b^3x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)-220\,b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-210\,b^2x+215\,\sqrt{2}\sqrt{b^2}\,\text{C}\left(\sqrt{2}\sqrt{b^2}x\right)}{80\,\pi^3b^7}$$

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/80*(8*pi^2*b^6*x^5 - 320*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)
 - 220*b^2*x*cos(1/2*pi*b^2*x^2)^2 - 210*b^2*x + 215*sqrt(2)*sqrt(b^2)*fres
nel_cos(sqrt(2)*sqrt(b^2)*x) - 40*(pi*b^4*x^3*cos(1/2*pi*b^2*x^2) + 2*(pi^2
*b^5*x^4 - 8*b)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}(bx)\, dx$$

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}(bx)\, dx$$

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^5\, \text{FresnelS}(b\,x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

```
[In] int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.95 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{x^4}{8b\pi}+\frac{\cos(b^2\pi x^2)}{b^5\pi^3}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{x^2\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] -1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3+3*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x
)/b^4/Pi^2-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2+3/8*I*x^2*hypergeom([1,
 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2-3/8*I*x^2*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)/b^3/Pi^2+x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1
/4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {6597, 3460, 3390, 30, 3377, 2718, 6589, 6581}

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2\pi^2b^5} + \frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\cos\left(\pi b^2x^2\right)}{\pi^3b^5} + \frac{3x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^2\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - \frac{x^4}{8\pi b}$$

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/8*x^4/(b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) + (3*x*Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) + (((
3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^3*
Pi^2) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2
])/(b^3*Pi^2) + (x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^2*Sin
[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{3\int x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^3 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} + \frac{x^3 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{3\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)\,dx}{b^4\pi^2} - \frac{3\int x\sin\left(b^2\pi x^2\right)\,dx}{2b^3\pi^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int x\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b\pi}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} \\
&\quad + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\
&\quad + \frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} \\
&\quad - \frac{\text{Subst}\left(\int x\,dx, x, x^2\right)}{4b\pi} + \frac{\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi} \\
&= -\frac{x^4}{8b\pi} + \frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} \\
&\quad + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\
&\quad + \frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^2\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} \\
&= -\frac{x^4}{8b\pi} + \frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} \\
&\quad - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\
&\quad - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^2\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [F]

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]

$$\int x^4\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\left(bx\right)dx$$

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Sympy [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x**4*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^4\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.96 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{5\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] -1/6*x^3/b/Pi+2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+x^2*FresnelS(b*x
)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b*
x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6597, 3472, 30, 3467, 3432, 6587}

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = -\frac{5\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2 b^4} + \frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - \frac{x^3}{6\pi b}$$

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/6*x^3/(b*Pi) + (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) - (5*Fre
snelS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &\quad - \frac{2\int x \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\ &= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b^4\pi^2} + \frac{x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &\quad - \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2} - \frac{\int x^2\,dx}{2b\pi} + \frac{\int x^2\cos\left(b^2\pi x^2\right)\,dx}{2b\pi}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b^4\pi^2} \\
&\quad + \frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\int\sin\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2} \\
&= -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{5\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} \\
&\quad + \frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$
\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx
$$

$$
= \frac{-4b^3\pi x^3 - 15\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 24\,\text{FresnelS}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 6bx\sin\left(b^2\pi x^2\right)}{24b^4\pi^2}
$$

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 24*FresnelS[b*x]*(2*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])
/(24*b^4*Pi^2)
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\text{FresnelS}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{b^3x^3}{6\pi}-\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	119

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))+1/6/Pi*b^3*x^3-1/2/Pi*(
1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = -\frac{4\,\pi b^4 x^3 - 48\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) + 15\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right) - 12\left(2\,\pi b^3 x^2\,\mathrm{S}\left(bx\right) + b^2 x\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\right) \mathrm{s}}{24\,\pi^2 b^5}$$

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/24*(4*pi*b^4*x^3 - 48*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 15*sqrt(2
)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(2*pi*b^3*x^2*fresnel_sin
(b*x) + b^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^2*b^5)
```

Sympy [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = \int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = \int x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right)\,dx$$

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^3\, \text{FresnelS}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

```
[In] int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.97 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{x^2}{4b\pi} - \frac{\text{FresnelS}(bx)^2}{2b^3\pi} + \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] -1/4*x^2/b/Pi-1/2*FresnelS(b*x)^2/b^3/Pi+x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2
)/b^2/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6597, 3460, 2714, 6575, 30}

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{\text{FresnelS}(bx)^2}{2\pi b^3} + \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\sin(\pi b^2x^2)}{4\pi^2b^3} - \frac{x^2}{4\pi b}$$

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/4*x^2/(b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi
*x^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
&\text{integral} \\
&= \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)}{b^3\pi} \\
&\quad - \frac{\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b\pi} \\
&= -\frac{x^2}{4b\pi} - \frac{\text{FresnelS}(bx)^2}{2b^3\pi} + \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = -\frac{x^2}{4b\pi}-\frac{\text{FresnelS}(bx)^2}{2b^3\pi}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] -1/4*x^2/(b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi
*x^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Maple [F]

$$\int x^2 \cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\,(bx)\,dx$$

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = -\frac{\pi b^2x^2+2\,\pi\,\text{S}\,(bx)^2-2\left(2\,\pi bx\,\text{S}\,(bx)+\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^2b^3}$$

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] -1/4*(pi*b^2*x^2 + 2*pi*fresnel_sin(b*x)^2 - 2*(2*pi*b*x*fresnel_sin(b*x) +
 cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$
$$=\begin{cases}-\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b}-\frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b}+\frac{x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2}+\frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3}-\frac{S^2(bx)}{2\pi b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) - x**2*cos(pi*b**2*x**2/2)
**2/(4*pi*b) + x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + sin(pi*b**2*
x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) - fresnels(b*x)**2/(2*pi*b**3),
Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\, dx = \int x^2\, \mathrm{FresnelS}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

```
[In] int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.98 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=-\frac{x}{2b\pi}+\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}+\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] -1/2*x/b/Pi+FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*FresnelC(b*x*2^(1/
2))/b^2/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6595, 3438, 3433}

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}+\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x}{2\pi b}$$

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/2*x/(b*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelS[b*x]*S
in[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\ &= \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\,dx}{b\pi} \\ &= -\frac{x}{2b\pi} + \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\int \cos\left(b^2\pi x^2\right)\,dx}{2b\pi} \\ &= -\frac{x}{2b\pi} + \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi} + \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \frac{-2bx+\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)+4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^2\pi}$$

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] (-2*b*x + Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/
2])/(4*b^2*Pi)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\frac{\text{FresnelS}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}-\frac{\frac{bx}{2}-\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$	52

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b/Pi*sin(1/2*b^2*Pi*x^2)-1/b/Pi*(1/2*b*x-1/4*2^(1/2)*Fresnel
C(b*x*2^(1/2))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = -\frac{2\,b^2x - 4\,b\,\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right) - \sqrt{2}\sqrt{b^2}\,\text{C}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,\pi b^3}$$

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] -1/4*(2*b^2*x - 4*b*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2) - sqrt(2)*sqrt(b^2
)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int x\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int x\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.99 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],-1/
2*I*b^2*Pi*x^2)+1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6581}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]

$$\int \cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\,(bx)\,dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Fricas [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)\,\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Sympy [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Maxima [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.100 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx=\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\left(bx\right)}{x}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)}{x}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)}{x}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx=\int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

3.101 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} - \frac{1}{2}b\pi\,\text{FresnelS}(bx)^2 + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

```
[Out] -cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x-1/2*b*Pi*FresnelS(b*x)^2+1/4*b*Si(b^2*
Pi*x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6599, 6575, 30, 3456}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x} + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right) - \frac{1}{2}\pi b\,\text{FresnelS}(bx)^2$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} + \frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx \\
&\quad - \left(b^2\pi\right)\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right) - (b\pi)\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right) \\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} - \frac{1}{2}b\pi\,\text{FresnelS}(bx)^2 + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} - \frac{1}{2}b\pi\,\text{FresnelS}(bx)^2 + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}\left(bx\right)}{x^2}dx$$

[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)

[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{2\,\pi bx\,\text{S}\left(bx\right)^2 - bx\,\text{Si}\left(\pi b^2x^2\right) + 4\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{4\,x}$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="fricas")

[Out] -1/4*(2*pi*b*x*fresnel_sin(b*x)^2 - b*x*sin_integral(pi*b^2*x^2) + 4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x))/x

Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^2}\,dx$$

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**2,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)

Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^2}\,dx$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^2, x)

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^2}\,dx$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)

[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)

3.102 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \frac{b^2\pi\,\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{4x} - \frac{1}{2}b^2\pi\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -1/2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2*
Pi*FresnelC(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(
1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Defer[Int][(FresnelS[b*
x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2x^2} + \frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad - \frac{1}{2}\left(b^2\pi\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{4x} \\
&\quad - \frac{1}{2}\left(b^2\pi\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx + \frac{1}{2}\left(b^3\pi\right)\int \cos\left(b^2\pi x^2\right)\,dx \\
&= \frac{b^2\pi\,\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2x^2} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{4x} - \frac{1}{2}\left(b^2\pi\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}\left(bx\right)}{x^3}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**3,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos \left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\,(bx)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

3.103 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{12x^2} - \frac{1}{3}b^2\pi\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] 1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/12*b
*sin(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Defer[Int][(FresnelS[b*x]*S
in[(b^2*Pi*x^2)/2])/x^2, x])/3
```

Rubi steps

$$\text{integral} = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x^3} + \frac{1}{6}b\int\frac{\sin(b^2\pi x^2)}{x^3}\,dx - \frac{1}{3}(b^2\pi)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x^3} + \frac{1}{12}b\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) - \frac{1}{3}(b^2\pi)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x^3} - \frac{b\sin(b^2\pi x^2)}{12x^2} - \frac{1}{3}(b^2\pi)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{12}(b^3\pi)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x^3} - \frac{b\sin(b^2\pi x^2)}{12x^2} - \frac{1}{3}(b^2\pi)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelS}\left(bx\right)}{x^4}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**4,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

3.104 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = {} & \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4x^4}\\ & -\frac{7b^4\pi^2\,\text{FresnelS}\left(\sqrt{2}bx\right)}{24\sqrt{2}}\\ & +\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}\\ & -\frac{1}{8}b^4\pi^2\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)\end{aligned}$$

```
[Out] 1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/4*cos(1/2*b^2*Pi*x^2)*Fresnel
S(b*x)/x^4+1/8*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/24*b*sin(b^2*
Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Uninte
grable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

```
[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) +
(b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(2
4*x^3) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4x^4}+\frac{1}{8}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad -\frac{1}{4}\left(b^2\pi\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= \frac{b^3\pi}{16x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4x^4}+\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} \\
&\quad -\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad +\frac{1}{12}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{8}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx \\
&= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4x^4}+\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} \\
&\quad -\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}-\frac{1}{8}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx \\
&\quad -\frac{1}{8}\left(b^5\pi^2\right)\int\sin\left(b^2\pi x^2\right)\,dx-\frac{1}{6}\left(b^5\pi^2\right)\int\sin\left(b^2\pi x^2\right)\,dx \\
&= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{4x^4}-\frac{7b^4\pi^2\,\text{FresnelS}\left(\sqrt{2}bx\right)}{24\sqrt{2}} \\
&\quad +\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}-\frac{1}{8}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx=\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\operatorname{FresnelS}\left(bx\right)}{x^5}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{S}\left(bx\right)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 3.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**5,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\,(bx)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\,(bx)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

3.105 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx &= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos(b^2\pi x^2)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5} \\ &\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x} \\ &\quad + \frac{1}{30}b^5\pi^3\,\text{FresnelS}(bx)^2 + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} \\ &\quad - \frac{b\sin(b^2\pi x^2)}{40x^4} - \frac{7}{120}b^5\pi^2\,\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

```
[Out] 1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/5*cos(1/2*b^2*Pi*x^2)*Fre
snelS(b*x)/x^5+1/15*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/30*b^5*P
i^3*FresnelS(b*x)^2-7/120*b^5*Pi^2*Si(b^2*Pi*x^2)+1/15*b^2*Pi*FresnelS(b*x)
*sin(1/2*b^2*Pi*x^2)/x^3-1/40*b*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

$= \{6599, 6591, 6575, 30, 3456, 3461, 3378, 3380, 3460\}$

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \frac{1}{30}\pi^3 b^5\,\text{FresnelS}(bx)^2 + \frac{\pi b^3}{60x^2} - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} + \frac{\pi b^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} - \frac{b\sin\left(\pi b^2 x^2\right)}{40x^4} - \frac{7}{120}\pi^2 b^5\,\text{Si}\left(b^2\pi x^2\right) + \frac{\pi^2 b^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{\pi b^3\cos\left(\pi b^2 x^2\right)}{24x^2}$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5}+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx \\
&\quad -\frac{1}{5}\left(b^2\pi\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= \frac{b^3\pi}{60x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5}+\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} \\
&\quad +\frac{1}{20}b\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx,x,x^2\right)+\frac{1}{30}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad -\frac{1}{15}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx
\end{aligned}
$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} + \frac{1}{60}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) + \frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{30}\left(b^5\pi^2\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx + \frac{1}{15}\left(b^6\pi^3\right)\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{1}{60}b^5\pi^2\text{Si}\left(b^2\pi x^2\right) - \frac{1}{60}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) - \frac{1}{40}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) + \frac{1}{15}\left(b^5\pi^3\right)\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x} + \frac{1}{30}b^5\pi^3\,\text{FresnelS}(bx)^2 + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x} + \frac{1}{30}b^5\pi^3\,\text{FresnelS}(bx)^2 + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
```

```
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelS}\left(bx\right)}{x^6}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)}{x^6}\,dx$$
$$= \frac{4\,\pi^3 b^5 x^5\,\mathrm{S}\left(bx\right)^2 - 7\,\pi^2 b^5 x^5\,\mathrm{Si}\left(\pi b^2 x^2\right) - 10\,\pi b^3 x^3\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 7\,\pi b^3 x^3 + 8\left(\pi^2 b^4 x^4 - 3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}}{120\,x^5}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/120*(4*pi^3*b^5*x^5*fresnel_sin(b*x)^2 - 7*pi^2*b^5*x^5*sin_integral(pi*b
^2*x^2) - 10*pi*b^3*x^3*cos(1/2*pi*b^2*x^2)^2 + 7*pi*b^3*x^3 + 8*(pi^2*b^4*
x^4 - 3)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 2*(4*pi*b^2*x^2*fresnel_sin
(b*x) - 3*b*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/x^5
```

Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^6}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**6,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\left(bx\right)}{x^6}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^6, x)
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\left(bx\right)}{x^6}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

3.106 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$
\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx &= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{7b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}} \\
&\quad - \frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right) \\
&\quad - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^2} \\
&\quad + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{60x^5} + \frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{1440x} \\
&\quad + \frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)
\end{aligned}
$$

```
[Out] 1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/6*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x^6+1/48*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2+1/24*
b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/60*b*sin(b^2*Pi*x^2)/x^5+67/
1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*2^(1
/2)+1/48*b^6*Pi^3*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

```
[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*Fr
esnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x
])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/
(1440*x) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/
48
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6} + \frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad - \frac{1}{6}\left(b^2\pi\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= \frac{b^3\pi}{144x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{60x^5} + \frac{1}{48}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx + \frac{1}{30}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad - \frac{1}{24}\left(b^4\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx \\
&= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^2} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{60x^5} - \frac{1}{96}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{72}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{48}\left(b^6\pi^3\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6} \\
&+ \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^2} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} \\
&- \frac{b\sin(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{1440x} \\
&+ \frac{1}{48}(b^6\pi^3)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx - \frac{1}{48}(b^7\pi^3)\int\cos\left(b^2\pi x^2\right)\,dx \\
&- \frac{1}{36}(b^7\pi^3)\int\cos\left(b^2\pi x^2\right)\,dx - \frac{1}{45}(2b^7\pi^3)\int\cos\left(b^2\pi x^2\right)\,dx \\
&= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{7b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}} \\
&- \frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6} \\
&+ \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^2} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} \\
&- \frac{b\sin(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{1440x} + \frac{1}{48}(b^6\pi^3)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}(bx)}{x^7}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\,(bx)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

Sympy [N/A]

Not integrable

Time = 11.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**7,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\,(bx)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

3.107 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx &= \frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{84}b^7\pi^3\,\text{CosIntegral}\left(b^2\pi x^2\right)\\ &\quad-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7}\\ &\quad+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x^3}\\ &\quad+\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}\\ &\quad-\frac{b\sin\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\sin\left(b^2\pi x^2\right)}{84x^2}\\ &\quad+\frac{1}{105}b^6\pi^3\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)\end{aligned}$$

```
[Out] 1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)/
x^4-1/7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/105*b^4*Pi^2*cos(1/2*b^2*Pi
*x^2)*FresnelS(b*x)/x^3+1/35*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1
/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2+1/105*b^6*Pi^3*
Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

```
[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInte
gral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*P
i^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Si
n[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[
b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/x^2, x])/105
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx \\
&\quad - \frac{1}{7}\left(b^2\pi\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx \\
&= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} \\
&\quad + \frac{1}{28}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) + \frac{1}{70}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx \\
&\quad - \frac{1}{35}\left(b^4\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx \\
&= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x^3} \\
&\quad + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b\sin\left(b^2\pi x^2\right)}{84x^6} \\
&\quad + \frac{1}{140}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&\quad + \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{210}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad + \frac{1}{105}\left(b^6\pi^3\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx
\end{aligned}$$

$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos(b^2\pi x^2)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b\sin(b^2\pi x^2)}{84x^6} - \frac{1}{420}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) - \frac{1}{280}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) - \frac{1}{168}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) + \frac{1}{105}(b^6\pi^3)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos(b^2\pi x^2)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b\sin(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2\sin(b^2\pi x^2)}{84x^2} + \frac{1}{105}(b^6\pi^3)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{420}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{280}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{168}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)$$

$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos(b^2\pi x^2)}{105x^4} - \frac{1}{84}b^7\pi^3\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b\sin(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2\sin(b^2\pi x^2)}{84x^2} + \frac{1}{105}(b^6\pi^3)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\operatorname{FresnelS}(bx)}{x^8}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{S}(bx)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

Sympy [N/A]

Not integrable

Time = 21.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**8,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

Mupad [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

3.108 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx &= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} \\
&+ \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^8} \\
&+ \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{192x^4} \\
&+ \frac{853b^8\pi^4\,\text{FresnelS}\left(\sqrt{2}bx\right)}{40320\sqrt{2}} \\
&+ \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\
&- \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} \\
&- \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} \\
&+ \frac{1}{384}b^8\pi^4\,\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}, x\right)
\end{aligned}$$

```
[Out] 1/480*b^3*Pi/x^5-1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/80
640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8+1/
192*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4+1/48*b^2*Pi*FresnelS(b*x
)*sin(1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/
x^2-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3+853/
80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(co
s(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

```
[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360
*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4
) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS
[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*P
i*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*
x])/x, x])/384
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx \\
&\quad - \frac{1}{8}\left(b^2\pi\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= \frac{b^3\pi}{480x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^8} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{56}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad - \frac{1}{48}\left(b^4\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx \\
&= \frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^8} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{192x^4} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} - \frac{1}{384}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{240}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad - \frac{1}{140}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx + \frac{1}{192}\left(b^6\pi^3\right)\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx
\end{aligned}$$

$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{192x^4} + \frac{b^2\pi\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b^6\pi^3\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} - \frac{1}{768}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{576}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{360}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{210}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx$$

$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{192x^4} + \frac{b^2\pi\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b^6\pi^3\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx + \frac{1}{384}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx + \frac{1}{288}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx + \frac{1}{180}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx + \frac{1}{105}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx$$

$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{192x^4} + \frac{853b^8\pi^4\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{40320\sqrt{2}} + \frac{b^2\pi\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b^6\pi^3\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}\left(bx\right)}{x^9}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\left(bx\right)}{x^9}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

Sympy [N/A]

Not integrable

Time = 38.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^9}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**9,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^9}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^9}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

Mupad [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^9} \, dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9} \, dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

3.109 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 278

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x} - \frac{b^9\pi^5\,\text{FresnelS}(bx)^2}{1890} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} + \frac{83b^9\pi^4\text{Si}(b^2\pi x^2)}{30240}$$

```
[Out] 1/756*b^3*Pi/x^6-1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/2
016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/9*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9+
1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5-1/945*b^8*Pi^4*cos(1/2
*b^2*Pi*x^2)*FresnelS(b*x)/x-1/1890*b^9*Pi^5*FresnelS(b*x)^2+83/30240*b^9*P
i^4*Si(b^2*Pi*x^2)+1/63*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*
b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/144*b*sin(b^2*Pi*x^2)/x^8+
67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6599, 6591, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{\pi^5 b^9\,\text{FresnelS}(bx)^2}{1890} - \frac{\pi^3 b^7}{3780x^2} + \frac{\pi b^3}{756x^6} - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} + \frac{\pi b^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{63x^7} - \frac{b\sin\left(\pi b^2 x^2\right)}{144x^8} + \frac{83\pi^4 b^9\,\text{Si}(b^2\pi x^2)}{30240} - \frac{\pi^4 b^8\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{945x} + \frac{5\pi^3 b^7\cos\left(\pi b^2 x^2\right)}{2016x^2} - \frac{\pi^3 b^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{945x^3} + \frac{67\pi^2 b^5\sin\left(\pi b^2 x^2\right)}{30240x^4} + \frac{\pi^2 b^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{315x^5} - \frac{11\pi b^3\cos\left(\pi b^2 x^2\right)}{3024x^6}$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{1}{18}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx - \frac{1}{9}\left(b^2\pi\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^5}\,dx, x, x^2\right) + \frac{1}{126}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{63}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b\sin\left(b^2\pi x^2\right)}{144x^8} + \frac{1}{252}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) + \frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) - \frac{1}{630}\left(b^5\pi^2\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{315}\left(b^6\pi^3\right)\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{144x^8} - \frac{\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^3}\,dx, x, x^2\right)}{1260} - \frac{1}{756}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{432}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{\left(b^7\pi^3\right)\int\frac{\cos(b^2\pi x^2)}{x^3}\,dx}{1890} + \frac{1}{945}\left(b^8\pi^4\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} - \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{3780} - \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{2520} - \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{1512} - \frac{1}{864}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right) + \frac{(b^9\pi^4)\int\frac{\sin(b^2\pi x^2)}{x}\,dx}{1890} - \frac{1}{945}(b^{10}\pi^5)\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} + \frac{b^9\pi^4\text{Si}(b^2\pi x^2)}{3780} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{3780} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{2520} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{1512} + \frac{1}{864}(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{945}(b^9\pi^5)\,\text{Subst}\left(\int x\,dx, x, \text{FresnelS}(bx)\right)$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x} - \frac{b^9\pi^5\,\text{FresnelS}(bx)^2}{1890} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} + \frac{83b^9\pi^4\text{Si}(b^2\pi x^2)}{30240}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^{10}}\,dx = \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{945x} - \frac{b^9\pi^5\operatorname{FresnelS}(bx)^2}{1890} + \frac{b^2\pi\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} + \frac{83b^9\pi^4\operatorname{Si}(b^2\pi x^2)}{30240}$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\operatorname{FresnelS}(bx)}{x^{10}}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.73

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx =$$
$$-\frac{16\,\pi^5b^9x^9\,\text{S}\,(bx)^2 - 83\,\pi^4b^9x^9\,\text{Si}\,(\pi b^2x^2) + 83\,\pi^3b^7x^7 - 150\,\pi b^3x^3 - 10\,(15\,\pi^3b^7x^7 - 22\,\pi b^3x^3)\cos\left(\frac{1}{2}\,\pi b^2\right.}{}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] -1/30240*(16*pi^5*b^9*x^9*fresnel_sin(b*x)^2 - 83*pi^4*b^9*x^9*sin_integral
(pi*b^2*x^2) + 83*pi^3*b^7*x^7 - 150*pi*b^3*x^3 - 10*(15*pi^3*b^7*x^7 - 22*
pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)^2 + 32*(pi^4*b^8*x^8 - 3*pi^2*b^4*x^4 + 105
)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 2*((67*pi^2*b^5*x^5 - 210*b*x)*cos
(1/2*pi*b^2*x^2) - 16*(pi^3*b^6*x^6 - 15*pi*b^2*x^2)*fresnel_sin(b*x))*sin(
1/2*pi*b^2*x^2))/x^9
```

Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**10,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^10, x)
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\operatorname{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

3.110 $\int x^7 \operatorname{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 124

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105\operatorname{FresnelC}(bx)}{8b^8\pi^4} + \frac{1}{8}x^8\operatorname{FresnelC}(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}$$

```
[Out] 105/8*x*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-7/8*x^5*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2-1
05/8*FresnelC(b*x)/b^8/Pi^4+1/8*x^8*FresnelC(b*x)+35/8*x^3*sin(1/2*b^2*Pi*x
^2)/b^5/Pi^3-1/8*x^7*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3467, 3466, 3433}

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = -\frac{105\operatorname{FresnelC}(bx)}{8\pi^4b^8} - \frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b} + \frac{105x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7} + \frac{35x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5} - \frac{7x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3} + \frac{1}{8}x^8\operatorname{FresnelC}(bx)$$

```
[In] Int[x^7*FresnelC[b*x],x]
```

```
[Out] (105*x*Cos[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2])/(8*b
^3*Pi^2) - (105*FresnelC[b*x])/(8*b^8*Pi^4) + (x^8*FresnelC[b*x])/8 + (35*x
^3*Sin[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*Sin[(b^2*Pi*x^2)/2])/(8*b*Pi)
```

<u>Rule 3433</u>

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^8\,\text{FresnelC}(bx) - \frac{1}{8}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{8}x^8\,\text{FresnelC}(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{7\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b\pi} \\
&= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8\,\text{FresnelC}(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^3\pi^2} \\
&= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8\,\text{FresnelC}(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} \\
&\quad - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^5\pi^3} \\
&= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8\,\text{FresnelC}(bx) \\
&\quad + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^7\pi^4} \\
&= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105\,\text{FresnelC}(bx)}{8b^8\pi^4} \\
&\quad + \frac{1}{8}x^8\,\text{FresnelC}(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = \frac{-7bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+(-105+b^8\pi^4x^8)\operatorname{FresnelC}(bx)+b^3\pi x^3(35-b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^8\pi^4}$$

```
[In] Integrate[x^7*FresnelC[b*x],x]
[Out] (-7*b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fr
esnelC[b*x] + b^3*Pi*x^3*(35 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{b\,x^{9}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{9}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{13}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{9}$
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^{8}x^{8}}{8}-\frac{b^{7}x^{7}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{8\pi}+\frac{-\frac{7b^{5}x^{5}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{8\pi}+\frac{7\left(\frac{5b^{3}x^{3}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}-\frac{15\left(-\frac{bx\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^{8}}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)b^{8}x^{8}}{8}-\frac{b^{7}x^{7}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{8\pi}+\frac{-\frac{7b^{5}x^{5}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{8\pi}+\frac{7\left(\frac{5b^{3}x^{3}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}-\frac{15\left(-\frac{bx\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^{8}}$
parts	$\frac{x^{8}\,\mathrm{FresnelC}(bx)}{8}-\frac{b\left(\frac{x^{7}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{b^{2}\pi}-\frac{7\left(-\frac{x^{5}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{b^{2}\pi}+\frac{\frac{5x^{3}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{b^{2}\pi}-\frac{15\left(-\frac{x\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{b^{2}\pi}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}\right.}{b^{2}\pi}}{b^{2}\pi}\right.}{b^{2}\pi}\right.}{8}$

```
[In] int(x^7*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/9*b*x^9*hypergeom([1/4,9/4],[1/2,5/4,13/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int x^{7}\,\mathrm{FresnelC}(bx)\,dx = -\frac{7\left(\pi^{2}b^{5}x^{5}-15\,bx\right)\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-\left(\pi^{4}b^{8}x^{8}-105\right)\mathrm{C}\left(bx\right)+\left(\pi^{3}b^{7}x^{7}-35\,\pi b^{3}x^{3}\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{8\,\pi^{4}b^{8}}$$

```
[In] integrate(x^7*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] -1/8*(7*(pi^2*b^5*x^5 - 15*b*x)*cos(1/2*pi*b^2*x^2) - (pi^4*b^8*x^8 - 105)*
fresnel_cos(b*x) + (pi^3*b^7*x^7 - 35*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2))/(pi^
4*b^8)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = \frac{45x^8C(bx)\,\Gamma\left(\frac{1}{4}\right)}{512\Gamma\left(\frac{13}{4}\right)} - \frac{45x^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi b\Gamma\left(\frac{13}{4}\right)} - \frac{315x^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{13}{4}\right)} + \frac{1575x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{13}{4}\right)} + \frac{4725x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^4b^7\Gamma\left(\frac{13}{4}\right)} - \frac{4725C(bx)\,\Gamma\left(\frac{1}{4}\right)}{512\pi^4b^8\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate(x**7*fresnelc(b*x),x)
```

```
[Out] 45*x**8*fresnelc(b*x)*gamma(1/4)/(512*gamma(13/4)) - 45*x**7*sin(pi*b**2*x*
*2/2)*gamma(1/4)/(512*pi*b*gamma(13/4)) - 315*x**5*cos(pi*b**2*x**2/2)*gamm
a(1/4)/(512*pi**2*b**3*gamma(13/4)) + 1575*x**3*sin(pi*b**2*x**2/2)*gamma(1
/4)/(512*pi**3*b**5*gamma(13/4)) + 4725*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(5
12*pi**4*b**7*gamma(13/4)) - 4725*fresnelc(b*x)*gamma(1/4)/(512*pi**4*b**8*
gamma(13/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = \frac{1}{8}\,x^8\,\mathrm{C}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(-(105i-105)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) + (105i+105)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right) + 28\left(\sqrt{\frac{1}{2}}\pi^3b^5x^5 - \right.}{16\,\pi^5b^8}$$

```
[In] integrate(x^7*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*x^8*fresnel_cos(b*x) - 1/16*sqrt(1/2)*(-(105*I - 105)*(1/4)^(1/4)*pi*er
f(sqrt(1/2*I*pi)*b*x) + (105*I + 105)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*
x) + 28*(sqrt(1/2)*pi^3*b^5*x^5 - 15*sqrt(1/2)*pi*b*x)*cos(1/2*pi*b^2*x^2)
+ 4*(sqrt(1/2)*pi^4*b^7*x^7 - 35*sqrt(1/2)*pi^2*b^3*x^3)*sin(1/2*pi*b^2*x^2
))/(pi^5*b^8)
```

Giac [F]

$$\int x^7 \operatorname{FresnelC}(bx)\, dx = \int x^7 \operatorname{C}(bx)\, dx$$

```
[In] integrate(x^7*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^7*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelC}(bx)\, dx = \int x^7 \operatorname{FresnelC}(b\,x)\, dx$$

```
[In] int(x^7*FresnelC(b*x),x)
[Out] int(x^7*FresnelC(b*x), x)
```

3.111 $\int x^6 \operatorname{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 109

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7\operatorname{FresnelC}(bx) + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}$$

```
[Out] 48/7*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/7*
x^7*FresnelC(b*x)+24/7*x^2*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/7*x^6*sin(1/2*b^2
*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3461, 3377, 2718}

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = -\frac{x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b} + \frac{48\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} + \frac{24x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5} - \frac{6x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3} + \frac{1}{7}x^7\operatorname{FresnelC}(bx)$$

```
[In] Int[x^6*FresnelC[b*x],x]
```

```
[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*
Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) -
 (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}x^7\,\text{FresnelC}(bx) - \frac{1}{7}b\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{7}x^7\,\text{FresnelC}(bx) - \frac{1}{14}b\text{Subst}\left(\int x^3\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{1}{7}x^7\,\text{FresnelC}(bx) - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{3\text{Subst}\left(\int x^2\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b\pi} \\
&= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx) \\
&\quad - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{12\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b^3\pi^2} \\
&= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx) + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} \\
&\quad - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} - \frac{24\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b^5\pi^3} \\
&= \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx) \\
&\quad + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int x^6 \operatorname{FresnelC}(bx)\, dx = -\frac{6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} + \frac{1}{7}x^7 \operatorname{FresnelC}(bx) - \frac{x^2(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3}$$

```
[In] Integrate[x^6*FresnelC[b*x],x]
[Out] (-6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) + (x^7*FresnelC[b
*x])/7 - (x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.24

method	result	size
meijerg	$\frac{b\,x^8 \operatorname{hypergeom}\left(\left[\frac{1}{4},2\right],\left[\frac{1}{2},\frac{5}{4},3\right],-\frac{x^4\pi^2b^4}{16}\right)}{8}$	26
derivativedivides	$\frac{\frac{\operatorname{FresnelC}(bx)b^7x^7}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$	107
default	$\frac{\frac{\operatorname{FresnelC}(bx)b^7x^7}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$	107
parts	$\frac{x^7\operatorname{FresnelC}(bx)}{7}-\frac{b\left(\frac{x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{6\left(-\frac{x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{4x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}}{b^2\pi}\right)}{b^2\pi}\right)}{7}$	112

```
[In] int(x^6*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/8*b*x^8*hypergeom([1/4,2],[1/2,5/4,3],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{\pi^4 b^7 x^7 \operatorname{C}(bx) - 6\,(\pi^2 b^4 x^4 - 8)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - (\pi^3 b^6 x^6 - 24\,\pi b^2 x^2)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{7\,\pi^4 b^7}$$

[In] integrate(x^6*fresnel_cos(b*x),x, algorithm="fricas")

[Out] 1/7*(pi^4*b^7*x^7*fresnel_cos(b*x) - 6*(pi^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2) - (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{x^7 C(bx)\,\Gamma\left(\frac{1}{4}\right)}{28\Gamma\left(\frac{5}{4}\right)} - \frac{x^6 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{28\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{3x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{14\pi^2 b^3\Gamma\left(\frac{5}{4}\right)} + \frac{6x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^3 b^5\Gamma\left(\frac{5}{4}\right)} + \frac{12\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^4 b^7\Gamma\left(\frac{5}{4}\right)}$$

[In] integrate(x**6*fresnelc(b*x),x)

[Out] x**7*fresnelc(b*x)*gamma(1/4)/(28*gamma(5/4)) - x**6*sin(pi*b**2*x**2/2)*gamma(1/4)/(28*pi*b*gamma(5/4)) - 3*x**4*cos(pi*b**2*x**2/2)*gamma(1/4)/(14*pi**2*b**3*gamma(5/4)) + 6*x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**3*b**5*gamma(5/4)) + 12*cos(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**4*b**7*gamma(5/4))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{1}{7}\,x^7 \operatorname{C}(bx) - \frac{6\,(\pi^2 b^4 x^4 - 8)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + (\pi^3 b^6 x^6 - 24\,\pi b^2 x^2)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{7\,\pi^4 b^7}$$

[In] integrate(x^6*fresnel_cos(b*x),x, algorithm="maxima")

[Out] 1/7*x^7*fresnel_cos(b*x) - 1/7*(6*(pi^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2) + (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)

Giac [F]

$$\int x^6 \,\mathrm{FresnelC}(bx)\, dx = \int x^6 \,\mathrm{C}\left(bx\right)\, dx$$

```
[In] integrate(x^6*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^6*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \,\mathrm{FresnelC}(bx)\, dx = \int x^6 \,\mathrm{FresnelC}(b\,x)\, dx$$

```
[In] int(x^6*FresnelC(b*x),x)
[Out] int(x^6*FresnelC(b*x), x)
```

3.112 $\int x^5 \, \text{FresnelC}(bx) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 99

$$\int x^5 \, \text{FresnelC}(bx) \, dx = -\frac{5x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 \, \text{FresnelC}(bx) - \frac{5 \, \text{FresnelS}(bx)}{2b^6\pi^3} + \frac{5x \sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}$$

```
[Out] -5/6*x^3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/6*x^6*FresnelC(b*x)-5/2*FresnelS(b*
x)/b^6/Pi^3+5/2*x*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/6*x^5*sin(1/2*b^2*Pi*x^2)/
b/Pi
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3467, 3466, 3432}

$$\int x^5 \, \text{FresnelC}(bx) \, dx = -\frac{5 \, \text{FresnelS}(bx)}{2\pi^3 b^6} - \frac{x^5 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6\pi b} + \frac{5x \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^3 b^5} - \frac{5x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6\pi^2 b^3} + \frac{1}{6}x^6 \, \text{FresnelC}(bx)$$

```
[In] Int[x^5*FresnelC[b*x],x]
```

```
[Out] (-5*x^3*Cos[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*FresnelC[b*x])/6 - (5*Fres
nelS[b*x])/(2*b^6*Pi^3) + (5*x*Sin[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*Sin
[(b^2*Pi*x^2)/2])/(6*b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\,\text{FresnelC}(bx) - \frac{1}{6}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{6}x^6\,\text{FresnelC}(bx) - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{6b\pi} \\
&= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6\,\text{FresnelC}(bx) - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^3\pi^2} \\
&= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6\,\text{FresnelC}(bx) + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} \\
&\quad - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} - \frac{5\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^5\pi^3} \\
&= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6\,\text{FresnelC}(bx) - \frac{5\,\text{FresnelS}(bx)}{2b^6\pi^3} + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{-5b^3\pi x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)+b^6\pi^3x^6\operatorname{FresnelC}(bx)-15\operatorname{FresnelS}(bx)+bx(15-b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^6\pi^3}$$

```
[In] Integrate[x^5*FresnelC[b*x],x]
[Out] (-5*b^3*Pi*x^3*Cos[(b^2*Pi*x^2)/2] + b^6*Pi^3*x^6*FresnelC[b*x] - 15*Fresne
lS[b*x] + b*x*(15 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

method	result	size
meijerg	$\frac{b\,x^7\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{7}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{11}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{7}$	26
derivativedivides	$\frac{\frac{\operatorname{FresnelC}(bx)b^6x^6}{6}-\frac{-\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{-\frac{5b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\operatorname{FresnelS}(bx)}{\pi}\right)}{6\pi}}{\pi}}{}}{b^6}$	97
default	$\frac{\frac{\operatorname{FresnelC}(bx)b^6x^6}{6}-\frac{-\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{-\frac{5b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\operatorname{FresnelS}(bx)}{\pi}\right)}{6\pi}}{\pi}}{}}{b^6}$	97
parts	$\frac{x^6\operatorname{FresnelC}(bx)}{6}-\frac{b\left(\frac{x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{5\left(-\frac{x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{3x\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{3\operatorname{FresnelS}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{b^2\sqrt{\pi}\,\sqrt{b^2\pi}}}{b^2\pi}\right)}{b^2\pi}\right)}{6}$	123

```
[In] int(x^5*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/7*b*x^7*hypergeom([1/4,7/4],[1/2,5/4,11/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{\pi^3 b^7 x^6\, \mathrm{C}\,(bx) - 5\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - (\pi^2 b^6 x^5 - 15\,b^2 x)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 15\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{b^2} x\right)}{6\,\pi^3 b^7}$$

```
[In] integrate(x^5*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/6*(pi^3*b^7*x^6*fresnel_cos(b*x) - 5*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - (pi
^2*b^6*x^5 - 15*b^2*x)*sin(1/2*pi*b^2*x^2) - 15*sqrt(b^2)*fresnel_sin(sqrt(
b^2)*x))/(pi^3*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{bx^7 \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{7}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{4}, \frac{7}{4} \\ \frac{1}{2}, \frac{5}{4}, \frac{11}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right) \Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**5*fresnelc(b*x),x)
```

```
[Out] b*x**7*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -pi**2*b**
4*x**4/16)/(16*gamma(5/4)*gamma(11/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{1}{6}\,x^6\,\mathrm{C}\,(bx) - \frac{\sqrt{\frac{1}{2}}\left(20\,\sqrt{\frac{1}{2}}\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + (15i + 15)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2} i\,\pi} bx\right) - (15i - 15)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2} i\,\pi} b\right.}{12\,\pi^4 b^6}$$

```
[In] integrate(x^5*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*fresnel_cos(b*x) - 1/12*sqrt(1/2)*(20*sqrt(1/2)*pi^2*b^3*x^3*cos(1/
2*pi*b^2*x^2) + (15*I + 15)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (15*I
- 15)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x) + 4*(sqrt(1/2)*pi^3*b^5*x^5 -
 15*sqrt(1/2)*pi*b*x)*sin(1/2*pi*b^2*x^2))/(pi^4*b^6)
```

Giac [F]

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \int x^5\,\mathrm{C}\left(bx\right)\,dx$$

```
[In] integrate(x^5*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^5*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \int x^5 \operatorname{FresnelC}(b\,x)\,dx$$

```
[In] int(x^5*FresnelC(b*x),x)
[Out] int(x^5*FresnelC(b*x), x)
```

3.113 $\int x^4 \operatorname{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 84

$$\int x^4 \operatorname{FresnelC}(bx)\,dx = -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\operatorname{FresnelC}(bx) + \frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}$$

```
[Out] -4/5*x^2*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)+8/5*sin(1/2*b^2
*Pi*x^2)/b^5/Pi^3-1/5*x^4*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3461, 3377, 2717}

$$\int x^4 \operatorname{FresnelC}(bx)\,dx = -\frac{x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b} + \frac{8\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5} - \frac{4x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3} + \frac{1}{5}x^5\operatorname{FresnelC}(bx)$$

```
[In] Int[x^4*FresnelC[b*x],x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 + (8*Sin[
(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\,\text{FresnelC}(bx) - \frac{1}{5}b\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{1}{5}x^5\,\text{FresnelC}(bx) - \frac{1}{10}b\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)\\
&= \frac{1}{5}x^5\,\text{FresnelC}(bx) - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{2\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b\pi}\\
&= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\,\text{FresnelC}(bx) - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{4\text{Subst}\left(\int\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b^3\pi^2}\\
&= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\,\text{FresnelC}(bx) + \frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int x^4\,\text{FresnelC}(bx)\,dx = -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\,\text{FresnelC}(bx) - \frac{(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}$$

```
[In] Integrate[x^4*FresnelC[b*x],x]
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 - ((-8 +
b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.31

method	result	size
meijerg	$\frac{b\,x^{6}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{1}{2},\frac{5}{4},\frac{5}{2}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{6}$	26
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^{5}x^{5}}{5}-\frac{b^{4}x^{4}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{5\pi}+\frac{-\frac{4b^{2}x^{2}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{5\pi}+\frac{8\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{5\pi^{2}}}{\pi}}{b^{5}}$	81
default	$\frac{\frac{\mathrm{FresnelC}(bx)b^{5}x^{5}}{5}-\frac{b^{4}x^{4}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{5\pi}+\frac{-\frac{4b^{2}x^{2}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{5\pi}+\frac{8\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{5\pi^{2}}}{\pi}}{b^{5}}$	81
parts	$\frac{x^{5}\,\mathrm{FresnelC}(bx)}{5}-\frac{b\left(\frac{x^{4}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}-\frac{4\left(-\frac{x^{2}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}+\frac{2\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{4}\pi^{2}}\right)}{b^{2}\pi}\right)}{5}$	83

```
[In] int(x^4*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/6*b*x^6*hypergeom([1/4,3/2],[1/2,5/4,5/2],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int x^{4}\,\mathrm{FresnelC}(bx)\,dx=\frac{\pi^{3}b^{5}x^{5}\,\mathrm{C}\left(bx\right)-4\,\pi b^{2}x^{2}\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-\left(\pi^{2}b^{4}x^{4}-8\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{5\,\pi^{3}b^{5}}$$

```
[In] integrate(x^4*fresnel_cos(b*x),x, algorithm="fricas")
[Out] 1/5*(pi^3*b^5*x^5*fresnel_cos(b*x) - 4*pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - (pi
^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.38

$$\int x^4 \operatorname{FresnelC}(bx)\, dx = \frac{x^5 C(bx)\, \Gamma\left(\frac{1}{4}\right)}{20\Gamma\left(\frac{5}{4}\right)} - \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{20\pi b \Gamma\left(\frac{5}{4}\right)} - \frac{x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{5\pi^2 b^3 \Gamma\left(\frac{5}{4}\right)} + \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \Gamma\left(\frac{1}{4}\right)}{5\pi^3 b^5 \Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(x**4*fresnelc(b*x),x)
```

```
[Out] x**5*fresnelc(b*x)*gamma(1/4)/(20*gamma(5/4)) - x**4*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(20*pi*b*gamma(5/4)) - x**2*cos(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**
2*b**3*gamma(5/4)) + 2*sin(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**3*b**5*gamma(5
/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4 \operatorname{FresnelC}(bx)\, dx = \frac{1}{5}\, x^5\, \mathrm{C}\left(bx\right) - \frac{4\, \pi b^2 x^2 \cos\left(\frac{1}{2}\, \pi b^2 x^2\right) + \left(\pi^2 b^4 x^4 - 8\right) \sin\left(\frac{1}{2}\, \pi b^2 x^2\right)}{5\, \pi^3 b^5}$$

```
[In] integrate(x^4*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*fresnel_cos(b*x) - 1/5*(4*pi*b^2*x^2*cos(1/2*pi*b^2*x^2) + (pi^2*b^
4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Giac [F]

$$\int x^4 \operatorname{FresnelC}(bx)\, dx = \int x^4\, \mathrm{C}\left(bx\right)\, dx$$

```
[In] integrate(x^4*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelC}(bx)\,dx = \int x^4 \operatorname{FresnelC}(b\,x)\,dx$$

```
[In] int(x^4*FresnelC(b*x),x)
[Out] int(x^4*FresnelC(b*x), x)
```

3.114 $\int x^3 \operatorname{FresnelC}(bx) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 74

$$\int x^3 \operatorname{FresnelC}(bx) \, dx = -\frac{3x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3 \operatorname{FresnelC}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}$$

```
[Out] -3/4*x*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+3/4*FresnelC(b*x)/b^4/Pi^2+1/4*x^4*Fres
nelC(b*x)-1/4*x^3*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3467, 3466, 3433}

$$\int x^3 \operatorname{FresnelC}(bx) \, dx = \frac{3 \operatorname{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 \operatorname{FresnelC}(bx)$$

```
[In] Int[x^3*FresnelC[b*x],x]
```

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{1}{4}b\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi} \\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^3\pi^2} \\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\operatorname{FresnelC}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\operatorname{FresnelC}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}$$

```
[In] Integrate[x^3*FresnelC[b*x],x]
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{-\frac{3\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)bx}{4}-\frac{b^{3}x^{3}\pi\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4}+\frac{\left(5x^{4}\pi^{2}b^{4}+15\right)\operatorname{FresnelC}\left(bx\right)}{20}}{b^{4}\pi^{2}}$	62
derivativedivides	$\frac{\frac{\operatorname{FresnelC}\left(bx\right)b^{4}x^{4}}{4}-\frac{b^{3}x^{3}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4\pi}+\frac{-\frac{3bx\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4\pi}+\frac{3\operatorname{FresnelC}\left(bx\right)}{4\pi}}{\pi}}{b^{4}}$	70
default	$\frac{\frac{\operatorname{FresnelC}\left(bx\right)b^{4}x^{4}}{4}-\frac{b^{3}x^{3}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4\pi}+\frac{-\frac{3bx\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4\pi}+\frac{3\operatorname{FresnelC}\left(bx\right)}{4\pi}}{\pi}}{b^{4}}$	70
parts	$\frac{x^{4}\operatorname{FresnelC}\left(bx\right)}{4}-\frac{b\left(\frac{x^{3}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}-\frac{3\left(-\frac{x\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}+\frac{\operatorname{FresnelC}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}\right)}{b^{2}\pi}\right)}{4}$	93

```
[In] int(x^3*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/Pi^2/b^4*(-3/8*cos(1/2*b^2*Pi*x^2)*b*x-1/8*b^3*x^3*Pi*sin(1/2*b^2*Pi*x^2)
+1/40*(5*Pi^2*b^4*x^4+15)*FresnelC(b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int x^{3}\operatorname{FresnelC}(bx)\,dx=-\frac{\pi b^{3}x^{3}\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)+3\,bx\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-\left(\pi^{2}b^{4}x^{4}+3\right)\mathrm{C}\left(bx\right)}{4\,\pi^{2}b^{4}}$$

```
[In] integrate(x^3*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] -1/4*(pi*b^3*x^3*sin(1/2*pi*b^2*x^2) + 3*b*x*cos(1/2*pi*b^2*x^2) - (pi^2*b^
4*x^4 + 3)*fresnel_cos(b*x))/(pi^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int x^3\,\mathrm{FresnelC}(bx)\,dx = \frac{5x^4C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)} - \frac{5x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi b\Gamma\left(\frac{9}{4}\right)} - \frac{15x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{9}{4}\right)} + \frac{15C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**3*fresnelc(b*x),x)
```

```
[Out] 5*x**4*fresnelc(b*x)*gamma(1/4)/(64*gamma(9/4)) - 5*x**3*sin(pi*b**2*x**2/2
)*gamma(1/4)/(64*pi*b*gamma(9/4)) - 15*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(64
*pi**2*b**3*gamma(9/4)) + 15*fresnelc(b*x)*gamma(1/4)/(64*pi**2*b**4*gamma(
9/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int x^3\,\mathrm{FresnelC}(bx)\,dx = \frac{1}{4}x^4\,\mathrm{C}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi^2b^3x^3\sin\left(\frac{1}{2}\pi b^2x^2\right) + 12\sqrt{\frac{1}{2}}\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right) + (3i-3)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - (3i+3)\right.}{8\,\pi^3b^4}$$

```
[In] integrate(x^3*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_cos(b*x) - 1/8*sqrt(1/2)*(4*sqrt(1/2)*pi^2*b^3*x^3*sin(1/2*
pi*b^2*x^2) + 12*sqrt(1/2)*pi*b*x*cos(1/2*pi*b^2*x^2) + (3*I - 3)*(1/4)^(1/
4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (3*I + 3)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi
)*b*x))/(pi^3*b^4)
```

Giac [F]

$$\int x^3\,\mathrm{FresnelC}(bx)\,dx = \int x^3\,\mathrm{C}\left(bx\right)\,dx$$

```
[In] integrate(x^3*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(bx) \, dx = \int x^3 \operatorname{FresnelC}(b\,x) \, dx$$

```
[In] int(x^3*FresnelC(b*x),x)
```

```
[Out] int(x^3*FresnelC(b*x), x)
```

3.115 $\int x^2 \operatorname{FresnelC}(bx)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x^2 \operatorname{FresnelC}(bx)\, dx = -\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3 \operatorname{FresnelC}(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

```
[Out] -2/3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)-1/3*x^2*sin(1/2*b^2
*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3377, 2718}

$$\int x^2 \operatorname{FresnelC}(bx)\, dx = -\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} - \frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{1}{3}x^3 \operatorname{FresnelC}(bx)$$

```
[In] Int[x^2*FresnelC[b*x],x]
```

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\,\text{FresnelC}(bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{3}x^3\,\text{FresnelC}(bx) - \frac{1}{6}b\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\,\text{FresnelC}(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\text{Subst}\left(\int\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{3b\pi} \\
&= -\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3\,\text{FresnelC}(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^2\,\text{FresnelC}(bx)\,dx = -\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3\,\text{FresnelC}(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

```
[In] Integrate[x^2*FresnelC[b*x],x]
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.44

method	result	size
meijerg	$\frac{b\,x^{4} \operatorname{hypergeom}\left(\left[\frac{1}{4},1\right],\left[\frac{1}{2},\frac{5}{4},2\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{4}$	26
parts	$\frac{x^{3} \operatorname{FresnelC}\left(b x\right)}{3}-\frac{b \left(\frac{x^{2} \sin \left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{b^{2} \pi}+\frac{2 \cos \left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{b^{4} \pi^{2}}\right)}{3}$	53
derivativedivides	$\frac{\frac{\operatorname{FresnelC}\left(b x\right) b^{3} x^{3}}{3}-\frac{b^{2} x^{2} \sin \left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{3 \pi}-\frac{2 \cos \left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{3 \pi^{2}}}{b^{3}}$	54
default	$\frac{\frac{\operatorname{FresnelC}\left(b x\right) b^{3} x^{3}}{3}-\frac{b^{2} x^{2} \sin \left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{3 \pi}-\frac{2 \cos \left(\frac{b^{2} \pi \,x^{2}}{2}\right)}{3 \pi^{2}}}{b^{3}}$	54

```
[In] int(x^2*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/4*b*x^4*hypergeom([1/4,1],[1/2,5/4,2],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \frac{\pi^2 b^3 x^3 \operatorname{C}(bx) - \pi b^2 x^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 2\,\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{3\,\pi^2 b^3}$$

```
[In] integrate(x^2*fresnel_cos(b*x),x, algorithm="fricas")
[Out] 1/3*(pi^2*b^3*x^3*fresnel_cos(b*x) - pi*b^2*x^2*sin(1/2*pi*b^2*x^2) - 2*cos
(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \frac{x^3 C(bx)\,\Gamma\left(\frac{1}{4}\right)}{12\Gamma\left(\frac{5}{4}\right)} - \frac{x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{6\pi^2 b^3\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(x**2*fresnelc(b*x),x)
[Out] x**3*fresnelc(b*x)*gamma(1/4)/(12*gamma(5/4)) - x**2*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(12*pi*b*gamma(5/4)) - cos(pi*b**2*x**2/2)*gamma(1/4)/(6*pi**2*b**
3*gamma(5/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \frac{1}{3}\,x^3\,\mathrm{C}\left(bx\right) - \frac{\pi b^2 x^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 2\,\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{3\,\pi^2 b^3}$$

```
[In] integrate(x^2*fresnel_cos(b*x),x, algorithm="maxima")
[Out] 1/3*x^3*fresnel_cos(b*x) - 1/3*(pi*b^2*x^2*sin(1/2*pi*b^2*x^2) + 2*cos(1/2*
pi*b^2*x^2))/(pi^2*b^3)
```

Giac [F]

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \int x^2\,\mathrm{C}\left(bx\right)\,dx$$

```
[In] integrate(x^2*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \int x^2\,\operatorname{FresnelC}(b\,x)\,dx$$

```
[In] int(x^2*FresnelC(b*x),x)
[Out] int(x^2*FresnelC(b*x), x)
```

3.116 $\int x \operatorname{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 49

$$\int x \operatorname{FresnelC}(bx)\,dx = \frac{1}{2}x^2 \operatorname{FresnelC}(bx) + \frac{\operatorname{FresnelS}(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

```
[Out] 1/2*x^2*FresnelC(b*x)+1/2*FresnelS(b*x)/b^2/Pi-1/2*x*sin(1/2*b^2*Pi*x^2)/b/
Pi
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3467, 3432}

$$\int x \operatorname{FresnelC}(bx)\,dx = \frac{\operatorname{FresnelS}(bx)}{2\pi b^2} - \frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{1}{2}x^2 \operatorname{FresnelC}(bx)$$

```
[In] Int[x*FresnelC[b*x],x]
```

```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
```

```
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\,\text{FresnelC}(bx) - \frac{1}{2}b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{1}{2}x^2\,\text{FresnelC}(bx) - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}\\ &= \frac{1}{2}x^2\,\text{FresnelC}(bx) + \frac{\text{FresnelS}(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x\,\text{FresnelC}(bx)\,dx = \frac{1}{2}x^2\,\text{FresnelC}(bx) + \frac{\text{FresnelS}(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

```
[In] Integrate[x*FresnelC[b*x],x]
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

method	result	size
meijerg	$\frac{b\,x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{7}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{3}$	26
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^{2}x^{2}}{2}-\frac{bx\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2\pi}+\frac{\mathrm{FresnelS}(bx)}{2\pi}}{b^{2}}$	44
default	$\frac{\frac{\mathrm{FresnelC}(bx)b^{2}x^{2}}{2}-\frac{bx\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2\pi}+\frac{\mathrm{FresnelS}(bx)}{2\pi}}{b^{2}}$	44
parts	$\frac{x^{2}\,\mathrm{FresnelC}(bx)}{2}-\frac{b\left(\frac{x\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}-\frac{\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}\right)}{2}$	64

```
[In] int(x*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x^3*hypergeom([1/4,3/4],[1/2,5/4,7/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int x\,\mathrm{FresnelC}(bx)\,dx=\frac{\pi b^{3}x^{2}\,\mathrm{C}\left(bx\right)-b^{2}x\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)+\sqrt{b^{2}}\,\mathrm{S}\left(\sqrt{b^{2}}x\right)}{2\,\pi b^{3}}$$

```
[In] integrate(x*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^3*x^2*fresnel_cos(b*x) - b^2*x*sin(1/2*pi*b^2*x^2) + sqrt(b^2)*fr
esnel_sin(sqrt(b^2)*x))/(pi*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x\,\mathrm{FresnelC}(bx)\,dx=\frac{bx^{3}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x*fresnelc(b*x),x)
```

```
[Out] b*x**3*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -pi**2*b**4
*x**4/16)/(16*gamma(5/4)*gamma(7/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int x\,\mathrm{FresnelC}(bx)\,dx = \frac{1}{2}\,x^2\,\mathrm{C}\,(bx) - \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right) - (i+1)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) + (i-1)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right)\right)}{4\,\pi^2b^2}$$

```
[In] integrate(x*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*fresnel_cos(b*x) - 1/4*sqrt(1/2)*(4*sqrt(1/2)*pi*b*x*sin(1/2*pi*b^2
*x^2) - (I + 1)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) + (I - 1)*(1/4)^(1/4
)*pi*erf(sqrt(-1/2*I*pi)*b*x))/(pi^2*b^2)
```

Giac [F]

$$\int x\,\mathrm{FresnelC}(bx)\,dx = \int x\,\mathrm{C}\,(bx)\,dx$$

```
[In] integrate(x*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\,\mathrm{FresnelC}(bx)\,dx = \int x\,\mathrm{FresnelC}(b\,x)\,dx$$

```
[In] int(x*FresnelC(b*x),x)
```

```
[Out] int(x*FresnelC(b*x), x)
```

3.117 $\int \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 27

$$\int \text{FresnelC}(bx)\,dx = x\,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[Out] x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6554}

$$\int \text{FresnelC}(bx)\,dx = x\,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}$$

```
[In] Int[FresnelC[b*x],x]
```

```
[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = x\,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)\,dx = x\,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[In] Integrate[FresnelC[b*x],x]
[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
parts	$x\ \text{FresnelC}\left(bx\right)-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{b\pi}$	26
derivativedivides	$\frac{\text{FresnelC}(bx)bx-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}}{b}$	28
default	$\frac{\text{FresnelC}(bx)bx-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}}{b}$	28
meijerg	$\frac{-\frac{4\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\sqrt{\pi}}+4\sqrt{\pi}\,bx\ \text{FresnelC}(bx)}{4b\sqrt{\pi}}$	36

```
[In] int(FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/b/Pi
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \text{FresnelC}(bx)\,dx = \frac{\pi bx\,\text{C}\left(bx\right) - \sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi b}$$

```
[In] integrate(fresnel_cos(b*x),x, algorithm="fricas")
[Out] (pi*b*x*fresnel_cos(b*x) - sin(1/2*pi*b^2*x^2))/(pi*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \text{FresnelC}(bx)\,dx = \frac{xC(bx)\,\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{4\pi b\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x),x)
[Out] x*fresnelc(b*x)*gamma(1/4)/(4*gamma(5/4)) - sin(pi*b**2*x**2/2)*gamma(1/4)/
(4*pi*b*gamma(5/4))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)\,dx = \frac{bx\,\text{C}\,(bx) - \frac{\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi}}{b}$$

```
[In] integrate(fresnel_cos(b*x),x, algorithm="maxima")
[Out] (b*x*fresnel_cos(b*x) - sin(1/2*pi*b^2*x^2)/pi)/b
```

Giac [F]

$$\int \text{FresnelC}(bx)\,dx = \int \text{C}\,(bx)\;dx$$

```
[In] integrate(fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx)\,dx = \int \text{FresnelC}(b\,x)\,dx$$

```
[In] int(FresnelC(b*x),x)
[Out] int(FresnelC(b*x), x)
```

3.118 $\int \frac{\text{FresnelC}(bx)}{x} dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\text{FresnelC}(bx)}{x} dx = \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)+1/2*b*x*hypergeo
m([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6560, 6493, 6495}

$$\int \frac{\text{FresnelC}(bx)}{x} dx = \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[In] Int[FresnelC[b*x]/x,x]
```

```
[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-1/2*I)*b^2*Pi*x^2])/2 + (b
*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2
```

Rule 6493

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

Rule 6495

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

Rule 6560

```
Int[FresnelC[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 - I)/4, Int[Erf[(Sqrt[P
i]/2)*(1 + I)*b*x]/x, x], x] + Dist[(1 + I)/4, Int[Erf[(Sqrt[Pi]/2)*(1 - I)
*b*x]/x, x], x] /; FreeQ[b, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\frac{1}{4}-\frac{i}{4}\right)\int\frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int\frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}\,dx\\ &= \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F]

$$\int\frac{\operatorname{FresnelC}(bx)}{x}\,dx=\int\frac{\operatorname{FresnelC}(bx)}{x}\,dx$$

```
[In] Integrate[FresnelC[b*x]/x,x]
[Out] Integrate[FresnelC[b*x]/x, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.33

method	result	size
meijerg	$bx\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)$	23

```
[In] int(FresnelC(b*x)/x,x,method=_RETURNVERBOSE)
[Out] b*x*hypergeom([1/4,1/4],[1/2,5/4,5/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [F]

$$\int \frac{\text{FresnelC}(bx)}{x} \, dx = \int \frac{\mathrm{C}\left(bx\right)}{x} \, dx$$

```
[In] integrate(fresnel_cos(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)/x, x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{\text{FresnelC}(bx)}{x} \, dx = \frac{bx\Gamma^2\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{5}{4}, \frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16\Gamma^2\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x,x)
```

```
[Out] b*x*gamma(1/4)**2*hyper((1/4, 1/4), (1/2, 5/4, 5/4), -pi**2*b**4*x**4/16)/(
16*gamma(5/4)**2)
```

Maxima [F]

$$\int \frac{\text{FresnelC}(bx)}{x} \, dx = \int \frac{\mathrm{C}\left(bx\right)}{x} \, dx$$

```
[In] integrate(fresnel_cos(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)/x, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x} \, dx = \int \frac{\mathrm{C}\left(bx\right)}{x} \, dx$$

```
[In] integrate(fresnel_cos(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x} \, dx = \int \frac{\text{FresnelC}(b\,x)}{x} \, dx$$

```
[In] int(FresnelC(b*x)/x,x)
[Out] int(FresnelC(b*x)/x, x)
```

3.119 $\int \frac{\text{FresnelC}(bx)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \frac{1}{2}b\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

```
[Out] 1/2*b*Ci(1/2*b^2*Pi*x^2)-FresnelC(b*x)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6562, 3457}

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \frac{1}{2}b\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

```
[In] Int[FresnelC[b*x]/x^2,x]
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelC}(bx)}{x} + b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$
$$= \frac{1}{2}b\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \frac{1}{2}b\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

```
[In] Integrate[FresnelC[b*x]/x^2,x]
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{b\,\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2} - \frac{\text{FresnelC}(bx)}{x}$	24
derivativedivides	$b\left(-\frac{\text{FresnelC}(bx)}{bx} + \frac{\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
default	$b\left(-\frac{\text{FresnelC}(bx)}{bx} + \frac{\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
meijerg	$\frac{b\sqrt{\pi}\left(-\frac{\pi^{\frac{3}{2}} x^4 b^4\,\text{hypergeom}\left(\left[1,1,\frac{5}{4}\right],\left[\frac{3}{2},2,2,\frac{9}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{10} + \frac{8\gamma-8\ln(2)-16+16\ln(x)+8\ln(\pi)+16\ln(b)}{\sqrt{\pi}}\right)}{16}$	66

```
[In] int(FresnelC(b*x)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b*Ci(1/2*b^2*Pi*x^2)-FresnelC(b*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \frac{bx\,\text{Ci}\left(\frac{1}{2}\,\pi b^2x^2\right) - 2\ \text{C}\,(bx)}{2\,x}$$

```
[In] integrate(fresnel_cos(b*x)/x^2,x, algorithm="fricas")
[Out] 1/2*(b*x*cos_integral(1/2*pi*b^2*x^2) - 2*fresnel_cos(b*x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(20) = 40.

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = -\frac{\pi^2b^5x^4\Gamma\left(\frac{5}{4}\right)\,{}_3F_4\left(\begin{matrix}1,1,\frac{5}{4}\\ \frac{3}{2},2,2,\frac{9}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{128\Gamma\left(\frac{9}{4}\right)} + \frac{b\log\left(b^4x^4\right)}{4}$$

```
[In] integrate(fresnelc(b*x)/x**2,x)
[Out] -pi**2*b**5*x**4*gamma(5/4)*hyper((1, 1, 5/4), (3/2, 2, 2, 9/4), -pi**2*b**
4*x**4/16)/(128*gamma(9/4)) + b*log(b**4*x**4)/4
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \frac{1}{4}\,b\left(\text{Ei}\left(\frac{1}{2}i\,\pi b^2x^2\right) + \text{Ei}\left(-\frac{1}{2}i\,\pi b^2x^2\right)\right) - \frac{\text{C}\,(bx)}{x}$$

```
[In] integrate(fresnel_cos(b*x)/x^2,x, algorithm="maxima")
[Out] 1/4*b*(Ei(1/2*I*pi*b^2*x^2) + Ei(-1/2*I*pi*b^2*x^2)) - fresnel_cos(b*x)/x
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^2}\, dx = \int \frac{\text{C}\,(bx)}{x^2}\, dx$$

```
[In] integrate(fresnel_cos(b*x)/x^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^2}\, dx = \int \frac{\text{FresnelC}(b\,x)}{x^2}\, dx$$

```
[In] int(FresnelC(b*x)/x^2,x)
[Out] int(FresnelC(b*x)/x^2, x)
```

3.120 $\int \frac{\text{FresnelC}(bx)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \frac{\text{FresnelC}(bx)}{x^3} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}b^2\pi \,\text{FresnelS}(bx)$$

```
[Out] -1/2*b*cos(1/2*b^2*Pi*x^2)/x-1/2*FresnelC(b*x)/x^2-1/2*b^2*Pi*FresnelS(b*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6562, 3469, 3432}

$$\int \frac{\text{FresnelC}(bx)}{x^3} dx = -\frac{1}{2}\pi b^2 \,\text{FresnelS}(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2}$$

```
[In] Int[FresnelC[b*x]/x^3,x]
[Out] -1/2*(b*Cos[(b^2*Pi*x^2)/2])/x - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
```

```
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{2x^2} + \frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}\left(b^3\pi\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}b^2\pi\,\text{FresnelS}(bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}b^2\pi\,\text{FresnelS}(bx)$$

```
[In] Integrate[FresnelC[b*x]/x^3,x]
[Out] -1/2*(b*Cos[(b^2*Pi*x^2)/2])/x - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{x}$	26
derivativedivides	$b^2\left(-\frac{\mathrm{FresnelC}(bx)}{2b^2x^2}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{2bx}-\frac{\pi\,\mathrm{FresnelS}(bx)}{2}\right)$	43
default	$b^2\left(-\frac{\mathrm{FresnelC}(bx)}{2b^2x^2}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{2bx}-\frac{\pi\,\mathrm{FresnelS}(bx)}{2}\right)$	43
parts	$-\frac{\mathrm{FresnelC}(bx)}{2x^2}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{x}-\frac{b^2\pi^{\frac{3}{2}}\,\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{2}$	61

```
[In] int(FresnelC(b*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/x*hypergeom([-1/4,1/4],[1/2,3/4,5/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int\frac{\mathrm{FresnelC}(bx)}{x^3}\,dx=-\frac{\pi\sqrt{b^2}bx^2\,\mathrm{S}\left(\sqrt{b^2}x\right)+bx\cos\left(\frac{1}{2}\pi b^2x^2\right)+\mathrm{C}\left(bx\right)}{2\,x^2}$$

```
[In] integrate(fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(pi*sqrt(b^2)*b*x^2*fresnel_sin(sqrt(b^2)*x) + b*x*cos(1/2*pi*b^2*x^2)
 + fresnel_cos(b*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int\frac{\mathrm{FresnelC}(bx)}{x^3}\,dx=\frac{b\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x**3,x)
```

```
[Out] b*gamma(-1/4)*gamma(1/4)*hyper((-1/4, 1/4), (1/2, 3/4, 5/4), -pi**2*b**4*x*
*4/16)/(16*x*gamma(3/4)*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = -\frac{\sqrt{\frac{1}{2}}\sqrt{\pi x^2}\left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{1}{2}i\pi b^2x^2\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^2}{16\,x}-\frac{\text{C}\left(bx\right)}{2\,x^2}$$

```
[In] integrate(fresnel_cos(b*x)/x^3,x, algorithm="maxima")
[Out] -1/16*sqrt(1/2)*sqrt(pi*x^2)*((I + 1)*sqrt(2)*gamma(-1/2, 1/2*I*pi*b^2*x^2)
 - (I - 1)*sqrt(2)*gamma(-1/2, -1/2*I*pi*b^2*x^2))*b^2/x - 1/2*fresnel_cos(
b*x)/x^2
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\text{C}\left(bx\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^3,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^3}\,dx$$

```
[In] int(FresnelC(b*x)/x^3,x)
[Out] int(FresnelC(b*x)/x^3, x)
```

3.121 $\int \frac{\text{FresnelC}(bx)}{x^4} dx$

Optimal result

Integrand size = 8, antiderivative size = 52

$$\int \frac{\text{FresnelC}(bx)}{x^4} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{\text{FresnelC}(bx)}{3x^3} - \frac{1}{12}b^3\pi \text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -1/6*b*cos(1/2*b^2*Pi*x^2)/x^2-1/3*FresnelC(b*x)/x^3-1/12*b^3*Pi*Si(1/2*b^2
*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3461, 3378, 3380}

$$\int \frac{\text{FresnelC}(bx)}{x^4} dx = -\frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^2} - \frac{1}{12}\pi b^3 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{3x^3}$$

```
[In] Int[FresnelC[b*x]/x^4,x]
```

```
[Out] -1/6*(b*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{3x^3} + \frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{\text{FresnelC}(bx)}{3x^3} + \frac{1}{6}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{\text{FresnelC}(bx)}{3x^3} - \frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{\text{FresnelC}(bx)}{3x^3} - \frac{1}{12}b^3\pi\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2} - \frac{\text{FresnelC}(bx)}{3x^3} - \frac{1}{12}b^3\pi\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[In] Integrate[FresnelC[b*x]/x^4,x]
[Out] -1/6*(b*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{2x^2}$	26
parts	$-\frac{\mathrm{FresnelC}(bx)}{3x^3}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{3}$	46
derivativedivides	$b^3\left(-\frac{\mathrm{FresnelC}(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
default	$b^3\left(-\frac{\mathrm{FresnelC}(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49

```
[In] int(FresnelC(b*x)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2*b/x^2*hypergeom([-1/2,1/4],[1/2,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\mathrm{FresnelC}(bx)}{x^4}\,dx=-\frac{\pi b^3x^3\,\mathrm{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)+2\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,\mathrm{C}\left(bx\right)}{12\,x^3}$$

```
[In] integrate(fresnel_cos(b*x)/x^4,x, algorithm="fricas")
[Out] -1/12*(pi*b^3*x^3*sin_integral(1/2*pi*b^2*x^2) + 2*b*x*cos(1/2*pi*b^2*x^2)
+ 4*fresnel_cos(b*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\mathrm{FresnelC}(bx)}{x^4}\,dx=-\frac{b\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2},\frac{1}{2},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{8x^2\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x**4,x)
[Out] -b*gamma(1/4)*hyper((-1/2, 1/4), (1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(8*x
**2*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = -\frac{1}{24}\left(i\,\pi\Gamma\left(-1,\frac{1}{2}i\,\pi b^2x^2\right) - i\,\pi\Gamma\left(-1,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^3 - \frac{\mathrm{C}\left(bx\right)}{3\,x^3}$$

```
[In] integrate(fresnel_cos(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/24*(I*pi*gamma(-1, 1/2*I*pi*b^2*x^2) - I*pi*gamma(-1, -1/2*I*pi*b^2*x^2)
)*b^3 - 1/3*fresnel_cos(b*x)/x^3
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = \int \frac{\mathrm{C}\left(bx\right)}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^4}\,dx$$

```
[In] int(FresnelC(b*x)/x^4,x)
```

```
[Out] int(FresnelC(b*x)/x^4, x)
```

3.122 $\int \frac{\text{FresnelC}(bx)}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\text{FresnelC}(bx)}{x^5} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{4x^4} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}$$

```
[Out] -1/12*b*cos(1/2*b^2*Pi*x^2)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)-1/4*FresnelC(b*
x)/x^4+1/12*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3469, 3468, 3433}

$$\int \frac{\text{FresnelC}(bx)}{x^5} dx = -\frac{1}{12}\pi^2 b^4\,\text{FresnelC}(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3} + \frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x} - \frac{\text{FresnelC}(bx)}{4x^4}$$

```
[In] Int[FresnelC[b*x]/x^5,x]
```

```
[Out] -1/12*(b*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{4x^4} + \frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{\text{FresnelC}(bx)}{4x^4} - \frac{1}{12}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{\text{FresnelC}(bx)}{4x^4} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}\left(b^5\pi^2\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{4x^4} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^5}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{4x^4} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}$$

```
[In] Integrate[FresnelC[b*x]/x^5,x]
```

```
[Out] -1/12*(b*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$b^{4}\left(-\frac{\operatorname{FresnelC}(bx)}{4b^{4}x^{4}}-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{12b^{3}x^{3}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}+\pi\operatorname{FresnelC}(bx)\right)}{12}\right)$	64
default	$b^{4}\left(-\frac{\operatorname{FresnelC}(bx)}{4b^{4}x^{4}}-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{12b^{3}x^{3}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}+\pi\operatorname{FresnelC}(bx)\right)}{12}\right)$	64
meijerg	$\frac{\pi^{2}b^{4}\left(-\frac{32\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3\pi^{2}x^{3}b^{3}}+\frac{32\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3\pi xb}-\frac{32\left(x^{4}\pi^{2}b^{4}+3\right)\operatorname{FresnelC}(bx)}{3\pi^{2}x^{4}b^{4}}\right)}{128}$	79
parts	$-\frac{\operatorname{FresnelC}(bx)}{4x^{4}}+\frac{b\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3x^{3}}-\frac{b^{2}\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{x}+\frac{b^{2}\pi^{\frac{3}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{\sqrt{b^{2}\pi}}\right)}{3}\right)}{4}$	82

```
[In] int(FresnelC(b*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-1/4*FresnelC(b*x)/b^4/x^4-1/12/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/12*Pi*(-
1/b/x*sin(1/2*b^2*Pi*x^2)+Pi*FresnelC(b*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{FresnelC}(bx)}{x^{5}}\,dx=\frac{\pi b^{3}x^{3}\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-bx\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-\left(\pi^{2}b^{4}x^{4}+3\right)\mathrm{C}\left(bx\right)}{12\,x^{4}}$$

```
[In] integrate(fresnel_cos(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(pi*b^3*x^3*sin(1/2*pi*b^2*x^2) - b*x*cos(1/2*pi*b^2*x^2) - (pi^2*b^4*
x^4 + 3)*fresnel_cos(b*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\text{FresnelC}(bx)}{x^5}\,dx = \frac{\pi^2 b^4 C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64\Gamma\left(\frac{5}{4}\right)} - \frac{\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x\Gamma\left(\frac{5}{4}\right)} + \frac{b\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64x^4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x**5,x)
[Out] pi**2*b**4*fresnelc(b*x)*gamma(-3/4)/(64*gamma(5/4)) - pi*b**3*sin(pi*b**2*
x**2/2)*gamma(-3/4)/(64*x*gamma(5/4)) + b*cos(pi*b**2*x**2/2)*gamma(-3/4)/(
64*x**3*gamma(5/4)) + 3*fresnelc(b*x)*gamma(-3/4)/(64*x**4*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelC}(bx)}{x^5}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{3}{2}}\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{1}{2}i\pi b^2x^2\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^4}{64\,x^3} - \frac{\mathrm{C}\left(bx\right)}{4\,x^4}$$

```
[In] integrate(fresnel_cos(b*x)/x^5,x, algorithm="maxima")
[Out] -1/64*sqrt(1/2)*(pi*x^2)^(3/2)*((I - 1)*sqrt(2)*gamma(-3/2, 1/2*I*pi*b^2*x^
2) - (I + 1)*sqrt(2)*gamma(-3/2, -1/2*I*pi*b^2*x^2))*b^4/x^3 - 1/4*fresnel_
cos(b*x)/x^4
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\mathrm{C}\left(bx\right)}{x^5}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^5,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^5} \, dx = \int \frac{\text{FresnelC}(b\,x)}{x^5} \, dx$$

```
[In] int(FresnelC(b*x)/x^5,x)
[Out] int(FresnelC(b*x)/x^5, x)
```

3.123 $\int \frac{\text{FresnelC}(bx)}{x^6} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 77

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2 \text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}$$

```
[Out] -1/80*b^5*Pi^2*Ci(1/2*b^2*Pi*x^2)-1/20*b*cos(1/2*b^2*Pi*x^2)/x^4-1/5*Fresne
lC(b*x)/x^5+1/40*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3461, 3378, 3383}

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = -\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4} - \frac{1}{80}\pi^2b^5 \text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2} - \frac{\text{FresnelC}(bx)}{5x^5}$$

```
[In] Int[FresnelC[b*x]/x^6,x]
```

```
[Out] -1/20*(b*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{5x^5} + \frac{1}{5}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{\text{FresnelC}(bx)}{5x^5} + \frac{1}{10}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{\text{FresnelC}(bx)}{5x^5} - \frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{\text{FresnelC}(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} \\
&\quad - \frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^6}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}$$

```
[In] Integrate[FresnelC[b*x]/x^6,x]
```

```
[Out] -1/20*(b*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
parts	$-\frac{\text{FresnelC}(bx)}{5x^5}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}+\frac{b^2\pi\,\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{5}$	68
derivativedivides	$b^5\left(-\frac{\text{FresnelC}(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71
default	$b^5\left(-\frac{\text{FresnelC}(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71
meijerg	$\frac{\pi^{\frac{5}{2}}b^5\left(\frac{\pi^{\frac{3}{2}}x^4b^4\,\text{hypergeom}\left(\left[1,1,\frac{9}{4}\right],\left[2,\frac{5}{2},3,\frac{13}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{54}-\frac{8\left(-\frac{19}{5}+2\gamma-2\ln(2)+4\ln(x)+2\ln(\pi)+4\ln(b)\right)}{5\sqrt{\pi}}-\frac{64}{\pi^{\frac{5}{2}}x^4b^4}\right)}{256}$	79

```
[In] int(FresnelC(b*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*FresnelC(b*x)/x^5+1/5*b*(-1/4/x^4*cos(1/2*b^2*Pi*x^2)-1/4*b^2*Pi*(-1/2
*sin(1/2*b^2*Pi*x^2)/x^2+1/4*b^2*Pi*Ci(1/2*b^2*Pi*x^2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = -\frac{\pi^2 b^5 x^5 \operatorname{Ci}\left(\frac{1}{2}\pi b^2 x^2\right) - 2\pi b^3 x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right) + 4\, bx \cos\left(\frac{1}{2}\pi b^2 x^2\right) + 16\, \operatorname{C}(bx)}{80\, x^5}$$

```
[In] integrate(fresnel_cos(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/80*(pi^2*b^5*x^5*cos_integral(1/2*pi*b^2*x^2) - 2*pi*b^3*x^3*sin(1/2*pi*
b^2*x^2) + 4*b*x*cos(1/2*pi*b^2*x^2) + 16*fresnel_cos(b*x))/x^5
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = \frac{\pi^4 b^9 x^4 \Gamma\left(\frac{9}{4}\right) {}_{3}F_{4}\left(\begin{matrix} 1, 1, \frac{9}{4} \\ 2, \frac{5}{2}, 3, \frac{13}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{6144\Gamma\left(\frac{13}{4}\right)} - \frac{\pi^2 b^5 \log\left(b^4 x^4\right)}{160} - \frac{b}{4x^4}$$

```
[In] integrate(fresnelc(b*x)/x**6,x)
```

```
[Out] pi**4*b**9*x**4*gamma(9/4)*hyper((1, 1, 9/4), (2, 5/2, 3, 13/4), -pi**2*b**
4*x**4/16)/(6144*gamma(13/4)) - pi**2*b**5*log(b**4*x**4)/160 - b/(4*x**4)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = \frac{1}{80} \left(\pi^2 \Gamma\left(-2, \frac{1}{2} i\, \pi b^2 x^2\right) + \pi^2 \Gamma\left(-2, -\frac{1}{2} i\, \pi b^2 x^2\right)\right) b^5 - \frac{\operatorname{C}(bx)}{5\, x^5}$$

```
[In] integrate(fresnel_cos(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/80*(pi^2*gamma(-2, 1/2*I*pi*b^2*x^2) + pi^2*gamma(-2, -1/2*I*pi*b^2*x^2))
*b^5 - 1/5*fresnel_cos(b*x)/x^5
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{C(bx)}{x^6}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^6,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^6}\,dx$$

```
[In] int(FresnelC(b*x)/x^6,x)
[Out] int(FresnelC(b*x)/x^6, x)
```

3.124 $\int \frac{\text{FresnelC}(bx)}{x^7} dx$

Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \frac{\text{FresnelC}(bx)}{x^7} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{\text{FresnelC}(bx)}{6x^6} + \frac{1}{90}b^6\pi^3\,\text{FresnelS}(bx) + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}$$

```
[Out] -1/30*b*cos(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x-1/6*Fre
snelC(b*x)/x^6+1/90*b^6*Pi^3*FresnelS(b*x)+1/90*b^3*Pi*sin(1/2*b^2*Pi*x^2)/
x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3469, 3468, 3432}

$$\int \frac{\text{FresnelC}(bx)}{x^7} dx = \frac{1}{90}\pi^3 b^6\,\text{FresnelS}(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{30x^5} + \frac{\pi^2 b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{90x} + \frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{90x^3} - \frac{\text{FresnelC}(bx)}{6x^6}$$

```
[In] Int[FresnelC[b*x]/x^7,x]
```

```
[Out] -1/30*(b*Cos[(b^2*Pi*x^2)/2])/x^5 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(90*x) -
 FresnelC[b*x]/(6*x^6) + (b^6*Pi^3*FresnelS[b*x])/90 + (b^3*Pi*Sin[(b^2*Pi*
x^2)/2])/(90*x^3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{6x^6}+\frac{1}{6}b\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{\text{FresnelC}(bx)}{6x^6}-\frac{1}{30}\left(b^3\pi\right)\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{\text{FresnelC}(bx)}{6x^6}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}\left(b^5\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{\text{FresnelC}(bx)}{6x^6}\\
&\quad+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}+\frac{1}{90}\left(b^7\pi^3\right)\int\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{\text{FresnelC}(bx)}{6x^6}\\
&\quad+\frac{1}{90}b^6\pi^3\,\text{FresnelS}(bx)+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{\text{FresnelC}(bx)}{x^7} dx = \frac{1}{90}\left(\frac{b(-3+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} - \frac{15\,\text{FresnelC}(bx)}{x^6} + b^6\pi^3\,\text{FresnelS}(bx) + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\right)$$

```
[In] Integrate[FresnelC[b*x]/x^7,x]
[Out] ((b*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^5 - (15*FresnelC[b*x])/x^6 +
 b^6*Pi^3*FresnelS[b*x] + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^3)/90
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{4}\right],\left[-\frac{1}{4},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{5x^5}$	26
derivativedivides	$b^6\left(-\frac{\mathrm{FresnelC}(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{FresnelS}(bx)\right)}{3}\right)}{30}\right)$	87
default	$b^6\left(-\frac{\mathrm{FresnelC}(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{FresnelS}(bx)\right)}{3}\right)}{30}\right)$	87
parts	$-\frac{\mathrm{FresnelC}(bx)}{6x^6}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{5x^5}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{x}-\frac{b^2\pi^{\frac{3}{2}}\,\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{3}\right)}{5}\right)}{6}$	105

```
[In] int(FresnelC(b*x)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/5*b/x^5*hypergeom([-5/4,1/4],[-1/4,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \frac{\mathrm{FresnelC}(bx)}{x^7}\,dx$$
$$=\frac{\pi^3\sqrt{b^2}b^5x^6\,\mathrm{S}\left(\sqrt{b^2}x\right)+\pi b^3x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)+\left(\pi^2b^5x^5-3\,bx\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-15\,\mathrm{C}\left(bx\right)}{90\,x^6}$$

```
[In] integrate(fresnel_cos(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] 1/90*(pi^3*sqrt(b^2)*b^5*x^6*fresnel_sin(sqrt(b^2)*x) + pi*b^3*x^3*sin(1/2*
pi*b^2*x^2) + (pi^2*b^5*x^5 - 3*b*x)*cos(1/2*pi*b^2*x^2) - 15*fresnel_cos(b
*x))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \frac{b\Gamma\left(-\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{5}{4}, \frac{1}{4} \\ -\frac{1}{4}, \frac{1}{2}, \frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16x^5\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x**7,x)
```

```
[Out] b*gamma(-5/4)*gamma(1/4)*hyper((-5/4, 1/4), (-1/4, 1/2, 5/4), -pi**2*b**4*x
**4/16)/(16*x**5*gamma(-1/4)*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{5}{2}}\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2}, \frac{1}{2}i\pi b^2x^2\right) + (i-1)\sqrt{2}\Gamma\left(-\frac{5}{2}, -\frac{1}{2}i\pi b^2x^2\right)\right)b^6}{192\,x^5} - \frac{\mathrm{C}\left(bx\right)}{6\,x^6}$$

```
[In] integrate(fresnel_cos(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] -1/192*sqrt(1/2)*(pi*x^2)^(5/2)*(-(I + 1)*sqrt(2)*gamma(-5/2, 1/2*I*pi*b^2*
x^2) + (I - 1)*sqrt(2)*gamma(-5/2, -1/2*I*pi*b^2*x^2))*b^6/x^5 - 1/6*fresne
l_cos(b*x)/x^6
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\text{C}\,(bx)}{x^7}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^7,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^7}\,dx$$

```
[In] int(FresnelC(b*x)/x^7,x)
[Out] int(FresnelC(b*x)/x^7, x)
```

3.125 $\int \frac{\text{FresnelC}(bx)}{x^8} dx$

Optimal result

Integrand size = 8, antiderivative size = 102

$$\int \frac{\text{FresnelC}(bx)}{x^8} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{\text{FresnelC}(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -1/42*b*cos(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^2-1/7*
FresnelC(b*x)/x^7+1/672*b^7*Pi^3*Si(1/2*b^2*Pi*x^2)+1/168*b^3*Pi*sin(1/2*b^
2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3461, 3378, 3380}

$$\int \frac{\text{FresnelC}(bx)}{x^8} dx = -\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6} + \frac{1}{672}\pi^3b^7\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2} + \frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4} - \frac{\text{FresnelC}(bx)}{7x^7}$$

```
[In] Int[FresnelC[b*x]/x^8,x]
```

```
[Out] -1/42*(b*Cos[(b^2*Pi*x^2)/2])/x^6 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(336*x^2
) - FresnelC[b*x]/(7*x^7) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(168*x^4) + (b^7*P
i^3*SinIntegral[(b^2*Pi*x^2)/2])/672
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{7x^7} + \frac{1}{7}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{\text{FresnelC}(bx)}{7x^7} + \frac{1}{14}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{\text{FresnelC}(bx)}{7x^7} - \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{\text{FresnelC}(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} \\
&\quad - \frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{\text{FresnelC}(bx)}{7x^7} \\
&\quad + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{\text{FresnelC}(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = \frac{1}{672}\left(\frac{2b(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} - \frac{96\,\text{FresnelC}(bx)}{x^7} + \frac{4b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} + b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\right)$$

```
[In] Integrate[FresnelC[b*x]/x^8,x]
```

```
[Out] ((2*b*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (96*FresnelC[b*x])/x^7
 + (4*b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^4 + b^7*Pi^3*SinIntegral[(b^2*Pi*x^2)/2
])/672
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.25

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{6x^6}$	26
parts	$-\frac{\mathrm{FresnelC}(bx)}{7x^7}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6x^6}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{7}$	90
derivativedivides	$b^7\left(-\frac{\mathrm{FresnelC}(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93
default	$b^7\left(-\frac{\mathrm{FresnelC}(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93

```
[In] int(FresnelC(b*x)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/6*b/x^6*hypergeom([-3/2,1/4],[-1/2,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{\mathrm{FresnelC}(bx)}{x^8}\,dx = \frac{\pi^3 b^7 x^7\,\mathrm{Si}\left(\frac{1}{2}\pi b^2 x^2\right)+4\,\pi b^3 x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)+2\left(\pi^2 b^5 x^5-8\,bx\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right)-96\ \mathrm{C}\left(bx\right)}{672\,x^7}$$

```
[In] integrate(fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] 1/672*(pi^3*b^7*x^7*sin_integral(1/2*pi*b^2*x^2) + 4*pi*b^3*x^3*sin(1/2*pi*
b^2*x^2) + 2*(pi^2*b^5*x^5 - 8*b*x)*cos(1/2*pi*b^2*x^2) - 96*fresnel_cos(b*
x))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = -\frac{b\Gamma\left(\frac{1}{4}\right)\,{}_2F_3\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2},\frac{1}{2},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{24x^6\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x**8,x)
```

```
[Out] -b*gamma(1/4)*hyper((-3/2, 1/4), (-1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(24
*x**6*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.47

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = -\frac{1}{224}\left(-i\,\pi^3\Gamma\left(-3,\frac{1}{2}i\,\pi b^2x^2\right)+i\,\pi^3\Gamma\left(-3,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^7-\frac{\mathrm{C}\,(bx)}{7\,x^7}$$

```
[In] integrate(fresnel_cos(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] -1/224*(-I*pi^3*gamma(-3, 1/2*I*pi*b^2*x^2) + I*pi^3*gamma(-3, -1/2*I*pi*b^
2*x^2))*b^7 - 1/7*fresnel_cos(b*x)/x^7
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\mathrm{C}\,(bx)}{x^8}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^8}\,dx$$

```
[In] int(FresnelC(b*x)/x^8,x)
```

```
[Out] int(FresnelC(b*x)/x^8, x)
```

3.126 $\int \frac{\text{FresnelC}(bx)}{x^9} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 119

$$\int \frac{\text{FresnelC}(bx)}{x^9} \, dx = -\frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}b^8\pi^4 \text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{8x^8} + \frac{b^3\pi \sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}$$

```
[Out] -1/56*b*cos(1/2*b^2*Pi*x^2)/x^7+1/840*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^3+1/84
0*b^8*Pi^4*FresnelC(b*x)-1/8*FresnelC(b*x)/x^8+1/280*b^3*Pi*sin(1/2*b^2*Pi*
x^2)/x^5-1/840*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3469, 3468, 3433}

$$\int \frac{\text{FresnelC}(bx)}{x^9} \, dx = \frac{1}{840}\pi^4 b^8 \text{FresnelC}(bx) - \frac{b \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} - \frac{\text{FresnelC}(bx)}{8x^8}$$

```
[In] Int[FresnelC[b*x]/x^9,x]
```

```
[Out] -1/56*(b*Cos[(b^2*Pi*x^2)/2])/x^7 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(840*x^3
) + (b^8*Pi^4*FresnelC[b*x])/840 - FresnelC[b*x]/(8*x^8) + (b^3*Pi*Sin[(b^2
*Pi*x^2)/2])/(280*x^5) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(840*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{8x^8} + \frac{1}{8}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{\text{FresnelC}(bx)}{8x^8} - \frac{1}{56}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{\text{FresnelC}(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{1}{280}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{\text{FresnelC}(bx)}{8x^8} \\
&\quad + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{1}{840}\left(b^7\pi^3\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{\text{FresnelC}(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} \\
&\quad - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{840}\left(b^9\pi^4\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}b^8\pi^4\,\text{FresnelC}(bx) \\
&\quad - \frac{\text{FresnelC}(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx$$
$$= \frac{bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+(-105+b^8\pi^4x^8)\,\text{FresnelC}(bx)+b^3\pi x^3(3-b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^8}$$

```
[In] Integrate[FresnelC[b*x]/x^9,x]
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fresn
elC[b*x] + b^3*Pi*x^3*(3 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

method	result
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{7}{4},\frac{1}{4}\right],\left[-\frac{3}{4},\frac{1}{2},\frac{5}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{7x^{7}}$
derivativedivides	$b^{8}\left(-\frac{\mathrm{FresnelC}(bx)}{8b^{8}x^{8}}-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{56b^{7}x^{7}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5b^{5}x^{5}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3b^{3}x^{3}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}+\pi\,\mathrm{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$
default	$b^{8}\left(-\frac{\mathrm{FresnelC}(bx)}{8b^{8}x^{8}}-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{56b^{7}x^{7}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5b^{5}x^{5}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3b^{3}x^{3}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}+\pi\,\mathrm{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$
	$b\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{7x^{7}}-\frac{b^{2}\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5x^{5}}+\frac{b^{2}\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3x^{3}}-\frac{b^{2}\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{x}+\frac{b^{2}\pi^{\frac{3}{2}}\,\mathrm{FresnelC}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right)}{\sqrt{b^{2}\pi}}\cdots\right)}{3}\cdots\right)}{5}\cdots\right)}{7}\cdots\right.$

```
[In] int(FresnelC(b*x)/x^9,x,method=_RETURNVERBOSE)
[Out] -1/7*b/x^7*hypergeom([-7/4,1/4],[-3/4,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \frac{(\pi^2 b^5 x^5 - 15\,bx)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + (\pi^4 b^8 x^8 - 105)\,\mathrm{C}\,(bx) - (\pi^3 b^7 x^7 - 3\,\pi b^3 x^3)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{840\,x^8}$$

```
[In] integrate(fresnel_cos(b*x)/x^9,x, algorithm="fricas")
[Out] 1/840*((pi^2*b^5*x^5 - 15*b*x)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 - 105)*f
resnel_cos(b*x) - (pi^3*b^7*x^7 - 3*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \frac{\pi^4 b^8 C(bx)\,\Gamma\left(-\frac{7}{4}\right)}{2560\Gamma\left(\frac{5}{4}\right)} - \frac{\pi^3 b^7 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x\Gamma\left(\frac{5}{4}\right)} + \frac{\pi^2 b^5 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^5\Gamma\left(\frac{5}{4}\right)} - \frac{3b\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{512x^7\Gamma\left(\frac{5}{4}\right)} - \frac{21C(bx)\,\Gamma\left(-\frac{7}{4}\right)}{512x^8\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(fresnelc(b*x)/x**9,x)
[Out] pi**4*b**8*fresnelc(b*x)*gamma(-7/4)/(2560*gamma(5/4)) - pi**3*b**7*sin(pi*
b**2*x**2/2)*gamma(-7/4)/(2560*x*gamma(5/4)) + pi**2*b**5*cos(pi*b**2*x**2/
2)*gamma(-7/4)/(2560*x**3*gamma(5/4)) + 3*pi*b**3*sin(pi*b**2*x**2/2)*gamma
(-7/4)/(2560*x**5*gamma(5/4)) - 3*b*cos(pi*b**2*x**2/2)*gamma(-7/4)/(512*x*
*7*gamma(5/4)) - 21*fresnelc(b*x)*gamma(-7/4)/(512*x**8*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{7}{2}}\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2},\frac{1}{2}i\pi b^2x^2\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{7}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^8}{512\,x^7} - \frac{\mathrm{C}\left(bx\right)}{8\,x^8}$$

```
[In] integrate(fresnel_cos(b*x)/x^9,x, algorithm="maxima")
[Out] -1/512*sqrt(1/2)*(pi*x^2)^(7/2)*(-(I - 1)*sqrt(2)*gamma(-7/2, 1/2*I*pi*b^2*
x^2) + (I + 1)*sqrt(2)*gamma(-7/2, -1/2*I*pi*b^2*x^2))*b^8/x^7 - 1/8*fresne
l_cos(b*x)/x^8
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\mathrm{C}\left(bx\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^9,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^9}\,dx$$

```
[In] int(FresnelC(b*x)/x^9,x)
[Out] int(FresnelC(b*x)/x^9, x)
```

3.127 $\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 127

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}-\frac{\text{FresnelC}(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}$$

```
[Out] 1/6912*b^9*Pi^4*Ci(1/2*b^2*Pi*x^2)-1/72*b*cos(1/2*b^2*Pi*x^2)/x^8+1/1728*b^
5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^4-1/9*FresnelC(b*x)/x^9+1/432*b^3*Pi*sin(1/2*b
^2*Pi*x^2)/x^6-1/3456*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3461, 3378, 3383}

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = -\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^4b^9\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}-\frac{\pi^3b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{\text{FresnelC}(bx)}{9x^9}$$

```
[In] Int[FresnelC[b*x]/x^10,x]
```

```
[Out] -1/72*(b*Cos[(b^2*Pi*x^2)/2])/x^8 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(1728*x^
4) + (b^9*Pi^4*CosIntegral[(b^2*Pi*x^2)/2])/6912 - FresnelC[b*x]/(9*x^9) +
(b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(432*x^6) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(34
56*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)}{9x^9}+\frac{1}{9}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx\\
&= -\frac{\text{FresnelC}(bx)}{9x^9}+\frac{1}{18}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{\text{FresnelC}(bx)}{9x^9}-\frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{\text{FresnelC}(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}\\
&\quad -\frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{\text{FresnelC}(bx)}{9x^9}\\
&\quad +\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)}{3456}
\end{aligned}
$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{\text{FresnelC}(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx, x, x^2\right)}{6912}$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)}{6912} - \frac{\text{FresnelC}(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \frac{\frac{4b(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} + b^9\pi^4\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{768\,\text{FresnelC}(bx)}{x^9} - \frac{2b^3\pi(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}}{6912}$$

```
[In] Integrate[FresnelC[b*x]/x^10,x]
[Out] ((4*b*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*CosIntegral[
(b^2*Pi*x^2)/2] - (768*FresnelC[b*x])/x^9 - (2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*S
in[(b^2*Pi*x^2)/2])/x^6)/6912
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
meijerg	$\frac{\pi^{\frac{9}{2}} b^9 \left(-\frac{\pi^{\frac{3}{2}} x^4 b^4 \operatorname{hypergeom}\left(\left[1,1,\frac{13}{4}\right],\left[2,\frac{7}{2},4,\frac{17}{4}\right],-\frac{x^4 \pi^2 b^4}{16}\right)}{585}+\frac{-\frac{332}{243}+\frac{16\gamma}{27}-\frac{16\ln(2)}{27}+\frac{32\ln(x)}{27}+\frac{16\ln(\pi)}{27}+\frac{32\ln(b)}{27}}{\sqrt{\pi}}-\frac{512}{\pi^{\frac{9}{2}} x^8 b^8}+\right.}{4096}$
parts	$-\frac{\operatorname{FresnelC}(bx)}{9x^9}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{8x^8}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6x^6}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}+\frac{b^2\pi \operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right.}{4}\right.}{6}\right.}{8}\right.}{9}$
derivativedivides	$b^9\left(-\frac{\operatorname{FresnelC}(bx)}{9b^9x^9}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi \operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right.}{72}\right.$
default	$b^9\left(-\frac{\operatorname{FresnelC}(bx)}{9b^9x^9}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi \operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right.}{72}\right.$

```
[In] int(FresnelC(b*x)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4096*Pi^(9/2)*b^9*(-1/585*Pi^(3/2)*x^4*b^4*hypergeom([1,1,13/4],[2,7/2,2,4,
17/4],-1/16*x^4*Pi^2*b^4)+8/27*(-83/18+2*gamma-2*ln(2)+4*ln(x)+2*ln(Pi)+4*l
n(b))/Pi^(1/2)-512/Pi^(9/2)/x^8/b^8+128/5/Pi^(5/2)/x^4/b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \frac{\pi^4 b^9 x^9 \operatorname{Ci}\left(\frac{1}{2}\pi b^2 x^2\right) + 4\left(\pi^2 b^5 x^5 - 24\,bx\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) - 2\left(\pi^3 b^7 x^7 - 8\,\pi b^3 x^3\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right) - 768\,\mathrm{C}\left(bx\right)}{6912\,x^9}$$

```
[In] integrate(fresnel_cos(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] 1/6912*(pi^4*b^9*x^9*cos_integral(1/2*pi*b^2*x^2) + 4*(pi^2*b^5*x^5 - 24*b*
x)*cos(1/2*pi*b^2*x^2) - 2*(pi^3*b^7*x^7 - 8*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2
) - 768*fresnel_cos(b*x))/x^9
```

Sympy [A] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = -\frac{\pi^6 b^{13} x^4 \Gamma\left(\frac{13}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{13}{4} \\ 2, \frac{7}{2}, 4, \frac{17}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{737280\Gamma\left(\frac{17}{4}\right)} + \frac{\pi^4 b^9 \log\left(b^4 x^4\right)}{13824} + \frac{\pi^2 b^5}{160x^4} - \frac{b}{8x^8}$$

```
[In] integrate(fresnelc(b*x)/x**10,x)
```

```
[Out] -pi**6*b**13*x**4*gamma(13/4)*hyper((1, 1, 13/4), (2, 7/2, 4, 17/4), -pi**2
*b**4*x**4/16)/(737280*gamma(17/4)) + pi**4*b**9*log(b**4*x**4)/13824 + pi*
*2*b**5/(160*x**4) - b/(8*x**8)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = -\frac{1}{576}\left(\pi^4\Gamma\left(-4, \frac{1}{2}i\,\pi b^2x^2\right) + \pi^4\Gamma\left(-4, -\frac{1}{2}i\,\pi b^2x^2\right)\right)b^9 - \frac{\mathrm{C}\left(bx\right)}{9\,x^9}$$

```
[In] integrate(fresnel_cos(b*x)/x^10,x, algorithm="maxima")
[Out] -1/576*(pi^4*gamma(-4, 1/2*I*pi*b^2*x^2) + pi^4*gamma(-4, -1/2*I*pi*b^2*x^2
))*b^9 - 1/9*fresnel_cos(b*x)/x^9
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\mathrm{C}\left(bx\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)/x^10,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^{10}}\,dx$$

```
[In] int(FresnelC(b*x)/x^10,x)
[Out] int(FresnelC(b*x)/x^10, x)
```

3.128 $\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 298

$$\begin{aligned}\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx = &-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} \\ &-\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} \\ &-\frac{(bc-ad)^4\operatorname{FresnelC}(a+bx)}{4b^4d} \\ &+\frac{3d^3\operatorname{FresnelC}(a+bx)}{4b^4\pi^2}+\frac{(c+dx)^4\operatorname{FresnelC}(a+bx)}{4d} \\ &+\frac{3d(bc-ad)^2\operatorname{FresnelS}(a+bx)}{2b^4\pi} \\ &-\frac{(bc-ad)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} \\ &-\frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} \\ &-\frac{d^2(bc-ad)(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} \\ &-\frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}\end{aligned}$$

```
[Out] -2*d^2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*cos(1/2*Pi
*(b*x+a)^2)/b^4/Pi^2-1/4*(-a*d+b*c)^4*FresnelC(b*x+a)/b^4/d+3/4*d^3*Fresnel
C(b*x+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelC(b*x+a)/d+3/2*d*(-a*d+b*c)^2*Fresne
lS(b*x+a)/b^4/Pi-(-a*d+b*c)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)
^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-d^2*(-a*d+b*c)*(b*x+a)^2*sin(1/2*Pi
*(b*x+a)^2)/b^4/Pi-1/4*d^3*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718, 3466}

$$\begin{aligned}\int (c+dx)^3 \text{FresnelC}(a+bx)\,dx = &-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}\\ &-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}\\ &-\frac{(bc-ad)^4\text{FresnelC}(a+bx)}{4b^4d}\\ &+\frac{3d(bc-ad)^2\text{FresnelS}(a+bx)}{2\pi b^4}\\ &-\frac{(bc-ad)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}\\ &-\frac{3d(a+bx)(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}\\ &+\frac{3d^3\text{FresnelC}(a+bx)}{4\pi^2b^4}-\frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4}\\ &-\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2b^4}\\ &+\frac{(c+dx)^4\text{FresnelC}(a+bx)}{4d}\end{aligned}$$

```
[In] Int[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*
Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - ((b*c - a*d)^4*FresnelC[a + b*x])/(
4*b^4*d) + (3*d^3*FresnelC[a + b*x])/(4*b^4*Pi^2) + ((c + d*x)^4*FresnelC[a
 + b*x])/(4*d) + (3*d*(b*c - a*d)^2*FresnelS[a + b*x])/(2*b^4*Pi) - ((b*c -
 a*d)^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*d*(b*c - a*d)^2*(a + b*x)*Si
n[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (d^2*(b*c - a*d)*(a + b*x)^2*Sin[(Pi*(a
 + b*x)^2)/2])/(b^4*Pi) - (d^3*(a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*
Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{b\int (c+dx)^4 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{4d}$$

$$= \frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{\operatorname{Subst}\left(\int \left(b^4c^4\left(1+\frac{ad(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{b^4c^4}\right)\cos\left(\frac{\pi x^2}{2}\right)+4b^3c^3d\left(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\right) \cdots\right.\right.}{}$$

$$= \frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{d^3\operatorname{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} - \frac{(d^2(bc-ad))\operatorname{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^4} - \frac{(3d(bc-ad)^2)\operatorname{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^4} - \frac{(bc-ad)^3\operatorname{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^4} - \frac{(bc-ad)^4\operatorname{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4d}$$

$$= -\frac{(bc-ad)^4 \operatorname{FresnelC}(a+bx)}{4b^4d} + \frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{(d^2(bc-ad))\operatorname{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{2b^4} - \frac{(bc-ad)^3\operatorname{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{2b^4} + \frac{(3d^3)\operatorname{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4\pi} + \frac{(3d(bc-ad)^2)\operatorname{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^4\pi}$$

$$= -\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4\,\mathrm{FresnelC}(a+bx)}{4b^4d} + \frac{(c+dx)^4\,\mathrm{FresnelC}(a+bx)}{4d} + \frac{3d(bc-ad)^2\,\mathrm{FresnelS}(a+bx)}{2b^4\pi} - \frac{(bc-ad)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{d^2(bc-ad)(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} + \frac{(3d^3)\,\mathrm{Subst}\left(\int\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4\pi^2} + \frac{(d^2(bc-ad))\,\mathrm{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{b^4\pi}$$

$$= -\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4\,\mathrm{FresnelC}(a+bx)}{4b^4d} + \frac{3d^3\,\mathrm{FresnelC}(a+bx)}{4b^4\pi^2} + \frac{(c+dx)^4\,\mathrm{FresnelC}(a+bx)}{4d} + \frac{3d(bc-ad)^2\,\mathrm{FresnelS}(a+bx)}{2b^4\pi} - \frac{(bc-ad)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{d^2(bc-ad)(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.42

$$\int (c+dx)^3\,\mathrm{FresnelC}(a+bx)\,dx = \frac{-8bcd^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 5ad^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 3bd^3x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + (4b^3c^3\pi^2(a+bx) + 6b^2c^2c}{}$$

```
[In] Integrate[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-8*b*c*d^2*Cos[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Cos[(Pi*(a + b*x)^2)/2] - 3*b
*d^3*x*Cos[(Pi*(a + b*x)^2)/2] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi
^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 +
b^4*Pi^2*x^4))*FresnelC[a + b*x] + 6*d*(b*c - a*d)^2*Pi*FresnelS[a + b*x] -
 4*b^3*c^3*Pi*Sin[(Pi*(a + b*x)^2)/2] + 6*a*b^2*c^2*d*Pi*Sin[(Pi*(a + b*x)^
2)/2] - 4*a^2*b*c*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + a^3*d^3*Pi*Sin[(Pi*(a +
b*x)^2)/2] - 6*b^3*c^2*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + 4*a*b^2*c*d^2*Pi*x*
Sin[(Pi*(a + b*x)^2)/2] - a^2*b*d^3*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - 4*b^3*c*
d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*d^3*Pi*x^2*Sin[(Pi*(a + b*x)^2)/
2] - b^3*d^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d}-\frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\text{FresnelC}(bx+a)}{\pi}\right)}{\pi}-\frac{4(ad-bc)}{}}{}}{}$
default	$\frac{\frac{\text{FresnelC}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d}-\frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\text{FresnelC}(bx+a)}{\pi}\right)}{\pi}-\frac{4(ad-bc)}{}}{}}{}$
parts	Expression too large to display

```
[In] int((d*x+c)^3*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*FresnelC(b*x+a)*(a*d-b*c-d*(b*x+a))^4/b^3/d-1/4/b^3/d*(d^4/Pi*(b*x
+a)^3*sin(1/2*Pi*(b*x+a)^2)-3*d^4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1
/Pi*FresnelC(b*x+a))-4*(a*d-b*c)*d^3/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-8*(
a*d-b*c)*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+6*(a*d-b*c)^2*d^2/Pi*(b*x+a)*sin(1/
2*Pi*(b*x+a)^2)-6*(a*d-b*c)^2*d^2/Pi*FresnelS(b*x+a)-4*(a*d-b*c)^3*d/Pi*sin
(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^4*FresnelC(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.26

$$\int (c+dx)^3 \,\text{FresnelC}(a+bx)\,dx$$
$$= \frac{6\,\pi(b^2c^2d-2\,abcd^2+a^2d^3)\sqrt{b^2}\,\text{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(\pi^2(4\,ab^3c^3-6\,a^2b^2c^2d+4\,a^3bcd^2-a^4d^3)+3\,d^3)\sqrt{b^2}\,\text{C}\left(\right.}{}$$

```
[In] integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(6*pi*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(b^2)*fresnel_sin(sqrt(b^
2)*(b*x + a)/b) + (pi^2*(4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 - a^
4*d^3) + 3*d^3)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) - (3*b^2*d^3*x
 + 8*b^2*c*d^2 - 5*a*b*d^3)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (
pi^2*b^5*d^3*x^4 + 4*pi^2*b^5*c*d^2*x^3 + 6*pi^2*b^5*c^2*d*x^2 + 4*pi^2*b^5
*c^3*x)*fresnel_cos(b*x + a) - (pi*b^4*d^3*x^3 + pi*(4*b^4*c*d^2 - a*b^3*d^
3)*x^2 + pi*(6*b^4*c^2*d - 4*a*b^3*c*d^2 + a^2*b^2*d^3)*x + pi*(4*b^4*c^3 -
 6*a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - a^3*b*d^3))*sin(1/2*pi*b^2*x^2 + pi*a*b*
x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [F]

$$\int (c+dx)^3 \text{FresnelC}(a+bx)\,dx = \int (c+dx)^3\, C(a+bx)\, dx$$

```
[In] integrate((d*x+c)**3*fresnelc(b*x+a),x)
[Out] Integral((c + d*x)**3*fresnelc(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^3 \text{FresnelC}(a+bx)\,dx = \int (dx+c)^3\, \text{C}\,(bx+a)\, dx$$

```
[In] integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^3*fresnel_cos(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^3 \text{FresnelC}(a+bx)\,dx = \int (dx+c)^3\, \text{C}\,(bx+a)\, dx$$

```
[In] integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \text{FresnelC}(a+bx)\,dx = \int \text{FresnelC}(a+b\,x)\,\left(c+d\,x\right)^3 dx$$

```
[In] int(FresnelC(a + b*x)*(c + d*x)^3,x)
[Out] int(FresnelC(a + b*x)*(c + d*x)^3, x)
```

3.129 $\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 194

$$\begin{aligned}\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = & -\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}-\frac{(bc-ad)^3\operatorname{FresnelC}(a+bx)}{3b^3d}\\ & +\frac{(c+dx)^3\operatorname{FresnelC}(a+bx)}{3d}\\ & +\frac{d(bc-ad)\operatorname{FresnelS}(a+bx)}{b^3\pi}\\ & -\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}\\ & -\frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}\\ & -\frac{d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}\end{aligned}$$

```
[Out] -2/3*d^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2-1/3*(-a*d+b*c)^3*FresnelC(b*x+a)/b^
3/d+1/3*(d*x+c)^3*FresnelC(b*x+a)/d+d*(-a*d+b*c)*FresnelS(b*x+a)/b^3/Pi-(-a
*d+b*c)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c)*(b*x+a)*sin(1/2*Pi*(b*x
+a)^2)/b^3/Pi-1/3*d^2*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718}

$$\begin{aligned}\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = &-\frac{(bc-ad)^3 \operatorname{FresnelC}(a+bx)}{3b^3d}\\ &+\frac{d(bc-ad)\operatorname{FresnelS}(a+bx)}{\pi b^3}\\ &-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}\\ &-\frac{d(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}\\ &-\frac{d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}\\ &-\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}+\frac{(c+dx)^3\operatorname{FresnelC}(a+bx)}{3d}\end{aligned}$$

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) - ((b*c - a*d)^3*FresnelC[a +
 b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelC[a + b*x])/(3*d) + (d*(b*c - a*d)*F
resnelS[a + b*x])/(b^3*Pi) - ((b*c - a*d)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*P
i) - (d*(b*c - a*d)*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (d^2*(a +
 b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^3\,\text{FresnelC}(a+bx)}{3d} - \frac{b\int (c+dx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{3d}$$

$$= \frac{(c+dx)^3\,\text{FresnelC}(a+bx)}{3d} - \frac{\text{Subst}\left(\int \left(b^3c^3\left(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\right)\cos\left(\frac{\pi x^2}{2}\right)+3b^2c^2d\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)x\cos\left(\frac{\pi x^2}{2}\right)+3bc\right.\right.}{3b^3d}$$

$$=\frac{(c+dx)^3\,\text{FresnelC}(a+bx)}{3d}-\frac{d^2\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3}$$
$$-\frac{(d(bc-ad))\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3}$$
$$-\frac{(bc-ad)^2\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3}$$
$$-\frac{(bc-ad)^3\text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3d}$$
$$=-\frac{(bc-ad)^3\,\text{FresnelC}(a+bx)}{3b^3d}+\frac{(c+dx)^3\,\text{FresnelC}(a+bx)}{3d}$$
$$-\frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}-\frac{d^2\text{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{6b^3}$$
$$-\frac{(bc-ad)^2\text{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^3}$$
$$+\frac{(d(bc-ad))\text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3\pi}$$
$$=-\frac{(bc-ad)^3\,\text{FresnelC}(a+bx)}{3b^3d}+\frac{(c+dx)^3\,\text{FresnelC}(a+bx)}{3d}$$
$$+\frac{d(bc-ad)\,\text{FresnelS}(a+bx)}{b^3\pi}-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$-\frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$-\frac{d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}+\frac{d^2\text{Subst}\left(\int \sin\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{3b^3\pi}$$
$$=-\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}-\frac{(bc-ad)^3\,\text{FresnelC}(a+bx)}{3b^3d}$$
$$+\frac{(c+dx)^3\,\text{FresnelC}(a+bx)}{3d}+\frac{d(bc-ad)\,\text{FresnelS}(a+bx)}{b^3\pi}$$
$$-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}-\frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$-\frac{d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.22

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = \frac{-2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi^2(3ab^2c^2-3a^2bcd+a^3d^2+b^3x(3c^2+3cdx+d^2x^2))\operatorname{FresnelC}(a+bx)+3d(b$$

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2
 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelC[a + b*x] + 3*d*(b*c - a*d)*P
i*FresnelS[a + b*x] - 3*b^2*c^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + 3*a*b*c*d*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - 3*b^2*c*d*Pi*
x*Sin[(Pi*(a + b*x)^2)/2] + a*b*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - b^2*d^2*
Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\operatorname{FresnelC}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{-\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{3(ad-bc)d^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{3b^2d}}{b}$
default	$\frac{-\frac{\operatorname{FresnelC}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{-\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{3(ad-bc)d^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{3b^2d}}{b}$
parts	$\frac{\operatorname{FresnelC}(bx+a)d^2x^3}{3}+\operatorname{FresnelC}(bx+a)\,dc\,x^2+\operatorname{FresnelC}(bx+a)\,c^2x+\frac{\operatorname{FresnelC}(bx+a)c^3}{3d}-$

```
[In] int((d*x+c)^2*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3*FresnelC(b*x+a)*(a*d-b*c-d*(b*x+a))^3/b^2/d+1/3/b^2/d*(-d^3/Pi*(b
*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+3*(a*d-b*c)*
```

```
d^2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-3*(a*d-b*c)*d^2/Pi*FresnelS(b*x+a)-3*(
a*d-b*c)^2*d/Pi*sin(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^3*FresnelC(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.28

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx$$
$$= \pi^2(3\,ab^2c^2 - 3\,a^2bcd + a^3d^2)\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2\,bd^2\cos\left(\frac{1}{2}\,\pi b^2x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right) + 3\,\pi(bcd - ad^2)\sqrt{b^2}\,\mathrm{S}$$

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2)*sqrt(b^2)*fresnel_cos(sqrt(
b^2)*(b*x + a)/b) - 2*b*d^2*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + 3
*pi*(b*c*d - a*d^2)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) + (pi^2*b^
4*d^2*x^3 + 3*pi^2*b^4*c*d*x^2 + 3*pi^2*b^4*c^2*x)*fresnel_cos(b*x + a) - (
pi*b^3*d^2*x^2 + pi*(3*b^3*c*d - a*b^2*d^2)*x + pi*(3*b^3*c^2 - 3*a*b^2*c*d
 + a^2*b*d^2))*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = \int (c+dx)^2\,C(a+bx)\;dx$$

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = \int (dx+c)^2\,\mathrm{C}\left(bx+a\right)\;dx$$

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = \int (dx+c)^2\, C\,(bx+a)\,\,dx$$

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = \int \operatorname{FresnelC}(a+b\,x)\,\,(c+d\,x)^2\,dx$$

```
[In] int(FresnelC(a + b*x)*(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)*(c + d*x)^2, x)
```

3.130 $\int (c+dx)\,\text{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 122

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx = -\frac{(bc-ad)^2\,\text{FresnelC}(a+bx)}{2b^2d} + \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d} + \frac{d\,\text{FresnelS}(a+bx)}{2b^2\pi} - \frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

```
[Out] -1/2*(-a*d+b*c)^2*FresnelC(b*x+a)/b^2/d+1/2*(d*x+c)^2*FresnelC(b*x+a)/d+1/2
*d*FresnelS(b*x+a)/b^2/Pi-(-a*d+b*c)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*d*(b*
x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432}

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx = -\frac{(bc-ad)^2\,\text{FresnelC}(a+bx)}{2b^2d} - \frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2} + \frac{d\,\text{FresnelS}(a+bx)}{2\pi b^2} - \frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2} + \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d}$$

```
[In] Int[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] -1/2*((b*c - a*d)^2*FresnelC[a + b*x])/(b^2*d) + ((c + d*x)^2*FresnelC[a +
b*x])/(2*d) + (d*FresnelS[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)*Sin[(Pi*(a +
b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
```

```
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d} - \frac{b\int (c+dx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{2d}$$

$$= \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\cos\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)x\cos\left(\frac{\pi x^2}{2}\right)+d^2x^2\cos\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,c}{2b^2d}$$

$$= \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d} - \frac{d\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2} - \frac{(bc-ad)\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^2} - \frac{(bc-ad)^2\text{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2d}$$

$$= -\frac{(bc-ad)^2\,\text{FresnelC}(a+bx)}{2b^2d} + \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d} - \frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{(bc-ad)\text{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^2} + \frac{d\text{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2\pi}$$

$$= -\frac{(bc-ad)^2\,\text{FresnelC}(a+bx)}{2b^2d} + \frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d} + \frac{d\,\text{FresnelS}(a+bx)}{2b^2\pi} - \frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.61

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx = \frac{-\pi(a+bx)(ad-b(2c+dx))\,\text{FresnelC}(a+bx)+d\,\text{FresnelS}(a+bx)+(-2bc+ad-bdx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right.}{2b^2\pi}$$

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] (-(Pi*(a + b*x)*(a*d - b*(2*c + d*x))*FresnelC[a + b*x]) + d*FresnelS[a + b
*x] + (-2*b*c + a*d - b*d*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\text{FresnelC}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\text{FresnelS}(bx+a)}{\pi}+\frac{(2ad-2bc)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
default	$\frac{-\frac{\text{FresnelC}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\text{FresnelS}(bx+a)}{\pi}+\frac{(2ad-2bc)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
parts	$\frac{\text{FresnelC}(bx+a)\,d\,x^2}{2}+\text{FresnelC}(bx+a)\,cx-\frac{b\left(\frac{dx\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{da\left(\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a\right.}{b^2\pi}\right.}{}\right.}{}$

```
[In] int((d*x+c)*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-FresnelC(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/2/b*(-d
/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+d/Pi*FresnelS(b*x+a)+(2*a*d-2*b*c)/Pi*sin
(1/2*Pi*(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx = \frac{\pi(2\,abc-a^2d)\sqrt{b^2}\,\text{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+\sqrt{b^2}d\,\text{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(\pi b^3dx^2+2\,\pi b^3cx)\,\text{C}\,(bx+a)-(b^2dx+2\,b^2c-}{2\,\pi b^3}$$

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*(2*a*b*c - a^2*d)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + sq
rt(b^2)*d*fresnel_sin(sqrt(b^2)*(b*x + a)/b) + (pi*b^3*d*x^2 + 2*pi*b^3*c*x
)*fresnel_cos(b*x + a) - (b^2*d*x + 2*b^2*c - a*b*d)*sin(1/2*pi*b^2*x^2 + p
i*a*b*x + 1/2*pi*a^2))/(pi*b^3)
```

Sympy [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int (c+dx)\,C(a+bx)\;dx$$

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x)
[Out] Integral((c + d*x)*fresnelc(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int (dx+c)\,\text{C}\,(bx+a)\;dx$$

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)*fresnel_cos(b*x + a), x)
```

Giac [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int (dx+c)\,\text{C}\,(bx+a)\;dx$$

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)*fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int \text{FresnelC}(a+b\,x)\;(c+d\,x)\;dx$$

```
[In] int(FresnelC(a + b*x)*(c + d*x),x)
[Out] int(FresnelC(a + b*x)*(c + d*x), x)
```

3.131 $\int \text{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(a+bx)\,\text{FresnelC}(a+bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6554}

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(a+bx)\,\text{FresnelC}(a+bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\,\text{FresnelC}(a+bx)}{b} - \frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \text{FresnelC}(a+bx)\,dx=\frac{a\,\text{FresnelC}(a+bx)}{b}+x\,\text{FresnelC}(a+bx)-\frac{\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)\sin\left(\frac{a^2\pi}{2}\right)}{b\pi}-\frac{\cos\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
default	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
parts	$x\ \text{FresnelC}\left(bx+a\right)-b\left(\frac{\sin\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelC}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	85

```
[In] int(FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(\pi bx+\pi a)\,\text{C}\,(bx+a)-\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
[Out] ((pi*b*x + pi*a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]

$$\int \text{FresnelC}(a+bx)\,dx = \int C(a+bx)\ dx$$

```
[In] integrate(fresnelc(b*x+a),x)
[Out] Integral(fresnelc(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(bx+a)\,\text{C}\,(bx+a)-\frac{\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Giac [F]

$$\int \text{FresnelC}(a+bx)\,dx = \int \text{C}\left(bx+a\right)\,dx$$

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(a+bx)\,dx = \int \text{FresnelC}(a+b\,x)\,dx$$

```
[In] int(FresnelC(a + b*x),x)
[Out] int(FresnelC(a + b*x), x)
```

3.132 $\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)}{c+dx}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx = \int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$

```
[In] Int[FresnelC[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx$$

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x),x]
[Out] Integrate[FresnelC[a + b*x]/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{dx+c}dx$$

```
[In] int(FresnelC(b*x+a)/(d*x+c),x)
[Out] int(FresnelC(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{C}\left(bx+a\right)}{dx+c}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{C(a+bx)}{c+dx}\,dx$$

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x)
[Out] Integral(fresnelc(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{C}\,(bx+a)}{dx+c}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{C}\,(bx+a)}{dx+c}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{FresnelC}(a+b\,x)}{c+d\,x}\,dx$$

```
[In] int(FresnelC(a + b*x)/(c + d*x),x)
[Out] int(FresnelC(a + b*x)/(c + d*x), x)
```

3.133 $\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx = \int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

```
[In] Int[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx = \int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x)^2,x]
[Out] Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx+a)}{(dx+c)^2} dx$$

```
[In] int(FresnelC(b*x+a)/(d*x+c)^2,x)
[Out] int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx = \int \frac{\text{C}\,(bx+a)}{(dx+c)^2} dx$$

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx = \int \frac{C(a+bx)}{(c+dx)^2} dx$$

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)**2,x)
[Out] Integral(fresnelc(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{C}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{C}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{FresnelC}(a+b\,x)}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(FresnelC(a + b*x)/(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)/(c + d*x)^2, x)
```

3.134 $\int x^3 \operatorname{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 227

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4\operatorname{FresnelC}(a+bx)}{4b^4} + \frac{3\operatorname{FresnelC}(a+bx)}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelC}(a+bx) + \frac{3a^2\operatorname{FresnelS}(a+bx)}{2b^4\pi} + \frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{a(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}$$

```
[Out] 2*a*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi
^2-1/4*a^4*FresnelC(b*x+a)/b^4+3/4*FresnelC(b*x+a)/b^4/Pi^2+1/4*x^4*Fresnel
C(b*x+a)+3/2*a^2*FresnelS(b*x+a)/b^4/Pi+a^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/
2*a^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi+a*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)/b^4/Pi-1/4*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules

used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718, 3466}

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = -\frac{a^4 \operatorname{FresnelC}(a+bx)}{4b^4} + \frac{a^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4} + \frac{3a^2 \operatorname{FresnelS}(a+bx)}{2\pi b^4} - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4} + \frac{3\operatorname{FresnelC}(a+bx)}{4\pi^2 b^4} + \frac{a(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4} + \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2 b^4} + \frac{1}{4}x^4 \operatorname{FresnelC}(a+bx)$$

```
[In] Int[x^3*FresnelC[a + b*x],x]
```

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2
)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x]
)/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b
^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*
(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*
Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\,\text{FresnelC}(a+bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx \\
&= \frac{1}{4}x^4\,\text{FresnelC}(a+bx) \\
&\quad - \frac{\text{Subst}\left(\int\left(a^4\cos\left(\frac{\pi x^2}{2}\right) - 4a^3x\cos\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\cos\left(\frac{\pi x^2}{2}\right) - 4ax^3\cos\left(\frac{\pi x^2}{2}\right) + x^4\cos\left(\frac{\pi x^2}{2}\right)\right)\right.}{4b^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}x^4 \operatorname{FresnelC}(a+bx) - \frac{\operatorname{Subst}\left(\int x^4 \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} \\
&\quad + \frac{a\operatorname{Subst}\left(\int x^3 \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^4} \\
&\quad - \frac{(3a^2)\operatorname{Subst}\left(\int x^2 \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^4} \\
&\quad + \frac{a^3\operatorname{Subst}\left(\int x \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^4} - \frac{a^4\operatorname{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} \\
&= -\frac{a^4 \operatorname{FresnelC}(a+bx)}{4b^4} + \frac{1}{4}x^4 \operatorname{FresnelC}(a+bx) \\
&\quad - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} \\
&\quad + \frac{a\operatorname{Subst}\left(\int x \cos\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{2b^4} + \frac{a^3\operatorname{Subst}\left(\int \cos\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{2b^4} \\
&\quad + \frac{3\operatorname{Subst}\left(\int x^2 \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4\pi} + \frac{(3a^2)\operatorname{Subst}\left(\int \sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^4\pi} \\
&= -\frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4 \operatorname{FresnelC}(a+bx)}{4b^4} + \frac{1}{4}x^4 \operatorname{FresnelC}(a+bx) \\
&\quad + \frac{3a^2 \operatorname{FresnelS}(a+bx)}{2b^4\pi} + \frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} \\
&\quad + \frac{a(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4\pi^2} - \frac{a\operatorname{Subst}\left(\int \sin\left(\frac{\pi x}{2}\right)\,dx, x, (a+bx)^2\right)}{b^4\pi} \\
&= \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4 \operatorname{FresnelC}(a+bx)}{4b^4} \\
&\quad + \frac{3\operatorname{FresnelC}(a+bx)}{4b^4\pi^2} + \frac{1}{4}x^4 \operatorname{FresnelC}(a+bx) + \frac{3a^2 \operatorname{FresnelS}(a+bx)}{2b^4\pi} \\
&\quad + \frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} \\
&\quad + \frac{a(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.73

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \frac{5a\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 3bx\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + (3 - a^4\pi^2 + b^4\pi^2x^4)\operatorname{FresnelC}(a+bx) + 6a^2\pi\operatorname{FresnelS}(a$$

```
[In] Integrate[x^3*FresnelC[a + b*x],x]
```

```
[Out] (5*a*Cos[(Pi*(a + b*x)^2)/2] - 3*b*x*Cos[(Pi*(a + b*x)^2)/2] + (3 - a^4*Pi^
2 + b^4*Pi^2*x^4)*FresnelC[a + b*x] + 6*a^2*Pi*FresnelS[a + b*x] + a^3*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*b*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*Pi*x^2*
Sin[(Pi*(a + b*x)^2)/2] - b^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx+a)b^4x^4}{4}-\frac{a^4\,\mathrm{FresnelC}(bx+a)}{4}+\frac{a^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3a^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{3a^2\,\mathrm{FresnelS}(bx+a)}{2\pi}+\frac{a(bx+a)^2\sin\left(\right.}{\pi}}{b^4}$
default	$\frac{\frac{\mathrm{FresnelC}(bx+a)b^4x^4}{4}-\frac{a^4\,\mathrm{FresnelC}(bx+a)}{4}+\frac{a^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3a^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{3a^2\,\mathrm{FresnelS}(bx+a)}{2\pi}+\frac{a(bx+a)^2\sin\left(\right.}{\pi}}{b^4}$
parts	$\frac{x^4\,\mathrm{FresnelC}(bx+a)}{4}-\frac{b\left(\frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{x\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right.}{b^2\pi}\right.}{}\right.}{}\right.}{}$

[In] int(x^3*FresnelC(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b^4*(1/4*FresnelC(b*x+a)*b^4*x^4-1/4*a^4*FresnelC(b*x+a)+a^3/Pi*sin(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*FresnelS(b*x+a)+a/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*cos(1/2*Pi*(b*x+a)^2)-1/4/Pi*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)+3/4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1/Pi*FresnelC(b*x+a)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.78

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \frac{\pi^2 b^5 x^4 \operatorname{C}(bx+a) + 6\,\pi a^2\sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2 a^4 - 3)\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (3\,b^2 x - 5\,ab)\cos\left(\frac{1}{2}\,\pi b^2 x^2 \right.}{4\,\pi^2 b^5}$$

```
[In] integrate(x^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(pi^2*b^5*x^4*fresnel_cos(b*x + a) + 6*pi*a^2*sqrt(b^2)*fresnel_sin(sqr
t(b^2)*(b*x + a)/b) - (pi^2*a^4 - 3)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x +
 a)/b) - (3*b^2*x - 5*a*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (p
i*b^4*x^3 - pi*a*b^3*x^2 + pi*a^2*b^2*x - pi*a^3*b)*sin(1/2*pi*b^2*x^2 + pi
*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [F]

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \int x^3 C(a+bx)\;dx$$

```
[In] integrate(x**3*fresnelc(b*x+a),x)
```

```
[Out] Integral(x**3*fresnelc(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.21

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \frac{1}{4}\,x^4\operatorname{C}(bx+a) + \frac{\left(16\left(-i\,\pi^2 e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)} + i\,\pi^2 e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)a^4 + 32\left(\pi\Gamma\left(2, \frac{1}{2}i\,\pi b^2x^2 + i\,\pi abx + \frac{1}{2}i\,\pi \right.\right.\right.}{}$$

```
[In] integrate(x^3*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_cos(b*x + a) + 1/32*(16*(-I*pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi
*a*b*x + 1/2*I*pi*a^2) + I*pi^2*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*p
i*a^2))*a^4 + 32*(pi*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)
```

```
 + pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 16*((-
I*pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi^2*e^(-1/2*I*
pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^3 + 2*(pi*gamma(2, 1/2*I*pi*b^2*
x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b
*x - 1/2*I*pi*a^2))*a)*b*x + (((I - 1)*sqrt(2)*pi^(5/2)*(erf(sqrt(1/2*I*pi*
b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) - (I + 1)*sqrt(2)*pi^(5/2)*(erf(
sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^4 + 12*(-(I +
1)*sqrt(2)*pi*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + (I
 - 1)*sqrt(2)*pi*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))
*a^2 + (4*I - 4)*sqrt(2)*gamma(5/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*p
i*a^2) - (4*I + 4)*sqrt(2)*gamma(5/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*
I*pi*a^2))*sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2))*b/(pi^3*b^6*x + pi^3
*a*b^5)
```

Giac [F]

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \int x^3\,\mathrm{C}\left(bx+a\right)\,dx$$

```
[In] integrate(x^3*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \int x^3\,\operatorname{FresnelC}(a+b\,x)\,dx$$

```
[In] int(x^3*FresnelC(a + b*x),x)
[Out] int(x^3*FresnelC(a + b*x), x)
```

3.135 $\int x^2 \,\mathrm{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 148

$$\int x^2 \,\mathrm{FresnelC}(a+bx)\,dx = -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} + \frac{a^3 \,\mathrm{FresnelC}(a+bx)}{3b^3} + \frac{1}{3}x^3 \,\mathrm{FresnelC}(a+bx) - \frac{a \,\mathrm{FresnelS}(a+bx)}{b^3\pi} - \frac{a^2 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}$$

```
[Out] -2/3*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2+1/3*a^3*FresnelC(b*x+a)/b^3+1/3*x^3*Fre
snelC(b*x+a)-a*FresnelS(b*x+a)/b^3/Pi-a^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*(b
*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-1/3*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/
Pi
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718}

$$\int x^2 \,\mathrm{FresnelC}(a+bx)\,dx = \frac{a^3 \,\mathrm{FresnelC}(a+bx)}{3b^3} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} - \frac{a \,\mathrm{FresnelS}(a+bx)}{\pi b^3} + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} - \frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3} - \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3} + \frac{1}{3}x^3 \,\mathrm{FresnelC}(a+bx)$$

```
[In] Int[x^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
 + (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\,\text{FresnelC}(a+bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx \\
&= \frac{1}{3}x^3\,\text{FresnelC}(a+bx) \\
&\quad - \frac{\text{Subst}\left(\int\left(-a^3\cos\left(\frac{\pi x^2}{2}\right)+3a^2x\cos\left(\frac{\pi x^2}{2}\right)-3ax^2\cos\left(\frac{\pi x^2}{2}\right)+x^3\cos\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a+bx\right)}{3b^3} \\
&= \frac{1}{3}x^3\,\text{FresnelC}(a+bx) - \frac{\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3} \\
&\quad + \frac{a\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3} \\
&\quad - \frac{a^2\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3} + \frac{a^3\text{Subst}\left(\int\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3} \\
&= \frac{a^3\,\text{FresnelC}(a+bx)}{3b^3} + \frac{1}{3}x^3\,\text{FresnelC}(a+bx) \\
&\quad + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{\text{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{6b^3} \\
&\quad - \frac{a^2\text{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^3} - \frac{a\text{Subst}\left(\int\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3\pi} \\
&= \frac{a^3\,\text{FresnelC}(a+bx)}{3b^3} + \frac{1}{3}x^3\,\text{FresnelC}(a+bx) - \frac{a\,\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&\quad - \frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{\text{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{3b^3\pi}
\end{aligned}$$

$$= -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} + \frac{a^3\,\text{FresnelC}(a+bx)}{3b^3} + \frac{1}{3}x^3\,\text{FresnelC}(a+bx) - \frac{a\,\text{FresnelS}(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int x^2\,\text{FresnelC}(a+bx)\,dx = -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - \pi^2(a^3+b^3x^3)\,\text{FresnelC}(a+bx) + 3a\pi\,\text{FresnelS}(a+bx) + a^2\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right) -}{3b^3\pi^2}$$

```
[In] Integrate[x^2*FresnelC[a + b*x],x]
```

```
[Out] -1/3*(2*Cos[(Pi*(a + b*x)^2)/2] - Pi^2*(a^3 + b^3*x^3)*FresnelC[a + b*x] +
3*a*Pi*FresnelS[a + b*x] + a^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Sin[(P
i*(a + b*x)^2)/2] + b^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx+a)b^3x^3}{3} + \frac{a^3\,\text{FresnelC}(bx+a)}{3} - \frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a\,\text{FresnelS}(bx+a)}{\pi} - \frac{(bx+a)^2\sin\left(\frac{\pi(bx-}{2}\right.}{3\pi}}{b^3}$
default	$\frac{\frac{\text{FresnelC}(bx+a)b^3x^3}{3} + \frac{a^3\,\text{FresnelC}(bx+a)}{3} - \frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a\,\text{FresnelS}(bx+a)}{\pi} - \frac{(bx+a)^2\sin\left(\frac{\pi(bx-}{2}\right.}{3\pi}}{b^3}$
parts	$\frac{x^3\,\text{FresnelC}(bx+a)}{3} - \frac{b\left(\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} - \frac{a\left(\frac{x\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} - \frac{a\left(\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} -\right.}{b}\right.}{b}\right.}{}$

```
[In] int(x^2*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelC(b*x+a)*b^3*x^3+1/3*a^3*FresnelC(b*x+a)-a^2/Pi*sin(1/2*P
i*(b*x+a)^2)+a/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-a/Pi*FresnelS(b*x+a)-1/3/Pi
*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2/3/Pi^2*cos(1/2*Pi*(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int x^2 \,\text{FresnelC}(a+bx)\,dx = \frac{\pi^2 b^4 x^3\,\text{C}\,(bx+a) + \pi^2 a^3 \sqrt{b^2}\,\text{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 3\,\pi a\sqrt{b^2}\,\text{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2\,b\cos\left(\frac{1}{2}\,\pi b^2x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right) -}{3\,\pi^2 b^4}$$

```
[In] integrate(x^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*b^4*x^3*fresnel_cos(b*x + a) + pi^2*a^3*sqrt(b^2)*fresnel_cos(sqr
t(b^2)*(b*x + a)/b) - 3*pi*a*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) -
 2*b*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (pi*b^3*x^2 - pi*a*b^2*x
 + pi*a^2*b)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

Sympy [F]

$$\int x^2 \,\text{FresnelC}(a+bx)\,dx = \int x^2 C(a+bx)\,dx$$

```
[In] integrate(x**2*fresnel_c(b*x+a),x)
```

```
[Out] Integral(x**2*fresnelc(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.86

$$\int x^2 \,\text{FresnelC}(a+bx)\,dx = \frac{1}{3}\,x^3\,\text{C}\,(bx+a) - \frac{\left(12\left(-i\,\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)} + i\,\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)a^3 + 4\left(3\left(-i\,\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)} + i\,\pi\right.\right.\right.}{}$$

```
[In] integrate(x^2*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*fresnel_cos(b*x + a) - 1/24*(12*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a
*b*x + 1/2*I*pi*a^2) + I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^
2))*a^3 + 4*(3*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*
pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 2*gamma(2, 1/2*
I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + 2*gamma(2, -1/2*I*pi*b^2*x^2 -
I*pi*a*b*x - 1/2*I*pi*a^2))*b*x + 8*a*(gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b
*x + 1/2*I*pi*a^2) + gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2
)) + sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2)*(((I - 1)*sqrt(2)*pi^(3/2)*
(erf(sqrt(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) - (I + 1)*sqr
t(2)*pi^(3/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1
))*a^3 + 6*(-(I + 1)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2
*I*pi*a^2) + (I - 1)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/
2*I*pi*a^2))*a))*b/(pi^2*b^5*x + pi^2*a*b^4)
```

Giac [F]

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx=\int x^2\,C\left(bx+a\right)\,dx$$

```
[In] integrate(x^2*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx=\int x^2\,\operatorname{FresnelC}(a+b\,x)\,dx$$

```
[In] int(x^2*FresnelC(a + b*x),x)
[Out] int(x^2*FresnelC(a + b*x), x)
```

3.136 $\int x \operatorname{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 95

$$\int x \operatorname{FresnelC}(a+bx)\,dx=-\frac{a^2 \operatorname{FresnelC}(a+bx)}{2b^2}+\frac{1}{2}x^2 \operatorname{FresnelC}(a+bx)+\frac{\operatorname{FresnelS}(a+bx)}{2b^2\pi}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

```
[Out] -1/2*a^2*FresnelC(b*x+a)/b^2+1/2*x^2*FresnelC(b*x+a)+1/2*FresnelS(b*x+a)/b^
2/Pi+a*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/P
i
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432}

$$\int x \operatorname{FresnelC}(a+bx)\,dx=-\frac{a^2 \operatorname{FresnelC}(a+bx)}{2b^2}+\frac{\operatorname{FresnelS}(a+bx)}{2\pi b^2}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2 \operatorname{FresnelC}(a+bx)$$

```
[In] Int[x*FresnelC[a + b*x],x]
```

```
[Out] -1/2*(a^2*FresnelC[a + b*x])/b^2 + (x^2*FresnelC[a + b*x])/2 + FresnelS[a +
 b*x]/(2*b^2*Pi) + (a*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - ((a + b*x)*Sin[(P
i*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\,\text{FresnelC}(a+bx)-\frac{1}{2}b\int x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelC}(a+bx)\\
&\quad-\frac{\text{Subst}\left(\int\left(a^2\cos\left(\frac{\pi x^2}{2}\right)-2ax\cos\left(\frac{\pi x^2}{2}\right)+x^2\cos\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a+bx\right)}{2b^2}\\
&= \frac{1}{2}x^2\,\text{FresnelC}(a+bx)-\frac{\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2}\\
&\quad+\frac{a\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^2}-\frac{a^2\text{Subst}\left(\int\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2}\\
&= -\frac{a^2\,\text{FresnelC}(a+bx)}{2b^2}+\frac{1}{2}x^2\,\text{FresnelC}(a+bx)-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}\\
&\quad+\frac{a\text{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right)\,dx,x,(a+bx)^2\right)}{2b^2}+\frac{\text{Subst}\left(\int\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2\pi}\\
&= -\frac{a^2\,\text{FresnelC}(a+bx)}{2b^2}+\frac{1}{2}x^2\,\text{FresnelC}(a+bx)+\frac{\text{FresnelS}(a+bx)}{2b^2\pi}\\
&\quad+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$
\int x\,\text{FresnelC}(a+bx)\,dx = \frac{(-a^2\pi+b^2\pi x^2)\,\text{FresnelC}(a+bx)+\text{FresnelS}(a+bx)+(a-bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}
$$

```
[In] Integrate[x*FresnelC[a + b*x],x]
```

```
[Out] ((-(a^2*Pi) + b^2*Pi*x^2)*FresnelC[a + b*x] + FresnelS[a + b*x] + (a - b*x)
*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\operatorname{FresnelC}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\operatorname{FresnelS}(bx+a)}{2\pi}}{b^2}$
default	$\frac{\operatorname{FresnelC}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\operatorname{FresnelS}(bx+a)}{2\pi}}{b^2}$
parts	$\frac{x^2\operatorname{FresnelC}(bx+a)}{2}-\frac{b\left(\frac{x\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\operatorname{FresnelC}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)}{b}-\frac{\mathrm{Fre}}{}\right.}{2}$

[In] int(x*FresnelC(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b^2*(FresnelC(b*x+a)*(-(b*x+a)*a+1/2*(b*x+a)^2)+a/Pi*sin(1/2*Pi*(b*x+a)^2
)-1/2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+1/2/Pi*FresnelS(b*x+a))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int x\operatorname{FresnelC}(a+bx)\,dx$$
$$=\frac{\pi b^3x^2\,\mathrm{C}\,(bx+a)-\pi a^2\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-(b^2x-ab)\sin\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)+\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{2\,\pi b^3}$$

[In] integrate(x*fresnel_cos(b*x+a),x, algorithm="fricas")

[Out] 1/2*(pi*b^3*x^2*fresnel_cos(b*x + a) - pi*a^2*sqrt(b^2)*fresnel_cos(sqrt(b^
2)*(b*x + a)/b) - (b^2*x - a*b)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)
 + sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b))/(pi*b^3)

Sympy [F]

$$\int x \operatorname{FresnelC}(a+bx)\,dx = \int xC(a+bx)\,dx$$

```
[In] integrate(x*fresnelc(b*x+a),x)
[Out] Integral(x*fresnelc(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.27

$$\int x \operatorname{FresnelC}(a+bx)\,dx = \frac{1}{2}x^2\,\mathrm{C}\left(bx+a\right) + \frac{\left(8\left(-i\,\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)} + i\,\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)abx + 8\left(-i\,\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)} + i\,\pi e^{(}\right.\right.}{}$$

```
[In] integrate(x*fresnel_cos(b*x+a),x, algorithm="maxima")
[Out] 1/2*x^2*fresnel_cos(b*x + a) + 1/16*(8*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*
b*x + 1/2*I*pi*a^2) + I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2
))*a*b*x + 8*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi
*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - sqrt(2*pi*b^2*x^2
 + 4*pi*a*b*x + 2*pi*a^2)*((-(I - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(1/2*I*pi*b^
2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I + 1)*sqrt(2)*pi^(3/2)*(erf(sq
rt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^2 + (2*I + 2)*sq
rt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) - (2*I - 2)*
sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)))*b/(pi^2
*b^4*x + pi^2*a*b^3)
```

Giac [F]

$$\int x \operatorname{FresnelC}(a+bx)\,dx = \int x\,\mathrm{C}\left(bx+a\right)\,dx$$

```
[In] integrate(x*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate(x*fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \operatorname{FresnelC}(a+bx)\,dx = \int x \operatorname{FresnelC}(a+b\,x)\;dx$$

```
[In] int(x*FresnelC(a + b*x),x)
[Out] int(x*FresnelC(a + b*x), x)
```

3.137 $\int \text{FresnelC}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \text{FresnelC}(a+bx)\,dx=\frac{(a+bx)\,\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6554}

$$\int \text{FresnelC}(a+bx)\,dx=\frac{(a+bx)\,\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral}=\frac{(a+bx)\,\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \text{FresnelC}(a+bx)\,dx=\frac{a\,\text{FresnelC}(a+bx)}{b}+x\,\text{FresnelC}(a+bx)-\frac{\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)\sin\left(\frac{a^2\pi}{2}\right)}{b\pi}-\frac{\cos\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
default	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
parts	$x\ \text{FresnelC}\left(bx+a\right)-b\left(\frac{\sin\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelC}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	85

```
[In] int(FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(\pi bx + \pi a)\,\text{C}\,(bx+a) - \sin\left(\frac{1}{2}\,\pi b^2x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right)}{\pi b}$$

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
[Out] ((pi*b*x + pi*a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]

$$\int \text{FresnelC}(a+bx)\,dx = \int C(a+bx)\;dx$$

```
[In] integrate(fresnelc(b*x+a),x)
[Out] Integral(fresnelc(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(bx+a)\,\text{C}\,(bx+a) - \frac{\sin\left(\frac{1}{2}\,\pi b^2x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Giac [F]

$$\int \text{FresnelC}(a+bx)\,dx=\int \mathrm{C}\left(bx+a\right)\,dx$$

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(a+bx)\,dx=\int \text{FresnelC}(a+b\,x)\,dx$$

```
[In] int(FresnelC(a + b*x),x)
[Out] int(FresnelC(a + b*x), x)
```

3.138 $\int \frac{\text{FresnelC}(a+bx)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(a+bx)}{x} dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x} dx = \int \frac{\text{FresnelC}(a+bx)}{x} dx$$

```
[In] Int[FresnelC[a + b*x]/x,x]
[Out] Defer[Int][FresnelC[a + b*x]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx = \int \frac{\text{FresnelC}(a+bx)}{x}\,dx$$

```
[In] Integrate[FresnelC[a + b*x]/x,x]
[Out] Integrate[FresnelC[a + b*x]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{x}dx$$

```
[In] int(FresnelC(b*x+a)/x,x)
[Out] int(FresnelC(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx = \int \frac{\text{C}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/x,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx = \int \frac{C(a+bx)}{x}\,dx$$

```
[In] integrate(fresnelc(b*x+a)/x,x)
[Out] Integral(fresnelc(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx=\int \frac{\text{C}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx=\int \frac{\text{C}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/x,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx=\int \frac{\text{FresnelC}(a+b\,x)}{x}\,dx$$

```
[In] int(FresnelC(a + b*x)/x,x)
[Out] int(FresnelC(a + b*x)/x, x)
```

3.139 $\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx = \int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx$$

```
[In] Int[FresnelC[a + b*x]/x^2,x]
[Out] Defer[Int][FresnelC[a + b*x]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx=\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[FresnelC[a + b*x]/x^2,x]
[Out] Integrate[FresnelC[a + b*x]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx+a)}{x^2}dx$$

```
[In] int(FresnelC(b*x+a)/x^2,x)
[Out] int(FresnelC(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx=\int \frac{\text{C}\,(bx+a)}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx=\int \frac{C(a+bx)}{x^2}\,dx$$

```
[In] integrate(fresnelc(b*x+a)/x**2,x)
[Out] Integral(fresnelc(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx=\int \frac{\mathrm{C}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx=\int \frac{\mathrm{C}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx=\int \frac{\text{FresnelC}(a+b\,x)}{x^2}\,dx$$

```
[In] int(FresnelC(a + b*x)/x^2,x)
[Out] int(FresnelC(a + b*x)/x^2, x)
```

3.140 $\int x^7 \operatorname{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 253

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx = -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{4b^3\pi^2} - \frac{105\operatorname{FresnelC}(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8\operatorname{FresnelC}(bx)^2 + \frac{35x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} - \frac{10\sin(b^2\pi x^2)}{b^8\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3}$$

```
[Out] -105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2+55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4-1
/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2+105/4*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/
b^7/Pi^4-7/4*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2-105/8*FresnelC(
b*x)^2/b^8/Pi^4+1/8*x^8*FresnelC(b*x)^2+35/4*x^3*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/b^5/Pi^3-1/4*x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-10*sin(b^2*
Pi*x^2)/b^8/Pi^5+5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6566, 6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717, 3390}

$$\begin{aligned}\int x^7 \text{FresnelC}(bx)^2 dx = & -\frac{105 \text{FresnelC}(bx)^2}{8\pi^4 b^8} - \frac{105x^2}{16\pi^4 b^6} - \frac{x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} \\ & + \frac{7x^6}{48\pi^2 b^2} - \frac{x^6 \cos(\pi b^2 x^2)}{16\pi^2 b^2} - \frac{10 \sin(\pi b^2 x^2)}{\pi^5 b^8} \\ & + \frac{105x \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^4 b^7} + \frac{55x^2 \cos(\pi b^2 x^2)}{16\pi^4 b^6} \\ & + \frac{35x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^3 b^5} + \frac{5x^4 \sin(\pi b^2 x^2)}{8\pi^3 b^4} \\ & - \frac{7x^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{8}x^8 \text{FresnelC}(bx)^2\end{aligned}$$

```
[In] Int[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx \\
&= \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} \\
&\quad + \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{8\pi} + \frac{7\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi} \\
&= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} + \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 \\
&\quad - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{4b^3\pi^2} \\
&\quad + \frac{7\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^2\pi^2} + \frac{\text{Subst}\left(\int x^3\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{16\pi} \\
&= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} + \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 \\
&\quad + \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} \\
&\quad - \frac{105\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^5\pi^3} - \frac{35\int x^3\sin\left(b^2\pi x^2\right)\,dx}{8b^4\pi^3} \\
&\quad + \frac{3\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{16b^2\pi^2} + \frac{7\text{Subst}\left(\int x^2\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{8b^2\pi^2} \\
&= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} \\
&\quad + \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 + \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} \\
&\quad - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3x^4\sin\left(b^2\pi x^2\right)}{16b^4\pi^3} \\
&\quad - \frac{105\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{4b^7\pi^4} - \frac{105\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^6\pi^4} \\
&\quad - \frac{3\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{8b^4\pi^3} - \frac{35\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{16b^4\pi^3} \\
&\quad + \frac{7\text{Subst}\left(\int x^2\,dx, x, x^2\right)}{16b^2\pi^2} + \frac{7\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{16b^2\pi^2}
\end{aligned}$$

$$= \frac{7x^6}{48b^2\pi^2} + \frac{41x^2\cos(b^2\pi x^2)}{16b^6\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} + \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 + \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3} - \frac{105\text{Subst}(\int x\,dx, x, \text{FresnelC}(bx))}{4b^8\pi^4} - \frac{3\text{Subst}(\int\cos(b^2\pi x)\,dx, x, x^2)}{8b^6\pi^4} - \frac{35\text{Subst}(\int\cos(b^2\pi x)\,dx, x, x^2)}{16b^6\pi^4} - \frac{105\text{Subst}\left(\int\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{8b^6\pi^4} - \frac{7\text{Subst}(\int x\sin(b^2\pi x)\,dx, x, x^2)}{8b^4\pi^3}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} - \frac{105\,\text{FresnelC}(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 + \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} - \frac{73\sin(b^2\pi x^2)}{8b^8\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3} - \frac{7\text{Subst}(\int\cos(b^2\pi x)\,dx, x, x^2)}{8b^6\pi^4}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} - \frac{105\,\text{FresnelC}(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8\,\text{FresnelC}(bx)^2 + \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} - \frac{10\sin(b^2\pi x^2)}{b^8\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.72

$$\int x^7\,\text{FresnelC}(bx)^2\,dx$$
$$= \frac{-315b^2\pi x^2 + 7b^6\pi^3x^6 - 3b^2\pi x^2(-55 + b^4\pi^2x^4)\cos(b^2\pi x^2) + 6\pi(-105 + b^8\pi^4x^8)\,\text{FresnelC}(bx)^2 - 12b\pi x}{}$$

```
[In] Integrate[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-315*b^2*Pi*x^2 + 7*b^6*Pi^3*x^6 - 3*b^2*Pi*x^2*(-55 + b^4*Pi^2*x^4)*Cos[b
^2*Pi*x^2] + 6*Pi*(-105 + b^8*Pi^4*x^8)*FresnelC[b*x]^2 - 12*b*Pi*x*Fresnel
```

```
C[b*x]*(7*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-35 + b^4*
Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 480*Sin[b^2*Pi*x^2] + 30*b^4*Pi^2*x^4*Sin[
b^2*Pi*x^2])/(48*b^8*Pi^5)
```

Maple [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx$$

```
[In] int(x^7*FresnelC(b*x)^2,x)
```

```
[Out] int(x^7*FresnelC(b*x)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx = \frac{5\,\pi^3 b^6 x^6 - 240\,\pi b^2 x^2 - 3\left(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 42\left(\pi^3 b^5 x^5 - 15\,\pi b x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) - \ldots}{24\,\pi \ldots}$$

```
[In] integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(5*pi^3*b^6*x^6 - 240*pi*b^2*x^2 - 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*co
s(1/2*pi*b^2*x^2)^2 - 42*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fre
snel_cos(b*x) - 3*(105*pi - pi^5*b^8*x^8)*fresnel_cos(b*x)^2 + 6*(5*(pi^2*b
^4*x^4 - 16)*cos(1/2*pi*b^2*x^2) - (pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*fresnel
_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^5*b^8)
```

Sympy [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx = \int x^{7} C^{2}\left(bx\right)\, dx$$

```
[In] integrate(x**7*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx = \int x^7\,\mathrm{C}\left(bx\right)^2\,dx$$

```
[In] integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^7*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx = \int x^7\,\mathrm{C}\left(bx\right)^2\,dx$$

```
[In] integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^7*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx = \int x^7 \operatorname{FresnelC}(b\,x)^2\,dx$$

```
[In] int(x^7*FresnelC(b*x)^2,x)
[Out] int(x^7*FresnelC(b*x)^2, x)
```

3.141 $\int x^6 \operatorname{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 239

$$\begin{aligned}\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = & -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4} \\ & - \frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{7b^7\pi^4} \\ & - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\operatorname{FresnelC}(bx)^2 \\ & - \frac{531\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{48x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} \\ & - \frac{2x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3}\end{aligned}$$

```
[Out] -48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2+21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4-1/14*x^5
*cos(b^2*Pi*x^2)/b^2/Pi^2+96/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^7/Pi^4-1
2/7*x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/7*x^7*FresnelC(b*x)^2+
48/7*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-2/7*x^6*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b/Pi+17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3-531/112*FresnelC
(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6566, 6590, 6598, 6596, 3439, 3433, 3466, 3473, 30, 3467}

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = -\frac{531 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}\pi^4 b^7} - \frac{48x}{7\pi^4 b^6} - \frac{2x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi b} + \frac{6x^5}{35\pi^2 b^2} - \frac{x^5 \cos\left(\pi b^2 x^2\right)}{14\pi^2 b^2} + \frac{96 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{21x \cos\left(\pi b^2 x^2\right)}{8\pi^4 b^6} + \frac{48x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} + \frac{17x^3 \sin\left(\pi b^2 x^2\right)}{28\pi^3 b^4} - \frac{12x^4 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^2 b^3} + \frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2$$

```
[In] Int[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) + (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (96*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(7*b^7*Pi^4) - (12*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*b^
3*Pi^2) + (x^7*FresnelC[b*x]^2)/7 - (531*FresnelC[Sqrt[2]*b*x])/(56*Sqrt[2]
*b^7*Pi^4) + (48*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (2*x
^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi) + (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
```

```
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{1}{7}(2b)\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

$$= \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{7\pi} + \frac{12\int x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b\pi}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{48\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{7b^3\pi^2} + \frac{5\int x^4\cos\left(b^2\pi x^2\right)\,dx}{14b^2\pi^2} + \frac{12\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b^2\pi^2}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 + \frac{48x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{5x^3\sin\left(b^2\pi x^2\right)}{28b^4\pi^3} - \frac{96\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b^5\pi^3} - \frac{15\int x^2\sin\left(b^2\pi x^2\right)\,dx}{28b^4\pi^3} - \frac{24\int x^2\sin\left(b^2\pi x^2\right)\,dx}{7b^4\pi^3} + \frac{6\int x^4\,dx}{7b^2\pi^2} + \frac{6\int x^4\cos\left(b^2\pi x^2\right)\,dx}{7b^2\pi^2}$$

$$= \frac{6x^5}{35b^2\pi^2} + \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 + \frac{48x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{17x^3\sin\left(b^2\pi x^2\right)}{28b^4\pi^3} - \frac{15\int\cos\left(b^2\pi x^2\right)\,dx}{56b^6\pi^4} - \frac{12\int\cos\left(b^2\pi x^2\right)\,dx}{7b^6\pi^4} - \frac{96\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b^6\pi^4} - \frac{9\int x^2\sin\left(b^2\pi x^2\right)\,dx}{7b^4\pi^3}$$

$$= \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{15\,\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} - \frac{6\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{7b^7\pi^4} + \frac{48x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{17x^3\sin\left(b^2\pi x^2\right)}{28b^4\pi^3} - \frac{9\int\cos\left(b^2\pi x^2\right)\,dx}{14b^6\pi^4} - \frac{96\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\,dx}{7b^6\pi^4}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4} - \frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{51\,\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} - \frac{6\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{7b^7\pi^4} + \frac{48x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3} - \frac{48\int\cos(b^2\pi x^2)\,dx}{7b^6\pi^4}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4} - \frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{51\,\text{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} - \frac{30\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{7b^7\pi^4} + \frac{48x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

$$\int x^6\,\text{FresnelC}(bx)^2\,dx$$

$$= \frac{80b^7\pi^4x^7\,\text{FresnelC}(bx)^2 - 2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) - 160\,\text{FresnelC}(bx)\left(6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\right.}{}$$

```
[In] Integrate[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (80*b^7*Pi^4*x^7*FresnelC[b*x]^2 - 2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160
*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[
b^2*Pi*x^2] + 2*(-960 + 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/
(560*b^7*Pi^4)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelC}(bx)\left(\frac{b^6 x^6 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} - \frac{6\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{7\pi}\right)}{}$
default	$\frac{\frac{\text{FresnelC}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelC}(bx)\left(\frac{b^6 x^6 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} - \frac{6\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{7\pi}\right)}{}$

```
[In] int(x^6*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/7*FresnelC(b*x)^2*b^7*x^7-2*FresnelC(b*x)*(1/7/Pi*b^6*x^6*sin(1/2*
b^2*Pi*x^2)-6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*b^5*x^5*Pi^2-
8*b*x)+6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2
*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/
2)))+1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b
^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*
2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int x^6 \,\text{FresnelC}(bx)^2 \, dx$$
$$= \frac{80\,\pi^4 b^8 x^7 \,\text{C}\,(bx)^2 + 136\,\pi^2 b^6 x^5 - 5310\,b^2 x - 20\,(4\,\pi^2 b^6 x^5 - 147\,b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 960\,(\pi^2 b^5 x^4 - 8\,b)\,\text{c}}{}$$

```
[In] integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/560*(80*pi^4*b^8*x^7*fresnel_cos(b*x)^2 + 136*pi^2*b^6*x^5 - 5310*b^2*x -
 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2)^2 - 960*(pi^2*b^5*x^4
- 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 2655*sqrt(2)*sqrt(b^2)*fresne
l_cos(sqrt(2)*sqrt(b^2)*x) + 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - 4*(pi^
3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^8
)
```

Sympy [F]

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 C^2(bx) \, dx$$

```
[In] integrate(x**6*fresnelc(b*x)**2,x)
[Out] Integral(x**6*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^6*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^6*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 \operatorname{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(x^6*FresnelC(b*x)^2,x)
[Out] int(x^6*FresnelC(b*x)^2, x)
```

3.142 $\int x^5 \,\text{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 265

$$
\begin{aligned}
\int x^5 \,\text{FresnelC}(bx)^2 \, dx = {} & \frac{5x^4}{24b^2\pi^2} + \frac{11\cos(b^2\pi x^2)}{6b^6\pi^4} - \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} \\
& - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{6}x^6\,\text{FresnelC}(bx)^2 \\
& - \frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^6\pi^3} - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\
& + \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} \\
& - \frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3}
\end{aligned}
$$

```
[Out] 5/24*x^4/b^2/Pi^2+11/6*cos(b^2*Pi*x^2)/b^6/Pi^4-1/12*x^4*cos(b^2*Pi*x^2)/b^
2/Pi^2-5/3*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/6*x^6*FresnelC(
b*x)^2-5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3-5/8*I*x^2*hypergeom([1, 1],
[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^4/Pi^3+5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/
2*I*b^2*Pi*x^2)/b^4/Pi^3+5*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/3
*x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+7/12*x^2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6566, 6590, 6598, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$\int x^5 \operatorname{FresnelC}(bx)^2\,dx = -\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}-\frac{5\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2\pi^3b^6}-\frac{x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{5x^4}{24\pi^2b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{12\pi^2b^2}+\frac{11\cos\left(\pi b^2x^2\right)}{6\pi^4b^6}+\frac{5x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^5}+\frac{7x^2\sin\left(\pi b^2x^2\right)}{12\pi^3b^4}-\frac{5x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{6}x^6\operatorname{FresnelC}(bx)^2$$

```
[In] Int[x^5*FresnelC[b*x]^2,x]
```

```
[Out] (5*x^4)/(24*b^2*Pi^2) + (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^
2) + (x^6*FresnelC[b*x]^2)/6 - (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3)
 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])
/(b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*
Pi*x^2])/(b^4*Pi^3) + (5*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^5*Pi^3) -
(x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi) + (7*x^2*Sin[b^2*Pi*x^2])/
(12*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
```

```
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= \frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}\\
&\quad+\frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{6\pi}+\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b\pi}\\
&= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2}+\frac{1}{6}x^6\,\text{FresnelC}(bx)^2\\
&\quad-\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}+\frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^3\pi^2}\\
&\quad+\frac{5\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b^2\pi^2}+\frac{\text{Subst}\left(\int x^2\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{12\pi}\\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2}-\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2}+\frac{1}{6}x^6\,\text{FresnelC}(bx)^2\\
&\quad+\frac{5x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3}-\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}\\
&\quad-\frac{5\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^5\pi^3}-\frac{5\int x\sin\left(b^2\pi x^2\right)\,dx}{2b^4\pi^3}\\
&\quad+\frac{\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx,x,x^2\right)}{6b^2\pi^2}+\frac{5\text{Subst}\left(\int x\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)}{6b^2\pi^2}\\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2}-\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2}+\frac{1}{6}x^6\,\text{FresnelC}(bx)^2\\
&\quad-\frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^6\pi^3}-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3}\\
&\quad+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3}+\frac{5x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3}\\
&\quad-\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}+\frac{x^2\sin\left(b^2\pi x^2\right)}{6b^4\pi^3}\\
&\quad-\frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{6b^4\pi^3}-\frac{5\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{4b^4\pi^3}\\
&\quad+\frac{5\text{Subst}\left(\int x\,dx,x,x^2\right)}{12b^2\pi^2}+\frac{5\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx,x,x^2\right)}{12b^2\pi^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{5x^4}{24b^2\pi^2} + \frac{17\cos(b^2\pi x^2)}{12b^6\pi^4} - \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2} \\
&\quad + \frac{1}{6}x^6\,\text{FresnelC}(bx)^2 - \frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^6\pi^3} - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\
&\quad + \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} \\
&\quad - \frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3} - \frac{5\text{Subst}\left(\int\sin(b^2\pi x)\,dx, x, x^2\right)}{12b^4\pi^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5x^4}{24b^2\pi^2} + \frac{11\cos(b^2\pi x^2)}{6b^6\pi^4} - \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2} \\
&\quad + \frac{1}{6}x^6\,\text{FresnelC}(bx)^2 - \frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^6\pi^3} \\
&\quad - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\
&\quad + \frac{5x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3}
\end{aligned}
$$

Mathematica [F]

$$\int x^5\,\text{FresnelC}(bx)^2\,dx = \int x^5\,\text{FresnelC}(bx)^2\,dx$$

```
[In] Integrate[x^5*FresnelC[b*x]^2,x]
[Out] Integrate[x^5*FresnelC[b*x]^2, x]
```

Maple [F]

$$\int x^5\,\text{FresnelC}(bx)^2\,dx$$

```
[In] int(x^5*FresnelC(b*x)^2,x)
[Out] int(x^5*FresnelC(b*x)^2,x)
```

Fricas [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 \operatorname{C}\left(bx\right)^2 \, dx$$

```
[In] integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="fricas")
[Out] integral(x^5*fresnel_cos(b*x)^2, x)
```

Sympy [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 C^2(bx) \, dx$$

```
[In] integrate(x**5*fresnelc(b*x)**2,x)
[Out] Integral(x**5*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 \operatorname{C}\left(bx\right)^2 \, dx$$

```
[In] integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^5*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 \operatorname{C}\left(bx\right)^2 \, dx$$

```
[In] integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^5*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 \operatorname{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(x^5*FresnelC(b*x)^2,x)
[Out] int(x^5*FresnelC(b*x)^2, x)
```

3.143 $\int x^4 \operatorname{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 177

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \frac{4x^3}{15b^2\pi^2} - \frac{x^3 \cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{8x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{5b^3\pi^2} + \frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{43 \operatorname{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} + \frac{16 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{11x \sin(b^2\pi x^2)}{20b^4\pi^3}$$

```
[Out] 4/15*x^3/b^2/Pi^2-1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-8/5*x^2*cos(1/2*b^2*Pi*
x^2)*FresnelC(b*x)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)^2+16/5*FresnelC(b*x)*sin(
1/2*b^2*Pi*x^2)/b^5/Pi^3-2/5*x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+11/
20*x*sin(b^2*Pi*x^2)/b^4/Pi^3-43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6566, 6590, 6598, 6588, 3432, 3473, 30, 3467, 3466}

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = -\frac{43 \operatorname{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}\pi^3b^5} - \frac{2x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b} + \frac{4x^3}{15\pi^2b^2} - \frac{x^3 \cos(\pi b^2x^2)}{10\pi^2b^2} + \frac{16 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5} + \frac{11x \sin(\pi b^2x^2)}{20\pi^3b^4} - \frac{8x^2 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3} + \frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2$$

```
[In] Int[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b
^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*F
resnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
 + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
```

```
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\,\text{FresnelC}(bx)^2-\frac{1}{5}(2b)\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= \frac{1}{5}x^5\,\text{FresnelC}(bx)^2-\frac{2x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}\\
&\quad+\frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{5\pi}+\frac{8\int x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{5b\pi}\\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2}-\frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5b^3\pi^2}+\frac{1}{5}x^5\,\text{FresnelC}(bx)^2\\
&\quad-\frac{2x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}+\frac{16\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{5b^3\pi^2}\\
&\quad+\frac{3\int x^2\cos\left(b^2\pi x^2\right)\,dx}{10b^2\pi^2}+\frac{8\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{5b^2\pi^2}\\
&= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2}-\frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5b^3\pi^2}+\frac{1}{5}x^5\,\text{FresnelC}(bx)^2\\
&\quad+\frac{16\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}-\frac{2x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}+\frac{3x\sin\left(b^2\pi x^2\right)}{20b^4\pi^3}\\
&\quad-\frac{3\int\sin\left(b^2\pi x^2\right)\,dx}{20b^4\pi^3}-\frac{8\int\sin\left(b^2\pi x^2\right)\,dx}{5b^4\pi^3}+\frac{4\int x^2\,dx}{5b^2\pi^2}+\frac{4\int x^2\cos\left(b^2\pi x^2\right)\,dx}{5b^2\pi^2}
\end{aligned}$$

$$= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5b^3\pi^2} + \frac{1}{5}x^5\,\text{FresnelC}(bx)^2 - \frac{3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} - \frac{4\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{5b^5\pi^3} + \frac{16\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{11x\sin(b^2\pi x^2)}{20b^4\pi^3} - \frac{2\int\sin(b^2\pi x^2)\,dx}{5b^4\pi^3}$$

$$= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5b^3\pi^2} + \frac{1}{5}x^5\,\text{FresnelC}(bx)^2 - \frac{3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} - \frac{\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^5\pi^3} + \frac{16\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{11x\sin(b^2\pi x^2)}{20b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int x^4\,\text{FresnelC}(bx)^2\,dx = \frac{32b^3\pi x^3 - 12b^3\pi x^3\cos(b^2\pi x^2) + 24b^5\pi^3x^5\,\text{FresnelC}(bx)^2 - 129\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) - 48\,\text{FresnelC}(bx)\,(4}{120b^5\pi^3}$$

```
[In] Integrate[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 - 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelC[b
*x]^2 - 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 48*FresnelC[b*x]*(4*b^2*Pi*x^2*
Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right)+\frac{4b^3x^3}{15\pi^2}+\frac{2bx\sin\left(b^2\pi x^2 \cdots\right.}{5\pi}}{b^5}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right)+\frac{4b^3x^3}{15\pi^2}+\frac{2bx\sin\left(b^2\pi x^2 \cdots\right.}{5\pi}}{b^5}$

```
[In] int(x^4*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*FresnelC(b*x)^2*b^5*x^5-2*FresnelC(b*x)*(1/5/Pi*b^4*x^4*sin(1/2*
b^2*Pi*x^2)-4/5/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3+4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))+1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int x^4\,\mathrm{FresnelC}(bx)^2\,dx$$
$$= \frac{24\,\pi^3 b^6 x^5\,\mathrm{C}\,(bx)^2 - 24\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 44\,\pi b^4 x^3 - 192\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\,(bx) - 129\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{\cdots}\right.}{120\,\pi^3 b^6}$$

```
[In] integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(24*pi^3*b^6*x^5*fresnel_cos(b*x)^2 - 24*pi*b^4*x^3*cos(1/2*pi*b^2*x^
2)^2 + 44*pi*b^4*x^3 - 192*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)
- 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 12*(11*b^2*x*cos
(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^4 - 8*b)*fresnel_cos(b*x))*sin(1/2*pi*b^2*
x^2))/(pi^3*b^6)
```

Sympy [F]

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 C^2(bx) \, dx$$

```
[In] integrate(x**4*fresnelc(b*x)**2,x)
[Out] Integral(x**4*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^4*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^4*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 \operatorname{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(x^4*FresnelC(b*x)^2,x)
[Out] int(x^4*FresnelC(b*x)^2, x)
```

3.144 $\int x^3 \operatorname{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 140

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos(b^2\pi x^2)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{2b^3\pi^2} + \frac{3\operatorname{FresnelC}(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelC}(bx)^2 - \frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

```
[Out] 3/8*x^2/b^2/Pi^2-1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2-3/2*x*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/b^3/Pi^2+3/4*FresnelC(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelC(b*x)^
2-1/2*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6566, 6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717}

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \frac{3\operatorname{FresnelC}(bx)^2}{4\pi^2b^4} - \frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{3x^2}{8\pi^2b^2} - \frac{x^2\cos(\pi b^2x^2)}{8\pi^2b^2} + \frac{\sin(\pi b^2x^2)}{2\pi^3b^4} - \frac{3x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{1}{4}x^4\operatorname{FresnelC}(bx)^2$$

```
[In] Int[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\,\text{FresnelC}(bx)^2 - \frac{1}{2}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= \frac{1}{4}x^4\,\text{FresnelC}(bx)^2 - \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}\\
&\quad + \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{4\pi} + \frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}\\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2b^3\pi^2} + \frac{1}{4}x^4\,\text{FresnelC}(bx)^2\\
&\quad - \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{2b^3\pi^2}\\
&\quad + \frac{3\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^2\pi^2} + \frac{\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{8\pi}\\
&= -\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2b^3\pi^2} + \frac{1}{4}x^4\,\text{FresnelC}(bx)^2\\
&\quad - \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\text{Subst}\left(\int x\,dx,x,\text{FresnelC}(bx)\right)}{2b^4\pi^2}\\
&\quad + \frac{\text{Subst}\left(\int\cos\left(b^2\pi x\right)\,dx,x,x^2\right)}{8b^2\pi^2} + \frac{3\text{Subst}\left(\int\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)}{4b^2\pi^2}
\end{aligned}$$

$$= \frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos(b^2\pi x^2)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2b^3\pi^2} + \frac{3\,\text{FresnelC}(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4\,\text{FresnelC}(bx)^2 - \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

$$\int x^3\,\text{FresnelC}(bx)^2\,dx = \frac{3b^2\pi x^2 - b^2\pi x^2\cos(b^2\pi x^2) + 2\pi(3+b^4\pi^2x^4)\,\text{FresnelC}(bx)^2 - 4b\pi x\,\text{FresnelC}(bx)\left(3\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2\right.}{8b^4\pi^3}$$

```
[In] Integrate[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*b^2*Pi*x^2 - b^2*Pi*x^2*Cos[b^2*Pi*x^2] + 2*Pi*(3 + b^4*Pi^2*x^4)*Fresne
lC[b*x]^2 - 4*b*Pi*x*FresnelC[b*x]*(3*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[
(b^2*Pi*x^2)/2]) + 4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Maple [F]

$$\int x^3\,\text{FresnelC}(bx)^2\,dx$$

```
[In] int(x^3*FresnelC(b*x)^2,x)
```

```
[Out] int(x^3*FresnelC(b*x)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int x^3\,\text{FresnelC}(bx)^2\,dx = -\frac{\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)^2 - 2\pi b^2x^2 + 6\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right)C(bx) - (3\pi + \pi^3b^4x^4)\,C(bx)^2 + 2\left(\pi^2b^3x^3\,C(bx)\right.}{4\pi^3b^4}$$

```
[In] integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 2*pi*b^2*x^2 + 6*pi*b*x*cos(1/2*pi
*b^2*x^2)*fresnel_cos(b*x) - (3*pi + pi^3*b^4*x^4)*fresnel_cos(b*x)^2 + 2*(
pi^2*b^3*x^3*fresnel_cos(b*x) - 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))
/(pi^3*b^4)
```

Sympy [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 C^2(bx) \, dx$$

```
[In] integrate(x**3*fresnelc(b*x)**2,x)
[Out] Integral(x**3*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^3*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^3*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 \operatorname{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(x^3*FresnelC(b*x)^2,x)
[Out] int(x^3*FresnelC(b*x)^2, x)
```

3.145 $\int x^2 \operatorname{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 124

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \frac{2x}{3b^2\pi^2} - \frac{x\cos(b^2\pi x^2)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{3}x^3\operatorname{FresnelC}(bx)^2 + \frac{5\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} - \frac{2x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

```
[Out] 2/3*x/b^2/Pi^2-1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2-4/3*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)^2-2/3*x^2*FresnelC(b*x)*sin(1/2*b^2
*Pi*x^2)/b/Pi+5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6566, 6590, 6596, 3439, 3433, 3466}

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \frac{5\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}\pi^2 b^3} - \frac{2x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} - \frac{x\cos(\pi b^2x^2)}{6\pi^2b^2} + \frac{2x}{3\pi^2b^2} - \frac{4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{1}{3}x^3\operatorname{FresnelC}(bx)^2$$

```
[In] Int[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)
/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqr
```

```
t[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])
/(3*b*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{1}{3}(2b)\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\,dx$$

$$= \frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{2x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int x^2 \sin\left(b^2\pi x^2\right) dx}{3\pi} + \frac{4\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{3b\pi}$$

$$= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{2x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int \cos\left(b^2\pi x^2\right) dx}{6b^2\pi^2} + \frac{4\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{3b^2\pi^2}$$

$$= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 + \frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} - \frac{2x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{4\int \left(\frac{1}{2} + \frac{1}{2}\cos\left(b^2\pi x^2\right)\right) dx}{3b^2\pi^2}$$

$$= \frac{2x}{3b^2\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 + \frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} - \frac{2x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{2\int \cos\left(b^2\pi x^2\right) dx}{3b^2\pi^2}$$

$$= \frac{2x}{3b^2\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 + \frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} + \frac{\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{3b^3\pi^2} - \frac{2x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \frac{-2bx\left(-4+\cos\left(b^2\pi x^2\right)\right) + 4b^3\pi^2x^3 \operatorname{FresnelC}(bx)^2 + 5\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right) - 8\operatorname{FresnelC}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi\right.\right.}{12b^3\pi^2}$$

```
[In] Integrate[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (-2*b*x*(-4 + Cos[b^2*Pi*x^2]) + 4*b^3*Pi^2*x^3*FresnelC[b*x]^2 + 5*Sqrt[2]
*FresnelC[Sqrt[2]*b*x] - 8*FresnelC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^
2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^3 x^3}{3}-2\,\mathrm{FresnelC}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right)+\frac{2bx}{3\pi^2}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}}{}}{3\pi}}{b^3}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^3 x^3}{3}-2\,\mathrm{FresnelC}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right)+\frac{2bx}{3\pi^2}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}}{}}{3\pi}}{b^3}$

```
[In] int(x^2*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelC(b*x)^2*b^3*x^3-2*FresnelC(b*x)*(1/3/Pi*b^2*x^2*sin(1/2*
b^2*Pi*x^2)+2/3/Pi^2*cos(1/2*b^2*Pi*x^2))+2/3*b*x/Pi^2+1/3/Pi^2*2^(1/2)*Fre
snelC(b*x*2^(1/2))+1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresn
elC(b*x*2^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x^2\,\mathrm{FresnelC}(bx)^2\,dx$$
$$= \frac{4\,\pi^2 b^4 x^3\,\mathrm{C}\left(bx\right)^2 - 8\,\pi b^3 x^2\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 4\,b^2 x\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 10\,b^2 x - 16\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) +}{12\,\pi^2 b^4}$$

```
[In] integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi^2*b^4*x^3*fresnel_cos(b*x)^2 - 8*pi*b^3*x^2*fresnel_cos(b*x)*sin
(1/2*pi*b^2*x^2) - 4*b^2*x*cos(1/2*pi*b^2*x^2)^2 + 10*b^2*x - 16*b*cos(1/2*
pi*b^2*x^2)*fresnel_cos(b*x) + 5*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt
(b^2)*x))/(pi^2*b^4)
```

Sympy [F]

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \int x^2 C^2(bx) \, dx$$

```
[In] integrate(x**2*fresnelc(b*x)**2,x)
[Out] Integral(x**2*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \int x^2 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^2*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \int x^2 \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \int x^2 \operatorname{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(x^2*FresnelC(b*x)^2,x)
[Out] int(x^2*FresnelC(b*x)^2, x)
```

3.146 $\int x \operatorname{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 144

$$\int x \operatorname{FresnelC}(bx)^2 \, dx = -\frac{\cos(b^2\pi x^2)}{4b^2\pi^2} + \frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2 + \frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^2\pi} + \frac{ix^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[Out] -1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^2*FresnelC(b*x)^2+1/2*FresnelC(b*x)*Fre
snelS(b*x)/b^2/Pi+1/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/Pi
-1/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/Pi-x*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6566, 6590, 6582, 3460, 2718}

$$\int x \operatorname{FresnelC}(bx)^2 \, dx = \frac{ix^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} + \frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2\pi b^2} - \frac{x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b} - \frac{\cos(\pi b^2 x^2)}{4\pi^2 b^2} + \frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2$$

```
[In] Int[x*FresnelC[b*x]^2,x]
```

```
[Out] -1/4*Cos[b^2*Pi*x^2]/(b^2*Pi^2) + (x^2*FresnelC[b*x]^2)/2 + (FresnelC[b*x]*
FresnelS[b*x])/(2*b^2*Pi) + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-1/2*I)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (
I/2)*b^2*Pi*x^2])/Pi - (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\,\text{FresnelC}(bx)^2 - b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelC}(bx)^2 - \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}\\
&\quad + \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2\pi} + \frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}x^2\,\mathrm{FresnelC}(bx)^2 + \frac{\mathrm{FresnelC}(bx)\,\mathrm{FresnelS}(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} \\
&\quad - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{x\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\mathrm{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4\pi} \\
&= -\frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{1}{2}x^2\,\mathrm{FresnelC}(bx)^2 \\
&\quad + \frac{\mathrm{FresnelC}(bx)\,\mathrm{FresnelS}(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} \\
&\quad - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{x\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}
\end{aligned}
$$

Mathematica [F]

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int x\,\mathrm{FresnelC}(bx)^2\,dx$$

```
[In] Integrate[x*FresnelC[b*x]^2,x]
[Out] Integrate[x*FresnelC[b*x]^2, x]
```

Maple [F]

$$\int x\,\mathrm{FresnelC}\left(bx\right)^2\,dx$$

```
[In] int(x*FresnelC(b*x)^2,x)
[Out] int(x*FresnelC(b*x)^2,x)
```

Fricas [F]

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int x\,\mathrm{C}\left(bx\right)^2\,dx$$

```
[In] integrate(x*fresnel_cos(b*x)^2,x, algorithm="fricas")
[Out] integral(x*fresnel_cos(b*x)^2, x)
```

Sympy [F]

$$\int x \operatorname{FresnelC}(bx)^2 \, dx = \int x C^2(bx) \, dx$$

```
[In] integrate(x*fresnelc(b*x)**2,x)
[Out] Integral(x*fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int x \operatorname{FresnelC}(bx)^2 \, dx = \int x \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x*fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int x \operatorname{FresnelC}(bx)^2 \, dx = \int x \operatorname{C}(bx)^2 \, dx$$

```
[In] integrate(x*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \operatorname{FresnelC}(bx)^2 \, dx = \int x \operatorname{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(x*FresnelC(b*x)^2,x)
[Out] int(x*FresnelC(b*x)^2, x)
```

3.147 $\int \text{FresnelC}(bx)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 54

$$\int \text{FresnelC}(bx)^2\,dx = x\,\text{FresnelC}(bx)^2 + \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[Out] x*FresnelC(b*x)^2-2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*FresnelS(b*x
*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6556, 12, 6588, 3432}

$$\int \text{FresnelC}(bx)^2\,dx = -\frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + x\,\text{FresnelC}(bx)^2 + \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}\pi b}$$

```
[In] Int[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\,\text{FresnelC}(bx)^2 - 2\int bx\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= x\,\text{FresnelC}(bx)^2 - (2b)\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= x\,\text{FresnelC}(bx)^2 - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int\sin\left(b^2\pi x^2\right)\,dx}{\pi}\\
&= x\,\text{FresnelC}(bx)^2 + \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^2\,dx = x\,\text{FresnelC}(bx)^2 + \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[In] Integrate[FresnelC[b*x]^2,x]
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx)^{2} bx-\frac{2\,\mathrm{FresnelC}(bx)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49
default	$\frac{\mathrm{FresnelC}(bx)^{2} bx-\frac{2\,\mathrm{FresnelC}(bx)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49

```
[In] int(FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(FresnelC(b*x)^2*b*x-2*FresnelC(b*x)/Pi*sin(1/2*b^2*Pi*x^2)+1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \mathrm{FresnelC}(bx)^2\,dx = \frac{2\,\pi b^2 x\,\mathrm{C}\,(bx)^2 - 4\,b\,\mathrm{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + \sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{2\,\pi b^2}$$

```
[In] integrate(fresnel_cos(b*x)^2,x, algorithm="fricas")
[Out] 1/2*(2*pi*b^2*x*fresnel_cos(b*x)^2 - 4*b*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^
2) + sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F]

$$\int \mathrm{FresnelC}(bx)^2\,dx = \int C^2(bx)\,dx$$

```
[In] integrate(fresnelc(b*x)**2,x)
[Out] Integral(fresnelc(b*x)**2, x)
```

Maxima [F]

$$\int \text{FresnelC}(bx)^2 \, dx = \int \text{C}\left(bx\right)^2 \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2, x)
```

Giac [F]

$$\int \text{FresnelC}(bx)^2 \, dx = \int \text{C}\left(bx\right)^2 \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx)^2 \, dx = \int \text{FresnelC}(b\,x)^2 \, dx$$

```
[In] int(FresnelC(b*x)^2,x)
[Out] int(FresnelC(b*x)^2, x)
```

3.148 $\int \frac{\text{FresnelC}(bx)^2}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx = \text{Int}\left(\frac{\text{FresnelC}(bx)^2}{x}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx = \int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

```
[In] Int[FresnelC[b*x]^2/x,x]
```

```
[Out] Defer[Int][FresnelC[b*x]^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx = \int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

```
[In] Integrate[FresnelC[b*x]^2/x,x]
[Out] Integrate[FresnelC[b*x]^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x}dx$$

```
[In] int(FresnelC(b*x)^2/x,x)
[Out] int(FresnelC(b*x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx = \int \frac{\text{C}\,(bx)^2}{x} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx = \int \frac{C^2(bx)}{x} \, dx$$

```
[In] integrate(fresnelc(b*x)**2/x,x)
[Out] Integral(fresnelc(b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\mathrm{C}\left(bx\right)^2}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\mathrm{C}\left(bx\right)^2}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\text{FresnelC}\left(b\,x\right)^2}{x}\,dx$$

```
[In] int(FresnelC(b*x)^2/x,x)
[Out] int(FresnelC(b*x)^2/x, x)
```

3.149 $\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx = -\frac{\text{FresnelC}(bx)^2}{x} + 2b\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}, x\right)$$

```
[Out] -FresnelC(b*x)^2/x+2*b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx = \int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

```
[In] Int[FresnelC[b*x]^2/x^2,x]
```

```
[Out] -(FresnelC[b*x]^2/x) + 2*b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x
, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelC}(bx)^2}{x} + (2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx=\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx$$

```
[In] Integrate[FresnelC[b*x]^2/x^2,x]
[Out] Integrate[FresnelC[b*x]^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^{2}}{x^{2}}dx$$

```
[In] int(FresnelC(b*x)^2/x^2,x)
[Out] int(FresnelC(b*x)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx=\int \frac{\text{C}\left(bx\right)^{2}}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx=\int \frac{C^{2}(bx)}{x^2}\,dx$$

```
[In] integrate(fresnelc(b*x)**2/x**2,x)
[Out] Integral(fresnelc(b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = \int \frac{\text{C}\,(bx)^2}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = \int \frac{\text{C}\,(bx)^2}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^2}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^2,x)
[Out] int(FresnelC(b*x)^2/x^2, x)
```

3.150 $\int \frac{\text{FresnelC}(bx)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = -\frac{\text{FresnelC}(bx)^2}{2x^2} + b\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}, x\right)$$

```
[Out] -1/2*FresnelC(b*x)^2/x^2+b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = \int \frac{\text{FresnelC}(bx)^2}{x^3} dx$$

```
[In] Int[FresnelC[b*x]^2/x^3,x]
```

```
[Out] -1/2*FresnelC[b*x]^2/x^2 + b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x^2, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelC}(bx)^2}{2x^2} + b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = \int \frac{\text{FresnelC}(bx)^2}{x^3} dx$$

```
[In] Integrate[FresnelC[b*x]^2/x^3,x]
[Out] Integrate[FresnelC[b*x]^2/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^{2}}{x^{3}}dx$$

```
[In] int(FresnelC(b*x)^2/x^3,x)
[Out] int(FresnelC(b*x)^2/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = \int \frac{\text{C}\left(bx\right)^{2}}{x^{3}} dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = \int \frac{C^{2}(bx)}{x^{3}} dx$$

```
[In] integrate(fresnelc(b*x)**2/x**3,x)
[Out] Integral(fresnelc(b*x)**2/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^3} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^3} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} \, dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^3} \, dx$$

```
[In] int(FresnelC(b*x)^2/x^3,x)
[Out] int(FresnelC(b*x)^2/x^3, x)
```

3.151 $\int \frac{\text{FresnelC}(bx)^2}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} dx = -\frac{b^2}{6x} - \frac{b^2\cos(b^2\pi x^2)}{6x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^2} - \frac{\text{FresnelC}(bx)^2}{3x^3} - \frac{b^3\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{3\sqrt{2}} - \frac{1}{3}b^3\pi\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)$$

```
[Out] -1/6*b^2/x-1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^2-1/3*FresnelC(b*x)^2/x^3-1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/3*
b^3*Pi*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} dx = \int \frac{\text{FresnelC}(bx)^2}{x^4} dx$$

```
[In] Int[FresnelC[b*x]^2/x^4,x]
```

```
[Out] -1/6*b^2/x - (b^2*Cos[b^2*Pi*x^2])/(6*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^2) - FresnelC[b*x]^2/(3*x^3) - (b^3*Pi*FresnelS[Sqrt[2]*b*x])/(3
*Sqrt[2]) - (b^3*Pi*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/3
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx\\
&= -\frac{b^2}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^2}-\frac{\text{FresnelC}(bx)^2}{3x^3}\\
&\quad+\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{3}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\
&= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^2}-\frac{\text{FresnelC}(bx)^2}{3x^3}\\
&\quad-\frac{1}{3}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx-\frac{1}{3}\left(b^4\pi\right)\int \sin\left(b^2\pi x^2\right)\,dx\\
&= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^2}-\frac{\text{FresnelC}(bx)^2}{3x^3}\\
&\quad-\frac{b^3\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{3\sqrt{2}}-\frac{1}{3}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx=\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$$

```
[In] Integrate[FresnelC[b*x]^2/x^4,x]
[Out] Integrate[FresnelC[b*x]^2/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^{2}}{x^{4}}dx$$

```
[In] int(FresnelC(b*x)^2/x^4,x)
[Out] int(FresnelC(b*x)^2/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{C^2(bx)}{x^4}\,dx$$

```
[In] integrate(fresnelc(b*x)**2/x**4,x)
[Out] Integral(fresnelc(b*x)**2/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{C}\,(bx)^2}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^4}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^4,x)
[Out] int(FresnelC(b*x)^2/x^4, x)
```

3.152 $\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 127

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = -\frac{b^2}{24x^2} - \frac{b^2 \cos(b^2\pi x^2)}{24x^2} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{6x^3} - \frac{1}{12}b^4\pi^2 \text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{4x^4} + \frac{b^3\pi \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{12}b^4\pi \text{Si}(b^2\pi x^2)$$

```
[Out] -1/24*b^2/x^2-1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b*cos(1/2*b^2*Pi*x^2)*Fresne
lC(b*x)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)^2-1/4*FresnelC(b*x)^2/x^4-1/12*b^4*
Pi*Si(b^2*Pi*x^2)+1/6*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6566, 6592, 6600, 6576, 30, 3456, 3461, 3378, 3380}

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = -\frac{1}{12}\pi^2 b^4 \text{FresnelC}(bx)^2 - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^3} - \frac{b^2}{24x^2} - \frac{b^2 \cos(\pi b^2 x^2)}{24x^2} - \frac{1}{12}\pi b^4 \text{Si}(b^2\pi x^2) + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{6x} - \frac{\text{FresnelC}(bx)^2}{4x^4}$$

```
[In] Int[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)^2}{4x^4}+\frac{1}{2}b\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx\\
&= -\frac{b^2}{24x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3}-\frac{\text{FresnelC}(bx)^2}{4x^4}\\
&\quad+\frac{1}{12}b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx-\frac{1}{6}\left(b^3\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b^2}{24x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3}-\frac{\text{FresnelC}(bx)^2}{4x^4}\\
&\quad+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x}+\frac{1}{24}b^2\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&\quad-\frac{1}{12}\left(b^4\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx-\frac{1}{6}\left(b^5\pi^2\right)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= -\frac{b^2}{24x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3}\\
&\quad-\frac{\text{FresnelC}(bx)^2}{4x^4}+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x}\\
&\quad-\frac{1}{24}b^4\pi\text{Si}\left(b^2\pi x^2\right)-\frac{1}{24}\left(b^4\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&\quad-\frac{1}{6}\left(b^4\pi^2\right)\text{Subst}\left(\int x\,dx,x,\text{FresnelC}(bx)\right)\\
&= -\frac{b^2}{24x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3}-\frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx)^2\\
&\quad-\frac{\text{FresnelC}(bx)^2}{4x^4}+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x}-\frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx = -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{4x^4} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)$$

```
[In] Integrate[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Maple [F]

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^5}dx$$

```
[In] int(FresnelC(b*x)^2/x^5,x)
```

```
[Out] int(FresnelC(b*x)^2/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx = -\frac{\pi b^4x^4\,\text{Si}\,(\pi b^2x^2) - 2\,\pi b^3x^3\,\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 2\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\,(bx) + (\pi^2b^4x}{12\,x^4}$$

```
[In] integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(pi*b^4*x^4*sin_integral(pi*b^2*x^2) - 2*pi*b^3*x^3*fresnel_cos(b*x)*
sin(1/2*pi*b^2*x^2) + b^2*x^2*cos(1/2*pi*b^2*x^2)^2 + 2*b*x*cos(1/2*pi*b^2*
x^2)*fresnel_cos(b*x) + (pi^2*b^4*x^4 + 3)*fresnel_cos(b*x)^2)/x^4
```

Sympy [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = \int \frac{C^2(bx)}{x^5} \, dx$$

```
[In] integrate(fresnelc(b*x)**2/x**5,x)
[Out] Integral(fresnelc(b*x)**2/x**5, x)
```

Maxima [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = \int \frac{\text{C}(bx)^2}{x^5} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^5, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = \int \frac{\text{C}(bx)^2}{x^5} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^5} \, dx$$

```
[In] int(FresnelC(b*x)^2/x^5,x)
[Out] int(FresnelC(b*x)^2/x^5, x)
```

3.153 $\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = &-\frac{b^2}{60x^3}-\frac{b^2\cos(b^2\pi x^2)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{10x^4}\\&-\frac{\text{FresnelC}(bx)^2}{5x^5}-\frac{7b^5\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}}\\&+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}+\frac{7b^4\pi\sin(b^2\pi x^2)}{120x}\\&-\frac{1}{20}b^5\pi^2\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)\end{aligned}$$

```
[Out] -1/60*b^2/x^3-1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/10*b*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/x^4-1/5*FresnelC(b*x)^2/x^5+1/20*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x^2+7/120*b^4*Pi*sin(b^2*Pi*x^2)/x-7/120*b^5*Pi^2*FresnelC(b*x*2^(1
/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx$$

```
[In] Int[FresnelC[b*x]^2/x^6,x]
```

```
[Out] -1/60*b^2/x^3 - (b^2*Cos[b^2*Pi*x^2])/(60*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(10*x^4) - FresnelC[b*x]^2/(5*x^5) - (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(20*x^2)
 + (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/x, x])/20
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)^2}{5x^5}+\frac{1}{5}(2b)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx\\
&= -\frac{b^2}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{10x^4}-\frac{\text{FresnelC}(bx)^2}{5x^5}\\
&\quad+\frac{1}{20}b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{10}\left(b^3\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\
&= -\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{10x^4}-\frac{\text{FresnelC}(bx)^2}{5x^5}\\
&\quad+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}-\frac{1}{40}\left(b^4\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx\\
&\quad-\frac{1}{30}\left(b^4\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{20}\left(b^5\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx\\
&= -\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{10x^4}\\
&\quad-\frac{\text{FresnelC}(bx)^2}{5x^5}+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}\\
&\quad+\frac{7b^4\pi\sin\left(b^2\pi x^2\right)}{120x}-\frac{1}{20}\left(b^5\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx\\
&\quad-\frac{1}{20}\left(b^6\pi^2\right)\int\cos\left(b^2\pi x^2\right)\,dx-\frac{1}{15}\left(b^6\pi^2\right)\int\cos\left(b^2\pi x^2\right)\,dx\\
&= -\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{10x^4}-\frac{\text{FresnelC}(bx)^2}{5x^5}\\
&\quad-\frac{7b^5\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}}+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}\\
&\quad+\frac{7b^4\pi\sin\left(b^2\pi x^2\right)}{120x}-\frac{1}{20}\left(b^5\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} \, dx = \int \frac{\text{FresnelC}(bx)^2}{x^6} \, dx$$

```
[In] Integrate[FresnelC[b*x]^2/x^6,x]
[Out] Integrate[FresnelC[b*x]^2/x^6, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^6} dx$$

```
[In] int(FresnelC(b*x)^2/x^6,x)
[Out] int(FresnelC(b*x)^2/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^6} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} \, dx = \int \frac{C^2(bx)}{x^6} \, dx$$

```
[In] integrate(fresnelc(b*x)**2/x**6,x)
[Out] Integral(fresnelc(b*x)**2/x**6, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{C}\,(bx)^2}{x^6}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^6, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{C}\,(bx)^2}{x^6}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^6, x)
```

Mupad [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^6}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^6,x)
[Out] int(FresnelC(b*x)^2/x^6, x)
```

3.154 $\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = -\frac{b^2}{120x^4} - \frac{b^2\cos(b^2\pi x^2)}{120x^4} - \frac{1}{72}b^6\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5} - \frac{\text{FresnelC}(bx)^2}{6x^6} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{b^4\pi\sin(b^2\pi x^2)}{72x^2} - \frac{1}{45}b^5\pi^2\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}, x\right)$$

```
[Out] -1/120*b^2/x^4-1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)-1/120*b^2*cos(b^2*Pi*x^2)/x^4-1
/15*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/6*FresnelC(b*x)^2/x^6+1/45*b^
3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3+1/72*b^4*Pi*sin(b^2*Pi*x^2)/x^2-
1/45*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

```
[In] Int[FresnelC[b*x]^2/x^7,x]
```

```
[Out] -1/120*b^2/x^4 - (b^2*Cos[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^5) - FresnelC[b
*x]^2/(6*x^6) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*Fr
esnelC[b*x])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)^2}{6x^6} + \frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx \\
&= -\frac{b^2}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5} - \frac{\text{FresnelC}(bx)^2}{6x^6} \\
&\quad + \frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{15}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b^2}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5} - \frac{\text{FresnelC}(bx)^2}{6x^6} \\
&\quad + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{1}{60}b^2\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&\quad - \frac{1}{90}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx \\
&= -\frac{b^2}{120x^4} - \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5} - \frac{\text{FresnelC}(bx)^2}{6x^6} \\
&\quad + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} - \frac{1}{180}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad - \frac{1}{120}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&\quad - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx \\
&= -\frac{b^2}{120x^4} - \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5} \\
&\quad - \frac{\text{FresnelC}(bx)^2}{6x^6} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} \\
&\quad + \frac{b^4\pi\sin\left(b^2\pi x^2\right)}{72x^2} - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx \\
&\quad - \frac{1}{180}\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&\quad - \frac{1}{120}\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x}\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{b^2}{120x^4} - \frac{b^2\cos(b^2\pi x^2)}{120x^4} - \frac{1}{72}b^6\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5} - \frac{\text{FresnelC}(bx)^2}{6x^6} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{b^4\pi\sin(b^2\pi x^2)}{72x^2} - \frac{1}{45}\left(b^5\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int\frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int\frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

```
[In] Integrate[FresnelC[b*x]^2/x^7,x]
[Out] Integrate[FresnelC[b*x]^2/x^7, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelC}\left(bx\right)^2}{x^7}dx$$

```
[In] int(FresnelC(b*x)^2/x^7,x)
[Out] int(FresnelC(b*x)^2/x^7,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int\frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int\frac{\text{C}\left(bx\right)^2}{x^7}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^7, x)
```

Sympy [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} \, dx = \int \frac{C^2(bx)}{x^7} \, dx$$

```
[In] integrate(fresnelc(b*x)**2/x**7,x)
[Out] Integral(fresnelc(b*x)**2/x**7, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^7} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^7, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^7} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^7, x)
```

Mupad [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^7}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^7,x)
[Out] int(FresnelC(b*x)^2/x^7, x)
```

3.155 $\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx &= -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}-\frac{b^2\cos(b^2\pi x^2)}{210x^5}+\frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x}\\ &\quad-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{168x^2}\\ &\quad-\frac{\text{FresnelC}(bx)^2}{7x^7}+\frac{b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{72\sqrt{2}}\\ &\quad+\frac{2}{315}\sqrt{2}b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4}\\ &\quad+\frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3}+\frac{1}{168}b^7\pi^3\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

```
[Out] -1/210*b^2/x^5+1/336*b^6*Pi^2/x-1/210*b^2*cos(b^2*Pi*x^2)/x^5+67/5040*b^6*P
i^2*cos(b^2*Pi*x^2)/x-1/21*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/168*b^
5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/7*FresnelC(b*x)^2/x^7+1/84*b
^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4+13/2520*b^4*Pi*sin(b^2*Pi*x^2)/
x^3+67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)+1/168*b^7*Pi^3*Unintegra
ble(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

```
[In] Int[FresnelC[b*x]^2/x^8,x]
```

```
[Out] -1/210*b^2/x^5 + (b^6*Pi^2)/(336*x) - (b^2*Cos[b^2*Pi*x^2])/(210*x^5) + (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(21*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(168*x^2) - Fresne
lC[b*x]^2/(7*x^7) + (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/(72*Sqrt[2]) + (2*Sqrt
[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(84*x^4) + (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) + (b^7*Pi^3*Def
er[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/168
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx \\
&= -\frac{b^2}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6} - \frac{\text{FresnelC}(bx)^2}{7x^7} \\
&\quad + \frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{21}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{b^2}{210x^5} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6} - \frac{\text{FresnelC}(bx)^2}{7x^7} \\
&\quad + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} - \frac{1}{168}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad - \frac{1}{105}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{84}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6} \\
&\quad + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{168x^2} - \frac{\text{FresnelC}(bx)^2}{7x^7} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} \\
&\quad + \frac{13b^4\pi\sin\left(b^2\pi x^2\right)}{2520x^3} - \frac{1}{336}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{252}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad - \frac{1}{315}\left(2b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{168}\left(b^7\pi^3\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos(b^2\pi x^2)}{210x^5} + \frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} \\
&\quad - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{168x^2} \\
&\quad - \frac{\text{FresnelC}(bx)^2}{7x^7} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} + \frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3} \\
&\quad + \frac{1}{168}(b^7\pi^3)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx + \frac{1}{168}(b^8\pi^3)\int \sin(b^2\pi x^2)\,dx \\
&\quad + \frac{1}{126}(b^8\pi^3)\int \sin(b^2\pi x^2)\,dx + \frac{1}{315}(4b^8\pi^3)\int \sin(b^2\pi x^2)\,dx \\
&= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos(b^2\pi x^2)}{210x^5} + \frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6} \\
&\quad + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{168x^2} - \frac{\text{FresnelC}(bx)^2}{7x^7} + \frac{b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{72\sqrt{2}} \\
&\quad + \frac{2}{315}\sqrt{2}b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} \\
&\quad + \frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3} + \frac{1}{168}(b^7\pi^3)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$
\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx
$$

```
[In] Integrate[FresnelC[b*x]^2/x^8,x]
[Out] Integrate[FresnelC[b*x]^2/x^8, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$
\int \frac{\text{FresnelC}(bx)^2}{x^8}dx
$$

```
[In] int(FresnelC(b*x)^2/x^8,x)
[Out] int(FresnelC(b*x)^2/x^8,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} \, dx = \int \frac{\text{C}\,(bx)^2}{x^8} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^8, x)
```

Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} \, dx = \int \frac{C^2(bx)}{x^8} \, dx$$

```
[In] integrate(fresnelc(b*x)**2/x**8,x)
[Out] Integral(fresnelc(b*x)**2/x**8, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} \, dx = \int \frac{\text{C}\,(bx)^2}{x^8} \, dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^8, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^8}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^8, x)
```

Mupad [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^8}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^8,x)
[Out] int(FresnelC(b*x)^2/x^8, x)
```

3.156 $\int \frac{\operatorname{FresnelC}(bx)^2}{x^9}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 242

$$\begin{aligned}\int \frac{\operatorname{FresnelC}(bx)^2}{x^9}\,dx = & -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos(b^2\pi x^2)}{336x^6}+\frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} \\ & -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{420x^3} \\ & +\frac{1}{840}b^8\pi^4\operatorname{FresnelC}(bx)^2-\frac{\operatorname{FresnelC}(bx)^2}{8x^8} \\ & +\frac{b^3\pi\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5}-\frac{b^7\pi^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} \\ & +\frac{b^4\pi\sin(b^2\pi x^2)}{420x^4}+\frac{1}{280}b^8\pi^3\operatorname{Si}(b^2\pi x^2)\end{aligned}$$

```
[Out] -1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2-1/336*b^2*cos(b^2*Pi*x^2)/x^6+1/336*b^6*
Pi^2*cos(b^2*Pi*x^2)/x^2-1/28*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/420
*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/840*b^8*Pi^4*FresnelC(b*x
)^2-1/8*FresnelC(b*x)^2/x^8+1/280*b^8*Pi^3*Si(b^2*Pi*x^2)+1/140*b^3*Pi*Fres
nelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/420*b^7*Pi^3*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x+1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6566, 6592, 6600, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$\int \frac{\text{FresnelC}(bx)^2}{x^9} dx = \frac{1}{840}\pi^4 b^8 \text{FresnelC}(bx)^2 + \frac{\pi^2 b^6}{1680x^2} - \frac{b \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{28x^7} - \frac{b^2}{336x^6} - \frac{b^2 \cos(\pi b^2 x^2)}{336x^6} + \frac{1}{280}\pi^3 b^8 \text{Si}\left(b^2 \pi x^2\right) - \frac{\pi^3 b^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{420x} + \frac{\pi^2 b^6 \cos(\pi b^2 x^2)}{336x^2} + \frac{\pi^2 b^5 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{420x^3} + \frac{\pi b^4 \sin(\pi b^2 x^2)}{420x^4} + \frac{\pi b^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{140x^5} - \frac{\text{FresnelC}(bx)^2}{8x^8}$$

```
[In] Int[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$$

$$= -\frac{b^2}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{28}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

$$= -\frac{b^2}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} + \frac{1}{112}b^2\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) - \frac{1}{280}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{140}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} - \frac{1}{560}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{336}\left(b^4\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{840}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx + \frac{1}{420}\left(b^7\pi^3\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} - \frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} + \frac{b^4\pi\sin\left(b^2\pi x^2\right)}{420x^4} - \frac{\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{1680} - \frac{\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{1120} - \frac{1}{672}\left(b^6\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) + \frac{1}{840}\left(b^8\pi^3\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx + \frac{1}{420}\left(b^9\pi^4\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} - \frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} + \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} + \frac{b^8\pi^3\text{Si}(b^2\pi x^2)}{1680} + \frac{(b^8\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{1680} + \frac{(b^8\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{1120} + \frac{1}{672}(b^8\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right) + \frac{1}{420}(b^8\pi^4)\,\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right)$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} + \frac{1}{840}b^8\pi^4\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} - \frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} + \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} + \frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$\int\frac{\text{FresnelC}(bx)^2}{x^9}\,dx = -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} + \frac{1}{840}b^8\pi^4\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} - \frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} + \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4} + \frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)$$

```
[In] Integrate[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
```

```
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Maple [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}dx$$

```
[In] int(FresnelC(b*x)^2/x^9,x)
[Out] int(FresnelC(b*x)^2/x^9,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = \frac{3\,\pi^3 b^8 x^8\,\text{Si}\left(\pi b^2 x^2\right) - 2\,\pi^2 b^6 x^6 + 5\left(\pi^2 b^6 x^6 - b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\left(\pi^2 b^5 x^5 - 15\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)}{840\,x^8}$$

```
[In] integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="fricas")
[Out] 1/840*(3*pi^3*b^8*x^8*sin_integral(pi*b^2*x^2) - 2*pi^2*b^6*x^6 + 5*(pi^2*b
^6*x^6 - b^2*x^2)*cos(1/2*pi*b^2*x^2)^2 + 2*(pi^2*b^5*x^5 - 15*b*x)*cos(1/2
*pi*b^2*x^2)*fresnel_cos(b*x) + (pi^4*b^8*x^8 - 105)*fresnel_cos(b*x)^2 + 2
*(2*pi*b^4*x^4*cos(1/2*pi*b^2*x^2) - (pi^3*b^7*x^7 - 3*pi*b^3*x^3)*fresnel_
cos(b*x))*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{C^2(bx)}{x^9}\,dx$$

```
[In] integrate(fresnelc(b*x)**2/x**9,x)
[Out] Integral(fresnelc(b*x)**2/x**9, x)
```

Maxima [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^9}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^9, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^9}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^9}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^9,x)
[Out] int(FresnelC(b*x)^2/x^9, x)
```

3.157 $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = &-\frac{b^2}{504x^7}+\frac{b^6\pi^2}{5184x^3}-\frac{b^2\cos(b^2\pi x^2)}{504x^7}\\ &+\frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{36x^8}\\ &+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{864x^4}-\frac{\text{FresnelC}(bx)^2}{9x^9}\\ &+\frac{853b^9\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{181440\sqrt{2}}+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6}\\ &-\frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^2}+\frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5}\\ &-\frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x}+\frac{b^9\pi^4\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)}{1728}\end{aligned}$$

```
[Out] -1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3-1/504*b^2*cos(b^2*Pi*x^2)/x^7+187/181440
*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/36*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+
1/864*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4-1/9*FresnelC(b*x)^2/x^
9+1/216*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6-1/1728*b^7*Pi^3*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5-853/362
880*b^8*Pi^3*sin(b^2*Pi*x^2)/x+853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/2))*2^
(1/2)+1/1728*b^9*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

```
[In] Int[FresnelC[b*x]^2/x^10,x]
```

```
[Out] -1/504*b^2/x^7 + (b^6*Pi^2)/(5184*x^3) - (b^2*Cos[b^2*Pi*x^2])/(504*x^7) +
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(36*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(864*x^4)
- FresnelC[b*x]^2/(9*x^9) + (853*b^9*Pi^4*FresnelC[Sqrt[2]*b*x])/(181440*Sq
rt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(216*x^6) - (b^7*Pi^3*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(1728*x^2) + (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) - (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/1728
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx \\
&= -\frac{b^2}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{36x^8} - \frac{\text{FresnelC}(bx)^2}{9x^9} \\
&\quad + \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{36}\left(b^3\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{b^2}{504x^7} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{36x^8} - \frac{\text{FresnelC}(bx)^2}{9x^9} \\
&\quad + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{1}{432}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad - \frac{1}{252}\left(b^4\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{216}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx \\
&= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{36x^8} \\
&\quad + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{864x^4} - \frac{\text{FresnelC}(bx)^2}{9x^9} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} \\
&\quad + \frac{19b^4\pi\sin\left(b^2\pi x^2\right)}{15120x^5} - \frac{\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx}{1728} - \frac{\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx}{1080} \\
&\quad - \frac{1}{630}\left(b^6\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx + \frac{1}{864}\left(b^7\pi^3\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx
\end{aligned}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos(b^2\pi x^2)}{504x^7} + \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}$$
$$- \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{864x^4}$$
$$- \frac{\operatorname{FresnelC}(bx)^2}{9x^9} + \frac{b^3\pi\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6}$$
$$- \frac{b^7\pi^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^2} + \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5}$$
$$+ \frac{(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx}{3456} + \frac{(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx}{2592} + \frac{(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx}{1620}$$
$$+ \frac{1}{945}(b^8\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^2}\,dx + \frac{(b^9\pi^4)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x}\,dx}{1728}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos(b^2\pi x^2)}{504x^7} + \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}$$
$$- \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{864x^4} - \frac{\operatorname{FresnelC}(bx)^2}{9x^9}$$
$$+ \frac{b^3\pi\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{b^7\pi^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^2}$$
$$+ \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5} - \frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x} + \frac{(b^9\pi^4)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x}\,dx}{1728}$$
$$+ \frac{(b^{10}\pi^4)\int\cos(b^2\pi x^2)\,dx}{1728} + \frac{(b^{10}\pi^4)\int\cos(b^2\pi x^2)\,dx}{1296}$$
$$+ \frac{1}{810}(b^{10}\pi^4)\int\cos\left(b^2\pi x^2\right)\,dx + \frac{1}{945}(2b^{10}\pi^4)\int\cos\left(b^2\pi x^2\right)\,dx$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos(b^2\pi x^2)}{504x^7} + \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}$$
$$- \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{864x^4}$$
$$- \frac{\operatorname{FresnelC}(bx)^2}{9x^9} + \frac{67b^9\pi^4\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{25920\sqrt{2}} + \frac{1}{945}\sqrt{2}b^9\pi^4\operatorname{FresnelC}\left(\sqrt{2}bx\right)$$
$$+ \frac{b^3\pi\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{b^7\pi^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^2}$$
$$+ \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5} - \frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x} + \frac{(b^9\pi^4)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x}\,dx}{1728}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

```
[In] Integrate[FresnelC[b*x]^2/x^10,x]
[Out] Integrate[FresnelC[b*x]^2/x^10, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^{10}}dx$$

```
[In] int(FresnelC(b*x)^2/x^10,x)
[Out] int(FresnelC(b*x)^2/x^10,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^{10}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^2/x^10, x)
```

Sympy [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{C^2(bx)}{x^{10}}\,dx$$

```
[In] integrate(fresnelc(b*x)**2/x**10,x)
[Out] Integral(fresnelc(b*x)**2/x**10, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{C}\,(bx)^2}{x^{10}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^2/x^10, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{C}\,(bx)^2}{x^{10}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^2/x^10, x)
```

Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^{10}}\,dx$$

```
[In] int(FresnelC(b*x)^2/x^10,x)
[Out] int(FresnelC(b*x)^2/x^10, x)
```

3.158 $\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 495

$$\begin{aligned}
\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx
&= \frac{2d^2x}{3b^2\pi^2} - \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} \\
&- \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelC}(a+bx)}{3b^3\pi^2} \\
&+ \frac{(bc-ad)^2(a+bx)\operatorname{FresnelC}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\operatorname{FresnelC}(a+bx)^2}{b^3} \\
&+ \frac{d^2(a+bx)^3\operatorname{FresnelC}(a+bx)^2}{3b^3} + \frac{5d^2\operatorname{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} \\
&+ \frac{d(bc-ad)\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{b^3\pi} + \frac{(bc-ad)^2\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&+ \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&- \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&- \frac{2(bc-ad)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&- \frac{2d(bc-ad)(a+bx)\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&- \frac{2d^2(a+bx)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}
\end{aligned}$$

```
[Out] 2/3*d^2*x/b^2/Pi^2-1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-1/6*d^2*(b*x
+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-4/3*d^2*cos(1/2*Pi*(b*x+a)^2)*FresnelC(b*x+a
)/b^3/Pi^2+(-a*d+b*c)^2*(b*x+a)*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^
```

```
2*FresnelC(b*x+a)^2/b^3+1/3*d^2*(b*x+a)^3*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c
)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hyper
geom([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(b*x+a)
^2*hypergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-2*(-a*d+b*c)^2*Fres
nelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2*d*(-a*d+b*c)*(b*x+a)*FresnelC(b*
x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2/3*d^2*(b*x+a)^2*FresnelC(b*x+a)*sin(1/2
*Pi*(b*x+a)^2)/b^3/Pi+5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^3/Pi^2*2^(1/2)+1
/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6568, 6556, 6588, 3432, 6566, 6590, 6582, 3460, 2718, 6596, 3439, 3433, 3466}

$$\begin{aligned}
&\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx \\
&= \frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3} \\
&- \frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3} \\
&+ \frac{d(bc-ad)\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{\pi b^3} \\
&+ \frac{d(a+bx)^2(bc-ad)\operatorname{FresnelC}(a+bx)^2}{b^3} + \frac{(a+bx)(bc-ad)^2\operatorname{FresnelC}(a+bx)^2}{b^3} \\
&- \frac{2d(a+bx)(bc-ad)\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} \\
&- \frac{2(bc-ad)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3} + \frac{(bc-ad)^2\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b^3} \\
&- \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2\pi^2 b^3} + \frac{d^2(a+bx)^3\operatorname{FresnelC}(a+bx)^2}{3b^3} \\
&+ \frac{5d^2\operatorname{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}\pi^2 b^3} - \frac{2d^2(a+bx)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3} \\
&- \frac{4d^2\operatorname{FresnelC}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6\pi^2 b^3} + \frac{2d^2x}{3\pi^2 b^2}
\end{aligned}$$

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) -
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x
)^2)/2]*FresnelC[a + b*x]])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC
[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a +
b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a +
```

```
b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*
Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-1/2*I)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*Hyper
geometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - (2*(b*c -
a*d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d*(b*c - a*
d)*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d^2*(
a + b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
```

```
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6568

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

integral

$$= \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\text{FresnelC}(x)^2+2bcd\left(1-\frac{ad}{bc}\right)x\,\text{FresnelC}(x)^2+d^2x^2\,\text{FresnelC}(x)^2\right)\,dx,x,\right.}{b^3}$$

$$= \frac{d^2\text{Subst}\left(\int x^2\,\text{FresnelC}(x)^2\,dx,x,a+bx\right)}{b^3} + \frac{(2d(bc-ad))\text{Subst}\left(\int x\,\text{FresnelC}(x)^2\,dx,x,a+bx\right)}{b^3} + \frac{(bc-ad)^2\text{Subst}\left(\int \text{FresnelC}(x)^2\,dx,x,a+bx\right)}{b^3}$$

$$= \frac{(bc-ad)^2(a+bx)\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\,\text{FresnelC}(a+bx)^2}{3b^3} - \frac{(2d^2)\,\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelC}(x)\,dx,x,a+bx\right)}{3b^3} - \frac{(2d(bc-ad))\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelC}(x)\,dx,x,a+bx\right)}{b^3} - \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelC}(x)\,dx,x,a+bx\right)}{b^3}$$

$$= \frac{(bc-ad)^2(a+bx)\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\,\text{FresnelC}(a+bx)^2}{3b^3} - \frac{2(bc-ad)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{2d(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{2d^2(a+bx)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{d^2\text{Subst}\left(\int x^2\sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{3b^3\pi} + \frac{(4d^2)\,\text{Subst}\left(\int x\,\text{FresnelC}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3\pi} + \frac{(d(bc-ad))\text{Subst}\left(\int x\sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b^3\pi} + \frac{(2d(bc-ad))\text{Subst}\left(\int \text{FresnelC}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b^3\pi} + \frac{(bc-ad)^2\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b^3\pi}$$

$$= -\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)}{3b^3\pi^2}$$
$$+ \frac{(bc-ad)^2(a+bx)\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelC}(a+bx)^2}{b^3}$$
$$+ \frac{d^2(a+bx)^3\,\text{FresnelC}(a+bx)^2}{3b^3} + \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi}$$
$$+ \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi}$$
$$+ \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi}$$
$$- \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi}$$
$$- \frac{2(bc-ad)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$- \frac{2d(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$- \frac{2d^2(a+bx)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}$$
$$+ \frac{d^2\text{Subst}\left(\int\cos\left(\pi x^2\right)\,dx,x,a+bx\right)}{6b^3\pi^2} + \frac{(4d^2)\,\text{Subst}\left(\int\cos^2\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{3b^3\pi^2}$$
$$+ \frac{(d(bc-ad))\text{Subst}\left(\int\sin(\pi x)\,dx,x,(a+bx)^2\right)}{2b^3\pi}$$

$$
\begin{aligned}
&= -\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} \\
&\quad - \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)}{3b^3\pi^2} + \frac{(bc-ad)^2(a+bx)\,\text{FresnelC}(a+bx)^2}{b^3} \\
&\quad + \frac{d(bc-ad)(a+bx)^2\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\,\text{FresnelC}(a+bx)^2}{3b^3} \\
&\quad + \frac{d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} + \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&\quad + \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{2(bc-ad)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&\quad - \frac{2d(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&\quad - \frac{2d^2(a+bx)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} \\
&\quad + \frac{(4d^2)\,\text{Subst}\left(\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(\pi x^2\right)\right)\,dx, x, a+bx\right)}{3b^3\pi^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2d^2x}{3b^2\pi^2} - \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} \\
&\quad - \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelC}(a+bx)}{3b^3\pi^2} + \frac{(bc-ad)^2(a+bx)\operatorname{FresnelC}(a+bx)^2}{b^3} \\
&\quad + \frac{d(bc-ad)(a+bx)^2\operatorname{FresnelC}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\operatorname{FresnelC}(a+bx)^2}{3b^3} \\
&\quad + \frac{d^2\operatorname{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} + \frac{d(bc-ad)\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{b^3\pi} \\
&\quad + \frac{(bc-ad)^2\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&\quad + \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&\quad - \frac{2(bc-ad)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&\quad - \frac{2d(bc-ad)(a+bx)\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&\quad - \frac{2d^2(a+bx)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} \\
&\quad + \frac{(2d^2)\operatorname{Subst}\left(\int\cos\left(\pi x^2\right)\,dx,x,a+bx\right)}{3b^3\pi^2}
\end{aligned}
$$

$$= \frac{2d^2x}{3b^2\pi^2} - \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}$$
$$- \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)}{3b^3\pi^2} + \frac{(bc-ad)^2(a+bx)\,\text{FresnelC}(a+bx)^2}{b^3}$$
$$+ \frac{d(bc-ad)(a+bx)^2\,\text{FresnelC}(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3\,\text{FresnelC}(a+bx)^2}{3b^3}$$
$$+ \frac{d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} + \frac{\sqrt{2}d^2\,\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{3b^3\pi^2}$$
$$+ \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{b^3\pi}$$
$$+ \frac{(bc-ad)^2\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi}$$
$$+ \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi}$$
$$- \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi}$$
$$- \frac{2(bc-ad)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$- \frac{2d(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$
$$- \frac{2d^2(a+bx)^2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}$$

Mathematica [F]

$$\int (c+dx)^2\,\text{FresnelC}(a+bx)^2\,dx = \int (c+dx)^2\,\text{FresnelC}(a+bx)^2\,dx$$

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2, x]
```

Maple [F]

$$\int (dx+c)^2\,\text{FresnelC}(bx+a)^2\,dx$$

```
[In] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
[Out] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

Fricas [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{C}(bx+a)^2 \, dx$$

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnel_cos(b*x + a)^2, x)
```

Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (c+dx)^2 C^2(a+bx) \, dx$$

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*fresnelc(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{C}(bx+a)^2 \, dx$$

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{C}(bx+a)^2 \, dx$$

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx = \int \operatorname{FresnelC}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

```
[In] int(FresnelC(a + b*x)^2*(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)^2*(c + d*x)^2, x)
```

3.159 $\int (c+dx)\, \text{FresnelC}(a+bx)^2\, dx$

Optimal result

Integrand size = 14, antiderivative size = 279

$$\begin{aligned}\int (c+dx)\, \text{FresnelC}(a+bx)^2\, dx = &-\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2} \\ &+\frac{(bc-ad)(a+bx)\, \text{FresnelC}(a+bx)^2}{b^2} \\ &+\frac{d(a+bx)^2\, \text{FresnelC}(a+bx)^2}{2b^2} \\ &+\frac{d\, \text{FresnelC}(a+bx)\, \text{FresnelS}(a+bx)}{2b^2\pi} \\ &+\frac{(bc-ad)\, \text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} \\ &+\frac{id(a+bx)^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} \\ &-\frac{id(a+bx)^2\, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} \\ &-\frac{2(bc-ad)\, \text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} \\ &-\frac{d(a+bx)\, \text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}\end{aligned}$$

```
[Out] -1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+(-a*d+b*c)*(b*x+a)*FresnelC(b*x+a)^2/b^2+
1/2*d*(b*x+a)^2*FresnelC(b*x+a)^2/b^2+1/2*d*FresnelC(b*x+a)*FresnelS(b*x+a)
/b^2/Pi+1/8*I*d*(b*x+a)^2*hypergeom([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^
2/Pi-1/8*I*d*(b*x+a)^2*hypergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^2/Pi
-2*(-a*d+b*c)*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-d*(b*x+a)*Fresne
lC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(-a*d+b*c)*FresnelS((b*x+a)*2^(1
/2))/b^2/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6568, 6556, 6588, 3432, 6566, 6590, 6582, 3460, 2718}

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2} - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2} + \frac{(a+bx)(bc-ad)\,\text{FresnelC}(a+bx)^2}{b^2} - \frac{2(bc-ad)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2} + \frac{(bc-ad)\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b^2} + \frac{d\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{2\pi b^2} + \frac{d(a+bx)^2\,\text{FresnelC}(a+bx)^2}{2b^2} - \frac{d(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2} - \frac{d\cos\left(\pi(a+bx)^2\right)}{4\pi^2 b^2}$$

```
[In] Int[(c + d*x)*FresnelC[a + b*x]^2,x]
```

```
[Out] -1/4*(d*Cos[Pi*(a + b*x)^2])/(b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a
 + b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[
a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a +
 b*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d
)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Fresne
lC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6568

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)\text{FresnelC}(x)^2 + dx\,\text{FresnelC}(x)^2\right)\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{d\text{Subst}\left(\int x\,\text{FresnelC}(x)^2\,dx, x, a+bx\right)}{b^2} + \frac{(bc-ad)\text{Subst}\left(\int \text{FresnelC}(x)^2\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelC}(a+bx)^2}{2b^2} - \frac{d\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelC}(x)\,dx, x, a+bx\right)}{b^2} - \frac{(2(bc-ad))\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelC}(x)\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelC}(a+bx)^2}{2b^2} - \frac{2(bc-ad)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{d(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d\text{Subst}\left(\int x\sin\left(\pi x^2\right)\,dx, x, a+bx\right)}{2b^2\pi} + \frac{d\text{Subst}\left(\int \text{FresnelC}(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{b^2\pi} + \frac{(bc-ad)\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx, x, a+bx\right)}{b^2\pi}$$

$$= \frac{(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelC}(a+bx)^2}{2b^2} + \frac{d\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{2b^2\pi} + \frac{(bc-ad)\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} + \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{2(bc-ad)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{d(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d\text{Subst}\left(\int \sin(\pi x)\,dx, x, (a+bx)^2\right)}{4b^2\pi}$$

$$= -\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2} + \frac{(bc-ad)(a+bx)\,\text{FresnelC}(a+bx)^2}{b^2} + \frac{d(a+bx)^2\,\text{FresnelC}(a+bx)^2}{2b^2} + \frac{d\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{2b^2\pi} + \frac{(bc-ad)\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} + \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} - \frac{2(bc-ad)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{d(a+bx)\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}$$

Mathematica [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx$$

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x]^2,x]
[Out] Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

Maple [F]

$$\int (dx+c)\,\text{FresnelC}\left(bx+a\right)^2\,dx$$

```
[In] int((d*x+c)*FresnelC(b*x+a)^2,x)
[Out] int((d*x+c)*FresnelC(b*x+a)^2,x)
```

Fricas [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \int (dx+c)\,\text{C}\left(bx+a\right)^2\,dx$$

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

Sympy [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \int (c+dx)\,C^2(a+bx)\;dx$$

```
[In] integrate((d*x+c)*fresnelc(b*x+a)**2,x)
[Out] Integral((c + d*x)*fresnelc(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \int (dx+c)\,\mathrm{C}\left(bx+a\right)^2\;dx$$

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \int (dx+c)\,\mathrm{C}\left(bx+a\right)^2\;dx$$

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx = \int \text{FresnelC}(a+b\,x)^2\,(c+d\,x)\;dx$$

```
[In] int(FresnelC(a + b*x)^2*(c + d*x),x)
[Out] int(FresnelC(a + b*x)^2*(c + d*x), x)
```

3.160 $\int \text{FresnelC}(a+bx)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \text{FresnelC}(a+bx)^2\,dx = \frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} + \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)^2/b-2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b/Pi+1/
2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6556, 6588, 3432}

$$\int \text{FresnelC}(a+bx)^2\,dx = \frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - \frac{2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b} + \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b}$$

```
[In] Int[FresnelC[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*
Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - 2\int (a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)\,dx \\
&= \frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\text{FresnelC}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - \frac{2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi} \\
&\quad + \frac{\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx, x, a+bx\right)}{b\pi} \\
&= \frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} + \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \text{FresnelC}(a+bx)^2\,dx = \frac{2\pi(a+bx)\,\text{FresnelC}(a+bx)^2 + \sqrt{2}\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right) - 4\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b\pi}$$

```
[In] Integrate[FresnelC[a + b*x]^2,x]
[Out] (2*Pi*(a + b*x)*FresnelC[a + b*x]^2 + Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)] -
 4*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(2*b*Pi)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx+a)^2(bx+a)-\frac{2\,\text{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\sqrt{2}\,\text{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60
default	$\frac{\text{FresnelC}(bx+a)^2(bx+a)-\frac{2\,\text{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\sqrt{2}\,\text{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60

```
[In] int(FresnelC(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(FresnelC(b*x+a)^2*(b*x+a)-2*FresnelC(b*x+a)/Pi*sin(1/2*Pi*(b*x+a)^2)+1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \text{FresnelC}(a+bx)^2\,dx = \frac{2\left(\pi b^2x+\pi ab\right)\text{C}\left(bx+a\right)^2-4\,b\,\text{C}\left(bx+a\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+\sqrt{2}\sqrt{b^2}\,\text{S}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{2\,\pi b^2}$$

```
[In] integrate(fresnel_cos(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(2*(pi*b^2*x + pi*a*b)*fresnel_cos(b*x + a)^2 - 4*b*fresnel_cos(b*x + a
)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + sqrt(2)*sqrt(b^2)*fresnel_s
in(sqrt(2)*sqrt(b^2)*(b*x + a)/b))/(pi*b^2)
```

Sympy [F]

$$\int \text{FresnelC}(a+bx)^2\,dx = \int C^2(a+bx)\;dx$$

```
[In] integrate(fresnelc(b*x+a)**2,x)
[Out] Integral(fresnelc(a + b*x)**2, x)
```

Maxima [F]

$$\int \text{FresnelC}(a+bx)^2\,dx = \int \text{C}\left(bx+a\right)^2\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)^2, x)
```

Giac [F]

$$\int \text{FresnelC}(a+bx)^2\,dx = \int \text{C}\left(bx+a\right)^2\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(a+bx)^2\,dx = \int \text{FresnelC}(a+b\,x)^2\,dx$$

```
[In] int(FresnelC(a + b*x)^2,x)
[Out] int(FresnelC(a + b*x)^2, x)
```

3.161 $\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)^2}{c+dx}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)^2/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx = \int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$$

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$$

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x),x]
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^{2}}{dx+c}dx$$

```
[In] int(FresnelC(b*x+a)^2/(d*x+c),x)
[Out] int(FresnelC(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{C}\left(bx+a\right)^{2}}{dx+c}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx=\int \frac{C^{2}(a+bx)}{c+dx}\,dx$$

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c),x)
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx = \int \frac{\mathrm{C}\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx = \int \frac{\mathrm{C}\left(bx+a\right)^2}{dx+c}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{FresnelC}(a+b\,x)^2}{c+d\,x}\,dx$$

```
[In] int(FresnelC(a + b*x)^2/(c + d*x),x)
[Out] int(FresnelC(a + b*x)^2/(c + d*x), x)
```

3.162 $\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx = \int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx$$

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^2}{\left(dx+c\right)^2}dx$$

```
[In] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
[Out] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{C}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{C^2(a+bx)}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\text{C}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\text{C}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx=\int \frac{\text{FresnelC}(a+b\,x)^2}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int(FresnelC(a + b*x)^2/(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)^2/(c + d*x)^2, x)
```

3.163 $\int x^2 \,\text{FresnelC}\,(d(a+b\log{(cx^n)}))\; dx$

Optimal result

Integrand size = 17, antiderivative size = 231

$$\begin{aligned}
&\int x^2 \,\text{FresnelC}\,(d(a+b\log{(cx^n)}))\; dx \\
&= \left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log{(cx^n)}\right)}{bd\sqrt{\pi}}\right) \\
&\quad -\left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}-iabd^2\pi-ib^2d^2\pi\log{(cx^n)}\right)}{bd\sqrt{\pi}}\right) \\
&\quad +\frac{1}{3}x^3\,\text{FresnelC}\,(d(a+b\log{(cx^n)}))
\end{aligned}$$

```
[Out] (1/12+1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+I
*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))-(1/12+1/1
2*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*d^2
*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*FresnelC(
d*(a+b*ln(c*x^n)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{6607, 4714, 2314, 2308, 2266, 2235, 2236\}$

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx$$
$$= \left(\frac{1}{12}+\frac{i}{12}\right)x^3(cx^n)^{-3/n}\,e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$-\left(\frac{1}{12}+\frac{i}{12}\right)x^3(cx^n)^{-3/n}\,e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\frac{1}{3}x^3\operatorname{FresnelC}(d(a+b\log(cx^n)))$$

```
[In] Int[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 + I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) - ((1/12 + I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelC[d*(a + b*Log[c*x^n])])/3
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{3}(bdn)\int x^2\cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)\,dx\\
&= \frac{1}{3}x^3\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{6}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx\\
&\quad -\frac{1}{6}(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx\\
&= \frac{1}{3}x^3\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{6}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\quad \left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{2-iabd^2n\pi}\,dx\\
&\quad -\frac{1}{6}\left(bdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{2+iabd^2n\pi}\,dx
\end{aligned}
$$

$$= \frac{1}{3}x^3 \operatorname{FresnelC}(d(a+b\log(cx^n)))$$
$$-\frac{1}{6}\left(bdx^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(3-iabd^2n\pi)x}{n}\right.\right.$$
$$\left.\left.-\frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right)$$
$$-\frac{1}{6}\left(bdx^3(cx^n)^{iabd^2\pi-\frac{3+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(3+iabd^2n\pi)x}{n}\right.\right.$$
$$\left.\left.+\frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right)$$

$$= \frac{1}{3}x^3 \operatorname{FresnelC}(d(a+b\log(cx^n)))$$
$$-\frac{1}{6}\left(bde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{i\left(\frac{3-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \log$$
$$-\frac{1}{6}\left(bde^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{iabd^2\pi-\frac{3+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{i\left(\frac{3+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \log$$

$$= \left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)$$
$$-\left(\frac{1}{12}\right.$$
$$\left.+\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)$$
$$+\frac{1}{3}x^3\operatorname{FresnelC}(d(a+b\log(cx^n)))$$

Mathematica [A] (verified)

Time = 4.61 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.38

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \frac{1}{12}x^3\Bigg(4\operatorname{FresnelC}(d(a+b\log(cx^n)))$$
$$+\sqrt[4]{-1}\sqrt{2}e^{\frac{1}{2}\left(-\frac{6a}{bn}-\frac{9i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}(cx^n)^{-3/n}\left(ie^{\frac{9i}{b^2d^2n^2\pi}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}\right.}{}\right.\right.$$

```
[In] Integrate[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
```

```
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(I*E^((9*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

Maple [F]

$$\int x^2 \operatorname{FresnelC}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

```
[In] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(187) = 374$.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.94

$$\begin{aligned}
\int x^2 \operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx &= \frac{1}{3}\,x^3\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right) \\
&-\frac{1}{6}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}-\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&-\frac{1}{6}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}+\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&+\frac{1}{6}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}-\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&-\frac{1}{6}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\,\log(c)}{n}-\frac{3\,a}{bn}+\frac{9i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)
\end{aligned}$$

```
[In] integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*fresnel_cos(b*d*log(c*x^n) + a*d) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*
log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*
log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2)) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) + 9/2*I/(
pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
 pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/6*I*pi*sqrt(b^
2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin
((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^
```

```
2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n -
3*a/(b*n) + 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi
*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
```

Sympy [F]

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\, dx = \int x^2 C(ad+bd\log(cx^n))\, dx$$

```
[In] integrate(x**2*fresnelc(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**2*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\, dx = \int x^2\, \mathrm{C}((b\log(cx^n)+a)d)\, dx$$

```
[In] integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x^2*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\, dx = \int x^2\, \mathrm{C}((b\log(cx^n)+a)d)\, dx$$

```
[In] integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\, dx = \int x^2 \operatorname{FresnelC}(d\,(a+b\,\ln(c\,x^n)))\, dx$$

```
[In] int(x^2*FresnelC(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*FresnelC(d*(a + b*log(c*x^n))), x)
```

3.164 $\int x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 227

$$\begin{aligned}&\int x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx\\&=\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)\\&\quad-\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)\\&\quad+\frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\end{aligned}$$

```
[Out] (1/8+1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(2
/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))-(1/8+
1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/P
i^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2*Fre
snelC(d*(a+b*ln(c*x^n)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used

$= \{6607, 4714, 2314, 2308, 2266, 2235, 2236\}$

$$\int x \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx$$
$$= \left(\frac{1}{8}+\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$-\left(\frac{1}{8}+\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+\frac{1}{2}x^2\operatorname{FresnelC}(d(a+b\log(cx^n)))$$

```
[In] Int[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 + I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) - ((1/8 + I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelC[d*(a + b*Log[c*x^n])])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{2}(bdn)\int x\cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{4}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx\\
&\quad-\frac{1}{4}(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx\\
&= \frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{4}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{1-iabd^2n\pi}\,dx\\
&\quad-\frac{1}{4}\left(bdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{1+iabd^2n\pi}\,dx
\end{aligned}
$$

$$= \frac{1}{2}x^2 \operatorname{FresnelC}(d(a + b\log(cx^n))) - \frac{1}{4}\left(bdx^2(cx^n)^{-iabd^2\pi - \frac{2-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi + \frac{(2-iabd^2n\pi)x}{n} - \frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right) - \frac{1}{4}\left(bdx^2(cx^n)^{iabd^2\pi - \frac{2+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{1}{2}ia^2d^2\pi + \frac{(2+iabd^2n\pi)x}{n} + \frac{1}{2}ib^2d^2\pi x^2\right)dx, x, \log(cx^n)\right)$$

$$= \frac{1}{2}x^2 \operatorname{FresnelC}(d(a + b\log(cx^n))) - \frac{1}{4}\left(bde^{-\frac{2(i+abd^2n\pi)}{b^2d^2n^2\pi}}x^2(cx^n)^{-iabd^2\pi - \frac{2-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\frac{i\left(\frac{2-iabd^2n\pi}{n} - ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \log\right. - \frac{1}{4}\left(bde^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{iabd^2\pi - \frac{2+iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{i\left(\frac{2+iabd^2n\pi}{n} + ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx, x, \log(c\right.$$

$$= \left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n} + iabd^2\pi + ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) - \left(\frac{1}{8} + \frac{i}{8}\right)e^{-\frac{2(i+abd^2n\pi)}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n} - iabd^2\pi - ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \frac{1}{2}x^2\operatorname{FresnelC}(d(a+b\log(cx^n)))$$

Mathematica [A] (verified)

Time = 4.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.40

$$\int x \operatorname{FresnelC}(d(a + b\log(cx^n)))\,dx = \frac{1}{8}x^2\left(4\operatorname{FresnelC}(d(a+b\log(cx^n))) + \sqrt[4]{-1}\sqrt{2}e^{-\frac{2a}{bn} - \frac{2i}{b^2d^2n^2\pi} - \frac{1}{2}ia^2d^2\pi + iabd^2\pi(n\log(x) - \log(cx^n)) - \frac{1}{2}ib^2d^2\pi(-n\log(x)+\log(cx^n))^2}(cx^n)^{-2/n}\left(ie^{\frac{4i}{b^2d^2n^2\pi}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\right.}{}\right.\right.\right.$$

```
[In] Integrate[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(I*E^((4*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
```

```
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

Maple [F]

$$\int x\ \mathrm{FresnelC}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

```
[In] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
[Out] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(187) = 374$.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.97

$$\begin{aligned}\int x\,\mathrm{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx = &-\frac{1}{4}\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n+2i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)\\&-\frac{1}{4}\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n-2i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)\\&+\frac{1}{4}i\,\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n+2i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)\\&-\frac{1}{4}i\,\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n-2i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)\\&+\frac{1}{2}x^{2}\,\mathrm{C}\left(bd\log\left(cx^{n}\right)+ad\right)\end{aligned}$$

```
[In] integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] -1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2)
)*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n +
 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*
log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*lo
g(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*
d^2*n^2)) + 1/4*I*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi
*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + p
i*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*I*pi*sqrt(b^2*
```

```
d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi
*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*x^2*fresnel_cos(b*d*log(c*x^n) + a*d)
```

Sympy [F]

$$\int x \,\mathrm{FresnelC}\left(d(a + b \log\left(cx^n\right))\right)\, dx = \int xC(ad + bd \log\left(cx^n\right))\, dx$$

```
[In] integrate(x*fresnelc(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x \,\mathrm{FresnelC}\left(d(a + b \log\left(cx^n\right))\right)\, dx = \int x \,\mathrm{C}\left((b \log\left(cx^n\right) + a)d\right)\, dx$$

```
[In] integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x \,\mathrm{FresnelC}\left(d(a + b \log\left(cx^n\right))\right)\, dx = \int x \,\mathrm{C}\left((b \log\left(cx^n\right) + a)d\right)\, dx$$

```
[In] integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \,\mathrm{FresnelC}\left(d(a + b \log\left(cx^n\right))\right)\, dx = \int x \,\mathrm{FresnelC}(d\left(a + b\,\ln\left(c\,x^n\right)\right))\, dx$$

```
[In] int(x*FresnelC(d*(a + b*log(c*x^n))),x)
[Out] int(x*FresnelC(d*(a + b*log(c*x^n))), x)
```

3.165 $\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 214

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) - \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) + x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)$$

```
[Out] (1/4+1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d
/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))-(1/4+1/4*I)*
x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/
exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelC(d*(a+b*ln(c*
x^n)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used

$= \{6604, 4712, 2312, 2308, 2266, 2235, 2236\}$

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$= \left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$-\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)$$
$$+x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)$$

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 + I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) - ((1/4 + I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelC[d*(a + b*Log[c*x^n])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2312

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Dist[(c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log[
```

```
F]), Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2
), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[F
]]
```

Rule 4712

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[1
/2, Int[E^((-I)*d*(a + b*Log[c*x^n])^2), x], x] + Dist[1/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 6604

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelC[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Cos[(Pi/2)*(d*(a +
 b*Log[c*x^n]))^2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-(bdn)\int\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx\\
&= x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-\frac{1}{2}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx\\
&\quad-\frac{1}{2}(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx\\
&= x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)\\
&\quad-\frac{1}{2}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)x^{-iabd^2n\pi}\,dx\\
&\quad-\frac{1}{2}\left(bdnx^{-iabd^2n\pi}(cx^n)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)x^{iabd^2n\pi}\,dx\\
&= x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)\\
&\quad-\frac{1}{2}\left(bdx(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(1-iabd^2n\pi)\,x}{n}\right.\right.\\
&\qquad\left.\left.-\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log(cx^n)\right)\\
&\quad-\frac{1}{2}\left(bdx(cx^n)^{iabd^2\pi-\frac{1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(1+iabd^2n\pi)\,x}{n}\right.\right.\\
&\qquad\left.\left.+\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log(cx^n)\right)
\end{aligned}$$

$$= x\,\text{FresnelC}\,(d(a+b\log(cx^n)))$$
$$-\frac{1}{2}\left(bde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{1-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx$$
$$-\frac{1}{2}\left(bde^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{iabd^2\pi-\frac{1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{1+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(c$$
$$= \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)$$
$$-\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)$$
$$+x\,\text{FresnelC}\,(d(a+b\log(cx^n)))$$

Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int\text{FresnelC}\,(d(a+b\log(cx^n)))\,dx = x\,\text{FresnelC}\,(d(a+b\log(cx^n)))$$
$$+\frac{\sqrt[4]{-1}e^{\frac{1}{2}\left(-\frac{2a}{bn}-\frac{i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}x(cx^n)^{-1/n}\left(ie^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\right.\right.}{}$$

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[(
(1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])
] + Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

Maple [F]

$$\int \text{FresnelC}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

[In] int(FresnelC(d*(a+b*ln(c*x^n))),x)

[Out] int(FresnelC(d*(a+b*ln(c*x^n))),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(176) = 352.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.08

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx = -\frac{1}{2}\,\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n+i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right) - \frac{1}{2}\,\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n-i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right) + \frac{1}{2}i\,\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n+i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right) - \frac{1}{2}i\,\pi\sqrt{b^{2}d^{2}n^{2}}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n-i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right) + x\,\mathrm{C}\left(bd\log\left(cx^{n}\right)+ad\right)$$

[In] integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] -1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + x*fresnel_cos(b*d*log(c*x^n) + a*d)

Sympy [F]

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int C(d(a+b\log\left(cx^n\right)))\,dx$$

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n))),x)
[Out] Integral(fresnelc(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{C}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{C}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{FresnelC}(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

```
[In] int(FresnelC(d*(a + b*log(c*x^n))),x)
[Out] int(FresnelC(d*(a + b*log(c*x^n))), x)
```

3.166 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\text{FresnelC}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bn} - \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{bdn\pi}$$

```
[Out] FresnelC(d*(a+b*ln(c*x^n)))*(a+b*ln(c*x^n))/b/n-sin(1/2*d^2*Pi*(a+b*ln(c*x^
n))^2)/b/d/n/Pi
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6554}

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx = \frac{(a+b\log(cx^n))\,\text{FresnelC}(d(a+b\log(cx^n)))}{bn} - \frac{\sin\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}$$

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a
+ b*Log[c*x^n])^2)/2]/(b*d*n*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \text{FresnelC}(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \text{FresnelC}(x)\,dx, x, ad+bd\log\left(cx^n\right)\right)}{bdn}$$
$$= \frac{\text{FresnelC}\left(ad+bd\log\left(cx^n\right)\right)\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\sin\left(\frac{1}{2}\pi\left(ad+bd\log\left(cx^n\right)\right)^2\right)}{bdn\pi}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int \frac{\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx$$
$$= \frac{a\,\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{bn} + \frac{\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)\log\left(cx^n\right)}{n}$$
$$- \frac{\cos\left(abd^2\pi\log\left(cx^n\right)+\frac{1}{2}b^2d^2\pi\log^2\left(cx^n\right)\right)\sin\left(\frac{1}{2}a^2d^2\pi\right)}{bdn\pi}$$
$$- \frac{\cos\left(\frac{1}{2}a^2d^2\pi\right)\sin\left(abd^2\pi\log\left(cx^n\right)+\frac{1}{2}b^2d^2\pi\log^2\left(cx^n\right)\right)}{bdn\pi}$$

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (a*FresnelC[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelC[d*(a + b*Log[c*x^n])]*
Log[c*x^n])/n - (Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2]*S
in[(a^2*d^2*Pi)/2])/(b*d*n*Pi) - (Cos[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\text{FresnelC}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))-\frac{\sin\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{ndb}$	64
default	$\frac{\text{FresnelC}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))-\frac{\sin\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{ndb}$	64

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/d/b*(FresnelC(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))-1/Pi*sin(1/2*Pi*(a
*d+b*d*ln(c*x^n))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.83

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(\pi bdn\log\left(x\right)+\pi bd\log\left(c\right)+\pi ad)\,\text{C}\left(bd\log\left(cx^n\right)+ad\right)-\sin\left(\frac{1}{2}\,\pi b^2d^2n^2\log\left(x\right)^2+\pi b^2d^2n\log\left(c\right)\log\left(x\right)+\cdots\right)}{\pi bdn}$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] ((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*fresnel_cos(b*d*log(c*x^n) + a*
d) - sin(1/2*pi*b^2*d^2*n^2*log(x)^2 + pi*b^2*d^2*n*log(c)*log(x) + 1/2*pi*
b^2*d^2*log(c)^2 + pi*a*b*d^2*n*log(x) + pi*a*b*d^2*log(c) + 1/2*pi*a^2*d^2
))/(pi*b*d*n)
```

Sympy [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \int \frac{C\left(ad+bd\log\left(cx^n\right)\right)}{x}\,dx$$

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(b\log\left(cx^n\right)+a)d\,\text{C}\left((b\log\left(cx^n\right)+a)d\right)-\frac{\sin\left(\frac{1}{2}\,\pi b^2d^2\log(cx^n)^2+\pi abd^2\log(cx^n)+\frac{1}{2}\,\pi a^2d^2\right)}{\pi}}{bdn}$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*fresnel_cos((b*log(c*x^n) + a)*d) - sin(1/2*pi*b^2*d^
2*log(c*x^n)^2 + pi*a*b*d^2*log(c*x^n) + 1/2*pi*a^2*d^2)/pi)/(b*d*n)
```

Giac [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\int \frac{\text{C}\left((b\log\left(cx^n\right)+a)d\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\int \frac{\text{FresnelC}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x}\,dx$$

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x, x)
```

3.167 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 217

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}$$

```
[Out] (1/4+1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2*
I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-(1/4+1/4*I)*ex
p(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*a*b
*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelC(d*(a+b*ln(c*x^n)))
/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

= {6607, 4714, 2314, 2308, 2266, 2235, 2236}

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx$$
$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(cx^n\right)^{\frac{1}{n}} e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(cx^n\right)^{\frac{1}{n}} e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{x} - \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}$$

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 + I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x - ((1/4 + I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelC[d*(a + b*Log[c*x^n])]/x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+(bdn)\int\frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)}{x^2}\,dx\\
&= -\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+\frac{1}{2}(bdn)\int\frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx\\
&\quad+\frac{1}{2}(bdn)\int\frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx\\
&= -\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+\frac{1}{2}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-2-iabd^2n\pi}\,dx\\
&\quad+\frac{1}{2}\left(bdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-2+iabd^2n\pi}\,dx
\end{aligned}$$

$$= -\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x} + \frac{\left(bd(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(-1-iabd^2n\pi)x}{n}-\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(cx\right.}{2x} + \frac{\left(bd(cx^n)^{iabd^2\pi-\frac{-1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(-1+iabd^2n\pi)x}{n}+\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(cx^n)\right)}{2x}$$

$$= -\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x} + \frac{\left(bde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{-1-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx^n)\right)}{2x} + \frac{\left(bde^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{iabd^2\pi-\frac{-1+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{-1+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx^n)\right)}{2x}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}$$

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\sqrt[4]{-1}\sqrt{2}e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\left(\text{erfi}\left(\frac{(-1)^{3/4}(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+ie^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+abd^2n\pi+b^2d^2n\pi\log(cx^n\right.}{bdn\sqrt{\pi}}\right.\right.}{4x}$$

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(
-1)*(Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n
*Sqrt[2*Pi])] + I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*P
i + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelC[d*(a + b*Log[
c*x^n])])/x
```

Maple [F]

$$\int \frac{\operatorname{FresnelC}(d(a+b\ln(c\,x^n)))}{x^2}dx$$

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{i}{2\,\pi b^2d}\right.}}{}$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
[Out] 1/2*(pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))
*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n +
I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n
 + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + p
i*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
+ I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*
fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I
)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/
n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) +
pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
 - 2*fresnel_cos(b*d*log(c*x^n) + a*d))/x
```

Sympy [F]

$$\int \frac{\operatorname{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{C(ad+bd\log(cx^n))}{x^2}\,dx$$

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**2,x)
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{C}\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{C}\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{FresnelC}(d\left(a + b\,\ln\left(c\,x^n\right)\right))}{x^2}\,dx$$

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x^2,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x^2, x)
```

3.168 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 228

$$\begin{aligned}\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx &= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} \\ &\quad - \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} \\ &\quad - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}\end{aligned}$$

```
[Out] (1/8+1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/2
+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2-(1/8+1/
8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n
))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*FresnelC(d*
(a+b*ln(c*x^n)))/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{6607, 4714, 2314, 2308, 2266, 2235, 2236\}$

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx$$
$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(cx^n\right)^{2/n} e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}} \operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2}$$
$$- \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(cx^n\right)^{2/n} e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{x^2}$$
$$- \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}$$

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 + I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 - ((1/8 + I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}(bdn)\int\frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)}{x^3}\,dx\\
&= -\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{4}(bdn)\int\frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx\\
&\quad+\frac{1}{4}(bdn)\int\frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx\\
&= -\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{4}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-3-iabd^2n\pi}\,dx\\
&\quad+\frac{1}{4}\left(bdnx^{-iabd^2n\pi}\left(cx^n\right)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi\right.\\
&\qquad\left.+\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x^{-3+iabd^2n\pi}\,dx
\end{aligned}$$

$$= -\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bd(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(-2-iabd^2n\pi)x}{n}-\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(cx^n)\right)}{4x^2} + \frac{\left(bd(cx^n)^{iabd^2\pi-\frac{-2+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(-2+iabd^2n\pi)x}{n}+\frac{1}{2}ib^2d^2\pi x^2\right)dx,x,\log(cx^n)\right)}{4x^2}$$

$$= -\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(bde^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{-2-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx^n)\right)}{4x^2} + \frac{\left(bde^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{iabd^2\pi-\frac{-2+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{-2+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)dx,x,\log(cx^n)\right)}{4x^2}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}}(cx^n)^{2/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\sqrt[4]{-1}e^{\frac{2\left(\frac{an}{b}-\frac{i}{b^2d^2\pi}+n(-n\log(x)+\log(cx^n))\right)}{n^2}}\left(\operatorname{erfi}\left(\frac{(-1)^{3/4}(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+ie^{\frac{4i}{b^2d^2n^2\pi}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(2i+abd^2n\pi+\ldots}{bdn\sqrt{\ldots}}\right.\right.}{4\sqrt{2}} - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}$$

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x
^n])))/n^2)*(Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[2*Pi])] + I*E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*
I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt[2]
- FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Maple [F]

$$\int \frac{\mathrm{FresnelC}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(183) = 366$.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.02

$$\int \frac{\mathrm{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \frac{\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}-\right.}}{}$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(pi*b^2*d^2*n
^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*
n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x^2*e^(
2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*
log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(p
i*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2
*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi
*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) - 2*fresnel_cos(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F]

$$\int \frac{\mathrm{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{C\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\text{C}\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\text{C}\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\text{FresnelC}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^3}\,dx$$

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x^3,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x^3, x)
```

3.169 $\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 280

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{i(1+m)\left(1+m-2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m-iabd^2n\pi-ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{(ex)^{1+m}\operatorname{FresnelC}(d(a+b\log(cx^n)))}{e(1+m)}$$

```
[Out] (1/4+1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)^
m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1
/2))/(1+m)/((c*x^n)^((1+m)/n))-(1/4+1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1+m-
I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m)*(1
+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*
FresnelC(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{6607, 4714, 2314, 2308, 2266, 2235, 2236\}$

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx$$
$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{m+1}$$
$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m(cx^n)^{-\frac{m+1}{n}}\exp\left(-\frac{i(m+1)(-2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-i\pi abd^2n-i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{m+1}$$
$$+\frac{(ex)^{m+1}\operatorname{FresnelC}(d(a+b\log(cx^n)))}{e(m+1)}$$

```
[In] Int[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 + I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) - ((1/4 + I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelC[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

integral

$$= \frac{(ex)^{1+m}\,\text{FresnelC}\,(d(a+b\log{(cx^n)}))}{e(1+m)} - \frac{(bdn)\int (ex)^m\cos\left(\frac{1}{2}d^2\pi(a+b\log{(cx^n)})^2\right)\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelC}\,(d(a+b\log{(cx^n)}))}{e(1+m)} - \frac{(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)} - \frac{(bdn)\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelC}\,(d(a+b\log{(cx^n)}))}{e(1+m)} - \frac{\left(bdnx^{-m+iabd^2n\pi}(ex)^m\,(cx^n)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2{(cx^n)}\right)x^{m-iabd^2n\pi}\,dx}{2(1+m)} - \frac{\left(bdnx^{-m-iabd^2n\pi}(ex)^m\,(cx^n)^{iabd^2\pi}\right)\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{1}{2}ib^2d^2\pi\log^2{(cx^n)}\right)x^{m+iabd^2n\pi}\,dx}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(1+m)} - \frac{\left(bdx(ex)^m\left(cx^n\right)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi+\frac{(1+m-iabd^2n\pi)x}{n}-\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log\left(\right.\right.}{2(1+m)} - \frac{\left(bdx(ex)^m\left(cx^n\right)^{iabd^2\pi-\frac{1+m+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{1}{2}ia^2d^2\pi+\frac{(1+m+iabd^2n\pi)x}{n}+\frac{1}{2}ib^2d^2\pi x^2\right)\,dx,x,\log\left(cx^n\right.\right.}{2(1+m)}$$

$$= \frac{(ex)^{1+m}\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(1+m)} - \frac{\left(bd\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(\frac{1+m-iabd^2n\pi}{n}-ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right.\right.}{2(1+m)} - \frac{\left(bd\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)^{iabd^2\pi-\frac{1+m+iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{i\left(\frac{1+m+iabd^2n\pi}{n}+ib^2d^2\pi x\right)^2}{2b^2d^2\pi}\right)\right.}{2(1+m)}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m-iabd^2n\pi-ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{(ex)^{1+m}\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(1+m)}$$

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int (ex)^m\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{(ex)^m\left((-1)^{3/4}\sqrt{2}e^{-\frac{(1+m)\left(i+im+2abd^2n\pi+2b^2d^2n\pi(-n\log(x)+\log(cx^n))\right)}{2b^2d^2n^2\pi}}x^{-m}\left(\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+im+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)-e\right.\right.}{4(1+m)}$$

```
[In] Integrate[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(((-1)^(3/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + b
^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] - E^((I*(1 + m)^2)/(b^2*d^2*n^2*
Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))
/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2*n
*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*FresnelC[d*
(a + b*Log[c*x^n])]))/(4*(1 + m))
```

Maple [F]

$$\int (ex)^m \operatorname{FresnelC}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

```
[In] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(310) = 620.

Time = 0.31 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.46

$$\int (ex)^m \operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx =$$

$$-\frac{\pi\sqrt{b^2d^2n^2}e^{\left(m\log(e)-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i\,m^2}{2\,\pi b^2d^2n^2}-\frac{i\,m}{\pi b^2d^2n^2}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,C\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\,m+}{\pi b^2d^2n^2}\right.}{}$$

```
[In] integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/2*(pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n
- a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b
^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*
a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*
n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I*m^2/
(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_c
os((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m - I)*s
qrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m
*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I
*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*l
og(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi
*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n
) - log(c)/n - a/(b*n) + 1/2*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2)
+ 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n
*log(c) + pi*a*b*d^2*n - I*m - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 2*x
*e^(m*log(e) + m*log(x))*fresnel_cos(b*d*log(c*x^n) + a*d))/(m + 1)
```

Sympy [F]

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx=\int (ex)^m C(ad+bd\log(cx^n))\,dx$$

```
[In] integrate((e*x)**m*fresnelc(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx=\int (ex)^m \mathrm{C}((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate((e*x)^m*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx=\int (ex)^m \mathrm{C}((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate((e*x)^m*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx=\int \operatorname{FresnelC}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.170 $\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx = -\frac{ie^c\,\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*I*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*hypergeom([
1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6572, 6511, 6510, 30}

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx = \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\,\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-1/8*I)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6510

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*
(Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
```

```
}, x] && EqQ[d, b^2]
```

Rule 6511

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6572

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&\quad+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{(ie^c)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\
&= -\frac{ie^c\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)
\end{aligned}
$$

Mathematica [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx$$

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F]

$$\int e^{c+\frac{ib^2\pi x^2}{2}} \operatorname{FresnelC}(bx)\, dx$$

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\, dx = \int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)} \operatorname{C}(bx)\, dx$$

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Sympy [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\, dx = e^c \int e^{\frac{i\pi b^2x^2}{2}} C(bx)\, dx$$

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\, dx = \int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)} \operatorname{C}(bx)\, dx$$

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx = \int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,C\left(bx\right)\,dx$$

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx = \int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\,\text{FresnelC}(b\,x)\,dx$$

```
[In] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
[Out] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

3.171 $\int e^{c-\frac{1}{2}ib^2\pi x^2} \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} \text{FresnelC}(bx)\,dx = -\frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b} + \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] -1/8*I*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*hypergeom(
[1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6572, 6508, 30, 6513}

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} \text{FresnelC}(bx)\,dx = \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-1/8*I)*E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6508

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*(
Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
```

```
x] && EqQ[d, -b^2]
```

Rule 6513

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6572

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&\quad+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\,dx \\
&= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{(ie^c)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)\right)}{4b} \\
&= -\frac{ie^c\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)
\end{aligned}
$$

Mathematica [F]

$$
\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{FresnelC}(bx)\,dx=\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{FresnelC}(bx)\,dx
$$

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F]

$$\int \mathrm{e}^{c-\frac{i b^{2} \pi x^{2}}{2}} \operatorname{FresnelC}(b x) d x$$

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\,dx = \int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)} \operatorname{C}(bx)\,dx$$

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Sympy [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\,dx = e^c \int e^{-\frac{i\pi b^2x^2}{2}} C(bx)\,dx$$

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\,dx = \int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)} \operatorname{C}(bx)\,dx$$

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{FresnelC}(bx)\,dx=\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\operatorname{C}(bx)\,dx$$

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{FresnelC}(bx)\,dx=\int e^{c-\frac{\Pi\,b^2\,x^2\,1\mathrm{i}}{2}}\operatorname{FresnelC}(b\,x)\,dx$$

```
[In] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
[Out] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

3.172 $\int \text{FresnelC}(bx) \sin \left(c + \frac{1}{2} b^2 \pi x^2\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \text{FresnelC}(bx) \sin \left(c + \frac{1}{2} b^2 \pi x^2\right) \, dx = \frac{\cos(c) \text{FresnelC}(bx) \text{FresnelS}(bx)}{2b} + \frac{1}{8} i b x^2 \cos(c) \, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2} i b^2 \pi x^2\right) - \frac{1}{8} i b x^2 \cos(c) \, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2} i b^2 \pi x^2\right) + \frac{\text{FresnelC}(bx)^2 \sin(c)}{2b}$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*cos(c)*hypergeom([1, 1
],[3/2, 2],-1/2*I*b^2*Pi*x^2)-1/8*I*b*x^2*cos(c)*hypergeom([1, 1],[3/2, 2],
1/2*I*b^2*Pi*x^2)+1/2*FresnelC(b*x)^2*sin(c)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6584, 6576, 30, 6582}

$$\int \text{FresnelC}(bx) \sin \left(c + \frac{1}{2} b^2 \pi x^2\right) \, dx = \frac{1}{8} i b x^2 \cos(c) \, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2} i b^2 \pi x^2\right) - \frac{1}{8} i b x^2 \cos(c) \, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2} i b^2 \pi x^2\right) + \frac{\cos(c) \text{FresnelC}(bx) \text{FresnelS}(bx)}{2b} + \frac{\sin(c) \text{FresnelC}(bx)^2}{2b}$$

```
[In] Int[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2
*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6584

```
Int[FresnelC[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \cos(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx+\sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx\\
&= \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\\
&\quad -\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\sin(c)\text{Subst}(\int x\,dx,x,\text{FresnelC}(bx))}{b}\\
&= \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\\
&\quad -\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)^2\sin(c)}{2b}
\end{aligned}
$$

Mathematica [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$$

```
[In] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int \text{FresnelC}\left(bx\right)\sin\left(c+\frac{b^2\pi\, x^2}{2}\right)dx$$

```
[In] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Sympy [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx = \int \sin\left(\frac{\pi b^2x^2}{2}+c\right)C(bx)\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

Maxima [F]

$$\int \text{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) \,dx = \int \text{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2 + c\right) \,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Giac [F]

$$\int \text{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) \,dx = \int \text{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2 + c\right) \,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) \,dx = \int \sin\left(\frac{\Pi\,b^2\,x^2}{2} + c\right)\,\text{FresnelC}(b\,x) \,dx$$

```
[In] int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
[Out] int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

3.173 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\cos(c)\,\text{FresnelC}(bx)^2}{2b} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)^2/b-1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*I
*b*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*x^2*hype
rgeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6578, 6576, 30, 6582}

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = -\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{\sin(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} + \frac{\cos(c)\,\text{FresnelC}(bx)^2}{2b}$$

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]*Sin
[c] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin
[c]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6578

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx - \sin(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= -\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) \\
&\quad + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{\cos(c)\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right)}{b} \\
&= \frac{\cos(c)\,\text{FresnelC}(bx)^2}{2b} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b} \\
&\quad - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)
\end{aligned}$$

Mathematica [F]

$$\int \cos \left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx=\int \cos \left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$$

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F]

$$\int \cos \left(c+\frac{b^2\pi\,x^2}{2}\right) \text{FresnelC}\left(bx\right)dx$$

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [F]

$$\int \cos \left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx=\int \cos \left(\frac{1}{2}\,\pi b^2x^2+c\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Sympy [F]

$$\int \cos \left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx=\int \cos \left(\frac{\pi b^2x^2}{2}+c\right)C(bx)\,dx$$

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

Maxima [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\text{C}\left(bx\right)\,dx$$

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\text{C}\left(bx\right)\,dx$$

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \cos\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\text{FresnelC}(b\,x)\,dx$$

```
[In] int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
[Out] int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

3.174 $\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2 \, dx = \frac{\text{FresnelC}(bx)^3}{3b}$$

```
[Out] 1/3*FresnelC(b*x)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6576, 30}

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2 \, dx = \frac{\text{FresnelC}(bx)^3}{3b}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
```

```
[Out] FresnelC[b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2\,dx, x, \text{FresnelC}(bx)\right)}{b}\\ &= \frac{\text{FresnelC}(bx)^3}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^2\,dx = \frac{\text{FresnelC}(bx)^3}{3b}$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
[Out] FresnelC[b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^3}{3b}$	12
default	$\frac{\text{FresnelC}(bx)^3}{3b}$	12

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*FresnelC(b*x)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^2\,dx = \frac{\text{C}\left(bx\right)^3}{3\,b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="fricas")
[Out] 1/3*fresnel_cos(b*x)^3/b
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \begin{cases} \frac{C^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**2,x)
[Out] Piecewise((fresnelc(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \frac{\text{C}\,(bx)^3}{3\,b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] 1/3*fresnel_cos(b*x)^3/b
```

Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\,(bx)^2\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \int \text{FresnelC}(b\,x)^2\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2), x)
```

3.175 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6576, 30}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}(\int x\,dx, x, \text{FresnelC}(bx))}{b}\\ &= \frac{\text{FresnelC}(bx)^2}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] FresnelC[b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelC}(bx)^2}{2b}$	12

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/2*FresnelC(b*x)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{C}\left(bx\right)^2}{2\,b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\mathrm{C}\,(bx)^2}{2\,b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\,(bx)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.176 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log(\text{FresnelC}(bx))}{b}$$

```
[Out] ln(FresnelC(b*x))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6576, 29}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log(\text{FresnelC}(bx))}{b}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
[Out] Log[FresnelC[b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \text{FresnelC}(bx)\right)}{b}$$
$$= \frac{\log(\text{FresnelC}(bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log(\text{FresnelC}(bx))}{b}$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
[Out] Log[FresnelC[b*x]]/b
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\text{FresnelC}(bx))}{b}$	10
default	$\frac{\ln(\text{FresnelC}(bx))}{b}$	10

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] ln(FresnelC(b*x))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log\left(\text{C}\left(bx\right)\right)}{b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="fricas")
[Out] log(fresnel_cos(b*x))/b
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \begin{cases} \frac{\log(C(bx))}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x),x)
[Out] Piecewise((log(fresnelc(b*x))/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log(\text{C}(bx))}{b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="maxima")
[Out] log(fresnel_cos(b*x))/b
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\text{C}(bx)}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelC}(b\,x)}\,dx$$

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x),x)
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x), x)
```

3.177 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelC}(bx)}$$

```
[Out] -1/b/FresnelC(b*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6576, 30}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelC}(bx)}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
[Out] -(1/(b*FresnelC[b*x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \text{FresnelC}(bx)\right)}{b} \\
&= -\frac{1}{b\,\text{FresnelC}(bx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelC}(bx)}
$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
[Out] -(1/(b*FresnelC[b*x]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{1}{b\ \text{FresnelC}(bx)}$	12
default	$-\frac{1}{b\ \text{FresnelC}(bx)}$	12

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/b/FresnelC(b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{C}\left(bx\right)}
$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="fricas")
[Out] -1/(b*fresnel_cos(b*x))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = \begin{cases} -\frac{1}{bC(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**2,x)
[Out] Piecewise((-1/(b*fresnelc(b*x)), Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{C}\,(bx)}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] -1/(b*fresnel_cos(b*x))
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{C}\,(bx)^2}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = \int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelC}\,(b\,x)^2}\,dx$$

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2,x)
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2, x)
```

3.178 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

```
[Out] -1/2/b/FresnelC(b*x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6576, 30}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelC[b*x]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3}\, dx, x, \text{FresnelC}(bx)\right)}{b}$$
$$= -\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\, dx = -\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
[Out] -1/2*1/(b*FresnelC[b*x]^2)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2b\,\text{FresnelC}(bx)^2}$	12
default	$-\frac{1}{2b\,\text{FresnelC}(bx)^2}$	12

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/FresnelC(b*x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\, dx = -\frac{1}{2\,b\,\text{C}\,(bx)^2}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="fricas")
[Out] -1/2/(b*fresnel_cos(b*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = \begin{cases} -\frac{1}{2bC^2(bx)} & \text{for } b\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**3,x)
[Out] Piecewise((-1/(2*b*fresnelc(b*x)**2), Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = -\frac{1}{2\,b\,\mathrm{C}\left(bx\right)^2}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="maxima")
[Out] -1/2/(b*fresnel_cos(b*x)^2)
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{C}\left(bx\right)^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = \int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\mathrm{FresnelC}\left(b\,x\right)^3}\,dx$$

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3,x)
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3, x)
```

3.179 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^n \, dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^n \, dx = \frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

```
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6576, 30}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^n \, dx = \frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^n\,dx, x, \text{FresnelC}(bx)\right)}{b}$$
$$= \frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$	18
default	$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$	18

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^n,x,method=_RETURNVERBOSE)
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\text{C}\left(bx\right)^n\text{C}\left(bx\right)}{bn+b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="fricas")
[Out] fresnel_cos(b*x)^n*fresnel_cos(b*x)/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \begin{cases} \tilde{\infty}x & \text{for } b = 0 \wedge n = -1 \\ 0^n x & \text{for } b = 0 \\ \frac{\log(C(bx))}{b} & \text{for } n = -1 \\ \frac{C(bx)C^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**n,x)
```

```
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (log(fresnelc(
b*x))/b, Eq(n, -1)), (fresnelc(b*x)*fresnelc(b*x)**n/(b*n + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\text{C}(bx)^{n+1}}{b(n+1)}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="maxima")
```

```
[Out] fresnel_cos(b*x)^(n + 1)/(b*(n + 1))
```

Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \int \text{C}(bx)^n \cos\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \int \text{FresnelC}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

3.180 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 231

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{105\,\text{FresnelC}(bx)^2}{2b^9\pi^4} - \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}$$

```
[Out] 105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2-55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*
x^6*cos(b^2*Pi*x^2)/b^3/Pi^2-105*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi
^4+7*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)^2/b
^9/Pi^4-35*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelC(b*x)
*sin(1/2*b^2*Pi*x^2)/b^2/Pi+40*sin(b^2*Pi*x^2)/b^9/Pi^5-5/2*x^4*sin(b^2*Pi*
x^2)/b^5/Pi^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717, 3390}

$$\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\frac{105\,\text{FresnelC}(bx)^2}{2\pi^4b^9}+\frac{105x^2}{4\pi^4b^7}-\frac{7x^6}{12\pi^2b^3}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{40\sin(\pi b^2x^2)}{\pi^5b^9}-\frac{105x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{55x^2\cos(\pi b^2x^2)}{4\pi^4b^7}-\frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{5x^4\sin(\pi b^2x^2)}{2\pi^3b^5}+\frac{7x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^6\cos(\pi b^2x^2)}{4\pi^2b^3}$$

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\text{integral} = \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^4\pi^2} - \frac{7\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} - \frac{\text{Subst}\left(\int x^3\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{35x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{105\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^6\pi^3} + \frac{35\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b^5\pi^3} - \frac{3\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} - \frac{7\text{Subst}\left(\int x^2\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{35x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3x^4\sin\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{105\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^8\pi^4} + \frac{105\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^7\pi^4} + \frac{3\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3} + \frac{35\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^5\pi^3} - \frac{7\text{Subst}\left(\int x^2\,dx, x, x^2\right)}{4b^3\pi^2} - \frac{7\text{Subst}\left(\int x^2\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2}$$

$$= -\frac{7x^6}{12b^3\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{35x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5x^4\sin\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{105\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right)}{b^9\pi^4} + \frac{3\text{Subst}\left(\int\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^7\pi^4} + \frac{35\text{Subst}\left(\int\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^7\pi^4} + \frac{105\text{Subst}\left(\int\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^7\pi^4} + \frac{7\text{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3}$$

$$
\begin{aligned}
&= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} \\
&\quad - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{105\,\text{FresnelC}(bx)^2}{2b^9\pi^4} - \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{73\sin(b^2\pi x^2)}{2b^9\pi^5} \\
&\quad - \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3} + \frac{7\text{Subst}\left(\int\cos(b^2\pi x)\,dx, x, x^2\right)}{2b^7\pi^4} \\
&= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} \\
&\quad - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{105\,\text{FresnelC}(bx)^2}{2b^9\pi^4} - \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx &= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} \\
&\quad + \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} \\
&\quad + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{105\,\text{FresnelC}(bx)^2}{2b^9\pi^4} \\
&\quad - \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad + \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}
\end{aligned}
$$

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
```

```
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Maple [F]

$$\int x^8 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)\, dx$$

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = -\frac{5\pi^3 b^6 x^6 - 240\pi b^2 x^2 - 3(\pi^3 b^6 x^6 - 55\pi b^2 x^2)\cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 42(\pi^3 b^5 x^5 - 15\pi bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{C}(bx}{6\pi^5 b^9}$$

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] -1/6*(5*pi^3*b^6*x^6 - 240*pi*b^2*x^2 - 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*co
s(1/2*pi*b^2*x^2)^2 - 42*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fre
snel_cos(b*x) - 315*pi*fresnel_cos(b*x)^2 + 6*(5*(pi^2*b^4*x^4 - 16)*cos(1/
2*pi*b^2*x^2) - (pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*fresnel_cos(b*x))*sin(1/2*
pi*b^2*x^2))/(pi^5*b^9)
```

Sympy [A] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.30

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \begin{cases} \frac{x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} - \frac{5x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{6\pi^2 b^3} - \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{3\pi^2 b^3} + \frac{7x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{5x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right)\cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{35x^3 \sin\left(\right.}{\pi} \\ 0 \end{cases}$$

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Piecewise((x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - 5*x**6*sin(pi
*b**2*x**2/2)**2/(6*pi**2*b**3) - x**6*cos(pi*b**2*x**2/2)**2/(3*pi**2*b**3
```

```
) + 7*x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 5*x**4*sin(pi*b
**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 35*x**3*sin(pi*b**2*x**2/2)*
fresnelc(b*x)/(pi**3*b**6) + 40*x**2*sin(pi*b**2*x**2/2)**2/(pi**4*b**7) +
25*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**4*b**7) - 105*x*cos(pi*b**2*x**2/2)*f
resnelc(b*x)/(pi**4*b**8) + 80*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi*
*5*b**9) + 105*fresnelc(b*x)**2/(2*pi**4*b**9), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^8\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.181 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 215

$$\begin{aligned}\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx =& \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos(b^2\pi x^2)}{16b^7\pi^4} \\ &+ \frac{x^5\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} \\ &+ \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\ &+ \frac{531\,\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\ &- \frac{24x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ &+ \frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{17x^3\sin(b^2\pi x^2)}{8b^5\pi^3}\end{aligned}$$

```
[Out] 24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2-147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*co
s(b^2*Pi*x^2)/b^3/Pi^2-48*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi^4+6*x^4*
cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2-24*x^2*FresnelC(b*x)*sin(1/2*b^2
*Pi*x^2)/b^6/Pi^3+x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-17/8*x^3*sin
(b^2*Pi*x^2)/b^5/Pi^3+531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6590, 6598, 6596, 3439, 3433, 3466, 3473, 30, 3467}

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{531\,\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4 b^8}+\frac{24x}{\pi^4 b^7}-\frac{3x^5}{5\pi^2 b^3}+\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4 b^8}-\frac{147x\cos\left(\pi b^2x^2\right)}{16\pi^4 b^7}-\frac{24x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3 b^6}-\frac{17x^3\sin\left(\pi b^2x^2\right)}{8\pi^3 b^5}+\frac{6x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2 b^4}+\frac{x^5\cos\left(\pi b^2x^2\right)}{4\pi^2 b^3}$$

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) - (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (48*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (531*FresnelC[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi) - (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &\quad -\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\ &= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &\quad -\frac{24\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx}{b^4\pi^2}-\frac{5\int x^4\cos\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2}-\frac{6\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{x^5 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^4\pi^2} - \frac{24x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5x^3 \sin\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{48 \int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx}{b^6\pi^3} \\
&\quad + \frac{15 \int x^2 \sin\left(b^2\pi x^2\right) \, dx}{8b^5\pi^3} + \frac{12 \int x^2 \sin\left(b^2\pi x^2\right) \, dx}{b^5\pi^3} - \frac{3 \int x^4 \, dx}{b^3\pi^2} - \frac{3 \int x^4 \cos\left(b^2\pi x^2\right) \, dx}{b^3\pi^2} \\
&= -\frac{3x^5}{5b^3\pi^2} - \frac{111x \cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^4\pi^2} - \frac{24x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{17x^3 \sin\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{15 \int \cos\left(b^2\pi x^2\right) \, dx}{16b^7\pi^4} \\
&\quad + \frac{6 \int \cos\left(b^2\pi x^2\right) \, dx}{b^7\pi^4} + \frac{48 \int \cos^2\left(\frac{1}{2}b^2\pi x^2\right) \, dx}{b^7\pi^4} + \frac{9 \int x^2 \sin\left(b^2\pi x^2\right) \, dx}{2b^5\pi^3} \\
&= -\frac{3x^5}{5b^3\pi^2} - \frac{147x \cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^4\pi^2} + \frac{15 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} + \frac{3\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{b^8\pi^4} \\
&\quad - \frac{24x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{17x^3 \sin\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{9 \int \cos\left(b^2\pi x^2\right) \, dx}{4b^7\pi^4} + \frac{48 \int \left(\frac{1}{2} + \frac{1}{2}\cos\left(b^2\pi x^2\right)\right) \, dx}{b^7\pi^4} \\
&= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x \cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^4\pi^2} + \frac{51 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\
&\quad + \frac{3\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{b^8\pi^4} - \frac{24x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{17x^3 \sin\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24 \int \cos\left(b^2\pi x^2\right) \, dx}{b^7\pi^4} \\
&= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x \cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^8\pi^4} \\
&\quad + \frac{6x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^4\pi^2} + \frac{51 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} + \frac{15\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{b^8\pi^4} \\
&\quad - \frac{24x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{17x^3 \sin\left(b^2\pi x^2\right)}{8b^5\pi^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.72

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$$
$$= \frac{2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 160\,\text{FresnelC}(bx)\left(6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2(-24+b^4\pi^2x^4)\sin\left(\right.\right.}{160b^8\pi^4}$$

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 160*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x
^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/
2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] - 2*(-960 + 24*b^4*P
i^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.47

method	result
default	$\frac{\text{FresnelC}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} - \frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7} - \frac{\frac{\frac{3}{5}b^5x^5\pi^2 - 24bx}{\pi^4} + \frac{3\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}}{}$

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*b^5*x^5*Pi^2-8*b*x)+3/Pi^4*(1/2*Pi*b^3*x^3*s
in(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(
b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/2/Pi^3*(-1/2*Pi*b^5*x^5*co
s(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x
^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx =$$
$$-\frac{136\,\pi^2 b^6 x^5 - 5310\,b^2 x - 20\left(4\,\pi^2 b^6 x^5 - 147\,b^2 x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 960\left(\pi^2 b^5 x^4 - 8\,b\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)}{160\,\pi}$$

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] -1/160*(136*pi^2*b^6*x^5 - 5310*b^2*x - 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos
(1/2*pi*b^2*x^2)^2 - 960*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_c
os(b*x) - 2655*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(17*
pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - 4*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel_c
os(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [F]

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^7\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)\,dx$$

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**7*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^7\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^7 \cos\left(\frac{1}{2}\pi b^2x^2\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^7\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.182 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 247

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{5x^4}{8b^3\pi^2} - \frac{11\cos(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7x^2\sin(b^2\pi x^2)}{4b^5\pi^3}$$

```
[Out] -5/8*x^4/b^3/Pi^2-11/2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*cos(b^2*Pi*x^2)/b^3
/Pi^2+5*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+15/2*FresnelC(b*x)*F
resnelS(b*x)/b^7/Pi^3+15/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^
2)/b^5/Pi^3-15/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/Pi^3
-15*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelC(b*x)*sin(1/2*
b^2*Pi*x^2)/b^2/Pi-7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6590, 6598, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5} + \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2\pi^3b^7} - \frac{5x^4}{8\pi^2b^3} + \frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{11\cos\left(\pi b^2x^2\right)}{2\pi^4b^7} - \frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} - \frac{7x^2\sin\left(\pi b^2x^2\right)}{4\pi^3b^5} + \frac{5x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^4\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*Hypergeom
etricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x
^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*
x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\text{integral} = \frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5\int x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^4\pi^2} - \frac{5\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} - \frac{\text{Subst}\left(\int x^2\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi}$$

$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{15\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^6\pi^3} + \frac{15\int x\sin\left(b^2\pi x^2\right)\,dx}{2b^5\pi^3} - \frac{\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2} - \frac{5\text{Subst}\left(\int x\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2}$$

$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{x^2\sin\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3} + \frac{15\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^5\pi^3} - \frac{5\text{Subst}\left(\int x\,dx, x, x^2\right)}{4b^3\pi^2} - \frac{5\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2}$$

$$= -\frac{5x^4}{8b^3\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7x^2\sin\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^5\pi^3}$$

$$= -\frac{5x^4}{8b^3\pi^2} - \frac{11\cos(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7x^2\sin(b^2\pi x^2)}{4b^5\pi^3}$$

Mathematica [F]

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F]

$$\int x^6\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\,(bx)\,dx$$

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [F]

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^6\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\,(bx)\;dx$$

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Sympy [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**6*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{C}(bx)\,dx$$

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{C}(bx)\,dx$$

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.183 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{2x^3}{3b^3\pi^2}+\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2}+\frac{43\,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3}-\frac{8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{11x\sin\left(b^2\pi x^2\right)}{8b^5\pi^3}$$

```
[Out] -2/3*x^3/b^3/Pi^2+1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+4*x^2*cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/b^4/Pi^2-8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^4*F
resnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3+43/
16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {6590, 6598, 6588, 3432, 3473, 30, 3467, 3466}

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{43\,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6} - \frac{2x^3}{3\pi^2b^3} + \frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} - \frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5} + \frac{4x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^3\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^
2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt
[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*Fre
snelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
```

```
m, n}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{4\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} + \frac{x^4\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{8\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx}{b^4\pi^2} - \frac{3\int x^2\cos\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2} - \frac{4\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} \\
&= \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} - \frac{8\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^4\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3x\sin\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{3\int\sin\left(b^2\pi x^2\right)\,dx}{8b^5\pi^3} \\
&\quad + \frac{4\int\sin\left(b^2\pi x^2\right)\,dx}{b^5\pi^3} - \frac{2\int x^2\,dx}{b^3\pi^2} - \frac{2\int x^2\cos\left(b^2\pi x^2\right)\,dx}{b^3\pi^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{3 \,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{2\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^6\pi^3} - \frac{8\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^4\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11x \sin\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{\int \sin\left(b^2\pi x^2\right) dx}{b^5\pi^3} \\
&= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3 \cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{11 \,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{2\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^6\pi^3} - \frac{8\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\
&\quad + \frac{x^4\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11x \sin\left(b^2\pi x^2\right)}{8b^5\pi^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$
\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx) \, dx
= \frac{-32b^3\pi x^3 + 12b^3\pi x^3 \cos\left(b^2\pi x^2\right) + 129\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 48\,\text{FresnelC}(bx) \left(4b^2\pi x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) + (-\right.}{48b^6\pi^3}
$$

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqrt
[2]*b*x] + 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*P
i^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$\frac{\frac{\text{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}+\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{\pi}-\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}+\frac{-\frac{\pi b^3x^3\cos}{2}}{}}{}}{b}$

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3+2/Pi
```

```
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))+1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$
$$=\frac{24\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-44\,\pi b^4x^3+192\,\pi b^3x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)+129\,\sqrt{2}\sqrt{b^2}\,\text{S}\left(\sqrt{2}\sqrt{b^2}x\right)-12\,(11\,b^2}{48\,\pi^3b^7}$$

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] 1/48*(24*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 44*pi*b^4*x^3 + 192*pi*b^3*x^2*
cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sq
rt(2)*sqrt(b^2)*x) - 12*(11*b^2*x*cos(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^4 - 8
*b)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]

$$\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x^5\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)\;dx$$

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)\;dx$$

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx = \int x^5 \cos\left(\frac{1}{2}\pi b^2x^2\right) \mathrm{C}\left(bx\right)\,dx$$

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx = \int x^5\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.184 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

```
[Out] -3/4*x^2/b^3/Pi^2+1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2+3*x*cos(1/2*b^2*Pi*x^2)*
FresnelC(b*x)/b^4/Pi^2-3/2*FresnelC(b*x)^2/b^5/Pi^2+x^3*FresnelC(b*x)*sin(1
/2*b^2*Pi*x^2)/b^2/Pi-sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717}

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{3\,\text{FresnelC}(bx)^2}{2\pi^2 b^5} - \frac{3x^2}{4\pi^2 b^3} + \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\sin(\pi b^2x^2)}{\pi^3 b^5} + \frac{3x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2 b^4} + \frac{x^2\cos(\pi b^2x^2)}{4\pi^2 b^3}$$

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
```

```
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{3\int x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} + \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{3\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx}{b^4\pi^2} - \frac{3\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^3\pi^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int x\sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi} \\
&= \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\operatorname{Subst}\left(\int x\,dx, x, \operatorname{FresnelC}(bx)\right)}{b^5\pi^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int \cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} - \frac{3\operatorname{Subst}\left(\int \cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^3\pi^2} \\
&= -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} \\
&\quad - \frac{3\operatorname{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\sin\left(b^2\pi x^2\right)}{b^5\pi^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Maple [F]

$$\int x^4\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}(bx)\,dx$$

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 - 2\,\pi b^2x^2 + 6\,\pi bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}(bx) - 3\,\pi\,\text{C}(bx)^2 + 2\left(\pi^2b^3x^3\,\text{C}(bx) - 2\,\cos\left(\frac{1}{2}\,\pi b\right.\right.}{2\,\pi^3b^5}$$

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 2*pi*b^2*x^2 + 6*pi*b*x*cos(1/2*pi*
b^2*x^2)*fresnel_cos(b*x) - 3*pi*fresnel_cos(b*x)^2 + 2*(pi^2*b^3*x^3*fresn
el_cos(b*x) - 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx$$
$$=\begin{cases}\frac{x^3\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2}-\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^2b^3}-\frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3}+\frac{3x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^2b^4}-\frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5}-\frac{3C^2(bx)}{2\pi^2b^5} & \text{for } b \\ 0 & \text{othe}\end{cases}$$

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - x**2*sin(pi*b
**2*x**2/2)**2/(pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(2*pi**2*b**3) +
3*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 2*sin(pi*b**2*x**2/2)*
cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnelc(b*x)**2/(2*pi**2*b**5), Ne(b,
 0)), (0, True))
```

Maxima [F]

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)\,dx$$

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)\,dx$$

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^4\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.185 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{5\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] -x/b^3/Pi^2+1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2+2*cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/b^4/Pi^2+x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-5/8*FresnelC(b*x*
2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6590, 6596, 3439, 3433, 3466}

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{5\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2 b^4} - \frac{x}{\pi^2 b^3} + \frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{x\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -(x/(b^3*Pi^2)) + (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (2*Cos[(b^2*Pi*x^2)/2]
*FresnelC[b*x])/(b^4*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2)
 + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\
&\quad - \frac{2\int x \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^2\pi} - \frac{\int x^2\sin\left(b^2\pi x^2\right)\, dx}{2b\pi} \\
&= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} \\
&\quad + \frac{x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \cos\left(b^2\pi x^2\right)\, dx}{4b^3\pi^2} - \frac{2\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right)\, dx}{b^3\pi^2} \\
&= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^4\pi^2} - \frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} \\
&\quad + \frac{x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2\int \left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\, dx}{b^3\pi^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad - \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int\cos\left(b^2\pi x^2\right)\,dx}{b^3\pi^2} \\
&= -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\
&\quad - \frac{5\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$
\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx
$$
$$
= \frac{2bx\left(-4+\cos\left(b^2\pi x^2\right)\right) - 5\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 8\,\text{FresnelC}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{8b^4\pi^2}
$$

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2*b*x*(-4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 8*Fresnel
C[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\text{FresnelC}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	114

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(b*x/Pi^2+1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))+1/2/Pi*(-1/2/P
i*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx$$
$$= \frac{8\,\pi b^3 x^2\,\mathrm{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,b^2 x\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 10\,b^2 x + 16\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\,(bx) - 5\,\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\right.}{8\,\pi^2 b^5}$$

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/8*(8*pi*b^3*x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) + 4*b^2*x*cos(1/2*pi
*b^2*x^2)^2 - 10*b^2*x + 16*b*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 5*sqrt
(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi^2*b^5)
```

Sympy [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx = \int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\;dx$$

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx = \int x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\,(bx)\;dx$$

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^3 \,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.186 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] 1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-1/2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x
^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*hypergeom([1
, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b/Pi+x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^
2/Pi
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6590, 6582, 3460, 2718}

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2\pi b^3} + \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\cos(\pi b^2x^2)}{4\pi^2b^3}$$

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b*Pi)
+ ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
+ (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\
&= -\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}\\
&\quad + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\\
&\quad - \frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi}
\end{aligned}$$

$$= \frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

Mathematica [F]

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F]

$$\int x^2\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}(bx)\,dx$$

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Fricas [F]

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{C}(bx)\,dx$$

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Sympy [F]

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int x^2\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**2*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Giac [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^2\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.187 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi} + \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*2
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6588, 3432}

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}$$

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^
2)/2])/(b^2*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
```

```
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= -\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi} + \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = -\frac{\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) - 4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^2\pi}$$

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] -1/4*(Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi} - \frac{\text{FresnelS}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$	45

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] (FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(1
/2))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\frac{4\,b\,\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\sqrt{2}\sqrt{b^2}\,\text{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,\pi b^3}$$

[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")

[Out] 1/4*(4*b*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) - sqrt(2)*sqrt(b^2)*fresnel_s
in(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)

Sympy [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)

[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)

Maxima [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)\,dx$$

[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")

[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)

Giac [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)\,dx$$

[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")

[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)

Mupad [F(-1)]

Timed out.

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x\, \text{FresnelC}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

```
[In] int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.188 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\frac{\text{FresnelC}(bx)^2}{2b}$$

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6576, 30}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\frac{\text{FresnelC}(bx)^2}{2b}$$

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}(\int x\,dx, x, \text{FresnelC}(bx))}{b} \\ &= \frac{\text{FresnelC}(bx)^2}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] FresnelC[b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelC}(bx)^2}{2b}$	12

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/2*FresnelC(b*x)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{C}\,(bx)^2}{2\,b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx = \begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx = \frac{\mathrm{C}\,(bx)^2}{2\,b}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\,(bx)\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx = \int \operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.189 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx=\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)}{x}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)}{x}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

3.190 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2} \, dx = \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2} \, dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2} \, dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^2}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^2}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^2}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**2,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{C}(bx)}{x^2}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{C}(bx)}{x^2}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

3.191 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^3} \, dx = -\frac{b}{4x} - \frac{b\cos\left(b^2\pi x^2\right)}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{2x^2} - \frac{b^2\pi \, \text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{1}{2}b^2\pi \text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)$$

```
[Out] -1/4*b/x-1/4*b*cos(b^2*Pi*x^2)/x-1/2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-
1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^3} \, dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^3} \, dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

```
[Out] -1/4*b/x - (b*Cos[b^2*Pi*x^2])/(4*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/
(2*x^2) - (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sqrt[2]) - (b^2*Pi*Defer[Int][(
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2x^2} + \frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad - \frac{1}{2}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&= -\frac{b}{4x} - \frac{b\cos\left(b^2\pi x^2\right)}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2x^2} \\
&\quad - \frac{1}{2}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx - \frac{1}{2}\left(b^3\pi\right)\int\sin\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b}{4x} - \frac{b\cos\left(b^2\pi x^2\right)}{4x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2x^2} \\
&\quad - \frac{b^2\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{1}{2}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^3}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx=\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\,(bx)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx=\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**3,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx=\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\,(bx)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^3}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

3.192 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b}{12x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3} - \frac{1}{6}b^3\pi^2\,\text{FresnelC}(bx)^2 + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} - \frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)$$

```
[Out] -1/12*b/x^2-1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^3-1/6*b^3*Pi^2*FresnelC(b*x)^2-1/6*b^3*Pi*Si(b^2*Pi*x^2)+1/3*b^2*Pi*Fre
snelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6592, 6600, 6576, 30, 3456, 3461, 3378, 3380}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = -\frac{1}{6}\pi^2b^3\,\text{FresnelC}(bx)^2 + \frac{\pi b^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x} - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3} - \frac{b\cos\left(\pi b^2x^2\right)}{12x^2} - \frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right) - \frac{b}{12x^2}$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -1/12*b/x^2 - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3} + \frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad - \frac{1}{3}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\
&= -\frac{b}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} \\
&\quad + \frac{1}{12}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{6}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx \\
&\quad - \frac{1}{3}\left(b^4\pi^2\right)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx \\
&= -\frac{b}{12x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} \\
&\quad - \frac{1}{12}b^3\pi\text{Si}\left(b^2\pi x^2\right) - \frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) \\
&\quad - \frac{1}{3}\left(b^3\pi^2\right)\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right) \\
&= -\frac{b}{12x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3} \\
&\quad - \frac{1}{6}b^3\pi^2\,\text{FresnelC}(bx)^2 + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} - \frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx &= -\frac{b}{12x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} \\
&\quad - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3} - \frac{1}{6}b^3\pi^2\,\text{FresnelC}(bx)^2 \\
&\quad + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} - \frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)
\end{aligned}$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -1/12*b/x^2 - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^4}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^4}\,dx = -\frac{\pi^2 b^3 x^3\,\mathrm{C}\left(bx\right)^2 + \pi b^3 x^3\,\mathrm{Si}\left(\pi b^2 x^2\right) - 2\,\pi b^2 x^2\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + bx\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\,\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}}{6\,x^3}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(pi^2*b^3*x^3*fresnel_cos(b*x)^2 + pi*b^3*x^3*sin_integral(pi*b^2*x^2)
 - 2*pi*b^2*x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) + b*x*cos(1/2*pi*b^2*x
^2)^2 + 2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x))/x^3
```

Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C\left(bx\right)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**4,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{C}(bx)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^4, x)
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{C}(bx)}{x^4}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^4}\,dx = \int \frac{\operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

3.193 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = & -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4x^4}\\ & -\frac{7b^4\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}}\\ & +\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}+\frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x}\\ & -\frac{1}{8}b^4\pi^2\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)\end{aligned}$$

```
[Out] -1/24*b/x^3-1/24*b*cos(b^2*Pi*x^2)/x^3-1/4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^4+1/8*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+7/48*b^3*Pi*sin(b^2*
Pi*x^2)/x-7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unintegr
able(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

```
[Out] -1/24*b/x^3 - (b*Cos[b^2*Pi*x^2])/(24*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelC[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4x^4} + \frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad - \frac{1}{4}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{b}{24x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4x^4} \\
&\quad + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} - \frac{1}{16}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad - \frac{1}{12}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{8}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx \\
&= -\frac{b}{24x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4x^4} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} \\
&\quad + \frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx \\
&\quad - \frac{1}{8}\left(b^5\pi^2\right)\int\cos\left(b^2\pi x^2\right)\,dx - \frac{1}{6}\left(b^5\pi^2\right)\int\cos\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b}{24x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4x^4} \\
&\quad - \frac{7b^4\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} \\
&\quad + \frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \mathrm{FresnelC}\left(bx\right)}{x^5}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{C}\left(bx\right)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**5,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\left(bx\right)}{x^5}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

3.194 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = &-\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{1}{24}b^5\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right)\\ &-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}\\ &+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2}\\ &-\frac{1}{15}b^4\pi^2\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)\end{aligned}$$

```
[Out] -1/40*b/x^4-1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)-1/40*b*cos(b^2*Pi*x^2)/x^4-1/5*cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+1/15*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi
*x^2)/x^3+1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(cos(1/
2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

```
[Out] -1/40*b/x^4 - (b*Cos[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*x^5) + (b^2*Pi*FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/15
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}+\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx\\
&\quad-\frac{1}{5}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}\\
&\quad+\frac{1}{20}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^3}\,dx,x,x^2\right)-\frac{1}{30}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx\\
&\quad-\frac{1}{15}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx\\
&= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}\\
&\quad+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}-\frac{1}{60}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&\quad-\frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&\quad-\frac{1}{15}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx\\
&= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}\\
&\quad+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2}-\frac{1}{15}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx\\
&\quad-\frac{1}{60}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&\quad-\frac{1}{40}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{1}{24}b^5\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right)\\
&\quad-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}\\
&\quad+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2}-\frac{1}{15}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^6}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^6}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)}{x^6}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

Sympy [N/A]

Not integrable

Time = 5.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^6}\,dx$$

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**6,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{C}(bx)}{x^6}\,dx$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{C}(bx)}{x^6}\,dx$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)

Mupad [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

3.195 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx &= -\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}-\frac{b\cos\left(b^2\pi x^2\right)}{60x^5} \\ &\quad+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6} \\ &\quad+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^2} \\ &\quad+\frac{7b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{144\sqrt{2}}+\frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) \\ &\quad+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4}+\frac{13b^3\pi\sin\left(b^2\pi x^2\right)}{720x^3} \\ &\quad+\frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

```
[Out] -1/60*b/x^5+1/96*b^5*Pi^2/x-1/60*b*cos(b^2*Pi*x^2)/x^5+67/1440*b^5*Pi^2*cos
(b^2*Pi*x^2)/x-1/6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/48*b^4*Pi^2*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2+1/24*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x^4+13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3+67/1440*b^6*Pi^3*FresnelS(b*x*2^
(1/2))*2^(1/2)+1/48*b^6*Pi^3*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)
/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

```
[Out] -1/60*b/x^5 + (b^5*Pi^2)/(96*x) - (b*Cos[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(6*x^6)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(48*x^2) + (7*b^6*Pi^3*Fresn
elS[Sqrt[2]*b*x])/(144*Sqrt[2]) + (Sqrt[2]*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/
45 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/x, x])/48
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6} + \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad - \frac{1}{6}\left(b^2\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{b}{60x^5} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6} \\
&\quad + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{1}{48}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad - \frac{1}{30}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{24}\left(b^4\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx \\
&= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^2} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} \\
&\quad + \frac{13b^3\pi\sin\left(b^2\pi x^2\right)}{720x^3} - \frac{1}{96}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{72}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad - \frac{1}{45}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{48}\left(b^6\pi^3\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2\cos(b^2\pi x^2)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^2} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} + \frac{13b^3\pi\sin(b^2\pi x^2)}{720x^3} + \frac{1}{48}(b^6\pi^3)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx + \frac{1}{48}(b^7\pi^3)\int\sin(b^2\pi x^2)\,dx + \frac{1}{36}(b^7\pi^3)\int\sin(b^2\pi x^2)\,dx + \frac{1}{45}(2b^7\pi^3)\int\sin(b^2\pi x^2)\,dx$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2\cos(b^2\pi x^2)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^2} + \frac{7b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{144\sqrt{2}} + \frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} + \frac{13b^3\pi\sin(b^2\pi x^2)}{720x^3} + \frac{1}{48}(b^6\pi^3)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{x^7}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

Sympy [N/A]

Not integrable

Time = 11.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**7,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^7}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

Mupad [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

3.196 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = &-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos(b^2\pi x^2)}{84x^6}\\&+\frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7}\\&+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3}\\&+\frac{1}{210}b^7\pi^4\,\text{FresnelC}(bx)^2\\&+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}\\&-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x}\\&+\frac{b^3\pi\sin(b^2\pi x^2)}{105x^4}+\frac{1}{70}b^7\pi^3\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

```
[Out] -1/84*b/x^6+1/420*b^5*Pi^2/x^2-1/84*b*cos(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*cos
(b^2*Pi*x^2)/x^2-1/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/105*b^4*Pi^2*c
os(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/210*b^7*Pi^4*FresnelC(b*x)^2+1/70*b^
7*Pi^3*Si(b^2*Pi*x^2)+1/35*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/1
05*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x+1/105*b^3*Pi*sin(b^2*Pi*x^2
)/x^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6592, 6600, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \frac{1}{210}\pi^4 b^7\,\text{FresnelC}(bx)^2 + \frac{\pi^2 b^5}{420x^2} - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7} + \frac{\pi b^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{35x^5} - \frac{b\cos\left(\pi b^2 x^2\right)}{84x^6} + \frac{1}{70}\pi^3 b^7\,\text{Si}\left(b^2\pi x^2\right) - \frac{\pi^3 b^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{105x} + \frac{\pi^2 b^5\cos\left(\pi b^2 x^2\right)}{84x^2} + \frac{\pi^2 b^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{105x^3} + \frac{\pi b^3\sin\left(\pi b^2 x^2\right)}{105x^4} - \frac{b}{84x^6}$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7} + \frac{1}{14}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx$$
$$-\frac{1}{7}\left(b^2\pi\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

$$= -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) - \frac{1}{70}\left(b^3\pi\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{35}\left(b^4\pi^2\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{1}{140}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{210}\left(b^5\pi^2\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx + \frac{1}{105}\left(b^6\pi^3\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x} + \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{420}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{280}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{168}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) + \frac{1}{210}\left(b^7\pi^3\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx + \frac{1}{105}\left(b^8\pi^4\right)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7}$$
$$+ \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}$$
$$- \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x} + \frac{b^3\pi\sin(b^2\pi x^2)}{105x^4}$$
$$+ \frac{1}{420}b^7\pi^3\text{Si}(b^2\pi x^2) + \frac{1}{420}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)$$
$$+ \frac{1}{280}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)$$
$$+ \frac{1}{168}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)$$
$$+ \frac{1}{105}(b^7\pi^4)\,\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right)$$
$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2}$$
$$- \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3}$$
$$+ \frac{1}{210}b^7\pi^4\,\text{FresnelC}(bx)^2 + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}$$
$$- \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x} + \frac{b^3\pi\sin(b^2\pi x^2)}{105x^4} + \frac{1}{70}b^7\pi^3\text{Si}(b^2\pi x^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos(b^2\pi x^2)}{84x^6}$$
$$+ \frac{b^5\pi^2\cos(b^2\pi x^2)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7}$$
$$+ \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3}$$
$$+ \frac{1}{210}b^7\pi^4\,\text{FresnelC}(bx)^2$$
$$+ \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}$$
$$- \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x}$$
$$+ \frac{b^3\pi\sin(b^2\pi x^2)}{105x^4} + \frac{1}{70}b^7\pi^3\text{Si}(b^2\pi x^2)$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^8}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^8}\,dx$$
$$= \frac{\pi^4 b^7 x^7\,\mathrm{C}\left(bx\right)^2 + 3\,\pi^3 b^7 x^7\,\mathrm{Si}\left(\pi b^2 x^2\right) - 2\,\pi^2 b^5 x^5 + 5\left(\pi^2 b^5 x^5 - bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\left(\pi^2 b^4 x^4 - 15\right)\cos\left(\frac{1}{2}\,\pi b\right.}{210\,x^7}$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] 1/210*(pi^4*b^7*x^7*fresnel_cos(b*x)^2 + 3*pi^3*b^7*x^7*sin_integral(pi*b^2
*x^2) - 2*pi^2*b^5*x^5 + 5*(pi^2*b^5*x^5 - b*x)*cos(1/2*pi*b^2*x^2)^2 + 2*(
pi^2*b^4*x^4 - 15)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*(2*pi*b^3*x^3*c
os(1/2*pi*b^2*x^2) - (pi^3*b^6*x^6 - 3*pi*b^2*x^2)*fresnel_cos(b*x))*sin(1/
2*pi*b^2*x^2))/x^7
```

Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C\left(bx\right)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**8,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^8, x)
```

Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^8}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

3.197 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx &= -\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}\\ &+\frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8}\\ &+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{192x^4}\\ &+\frac{853b^8\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}\\ &+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6}\\ &-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2}\\ &+\frac{19b^3\pi\sin\left(b^2\pi x^2\right)}{3360x^5}-\frac{853b^7\pi^3\sin\left(b^2\pi x^2\right)}{80640x}\\ &+\frac{1}{384}b^8\pi^4\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)\end{aligned}$$

```
[Out] -1/112*b/x^7+1/1152*b^5*Pi^2/x^3-1/112*b*cos(b^2*Pi*x^2)/x^7+187/40320*b^5*
Pi^2*cos(b^2*Pi*x^2)/x^3-1/8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+1/192*b^
4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4+1/48*b^2*Pi*FresnelC(b*x)*sin(
1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19
/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5-853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x+853/8
0640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

```
[Out] -1/112*b/x^7 + (b^5*Pi^2)/(1152*x^3) - (b*Cos[b^2*Pi*x^2])/(112*x^7) + (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(192*x^4) + (853*b
^8*Pi^4*FresnelC[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelC[b*x]*Sin[
(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3
84*x^2) + (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) - (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x, x])/384
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8} + \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx \\
&\quad - \frac{1}{8}\left(b^2\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{b}{112x^7} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8} \\
&\quad + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{1}{96}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad - \frac{1}{56}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{48}\left(b^4\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx \\
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{192x^4} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\
&\quad + \frac{19b^3\pi\sin\left(b^2\pi x^2\right)}{3360x^5} - \frac{1}{384}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{240}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad - \frac{1}{140}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx + \frac{1}{192}\left(b^6\pi^3\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos(b^2\pi x^2)}{112x^7} + \frac{187b^5\pi^2\cos(b^2\pi x^2)}{40320x^3} \\
&\quad - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{192x^4} \\
&\quad + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} \\
&\quad + \frac{19b^3\pi\sin(b^2\pi x^2)}{3360x^5} + \frac{1}{768}(b^7\pi^3)\int \frac{\sin(b^2\pi x^2)}{x^2}\,dx \\
&\quad + \frac{1}{576}(b^7\pi^3)\int \frac{\sin(b^2\pi x^2)}{x^2}\,dx + \frac{1}{360}(b^7\pi^3)\int \frac{\sin(b^2\pi x^2)}{x^2}\,dx \\
&\quad + \frac{1}{210}(b^7\pi^3)\int \frac{\sin(b^2\pi x^2)}{x^2}\,dx + \frac{1}{384}(b^8\pi^4)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos(b^2\pi x^2)}{112x^7} + \frac{187b^5\pi^2\cos(b^2\pi x^2)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{192x^4} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\
&\quad - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} + \frac{19b^3\pi\sin(b^2\pi x^2)}{3360x^5} \\
&\quad - \frac{853b^7\pi^3\sin(b^2\pi x^2)}{80640x} + \frac{1}{384}(b^8\pi^4)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx \\
&\quad + \frac{1}{384}(b^9\pi^4)\int \cos\left(b^2\pi x^2\right)\,dx + \frac{1}{288}(b^9\pi^4)\int \cos\left(b^2\pi x^2\right)\,dx \\
&\quad + \frac{1}{180}(b^9\pi^4)\int \cos\left(b^2\pi x^2\right)\,dx + \frac{1}{105}(b^9\pi^4)\int \cos\left(b^2\pi x^2\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos(b^2\pi x^2)}{112x^7} + \frac{187b^5\pi^2\cos(b^2\pi x^2)}{40320x^3} \\
&\quad - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{192x^4} \\
&\quad + \frac{853b^8\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\
&\quad - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} + \frac{19b^3\pi\sin(b^2\pi x^2)}{3360x^5} \\
&\quad - \frac{853b^7\pi^3\sin(b^2\pi x^2)}{80640x} + \frac{1}{384}(b^8\pi^4)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^9}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx=\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^9}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

Sympy [N/A]

Not integrable

Time = 36.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^9}\,dx$$

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**9,x)

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right)}{x^9}\,dx$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="maxima")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right)}{x^9}\,dx$$

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)

Mupad [N/A]

Not integrable

Time = 4.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

3.198 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = &-\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4}\\ &+\frac{5b^9\pi^4\,\text{CosIntegral}\left(b^2\pi x^2\right)}{2016}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9}\\ &+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{315x^5}\\ &+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7}\\ &-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3}\\ &+\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6}-\frac{5b^7\pi^3\sin\left(b^2\pi x^2\right)}{2016x^2}\\ &+\frac{1}{945}b^8\pi^4\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)\end{aligned}$$

```
[Out] -1/144*b/x^8+1/2520*b^5*Pi^2/x^4+5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)-1/144*b*cos
(b^2*Pi*x^2)/x^8+67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/9*cos(1/2*b^2*Pi*x
^2)*FresnelC(b*x)/x^9+1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+
1/63*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*b^6*Pi^3*FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x^3+11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6-5/2016*b^7*P
i^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*Fre
snelC(b*x)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$$

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

```
[Out] -1/144*b/x^8 + (b^5*Pi^2)/(2520*x^4) - (b*Cos[b^2*Pi*x^2])/(144*x^8) + (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) + (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(315*x^5) + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(63*x^7) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) + (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) - (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/945
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9} + \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}\,dx \\
&\quad - \frac{1}{9}\left(b^2\pi\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx \\
&= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} \\
&\quad + \frac{1}{36}b\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^5}\,dx, x, x^2\right) - \frac{1}{126}\left(b^3\pi\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx \\
&\quad - \frac{1}{63}\left(b^4\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx \\
&= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9} \\
&\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{315x^5} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} \\
&\quad - \frac{1}{252}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&\quad - \frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) - \frac{1}{630}\left(b^5\pi^2\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx \\
&\quad + \frac{1}{315}\left(b^6\pi^3\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx
\end{aligned}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos(b^2\pi x^2)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{315x^5} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} + \frac{11b^3\pi\sin(b^2\pi x^2)}{3024x^6} - \frac{(b^5\pi^2)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^3}\,dx, x, x^2\right)}{1260} - \frac{1}{756}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^3}\,dx, x, x^2\right) - \frac{1}{432}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^3}\,dx, x, x^2\right) + \frac{(b^7\pi^3)\int\frac{\sin(b^2\pi x^2)}{x^3}\,dx}{1890} + \frac{1}{945}(b^8\pi^4)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\cos(b^2\pi x^2)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{315x^5} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} + \frac{11b^3\pi\sin(b^2\pi x^2)}{3024x^6} + \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{3780} + \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{2520} + \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{1512} + \frac{1}{864}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) + \frac{1}{945}(b^8\pi^4)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\cos(b^2\pi x^2)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{315x^5} + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} + \frac{11b^3\pi\sin(b^2\pi x^2)}{3024x^6} - \frac{5b^7\pi^3\sin(b^2\pi x^2)}{2016x^2} + \frac{1}{945}(b^8\pi^4)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)}{3780} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)}{2520} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)}{1512} + \frac{1}{864}(b^9\pi^4)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\cos(b^2\pi x^2)}{30240x^4} + \frac{5b^9\pi^4 \operatorname{CosIntegral}(b^2\pi x^2)}{2016} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{315x^5} + \frac{b^2\pi \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} + \frac{11b^3\pi\sin(b^2\pi x^2)}{3024x^6} - \frac{5b^7\pi^3\sin(b^2\pi x^2)}{2016x^2} + \frac{1}{945}(b^8\pi^4)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^{10}}\,dx$$

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\operatorname{FresnelC}(bx)}{x^{10}}dx$$

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\operatorname{C}(bx)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

Sympy [N/A]

Not integrable

Time = 65.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**10,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)}{x^{10}}\,dx$$

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

Mupad [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

3.199 $\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \text{Int}\left(\text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right), x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

```
[In] Int[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Defer[Int][FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Rubi steps

$$\text{integral} = \int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

```
[In] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \text{FresnelC}\left(bx\right)^n \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

```
[In] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{C}\left(bx\right)^n \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

```
[In] integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \sin\left(\frac{\pi b^2 x^2}{2}\right) C^n(bx)\, dx$$

```
[In] integrate(fresnelc(b*x)**n*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)**n, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{C}(bx)^n \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{C}(bx)^n \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelC}(b\,x)^n \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

3.200 $\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40\cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} + \frac{105\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^9\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] -35/8*x^4/b^5/Pi^3+1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^
2*Pi*x^2)/b^5/Pi^3+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^7*co
s(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+105/2*FresnelC(b*x)*FresnelS(b*x)/b^
9/Pi^4+105/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7/Pi^4-10
5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/Pi^4-105*x*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2
)/b^4/Pi^2-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2*Pi*x^2)/b^3/Pi
^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6598, 6590, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7} + \frac{105\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2\pi^4b^9} - \frac{35x^4}{8\pi^3b^5} - \frac{x^7\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{40\cos\left(\pi b^2x^2\right)}{\pi^5b^9} - \frac{105x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8} - \frac{55x^2\sin\left(\pi b^2x^2\right)}{4\pi^4b^7} + \frac{35x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{5x^4\cos\left(\pi b^2x^2\right)}{2\pi^3b^5} + \frac{7x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^6\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} + \frac{x^8}{16\pi b}$$

```
[In] Int[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-35*x^4)/(8*b^5*Pi^3) + x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) +
(5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (105
*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) + (((105*I)/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^7*Pi^4) - (((105*I)/8)*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (105*x*
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(b^4*Pi^2) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Si
n[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
```

```
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} \\
&\quad + \frac{7\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx}{b^2\pi} + \frac{\int x^7 \cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= -\frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} + \frac{7x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad - \frac{35\int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} - \frac{7\int x^5 \sin\left(b^2\pi x^2\right)\,dx}{2b^3\pi^2} \\
&\quad + \frac{\text{Subst}\left(\int x^3 \cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b\pi} \\
&= \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} \\
&\quad + \frac{7x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{105\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx}{b^6\pi^3} \\
&\quad - \frac{35\int x^3 \cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^5\pi^3} - \frac{7\text{Subst}\left(\int x^2 \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} \\
&\quad + \frac{\text{Subst}\left(\int x^3\,dx, x, x^2\right)}{4b\pi} + \frac{\text{Subst}\left(\int x^3 \cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi} \\
&= \frac{x^8}{16b\pi} + \frac{7x^4 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{35x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{105x \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\
&\quad + \frac{7x^5 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^6 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \\
&\quad + \frac{105\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^8\pi^4} \\
&\quad + \frac{105\int x \sin\left(b^2\pi x^2\right)\,dx}{2b^7\pi^4} - \frac{7\text{Subst}\left(\int x \cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3} \\
&\quad - \frac{35\text{Subst}\left(\int x \cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b^5\pi^3} - \frac{3\text{Subst}\left(\int x^2 \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^8}{16b\pi} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} \\
&\quad + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} \\
&\quad - \frac{105x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad - \frac{7x^2\sin\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^6\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{7\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{2b^7\pi^4} \\
&\quad + \frac{105\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{4b^7\pi^4} - \frac{3\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx,x,x^2\right)}{2b^5\pi^3} \\
&\quad - \frac{35\text{Subst}\left(\int x\,dx,x,x^2\right)}{4b^5\pi^3} - \frac{35\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx,x,x^2\right)}{4b^5\pi^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{119\cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} \\
&\quad + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\
&\quad + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} \\
&\quad - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\
&\quad + \frac{7x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{55x^2\sin\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \\
&\quad + \frac{3\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{2b^7\pi^4} + \frac{35\text{Subst}\left(\int\sin\left(b^2\pi x\right)\,dx,x,x^2\right)}{4b^7\pi^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} \\
&\quad + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\
&\quad + \frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} \\
&\quad - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\
&\quad + \frac{7x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{55x^2\sin\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

```
[In] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

```
[In] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) dx$$

```
[In] integrate(x**8*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**8*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{FresnelC}(b\,x) \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.201 $\int x^7 \operatorname{FresnelC}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 218

$$\begin{aligned}\int x^7 \operatorname{FresnelC}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) dx = &-\frac{4x^3}{b^5\pi^3}+\frac{x^7}{14b\pi}+\frac{17x^3\cos(b^2\pi x^2)}{8b^5\pi^3} \\ &+\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} \\ &-\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} \\ &+\frac{531\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\ &-\frac{48\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\ &+\frac{6x^4\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\ &-\frac{147x\sin(b^2\pi x^2)}{16b^7\pi^4}+\frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

```
[Out] -4*x^3/b^5/Pi^3+1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3+24*x^2*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-48*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+6*x^4*FresnelC(b*x)*
sin(1/2*b^2*Pi*x^2)/b^4/Pi^2-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*sin(
b^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6598, 6590, 6588, 3432, 3473, 30, 3467, 3466}

$$\int x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = \frac{531\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}\pi^4b^8} - \frac{4x^3}{\pi^3b^5} - \frac{x^6\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{48\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8} - \frac{147x\sin\left(\pi b^2x^2\right)}{16\pi^4b^7} + \frac{24x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{17x^3\cos\left(\pi b^2x^2\right)}{8\pi^3b^5} + \frac{6x^4\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^5\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} + \frac{x^7}{14\pi b}$$

```
[In] Int[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
 + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\
&\quad + \frac{6\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^2\pi} + \frac{\int x^6\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{6x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad - \frac{24\int x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} \\
&\quad - \frac{3\int x^4\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2} + \frac{\int x^6\,dx}{2b\pi} + \frac{\int x^6\cos\left(b^2\pi x^2\right)\,dx}{2b\pi}
\end{aligned}
$$

$$= \frac{x^7}{14b\pi} + \frac{3x^3\cos(b^2\pi x^2)}{2b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}$$
$$- \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{6x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$
$$+ \frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2} - \frac{48\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^6\pi^3}$$
$$- \frac{9\int x^2\cos(b^2\pi x^2)\,dx}{2b^5\pi^3} - \frac{24\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^5\pi^3} - \frac{5\int x^4\sin(b^2\pi x^2)\,dx}{4b^3\pi^2}$$

$$= \frac{x^7}{14b\pi} + \frac{17x^3\cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}$$
$$- \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{48\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$
$$+ \frac{6x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{9x\sin(b^2\pi x^2)}{4b^7\pi^4}$$
$$+ \frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2} + \frac{9\int\sin(b^2\pi x^2)\,dx}{4b^7\pi^4} + \frac{24\int\sin(b^2\pi x^2)\,dx}{b^7\pi^4}$$
$$- \frac{15\int x^2\cos(b^2\pi x^2)\,dx}{8b^5\pi^3} - \frac{12\int x^2\,dx}{b^5\pi^3} - \frac{12\int x^2\cos(b^2\pi x^2)\,dx}{b^5\pi^3}$$

$$= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}$$
$$- \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{9\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^8\pi^4} + \frac{12\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^8\pi^4}$$
$$- \frac{48\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{6x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$
$$- \frac{147x\sin(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2} + \frac{15\int\sin(b^2\pi x^2)\,dx}{16b^7\pi^4} + \frac{6\int\sin(b^2\pi x^2)\,dx}{b^7\pi^4}$$

$$= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}$$
$$- \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{51\,\text{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4}$$
$$+ \frac{15\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{b^8\pi^4} - \frac{48\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$
$$+ \frac{6x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{147x\sin(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int x^7 \,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{-896b^3\pi x^3 + 16b^7\pi^3x^7 + 476b^3\pi x^3\cos\left(b^2\pi x^2\right) + 3717\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) - 224\,\text{FresnelC}(bx)\left(b^2\pi x^2(-24\right.}{224b^8\pi}$$

```
[In] Integrate[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-896*b^3*Pi*x^3 + 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*
Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 224*FresnelC[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi
^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) -
2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.48

method	result
default	$\frac{\text{FresnelC}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}}{\pi}\right)}{b^7} - \frac{-\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos\left(b^{\ldots}\right.}{2}}{}}{}$

```
[In] int(x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(-1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi*
b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*F
resnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5
*x^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi
*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b
^2*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{16\,\pi^3b^8x^7 + 952\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 - 1372\,\pi b^4x^3 - 224\,(\pi^3b^7x^6 - 24\,\pi b^3x^2)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right) + 3717}{224\,\pi}$$

```
[In] integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/224*(16*pi^3*b^8*x^7 + 952*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 1372*pi*b^4
*x^3 - 224*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b
*x) + 3717*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 28*((4*pi^2
*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2) + 48*(pi^2*b^5*x^4 - 8*b)*fresnel
_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [F]

$$\int x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^7\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

```
[In] integrate(x**7*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^7\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^7*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.202 $\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} - \frac{15\operatorname{FresnelC}(bx)^2}{2b^7\pi^3} + \frac{5x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{11\sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] -15/4*x^2/b^5/Pi^3+1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+15*x*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-15/2*FresnelC(b*x)^2/b^7/Pi^3+5*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x
^2)/b^4/Pi^2-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {6598, 6590, 6576, 30, 3461, 2714, 3460, 3377, 2717, 3390}

$$\int x^6 \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{15\,\text{FresnelC}(bx)^2}{2\pi^3 b^7} - \frac{15x^2}{4\pi^3 b^5} - \frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{11\sin\left(\pi b^2 x^2\right)}{2\pi^4 b^7} + \frac{15x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{7x^2\cos\left(\pi b^2 x^2\right)}{4\pi^3 b^5} + \frac{5x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{x^4\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^6}{12\pi b}$$

```
[In] Int[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
```

```
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\ &\quad+\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^2\pi}+\frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\end{aligned}$$

$$= -\frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} + \frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15 \int x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^4\pi^2} - \frac{5 \int x^3 \sin\left(b^2\pi x^2\right) dx}{2b^3\pi^2} + \frac{\text{Subst}\left(\int x^2 \cos^2\left(\frac{1}{2}b^2\pi x\right) dx, x, x^2\right)}{2b\pi}$$

$$= \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} + \frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15 \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx}{b^6\pi^3} - \frac{15 \int x \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{b^5\pi^3} - \frac{5\text{Subst}\left(\int x \sin\left(b^2\pi x\right) dx, x, x^2\right)}{4b^3\pi^2} + \frac{\text{Subst}\left(\int x^2\, dx, x, x^2\right)}{4b\pi} + \frac{\text{Subst}\left(\int x^2 \cos\left(b^2\pi x\right) dx, x, x^2\right)}{4b\pi}$$

$$= \frac{x^6}{12b\pi} + \frac{5x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} + \frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{15\text{Subst}(\int x\, dx, x, \text{FresnelC}(bx))}{b^7\pi^3} - \frac{5\text{Subst}\left(\int \cos\left(b^2\pi x\right) dx, x, x^2\right)}{4b^5\pi^3} - \frac{15\text{Subst}\left(\int \cos^2\left(\frac{1}{2}b^2\pi x\right) dx, x, x^2\right)}{2b^5\pi^3} - \frac{\text{Subst}\left(\int x \sin\left(b^2\pi x\right) dx, x, x^2\right)}{2b^3\pi^2}$$

$$= -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{15 \text{FresnelC}(bx)^2}{2b^7\pi^3} + \frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{5 \sin\left(b^2\pi x^2\right)}{b^7\pi^4} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\text{Subst}\left(\int \cos\left(b^2\pi x\right) dx, x, x^2\right)}{2b^5\pi^3}$$

$$= -\frac{15x^2}{4b^5\pi^3} + \frac{x^6}{12b\pi} + \frac{7x^2 \cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{15x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{15 \text{FresnelC}(bx)^2}{2b^7\pi^3} + \frac{5x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{11 \sin\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^4 \sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00

$$\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} - \frac{15\operatorname{FresnelC}(bx)^2}{2b^7\pi^3} + \frac{5x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{11\sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[In] Integrate[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Maple [F]

$$\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

```
[In] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.76

$$\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\pi^3b^6x^6+42\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-66\,\pi b^2x^2-12\left(\pi^3b^5x^5-15\,\pi bx\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)-90\,\pi\,\mathrm{C}\left(bx\right)^2+}{12\,\pi^4b^7}$$

```
[In] integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] 1/12*(pi^3*b^6*x^6 + 42*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 66*pi*b^2*x^2 -
12*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 90*pi*
fresnel_cos(b*x)^2 + 6*(10*pi^2*b^3*x^3*fresnel_cos(b*x) + (pi^2*b^4*x^4 -
22)*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \begin{cases} \frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right)\cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{11x^2 \sin^2\left(\frac{\pi b^2}{2}\right.}{2\pi^3 b^5} \\ 0 \end{cases}$$

```
[In] integrate(x**6*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) + x**6*cos(pi*b**2*x**2/2)
**2/(12*pi*b) - x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + x**4*sin
(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*sin(pi*b**2*x*
*2/2)*fresnelc(b*x)/(pi**2*b**4) - 11*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**3*
b**5) - 2*x**2*cos(pi*b**2*x**2/2)**2/(pi**3*b**5) + 15*x*cos(pi*b**2*x**2/
2)*fresnelc(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/
(pi**4*b**7) - 15*fresnelc(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Maxima [F]

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{FresnelC}(b\,x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

```
[In] int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.203 $\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\begin{aligned}\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = & -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos(b^2\pi x^2)}{8b^5\pi^3} \\ & + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} \\ & - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} \\ & - \frac{43\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} \\ & + \frac{4x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^3\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

```
[Out] -4*x/b^5/Pi^3+1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+8*cos(1/2*b^2*P
i*x^2)*FresnelC(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+
4*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x^3*sin(b^2*Pi*x^2)/b^
3/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

= {6598, 6590, 6596, 3439, 3433, 3466, 3473, 30, 3467}

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{43\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}\pi^3b^6} - \frac{4x}{\pi^3b^5} - \frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{8\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6} + \frac{11x\cos\left(\pi b^2x^2\right)}{8\pi^3b^5} + \frac{4x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^3\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} + \frac{x^5}{10\pi b}$$

```
[In] Int[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x)/(b^5*Pi^3) + x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) + (
8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*
FresnelC[b*x])/(b^2*Pi) - (43*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) +
 (4*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^3*Sin[b^2*Pi*x^2
])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\
&\quad+\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^2\pi}+\frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{4x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad-\frac{8\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} \\
&\quad-\frac{2\int x^2\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2}+\frac{\int x^4\,dx}{2b\pi}+\frac{\int x^4\cos\left(b^2\pi x^2\right)\,dx}{2b\pi}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{4x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^3\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \\
&\quad - \frac{\int\cos\left(b^2\pi x^2\right)\,dx}{b^5\pi^3} - \frac{8\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^5\pi^3} - \frac{3\int x^2\sin\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2} \\
&= \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{\text{FresnelC}\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^6\pi^3} \\
&\quad + \frac{4x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^3\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} \\
&\quad - \frac{3\int\cos\left(b^2\pi x^2\right)\,dx}{8b^5\pi^3} - \frac{8\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{b^5\pi^3} \\
&= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{11\,\text{FresnelC}\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} \\
&\quad + \frac{4x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^3\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{4\int\cos\left(b^2\pi x^2\right)\,dx}{b^5\pi^3} \\
&= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} \\
&\quad - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{11\,\text{FresnelC}\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} \\
&\quad - \frac{2\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right)}{b^6\pi^3} + \frac{4x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^3\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

$$
\int x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx
$$

$$
= \frac{-215\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right) - 80\,\text{FresnelC}(bx)\left(\left(-8+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 4b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx(}{80b^6\pi^3}
$$

```
[In] Integrate[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 80*FresnelC[b*x]*((-8 + b^4*Pi^2*x^4)
*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(-160 + 4*
b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6
*Pi^3)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.27

method	result
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(-\frac{b^{4}x^{4}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{\frac{4b^{2}x^{2}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi^{2}}}{\pi}\right)}{b^{5}}-\frac{-\frac{\frac{1}{5}b^{5}x^{5}\pi^{2}-8bx}{2\pi^{3}}+\frac{-\frac{bx\cos\left(b^{2}\pi x^{2}\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^{2}}-\frac{\pi b^{3}x \cdots}{\cdots}}{\cdots}}{b}$

```
[In] int(x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(-1/2/Pi^3*(1/5*b^5*x^
5*Pi^2-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b
*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x
*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int x^{5}\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)\,dx$$
$$=\frac{8\,\pi^{2}b^{6}x^{5}+220\,b^{2}x\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)^{2}-430\,b^{2}x-80\left(\pi^{2}b^{5}x^{4}-8\,b\right)\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{C}\left(bx\right)-215\,\sqrt{2}\sqrt{b^{2}}\,\mathrm{C}\left(\sqrt{2}\sqrt{\cdots}\right.\cdots}{80\,\pi^{3}b^{7}}$$

```
[In] integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/80*(8*pi^2*b^6*x^5 + 220*b^2*x*cos(1/2*pi*b^2*x^2)^2 - 430*b^2*x - 80*(pi
^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 215*sqrt(2)*sqrt(b
^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(pi*b^4*x^3*cos(1/2*pi*b^2*x^2) +
 8*pi*b^3*x^2*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]

$$\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx) \, dx$$

```
[In] integrate(x**5*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^5*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^5*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{FresnelC}(b\,x) \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.204 $\int x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = \frac{x^4}{8b\pi}+\frac{\cos(b^2\pi x^2)}{b^5\pi^3}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x^2\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] 1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3-x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)
/b^2/Pi-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2-3/8*I*x^2*hypergeom([1, 1]
,[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2+3/8*I*x^2*hypergeom([1, 1],[3/2, 2],1
/2*I*b^2*Pi*x^2)/b^3/Pi^2+3*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/
4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

= {6598, 6590, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$\int x^4 \,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2\pi^2b^5} - \frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\cos\left(\pi b^2x^2\right)}{\pi^3b^5} + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x^2\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} + \frac{x^4}{8\pi b}$$

```
[In] Int[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^3*Pi^2)
 + (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b
^3*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^
2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}\\&\quad+\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^2\pi}+\frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\end{aligned}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2} - \frac{3\int x\sin\left(b^2\pi x^2\right)\,dx}{2b^3\pi^2} + \frac{\text{Subst}\left(\int x\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{2b\pi}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2} + \frac{\text{Subst}\left(\int x\,dx, x, x^2\right)}{4b\pi} + \frac{\text{Subst}\left(\int x\cos\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b\pi}$$

$$= \frac{x^4}{8b\pi} + \frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^2\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\text{Subst}\left(\int \sin\left(b^2\pi x\right)\,dx, x, x^2\right)}{4b^3\pi^2}$$

$$= \frac{x^4}{8b\pi} + \frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^2\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [F]

$$\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

```
[In] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

```
[In] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4\, \mathrm{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

```
[In] integrate(x**4*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**4*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4\, \mathrm{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.205 $\int x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{x^3}{6b\pi} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{5 \text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x \sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] 1/6*x^3/b/Pi-x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+2*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b*x
*2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6598, 6588, 3432, 3473, 30, 3467}

$$\int x^3 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{5 \text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}\pi^2 b^4} - \frac{x^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4} + \frac{x \sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^3}{6\pi b}$$

```
[In] Int[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^3/(6*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (5*Fresne
lS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2
])/(b^4*Pi^2) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\
&\quad+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^2\pi}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&\quad-\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2}+\frac{\int x^2\,dx}{2b\pi}+\frac{\int x^2\cos\left(b^2\pi x^2\right)\,dx}{2b\pi}
\end{aligned}$$

$$= \frac{x^3}{6b\pi} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b^4\pi^2} + \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{4b^3\pi^2}$$

$$= \frac{x^3}{6b\pi} - \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{b^2\pi} - \frac{5\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3 \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= \frac{4b^3\pi x^3 - 15\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) - 24\,\text{FresnelC}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 6bx\sin\left(b^2\pi x^2\right.}{24b^4\pi^2}$$

```
[In] Integrate[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 24*FresnelC[b*x]*(b^2*Pi
*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])/
(24*b^4*Pi^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\text{FresnelC}(bx)\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{2\pi^2}-\frac{b^3x^3}{6\pi}-\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	120

```
[In] int(x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))-1/6/Pi*b^3*x^3-1/2/Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \frac{4\,\pi b^4 x^3 - 24\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{C}\left(bx\right) - 15\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right) + 12\,\left(b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,b\,\mathrm{C}\left(bx\right)\right) \sin}{24\,\pi^2 b^5}$$

```
[In] integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*(4*pi*b^4*x^3 - 24*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 1
5*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 12*(b^2*x*cos(1/2*pi
*b^2*x^2) + 4*b*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^2*b^5)
```

Sympy [F]

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

```
[In] integrate(x**3*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3\, \mathrm{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

```
[In] integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^3*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

```
[In] int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.206 $\int x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{x^2}{4b\pi} - \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{\text{FresnelC}(bx)^2}{2b^3\pi} + \frac{\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] 1/4*x^2/b/Pi-x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/2*FresnelC(b*x)^2
/b^3/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6598, 6576, 30, 3461, 2714}

$$\int x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{FresnelC}(bx)^2}{2\pi b^3} - \frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\sin(\pi b^2x^2)}{4\pi^2b^3} + \frac{x^2}{4\pi b}$$

```
[In] Int[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\
&\quad+\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx}{b^2\pi}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{Subst}\left(\int x\,dx,x,\text{FresnelC}(bx)\right)}{b^3\pi} \\
&\quad+\frac{\text{Subst}\left(\int\cos^2\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)}{2b\pi} \\
&= \frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{FresnelC}(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{FresnelC}(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

[In] Integrate[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)

Maple [F]

$$\int x^2\,\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

[In] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\pi b^2x^2-4\,\pi bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)+2\,\pi\,\text{C}\left(bx\right)^2+2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^2b^3}$$

[In] integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] 1/4*(pi*b^2*x^2 - 4*pi*b*x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*pi*fresnel_cos(b*x)^2 + 2*cos(1/2*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^2*b^3)

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \begin{cases} \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{4\pi b} + \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{4\pi b} - \frac{x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)\cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{C^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) + x**2*cos(pi*b**2*x**2/2)*
*2/(4*pi*b) - x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + sin(pi*b**2*x
**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + fresnelc(b*x)**2/(2*pi*b**3), N
e(b, 0)), (0, True))
```

Maxima [F]

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

```
[In] integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int x^2 \operatorname{FresnelC}(b\,x) \, \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) \, dx$$

```
[In] int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.207 $\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{b^2\pi} + \frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}$$

```
[Out] 1/2*x/b/Pi-cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/4*FresnelC(b*x*2^(1/2
))/b^2/Pi*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6596, 3439, 3433}

$$\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{\operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2} + \frac{x}{2\pi b}$$

```
[In] Int[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x/(2*b*Pi) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[Sqrt[2
]*b*x]/(2*Sqrt[2]*b^2*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\,dx}{b\pi}\\ &= \frac{x}{2b\pi}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\int\cos\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= \frac{x}{2b\pi}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{2bx-4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)+\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4b^2\pi}$$

```
[In] Integrate[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] (2*b*x - 4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x] + Sqrt[2]*FresnelC[Sqrt[2]*b*x
])/(4*b^2*Pi)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{\text{FresnelC}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}+\frac{\frac{bx}{2}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$	52

```
[In] int(x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-FresnelC(b*x)/b/Pi*cos(1/2*b^2*Pi*x^2)+1/b/Pi*(1/2*b*x+1/4*2^(1/2)*Fresne
lC(b*x*2^(1/2))))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{2\, b^2 x - 4\, b \cos\left(\frac{1}{2}\, \pi b^2 x^2\right) \operatorname{C}(bx) + \sqrt{2}\sqrt{b^2}\, \operatorname{C}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\, \pi b^3}$$

```
[In] integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x - 4*b*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + sqrt(2)*sqrt(b^2)
*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]

$$\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int x \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x\, \operatorname{C}(bx) \sin\left(\frac{1}{2}\, \pi b^2 x^2\right) dx$$

```
[In] integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int x \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int x \,\text{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) \, dx$$

```
[In] integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx = \int x \,\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) \, dx$$

```
[In] int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.208 $\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],-1/
2*I*b^2*Pi*x^2)-1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6582}

$$\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}$$

```
[In] Int[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\\ &\quad - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F]

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

```
[In] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]

$$\int \text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Fricas [F]

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \mathrm{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Maxima [F]

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Giac [F]

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

```
[In] int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.209 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\sin \left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\mathrm{C}\left(bx\right) \sin \left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\mathrm{C}\left(bx\right) \sin \left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

3.210 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \frac{1}{2}b\pi\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

```
[Out] 1/2*b*Pi*FresnelC(b*x)^2+1/4*b*Si(b^2*Pi*x^2)-FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6600, 6576, 30, 3456}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x} + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right) + \frac{1}{2}\pi b\,\text{FresnelC}(bx)^2$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}\,dx \\
&\quad+\left(b^2\pi\right)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx \\
&= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+(b\pi)\text{Subst}\left(\int x\,dx,x,\text{FresnelC}(bx)\right) \\
&= \frac{1}{2}b\pi\,\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$
\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx=\frac{1}{2}b\pi\,\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)
$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Maple [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^2}dx$$

[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)

[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \frac{2\,\pi bx\,\text{C}\left(bx\right)^2 + bx\,\text{Si}\left(\pi b^2x^2\right) - 4\,\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,x}$$

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")

[Out] 1/4*(2*pi*b*x*fresnel_cos(b*x)^2 + b*x*sin_integral(pi*b^2*x^2) - 4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2))/x

Sympy [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^2}\,dx$$

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)

Maxima [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")

[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

Giac [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{C\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")

[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)

[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)

3.211 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \frac{b^2\pi\,\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{4x} + \frac{1}{2}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

```
[Out] -1/2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2*
Pi*FresnelC(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx\\&\quad+\frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx\\&= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}\\&\quad+\frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx+\frac{1}{2}\left(b^3\pi\right)\int\cos\left(b^2\pi x^2\right)\,dx\\&= \frac{b^2\pi\,\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}\\&\quad-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx=\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^3}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx=\int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx=\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^3}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx=\int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

3.212 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{12x^2} + \frac{1}{3}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}, x\right)$$

```
[Out] 1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/12*b
*sin(b^2*Pi*x^2)/x^2+1/3*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b\int\frac{\sin(b^2\pi x^2)}{x^3}\,dx \\
&\quad + \frac{1}{3}(b^2\pi)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx \\
&= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{12}b\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) \\
&\quad + \frac{1}{3}(b^2\pi)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx \\
&= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{b\sin(b^2\pi x^2)}{12x^2} \\
&\quad + \frac{1}{3}(b^2\pi)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx \\
&\quad + \frac{1}{12}(b^3\pi)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right) \\
&= \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} \\
&\quad - \frac{b\sin(b^2\pi x^2)}{12x^2} + \frac{1}{3}(b^2\pi)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^4}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^4}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

3.213 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = -\frac{b^3\pi}{16x} - \frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^2} - \frac{7b^4\pi^2\,\text{FresnelS}\left(\sqrt{2}bx\right)}{24\sqrt{2}} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{b\sin\left(b^2\pi x^2\right)}{24x^3} - \frac{1}{8}b^4\pi^2\,\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/x^2-1/4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/24*b*sin(b^2
*Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unint
egrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -1/16*(b^3*Pi)/x - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (b^2*Pi*Cos[(b^2*Pi*
x^2)/2]*FresnelC[b*x])/(8*x^2) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqr
t[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (b*Sin[b^2*Pi*x^2])/(
24*x^3) - (b^4*Pi^2*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{8}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad+\frac{1}{4}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx \\
&= -\frac{b^3\pi}{16x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^2}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} \\
&\quad-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad+\frac{1}{12}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{8}\left(b^4\pi^2\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^2} \\
&\quad-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3} \\
&\quad-\frac{1}{8}\left(b^4\pi^2\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx \\
&\quad-\frac{1}{8}\left(b^5\pi^2\right)\int\sin\left(b^2\pi x^2\right)\,dx-\frac{1}{6}\left(b^5\pi^2\right)\int\sin\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^2}-\frac{7b^4\pi^2\,\text{FresnelS}\left(\sqrt{2}bx\right)}{24\sqrt{2}} \\
&\quad-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}-\frac{1}{8}\left(b^4\pi^2\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx=\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\mathrm{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{5}}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)}{x^{5}}\,dx=\int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{5}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Sympy [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)}{x^{5}}\,dx=\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C(bx)}{x^{5}}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^5}\,dx$$

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")

[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^5}\,dx$$

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")

[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)

Mupad [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelC}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)

[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)

3.214 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = &-\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} \\ &- \frac{1}{30}b^5\pi^3\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} \\ &+ \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} \\ &- \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

```
[Out] -1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/15*b^2*Pi*cos(1/2*b^2*Pi
*x^2)*FresnelC(b*x)/x^3-1/30*b^5*Pi^3*FresnelC(b*x)^2-7/120*b^5*Pi^2*Si(b^2
*Pi*x^2)-1/5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5+1/15*b^4*Pi^2*FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x-1/40*b*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

$= \{6600, 6592, 6576, 30, 3456, 3461, 3378, 3380, 3460\}$

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{1}{30}\pi^3 b^5 \,\text{FresnelC}(bx)^2 - \frac{\pi b^3}{60x^2} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} - \frac{\pi b^2 \,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3} - \frac{b\sin\left(\pi b^2 x^2\right)}{40x^4} - \frac{7}{120}\pi^2 b^5 \text{Si}\left(b^2\pi x^2\right) + \frac{\pi^2 b^4 \,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x} - \frac{\pi b^3\cos\left(\pi b^2 x^2\right)}{24x^2}$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/60*(b^3*Pi)/x^2 - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx\\&\quad+\frac{1}{5}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx\end{aligned}$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{20}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) + \frac{1}{30}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{15}\left(b^4\pi^2\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} + \frac{1}{60}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) + \frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\,dx, x, x^2\right) - \frac{1}{30}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx - \frac{1}{15}\left(b^6\pi^3\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{1}{60}b^5\pi^2\text{Si}\left(b^2\pi x^2\right) - \frac{1}{60}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) - \frac{1}{40}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x}\,dx, x, x^2\right) - \frac{1}{15}\left(b^5\pi^3\right)\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right)$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} - \frac{1}{30}b^5\pi^3\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{b^3\pi}{60x^2}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3}-\frac{1}{30}b^5\pi^3\,\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x}-\frac{b\sin\left(b^2\pi x^2\right)}{40x^4}-\frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/60*(b^3*Pi)/x^2 - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Maple [F]

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^6}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{4\,\pi^3b^5x^5\,\text{C}\left(bx\right)^2+7\,\pi^2b^5x^5\,\text{Si}\left(\pi b^2x^2\right)+10\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-3\,\pi b^3x^3+8\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)\cdots}{120\,x^5}$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(4*pi^3*b^5*x^5*fresnel_cos(b*x)^2 + 7*pi^2*b^5*x^5*sin_integral(pi*
b^2*x^2) + 10*pi*b^3*x^3*cos(1/2*pi*b^2*x^2)^2 - 3*pi*b^3*x^3 + 8*pi*b^2*x^
2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*(3*b*x*cos(1/2*pi*b^2*x^2) - 4*(
pi^2*b^4*x^4 - 3)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/x^5
```

Sympy [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^6}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

Maxima [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

3.215 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = & -\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} \\ & -\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4} \\ & -\frac{7b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right) \\ & -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} \\ & +\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} \\ & -\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{1440x} \\ & -\frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)\end{aligned}$$

```
[Out] -1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/24*b^2*Pi*cos(1/2*b^2
*Pi*x^2)*FresnelC(b*x)/x^4-1/6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b
^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/60*b*sin(b^2*Pi*x^2)/x^5+67
/1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*2^(
1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -1/144*(b^3*Pi)/x^3 - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (b^2*Pi*Cos[(
b^2*Pi*x^2)/2]*FresnelC[b*x])/(24*x^4) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])
/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(48*x^2) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])
/(1440*x) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])
/48
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad +\frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx \\
&= -\frac{b^3\pi}{144x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} \\
&\quad -\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}+\frac{1}{48}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{30}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad -\frac{1}{24}\left(b^4\pi^2\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= -\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4} \\
&\quad -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} \\
&\quad -\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}-\frac{1}{96}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{72}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx \\
&\quad -\frac{1}{45}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{48}\left(b^6\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4} \\
&\quad - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} \\
&\quad - \frac{b\sin(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{1440x} \\
&\quad - \frac{1}{48}\left(b^6\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx - \frac{1}{48}\left(b^7\pi^3\right)\int\cos\left(b^2\pi x^2\right)\,dx \\
&\quad - \frac{1}{36}\left(b^7\pi^3\right)\int\cos\left(b^2\pi x^2\right)\,dx - \frac{1}{45}\left(2b^7\pi^3\right)\int\cos\left(b^2\pi x^2\right)\,dx \\
&= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4} \\
&\quad - \frac{7b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right) \\
&\quad - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2} \\
&\quad - \frac{b\sin(b^2\pi x^2)}{60x^5} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{1440x} - \frac{1}{48}\left(b^6\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^7}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Sympy [N/A]

Not integrable

Time = 11.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^7}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{C\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^7}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

3.216 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = &-\frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{84}b^7\pi^3\,\text{CosIntegral}\left(b^2\pi x^2\right)\\ &-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5}\\ &-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}\\ &+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3}\\ &-\frac{b\sin\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\sin\left(b^2\pi x^2\right)}{84x^2}\\ &-\frac{1}{105}b^6\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)\end{aligned}$$

```
[Out] -1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)
/x^4-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/7*FresnelC(b*x)*si
n(1/2*b^2*Pi*x^2)/x^7+1/105*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-
1/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2-1/105*b^6*Pi^3
*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} \, dx = \int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} \, dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/280*(b^3*Pi)/x^4 - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInt
egral[b^2*Pi*x^2])/84 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(35*x^5)
 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*FresnelC[b*x]*Si
n[(b^2*Pi*x^2)/2])/(105*x^3) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin
[b^2*Pi*x^2])/(84*x^2) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/x^2, x])/105
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx \\
&\quad + \frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} \\
&\quad + \frac{1}{28}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^4}\,dx, x, x^2\right) + \frac{1}{70}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx \\
&\quad - \frac{1}{35}\left(b^4\pi^2\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx \\
&= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5} \\
&\quad - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} \\
&\quad - \frac{b\sin\left(b^2\pi x^2\right)}{84x^6} + \frac{1}{140}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&\quad + \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{210}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx \\
&\quad - \frac{1}{105}\left(b^6\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx
\end{aligned}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos(b^2\pi x^2)}{105x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b\sin(b^2\pi x^2)}{84x^6} - \frac{1}{420}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) - \frac{1}{280}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) - \frac{1}{168}(b^5\pi^2)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^2}\,dx, x, x^2\right) - \frac{1}{105}(b^6\pi^3)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos(b^2\pi x^2)}{105x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b\sin(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2\sin(b^2\pi x^2)}{84x^2} - \frac{1}{105}(b^6\pi^3)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx - \frac{1}{420}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{280}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right) - \frac{1}{168}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x}\,dx, x, x^2\right)$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos(b^2\pi x^2)}{105x^4} - \frac{1}{84}b^7\pi^3\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b\sin(b^2\pi x^2)}{84x^6} + \frac{b^5\pi^2\sin(b^2\pi x^2)}{84x^2} - \frac{1}{105}(b^6\pi^3)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^8}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Sympy [N/A]

Not integrable

Time = 20.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^8}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Mupad [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

3.217 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = &-\frac{b^3\pi}{480x^5}+\frac{b^7\pi^3}{768x}-\frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5}\\ &+\frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6}\\ &+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{384x^2}\\ &+\frac{853b^8\pi^4\,\text{FresnelS}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}\\ &+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4}\\ &-\frac{b\sin\left(b^2\pi x^2\right)}{112x^7}+\frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3}\\ &+\frac{1}{384}b^8\pi^4\,\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

```
[Out] -1/480*b^3*Pi/x^5+1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/8
0640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*
x)/x^6+1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/8*FresnelC(b*
x)*sin(1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)
/x^4-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3+853
/80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(F
resnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -1/480*(b^3*Pi)/x^5 + (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(336
0*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (b^2*Pi*Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/(48*x^6) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(384*x^2) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(192*x^4) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*
Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/x, x])/384
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}+\frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx \\
&\quad +\frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx \\
&= -\frac{b^3\pi}{480x^5}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} \\
&\quad -\frac{b\sin\left(b^2\pi x^2\right)}{112x^7}+\frac{1}{96}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx+\frac{1}{56}\left(b^3\pi\right)\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx \\
&\quad -\frac{1}{48}\left(b^4\pi^2\right)\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx \\
&= -\frac{b^3\pi}{480x^5}-\frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6} \\
&\quad -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} \\
&\quad -\frac{b\sin\left(b^2\pi x^2\right)}{112x^7}-\frac{1}{384}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{240}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx \\
&\quad -\frac{1}{140}\left(b^5\pi^2\right)\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{192}\left(b^6\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx
\end{aligned}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6}$$
$$+ \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{384x^2} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}$$
$$+ \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7}$$
$$+ \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} - \frac{1}{768}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx$$
$$- \frac{1}{576}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx - \frac{1}{360}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx$$
$$- \frac{1}{210}\left(b^7\pi^3\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x}$$
$$- \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{384x^2}$$
$$- \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7}$$
$$+ \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$
$$+ \frac{1}{384}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx + \frac{1}{288}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx$$
$$+ \frac{1}{180}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx + \frac{1}{105}\left(b^9\pi^4\right)\int\sin\left(b^2\pi x^2\right)\,dx$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x}$$
$$- \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{384x^2}$$
$$+ \frac{853b^8\pi^4\,\text{FresnelS}\left(\sqrt{2}bx\right)}{40320\sqrt{2}} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}$$
$$+ \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7}$$
$$+ \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} + \frac{1}{384}\left(b^8\pi^4\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^9}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Sympy [N/A]

Not integrable

Time = 36.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^9}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

3.218 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 278

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx &= -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}\\ &+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}\\ &+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3}\\ &+\frac{b^9\pi^5\,\text{FresnelC}(bx)^2}{1890}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}\\ &+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}\\ &-\frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x}-\frac{b\sin\left(b^2\pi x^2\right)}{144x^8}\\ &+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{30240x^4}+\frac{83b^9\pi^4\text{Si}\left(b^2\pi x^2\right)}{30240}\end{aligned}$$

```
[Out] -1/756*b^3*Pi/x^6+1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/
2016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/63*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(
b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/1890*b^9*Pi
^5*FresnelC(b*x)^2+83/30240*b^9*Pi^4*Si(b^2*Pi*x^2)-1/9*FresnelC(b*x)*sin(1
/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/9
45*b^8*Pi^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x-1/144*b*sin(b^2*Pi*x^2)/x^8
+67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6600, 6592, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \frac{\pi^5 b^9\,\text{FresnelC}(bx)^2}{1890}+\frac{\pi^3 b^7}{3780x^2}-\frac{\pi b^3}{756x^6}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{\pi b^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\sin\left(\pi b^2x^2\right)}{144x^8}+\frac{83\pi^4 b^9\text{Si}(b^2\pi x^2)}{30240}-\frac{\pi^4 b^8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{5\pi^3 b^7\cos\left(\pi b^2x^2\right)}{2016x^2}+\frac{\pi^3 b^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3}+\frac{67\pi^2 b^5\sin\left(\pi b^2x^2\right)}{30240x^4}+\frac{\pi^2 b^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{11\pi b^3\cos\left(\pi b^2x^2\right)}{3024x^6}$$

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -1/756*(b^3*Pi)/x^6 + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{1}{18}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx\\
&\quad+\frac{1}{9}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx\\
&= -\frac{b^3\pi}{756x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}\\
&\quad+\frac{1}{36}b\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^5}\,dx,x,x^2\right)+\frac{1}{126}\left(b^3\pi\right)\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx\\
&\quad-\frac{1}{63}\left(b^4\pi^2\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b^3\pi}{756x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}\\
&\quad-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}\\
&\quad-\frac{b\sin\left(b^2\pi x^2\right)}{144x^8}+\frac{1}{252}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\
&\quad+\frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int\frac{\cos\left(b^2\pi x\right)}{x^4}\,dx,x,x^2\right)-\frac{1}{630}\left(b^5\pi^2\right)\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx\\
&\quad-\frac{1}{315}\left(b^6\pi^3\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx\\
&= -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}\\
&\quad+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}\\
&\quad+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}-\frac{b\sin\left(b^2\pi x^2\right)}{144x^8}\\
&\quad-\frac{\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x^3}\,dx,x,x^2\right)}{1260}-\frac{1}{756}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&\quad-\frac{1}{432}\left(b^5\pi^2\right)\text{Subst}\left(\int\frac{\sin\left(b^2\pi x\right)}{x^3}\,dx,x,x^2\right)-\frac{\left(b^7\pi^3\right)\int\frac{\cos(b^2\pi x^2)}{x^3}\,dx}{1890}\\
&\quad+\frac{1}{945}\left(b^8\pi^4\right)\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx
\end{aligned}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} - \frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} - \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{3780} - \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{2520} - \frac{(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right)}{1512} - \frac{1}{864}(b^7\pi^3)\,\text{Subst}\left(\int\frac{\cos(b^2\pi x)}{x^2}\,dx, x, x^2\right) + \frac{(b^9\pi^4)\int\frac{\sin(b^2\pi x^2)}{x}\,dx}{1890} + \frac{1}{945}(b^{10}\pi^5)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} - \frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} + \frac{b^9\pi^4\text{Si}(b^2\pi x^2)}{3780} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{3780} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{2520} + \frac{(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right)}{1512} + \frac{1}{864}(b^9\pi^4)\,\text{Subst}\left(\int\frac{\sin(b^2\pi x)}{x}\,dx, x, x^2\right) + \frac{1}{945}(b^9\pi^5)\,\text{Subst}\left(\int x\,dx, x, \text{FresnelC}(bx)\right)$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} + \frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3} + \frac{b^9\pi^5\,\text{FresnelC}(bx)^2}{1890} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} - \frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x} - \frac{b\sin(b^2\pi x^2)}{144x^8} + \frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4} + \frac{83b^9\pi^4\text{Si}(b^2\pi x^2)}{30240}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6} +\frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7} +\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3} +\frac{b^9\pi^5\,\text{FresnelC}(bx)^2}{1890}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} +\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} -\frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x}-\frac{b\sin(b^2\pi x^2)}{144x^8} +\frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4}+\frac{83b^9\pi^4\text{Si}(b^2\pi x^2)}{30240}$$

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -1/756*(b^3*Pi)/x^6 + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Maple [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^{10}}dx$$

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.73

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$
$$= \frac{16\,\pi^5 b^9 x^9\,\mathrm{C}\left(bx\right)^2 + 83\,\pi^4 b^9 x^9\,\mathrm{Si}\left(\pi b^2 x^2\right) - 67\,\pi^3 b^7 x^7 + 70\,\pi b^3 x^3 + 10\left(15\,\pi^3 b^7 x^7 - 22\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2}{}$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/30240*(16*pi^5*b^9*x^9*fresnel_cos(b*x)^2 + 83*pi^4*b^9*x^9*sin_integral(
pi*b^2*x^2) - 67*pi^3*b^7*x^7 + 70*pi*b^3*x^3 + 10*(15*pi^3*b^7*x^7 - 22*pi
*b^3*x^3)*cos(1/2*pi*b^2*x^2)^2 + 32*(pi^3*b^6*x^6 - 15*pi*b^2*x^2)*cos(1/2
*pi*b^2*x^2)*fresnel_cos(b*x) + 2*((67*pi^2*b^5*x^5 - 210*b*x)*cos(1/2*pi*b
^2*x^2) - 16*(pi^4*b^8*x^8 - 3*pi^2*b^4*x^4 + 105)*fresnel_cos(b*x))*sin(1/
2*pi*b^2*x^2))/x^9
```

Sympy [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^{10}}\,dx$$

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

Maxima [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Giac [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```