Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

0-Independent-test-suites/10-Timofeev-Problems

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [705]. This is test number [10].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (705)	0.00 (0)
Rubi	99.86 (704)	0.14 (1)
Fricas	93.90 (662)	6.10 (43)
Maple	93.05 (656)	6.95 (49)
Giac	83.69 (590)	16.31 (115)
Maxima	80.14 (565)	19.86 (140)
Mupad	76.88 (542)	23.12 (163)
Sympy	65.25 (460)	34.75 (245)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

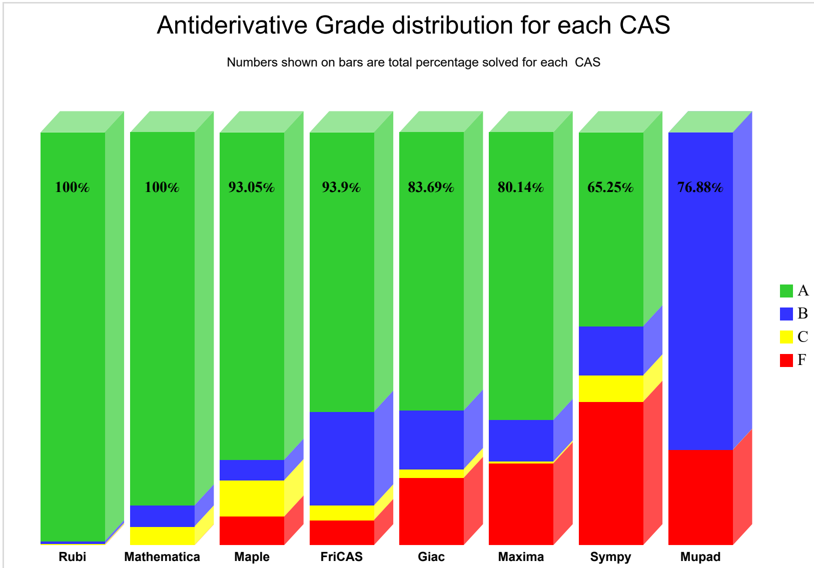
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

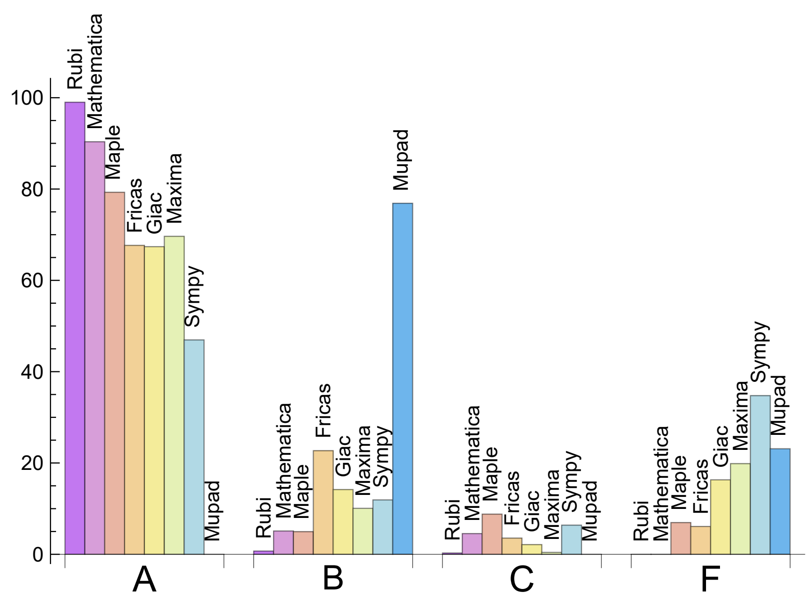
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.865	0.709	0.284	0.142
Mathematica	90.355	5.106	4.539	0.000
Maple	79.291	4.965	8.794	6.950
Maxima	69.645	10.071	0.426	19.858
Fricas	67.660	22.695	3.546	6.099
Giac	67.376	14.184	2.128	16.312
Sympy	46.950	11.915	6.383	34.752
Mupad	0.000	76.879	0.000	23.121

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Fricas	43	79.07	11.63	9.30
Maple	49	100.00	0.00	0.00
Giac	115	94.78	4.35	0.87
Maxima	140	87.86	2.86	9.29
Mupad	163	0.00	100.00	0.00
Sympy	245	75.51	22.86	1.63

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Rubi	0.25
Giac	0.30
Mupad	0.33
Mathematica	0.35
Fricas	0.41
Maple	0.91
Sympy	1.76

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	42.33	1.06	28.00	0.85
Mathematica	51.15	1.12	37.00	1.00
Rubi	52.37	1.09	40.00	1.00
Maxima	58.40	1.48	33.00	0.91
Giac	61.68	1.44	35.00	0.93
Sympy	120.98	2.49	36.00	1.04
Fricas	234.57	2.44	40.50	1.12
Maple	267.75	2.02	30.00	0.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

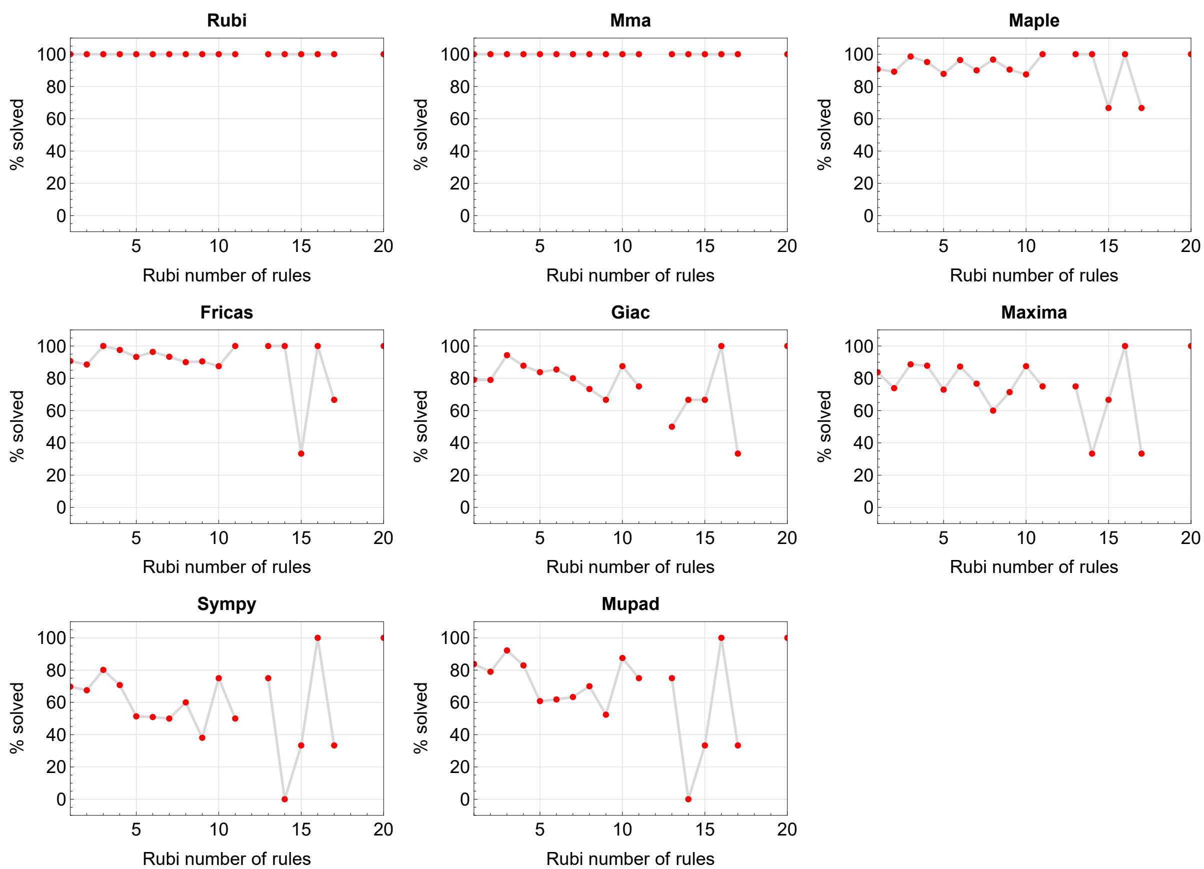


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

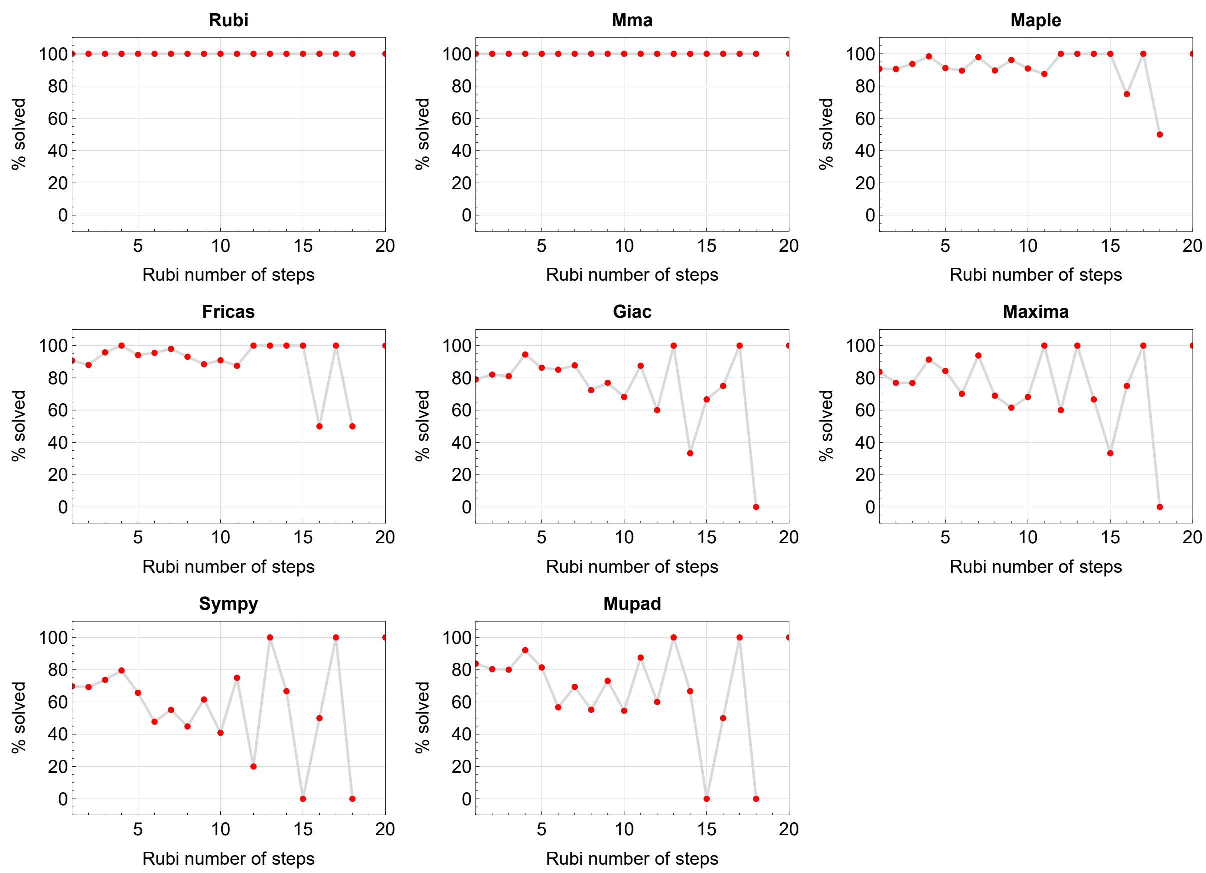


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

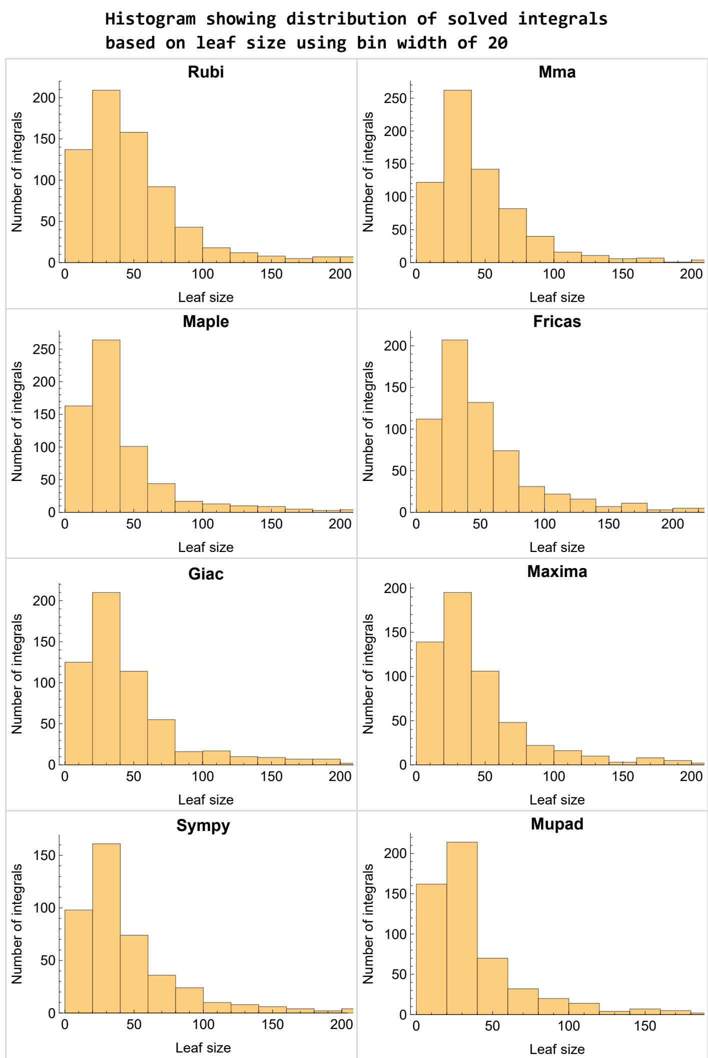


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

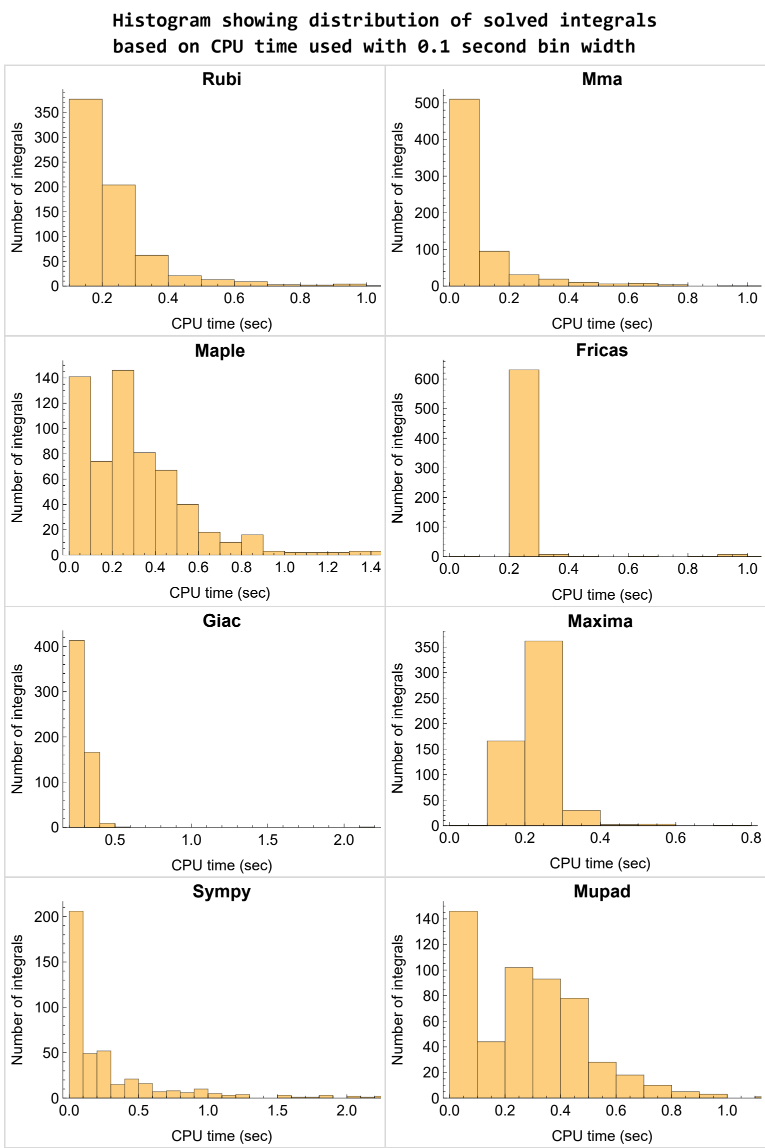


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

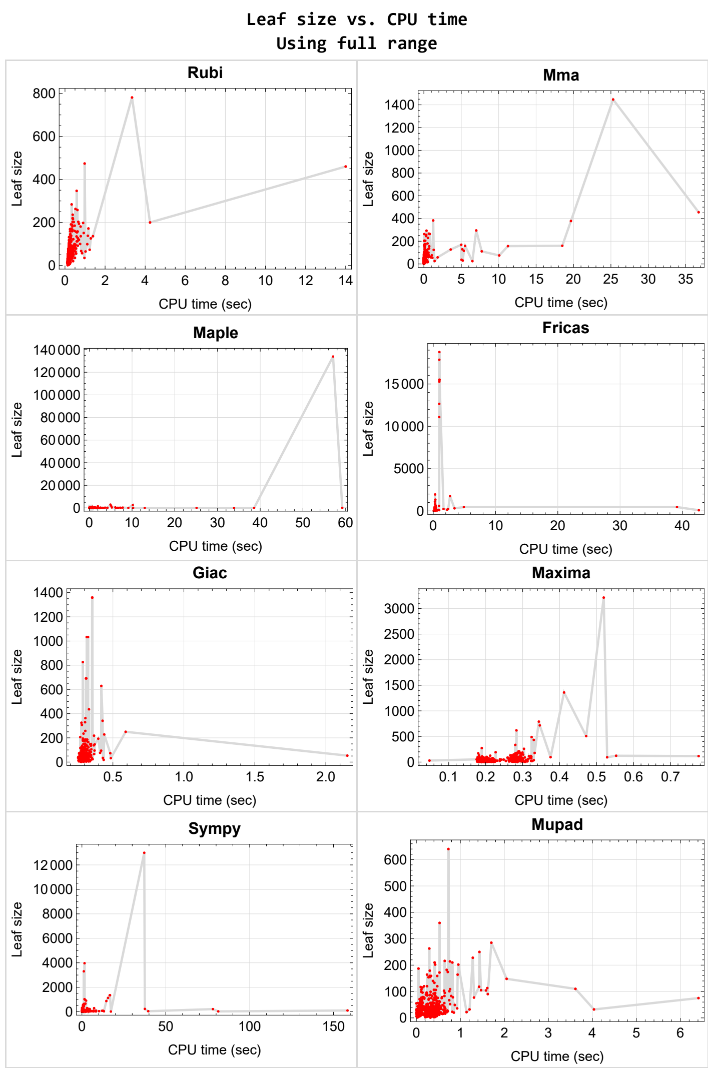


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {47, 209, 210, 212, 213, 221, 298, 306, 396, 398, 411, 416, 449, 497, 499, 501, 533, 592, 684}

Mathematica {417, 426, 446}

Maple {217, 294, 298, 313, 319, 413, 416, 417, 643, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 704}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

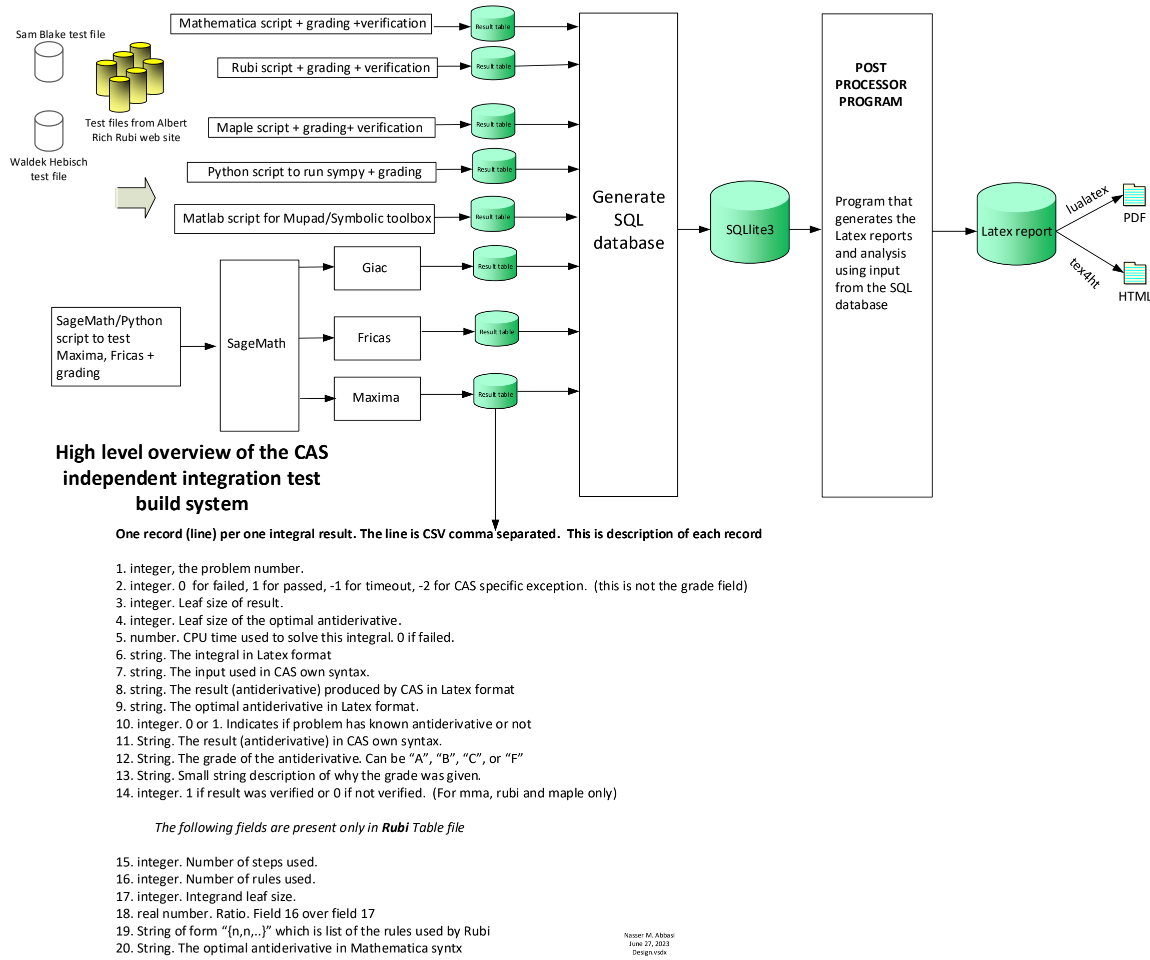
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53,
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B grade { 154, 335, 371, 417, 695 }

C grade { 579, 604 }

F normal fail { 222 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 240, 241, 242, 243, 244, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 438, 441, 442, 443, 447, 449, 450, 451, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 574, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

B grade { 4, 41, 52, 53, 56, 76, 99, 236, 237, 249, 254, 311, 322, 323, 338, 357, 361, 384, 389, 439, 444, 445, 456, 488, 553, 554, 555, 557, 559, 573, 575, 579, 592, 622, 623, 689 }

C grade { 37, 39, 50, 113, 193, 198, 222, 245, 246, 247, 248, 312, 328, 343, 365, 382, 399, 401, 416, 417, 424, 426, 434, 440, 446, 448, 452, 677, 678, 703, 704, 705 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 218, 219, 220, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 299, 300, 301, 307, 308, 309, 310, 311, 312, 314, 315, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 412, 419, 420, 421, 422, 425, 426, 428, 430, 439, 440, 441, 448, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 663, 666, 667, 668, 669, 670, 671, 673, 675, 676, 677, 678, 679, 680, 681, 682, 695, 696, 697, 698, 700, 701, 702 }

B grade { 13, 246, 247, 248, 253, 260, 367, 374, 383, 413, 416, 423, 424, 427, 429, 431, 432, 433, 434, 435, 436, 437, 438, 456, 457, 586, 587, 588, 592, 595, 661, 662, 665, 672, 674 }

C grade { 41, 119, 135, 136, 137, 138, 139, 142, 143, 144, 174, 177, 214, 217, 226, 228, 232, 249, 294, 295, 297, 298, 302, 303, 304, 305, 306, 313, 316, 319, 387, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 417, 492, 523, 589, 643, 658, 664, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 704 }

F normal fail { 67, 126, 133, 145, 193, 198, 221, 222, 328, 329, 352, 414, 415, 418, 442, 443, 444, 445, 446, 447, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 632, 699, 703, 705 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 227, 230, 231, 232, 233, 234, 238, 241, 243, 250, 251, 252, 258, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 312, 313, 316, 318, 320, 325, 326, 327, 330, 331, 332, 333, 334, 336, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 356, 359, 362, 364, 366, 368, 370, 373, 374, 375, 376, 378, 380, 381, 385, 386, 387, 395, 396, 398, 409, 412, 414, 415, 418, 419, 420, 422, 425, 426, 428, 436, 437, 439, 440, 441, 448, 450, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 583, 585, 590, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 705 }

B grade { 3, 4, 5, 19, 30, 31, 36, 37, 41, 53, 54, 55, 56, 57, 62, 63, 76, 99, 149, 159, 161, 165, 180, 187, 195, 196, 197, 202, 223, 224, 226, 228, 229, 235, 236, 237, 239, 240, 242, 244, 248, 249, 253, 254, 255, 256, 257, 259, 268, 269, 276, 281, 283, 284, 303, 311, 314, 319, 321, 322, 323, 324, 328, 335, 337, 338, 340, 342, 343, 353, 354, 355, 357, 358, 360, 361, 363, 365, 367, 369, 371, 372, 377, 379, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 400, 402, 403, 404, 405, 406, 407, 408, 410, 411, 423, 424, 427, 429, 430, 431, 432, 433, 434, 435, 438, 443, 447, 451, 452, 453, 456, 457, 490, 491, 505, 510, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 584, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 625, 635, 661, 662, 701, 702, 703, 704 }

C grade { 135, 136, 137, 138, 139, 142, 143, 144, 174, 177, 221, 222, 245, 246, 247, 298, 315, 317, 397, 399, 401, 413, 416, 421, 632 }

F normal fail { 126, 133, 145, 193, 198, 352, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698 }

F(-1) timedout fail { 442, 444, 445, 449, 455 }

F(-2) exception fail { 329, 417, 446, 543 }

2.1.5 Maxima

A grade { 2, 3, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 229, 235, 236, 237, 243, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 284, 285, 286, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 310, 312, 313, 316, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 370, 371, 372, 375, 376, 377, 378, 379, 380, 381, 382, 392, 396, 397, 398, 400, 401, 412, 414, 415, 418, 419, 420, 422, 424, 428, 439, 444, 445, 448, 450, 453, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 585, 590, 591, 598, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 685, 686, 687, 690, 692, 694, 697, 698, 700, 702, 704, 705 }

B grade { 1, 4, 5, 8, 13, 19, 31, 41, 62, 76, 81, 82, 86, 128, 159, 240, 241, 257, 311, 318, 322, 323, 367, 369, 373, 374, 384, 385, 386, 387, 388, 389, 393, 394, 399, 423, 425, 429, 430, 431, 433, 437, 449, 451, 456, 474, 482, 488, 489, 491, 492, 493, 577, 578, 579, 580, 581, 584, 586, 588, 589, 593, 594, 596, 597, 599, 601, 643, 691, 695, 696 }

C grade { 421, 426, 457 }

F normal fail { 126, 133, 145, 193, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 279, 281, 283, 287, 288, 289, 290, 291, 306, 307, 308, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 390, 391, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 432, 434, 435, 438, 440, 441, 442, 443, 447, 452, 454, 455, 473, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 592, 595, 637, 650, 657, 665, 668, 672, 674, 675, 683, 684, 688, 689, 693, 699, 701, 703 }

F(-1) timedout fail { 411, 427, 436, 446 }

F(-2) exception fail { 69, 149, 194, 195, 196, 197, 487, 494, 495, 583, 587, 603, 621 }

2.1.6 Giac

A grade { 2, 6, 7, 8, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 227, 229, 230, 231, 241, 250, 251, 252, 253, 260, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 282, 284, 285, 286, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 312, 313, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 390, 391, 392, 393, 395, 396, 397, 398, 400, 401, 419, 420, 422, 423, 424, 425, 428, 429, 430, 431, 433, 440, 441, 442, 443, 444, 445, 450, 451, 452, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 492, 493, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 574, 583, 584, 585, 587, 589, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 618, 619, 621, 622, 624, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 676, 677, 686, 687, 688, 691, 696, 697, 700, 701, 705 }

B grade { 1, 3, 4, 5, 9, 13, 19, 22, 52, 54, 55, 56, 58, 73, 99, 128, 197, 220, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 265, 266, 274, 279, 280, 281, 283, 287, 308, 311, 322, 323, 338, 342, 343, 353, 357, 365, 367, 374, 383, 384, 388, 389, 394, 438, 447, 449, 456, 474, 476, 487, 488, 489, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 586, 588, 590, 591, 595, 596, 597, 617, 620, 635, 645, 649, 658, 666, 685, 690, 702, 704 }

C grade { 79, 421, 437, 457, 494, 495, 503, 504, 505, 508, 509, 510, 537, 695, 703 }

F normal fail { 126, 133, 145, 154, 193, 198, 221, 223, 224, 225, 226, 228, 232, 233, 234, 291, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 432, 434, 435, 436, 439, 446, 448, 453, 473, 490, 491, 500, 506, 511, 516, 521, 529, 532, 533, 540, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 594, 608, 615, 616, 632, 636, 648, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 698, 699 }

F(-1) timedout fail { 222, 249, 623, 625, 626 }

F(-2) exception fail { 86 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 266, 268, 269, 271, 272, 273, 274, 282, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 306, 307, 308, 309, 311, 312, 313, 318, 322, 323, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 401, 408, 409, 410, 412, 414, 415, 419, 422, 425, 430, 431, 437, 439, 440, 441, 442, 443, 444, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 647, 648, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 696, 697, 700, 704 }

C grade { }

F normal fail { }

F(-1) timedout fail { 69, 86, 126, 133, 145, 193, 198, 221, 222, 226, 227, 228, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 257, 264, 265, 267, 270, 275, 276, 277, 278, 279, 280, 281, 283, 287, 288, 290, 291, 304, 305, 310, 314, 315, 316, 317, 319, 320, 321, 324, 325, 327, 328, 329, 394, 395, 396, 402, 403, 404, 405, 406, 407, 411, 413, 416, 417, 418, 420, 421, 423, 424, 426, 427, 428, 429, 432, 433, 434, 435, 436, 438, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 474, 490, 491, 492, 493, 500, 506, 511, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 592, 595, 616, 641, 645, 646, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 672, 674, 675, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 705 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 5, 6, 7, 9, 10, 11, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 53, 54, 55, 56, 57, 60, 63, 65, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 235, 236, 237, 240, 241, 251, 261, 262, 263, 266, 267, 271, 272, 293, 300, 322, 323, 330, 331, 332, 333, 334, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 361, 363, 364, 365, 366, 367, 368, 371, 372, 377, 385, 387, 441, 458, 459, 460, 463, 464, 465, 466, 471, 472, 476, 479, 480, 483, 484, 485, 486, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 528, 534, 535, 537, 538, 539, 541, 542, 543, 546, 564, 565, 566, 567, 568, 569, 570, 571, 575, 577, 578, 582, 584, 596, 597, 598, 599, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 641, 642, 644, 646, 647, 648, 651, 652, 653, 654, 656, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 671, 676, 677, 679, 680, 681, 699, 700 }

B grade { 1, 3, 4, 8, 12, 13, 19, 21, 30, 31, 36, 41, 48, 62, 89, 149, 159, 194, 195, 196, 197, 211, 214, 252, 253, 254, 273, 299, 312, 335, 340, 353, 354, 369, 370, 376, 378, 379, 382, 383, 388, 389, 396, 467, 468, 475, 487, 488, 489, 493, 494, 503, 504, 505, 508, 509, 510, 527, 530, 531, 547, 548, 572, 573, 574, 576, 580, 583, 585, 587, 588, 589, 590, 591, 602, 603, 604, 608, 609, 621, 655, 670, 697, 704 }

C grade { 2, 50, 51, 52, 79, 114, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 145, 175, 207, 215, 217, 250, 292, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 313, 316, 461, 462, 470, 477, 536, 544, 545, 586, 616 }

F normal fail { 58, 59, 61, 66, 69, 193, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 264, 265, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 360, 362, 373, 374, 380, 381, 384, 386, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 413, 427, 429, 430, 431, 434, 435, 436, 437, 439, 440, 442, 443, 444, 445, 448, 451, 452, 453, 456, 457, 469, 478, 481, 482, 490, 491, 492, 500, 506, 511, 516, 521, 529, 532, 533, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 579, 581, 592, 594, 595, 600, 601, 605, 606, 607, 622, 623, 624, 625, 626, 627, 633, 643, 645, 649, 650, 658, 665, 672, 678, 682, 683, 690, 692, 694, 695, 696, 698, 701, 705 }

F(-1) timedout fail { 64, 198, 216, 222, 260, 318, 375, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 432, 433, 438, 446, 447, 449, 450, 454, 455, 473, 474, 593, 640, 657, 684, 685, 686, 687, 688, 689, 691, 693, 702, 703 }

F(-2) exception fail { $495, 673, 674, 675$ }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	26	31	25	20	33	14
N.S.	1	1.00	1.00	1.86	2.21	1.79	1.43	2.36	1.00
time (sec)	N/A	0.120	0.005	0.189	0.199	0.254	0.058	0.280	0.055

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	26	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.86	1.00	1.00
time (sec)	N/A	0.116	0.004	0.220	0.283	0.243	0.054	0.276	0.041

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	17	26	27	40	11
N.S.	1	1.00	1.00	1.38	1.31	2.00	2.08	3.08	0.85
time (sec)	N/A	0.142	0.003	0.125	0.182	0.297	0.067	0.279	0.252

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	8	19	23	22	21	7
N.S.	1	1.00	2.09	0.73	1.73	2.09	2.00	1.91	0.64
time (sec)	N/A	0.137	0.033	0.135	0.192	0.259	0.059	0.271	0.079

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	27	29	22	29	24
N.S.	1	1.00	1.00	1.40	1.80	1.93	1.47	1.93	1.60
time (sec)	N/A	0.147	0.009	0.293	0.214	0.255	0.092	0.280	0.243

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	4	4	3	4	12
N.S.	1	1.00	1.00	1.50	2.00	2.00	1.50	2.00	6.00
time (sec)	N/A	0.141	0.002	0.069	0.194	0.254	0.020	0.313	0.331

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	6	5	6	6
N.S.	1	1.00	1.00	1.25	1.50	1.50	1.25	1.50	1.50
time (sec)	N/A	0.145	0.002	0.052	0.196	0.261	0.037	0.284	0.253

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	27	4	7	4	4
N.S.	1	1.00	1.00	0.83	4.50	0.67	1.17	0.67	0.67
time (sec)	N/A	0.168	0.026	0.102	0.231	0.248	0.319	0.293	0.205

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.152	0.003	0.059	0.192	0.247	0.086	0.287	0.211

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.157	0.002	0.069	0.204	0.240	0.163	0.285	0.002

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	12	15	14	13
N.S.	1	1.00	1.00	1.08	1.08	1.00	1.25	1.17	1.08
time (sec)	N/A	0.172	0.048	0.237	0.226	0.252	0.092	0.306	0.220

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	31	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.00	2.07	1.00	1.00
time (sec)	N/A	0.183	0.013	0.692	0.288	0.282	0.222	0.288	0.049

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	33	33	26	44	35	15
N.S.	1	1.00	1.00	2.20	2.20	1.73	2.93	2.33	1.00
time (sec)	N/A	0.183	0.013	0.500	0.212	0.252	0.252	0.275	0.204

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	21	32	17	48
N.S.	1	1.00	1.00	1.06	1.00	1.24	1.88	1.00	2.82
time (sec)	N/A	0.207	0.012	6.745	0.224	0.265	0.872	0.298	0.597

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	20	23	34	21	48
N.S.	1	1.00	1.00	1.05	1.05	1.21	1.79	1.11	2.53
time (sec)	N/A	0.203	0.013	6.177	0.226	0.275	0.914	0.280	0.592

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	18	18	34	18	58
N.S.	1	1.22	1.22	1.06	1.00	1.00	1.89	1.00	3.22
time (sec)	N/A	0.204	0.018	10.257	0.214	0.265	0.872	0.279	0.462

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	19	19	32	20	58
N.S.	1	1.22	1.22	1.06	1.06	1.06	1.78	1.11	3.22
time (sec)	N/A	0.204	0.016	9.131	0.207	0.270	0.937	0.287	0.470

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	21	19	14	13	31	61	46	26
N.S.	1	0.51	0.46	0.34	0.32	0.76	1.49	1.12	0.63
time (sec)	N/A	0.171	0.109	0.151	0.306	0.271	0.223	0.299	0.257

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	15	15	15	16	15
N.S.	1	1.00	1.00	1.00	2.50	2.50	2.50	2.67	2.50
time (sec)	N/A	0.150	0.001	0.033	0.221	0.251	0.048	0.284	0.002

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.138	0.001	0.025	0.197	0.243	0.035	0.271	0.206

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.150	0.005	0.103	0.285	0.268	0.068	0.293	0.411

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	7	8	7	7	7	22	7
N.S.	1	1.00	0.78	0.89	0.78	0.78	0.78	2.44	0.78
time (sec)	N/A	0.148	0.034	0.040	0.210	0.235	0.039	0.325	0.261

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.146	0.035	0.033	0.201	0.241	0.041	0.285	0.278

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	37	25	22	22	24	22	23	24
N.S.	1	1.48	1.00	0.88	0.88	0.96	0.88	0.92	0.96
time (sec)	N/A	0.169	0.026	0.066	0.209	0.248	0.151	0.294	0.047

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	38	30	21	20	22	31	21	22
N.S.	1	1.27	1.00	0.70	0.67	0.73	1.03	0.70	0.73
time (sec)	N/A	0.280	0.034	0.027	0.196	0.237	0.933	0.274	0.036

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	10	10	8	11	8
N.S.	1	1.00	1.00	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.137	0.003	0.155	0.200	0.244	0.024	0.287	0.086

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	30	47	24	36	40	42	37	47
N.S.	1	0.64	1.00	0.51	0.77	0.85	0.89	0.79	1.00
time (sec)	N/A	0.142	0.034	0.194	0.284	0.248	0.045	0.274	0.138

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	23	23	27	21	21
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.90	0.70	0.70
time (sec)	N/A	0.137	0.016	0.190	0.289	0.255	0.044	0.278	0.042

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	24	22	17	13
N.S.	1	1.00	1.00	0.67	0.62	1.14	1.05	0.81	0.62
time (sec)	N/A	0.162	0.052	0.249	0.199	0.246	0.128	0.301	0.190

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	24	26	11	11
N.S.	1	1.00	1.00	0.80	0.73	1.60	1.73	0.73	0.73
time (sec)	N/A	0.164	0.002	0.237	0.210	0.254	0.127	0.282	0.059

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	25	21	20	186	89	138	18	118
N.S.	1	1.19	1.00	0.95	8.86	4.24	6.57	0.86	5.62
time (sec)	N/A	0.255	0.088	0.240	0.299	0.273	0.560	0.286	1.426

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.150	0.002	0.059	0.197	0.245	0.022	0.269	0.029

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.152	0.002	0.063	0.198	0.241	0.016	0.271	0.027

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	12
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	1.50
time (sec)	N/A	0.157	0.002	0.125	0.203	0.249	0.017	0.303	0.030

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	15	14	22	15	14	11
N.S.	1	1.00	1.00	1.36	1.27	2.00	1.36	1.27	1.00
time (sec)	N/A	0.178	0.010	0.308	0.216	0.238	0.033	0.290	0.099

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	6	11	9	18	12	9	6
N.S.	1	1.00	0.86	1.57	1.29	2.57	1.71	1.29	0.86
time (sec)	N/A	0.176	0.003	0.287	0.201	0.247	0.024	0.293	0.254

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	28	17	12	23	19	18	10
N.S.	1	1.00	2.00	1.21	0.86	1.64	1.36	1.29	0.71
time (sec)	N/A	0.162	0.017	0.052	0.273	0.240	0.025	0.435	0.232

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	19	12	20	17	22	18
N.S.	1	1.00	1.62	1.19	0.75	1.25	1.06	1.38	1.12
time (sec)	N/A	0.201	0.017	0.059	0.287	0.247	0.058	0.304	0.331

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	12	29	11	12	19	10	12	12
N.S.	1	1.20	2.90	1.10	1.20	1.90	1.00	1.20	1.20
time (sec)	N/A	0.195	0.014	0.183	0.274	0.240	0.143	0.291	0.285

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	25	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.79	0.71	1.00	1.00
time (sec)	N/A	0.275	0.009	0.438	0.279	0.248	0.325	0.296	0.297

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	15	28	24	29	12	12
N.S.	1	1.00	2.27	1.36	2.55	2.18	2.64	1.09	1.09
time (sec)	N/A	0.198	0.046	0.101	0.290	0.251	0.230	0.292	0.243

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	31	17	23	14	8	12	10
N.S.	1	1.00	1.94	1.06	1.44	0.88	0.50	0.75	0.62
time (sec)	N/A	0.215	0.094	0.093	0.272	0.268	0.211	0.292	0.180

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	36	33	19	18	22	19	18	18
N.S.	1	1.44	1.32	0.76	0.72	0.88	0.76	0.72	0.72
time (sec)	N/A	0.175	0.033	0.052	0.197	0.242	0.046	0.286	0.238

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	13	12	15	8
N.S.	1	1.00	1.00	0.67	0.62	0.62	0.57	0.71	0.38
time (sec)	N/A	0.152	0.004	0.224	0.208	0.249	0.042	0.292	0.242

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	16	14	11	10	10	10	10	10
N.S.	1	1.14	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.153	0.008	0.078	0.271	0.228	0.043	0.277	0.068

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	29	35	45	42	44	34
N.S.	1	1.00	0.92	0.57	0.69	0.88	0.82	0.86	0.67
time (sec)	N/A	0.208	0.027	0.283	0.271	0.260	0.050	0.277	0.129

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	22	33	26	25	25	29	25	25
N.S.	1	0.67	1.00	0.79	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.155	0.022	0.136	0.200	0.232	0.056	0.290	0.246

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	18	19
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.62	0.66
time (sec)	N/A	0.143	0.025	0.181	0.289	0.244	0.377	0.284	0.217

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	31	20	17	22	19	37	19	20
N.S.	1	1.15	0.74	0.63	0.81	0.70	1.37	0.70	0.74
time (sec)	N/A	0.150	0.019	0.178	0.284	0.236	0.135	0.300	0.035

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	28	19	18	25	29	10	18
N.S.	1	1.00	1.27	0.86	0.82	1.14	1.32	0.45	0.82
time (sec)	N/A	0.140	0.127	0.256	0.281	0.243	0.498	0.309	0.069

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	12	26	22	20	24
N.S.	1	1.00	1.00	0.95	0.55	1.18	1.00	0.91	1.09
time (sec)	N/A	0.143	0.026	0.438	0.286	0.261	0.498	0.287	0.311

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	53	37	34	25	22	43	21
N.S.	1	1.00	2.30	1.61	1.48	1.09	0.96	1.87	0.91
time (sec)	N/A	0.143	0.051	0.208	0.210	0.243	0.522	0.305	0.554

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	49	35	12	40	7	37	26
N.S.	1	1.00	2.33	1.67	0.57	1.90	0.33	1.76	1.24
time (sec)	N/A	0.158	0.041	0.221	0.201	0.245	0.508	0.310	0.107

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	7	8	30	8	26	6
N.S.	1	1.00	1.75	0.58	0.67	2.50	0.67	2.17	0.50
time (sec)	N/A	0.129	0.082	0.250	0.275	0.246	0.229	0.292	0.243

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	21	33	15	16	38	20	53	26
N.S.	1	1.11	1.74	0.79	0.84	2.00	1.05	2.79	1.37
time (sec)	N/A	0.137	0.086	0.796	0.286	0.264	0.260	0.298	0.382

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	16	5	25	6
N.S.	1	1.00	5.00	0.88	0.75	2.00	0.62	3.12	0.75
time (sec)	N/A	0.124	0.044	0.424	0.308	0.244	0.192	0.286	0.219

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	22	21	43	22	21	40
N.S.	1	1.00	1.33	0.81	0.78	1.59	0.81	0.78	1.48
time (sec)	N/A	0.146	0.128	0.254	0.323	0.278	0.358	0.289	0.419

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	25	33	39	0	71	28
N.S.	1	1.00	0.94	0.78	1.03	1.22	0.00	2.22	0.88
time (sec)	N/A	0.144	0.108	0.250	0.300	0.258	0.000	0.305	0.427

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	16	17	17	0	28	19
N.S.	1	1.10	1.00	0.76	0.81	0.81	0.00	1.33	0.90
time (sec)	N/A	0.136	0.139	0.276	0.298	0.245	0.000	0.318	0.301

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	54	21	33	39	22	31	22
N.S.	1	1.00	1.93	0.75	1.18	1.39	0.79	1.11	0.79
time (sec)	N/A	0.296	0.160	0.216	0.221	0.271	0.299	0.292	0.361

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	18	17	14	13	31	0	46	26
N.S.	1	0.49	0.46	0.38	0.35	0.84	0.00	1.24	0.70
time (sec)	N/A	0.173	0.541	0.155	0.313	0.258	0.000	0.285	0.294

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	12	21	11	47	32	27	11	10
N.S.	1	0.86	1.50	0.79	3.36	2.29	1.93	0.79	0.71
time (sec)	N/A	0.199	0.064	0.200	0.221	0.255	0.499	0.273	0.281

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	3	9	4	15	45	15	17	3
N.S.	1	0.27	0.82	0.36	1.36	4.09	1.36	1.55	0.27
time (sec)	N/A	0.187	0.007	0.063	0.218	0.252	0.061	0.280	0.392

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	19	19	15	14	14	0	14	14
N.S.	1	1.06	1.06	0.83	0.78	0.78	0.00	0.78	0.78
time (sec)	N/A	0.213	0.021	0.184	0.213	0.267	0.000	0.269	0.424

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	15	17	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.83	0.94	0.78	0.78
time (sec)	N/A	0.206	0.024	0.257	0.216	0.269	0.795	0.259	0.376

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	0	15	37
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.88	2.18
time (sec)	N/A	0.152	0.036	0.132	0.296	0.251	0.000	0.279	0.355

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	0	18	16	19	16	16
N.S.	1	1.00	1.10	0.00	0.90	0.80	0.95	0.80	0.80
time (sec)	N/A	0.162	0.039	0.000	0.215	0.256	0.281	0.273	0.411

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.167	0.008	0.546	0.221	0.256	0.833	0.268	0.296

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	38	0	15	0
N.S.	1	1.00	1.00	0.90	0.00	0.90	0.00	0.36	0.00
time (sec)	N/A	0.190	0.049	0.247	0.000	0.256	0.000	0.285	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.160	0.009	0.168	0.305	0.254	0.411	0.260	0.394

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.160	0.000	0.021	0.211	0.239	0.040	0.287	0.034

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	13	11	15	13	9
N.S.	1	1.00	1.00	0.65	0.76	0.65	0.88	0.76	0.53
time (sec)	N/A	0.138	0.003	0.034	0.210	0.252	0.040	0.269	0.229

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	39	38	29	28	28	26	70	40
N.S.	1	1.08	1.06	0.81	0.78	0.78	0.72	1.94	1.11
time (sec)	N/A	0.201	0.009	0.108	0.212	0.257	0.048	0.285	0.413

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	15	18	17	15	21
N.S.	1	1.00	1.00	0.89	0.79	0.95	0.89	0.79	1.11
time (sec)	N/A	0.173	0.001	0.302	0.204	0.262	0.022	0.276	0.002

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	44	30	23	18	25	31	22	26
N.S.	1	1.29	0.88	0.68	0.53	0.74	0.91	0.65	0.76
time (sec)	N/A	0.260	0.001	0.318	0.207	0.255	0.017	0.270	0.024

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	31	71	26	42	69	46	38	33
N.S.	1	1.19	2.73	1.00	1.62	2.65	1.77	1.46	1.27
time (sec)	N/A	0.238	0.014	0.329	0.210	0.250	0.069	0.269	0.261

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	12	11	17	17	11	11
N.S.	1	1.00	0.61	0.52	0.48	0.74	0.74	0.48	0.48
time (sec)	N/A	0.151	0.015	0.125	0.207	0.248	0.163	0.277	0.022

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.156	0.053	0.132	0.206	0.257	0.093	0.270	0.030

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	21	24	20	107	329	20
N.S.	1	1.00	0.65	0.68	0.77	0.65	3.45	10.61	0.65
time (sec)	N/A	0.162	0.022	0.161	0.215	0.243	0.269	0.304	0.030

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	10	13	15	13	13
N.S.	1	1.00	1.00	0.65	0.59	0.76	0.88	0.76	0.76
time (sec)	N/A	0.146	0.003	0.073	0.207	0.259	0.127	0.271	0.240

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	94	22	15	12	35
N.S.	1	1.00	1.00	1.08	7.83	1.83	1.25	1.00	2.92
time (sec)	N/A	0.190	0.019	0.565	0.286	0.246	17.259	0.282	0.506

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	23	13
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.53	0.87
time (sec)	N/A	0.205	0.006	0.031	0.296	0.248	0.073	0.270	0.026

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	38	20
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91
time (sec)	N/A	0.168	0.001	0.013	0.290	0.268	0.980	0.267	0.023

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	25	24	23	23	22	23	22
N.S.	1	1.04	1.00	0.96	0.92	0.92	0.88	0.92	0.88
time (sec)	N/A	0.204	0.003	0.040	0.284	0.246	0.074	0.284	0.041

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.83	0.83
time (sec)	N/A	0.262	0.019	0.234	0.292	0.249	0.108	0.282	0.294

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	54	49	45	78	30	58	0	0
N.S.	1	1.42	1.29	1.18	2.05	0.79	1.53	0.00	0.00
time (sec)	N/A	0.232	0.056	0.053	0.300	0.257	4.078	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	21	21	22	21	21
N.S.	1	1.00	1.00	0.84	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.154	0.003	0.161	0.209	0.235	0.018	0.286	0.046

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	29	29	34	29	29
N.S.	1	1.00	1.00	0.74	0.74	0.74	0.87	0.74	0.74
time (sec)	N/A	0.168	0.001	0.238	0.201	0.232	0.021	0.274	0.036

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	23	27	78	23	23
N.S.	1	1.00	1.00	1.26	1.00	1.17	3.39	1.00	1.00
time (sec)	N/A	0.158	0.037	0.865	0.212	0.248	12.029	0.279	0.817

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	21	22	22	20	23	20
N.S.	1	1.00	0.90	0.70	0.73	0.73	0.67	0.77	0.67
time (sec)	N/A	0.149	0.010	0.168	0.214	0.258	0.026	0.282	0.037

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	27	26	32	34	26	32
N.S.	1	1.00	0.85	0.66	0.63	0.78	0.83	0.63	0.78
time (sec)	N/A	0.152	0.013	0.182	0.298	0.238	0.036	0.278	0.044

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	15	15	14	17	8
N.S.	1	1.10	1.00	0.67	0.71	0.71	0.67	0.81	0.38
time (sec)	N/A	0.152	0.004	0.226	0.220	0.244	0.042	0.268	0.095

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	32	29	28	28	36	28	30
N.S.	1	0.97	0.97	0.88	0.85	0.85	1.09	0.85	0.91
time (sec)	N/A	0.168	0.008	0.710	0.291	0.255	0.047	0.273	0.046

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.157	0.005	0.301	0.284	0.239	0.045	0.283	0.048

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	39	38	38	46	38	40
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.98	0.81	0.85
time (sec)	N/A	0.232	0.019	0.411	0.293	0.263	0.046	0.275	0.284

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.80	0.68
time (sec)	N/A	0.200	0.007	0.039	0.202	0.239	0.054	0.279	0.261

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	26	25	25	26	28	25
N.S.	1	1.00	0.88	0.79	0.76	0.76	0.79	0.85	0.76
time (sec)	N/A	0.211	0.019	0.053	0.213	0.242	0.117	0.276	0.109

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	64	37	26	25	25	29	29	29
N.S.	1	1.73	1.00	0.70	0.68	0.68	0.78	0.78	0.78
time (sec)	N/A	0.258	0.008	0.047	0.203	0.256	0.077	0.290	0.064

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	69	26	43	51	60	59	25
N.S.	1	1.00	2.23	0.84	1.39	1.65	1.94	1.90	0.81
time (sec)	N/A	0.154	0.020	0.060	0.287	0.251	0.289	0.291	0.098

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	26	25	25	26	29	15
N.S.	1	1.00	1.00	0.63	0.61	0.61	0.63	0.71	0.37
time (sec)	N/A	0.168	0.008	0.259	0.206	0.242	0.075	0.273	0.272

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	17	22	29	17	18	22
N.S.	1	1.00	0.64	0.68	0.88	1.16	0.68	0.72	0.88
time (sec)	N/A	0.152	0.011	0.180	0.206	0.227	0.040	0.284	0.205

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	39	31	45	32
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.25	0.89
time (sec)	N/A	0.160	0.016	0.171	0.204	0.252	0.030	0.278	0.038

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	37	25	30	45	31	27	30
N.S.	1	1.00	0.90	0.61	0.73	1.10	0.76	0.66	0.73
time (sec)	N/A	0.203	0.022	0.042	0.216	0.244	0.064	0.266	0.066

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	29	45	26	37	29
N.S.	1	1.00	1.00	1.04	1.07	1.67	0.96	1.37	1.07
time (sec)	N/A	0.196	0.018	0.193	0.206	0.271	0.052	0.294	0.269

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	44	47	31	29	45	32	35	31
N.S.	1	1.13	1.21	0.79	0.74	1.15	0.82	0.90	0.79
time (sec)	N/A	0.206	0.019	0.211	0.220	0.252	0.063	0.282	0.054

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	40	65	37	40	40
N.S.	1	1.00	0.87	0.76	0.87	1.41	0.80	0.87	0.87
time (sec)	N/A	0.208	0.018	0.049	0.206	0.239	0.068	0.290	0.245

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	23	22	24	29
N.S.	1	1.00	1.00	0.83	0.79	0.79	0.76	0.83	1.00
time (sec)	N/A	0.179	0.008	0.072	0.313	0.237	0.055	0.268	0.052

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	22	23	27
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.81	0.85	1.00
time (sec)	N/A	0.160	0.009	0.233	0.326	0.242	0.080	0.292	0.051

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	26	25	25	29	27	17
N.S.	1	1.00	1.33	1.08	1.04	1.04	1.21	1.12	0.71
time (sec)	N/A	0.142	0.012	0.049	0.309	0.242	0.061	0.279	0.065

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	32	44	39	33	49
N.S.	1	1.00	1.00	0.80	0.78	1.07	0.95	0.80	1.20
time (sec)	N/A	0.240	0.025	0.053	0.312	0.238	0.063	0.291	0.336

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	36	57	37	62	38
N.S.	1	1.00	0.87	0.76	0.78	1.24	0.80	1.35	0.83
time (sec)	N/A	0.231	0.021	0.276	0.330	0.241	0.089	0.288	0.223

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	31	36	59	36	32	42
N.S.	1	1.00	0.79	0.66	0.77	1.26	0.77	0.68	0.89
time (sec)	N/A	0.204	0.028	0.253	0.295	0.245	0.076	0.279	0.050

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	71	73	54	53	53	70	53	47
N.S.	1	1.06	1.09	0.81	0.79	0.79	1.04	0.79	0.70
time (sec)	N/A	0.214	0.064	0.030	0.295	0.241	0.086	0.294	0.116

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	67	46	54	58	306	64	73
N.S.	1	1.00	1.40	0.96	1.12	1.21	6.38	1.33	1.52
time (sec)	N/A	0.227	0.020	0.244	0.298	0.239	0.286	0.297	0.181

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	46	38	43	66	46	39	44
N.S.	1	1.00	0.79	0.66	0.74	1.14	0.79	0.67	0.76
time (sec)	N/A	0.226	0.027	0.369	0.285	0.239	0.078	0.284	0.229

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	35	44	35	35
N.S.	1	1.00	0.92	0.71	0.69	0.69	0.86	0.69	0.69
time (sec)	N/A	0.409	0.029	0.316	0.291	0.243	0.205	0.271	0.301

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	41	34	33	33	32	33	33
N.S.	1	1.10	1.00	0.83	0.80	0.80	0.78	0.80	0.80
time (sec)	N/A	0.410	0.010	0.243	0.198	0.245	0.076	0.277	0.062

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	58	52	51	49	45	73	50	64
N.S.	1	1.04	0.93	0.91	0.88	0.80	1.30	0.89	1.14
time (sec)	N/A	0.194	0.016	0.226	0.280	0.240	0.058	0.308	0.463

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	58	50	43	49	43	71	50	68
N.S.	1	1.04	0.89	0.77	0.88	0.77	1.27	0.89	1.21
time (sec)	N/A	0.197	0.010	0.207	0.287	0.238	0.050	0.280	0.123

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83
time (sec)	N/A	0.124	0.003	0.193	0.193	0.236	0.038	0.287	0.032

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	19	22	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.86	1.00	0.82
time (sec)	N/A	0.141	0.006	0.200	0.201	0.230	0.096	0.325	0.291

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	72	60	60	57	53	83	58	88
N.S.	1	1.14	0.95	0.95	0.90	0.84	1.32	0.92	1.40
time (sec)	N/A	0.221	0.019	0.230	0.276	0.238	0.071	0.296	0.319

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	74	68	60	57	62	80	58	86
N.S.	1	1.14	1.05	0.92	0.88	0.95	1.23	0.89	1.32
time (sec)	N/A	0.216	0.021	0.234	0.285	0.241	0.091	0.299	0.301

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	34	33	34	31	33	29	40	29
N.S.	1	1.03	1.00	1.03	0.94	1.00	0.88	1.21	0.88
time (sec)	N/A	0.176	0.007	0.207	0.198	0.235	0.125	0.271	0.086

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	88	74	66	66	68	90	67	99
N.S.	1	1.21	1.01	0.90	0.90	0.93	1.23	0.92	1.36
time (sec)	N/A	0.235	0.017	0.227	0.284	0.242	0.098	0.273	0.112

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	0.00
time (sec)	N/A	0.149	0.142	0.000	0.000	0.000	1.229	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	33	32	26	37	34	18
N.S.	1	1.00	1.41	1.22	1.19	0.96	1.37	1.26	0.67
time (sec)	N/A	0.145	0.012	0.249	0.288	0.244	0.065	0.277	0.093

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	29	26	24	30	13
N.S.	1	1.00	1.00	1.80	1.93	1.73	1.60	2.00	0.87
time (sec)	N/A	0.135	0.005	0.205	0.191	0.239	0.063	0.287	0.054

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	24	23	25	20	19	26	20
N.S.	1	1.17	1.00	0.96	1.04	0.83	0.79	1.08	0.83
time (sec)	N/A	0.142	0.008	0.203	0.196	0.231	0.120	0.284	0.338

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	40	46	41	40	36	44	42	31
N.S.	1	1.14	1.31	1.17	1.14	1.03	1.26	1.20	0.89
time (sec)	N/A	0.157	0.012	0.256	0.285	0.243	0.089	0.285	0.270

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	25	50	42	37	41	34	38	22
N.S.	1	0.96	1.92	1.62	1.42	1.58	1.31	1.46	0.85
time (sec)	N/A	0.147	0.009	0.216	0.210	0.236	0.098	0.275	0.260

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	42	48	41	40	45	48	42	31
N.S.	1	1.14	1.30	1.11	1.08	1.22	1.30	1.14	0.84
time (sec)	N/A	0.153	0.015	0.254	0.292	0.242	0.104	0.281	0.081

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	95	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.11	0.00	0.00
time (sec)	N/A	0.150	0.234	0.000	0.000	0.000	0.497	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	29	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.93	0.87	0.87
time (sec)	N/A	0.136	0.005	0.237	0.278	0.245	0.068	0.322	0.262

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	114	79	24	98	96	19	114	33
N.S.	1	1.05	0.72	0.22	0.90	0.88	0.17	1.05	0.30
time (sec)	N/A	0.271	0.034	0.232	0.295	0.243	0.059	0.301	0.112

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	202	204	55	180	11094	39	177	174
N.S.	1	1.00	1.01	0.27	0.90	55.19	0.19	0.88	0.87
time (sec)	N/A	0.423	0.183	0.230	0.293	0.962	0.058	0.290	0.714

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	203	204	60	160	18781	41	177	182
N.S.	1	1.01	1.01	0.30	0.80	93.44	0.20	0.88	0.91
time (sec)	N/A	0.386	0.067	0.211	0.285	0.985	0.054	0.304	0.690

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	220	204	58	160	12656	41	177	202
N.S.	1	1.09	1.01	0.29	0.80	62.97	0.20	0.88	1.00
time (sec)	N/A	0.373	0.072	0.210	0.298	0.964	0.062	0.316	0.954

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	202	204	73	180	17865	39	177	202
N.S.	1	1.00	1.01	0.36	0.90	88.88	0.19	0.88	1.00
time (sec)	N/A	0.370	0.041	0.213	0.285	0.972	0.062	0.288	0.427

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83
time (sec)	N/A	0.127	0.003	0.204	0.196	0.227	0.042	0.298	0.257

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	19	22	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.86	1.00	0.82
time (sec)	N/A	0.139	0.007	0.205	0.201	0.229	0.119	0.291	0.342

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	216	172	76	192	15275	48	185	210
N.S.	1	1.03	0.82	0.36	0.92	73.09	0.23	0.89	1.00
time (sec)	N/A	0.389	0.180	0.233	0.290	0.977	0.080	0.317	0.415

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	236	174	78	173	15499	51	185	210
N.S.	1	1.12	0.82	0.37	0.82	73.45	0.24	0.88	1.00
time (sec)	N/A	0.384	0.171	0.227	0.290	1.018	0.097	0.303	0.818

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	219	175	81	172	15501	51	185	214
N.S.	1	1.04	0.83	0.38	0.82	73.46	0.24	0.88	1.01
time (sec)	N/A	0.387	0.129	0.226	0.283	0.973	0.091	0.320	0.763

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	0.00
time (sec)	N/A	0.151	0.263	0.000	0.000	0.000	8.408	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	21	10	27	9	8	27	9
N.S.	1	1.00	0.60	0.29	0.77	0.26	0.23	0.77	0.26
time (sec)	N/A	0.488	0.008	0.240	0.282	0.239	0.051	0.269	0.034

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	65	51	44	54	71	63	44	45
N.S.	1	1.08	0.85	0.73	0.90	1.18	1.05	0.73	0.75
time (sec)	N/A	0.179	0.031	0.812	0.274	0.251	0.072	0.285	0.081

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	53	30	28	38	52	36	28	27
N.S.	1	1.23	0.70	0.65	0.88	1.21	0.84	0.65	0.63
time (sec)	N/A	0.162	0.014	0.234	0.274	0.253	0.059	0.287	0.227

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	96	88	103	0	448	323	92	159
N.S.	1	1.07	0.98	1.14	0.00	4.98	3.59	1.02	1.77
time (sec)	N/A	0.222	0.085	0.536	0.000	0.267	0.541	0.282	0.336

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	38	35	33	50	31	33	41
N.S.	1	1.11	1.00	0.92	0.87	1.32	0.82	0.87	1.08
time (sec)	N/A	0.212	0.015	0.322	0.278	0.241	0.050	0.289	0.218

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	65	49	41	42	58	53	43	60
N.S.	1	1.14	0.86	0.72	0.74	1.02	0.93	0.75	1.05
time (sec)	N/A	0.198	0.024	0.205	0.280	0.238	0.063	0.277	0.090

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	44	33	29	31	40	32	28	29
N.S.	1	1.22	0.92	0.81	0.86	1.11	0.89	0.78	0.81
time (sec)	N/A	0.170	0.017	0.234	0.268	0.226	0.063	0.278	0.217

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	55	78	39	40	40	46	40	52
N.S.	1	1.12	1.59	0.80	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.201	0.017	0.221	0.282	0.246	0.056	0.293	0.101

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	39	13	14	14	13	14	0	14
N.S.	1	3.00	1.00	1.08	1.08	1.00	1.08	0.00	1.08
time (sec)	N/A	0.209	0.028	0.290	0.204	0.246	0.784	0.000	0.370

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	48	38	35	34	34	42	34	34
N.S.	1	1.17	0.93	0.85	0.83	0.83	1.02	0.83	0.83
time (sec)	N/A	0.192	0.013	0.053	0.280	0.245	0.048	0.429	0.228

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	32	25	24	24	24	25	22
N.S.	1	1.12	1.00	0.78	0.75	0.75	0.75	0.78	0.69
time (sec)	N/A	0.177	0.007	0.051	0.189	0.249	0.049	0.311	0.087

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	25	18	17	17	15	19	16
N.S.	1	1.16	1.00	0.72	0.68	0.68	0.60	0.76	0.64
time (sec)	N/A	0.173	0.006	0.048	0.191	0.242	0.040	0.303	0.614

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	22	32	46	29	23	22
N.S.	1	1.00	0.67	0.61	0.89	1.28	0.81	0.64	0.61
time (sec)	N/A	0.180	0.015	0.197	0.188	0.240	0.043	0.303	0.042

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	24	22	72	72	70	22	29
N.S.	1	1.00	0.53	0.49	1.60	1.60	1.56	0.49	0.64
time (sec)	N/A	0.160	0.008	0.198	0.192	0.232	0.061	0.284	0.100

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	41	34	48	67	42	55	43
N.S.	1	1.00	0.75	0.62	0.87	1.22	0.76	1.00	0.78
time (sec)	N/A	0.220	0.014	0.191	0.188	0.238	0.051	0.274	0.053

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	38	59	41	43	31
N.S.	1	1.00	1.06	0.97	1.06	1.64	1.14	1.19	0.86
time (sec)	N/A	0.159	0.024	0.207	0.197	0.240	0.056	0.287	0.043

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	31	48	29	40	34
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.83	1.14	0.97
time (sec)	N/A	0.157	0.019	0.237	0.188	0.250	0.053	0.312	0.257

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	59	46	40	50	85	51	42	45
N.S.	1	0.97	0.75	0.66	0.82	1.39	0.84	0.69	0.74
time (sec)	N/A	0.185	0.025	0.243	0.205	0.246	0.069	0.284	0.137

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	58	36	36	44	62	42	34	39
N.S.	1	1.14	0.71	0.71	0.86	1.22	0.82	0.67	0.76
time (sec)	N/A	0.177	0.018	0.546	0.282	0.240	0.071	0.277	0.254

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	60	105	58	47	55
N.S.	1	1.00	1.00	0.83	1.11	1.94	1.07	0.87	1.02
time (sec)	N/A	0.175	0.018	0.196	0.191	0.235	0.069	0.279	0.060

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	47	53	32	40	53	49	48	33
N.S.	1	1.31	1.47	0.89	1.11	1.47	1.36	1.33	0.92
time (sec)	N/A	0.166	0.040	0.226	0.275	0.244	0.036	0.301	0.273

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	67	40	31	41	59	39	31	40
N.S.	1	1.16	0.69	0.53	0.71	1.02	0.67	0.53	0.69
time (sec)	N/A	0.165	0.021	0.237	0.291	0.241	0.051	0.281	0.077

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	46	43	34	36	45	42	36	36
N.S.	1	1.07	1.00	0.79	0.84	1.05	0.98	0.84	0.84
time (sec)	N/A	0.159	0.037	0.856	0.289	0.244	0.061	0.279	0.222

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	69	62	37	47	68	58	51	34
N.S.	1	1.60	1.44	0.86	1.09	1.58	1.35	1.19	0.79
time (sec)	N/A	0.209	0.041	0.313	0.291	0.232	0.061	0.289	0.262

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	46	25	27	30	38	24	40	32
N.S.	1	1.24	0.68	0.73	0.81	1.03	0.65	1.08	0.86
time (sec)	N/A	0.159	0.013	0.247	0.284	0.239	0.059	0.309	0.039

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	43	26	29	28	38	26	32	28
N.S.	1	1.34	0.81	0.91	0.88	1.19	0.81	1.00	0.88
time (sec)	N/A	0.176	0.017	0.183	0.205	0.230	0.047	0.485	0.223

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	19	20	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.46	1.54	0.85	0.85
time (sec)	N/A	0.128	0.005	0.217	0.205	0.230	0.115	0.275	0.229

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	46	45	57	81	51	56	52
N.S.	1	1.02	0.85	0.83	1.06	1.50	0.94	1.04	0.96
time (sec)	N/A	0.180	0.027	0.221	0.195	0.245	0.219	0.310	0.115

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	182	134	75	147	227	66	150	76
N.S.	1	1.16	0.85	0.48	0.94	1.45	0.42	0.96	0.48
time (sec)	N/A	0.361	0.112	0.273	0.275	0.239	0.205	0.294	0.148

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	80	75	53	60	78	78	50	53
N.S.	1	1.25	1.17	0.83	0.94	1.22	1.22	0.78	0.83
time (sec)	N/A	0.184	0.051	0.279	0.288	0.236	0.226	0.294	0.263

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	43	33	29	34	45	27	30	29
N.S.	1	1.10	0.85	0.74	0.87	1.15	0.69	0.77	0.74
time (sec)	N/A	0.164	0.016	0.215	0.190	0.225	0.058	0.320	0.051

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	347	293	47	335	1751	37	250	285
N.S.	1	1.09	0.92	0.15	1.05	5.49	0.12	0.78	0.89
time (sec)	N/A	0.572	0.351	0.247	0.280	2.695	0.113	0.590	1.704

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	75	65	39	56	73	58	55	41
N.S.	1	1.27	1.10	0.66	0.95	1.24	0.98	0.93	0.69
time (sec)	N/A	0.162	0.074	0.222	0.301	0.232	0.070	0.292	0.275

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	44	33	29	31	40	32	28	29
N.S.	1	1.22	0.92	0.81	0.86	1.11	0.89	0.78	0.81
time (sec)	N/A	0.174	0.016	0.228	0.292	0.237	0.059	0.301	0.004

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	38	65	36	35	33
N.S.	1	1.00	0.84	0.82	1.00	1.71	0.95	0.92	0.87
time (sec)	N/A	0.218	0.025	0.050	0.221	0.226	0.057	0.288	0.070

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	69	65	50	50	73	60	60	63
N.S.	1	1.08	1.02	0.78	0.78	1.14	0.94	0.94	0.98
time (sec)	N/A	0.272	0.032	0.225	0.301	0.238	0.098	0.307	0.111

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	66	63	54	59	97	68	72	73
N.S.	1	1.05	1.00	0.86	0.94	1.54	1.08	1.14	1.16
time (sec)	N/A	0.291	0.039	0.471	0.279	0.248	0.083	0.303	0.284

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	32	36	55	34	33	25
N.S.	1	1.00	1.07	0.74	0.84	1.28	0.79	0.77	0.58
time (sec)	N/A	0.160	0.020	0.226	0.198	0.234	0.068	0.290	0.059

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	31	20	18	43	43	39	18	18
N.S.	1	1.15	0.74	0.67	1.59	1.59	1.44	0.67	0.67
time (sec)	N/A	0.155	0.011	0.210	0.202	0.229	0.067	0.302	0.119

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	44	34	39	55	36	40	34
N.S.	1	1.09	0.96	0.74	0.85	1.20	0.78	0.87	0.74
time (sec)	N/A	0.166	0.018	0.200	0.185	0.225	0.045	0.306	0.220

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	50	39	35	37	56	34	43	33
N.S.	1	1.14	0.89	0.80	0.84	1.27	0.77	0.98	0.75
time (sec)	N/A	0.167	0.024	0.200	0.189	0.230	0.061	0.288	0.050

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	60	105	58	47	55
N.S.	1	1.00	1.00	0.83	1.11	1.94	1.07	0.87	1.02
time (sec)	N/A	0.173	0.009	0.187	0.186	0.233	0.069	0.278	0.002

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	39	31	45	32
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.25	0.89
time (sec)	N/A	0.157	0.015	0.182	0.182	0.237	0.025	0.299	0.002

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	38	38	39	39	37
N.S.	1	1.00	0.93	0.86	0.86	0.86	0.89	0.89	0.84
time (sec)	N/A	0.188	0.013	0.074	0.182	0.241	0.021	0.299	0.049

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	91	89	91	91	100	98	88
N.S.	1	1.00	0.95	0.93	0.95	0.95	1.04	1.02	0.92
time (sec)	N/A	0.263	0.030	0.362	0.194	0.234	0.022	0.277	0.218

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	167	165	171	171	189	188	164
N.S.	1	1.00	1.00	0.99	1.02	1.02	1.13	1.13	0.98
time (sec)	N/A	0.364	0.032	0.342	0.180	0.223	0.030	0.288	0.242

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	263	264	273	273	313	307	263
N.S.	1	1.00	1.00	1.00	1.04	1.04	1.19	1.17	1.00
time (sec)	N/A	0.524	0.061	0.350	0.189	0.246	0.036	0.282	0.294

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	267	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.673	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	71	66	63	0	203	246	60	155
N.S.	1	1.09	1.02	0.97	0.00	3.12	3.78	0.92	2.38
time (sec)	N/A	0.203	0.046	0.463	0.000	0.248	0.401	0.293	0.308

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	95	88	103	0	447	323	92	159
N.S.	1	1.07	0.99	1.16	0.00	5.02	3.63	1.03	1.79
time (sec)	N/A	0.207	0.073	0.464	0.000	0.259	0.522	0.292	0.304

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	143	127	155	0	1104	622	194	360
N.S.	1	1.10	0.98	1.19	0.00	8.49	4.78	1.49	2.77
time (sec)	N/A	0.249	0.127	0.457	0.000	0.269	0.996	0.289	0.528

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	191	168	206	0	1950	1027	363	640
N.S.	1	1.10	0.97	1.19	0.00	11.27	5.94	2.10	3.70
time (sec)	N/A	0.280	0.190	0.478	0.000	0.298	1.528	0.306	0.728

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	264	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.791	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	44	31	41	52	46	46	36
N.S.	1	1.04	0.90	0.63	0.84	1.06	0.94	0.94	0.73
time (sec)	N/A	0.187	0.026	0.263	0.262	0.232	0.041	0.294	0.176

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	93	70	56	67	97	76	61	53
N.S.	1	1.52	1.15	0.92	1.10	1.59	1.25	1.00	0.87
time (sec)	N/A	0.226	0.051	0.261	0.278	0.239	0.066	0.290	0.249

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	35	33	28	30	45	31	32	25
N.S.	1	0.97	0.92	0.78	0.83	1.25	0.86	0.89	0.69
time (sec)	N/A	0.163	0.017	0.248	0.182	0.230	0.050	0.282	0.052

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	81	87	60	90	165	88	62	65
N.S.	1	0.93	1.00	0.69	1.03	1.90	1.01	0.71	0.75
time (sec)	N/A	0.214	0.027	0.262	0.194	0.245	0.092	0.305	0.095

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	70	62	69	116	80	67	77
N.S.	1	1.06	0.86	0.77	0.85	1.43	0.99	0.83	0.95
time (sec)	N/A	0.227	0.035	0.849	0.263	0.238	0.102	0.311	0.115

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	75	87	60	90	165	90	62	90
N.S.	1	0.72	0.84	0.58	0.87	1.59	0.87	0.60	0.87
time (sec)	N/A	0.218	0.025	0.253	0.179	0.246	0.076	0.299	0.242

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	101	99	64	94	173	90	66	85
N.S.	1	0.99	0.97	0.63	0.92	1.70	0.88	0.65	0.83
time (sec)	N/A	0.235	0.052	0.284	0.178	0.236	0.091	0.292	0.246

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	40	35	39	39	37	47	37
N.S.	1	1.10	1.00	0.88	0.98	0.98	0.92	1.18	0.92
time (sec)	N/A	0.171	0.006	0.179	0.189	0.242	0.114	0.293	0.249

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	102	62	55	83	113	102	58	79
N.S.	1	1.23	0.75	0.66	1.00	1.36	1.23	0.70	0.95
time (sec)	N/A	0.190	0.025	0.292	0.261	0.233	0.323	0.288	0.149

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	43	32	31	37	42	31	31
N.S.	1	1.08	0.88	0.65	0.63	0.76	0.86	0.63	0.63
time (sec)	N/A	0.236	0.025	0.203	0.207	0.234	0.235	0.277	0.217

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	32	31	31	48	31	31
N.S.	1	1.00	1.00	0.58	0.56	0.56	0.87	0.56	0.56
time (sec)	N/A	0.545	0.193	0.216	0.195	0.241	1.214	0.312	0.066

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	16	22	19	18	18	19	18	18
N.S.	1	0.73	1.00	0.86	0.82	0.82	0.86	0.82	0.82
time (sec)	N/A	0.153	0.016	0.092	0.192	0.236	0.052	0.279	0.134

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	16	18	13	12	11	22	12	11
N.S.	1	1.07	1.20	0.87	0.80	0.73	1.47	0.80	0.73
time (sec)	N/A	0.142	0.017	0.040	0.217	0.233	0.486	0.275	0.031

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	27	24	21	20	19	20	21	19
N.S.	1	1.08	0.96	0.84	0.80	0.76	0.80	0.84	0.76
time (sec)	N/A	0.167	0.021	0.179	0.189	0.233	0.056	0.313	0.252

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	23	31	26	25	25	27	26	25
N.S.	1	0.70	0.94	0.79	0.76	0.76	0.82	0.79	0.76
time (sec)	N/A	0.180	0.356	0.065	0.190	0.239	0.525	0.309	0.291

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	24	17	19	19	134	19	16
N.S.	1	1.14	0.83	0.59	0.66	0.66	4.62	0.66	0.55
time (sec)	N/A	0.149	0.033	0.201	0.184	0.232	0.678	0.276	0.575

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	56	36	30	55	63	3966	49	43
N.S.	1	1.08	0.69	0.58	1.06	1.21	76.27	0.94	0.83
time (sec)	N/A	0.148	0.054	0.214	0.186	0.237	1.557	0.316	0.049

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	132	70	59	111	125	0	104	96
N.S.	1	1.12	0.59	0.50	0.94	1.06	0.00	0.88	0.81
time (sec)	N/A	0.202	0.094	0.310	0.190	0.240	0.000	0.299	0.233

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	104	118	90	85	105	86	12993	82	120
N.S.	1	1.13	0.87	0.82	1.01	0.83	124.93	0.79	1.15
time (sec)	N/A	0.197	0.242	0.642	0.291	0.254	37.158	0.314	0.244

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	57	64	39	43	32	0	29	43
N.S.	1	1.50	1.68	1.03	1.13	0.84	0.00	0.76	1.13
time (sec)	N/A	0.152	0.094	0.187	0.279	0.234	0.000	0.311	0.219

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	131	99	141	130	73	0	103	140
N.S.	1	1.42	1.08	1.53	1.41	0.79	0.00	1.12	1.52
time (sec)	N/A	0.200	0.380	0.161	0.294	0.238	0.000	0.311	0.377

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	64	0	65	0	74	95
N.S.	1	1.00	0.87	1.19	0.00	1.20	0.00	1.37	1.76
time (sec)	N/A	0.218	0.119	0.276	0.000	0.245	0.000	0.303	0.646

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	474	157	0	0	367	0	0	0
N.S.	1	1.56	0.52	0.00	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.923	11.239	0.000	0.000	0.263	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	F	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	0	455	0	0	785	0	0	0
N.S.	1	0.00	1.56	0.00	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	0.000	36.725	0.000	0.000	0.289	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	27	25	22	0	47	0	0	25
N.S.	1	1.08	1.00	0.88	0.00	1.88	0.00	0.00	1.00
time (sec)	N/A	0.188	0.022	0.229	0.000	0.234	0.000	0.000	0.259

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	28	25	22	0	69	0	0	25
N.S.	1	1.12	1.00	0.88	0.00	2.76	0.00	0.00	1.00
time (sec)	N/A	0.205	0.022	0.830	0.000	0.234	0.000	0.000	0.298

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	70	30	27	0	77	0	0	30
N.S.	1	1.32	0.57	0.51	0.00	1.45	0.00	0.00	0.57
time (sec)	N/A	0.210	0.051	1.448	0.000	0.226	0.000	0.000	0.257

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	114	123	370	0	128	0	0	0
N.S.	1	1.70	1.84	5.52	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.217	0.132	0.532	0.000	0.233	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	94	96	86	0	142	0	83	0
N.S.	1	0.77	0.79	0.70	0.00	1.16	0.00	0.68	0.00
time (sec)	N/A	0.487	0.120	0.762	0.000	0.250	0.000	0.344	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	284	229	1374	0	280	0	0	0
N.S.	1	1.89	1.53	9.16	0.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.323	0.550	5.166	0.000	0.244	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	26	51	64	0	23	27
N.S.	1	1.00	0.91	0.60	1.19	1.49	0.00	0.53	0.63
time (sec)	N/A	0.152	0.231	0.276	0.224	0.236	0.000	0.287	0.052

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	62	0	34	0
N.S.	1	1.00	0.88	0.83	0.00	1.48	0.00	0.81	0.00
time (sec)	N/A	0.170	0.025	0.154	0.000	0.242	0.000	0.275	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	106	58	71	0	138	0	75	0
N.S.	1	0.76	0.42	0.51	0.00	0.99	0.00	0.54	0.00
time (sec)	N/A	0.202	0.064	0.165	0.000	0.242	0.000	0.281	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	98	123	446	0	128	0	0	0
N.S.	1	1.31	1.64	5.95	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.187	0.131	0.566	0.000	0.238	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	27	23	20	0	22	0	0	24
N.S.	1	0.93	0.79	0.69	0.00	0.76	0.00	0.00	0.83
time (sec)	N/A	0.158	0.020	0.070	0.000	0.231	0.000	0.000	0.054

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	33	29	0	44	0	0	37
N.S.	1	1.00	0.36	0.32	0.00	0.48	0.00	0.00	0.40
time (sec)	N/A	0.191	0.051	0.083	0.000	0.255	0.000	0.000	0.092

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	32	15	16	33	20	36	16
N.S.	1	1.00	1.68	0.79	0.84	1.74	1.05	1.89	0.84
time (sec)	N/A	0.141	0.096	0.348	0.273	0.239	0.253	0.295	0.238

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	12	23	5	8	29	3	24	4
N.S.	1	1.50	2.88	0.62	1.00	3.62	0.38	3.00	0.50
time (sec)	N/A	0.135	0.082	0.293	0.279	0.268	0.245	0.275	0.237

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	33	12	11	40	12	31	11
N.S.	1	1.00	2.75	1.00	0.92	3.33	1.00	2.58	0.92
time (sec)	N/A	0.132	0.093	0.256	0.263	0.247	0.256	0.283	0.264

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	24	0	23	0	51	79
N.S.	1	1.00	1.06	0.77	0.00	0.74	0.00	1.65	2.55
time (sec)	N/A	0.134	0.144	0.582	0.000	0.244	0.000	0.286	0.583

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	40	22	0	54	0	57	61
N.S.	1	1.00	1.29	0.71	0.00	1.74	0.00	1.84	1.97
time (sec)	N/A	0.142	0.137	0.245	0.000	0.240	0.000	0.273	0.533

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	112	48	37	42	78
N.S.	1	1.00	1.00	0.88	4.67	2.00	1.54	1.75	3.25
time (sec)	N/A	0.145	0.060	0.339	0.274	0.240	2.274	0.266	0.831

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	101	32	24	20	83
N.S.	1	1.00	1.00	0.84	4.04	1.28	0.96	0.80	3.32
time (sec)	N/A	0.146	0.039	0.458	0.287	0.244	2.288	0.288	0.792

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	37	0	72	0	74	107
N.S.	1	1.00	1.33	0.86	0.00	1.67	0.00	1.72	2.49
time (sec)	N/A	0.164	0.164	0.598	0.000	0.249	0.000	0.312	0.115

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	49	54	74	0	109	0
N.S.	1	1.00	1.00	0.79	0.87	1.19	0.00	1.76	0.00
time (sec)	N/A	0.206	0.232	0.556	0.271	0.256	0.000	0.316	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	81	90	69	0	151	0	164	0
N.S.	1	0.99	1.10	0.84	0.00	1.84	0.00	2.00	0.00
time (sec)	N/A	0.322	0.373	0.783	0.000	0.252	0.000	0.323	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	102	53	0	237	0	165	0
N.S.	1	1.00	1.62	0.84	0.00	3.76	0.00	2.62	0.00
time (sec)	N/A	0.235	0.198	1.438	0.000	0.254	0.000	0.326	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	103	128	0	105	0	152	0
N.S.	1	1.00	1.84	2.29	0.00	1.88	0.00	2.71	0.00
time (sec)	N/A	0.197	0.184	1.034	0.000	0.253	0.000	0.304	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	123	158	0	229	0	257	0
N.S.	1	1.00	1.76	2.26	0.00	3.27	0.00	3.67	0.00
time (sec)	N/A	0.221	0.266	1.649	0.000	0.260	0.000	0.306	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	124	192	0	1339	0	629	0
N.S.	1	1.00	1.55	2.40	0.00	16.74	0.00	7.86	0.00
time (sec)	N/A	0.286	1.314	0.941	0.000	0.337	0.000	0.418	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	85	82	0	81	0	0	0
N.S.	1	1.00	2.24	2.16	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.159	0.574	0.537	0.000	0.234	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	75	57	40	48	42	153	36	35
N.S.	1	1.15	0.88	0.62	0.74	0.65	2.35	0.55	0.54
time (sec)	N/A	0.167	0.174	0.287	0.275	0.245	4.218	0.279	0.031

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	28	25	37	31	41	51	25
N.S.	1	1.10	0.57	0.51	0.76	0.63	0.84	1.04	0.51
time (sec)	N/A	0.156	0.050	0.234	0.280	0.263	1.065	0.289	0.035

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	28	25	37	44	139	26	187
N.S.	1	1.10	0.57	0.51	0.76	0.90	2.84	0.53	3.82
time (sec)	N/A	0.144	0.058	0.218	0.186	0.241	2.934	0.298	0.048

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	354	10	62	58	11	154
N.S.	1	1.00	1.00	29.50	0.83	5.17	4.83	0.92	12.83
time (sec)	N/A	0.216	0.020	0.075	0.188	0.243	0.738	0.272	0.275

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	26	11	10	37	29	22	24
N.S.	1	1.00	2.17	0.92	0.83	3.08	2.42	1.83	2.00
time (sec)	N/A	0.149	0.163	0.224	0.274	0.250	4.842	0.300	0.234

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	48	23	0	67	0	64	77
N.S.	1	1.00	1.78	0.85	0.00	2.48	0.00	2.37	2.85
time (sec)	N/A	0.144	0.125	0.266	0.000	0.250	0.000	0.296	0.180

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	38	0	83	0	101	117
N.S.	1	1.00	1.15	0.79	0.00	1.73	0.00	2.10	2.44
time (sec)	N/A	0.147	0.141	0.297	0.000	0.242	0.000	0.301	0.099

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	54	57	107	77	0	72	0
N.S.	1	1.00	1.32	1.39	2.61	1.88	0.00	1.76	0.00
time (sec)	N/A	0.160	0.191	0.454	0.271	0.234	0.000	0.324	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	50	0	50	0	123	115
N.S.	1	1.00	1.00	1.06	0.00	1.06	0.00	2.62	2.45
time (sec)	N/A	0.160	0.234	0.621	0.000	0.238	0.000	0.294	0.137

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	108	82	0	160	0	135	159
N.S.	1	1.00	1.23	0.93	0.00	1.82	0.00	1.53	1.81
time (sec)	N/A	0.400	0.333	1.450	0.000	0.248	0.000	0.319	0.474

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	106	654	0	170	0	176	216
N.S.	1	1.00	0.78	4.81	0.00	1.25	0.00	1.29	1.59
time (sec)	N/A	1.376	0.377	0.040	0.000	0.244	0.000	0.300	0.645

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	66	68	44	63	51	78	45	56
N.S.	1	1.03	1.06	0.69	0.98	0.80	1.22	0.70	0.88
time (sec)	N/A	0.191	0.133	0.470	0.275	0.240	0.252	0.280	0.106

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	91	76	57	81	60	92	54	75
N.S.	1	1.02	0.85	0.64	0.91	0.67	1.03	0.61	0.84
time (sec)	N/A	0.216	0.141	0.306	0.269	0.246	0.252	0.315	0.340

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	118	84	65	101	68	102	63	96
N.S.	1	1.04	0.74	0.58	0.89	0.60	0.90	0.56	0.85
time (sec)	N/A	0.242	0.153	0.323	0.276	0.238	0.275	0.311	0.139

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	35	54	62	0	71	0
N.S.	1	1.00	1.59	0.71	1.10	1.27	0.00	1.45	0.00
time (sec)	N/A	0.186	0.185	0.468	0.277	0.252	0.000	0.321	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	50	66	64	0	147	0
N.S.	1	1.00	0.99	0.63	0.84	0.81	0.00	1.86	0.00
time (sec)	N/A	0.217	0.325	0.563	0.268	0.238	0.000	0.301	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	10	11	18	14	34	12
N.S.	1	1.00	1.67	0.83	0.92	1.50	1.17	2.83	1.00
time (sec)	N/A	0.138	0.071	0.408	0.275	0.240	0.201	0.298	0.265

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	58	47	33	48	39	41	39	0
N.S.	1	1.09	0.89	0.62	0.91	0.74	0.77	0.74	0.00
time (sec)	N/A	0.181	0.107	0.418	0.278	0.232	0.237	0.329	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	22	34	0	15	13
N.S.	1	1.00	1.00	0.84	1.16	1.79	0.00	0.79	0.68
time (sec)	N/A	0.128	0.132	0.431	0.188	0.233	0.000	0.298	0.032

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	22	28	0	13	15
N.S.	1	1.00	1.00	0.82	1.29	1.65	0.00	0.76	0.88
time (sec)	N/A	0.130	0.129	0.428	0.189	0.234	0.000	0.308	0.026

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	47	33	47	64	0	38	0
N.S.	1	1.09	0.84	0.59	0.84	1.14	0.00	0.68	0.00
time (sec)	N/A	0.184	0.155	0.422	0.267	0.238	0.000	0.290	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	75	52	38	56	44	46	44	61
N.S.	1	1.15	0.80	0.58	0.86	0.68	0.71	0.68	0.94
time (sec)	N/A	0.193	0.096	0.419	0.271	0.234	0.232	0.310	0.133

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	52	38	56	44	85	44	43
N.S.	1	1.09	0.95	0.69	1.02	0.80	1.55	0.80	0.78
time (sec)	N/A	0.169	0.153	0.424	0.271	0.257	0.308	0.306	0.233

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	84	62	48	77	54	168	54	56
N.S.	1	1.14	0.84	0.65	1.04	0.73	2.27	0.73	0.76
time (sec)	N/A	0.187	0.255	0.420	0.274	0.246	0.451	0.288	0.074

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	31	37	52	0	67	31
N.S.	1	1.00	0.87	0.82	0.97	1.37	0.00	1.76	0.82
time (sec)	N/A	0.153	0.108	0.428	0.282	0.240	0.000	0.290	0.029

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	60	42	42	50	63	0	84	0
N.S.	1	1.05	0.74	0.74	0.88	1.11	0.00	1.47	0.00
time (sec)	N/A	0.176	0.120	0.428	0.283	0.256	0.000	0.290	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	65	45	41	58	94	0	80	0
N.S.	1	1.05	0.73	0.66	0.94	1.52	0.00	1.29	0.00
time (sec)	N/A	0.189	0.189	0.437	0.270	0.251	0.000	0.303	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	87	52	46	71	107	0	117	0
N.S.	1	1.10	0.66	0.58	0.90	1.35	0.00	1.48	0.00
time (sec)	N/A	0.213	0.201	0.517	0.279	0.239	0.000	0.287	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	22	25	30	0	32	0
N.S.	1	1.00	0.82	1.00	1.14	1.36	0.00	1.45	0.00
time (sec)	N/A	0.140	0.115	0.422	0.269	0.238	0.000	0.310	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	85	69	0	110	0	147	0
N.S.	1	1.00	0.99	0.80	0.00	1.28	0.00	1.71	0.00
time (sec)	N/A	0.420	0.281	0.453	0.000	0.262	0.000	0.339	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	67	71	56	61	92	0	149	0
N.S.	1	1.08	1.15	0.90	0.98	1.48	0.00	2.40	0.00
time (sec)	N/A	0.229	0.270	0.481	0.284	0.250	0.000	0.346	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	84	64	0	174	0	235	0
N.S.	1	1.07	1.11	0.84	0.00	2.29	0.00	3.09	0.00
time (sec)	N/A	0.273	0.611	0.836	0.000	0.243	0.000	0.293	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	21	28	22	0	30	19
N.S.	1	1.00	0.72	0.58	0.78	0.61	0.00	0.83	0.53
time (sec)	N/A	0.247	0.127	0.374	0.279	0.243	0.000	0.291	0.307

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	63	0	147	0	148	0
N.S.	1	1.00	1.09	0.72	0.00	1.69	0.00	1.70	0.00
time (sec)	N/A	0.366	0.539	1.822	0.000	0.251	0.000	0.300	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	39	38	76	98	0	33	69
N.S.	1	1.09	0.67	0.66	1.31	1.69	0.00	0.57	1.19
time (sec)	N/A	0.174	0.313	0.441	0.186	0.238	0.000	0.287	0.279

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	73	0	27	29
N.S.	1	1.00	0.70	0.64	1.26	1.55	0.00	0.57	0.62
time (sec)	N/A	0.155	0.217	0.325	0.177	0.246	0.000	0.293	0.057

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	51	0	39	29
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.83	0.62
time (sec)	N/A	0.161	0.227	0.369	0.189	0.244	0.000	0.320	0.238

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	43	40	0	39	0	60	0
N.S.	1	1.00	1.48	1.38	0.00	1.34	0.00	2.07	0.00
time (sec)	N/A	0.211	0.291	0.072	0.000	0.255	0.000	0.292	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	59	54	52	0	63	0	66	0
N.S.	1	1.31	1.20	1.16	0.00	1.40	0.00	1.47	0.00
time (sec)	N/A	0.188	0.133	0.082	0.000	0.248	0.000	0.283	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	64	55	0	54	0	54	71
N.S.	1	1.00	0.81	0.70	0.00	0.68	0.00	0.68	0.90
time (sec)	N/A	0.297	0.206	0.049	0.000	0.244	0.000	0.279	0.074

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	80	0	99	0	105	0
N.S.	1	1.00	0.90	1.00	0.00	1.24	0.00	1.31	0.00
time (sec)	N/A	0.460	0.241	0.438	0.000	0.255	0.000	0.304	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	159	140	0	155	0	0	0
N.S.	1	1.00	1.01	0.89	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.624	5.512	0.023	0.000	0.253	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	31	30	38	28	131	29	29
N.S.	1	0.95	0.76	0.73	0.93	0.68	3.20	0.71	0.71
time (sec)	N/A	0.145	0.047	0.273	0.277	0.240	1.881	0.287	0.045

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	17	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.31	0.77	0.85	0.85
time (sec)	N/A	0.130	0.033	0.023	0.192	0.246	0.231	0.298	0.480

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	71	78	85	67	76	80	541	76	90
N.S.	1	1.10	1.20	0.94	1.07	1.13	7.62	1.07	1.27
time (sec)	N/A	0.187	0.127	0.589	0.273	0.249	1.294	0.334	0.228

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	18	28	29	178	49	23
N.S.	1	1.00	0.58	0.45	0.70	0.72	4.45	1.22	0.58
time (sec)	N/A	0.151	0.027	0.315	0.189	0.236	0.877	0.282	0.217

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	59	48	53	52	52	51	53	36
N.S.	1	1.23	1.00	1.10	1.08	1.08	1.06	1.10	0.75
time (sec)	N/A	0.172	0.049	0.289	0.267	0.252	1.018	2.151	0.928

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	36	20	45	25	3303	63	45
N.S.	1	1.03	0.52	0.29	0.65	0.36	47.87	0.91	0.65
time (sec)	N/A	0.172	0.028	0.292	0.183	0.238	1.122	0.270	0.332

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	C	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	171	120	85	157	120	44	142	77
N.S.	1	0.89	0.62	0.44	0.81	0.62	0.23	0.74	0.40
time (sec)	N/A	0.305	0.358	0.329	0.280	0.263	2.753	0.311	1.310

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	26	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69
time (sec)	N/A	0.133	0.014	0.342	0.188	0.248	0.115	0.298	0.306

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.69
time (sec)	N/A	0.129	0.018	0.302	0.177	0.236	0.181	0.294	0.345

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	65	82	67	64	66	42	65	76
N.S.	1	1.10	1.39	1.14	1.08	1.12	0.71	1.10	1.29
time (sec)	N/A	0.176	0.085	7.366	0.269	0.240	0.467	0.298	0.485

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	83	76	66	79	34	67	92
N.S.	1	1.09	1.19	1.09	0.94	1.13	0.49	0.96	1.31
time (sec)	N/A	0.188	0.126	6.638	0.270	0.240	0.681	0.273	0.392

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	76	68	20	83	146	41	83	18
N.S.	1	1.12	1.00	0.29	1.22	2.15	0.60	1.22	0.26
time (sec)	N/A	0.186	0.235	2.963	0.273	2.234	0.547	0.287	0.484

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	103	82	19	130	105	41	110	0
N.S.	1	1.11	0.88	0.20	1.40	1.13	0.44	1.18	0.00
time (sec)	N/A	0.204	0.389	2.229	0.269	0.257	0.916	0.313	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	103	82	20	129	105	39	109	0
N.S.	1	1.11	0.88	0.22	1.39	1.13	0.42	1.17	0.00
time (sec)	N/A	0.198	0.360	2.187	0.270	0.254	1.028	0.291	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	105	137	15	0	99	0	69	29
N.S.	1	1.13	1.47	0.16	0.00	1.06	0.00	0.74	0.31
time (sec)	N/A	0.234	0.704	2.615	0.000	0.349	0.000	0.297	0.402

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	127	69	51	0	87	0	66	27
N.S.	1	1.01	0.55	0.40	0.00	0.69	0.00	0.52	0.21
time (sec)	N/A	0.284	0.196	0.422	0.000	42.644	0.000	0.368	0.325

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	41	28	0	49	0	70	35
N.S.	1	1.00	1.21	0.82	0.00	1.44	0.00	2.06	1.03
time (sec)	N/A	0.164	0.254	0.856	0.000	0.261	0.000	0.301	0.553

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	67	55	41	0	47	0	49	43
N.S.	1	1.16	0.95	0.71	0.00	0.81	0.00	0.84	0.74
time (sec)	N/A	0.190	0.116	1.102	0.000	0.237	0.000	0.285	0.331

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	80	57	52	52	81	0	112	0
N.S.	1	1.13	0.80	0.73	0.73	1.14	0.00	1.58	0.00
time (sec)	N/A	0.213	0.137	0.217	0.270	0.245	0.000	0.285	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	52	23	52	59	0	69	49
N.S.	1	1.00	2.48	1.10	2.48	2.81	0.00	3.29	2.33
time (sec)	N/A	0.167	0.135	0.208	0.175	0.258	0.000	0.332	0.874

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	75	14	13	13	36	13	21
N.S.	1	1.00	4.41	0.82	0.76	0.76	2.12	0.76	1.24
time (sec)	N/A	0.141	10.073	0.326	0.180	0.243	0.183	0.310	0.387

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	116	34	35	221	34	34
N.S.	1	1.00	0.87	2.52	0.74	0.76	4.80	0.74	0.74
time (sec)	N/A	0.333	0.061	0.358	0.183	0.236	4.670	0.277	0.449

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	110	93	0	232	0	0	0
N.S.	1	1.00	1.41	1.19	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.166	0.357	3.120	0.000	1.651	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	146	76	100	0	225	0	0	0
N.S.	1	1.04	0.54	0.71	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.266	0.297	1.813	0.000	2.394	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	90	29	94	86	71	0	0
N.S.	1	1.03	1.43	0.46	1.49	1.37	1.13	0.00	0.00
time (sec)	N/A	0.163	0.264	1.628	0.272	0.247	0.966	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	79	74	87	0	313	0	0	0
N.S.	1	1.07	1.00	1.18	0.00	4.23	0.00	0.00	0.00
time (sec)	N/A	0.188	0.412	2.648	0.000	3.411	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	64	26	23	73	40	0	0	23
N.S.	1	1.33	0.54	0.48	1.52	0.83	0.00	0.00	0.48
time (sec)	N/A	0.171	1.437	0.371	0.185	0.242	0.000	0.000	0.282

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	90	180	2517	0	458	0	0	0
N.S.	1	1.00	2.00	27.97	0.00	5.09	0.00	0.00	0.00
time (sec)	N/A	0.190	0.490	10.174	0.000	4.911	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	0	18	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.175	0.006	0.704	0.000	0.263	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	0	42	0	0	0
N.S.	1	1.00	1.00	0.96	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.174	0.004	0.887	0.000	0.275	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	21	41	18	57	49	14	51	17
N.S.	1	1.31	2.56	1.12	3.56	3.06	0.88	3.19	1.06
time (sec)	N/A	0.165	0.092	0.320	0.268	0.239	3.626	0.289	0.152

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	19	39	18	57	47	14	51	17
N.S.	1	1.19	2.44	1.12	3.56	2.94	0.88	3.19	1.06
time (sec)	N/A	0.167	0.091	0.317	0.266	0.243	2.135	0.277	0.252

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	0	45	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.182	0.261	1.207	0.000	0.268	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	13	0	0	0
N.S.	1	1.00	1.00	0.93	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.175	0.200	0.861	0.000	0.268	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	22	14	0	0	14
N.S.	1	1.00	1.00	0.94	1.38	0.88	0.00	0.00	0.88
time (sec)	N/A	0.147	0.448	0.314	0.238	0.257	0.000	0.000	0.057

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	78	0	252	0	0	0
N.S.	1	1.00	0.88	1.05	0.00	3.41	0.00	0.00	0.00
time (sec)	N/A	0.310	0.769	2.507	0.000	0.383	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	96	0	0	62	0	0	0
N.S.	1	1.00	4.36	0.00	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.194	0.366	0.000	0.000	0.631	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.153	0.002	0.055	0.223	0.248	0.016	0.266	0.201

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	10	8	9	9
N.S.	1	1.00	1.00	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.168	0.001	0.411	0.206	0.251	0.019	0.294	0.035

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	22	17	16	19	24	16	16
N.S.	1	1.21	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.189	0.002	0.382	0.199	0.242	0.018	0.316	0.037

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	44	30	23	24	25	36	22	22
N.S.	1	1.29	0.88	0.68	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.239	0.003	0.388	0.200	0.244	0.016	0.301	0.039

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	59	38	29	30	31	48	28	28
N.S.	1	1.34	0.86	0.66	0.68	0.70	1.09	0.64	0.64
time (sec)	N/A	0.288	0.011	0.447	0.211	0.250	0.019	0.283	0.029

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	47	21	15	23	37	99	14	20
N.S.	1	2.35	1.05	0.75	1.15	1.85	4.95	0.70	1.00
time (sec)	N/A	0.207	0.052	0.442	0.202	0.257	0.128	0.275	0.323

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	33	31	22	23	22	39	23	22
N.S.	1	1.06	1.00	0.71	0.74	0.71	1.26	0.74	0.71
time (sec)	N/A	0.172	0.054	0.591	0.198	0.251	0.086	0.357	0.269

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	18	20	37	32	20	27
N.S.	1	1.00	1.29	0.86	0.95	1.76	1.52	0.95	1.29
time (sec)	N/A	0.163	0.012	0.418	0.213	0.246	0.018	0.285	0.224

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	46	95	32	54	93	60	112	44
N.S.	1	1.28	2.64	0.89	1.50	2.58	1.67	3.11	1.22
time (sec)	N/A	0.305	0.016	0.460	0.196	0.257	0.074	0.305	0.274

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	37	33	40	66	33	51
N.S.	1	1.00	1.00	0.90	0.80	0.98	1.61	0.80	1.24
time (sec)	N/A	0.185	0.011	0.688	0.190	0.242	0.019	0.275	0.230

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	40	51	70	388	53	35
N.S.	1	1.00	1.00	1.00	1.28	1.75	9.70	1.32	0.88
time (sec)	N/A	0.218	0.016	0.506	0.195	0.252	0.526	0.298	0.657

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	19	18	18	31	18	18
N.S.	1	1.00	1.09	0.86	0.82	0.82	1.41	0.82	0.82
time (sec)	N/A	0.232	0.001	0.029	0.266	0.256	0.025	0.291	0.035

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	26	22	40	19	37	26
N.S.	1	1.00	1.15	1.30	1.10	2.00	0.95	1.85	1.30
time (sec)	N/A	0.238	0.013	0.058	0.184	0.254	0.042	0.292	0.274

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	25	30	70	20	53	24
N.S.	1	1.00	1.25	0.78	0.94	2.19	0.62	1.66	0.75
time (sec)	N/A	0.213	0.022	0.075	0.264	0.251	0.067	0.302	0.090

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	76	46	35	24	37	56	34	38
N.S.	1	1.36	0.82	0.62	0.43	0.66	1.00	0.61	0.68
time (sec)	N/A	0.390	0.041	0.444	0.177	0.252	0.018	0.283	0.045

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	55	26	25	25	27	25	25
N.S.	1	1.00	1.67	0.79	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.190	0.102	0.252	0.181	0.252	0.022	0.275	0.229

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	46	37	40	38	44	42	36
N.S.	1	1.04	1.00	0.80	0.87	0.83	0.96	0.91	0.78
time (sec)	N/A	0.195	0.014	0.529	0.177	0.265	0.038	0.277	0.035

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	53	28	37	55	44	26	27
N.S.	1	1.00	1.29	0.68	0.90	1.34	1.07	0.63	0.66
time (sec)	N/A	0.198	0.052	0.705	0.197	0.239	0.023	0.292	0.125

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	14	11	10	19	14	10	18
N.S.	1	1.21	0.58	0.46	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.209	0.002	0.066	0.179	0.255	0.020	0.272	0.056

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	61	22	17	16	31	31	16	32
N.S.	1	1.33	0.48	0.37	0.35	0.67	0.67	0.35	0.70
time (sec)	N/A	0.331	0.002	0.418	0.179	0.250	0.021	0.311	0.040

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	93	30	23	24	43	46	22	44
N.S.	1	1.37	0.44	0.34	0.35	0.63	0.68	0.32	0.65
time (sec)	N/A	0.470	0.012	0.481	0.192	0.260	0.021	0.273	0.028

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	125	38	29	30	55	61	28	56
N.S.	1	1.39	0.42	0.32	0.33	0.61	0.68	0.31	0.62
time (sec)	N/A	0.633	0.040	0.543	0.182	0.279	0.021	0.277	0.031

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	58	0	0	0	0	0	52
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.76
time (sec)	N/A	0.209	0.065	0.000	0.000	0.000	0.000	0.000	0.766

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	47	25	41	71	54	132	24
N.S.	1	1.00	1.47	0.78	1.28	2.22	1.69	4.12	0.75
time (sec)	N/A	0.189	0.037	0.479	0.193	0.258	0.569	0.305	0.339

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	17	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	2.12	0.75	0.75
time (sec)	N/A	0.164	0.003	0.170	0.183	0.242	0.018	0.338	0.029

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	14	22	15	14	11
N.S.	1	1.00	1.00	0.91	1.27	2.00	1.36	1.27	1.00
time (sec)	N/A	0.173	0.002	0.122	0.188	0.244	0.036	0.302	0.349

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.155	0.002	0.270	0.182	0.247	0.020	0.279	0.343

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	31	71	36	38	68	41	47	24
N.S.	1	1.19	2.73	1.38	1.46	2.62	1.58	1.81	0.92
time (sec)	N/A	0.260	0.034	0.134	0.194	0.247	0.068	0.264	0.337

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	14	13
N.S.	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.76
time (sec)	N/A	0.189	0.002	0.165	0.184	0.247	0.038	0.280	0.072

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	20	19	20	39	20	22
N.S.	1	1.00	0.77	0.65	0.61	0.65	1.26	0.65	0.71
time (sec)	N/A	0.187	0.077	0.332	0.193	0.248	13.120	0.293	1.142

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	22	14	13	27	0	13	32
N.S.	1	1.00	1.05	0.67	0.62	1.29	0.00	0.62	1.52
time (sec)	N/A	0.183	0.063	0.227	0.188	0.252	0.000	0.299	1.209

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	48	95	52	54	93	56	44	57
N.S.	1	1.26	2.50	1.37	1.42	2.45	1.47	1.16	1.50
time (sec)	N/A	0.349	0.037	0.230	0.185	0.254	0.066	0.296	0.316

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	77	74	76	74	82	0	95	75
N.S.	1	1.01	0.97	1.00	0.97	1.08	0.00	1.25	0.99
time (sec)	N/A	0.323	0.034	0.777	0.191	0.258	0.000	0.371	6.411

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	100	127	76	115	178	100	149	133
N.S.	1	1.14	1.44	0.86	1.31	2.02	1.14	1.69	1.51
time (sec)	N/A	0.748	3.584	59.217	0.277	0.287	158.344	0.293	0.410

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	74	46	54	54	148	54	94
N.S.	1	1.09	1.06	0.66	0.77	0.77	2.11	0.77	1.34
time (sec)	N/A	0.320	0.326	0.690	0.193	0.267	0.205	0.286	0.428

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	60	33	36	71	39	75	75
N.S.	1	1.00	1.82	1.00	1.09	2.15	1.18	2.27	2.27
time (sec)	N/A	0.366	0.048	0.125	0.281	0.262	0.219	0.296	0.576

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	18	20	21	14	26	24	14	26
N.S.	1	1.12	1.25	1.31	0.88	1.62	1.50	0.88	1.62
time (sec)	N/A	0.215	0.039	33.879	0.284	0.268	7.045	0.321	0.341

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	22	21	27	24	23	10
N.S.	1	1.00	1.00	1.83	1.75	2.25	2.00	1.92	0.83
time (sec)	N/A	0.199	0.015	0.622	0.183	0.274	0.429	0.276	0.291

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.163	0.007	0.211	0.186	0.248	0.120	0.270	0.255

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	45	26	31	54	43	75	38	33
N.S.	1	1.73	1.00	1.19	2.08	1.65	2.88	1.46	1.27
time (sec)	N/A	0.208	0.056	38.577	0.186	0.280	6.979	0.271	0.258

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	30	31	139	28	36
N.S.	1	1.00	1.00	0.76	0.79	0.82	3.66	0.74	0.95
time (sec)	N/A	0.202	0.013	0.530	0.198	0.257	1.594	0.287	0.321

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	42	20	35	33	43	22	21	21
N.S.	1	2.10	1.00	1.75	1.65	2.15	1.10	1.05	1.05
time (sec)	N/A	0.237	0.015	25.099	0.193	0.282	8.365	0.277	0.070

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	16	12	19	19	25	14	13	35
N.S.	1	1.33	1.00	1.58	1.58	2.08	1.17	1.08	2.92
time (sec)	N/A	0.192	0.013	2.118	0.195	0.280	2.490	0.274	0.372

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	41	39	35	208	64	0	75	47
N.S.	1	0.64	0.61	0.55	3.25	1.00	0.00	1.17	0.73
time (sec)	N/A	0.256	0.065	0.441	0.291	0.286	0.000	0.289	0.368

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	21	17	27	92	17	0	24	25
N.S.	1	1.91	1.55	2.45	8.36	1.55	0.00	2.18	2.27
time (sec)	N/A	0.192	0.016	0.882	0.184	0.278	0.000	0.284	0.716

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	11	7	12	9	11	0	9	5
N.S.	1	1.57	1.00	1.71	1.29	1.57	0.00	1.29	0.71
time (sec)	N/A	0.219	0.125	12.969	0.273	0.277	0.000	0.273	0.290

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	34	33	43	27	38	107	78	23
N.S.	1	0.64	0.62	0.81	0.51	0.72	2.02	1.47	0.43
time (sec)	N/A	0.199	0.091	0.629	0.263	0.277	4.582	0.281	0.241

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	55	22	20	39	66	39	37	17
N.S.	1	1.67	0.67	0.61	1.18	2.00	1.18	1.12	0.52
time (sec)	N/A	0.220	0.062	0.188	0.265	0.277	0.264	0.310	0.087

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	9	9	8	7	21	219	20	16
N.S.	1	0.33	0.33	0.30	0.26	0.78	8.11	0.74	0.59
time (sec)	N/A	0.176	0.111	0.809	0.273	0.280	4.411	0.287	0.321

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	43	41	31	28	46	102	29	38
N.S.	1	1.54	1.46	1.11	1.00	1.64	3.64	1.04	1.36
time (sec)	N/A	0.227	0.046	0.191	0.264	0.265	0.231	0.277	0.347

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	45	58	39	38	61	0	78	120
N.S.	1	0.67	0.87	0.58	0.57	0.91	0.00	1.16	1.79
time (sec)	N/A	0.215	0.204	0.324	0.272	0.270	0.000	0.278	0.476

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	37	54	29	28	58	0	61	77
N.S.	1	0.67	0.98	0.53	0.51	1.05	0.00	1.11	1.40
time (sec)	N/A	0.190	1.136	0.207	0.269	0.273	0.000	0.311	0.413

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	52	58	45	44	77	252	39	48
N.S.	1	1.24	1.38	1.07	1.05	1.83	6.00	0.93	1.14
time (sec)	N/A	0.429	1.840	0.093	0.265	0.259	0.238	0.286	0.365

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	20	0	19	76	21	7
N.S.	1	1.00	1.00	2.22	0.00	2.11	8.44	2.33	0.78
time (sec)	N/A	0.181	0.011	0.897	0.000	0.259	1.799	0.277	0.395

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	35	13	129	33	0	49	12
N.S.	1	1.00	2.33	0.87	8.60	2.20	0.00	3.27	0.80
time (sec)	N/A	0.181	0.089	0.463	0.274	0.274	0.000	0.310	0.129

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	15	28	21	128	26	22	20	9
N.S.	1	0.88	1.65	1.24	7.53	1.53	1.29	1.18	0.53
time (sec)	N/A	0.205	0.032	0.974	0.267	0.270	0.465	0.277	0.275

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	21	18	81	19	0	24	15
N.S.	1	1.29	1.00	0.86	3.86	0.90	0.00	1.14	0.71
time (sec)	N/A	0.213	0.020	2.367	0.268	0.266	0.000	0.282	0.122

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	21	27	129	19	17	24	17
N.S.	1	1.29	1.00	1.29	6.14	0.90	0.81	1.14	0.81
time (sec)	N/A	0.186	0.013	0.428	0.267	0.261	0.448	0.307	0.316

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	48	27
N.S.	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.191	0.028	0.322	0.281	0.258	3.242	0.290	0.501

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	25	70	28	171	50	294	48	27
N.S.	1	0.96	2.69	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.203	0.073	0.596	0.283	0.269	7.256	0.280	0.355

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	25	22	0	34	0	17	21
N.S.	1	1.00	1.56	1.38	0.00	2.12	0.00	1.06	1.31
time (sec)	N/A	0.159	0.017	0.514	0.000	0.257	0.000	0.309	0.276

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	31	0	35	0	29	23
N.S.	1	1.00	1.59	1.82	0.00	2.06	0.00	1.71	1.35
time (sec)	N/A	0.162	0.017	0.391	0.000	0.255	0.000	0.316	0.276

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	16	9	41	55	0	41	13
N.S.	1	1.00	0.59	0.33	1.52	2.04	0.00	1.52	0.48
time (sec)	N/A	0.172	0.012	0.156	0.310	0.253	0.000	0.275	0.061

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	17	101	58	0	16	28
N.S.	1	1.00	1.10	0.57	3.37	1.93	0.00	0.53	0.93
time (sec)	N/A	0.185	0.039	0.356	0.324	0.249	0.000	0.344	0.269

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	61	52	433	107	0	100	0
N.S.	1	1.00	1.15	0.98	8.17	2.02	0.00	1.89	0.00
time (sec)	N/A	0.241	0.188	0.510	0.330	0.268	0.000	0.308	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	47	0	71	0	72	0
N.S.	1	1.00	1.04	0.64	0.00	0.97	0.00	0.99	0.00
time (sec)	N/A	0.293	1.068	0.487	0.000	0.262	0.000	0.302	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	43	36	42	43	40	230	43	0
N.S.	1	0.78	0.65	0.76	0.78	0.73	4.18	0.78	0.00
time (sec)	N/A	0.398	0.097	0.493	0.181	0.269	37.509	0.325	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	110	40	49	80	73	0	80	65
N.S.	1	1.12	0.41	0.50	0.82	0.74	0.00	0.82	0.66
time (sec)	N/A	0.305	0.074	0.108	0.277	0.259	0.000	0.272	0.129

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	45	69	53	52	54	0	52	67
N.S.	1	0.79	1.21	0.93	0.91	0.95	0.00	0.91	1.18
time (sec)	N/A	0.242	0.062	0.096	0.269	0.259	0.000	0.332	0.747

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	73	130	3213	151	0	0	63
N.S.	1	1.06	0.84	1.49	36.93	1.74	0.00	0.00	0.72
time (sec)	N/A	0.418	0.113	0.287	0.519	0.256	0.000	0.000	0.468

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	31	30	82	0	31	105
N.S.	1	1.00	0.75	0.78	0.75	2.05	0.00	0.78	2.62
time (sec)	N/A	0.316	5.196	0.447	0.181	0.260	0.000	0.289	1.470

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	154	62	94	117	603	0	111	228
N.S.	1	1.83	0.74	1.12	1.39	7.18	0.00	1.32	2.71
time (sec)	N/A	0.598	0.319	0.083	0.279	0.933	0.000	0.293	1.283

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	266	0	137	0	0	0
N.S.	1	1.00	1.00	8.58	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.173	0.059	0.375	0.000	0.279	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	0
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.178	0.044	0.411	0.000	0.265	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	50	41	171	0	151	0	0	0
N.S.	1	1.11	0.91	3.80	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.235	0.050	2.493	0.000	0.273	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	396	0	76	0	0	0
N.S.	1	1.00	0.91	8.43	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.261	0.095	5.267	0.000	0.279	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	69	50	510	0	181	0	0	0
N.S.	1	1.13	0.82	8.36	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.416	0.133	6.089	0.000	0.278	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	68	56	1108	0	205	0	0	0
N.S.	1	1.11	0.92	18.16	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.398	0.107	0.729	0.000	0.268	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	508	0	39	0	0	18
N.S.	1	1.00	1.00	31.75	0.00	2.44	0.00	0.00	1.12
time (sec)	N/A	0.171	0.062	0.654	0.000	0.269	0.000	0.000	0.621

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	2946	0	32	0	0	20
N.S.	1	1.00	0.65	95.03	0.00	1.03	0.00	0.00	0.65
time (sec)	N/A	0.237	0.058	4.941	0.000	0.262	0.000	0.000	0.429

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	31	24	121	0	43	0	0	29
N.S.	1	1.07	0.83	4.17	0.00	1.48	0.00	0.00	1.00
time (sec)	N/A	0.283	0.050	0.375	0.000	0.251	0.000	0.000	0.511

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	65	111	761	0	136	0	0	0
N.S.	1	0.96	1.63	11.19	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	1.007	7.736	1.306	0.000	0.285	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	31	19	16	6	15	0	0	15
N.S.	1	1.63	1.00	0.84	0.32	0.79	0.00	0.00	0.79
time (sec)	N/A	0.252	0.010	0.876	0.300	0.257	0.000	0.000	0.594

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	92	161	66	299	0	628	0	0	0
N.S.	1	1.75	0.72	3.25	0.00	6.83	0.00	0.00	0.00
time (sec)	N/A	0.468	0.132	7.826	0.000	0.416	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	53	63	0	13	29	0	0	32
N.S.	1	1.13	1.34	0.00	0.28	0.62	0.00	0.00	0.68
time (sec)	N/A	0.231	0.175	0.000	0.280	0.247	0.000	0.000	4.036

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	61	35	0	77	101	0	0	110
N.S.	1	0.87	0.50	0.00	1.10	1.44	0.00	0.00	1.57
time (sec)	N/A	0.341	0.062	0.000	0.288	0.269	0.000	0.000	3.615

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	172	75	247	0	479	0	0	0
N.S.	1	1.59	0.69	2.29	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	1.163	0.502	3.679	0.000	0.459	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	364	781	378	133928	0	0	0	0	0
N.S.	1	2.15	1.04	367.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.303	19.658	57.052	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	151	58	0	60	56	0	0	0
N.S.	1	1.21	0.46	0.00	0.48	0.45	0.00	0.00	0.00
time (sec)	N/A	0.925	0.937	0.000	0.315	0.266	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	83	61	56	55	108	0	55	43
N.S.	1	1.14	0.84	0.77	0.75	1.48	0.00	0.75	0.59
time (sec)	N/A	0.216	0.168	1.084	0.290	0.287	0.000	0.304	0.137

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	79	48	54	53	88	0	41	0
N.S.	1	1.14	0.70	0.78	0.77	1.28	0.00	0.59	0.00
time (sec)	N/A	0.227	0.082	2.151	0.271	0.263	0.000	0.292	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	61	47	36	123	0	41	0
N.S.	1	1.09	1.05	0.81	0.62	2.12	0.00	0.71	0.00
time (sec)	N/A	0.221	0.081	0.261	0.285	0.264	0.000	0.285	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	37	44	43	51	0	30	28
N.S.	1	1.09	0.67	0.80	0.78	0.93	0.00	0.55	0.51
time (sec)	N/A	0.221	0.130	0.336	0.209	0.298	0.000	0.293	0.705

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	58	716	100	0	38	0
N.S.	1	1.00	1.00	1.49	18.36	2.56	0.00	0.97	0.00
time (sec)	N/A	0.231	0.110	0.742	0.346	0.288	0.000	0.287	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	53	63	105	69	131	0	40	0
N.S.	1	1.10	1.31	2.19	1.44	2.73	0.00	0.83	0.00
time (sec)	N/A	0.254	0.453	0.493	0.293	0.272	0.000	0.286	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	28	38	192	42	0	33	28
N.S.	1	1.08	0.57	0.78	3.92	0.86	0.00	0.67	0.57
time (sec)	N/A	0.299	0.096	0.356	0.227	0.271	0.000	0.274	0.627

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	119	295	131	115	127	0	0	0
N.S.	1	1.07	2.66	1.18	1.04	1.14	0.00	0.00	0.00
time (sec)	N/A	0.701	7.001	1.760	0.317	0.876	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	156	233	0	195	0	0	0
N.S.	1	1.00	1.39	2.08	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.590	0.719	1.150	0.000	0.298	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	29	38	25	26	0	35	0
N.S.	1	1.06	0.88	1.15	0.76	0.79	0.00	1.06	0.00
time (sec)	N/A	0.219	0.077	0.471	0.189	0.277	0.000	0.289	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	32	62	488	77	0	27	0
N.S.	1	1.12	0.97	1.88	14.79	2.33	0.00	0.82	0.00
time (sec)	N/A	0.179	0.040	0.472	0.324	0.267	0.000	0.289	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	72	49	55	790	103	0	48	29
N.S.	1	1.31	0.89	1.00	14.36	1.87	0.00	0.87	0.53
time (sec)	N/A	0.203	0.209	0.401	0.344	0.281	0.000	0.302	0.486

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	39	90	39	0	22	12
N.S.	1	1.00	1.00	2.44	5.62	2.44	0.00	1.38	0.75
time (sec)	N/A	0.168	0.058	0.582	0.306	0.255	0.000	0.293	0.437

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	68	49	100	0	118	0	0	0
N.S.	1	1.39	1.00	2.04	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.232	0.127	0.603	0.000	0.279	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	157	62	180	1359	163	0	55	0
N.S.	1	1.80	0.71	2.07	15.62	1.87	0.00	0.63	0.00
time (sec)	N/A	0.543	0.426	2.613	0.412	0.291	0.000	0.287	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	115	217	0	130	0	0	0
N.S.	1	1.04	1.69	3.19	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.230	0.266	7.204	0.000	0.269	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	79	124	0	115	0	0	0
N.S.	1	1.00	1.98	3.10	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.194	0.137	1.365	0.000	0.275	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	100	131	151	0	97	0	0	0
N.S.	1	1.06	1.39	1.61	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.448	0.600	4.208	0.000	0.332	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	36	67	63	33	0	135	20
N.S.	1	1.05	0.92	1.72	1.62	0.85	0.00	3.46	0.51
time (sec)	N/A	0.293	0.093	2.726	0.205	0.270	0.000	0.283	0.853

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	129	1498	0	257	0	121	0
N.S.	1	1.00	1.77	20.52	0.00	3.52	0.00	1.66	0.00
time (sec)	N/A	1.222	5.136	2.001	0.000	0.347	0.000	0.328	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	116	62	47	84	0	0	113
N.S.	1	1.00	2.04	1.09	0.82	1.47	0.00	0.00	1.98
time (sec)	N/A	0.843	5.350	3.319	0.282	0.275	0.000	0.000	1.612

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	76	49	61	0	50	0	52	90
N.S.	1	1.15	0.74	0.92	0.00	0.76	0.00	0.79	1.36
time (sec)	N/A	0.222	0.081	0.074	0.000	0.275	0.000	0.271	1.623

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	63	71	41	0	76	46	36	172
N.S.	1	1.17	1.31	0.76	0.00	1.41	0.85	0.67	3.19
time (sec)	N/A	0.212	0.661	0.056	0.000	0.283	2.317	0.268	0.525

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	138	105	0	0	0	0	186	250
N.S.	1	1.04	0.79	0.00	0.00	0.00	0.00	1.40	1.88
time (sec)	N/A	0.308	0.157	0.000	0.000	0.000	0.000	0.308	1.433

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	91	69	0	0	117	0	79	101
N.S.	1	1.32	1.00	0.00	0.00	1.70	0.00	1.14	1.46
time (sec)	N/A	0.247	0.171	0.000	0.000	0.628	0.000	0.281	0.750

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F(-1)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	59	256	0	71	0	0	73	46
N.S.	1	1.13	4.92	0.00	1.37	0.00	0.00	1.40	0.88
time (sec)	N/A	0.254	0.323	0.000	0.281	0.000	0.000	0.482	0.438

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	61	245	0	74	0	0	76	0
N.S.	1	1.13	4.54	0.00	1.37	0.00	0.00	1.41	0.00
time (sec)	N/A	0.258	0.302	0.000	0.287	0.000	0.000	0.410	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-1)	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	200	1447	0	0	0	0	0	0
N.S.	1	1.50	10.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.486	25.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	138	169	0	0	271	0	193	0
N.S.	1	1.38	1.69	0.00	0.00	2.71	0.00	1.93	0.00
time (sec)	N/A	1.128	4.996	0.000	0.000	0.303	0.000	0.397	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	114	47	127	136	185	0	0	0
N.S.	1	1.02	0.42	1.13	1.21	1.65	0.00	0.00	0.00
time (sec)	N/A	0.325	0.038	0.688	0.283	0.285	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	159	154	0	145	0	0	146	0
N.S.	1	1.67	1.62	0.00	1.53	0.00	0.00	1.54	0.00
time (sec)	N/A	0.341	0.103	0.000	0.301	0.000	0.000	0.372	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	53	30	0	37	46	0	37	0
N.S.	1	1.08	0.61	0.00	0.76	0.94	0.00	0.76	0.00
time (sec)	N/A	0.264	0.358	0.000	0.199	0.307	0.000	0.281	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	26	86	35	0	25	43
N.S.	1	1.00	1.00	1.30	4.30	1.75	0.00	1.25	2.15
time (sec)	N/A	0.403	0.271	0.184	0.259	0.266	0.000	0.329	0.635

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	27	35	97	0	0	93	0	40	0
N.S.	1	1.30	3.59	0.00	0.00	3.44	0.00	1.48	0.00
time (sec)	N/A	0.977	0.422	0.000	0.000	0.956	0.000	0.310	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	126	89	0	100	461	0	0	0
N.S.	1	1.25	0.88	0.00	0.99	4.56	0.00	0.00	0.00
time (sec)	N/A	1.279	0.452	0.000	0.296	39.137	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	37	20	0	0	26	0	25	0
N.S.	1	1.48	0.80	0.00	0.00	1.04	0.00	1.00	0.00
time (sec)	N/A	0.215	0.081	0.000	0.000	0.271	0.000	0.293	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	189	153	0	0	0	0	120	0
N.S.	1	1.85	1.50	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.352	0.106	0.000	0.000	0.000	0.000	0.275	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	18	45	78	259	50	0	85	0
N.S.	1	1.06	2.65	4.59	15.24	2.94	0.00	5.00	0.00
time (sec)	N/A	0.212	0.078	0.693	0.304	0.284	0.000	0.291	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	39	52	143	507	115	0	138	0
N.S.	1	1.22	1.62	4.47	15.84	3.59	0.00	4.31	0.00
time (sec)	N/A	0.201	0.088	6.694	0.472	0.267	0.000	0.365	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	37	31	24	27	30	24	32	24
N.S.	1	1.19	1.00	0.77	0.87	0.97	0.77	1.03	0.77
time (sec)	N/A	0.152	0.004	0.288	0.203	0.249	0.048	0.275	0.341

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	49	39	29	30	32	32	30	30
N.S.	1	1.26	1.00	0.74	0.77	0.82	0.82	0.77	0.77
time (sec)	N/A	0.144	0.010	0.282	0.281	0.266	0.055	0.263	0.036

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	64	40	34	46	73	41	42	44
N.S.	1	1.10	0.69	0.59	0.79	1.26	0.71	0.72	0.76
time (sec)	N/A	0.176	0.012	0.304	0.211	0.262	0.061	0.256	0.085

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	54	46	35	33	65	984	35	34
N.S.	1	1.04	0.88	0.67	0.63	1.25	18.92	0.67	0.65
time (sec)	N/A	0.150	0.045	0.420	0.282	0.253	1.810	0.263	0.550

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	71	49	41	42	47	165	46	46
N.S.	1	1.16	0.80	0.67	0.69	0.77	2.70	0.75	0.75
time (sec)	N/A	0.161	0.039	0.434	0.278	0.255	3.434	0.273	0.488

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	15	15	14	24
N.S.	1	1.00	0.94	0.94	0.88	0.94	0.94	0.88	1.50
time (sec)	N/A	0.123	0.002	0.014	0.209	0.259	0.016	0.258	0.498

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	42	36	30	35	49	32	36	35
N.S.	1	0.91	0.78	0.65	0.76	1.07	0.70	0.78	0.76
time (sec)	N/A	0.158	0.010	0.253	0.201	0.252	0.042	0.267	0.059

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	23	21	36	36	32	21	36
N.S.	1	1.10	0.58	0.52	0.90	0.90	0.80	0.52	0.90
time (sec)	N/A	0.176	0.006	0.292	0.216	0.253	0.050	0.267	0.070

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	31	25	17	19	21	41	19	20
N.S.	1	1.15	0.93	0.63	0.70	0.78	1.52	0.70	0.74
time (sec)	N/A	0.144	0.016	0.299	0.197	0.240	2.710	0.261	0.439

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	25	20	28	33	82	26	21
N.S.	1	1.11	0.66	0.53	0.74	0.87	2.16	0.68	0.55
time (sec)	N/A	0.150	0.019	0.295	0.193	0.245	0.794	0.261	0.461

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	24	20	25	34	61	19	99
N.S.	1	1.00	0.73	0.61	0.76	1.03	1.85	0.58	3.00
time (sec)	N/A	0.128	0.031	0.250	0.203	0.238	0.856	0.275	0.323

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	26	23	51	61	0	21	22
N.S.	1	1.00	0.60	0.53	1.19	1.42	0.00	0.49	0.51
time (sec)	N/A	0.143	0.153	0.351	0.192	0.250	0.000	0.283	0.314

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	28	23	35	40	153	62	24
N.S.	1	1.11	0.60	0.49	0.74	0.85	3.26	1.32	0.51
time (sec)	N/A	0.139	0.036	0.291	0.284	0.247	1.175	0.278	0.459

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	16	16	15	24	16	17
N.S.	1	1.00	0.68	0.57	0.57	0.54	0.86	0.57	0.61
time (sec)	N/A	0.136	0.014	0.254	0.195	0.248	0.808	0.267	0.039

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	58	49	47	53	74	49	58	51
N.S.	1	1.12	0.94	0.90	1.02	1.42	0.94	1.12	0.98
time (sec)	N/A	0.167	0.018	0.267	0.202	0.242	0.072	0.277	0.362

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	21	0	0	15
N.S.	1	1.00	1.00	0.88	0.00	0.84	0.00	0.00	0.60
time (sec)	N/A	0.171	6.487	0.260	0.000	0.390	0.000	0.000	0.501

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	35	28	79	69	0	98	0
N.S.	1	1.00	0.70	0.56	1.58	1.38	0.00	1.96	0.00
time (sec)	N/A	0.147	0.191	0.336	0.284	0.247	0.000	0.320	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	40	22	21	41	49	29	54
N.S.	1	1.00	1.67	0.92	0.88	1.71	2.04	1.21	2.25
time (sec)	N/A	0.129	0.054	0.323	0.286	0.241	0.181	0.273	0.325

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	32	45	45	32	77	33
N.S.	1	1.00	1.15	0.80	1.12	1.12	0.80	1.92	0.82
time (sec)	N/A	0.138	0.056	0.473	0.274	0.264	0.376	0.275	0.042

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	28	25	37	44	291	35	179
N.S.	1	1.10	0.57	0.51	0.76	0.90	5.94	0.71	3.65
time (sec)	N/A	0.138	0.169	0.298	0.180	0.275	2.995	0.284	0.305

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	31	28	59	47	0	35	29
N.S.	1	1.00	0.66	0.60	1.26	1.00	0.00	0.74	0.62
time (sec)	N/A	0.144	0.135	0.408	0.189	0.244	0.000	0.274	0.358

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	32	34	32	32
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.94	0.89	0.89
time (sec)	N/A	0.158	0.002	0.381	0.184	0.261	0.021	0.305	0.034

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	29	29	34	29	29
N.S.	1	1.00	1.00	0.77	0.74	0.74	0.87	0.74	0.74
time (sec)	N/A	0.155	0.001	0.327	0.182	0.223	0.021	0.265	0.030

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	73	0	27	29
N.S.	1	1.00	0.70	0.64	1.26	1.55	0.00	0.57	0.62
time (sec)	N/A	0.150	0.195	0.377	0.182	0.253	0.000	0.279	0.414

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	30	76	73	0	41	29
N.S.	1	1.00	0.73	0.67	1.69	1.62	0.00	0.91	0.64
time (sec)	N/A	0.168	0.310	0.372	0.185	0.234	0.000	0.295	0.184

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	113	67	56	55	57	112	55	69
N.S.	1	1.36	0.81	0.67	0.66	0.69	1.35	0.66	0.83
time (sec)	N/A	0.619	0.039	0.530	0.195	0.242	0.368	0.278	0.423

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	92	51	50	49	52	92	49	59
N.S.	1	1.26	0.70	0.68	0.67	0.71	1.26	0.67	0.81
time (sec)	N/A	0.632	0.067	0.531	0.193	0.244	0.254	0.312	0.375

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	181	70	67	66	72	192	66	88
N.S.	1	1.72	0.67	0.64	0.63	0.69	1.83	0.63	0.84
time (sec)	N/A	0.767	0.059	0.842	0.197	0.257	0.547	0.265	0.418

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	48	39	32	35	36	53	35	40
N.S.	1	1.09	0.89	0.73	0.80	0.82	1.20	0.80	0.91
time (sec)	N/A	0.296	0.074	0.448	0.201	0.244	0.178	0.255	0.097

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	47	0	45	507	206	56
N.S.	1	1.00	1.00	1.42	0.00	1.36	15.36	6.24	1.70
time (sec)	N/A	0.296	0.025	0.333	0.000	0.249	0.683	0.270	0.635

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	104	43	619	47	551	341	35
N.S.	1	1.00	3.47	1.43	20.63	1.57	18.37	11.37	1.17
time (sec)	N/A	0.191	0.114	0.295	0.283	0.242	0.643	0.427	0.561

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	132	15	128	53	16
N.S.	1	1.00	1.00	0.81	8.25	0.94	8.00	3.31	1.00
time (sec)	N/A	0.191	0.024	0.381	0.182	0.243	0.410	0.273	0.384

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	74	57	57	66	113	0	0	0
N.S.	1	1.19	0.92	0.92	1.06	1.82	0.00	0.00	0.00
time (sec)	N/A	0.391	0.023	0.305	0.297	0.264	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	66	54	59	210	138	0	0	0
N.S.	1	1.12	0.92	1.00	3.56	2.34	0.00	0.00	0.00
time (sec)	N/A	0.339	0.011	0.149	0.289	0.258	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	44	78	13	0	10	0
N.S.	1	1.00	1.17	3.67	6.50	1.08	0.00	0.83	0.00
time (sec)	N/A	0.229	0.228	3.049	0.278	0.249	0.000	0.276	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	20	69	19	66	39	0
N.S.	1	1.00	0.95	1.00	3.45	0.95	3.30	1.95	0.00
time (sec)	N/A	0.224	0.288	0.505	0.199	0.246	0.716	0.286	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	22	42	325	22
N.S.	1	1.00	1.00	1.05	0.00	1.00	1.91	14.77	1.00
time (sec)	N/A	0.165	0.021	0.079	0.000	0.237	0.307	0.278	0.415

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	41	46	42	0	52	0	436	34
N.S.	1	1.21	1.35	1.24	0.00	1.53	0.00	12.82	1.00
time (sec)	N/A	0.359	0.041	0.087	0.000	0.249	0.000	0.331	0.446

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	11	7	7	4
N.S.	1	1.00	1.00	0.89	0.78	1.22	0.78	0.78	0.44
time (sec)	N/A	0.132	0.008	0.027	0.187	0.228	0.033	0.272	0.050

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	25	22	17	16	21	17	24	8
N.S.	1	1.14	1.00	0.77	0.73	0.95	0.77	1.09	0.36
time (sec)	N/A	0.168	0.010	0.035	0.200	0.245	0.042	0.262	0.055

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	24	23	24	24	25	23
N.S.	1	1.00	0.97	0.77	0.74	0.77	0.77	0.81	0.74
time (sec)	N/A	0.161	0.027	0.046	0.196	0.231	0.054	0.272	0.349

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	37	29	28	31	31	36	28
N.S.	1	1.06	1.03	0.81	0.78	0.86	0.86	1.00	0.78
time (sec)	N/A	0.169	0.041	0.049	0.199	0.248	0.067	0.285	0.335

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	52	45	0	0	0	0	0	0
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	40	34	44	41	39	53	46	41
N.S.	1	0.93	0.79	1.02	0.95	0.91	1.23	1.07	0.95
time (sec)	N/A	0.185	0.061	0.181	0.203	0.249	0.114	0.281	0.499

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	27	26	29	27	26
N.S.	1	1.00	1.00	1.00	1.00	0.96	1.07	1.00	0.96
time (sec)	N/A	0.147	0.013	0.040	0.228	0.244	0.059	0.281	0.400

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	46	53	55	51	62	250	691	68
N.S.	1	0.87	1.00	1.04	0.96	1.17	4.72	13.04	1.28
time (sec)	N/A	0.276	0.043	0.070	0.192	0.255	0.461	0.313	0.410

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	68	65	84	77	130	665	1033	81
N.S.	1	0.86	0.82	1.06	0.97	1.65	8.42	13.08	1.03
time (sec)	N/A	0.298	0.072	0.117	0.193	0.244	2.049	0.315	0.439

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	83	80	109	99	205	1350	1359	106
N.S.	1	0.85	0.82	1.11	1.01	2.09	13.78	13.87	1.08
time (sec)	N/A	0.318	0.077	0.118	0.202	0.248	16.753	0.354	0.443

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	80	73	0	0	0	0	0	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	28	28	29	28	27
N.S.	1	1.00	1.00	1.00	1.00	1.00	1.04	1.00	0.96
time (sec)	N/A	0.141	0.010	0.036	0.227	0.242	0.059	0.268	0.330

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	46	53	55	51	64	248	691	69
N.S.	1	0.87	1.00	1.04	0.96	1.21	4.68	13.04	1.30
time (sec)	N/A	0.274	0.064	0.051	0.208	0.239	0.444	0.310	0.384

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	68	66	84	77	131	663	1033	81
N.S.	1	0.86	0.84	1.06	0.97	1.66	8.39	13.08	1.03
time (sec)	N/A	0.300	0.103	0.076	0.206	0.267	2.051	0.325	0.441

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	83	80	109	99	207	1348	1359	106
N.S.	1	0.85	0.82	1.11	1.01	2.11	13.76	13.87	1.08
time (sec)	N/A	0.313	0.107	0.082	0.204	0.245	16.580	0.355	0.403

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	82	75	0	0	0	0	0	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	19	15	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.27	1.00	1.00	1.00
time (sec)	N/A	0.131	0.008	0.028	0.198	0.234	0.036	0.286	0.360

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	35	31	31	29	44	30	26
N.S.	1	0.97	1.06	0.94	0.94	0.88	1.33	0.91	0.79
time (sec)	N/A	0.156	0.017	0.029	0.194	0.240	0.049	0.287	0.352

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	42	45	41	46	39	70	40	34
N.S.	1	0.84	0.90	0.82	0.92	0.78	1.40	0.80	0.68
time (sec)	N/A	0.161	0.020	0.042	0.196	0.254	0.061	0.292	0.359

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	50	53	50	61	47	87	48	42
N.S.	1	0.77	0.82	0.77	0.94	0.72	1.34	0.74	0.65
time (sec)	N/A	0.164	0.022	0.042	0.193	0.253	0.067	0.329	0.366

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	55
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.38
time (sec)	N/A	0.156	0.040	0.000	0.000	0.000	0.000	0.000	0.361

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	21	19	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.31	1.19	1.00	1.00
time (sec)	N/A	0.132	0.005	0.023	0.194	0.229	0.037	0.307	0.338

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	35	31	31	29	44	30	27
N.S.	1	0.97	1.06	0.94	0.94	0.88	1.33	0.91	0.82
time (sec)	N/A	0.159	0.018	0.030	0.221	0.237	0.050	0.300	0.345

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	42	45	41	46	39	70	40	35
N.S.	1	0.84	0.90	0.82	0.92	0.78	1.40	0.80	0.70
time (sec)	N/A	0.161	0.021	0.042	0.199	0.228	0.064	0.346	0.350

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	50	53	50	61	47	87	48	43
N.S.	1	0.77	0.82	0.77	0.94	0.72	1.34	0.74	0.66
time (sec)	N/A	0.165	0.023	0.046	0.200	0.241	0.071	0.299	0.352

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	57
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.160	0.040	0.000	0.000	0.000	0.000	0.000	0.342

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	30	38	23	23	22	15	26	22
N.S.	1	1.25	1.58	0.96	0.96	0.92	0.62	1.08	0.92
time (sec)	N/A	0.149	0.019	0.030	0.202	0.242	0.048	0.269	0.359

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	119	97	26	100	311	22	116	104
N.S.	1	1.19	0.97	0.26	1.00	3.11	0.22	1.16	1.04
time (sec)	N/A	0.278	0.081	0.060	0.277	0.262	0.075	0.271	1.583

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	15	15	12	11	11	8	11	11
N.S.	1	1.25	1.25	1.00	0.92	0.92	0.67	0.92	0.92
time (sec)	N/A	0.166	0.014	0.032	0.193	0.252	0.030	0.279	0.049

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	54	45	38	37	39	22	37	39
N.S.	1	1.15	0.96	0.81	0.79	0.83	0.47	0.79	0.83
time (sec)	N/A	0.217	0.043	0.044	0.299	0.245	0.067	0.286	0.426

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	29	28	28	48	29	28
N.S.	1	1.00	0.95	0.74	0.72	0.72	1.23	0.74	0.72
time (sec)	N/A	0.222	0.049	0.053	0.286	0.252	0.094	0.278	0.094

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	32	37	94	32	29
N.S.	1	1.00	1.00	1.10	1.07	1.23	3.13	1.07	0.97
time (sec)	N/A	0.173	0.046	0.075	0.190	0.243	0.421	0.288	0.457

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	54	57	56	56	61	57	33
N.S.	1	1.02	1.00	1.06	1.04	1.04	1.13	1.06	0.61
time (sec)	N/A	0.170	0.030	0.066	0.268	0.236	0.535	0.293	0.408

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	75
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.27
time (sec)	N/A	0.176	0.036	0.000	0.000	0.000	0.000	0.000	0.481

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	22	15	7	18	37	18	14
N.S.	1	1.00	1.22	0.83	0.39	1.00	2.06	1.00	0.78
time (sec)	N/A	0.160	0.028	0.037	0.205	0.242	0.257	0.272	0.492

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	24	17	20	20	37	20	16
N.S.	1	1.00	1.20	0.85	1.00	1.00	1.85	1.00	0.80
time (sec)	N/A	0.162	0.029	0.037	0.205	0.237	0.274	0.277	0.520

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	0	39	46	0	0	0
N.S.	1	1.00	1.05	0.00	0.98	1.15	0.00	0.00	0.00
time (sec)	N/A	0.223	0.113	0.000	0.277	0.237	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	57	54	0	0	0	0	0	0
N.S.	1	1.54	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	75	44	37	49	30	49	65	30
N.S.	1	1.03	0.60	0.51	0.67	0.41	0.67	0.89	0.41
time (sec)	N/A	0.200	0.040	0.032	0.207	0.249	0.915	0.322	0.136

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	50	23	21	19	19	20	19	21
N.S.	1	1.14	0.52	0.48	0.43	0.43	0.45	0.43	0.48
time (sec)	N/A	0.218	0.015	0.029	0.197	0.247	0.031	0.276	0.027

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	26	27	7	23	32	27	22
N.S.	1	1.05	0.67	0.69	0.18	0.59	0.82	0.69	0.56
time (sec)	N/A	0.193	0.024	0.068	0.222	0.247	0.732	0.272	0.364

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	53	29	28	27	27	37	826	27
N.S.	1	1.20	0.66	0.64	0.61	0.61	0.84	18.77	0.61
time (sec)	N/A	0.197	0.015	0.033	0.200	0.229	0.048	0.288	0.058

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	18	9	8	18	9
N.S.	1	1.00	1.00	0.83	1.50	0.75	0.67	1.50	0.75
time (sec)	N/A	0.176	0.021	0.036	0.205	0.240	0.033	0.278	0.058

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	31	25	25	33	24	40	26
N.S.	1	1.12	0.97	0.78	0.78	1.03	0.75	1.25	0.81
time (sec)	N/A	0.216	0.074	0.043	0.282	0.237	0.046	0.285	0.422

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	21	12	0	0	12
N.S.	1	1.00	1.00	1.33	1.40	0.80	0.00	0.00	0.80
time (sec)	N/A	0.180	0.176	0.267	0.254	0.242	0.000	0.000	0.522

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.156	0.030	0.105	0.204	0.246	0.172	0.291	0.040

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	51	26	18	39	21	29	39	19
N.S.	1	1.46	0.74	0.51	1.11	0.60	0.83	1.11	0.54
time (sec)	N/A	0.341	0.072	0.257	0.201	0.250	0.235	0.269	0.111

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	70	37	32	31	0	70	39	33
N.S.	1	1.23	0.65	0.56	0.54	0.00	1.23	0.68	0.58
time (sec)	N/A	0.208	0.079	0.247	0.290	0.000	0.462	0.264	0.044

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	39	37	45	37	265	43	37
N.S.	1	1.00	0.72	0.69	0.83	0.69	4.91	0.80	0.69
time (sec)	N/A	0.201	0.027	0.262	0.214	0.251	0.434	0.268	0.058

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	81	64	60	73	65	638	63	47
N.S.	1	0.99	0.78	0.73	0.89	0.79	7.78	0.77	0.57
time (sec)	N/A	0.244	0.122	0.336	0.218	0.249	1.216	0.283	0.063

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	99	36	28	27	42	76	33	39
N.S.	1	1.25	0.46	0.35	0.34	0.53	0.96	0.42	0.49
time (sec)	N/A	0.262	0.056	0.381	0.193	0.244	0.444	0.272	0.384

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	19	27	40	70	24	18
N.S.	1	1.00	0.58	0.53	0.75	1.11	1.94	0.67	0.50
time (sec)	N/A	0.192	0.033	0.191	0.208	0.243	0.522	0.277	0.432

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	19	27	50	99	24	18
N.S.	1	1.00	0.58	0.53	0.75	1.39	2.75	0.67	0.50
time (sec)	N/A	0.191	0.040	0.266	0.192	0.255	0.536	0.280	0.452

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	85	97	0	0	0	0	0	0
N.S.	1	1.47	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	90	0	0	0	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	81	66	0	0	0	0	0	0
N.S.	1	1.59	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	84	0	0	0	0	0	0
N.S.	1	1.00	3.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.614	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.622	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	11	22	12	0	10	8
N.S.	1	1.00	0.80	0.73	1.47	0.80	0.00	0.67	0.53
time (sec)	N/A	0.159	0.106	0.243	0.267	0.251	0.000	0.289	0.470

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	46	100	0	0	0	0	0	0
N.S.	1	1.12	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	8	22	11	0	7	7
N.S.	1	1.00	1.00	0.67	1.83	0.92	0.00	0.58	0.58
time (sec)	N/A	0.158	0.098	0.181	0.272	0.250	0.000	0.286	0.464

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	45	87	0	0	0	0	0	0
N.S.	1	1.07	2.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.435	0.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	49	72	0	0	0	0	0	0
N.S.	1	1.07	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	13	19	22	24	0	20	24
N.S.	1	1.00	0.93	1.36	1.57	1.71	0.00	1.43	1.71
time (sec)	N/A	0.151	0.046	0.251	0.273	0.249	0.000	0.290	0.502

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	48	73	0	0	0	0	0	0
N.S.	1	1.12	1.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.362	0.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	22	24	0	21	20
N.S.	1	1.00	1.00	1.38	1.69	1.85	0.00	1.62	1.54
time (sec)	N/A	0.156	0.044	0.248	0.276	0.252	0.000	0.297	0.528

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	16	17	17	27	15	17
N.S.	1	1.00	0.60	0.53	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.189	0.013	0.085	0.202	0.245	0.149	0.296	0.068

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	25	22	26	26	48	25	21
N.S.	1	1.00	0.50	0.44	0.52	0.52	0.96	0.50	0.42
time (sec)	N/A	0.269	0.027	0.110	0.208	0.241	0.285	0.296	0.404

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	38	31	33	37	80	33	39
N.S.	1	1.07	0.51	0.41	0.44	0.49	1.07	0.44	0.52
time (sec)	N/A	0.305	0.028	0.130	0.214	0.255	0.373	0.282	0.456

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	258	72	59	77	72	202	73	83
N.S.	1	1.38	0.39	0.32	0.41	0.39	1.08	0.39	0.44
time (sec)	N/A	0.622	0.099	0.345	0.219	0.250	1.006	0.262	0.359

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	40	40	41	41	85	39	51
N.S.	1	1.06	0.46	0.46	0.47	0.47	0.98	0.45	0.59
time (sec)	N/A	0.325	0.056	0.128	0.197	0.241	0.295	0.270	0.447

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	76	78	77	72	202	73	83
N.S.	1	1.00	0.41	0.42	0.42	0.39	1.09	0.39	0.45
time (sec)	N/A	0.477	0.147	0.859	0.201	0.259	1.020	0.290	0.639

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.137	0.002	0.025	0.187	0.243	0.064	0.286	0.020

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.137	0.002	0.029	0.215	0.234	0.066	0.287	0.025

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00
time (sec)	N/A	0.142	0.000	0.045	0.221	0.243	0.046	0.285	0.021

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	18	12	12	3
N.S.	1	1.00	2.33	1.33	1.00	6.00	4.00	4.00	1.00
time (sec)	N/A	0.144	0.000	0.064	0.199	0.252	0.133	0.285	0.030

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	8	7	5	5
N.S.	1	1.00	1.00	1.33	1.00	2.67	2.33	1.67	1.67
time (sec)	N/A	0.143	0.002	0.195	0.189	0.236	0.178	0.328	0.023

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	17	6	5	17	5	14	5
N.S.	1	1.00	3.40	1.20	1.00	3.40	1.00	2.80	1.00
time (sec)	N/A	0.143	0.013	0.035	0.201	0.247	0.121	0.286	0.013

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	10	24	24	10
N.S.	1	1.00	1.00	0.79	1.14	0.71	1.71	1.71	0.71
time (sec)	N/A	0.151	0.006	0.079	0.195	0.236	0.064	0.279	0.028

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	18	35	42	29	37	15
N.S.	1	1.00	1.21	0.95	1.84	2.21	1.53	1.95	0.79
time (sec)	N/A	0.161	0.012	0.490	0.213	0.236	0.182	0.290	0.029

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	38	68	10	26	12
N.S.	1	1.00	1.14	0.93	2.71	4.86	0.71	1.86	0.86
time (sec)	N/A	0.186	0.005	0.037	0.219	0.242	0.078	0.284	0.072

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	24	47	11	45	211	0	45	16
N.S.	1	1.50	2.94	0.69	2.81	13.19	0.00	2.81	1.00
time (sec)	N/A	0.191	0.012	0.373	0.211	0.230	0.000	0.272	0.392

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	31	26	21	61	461	422	60	22
N.S.	1	1.19	1.00	0.81	2.35	17.73	16.23	2.31	0.85
time (sec)	N/A	0.250	0.004	0.965	0.284	0.245	1.168	0.277	0.074

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	24	18	17	35	257	0	43	35
N.S.	1	1.33	1.00	0.94	1.94	14.28	0.00	2.39	1.94
time (sec)	N/A	0.176	0.006	6.589	0.293	0.246	0.000	0.279	0.442

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	20	0	359	41	0	120
N.S.	1	1.00	0.87	0.65	0.00	11.58	1.32	0.00	3.87
time (sec)	N/A	0.186	0.054	0.200	0.000	0.237	39.538	0.000	0.177

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	0	175	126	32	43
N.S.	1	1.02	1.00	0.88	0.00	4.27	3.07	0.78	1.05
time (sec)	N/A	0.195	0.031	0.102	0.000	0.249	1.829	0.274	0.183

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	15	49	58	14	14	14
N.S.	1	1.00	0.64	0.60	1.96	2.32	0.56	0.56	0.56
time (sec)	N/A	0.204	0.009	0.057	0.207	0.229	0.163	0.280	0.395

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	42	41	42	146	43	35
N.S.	1	1.00	1.26	1.08	1.05	1.08	3.74	1.10	0.90
time (sec)	N/A	0.270	0.086	0.050	0.201	0.247	0.223	0.272	0.146

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	33	31	98	76	288	1129	79	109
N.S.	1	1.06	1.00	3.16	2.45	9.29	36.42	2.55	3.52
time (sec)	N/A	0.195	0.452	0.316	0.320	0.250	15.576	0.273	0.766

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	37	35	74	0	388	874	50	106
N.S.	1	1.06	1.00	2.11	0.00	11.09	24.97	1.43	3.03
time (sec)	N/A	0.203	0.077	0.219	0.000	0.270	14.555	0.273	0.399

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	46	69	113	908	48	63
N.S.	1	1.00	0.96	1.84	2.76	4.52	36.32	1.92	2.52
time (sec)	N/A	0.176	0.270	0.447	0.281	0.257	2.479	0.271	0.519

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	37	44	70	74	102	22	22
N.S.	1	1.00	1.12	1.33	2.12	2.24	3.09	0.67	0.67
time (sec)	N/A	0.308	5.033	0.312	0.286	0.248	0.518	0.291	0.467

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	42	44	116	48	40
N.S.	1	1.00	1.00	0.77	1.40	1.47	3.87	1.60	1.33
time (sec)	N/A	0.199	0.055	2.092	0.194	0.249	0.939	0.284	0.502

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	42	111	139	48	40
N.S.	1	1.00	1.00	0.77	1.40	3.70	4.63	1.60	1.33
time (sec)	N/A	0.201	0.038	2.012	0.275	0.242	0.938	0.287	0.559

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	67	160	167	0	376	0	90	0
N.S.	1	0.97	2.32	2.42	0.00	5.45	0.00	1.30	0.00
time (sec)	N/A	0.808	18.500	1.328	0.000	0.260	0.000	0.335	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	26	30	125	474	0	38	57
N.S.	1	1.00	0.70	0.81	3.38	12.81	0.00	1.03	1.54
time (sec)	N/A	0.201	0.070	0.180	0.212	0.260	0.000	0.322	0.143

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	21	30	177	161	0	0	47
N.S.	1	1.00	0.72	1.03	6.10	5.55	0.00	0.00	1.62
time (sec)	N/A	0.269	0.038	0.236	0.332	0.262	0.000	0.000	0.525

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	63	0	482	0	58	0
N.S.	1	1.00	1.00	4.20	0.00	32.13	0.00	3.87	0.00
time (sec)	N/A	0.174	0.009	0.490	0.000	0.258	0.000	0.299	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	49	93	17	51	21
N.S.	1	1.00	1.00	1.50	3.06	5.81	1.06	3.19	1.31
time (sec)	N/A	0.200	0.015	0.043	0.313	0.254	0.064	0.299	0.431

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	53	95	22	53	27
N.S.	1	1.00	1.00	1.75	3.31	5.94	1.38	3.31	1.69
time (sec)	N/A	0.209	0.015	0.067	0.238	0.250	0.263	0.280	0.399

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	14	13	20	26	11	16
N.S.	1	1.00	1.15	0.70	0.65	1.00	1.30	0.55	0.80
time (sec)	N/A	0.259	0.043	0.418	0.198	0.253	0.187	0.311	0.063

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	16	35	20	12	14	17
N.S.	1	1.00	1.33	1.07	2.33	1.33	0.80	0.93	1.13
time (sec)	N/A	0.266	0.061	0.186	0.193	0.240	0.149	0.293	0.342

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	22	75	0	24	24
N.S.	1	1.00	1.00	0.75	1.10	3.75	0.00	1.20	1.20
time (sec)	N/A	0.153	0.009	0.514	0.201	0.242	0.000	0.288	0.373

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	9	75	102	0	10	19
N.S.	1	1.00	1.00	0.69	5.77	7.85	0.00	0.77	1.46
time (sec)	N/A	0.148	0.008	0.619	0.202	0.250	0.000	0.286	0.348

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	16	12	6	6
N.S.	1	1.00	1.00	0.78	0.67	1.78	1.33	0.67	0.67
time (sec)	N/A	0.142	0.001	0.596	0.194	0.228	0.211	0.275	0.365

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	19	18	13	0	25	32	16	14
N.S.	1	1.46	1.38	1.00	0.00	1.92	2.46	1.23	1.08
time (sec)	N/A	0.186	0.020	0.555	0.000	0.242	0.225	0.287	0.112

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	4	1	2	1	1	10	1	1
N.S.	1	4.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.137	0.000	0.509	0.199	0.221	0.174	0.277	0.320

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	20	36	17	16	26	0	17	16
N.S.	1	0.91	1.64	0.77	0.73	1.18	0.00	0.77	0.73
time (sec)	N/A	0.168	0.021	0.092	0.205	0.242	0.000	0.285	0.064

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	18	12	11	21	0	11	11
N.S.	1	1.00	1.38	0.92	0.85	1.62	0.00	0.85	0.85
time (sec)	N/A	0.178	0.018	0.187	0.197	0.232	0.000	0.279	0.351

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	12	11	22	0	11	11
N.S.	1	1.00	1.33	0.80	0.73	1.47	0.00	0.73	0.73
time (sec)	N/A	0.185	0.018	0.234	0.204	0.237	0.000	0.286	0.050

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	26	25	56	0	32
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	1.23
time (sec)	N/A	0.149	0.007	0.030	0.200	0.241	0.275	0.000	0.450

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	49	30	41	42	45	155	84	43
N.S.	1	1.17	0.71	0.98	1.00	1.07	3.69	2.00	1.02
time (sec)	N/A	0.187	0.008	0.045	0.203	0.247	0.355	0.289	0.396

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	39	21	23	22	17	34	22	17
N.S.	1	1.15	0.62	0.68	0.65	0.50	1.00	0.65	0.50
time (sec)	N/A	0.167	0.004	0.085	0.198	0.247	0.724	0.275	0.037

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	46	28	25	25	25	22	24	21
N.S.	1	1.64	1.00	0.89	0.89	0.89	0.79	0.86	0.75
time (sec)	N/A	0.175	0.004	0.024	0.204	0.256	0.047	0.300	0.375

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	70	81	58	69	69	71	71	71
N.S.	1	1.04	1.21	0.87	1.03	1.03	1.06	1.06	1.06
time (sec)	N/A	0.199	0.012	0.302	0.204	0.233	0.062	0.285	0.439

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	36	23	26	23	20
N.S.	1	1.00	1.00	1.04	1.57	1.00	1.13	1.00	0.87
time (sec)	N/A	0.148	0.004	0.021	0.205	0.238	0.049	0.281	0.422

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	66	48	51	52	51
N.S.	1	1.00	1.00	0.80	1.10	0.80	0.85	0.87	0.85
time (sec)	N/A	0.273	0.008	0.030	0.211	0.236	0.069	0.299	0.458

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	31	8	34	32	0	29
N.S.	1	1.00	1.00	0.72	0.19	0.79	0.74	0.00	0.67
time (sec)	N/A	0.266	0.025	0.036	0.225	0.236	0.280	0.000	0.330

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	32	25	0	177	0	0
N.S.	1	1.00	1.03	1.10	0.86	0.00	6.10	0.00	0.00
time (sec)	N/A	0.176	0.004	0.356	0.197	0.000	3.779	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	138	35
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	4.76	1.21
time (sec)	N/A	0.155	0.011	0.278	0.195	0.248	0.109	0.307	0.475

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	22
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.83
time (sec)	N/A	0.148	0.004	0.036	0.188	0.247	0.369	0.314	0.385

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	36	19	19
N.S.	1	1.00	1.00	1.05	1.00	1.16	1.89	1.00	1.00
time (sec)	N/A	0.160	0.007	0.063	0.189	0.243	0.465	0.287	0.501

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	30	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	2.73	1.00
time (sec)	N/A	0.158	0.020	0.029	0.197	0.230	0.048	0.269	0.345

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	0	27	71	22	22
N.S.	1	1.00	1.00	1.04	0.00	1.17	3.09	0.96	0.96
time (sec)	N/A	0.167	0.007	0.101	0.000	0.242	5.322	0.264	0.487

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	7	18	0	18	14
N.S.	1	1.00	2.88	0.94	0.44	1.12	0.00	1.12	0.88
time (sec)	N/A	0.175	0.014	0.026	0.202	0.230	0.000	0.282	0.473

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	50	17	20	20	0	0	16
N.S.	1	1.00	2.78	0.94	1.11	1.11	0.00	0.00	0.89
time (sec)	N/A	0.175	0.015	0.028	0.194	0.236	0.000	0.000	0.449

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	7	25	0	10	16
N.S.	1	1.00	1.00	0.94	0.39	1.39	0.00	0.56	0.89
time (sec)	N/A	0.174	0.015	0.027	0.286	0.248	0.000	0.301	0.656

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	37	13	44	0	0	27
N.S.	1	1.00	1.00	1.68	0.59	2.00	0.00	0.00	1.23
time (sec)	N/A	0.214	0.011	0.027	0.192	0.245	0.000	0.000	0.638

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	37	27	0	0	22
N.S.	1	1.00	1.00	1.62	1.54	1.12	0.00	0.00	0.92
time (sec)	N/A	0.228	0.013	0.026	0.185	0.256	0.000	0.000	0.689

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	43	13	27	0	21	25
N.S.	1	1.00	1.00	1.87	0.57	1.17	0.00	0.91	1.09
time (sec)	N/A	0.238	0.012	0.024	0.279	0.249	0.000	0.332	0.659

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.137	0.004	0.027	0.194	0.246	0.065	0.331	0.390

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	22	20	21	15	20	24	20	15
N.S.	1	1.10	1.00	1.05	0.75	1.00	1.20	1.00	0.75
time (sec)	N/A	0.183	0.005	0.046	0.191	0.243	0.094	0.290	0.500

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	29	30	22	29	36	29	29
N.S.	1	1.14	1.00	1.03	0.76	1.00	1.24	1.00	1.00
time (sec)	N/A	0.198	0.005	0.057	0.193	0.239	0.124	0.297	0.409

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	44	38	39	29	38	48	38	38
N.S.	1	1.16	1.00	1.03	0.76	1.00	1.26	1.00	1.00
time (sec)	N/A	0.220	0.007	0.060	0.194	0.256	0.151	0.309	0.420

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	29	15	24	0	24
N.S.	1	1.00	1.00	0.00	1.21	0.62	1.00	0.00	1.00
time (sec)	N/A	0.188	0.023	0.000	0.048	0.074	0.703	0.000	0.479

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	0	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.00	1.25	1.00
time (sec)	N/A	0.168	0.012	0.197	0.191	0.244	0.000	0.275	0.433

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	10	9	12	7	9	9
N.S.	1	1.00	1.00	1.43	1.29	1.71	1.00	1.29	1.29
time (sec)	N/A	0.239	0.003	0.663	0.207	0.253	0.681	0.297	0.430

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	46	10	38	8
N.S.	1	1.00	1.00	1.09	1.00	4.18	0.91	3.45	0.73
time (sec)	N/A	0.170	0.006	4.227	0.192	0.233	0.195	0.290	0.374

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	0	16
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.00	1.78
time (sec)	N/A	0.159	0.004	1.256	0.190	0.232	1.632	0.000	0.514

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.142	0.014	0.017	0.000	0.251	2.612	0.307	0.083

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	30	27	26	25	26	26	27	25
N.S.	1	0.86	0.77	0.74	0.71	0.74	0.74	0.77	0.71
time (sec)	N/A	0.162	0.009	0.046	0.191	0.234	0.063	0.315	0.068

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	24	20	26	27	20	24
N.S.	1	1.00	0.75	0.75	0.62	0.81	0.84	0.62	0.75
time (sec)	N/A	0.220	0.019	0.083	0.208	0.237	81.280	0.290	0.438

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	42	43	42	42	0	43	31
N.S.	1	0.98	0.81	0.83	0.81	0.81	0.00	0.83	0.60
time (sec)	N/A	0.216	0.021	0.052	0.196	0.259	0.000	0.278	0.613

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	35	30	26	25	24	42	26	0
N.S.	1	1.17	1.00	0.87	0.83	0.80	1.40	0.87	0.00
time (sec)	N/A	0.239	0.006	0.384	0.196	0.239	0.662	0.302	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	35	29	27	25	39	46	26	148
N.S.	1	1.17	0.97	0.90	0.83	1.30	1.53	0.87	4.93
time (sec)	N/A	0.260	0.047	10.178	0.204	0.245	10.177	0.293	2.051

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	28	33	32	164	56	32	0	43	39
N.S.	1	1.18	1.14	5.86	2.00	1.14	0.00	1.54	1.39
time (sec)	N/A	0.215	0.078	0.668	0.204	0.245	0.000	0.307	0.644

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	66	56	55	86	53	88	36	164
N.S.	1	1.10	0.93	0.92	1.43	0.88	1.47	0.60	2.73
time (sec)	N/A	0.487	0.108	0.592	0.273	0.250	2.715	0.351	0.938

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	71	52	52	51	44	0	104	0
N.S.	1	1.09	0.80	0.80	0.78	0.68	0.00	1.60	0.00
time (sec)	N/A	0.325	0.022	0.122	0.273	0.256	0.000	0.304	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	42	37	47	36	54	57	0
N.S.	1	1.07	0.69	0.61	0.77	0.59	0.89	0.93	0.00
time (sec)	N/A	0.310	0.014	0.392	0.279	0.256	0.135	0.305	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	64	37	42	44	36	44	41	41
N.S.	1	1.21	0.70	0.79	0.83	0.68	0.83	0.77	0.77
time (sec)	N/A	0.523	0.010	0.096	0.285	0.242	0.138	0.286	0.413

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	79	56	48	64	53	53	0	44
N.S.	1	1.30	0.92	0.79	1.05	0.87	0.87	0.00	0.72
time (sec)	N/A	0.555	0.011	0.139	0.278	0.247	0.230	0.000	0.109

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	69	42	56	95	35	0	106	0
N.S.	1	1.10	0.67	0.89	1.51	0.56	0.00	1.68	0.00
time (sec)	N/A	0.390	0.026	0.108	0.375	0.264	0.000	0.306	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	203	92	174	0	77	0	137	0
N.S.	1	1.37	0.62	1.18	0.00	0.52	0.00	0.93	0.00
time (sec)	N/A	0.652	0.041	0.464	0.000	0.261	0.000	0.313	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	30	26	31	27	0
N.S.	1	1.00	0.88	0.91	0.88	0.76	0.91	0.79	0.00
time (sec)	N/A	0.219	0.006	0.322	0.271	0.256	0.535	0.307	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	31	27	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	0.91	0.79	0.00
time (sec)	N/A	0.212	0.011	0.302	0.280	0.257	0.607	0.320	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	33	26	28	22	27	37	22	0
N.S.	1	1.10	0.87	0.93	0.73	0.90	1.23	0.73	0.00
time (sec)	N/A	0.187	0.016	0.385	0.275	0.241	0.162	0.325	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	76	42	54	50	39	53	50	0
N.S.	1	1.29	0.71	0.92	0.85	0.66	0.90	0.85	0.00
time (sec)	N/A	0.310	0.019	0.439	0.280	0.245	0.280	0.320	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	38	35	37	27	37	63	34	0
N.S.	1	1.03	0.95	1.00	0.73	1.00	1.70	0.92	0.00
time (sec)	N/A	0.196	0.011	0.403	0.293	0.256	0.378	0.294	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	64	47	77	49	47	88	60	0
N.S.	1	1.05	0.77	1.26	0.80	0.77	1.44	0.98	0.00
time (sec)	N/A	0.251	0.032	0.500	0.296	0.264	6.920	0.305	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	97	119	155	0	0	0	0	0
N.S.	1	1.02	1.25	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.204	0.591	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	36	201	35	44	0	135	0
N.S.	1	1.02	0.88	4.90	0.85	1.07	0.00	3.29	0.00
time (sec)	N/A	0.225	0.023	0.589	0.274	0.260	0.000	0.345	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	32	32	26	26	27	0
N.S.	1	1.00	0.82	0.94	0.94	0.76	0.76	0.79	0.00
time (sec)	N/A	0.238	0.007	0.296	0.276	0.242	0.105	0.327	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	66	43	54	52	39	53	50	0
N.S.	1	1.08	0.70	0.89	0.85	0.64	0.87	0.82	0.00
time (sec)	N/A	0.329	0.014	0.453	0.285	0.255	0.212	0.328	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	46	25	44	24	27	0
N.S.	1	1.00	1.00	2.42	1.32	2.32	1.26	1.42	0.00
time (sec)	N/A	0.179	0.009	0.306	0.279	0.254	3.930	0.300	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	32	47	25	44	24	27	0
N.S.	1	1.00	1.88	2.76	1.47	2.59	1.41	1.59	0.00
time (sec)	N/A	0.174	0.027	0.316	0.276	0.259	4.144	0.292	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	64	45	63	48	61	78	54	0
N.S.	1	1.03	0.73	1.02	0.77	0.98	1.26	0.87	0.00
time (sec)	N/A	0.243	0.046	0.319	0.295	0.266	12.590	0.316	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	40	102	45	57	37	40	0
N.S.	1	1.00	1.11	2.83	1.25	1.58	1.03	1.11	0.00
time (sec)	N/A	0.220	0.034	0.632	0.279	0.263	6.021	0.295	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	112	430	0	0	0	0	0
N.S.	1	1.00	1.81	6.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.156	0.691	0.000	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	38	43	42	36	49	95	0
N.S.	1	1.02	0.70	0.80	0.78	0.67	0.91	1.76	0.00
time (sec)	N/A	0.273	0.025	0.269	0.276	0.251	5.063	0.333	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	71	50	59	52	41	78	53	0
N.S.	1	1.08	0.76	0.89	0.79	0.62	1.18	0.80	0.00
time (sec)	N/A	0.274	0.031	0.470	0.278	0.245	0.309	0.304	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	78	60	69	0	49	66	60	0
N.S.	1	1.07	0.82	0.95	0.00	0.67	0.90	0.82	0.00
time (sec)	N/A	0.516	0.015	0.342	0.000	0.253	0.205	0.305	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	37	21	22	26	19	31	26	21
N.S.	1	1.16	0.66	0.69	0.81	0.59	0.97	0.81	0.66
time (sec)	N/A	0.177	0.012	0.326	0.276	0.258	0.205	0.298	0.090

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	54	36	37	39	38	88	34	26
N.S.	1	1.23	0.82	0.84	0.89	0.86	2.00	0.77	0.59
time (sec)	N/A	0.185	0.013	0.342	0.279	0.244	0.312	0.282	0.453

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.83	0.83
time (sec)	N/A	0.258	0.003	0.281	0.278	0.251	0.110	0.318	0.368

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	66	57	126	0	0	0	0	0
N.S.	1	0.99	0.85	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.018	0.444	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	40	26	0	0	23
N.S.	1	1.00	0.82	0.85	1.18	0.76	0.00	0.00	0.68
time (sec)	N/A	0.236	0.017	0.509	0.293	0.240	0.000	0.000	0.072

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	84	64	137	0	0	0	0	0
N.S.	1	1.06	0.81	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	0.043	0.349	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	99	70	147	0	0	0	0	0
N.S.	1	1.11	0.79	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.078	0.137	0.539	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	39	22	23	21	26	19	25	22
N.S.	1	1.77	1.00	1.05	0.95	1.18	0.86	1.14	1.00
time (sec)	N/A	0.251	0.003	0.080	0.300	0.243	0.109	0.285	0.070

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	34	59	30	31	26	34	31	30
N.S.	1	1.10	1.90	0.97	1.00	0.84	1.10	1.00	0.97
time (sec)	N/A	0.185	0.006	0.076	0.294	0.245	0.216	0.276	0.378

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	81	79	71	0	0	0	53
N.S.	1	1.00	1.29	1.25	1.13	0.00	0.00	0.00	0.84
time (sec)	N/A	0.247	0.004	0.355	0.323	0.000	0.000	0.000	0.513

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	33	28	25	27	29	22	0	24
N.S.	1	1.18	1.00	0.89	0.96	1.04	0.79	0.00	0.86
time (sec)	N/A	0.280	0.006	0.342	0.281	0.258	0.154	0.000	0.089

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	44	38	34	36	38	32	0	31
N.S.	1	1.13	0.97	0.87	0.92	0.97	0.82	0.00	0.79
time (sec)	N/A	0.342	0.011	0.090	0.270	0.242	0.185	0.000	0.073

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	79	56	57	71	54	61	0	51
N.S.	1	1.32	0.93	0.95	1.18	0.90	1.02	0.00	0.85
time (sec)	N/A	0.326	0.011	0.123	0.285	0.255	0.236	0.000	0.117

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	92	47	78	94	51	0	0	56
N.S.	1	1.16	0.59	0.99	1.19	0.65	0.00	0.00	0.71
time (sec)	N/A	0.428	0.034	0.555	0.277	0.238	0.000	0.000	0.404

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	91	116	194	0	0	0	0	0
N.S.	1	0.85	1.08	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.163	0.658	0.000	0.000	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	106	128	86	274	0	51	0	0	0
N.S.	1	1.21	0.81	2.58	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.631	0.232	0.504	0.000	0.253	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	42	48	85	27	23	0	75	0
N.S.	1	1.02	1.17	2.07	0.66	0.56	0.00	1.83	0.00
time (sec)	N/A	0.206	0.029	0.485	0.281	0.258	0.000	0.338	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	66	67	127	48	75	0	58	0
N.S.	1	1.02	1.03	1.95	0.74	1.15	0.00	0.89	0.00
time (sec)	N/A	0.189	0.078	0.698	0.249	0.271	0.000	0.347	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	52	61	134	46	68	0	53	0
N.S.	1	1.02	1.20	2.63	0.90	1.33	0.00	1.04	0.00
time (sec)	N/A	0.217	0.070	0.493	0.243	0.253	0.000	0.333	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	82	65	75	101	0	69	0	64	0
N.S.	1	0.79	0.91	1.23	0.00	0.84	0.00	0.78	0.00
time (sec)	N/A	0.264	0.177	0.361	0.000	0.266	0.000	0.324	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	175	198	383	246	0	0	0	0	0
N.S.	1	1.13	2.19	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.906	1.248	0.832	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	23	23	35	56	17	16	0	50	0
N.S.	1	1.00	1.52	2.43	0.74	0.70	0.00	2.17	0.00
time (sec)	N/A	0.194	0.026	0.352	0.295	0.248	0.000	0.316	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	89	79	204	123	81	0	105	0
N.S.	1	1.27	1.13	2.91	1.76	1.16	0.00	1.50	0.00
time (sec)	N/A	0.276	0.086	0.721	0.553	0.255	0.000	0.348	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	89	76	102	58	37	0	0	0
N.S.	1	1.20	1.03	1.38	0.78	0.50	0.00	0.00	0.00
time (sec)	N/A	0.490	0.040	0.329	0.286	0.247	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	194	84	114	0	59	0	0	0
N.S.	1	1.46	0.63	0.86	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.703	0.176	0.529	0.000	0.251	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	135	92	147	93	57	0	0	0
N.S.	1	1.23	0.84	1.34	0.85	0.52	0.00	0.00	0.00
time (sec)	N/A	0.589	0.052	0.540	0.528	0.259	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	136	99	86	103	51	0	94	0
N.S.	1	2.47	1.80	1.56	1.87	0.93	0.00	1.71	0.00
time (sec)	N/A	0.776	0.107	0.078	0.294	0.267	0.000	0.417	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	71	66	89	58	0	49	36
N.S.	1	1.00	1.78	1.65	2.22	1.45	0.00	1.22	0.90
time (sec)	N/A	0.223	0.054	0.079	0.283	0.261	0.000	0.320	0.426

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	35	32	31	50	153	32	31
N.S.	1	1.15	0.90	0.82	0.79	1.28	3.92	0.82	0.79
time (sec)	N/A	0.207	0.021	0.356	0.289	0.248	0.235	0.296	0.408

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	105	102	118	0	0	0	0
N.S.	1	1.00	0.86	0.84	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.019	0.391	0.322	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	14	27	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.50	0.96	0.00	0.00
time (sec)	N/A	0.222	0.012	0.000	0.000	0.256	0.434	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	26	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.195	0.017	0.294	0.199	0.250	0.566	0.278	0.637

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	62	61	70	0	90	0	58	0
N.S.	1	1.09	1.07	1.23	0.00	1.58	0.00	1.02	0.00
time (sec)	N/A	0.193	0.069	0.286	0.000	0.249	0.000	0.301	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	88	72	76	116	125	0	228	0
N.S.	1	1.07	0.88	0.93	1.41	1.52	0.00	2.78	0.00
time (sec)	N/A	0.261	0.053	0.306	0.776	0.262	0.000	0.439	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	52	66	0	0	519	0	218	0
N.S.	1	1.06	1.35	0.00	0.00	10.59	0.00	4.45	0.00
time (sec)	N/A	0.352	0.153	0.000	0.000	0.282	0.000	0.367	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	41	144	850	54	423	214	70	103
N.S.	1	1.14	4.00	23.61	1.50	11.75	5.94	1.94	2.86
time (sec)	N/A	0.311	0.116	0.367	0.288	0.259	78.092	0.285	0.629

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	109	0	16	26	0	29	0
N.S.	1	1.00	3.89	0.00	0.57	0.93	0.00	1.04	0.00
time (sec)	N/A	0.278	0.400	0.000	0.296	0.255	0.000	0.316	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [342] had the largest ratio of [2.75000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	14	0.071
2	A	1	1	1.00	13	0.077
3	A	2	2	1.00	5	0.400
4	A	3	3	1.00	10	0.300
5	A	3	3	1.00	12	0.250
6	A	4	3	1.00	5	0.600
7	A	5	4	1.00	5	0.800
8	A	4	3	1.00	7	0.429
9	A	2	2	1.00	6	0.333
10	A	2	2	1.00	8	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	17	0.176
13	A	4	3	1.00	18	0.167
14	A	5	4	1.00	19	0.211
15	A	5	4	1.00	20	0.200
16	A	5	4	1.22	19	0.211
17	A	5	4	1.22	20	0.200
18	A	4	3	0.51	10	0.300
19	A	3	2	1.00	13	0.154
20	A	3	2	1.00	8	0.250
21	A	3	2	1.00	12	0.167
22	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	3	2	1.00	14	0.143
24	A	4	3	1.48	16	0.188
25	A	6	5	1.27	22	0.227
26	A	2	2	1.00	13	0.154
27	A	4	4	0.64	15	0.267
28	A	3	3	1.00	15	0.200
29	A	2	2	1.00	9	0.222
30	A	2	2	1.00	9	0.222
31	A	5	5	1.19	10	0.500
32	A	3	3	1.00	4	0.750
33	A	3	3	1.00	4	0.750
34	A	4	3	1.00	7	0.429
35	A	5	4	1.00	7	0.571
36	A	5	4	1.00	9	0.444
37	A	3	3	1.00	8	0.375
38	A	4	4	1.00	8	0.500
39	A	5	4	1.20	9	0.444
40	A	7	7	1.00	9	0.778
41	A	4	4	1.00	9	0.444
42	A	4	4	1.00	11	0.364
43	A	5	4	1.44	19	0.211
44	A	2	2	1.00	10	0.200
45	A	4	3	1.14	16	0.188
46	A	2	2	1.00	14	0.143
47	A	5	4	0.67	11	0.364
48	A	3	2	1.00	13	0.154
49	A	4	3	1.15	13	0.231
50	A	4	3	1.00	15	0.200
51	A	4	3	1.00	17	0.176
52	A	4	3	1.00	17	0.176
53	A	4	3	1.00	15	0.200
54	A	3	2	1.00	12	0.167
55	A	3	2	1.11	14	0.143
56	A	3	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	3	1.00	18	0.167
58	A	3	2	1.00	16	0.125
59	A	1	1	1.10	18	0.056
60	A	3	3	1.00	17	0.176
61	A	4	3	0.49	10	0.300
62	A	5	4	0.86	11	0.364
63	A	4	3	0.27	17	0.176
64	A	5	4	1.06	19	0.211
65	A	5	4	1.00	19	0.211
66	A	4	3	1.00	11	0.273
67	A	4	3	1.00	17	0.176
68	A	1	1	1.00	12	0.083
69	A	1	1	1.00	24	0.042
70	A	1	1	1.00	16	0.062
71	A	2	2	1.00	6	0.333
72	A	1	1	1.00	6	0.167
73	A	5	5	1.08	12	0.417
74	A	4	3	1.00	4	0.750
75	A	7	7	1.29	9	0.778
76	A	6	6	1.19	4	1.500
77	A	1	1	1.00	8	0.125
78	A	1	1	1.00	10	0.100
79	A	1	1	1.00	6	0.167
80	A	1	1	1.00	3	0.333
81	A	5	5	1.00	8	0.625
82	A	5	5	1.00	6	0.833
83	A	5	4	1.00	6	0.667
84	A	3	3	1.04	4	0.750
85	A	4	4	1.00	13	0.308
86	A	7	6	1.42	12	0.500
87	A	2	2	1.00	11	0.182
88	A	2	2	1.00	16	0.125
89	A	1	1	1.00	15	0.067
90	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	2	2	1.00	13	0.154
92	A	2	2	1.10	12	0.167
93	A	6	5	0.97	16	0.312
94	A	2	2	1.00	14	0.143
95	A	3	3	1.00	29	0.103
96	A	3	3	1.00	19	0.158
97	A	2	2	1.00	39	0.051
98	A	8	7	1.73	21	0.333
99	A	2	2	1.00	20	0.100
100	A	2	2	1.00	21	0.095
101	A	2	2	1.00	11	0.182
102	A	2	2	1.00	9	0.222
103	A	2	2	1.00	25	0.080
104	A	2	2	1.00	24	0.083
105	A	3	3	1.13	19	0.158
106	A	3	3	1.00	19	0.158
107	A	2	2	1.00	19	0.105
108	A	2	2	1.00	16	0.125
109	A	3	3	1.00	14	0.214
110	A	3	3	1.00	33	0.091
111	A	2	2	1.00	21	0.095
112	A	2	2	1.00	21	0.095
113	A	7	6	1.06	10	0.600
114	A	3	3	1.00	17	0.176
115	A	2	2	1.00	26	0.077
116	A	2	2	1.00	29	0.069
117	A	4	3	1.10	30	0.100
118	A	8	7	1.04	9	0.778
119	A	8	7	1.04	11	0.636
120	A	1	1	1.00	13	0.077
121	A	5	4	1.18	13	0.308
122	A	9	8	1.14	13	0.615
123	A	9	8	1.14	13	0.615
124	A	4	3	1.03	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
125	A	10	9	1.21	13	0.692
126	A	1	1	1.00	15	0.067
127	A	3	3	1.00	11	0.273
128	A	3	2	1.00	13	0.154
129	A	5	4	1.17	15	0.267
130	A	4	4	1.14	15	0.267
131	A	4	3	0.96	15	0.200
132	A	4	4	1.14	15	0.267
133	A	1	1	1.00	17	0.059
134	A	3	2	1.00	11	0.182
135	A	9	8	1.05	13	0.615
136	A	9	8	1.00	9	0.889
137	A	9	8	1.01	11	0.727
138	A	10	9	1.09	13	0.692
139	A	9	8	1.00	13	0.615
140	A	1	1	1.00	13	0.077
141	A	5	4	1.18	13	0.308
142	A	10	9	1.03	13	0.692
143	A	11	10	1.12	13	0.769
144	A	10	9	1.04	13	0.692
145	A	1	1	1.00	15	0.067
146	A	2	2	1.00	13	0.154
147	A	5	4	1.08	10	0.400
148	A	5	5	1.23	16	0.312
149	A	4	3	1.07	19	0.158
150	A	8	7	1.11	26	0.269
151	A	9	8	1.14	7	1.143
152	A	6	6	1.22	18	0.333
153	A	9	8	1.12	9	0.889
154	B	6	5	3.00	18	0.278
155	A	7	6	1.17	18	0.333
156	A	5	4	1.12	16	0.250
157	A	4	3	1.16	16	0.188
158	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	2	2	1.00	9	0.222
160	A	3	3	1.00	15	0.200
161	A	2	2	1.00	11	0.182
162	A	2	2	1.00	11	0.182
163	A	2	2	0.97	10	0.200
164	A	5	4	1.14	10	0.400
165	A	2	2	1.00	11	0.182
166	A	4	4	1.31	11	0.364
167	A	5	5	1.16	11	0.455
168	A	4	3	1.07	18	0.167
169	A	3	3	1.60	18	0.167
170	A	5	4	1.24	11	0.364
171	A	4	3	1.34	21	0.143
172	A	1	1	1.00	13	0.077
173	A	4	3	1.02	13	0.231
174	A	12	11	1.16	13	0.846
175	A	6	5	1.25	13	0.385
176	A	4	3	1.10	13	0.231
177	A	11	10	1.09	13	0.769
178	A	5	5	1.27	16	0.312
179	A	6	6	1.22	18	0.333
180	A	2	2	1.00	44	0.045
181	A	4	4	1.08	16	0.250
182	A	4	4	1.05	22	0.182
183	A	2	2	1.00	15	0.133
184	A	4	3	1.15	13	0.231
185	A	4	3	1.09	13	0.231
186	A	2	2	1.14	11	0.182
187	A	2	2	1.00	11	0.182
188	A	2	2	1.00	9	0.222
189	A	2	2	1.00	17	0.118
190	A	2	2	1.00	19	0.105
191	A	2	2	1.00	19	0.105
192	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	2	2	1.00	19	0.105
194	A	6	5	1.09	19	0.263
195	A	4	3	1.07	19	0.158
196	A	5	4	1.10	19	0.211
197	A	6	5	1.10	19	0.263
198	A	2	2	1.00	21	0.095
199	A	2	2	1.04	14	0.143
200	A	4	4	1.52	18	0.222
201	A	2	2	0.97	14	0.143
202	A	2	2	0.93	10	0.200
203	A	4	4	1.06	16	0.250
204	A	2	2	0.72	14	0.143
205	A	2	2	0.99	20	0.100
206	A	4	3	1.10	14	0.214
207	A	7	6	1.23	13	0.462
208	A	5	4	1.08	24	0.167
209	A	9	8	1.00	33	0.242
210	A	5	4	0.73	11	0.364
211	A	3	2	1.07	13	0.154
212	A	6	5	1.08	21	0.238
213	A	7	6	0.70	19	0.316
214	A	4	3	1.14	17	0.176
215	A	6	5	1.08	11	0.455
216	A	10	9	1.12	13	0.692
217	A	9	8	1.13	11	0.727
218	A	4	3	1.50	15	0.200
219	A	6	5	1.42	19	0.263
220	A	8	7	1.00	27	0.259
221	A	18	17	1.56	52	0.327
222	F	0	0	N/A	0.000	N/A
223	A	2	2	1.08	15	0.133
224	A	2	2	1.12	15	0.133
225	A	3	3	1.32	15	0.200

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	4	4	1.70	13	0.308
227	A	10	9	0.77	19	0.474
228	A	9	8	1.89	17	0.471
229	A	2	2	1.00	12	0.167
230	A	5	4	1.00	17	0.235
231	A	8	7	0.76	17	0.412
232	A	3	3	1.31	17	0.176
233	A	3	3	0.93	17	0.176
234	A	5	5	1.00	17	0.294
235	A	3	2	1.00	14	0.143
236	A	3	2	1.50	14	0.143
237	A	3	2	1.00	14	0.143
238	A	3	2	1.00	19	0.105
239	A	3	2	1.00	19	0.105
240	A	4	3	1.00	22	0.136
241	A	4	3	1.00	22	0.136
242	A	5	4	1.00	17	0.235
243	A	7	6	1.00	21	0.286
244	A	10	9	0.99	20	0.450
245	A	7	6	1.00	24	0.250
246	A	6	5	1.00	21	0.238
247	A	6	5	1.00	30	0.167
248	A	6	5	1.00	32	0.156
249	A	3	2	1.00	30	0.067
250	A	4	4	1.15	15	0.267
251	A	3	3	1.10	13	0.231
252	A	3	3	1.10	11	0.273
253	A	4	3	1.00	20	0.150
254	A	3	3	1.00	18	0.167
255	A	5	4	1.00	17	0.235
256	A	4	3	1.00	17	0.176
257	A	7	6	1.00	20	0.300
258	A	5	4	1.00	24	0.167
259	A	2	2	1.00	33	0.061

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	2	2	1.00	44	0.045
261	A	5	4	1.03	16	0.250
262	A	6	5	1.02	18	0.278
263	A	7	6	1.04	18	0.333
264	A	7	6	1.00	19	0.316
265	A	6	5	1.00	24	0.208
266	A	3	2	1.00	10	0.200
267	A	6	5	1.09	14	0.357
268	A	1	1	1.00	10	0.100
269	A	1	1	1.00	12	0.083
270	A	6	5	1.09	14	0.357
271	A	7	6	1.15	14	0.429
272	A	5	4	1.09	10	0.400
273	A	6	5	1.14	10	0.500
274	A	4	3	1.00	14	0.214
275	A	6	5	1.05	14	0.357
276	A	6	5	1.05	14	0.357
277	A	8	7	1.10	14	0.500
278	A	3	2	1.00	16	0.125
279	A	3	3	1.00	22	0.136
280	A	8	7	1.08	18	0.389
281	A	9	8	1.07	28	0.286
282	A	5	5	1.00	34	0.147
283	A	2	2	1.00	24	0.083
284	A	3	3	1.09	12	0.250
285	A	2	2	1.00	14	0.143
286	A	2	2	1.00	14	0.143
287	A	2	2	1.00	16	0.125
288	A	4	3	1.31	14	0.214
289	A	2	2	1.00	23	0.087
290	A	2	2	1.00	29	0.069
291	A	2	2	1.00	31	0.065
292	A	5	4	0.95	11	0.364
293	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
294	A	7	6	1.10	13	0.462
295	A	2	2	1.00	13	0.154
296	A	8	7	1.23	17	0.412
297	A	4	3	1.03	15	0.200
298	A	14	13	0.89	17	0.765
299	A	1	1	1.00	13	0.077
300	A	1	1	1.00	13	0.077
301	A	6	5	1.10	15	0.333
302	A	7	6	1.09	13	0.462
303	A	6	5	1.12	15	0.333
304	A	7	6	1.11	15	0.400
305	A	7	6	1.11	15	0.400
306	A	7	6	1.13	13	0.462
307	A	9	8	1.01	13	0.615
308	A	5	4	1.00	22	0.182
309	A	6	5	1.16	16	0.312
310	A	6	5	1.13	18	0.278
311	A	5	4	1.00	23	0.174
312	A	1	1	1.00	23	0.043
313	A	2	2	1.00	39	0.051
314	A	1	1	1.00	17	0.059
315	A	10	9	1.04	17	0.529
316	A	2	2	1.03	15	0.133
317	A	6	5	1.07	17	0.294
318	A	4	4	1.33	17	0.235
319	A	5	4	1.00	32	0.125
320	A	3	2	1.00	24	0.083
321	A	3	2	1.00	24	0.083
322	A	7	6	1.31	18	0.333
323	A	8	7	1.19	18	0.389
324	A	3	2	1.00	27	0.074
325	A	3	2	1.00	27	0.074
326	A	1	1	1.00	21	0.048
327	A	1	1	1.00	44	0.023

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	3	2	1.00	27	0.074
329	A	3	2	1.00	31	0.065
330	A	3	3	1.00	4	0.750
331	A	4	3	1.00	4	0.750
332	A	5	5	1.21	4	1.250
333	A	7	7	1.29	4	1.750
334	A	9	9	1.34	4	2.250
335	B	4	4	2.35	14	0.286
336	A	5	4	1.06	14	0.286
337	A	4	3	1.00	4	0.750
338	A	8	8	1.28	4	2.000
339	A	4	3	1.00	4	0.750
340	A	4	4	1.00	12	0.333
341	A	7	7	1.00	4	1.750
342	A	11	11	1.00	4	2.750
343	A	5	5	1.00	14	0.357
344	A	11	11	1.36	9	1.222
345	A	5	4	1.00	9	0.444
346	A	6	5	1.04	7	0.714
347	A	5	4	1.00	9	0.444
348	A	5	5	1.21	9	0.556
349	A	9	9	1.33	9	1.000
350	A	13	13	1.37	9	1.444
351	A	17	17	1.39	9	1.889
352	A	2	2	1.00	13	0.154
353	A	5	4	1.00	23	0.174
354	A	4	3	1.00	9	0.333
355	A	5	4	1.00	7	0.571
356	A	4	3	1.00	7	0.429
357	A	6	6	1.19	9	0.667
358	A	7	6	1.00	9	0.667
359	A	5	4	1.00	11	0.364
360	A	5	4	1.00	11	0.364
361	A	8	8	1.26	9	0.889

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
362	A	6	6	1.01	29	0.207
363	A	9	8	1.14	22	0.364
364	A	3	3	1.09	17	0.176
365	A	10	10	1.00	17	0.588
366	A	5	4	1.12	9	0.444
367	A	5	4	1.00	7	0.571
368	A	2	2	1.00	7	0.286
369	A	7	6	1.73	9	0.667
370	A	3	3	1.00	9	0.333
371	B	6	5	2.10	9	0.556
372	A	6	5	1.33	9	0.556
373	A	10	9	0.64	11	0.818
374	A	7	6	1.91	9	0.667
375	A	7	6	1.57	16	0.375
376	A	5	4	0.64	14	0.286
377	A	5	4	1.67	12	0.333
378	A	4	3	0.33	16	0.188
379	A	7	6	1.54	10	0.600
380	A	6	5	0.67	9	0.556
381	A	5	4	0.67	9	0.444
382	A	10	10	1.24	21	0.476
383	A	4	3	1.00	9	0.333
384	A	4	3	1.00	7	0.429
385	A	6	5	0.88	9	0.556
386	A	7	6	1.29	9	0.667
387	A	7	6	1.29	7	0.857
388	A	5	4	1.00	7	0.571
389	A	6	5	0.96	9	0.556
390	A	2	2	1.00	10	0.200
391	A	2	2	1.00	12	0.167
392	A	4	3	1.00	10	0.300
393	A	4	3	1.00	12	0.250
394	A	6	5	1.00	12	0.417
395	A	6	6	1.00	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	8	7	0.78	32	0.219
397	A	12	11	1.12	6	1.833
398	A	12	11	0.79	8	1.375
399	A	10	9	1.06	12	0.750
400	A	4	3	1.00	32	0.094
401	A	6	5	1.83	13	0.385
402	A	2	2	1.00	11	0.182
403	A	2	2	1.00	11	0.182
404	A	4	4	1.11	11	0.364
405	A	3	3	1.00	16	0.188
406	A	8	8	1.13	13	0.615
407	A	8	8	1.11	13	0.615
408	A	2	2	1.00	13	0.154
409	A	4	4	1.00	13	0.308
410	A	6	6	1.07	11	0.545
411	A	8	7	0.96	35	0.200
412	A	7	6	1.63	11	0.545
413	A	14	13	1.75	11	1.182
414	A	7	6	1.13	13	0.462
415	A	6	5	0.87	13	0.385
416	A	7	6	1.59	27	0.222
417	B	5	4	2.15	41	0.098
418	A	8	7	1.21	28	0.250
419	A	8	7	1.14	15	0.467
420	A	7	6	1.14	18	0.333
421	A	7	6	1.09	20	0.300
422	A	6	5	1.09	20	0.250
423	A	5	4	1.00	19	0.211
424	A	7	6	1.10	22	0.273
425	A	6	5	1.08	23	0.217
426	A	3	3	1.07	33	0.091
427	A	6	5	1.00	39	0.128
428	A	7	6	1.06	17	0.353
429	A	5	4	1.12	11	0.364

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
430	A	7	6	1.31	11	0.545
431	A	2	2	1.00	11	0.182
432	A	9	8	1.39	13	0.615
433	A	15	14	1.80	28	0.500
434	A	10	9	1.04	12	0.750
435	A	6	5	1.00	12	0.417
436	A	15	14	1.06	22	0.636
437	A	7	6	1.05	23	0.261
438	A	6	5	1.00	31	0.161
439	A	5	4	1.00	48	0.083
440	A	9	8	1.15	15	0.533
441	A	8	7	1.17	15	0.467
442	A	8	7	1.04	19	0.368
443	A	9	8	1.32	15	0.533
444	A	11	10	1.13	19	0.526
445	A	10	9	1.13	20	0.450
446	A	5	4	1.50	61	0.066
447	A	10	9	1.38	29	0.310
448	A	10	9	1.02	20	0.450
449	A	16	15	1.67	15	1.000
450	A	6	5	1.08	19	0.263
451	A	4	3	1.00	33	0.091
452	A	5	4	1.30	31	0.129
453	A	5	4	1.25	52	0.077
454	A	6	5	1.48	15	0.333
455	A	6	5	1.85	15	0.333
456	A	6	5	1.06	11	0.455
457	A	8	7	1.22	11	0.636
458	A	4	3	1.19	11	0.273
459	A	4	4	1.26	11	0.364
460	A	4	3	1.10	11	0.273
461	A	6	5	1.04	13	0.385
462	A	7	6	1.16	13	0.462
463	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
464	A	5	4	0.91	11	0.364
465	A	5	4	1.10	19	0.211
466	A	4	3	1.15	13	0.231
467	A	4	3	1.11	13	0.231
468	A	2	2	1.00	11	0.182
469	A	2	2	1.00	12	0.167
470	A	3	3	1.11	13	0.231
471	A	2	2	1.00	13	0.154
472	A	4	3	1.12	11	0.273
473	A	3	3	1.00	23	0.130
474	A	2	2	1.00	19	0.105
475	A	2	2	1.00	15	0.133
476	A	3	3	1.00	15	0.200
477	A	3	3	1.10	11	0.273
478	A	2	2	1.00	14	0.143
479	A	2	2	1.00	12	0.167
480	A	2	2	1.00	16	0.125
481	A	2	2	1.00	20	0.100
482	A	3	3	1.00	25	0.120
483	A	16	15	1.36	8	1.875
484	A	16	16	1.26	8	2.000
485	A	20	20	1.72	8	2.500
486	A	7	7	1.09	10	0.700
487	A	10	10	1.00	10	1.000
488	A	2	2	1.00	8	0.250
489	A	5	4	1.00	8	0.500
490	A	11	10	1.19	8	1.250
491	A	10	9	1.12	6	1.500
492	A	3	2	1.00	18	0.111
493	A	5	4	1.00	15	0.267
494	A	2	2	1.00	11	0.182
495	A	3	3	1.21	22	0.136
496	A	1	1	1.00	11	0.091
497	A	5	4	1.14	13	0.308

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
498	A	5	4	1.00	13	0.308
499	A	5	4	1.06	13	0.308
500	A	5	4	1.08	13	0.308
501	A	5	4	0.93	15	0.267
502	A	1	1	1.00	11	0.091
503	A	4	3	0.87	13	0.231
504	A	4	3	0.86	13	0.231
505	A	4	3	0.85	13	0.231
506	A	2	2	1.11	13	0.154
507	A	1	1	1.00	13	0.077
508	A	4	3	0.87	15	0.200
509	A	4	3	0.86	15	0.200
510	A	4	3	0.85	15	0.200
511	A	2	2	1.11	15	0.133
512	A	1	1	1.00	7	0.143
513	A	4	3	0.97	9	0.333
514	A	4	3	0.84	9	0.333
515	A	4	3	0.77	9	0.333
516	A	3	2	1.00	9	0.222
517	A	1	1	1.00	9	0.111
518	A	4	3	0.97	11	0.273
519	A	4	3	0.84	11	0.273
520	A	4	3	0.77	11	0.273
521	A	3	2	1.00	11	0.182
522	A	5	4	1.25	11	0.364
523	A	10	9	1.19	15	0.600
524	A	5	4	1.25	13	0.308
525	A	7	6	1.15	24	0.250
526	A	5	4	1.00	29	0.138
527	A	3	2	1.00	21	0.095
528	A	7	6	1.02	15	0.400
529	A	3	2	1.00	15	0.133
530	A	4	3	1.00	17	0.176
531	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
532	A	5	4	1.00	39	0.103
533	A	6	5	1.54	17	0.294
534	A	4	3	1.03	21	0.143
535	A	4	4	1.14	11	0.364
536	A	3	3	1.05	11	0.273
537	A	3	3	1.20	9	0.333
538	A	2	2	1.00	12	0.167
539	A	7	6	1.12	13	0.462
540	A	1	1	1.00	25	0.040
541	A	1	1	1.00	10	0.100
542	A	5	4	1.46	21	0.190
543	A	2	2	1.23	16	0.125
544	A	2	2	1.00	10	0.200
545	A	2	2	0.99	10	0.200
546	A	3	3	1.25	16	0.188
547	A	2	2	1.00	14	0.143
548	A	2	2	1.00	22	0.091
549	A	2	2	1.47	10	0.200
550	A	1	1	1.00	10	0.100
551	A	2	2	1.59	10	0.200
552	A	2	2	1.00	10	0.200
553	A	2	2	1.00	12	0.167
554	A	2	2	1.00	10	0.200
555	A	2	2	1.00	12	0.167
556	A	1	1	1.00	18	0.056
557	A	5	5	1.12	16	0.312
558	A	1	1	1.00	14	0.071
559	A	5	5	1.07	16	0.312
560	A	6	6	1.07	18	0.333
561	A	1	1	1.00	16	0.062
562	A	5	5	1.12	14	0.357
563	A	1	1	1.00	16	0.062
564	A	2	2	1.00	7	0.286
565	A	4	4	1.00	9	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	4	4	1.07	11	0.364
567	A	4	4	1.38	15	0.267
568	A	4	4	1.06	13	0.308
569	A	2	2	1.00	17	0.118
570	A	2	2	1.00	2	1.000
571	A	3	3	1.00	2	1.500
572	A	3	3	1.00	2	1.500
573	A	3	3	1.00	2	1.500
574	A	2	2	1.00	2	1.000
575	A	3	3	1.00	2	1.500
576	A	3	3	1.00	4	0.750
577	A	5	4	1.00	4	1.000
578	A	7	7	1.00	4	1.750
579	C	7	7	1.50	4	1.750
580	A	6	6	1.19	4	1.500
581	A	7	6	1.33	7	0.857
582	A	6	5	1.00	11	0.455
583	A	4	3	1.02	8	0.375
584	A	4	4	1.00	6	0.667
585	A	5	5	1.00	8	0.625
586	A	4	3	1.06	14	0.214
587	A	4	3	1.06	15	0.200
588	A	5	4	1.00	10	0.400
589	A	5	4	1.00	23	0.174
590	A	3	3	1.00	11	0.273
591	A	4	4	1.00	15	0.267
592	A	9	8	0.97	31	0.258
593	A	6	5	1.00	15	0.333
594	A	8	7	1.00	21	0.333
595	A	4	3	1.00	11	0.273
596	A	8	8	1.00	6	1.333
597	A	8	8	1.00	6	1.333
598	A	3	3	1.00	16	0.188
599	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
600	A	4	3	1.00	10	0.300
601	A	4	3	1.00	10	0.300
602	A	3	2	1.00	13	0.154
603	A	2	2	1.46	13	0.154
604	C	3	2	4.00	11	0.182
605	A	5	4	0.91	12	0.333
606	A	5	4	1.00	14	0.286
607	A	5	4	1.00	18	0.222
608	A	1	1	1.00	6	0.167
609	A	2	2	1.17	8	0.250
610	A	2	2	1.15	10	0.200
611	A	4	4	1.64	8	0.500
612	A	4	4	1.04	10	0.400
613	A	1	1	1.00	14	0.071
614	A	2	2	1.00	16	0.125
615	A	6	5	1.00	8	0.625
616	A	2	2	1.00	10	0.200
617	A	2	2	1.00	10	0.200
618	A	3	2	1.00	8	0.250
619	A	3	2	1.00	12	0.167
620	A	3	2	1.00	12	0.167
621	A	3	2	1.00	14	0.143
622	A	4	3	1.00	16	0.188
623	A	4	3	1.00	18	0.167
624	A	4	3	1.00	18	0.167
625	A	5	4	1.00	20	0.200
626	A	5	4	1.00	22	0.182
627	A	5	4	1.00	22	0.182
628	A	1	1	1.00	7	0.143
629	A	4	3	1.10	9	0.333
630	A	5	4	1.14	9	0.444
631	A	6	5	1.16	9	0.556
632	A	4	3	1.00	9	0.333
633	A	4	3	1.00	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
634	A	6	5	1.00	8	0.625
635	A	4	4	1.00	6	0.667
636	A	3	2	1.00	6	0.333
637	A	3	3	1.00	14	0.214
638	A	4	4	0.86	8	0.500
639	A	5	4	1.00	20	0.200
640	A	7	6	0.98	14	0.429
641	A	6	5	1.17	8	0.625
642	A	6	5	1.17	8	0.625
643	A	5	5	1.18	14	0.357
644	A	11	11	1.10	12	0.917
645	A	5	5	1.09	8	0.625
646	A	5	5	1.07	8	0.625
647	A	11	10	1.21	8	1.250
648	A	14	13	1.30	8	1.625
649	A	10	9	1.10	8	1.125
650	A	15	14	1.37	8	1.750
651	A	3	3	1.00	14	0.214
652	A	3	3	1.00	14	0.214
653	A	2	2	1.10	15	0.133
654	A	6	6	1.29	14	0.429
655	A	3	3	1.03	15	0.200
656	A	4	4	1.05	17	0.235
657	A	10	9	1.02	17	0.529
658	A	5	4	1.02	17	0.235
659	A	3	3	1.00	17	0.176
660	A	5	5	1.08	17	0.294
661	A	2	2	1.00	15	0.133
662	A	2	2	1.00	15	0.133
663	A	4	4	1.03	14	0.286
664	A	3	3	1.00	17	0.176
665	A	8	7	1.00	17	0.412
666	A	4	4	1.02	17	0.235
667	A	7	6	1.08	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
668	A	6	6	1.07	19	0.316
669	A	3	3	1.16	11	0.273
670	A	4	4	1.23	11	0.364
671	A	4	4	1.00	13	0.308
672	A	9	8	0.99	13	0.615
673	A	2	2	1.00	13	0.154
674	A	9	8	1.06	13	0.615
675	A	18	17	1.11	13	1.308
676	A	9	8	1.77	11	0.727
677	A	3	3	1.10	11	0.273
678	A	2	2	1.00	13	0.154
679	A	8	7	1.18	13	0.538
680	A	9	8	1.13	8	1.000
681	A	10	9	1.32	13	0.692
682	A	4	4	1.16	15	0.267
683	A	9	8	0.85	15	0.533
684	A	12	11	1.21	15	0.733
685	A	4	4	1.02	15	0.267
686	A	4	4	1.02	12	0.333
687	A	4	4	1.02	15	0.267
688	A	6	5	0.79	15	0.333
689	A	16	15	1.13	15	1.000
690	A	2	2	1.00	15	0.133
691	A	6	6	1.27	15	0.400
692	A	7	6	1.20	17	0.353
693	A	12	11	1.46	17	0.647
694	A	8	7	1.23	17	0.412
695	B	8	7	2.47	16	0.438
696	A	5	4	1.00	16	0.250
697	A	4	4	1.15	8	0.500
698	A	7	6	1.00	13	0.462
699	A	3	2	1.00	24	0.083
700	A	2	2	1.00	21	0.095
701	A	6	5	1.09	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
702	A	6	6	1.07	10	0.600
703	A	8	7	1.06	8	0.875
704	A	9	8	1.14	10	0.800
705	A	6	5	1.00	7	0.714

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{a^2-b^2x^2}\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

output
```
arctanh(b*x/a)/a/b
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

input
```
Integrate[(a^2 - b^2*x^2)^(-1),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b)
```

3.1.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2-b^2x^2} \, dx$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

input
```
Int[(a^2 - b^2*x^2)^(-1),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b)
```

3.1.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.1.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$-\frac{\ln(bx-a)-\ln(bx+a)}{2ab}$	26
default	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ab}$	31
norman	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ab}$	31
risch	$-\frac{\ln(-bx+a)}{2ba}+\frac{\ln(bx+a)}{2ab}$	31

input
```
int(1/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(b*x-a)-ln(b*x+a))/a/b
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{a^2-b^2x^2}\,dx = \frac{\log(bx+a)-\log(bx-a)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2-b^2x^2}\,dx = -\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

input
```
integrate(1/(-b**2*x**2+a**2),x)
```

output
```
-(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{a^2-b^2x^2}\,dx = \frac{\log(bx+a)}{2\,ab}-\frac{\log(bx-a)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{a^2 - b^2 x^2}\, dx = \frac{\log\left(|bx + a|\right)}{2\,ab} - \frac{\log\left(|bx - a|\right)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x + a))/(a*b) - 1/2*log(abs(b*x - a))/(a*b)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 - b^2 x^2}\, dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b}$$

input
```
int(1/(a^2 - b^2*x^2),x)
```

output
```
atanh((b*x)/a)/(a*b)
```

3.2 $\int \frac{1}{a^2+b^2x^2} dx$

3.2.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{a^2+b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

output
```
arctan(b*x/a)/a/b
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
Integrate[(a^2 + b^2*x^2)^(-1),x]
```

output
```
ArcTan[(b*x)/a]/(a*b)
```

3.2.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+b^2x^2}\,dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
Int[(a^2 + b^2*x^2)^(-1),x]
```

output
```
ArcTan[(b*x)/a]/(a*b)
```

3.2.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.2.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15
risch	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15
parallelrisch	$-\frac{i\ln(-ia+bx)-i\ln(ia+bx)}{2ab}$	34

input
```
int(1/(b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b*x/a)/a/b
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
integrate(1/(b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
arctan(b*x/a)/(a*b)
```

3.2.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{-\frac{i\log\left(-\frac{ia}{b}+x\right)}{2}+\frac{i\log\left(\frac{ia}{b}+x\right)}{2}}{ab}$$

input
```
integrate(1/(b**2*x**2+a**2),x)
```

output
```
(-I*log(-I*a/b + x)/2 + I*log(I*a/b + x)/2)/(a*b)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
integrate(1/(b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
arctan(b*x/a)/(a*b)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2}\, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
integrate(1/(b^2*x^2+a^2),x, algorithm="giac")
```

output
```
arctan(b*x/a)/(a*b)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2}\, dx = \frac{\operatorname{atan}\left(\frac{b\,x}{a}\right)}{a\,b}$$

input
```
int(1/(a^2 + b^2*x^2),x)
```

output
```
atan((b*x)/a)/(a*b)
```

3.3 $\int \sec(2ax)\,dx$

3.3.1 Optimal result

Integrand size = 5, antiderivative size = 13

$$\int \sec(2ax)\,dx = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

output
```
1/2*arctanh(sin(2*a*x))/a
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sec(2ax)\,dx = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

input
```
Integrate[Sec[2*a*x],x]
```

output
```
ArcTanh[Sin[2*a*x]]/(2*a)
```

3.3.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(2ax)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2ax+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

input
```
Int[Sec[2*a*x],x]
```

output
```
ArcTanh[Sin[2*a*x]]/(2*a)
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.3.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$	18
default	$\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$	18
parallelrisch	$\frac{-\ln(\tan(ax)-1)+\ln(\tan(ax)+1)}{2a}$	23
norman	$-\frac{\ln(\tan(ax)-1)}{2a}+\frac{\ln(\tan(ax)+1)}{2a}$	26
risch	$\frac{\ln(e^{2iax}+i)}{2a}-\frac{\ln(e^{2iax}-i)}{2a}$	32

input
```
int(sec(2*a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*ln(sec(2*a*x)+tan(2*a*x))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \sec(2ax)\,dx = \frac{\log(\sin(2\,ax)+1)-\log(-\sin(2\,ax)+1)}{4\,a}$$

input
```
integrate(sec(2*a*x),x, algorithm="fricas")
```

output
```
1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \sec(2ax)\,dx = \begin{cases} \frac{-\frac{\log(\sin(2ax)-1)}{2}+\frac{\log(\sin(2ax)+1)}{2}}{2a} & \text{for } a\neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(sec(2*a*x),x)
```

output
```
Piecewise(((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a), Ne(a, 0
)), (x, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sec(2ax)\,dx = \frac{\log\left(\sec\left(2\,ax\right) + \tan\left(2\,ax\right)\right)}{2\,a}$$

input
```
integrate(sec(2*a*x),x, algorithm="maxima")
```

output
```
1/2*log(sec(2*a*x) + tan(2*a*x))/a
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \sec(2ax)\,dx = \frac{\log\left(\left|\frac{1}{\sin\left(2\,ax\right)} + \sin\left(2\,ax\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin\left(2\,ax\right)} + \sin\left(2\,ax\right) - 2\right|\right)}{8\,a}$$

input
```
integrate(sec(2*a*x),x, algorithm="giac")
```

output
```
1/8*(log(abs(1/sin(2*a*x) + sin(2*a*x) + 2)) - log(abs(1/sin(2*a*x) + sin(
2*a*x) - 2)))/a
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sec(2ax)\,dx = \frac{\operatorname{atanh}(\sin(2\,a\,x))}{2\,a}$$

input
```
int(1/cos(2*a*x),x)
```

output
```
atanh(sin(2*a*x))/(2*a)
```

3.4 $\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx$

3.4.1 Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx=-\frac{3}{4}\text{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

output
```
-3/4*arctanh(cos(1/3*x))
```

3.4.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx=\frac{1}{4}\left(-3\log\left(\cos\left(\frac{x}{6}\right)\right)+3\log\left(\sin\left(\frac{x}{6}\right)\right)\right)$$

input
```
Integrate[Csc[x/3]/4,x]
```

output
```
(-3*Log[Cos[x/6]] + 3*Log[Sin[x/6]])/4
```

3.4.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \csc\left(\frac{x}{3}\right)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \csc\left(\frac{x}{3}\right)dx$$

$$\Big\downarrow \; 4257$$

$$-\frac{3}{4}\text{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

input
```
Int[Csc[x/3]/4,x]
```

output
```
(-3*ArcTanh[Cos[x/3]])/4
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.4.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
norman	$\frac{3\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$	8
parallelrisch	$\frac{3\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$	8
derivativedivides	$\frac{3\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$	15
default	$\frac{3\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$	15
risch	$\frac{3\ln\left(e^{\frac{ix}{3}}-1\right)}{4}-\frac{3\ln\left(e^{\frac{ix}{3}}+1\right)}{4}$	22

input
```
int(1/4/sin(1/3*x),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(tan(1/6*x))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)+\frac{3}{8}\log\left(-\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)$$

input
```
integrate(1/4/sin(1/3*x),x, algorithm="fricas")
```

output
```
-3/8*log(1/2*cos(1/3*x) + 1/2) + 3/8*log(-1/2*cos(1/3*x) + 1/2)
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\log\left(\cos\left(\frac{x}{3}\right)-1\right)}{8} - \frac{3\log\left(\cos\left(\frac{x}{3}\right)+1\right)}{8}$$

input
```
integrate(1/4/sin(1/3*x),x)
```

output
```
3*log(cos(x/3) - 1)/8 - 3*log(cos(x/3) + 1)/8
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right) + \frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)-1\right)$$

input
```
integrate(1/4/sin(1/3*x),x, algorithm="maxima")
```

output
```
-3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right) + \frac{3}{8}\log\left(-\cos\left(\frac{1}{3}x\right)+1\right)$$

input
```
integrate(1/4/sin(1/3*x),x, algorithm="giac")
```

output
```
-3/8*log(cos(1/3*x) + 1) + 3/8*log(-cos(1/3*x) + 1)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\,\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$$

input
```
int(1/(4*sin(x/3)),x)
```

output
```
(3*log(tan(x/6)))/4
```

3.5 $\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$

3.5.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{2}\text{arctanh}\Big(\sin\left(\frac{\pi}{4}+2x\right)\Big)$$

output
```
-1/2*arctanh(sin(1/4*Pi+2*x))
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{2}\text{arctanh}\Big(\sin\left(\frac{\pi}{4}+2x\right)\Big)$$

input
```
Integrate[-Sec[Pi/4 + 2*x],x]
```

output
```
-1/2*ArcTanh[Sin[Pi/4 + 2*x]]
```

3.5.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\sec\left(2x+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \ 25$$
$$-\int \sec\left(2x+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \ 3042$$
$$-\int \csc\left(2x+\frac{3\pi}{4}\right)dx$$
$$\Big\downarrow \ 4257$$
$$-\frac{1}{2}\text{arctanh}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

input
```
Int[-Sec[Pi/4 + 2*x],x]
```

output
```
-1/2*ArcTanh[Sin[Pi/4 + 2*x]]
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.5.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	21
default	$-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$	21
norman	$\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$	24
parallelrisch	$-\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)-1}}\right)-\ln\left(\sqrt{\tan\left(\frac{\pi}{8}+x\right)+1}\right)$	28
risch	$-\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+8x)}{4}}+i\right)}{2}+\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+8x)}{4}}-i\right)}{2}$	32

input
```
int(-1/cos(1/4*Pi+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\,\log\left(\sin\left(\frac{1}{4}\,\pi+2\,x\right)+1\right)+\frac{1}{4}\,\log\left(-\sin\left(\frac{1}{4}\,\pi+2\,x\right)+1\right)$$

input
```
integrate(-1/cos(1/4*pi+2*x),x, algorithm="fricas")
```

output
```
-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2} - \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}$$

input
```
integrate(-1/cos(1/4*pi+2*x),x)
```

output
```
log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2\,x\right)+1\right) + \frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2\,x\right)-1\right)$$

input
```
integrate(-1/cos(1/4*pi+2*x),x, algorithm="maxima")
```

output
```
-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2\,x\right)+1\right) + \frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2\,x\right)+1\right)$$

input
```
integrate(-1/cos(1/4*pi+2*x),x, algorithm="giac")
```

output
```
-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\ln\left(\frac{\sin\left(\frac{\Pi}{4}+2\,x\right)+1}{\cos\left(\frac{\Pi}{4}+2\,x\right)}\right)}{2}$$

input
```
int(-1/cos(Pi/4 + 2*x),x)
```

output
```
-log((sin(Pi/4 + 2*x) + 1)/cos(Pi/4 + 2*x))/2
```

3.6 $\int \sec(x)\tan(x)\,dx$

3.6.1 Optimal result

Integrand size = 5, antiderivative size = 2

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

output
```
sec(x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

input
```
Integrate[Sec[x]*Tan[x],x]
```

output
```
Sec[x]
```

3.6.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec(x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \tan(x)\sec(x)dx$$
$$\Big\downarrow \; 3086$$
$$\int 1d\sec(x)$$
$$\Big\downarrow \; 24$$
$$\sec(x)$$

input
```
Int[Sec[x]*Tan[x],x]
```

output
```
Sec[x]
```

3.6.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.6.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\sec(x)$	3
default	$\sec(x)$	3
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+1}$	17

input
```
int(sec(x)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
sec(x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x, algorithm="fricas")
```

output
```
1/cos(x)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x)
```

output
```
1/cos(x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x, algorithm="maxima")
```

output
```
1/cos(x)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x, algorithm="giac")
```

output
```
1/cos(x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \sec(x)\tan(x)\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

input
```
int(tan(x)/cos(x),x)
```

output
```
-2/(tan(x/2)^2 - 1)
```

3.7 $\int \cot(x)\csc(x)\,dx$

3.7.1 Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

output
```
-csc(x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

input
```
Integrate[Cot[x]*Csc[x],x]
```

output
```
-Csc[x]
```

3.7.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\int 1d\csc(x)$$

$$\Big\downarrow \text{24}$$

$$-\csc(x)$$

input
```
Int[Cot[x]*Csc[x],x]
```

output
```
-Csc[x]
```

3.7.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.7.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$-\csc\left(x\right)$	5
default	$-\csc\left(x\right)$	5
risch	$-\frac{2 i \,{\mathrm e}^{i x}}{{\mathrm e}^{2 i x}-1}$	18

input
```
int(csc(x)*cot(x),x,method=_RETURNVERBOSE)
```

output
```
-csc(x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin\left(x\right)}$$

input
```
integrate(cot(x)*csc(x),x, algorithm="fricas")
```

output
```
-1/sin(x)
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin{(x)}}$$

input
```
integrate(cot(x)*csc(x),x)
```

output
```
-1/sin(x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin{(x)}}$$

input
```
integrate(cot(x)*csc(x),x, algorithm="maxima")
```

output
```
-1/sin(x)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin{(x)}}$$

input
```
integrate(cot(x)*csc(x),x, algorithm="giac")
```

output
```
-1/sin(x)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

input
```
int(cot(x)/sin(x),x)
```

output
```
-1/sin(x)
```

3.8 $\int \csc(2x)\tan(x)\,dx$

3.8.1 Optimal result

Integrand size = 7, antiderivative size = 6

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

output
```
1/2*tan(x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

input
```
Integrate[Csc[2*x]*Tan[x],x]
```

output
```
Tan[x]/2
```

3.8.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4889, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\csc(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\sin(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{2}d\tan(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(x)}{2}$$

input
```
Int[Csc[2*x]*Tan[x],x]
```

output
```
Tan[x]/2
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.8.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\tan(x)}{2}$	5
default	$\frac{\tan(x)}{2}$	5
norman	$\frac{\tan(x)}{2}$	5
parallelrisch	$\frac{\tan(x)}{2}$	5
risch	$\frac{i}{e^{2ix}+1}$	13

input
```
int(tan(x)/sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{1}{2}\,\tan\left(x\right)$$

input
```
integrate(tan(x)/sin(2*x),x, algorithm="fricas")
```

output
```
1/2*tan(x)
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \csc(2x)\tan(x)\,dx = \frac{\sin(x)}{2\cos(x)}$$

input
```
integrate(tan(x)/sin(2*x),x)
```

output
```
sin(x)/(2*cos(x))
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(4) = 8$.

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \csc(2x)\tan(x)\,dx = \frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

input
```
integrate(tan(x)/sin(2*x),x, algorithm="maxima")
```

output
```
sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{1}{2}\tan(x)$$

input
```
integrate(tan(x)/sin(2*x),x, algorithm="giac")
```

output
```
1/2*tan(x)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

input
```
int(tan(x)/sin(2*x),x)
```

output
```
tan(x)/2
```

3.9 $\int \frac{1}{1+\cos(x)} \, dx$

3.9.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)} \, dx = \frac{\sin(x)}{1+\cos(x)}$$

output
```
sin(x)/(1+cos(x))
```

3.9.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)} \, dx = \tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x])^(-1),x]
```

output
```
Tan[x/2]
```

3.9.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+1}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \; 3127$$

$$\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[(1 + Cos[x])^(-1),x]
```

output
```
Sin[x]/(1 + Cos[x])
```

3.9.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
parallelrisch	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

input
```
int(1/(cos(x)+1),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(cos(x) + 1)
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
integrate(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(cos(x) + 1)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(cos(x) + 1),x)
```

output
```
tan(x/2)
```

3.10 $\int \frac{1}{1-\cos(x)}\,dx$

3.10.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

output
```
-sin(x)/(1-cos(x))
```

3.10.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 - Cos[x])^(-1),x]
```

output
```
-Cot[x/2]
```

3.10.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos(x)} \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 3127$$
$$-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[(1 - Cos[x])^(-1),x]
```

output
```
-(Sin[x]/(1 - Cos[x]))
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.10.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

input
```
int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(1/2*x)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)),x)
```

output
```
-1/tan(x/2)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int(-1/(cos(x) - 1),x)
```

output
```
-cot(x/2)
```

3.11 $\int \frac{\sin(x)}{a-b\cos(x)}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log(a-b\cos(x))}{b}$$

output
```
ln(a-b*cos(x))/b
```

3.11.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log(a-b\cos(x))}{b}$$

input
```
Integrate[Sin[x]/(a - b*Cos[x]),x]
```

output
```
Log[a - b*Cos[x]]/b
```

3.11.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a-b\cos(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a+b\sin\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3147}$$
$$\frac{\int \frac{1}{a-b\cos(x)}d(-b\cos(x))}{b}$$
$$\Big\downarrow \text{16}$$
$$\frac{\log(a-b\cos(x))}{b}$$

input
```
Int[Sin[x]/(a - b*Cos[x]),x]
```

output
```
Log[a - b*Cos[x]]/b
```

3.11.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(a-b\cos(x))}{b}$	13
default	$\frac{\ln(a-b\cos(x))}{b}$	13
parallelrisch	$\frac{-\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{a-b\cos(x)}{\cos(x)+1}\right)}{b}$	30
risch	$-\frac{ix}{b}+\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2a\,\mathrm{e}^{ix}}{b}+1\right)}{b}$	32
norman	$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+b\left(\tan^2\left(\frac{x}{2}\right)\right)+a-b\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$	42

input
```
int(sin(x)/(a-b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a-b*cos(x))/b
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log\left(-b\cos\left(x\right)+a\right)}{b}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x, algorithm="fricas")
```

output
```
log(-b*cos(x) + a)/b
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \begin{cases} \frac{\log\left(-\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x)
```

output
```
Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log\left(b\cos\left(x\right) - a\right)}{b}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x, algorithm="maxima")
```

output
```
log(b*cos(x) - a)/b
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log\left(\left|b\cos\left(x\right) - a\right|\right)}{b}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x, algorithm="giac")
```

output
```
log(abs(b*cos(x) - a))/b
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\ln\left(b\,\cos\left(x\right) - a\right)}{b}$$

input
```
int(sin(x)/(a - b*cos(x)),x)
```

output
```
log(b*cos(x) - a)/b
```

3.12 $\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx$

3.12.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

output
```
arctan(b*sin(x)/a)/a/b
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
ArcTan[(b*Sin[x])/a]/(a*b)
```

3.12.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3669, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{a^2+b^2\sin(x)^2}dx$$
$$\Big\downarrow \text{3669}$$
$$\int \frac{1}{a^2+b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
ArcTan[(b*Sin[x])/a]/(a*b)
```

3.12.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.12.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$	16
default	$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$	16
parallelrisch	$-\frac{i\left(-\ln\left(\frac{-ib\sin(x)+a}{\cos(x)+1}\right)+\ln\left(\frac{ib\sin(x)+a}{\cos(x)+1}\right)\right)}{2ab}$	45
risch	$-\frac{i\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}-1\right)}{2ab}+\frac{i\ln\left(e^{2ix}-\frac{2a\,e^{ix}}{b}-1\right)}{2ab}$	58

input
```
int(cos(x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b*sin(x)/a)/a/b
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
arctan(b*sin(x)/a)/(a*b)
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a=0 \wedge b=0 \\ -\frac{1}{b^2\sin(x)} & \text{for } a=0 \\ \frac{\sin(x)}{a^2} & \text{for } b=0 \\ \frac{\operatorname{atan}\left(\frac{b\sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)/(a**2+b**2*sin(x)**2),x)
```

output
```
Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sin(x)), Eq(a, 0)),
 (sin(x)/a**2, Eq(b, 0)), (atan(b*sin(x)/a)/(a*b), True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(b*sin(x)/a)/(a*b)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

output
```
arctan(b*sin(x)/a)/(a*b)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{a\, b}$$

input
```
int(cos(x)/(b^2*sin(x)^2 + a^2),x)
```

output
```
atan((b*sin(x))/a)/(a*b)
```

3.13 $\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$

3.13.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

output
```
arctanh(b*sin(x)/a)/a/b
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
ArcTanh[(b*Sin[x])/a]/(a*b)
```

3.13.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3669, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{a^2-b^2\sin(x)^2}dx$$
$$\Big\downarrow \text{3669}$$
$$\int \frac{1}{a^2-b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{221}$$
$$\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Int[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
ArcTanh[(b*Sin[x])/a]/(a*b)
```

3.13.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.13.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

method	result	size
derivativedivides	$\frac{\ln(a+b\sin(x))}{2ba}-\frac{\ln(-b\sin(x)+a)}{2ba}$	33
default	$\frac{\ln(a+b\sin(x))}{2ba}-\frac{\ln(-b\sin(x)+a)}{2ba}$	33
parallelrisch	$\frac{-\ln\left(\frac{-b\sin(x)+a}{\cos(x)+1}\right)+\ln\left(\frac{a+b\sin(x)}{\cos(x)+1}\right)}{2ab}$	41
norman	$-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)+a\right)}{2ba}+\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}{2ba}$	54
risch	$\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{2ba}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{b}-1\right)}{2ba}$	58

input
```
int(cos(x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/a*ln(a+b*sin(x))-1/2/b/a*ln(-b*sin(x)+a)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\log\left(b\sin\left(x\right)+a\right)-\log\left(-b\sin\left(x\right)+a\right)}{2\,ab}$$

input
```
integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
1/2*(log(b*sin(x) + a) - log(-b*sin(x) + a))/(a*b)
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b^2\sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ -\frac{\log\left(-\frac{a}{b}+\sin(x)\right)}{2ab} + \frac{\log\left(\frac{a}{b}+\sin(x)\right)}{2ab} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)/(a**2-b**2*sin(x)**2),x)
```

output
```
Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (1/(b**2*sin(x)), Eq(a, 0)),
(sin(x)/a**2, Eq(b, 0)), (-log(-a/b + sin(x))/(2*a*b) + log(a/b + sin(x))/
(2*a*b), True))
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\log(b\sin(x) + a)}{2\,ab} - \frac{\log(b\sin(x) - a)}{2\,ab}$$

input
```
integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(b*sin(x) + a)/(a*b) - 1/2*log(b*sin(x) - a)/(a*b)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\log\left(|b\sin\left(x\right) + a|\right)}{2\,ab} - \frac{\log\left(|b\sin\left(x\right) - a|\right)}{2\,ab}$$

input
```
integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*sin(x) + a))/(a*b) - 1/2*log(abs(b*sin(x) - a))/(a*b)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,\sin(x)}{a}\right)}{a\,b}$$

input
```
int(-cos(x)/(b^2*sin(x)^2 - a^2),x)
```

output
```
atanh((b*sin(x))/a)/(a*b)
```

3.14 $\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$

3.14.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

output
```
ln(a^2+b^2*sin(x)^2)/b^2
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
Log[a^2 + b^2*Sin[x]^2]/b^2
```

3.14.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(2x)}{a^2+b^2\sin(x)^2}dx$$
$$\Big\downarrow \text{ 4878}$$
$$\int \frac{2\sin(x)}{a^2+b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{ 27}$$
$$2\int \frac{\sin(x)}{a^2+b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{ 240}$$
$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
Log[a^2 + b^2*Sin[x]^2]/b^2
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.14.4 Maple [A] (verified)

Time = 6.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln\left(a^2+b^2\left(\sin^2\left(x\right)\right)\right)}{b^2}$	18
default	$\frac{\ln\left(a^2+b^2\left(\sin^2\left(x\right)\right)\right)}{b^2}$	18
risch	$-\frac{2ix}{b^2}+\frac{\ln\left(\mathrm{e}^{4ix}-\frac{2\left(2a^2+b^2\right)\mathrm{e}^{2ix}}{b^2}+1\right)}{b^2}$	40

input
```
int(sin(2*x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(a^2+b^2*sin(x)^2)/b^2
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(-b^2\cos\left(x\right)^2+a^2+b^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
log(-b^2*cos(x)^2 + a^2 + b^2)/b^2
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{\sin(2x)}{a^2 + b^2\sin^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log(a^2+b^2\sin^2(x))}{2b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*sin(x)**2
)/(2*b**2), True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\sin^2(x)}\,dx = \frac{\log\left(b^2\sin(x)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
log(b^2*sin(x)^2 + a^2)/b^2
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\sin^2(x)}\,dx = \frac{\log\left(b^2\sin(x)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

output
```
log(b^2*sin(x)^2 + a^2)/b^2
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{\operatorname{atan}\left(\frac{b^2 \sin(x)^2}{a^2 \cos(x)^2\, 2\mathrm{i} + a^2 \sin(x)^2\, 2\mathrm{i} + b^2 \sin(x)^2\, 1\mathrm{i}}\right) 2\mathrm{i}}{b^2}$$

input
```
int(sin(2*x)/(b^2*sin(x)^2 + a^2),x)
```

output
```
(atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i + b^2*sin(x)^2*1i)
)*2i)/b^2
```

3.15 $\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx = -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

output
```
-ln(a^2-b^2*sin(x)^2)/b^2
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx = -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
-(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

3.15.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2x)}{a^2-b^2\sin(x)^2}dx$$
$$\Big\downarrow \text{4878}$$
$$\int \frac{2\sin(x)}{a^2-b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{27}$$
$$2\int \frac{\sin(x)}{a^2-b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{240}$$
$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
-(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.15.4 Maple [A] (verified)

Time = 6.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\ln\left(a^{2}-b^{2}\left(\sin^{2}\left(x\right)\right)\right)}{b^{2}}$	20
default	$-\frac{\ln\left(a^{2}-b^{2}\left(\sin^{2}\left(x\right)\right)\right)}{b^{2}}$	20
risch	$\frac{2ix}{b^{2}}-\frac{\ln\left({\mathrm e}^{4ix}+\frac{2\left(2a^{2}-b^{2}\right){\mathrm e}^{2ix}}{b^{2}}+1\right)}{b^{2}}$	43

input
```
int(sin(2*x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a^2-b^2*sin(x)^2)/b^2
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx=-\frac{\log\left(b^2\cos\left(x\right)^2+a^2-b^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
-log(b^2*cos(x)^2 + a^2 - b^2)/b^2
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)}\, dx = 2 \left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log(a^2 - b^2 \sin^2(x))}{2b^2} & \text{otherwise} \end{cases} \right)$$

input
```
integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2*sin(x)**
2)/(2*b**2), True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)}\, dx = -\frac{\log\left(b^2 \sin(x)^2 - a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
-log(b^2*sin(x)^2 - a^2)/b^2
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)}\, dx = -\frac{\log\left(\left|b^2 \sin(x)^2 - a^2\right|\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
```

output
```
-log(abs(b^2*sin(x)^2 - a^2))/b^2
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)} \, dx = \frac{\operatorname{atan}\left(\frac{b^2 \sin(x)^2}{a^2 \cos(x)^2 \, 2\mathrm{i} + a^2 \sin(x)^2 \, 2\mathrm{i} - b^2 \sin(x)^2 \, 1\mathrm{i}}\right) 2\mathrm{i}}{b^2}$$

input
```
int(-sin(2*x)/(b^2*sin(x)^2 - a^2),x)
```

output
```
(atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i - b^2*sin(x)^2*1i)
)*2i)/b^2
```

3.16 $\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$

3.16.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx=-\frac{\log\left(a^2+b^2\cos^2(x)\right)}{b^2}$$

output
```
-ln(a^2+b^2*cos(x)^2)/b^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx=-\frac{\log\left(a^2+b^2-b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

output
```
-(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

3.16.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2x)}{a^2+b^2\cos(x)^2}dx$$
$$\Big\downarrow \text{4878}$$
$$\int \frac{2\sin(x)}{a^2-b^2\sin^2(x)+b^2}d\sin(x)$$
$$\Big\downarrow \text{27}$$
$$2\int \frac{\sin(x)}{a^2+b^2-b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{240}$$
$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

output
```
-(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.16.4 Maple [A] (verified)

Time = 10.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(a^2+b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
default	$-\frac{\ln\left(a^2+b^2\left(\cos^2(x)\right)\right)}{b^2}$	19
risch	$\frac{2ix}{b^2}-\frac{\ln\left(\mathrm{e}^{4ix}+\frac{2\left(2a^2+b^2\right)\mathrm{e}^{2ix}}{b^2}+1\right)}{b^2}$	41

input
```
int(sin(2*x)/(a^2+b^2*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a^2+b^2*cos(x)^2)/b^2
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx=-\frac{\log\left(b^2\cos{(x)}^2+a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="fricas")
```

output
```
-log(b^2*cos(x)^2 + a^2)/b^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log(a^2+b^2\cos^2(x))}{2b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2*cos(x)**
2)/(2*b**2), True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos(x)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="maxima")
```

output
```
-log(b^2*cos(x)^2 + a^2)/b^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos(x)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="giac")
```

output
```
-log(b^2*cos(x)^2 + a^2)/b^2
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{b^2}{2\,a^2+b^2\cos(x)^2+b^2}-\frac{b^2\cos(x)^2}{2\,a^2+b^2\cos(x)^2+b^2}\right)}{b^2}$$

input
```
int(sin(2*x)/(b^2*cos(x)^2 + a^2),x)
```

output
```
(2*atanh(b^2/(b^2*cos(x)^2 + 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 +
 2*a^2 + b^2)))/b^2
```

3.17 $\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$

3.17.1 Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx = \frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

output
```
ln(a^2-b^2*cos(x)^2)/b^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx = \frac{\log\left(a^2-b^2+b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

output
```
Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

3.17.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(2x)}{a^2-b^2\cos(x)^2}dx$$
$$\Big\downarrow \text{ 4878}$$
$$\int \frac{2\sin(x)}{a^2+b^2\sin^2(x)-b^2}d\sin(x)$$
$$\Big\downarrow \text{ 27}$$
$$2\int \frac{\sin(x)}{a^2-b^2+b^2\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{ 240}$$
$$\frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

output
```
Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.17.4 Maple [A] (verified)

Time = 9.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln\left(a^{2}-b^{2}\left(\cos^{2}\left(x\right)\right)\right)}{b^{2}}$	19
default	$\frac{\ln\left(a^{2}-b^{2}\left(\cos^{2}\left(x\right)\right)\right)}{b^{2}}$	19
risch	$-\frac{2ix}{b^{2}}+\frac{\ln\left({\mathrm e}^{4ix}-\frac{2\left(2a^{2}-b^{2}\right){\mathrm e}^{2ix}}{b^{2}}+1\right)}{b^{2}}$	42

input
```
int(sin(2*x)/(a^2-b^2*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(a^2-b^2*cos(x)^2)/b^2
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx=\frac{\log\left(b^2\cos\left(x\right)^2-a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="fricas")
```

output
```
log(b^2*cos(x)^2 - a^2)/b^2
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log(a^2 - b^2\cos^2(x))}{2b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*cos(x)**2
)/(2*b**2), True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = \frac{\log\left(b^2\cos(x)^2 - a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="maxima")
```

output
```
log(b^2*cos(x)^2 - a^2)/b^2
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = \frac{\log\left(\left|b^2\cos(x)^2 - a^2\right|\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="giac")
```

output
```
log(abs(b^2*cos(x)^2 - a^2))/b^2
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b^2}{-2\,a^2+b^2\cos(x)^2+b^2} - \frac{b^2\cos(x)^2}{-2\,a^2+b^2\cos(x)^2+b^2}\right)}{b^2}$$

input
```
int(-sin(2*x)/(b^2*cos(x)^2 - a^2),x)
```

output
```
-(2*atanh(b^2/(b^2*cos(x)^2 - 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2
- 2*a^2 + b^2)))/b^2
```

3.18 $\int \frac{1}{4-\cos^2(x)} \, dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{1}{4-\cos^2(x)} \, dx = \frac{x}{2\sqrt{3}} + \frac{\arctan\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}$$

output
```
1/6*x*3^(1/2)+1/6*arctan(cos(x)*sin(x)/(3+sin(x)^2+2*3^(1/2)))*3^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{1}{4-\cos^2(x)} \, dx = \frac{\arctan\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[(4 - Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])
```

3.18.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-\cos^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{4-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 3660$$

$$-\int \frac{1}{3\cot^2(x)+4}d\cot(x)$$

$$\Big\downarrow \ 216$$

$$-\frac{\arctan\left(\frac{1}{2}\sqrt{3}\cot(x)\right)}{2\sqrt{3}}$$

input
```
Int[(4 - Cos[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[(Sqrt[3]*Cot[x])/2]/Sqrt[3]
```

3.18.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.18.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{2\tan(x)\sqrt{3}}{3}\right)}{6}$	14
risch	$\frac{i\sqrt{3}\,\ln\left(e^{2ix}-4\sqrt{3}-7\right)}{12}-\frac{i\sqrt{3}\,\ln\left(e^{2ix}+4\sqrt{3}-7\right)}{12}$	40

input
```
int(1/(4-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(2/3*tan(x)*3^(1/2))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{4-\cos^2(x)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{7\,\sqrt{3}\cos\left(x\right)^2-4\,\sqrt{3}}{12\,\cos\left(x\right)\sin\left(x\right)}\right)$$

input
```
integrate(1/(4-cos(x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x)*sin(x))
)
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(4-cos(x)**2),x)
```

output
```
sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6 + sqrt(3)
*(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\tan\left(x\right)\right)$$

input
```
integrate(1/(4-cos(x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{1}{6}\,\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2\,x\right)-2\,\sin\left(2\,x\right)}{\sqrt{3}\cos\left(2\,x\right)+\sqrt{3}-2\,\cos\left(2\,x\right)+2}\right)\right)$$

input
```
integrate(1/(4-cos(x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)*cos(2*x)
 + sqrt(3) - 2*cos(2*x) + 2)))
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\,(x-\operatorname{atan}(\tan(x)))}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tan(x)}{3}\right)}{6}$$

input
```
int(-1/(cos(x)^2 - 4),x)
```

output
```
(3^(1/2)*(x - atan(tan(x))))/6 + (3^(1/2)*atan((2*3^(1/2)*tan(x))/3))/6
```

3.19 $\int \frac{e^x}{-1+e^{2x}}\,dx$

3.19.1 Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\text{arctanh}(e^x)$$

output
```
-arctanh(exp(x))
```

3.19.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\text{arctanh}(e^x)$$

input
```
Integrate[E^x/(-1 + E^(2*x)),x]
```

output
```
-ArcTanh[E^x]
```

3.19.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2679, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}-1}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\int \frac{1}{e^{2x}-1}de^x$$

$$\Big\downarrow \text{220}$$

$$-\text{arctanh}(e^x)$$

input
```
Int[E^x/(-1 + E^(2*x)),x]
```

output
```
-ArcTanh[E^x]
```

3.19.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.19.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
norman	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16
risch	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16

input
```
int(exp(x)/(exp(2*x)-1),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(exp(x))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\log\left(e^x-1\right)}{2}-\frac{\log\left(e^x+1\right)}{2}$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2} - \frac{\ln\left(\mathrm{e}^x+1\right)}{2}$$

input
```
int(exp(x)/(exp(2*x) - 1),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.20 $\int \frac{1}{x \log(x)} \, dx$

3.20.1 Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

output
```
ln(ln(x))
```

3.20.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} \, dx = \log(\log(x))$$

input
```
Integrate[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

3.20.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)}dx$$

$$\Big\downarrow \text{ 2739}$$

$$\int \frac{1}{\log(x)}d\log(x)$$

$$\Big\downarrow \text{ 14}$$

$$\log(\log(x))$$

input
```
Int[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

3.20.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.20.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4
parallelrisch	$\ln(\ln(x))$	4

input
```
int(1/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="fricas")
```

output
```
log(log(x))
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/ln(x),x)
```

output
```
log(log(x))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="maxima")
```

output
```
log(log(x))
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\log(x)}\,dx = \log(|\log(x)|)$$

input
```
integrate(1/x/log(x),x, algorithm="giac")
```

output
```
log(abs(log(x)))
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \ln(\ln(x))$$

input
```
int(1/(x*log(x)),x)
```

output
```
log(log(x))
```

3.21 $\int \frac{1}{x(1+\log^2(x))} dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x(1+\log^2(x))} dx = \arctan(\log(x))$$

output
```
arctan(ln(x))
```

3.21.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+\log^2(x))} dx = \arctan(\log(x))$$

input
```
Integrate[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

3.21.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\log^2(x)+1\right)}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\int \frac{1}{\log^2(x)+1}d\log(x)$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(\log(x))$$

input
```
Int[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

3.21.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.21.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

input
```
int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(ln(x))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+\log^2(x))}\,dx = \arctan(\log(x))$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

output
```
arctan(log(x))
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x(1+\log^2(x))}\,dx = \mathrm{RootSum}\left(4z^2+1,\left(i\mapsto i\log(2i+\log(x))\right)\right)$$

input
```
integrate(1/x/(1+ln(x)**2),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

output
```
arctan(log(x))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

output
```
arctan(log(x))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \mathrm{atan}(\ln\left(x\right))$$

input
```
int(1/(x*(log(x)^2 + 1)),x)
```

output
```
atan(log(x))
```

3.22 $\int \frac{1}{x(1-\log(x))}\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log(1-\log(x))$$

output
```
-ln(1-ln(x))
```

3.22.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log(-1+\log(x))$$

input
```
Integrate[1/(x*(1 - Log[x])),x]
```

output
```
-Log[-1 + Log[x]]
```

3.22.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-\log(x))} \, dx$$

$$\Big\downarrow \text{2739}$$

$$-\int \frac{1}{1-\log(x)} d(1-\log(x))$$

$$\Big\downarrow \text{14}$$

$$-\log(1-\log(x))$$

input
```
Int[1/(x*(1 - Log[x])),x]
```

output
```
-Log[1 - Log[x]]
```

3.22.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.22.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
norman	$-\ln(-1+\ln(x))$	8
risch	$-\ln(-1+\ln(x))$	8
parallelrisch	$-\ln(-1+\ln(x))$	8
derivativedivides	$-\ln(1-\ln(x))$	10
default	$-\ln(1-\ln(x))$	10

input
```
int(1/x/(1-ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+ln(x))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

input
```
integrate(1/x/(1-log(x)),x, algorithm="fricas")
```

output
```
-log(log(x) - 1)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

input
```
integrate(1/x/(1-ln(x)),x)
```

output
```
-log(log(x) - 1)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

input
```
integrate(1/x/(1-log(x)),x, algorithm="maxima")
```

output
```
-log(log(x) - 1)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \frac{1}{x(1-\log(x))}\,dx = -\frac{1}{2}\log\left(\frac{1}{4}\,\pi^2(\mathrm{sgn}(x)-1)^2+\left(\log\left(|x|\right)-1\right)^2\right)$$

input
```
integrate(1/x/(1-log(x)),x, algorithm="giac")
```

output
```
-1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) - 1)^2)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\ln\left(\ln\left(x\right)-1\right)$$

input
```
int(-1/(x*(log(x) - 1)),x)
```

output
```
-log(log(x) - 1)
```

3.23 $\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 9

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(1+\log\left(\frac{x}{a}\right)\right)$$

output
```
ln(1+ln(x/a))
```

3.23.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(1+\log\left(\frac{x}{a}\right)\right)$$

input
```
Integrate[1/(x*(1 + Log[x/a])),x]
```

output
```
Log[1 + Log[x/a]]
```

3.23.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\log\left(\frac{x}{a}\right)+1\right)}dx$$

$$\Big\downarrow \text{2739}$$

$$\int \frac{1}{\log\left(\frac{x}{a}\right)+1}d\left(\log\left(\frac{x}{a}\right)+1\right)$$

$$\Big\downarrow \text{14}$$

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
Int[1/(x*(1 + Log[x/a])),x]
```

output
```
Log[1 + Log[x/a]]
```

3.23.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.23.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
default	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
norman	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
risch	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10
parallelrisch	$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$	10

input
```
int(1/x/(1+ln(x/a)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+ln(x/a))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+log(x/a)),x, algorithm="fricas")
```

output
```
log(log(x/a) + 1)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+ln(x/a)),x)
```

output
```
log(log(x/a) + 1)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+log(x/a)),x, algorithm="maxima")
```

output
```
log(log(x/a) + 1)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+log(x/a)),x, algorithm="giac")
```

output
```
log(log(x/a) + 1)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \ln\left(\ln\left(\frac{x}{a}\right)+1\right)$$

input
```
int(1/(x*(log(x/a) + 1)),x)
```

output
```
log(log(x/a) + 1)
```

3.24 $\int \frac{(1-\sqrt{x}+x)^2}{x^2} \, dx$

3.24.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{(1-\sqrt{x}+x)^2}{x^2} \, dx = -\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3\log(x)$$

output
```
-1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(1-\sqrt{x}+x)^2}{x^2} \, dx = -\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3\log(x)$$

input
```
Integrate[(1 - Sqrt[x] + x)^2/x^2,x]
```

output
```
-x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

3.24.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{x}+1\right)^2}{x^2}dx$$

$$\Big\downarrow \text{ 1693}$$

$$2\int \frac{\left(x-\sqrt{x}+1\right)^2}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1140}$$

$$2\int \left(\sqrt{x}-2+\frac{3}{\sqrt{x}}-\frac{2}{x}+\frac{1}{x^{3/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{x}{2}-2\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{1}{2x}+3\log\left(\sqrt{x}\right)\right)$$

input
```
Int[(1 - Sqrt[x] + x)^2/x^2,x]
```

output
```
2*(-1/2*1/x + 2/Sqrt[x] - 2*Sqrt[x] + x/2 + 3*Log[Sqrt[x]])
```

3.24.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{1}{x}+x+3\ln\left(x\right)+\frac{4}{\sqrt{x}}-4\sqrt{x}$	22
default	$-\frac{1}{x}+x+3\ln\left(x\right)+\frac{4}{\sqrt{x}}-4\sqrt{x}$	22
trager	$\frac{(-1+x)(1+x)}{x}-\frac{4(-1+x)}{\sqrt{x}}-3\ln\left(\frac{1}{x}\right)$	26

input
```
int((1+x-x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=\frac{x^2+6\,x\log\left(\sqrt{x}\right)-4\,(x-1)\sqrt{x}-1}{x}$$

input
```
integrate((1+x-x^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
(x^2 + 6*x*log(sqrt(x)) - 4*(x - 1)*sqrt(x) - 1)/x
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=-4\sqrt{x}+x+3\log\left(x\right)-\frac{1}{x}+\frac{4}{\sqrt{x}}$$

input
```
integrate((1+x-x**(1/2))**2/x**2,x)
```

output
```
-4*sqrt(x) + x + 3*log(x) - 1/x + 4/sqrt(x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x-4\sqrt{x}+\frac{4\sqrt{x}-1}{x}+3\log\left(x\right)$$

input
```
integrate((1+x-x^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(x)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x-4\sqrt{x}+\frac{4\sqrt{x}-1}{x}+3\log\left(|x|\right)$$

input
```
integrate((1+x-x^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(abs(x))
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x+6\ln\left(\sqrt{x}\right)+\frac{4\sqrt{x}-1}{x}-4\sqrt{x}$$

input
```
int((x - x^(1/2) + 1)^2/x^2,x)
```

output
```
x + 6*log(x^(1/2)) + (4*x^(1/2) - 1)/x - 4*x^(1/2)
```

3.25 $\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx$

3.25.1 Optimal result

Integrand size = 22, antiderivative size = 30

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)$$

output
```
-3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)$$

input
```
Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

output
```
4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

3.25.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {10, 7267, 25, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2-x^{2/3}\right)\left(x+\sqrt{x}\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{\left(\sqrt{x}+1\right)\left(2-x^{2/3}\right)}{x}dx$$

$$\Big\downarrow \text{7267}$$

$$-6\int -\frac{\left(\sqrt{x}+1\right)\left(2-x^{2/3}\right)}{\sqrt[6]{x}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{25}$$

$$6\int \frac{\left(\sqrt{x}+1\right)\left(2-x^{2/3}\right)}{\sqrt[6]{x}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{2360}$$

$$6\int \left(-x-\sqrt{x}+2\sqrt[3]{x}+\frac{2}{\sqrt[6]{x}}\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{2009}$$

$$-6\left(\frac{x^{7/6}}{7}+\frac{x^{2/3}}{4}-\frac{2\sqrt{x}}{3}-2\log\left(\sqrt[6]{x}\right)\right)$$

input
```
Int[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

output
```
-6*((-2*Sqrt[x])/3 + x^(2/3)/4 + x^(7/6)/7 - 2*Log[x^(1/6)])
```

3.25.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.25.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}+2\ln\left(x\right)+4\sqrt{x}$	21
default	$-\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}+2\ln\left(x\right)+4\sqrt{x}$	21

input
```
int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} - \frac{3}{2}\,x^{\frac{2}{3}} + 4\,\sqrt{x} + 12\,\log\left(x^{\frac{1}{6}}\right)$$

input
```
integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
-6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx = -\frac{6x^{\frac{7}{6}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + 4\sqrt{x} + 6\log\left(\sqrt[3]{x}\right)$$

input
```
integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)
```

output
```
-6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 6*log(x**(1/3))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} - \frac{3}{2}\,x^{\frac{2}{3}} + 4\,\sqrt{x} + 2\,\log\left(x\right)$$

input
```
integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 2\log(|x|)$$

input
```
integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
-6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(abs(x))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = 12\ln\left(x^{1/6}\right) + 4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

input
```
int(-((x^(2/3) - 2)*(x + x^(1/2)))/x^(3/2),x)
```

output
```
12*log(x^(1/6)) + 4*x^(1/2) - (3*x^(2/3))/2 - (6*x^(7/6))/7
```

3.26 $\int \frac{-1+2x}{3+2x}\,dx$

3.26.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\log(3+2x)$$

output
```
x-2*ln(3+2*x)
```

3.26.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\log(3+2x)$$

input
```
Integrate[(-1 + 2*x)/(3 + 2*x),x]
```

output
```
x - 2*Log[3 + 2*x]
```

3.26.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-1}{2x+3} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(1-\frac{4}{2x+3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$x-2\log(2x+3)$$

input
```
Int[(-1 + 2*x)/(3 + 2*x),x]
```

output
```
x - 2*Log[3 + 2*x]
```

3.26.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$x-2\ln\left(\frac{3}{2}+x\right)$	9
default	$x-2\ln\left(3+2x\right)$	11
norman	$x-2\ln\left(3+2x\right)$	11
meijerg	$-2\ln\left(1+\frac{2x}{3}\right)+x$	11
risch	$x-2\ln\left(3+2x\right)$	11

input
```
int((2*x-1)/(3+2*x),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(3/2+x)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\log\left(2\,x+3\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x, algorithm="fricas")
```

output
```
x - 2*log(2*x + 3)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\log\left(2x+3\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x)
```

output
```
x - 2*log(2*x + 3)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\log\left(2\,x+3\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x, algorithm="maxima")
```

output
```
x - 2*log(2*x + 3)
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\log\left(|2\,x+3|\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x, algorithm="giac")
```

output
```
x - 2*log(abs(2*x + 3))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\ln\left(x+\frac{3}{2}\right)$$

input
```
int((2*x - 1)/(2*x + 3),x)
```

output
```
x - 2*log(x + 3/2)
```

3.27 $\int \frac{-5+2x}{-2+3x^2} \, dx$

3.27.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{-5+2x}{-2+3x^2} \, dx = \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)$$

output
```
1/12*ln(-3*x+6^(1/2))*(4-5*6^(1/2))+1/12*ln(3*x+6^(1/2))*(4+5*6^(1/2))
```

3.27.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{-2+3x^2} \, dx = \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)$$

input
```
Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]
```

output
```
((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3
*x])/12
```

3.27.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {452, 25, 220, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-5}{3x^2-2}\,dx$$

$$\Big\downarrow \; 452$$

$$2\int -\frac{x}{2-3x^2}dx-5\int \frac{1}{3x^2-2}dx$$

$$\Big\downarrow \; 25$$

$$-2\int \frac{x}{2-3x^2}dx-5\int \frac{1}{3x^2-2}dx$$

$$\Big\downarrow \; 220$$

$$\frac{5\text{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}-2\int \frac{x}{2-3x^2}dx$$

$$\Big\downarrow \; 240$$

$$\frac{5\text{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}+\frac{1}{3}\log\left(2-3x^2\right)$$

input
```
Int[(-5 + 2*x)/(-2 + 3*x^2),x]
```

output
```
(5*ArcTanh[Sqrt[3/2]*x])/Sqrt[6] + Log[2 - 3*x^2]/3
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\ln(3x^2-2)}{3}+\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)}{6}$	24
meijerg	$\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(1-\frac{3x^2}{2}\right)}{3}$	27
risch	$\frac{\ln\left(3x+\sqrt{6}\right)}{3}+\frac{5\ln\left(3x+\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(3x-\sqrt{6}\right)}{3}-\frac{5\ln\left(3x-\sqrt{6}\right)\sqrt{6}}{12}$	52

input
```
int((-5+2*x)/(3*x^2-2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{-5+2x}{-2+3x^2}\,dx=\frac{5}{12}\,\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}x+2}{3\,x^2-2}\right)+\frac{1}{3}\,\log\left(3\,x^2-2\right)$$

input
```
integrate((-5+2*x)/(3*x^2-2),x, algorithm="fricas")
```

output
```
5/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2)) + 1/3*log(3*x^2 -
2)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \left(\frac{1}{3}-\frac{5\sqrt{6}}{12}\right)\log\left(x-\frac{\sqrt{6}}{3}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(x+\frac{\sqrt{6}}{3}\right)$$

input
```
integrate((-5+2*x)/(3*x**2-2),x)
```

output
```
(1/3 - 5*sqrt(6)/12)*log(x - sqrt(6)/3) + (1/3 + 5*sqrt(6)/12)*log(x + sqr
t(6)/3)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{-2+3x^2}\,dx = -\frac{5}{12}\sqrt{6}\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)+\frac{1}{3}\log\left(3\,x^2-2\right)$$

input
```
integrate((-5+2*x)/(3*x^2-2),x, algorithm="maxima")
```

output
```
-5/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6))) + 1/3*log(3*x^2 - 2)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{1}{12}\left(5\sqrt{6}+4\right)\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\left(5\sqrt{6}-4\right)\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

input
```
integrate((-5+2*x)/(3*x^2-2),x, algorithm="giac")
```

output
```
1/12*(5*sqrt(6) + 4)*log(abs(x + 1/3*sqrt(6))) - 1/12*(5*sqrt(6) - 4)*log(
abs(x - 1/3*sqrt(6)))
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{\ln\left(x-\frac{\sqrt{6}}{3}\right)}{3} + \frac{\ln\left(x+\frac{\sqrt{6}}{3}\right)}{3} - \frac{5\sqrt{6}\,\ln\left(x-\frac{\sqrt{6}}{3}\right)}{12} + \frac{5\sqrt{6}\,\ln\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

input
```
int((2*x - 5)/(3*x^2 - 2),x)
```

output
```
log(x - 6^(1/2)/3)/3 + log(x + 6^(1/2)/3)/3 - (5*6^(1/2)*log(x - 6^(1/2)/3
))/12 + (5*6^(1/2)*log(x + 6^(1/2)/3))/12
```

3.28 $\int \frac{-5+2x}{2+3x^2}\,dx$

3.28.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{-5+2x}{2+3x^2}\,dx = -\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}+\frac{1}{3}\log\left(2+3x^2\right)$$

output
```
1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{2+3x^2}\,dx = -\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}+\frac{1}{3}\log\left(2+3x^2\right)$$

input
```
Integrate[(-5 + 2*x)/(2 + 3*x^2),x]
```

output
```
(-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

3.28.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-5}{3x^2+2}dx$$

$$\Big\downarrow \text{ 452}$$

$$2\int \frac{x}{3x^2+2}dx-5\int \frac{1}{3x^2+2}dx$$

$$\Big\downarrow \text{ 216}$$

$$2\int \frac{x}{3x^2+2}dx-\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Int[(-5 + 2*x)/(2 + 3*x^2),x]
```

output
```
(-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

3.28.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

3.28.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\ln(3x^2+2)}{3}-\frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	24
risch	$\frac{\ln(9x^2+6)}{3}-\frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	24
meijerg	$-\frac{5\sqrt{6}\,\arctan\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(1+\frac{3x^2}{2}\right)}{3}$	27

input
```
int((-5+2*x)/(3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2}\,dx=-\frac{5}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)+\frac{1}{3}\,\log\left(3\,x^2+2\right)$$

input
```
integrate((-5+2*x)/(3*x^2+2),x, algorithm="fricas")
```

output
```
-5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{-5+2x}{2+3x^2}\,dx = \frac{\log\left(x^2+\frac{2}{3}\right)}{3} - \frac{5\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

input
```
integrate((-5+2*x)/(3*x**2+2),x)
```

output
```
log(x**2 + 2/3)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2}\,dx = -\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{3}\log\left(3x^2+2\right)$$

input
```
integrate((-5+2*x)/(3*x^2+2),x, algorithm="maxima")
```

output
```
-5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{-5+2x}{2+3x^2}\,dx = -\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{3}\log\left(x^2+\frac{2}{3}\right)$$

input
```
integrate((-5+2*x)/(3*x^2+2),x, algorithm="giac")
```

output
```
-5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(x^2 + 2/3)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{-5+2x}{2+3x^2}\,dx = \frac{\ln\left(x^2+\frac{2}{3}\right)}{3} - \frac{5\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

input
```
int((2*x - 5)/(3*x^2 + 2),x)
```

output
```
log(x^2 + 2/3)/3 - (5*6^(1/2)*atan((6^(1/2)*x)/2))/6
```

3.29 $\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$

3.29.1 Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx=\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

output
```
2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

3.29.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx=\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

input
```
Integrate[Sin[x/4]*Sin[x],x]
```

output
```
(2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

3.29.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\frac{x}{4}\right)\sin(x)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

input
```
Int[Sin[x/4]*Sin[x],x]
```

output
```
(2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

3.29.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.29.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
parallelrisch	$\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
derivativedivides	$-\frac{32\left(\sin^5\left(\frac{x}{4}\right)\right)}{5}+\frac{16\left(\sin^3\left(\frac{x}{4}\right)\right)}{3}$	18
default	$-\frac{32\left(\sin^5\left(\frac{x}{4}\right)\right)}{5}+\frac{16\left(\sin^3\left(\frac{x}{4}\right)\right)}{3}$	18
norman	$\frac{-\frac{8\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{8}\right)\right)}{15}+\frac{32\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{8}\right)}{15}+\frac{8\tan\left(\frac{x}{2}\right)}{15}-\frac{32\tan\left(\frac{x}{8}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{8}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	59

input
```
int(sin(1/4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx=-\frac{16}{15}\left(6\cos\left(\frac{1}{4}x\right)^4-7\cos\left(\frac{1}{4}x\right)^2+1\right)\sin\left(\frac{1}{4}x\right)$$

input
```
integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
```

output
```
-16/15*(6*cos(1/4*x)^4 - 7*cos(1/4*x)^2 + 1)*sin(1/4*x)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx=-\frac{16\sin\left(\frac{x}{4}\right)\cos(x)}{15}+\frac{4\sin(x)\cos\left(\frac{x}{4}\right)}{15}$$

input
```
integrate(sin(1/4*x)*sin(x),x)
```

output
```
-16*sin(x/4)*cos(x)/15 + 4*sin(x)*cos(x/4)/15
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{2}{5}\sin\left(\frac{5}{4}x\right) + \frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

input
```
integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
```

output
```
-2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{32}{5}\sin\left(\frac{1}{4}x\right)^5 + \frac{16}{3}\sin\left(\frac{1}{4}x\right)^3$$

input
```
integrate(sin(1/4*x)*sin(x),x, algorithm="giac")
```

output
```
-32/5*sin(1/4*x)^5 + 16/3*sin(1/4*x)^3
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = \frac{2\sin\left(\frac{3x}{4}\right)}{3} - \frac{2\sin\left(\frac{5x}{4}\right)}{5}$$

input
```
int(sin(x/4)*sin(x),x)
```

output
```
(2*sin((3*x)/4))/3 - (2*sin((5*x)/4))/5
```

3.30 $\int \cos(3x)\cos(4x)\,dx$

3.30.1 Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

output
```
1/2*sin(x)+1/14*sin(7*x)
```

3.30.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

input
```
Integrate[Cos[3*x]*Cos[4*x],x]
```

output
```
Sin[x]/2 + Sin[7*x]/14
```

3.30.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(3x)\cos(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(3x)\cos(4x)dx$$

$$\Big\downarrow \text{4771}$$

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

input
```
Int[Cos[3*x]*Cos[4*x],x]
```

output
```
Sin[x]/2 + Sin[7*x]/14
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
parallelrisch	$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$	12
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{7}+\frac{6\left(\tan^2(2x)\right)\tan\left(\frac{3x}{2}\right)}{7}+\frac{8\tan(2x)}{7}-\frac{6\tan\left(\frac{3x}{2}\right)}{7}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

input
```
int(cos(3*x)*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)+1/14*sin(7*x)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{7}\left(32\cos(x)^6-40\cos(x)^4+12\cos(x)^2+3\right)\sin(x)$$

input
```
integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

output
```
1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x)\cos(4x)\,dx = -\frac{3\sin(3x)\cos(4x)}{7}+\frac{4\sin(4x)\cos(3x)}{7}$$

input
```
integrate(cos(3*x)*cos(4*x),x)
```

output
```
-3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
```

output
```
1/14*sin(7*x) + 1/2*sin(x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
```

output
```
1/14*sin(7*x) + 1/2*sin(x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(7x)}{14} + \frac{\sin(x)}{2}$$

input
```
int(cos(3*x)*cos(4*x),x)
```

output
```
sin(7*x)/14 + sin(x)/2
```

3.31 $\int -\tan(a-x)\tan(x)\,dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 21

$$\int -\tan(a-x)\tan(x)\,dx = -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))$$

output
```
-x-cot(a)*ln(cos(x))+cot(a)*ln(cos(a-x))
```

3.31.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int -\tan(a-x)\tan(x)\,dx = -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))$$

input
```
Integrate[-(Tan[a - x]*Tan[x]),x]
```

output
```
-x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

3.31.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {25, 5123, 5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)(-\tan(a-x))\,dx$$
$$\Big\downarrow \; 25$$
$$-\int \tan(a-x)\tan(x)dx$$
$$\Big\downarrow \; 5123$$
$$\cos(a)\int \sec(a-x)\sec(x)dx - x$$
$$\Big\downarrow \; 5121$$
$$\cos(a)\left(\csc(a)\int \tan(a-x)dx + \csc(a)\int \tan(x)dx\right) - x$$
$$\Big\downarrow \; 3042$$
$$\cos(a)\left(\csc(a)\int \tan(a-x)dx + \csc(a)\int \tan(x)dx\right) - x$$
$$\Big\downarrow \; 3956$$
$$\cos(a)(\csc(a)\log(\cos(a-x)) - \csc(a)\log(\cos(x))) - x$$

input
```
Int[-(Tan[a - x]*Tan[x]),x]
```

output
```
-x + Cos[a]*(Csc[a]*Log[Cos[a - x]] - Csc[a]*Log[Cos[x]])
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

rule 5123
```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[(b/d)*Cos[(b*c - a*d)/d]   Int[Sec[a + b*x]*Sec[c + d*x],
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln(1+\tan(a)\tan(x))}{\tan(a)} - \arctan(\tan(x))$	20
default	$\frac{\ln(1+\tan(a)\tan(x))}{\tan(a)} - \arctan(\tan(x))$	20
risch	$-x + \frac{i\ln(e^{2ia}+e^{2ix})e^{2ia}}{e^{2ia}-1} + \frac{i\ln(e^{2ia}+e^{2ix})}{e^{2ia}-1} - \frac{i\ln(e^{2ix}+1)e^{2ia}}{e^{2ia}-1} - \frac{i\ln(e^{2ix}+1)}{e^{2ia}-1}$	103

input
```
int(-tan(x)*tan(a-x),x,method=_RETURNVERBOSE)
```

output
```
1/tan(a)*ln(1+tan(a)*tan(x))-arctan(tan(x))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.24

$$\int -\tan(a-x)\tan(x)\,dx$$
$$= \frac{(\cos(2a)+1)\log\left(-\frac{(\cos(2a)-1)\tan(x)^2-2\sin(2a)\tan(x)-\cos(2a)-1}{(\cos(2a)+1)\tan(x)^2+\cos(2a)+1}\right)-(\cos(2a)+1)\log\left(\frac{1}{\tan(x)^2+1}\right)-2x\sin(2}{2\sin(2a)}$$

input
```
integrate(-tan(x)*tan(a-x),x, algorithm="fricas")
```

output
```
1/2*((cos(2*a) + 1)*log(-((cos(2*a) - 1)*tan(x)^2 - 2*sin(2*a)*tan(x) - co
s(2*a) - 1)/((cos(2*a) + 1)*tan(x)^2 + cos(2*a) + 1)) - (cos(2*a) + 1)*log
(1/(tan(x)^2 + 1)) - 2*x*sin(2*a))/sin(2*a)
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.57

$$\int -\tan(a-x)\tan(x)\,dx$$
$$= -\left(\begin{cases}\frac{2x\tan(a)}{2\tan^2(a)+2}-\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^2(a)+2}+\frac{\log(\tan^2(x)+1)}{2\tan^2(a)+2} & \text{for } a\neq 0\\ \frac{\log(\tan^2(x)+1)}{2} & \text{otherwise}\end{cases}\right)\tan(a)$$
$$+\begin{cases}-\frac{2x\tan(a)}{2\tan^3(a)+2\tan(a)}+\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^3(a)+2\tan(a)}+\frac{\log(\tan^2(x)+1)\tan^2(a)}{2\tan^3(a)+2\tan(a)} & \text{for } a\neq 0\\ -x+\tan(x) & \text{otherwise}\end{cases}$$

input
```
integrate(-tan(x)*tan(a-x),x)
```

output
```
-Piecewise((2*x*tan(a)/(2*tan(a)**2 + 2) - 2*log(tan(x) + 1/tan(a))/(2*tan
(a)**2 + 2) + log(tan(x)**2 + 1)/(2*tan(a)**2 + 2), Ne(a, 0)), (log(tan(x)
**2 + 1)/2, True))*tan(a) + Piecewise((-2*x*tan(a)/(2*tan(a)**3 + 2*tan(a)
) + 2*log(tan(x) + 1/tan(a))/(2*tan(a)**3 + 2*tan(a)) + log(tan(x)**2 + 1)
*tan(a)**2/(2*tan(a)**3 + 2*tan(a)), Ne(a, 0)), (-x + tan(x), True))
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(21) = 42.

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 8.86

$$\int -\tan(a-x)\tan(x)\,dx =$$

$$-\frac{\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}-2\,\cos\left(2\,a\right)+1\right)x+\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}-1\right)\arctan\left(\sin\left(2\,a\right)+\sin\left(2\,x\right),\mathrm{co}\right.}{}$$

input
```
integrate(-tan(x)*tan(a-x),x, algorithm="maxima")
```

output
```
-((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + sin(2*a)^2
- 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) - (cos(2*a)^2 + sin
(2*a)^2 - 1)*arctan2(sin(2*x), cos(2*x) + 1) - log(cos(2*a)^2 + 2*cos(2*a)
*cos(2*x) + cos(2*x)^2 + sin(2*a)^2 + 2*sin(2*a)*sin(2*x) + sin(2*x)^2)*si
n(2*a) + log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*sin(2*a))/(cos(2*a)
^2 + sin(2*a)^2 - 2*cos(2*a) + 1)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int -\tan(a-x)\tan(x)\,dx = -x + \frac{\log\left(\left|\tan\left(a\right)\tan\left(x\right)+1\right|\right)}{\tan\left(a\right)}$$

input
```
integrate(-tan(x)*tan(a-x),x, algorithm="giac")
```

output
```
-x + log(abs(tan(a)*tan(x) + 1))/tan(a)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 5.62

$$\int -\tan(a-x)\tan(x)\,dx = -x - \frac{\frac{\sin(2a)\ln\left(\sin(2a+x)^2\,2\mathrm{i}+\sin(2a)^2\,2\mathrm{i}-\sin(x)^2\,2\mathrm{i}+\sin(4a)-\sin(2x)+\sin(4a+2x)\right)}{2} - \frac{\sin(2a)\ln\left(\sin(2a)\left(2\sin(a)^2-1\right)-\sin(2a)^2\,1\mathrm{i}+\sin(\right.}{2}}{\sin(a)^2}$$

input
```
int(-tan(a - x)*tan(x),x)
```

output
```
- x - ((sin(2*a)*log(sin(4*a) - sin(2*x) + sin(4*a + 2*x) - sin(x)^2*2i +
sin(2*a + x)^2*2i + sin(2*a)^2*2i))/2 - (sin(2*a)*log(sin(2*a)*(2*sin(a)^2
 - 1) - sin(2*a)^2*1i + sin(2*a)*(2*sin(x)^2 - 1) - sin(2*a)*sin(2*x)*1i))
/2)/sin(a)^2
```

3.32 $\int \sin^2(x)\,dx$

3.32.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.32.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

input
```
Integrate[Sin[x]^2,x]
```

output
```
x/2 - Sin[2*x]/4
```

3.32.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2} - \frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Sin[x]^2,x]
```

output
```
x/2 - (Cos[x]*Sin[x])/2
```

3.32.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.32.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

input
```
integrate(sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*cos(x)*sin(x) + 1/2*x
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(sin(x)**2,x)
```

output
```
x/2 - sin(x)*cos(x)/2
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

input
```
int(sin(x)^2,x)
```

output
```
x/2 - sin(2*x)/4
```

3.33 $\int \cos^2(x)\,dx$

3.33.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.33.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

3.33.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.33.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.33.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	45

input
```
int(cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

input
```
integrate(cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(cos(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

3.34 $\int \cos^3(x)\sin(x)\,dx$

3.34.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos^4(x)$$

output
```
-1/4*cos(x)^4
```

3.34.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos^4(x)$$

input
```
Integrate[Cos[x]^3*Sin[x],x]
```

output
```
-1/4*Cos[x]^4
```

3.34.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(x)\cos(x)^3dx$$
$$\Big\downarrow \text{3045}$$
$$-\int \cos^3(x)d\cos(x)$$
$$\Big\downarrow \text{15}$$
$$-\frac{1}{4}\cos^4(x)$$

input
```
Int[Cos[x]^3*Sin[x],x]
```

output
```
-1/4*Cos[x]^4
```

3.34.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.34.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{(\cos^4(x))}{4}$	7
default	$-\frac{(\cos^4(x))}{4}$	7
risch	$-\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{32}+\frac{5}{32}-\frac{\cos(2x)}{8}$	15
norman	$\frac{2(\tan^2(\frac{x}{2}))+2(\tan^6(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^4}$	29
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{8}+\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(4x)}{\sqrt{\pi}}\right)}{32}$	38

input
```
int(cos(x)^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x)^4
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos(x)^4$$

input
```
integrate(cos(x)^3*sin(x),x, algorithm="fricas")
```

output
```
-1/4*cos(x)^4
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \cos^3(x)\sin(x)\,dx = -\frac{\cos^4(x)}{4}$$

input
```
integrate(cos(x)**3*sin(x),x)
```

output
```
-cos(x)**4/4
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos(x)^4$$

input
```
integrate(cos(x)^3*sin(x),x, algorithm="maxima")
```

output
```
-1/4*cos(x)^4
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos(x)^4$$

input
```
integrate(cos(x)^3*sin(x),x, algorithm="giac")
```

output
```
-1/4*cos(x)^4
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \cos^3(x)\sin(x)\,dx = -\frac{\sin(x)^2\left(\sin(x)^2-2\right)}{4}$$

input
```
int(cos(x)^3*sin(x),x)
```

output
```
-(sin(x)^2*(sin(x)^2 - 2))/4
```

3.35 $\int \cot^3(x)\csc(x)\,dx$

3.35.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

output
```
-1/3/sin(x)^3+1/sin(x)
```

3.35.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

input
```
Integrate[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

3.35.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 3086$$

$$-\int \left(\csc^2(x)-1\right)d\csc(x)$$

$$\Big\downarrow \; 2009$$

$$\csc(x)-\frac{\csc^3(x)}{3}$$

input
```
Int[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.35.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
parallelrisch	$-\frac{(\csc^3(x))(-1+3\cos(2x))}{6}$	15
default	$-\frac{\cos^4(x)}{3\sin(x)^3}+\frac{\cos^4(x)}{3\sin(x)}+\frac{(2+\cos^2(x))\sin(x)}{3}$	32
norman	$\frac{-\frac{1}{24}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34
risch	$\frac{2i(3\,e^{5ix}-2\,e^{3ix}+3\,e^{ix})}{3(e^{2ix}-1)^3}$	35

input
```
int(cos(x)^3/sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csc(x)^3*(-1+3*cos(2*x))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\cos{(x)}^2-2}{3\,\left(\cos{(x)}^2-1\right)\sin{(x)}}$$

input
```
integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\csc(x)\,dx = -\frac{1-3\sin^2(x)}{3\sin^3(x)}$$

input
```
integrate(cos(x)**3/sin(x)**4,x)
```

output
```
-(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin(x)^2-1}{3\,\sin(x)^3}$$

input
```
integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin(x)^2-1}{3\,\sin(x)^3}$$

input
```
integrate(cos(x)^3/sin(x)^4,x, algorithm="giac")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

input
```
int(cos(x)^3/sin(x)^4,x)
```

output
```
(sin(x)^2 - 1/3)/sin(x)^3
```

3.36 $\int \csc^2(x)\sec^2(x)\,dx$

3.36.1 Optimal result

Integrand size = 9, antiderivative size = 7

$$\int \csc^2(x)\sec^2(x)\,dx = -\cot(x)+\tan(x)$$

output
```
-cot(x)+tan(x)
```

3.36.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2x)$$

input
```
Integrate[Csc[x]^2*Sec[x]^2,x]
```

output
```
-2*Cot[2*x]
```

3.36.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^2\sec(x)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\int \left(\tan^2(x)+1\right)\cot^2(x)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\cot^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\tan(x)-\cot(x)$$

input
```
Int[Csc[x]^2*Sec[x]^2,x]
```

output
```
-Cot[x] + Tan[x]
```

3.36.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.36.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

method	result	size
parallelrisch	$-2\cot\left(x\right)+\sec\left(x\right)\csc\left(x\right)$	11
default	$\frac{1}{\cos\left(x\right)\sin\left(x\right)}-2\cot\left(x\right)$	15
risch	$-\frac{4i}{\left(e^{2ix}+1\right)\left(e^{2ix}-1\right)}$	22
norman	$\frac{\frac{1}{2}-3\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}}{\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}$	36

input
```
int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*cot(x)+sec(x)*csc(x)
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \csc^2(x)\sec^2(x)\,dx=-\frac{2\cos\left(x\right)^2-1}{\cos\left(x\right)\sin\left(x\right)}$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

output
```
-(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{2\cos(2x)}{\sin(2x)}$$

input
```
integrate(1/cos(x)**2/sin(x)**2,x)
```

output
```
-2*cos(2*x)/sin(2*x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
```

output
```
-1/tan(x) + tan(x)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
```

output
```
-1/tan(x) + tan(x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2x)$$

input
```
int(1/(cos(x)^2*sin(x)^2),x)
```

output
```
-2*cot(2*x)
```

3.37 $\int \cot^2\left(\frac{3x}{4}\right) dx$

3.37.1 Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \cot^2\left(\frac{3x}{4}\right) dx = -x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

output
```
-x-4/3*cot(3/4*x)
```

3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \cot^2\left(\frac{3x}{4}\right) dx = -\frac{4}{3}\cot\left(\frac{3x}{4}\right) \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2\left(\frac{3x}{4}\right)\right)$$

input
```
Integrate[Cot[(3*x)/4]^2,x]
```

output
```
(-4*Cot[(3*x)/4]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[(3*x)/4]^2])/3
```

3.37.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2\left(\frac{3x}{4}\right) dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan\left(\frac{3x}{4}+\frac{\pi}{2}\right)^2 dx$$
$$\Big\downarrow \text{3954}$$
$$-\int 1dx - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$
$$\Big\downarrow \text{24}$$
$$-x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

input
```
Int[Cot[(3*x)/4]^2,x]
```

output
```
-x - (4*Cot[(3*x)/4])/3
```

3.37.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.37.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
norman	$\frac{-\frac{4}{3}-x\tan\left(\frac{3x}{4}\right)}{\tan\left(\frac{3x}{4}\right)}$	17
risch	$-x-\frac{8i}{3\left(\mathrm{e}^{\frac{3ix}{2}}-1\right)}$	17
derivativedivides	$-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18
default	$-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18
parallelrisch	$\frac{-3x\tan\left(\frac{3x}{4}\right)-4}{3\tan\left(\frac{3x}{4}\right)}$	18

input
```
int(cot(3/4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4/3-x*tan(3/4*x))/tan(3/4*x)
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx=-\frac{3\,x\sin\left(\frac{3}{2}\,x\right)+4\,\cos\left(\frac{3}{2}\,x\right)+4}{3\,\sin\left(\frac{3}{2}\,x\right)}$$

input
```
integrate(cot(3/4*x)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4\cos\left(\frac{3x}{4}\right)}{3\sin\left(\frac{3x}{4}\right)}$$

input
```
integrate(cot(3/4*x)**2,x)
```

output
```
-x - 4*cos(3*x/4)/(3*sin(3*x/4))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4}{3\,\tan\left(\frac{3}{4}\,x\right)}$$

input
```
integrate(cot(3/4*x)^2,x, algorithm="maxima")
```

output
```
-x - 4/3/tan(3/4*x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{2}{3\,\tan\left(\frac{3}{8}\,x\right)} + \frac{2}{3}\,\tan\left(\frac{3}{8}\,x\right)$$

input
```
integrate(cot(3/4*x)^2,x, algorithm="giac")
```

output
```
-x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4\cot\left(\frac{3x}{4}\right)}{3}$$

input
```
int(cot((3*x)/4)^2,x)
```

output
```
- x - (4*cot((3*x)/4))/3
```

3.38 $\int (1+\tan(2x))^2 \, dx$

3.38.1 Optimal result

Integrand size = 8, antiderivative size = 16

$$\int (1+\tan(2x))^2 \, dx = -\log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

output
```
-ln(cos(2*x))+1/2*tan(2*x)
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (1+\tan(2x))^2 \, dx = x - \frac{1}{2}\arctan(\tan(2x)) - \log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

input
```
Integrate[(1 + Tan[2*x])^2,x]
```

output
```
x - ArcTan[Tan[2*x]]/2 - Log[Cos[2*x]] + Tan[2*x]/2
```

3.38.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(2x)+1)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(2x)+1)^2dx$$

$$\Big\downarrow \text{3958}$$

$$2\int \tan(2x)dx+\frac{1}{2}\tan(2x)$$

$$\Big\downarrow \text{3042}$$

$$2\int \tan(2x)dx+\frac{1}{2}\tan(2x)$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

input
```
Int[(1 + Tan[2*x])^2,x]
```

output
```
-Log[Cos[2*x]] + Tan[2*x]/2
```

3.38.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

3.38.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
default	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
norman	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
parallelrisch	$\frac{\tan(2x)}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	19
parts	$x+\frac{\tan(2x)}{2}-\frac{\arctan(\tan(2x))}{2}+\frac{\ln(1+\tan^2(2x))}{2}$	27
risch	$2ix+\frac{i}{e^{4ix}+1}-\ln\left(e^{4ix}+1\right)$	28

input
```
int((1+tan(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (1+\tan(2x))^2\,dx = -\frac{1}{2}\log\left(\frac{1}{\tan(2x)^2+1}\right)+\frac{1}{2}\tan(2x)$$

input
```
integrate((1+tan(2*x))^2,x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (1+\tan(2x))^2\,dx = \frac{\log\left(\tan^2\left(2x\right)+1\right)}{2} + \frac{\tan\left(2x\right)}{2}$$

input
```
integrate((1+tan(2*x))**2,x)
```

output
```
log(tan(2*x)**2 + 1)/2 + tan(2*x)/2
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1+\tan(2x))^2\,dx = \log\left(\sec\left(2\,x\right)\right) + \frac{1}{2}\,\tan\left(2\,x\right)$$

input
```
integrate((1+tan(2*x))^2,x, algorithm="maxima")
```

output
```
log(sec(2*x)) + 1/2*tan(2*x)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (1+\tan(2x))^2\,dx = -\frac{1}{2}\,\log\left(\frac{4}{\tan\left(2\,x\right)^2+1}\right) + \frac{1}{2}\,\tan\left(2\,x\right)$$

input
```
integrate((1+tan(2*x))^2,x, algorithm="giac")
```

output
```
-1/2*log(4/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (1 + \tan(2x))^2 \, dx = \frac{\tan(2\,x)}{2} + \frac{\ln\left(\tan(2\,x)^2 + 1\right)}{2}$$

input
```
int((tan(2*x) + 1)^2,x)
```

output
```
tan(2*x)/2 + log(tan(2*x)^2 + 1)/2
```

3.39 $\int (-\cot(x)+\tan(x))^2\,dx$

3.39.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int (-\cot(x)+\tan(x))^2\,dx = -4x-\cot(x)+\tan(x)$$

output
```
-4*x-cot(x)+tan(x)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int (-\cot(x)+\tan(x))^2\,dx = -2x-\arctan(\tan(x)) - \cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)+\tan(x)$$

input
```
Integrate[(-Cot[x] + Tan[x])^2,x]
```

output
```
-2*x - ArcTan[Tan[x]] - Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2]
+ Tan[x]
```

3.39.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4853, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)-\cot(x))^2 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\tan(x)-\cot(x))^2 dx$$

$$\Big\downarrow \text{ 4853}$$

$$\int \frac{(1-\tan^2(x))^2\cot^2(x)}{\tan^2(x)+1} d\tan(x)$$

$$\Big\downarrow \text{ 364}$$

$$\int \left(-\frac{4}{\tan^2(x)+1}+\cot^2(x)+1\right) d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-4\arctan(\tan(x))+\tan(x)-\cot(x)$$

input
```
Int[(-Cot[x] + Tan[x])^2,x]
```

output
```
-4*ArcTan[Tan[x]] - Cot[x] + Tan[x]
```

3.39.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

3.39.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-4x-\cot(x)+\tan(x)$	11
parallelrisch	$-4x-\cot(x)+\tan(x)$	11
norman	$\frac{-1+\tan^2(x)-4x\tan(x)}{\tan(x)}$	17
parts	$-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))+\tan(x)-\arctan(\tan(x))-2x$	24
risch	$-4x-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	26

input
```
int((-cot(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-4*x-cot(x)+tan(x)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int(-\cot(x)+\tan(x))^2\,dx=-\frac{4\,x\tan(x)-\tan(x)^2+1}{\tan(x)}$$

input
```
integrate((-cot(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-(4*x*tan(x) - tan(x)^2 + 1)/tan(x)
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (-\cot(x)+\tan(x))^2\,dx = -4x+\tan(x)-\frac{1}{\tan(x)}$$

input
```
integrate((-cot(x)+tan(x))**2,x)
```

output
```
-4*x + tan(x) - 1/tan(x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x)+\tan(x))^2\,dx = -4\,x-\frac{1}{\tan(x)}+\tan(x)$$

input
```
integrate((-cot(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-4*x - 1/tan(x) + tan(x)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x)+\tan(x))^2\,dx = -4\,x-\frac{1}{\tan(x)}+\tan(x)$$

input
```
integrate((-cot(x)+tan(x))^2,x, algorithm="giac")
```

output
```
-4*x - 1/tan(x) + tan(x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x)+\tan(x))^2\,dx=\tan(x)-4\,x-\frac{1}{\tan\left(x\right)}$$

input
```
int((cot(x) - tan(x))^2,x)
```

output
```
tan(x) - 4*x - 1/tan(x)
```

3.40 $\int (-\sec(x)+\tan(x))^2\,dx$

3.40.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int (-\sec(x)+\tan(x))^2\,dx = -x-\frac{2\cos(x)}{1+\sin(x)}$$

output
```
-x-2*cos(x)/(1+sin(x))
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -\arctan(\tan(x))-2\sec(x)+2\tan(x)$$

input
```
Integrate[(-Sec[x] + Tan[x])^2,x]
```

output
```
-ArcTan[Tan[x]] - 2*Sec[x] + 2*Tan[x]
```

3.40.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)-\sec(x))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\tan(x)-\sec(x))^2dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int (\sin(x)-1)^2\sec^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(\sin(x)-1)^2}{\cos(x)^2}dx$$

$$\Big\downarrow \text{ 3149}$$

$$\int \frac{\cos^2(x)}{(\sin(x)+1)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^2}{(\sin(x)+1)^2}dx$$

$$\Big\downarrow \text{ 3159}$$

$$-\int 1dx-\frac{2\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{ 24}$$

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

input
```
Int[(-Sec[x] + Tan[x])^2,x]
```

output
```
-x - (2*Cos[x])/(1 + Sin[x])
```

3.40.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.40.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-x+2\tan\left(x\right)-\frac{2}{\cos\left(x\right)}$	15
parts	$2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)-2\sec\left(x\right)$	15
risch	$-x-\frac{4}{i+{\mathrm e}^{i x}}$	17

input
```
int((-sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x+2*tan(x)-2/cos(x)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int (-\sec(x)+\tan(x))^2\,dx = -\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

input
```
integrate((-sec(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (-\sec(x)+\tan(x))^2\,dx = -x+2\tan(x)-2\sec(x)$$

input
```
integrate((-sec(x)+tan(x))**2,x)
```

output
```
-x + 2*tan(x) - 2*sec(x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x-\frac{2}{\cos(x)}+2\,\tan(x)$$

input
```
integrate((-sec(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-x - 2/cos(x) + 2*tan(x)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x) + \tan(x))^2 \, dx = -x - \frac{4}{\tan\left(\frac{1}{2}x\right) + 1}$$

input
```
integrate((-sec(x)+tan(x))^2,x, algorithm="giac")
```

output
```
-x - 4/(tan(1/2*x) + 1)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x) + \tan(x))^2 \, dx = -x - \frac{4}{\tan\left(\frac{x}{2}\right) + 1}$$

input
```
int((tan(x) - 1/cos(x))^2,x)
```

output
```
- x - 4/(tan(x/2) + 1)
```

3.41 $\int \frac{\sin(x)}{1+\sin(x)}\,dx$

3.41.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x+\frac{\cos(x)}{1+\sin(x)}$$

output
```
x+cos(x)/(1+sin(x))
```

3.41.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sin[x]/(1 + Sin[x]),x]
```

output
```
x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

3.41.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3214}$$
$$x-\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3042}$$
$$x-\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3127}$$
$$x+\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[Sin[x]/(1 + Sin[x]),x]
```

output
```
x + Cos[x]/(1 + Sin[x])
```

3.41.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.41.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
risch	$x+\frac{2}{i+{\mathrm e}^{i x}}$	15
default	$2 \arctan \left(\tan \left(\frac{x}{2}\right)\right)+\frac{2}{1+\tan \left(\frac{x}{2}\right)}$	19
parallelrisch	$\frac{x \tan \left(\frac{x}{2}\right)+2+x}{1+\tan \left(\frac{x}{2}\right)}$	19
norman	$\frac{x+x \tan \left(\frac{x}{2}\right)+x \left(\tan^{2}\left(\frac{x}{2}\right)\right)+2 \left(\tan^{2}\left(\frac{x}{2}\right)\right)+\left(\tan^{3}\left(\frac{x}{2}\right)\right) x+2}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right) \left(1+\tan \left(\frac{x}{2}\right)\right)}$	53

input
```
int(sin(x)/(sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
x+2/(I+exp(I*x))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx=\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

input
```
integrate(sin(x)/(1+sin(x)),x, algorithm="fricas")
```

output
```
((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1} + \frac{x}{\tan\left(\frac{x}{2}\right)+1} + \frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
integrate(sin(x)/(1+sin(x)),x)
```

output
```
x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) + 2/(tan(x/2) + 1)
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{2}{\frac{\sin(x)}{\cos(x)+1}+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(sin(x)/(1+sin(x)),x, algorithm="maxima")
```

output
```
2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x + \frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

input
```
integrate(sin(x)/(1+sin(x)),x, algorithm="giac")
```

output
```
x + 2/(tan(1/2*x) + 1)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x + \frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(sin(x)/(sin(x) + 1),x)
```

output
```
x + 2/(tan(x/2) + 1)
```

3.42 $\int \frac{\cos(x)}{1-\cos(x)}\,dx$

3.42.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x-\frac{\sin(x)}{1-\cos(x)}$$

output
```
-x-sin(x)/(1-cos(x))
```

3.42.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -\csc(x)\left(1+\cos(x)+2\arcsin\left(\sqrt{\sin^2\left(\frac{x}{2}\right)}\right)\sqrt{\sin^2(x)}\right)$$

input
```
Integrate[Cos[x]/(1 - Cos[x]),x]
```

output
```
-(Csc[x]*(1 + Cos[x] + 2*ArcSin[Sqrt[Sin[x/2]^2]]*Sqrt[Sin[x]^2]))
```

3.42.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\int \frac{1}{1-\cos(x)}dx-x$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx-x$$

$$\Big\downarrow \text{ 3127}$$

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[Cos[x]/(1 - Cos[x]),x]
```

output
```
-x - Sin[x]/(1 - Cos[x])
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}$	17
risch	$-x-\frac{2i}{\mathrm{e}^{ix}-1}$	17
parallelrisch	$\frac{-x\tan\left(\frac{x}{2}\right)-1}{\tan\left(\frac{x}{2}\right)}$	17
norman	$\frac{-1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)-x\tan\left(\frac{x}{2}\right)-\left(\tan^{3}\left(\frac{x}{2}\right)\right)x}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	44

input
```
int(cos(x)/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*arctan(tan(1/2*x))-1/tan(1/2*x)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int\frac{\cos(x)}{1-\cos(x)}\,dx=-\frac{x\sin\left(x\right)+\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(cos(x)/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(x*sin(x) + cos(x) + 1)/sin(x)
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(cos(x)/(1-cos(x)),x)
```

output
```
-x - 1/tan(x/2)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(cos(x)/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x - \frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(cos(x)/(1-cos(x)),x, algorithm="giac")
```

output
```
-x - 1/tan(1/2*x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x-\cot\left(\frac{x}{2}\right)$$

input
```
int(-cos(x)/(cos(x) - 1),x)
```

output
```
- x - cot(x/2)
```

3.43 $\int e^{-x/2}\left(-1+e^{x/2}\right)^3 dx$

3.43.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3 dx = 2e^{-x/2}-6e^{x/2}+e^x+3x$$

output
```
2/exp(1/2*x)-6*exp(1/2*x)+exp(x)+3*x
```

3.43.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3 dx = e^{-x/2}\left(2-6e^x+e^{3x/2}\right)+6\log\left(e^{x/2}\right)$$

input
```
Integrate[(-1 + E^(x/2))^3/E^(x/2),x]
```

output
```
(2 - 6*E^x + E^((3*x)/2))/E^(x/2) + 6*Log[E^(x/2)]
```

3.43.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2678, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x/2}\left(e^{x/2}-1\right)^3 dx$$

$$\Big\downarrow \text{2678}$$

$$2\int -e^{-x}\left(1-e^{x/2}\right)^3 de^{x/2}$$

$$\Big\downarrow \text{25}$$

$$-2\int e^{-x}\left(1-e^{x/2}\right)^3 de^{x/2}$$

$$\Big\downarrow \text{49}$$

$$-2\int \left(3+e^{-x}-3e^{-x/2}-e^{x/2}\right) de^{x/2}$$

$$\Big\downarrow \text{2009}$$

$$2\left(e^{-x/2}-3e^{x/2}+\frac{e^x}{2}+3\log\left(e^{x/2}\right)\right)$$

input
```
Int[(-1 + E^(x/2))^3/E^(x/2),x]
```

output
```
2*(E^(-1/2*x) - 3*E^(x/2) + E^x/2 + 3*Log[E^(x/2)])
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

3.43.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$e^{x}+3x-6\,e^{\frac{x}{2}}+2\,e^{-\frac{x}{2}}$	19
parts	$e^{x}+3x-6\,e^{\frac{x}{2}}+2\,e^{-\frac{x}{2}}$	25
derivativedivides	$e^{x}-6\,e^{\frac{x}{2}}+6\ln\left(e^{\frac{x}{2}}\right)+2\,e^{-\frac{x}{2}}$	29
default	$e^{x}-6\,e^{\frac{x}{2}}+6\ln\left(e^{\frac{x}{2}}\right)+2\,e^{-\frac{x}{2}}$	29
norman	$\left(2+e^{\frac{3x}{2}}-6\,e^{x}+3x\,e^{\frac{x}{2}}\right)e^{-\frac{x}{2}}$	31
parallelrisch	$-\left(-2-e^{\frac{3x}{2}}-6\ln\left(e^{\frac{x}{2}}\right)e^{\frac{x}{2}}+6\,e^{x}\right)e^{-\frac{x}{2}}$	38

input
```
int((-1+exp(1/2*x))^3/exp(1/2*x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+3*x-6*exp(1/2*x)+2*exp(-1/2*x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=\left(3\,xe^{\left(\frac{1}{2}\,x\right)}+e^{\left(\frac{3}{2}\,x\right)}-6\,e^{x}+2\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="fricas")
```

output
```
(3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = 3x - 6e^{\frac{x}{2}} + e^x + 2e^{-\frac{x}{2}}$$

input
```
integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
```

output
```
3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = 3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^x$$

input
```
integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="maxima")
```

output
```
3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = 3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^x$$

input
```
integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="giac")
```

output
```
3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = 3\,x+2\,\mathrm{e}^{-\frac{x}{2}}-6\,\mathrm{e}^{x/2}+\mathrm{e}^{x}$$

input
```
int(exp(-x/2)*(exp(x/2) - 1)^3,x)
```

output
```
3*x + 2*exp(-x/2) - 6*exp(x/2) + exp(x)
```

3.44 $\int \frac{1}{5-6x+x^2} dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{5-6x+x^2} dx = -\frac{1}{4}\log(1-x)+\frac{1}{4}\log(5-x)$$

output
```
-1/4*ln(1-x)+1/4*ln(5-x)
```

3.44.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{5-6x+x^2} dx = -\frac{1}{4}\log(1-x)+\frac{1}{4}\log(5-x)$$

input
```
Integrate[(5 - 6*x + x^2)^(-1),x]
```

output
```
-1/4*Log[1 - x] + Log[5 - x]/4
```

3.44.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-6x+5}\,dx$$

$$\Big\downarrow \text{1081}$$

$$\int \left(\frac{1}{4(1-x)}-\frac{1}{4(5-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

input
```
Int[(5 - 6*x + x^2)^(-1),x]
```

output
```
-1/4*Log[1 - x] + Log[5 - x]/4
```

3.44.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.44.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14
norman	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14
risch	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14
parallelrisch	$-\frac{\ln(-1+x)}{4}+\frac{\ln(x-5)}{4}$	14

input
```
int(1/(x^2-6*x+5),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(-1+x)+1/4*ln(x-5)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\,\log\left(x-1\right)+\frac{1}{4}\,\log\left(x-5\right)$$

input
```
integrate(1/(x^2-6*x+5),x, algorithm="fricas")
```

output
```
-1/4*log(x - 1) + 1/4*log(x - 5)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{5-6x+x^2}\,dx = \frac{\log\left(x-5\right)}{4}-\frac{\log\left(x-1\right)}{4}$$

input
```
integrate(1/(x**2-6*x+5),x)
```

output
```
log(x - 5)/4 - log(x - 1)/4
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(x-1) + \frac{1}{4}\log(x-5)$$

input
```
integrate(1/(x^2-6*x+5),x, algorithm="maxima")
```

output
```
-1/4*log(x - 1) + 1/4*log(x - 5)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(|x-1|) + \frac{1}{4}\log(|x-5|)$$

input
```
integrate(1/(x^2-6*x+5),x, algorithm="giac")
```

output
```
-1/4*log(abs(x - 1)) + 1/4*log(abs(x - 5))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x}{2}-\frac{3}{2}\right)}{2}$$

input
```
int(1/(x^2 - 6*x + 5),x)
```

output
```
-atanh(x/2 - 3/2)/2
```

3.45 $\int \frac{x^2}{13-6x^3+x^6} \, dx$

3.45.1 Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{x^2}{13-6x^3+x^6} \, dx = \frac{1}{6}\arctan\left(\frac{1}{2}\left(-3+x^3\right)\right)$$

output
```
1/6*arctan(1/2*x^3-3/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{13-6x^3+x^6} \, dx = \frac{1}{6}\arctan\left(\frac{1}{2}\left(-3+x^3\right)\right)$$

input
```
Integrate[x^2/(13 - 6*x^3 + x^6),x]
```

output
```
ArcTan[(-3 + x^3)/2]/6
```

3.45.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6-6x^3+13}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{x^6-6x^3+13}dx^3$$

$$\Big\downarrow \text{1083}$$

$$-\frac{2}{3}\int \frac{1}{-x^6-16}d(2x^3-6)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\arctan\left(\frac{1}{4}(2x^3-6)\right)$$

input
```
Int[x^2/(13 - 6*x^3 + x^6),x]
```

output
```
ArcTan[(-6 + 2*x^3)/4]/6
```

3.45.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.45.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$	11
parallelrisch	$\frac{i\ln\left(x^3+2i-3\right)}{12}-\frac{i\ln\left(x^3-2i-3\right)}{12}$	24

input
```
int(x^2/(x^6-6*x^3+13),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/2*x^3-3/2)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\arctan\left(\frac{1}{2}\,x^3-\frac{3}{2}\right)$$

input
```
integrate(x^2/(x^6-6*x^3+13),x, algorithm="fricas")
```

output
```
1/6*arctan(1/2*x^3 - 3/2)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx = \frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

input
```
integrate(x**2/(x**6-6*x**3+13),x)
```

output
```
atan(x**3/2 - 3/2)/6
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx = \frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

input
```
integrate(x^2/(x^6-6*x^3+13),x, algorithm="maxima")
```

output
```
1/6*arctan(1/2*x^3 - 3/2)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx = \frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

input
```
integrate(x^2/(x^6-6*x^3+13),x, algorithm="giac")
```

output
```
1/6*arctan(1/2*x^3 - 3/2)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx = \frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

input
```
int(x^2/(x^6 - 6*x^3 + 13),x)
```

output
```
atan(x^3/2 - 3/2)/6
```

3.46 $\int \frac{2+x}{-1-4x+x^2} dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{2+x}{-1-4x+x^2} dx = \frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(2-\sqrt{5}-x\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)$$

output
```
1/10*ln(2-x-5^(1/2))*(5-4*5^(1/2))+1/10*ln(2-x+5^(1/2))*(5+4*5^(1/2))
```

3.46.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{-1-4x+x^2} dx = \frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-2+\sqrt{5}+x\right)$$

input
```
Integrate[(2 + x)/(-1 - 4*x + x^2),x]
```

output
```
((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt
[5] + x])/10
```

3.46.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^2-4x-1}\,dx$$

$$\Big\downarrow \ 1141$$

$$\int \left(-\frac{5+4\sqrt{5}}{10\left(-x+\sqrt{5}+2\right)}-\frac{5-4\sqrt{5}}{10\left(-x-\sqrt{5}+2\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

input
```
Int[(2 + x)/(-1 - 4*x + x^2),x]
```

output
```
((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[
5] - x])/10
```

3.46.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.46.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\ln(x^2-4x-1)}{2}-\frac{4\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-4+2x)\sqrt{5}}{10}\right)}{5}$	29
risch	$\frac{\ln\left(x-2-\sqrt{5}\right)}{2}+\frac{2\ln\left(x-2-\sqrt{5}\right)\sqrt{5}}{5}+\frac{\ln\left(x-2+\sqrt{5}\right)}{2}-\frac{2\ln\left(x-2+\sqrt{5}\right)\sqrt{5}}{5}$	48

input
```
int((2+x)/(x^2-4*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(-4+2*x)*5^(1/2))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{x^2-2\sqrt{5}(x-2)-4x+9}{x^2-4x-1}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

input
```
integrate((2+x)/(x^2-4*x-1),x, algorithm="fricas")
```

output
```
2/5*sqrt(5)*log((x^2 - 2*sqrt(5)*(x - 2) - 4*x + 9)/(x^2 - 4*x - 1)) + 1/2
*log(x^2 - 4*x - 1)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \left(\frac{1}{2}-\frac{2\sqrt{5}}{5}\right)\log\left(x-2+\sqrt{5}\right)+\left(\frac{1}{2}+\frac{2\sqrt{5}}{5}\right)\log\left(x-\sqrt{5}-2\right)$$

input
```
integrate((2+x)/(x**2-4*x-1),x)
```

output
```
(1/2 - 2*sqrt(5)/5)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*log(x - sqr
t(5) - 2)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{x-\sqrt{5}-2}{x+\sqrt{5}-2}\right)+\frac{1}{2}\log\left(x^2-4\,x-1\right)$$

input
```
integrate((2+x)/(x^2-4*x-1),x, algorithm="maxima")
```

output
```
2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^2 - 4*x -
 1)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{\left|2\,x-2\,\sqrt{5}-4\right|}{\left|2\,x+2\,\sqrt{5}-4\right|}\right)+\frac{1}{2}\log\left(\left|x^2-4\,x-1\right|\right)$$

input
```
integrate((2+x)/(x^2-4*x-1),x, algorithm="giac")
```

output
```
2/5*sqrt(5)*log(abs(2*x - 2*sqrt(5) - 4)/abs(2*x + 2*sqrt(5) - 4)) + 1/2*l
og(abs(x^2 - 4*x - 1))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \ln\left(x-\sqrt{5}-2\right)\left(\frac{2\sqrt{5}}{5}+\frac{1}{2}\right)-\ln\left(x+\sqrt{5}-2\right)\left(\frac{2\sqrt{5}}{5}-\frac{1}{2}\right)$$

input
```
int(-(x + 2)/(4*x - x^2 + 1),x)
```

output
```
log(x - 5^(1/2) - 2)*((2*5^(1/2))/5 + 1/2) - log(x + 5^(1/2) - 2)*((2*5^(1
/2))/5 - 1/2)
```

3.47 $\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$

3.47.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx=-3\sqrt[3]{1+x}+\frac{3}{2}(1+x)^{2/3}+3\log\left(1+\sqrt[3]{1+x}\right)$$

output
```
-3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))
```

3.47.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx=\frac{3}{2}\sqrt[3]{1+x}\left(-2+\sqrt[3]{1+x}\right)+3\log\left(1+\sqrt[3]{1+x}\right)$$

input
```
Integrate[(1 + (1 + x)^(1/3))^(-1),x]
```

output
```
(3*(1 + x)^(1/3)*(-2 + (1 + x)^(1/3)))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

3.47.3 Rubi [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x+1}+1} \, dx$$

$$\Big\downarrow \text{239}$$

$$\int \frac{1}{\sqrt[3]{x+1}+1} d(x+1)$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{(x+1)^{2/3}}{x+2} d\sqrt[3]{x+1}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x+\frac{1}{x+2}\right) d\sqrt[3]{x+1}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-x+\frac{1}{2}(x+1)^{2/3}+\log(x+2)-1\right)$$

input
```
Int[(1 + (1 + x)^(1/3))^(-1),x]
```

output
```
3*(-1 - x + (1 + x)^(2/3)/2 + Log[2 + x])
```

3.47.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.47.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+3\ln\left(1+(1+x)^{\frac{1}{3}}\right)$
trager	$-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+\ln\left(-3(1+x)^{\frac{2}{3}}-3(1+x)^{\frac{1}{3}}-x-2\right)$
default	$\ln(2+x)+\frac{3(1+x)^{\frac{2}{3}}}{2}+2\ln\left(1+(1+x)^{\frac{1}{3}}\right)-\ln\left((1+x)^{\frac{2}{3}}-(1+x)^{\frac{1}{3}}+1\right)-3(1+x)^{\frac{1}{3}}$

input
```
int(1/(1+(1+x)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx=\frac{3}{2}\,(x+1)^{\frac{2}{3}}-3\,(x+1)^{\frac{1}{3}}+3\,\log\left((x+1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/3)),x, algorithm="fricas")
```

output
```
3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3(x+1)^{\frac{2}{3}}}{2} - 3\sqrt[3]{x+1} + 3\log\left(\sqrt[3]{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)**(1/3)),x)
```

output
```
3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/3)),x, algorithm="maxima")
```

output
```
3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/3)),x, algorithm="giac")
```

output
```
3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = 3\ln\left((x+1)^{1/3}+1\right) - 3\,(x+1)^{1/3} + \frac{3\,(x+1)^{2/3}}{2}$$

input
```
int(1/((x + 1)^(1/3) + 1),x)
```

output
```
3*log((x + 1)^(1/3) + 1) - 3*(x + 1)^(1/3) + (3*(x + 1)^(2/3))/2
```

3.48 $\int \frac{1}{\sqrt{x}(b+ax)}\,dx$

3.48.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
2*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(b + a*x)),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

3.48.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(ax+b)}\,dx$$

$$\Big\downarrow \; 73$$

$$2\int \frac{1}{b+ax}d\sqrt{x}$$

$$\Big\downarrow \; 218$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*(b + a*x)),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

3.48.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.48.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

input
```
int(1/(a*x+b)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

input
```
integrate(1/(a*x+b)/x^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqr
t(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b)/x**(1/2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2
/(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(-b/a))/(a*sqrt(-b/a)) - log(s
qrt(x) + sqrt(-b/a))/(a*sqrt(-b/a)), True))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a*x+b)/x^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a*x+b)/x^(1/2),x, algorithm="giac")
```

output
```
2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(b + a*x)),x)
```

output
```
(2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))
```

3.49 $\int x^3\sqrt{1+x^2}\,dx$

3.49.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3\sqrt{1+x^2}\,dx=-\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}$$

output
```
-1/3*(x^2+1)^(3/2)+1/5*(x^2+1)^(5/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx=\frac{1}{15}\left(1+x^2\right)^{3/2}\left(-2+3x^2\right)$$

input
```
Integrate[x^3*Sqrt[1 + x^2],x]
```

output
```
((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

3.49.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2\sqrt{x^2+1}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\left(x^2+1\right)^{3/2}-\sqrt{x^2+1}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(x^2+1\right)^{5/2}-\frac{2}{3}\left(x^2+1\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[1 + x^2],x]
```

output
```
((-2*(1 + x^2)^(3/2))/3 + (2*(1 + x^2)^(5/2))/5)/2
```

3.49.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.49.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}(3x^2-2)}{15}$	17
pseudoelliptic	$\frac{(x^2+1)^{\frac{3}{2}}(3x^2-2)}{15}$	17
risch	$\frac{(3x^4+x^2-2)\sqrt{x^2+1}}{15}$	20
trager	$\left(\frac{1}{5}x^4+\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{x^2+1}$	21
default	$\frac{x^2(x^2+1)^{\frac{3}{2}}}{5}-\frac{2(x^2+1)^{\frac{3}{2}}}{15}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(x^2+1)^{\frac{3}{2}}(-3x^2+2)}{15}}{4\sqrt{\pi}}$	31

input
```
int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{15}\left(3\,x^4+x^2-2\right)\sqrt{x^2+1}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^3\sqrt{1+x^2}\,dx = \frac{x^4\sqrt{x^2+1}}{5} + \frac{x^2\sqrt{x^2+1}}{15} - \frac{2\sqrt{x^2+1}}{15}$$

input
```
integrate(x**3*(x**2+1)**(1/2),x)
```

output
```
x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2 - \frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}} - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx=\sqrt{x^2+1}\left(\frac{x^4}{5}+\frac{x^2}{15}-\frac{2}{15}\right)$$

input
```
int(x^3*(x^2 + 1)^(1/2),x)
```

output
```
(x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)
```

3.50 $\int \frac{x}{\sqrt{a^4-x^4}}\,dx$

3.50.1 Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

output
```
1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\frac{1}{2}i\log\left(ix^2+\sqrt{a^4-x^4}\right)$$

input
```
Integrate[x/Sqrt[a^4 - x^4],x]
```

output
```
(-1/2*I)*Log[I*x^2 + Sqrt[a^4 - x^4]]
```

3.50.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^4-x^4}}dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2}\int \frac{1}{\sqrt{a^4-x^4}}dx^2$$

$$\Big\downarrow \ 224$$

$$\frac{1}{2}\int \frac{1}{x^4+1}d\frac{x^2}{\sqrt{a^4-x^4}}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

input
```
Int[x/Sqrt[a^4 - x^4],x]
```

output
```
ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

3.50.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.50.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$	19
elliptic	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$	19
pseudoelliptic	$-\frac{i\ln\left(ix^2+\sqrt{a^4-x^4}\right)}{2}$	23

input
```
int(x/(a^4-x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\arctan\left(-\frac{a^2-\sqrt{a^4-x^4}}{x^2}\right)$$

input
```
integrate(x/(a^4-x^4)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)
```

3.50.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{a^4 - x^4}}\,dx = \begin{cases} -\frac{i\,\mathrm{acosh}\left(\frac{x^2}{a^2}\right)}{2} & \text{for } \left|\frac{x^4}{a^4}\right| > 1 \\ \frac{\mathrm{asin}\left(\frac{x^2}{a^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a**4-x**4)**(1/2),x)
```

output
```
Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4/a**4) > 1), (asin(x**2/a**2)/2,
 True))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a^4 - x^4}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{a^4 - x^4}}{x^2}\right)$$

input
```
integrate(x/(a^4-x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan(sqrt(a^4 - x^4)/x^2)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \frac{x}{\sqrt{a^4 - x^4}}\,dx = \frac{1}{2}\arcsin\left(\frac{x^2}{a^2}\right)$$

input
```
integrate(x/(a^4-x^4)^(1/2),x, algorithm="giac")
```

output
```
1/2*arcsin(x^2/a^2)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a^4 - x^4}}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$$

input
```
int(x/(a^4 - x^4)^(1/2),x)
```

output
```
atan(x^2/(a^4 - x^4)^(1/2))/2
```

3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$

3.51.1 Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

output
```
arctan((-a^2+x^2)^(1/2)/a)/a
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Sqrt[-a^2 + x^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

3.51.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^2-a^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^2-a^2}}dx^2$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{a^2+x^4}d\sqrt{x^2-a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Sqrt[-a^2 + x^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

3.51.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.51.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$	21
default	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}}$	41

input
```
int(1/(-a^2+x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
arctan((-a^2+x^2)^(1/2)/a)/a
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{2\arctan\left(-\frac{x-\sqrt{-a^2+x^2}}{a}\right)}{a}$$

input
```
integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(-(x - sqrt(-a^2 + x^2))/a)/a
```

3.51.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\mathrm{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-a**2+x**2)**(1/2),x)
```

output
```
Piecewise((I*acosh(a/x)/a, Abs(a**2/x**2) > 1), (-asin(a/x)/a, True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = -\frac{\arcsin\left(\frac{a}{|x|}\right)}{a}$$

input
```
integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(a/abs(x))/a
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

input
```
integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(-a^2 + x^2)/a)/a
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*(x^2 - a^2)^(1/2)),x)
```

output
```
atan((x^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$

3.52.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

output
```
-arctanh((a^2-x^2)^(1/2)/a)/a
```

3.52.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2-x^2}\right)}{2a} + \frac{\log\left(-a^2+a\sqrt{a^2-x^2}\right)}{2a}$$

input
```
Integrate[1/(x*Sqrt[a^2 - x^2]),x]
```

output
```
-1/2*Log[a + Sqrt[a^2 - x^2]]/a + Log[-a^2 + a*Sqrt[a^2 - x^2]]/(2*a)
```

3.52.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2-x^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2-x^2}}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{a^2-x^4}d\sqrt{a^2-x^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Sqrt[a^2 - x^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

3.52.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.52.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

method	result	size
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-x^2}}{x}\right)}{\sqrt{a^2}}$	37
pseudoelliptic	$\frac{\ln\left(-a+\sqrt{a^2-x^2}\right)-\ln\left(a+\sqrt{a^2-x^2}\right)}{2a}$	39

input
```
int(1/x/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-x^2)^(1/2))/x)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

input
```
integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
log(-(a - sqrt(a^2 - x^2))/x)/a
```

3.52.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a**2-x**2)**(1/2),x)
```

output
```
Piecewise((-acosh(a/x)/a, Abs(a**2/x**2) > 1), (I*asin(a/x)/a, True))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(\frac{2a^2}{|x|}+\frac{2\sqrt{a^2-x^2}a}{|x|}\right)}{a}$$

input
```
integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*a^2/abs(x) + 2*sqrt(a^2 - x^2)*a/abs(x))/a
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{2a} + \frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{2a}$$

input
```
integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(a + sqrt(a^2 - x^2)))/a + 1/2*log(abs(-a + sqrt(a^2 - x^2)))/
a
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

input
```
int(1/(x*(a^2 - x^2)^(1/2)),x)
```

output
```
-atanh((a^2 - x^2)^(1/2)/a)/a
```

3.53 $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$

3.53.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

output
```
-arctanh((a^2+x^2)^(1/2)/a)/a
```

3.53.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2a} + \frac{\log\left(-a^2+a\sqrt{a^2+x^2}\right)}{2a}$$

input
```
Integrate[1/(x*Sqrt[a^2 + x^2]),x]
```

output
```
-1/2*Log[a + Sqrt[a^2 + x^2]]/a + Log[-a^2 + a*Sqrt[a^2 + x^2]]/(2*a)
```

3.53.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2+x^2}}dx^2$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{x^4-a^2}d\sqrt{a^2+x^2}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Sqrt[a^2 + x^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

3.53.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.53.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+x^2}}{x}\right)}{\sqrt{a^2}}$	35
pseudoelliptic	$\frac{\ln\left(-a+\sqrt{a^2+x^2}\right)-\ln\left(a+\sqrt{a^2+x^2}\right)}{2a}$	35

input
```
int(1/x/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+x^2)^(1/2))/x)
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a-x+\sqrt{a^2+x^2}\right)-\log\left(-a-x+\sqrt{a^2+x^2}\right)}{a}$$

input
```
integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
-(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.33

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

input
```
integrate(1/x/(a**2+x**2)**(1/2),x)
```

output
```
-asinh(a/x)/a
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{|x|}\right)}{a}$$

input
```
integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/abs(x))/a
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2\,a} + \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{2\,a}$$

input
```
integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(a + sqrt(a^2 + x^2))/a + 1/2*log(-a + sqrt(a^2 + x^2))/a
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a^2+x^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

input

```
int(1/(x*(a^2 + x^2)^(1/2)),x)
```

output

```
atan((a^2 + x^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

3.54 $\int \frac{1}{\sqrt{2+x-x^2}} \, dx$

3.54.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{\sqrt{2+x-x^2}} \, dx = -\arcsin \left(\frac{1}{3}(1-2x)\right)$$

output
```
arcsin(-1/3+2/3*x)
```

3.54.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{2+x-x^2}} \, dx = -2 \arctan \left(\frac{\sqrt{2+x-x^2}}{1+x}\right)$$

input
```
Integrate[1/Sqrt[2 + x - x^2],x]
```

output
```
-2*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

3.54.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2+x+2}}dx$$
$$\Big\downarrow \text{ 1090}$$
$$-\frac{1}{3}\int \frac{1}{\sqrt{1-\frac{1}{9}(1-2x)^2}}d(1-2x)$$
$$\Big\downarrow \text{ 223}$$
$$-\arcsin\left(\frac{1}{3}(1-2x)\right)$$

input
```
Int[1/Sqrt[2 + x - x^2],x]
```

output
```
-ArcSin[(1 - 2*x)/3]
```

3.54.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x+2}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	37

input
```
int(1/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-1/3+2/3*x)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2+x+2}(2\,x-1)}{2\,(x^2-x-2)}\right)$$

input
```
integrate(1/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2x}{3}-\frac{1}{3}\right)$$

input
```
integrate(1/(-x**2+x+2)**(1/2),x)
```

output
```
asin(2*x/3 - 1/3)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arcsin\left(-\frac{2}{3}\,x+\frac{1}{3}\right)$$

input
```
integrate(1/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-2/3*x + 1/3)
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \frac{1}{4}\,\sqrt{-x^2+x+2}(2\,x-1)+\frac{9}{8}\,\arcsin\left(\frac{2}{3}\,x-\frac{1}{3}\right)$$

input
```
integrate(1/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x + 2)*(2*x - 1) + 9/8*arcsin(2/3*x - 1/3)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \mathrm{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)$$

input
```
int(1/(x - x^2 + 2)^(1/2),x)
```

output
```
asin((2*x)/3 - 1/3)
```

3.55 $\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

output
```
-1/3*arcsinh(1/11*(2-3*x)*11^(1/2))*3^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = -\frac{\log\left(2-3x+\sqrt{3}\sqrt{5-4x+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]
```

output
```
-(Log[2 - 3*x + Sqrt[3]*Sqrt[5 - 4*x + 3*x^2]]/Sqrt[3])
```

3.55.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2-4x+5}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{1}{44}(6x-4)^2+1}}d(6x-4)}{2\sqrt{33}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{6x-4}{2\sqrt{11}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[5 - 4*x + 3*x^2],x]
```

output
```
ArcSinh[(-4 + 6*x)/(2*Sqrt[11])]/Sqrt[3]
```

3.55.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{11}\,\left(x-\frac{2}{3}\right)}{11}\right)}{3}$	15
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2-4x+5}-2\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{3}$	42

input
```
int(1/(3*x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-2\sqrt{3}\sqrt{3x^2-4x+5}(3x-2)-18x^2+24x-19\right)$$

input
```
integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-2*sqrt(3)*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 18*x^2 + 24*x
 - 19)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{11}\left(x-\frac{2}{3}\right)}{11}\right)}{3}$$

input
```
integrate(1/(3*x**2-4*x+5)**(1/2),x)
```

output
```
sqrt(3)*asinh(3*sqrt(11)*(x - 2/3)/11)/3
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{11}\sqrt{11}(3x-2)\right)$$

input
```
integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(1/11*sqrt(11)*(3*x - 2))
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{6}\sqrt{3x^2-4x+5}(3x-2) - \frac{11}{18}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-4x+5}\right)+2\right)$$

input
```
integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 11/18*sqrt(3)*log(-sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - 4*x + 5)) + 2)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x-\frac{2}{3}\right)+\sqrt{3x^2-4x+5}\right)}{3}$$

input
```
int(1/(3*x^2 - 4*x + 5)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x - 2/3) + (3*x^2 - 4*x + 5)^(1/2)))/3
```

3.56 $\int \frac{1}{\sqrt{x-x^2}}\,dx$

3.56.1 Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -\arcsin(1-2x)$$

output
```
arcsin(-1+2*x)
```

3.56.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/Sqrt[x - x^2],x]
```

output
```
(-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

3.56.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)$$

$$\Big\downarrow \text{223}$$

$$-\arcsin(1-2x)$$

input
```
Int[1/Sqrt[x - x^2],x]
```

output
```
-ArcSin[1 - 2*x]
```

3.56.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\arcsin\left(2x-1\right)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-x(-1+x)}}{x}\right)$	16
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	36

input
```
int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(2*x-1)
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + x)/x)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \operatorname{asin}\left(2x-1\right)$$

input
```
integrate(1/(-x**2+x)**(1/2),x)
```

output
```
asin(2*x - 1)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \arcsin\left(2\,x-1\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(2*x - 1)
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \frac{1}{4}\,\sqrt{-x^2+x}(2\,x-1) + \frac{1}{8}\,\arcsin\left(2\,x-1\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \mathrm{asin}(2\,x-1)$$

input
```
int(1/(x - x^2)^(1/2),x)
```

output
```
asin(2*x - 1)
```

3.57 $\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx$

3.57.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{2+x-x^2}-2\arcsin\left(\frac{1}{3}(1-2x)\right)$$

output
```
2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{2+x-x^2}-4\arctan\left(\frac{\sqrt{2+x-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

output
```
-2*Sqrt[2 + x - x^2] - 4*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

3.57.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{\sqrt{-x^2+x+2}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$2\int \frac{1}{\sqrt{-x^2+x+2}}dx-2\sqrt{-x^2+x+2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{9}(1-2x)^2}}d(1-2x)-2\sqrt{-x^2+x+2}$$

$$\Big\downarrow \text{ 223}$$

$$-2\arcsin\left(\frac{1}{3}(1-2x)\right)-2\sqrt{-x^2+x+2}$$

input
```
Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

output
```
-2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

3.57.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.57.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result
default	$2 \arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)-2\sqrt{-x^2+x+2}$
risch	$\frac{2x^2-2x-4}{\sqrt{-x^2+x+2}}+2 \arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$
trager	$-2\sqrt{-x^2+x+2}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x+2}+\mathrm{Root}\right.$

input
```
int((1+2*x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2}-2\arctan\left(\frac{\sqrt{-x^2+x+2}(2x-1)}{2(x^2-x-2)}\right)$$

input
```
integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-x^2 + x + 2) - 2*arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x
 - 2))
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2}+2\,\mathrm{asin}\left(\frac{2x}{3}-\frac{1}{3}\right)$$

input
```
integrate((1+2*x)/(-x**2+x+2)**(1/2),x)
```

output
```
-2*sqrt(-x**2 + x + 2) + 2*asin(2*x/3 - 1/3)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\,\sqrt{-x^2+x+2}-2\,\arcsin\left(-\frac{2}{3}\,x+\frac{1}{3}\right)$$

input
```
integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\,\sqrt{-x^2+x+2}+2\,\arcsin\left(\frac{2}{3}\,x-\frac{1}{3}\right)$$

input
```
integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-x^2 + x + 2) + 2*arcsin(2/3*x - 1/3)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)-2\,\sqrt{-x^2+x+2}-\ln\left(x\,1\mathrm{i}+\sqrt{-x^2+x+2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((2*x + 1)/(x - x^2 + 2)^(1/2),x)
```

output
```
asin((2*x)/3 - 1/3) - log(x*1i + (x - x^2 + 2)^(1/2) - 1i/2)*1i - 2*(x - x
^2 + 2)^(1/2)
```

3.58 $\int \frac{1}{x\sqrt{2+x-x^2}}\,dx$

3.58.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2+x-x^2}}{-2+x}\right)$$

input
```
Integrate[1/(x*Sqrt[2 + x - x^2]),x]
```

output
```
Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[2 + x - x^2])/(-2 + x)]
```

3.58.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-x^2+x+2}}\,dx$$
$$\Big\downarrow \text{ 1154}$$
$$-2\int \frac{1}{8-\frac{(x+4)^2}{-x^2+x+2}}d\frac{x+4}{\sqrt{-x^2+x+2}}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{\operatorname{arctanh}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[2 + x - x^2]),x]
```

output
```
-(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

3.58.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.58.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(4+x)\sqrt{2}}{4\sqrt{-x^2+x+2}}\right)\sqrt{2}}{2}$	25
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2+x+2}+4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{2}$	43

input
```
int(1/x/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{-x^2+x+2}(x+4)+7x^2-16x-32}{x^2}\right)$$

input
```
integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 - 16*x - 32
)/x^2)
```

3.58.6 Sympy [F]

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \int \frac{1}{x\sqrt{-(x-2)(x+1)}}\,dx$$

input
```
integrate(1/x/(-x**2+x+2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2+x+2}}{|x|}+\frac{4}{|x|}+1\right)$$

input
```
integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) + 1)
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}\right)$$

input
```
integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) -
 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6))
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{x+2\sqrt{2}\sqrt{-x^2+x+2}+4}{x}\right)}{2}$$

input
```
int(1/(x*(x - x^2 + 2)^(1/2)),x)
```

output
```
-(2^(1/2)*log((x + 2*2^(1/2)*(x - x^2 + 2)^(1/2) + 4)/x))/2
```

3.59 $\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx$

3.59.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx=\frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

output
```
2/3*(-x^2+x+2)^(1/2)/(-2+x)
```

3.59.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx=\frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

input
```
Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

output
```
(2*Sqrt[2 + x - x^2])/(3*(-2 + x))
```

3.59.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-2)\sqrt{-x^2+x+2}}\,dx$$

$$\Big\downarrow \; 1123$$

$$-\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

input
```
Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

output
```
(-2*Sqrt[2 + x - x^2])/(3*(2 - x))
```

3.59.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
risch	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
trager	$\frac{2\sqrt{-x^2+x+2}}{3(-2+x)}$	18
default	$\frac{2\sqrt{-(-2+x)^2+6-3x}}{3(-2+x)}$	22

input
```
int(1/(-2+x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+x)/(-x^2+x+2)^(1/2)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3\,(x-2)}$$

input
```
integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

3.59.6 Sympy [F]

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-2)\,(x+1)}\,(x-2)}\,dx$$

input
```
integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3\,(x-2)}$$

input
```
integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = -\frac{4}{3\left(\frac{2\sqrt{-x^2+x+2}-3}{2x-1}+1\right)}$$

input
```
integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
-4/3/((2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) + 1)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

input
```
int(1/((x - 2)*(x - x^2 + 2)^(1/2)),x)
```

output
```
(2*(x - x^2 + 2)^(1/2))/(3*(x - 2))
```

3.60 $\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$

3.60.1 Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\text{arctanh}(\cos(x)) - \frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)}$$

output
```
-arctanh(cos(x))-1/(1-cos(x))-3*sin(x)/(1-cos(x))
```

3.60.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \frac{1}{2}\csc^2\left(\frac{x}{2}\right)\left(-1-\log\left(\cos\left(\frac{x}{2}\right)\right) + \cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right) - 3\sin(x)\right)$$

input
```
Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

output
```
(Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) +
 Log[Sin[x/2]] - 3*Sin[x]))/2
```

3.60.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3\sin(x)+2)\csc(x)}{1-\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3\sin(x)+2}{\sin(x)(1-\cos(x))}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(-\frac{3}{\cos(x)-1}-\frac{2\csc(x)}{\cos(x)-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{arctanh}(\cos(x))-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}$$

input
```
Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

output
```
-ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.60.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{\left(\cot^{2}\left(\frac{x}{2}\right)\right)}{2}-3\cot\left(\frac{x}{2}\right)$	21
default	$-\frac{3}{\tan\left(\frac{x}{2}\right)}+\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{2\tan\left(\frac{x}{2}\right)^{2}}$	23
risch	$\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)\left(10\,\mathrm{e}^{ix}-9+3i\right)}{\left(\mathrm{e}^{ix}-1\right)^{2}}-\ln\left(\mathrm{e}^{ix}+1\right)+\ln\left(\mathrm{e}^{ix}-1\right)$	44
norman	$\frac{-\frac{1}{2}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-3\left(\tan^{3}\left(\frac{x}{2}\right)\right)-3\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^{2}}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	48

input
```
int((2+3*sin(x))/(1-cos(x))/sin(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*x))-1/2*cot(1/2*x)^2-3*cot(1/2*x)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$$
$$=-\frac{(\cos(x)-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-6\sin(x)-2}{2(\cos(x)-1)}$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="fricas")
```

output
```
-1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*cos(x) +
1/2) - 6*sin(x) - 2)/(cos(x) - 1)
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \log\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3}{\tan\left(\frac{x}{2}\right)} - \frac{1}{2\tan^2\left(\frac{x}{2}\right)}$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

output
```
log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)+1)^2}{2\sin(x)^2} - \frac{3(\cos(x)+1)}{\sin(x)} + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)/(cos(x)
+ 1))
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{3\tan\left(\frac{1}{2}x\right)^2 + 6\tan\left(\frac{1}{2}x\right) + 1}{2\tan\left(\frac{1}{2}x\right)^2} + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="giac")
```

output
```
-1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + log(abs(tan(1/2*x)
))
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3\tan\left(\frac{x}{2}\right)+\frac{1}{2}}{\tan\left(\frac{x}{2}\right)^2}$$

input
```
int(-(3*sin(x) + 2)/(sin(x)*(cos(x) - 1)),x)
```

output
```
log(tan(x/2)) - (3*tan(x/2) + 1/2)/tan(x/2)^2
```

3.61 $\int \frac{1}{2+3\cos^2(x)}\,dx$

3.61.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{1}{2+3\cos^2(x)}\,dx=\frac{x}{\sqrt{10}}-\frac{\arctan\left(\frac{3\cos(x)\sin(x)}{2+\sqrt{10}+3\cos^2(x)}\right)}{\sqrt{10}}$$

output
```
1/10*x*10^(1/2)-1/10*arctan(3*cos(x)*sin(x)/(2+3*cos(x)^2+10^(1/2)))*10^(1
/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+3\cos^2(x)}\,dx=\frac{\arctan\left(\sqrt{\frac{2}{5}}\tan(x)\right)}{\sqrt{10}}$$

input
```
Integrate[(2 + 3*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]
```

3.61.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\cos^2(x)+2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{3\sin\left(x+\frac{\pi}{2}\right)^2+2}dx$$
$$\Big\downarrow \text{ 3660}$$
$$-\int \frac{1}{5\cot^2(x)+2}d\cot(x)$$
$$\Big\downarrow \text{ 216}$$
$$-\frac{\arctan\left(\sqrt{\frac{5}{2}}\cot(x)\right)}{\sqrt{10}}$$

input
```
Int[(2 + 3*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[Sqrt[5/2]*Cot[x]]/Sqrt[10])
```

3.61.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.61.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.38

method	result	size
default	$\frac{\sqrt{10}\,\arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)}{10}$	14
risch	$\frac{i\sqrt{10}\,\ln\left(\mathrm{e}^{2ix}+\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}-\frac{i\sqrt{10}\,\ln\left(\mathrm{e}^{2ix}-\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}$	40

input
```
int(1/(3*cos(x)^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{2+3\cos^2(x)}\,dx = -\frac{1}{20}\sqrt{10}\arctan\left(\frac{7\sqrt{10}\cos(x)^2-2\sqrt{10}}{20\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(2+3*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(cos(x)*sin(
x)))
```

3.61.6 Sympy [F]

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \int \frac{1}{3\cos^2{(x)}+2}\,dx$$

input
```
integrate(1/(2+3*cos(x)**2),x)
```

output
```
Integral(1/(3*cos(x)**2 + 2), x)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.35

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{1}{10}\,\sqrt{10}\arctan\left(\frac{1}{5}\,\sqrt{10}\tan{(x)}\right)$$

input
```
integrate(1/(2+3*cos(x)^2),x, algorithm="maxima")
```

output
```
1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{1}{10}\,\sqrt{10}\left(x+\arctan\left(-\frac{\sqrt{10}\sin{(2\,x)}-2\,\sin{(2\,x)}}{\sqrt{10}\cos{(2\,x)}+\sqrt{10}-2\,\cos{(2\,x)}+2}\right)\right)$$

input
```
integrate(1/(2+3*cos(x)^2),x, algorithm="giac")
```

output
```
1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt(10)*cos(
2*x) + sqrt(10) - 2*cos(2*x) + 2)))
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3\cos^2(x)}\,dx=\frac{\sqrt{10}\,(x-\operatorname{atan}(\tan(x)))}{10}+\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\tan(x)}{5}\right)}{10}$$

input
```
int(1/(3*cos(x)^2 + 2),x)
```

output
```
(10^(1/2)*(x - atan(tan(x))))/10 + (10^(1/2)*atan((10^(1/2)*tan(x))/5))/10
```

3.62 $\int \csc(2x)(1-\tan(x))\,dx$

3.62.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\log(\tan(x)) - \frac{\tan(x)}{2}$$

output
```
1/2*ln(tan(x))-1/2*tan(x)
```

3.62.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \csc(2x)(1-\tan(x))\,dx = -\frac{1}{2}\log(\cos(x)) + \frac{1}{2}\log(\sin(x)) - \frac{\tan(x)}{2}$$

input
```
Integrate[Csc[2*x]*(1 - Tan[x]),x]
```

output
```
-1/2*Log[Cos[x]] + Log[Sin[x]]/2 - Tan[x]/2
```

3.62.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4889, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\tan(x))\csc(2x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1-\tan(x)}{\sin(2x)}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{1}{2}(\cot(x)-1)d\tan(x)$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{2}\int (\cot(x)-1)d\tan(x)$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}(\log(\tan(x))-\tan(x))$$

input
```
Int[Csc[2*x]*(1 - Tan[x]),x]
```

output
```
(Log[Tan[x]] - Tan[x])/2
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.62.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
default	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
norman	$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$	11
parallelrisch	$\ln\left(\sqrt{\tan}(x)\right)-\frac{\tan(x)}{2}$	11
risch	$-\frac{i}{e^{2ix}+1}+\frac{\ln(e^{2ix}-1)}{2}-\frac{\ln(e^{2ix}+1)}{2}$	34

input
```
int((1-tan(x))/sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tan(x))-1/2*tan(x)
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{4}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{1}{\tan(x)^2+1}\right)-\frac{1}{2}\tan(x)$$

input
```
integrate((1-tan(x))/sin(2*x),x, algorithm="fricas")
```

output
```
1/4*log(tan(x)^2/(tan(x)^2 + 1)) - 1/4*log(1/(tan(x)^2 + 1)) - 1/2*tan(x)
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\log(\cos(2x)-1)}{4} - \frac{\log(\cos(2x)+1)}{4} - \frac{\sin(x)}{2\cos(x)}$$

input
```
integrate((1-tan(x))/sin(2*x),x)
```

output
```
log(cos(2*x) - 1)/4 - log(cos(2*x) + 1)/4 - sin(x)/(2*cos(x))
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.36

$$\int \csc(2x)(1-\tan(x))\,dx = -\frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1} - \frac{1}{4}\log(\cos(2x)+1) + \frac{1}{4}\log(\cos(2x)-1)$$

input
```
integrate((1-tan(x))/sin(2*x),x, algorithm="maxima")
```

output
```
-sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/4*log(cos(2*x) +
1) + 1/4*log(cos(2*x) - 1)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\,\log\left(|\tan\left(x\right)|\right) - \frac{1}{2}\,\tan\left(x\right)$$

input
```
integrate((1-tan(x))/sin(2*x),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(x))) - 1/2*tan(x)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\ln\left(\tan(x)\right)}{2} - \frac{\tan(x)}{2}$$

input
```
int(-(tan(x) - 1)/sin(2*x),x)
```

output
```
log(tan(x))/2 - tan(x)/2
```

3.63 $\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$

3.63.1 Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\text{arctanh}(2\cos(x)\sin(x))$$

output
```
1/2*arctanh(2*cos(x)*sin(x))
```

3.63.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\text{arctanh}(\sin(2x))$$

input
```
Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

output
```
ArcTanh[Sin[2*x]]/2
```

3.63.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4853, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)+1}{1-\tan^2(x)} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(x)^2+1}{1-\tan(x)^2}dx$$

$$\Big\downarrow \; 4853$$

$$\int \frac{1}{1-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \; 219$$

$$\text{arctanh}(\tan(x))$$

input
```
Int[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

output
```
ArcTanh[Tan[x]]
```

3.63.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

3.63.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.36

method	result	size
derivativedivides	$\operatorname{arctanh}(\tan(x))$	4
default	$\operatorname{arctanh}(\tan(x))$	4
norman	$-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(\tan(x)+1)}{2}$	16
parallelrisch	$-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(\tan(x)+1)}{2}$	16
risch	$-\frac{\ln(\mathrm{e}^{2ix}-i)}{2}+\frac{\ln(\mathrm{e}^{2ix}+i)}{2}$	24

input
```
int((1+tan(x)^2)/(1-tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(tan(x))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx=\frac{1}{4}\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{\tan(x)^2-2\tan(x)+1}{\tan(x)^2+1}\right)$$

input
```
integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="fricas")
```

output
```
1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - 2*
tan(x) + 1)/(tan(x)^2 + 1))
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = -\frac{\log(\tan(x)-1)}{2} + \frac{\log(\tan(x)+1)}{2}$$

input
```
integrate((1+tan(x)**2)/(1-tan(x)**2),x)
```

output
```
-log(tan(x) - 1)/2 + log(tan(x) + 1)/2
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log(\tan(x)+1) - \frac{1}{2}\log(\tan(x)-1)$$

input
```
integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log(|\tan(x)+1|) - \frac{1}{2}\log(|\tan(x)-1|)$$

input
```
integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \operatorname{atanh}(\tan(x))$$

input
```
int(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x)
```

output
```
atanh(tan(x))
```

3.64 $\int (a^2-4\cos^2(x))^{3/4}\sin(2x)\,dx$

3.64.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int (a^2-4\cos^2(x))^{3/4}\sin(2x)\,dx=\frac{1}{7}(a^2-4\cos^2(x))^{7/4}$$

output
```
1/7*(a^2-4*cos(x)^2)^(7/4)
```

3.64.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int (a^2-4\cos^2(x))^{3/4}\sin(2x)\,dx=\frac{1}{7}(-4+a^2+4\sin^2(x))^{7/4}$$

input
```
Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

output
```
(-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

3.64.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\left(a^2-4\cos^2(x)\right)^{3/4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(2x)\left(a^2-4\cos(x)^2\right)^{3/4}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int 2\sin(x)\left(a^2+4\sin^2(x)-4\right)^{3/4}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \sin(x)\left(a^2+4\sin^2(x)-4\right)^{3/4}d\sin(x)$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

input
```
Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

output
```
(-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.64.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{(a^2-4(\cos^2(x)))^{\frac{7}{4}}}{7}$	15
default	$\frac{(a^2-4(\cos^2(x)))^{\frac{7}{4}}}{7}$	15

input
```
int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*(a^2-4*cos(x)^2)^(7/4)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\cos(x)^2\right)^{\frac{7}{4}}$$

input
```
integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")
```

output
```
1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\text{Timed out}$$

input
```
integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)
```

output
```
Timed out
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\,\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

input
```
integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
```

output
```
1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\,\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

input
```
integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="giac")
```

output
```
1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{\left(a^2-4\cos\left(x\right)^2\right)^{7/4}}{7}$$

input
```
int(sin(2*x)*(a^2 - 4*cos(x)^2)^(3/4),x)
```

output
```
(a^2 - 4*cos(x)^2)^(7/4)/7
```

3.65 $\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

output
```
-3/8*(a^2-4*sin(x)^2)^(2/3)
```

3.65.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

input
```
Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

output
```
(-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

3.65.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin(x)^2}}dx$$

$$\Big\downarrow \; 4878$$

$$\int \frac{2\sin(x)}{\sqrt[3]{a^2-4\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \; 27$$

$$2\int \frac{\sin(x)}{\sqrt[3]{a^2-4\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \; 241$$

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

input
```
Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

output
```
(-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.65.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{3\left(a^{2}-4\left(\sin^{2}\left(x\right)\right)\right)^{\frac{2}{3}}}{8}$	15
default	$-\frac{3\left(a^{2}-4\left(\sin^{2}\left(x\right)\right)\right)^{\frac{2}{3}}}{8}$	15

input
```
int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*(a^2-4*sin(x)^2)^(2/3)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2+4\cos(x)^2-4\right)^{\frac{2}{3}}$$

input
```
integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/8*(a^2 + 4*cos(x)^2 - 4)^(2/3)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3\left(a^2-4\sin^2(x)\right)^{\frac{2}{3}}}{8}$$

input
```
integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
```

output
```
-3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\sin(x)^2\right)^{\frac{2}{3}}$$

input
```
integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="maxima")
```

output
```
-3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\,\sin\left(x\right)^2\right)^{\frac{2}{3}}$$

input
```
integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="giac")
```

output
```
-3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3\left(a^2-4\sin\left(x\right)^2\right)^{2/3}}{8}$$

input
```
int(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x)
```

output
```
-(3*(a^2 - 4*sin(x)^2)^(2/3))/8
```

3.66 $\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx$

3.66.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

output
```
arctan((-1+a^(2*x))^(1/2))/ln(a)
```

3.66.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

input
```
Integrate[1/Sqrt[-1 + a^(2*x)],x]
```

output
```
ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

3.66.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^{2x}-1}}dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{a^{-2x}}{\sqrt{a^{2x}-1}}da^{2x}}{2\log(a)}$$
$$\Big\downarrow \text{73}$$
$$\frac{\int \frac{1}{a^{4x}+1}d\sqrt{a^{2x}-1}}{\log(a)}$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

input
```
Int[1/Sqrt[-1 + a^(2*x)],x]
```

output
```
ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

3.66.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.66.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$	16
default	$\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$	16

input
```
int(1/(-1+a^(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((-1+a^(2*x))^(1/2))/ln(a)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

input
```
integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(a^(2*x) - 1))/log(a)
```

3.66.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \int \frac{1}{\sqrt{a^{2x}-1}}\,dx$$

input
```
integrate(1/(-1+a**(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a**(2*x) - 1), x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

input
```
integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(a^(2*x) - 1))/log(a)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

input
```
integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(a^(2*x) - 1))/log(a)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = -\frac{a^x \operatorname{asin}\left(\frac{1}{a^x}\right)\sqrt{1-\frac{1}{a^{2x}}}}{\ln(a)\sqrt{a^{2x}-1}}$$

input
```
int(1/(a^(2*x) - 1)^(1/2),x)
```

output
```
-(a^x*asin(1/a^x)*(1 - 1/a^(2*x))^(1/2))/(log(a)*(a^(2*x) - 1)^(1/2))
```

3.67 $\int \frac{e^{x/2}}{\sqrt{-1+e^x}} dx$

3.67.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}} dx = 2\text{arctanh}\left(\frac{e^{x/2}}{\sqrt{-1+e^x}}\right)$$

output
```
2*arctanh(exp(1/2*x)/(-1+exp(x))^(1/2))
```

3.67.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}} dx = -2\log\left(-e^{x/2}+\sqrt{-1+e^x}\right)$$

input
```
Integrate[E^(x/2)/Sqrt[-1 + E^x],x]
```

output
```
-2*Log[-E^(x/2) + Sqrt[-1 + E^x]]
```

3.67.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x/2}}{\sqrt{e^x-1}}dx$$

$$\Big\downarrow \; 2679$$

$$2\int \frac{1}{\sqrt{-1+e^x}}de^{x/2}$$

$$\Big\downarrow \; 224$$

$$2\int \frac{1}{1-e^x}d\frac{e^{x/2}}{\sqrt{-1+e^x}}$$

$$\Big\downarrow \; 219$$

$$2\text{arctanh}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

input
```
Int[E^(x/2)/Sqrt[-1 + E^x],x]
```

output
```
2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

3.67.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.67.4 Maple [F]

$$\int \frac{\mathrm{e}^{\frac{x}{2}}}{\sqrt{-1+\mathrm{e}^{x}}}dx$$

input
```
int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

output
```
int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\,\log\left(\sqrt{e^x-1}-e^{\left(\frac{1}{2}\,x\right)}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*log(sqrt(e^x - 1) - e^(1/2*x))
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\log\left(2\sqrt{e^x-1}+2e^{\frac{x}{2}}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)
```

output
```
2*log(2*sqrt(exp(x) - 1) + 2*exp(x/2))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\,\log\left(2\,\sqrt{e^x-1}+2\,e^{\left(\frac{1}{2}\,x\right)}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="maxima")
```

output
```
2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\,\log\left(-\sqrt{e^x-1}+e^{\left(\frac{1}{2}\,x\right)}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="giac")
```

output
```
-2*log(-sqrt(e^x - 1) + e^(1/2*x))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = \ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^x}\,\sqrt{\mathrm{e}^x-1}-\frac{1}{2}\right)$$

input
```
int(exp(x/2)/(exp(x) - 1)^(1/2),x)
```

output
```
log(exp(x) + exp(x)^(1/2)*(exp(x) - 1)^(1/2) - 1/2)
```

3.68 $\int \frac{\arctan(x)^n}{1+x^2} dx$

3.68.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx=\frac{\arctan(x)^{1+n}}{1+n}$$

output
```
arctan(x)^(1+n)/(1+n)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx=\frac{\arctan(x)^{1+n}}{1+n}$$

input
```
Integrate[ArcTan[x]^n/(1 + x^2),x]
```

output
```
ArcTan[x]^(1 + n)/(1 + n)
```

3.68.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)^n}{x^2+1} \, dx$$
$$\Big\downarrow \text{ 5419}$$
$$\frac{\arctan(x)^{n+1}}{n+1}$$

input
```
Int[ArcTan[x]^n/(1 + x^2),x]
```

output
```
ArcTan[x]^(1 + n)/(1 + n)
```

3.68.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.68.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\arctan(x)^{1+n}}{1+n}$	13
default	$\frac{\arctan(x)^{1+n}}{1+n}$	13
risch	$\frac{i(\ln(-ix+1)-\ln(ix+1))\left(\frac{i(\ln(-ix+1)-\ln(ix+1))}{2}\right)^n}{2+2n}$	48

input
```
int(arctan(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)^(1+n)/(1+n)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^n \arctan(x)}{n+1}$$

input
```
integrate(arctan(x)^n/(x^2+1),x, algorithm="fricas")
```

output
```
arctan(x)^n*arctan(x)/(n + 1)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \begin{cases} \frac{\operatorname{atan}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{atan}(x)) & \text{otherwise} \end{cases}$$

input
```
integrate(atan(x)**n/(x**2+1),x)
```

output
```
Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^{n+1}}{n+1}$$

input
```
integrate(arctan(x)^n/(x^2+1),x, algorithm="maxima")
```

output
```
arctan(x)^(n + 1)/(n + 1)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan{(x)}^{n+1}}{n+1}$$

input
```
integrate(arctan(x)^n/(x^2+1),x, algorithm="giac")
```

output
```
arctan(x)^(n + 1)/(n + 1)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\mathrm{atan}(x)^{n+1}}{n+1}$$

input
```
int(atan(x)^n/(x^2 + 1),x)
```

output
```
atan(x)^(n + 1)/(n + 1)
```

3.69 $\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

3.69.1 Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx=\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

output
```
2/5*a*arcsin(x/a)^(5/2)*(1-x^2/a^2)^(1/2)/(a^2-x^2)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx=\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

input
```
Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

output
```
(2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

3.69.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$
$$\Big\downarrow \ 5152$$
$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

input
```
Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

output
```
(2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

3.69.3.1 Defintions of rubi rules used

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

3.69.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\,a}{5\sqrt{a^2-x^2}}$	38

input
```
int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arcsin(x/a)^(5/2)/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)*a
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2}{5}\sqrt{-\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)}\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2$$

input
```
integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(-arctan(-x/sqrt(a^2 - x^2)))*arctan(-x/sqrt(a^2 - x^2))^2
```

3.69.6 Sympy [F]

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\operatorname{asin}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-(-a+x)(a+x)}}\,dx$$

input
```
integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

output
```
Integral(asin(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

3.69.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2\,|a|\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}}{5\,a}$$

input
```
integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
2/5*abs(a)*arcsin(x/a)^(5/2)/a
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\mathrm{asin}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

input
```
int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
```

output
```
int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

3.70 $\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx$

3.70.1 Optimal result

Integrand size = 16, antiderivative size = 8

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx=\frac{1}{2\arccos(x)^2}$$

output
```
1/2/arccos(x)^2
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx=\frac{1}{2\arccos(x)^2}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

output
```
1/(2*ArcCos[x]^2)
```

3.70.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{2\arccos(x)^2}$$

input
```
Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

output
```
1/(2*ArcCos[x]^2)
```

3.70.3.1 Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

3.70.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{1}{2\arccos(x)^2}$	7
default	$\frac{1}{2\arccos(x)^2}$	7

input
```
int(1/arccos(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/arccos(x)^2
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos{(x)}^2}$$

input
```
integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2/arccos(x)^2
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\operatorname{acos}^2{(x)}}$$

input
```
integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)
```

output
```
1/(2*acos(x)**2)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos{(x)}^2}$$

input
```
integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2/arccos(x)^2
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos{(x)}^2}$$

input
```
integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2/arccos(x)^2
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\,\mathrm{acos}\,(x)^2}$$

input
```
int(1/(acos(x)^3*(1 - x^2)^(1/2)),x)
```

output
```
1/(2*acos(x)^2)
```

3.71 $\int x \log^2(x)\,dx$

3.71.1 Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

3.71.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x \log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

input
```
Integrate[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

3.71.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2(x) \, dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{2}x^2 \log^2(x) - \int x \log(x)dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

input
```
Int[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

3.71.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

input
```
int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="fricas")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{x^2\log\left(x\right)^2}{2} - \frac{x^2\log\left(x\right)}{2} + \frac{x^2}{4}$$

input
```
integrate(x*ln(x)**2,x)
```

output
```
x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{1}{4}\left(2\,\log\left(x\right)^2 - 2\,\log\left(x\right) + 1\right)x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="maxima")
```

output
```
1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{x^2\,\left(2\ln\left(x\right)^2 - 2\,\ln\left(x\right) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.72 $\int \frac{\log(x)}{x^5} \, dx$

3.72.1 Optimal result

Integrand size = 6, antiderivative size = 17

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

output
```
-1/16/x^4-1/4*ln(x)/x^4
```

3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

input
```
Integrate[Log[x]/x^5,x]
```

output
```
-1/16*1/x^4 - Log[x]/(4*x^4)
```

3.72.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x^5} \, dx$$

$$\Big\downarrow \; 2741$$

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

input
```
Int[Log[x]/x^5,x]
```

output
```
-1/16*1/x^4 - Log[x]/(4*x^4)
```

3.72.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.72.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
norman	$\frac{-\frac{1}{16}-\frac{\ln(x)}{4}}{x^4}$	11
parallelrisch	$-\frac{1+4\ln(x)}{16x^4}$	12
default	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14
risch	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14
parts	$-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$	14

input
```
int(ln(x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/16-1/4*ln(x))/x^4
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{4\,\log(x)+1}{16\,x^4}$$

input
```
integrate(log(x)/x^5,x, algorithm="fricas")
```

output
```
-1/16*(4*log(x) + 1)/x^4
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

input
```
integrate(ln(x)/x**5,x)
```

output
```
-log(x)/(4*x**4) - 1/(16*x**4)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

input
```
integrate(log(x)/x^5,x, algorithm="maxima")
```

output
```
-1/4*log(x)/x^4 - 1/16/x^4
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

input
```
integrate(log(x)/x^5,x, algorithm="giac")
```

output
```
-1/4*log(x)/x^4 - 1/16/x^4
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\ln(x) + \frac{1}{4}}{4\,x^4}$$

input
```
int(log(x)/x^5,x)
```

output
```
-(log(x) + 1/4)/(4*x^4)
```

3.73 $\int x^2 \log\left(\frac{-1+x}{x}\right) dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = -\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3}\log(-1+x) + \frac{1}{3}x^3 \log\left(\frac{-1+x}{x}\right)$$

output
```
-1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

3.73.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = -\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3}\log(1-x) + \frac{1}{3}x^3 \log\left(\frac{-1+x}{x}\right)$$

input
```
Integrate[x^2*Log[(-1 + x)/x],x]
```

output
```
-1/3*x - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3
```

3.73.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2911, 2905, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log\left(\frac{x-1}{x}\right) dx$$
$$\Big\downarrow \text{2911}$$
$$\int x^2 \log\left(1-\frac{1}{x}\right) dx$$
$$\Big\downarrow \text{2905}$$
$$\frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right)-\frac{1}{3}\int \frac{x}{1-\frac{1}{x}}dx$$
$$\Big\downarrow \text{795}$$
$$\frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right)-\frac{1}{3}\int \frac{x^2}{x-1}dx$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right)-\frac{1}{3}\int \left(x+\frac{1}{x-1}+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right)+\frac{1}{3}\left(-\frac{x^2}{2}-x-\log(1-x)\right)$$

input
```
Int[x^2*Log[(-1 + x)/x],x]
```

output
```
(x^3*Log[1 - x^(-1)])/3 + (-x - x^2/2 - Log[1 - x])/3
```

3.73.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

3.73.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{x}{3}-\frac{x^{2}}{6}-\frac{\ln \left(-1+x \right)}{3}+\frac{x^{3} \ln \left(\frac{-1+x}{x}\right)}{3}$	29
parts	$-\frac{x}{3}-\frac{x^{2}}{6}-\frac{\ln \left(-1+x \right)}{3}+\frac{x^{3} \ln \left(\frac{-1+x}{x}\right)}{3}$	29
parallelrisch	$\frac{x^{3} \ln \left(\frac{-1+x}{x}\right)}{3}-\frac{1}{3}-\frac{x^{2}}{6}-\frac{\ln \left(x \right)}{3}-\frac{x}{3}-\frac{\ln \left(\frac{-1+x}{x}\right)}{3}$	38
derivativedivides	$-\frac{x^{2}}{6}-\frac{x}{3}+\frac{\ln \left(-\frac{1}{x}\right)}{3}+\frac{\ln \left(1-\frac{1}{x}\right) \left(1-\frac{1}{x}\right) \left(\left(1-\frac{1}{x}\right)^{2}+\frac{3}{x}\right) x^{3}}{3}$	53
default	$-\frac{x^{2}}{6}-\frac{x}{3}+\frac{\ln \left(-\frac{1}{x}\right)}{3}+\frac{\ln \left(1-\frac{1}{x}\right) \left(1-\frac{1}{x}\right) \left(\left(1-\frac{1}{x}\right)^{2}+\frac{3}{x}\right) x^{3}}{3}$	53

input
```
int(x^2*ln((-1+x)/x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^2 \log \left(\frac{-1+x}{x}\right) \, dx = \frac{1}{3} \, x^3 \log \left(\frac{x-1}{x}\right) - \frac{1}{6} \, x^2 - \frac{1}{3} \, x - \frac{1}{3} \, \log \left(x-1\right)$$

input
```
integrate(x^2*log((-1+x)/x),x, algorithm="fricas")
```

output
```
1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^2 \log \left(\frac{-1+x}{x}\right) \, dx = \frac{x^3 \log \left(\frac{x-1}{x}\right)}{3} - \frac{x^2}{6} - \frac{x}{3} - \frac{\log \left(x-1\right)}{3}$$

input
```
integrate(x**2*ln((-1+x)/x),x)
```

output
```
x**3*log((x - 1)/x)/3 - x**2/6 - x/3 - log(x - 1)/3
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^2 \log \left(\frac{-1+x}{x}\right) \, dx = \frac{1}{3} \, x^3 \log \left(\frac{x-1}{x}\right) - \frac{1}{6} \, x^2 - \frac{1}{3} \, x - \frac{1}{3} \, \log \left(x-1\right)$$

input
```
integrate(x^2*log((-1+x)/x),x, algorithm="maxima")
```

output
```
1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

3.73.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int x^2 \log\left(\frac{-1+x}{x}\right)\,dx = \frac{\frac{2(x-1)}{x}-3}{6\left(\frac{x-1}{x}-1\right)^2} - \frac{\log\left(\frac{x-1}{x}\right)}{3\left(\frac{x-1}{x}-1\right)^3} - \frac{1}{3}\log\left(\frac{|x-1|}{|x|}\right) + \frac{1}{3}\log\left(\left|\frac{x-1}{x}-1\right|\right)$$

input
```
integrate(x^2*log((-1+x)/x),x, algorithm="giac")
```

output
```
1/6*(2*(x - 1)/x - 3)/((x - 1)/x - 1)^2 - 1/3*log((x - 1)/x)/((x - 1)/x -
1)^3 - 1/3*log(abs(x - 1)/abs(x)) + 1/3*log(abs((x - 1)/x - 1))
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^2 \log\left(\frac{-1+x}{x}\right)\,dx = \frac{x^3\,\ln\left(\frac{x-1}{x}\right)}{3} - \frac{\ln\left(x\,(x-1)\right)}{6} - \frac{\ln\left(\frac{x-1}{x}\right)}{6} - \frac{x}{3} - \frac{x^2}{6}$$

input
```
int(x^2*log((x - 1)/x),x)
```

output
```
(x^3*log((x - 1)/x))/3 - log(x*(x - 1))/6 - log((x - 1)/x)/6 - x/3 - x^2/6
```

3.74 $\int \cos^5(x)\,dx$

3.74.1 Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

output
```
sin(x)-2/3*sin(x)^3+1/5*sin(x)^5
```

3.74.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

input
```
Integrate[Cos[x]^5,x]
```

output
```
Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

3.74.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{3113}$$

$$-\int \left(\sin^4(x)-2\sin^2(x)+1\right)d(-\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

input
```
Int[Cos[x]^5,x]
```

output
```
Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.74.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{5}$	17
risch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18
parallelrisch	$\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$	18

input
```
int(cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos^5(x)\,dx = \frac{1}{15}\left(3\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+8\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^5,x, algorithm="fricas")
```

output
```
1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos^5(x)\,dx = \frac{\sin^5\left(x\right)}{5}-\frac{2\sin^3\left(x\right)}{3}+\sin\left(x\right)$$

input
```
integrate(cos(x)**5,x)
```

output
```
sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^5,x, algorithm="maxima")
```

output
```
1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^5,x, algorithm="giac")
```

output
```
1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos^5(x)\,dx = \frac{\sin(x)\,\cos(x)^4}{5} + \frac{4\,\sin(x)\,\cos(x)^2}{15} + \frac{8\,\sin(x)}{15}$$

input
```
int(cos(x)^5,x)
```

output
```
(8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5
```

3.75 $\int \cos^4(x)\sin^2(x)\,dx$

3.75.1 Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \cos^4(x)\sin^2(x)\,dx=\frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)$$

output
```
1/16*x+1/16*cos(x)*sin(x)+1/24*cos(x)^3*sin(x)-1/6*cos(x)^5*sin(x)
```

3.75.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^4(x)\sin^2(x)\,dx=\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

input
```
Integrate[Cos[x]^4*Sin[x]^2,x]
```

output
```
x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

3.75.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2\cos(x)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{6}\int \cos^4(x)dx-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

input
```
Int[Cos[x]^4*Sin[x]^2,x]
```

output
```
-1/6*(Cos[x]^5*Sin[x]) + ((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/
2))/4)/6
```

3.75.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.75.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x}{16}-\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}+\frac{\sin(2x)}{64}$
parallelrisch	$\frac{x}{16}-\frac{\sin(6x)}{192}-\frac{\sin(4x)}{64}+\frac{\sin(2x)}{64}$
default	$-\frac{(\cos^5(x))\sin(x)}{6}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{24}+\frac{x}{16}$
norman	$\frac{\frac{x}{16}+\frac{47\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{13\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{13\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}-\frac{47\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

input
```
int(sin(x)^2*cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16*x-1/192*sin(6*x)-1/64*sin(4*x)+1/64*sin(2*x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\sin^2(x)\,dx = -\frac{1}{48}\left(8\cos(x)^5 - 2\cos(x)^3 - 3\cos(x)\right)\sin(x) + \frac{1}{16}\,x$$

input
```
integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

input
```
integrate(cos(x)**4*sin(x)**2,x)
```

output
```
x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{48}\sin(2\,x)^3 + \frac{1}{16}\,x - \frac{1}{64}\sin(4\,x)$$

input
```
integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
```

output
```
1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) - \frac{1}{64}\,\sin\left(4\,x\right) + \frac{1}{64}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
```

output
```
1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^2(x)\,dx = \left(\frac{\cos\left(x\right)^3}{6} + \frac{\cos\left(x\right)}{8}\right)\,\sin\left(x\right)^3 - \frac{\cos\left(x\right)\,\sin\left(x\right)}{16} + \frac{x}{16}$$

input
```
int(cos(x)^4*sin(x)^2,x)
```

output
```
x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)
```

3.76 $\int \csc^5(x)\,dx$

3.76.1 Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \csc^5(x)\,dx = -\frac{3}{8}\text{arctanh}(\cos(x)) - \frac{3}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)$$

output
```
-3/8*arctanh(cos(x))-3/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \csc^5(x)\,dx = -\frac{3}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{3}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]^5,x]
```

output
```
(-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]
])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64
```

3.76.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

input
```
Int[Csc[x]^5,x]
```

output
```
-1/4*(Cot[x]*Csc[x]^3) + (3*(-1/2*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/2))/4
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.76.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\left(-\frac{\left(\csc^{3}\left(x\right)\right)}{4}-\frac{3\csc\left(x\right)}{8}\right)\cot\left(x\right)+\frac{3\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)}{8}$	26
parallelrisch	$\ln\left(\left(\csc\left(x\right)-\cot\left(x\right)\right)^{\frac{3}{8}}\right)+\frac{3\left(\cot^{3}\left(x\right)\right)\csc\left(x\right)}{8}-\frac{5\left(\csc^{3}\left(x\right)\right)\cot\left(x\right)}{8}$	28
norman	$\frac{-\frac{1}{64}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{8}+\frac{\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{64}}{\tan\left(\frac{x}{2}\right)^{4}}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$	42
risch	$\frac{3\,\mathrm{e}^{7ix}-11\,\mathrm{e}^{5ix}-11\,\mathrm{e}^{3ix}+3\,\mathrm{e}^{ix}}{4\left(\mathrm{e}^{2ix}-1\right)^{4}}-\frac{3\ln\left(\mathrm{e}^{ix}+1\right)}{8}+\frac{3\ln\left(\mathrm{e}^{ix}-1\right)}{8}$	62

input
```
int(1/sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int \csc^5(x)\,dx$$
$$= \frac{6\cos(x)^3 - 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(\right.}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

input
```
integrate(1/sin(x)^5,x, algorithm="fricas")
```

output
```
1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3
*(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(cos(x)^4
 - 2*cos(x)^2 + 1)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \csc^5(x)\,dx = \frac{3\cos^3(x) - 5\cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} + \frac{3\log(\cos(x) - 1)}{16} - \frac{3\log(\cos(x) + 1)}{16}$$

input
```
integrate(1/sin(x)**5,x)
```

output
```
(3*cos(x)**3 - 5*cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) + 3*log(cos(x) -
 1)/16 - 3*log(cos(x) + 1)/16
```

3.76.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \csc^5(x)\,dx = \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)} - \frac{3}{16}\log(\cos(x) + 1) + \frac{3}{16}\log(\cos(x) - 1)$$

input
```
integrate(1/sin(x)^5,x, algorithm="maxima")
```

output
```
1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3/16*log(cos(x)
+ 1) + 3/16*log(cos(x) - 1)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \csc^5(x)\,dx = \frac{3\,\cos{(x)}^3 - 5\,\cos{(x)}}{8\,\left(\cos{(x)}^2 - 1\right)^2} - \frac{3}{16}\,\log{(\cos{(x)} + 1)} + \frac{3}{16}\,\log{(-\cos{(x)} + 1)}$$

input
```
integrate(1/sin(x)^5,x, algorithm="giac")
```

output
```
1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^2 - 1)^2 - 3/16*log(cos(x) + 1) + 3/16
*log(-cos(x) + 1)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \csc^5(x)\,dx = -\frac{3\,\mathrm{atanh}(\cos{(x)})}{8} - \frac{\frac{5\,\cos(x)}{8} - \frac{3\cos(x)^3}{8}}{\cos{(x)}^4 - 2\cos{(x)}^2 + 1}$$

input
```
int(1/sin(x)^5,x)
```

output
```
- (3*atanh(cos(x)))/8 - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(cos(x)^4 - 2*cos(
x)^2 + 1)
```

3.77 $\int e^{-x}\sin(x)\,dx$

3.77.1 Optimal result

Integrand size = 8, antiderivative size = 23

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}e^{-x}\cos(x) - \frac{1}{2}e^{-x}\sin(x)$$

output
```
-1/2*cos(x)/exp(x)-1/2*sin(x)/exp(x)
```

3.77.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}e^{-x}(\cos(x)+\sin(x))$$

input
```
Integrate[Sin[x]/E^x,x]
```

output
```
-1/2*(Cos[x] + Sin[x])/E^x
```

3.77.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\sin(x)\,dx$$

$$\Big\downarrow \text{4932}$$

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

input
```
Int[Sin[x]/E^x,x]
```

output
```
-1/2*Cos[x]/E^x - Sin[x]/(2*E^x)
```

3.77.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.77.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$-\frac{(\cos(x)+\sin(x))\mathrm{e}^{-x}}{2}$	12
default	$-\frac{\mathrm{e}^{-x}\cos(x)}{2}-\frac{\mathrm{e}^{-x}\sin(x)}{2}$	18
norman	$\frac{\left(-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)\right)\mathrm{e}^{-x}}{1+\tan^2\left(\frac{x}{2}\right)}$	32
risch	$-\frac{\mathrm{e}^{(-1+i)x}}{4}+\frac{i\mathrm{e}^{(-1+i)x}}{4}-\frac{\mathrm{e}^{(-1-i)x}}{4}-\frac{i\mathrm{e}^{(-1-i)x}}{4}$	36

input
```
int(sin(x)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(cos(x)+sin(x))*exp(-x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\cos(x)\,e^{(-x)} - \frac{1}{2}\,e^{(-x)}\sin(x)$$

input
```
integrate(sin(x)/exp(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)*e^(-x) - 1/2*e^(-x)*sin(x)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-x}\sin(x)\,dx = -\frac{e^{-x}\sin(x)}{2} - \frac{e^{-x}\cos(x)}{2}$$

input
```
integrate(sin(x)/exp(x),x)
```

output
```
-exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\,(\cos(x) + \sin(x))e^{(-x)}$$

input
```
integrate(sin(x)/exp(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) + sin(x))*e^(-x)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\left(\cos\left(x\right) + \sin\left(x\right)\right)e^{(-x)}$$

input
```
integrate(sin(x)/exp(x),x, algorithm="giac")
```

output
```
-1/2*(cos(x) + sin(x))*e^(-x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{e^{-x}\left(\cos\left(x\right) + \sin\left(x\right)\right)}{2}$$

input
```
int(exp(-x)*sin(x),x)
```

output
```
-(exp(-x)*(cos(x) + sin(x)))/2
```

3.78 $\int e^{2x} \sin(3x)\,dx$

3.78.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{2x} \sin(3x)\,dx = -\frac{3}{13}e^{2x}\cos(3x) + \frac{2}{13}e^{2x}\sin(3x)$$

output
```
-3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

3.78.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{2x} \sin(3x)\,dx = \frac{1}{13}e^{2x}(-3\cos(3x) + 2\sin(3x))$$

input
```
Integrate[E^(2*x)*Sin[3*x],x]
```

output
```
(E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13
```

3.78.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}\sin(3x)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{2}{13}e^{2x}\sin(3x)-\frac{3}{13}e^{2x}\cos(3x)$$

input
```
Int[E^(2*x)*Sin[3*x],x]
```

output
```
(-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

3.78.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.78.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{\mathrm{e}^{2x}\left(-3\cos\left(3x\right)+2\sin\left(3x\right)\right)}{13}$	20
default	$-\frac{3\,\mathrm{e}^{2x}\cos\left(3x\right)}{13}+\frac{2\,\mathrm{e}^{2x}\sin\left(3x\right)}{13}$	22
risch	$-\frac{3\,\mathrm{e}^{(2+3i)x}}{26}-\frac{i\mathrm{e}^{(2+3i)x}}{13}-\frac{3\,\mathrm{e}^{(2-3i)x}}{26}+\frac{i\mathrm{e}^{(2-3i)x}}{13}$	36
norman	$\frac{\frac{4\,\mathrm{e}^{2x}\tan\left(\frac{3x}{2}\right)}{13}+\frac{3\,\mathrm{e}^{2x}\left(\tan^{2}\left(\frac{3x}{2}\right)\right)}{13}-\frac{3\,\mathrm{e}^{2x}}{13}}{1+\tan^{2}\left(\frac{3x}{2}\right)}$	41

input
```
int(exp(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/13*exp(2*x)*(-3*cos(3*x)+2*sin(3*x))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{2x}\sin(3x)\,dx = -\frac{3}{13}\,\cos\left(3\,x\right)e^{(2\,x)} + \frac{2}{13}\,e^{(2\,x)}\sin\left(3\,x\right)$$

input
```
integrate(exp(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
-3/13*cos(3*x)*e^(2*x) + 2/13*e^(2*x)*sin(3*x)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{2x}\sin(3x)\,dx = \frac{2e^{2x}\sin\left(3x\right)}{13} - \frac{3e^{2x}\cos\left(3x\right)}{13}$$

input
```
integrate(exp(2*x)*sin(3*x),x)
```

output
```
2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{(2\,x)}$$

input
```
integrate(exp(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{(2\,x)}$$

input
```
integrate(exp(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
-1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{\mathrm{e}^{2\,x}\,\left(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right)\right)}{13}$$

input
```
int(sin(3*x)*exp(2*x),x)
```

output
```
-(exp(2*x)*(3*cos(3*x) - 2*sin(3*x)))/13
```

3.79 $\int a^x \cos(x) \, dx$

3.79.1 Optimal result

Integrand size = 6, antiderivative size = 31

$$\int a^x \cos(x) \, dx = \frac{a^x \cos(x) \log(a)}{1 + \log^2(a)} + \frac{a^x \sin(x)}{1 + \log^2(a)}$$

output
```
a^x*cos(x)*ln(a)/(1+ln(a)^2)+a^x*sin(x)/(1+ln(a)^2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x) \, dx = \frac{a^x (\cos(x) \log(a) + \sin(x))}{1 + \log^2(a)}$$

input
```
Integrate[a^x*Cos[x],x]
```

output
```
(a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)
```

3.79.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x \cos(x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{a^x \sin(x)}{\log^2(a)+1}+\frac{a^x \log(a)\cos(x)}{\log^2(a)+1}$$

input
```
Int[a^x*Cos[x],x]
```

output
```
(a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

3.79.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.79.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{a^{x} \left(\cos \left(x \right) \ln \left(a \right)+\sin \left(x \right)\right)}{1+\ln \left(a \right)^{2}}$	21
risch	$\frac{a^{x} \cos \left(x \right) \ln \left(a \right)}{1+\ln \left(a \right)^{2}}+\frac{a^{x} \sin \left(x \right)}{1+\ln \left(a \right)^{2}}$	32
norman	$\frac{\frac{\ln \left(a \right) {\mathrm e}^{x \ln \left(a \right)}}{1+\ln \left(a \right)^{2}}+\frac{2 \,{\mathrm e}^{x \ln \left(a \right)} \tan \left(\frac{x}{2}\right)}{1+\ln \left(a \right)^{2}}-\frac{\ln \left(a \right) {\mathrm e}^{x \ln \left(a \right)} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\ln \left(a \right)^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	71

input
```
int(a^x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
a^x*(cos(x)*ln(a)+sin(x))/(1+ln(a)^2)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\, dx = \frac{(\cos(x)\log(a) + \sin(x))a^x}{\log(a)^2 + 1}$$

input
```
integrate(a^x*cos(x),x, algorithm="fricas")
```

output
```
(cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)
```

3.79.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int a^x \cos(x)\, dx = \begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for } a = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for } a = e^{i} \\ \frac{a^x\log(a)\cos(x)}{\log(a)^2+1} + \frac{a^x\sin(x)}{\log(a)^2+1} & \text{otherwise} \end{cases}$$

input
```
integrate(a**x*cos(x),x)
```

output
```
Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos
(x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*
exp(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)*cos(x)/(log(a)**2 + 1) + a
**x*sin(x)/(log(a)**2 + 1), True))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int a^x \cos(x)\,dx = \frac{a^x \cos(x)\log(a) + a^x \sin(x)}{\log(a)^2 + 1}$$

input
```
integrate(a^x*cos(x),x, algorithm="maxima")
```

output
```
(a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)
```

3.79.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 10.61

$$\begin{aligned}\int a^x \cos(x)\,dx &= |a|^x\left(\frac{2\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(a) - \frac{1}{2}\pi x + x\right)\log(|a|)}{(\pi - \pi\mathrm{sgn}(a) - 2)^2 + 4\log(|a|)^2} - \frac{(\pi - \pi\mathrm{sgn}(a) - 2)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(a) - \frac{1}{2}\pi x + x\right)}{(\pi - \pi\mathrm{sgn}(a) - 2)^2 + 4\log(|a|)^2}\right)\\
&+ |a|^x\left(\frac{2\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(a) - \frac{1}{2}\pi x - x\right)\log(|a|)}{(\pi - \pi\mathrm{sgn}(a) + 2)^2 + 4\log(|a|)^2} - \frac{(\pi - \pi\mathrm{sgn}(a) + 2)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(a) - \frac{1}{2}\pi x - x\right)}{(\pi - \pi\mathrm{sgn}(a) + 2)^2 + 4\log(|a|)^2}\right)\\
&+ i|a|^x\left(\frac{ie^{\left(\frac{1}{2}i\pi x\mathrm{sgn}(a) - \frac{1}{2}i\pi x + ix\right)}}{-2i\pi + 2i\pi\mathrm{sgn}(a) + 4\log(|a|) + 4i} - \frac{ie^{\left(-\frac{1}{2}i\pi x\mathrm{sgn}(a) + \frac{1}{2}i\pi x - ix\right)}}{2i\pi - 2i\pi\mathrm{sgn}(a) + 4\log(|a|) - 4i}\right)\\
&+ i|a|^x\left(\frac{ie^{\left(\frac{1}{2}i\pi x\mathrm{sgn}(a) - \frac{1}{2}i\pi x - ix\right)}}{-2i\pi + 2i\pi\mathrm{sgn}(a) + 4\log(|a|) - 4i} - \frac{ie^{\left(-\frac{1}{2}i\pi x\mathrm{sgn}(a) + \frac{1}{2}i\pi x + ix\right)}}{2i\pi - 2i\pi\mathrm{sgn}(a) + 4\log(|a|) + 4i}\right)\end{aligned}$$

input
```
integrate(a^x*cos(x),x, algorithm="giac")
```

output
```
abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x + x)*log(abs(a))/((pi - pi*sgn(
a) - 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) - 2)*sin(1/2*pi*x*sgn(a) -
1/2*pi*x + x)/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2)) + abs(a)^x*(2*co
s(1/2*pi*x*sgn(a) - 1/2*pi*x - x)*log(abs(a))/((pi - pi*sgn(a) + 2)^2 + 4*
log(abs(a))^2) - (pi - pi*sgn(a) + 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x - x)/
((pi - pi*sgn(a) + 2)^2 + 4*log(abs(a))^2)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*
sgn(a) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I)
 - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4
*log(abs(a)) - 4*I)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x - I
*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I) - I*e^(-1/2*I*pi*x*sgn
(a) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I))
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{a^x\,(\sin(x) + \ln(a)\,\cos(x))}{\ln(a)^2 + 1}$$

input
```
int(a^x*cos(x),x)
```

output
```
(a^x*(sin(x) + log(a)*cos(x)))/(log(a)^2 + 1)
```

3.80 $\int \cos(\log(x))\,dx$

3.80.1 Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

output
```
1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

3.80.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

input
```
Integrate[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

3.80.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\log(x))\,dx$$

$$\Big\downarrow \text{4979}$$

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

input
```
Int[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

3.80.3.1 Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

3.80.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$	11
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

input
```
int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(cos(ln(x))+sin(ln(x)))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(cos(log(x)),x, algorithm="fricas")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x))\,dx = \frac{x\sin\left(\log\left(x\right)\right)}{2} + \frac{x\cos\left(\log\left(x\right)\right)}{2}$$

input
```
integrate(cos(ln(x)),x)
```

output
```
x*sin(log(x))/2 + x*cos(log(x))/2
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x(\cos\left(\log\left(x\right)\right) + \sin\left(\log\left(x\right)\right))$$

input
```
integrate(cos(log(x)),x, algorithm="maxima")
```

output
```
1/2*x*(cos(log(x)) + sin(log(x)))
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(cos(log(x)),x, algorithm="giac")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln\left(x\right)\right)}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(2^(1/2)*x*sin(pi/4 + log(x)))/2
```

3.81 $\int \log(\cos(x))\sec^2(x)\,dx$

3.81.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \tan(x) + \log(\cos(x))\tan(x)$$

output
```
-x+tan(x)+ln(cos(x))*tan(x)
```

3.81.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \tan(x) + \log(\cos(x))\tan(x)$$

input
```
Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

3.81.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3034, 25, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x) \log(\cos(x)) \, dx$$

$$\Big\downarrow \text{3034}$$

$$\tan(x) \log(\cos(x)) - \int -\tan^2(x) dx$$

$$\Big\downarrow \text{25}$$

$$\int \tan^2(x) dx + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2 dx + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{3954}$$

$$-\int 1 dx + \tan(x) + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{24}$$

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

input
```
Int[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

3.81.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.81.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result
parallelrisch	$-x+\tan\left(x\right)+\ln\left(\cos\left(x\right)\right)\tan\left(x\right)$
norman	$\frac{x-x\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}$
default	$-4i\left(\frac{\frac{\mathrm{e}^{2ix}\ln\left(\left(\mathrm{e}^{2ix}+1\right)\mathrm{e}^{-ix}\right)}{2}-\frac{1}{2}}{\mathrm{e}^{2ix}+1}-\frac{\ln\left(\mathrm{e}^{2ix}+1\right)}{4}+\frac{\ln\left(2\right)}{2\,\mathrm{e}^{2ix}+2}\right)$
risch	$-\frac{2i\ln\left(\mathrm{e}^{ix}\right)}{\mathrm{e}^{2ix}+1}+\frac{-\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}+1\right)\right)\mathrm{csgn}\left(i\cos\left(x\right)\right)^{2}+\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}+1\right)\right)\mathrm{csgn}\left(i\cos\left(x\right)\right)\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)+\pi\ \mathrm{csgn}\left(i\cos\left(x\right)\right)^{3}-\pi\ \mathrm{csg}}{\mathrm{e}^{2ix}+1}$

input
```
int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(x)+ln(cos(x))*tan(x)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(ln(cos(x))*sec(x)**2,x)
```

output
```
-x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 7.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

output
```
-2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x
)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(
x) + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = \log(\cos(x))\tan(x) - x + \tan(x)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

output
```
log(cos(x))*tan(x) - x + tan(x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \log(\cos(x))\sec^2(x)\,dx = \tan(x) - 2x + \ln(\cos(x))\ \tan(x) + \ln(\cos(x))\ 1\mathrm{i} - \ln(\cos(2x) + 1 + \sin(2x)\ 1\mathrm{i})\ 1\mathrm{i}$$

input
```
int(log(cos(x))/cos(x)^2,x)
```

output
```
log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(c
os(x))*tan(x)
```

3.82 $\int x \tan^2(x)\,dx$

3.82.1 Optimal result

Integrand size = 6, antiderivative size = 15

$$\int x \tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

output
```
-1/2*x^2+ln(cos(x))+x*tan(x)
```

3.82.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x \tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

input
```
Integrate[x*Tan[x]^2,x]
```

output
```
-1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

3.82.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 4203, 15, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x \tan(x)^2 dx$$

$$\Big\downarrow \text{4203}$$

$$-\int x dx - \int \tan(x) dx + x \tan(x)$$

$$\Big\downarrow \text{15}$$

$$-\int \tan(x) dx - \frac{x^2}{2} + x \tan(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan(x) dx - \frac{x^2}{2} + x \tan(x)$$

$$\Big\downarrow \text{3956}$$

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

input
```
Int[x*Tan[x]^2,x]
```

output
```
-1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

3.82.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.82.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

method	result	size
norman	$x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	20
parallelrisch	$x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	20
risch	$-\frac{x^{2}}{2}-2ix+\frac{2ix}{{\mathrm e}^{2ix}+1}+\ln\left({\mathrm e}^{2ix}+1\right)$	32

input
```
int(x*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan\left(x\right) + \frac{1}{2}\,\log\left(\frac{1}{\tan\left(x\right)^2 + 1}\right)$$

input
```
integrate(x*tan(x)^2,x, algorithm="fricas")
```

output
```
-1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + x\tan\left(x\right) - \frac{\log\left(\tan^2\left(x\right) + 1\right)}{2}$$

input
```
integrate(x*tan(x)**2,x)
```

output
```
-x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.13

$$\int x\tan^2(x)\,dx =$$

$$-\frac{x^2\cos\left(2\,x\right)^2 + x^2\sin\left(2\,x\right)^2 + 2\,x^2\cos\left(2\,x\right) + x^2 - \left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)\log\left(\cos\left(2\,x\right.\right.}{2\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)}$$

input
```
integrate(x*tan(x)^2,x, algorithm="maxima")
```

output
```
-1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan\left(x\right) + \frac{1}{2}\,\log\left(\frac{4}{\tan\left(x\right)^2+1}\right)$$

input
```
integrate(x*tan(x)^2,x, algorithm="giac")
```

output
```
-1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\tan^2(x)\,dx = \ln\left(\cos\left(x\right)\right) + x\tan(x) - \frac{x^2}{2}$$

input
```
int(x*tan(x)^2,x)
```

output
```
log(cos(x)) + x*tan(x) - x^2/2
```

3.83 $\int \frac{\arcsin(x)}{x^2} dx$

3.83.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \frac{\arcsin(x)}{x^2} dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-arcsin(x)/x-arctanh((-x^2+1)^(1/2))
```

3.83.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2} dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

input
```
Integrate[ArcSin[x]/x^2,x]
```

output
```
-(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

3.83.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5138, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{x^2} \, dx$$

$$\Big\downarrow \text{5138}$$

$$\int \frac{1}{x\sqrt{1-x^2}}dx-\frac{\arcsin(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2-\frac{\arcsin(x)}{x}$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{1-x^4}d\sqrt{1-x^2}-\frac{\arcsin(x)}{x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arcsin(x)}{x}-\text{arctanh}\left(\sqrt{1-x^2}\right)$$

input
```
Int[ArcSin[x]/x^2,x]
```

output
```
-(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

3.83.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.83.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21
parts	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21

input
```
int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin\left(x\right)}{2\,x}$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))
/x
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

input
```
integrate(asin(x)/x**2,x)
```

output
```
Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

output
```
-arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="giac")
```

output
```
-arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\mathrm{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{\mathrm{asin}(x)}{x}$$

input
```
int(asin(x)/x^2,x)
```

output
```
- atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

3.84 $\int \arcsin(x)^2 \, dx$

3.84.1 Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2$$

output
```
-2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2$$

input
```
Integrate[ArcSin[x]^2,x]
```

output
```
-2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

3.84.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5130, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(x)^2\,dx$$

$$\Big\downarrow \text{5130}$$

$$x\arcsin(x)^2-2\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5182}$$

$$x\arcsin(x)^2-2\left(\int 1dx-\sqrt{1-x^2}\arcsin(x)\right)$$

$$\Big\downarrow \text{24}$$

$$x\arcsin(x)^2-2\left(x-\sqrt{1-x^2}\arcsin(x)\right)$$

input
```
Int[ArcSin[x]^2,x]
```

output
```
x*ArcSin[x]^2 - 2*(x - Sqrt[1 - x^2]*ArcSin[x])
```

3.84.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.84.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-2x + x \arcsin\left(x\right)^{2} + 2 \arcsin\left(x\right) \sqrt{-x^{2}+1}$	24

input
```
int(arcsin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2 \, dx = x \arcsin\left(x\right)^{2} + 2 \sqrt{-x^{2}+1} \arcsin\left(x\right) - 2 \, x$$

input
```
integrate(arcsin(x)^2,x, algorithm="fricas")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2 \, dx = x \operatorname{asin}^{2}\left(x\right) - 2x + 2\sqrt{1 - x^{2}} \operatorname{asin}\left(x\right)$$

input
```
integrate(asin(x)**2,x)
```

output
```
x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin{(x)}^2 + 2\sqrt{-x^2+1}\arcsin{(x)} - 2\,x$$

input
```
integrate(arcsin(x)^2,x, algorithm="maxima")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin{(x)}^2 + 2\sqrt{-x^2+1}\arcsin{(x)} - 2\,x$$

input
```
integrate(arcsin(x)^2,x, algorithm="giac")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = 2\,\mathrm{asin}(x)\,\sqrt{1-x^2} + x\,\left(\mathrm{asin}(x)^2 - 2\right)$$

input
```
int(asin(x)^2,x)
```

output
```
2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

3.85 $\int \frac{x^2 \arctan(x)}{1+x^2} \, dx$

3.85.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

output
```
x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

3.85.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

3.85.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{ 5451}$$

$$\int \arctan(x)dx - \int \frac{\arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{ 5345}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx - \int \frac{x}{x^2+1}dx + x\arctan(x)$$

$$\Big\downarrow \text{ 240}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx + x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{ 5419}$$

$$-\frac{1}{2}\arctan(x)^2 + x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

3.85.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.85.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$x \arctan\left(x\right)-\frac{\arctan\left(x\right)^{2}}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	20
parallelrisch	$x \arctan\left(x\right)-\frac{\arctan\left(x\right)^{2}}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	20
parts	$x \arctan\left(x\right)-\frac{\arctan\left(x\right)^{2}}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	20
risch	$\frac{\ln\left(ix+1\right)^{2}}{8}+\frac{i\left(-x+\frac{i\ln\left(-ix+1\right)}{2}\right)\ln\left(ix+1\right)}{2}+\frac{\ln\left(-ix+1\right)^{2}}{8}+\frac{ix\ln\left(-ix+1\right)}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	67

input
```
int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2\arctan(x)}{1+x^2}\,dx=x\arctan\left(x\right)-\frac{1}{2}\arctan\left(x\right)^{2}-\frac{1}{2}\log\left(x^{2}+1\right)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

input
```
integrate(x**2*atan(x)/(x**2+1),x)
```

output
```
x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = (x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
```

output
```
(x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{\operatorname{atan}(x)^2}{2} + x \operatorname{atan}(x) - \frac{\ln(x^2+1)}{2}$$

input
```
int((x^2*atan(x))/(x^2 + 1),x)
```

output
```
x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

3.86 $\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx$

3.86.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = (1+x)\left(\sqrt{\frac{1}{1+x}}\sqrt{\frac{x}{1+x}} + \arccos\left(\sqrt{\frac{x}{1+x}}\right)\right)$$

output
```
(1+x)*(arccos((x/(1+x))^(1/2))+(1/(1+x))^(1/2)*(x/(1+x))^(1/2))
```

3.86.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = x\arccos\left(\sqrt{\frac{x}{1+x}}\right) + \frac{\sqrt{\frac{x}{(1+x)^2}}(1+x)\left(\sqrt{x} - \arctan\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[ArcCos[Sqrt[x/(1 + x)]],x]
```

output
```
x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] - ArcTan[S
qrt[x]]))/Sqrt[x]
```

3.86.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5340, 27, 7270, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(\sqrt{\frac{x}{x+1}}\right)dx$$

$$\Big\downarrow \text{ 5340}$$

$$\int \frac{1}{2}\sqrt{\frac{x}{(x+1)^2}}dx+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \sqrt{\frac{x}{(x+1)^2}}dx+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{ 7270}$$

$$\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\int \frac{\sqrt{x}}{x+1}dx}{2\sqrt{x}}+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(2\sqrt{x}-\int \frac{1}{\sqrt{x}(x+1)}dx\right)}{2\sqrt{x}}+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(2\sqrt{x}-2\int \frac{1}{x+1}d\sqrt{x}\right)}{2\sqrt{x}}+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{ 216}$$

$$x\arccos\left(\sqrt{\frac{x}{x+1}}\right)+\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(2\sqrt{x}-2\arctan\left(\sqrt{x}\right)\right)}{2\sqrt{x}}$$

input
```
Int[ArcCos[Sqrt[x/(1 + x)]],x]
```

output
```
x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(2*Sqrt[x] - 2*ArcT
an[Sqrt[x]]))/(2*Sqrt[x])
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5340
```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.86.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

method	result	size
default	$x\arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,(\arctan(\sqrt{x})-\sqrt{x})}{\sqrt{\frac{x}{1+x}}}$	45
parts	$x\arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,(\arctan(\sqrt{x})-\sqrt{x})}{\sqrt{\frac{x}{1+x}}}$	45

input
```
int(arccos((x/(1+x))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccos((x/(1+x))^(1/2))-1/(x/(1+x))^(1/2)*x^(1/2)*(1/(1+x))^(1/2)*(arcta
n(x^(1/2))-x^(1/2))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx=(x+1)\arccos\left(\sqrt{\frac{x}{x+1}}\right)+\sqrt{x+1}\sqrt{\frac{x}{x+1}}$$

input
```
integrate(arccos((x/(1+x))^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arccos(sqrt(x/(x + 1))) + sqrt(x + 1)*sqrt(x/(x + 1))
```

3.86.6 Sympy [A] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx=x\,\mathrm{acos}\left(\sqrt{\frac{x}{x+1}}\right)-\begin{cases}-\frac{\sqrt{\frac{x}{x+1}}}{\sqrt{-\frac{x}{x+1}+1}}+\mathrm{asin}\left(\sqrt{\frac{x}{x+1}}\right) & \text{for } \sqrt{\frac{x}{x+1}}>-1\wedge\sqrt{\frac{x}{x+1}}<1\end{cases}$$

input
```
integrate(acos((x/(1+x))**(1/2)),x)
```

output
```
x*acos(sqrt(x/(x + 1))) - Piecewise((-sqrt(x/(x + 1))/sqrt(-x/(x + 1) + 1)
 + asin(sqrt(x/(x + 1))), (sqrt(x/(x + 1)) > -1) & (sqrt(x/(x + 1)) < 1)))
```

3.86.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = -\frac{\arccos\left(\sqrt{\frac{x}{x+1}}\right)}{\frac{x}{x+1}-1} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}+1\right)} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}-1\right)}$$

input
```
integrate(arccos((x/(1+x))^(1/2)),x, algorithm="maxima")
```

output
```
-arccos(sqrt(x/(x + 1)))/(x/(x + 1) - 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(
x/(x + 1)) + 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x/(x + 1)) - 1)
```

3.86.8 Giac [F(-2)]

Exception generated.

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos((x/(1+x))^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx = \int \mathrm{acos}\left(\sqrt{\frac{x}{x+1}}\right)\,dx$$

input
```
int(acos((x/(x + 1))^(1/2)),x)
```

output
```
int(acos((x/(x + 1))^(1/2)), x)
```

3.87 $\int (2x+3x^2)^3 \, dx$

3.87.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int (2x+3x^2)^3 \, dx = 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

output
```
2*x^4+36/5*x^5+9*x^6+27/7*x^7
```

3.87.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (2x+3x^2)^3 \, dx = 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

input
```
Integrate[(2*x + 3*x^2)^3,x]
```

output
```
2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

3.87.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+2x\right)^3 dx$$

$$\Big\downarrow \text{1080}$$

$$\int \left(27x^6+54x^5+36x^4+8x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

input
```
Int[(2*x + 3*x^2)^3,x]
```

output
```
2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

3.87.3.1 Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^4(135x^3+315x^2+252x+70)}{35}$	21
default	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22
norman	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22
risch	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22
parallelrisch	$2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$	22

input
```
int((3*x^2+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/35*x^4*(135*x^3+315*x^2+252*x+70)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (2x+3x^2)^3\,dx=\frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

input
```
integrate((3*x^2+2*x)^3,x, algorithm="fricas")
```

output
```
27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (2x+3x^2)^3\,dx=\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

input
```
integrate((3*x**2+2*x)**3,x)
```

output
```
27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (2x+3x^2)^3\,dx = \frac{27}{7}\,x^7 + 9\,x^6 + \frac{36}{5}\,x^5 + 2\,x^4$$

input
```
integrate((3*x^2+2*x)^3,x, algorithm="maxima")
```

output
```
27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (2x+3x^2)^3\,dx = \frac{27}{7}\,x^7 + 9\,x^6 + \frac{36}{5}\,x^5 + 2\,x^4$$

input
```
integrate((3*x^2+2*x)^3,x, algorithm="giac")
```

output
```
27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (2x+3x^2)^3\,dx = \frac{27\,x^7}{7} + 9\,x^6 + \frac{36\,x^5}{5} + 2\,x^4$$

input
```
int((2*x + 3*x^2)^3,x)
```

output
```
2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

3.88 $\int (-1+x)\left(-1+2x+3x^2\right)^2 \, dx$

3.88.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int (-1+x)\left(-1+2x+3x^2\right)^2 \, dx = -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

output
```
-x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6
```

3.88.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (-1+x)\left(-1+2x+3x^2\right)^2 \, dx = -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

input
```
Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

output
```
-x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

3.88.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-1)\left(3x^2+2x-1\right)^2\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(9x^5+3x^4-14x^3-2x^2+5x-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

input
```
Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

output
```
-x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

3.88.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)}{30}$	29
default	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
norman	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
risch	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30
parallelrisch	$-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$	30

input
```
int((-1+x)*(3*x^2+2*x-1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(45*x^5+18*x^4-105*x^3-20*x^2+75*x-30)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

input
```
integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="fricas")
```

output
```
3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

input
```
integrate((-1+x)*(3*x**2+2*x-1)**2,x)
```

output
```
3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

input
```
integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="maxima")
```

output
```
3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

input
```
integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="giac")
```

output
```
3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3\,x^6}{2}+\frac{3\,x^5}{5}-\frac{7\,x^4}{2}-\frac{2\,x^3}{3}+\frac{5\,x^2}{2}-x$$

input
```
int((x - 1)*(2*x + 3*x^2 - 1)^2,x)
```

output
```
(5*x^2)/2 - x - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

3.89 $\int x^{-1+k}(a+bx^k)^n\,dx$

3.89.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(a+bx^k)^{1+n}}{bk(1+n)}$$

output
```
(a+b*x^k)^(1+n)/b/k/(1+n)
```

3.89.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(a+bx^k)^{1+n}}{bk(1+n)}$$

input
```
Integrate[x^(-1 + k)*(a + b*x^k)^n,x]
```

output
```
(a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

3.89.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{k-1}\left(a+bx^k\right)^n\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

input
```
Int[x^(-1 + k)*(a + b*x^k)^n,x]
```

output
```
(a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

3.89.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.89.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
risch	$\frac{(a+b\,x^k)\left(a+b\,x^k\right)^n}{b(1+n)k}$	29

input
```
int(x^(-1+k)*(a+b*x^k)^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(bx^k+a\right)\left(bx^k+a\right)^n}{bkn+bk}$$

input
```
integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="fricas")
```

output
```
(b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(15) = 30.

Time = 12.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\begin{cases}\frac{\log(x)}{a} & \text{for } b=0\wedge k=0\wedge n=-1\\ \frac{a^n x x^{k-1}}{k} & \text{for } b=0\\ (a+b)^n\log(x) & \text{for } k=0\\ \frac{\log\left(\frac{a}{b}+x^k\right)}{bk} & \text{for } n=-1\\ \frac{a\left(a+bx^k\right)^n}{bkn+bk}+\frac{bx^k\left(a+bx^k\right)^n}{bkn+bk} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(k, 0) & Eq(n, -1)), (a**n*x*x**(k - 1)/
k, Eq(b, 0)), ((a + b)**n*log(x), Eq(k, 0)), (log(a/b + x**k)/(b*k), Eq(n,
 -1)), (a*(a + b*x**k)**n/(b*k*n + b*k) + b*x**k*(a + b*x**k)**n/(b*k*n +
b*k), True))
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(bx^k+a)^{n+1}}{bk(n+1)}$$

input
```
integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="maxima")
```

output
```
(b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(bx^k+a)^{n+1}}{bk(n+1)}$$

input
```
integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="giac")
```

output
```
(b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(a+b\,x^k)^{n+1}}{b\,k\,(n+1)}$$

input
```
int(x^(k - 1)*(a + b*x^k)^n,x)
```

output
```
(a + b*x^k)^(n + 1)/(b*k*(n + 1))
```

3.90 $\int \frac{x^3}{1+2x} dx$

3.90.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{x^3}{1+2x} dx = \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16}\log(1+2x)$$

output
```
1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)
```

3.90.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{1+2x} dx = \frac{1}{96}\left(11 + 12x - 12x^2 + 16x^3 - 6\log(1+2x)\right)$$

input
```
Integrate[x^3/(1 + 2*x),x]
```

output
```
(11 + 12*x - 12*x^2 + 16*x^3 - 6*Log[1 + 2*x])/96
```

3.90.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{2x+1}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{x^2}{2}-\frac{x}{4}-\frac{1}{8(2x+1)}+\frac{1}{8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{1}{16}\log(2x+1)$$

input
```
Int[x^3/(1 + 2*x),x]
```

output
```
x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

3.90.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.90.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{\ln\left(x+\frac{1}{2}\right)}{16}$	21
default	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23
norman	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23
meijerg	$\frac{x(16x^2-12x+12)}{96}-\frac{\ln(1+2x)}{16}$	23
risch	$\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$	23

input
```
int(x^3/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3-1/8*x^2+1/8*x-1/16*ln(x+1/2)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log\left(2\,x+1\right)$$

input
```
integrate(x^3/(1+2*x),x, algorithm="fricas")
```

output
```
1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+2x}\,dx = \frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{\log\left(2x+1\right)}{16}$$

input
```
integrate(x**3/(1+2*x),x)
```

output
```
x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log\left(2\,x+1\right)$$

input
```
integrate(x^3/(1+2*x),x, algorithm="maxima")
```

output
```
1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log\left(|2\,x+1|\right)$$

input
```
integrate(x^3/(1+2*x),x, algorithm="giac")
```

output
```
1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(abs(2*x + 1))
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+2x}\,dx = \frac{x}{8} - \frac{\ln\left(x+\frac{1}{2}\right)}{16} - \frac{x^2}{8} + \frac{x^3}{6}$$

input
```
int(x^3/(2*x + 1),x)
```

output
```
x/8 - log(x + 1/2)/16 - x^2/8 + x^3/6
```

3.91 $\int \frac{x^6}{2+3x^2} dx$

3.91.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x^6}{2+3x^2} dx = \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)$$

output
```
4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{2+3x^2} dx = \frac{1}{405}\left(60x - 30x^3 + 27x^5 - 20\sqrt{6}\arctan\left(\sqrt{\frac{3}{2}}x\right)\right)$$

input
```
Integrate[x^6/(2 + 3*x^2),x]
```

output
```
(60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405
```

3.91.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{3x^2+2} \, dx$$
$$\Big\downarrow \text{254}$$
$$\int \left(\frac{x^4}{3}-\frac{2x^2}{9}-\frac{8}{27(3x^2+2)}+\frac{4}{27}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{4}{27}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)+\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}$$

input
```
Int[x^6/(2 + 3*x^2),x]
```

output
```
(4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

3.91.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{4x}{27}-\frac{2x^{3}}{27}+\frac{x^{5}}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$	27
risch	$\frac{4x}{27}-\frac{2x^{3}}{27}+\frac{x^{5}}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$	27
meijerg	$\frac{2\sqrt{2}\,\sqrt{3}\left(\frac{x\sqrt{2}\,\sqrt{3}\left(\frac{189}{4}x^{4}-\frac{105}{2}x^{2}+105\right)}{105}-2\arctan\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{81}$	43

input
```
int(x^6/(3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{2+3x^2}\,dx=\frac{1}{15}\,x^{5}-\frac{2}{27}\,x^{3}-\frac{4}{81}\,\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{3}\sqrt{2}x\right)+\frac{4}{27}\,x$$

input
```
integrate(x^6/(3*x^2+2),x, algorithm="fricas")
```

output
```
1/15*x^5 - 2/27*x^3 - 4/81*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) +
 4/27*x
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^6}{2+3x^2}\,dx=\frac{x^{5}}{15}-\frac{2x^{3}}{27}+\frac{4x}{27}-\frac{4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}$$

input
```
integrate(x**6/(3*x**2+2),x)
```

output
```
x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

input
```
integrate(x^6/(3*x^2+2),x, algorithm="maxima")
```

output
```
1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

input
```
integrate(x^6/(3*x^2+2),x, algorithm="giac")
```

output
```
1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{4\,x}{27} - \frac{2\,x^3}{27} + \frac{x^5}{15} - \frac{4\,\sqrt{2}\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{81}$$

input
```
int(x^6/(3*x^2 + 2),x)
```

output
```
(4*x)/27 - (2*x^3)/27 + x^5/15 - (4*2^(1/2)*3^(1/2)*atan((2^(1/2)*3^(1/2)*
x)/2))/81
```

3.92 $\int \frac{1}{2-7x+3x^2} \, dx$

3.92.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{2-7x+3x^2} \, dx = -\frac{1}{5}\log(1-3x)+\frac{1}{5}\log(2-x)$$

output
```
-1/5*ln(1-3*x)+1/5*ln(2-x)
```

3.92.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-7x+3x^2} \, dx = -\frac{1}{5}\log(1-3x)+\frac{1}{5}\log(2-x)$$

input
```
Integrate[(2 - 7*x + 3*x^2)^(-1),x]
```

output
```
-1/5*Log[1 - 3*x] + Log[2 - x]/5
```

3.92.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2-7x+2}dx$$
$$\Big\downarrow \text{1081}$$
$$3\int \left(\frac{1}{5(1-3x)}-\frac{1}{15(2-x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$3\left(\frac{1}{15}\log(2-x)-\frac{1}{15}\log(1-3x)\right)$$

input
```
Int[(2 - 7*x + 3*x^2)^(-1),x]
```

output
```
3*(-1/15*Log[1 - 3*x] + Log[2 - x]/15)
```

3.92.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(-2+x)}{5}-\frac{\ln\left(x-\frac{1}{3}\right)}{5}$	14
default	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16
norman	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16
risch	$-\frac{\ln(-1+3x)}{5}+\frac{\ln(-2+x)}{5}$	16

input
```
int(1/(3*x^2-7*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(-2+x)-1/5*ln(x-1/3)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\,\log\left(3\,x-1\right)+\frac{1}{5}\,\log\left(x-2\right)$$

input
```
integrate(1/(3*x^2-7*x+2),x, algorithm="fricas")
```

output
```
-1/5*log(3*x - 1) + 1/5*log(x - 2)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{2-7x+3x^2}\,dx = \frac{\log\left(x-2\right)}{5}-\frac{\log\left(x-\frac{1}{3}\right)}{5}$$

input
```
integrate(1/(3*x**2-7*x+2),x)
```

output
```
log(x - 2)/5 - log(x - 1/3)/5
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(3x-1) + \frac{1}{5}\log(x-2)$$

input
```
integrate(1/(3*x^2-7*x+2),x, algorithm="maxima")
```

output
```
-1/5*log(3*x - 1) + 1/5*log(x - 2)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(|3x-1|) + \frac{1}{5}\log(|x-2|)$$

input
```
integrate(1/(3*x^2-7*x+2),x, algorithm="giac")
```

output
```
-1/5*log(abs(3*x - 1)) + 1/5*log(abs(x - 2))
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{6x}{5}-\frac{7}{5}\right)}{5}$$

input
```
int(1/(3*x^2 - 7*x + 2),x)
```

output
```
-(2*atanh((6*x)/5 - 7/5))/5
```

3.93 $\int \frac{-1+3x}{1-x+x^2}\,dx$

3.93.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{-1+3x}{1-x+x^2}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

output
```
3/2*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{-1+3x}{1-x+x^2}\,dx=\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(-1 + 3*x)/(1 - x + x^2),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

3.93.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x-1}{x^2-x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{3}{2}\int -\frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{1103}$$

$$\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
Int[(-1 + 3*x)/(1 - x + x^2),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.93.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{3\ln\left(4x^2-4x+4\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31

input
```
int((-1+3*x)/(x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((-1+3*x)/(x^2-x+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-1+3*x)/(x**2-x+1),x)
```

output
```
3*log(x**2 - x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((-1+3*x)/(x^2-x+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((-1+3*x)/(x^2-x+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{3\ln\left(x^2-x+1\right)}{2} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((3*x - 1)/(x^2 - x + 1),x)
```

output
```
(3*log(x^2 - x + 1))/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.94 $\int \frac{x^2}{5+2x+x^2} \, dx$

3.94.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x^2}{5+2x+x^2} \, dx = x - \frac{3}{2} \arctan\left(\frac{1+x}{2}\right) - \log\left(5+2x+x^2\right)$$

output
```
x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

3.94.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{5+2x+x^2} \, dx = x - \frac{3}{2} \arctan\left(\frac{1+x}{2}\right) - \log\left(5+2x+x^2\right)$$

input
```
Integrate[x^2/(5 + 2*x + x^2),x]
```

output
```
x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

3.94.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+2x+5}\,dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(1-\frac{2x+5}{x^2+2x+5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\arctan\left(\frac{x+1}{2}\right)-\log\left(x^2+2x+5\right)+x$$

input
```
Int[x^2/(5 + 2*x + x^2),x]
```

output
```
x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

3.94.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^{2}+2x+5\right)$	22
risch	$x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^{2}+2x+5\right)$	22
parallelrisch	$x-\ln\left(x+1-2i\right)+\frac{3i\ln\left(x+1-2i\right)}{4}-\ln\left(x+1+2i\right)-\frac{3i\ln\left(x+1+2i\right)}{4}$	37

input
```
int(x^2/(x^2+2*x+5),x,method=_RETURNVERBOSE)
```

output
```
x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x-\frac{3}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)-\log\left(x^2+2x+5\right)$$

input
```
integrate(x^2/(x^2+2*x+5),x, algorithm="fricas")
```

output
```
x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{5+2x+x^2}\,dx = x-\log\left(x^2+2x+5\right)-\frac{3\operatorname{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
integrate(x**2/(x**2+2*x+5),x)
```

output
```
x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \frac{3}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right) - \log\left(x^2+2x+5\right)$$

input
```
integrate(x^2/(x^2+2*x+5),x, algorithm="maxima")
```

output
```
x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \frac{3}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right) - \log\left(x^2+2x+5\right)$$

input
```
integrate(x^2/(x^2+2*x+5),x, algorithm="giac")
```

output
```
x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \ln\left(x^2+2x+5\right) - \frac{3\,\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
int(x^2/(2*x + x^2 + 5),x)
```

output
```
x - log(2*x + x^2 + 5) - (3*atan(x/2 + 1/2))/2
```

3.95 $\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx$

3.95.1 Optimal result

Integrand size = 29, antiderivative size = 47

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx = -\frac{x^2}{2}+x^3-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)$$

output
```
-1/2*x^2+x^3+1/4*ln(2*x^2-x+1)-1/14*arctan(1/7*(-4*x+1)*7^(1/2))*7^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx = -\frac{x^2}{2}+x^3+\frac{\arctan\left(\frac{-1+4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)$$

input
```
Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

output
```
-1/2*x^2 + x^3 + ArcTan[(-1 + 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^
2]/4
```

3.95.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2028, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x^4-5x^3+4x^2}{2x^2-x+1}dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x^2 (6x^2-5x+4)}{2x^2-x+1}dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(3x^2+\frac{x}{2x^2-x+1}-x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)$$

input
```
Int[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

output
```
-1/2*x^2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2
]/4
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.95.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
default	$x^{3}-\frac{x^{2}}{2}+\frac{\ln\left(2x^{2}-x+1\right)}{4}+\frac{\sqrt{7}\,\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$	39
risch	$x^{3}-\frac{x^{2}}{2}+\frac{\ln\left(16x^{2}-8x+8\right)}{4}+\frac{\sqrt{7}\,\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$	39

input
```
int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
x^3-1/2*x^2+1/4*ln(2*x^2-x+1)+1/14*7^(1/2)*arctan(1/7*(-1+4*x)*7^(1/2))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="fricas")
```

output
```
x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
 - x + 1)
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{x^2}{2}+\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{14}$$

input
```
integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
```

output
```
x**3 - x**2/2 + log(x**2 - x/2 + 1/2)/4 + sqrt(7)*atan(4*sqrt(7)*x/7 - sqr
t(7)/7)/14
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="maxima")
```

output
```
x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
 - x + 1)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="giac")
```

output
```
x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
 - x + 1)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=\frac{\ln\left(2\,x^2-x+1\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\,\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{14}-\frac{x^2}{2}+x^3$$

input
```
int((4*x^2 - 5*x^3 + 6*x^4)/(2*x^2 - x + 1),x)
```

output
```
log(2*x^2 - x + 1)/4 + (7^(1/2)*atan((4*7^(1/2)*x)/7 - 7^(1/2)/7))/14 - x^
2/2 + x^3
```

3.96 $\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx$

3.96.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx = \frac{1}{2}\log(2-x) + \frac{\log(x)}{6} + \frac{1}{3}\log(3+x)$$

output
```
1/2*ln(2-x)+1/6*ln(x)+1/3*ln(3+x)
```

3.96.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx = \frac{1}{2}\log(2-x) + \frac{\log(x)}{6} + \frac{1}{3}\log(3+x)$$

input
```
Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

output
```
Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

3.96.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-1}{x^3+x^2-6x}dx$$
$$\Big\downarrow \ 2026$$
$$\int \frac{x^2+x-1}{x(x^2+x-6)}dx$$
$$\Big\downarrow \ 2159$$
$$\int \left(\frac{1}{6x}+\frac{1}{3(x+3)}+\frac{1}{2(x-2)}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

input
```
Int[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

output
```
Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.96.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18
norman	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18
risch	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18
parallelrisch	$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$	18

input
```
int((x^2+x-1)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x)+1/3*ln(3+x)+1/2*ln(-2+x)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

input
```
integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="fricas")
```

output
```
1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{\log(x)}{6}+\frac{\log(x-2)}{2}+\frac{\log(x+3)}{3}$$

input
```
integrate((x**2+x-1)/(x**3+x**2-6*x),x)
```

output
```
log(x)/6 + log(x - 2)/2 + log(x + 3)/3
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

input
```
integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="maxima")
```

output
```
1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(|x+3|)+\frac{1}{2}\log(|x-2|)+\frac{1}{6}\log(|x|)$$

input
```
integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="giac")
```

output
```
1/3*log(abs(x + 3)) + 1/2*log(abs(x - 2)) + 1/6*log(abs(x))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{\ln(x-2)}{2}+\frac{\ln(x+3)}{3}+\frac{\ln(x)}{6}$$

input
```
int((x + x^2 - 1)/(x^2 - 6*x + x^3),x)
```

output
```
log(x - 2)/2 + log(x + 3)/3 + log(x)/6
```

3.97 $\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$

3.97.1 Optimal result

Integrand size = 39, antiderivative size = 33

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

output
```
9/2*ln(a-x)-17*ln(2*a-x)+35/2*ln(3*a-x)
```

3.97.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{35}{2}\log(-3a+x)-17\log(-2a+x)+\frac{9}{2}\log(-a+x)$$

input
```
Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

output
```
(35*Log[-3*a + x])/2 - 17*Log[-2*a + x] + (9*Log[-a + x])/2
```

3.97.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$$

$$\Big\downarrow \ 2462$$

$$\int \left(\frac{17}{2a-x}-\frac{35}{2(3a-x)}-\frac{9}{2(a-x)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

input
```
Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

output
```
(9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.97.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{35\ln(-3a+x)}{2}-17\ln(x-2a)+\frac{9\ln(-a+x)}{2}$	26
parallelrisch	$\frac{35\ln(-3a+x)}{2}-17\ln(x-2a)+\frac{9\ln(-a+x)}{2}$	26
default	$\frac{9\ln(a-x)}{2}-17\ln(-x+2a)+\frac{35\ln(3a-x)}{2}$	30
norman	$\frac{9\ln(a-x)}{2}-17\ln(-x+2a)+\frac{35\ln(3a-x)}{2}$	30

input
```
int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x,method=_RETURNVER
BOSE)
```

output
```
35/2*ln(-3*a+x)-17*ln(x-2*a)+9/2*ln(-a+x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{9}{2}\log\left(-a+x\right)-17\log\left(-2\,a+x\right)+\frac{35}{2}\log\left(-3\,a+x\right)$$

input
```
integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm=
"fricas")
```

output
```
9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{35\log\left(-3a+x\right)}{2}-17\log\left(-2a+x\right)+\frac{9\log\left(-a+x\right)}{2}$$

input
```
integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
```

output
```
35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{9}{2}\,\log\left(-a+x\right)-17\,\log\left(-2\,a+x\right)+\frac{35}{2}\,\log\left(-3\,a+x\right)$$

input
```
integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm=
"maxima")
```

output
```
9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{9}{2}\,\log\left(|-a+x|\right)-17\,\log\left(|-2\,a+x|\right)+\frac{35}{2}\,\log\left(|-3\,a+x|\right)$$

input
```
integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm=
"giac")
```

output
```
9/2*log(abs(-a + x)) - 17*log(abs(-2*a + x)) + 35/2*log(abs(-3*a + x))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx = \frac{9\,\ln\left(x-a\right)}{2}-17\,\ln\left(x-2\,a\right)+\frac{35\,\ln\left(x-3\,a\right)}{2}$$

input
```
int(-(11*a^2 - 7*a*x + 5*x^2)/(6*a*x^2 - 11*a^2*x + 6*a^3 - x^3),x)
```

output
```
(9*log(x - a))/2 - 17*log(x - 2*a) + (35*log(x - 3*a))/2
```

3.98 $\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(1+x)-\frac{2}{3}\log(2+x)$$

output
```
-1/3*ln(1-x)+1/3*ln(2-x)+2/3*ln(1+x)-2/3*ln(2+x)
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(1+x)-\frac{2}{3}\log(2+x)$$

input
```
Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
-1/3*Log[1 - x] + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

3.98.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2202, 25, 1432, 1081, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-x+2}{x^4-5x^2+4} \, dx$$
$$\Big\downarrow \text{2202}$$
$$\int -\frac{x}{x^4-5x^2+4}dx+\int \frac{x^2+2}{x^4-5x^2+4}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{x^2+2}{x^4-5x^2+4}dx-\int \frac{x}{x^4-5x^2+4}dx$$
$$\Big\downarrow \text{1432}$$
$$\int \frac{x^2+2}{x^4-5x^2+4}dx-\frac{1}{2}\int \frac{1}{x^4-5x^2+4}dx^2$$
$$\Big\downarrow \text{1081}$$
$$\int \frac{x^2+2}{x^4-5x^2+4}dx-\frac{1}{2}\int \left(\frac{1}{3 (1-x^2)}-\frac{1}{3 (4-x^2)}\right)dx^2$$
$$\Big\downarrow \text{1475}$$
$$\frac{1}{2}\int \frac{1}{x^2-3x+2}dx+\frac{1}{2}\int \frac{1}{x^2+3x+2}dx-\frac{1}{2}\int \left(\frac{1}{3 (1-x^2)}-\frac{1}{3 (4-x^2)}\right)dx^2$$
$$\Big\downarrow \text{1081}$$
$$-\frac{1}{2}\int \left(\frac{1}{3 (1-x^2)}-\frac{1}{3 (4-x^2)}\right)dx^2+\frac{1}{2}\int \left(\frac{1}{x-2}+\frac{1}{1-x}\right)dx+\frac{1}{2}\int \left(\frac{1}{x+1}+\frac{1}{-x-2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{1}{3}\log \left(1-x^2\right)-\frac{1}{3}\log \left(4-x^2\right)\right)+\frac{1}{2}(\log(2-x)-\log(1-x))+\frac{1}{2}(\log(x+1)-\log(x+2))$$

input
```
Int[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
(-Log[1 - x] + Log[2 - x])/2 + (Log[1 + x] - Log[2 + x])/2 + (Log[1 - x^2]
/3 - Log[4 - x^2]/3)/2
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.98.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26
norman	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26
risch	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26
parallelrisch	$-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}+\frac{\ln(-2+x)}{3}$	26

input
```
int((x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(-1+x)-2/3*ln(2+x)+2/3*ln(1+x)+1/3*ln(-2+x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

input
```
integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
-2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{3}-\frac{\log(x-1)}{3}+\frac{2\log(x+1)}{3}-\frac{2\log(x+2)}{3}$$

input
```
integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

output
```
log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

input
```
integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
-2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(|x+2|)+\frac{2}{3}\log(|x+1|)-\frac{1}{3}\log(|x-1|)+\frac{1}{3}\log(|x-2|)$$

input
```
integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
-2/3*log(abs(x + 2)) + 2/3*log(abs(x + 1)) - 1/3*log(abs(x - 1)) + 1/3*log
(abs(x - 2))
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{64}{3\,(24\,x-16)}-\frac{5}{3}\right)}{3}+\frac{4\,\mathrm{atanh}\left(\frac{128}{3\,(48\,x+32)}+\frac{5}{3}\right)}{3}$$

input
```
int((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x)
```

output
```
(2*atanh(64/(3*(24*x - 16)) - 5/3))/3 + (4*atanh(128/(3*(48*x + 32)) + 5/3
))/3
```

3.99 $\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx$

3.99.1 Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

3.99.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(31) = 62.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx=\frac{1}{12}\left(3\sqrt{2}\log\left(\sqrt{2}-x\right)+2\sqrt{3}\log\left(\sqrt{3}-x\right)-3\sqrt{2}\log\left(\sqrt{2}+x\right)-2\sqrt{3}\log\left(\sqrt{3}+x\right)\right)$$

input
```
Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

output
```
(3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt[2]*Log[S
qrt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12
```

3.99.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-5}{x^4-5x^2+6}\,dx$$
$$\Big\downarrow \; 1480$$
$$\int \frac{1}{x^2-3}dx+\int \frac{1}{x^2-2}dx$$
$$\Big\downarrow \; 220$$
$$-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

output
```
-(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

3.99.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.99.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	26
risch	$\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\right)}{6}+\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(x+\sqrt{2}\right)}{4}$	50

input
```
int((2*x^2-5)/(x^4-5*x^2+6),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)$$

input
```
integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) + 1/6*sqrt(3)*log((x^2
- 2*sqrt(3)*x + 3)/(x^2 - 3))
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{\sqrt{2}\log\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\log\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{3}\log\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\log\left(x+\sqrt{3}\right)}{6}$$

input
```
integrate((2*x**2-5)/(x**4-5*x**2+6),x)
```

output
```
sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)*log(x -
sqrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)$$

input
```
integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log((x - sqrt(2
))/(x + sqrt(2)))
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{|2\,x-2\,\sqrt{3}|}{|2\,x+2\,\sqrt{3}|}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}|}{|2\,x+2\,\sqrt{2}|}\right)$$

input
```
integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + 1/4*sqrt(2)*l
og(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)))
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx=-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$$

input
```
int((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x)
```

output
```
- (2^(1/2)*atanh((2^(1/2)*x)/2))/2 - (3^(1/2)*atanh((3^(1/2)*x)/3))/3
```

3.100 $\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} \, dx$

3.100.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} \, dx = -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x) -\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

output
```
-1/6*ln(1-x)+1/2*ln(2-x)-1/2*ln(3-x)+1/6*ln(4-x)
```

3.100.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} \, dx = -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x) -\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

input
```
Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
-1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

3.100.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-4)(x-3)(x-2)(x-1)}\,dx$$

$$\Big\downarrow \ 198$$

$$\int \left(-\frac{1}{2(x-3)}+\frac{1}{2(x-2)}-\frac{1}{6(x-1)}+\frac{1}{6(x-4)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

input
```
Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
-1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

3.100.3.1 Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.100.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{\ln(-1+x)}{6}-\frac{\ln(-3+x)}{2}+\frac{\ln(x-4)}{6}+\frac{\ln(-2+x)}{2}$	26
norman	$-\frac{\ln(-1+x)}{6}-\frac{\ln(-3+x)}{2}+\frac{\ln(x-4)}{6}+\frac{\ln(-2+x)}{2}$	26
risch	$-\frac{\ln(-1+x)}{6}-\frac{\ln(-3+x)}{2}+\frac{\ln(x-4)}{6}+\frac{\ln(-2+x)}{2}$	26
parallelrisch	$-\frac{\ln(-1+x)}{6}-\frac{\ln(-3+x)}{2}+\frac{\ln(x-4)}{6}+\frac{\ln(-2+x)}{2}$	26

input
```
int(1/(x-4)/(-3+x)/(-2+x)/(-1+x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(-1+x)-1/2*ln(-3+x)+1/6*ln(x-4)+1/2*ln(-2+x)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(x-1)+\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x-3)+\frac{1}{6}\log(x-4)$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

output
```
-1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \frac{\log(x-4)}{6}-\frac{\log(x-3)}{2}+\frac{\log(x-2)}{2}-\frac{\log(x-1)}{6}$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
log(x - 4)/6 - log(x - 3)/2 + log(x - 2)/2 - log(x - 1)/6
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log{(x-1)} + \frac{1}{2}\log{(x-2)} - \frac{1}{2}\log{(x-3)} + \frac{1}{6}\log{(x-4)}$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

output
```
-1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log{(|x-1|)} + \frac{1}{2}\log{(|x-2|)} - \frac{1}{2}\log{(|x-3|)} + \frac{1}{6}\log{(|x-4|)}$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

output
```
-1/6*log(abs(x - 1)) + 1/2*log(abs(x - 2)) - 1/2*log(abs(x - 3)) + 1/6*log
(abs(x - 4))
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.37

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \operatorname{atanh}(2\,x-5)-\frac{\operatorname{atanh}\left(\frac{2\,x}{3}-\frac{5}{3}\right)}{3}$$

input
```
int(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x)
```

output
```
atanh(2*x - 5) - atanh((2*x)/3 - 5/3)/3
```

3.101 $\int \frac{1+x^2}{(-1+x)^3}\,dx$

3.101.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{1}{(1-x)^2}+\frac{2}{1-x}+\log(1-x)$$

output
```
-1/(1-x)^2+2/(1-x)+ln(1-x)
```

3.101.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{1-2x}{(-1+x)^2}+\log(-1+x)$$

input
```
Integrate[(1 + x^2)/(-1 + x)^3,x]
```

output
```
(1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

3.101.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(x-1)^3}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{2}{(x-1)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{1-x}-\frac{1}{(1-x)^2}+\log(1-x)$$

input
```
Int[(1 + x^2)/(-1 + x)^3,x]
```

output
```
-(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

3.101.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.101.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
norman	$\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$	17
risch	$\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$	17
default	$\ln(-1+x)-\frac{2}{-1+x}-\frac{1}{(-1+x)^2}$	20
parallelrisch	$\frac{\ln(-1+x)x^2+1-2\ln(-1+x)x+\ln(-1+x)-2x}{(-1+x)^2}$	31
meijerg	$-\frac{x(2-x)}{2(1-x)^2}+\frac{x(-9x+6)}{6(1-x)^2}+\ln(1-x)$	38

input
```
int((x^2+1)/(-1+x)^3,x,method=_RETURNVERBOSE)
```

output
```
(1-2*x)/(-1+x)^2+ln(-1+x)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{(x^2-2\,x+1)\log(x-1)-2\,x+1}{x^2-2\,x+1}$$

input
```
integrate((x^2+1)/(-1+x)^3,x, algorithm="fricas")
```

output
```
((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{1-2x}{x^2-2x+1}+\log(x-1)$$

input
```
integrate((x**2+1)/(-1+x)**3,x)
```

output
```
(1 - 2*x)/(x**2 - 2*x + 1) + log(x - 1)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{2\,x-1}{x^2-2\,x+1} + \log\left(x-1\right)$$

input
```
integrate((x^2+1)/(-1+x)^3,x, algorithm="maxima")
```

output
```
-(2*x - 1)/(x^2 - 2*x + 1) + log(x - 1)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{2\,x-1}{(x-1)^2} + \log\left(|x-1|\right)$$

input
```
integrate((x^2+1)/(-1+x)^3,x, algorithm="giac")
```

output
```
-(2*x - 1)/(x - 1)^2 + log(abs(x - 1))
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \ln\left(x-1\right) - \frac{2\,x-1}{x^2-2\,x+1}$$

input
```
int((x^2 + 1)/(x - 1)^3,x)
```

output
```
log(x - 1) - (2*x - 1)/(x^2 - 2*x + 1)
```

3.102 $\int \frac{x^5}{(3+x)^2} \, dx$

3.102.1 Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{x^5}{(3+x)^2} \, dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x)$$

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

3.102.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(3+x)^2} \, dx = \frac{1}{4}\left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x}\right) + 405\log(3+x)$$

input
```
Integrate[x^5/(3 + x)^2,x]
```

output
```
(-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

3.102.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x+3)^2} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(x^3-6x^2+27x+\frac{405}{x+3}-\frac{243}{(x+3)^2}-108\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^4}{4}-2x^3+\frac{27x^2}{2}-108x+\frac{243}{x+3}+405\log(x+3)$$

input
```
Int[x^5/(3 + x)^2,x]
```

output
```
-108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

3.102.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.102.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$-108x+\frac{27x^{2}}{2}-2x^{3}+\frac{x^{4}}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$	33
risch	$-108x+\frac{27x^{2}}{2}-2x^{3}+\frac{x^{4}}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$	33
norman	$\frac{-\frac{135}{2}x^{2}+\frac{15}{2}x^{3}-\frac{5}{4}x^{4}+\frac{1}{4}x^{5}+1215}{3+x}+405\ln\left(3+x\right)$	36
meijerg	$-\frac{9x\left(-\frac{1}{27}x^{4}+\frac{5}{27}x^{3}-\frac{10}{9}x^{2}+10x+60\right)}{4\left(1+\frac{x}{3}\right)}+405\ln\left(1+\frac{x}{3}\right)$	40
parallelrisch	$\frac{x^{5}-5x^{4}+30x^{3}+1620\ln(3+x)x-270x^{2}+4860+4860\ln(3+x)}{12+4x}$	41

input
```
int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int\frac{x^{5}}{(3+x)^{2}}\,dx=\frac{x^{5}-5\,x^{4}+30\,x^{3}-270\,x^{2}+1620\,(x+3)\log\left(x+3\right)-1296\,x+972}{4\,(x+3)}$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

output
```
1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 9
72)/(x + 3)
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int\frac{x^{5}}{(3+x)^{2}}\,dx=\frac{x^{4}}{4}-2x^{3}+\frac{27x^{2}}{2}-108x+405\log\left(x+3\right)+\frac{243}{x+3}$$

input
```
integrate(x**5/(3+x)**2,x)
```

output
```
x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log\left(x+3\right)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(3+x)^2}\,dx = -\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log\left(|x+3|\right)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="giac")
```

output
```
-1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x
+ 3) + 405*log(abs(x + 3))
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = 405\,\ln\left(x+3\right) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

input
```
int(x^5/(x + 3)^2,x)
```

output
```
405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

3.103 $\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx$

3.103.1 Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx=-\frac{133}{8(3-x)^2}+\frac{407}{16(3-x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)$$

output
```
-133/8/(3-x)^2+407/16/(3-x)+313/64*ln(3-x)+7/64*ln(1+x)
```

3.103.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx=-\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)$$

input
```
Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

output
```
-133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (7*Log[1 +
 x])/64
```

3.103.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^3-2}{x^4-8x^3+18x^2-27}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{7}{64(x+1)}+\frac{313}{64(x-3)}+\frac{407}{16(x-3)^2}+\frac{133}{4(x-3)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

input
```
Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

output
```
-133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x
])/64
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.103.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

method	result	size
norman	$\frac{-\frac{407x}{16}+\frac{955}{16}}{(-3+x)^2}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	25
default	$\frac{7\ln(1+x)}{64}-\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313\ln(-3+x)}{64}$	28
risch	$\frac{-\frac{407x}{16}+\frac{955}{16}}{x^2-6x+9}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$	30
parallelrisch	$\frac{7\ln(1+x)x^2+313\ln(-3+x)x^2+3820-42\ln(1+x)x-1878\ln(-3+x)x+63\ln(1+x)+2817\ln(-3+x)-1628x}{64x^2-384x+576}$	62

input
```
int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x,method=_RETURNVERBOSE)
```

output
```
(-407/16*x+955/16)/(-3+x)^2+313/64*ln(-3+x)+7/64*ln(1+x)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx$$
$$=\frac{7\,(x^2-6\,x+9)\log(x+1)+313\,(x^2-6\,x+9)\log(x-3)-1628\,x+3820}{64\,(x^2-6\,x+9)}$$

input
```
integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="fricas")
```

output
```
1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x - 3) - 1628
*x + 3820)/(x^2 - 6*x + 9)
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{955-407x}{16x^2-96x+144} + \frac{313\log(x-3)}{64} + \frac{7\log(x+1)}{64}$$

input
```
integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
```

output
```
(955 - 407*x)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log(x + 1)/64
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{407\,x-955}{16\,(x^2-6\,x+9)} + \frac{7}{64}\log(x+1) + \frac{313}{64}\log(x-3)$$

input
```
integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="maxima")
```

output
```
-1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*log(x - 3)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{407\,x-955}{16\,(x-3)^2} + \frac{7}{64}\log(|x+1|) + \frac{313}{64}\log(|x-3|)$$

input
```
integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="giac")
```

output
```
-1/16*(407*x - 955)/(x - 3)^2 + 7/64*log(abs(x + 1)) + 313/64*log(abs(x -
3))
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{7\,\ln\left(x+1\right)}{64}+\frac{313\,\ln\left(x-3\right)}{64}-\frac{\frac{407\,x}{16}-\frac{955}{16}}{x^2-6\,x+9}$$

input
```
int((5*x^3 - 2)/(18*x^2 - 8*x^3 + x^4 - 27),x)
```

output
```
(7*log(x + 1))/64 + (313*log(x - 3))/64 - ((407*x)/16 - 955/16)/(x^2 - 6*x
 + 9)
```

3.104 $\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx$

3.104.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx = \frac{99}{3+x}+\frac{181}{4+x}+264\log(3+x)-263\log(4+x)$$

output
```
99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

3.104.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx = \frac{99}{3+x}+\frac{181}{4+x}+264\log(3+x)-263\log(4+x)$$

input
```
Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

output
```
99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

3.104.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-6x^2+3x-9}{(x+3)^2(x+4)^2}dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(-\frac{263}{x+4}-\frac{181}{(x+4)^2}+\frac{264}{x+3}-\frac{99}{(x+3)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

input
```
Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

output
```
99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.104.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{99}{3+x}+\frac{181}{4+x}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	28
norman	$\frac{280x+939}{(4+x)(3+x)}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	30
risch	$\frac{280x+939}{(4+x)(3+x)}+264\ln\left(3+x\right)-263\ln\left(4+x\right)$	30
parallelrisch	$\frac{264\ln(3+x)x^{2}-263\ln(4+x)x^{2}+939+1848\ln(3+x)x-1841\ln(4+x)x+3168\ln(3+x)-3156\ln(4+x)+280x}{(3+x)(4+x)}$	61

input
```
int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x,method=_RETURNVERBOSE)
```

output
```
99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = -\frac{263\,(x^2+7\,x+12)\log\left(x+4\right)-264\,(x^2+7\,x+12)\log\left(x+3\right)-280\,x-939}{x^2+7\,x+12}$$

input
```
integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="fricas")
```

output
```
-(263*(x^2 + 7*x + 12)*log(x + 4) - 264*(x^2 + 7*x + 12)*log(x + 3) - 280*
x - 939)/(x^2 + 7*x + 12)
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{280x+939}{x^2+7x+12}+264\log\left(x+3\right)-263\log\left(x+4\right)$$

input
```
integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)
```

output
```
(280*x + 939)/(x**2 + 7*x + 12) + 264*log(x + 3) - 263*log(x + 4)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{280\,x+939}{x^2+7\,x+12} - 263\,\log\left(x+4\right) + 264\,\log\left(x+3\right)$$

input
```
integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="maxima")
```

output
```
(280*x + 939)/(x^2 + 7*x + 12) - 263*log(x + 4) + 264*log(x + 3)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{181}{x+4} - \frac{99}{\frac{1}{x+4}-1} + \log\left(|x+4|\right) + 264\,\log\left(\left|-\frac{1}{x+4}+1\right|\right)$$

input
```
integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="giac")
```

output
```
181/(x + 4) - 99/(1/(x + 4) - 1) + log(abs(x + 4)) + 264*log(abs(-1/(x + 4
) + 1))
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = 264\,\ln\left(x+3\right) - 263\,\ln\left(x+4\right) + \frac{280\,x+939}{x^2+7\,x+12}$$

input
```
int((3*x - 6*x^2 + x^3 - 9)/((x + 3)^2*(x + 4)^2),x)
```

output
```
264*log(x + 3) - 263*log(x + 4) + (280*x + 939)/(7*x + x^2 + 12)
```

3.105 $\int \frac{2+x^2+x^3}{x(-1+x^2)^2} dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx=\frac{3+x}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(1+x)$$

output
```
1/2*(3+x)/(-x^2+1)-3/4*ln(1-x)+2*ln(x)-5/4*ln(1+x)
```

3.105.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx=\frac{1}{4}\left(-\frac{2}{-1+x}-\frac{4}{-1+x^2}+\log(1-x)+8\log(x)-\log(1+x)-4\log\left(1-x^2\right)\right)$$

input
```
Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

output
```
(-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x] - 4*Log[1
 - x^2])/4
```

3.105.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2336, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+2}{x\left(x^2-1\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{1}{2}\int \frac{4-x}{x\left(1-x^2\right)}dx+\frac{x+3}{2\left(1-x^2\right)}$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{2}\int \left(\frac{4}{x}-\frac{5}{2(x+1)}-\frac{3}{2(x-1)}\right)dx+\frac{x+3}{2\left(1-x^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x+3}{2\left(1-x^2\right)}+\frac{1}{2}\left(-\frac{3}{2}\log(1-x)+4\log(x)-\frac{5}{2}\log(x+1)\right)$$

input
```
Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

output
```
(3 + x)/(2*(1 - x^2)) + ((-3*Log[1 - x])/2 + 4*Log[x] - (5*Log[1 + x])/2)/
2
```

3.105.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.105.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$	31
risch	$\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$	31
default	$-\frac{1}{-1+x}-\frac{3\ln(-1+x)}{4}+2\ln(x)+\frac{1}{2x+2}-\frac{5\ln(1+x)}{4}$	32
parallelrisch	$\frac{8x^2\ln(x)-3\ln(-1+x)x^2-5\ln(1+x)x^2-6-8\ln(x)+3\ln(-1+x)+5\ln(1+x)-2x}{4x^2-4}$	56
meijerg	$\frac{i\left(-\frac{ix}{-x^2+1}+i\operatorname{arctanh}(x)\right)}{2}+\frac{3x^2}{-2x^2+2}+1+2\ln(x)+i\pi-\ln(-x^2+1)$	71

input
```
int((x^3+x^2+2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x-3/2)/(x^2-1)+2*ln(x)-3/4*ln(-1+x)-5/4*ln(1+x)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{2+x^2+x^3}{x(-1+x^2)^2}\,dx = -\frac{5\,(x^2-1)\log(x+1)+3\,(x^2-1)\log(x-1)-8\,(x^2-1)\log(x)+2\,x+6}{4\,(x^2-1)}$$

input
```
integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="fricas")
```

output
```
-1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 - 1)*log(x)
 + 2*x + 6)/(x^2 - 1)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = \frac{-x-3}{2x^2-2} + 2\log\left(x\right) - \frac{3\log\left(x-1\right)}{4} - \frac{5\log\left(x+1\right)}{4}$$

input
```
integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
```

output
```
(-x - 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/4
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = -\frac{x+3}{2\left(x^2-1\right)} - \frac{5}{4}\log\left(x+1\right) - \frac{3}{4}\log\left(x-1\right) + 2\log\left(x\right)$$

input
```
integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(x)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = -\frac{x+3}{2\left(x+1\right)\left(x-1\right)} - \frac{5}{4}\log\left(\left|x+1\right|\right) - \frac{3}{4}\log\left(\left|x-1\right|\right) + 2\log\left(\left|x\right|\right)$$

input
```
integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="giac")
```

output
```
-1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*log(abs(x + 1)) - 3/4*log(abs(x - 1))
 + 2*log(abs(x))
```

3.105.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = 2\,\ln\left(x\right)-\frac{5\,\ln\left(x+1\right)}{4}-\frac{3\,\ln\left(x-1\right)}{4}-\frac{\frac{x}{2}+\frac{3}{2}}{x^2-1}$$

input
```
int((x^2 + x^3 + 2)/(x*(x^2 - 1)^2),x)
```

output
```
2*log(x) - (5*log(x + 1))/4 - (3*log(x - 1))/4 - (x/2 + 3/2)/(x^2 - 1)
```

3.106 $\int \frac{1}{x^3-x^4-x^5+x^6}\,dx$

3.106.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx=\frac{1}{2(1-x)}-\frac{1}{2x^2}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(1+x)$$

output
```
1/2/(1-x)-1/2/x^2-1/x-7/4*ln(1-x)+2*ln(x)-1/4*ln(1+x)
```

3.106.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx=\frac{1}{4}\left(-\frac{2}{-1+x}-\frac{2}{x^2}-\frac{4}{x}-7\log(1-x)+8\log(x)-\log(1+x)\right)$$

input
```
Integrate[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

output
```
(-2/(-1 + x) - 2/x^2 - 4/x - 7*Log[1 - x] + 8*Log[x] - Log[1 + x])/4
```

3.106.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2026, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-x^5-x^4+x^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^3 \left(x^3-x^2-x+1\right)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-\frac{1}{4(x+1)}-\frac{7}{4(x-1)}+\frac{1}{2(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

input
```
Int[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

output
```
1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 +
 x]/4
```

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.106.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{1}{2(-1+x)}-\frac{7\ln(-1+x)}{4}-\frac{1}{2x^2}-\frac{1}{x}+2\ln(x)-\frac{\ln(1+x)}{4}$	35
norman	$\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln(x)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	37
risch	$\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln(x)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$	37
parallelrisch	$\frac{8x^3\ln(x)-7\ln(-1+x)x^3-\ln(1+x)x^3+2-8x^2\ln(x)+7\ln(-1+x)x^2+\ln(1+x)x^2-6x^2+2x}{4x^2(-1+x)}$	70

input
```
int(1/(x^6-x^5-x^4+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+x)-7/4*ln(-1+x)-1/2/x^2-1/x+2*ln(x)-1/4*ln(1+x)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = -\frac{6\,x^2+(x^3-x^2)\log(x+1)+7\,(x^3-x^2)\log(x-1)-8\,(x^3-x^2)\log(x)-2\,x-2}{4\,(x^3-x^2)}$$

input
```
integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="fricas")
```

output
```
-1/4*(6*x^2 + (x^3 - x^2)*log(x + 1) + 7*(x^3 - x^2)*log(x - 1) - 8*(x^3 -
 x^2)*log(x) - 2*x - 2)/(x^3 - x^2)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = 2\log(x) - \frac{7\log(x-1)}{4} - \frac{\log(x+1)}{4} + \frac{-3x^2+x+1}{2x^3-2x^2}$$

input
```
integrate(1/(x**6-x**5-x**4+x**3),x)
```

output
```
2*log(x) - 7*log(x - 1)/4 - log(x + 1)/4 + (-3*x**2 + x + 1)/(2*x**3 - 2*x
**2)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = -\frac{3\,x^2-x-1}{2\,(x^3-x^2)} - \frac{1}{4}\,\log(x+1) - \frac{7}{4}\,\log(x-1) + 2\,\log(x)$$

input
```
integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="maxima")
```

output
```
-1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*log(x + 1) - 7/4*log(x - 1) + 2*log
(x)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = -\frac{3\,x^2-x-1}{2\,(x-1)x^2} - \frac{1}{4}\,\log(|x+1|) - \frac{7}{4}\,\log(|x-1|) + 2\,\log(|x|)$$

input
```
integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="giac")
```

output
```
-1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*log(abs(x + 1)) - 7/4*log(abs(x -
 1)) + 2*log(abs(x))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6} \, dx = 2 \ln(x) - \frac{\ln(x+1)}{4} - \frac{7 \ln(x-1)}{4} - \frac{-\frac{3x^2}{2} + \frac{x}{2} + \frac{1}{2}}{x^2 - x^3}$$

input
```
int(1/(x^3 - x^4 - x^5 + x^6),x)
```

output
```
2*log(x) - log(x + 1)/4 - (7*log(x - 1))/4 - (x/2 - (3*x^2)/2 + 1/2)/(x^2
- x^3)
```

3.107 $\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx$

3.107.1 Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
x+1/2*x^2-arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

3.107.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

output
```
x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

3.107.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^3-x^2+x-1} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-x-1}{x^2+1}+x+\frac{1}{x-1}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)$$

input
```
Int[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

output
```
x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.107.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$x+\frac{x^{2}}{2}+\ln\left(-1+x\right)-\frac{\ln\left(x^{2}+1\right)}{2}-\arctan\left(x\right)$	24
risch	$x+\frac{x^{2}}{2}+\ln\left(-1+x\right)-\frac{\ln\left(x^{2}+1\right)}{2}-\arctan\left(x\right)$	24
parallelrisch	$\frac{x^{2}}{2}+x+\ln\left(-1+x\right)-\frac{\ln\left(x-i\right)}{2}+\frac{i\ln\left(x-i\right)}{2}-\frac{\ln\left(x+i\right)}{2}-\frac{i\ln\left(x+i\right)}{2}$	42

input
```
int((x^4+1)/(x^3-x^2+x-1),x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+ln(-1+x)-1/2*ln(x^2+1)-arctan(x)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{1}{2}\,x^{2}+x-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)+\log\left(x-1\right)$$

input
```
integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="fricas")
```

output
```
1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{x^{2}}{2}+x+\log\left(x-1\right)-\frac{\log\left(x^{2}+1\right)}{2}-\operatorname{atan}\left(x\right)$$

input
```
integrate((x**4+1)/(x**3-x**2+x-1),x)
```

output
```
x**2/2 + x + log(x - 1) - log(x**2 + 1)/2 - atan(x)
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(x-1\right)$$

input
```
integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="maxima")
```

output
```
1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x-1|\right)$$

input
```
integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="giac")
```

output
```
1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x - 1))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x + \ln\left(x-1\right) + \frac{x^2}{2} + \ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x^4 + 1)/(x - x^2 + x^3 - 1),x)
```

output
```
x + log(x - 1) - log(x - 1i)*(1/2 - 1i/2) - log(x + 1i)*(1/2 + 1i/2) + x^2
/2
```

3.108 $\int \frac{1}{x(1+x)(1+x^2)}\,dx$

3.108.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx=-\frac{\arctan(x)}{2}+\log(x)-\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

output
```
-1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

3.108.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx=-\frac{\arctan(x)}{2}+\log(x)-\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[1/(x*(1 + x)*(1 + x^2)),x]
```

output
```
-1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

3.108.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)\left(x^2+1\right)}dx$$

$$\Big\downarrow \ 615$$

$$\int \left(\frac{-x-1}{2\left(x^2+1\right)}+\frac{1}{x}-\frac{1}{2(x+1)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)$$

input
```
Int[1/(x*(1 + x)*(1 + x^2)),x]
```

output
```
-1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

3.108.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.108.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	22
risch	$-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	22
parallelrisch	$\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x-i)}{4}+\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}-\frac{i\ln(x+i)}{4}$	40

input
```
int(1/x/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = -\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x+1)+\log(x)$$

input
```
integrate(1/x/(1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = \log(x)-\frac{\log(x+1)}{2}-\frac{\log(x^2+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/x/(1+x)/(x**2+1),x)
```

output
```
log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(x+1) + \log(x)$$

input
```
integrate(1/x/(1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(|x+1|) + \log(|x|)$$

input
```
integrate(1/x/(1+x)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1)) + log(abs(x))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = \ln(x) - \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x*(x^2 + 1)*(x + 1)),x)
```

output
```
log(x) - log(x - 1i)*(1/4 - 1i/4) - log(x + 1i)*(1/4 + 1i/4) - log(x + 1)/
2
```

3.109 $\int \frac{x^2}{-2+x^2+x^4} dx$

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{x^2}{-2+x^2+x^4} dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{\operatorname{arctanh}(x)}{3}$$

output
```
-1/3*arctanh(x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{-2+x^2+x^4} dx = \frac{1}{6}\left(2\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + \log(1-x) - \log(1+x)\right)$$

input
```
Integrate[x^2/(-2 + x^2 + x^4),x]
```

output
```
(2*Sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6
```

3.109.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1450, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+x^2-2}\,dx$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{3}\int \frac{1}{x^2-1}dx+\frac{2}{3}\int \frac{1}{x^2+2}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\int \frac{1}{x^2-1}dx+\frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{\text{arctanh}(x)}{3}$$

input
```
Int[x^2/(-2 + x^2 + x^4),x]
```

output
```
(Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

3.109.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

3.109.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\ln(-1+x)}{6}+\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}-\frac{\ln(1+x)}{6}$	26
risch	$\frac{\ln(-1+x)}{6}+\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}-\frac{\ln(1+x)}{6}$	26

input
```
int(x^2/(x^4+x^2-2),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(-1+x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)-1/6*ln(1+x)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{1}{6}\,\log\left(x+1\right) + \frac{1}{6}\,\log\left(x-1\right)$$

input
```
integrate(x^2/(x^4+x^2-2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

input
```
integrate(x**2/(x**4+x**2-2),x)
```

output
```
log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

input
```
integrate(x^2/(x^4+x^2-2),x, algorithm="maxima")
```

output
```
1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(|x+1|) + \frac{1}{6}\log(|x-1|)$$

input
```
integrate(x^2/(x^4+x^2-2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x -
1))
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{-2+x^2+x^4}\,dx=\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}-\frac{\mathrm{atanh}(x)}{3}$$

input
```
int(x^2/(x^2 + x^4 - 2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x)/2))/3 - atanh(x)/3
```

3.110 $\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} dx$

3.110.1 Optimal result

Integrand size = 33, antiderivative size = 41

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)$$

output
```
1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)$$

input
```
Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

output
```
(1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
 x^2])/3
```

3.110.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2028, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+4x^2+6x}{x^4+2x^3+3x^2+4x+2}dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x(x^2+4x+6)}{x^4+2x^3+3x^2+4x+2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{4(x+2)}{3(x^2+2)}-\frac{1}{3(x+1)}-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)$$

input
```
Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

output
```
(1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
 x^2])/3
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.110.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33
risch	$\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33

input
```
int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+2(x+1)\log\left(x^2+2\right)-(x+1)\log(x+1)+3}{3(x+1)}$$

input
```
integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2 + 2) - (x
 + 1)*log(x + 1) + 3)/(x + 1)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = -\frac{\log{(x+1)}}{3}+\frac{2\log{(x^2+2)}}{3}+\frac{4\sqrt{2}\operatorname{atan}{\left(\frac{\sqrt{2}x}{2}\right)}}{3}+\frac{1}{x+1}$$

input
```
integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
```

output
```
-log(x + 1)/3 + 2*log(x**2 + 2)/3 + 4*sqrt(2)*atan(sqrt(2)*x/2)/3 + 1/(x +
 1)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(x+1\right)$$

input
```
integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="maxima")
```

output
```
4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log
(x + 1)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(|x+1|\right)$$

input
```
integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="giac")
```

output
```
4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log
(abs(x + 1))
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{x+1} - \frac{\ln(x+1)}{3} - \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right) + \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)$$

input
```
int((6*x + 4*x^2 + x^3)/(4*x + 3*x^2 + 2*x^3 + x^4 + 2),x)
```

output
```
1/(x + 1) - log(x + 1)/3 - log(x - 2^(1/2)*1i)*((2^(1/2)*2i)/3 - 2/3) + lo
g(x + 2^(1/2)*1i)*((2^(1/2)*2i)/3 + 2/3)
```

3.111 $\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx$

3.111.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{2}{5(1+2x)}+\frac{\arctan(x)}{50}-\frac{1}{2}\log(1+x)+\frac{16}{25}\log(1+2x)-\frac{7}{100}\log\left(1+x^2\right)$$

output
```
2/5/(1+2*x)+1/50*arctan(x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x^2+1)
```

3.111.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{1}{100}\left(\frac{40}{1+2x}+2\arctan(x)-50\log(1+x)+64\log(1+2x)-7\log\left(1+x^2\right)\right)$$

input
```
Integrate[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

output
```
(40/(1 + 2*x) + 2*ArcTan[x] - 50*Log[1 + x] + 64*Log[1 + 2*x] - 7*Log[1 +
x^2])/100
```

3.111.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)(2x+1)^2(x^2+1)}dx$$

$$\Big\downarrow \text{2348}$$

$$\int \left(\frac{1-7x}{50(x^2+1)}-\frac{1}{2(x+1)}+\frac{32}{25(2x+1)}-\frac{4}{5(2x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(x)}{50}-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)$$

input
```
Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

output
```
2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*
Log[1 + x^2])/100
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.111.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result
risch	$\frac{1}{5x+\frac{5}{2}}-\frac{\ln(1+x)}{2}-\frac{7\ln(x^2+1)}{100}+\frac{\arctan(x)}{50}+\frac{16\ln(1+2x)}{25}$
default	$\frac{2}{5(1+2x)}+\frac{\arctan(x)}{50}-\frac{\ln(1+x)}{2}+\frac{16\ln(1+2x)}{25}-\frac{7\ln(x^2+1)}{100}$
parallelrisch	$-\frac{2i\ln(x-i)x-2i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)+100\ln(1+x)x-128\ln\left(x+\frac{1}{2}\right)x+14\ln(x-i)x+14\ln(x+i)x-40+50\ln(1+x)-}{100(1+2x)}$

input
```
int(x/(1+x)/(1+2*x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5/(x+1/2)-1/2*ln(1+x)-7/100*ln(x^2+1)+1/50*arctan(x)+16/25*ln(1+2*x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx$$
$$=\frac{2\,(2\,x+1)\arctan(x)-7\,(2\,x+1)\log(x^2+1)+64\,(2\,x+1)\log(2\,x+1)-50\,(2\,x+1)\log(x+1)+40}{100\,(2\,x+1)}$$

input
```
integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/100*(2*(2*x + 1)*arctan(x) - 7*(2*x + 1)*log(x^2 + 1) + 64*(2*x + 1)*log
(2*x + 1) - 50*(2*x + 1)*log(x + 1) + 40)/(2*x + 1)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx=\frac{16\log\left(x+\frac{1}{2}\right)}{25}-\frac{\log(x+1)}{2}-\frac{7\log(x^2+1)}{100}+\frac{\operatorname{atan}(x)}{50}+\frac{2}{10x+5}$$

input
```
integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

output
```
16*log(x + 1/2)/25 - log(x + 1)/2 - 7*log(x**2 + 1)/100 + atan(x)/50 + 2/(
10*x + 5)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{2}{5\,(2\,x+1)} + \frac{1}{50}\,\arctan\left(x\right) - \frac{7}{100}\,\log\left(x^2+1\right) + \frac{16}{25}\,\log\left(2\,x+1\right) - \frac{1}{2}\,\log\left(x+1\right)$$

input
```
integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(2*x + 1) -
 1/2*log(x + 1)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{2}{5\,(2\,x+1)} + \frac{1}{50}\,\arctan\left(-\frac{5}{2\,(2\,x+1)} + \frac{1}{2}\right) - \frac{7}{100}\,\log\left(-\frac{2}{2\,x+1} + \frac{5}{(2\,x+1)^2} + 1\right) - \frac{1}{2}\,\log\left(\left|-\frac{1}{2\,x+1} - 1\right|\right)$$

input
```
integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="giac")
```

output
```
2/5/(2*x + 1) + 1/50*arctan(-5/2/(2*x + 1) + 1/2) - 7/100*log(-2/(2*x + 1)
 + 5/(2*x + 1)^2 + 1) - 1/2*log(abs(-1/(2*x + 1) - 1))
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)}\,dx = \frac{16\ln\left(x+\frac{1}{2}\right)}{25} - \frac{\ln(x+1)}{2} + \frac{1}{5\left(x+\frac{1}{2}\right)} + \ln(x-\mathrm{i})\left(-\frac{7}{100}-\frac{1}{100}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{7}{100}+\frac{1}{100}\mathrm{i}\right)$$

input
```
int(x/((2*x + 1)^2*(x^2 + 1)*(x + 1)),x)
```

output
```
(16*log(x + 1/2))/25 - log(x + 1)/2 - log(x - 1i)*(7/100 + 1i/100) - log(x
 + 1i)*(7/100 - 1i/100) + 1/(5*(x + 1/2))
```

3.112 $\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx$

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx=-\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\arctan(x)-\frac{3}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)$$

output
```
-1/2/(1-x)^2+5/2/(1-x)-arctan(x)-3/2*ln(1-x)+3/4*ln(x^2+1)
```

3.112.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx=\frac{1}{4}\left(-\frac{2}{(-1+x)^2}-\frac{10}{-1+x}-4\arctan(x)-6\log(-1+x)+3\log\left(1+x^2\right)\right)$$

input
```
Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
(-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Log[1 + x^2
])/4
```

3.112.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+x-2}{(x-1)^3(x^2+1)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{3x-2}{2(x^2+1)}-\frac{3}{2(x-1)}+\frac{5}{2(x-1)^2}+\frac{1}{(x-1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)+\frac{3}{4}\log(x^2+1)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)$$

input
```
Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
-1/2*1/(1 - x)^2 + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1
 + x^2])/4
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.112.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{5x}{2}+2}{(-1+x)^2}+\frac{3\ln(x^2+1)}{4}-\arctan(x)-\frac{3\ln(-1+x)}{2}$
default	$-\frac{1}{2(-1+x)^2}-\frac{5}{2(-1+x)}-\frac{3\ln(-1+x)}{2}+\frac{3\ln(x^2+1)}{4}-\arctan(x)$
parallelrisch	$-\frac{-2i\ln(x-i)x^2+2i\ln(x+i)+6\ln(-1+x)x^2+2i\ln(x+i)x^2-3x^2\ln(x-i)+4i\ln(x-i)x-3\ln(x+i)x^2-3-12\ln(-1+x)x-2i\ln(}{4(-1+x)^2}$

input
```
int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(-5/2*x+2)/(-1+x)^2+3/4*ln(x^2+1)-arctan(x)-3/2*ln(-1+x)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{4(x^2-2x+1)\arctan(x)-3(x^2-2x+1)\log(x^2+1)+6(x^2-2x+1)\log(x-1)+10x-8}{4(x^2-2x+1)}$$

input
```
integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

output
```
-1/4*(4*(x^2 - 2*x + 1)*arctan(x) - 3*(x^2 - 2*x + 1)*log(x^2 + 1) + 6*(x^
2 - 2*x + 1)*log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = \frac{4-5x}{2x^2-4x+2}-\frac{3\log(x-1)}{2}+\frac{3\log(x^2+1)}{4}-\operatorname{atan}(x)$$

input
```
integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
```

output
```
(4 - 5*x)/(2*x**2 - 4*x + 2) - 3*log(x - 1)/2 + 3*log(x**2 + 1)/4 - atan(x
)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{5\,x-4}{2\,(x^2-2\,x+1)} - \arctan\left(x\right) + \frac{3}{4}\,\log\left(x^2+1\right) - \frac{3}{2}\,\log\left(x-1\right)$$

input
```
integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*(5*x - 4)/(x^2 - 2*x + 1) - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(x
- 1)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{5\,x-4}{2\,(x-1)^2} - \arctan\left(x\right) + \frac{3}{4}\,\log\left(x^2+1\right) - \frac{3}{2}\,\log\left(|x-1|\right)$$

input
```
integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(abs(x -
1))
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{3\,\ln\left(x-1\right)}{2} - \frac{\frac{5\,x}{2}-2}{x^2-2\,x+1} + \ln\left(x-\mathrm{i}\right)\,\left(\frac{3}{4}+\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(\frac{3}{4}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + 3*x^2 - 2)/((x^2 + 1)*(x - 1)^3),x)
```

output
```
log(x - 1i)*(3/4 + 1i/2) - (3*log(x - 1))/2 + log(x + 1i)*(3/4 - 1i/2) - (
(5*x)/2 - 2)/(x^2 - 2*x + 1)
```

3.112. $\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx$

3.113 $\int \frac{1}{1+x^2+x^4}\,dx$

3.113.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{1+x^2+x^4}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)$$

output
```
-1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1
/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-\sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

input
```
Integrate[(1 + x^2 + x^4)^(-1),x]
```

output
```
(I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]
*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

3.113.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[(1 + x^2 + x^4)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x + x^2]/2)/2 + (ArcTan[(1 +
 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

3.113.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	54
risch	$\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(4x^2+4x+4)}{4}-\frac{\ln(4x^2-4x+4)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	60

input
```
int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1
/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**4+x**2+1),x)
```

output
```
-log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{4}\,\log\left(x^2+x+1\right)-\frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{4}\,\log\left(x^2+x+1\right)-\frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atanh}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^2 + x^4 + 1),x)
```

output
```
atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

3.114 $\int \frac{3+2x^3}{-9x+x^5} dx$

3.114.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{3+2x^3}{-9x+x^5} dx = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{12}\log\left(9-x^4\right)$$

output
```
-1/3*ln(x)+1/12*ln(-x^4+9)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*arctanh(1
/3*x*3^(1/2))*3^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{3+2x^3}{-9x+x^5} dx = \frac{1}{12}\left(4\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) - 4\log(x) + 2\sqrt{3}\log\left(3-\sqrt{3}x\right) - 2\sqrt{3}\log\left(3+\sqrt{3}x\right) + \log\left(9-x^4\right)\right)$$

input
```
Integrate[(3 + 2*x^3)/(-9*x + x^5),x]
```

output
```
(4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[3]*x] - 2
*Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

3.114.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2026, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3}{x^5-9x} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{2x^3+3}{x(x^4-9)}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{3}{(x^4-9)x}+\frac{2x^2}{x^4-9}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}$$

input
```
Int[(3 + 2*x^3)/(-9*x + x^5),x]
```

output
```
ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9
- x^4]/12
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

3.114.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(x^2+3)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln(x)}{3}+\frac{\ln(x^2-3)}{12}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	46
risch	$\frac{\ln(x^2+3)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(x-\sqrt{3}\right)}{12}+\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\right)}{6}+\frac{\ln\left(x+\sqrt{3}\right)}{12}-\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\right)}{6}-\frac{\ln(x)}{3}$	68
meijerg	$-\frac{\ln(x)}{3}+\frac{\ln(3)}{6}-\frac{i\pi}{12}+\frac{\ln\left(1-\frac{x^4}{9}\right)}{12}+\frac{x^3\sqrt{3}\left(\ln\left(1-\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)-\ln\left(1+\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)+2\arctan\left(\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)\right)}{6(x^4)^{\frac{3}{4}}}$	79

input
```
int((2*x^3+3)/(x^5-9*x),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(x^2+3)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*ln(x)+1/12*ln(x^2-3)-
1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{3+2x^3}{-9x+x^5}\,dx=\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log\left(x\right)$$

input
```
integrate((2*x^3+3)/(x^5-9*x),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3
)/(x^2 - 3)) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

3.114.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.38

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = -\frac{\log(x)}{3} + \left(\frac{1}{12} + \frac{\sqrt{3}i}{6}\right)\log\left(x + \frac{17413}{11544} - \frac{943\sqrt{3}i}{5772} + \frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^3}{481} + \frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^2}{481} - \frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^4}{481}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{6}\right)\log\left(x + \frac{17413}{11544} - \frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^4}{481} + \frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^2}{481} + \frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^3}{481} + \frac{943\sqrt{3}i}{5772}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}}{6}\right)\log\left(x - \frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^4}{481} + \frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^3}{481} + \frac{943\sqrt{3}}{5772} + \frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^2}{481} + \frac{17413}{11544}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}}{6}\right)\log\left(x - \frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^4}{481} - \frac{943\sqrt{3}}{5772} + \frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^3}{481} + \frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^2}{481} + \frac{17413}{11544}\right)$$

input
```
integrate((2*x**3+3)/(x**5-9*x),x)
```

output
```
-log(x)/3 + (1/12 + sqrt(3)*I/6)*log(x + 17413/11544 - 943*sqrt(3)*I/5772
+ 1368*(1/12 + sqrt(3)*I/6)**3/481 + 4158*(1/12 + sqrt(3)*I/6)**2/481 - 10
8000*(1/12 + sqrt(3)*I/6)**4/481) + (1/12 - sqrt(3)*I/6)*log(x + 17413/115
44 - 108000*(1/12 - sqrt(3)*I/6)**4/481 + 4158*(1/12 - sqrt(3)*I/6)**2/481
 + 1368*(1/12 - sqrt(3)*I/6)**3/481 + 943*sqrt(3)*I/5772) + (1/12 - sqrt(3
)/6)*log(x - 108000*(1/12 - sqrt(3)/6)**4/481 + 1368*(1/12 - sqrt(3)/6)**3
/481 + 943*sqrt(3)/5772 + 4158*(1/12 - sqrt(3)/6)**2/481 + 17413/11544) +
(1/12 + sqrt(3)/6)*log(x - 108000*(1/12 + sqrt(3)/6)**4/481 - 943*sqrt(3)/
5772 + 1368*(1/12 + sqrt(3)/6)**3/481 + 4158*(1/12 + sqrt(3)/6)**2/481 + 1
7413/11544)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + \frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log\left(x\right)$$

input
```
integrate((2*x^3+3)/(x^5-9*x),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqr
t(3))) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}\right|}{\left|2\,x+2\,\sqrt{3}\right|}\right) + \frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(\left|x^2-3\right|\right)-\frac{1}{3}\log\left(\left|x\right|\right)$$

input
```
integrate((2*x^3+3)/(x^5-9*x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/a
bs(2*x + 2*sqrt(3))) + 1/12*log(x^2 + 3) + 1/12*log(abs(x^2 - 3)) - 1/3*lo
g(abs(x))
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \ln\left(x-\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{12}\right)-\ln\left(x+\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{12}\right)-\frac{\ln(x)}{3} - \ln\left(x-\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(-(2*x^3 + 3)/(9*x - x^5),x)
```

output
```
log(x - 3^(1/2))*(3^(1/2)/6 + 1/12) - log(x + 3^(1/2))*(3^(1/2)/6 - 1/12)
- log(x)/3 - log(x - 3^(1/2)*1i)*((3^(1/2)*1i)/6 - 1/12) + log(x + 3^(1/2)
*1i)*((3^(1/2)*1i)/6 + 1/12)
```

3.115 $\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$

3.115.1 Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{3}{16}\arctan\left(1-\frac{x}{2}\right) - \frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)$$

output
```
-83/4/(4-x)^2+41/4/(4-x)+3/16*arctan(-1+1/2*x)-45/16*ln(4-x)+45/32*ln(x^2-
4*x+8)
```

3.115.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = \frac{1}{32}\left(-\frac{664}{(-4+x)^2}-\frac{328}{-4+x}+6\arctan\left(\frac{1}{2}(-2+x)\right) - 90\log(-4+x)+45\log\left(8-4x+x^2\right)\right)$$

input
```
Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

output
```
(-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-4 + x] +
45*Log[8 - 4*x + x^2])/32
```

3.115.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^3+8x-20}{(x-4)^3 (x^2-4x+8)} \, dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{3(15x-28)}{16 (x^2-4x+8)}-\frac{45}{16(x-4)}+\frac{41}{4(x-4)^2}+\frac{83}{2(x-4)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3}{16}\arctan\left(1-\frac{x}{2}\right)+\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)$$

input
```
Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

output
```
-83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 -
x])/16 + (45*Log[8 - 4*x + x^2])/32
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.115.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{41x}{4}+\frac{81}{4}}{(x-4)^2}-\frac{45\ln(x-4)}{16}+\frac{45\ln(x^2-4x+8)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}$
default	$\frac{45\ln(x^2-4x+8)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}-\frac{83}{4(x-4)^2}-\frac{41}{4(x-4)}-\frac{45\ln(x-4)}{16}$
parallelrisch	$-\frac{-96i\ln(x-2+2i)-48i\ln(x-2-2i)x+96i\ln(x-2-2i)+48i\ln(x-2+2i)x+180\ln(x-4)x^2-90\ln(x-2-2i)x^2-90\ln(x-2+2i}{}$

input
```
int((5*x^3+8*x-20)/(x-4)^3/(x^2-4*x+8),x,method=_RETURNVERBOSE)
```

output
```
(-41/4*x+81/4)/(x-4)^2-45/16*ln(x-4)+45/32*ln(x^2-4*x+8)+3/16*arctan(-1+1/
2*x)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\,(8-4x+x^2)}\,dx$$
$$=\frac{6\,(x^2-8\,x+16)\arctan\left(\frac{1}{2}\,x-1\right)+45\,(x^2-8\,x+16)\log\left(x^2-4\,x+8\right)-90\,(x^2-8\,x+16)\log\left(x-4\right)}{32\,(x^2-8\,x+16)}$$

input
```
integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="fricas")
```

output
```
1/32*(6*(x^2 - 8*x + 16)*arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*log(x^2 -
 4*x + 8) - 90*(x^2 - 8*x + 16)*log(x - 4) - 328*x + 648)/(x^2 - 8*x + 16)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = \frac{81-41x}{4x^2-32x+64} - \frac{45\log(x-4)}{16} + \frac{45\log(x^2-4x+8)}{32} + \frac{3\,\mathrm{atan}\left(\frac{x}{2}-1\right)}{16}$$

input
```
integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
```

output
```
(81 - 41*x)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**2 - 4*x +
8)/32 + 3*atan(x/2 - 1)/16
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{41\,x-81}{4\,(x^2-8\,x+16)} + \frac{3}{16}\arctan\left(\frac{1}{2}\,x-1\right) + \frac{45}{32}\log\left(x^2-4\,x+8\right) - \frac{45}{16}\log\left(x-4\right)$$

input
```
integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="maxima")
```

output
```
-1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2
 - 4*x + 8) - 45/16*log(x - 4)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{41\,x-81}{4\,(x-4)^2} + \frac{3}{16}\arctan\left(\frac{1}{2}\,x-1\right) + \frac{45}{32}\log\left(x^2-4\,x+8\right) - \frac{45}{16}\log\left(|x-4|\right)$$

input
```
integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="giac")
```

output
```
-1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2 - 4*x
+ 8) - 45/16*log(abs(x - 4))
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\,(8-4x+x^2)}\,dx = -\frac{45\,\ln(x-4)}{16} - \frac{\frac{41\,x}{4}-\frac{81}{4}}{x^2-8\,x+16} + \ln(x-2-2\mathrm{i})\,\left(\frac{45}{32}-\frac{3}{32}\mathrm{i}\right) + \ln(x-2+2\mathrm{i})\,\left(\frac{45}{32}+\frac{3}{32}\mathrm{i}\right)$$

input
```
int((8*x + 5*x^3 - 20)/((x - 4)^3*(x^2 - 4*x + 8)),x)
```

output
```
log(x - (2 + 2i))*(45/32 - 3i/32) - (45*log(x - 4))/16 + log(x - (2 - 2i))
*(45/32 + 3i/32) - ((41*x)/4 - 81/4)/(x^2 - 8*x + 16)
```

3.116 $\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx$

3.116.1 Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ar
ctan(1/3*x*3^(1/2))*3^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\left(-\arctan\left(\frac{x}{2}\right)+2\arctan(x)-3\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+2\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right)\right)$$

input
```
Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
(-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqrt[3]*ArcT
an[x/Sqrt[3]])/12
```

3.116.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{1}{2(x^2+2)}+\frac{1}{2(x^2+3)}-\frac{1}{6(x^2+4)}+\frac{1}{6(x^2+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
-1/12*ArcTan[x/2] + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x
/Sqrt[3]]/(2*Sqrt[3])
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan\left(x\right)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan\left(x\right)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36

input
```
int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ar
ctan(1/3*x*3^(1/2))*3^(1/2)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan\left(x\right)$$

input
```
integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/
12*arctan(1/2*x) + 1/6*arctan(x)
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12}+\frac{\operatorname{atan}\left(x\right)}{6}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

input
```
integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

output
```
-atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)*atan(sqr
t(3)*x/3)/6
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/
12*arctan(1/2*x) + 1/6*arctan(x)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/
12*arctan(1/2*x) + 1/6*arctan(x)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\operatorname{atan}(x)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)}{6}$$

input
```
int(1/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

output
```
atan(x)/6 - atan(x/2)/12 - (2^(1/2)*atan((2^(1/2)*x)/2))/4 + (3^(1/2)*atan
((3^(1/2)*x)/3))/6
```

3.117 $\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx$

3.117.1 Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\log\left(1+x^2\right)-\frac{1}{4}\log\left(2+x^2\right)+\frac{1}{4}\log\left(3+x^2\right)-\frac{1}{12}\log\left(4+x^2\right)$$

output
```
1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

3.117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\log\left(1+x^2\right)-\frac{1}{4}\log\left(2+x^2\right)+\frac{1}{4}\log\left(3+x^2\right)-\frac{1}{12}\log\left(4+x^2\right)$$

input
```
Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

3.117.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7245, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}dx$$

$$\Big\downarrow \text{7245}$$

$$\frac{1}{2}\int \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}dx^2$$

$$\Big\downarrow \text{198}$$

$$\frac{1}{2}\int \left(-\frac{1}{2(x^2+2)}+\frac{1}{2(x^2+3)}-\frac{1}{6(x^2+4)}+\frac{1}{6(x^2+1)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{6}\log(x^2+1)-\frac{1}{2}\log(x^2+2)+\frac{1}{2}\log(x^2+3)-\frac{1}{6}\log(x^2+4)\right)$$

input
```
Int[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
(Log[1 + x^2]/6 - Log[2 + x^2]/2 + Log[3 + x^2]/2 - Log[4 + x^2]/6)/2
```

3.117.3.1 Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7245
```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((e_.) + (f_.)*(w_))^(p_.)*((a_.) + (b_
.)*(y_))^(m_.)*((g_.) + (h_.)*(z_))^(q_.), x_Symbol] :> With[{r = Derivativ
eDivides[y, u, x]}, Simp[r   Subst[Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*
(g + h*x)^q, x], x, y], x] /;  !FalseQ[r]] /; FreeQ[{a, b, c, d, e, f, g, h
, m, n, p, q}, x] && EqQ[v, y] && EqQ[w, y] && EqQ[z, y]
```

3.117.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$	34
norman	$\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$	34
risch	$\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$	34
parallelrisch	$\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$	34

input
```
int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

output
```
-1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 +
1)
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\log(x^2+1)}{12} - \frac{\log(x^2+2)}{4} + \frac{\log(x^2+3)}{4} - \frac{\log(x^2+4)}{12}$$

input
```
integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

output
```
log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 + 4)/12
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right) + \frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right) + \frac{1}{12}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

output
```
-1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 +
1)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right) + \frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right) + \frac{1}{12}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

output
```
-1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 +
1)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{3072}{5\,(1280\,x^2+3072)}-\frac{1}{5}\right)}{2} - \frac{\operatorname{atanh}\left(\frac{1024}{405\,\left(\frac{640\,x^2}{243}+\frac{1024}{243}\right)}-\frac{3}{5}\right)}{6}$$

input
```
int(x/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

output
```
atanh(3072/(5*(1280*x^2 + 3072)) - 1/5)/2 - atanh(1024/(405*((640*x^2)/243
 + 1024/243)) - 3/5)/6
```

3.118 $\int \frac{1}{a^3+x^3}\,dx$

3.118.1 Optimal result

Integrand size = 9, antiderivative size = 56

$$\int \frac{1}{a^3+x^3}\,dx=-\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}+\frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}$$

output
```
1/3*ln(a+x)/a^2-1/6*ln(a^2-a*x+x^2)/a^2-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/
a^2*3^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)+2\log(a+x)-\log\left(a^2-ax+x^2\right)}{6a^2}$$

input
```
Integrate[(a^3 + x^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^2 - a*x +
 x^2])/(6*a^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^3+x^3}\,dx$$

$$\Big\downarrow \ 750$$

$$\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\int \frac{1}{a+x}dx}{3a^2}$$

$$\Big\downarrow \ 16$$

$$\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx+3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{-\frac{1}{2}\log\left(a^2-ax+x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}$$

input
```
Int[(a^3 + x^3)^(-1),x]
```

output
```
Log[a + x]/(3*a^2) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/Sqrt[3]]) - Log[a^2 -
 a*x + x^2]/2)/(3*a^2)
```

3.118.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.118.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{-\frac{\ln\left(a^{2}-ax+x^{2}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^{2}}+\frac{\ln(a+x)}{3a^{2}}$	51
risch	$-\frac{\ln\left(4a^{2}-4ax+4x^{2}\right)}{6a^{2}}+\frac{\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^{2}}+\frac{\ln(a+x)}{3a^{2}}$	56

input
```
int(1/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a))+1/3*
ln(a+x)/a^2
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{1}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log\left(a^2-ax+x^2\right)+2\log\left(a+x\right)}{6a^2}$$

input
```
integrate(1/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a^2 - a*x + x^2) + 2
*log(a + x))/a^2
```

3.118.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{a^3+x^3}\,dx=\frac{\frac{\log(a+x)}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)+x\right)+\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)+x\right)}{a^2}$$

input
```
integrate(1/(a**3+x**3),x)
```

output
```
(log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(3*a*(-1/6 - sqrt(3)*I/6) + x) + (
-1/6 + sqrt(3)*I/6)*log(3*a*(-1/6 + sqrt(3)*I/6) + x))/a**2
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log\left(a^2-ax+x^2\right)}{6a^2} + \frac{\log\left(a+x\right)}{3a^2}$$

input
```
integrate(1/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2
)/a^2 + 1/3*log(a + x)/a^2
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log\left(a^2-ax+x^2\right)}{6a^2} + \frac{\log\left(|a+x|\right)}{3a^2}$$

input
```
integrate(1/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2
)/a^2 + 1/3*log(abs(a + x))/a^2
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\ln(a+x)}{3\,a^2} + \frac{\ln\left(x+\frac{a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2} - \frac{\ln\left(x-\frac{a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2}$$

input
```
int(1/(a^3 + x^3),x)
```

output
```
log(a + x)/(3*a^2) + (log(x + (a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6
*a^2) - (log(x - (a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^2)
```

3.119 $\int \frac{x}{a^3+x^3} \, dx$

3.119.1 Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{x}{a^3+x^3} \, dx = -\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a} - \frac{\log(a+x)}{3a} + \frac{\log\left(a^2-ax+x^2\right)}{6a}$$

output
```
-1/3*ln(a+x)/a+1/6*ln(a^2-a*x+x^2)/a-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/a*3
^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{a^3+x^3} \, dx = \frac{2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2\log(a+x) + \log\left(a^2-ax+x^2\right)}{6a}$$

input
```
Integrate[x/(a^3 + x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
 x^2])/(6*a)
```

3.119.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^3+x^3}\,dx$$

$$\Big\downarrow \text{821}$$

$$\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\int \frac{1}{a+x}dx}{3a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{1}{2}\log\left(a^2-ax+x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}$$

input
```
Int[x/(a^3 + x^3),x]
```

output
```
-1/3*Log[a + x]/a + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/Sqrt[3]]) + Log[a^2 -
a*x + x^2]/2)/(3*a)
```

3.119.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\ln(a+x)}{3a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^2a^2-a\_Z+1)}\_R\ln\left(a^2\_R-a+x\right)\right)}{3}$	43
default	$\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a}-\frac{\ln(a+x)}{3a}$	51

input
```
int(x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(a+x)/a+1/3*sum(_R*ln(_R*a^2-a+x),_R=RootOf(_Z^2*a^2-_Z*a+1))
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{x}{a^3+x^3}\,dx = \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+\log\left(a^2-ax+x^2\right)-2\log\left(a+x\right)}{6a}$$

input
```
integrate(x/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + log(a^2 - a*x + x^2) - 2
*log(a + x))/a
```

3.119.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{x}{a^3+x^3}\,dx = \frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a}$$

input
```
integrate(x/(a**3+x**3),x)
```

output
```
(-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) +
 (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a} + \frac{\log\left(a^2-ax+x^2\right)}{6\,a} - \frac{\log\left(a+x\right)}{3\,a}$$

input
```
integrate(x/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/
a - 1/3*log(a + x)/a
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a} + \frac{\log\left(a^2-ax+x^2\right)}{6\,a} - \frac{\log\left(|a+x|\right)}{3\,a}$$

input
```
integrate(x/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/
a - 1/3*log(abs(a + x))/a
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{x}{a^3+x^3}\,dx = -\frac{\ln(a+x)}{3\,a} - \frac{\ln\left(x+\frac{a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a} + \frac{\ln\left(x+\frac{a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a}$$

input
```
int(x/(a^3 + x^3),x)
```

output
```
(log(x + (a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a) - (log(x + (a*(
3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a) - log(a + x)/(3*a)
```

3.120 $\int \frac{x^2}{a^3+x^3} dx$

3.120.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^2}{a^3+x^3} dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

output
```
1/3*ln(a^3+x^3)
```

3.120.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^3+x^3} dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

input
```
Integrate[x^2/(a^3 + x^3),x]
```

output
```
Log[a^3 + x^3]/3
```

3.120.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^3+x^3} \, dx$$
$$\Big\downarrow \; 792$$
$$\frac{1}{3} \log \left(a^3+x^3\right)$$

input
```
Int[x^2/(a^3 + x^3),x]
```

output
```
Log[a^3 + x^3]/3
```

3.120.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

3.120.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln\left(a^3+x^3\right)}{3}$	11
default	$\frac{\ln\left(a^3+x^3\right)}{3}$	11
risch	$\frac{\ln\left(a^3+x^3\right)}{3}$	11
norman	$\frac{\ln(a+x)}{3}+\frac{\ln\left(a^2-ax+x^2\right)}{3}$	22
parallelrisch	$\frac{\ln(a+x)}{3}+\frac{\ln\left(a^2-ax+x^2\right)}{3}$	22

input
```
int(x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a^3+x^3)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

input
```
integrate(x^2/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/3*log(a^3 + x^3)
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{\log\left(a^3+x^3\right)}{3}$$

input
```
integrate(x**2/(a**3+x**3),x)
```

output
```
log(a**3 + x**3)/3
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

input
```
integrate(x^2/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*log(a^3 + x^3)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{a^3 + x^3}\, dx = \frac{1}{3} \log \left(\left|a^3 + x^3\right|\right)$$

input
```
integrate(x^2/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*log(abs(a^3 + x^3))
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3 + x^3}\, dx = \frac{\ln \left(a^3 + x^3\right)}{3}$$

input
```
int(x^2/(a^3 + x^3),x)
```

output
```
log(a^3 + x^3)/3
```

3.121 $\int \frac{1}{x(a^3+x^3)}\,dx$

3.121.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x\,(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

output
```
ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

3.121.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

input
```
Integrate[1/(x*(a^3 + x^3)),x]
```

output
```
Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

3.121.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^3+x^3\right)}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{1}{x^3\left(a^3+x^3\right)}dx^3$$

$$\Big\downarrow \ 47$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3}dx^3}{a^3}-\frac{\int \frac{1}{a^3+x^3}dx^3}{a^3}\right)$$

$$\Big\downarrow \ 14$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a^3}-\frac{\int \frac{1}{a^3+x^3}dx^3}{a^3}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a^3}-\frac{\log\left(a^3+x^3\right)}{a^3}\right)$$

input
```
Int[1/(x*(a^3 + x^3)),x]
```

output
```
(Log[x^3]/a^3 - Log[a^3 + x^3]/a^3)/3
```

3.121.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.121.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\ln(x)}{a^3}-\frac{\ln(a^3+x^3)}{3a^3}$	21
parallelrisch	$\frac{3\ln(x)-\ln(a+x)-\ln(a^2-ax+x^2)}{3a^3}$	31
default	$\frac{\ln(x)}{a^3}-\frac{\ln(a^2-ax+x^2)}{3a^3}-\frac{\ln(a+x)}{3a^3}$	34
norman	$\frac{\ln(x)}{a^3}-\frac{\ln(a^2-ax+x^2)}{3a^3}-\frac{\ln(a+x)}{3a^3}$	34

input
```
int(1/x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(a^3+x^3)}\,dx = -\frac{\log\left(a^3+x^3\right)-3\,\log\left(x\right)}{3\,a^3}$$

input
```
integrate(1/x/(a^3+x^3),x, algorithm="fricas")
```

output
```
-1/3*(log(a^3 + x^3) - 3*log(x))/a^3
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\,(a^3+x^3)}\,dx = \frac{\log\left(x\right)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

input
```
integrate(1/x/(a**3+x**3),x)
```

output
```
log(x)/a**3 - log(a**3 + x**3)/(3*a**3)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\,(a^3+x^3)}\,dx = -\frac{\log\left(a^3+x^3\right)}{3\,a^3}+\frac{\log\left(x^3\right)}{3\,a^3}$$

input
```
integrate(1/x/(a^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*log(a^3 + x^3)/a^3 + 1/3*log(x^3)/a^3
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^3+x^3\right)}\,dx = -\frac{\log\left(\left|a^3+x^3\right|\right)}{3\,a^3}+\frac{\log\left(\left|x\right|\right)}{a^3}$$

input
```
integrate(1/x/(a^3+x^3),x, algorithm="giac")
```

output
```
-1/3*log(abs(a^3 + x^3))/a^3 + log(abs(x))/a^3
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a^3+x^3\right)}\,dx = -\frac{\ln\left(a^3+x^3\right)-3\,\ln\left(x\right)}{3\,a^3}$$

input
```
int(1/(x*(a^3 + x^3)),x)
```

output
```
-(log(a^3 + x^3) - 3*log(x))/(3*a^3)
```

3.122 $\int \frac{1}{x^2(a^3+x^3)}\,dx$

3.122.1 Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{1}{a^3x}+\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}+\frac{\log(a+x)}{3a^4}-\frac{\log(a^2-ax+x^2)}{6a^4}$$

output
```
-1/a^3/x+1/3*ln(a+x)/a^4-1/6*ln(a^2-a*x+x^2)/a^4+1/3*arctan(1/3*(a-2*x)/a*
3^(1/2))/a^4*3^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{6a+2\sqrt{3}x\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)-2x\log(a+x)+x\log(a^2-ax+x^2)}{6a^4x}$$

input
```
Integrate[1/(x^2*(a^3 + x^3)),x]
```

output
```
-1/6*(6*a + 2*Sqrt[3]*x*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*x*Log[a + x] +
x*Log[a^2 - a*x + x^2])/(a^4*x)
```

3.122.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {847, 821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^3+x^3)} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{\int \frac{x}{a^3+x^3}dx}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{821}$$

$$-\frac{\frac{\int \frac{a+x}{a^2-x a+x^2}dx}{3 a}-\frac{\int \frac{1}{a+x}dx}{3 a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{\int \frac{a+x}{a^2-x a+x^2}dx}{3 a}-\frac{\log (a+x)}{3 a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{\frac{3}{2} a \int \frac{1}{a^2-x a+x^2}dx+\frac{1}{2} \int -\frac{a-2 x}{a^2-x a+x^2}dx}{3 a}-\frac{\log (a+x)}{3 a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{3}{2} a \int \frac{1}{a^2-x a+x^2}dx-\frac{1}{2} \int \frac{a-2 x}{a^2-x a+x^2}dx}{3 a}-\frac{\log (a+x)}{3 a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{3 \int \frac{1}{-\left(1-\frac{2 x}{a}\right)^2-3}d\left(1-\frac{2 x}{a}\right)-\frac{1}{2} \int \frac{a-2 x}{a^2-x a+x^2}dx}{3 a}-\frac{\log (a+x)}{3 a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{-\frac{1}{2} \int \frac{a-2 x}{a^2-x a+x^2}dx-\sqrt{3} \arctan \left(\frac{1-\frac{2 x}{a}}{\sqrt{3}}\right)}{3 a}-\frac{\log (a+x)}{3 a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{a^3x}-\frac{\frac{\frac{1}{2}\log(a^2-ax+x^2)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}$$

input
```
Int[1/(x^2*(a^3 + x^3)),x]
```

output
```
-(1/(a^3*x)) - (-1/3*Log[a + x]/a + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/Sqrt[3
]]) + Log[a^2 - a*x + x^2]/2)/(3*a))/a^3
```

3.122.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.122.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{a^3 x}+\frac{-\frac{\ln\left(a^2-ax+x^2\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^4}+\frac{\ln(a+x)}{3a^4}$	60
risch	$-\frac{1}{a^3 x}-\frac{\ln(4a^2-4ax+4x^2)}{6a^4}+\frac{\arctan\left(\frac{(a-2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3a^4}+\frac{\ln(-a-x)}{3a^4}$	66

input
```
int(1/x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/x+1/3/a^4*(-1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)
/a))+1/3*ln(a+x)/a^4
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx = -\frac{2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+x\log\left(a^2-ax+x^2\right)-2x\log(a+x)+6a}{6a^4x}$$

input
```
integrate(1/x^2/(a^3+x^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + x*log(a^2 - a*x + x^2
) - 2*x*log(a + x) + 6*a)/(a^4*x)
```

3.122.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2 (a^3 + x^3)} \, dx = -\frac{1}{a^3 x} + \frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a^4}$$

input
```
integrate(1/x**2/(a**3+x**3),x)
```

output
```
-1/(a**3*x) + (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(9*a*(-1/6 - sqrt(3)
*I/6)**2 + x) + (-1/6 + sqrt(3)*I/6)*log(9*a*(-1/6 + sqrt(3)*I/6)**2 + x))
/a**4
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 (a^3 + x^3)} \, dx = -\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log(a^2 - ax + x^2)}{6a^4} + \frac{\log(a+x)}{3a^4} - \frac{1}{a^3 x}$$

input
```
integrate(1/x^2/(a^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^
2)/a^4 + 1/3*log(a + x)/a^4 - 1/(a^3*x)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a^3 + x^3)} \, dx = -\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log(a^2 - ax + x^2)}{6a^4} + \frac{\log(|a+x|)}{3a^4} - \frac{1}{a^3 x}$$

input
```
integrate(1/x^2/(a^3+x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^
2)/a^4 + 1/3*log(abs(a + x))/a^4 - 1/(a^3*x)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2\,(a^3+x^3)}\,dx = \frac{\ln(a+x)}{3\,a^4} - \frac{1}{a^3\,x} + \frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2 a^4}{4} + x\,a^3\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^4} - \frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2 a^4}{4} + x\,a^3\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^4}$$

input
```
int(1/(x^2*(a^3 + x^3)),x)
```

output
```
log(a + x)/(3*a^4) - 1/(a^3*x) + (log(a^3*x + (a^4*(3^(1/2)*1i - 1)^2)/4)*
(3^(1/2)*1i - 1))/(6*a^4) - (log(a^3*x + (a^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1
/2)*1i + 1))/(6*a^4)
```

3.123 $\int \frac{1}{x^3(a^3+x^3)}\,dx$

3.123.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{1}{2a^3x^2}+\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

output
```
-1/2/a^3/x^2-1/3*ln(a+x)/a^5+1/6*ln(a^2-a*x+x^2)/a^5+1/3*arctan(1/3*(a-2*x
)/a*3^(1/2))/a^5*3^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{1}{2a^3x^2}-\frac{\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}$$

input
```
Integrate[1/(x^3*(a^3 + x^3)),x]
```

output
```
-1/2*1/(a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a +
x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

3.123.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {847, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^3+x^3)} \, dx$$

$$\Big\downarrow \; 847$$

$$-\frac{\int \frac{1}{a^3+x^3}dx}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 750$$

$$-\frac{\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\int \frac{1}{a+x}dx}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 16$$

$$-\frac{\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 1142$$

$$-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx+3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{1}{2a^3x^2}-\frac{\frac{-\frac{1}{2}\log(a^2-ax+x^2)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}$$

input
```
Int[1/(x^3*(a^3 + x^3)),x]
```

output
```
-1/2*1/(a^3*x^2) - (Log[a + x]/(3*a^2) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/S
qrt[3]]) - Log[a^2 - a*x + x^2]/2)/(3*a^2))/a^3
```

3.123.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.123.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^5}-\frac{1}{2a^3x^2}-\frac{\ln(a+x)}{3a^5}$	60
risch	$-\frac{1}{2a^3x^2}-\frac{\ln(a+x)}{3a^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^2-a^5\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{15}-3\right)x-a^6\_R\right)\right)}{3}$	62

input
```
int(1/x^3/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^5*(1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a))-1/2/a
^3/x^2-1/3*ln(a+x)/a^5
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx$$
$$=-\frac{2\sqrt{3}x^2\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-x^2\log\left(a^2-ax+x^2\right)+2x^2\log\left(a+x\right)+3a^2}{6a^5x^2}$$

input
```
integrate(1/x^3/(a^3+x^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x^2*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - x^2*log(a^2 - a*x +
 x^2) + 2*x^2*log(a + x) + 3*a^2)/(a^5*x^2)
```

3.123.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{1}{2a^3x^2} + \frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^5}$$

input
```
integrate(1/x**3/(a**3+x**3),x)
```

output
```
-1/(2*a**3*x**2) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(-3*a*(1/6 - sq
rt(3)*I/6) + x) + (1/6 + sqrt(3)*I/6)*log(-3*a*(1/6 + sqrt(3)*I/6) + x))/a
**5
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5} - \frac{\log(a+x)}{3a^5} - \frac{1}{2a^3x^2}$$

input
```
integrate(1/x^3/(a^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^
2)/a^5 - 1/3*log(a + x)/a^5 - 1/2/(a^3*x^2)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5} - \frac{\log(|a+x|)}{3a^5} - \frac{1}{2a^3x^2}$$

input
```
integrate(1/x^3/(a^3+x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^
2)/a^5 - 1/3*log(abs(a + x))/a^5 - 1/2/(a^3*x^2)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx = -\frac{\ln\left(a+x\right)}{3\,a^5}-\frac{1}{2\,a^3\,x^2}-\frac{\ln\left(\frac{3\,a^7\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,a^6\,x\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^5}+\frac{\ln\left(\frac{3\,a^7\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}-3\,a^6\,x\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^5}$$

input
```
int(1/(x^3*(a^3 + x^3)),x)
```

output
```
(log((3*a^7*(3^(1/2)*1i + 1))/2 - 3*a^6*x)*(3^(1/2)*1i + 1))/(6*a^5) - 1/(
2*a^3*x^2) - (log((3*a^7*(3^(1/2)*1i - 1))/2 + 3*a^6*x)*(3^(1/2)*1i - 1))/
(6*a^5) - log(a + x)/(3*a^5)
```

3.124 $\int \frac{1}{x^4(a^3+x^3)}\,dx$

3.124.1 Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=-\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

output
```
-1/3/a^3/x^3-ln(x)/a^6+1/3*ln(a^3+x^3)/a^6
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=-\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

input
```
Integrate[1/(x^4*(a^3 + x^3)),x]
```

output
```
-1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

3.124.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^3+x^3)} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3} \int \frac{1}{x^6 (a^3+x^3)}dx^3$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{3} \int \left(-\frac{1}{a^6 x^3}+\frac{1}{a^3 x^6}+\frac{1}{a^6 (a^3+x^3)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{\log (x^3)}{a^6}-\frac{1}{a^3 x^3}+\frac{\log (a^3+x^3)}{a^6}\right)$$

input
```
Int[1/(x^4*(a^3 + x^3)),x]
```

output
```
(-(1/(a^3*x^3)) - Log[x^3]/a^6 + Log[a^3 + x^3]/a^6)/3
```

3.124.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.124.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln(-a^3-x^3)}{3a^6}$	34
default	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln(a^2-ax+x^2)}{3a^6}+\frac{\ln(a+x)}{3a^6}$	43
norman	$-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln(a^2-ax+x^2)}{3a^6}+\frac{\ln(a+x)}{3a^6}$	43
parallelrisch	$-\frac{3x^3\ln(x)-\ln(a+x)x^3-\ln(a^2-ax+x^2)x^3+a^3}{3a^6x^3}$	46

input
```
int(1/x^4/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^3/x^3-ln(x)/a^6+1/3/a^6*ln(-a^3-x^3)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a^3+x^3)}\,dx=\frac{x^3\log(a^3+x^3)-3\,x^3\log(x)-a^3}{3\,a^6x^3}$$

input
```
integrate(1/x^4/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4(a^3+x^3)}\,dx=-\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log(a^3+x^3)}{3a^6}$$

input
```
integrate(1/x**4/(a**3+x**3),x)
```

output
```
-1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (a^3 + x^3)} \, dx = \frac{\log(a^3 + x^3)}{3\,a^6} - \frac{\log(x^3)}{3\,a^6} - \frac{1}{3\,a^3 x^3}$$

input
```
integrate(1/x^4/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (a^3 + x^3)} \, dx = \frac{\log(|a^3 + x^3|)}{3\,a^6} - \frac{\log(|x|)}{a^6} - \frac{a^3 - x^3}{3\,a^6 x^3}$$

input
```
integrate(1/x^4/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*log(abs(a^3 + x^3))/a^6 - log(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^6*x^3)
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a^3 + x^3)} \, dx = \frac{\ln(a^3 + x^3)}{3\,a^6} - \frac{\ln(x)}{a^6} - \frac{1}{3\,a^3\,x^3}$$

input
```
int(1/(x^4*(a^3 + x^3)),x)
```

output
```
log(a^3 + x^3)/(3*a^6) - log(x)/a^6 - 1/(3*a^3*x^3)
```

3.125 $\int \frac{1}{x^5(a^3+x^3)}\,dx$

3.125.1 Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx=-\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

output
```
-1/4/a^3/x^4+1/a^6/x-1/3*ln(a+x)/a^7+1/6*ln(a^2-a*x+x^2)/a^7-1/3*arctan(1/
3*(a-2*x)/a*3^(1/2))/a^7*3^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx=-\frac{1}{4a^3x^4}+\frac{1}{a^6x}+\frac{\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

input
```
Integrate[1/(x^5*(a^3 + x^3)),x]
```

output
```
-1/4*1/(a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7
) - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

3.125.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {847, 847, 821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a^3+x^3)} \, dx$$

$$\Big\downarrow \; 847$$

$$-\frac{\int \frac{1}{x^2(a^3+x^3)}dx}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 847$$

$$-\frac{-\frac{\int \frac{x}{a^3+x^3}dx}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 821$$

$$-\frac{-\frac{\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\int \frac{1}{a+x}dx}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 16$$

$$-\frac{-\frac{\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 1142$$

$$-\frac{-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 1082$$

$$-\frac{-\frac{\frac{3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \ 217$$

$$-\frac{-\frac{\frac{-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \ 1103$$

$$-\frac{1}{4a^3x^4}-\frac{-\frac{1}{a^3x}-\frac{\frac{\frac{1}{2}\log\left(a^2-ax+x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}}{a^3}$$

input
```
Int[1/(x^5*(a^3 + x^3)),x]
```

output
```
-1/4*1/(a^3*x^4) - (-(1/(a^3*x)) - (-1/3*Log[a + x]/a + (-(Sqrt[3]*ArcTan[
(1 - (2*x)/a)/Sqrt[3]]) + Log[a^2 - a*x + x^2]/2)/(3*a))/a^3)/a^3
```

3.125.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.125.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4a^3x^4}+\frac{1}{a^6x}+\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3a^7}-\frac{\ln(a+x)}{3a^7}$	66
risch	$\frac{\frac{x^3}{a^6}-\frac{1}{4a^3}}{x^4}-\frac{\ln(a+x)}{3a^7}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{21}-3\right)x+a^{15}\_R^2\right)\right)}{3}$	72

input
```
int(1/x^5/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^3/x^4+1/a^6/x+1/3/a^7*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(-a+2
*x)*3^(1/2)/a))-1/3*ln(a+x)/a^7
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx = \frac{4\sqrt{3}x^4\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+2x^4\log\left(a^2-ax+x^2\right)-4x^4\log\left(a+x\right)-3a^4+12ax^3}{12a^7x^4}$$

input
```
integrate(1/x^5/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 2*x^4*log(a^2 - a*x
 + x^2) - 4*x^4*log(a + x) - 3*a^4 + 12*a*x^3)/(a^7*x^4)
```

3.125.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx = \frac{-a^3+4x^3}{4a^6x^4} + \frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^7}$$

input
```
integrate(1/x**5/(a**3+x**3),x)
```

output
```
(-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(
9*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(
3)*I/6)**2 + x))/a**7
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5(a^3+x^3)}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7} - \frac{\log(a+x)}{3a^7} - \frac{a^3-4x^3}{4a^6x^4}$$

input
```
integrate(1/x^5/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2
)/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5(a^3+x^3)}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7} - \frac{\log(|a+x|)}{3a^7} - \frac{a^3-4x^3}{4a^6x^4}$$

input
```
integrate(1/x^5/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2
)/a^7 - 1/3*log(abs(a + x))/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5(a^3+x^3)}\,dx = -\frac{\frac{1}{4a^3}-\frac{x^3}{a^6}}{x^4} - \frac{\ln(a+x)}{3a^7} - \frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2 a^7}{4}+x\,a^6\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6a^7} + \frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2 a^7}{4}+x\,a^6\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6a^7}$$

input
```
int(1/(x^5*(a^3 + x^3)),x)
```

output
```
(log(a^6*x + (a^7*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^7) - log(a
 + x)/(3*a^7) - (log(a^6*x + (a^7*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))
/(6*a^7) - (1/(4*a^3) - x^3/a^6)/x^4
```

3.125. $\int \frac{1}{x^5(a^3+x^3)}\,dx$

3.126 $\int \frac{x^{-m}}{a^3+x^3}\,dx$

3.126.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

output
```
x^(1-m)*hypergeom([1, 1/3-1/3*m],[4/3-1/3*m],-x^3/a^3)/a^3/(1-m)
```

3.126.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=-\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{3}-\frac{m}{3},\frac{4}{3}-\frac{m}{3},-\frac{x^3}{a^3}\right)}{a^3(-1+m)}$$

input
```
Integrate[1/(x^m*(a^3 + x^3)),x]
```

output
```
-((x^(1 - m)*Hypergeometric2F1[1, 1/3 - m/3, 4/3 - m/3, -(x^3/a^3)])/(a^3*
(-1 + m)))
```

3.126.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-m}}{a^3+x^3} \, dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{1-m} \text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

input
```
Int[1/(x^m*(a^3 + x^3)),x]
```

output
```
(x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)])/(a^3*(1
 - m))
```

3.126.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

3.126.4 Maple [F]

$$\int \frac{x^{-m}}{a^3+x^3}dx$$

input
```
int(1/(x^m)/(a^3+x^3),x)
```

output
```
int(1/(x^m)/(a^3+x^3),x)
```

3.126.5 Fricas [F]

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\int \frac{1}{(a^3+x^3)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^3+x^3),x, algorithm="fricas")
```

output
```
integral(1/((a^3 + x^3)*x^m), x)
```

3.126.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=-\frac{mx^{1-m}\Phi\left(\frac{x^3e^{i\pi}}{a^3},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^3\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}+\frac{x^{1-m}\Phi\left(\frac{x^3e^{i\pi}}{a^3},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^3\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}$$

input
```
integrate(1/(x**m)/(a**3+x**3),x)
```

output
```
-m*x**(1 - m)*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3
- m/3)/(9*a**3*gamma(4/3 - m/3)) + x**(1 - m)*lerchphi(x**3*exp_polar(I*pi
)/a**3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*gamma(4/3 - m/3))
```

3.126.7 Maxima [F]

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\int \frac{1}{(a^3+x^3)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^3+x^3),x, algorithm="maxima")
```

output
```
integrate(1/((a^3 + x^3)*x^m), x)
```

3.126.8 Giac [F]

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{(a^3 + x^3)x^m} \, dx$$

input
```
integrate(1/(x^m)/(a^3+x^3),x, algorithm="giac")
```

output
```
integrate(1/((a^3 + x^3)*x^m), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{x^m (a^3 + x^3)} \, dx$$

input
```
int(1/(x^m*(a^3 + x^3)),x)
```

output
```
int(1/(x^m*(a^3 + x^3)), x)
```

3.127 $\int \frac{1}{a^4-x^4} \, dx$

3.127.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{a^4-x^4} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{2a^3} + \frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^3}$$

output
```
1/2*arctan(x/a)/a^3+1/2*arctanh(x/a)/a^3
```

3.127.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{a^4-x^4} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{2a^3} - \frac{\log(a-x)}{4a^3} + \frac{\log(a+x)}{4a^3}$$

input
```
Integrate[(a^4 - x^4)^(-1),x]
```

output
```
ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)
```

3.127.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^4-x^4}\,dx$$

$$\Big\downarrow \ 756$$

$$\frac{\int \frac{1}{a^2-x^2}dx}{2a^2}+\frac{\int \frac{1}{a^2+x^2}dx}{2a^2}$$

$$\Big\downarrow \ 216$$

$$\frac{\int \frac{1}{a^2-x^2}dx}{2a^2}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\text{arctanh}\left(\frac{x}{a}\right)}{2a^3}$$

input
```
Int[(a^4 - x^4)^(-1),x]
```

output
```
ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

3.127.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.127.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\ln(a-x)}{4a^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\ln(a+x)}{4a^3}$	33
parallelrisch	$-\frac{i\ln(-ia+x)-i\ln(ia+x)+\ln(-a+x)-\ln(a+x)}{4a^3}$	39
risch	$\frac{\ln(a+x)}{4a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^6\_Z^2+1\right)}\_R\ln\left(\_R\,a^4+x\right)\right)}{4}-\frac{\ln(-a+x)}{4a^3}$	47

input
```
int(1/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^3*ln(a-x)+1/2*arctan(x/a)/a^3+1/4*ln(a+x)/a^3
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{a^4-x^4}\,dx=\frac{2\,\arctan\left(\frac{x}{a}\right)+\log\left(a+x\right)-\log\left(-a+x\right)}{4\,a^3}$$

input
```
integrate(1/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/4*(2*arctan(x/a) + log(a + x) - log(-a + x))/a^3
```

3.127.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{a^4 - x^4}\,dx = -\frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} + \frac{i\log(-ia+x)}{4} - \frac{i\log(ia+x)}{4}}{a^3}$$

input
```
integrate(1/(a**4-x**4),x)
```

output
```
-(log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*log(I*a + x)/4)/a**
3
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\log(a+x)}{4\,a^3} - \frac{\log(-a+x)}{4\,a^3}$$

input
```
integrate(1/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/2*arctan(x/a)/a^3 + 1/4*log(a + x)/a^3 - 1/4*log(-a + x)/a^3
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\log(|a+x|)}{4\,a^3} - \frac{\log(|-a+x|)}{4\,a^3}$$

input
```
integrate(1/(a^4-x^4),x, algorithm="giac")
```

output
```
1/2*arctan(x/a)/a^3 + 1/4*log(abs(a + x))/a^3 - 1/4*log(abs(-a + x))/a^3
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x}{a}\right) + \operatorname{atanh}\left(\frac{x}{a}\right)}{2\,a^3}$$

input
```
int(1/(a^4 - x^4),x)
```

output
```
(atan(x/a) + atanh(x/a))/(2*a^3)
```

3.128 $\int \frac{x}{a^4-x^4} dx$

3.128.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{a^4-x^4} dx = \frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

output
```
1/2*arctanh(x^2/a^2)/a^2
```

3.128.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^4-x^4} dx = \frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Integrate[x/(a^4 - x^4),x]
```

output
```
ArcTanh[x^2/a^2]/(2*a^2)
```

3.128.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^4 - x^4} \, dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2} \int \frac{1}{a^4 - x^4} dx^2$$

$$\Big\downarrow \ 219$$

$$\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Int[x/(a^4 - x^4),x]
```

output
```
ArcTanh[x^2/a^2]/(2*a^2)
```

3.128.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.128.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

method	result	size
parallelrisch	$-\frac{\ln(-a+x)+\ln(a+x)-\ln(a^2+x^2)}{4a^2}$	27
default	$\frac{\ln(a^2+x^2)}{4a^2}-\frac{\ln(a^2-x^2)}{4a^2}$	30
risch	$-\frac{\ln(-a^2+x^2)}{4a^2}+\frac{\ln(a^2+x^2)}{4a^2}$	30
norman	$-\frac{\ln(a-x)}{4a^2}-\frac{\ln(a+x)}{4a^2}+\frac{\ln(a^2+x^2)}{4a^2}$	35

input
```
int(x/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(ln(-a+x)+ln(a+x)-ln(a^2+x^2))/a^2
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{x}{a^4-x^4}\,dx=\frac{\log(a^2+x^2)-\log(-a^2+x^2)}{4\,a^2}$$

input
```
integrate(x/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

3.128.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{x}{a^4-x^4}\,dx=-\frac{\frac{\log(-a^2+x^2)}{4}-\frac{\log(a^2+x^2)}{4}}{a^2}$$

input
```
integrate(x/(a**4-x**4),x)
```

output
```
-(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{a^4 - x^4}\,dx = \frac{\log(a^2 + x^2)}{4\,a^2} - \frac{\log(-a^2 + x^2)}{4\,a^2}$$

input
```
integrate(x/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{x}{a^4 - x^4}\,dx = \frac{\log(a^2 + x^2)}{4\,a^2} - \frac{\log(|-a^2 + x^2|)}{4\,a^2}$$

input
```
integrate(x/(a^4-x^4),x, algorithm="giac")
```

output
```
1/4*log(a^2 + x^2)/a^2 - 1/4*log(abs(-a^2 + x^2))/a^2
```

3.128.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 - x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

input
```
int(x/(a^4 - x^4),x)
```

output
```
atanh(x^2/a^2)/(2*a^2)
```

3.129 $\int \frac{1}{x(a^4-x^4)}\,dx$

3.129.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{x(a^4-x^4)}\,dx = \frac{\log(x)}{a^4} - \frac{\log(a^4-x^4)}{4a^4}$$

output
```
ln(x)/a^4-1/4*ln(a^4-x^4)/a^4
```

3.129.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a^4-x^4)}\,dx = \frac{\log(x)}{a^4} - \frac{\log(-a^4+x^4)}{4a^4}$$

input
```
Integrate[1/(x*(a^4 - x^4)),x]
```

output
```
Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)
```

3.129.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^4-x^4\right)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(a^4-x^4\right)}dx^4$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^4}dx^4}{a^4}+\frac{\int \frac{1}{a^4-x^4}dx^4}{a^4}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{a^4-x^4}dx^4}{a^4}+\frac{\log\left(x^4\right)}{a^4}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a^4}-\frac{\log\left(a^4-x^4\right)}{a^4}\right)$$

input
```
Int[1/(x*(a^4 - x^4)),x]
```

output
```
(Log[x^4]/a^4 - Log[a^4 - x^4]/a^4)/4
```

3.129.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, 
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.129.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\ln(x)}{a^4} - \frac{\ln(-a^4+x^4)}{4a^4}$	23
parallelrisch	$\frac{4\ln(x)-\ln(-a+x)-\ln(a+x)-\ln(a^2+x^2)}{4a^4}$	35
default	$-\frac{\ln(a-x)}{4a^4} - \frac{\ln(a^2+x^2)}{4a^4} + \frac{\ln(x)}{a^4} - \frac{\ln(a+x)}{4a^4}$	41
norman	$-\frac{\ln(a-x)}{4a^4} - \frac{\ln(a^2+x^2)}{4a^4} + \frac{\ln(x)}{a^4} - \frac{\ln(a+x)}{4a^4}$	41

input
```
int(1/x/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^4-1/4/a^4*ln(-a^4+x^4)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=-\frac{\log\left(-a^4+x^4\right)-4\,\log\left(x\right)}{4\,a^4}$$

input
```
integrate(1/x/(a^4-x^4),x, algorithm="fricas")
```

output
```
-1/4*(log(-a^4 + x^4) - 4*log(x))/a^4
```

3.129.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=\frac{\log\left(x\right)}{a^4}-\frac{\log\left(-a^4+x^4\right)}{4a^4}$$

input
```
integrate(1/x/(a**4-x**4),x)
```

output
```
log(x)/a**4 - log(-a**4 + x**4)/(4*a**4)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=-\frac{\log\left(-a^4+x^4\right)}{4\,a^4}+\frac{\log\left(x^4\right)}{4\,a^4}$$

input
```
integrate(1/x/(a^4-x^4),x, algorithm="maxima")
```

output
```
-1/4*log(-a^4 + x^4)/a^4 + 1/4*log(x^4)/a^4
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\,(a^4-x^4)}\,dx = \frac{\log\left(x^4\right)}{4\,a^4} - \frac{\log\left(\left|-a^4+x^4\right|\right)}{4\,a^4}$$

input
```
integrate(1/x/(a^4-x^4),x, algorithm="giac")
```

output
```
1/4*log(x^4)/a^4 - 1/4*log(abs(-a^4 + x^4))/a^4
```

3.129.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\,(a^4-x^4)}\,dx = -\frac{\ln\left(x^4-a^4\right)-4\,\ln\left(x\right)}{4\,a^4}$$

input
```
int(1/(x*(a^4 - x^4)),x)
```

output
```
-(log(x^4 - a^4) - 4*log(x))/(4*a^4)
```

3.130 $\int \frac{1}{x^2(a^4-x^4)}\,dx$

3.130.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx = -\frac{1}{a^4x}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^5}$$

output
```
-1/a^4/x-1/2*arctan(x/a)/a^5+1/2*arctanh(x/a)/a^5
```

3.130.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx = -\frac{1}{a^4x}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}-\frac{\log(a-x)}{4a^5}+\frac{\log(a+x)}{4a^5}$$

input
```
Integrate[1/(x^2*(a^4 - x^4)),x]
```

output
```
-(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a + x]/(4*a^
5)
```

3.130.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {847, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^4-x^4)} \, dx$$

$$\Big\downarrow \; 847$$

$$\frac{\int \frac{x^2}{a^4-x^4}dx}{a^4}-\frac{1}{a^4 x}$$

$$\Big\downarrow \; 827$$

$$\frac{\frac{1}{2}\int \frac{1}{a^2-x^2}dx-\frac{1}{2}\int \frac{1}{a^2+x^2}dx}{a^4}-\frac{1}{a^4 x}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{2}\int \frac{1}{a^2-x^2}dx-\frac{\arctan\left(\frac{x}{a}\right)}{2a}}{a^4}-\frac{1}{a^4 x}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{x}{a}\right)}{2a}-\frac{\arctan\left(\frac{x}{a}\right)}{2a}}{a^4}-\frac{1}{a^4 x}$$

input
```
Int[1/(x^2*(a^4 - x^4)),x]
```

output
```
-(1/(a^4*x)) + (-1/2*ArcTan[x/a]/a + ArcTanh[x/a]/(2*a))/a^4
```

3.130.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

3.130.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{1}{a^4 x}-\frac{\ln(a-x)}{4a^5}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}+\frac{\ln(a+x)}{4a^5}$	41
risch	$-\frac{1}{a^4 x}-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5}-\frac{\ln(-a+x)}{4a^5}+\frac{\ln(a+x)}{4a^5}$	41
parallelrisch	$-\frac{-i\ln(-ia+x)x+i\ln(ia+x)x+\ln(-a+x)x-\ln(a+x)x+4a}{4a^5 x}$	50

input
```
int(1/x^2/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/a^4/x-1/4/a^5*ln(a-x)-1/2*arctan(x/a)/a^5+1/4*ln(a+x)/a^5
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = -\frac{2\,x \arctan\left(\frac{x}{a}\right) - x \log(a + x) + x \log(-a + x) + 4\,a}{4\,a^5 x}$$

input
```
integrate(1/x^2/(a^4-x^4),x, algorithm="fricas")
```

output
```
-1/4*(2*x*arctan(x/a) - x*log(a + x) + x*log(-a + x) + 4*a)/(a^5*x)
```

3.130.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = -\frac{1}{a^4 x} - \frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} - \frac{i \log(-ia+x)}{4} + \frac{i \log(ia+x)}{4}}{a^5}$$

input
```
integrate(1/x**2/(a**4-x**4),x)
```

output
```
-1/(a**4*x) - (log(-a + x)/4 - log(a + x)/4 - I*log(-I*a + x)/4 + I*log(I*
a + x)/4)/a**5
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = -\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5} + \frac{\log(a + x)}{4\,a^5} - \frac{\log(-a + x)}{4\,a^5} - \frac{1}{a^4 x}$$

input
```
integrate(1/x^2/(a^4-x^4),x, algorithm="maxima")
```

output
```
-1/2*arctan(x/a)/a^5 + 1/4*log(a + x)/a^5 - 1/4*log(-a + x)/a^5 - 1/(a^4*x
)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = -\frac{\arctan\left(\frac{x}{a}\right)}{2 a^5} + \frac{\log(|a + x|)}{4 a^5} - \frac{\log(|-a + x|)}{4 a^5} - \frac{1}{a^4 x}$$

input
```
integrate(1/x^2/(a^4-x^4),x, algorithm="giac")
```

output
```
-1/2*arctan(x/a)/a^5 + 1/4*log(abs(a + x))/a^5 - 1/4*log(abs(-a + x))/a^5
- 1/(a^4*x)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = \frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2 a^5} - \frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2 a^5} - \frac{1}{a^4 x}$$

input
```
int(1/(x^2*(a^4 - x^4)),x)
```

output
```
atanh(x/a)/(2*a^5) - atan(x/a)/(2*a^5) - 1/(a^4*x)
```

3.131 $\int \frac{1}{x^3(a^4-x^4)}\,dx$

3.131.1 Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{1}{x^3(a^4-x^4)}\,dx = -\frac{1}{2a^4x^2}+\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^6}$$

output
```
-1/2/a^4/x^2+1/2*arctanh(x^2/a^2)/a^6
```

3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^3(a^4-x^4)}\,dx = -\frac{1}{2a^4x^2}-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}+\frac{\log(a^2+x^2)}{4a^6}$$

input
```
Integrate[1/(x^3*(a^4 - x^4)),x]
```

output
```
-1/2*1/(a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2
]/(4*a^6)
```

3.131.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {807, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^4 - x^4)} dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^4 (a^4 - x^4)} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{a^4 - x^4} dx^2}{a^4} - \frac{1}{a^4 x^2} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{\text{arctanh}\left(\frac{x^2}{a^2}\right)}{a^6} - \frac{1}{a^4 x^2} \right)$$

input
```
Int[1/(x^3*(a^4 - x^4)),x]
```

output
```
(-(1/(a^4*x^2)) + ArcTanh[x^2/a^2]/a^6)/2
```

3.131.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.131.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

method	result	size
risch	$-\frac{1}{2a^4x^2}+\frac{\ln(-a^2-x^2)}{4a^6}-\frac{\ln(a^2-x^2)}{4a^6}$	42
default	$-\frac{\ln(a-x)}{4a^6}+\frac{\ln(a^2+x^2)}{4a^6}-\frac{1}{2a^4x^2}-\frac{\ln(a+x)}{4a^6}$	43
norman	$-\frac{\ln(a-x)}{4a^6}+\frac{\ln(a^2+x^2)}{4a^6}-\frac{1}{2a^4x^2}-\frac{\ln(a+x)}{4a^6}$	43
parallelrisch	$-\frac{\ln(-a+x)x^2+\ln(a+x)x^2-x^2\ln(a^2+x^2)+2a^2}{4a^6x^2}$	46

input
```
int(1/x^3/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^4/x^2+1/4/a^6*ln(-a^2-x^2)-1/4/a^6*ln(a^2-x^2)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3(a^4-x^4)}\,dx = \frac{x^2\log(a^2+x^2)-x^2\log(-a^2+x^2)-2a^2}{4a^6x^2}$$

input
```
integrate(1/x^3/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/4*(x^2*log(a^2 + x^2) - x^2*log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = -\frac{1}{2a^4x^2} - \frac{\frac{\log(-a^2+x^2)}{4} - \frac{\log(a^2+x^2)}{4}}{a^6}$$

input
```
integrate(1/x**3/(a**4-x**4),x)
```

output
```
-1/(2*a**4*x**2) - (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**6
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{\log(a^2 + x^2)}{4 a^6} - \frac{\log(-a^2 + x^2)}{4 a^6} - \frac{1}{2 a^4 x^2}$$

input
```
integrate(1/x^3/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/4*log(a^2 + x^2)/a^6 - 1/4*log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{\log(a^2 + x^2)}{4 a^6} - \frac{\log(|-a^2 + x^2|)}{4 a^6} - \frac{1}{2 a^4 x^2}$$

input
```
integrate(1/x^3/(a^4-x^4),x, algorithm="giac")
```

output
```
1/4*log(a^2 + x^2)/a^6 - 1/4*log(abs(-a^2 + x^2))/a^6 - 1/2/(a^4*x^2)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx = \frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^6} - \frac{1}{2\,a^4\,x^2}$$

input
```
int(1/(x^3*(a^4 - x^4)),x)
```

output
```
atanh(x^2/a^2)/(2*a^6) - 1/(2*a^4*x^2)
```

3.132 $\int \frac{1}{x^4(a^4-x^4)}\,dx$

3.132.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=-\frac{1}{3a^4x^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^7}$$

output
```
-1/3/a^4/x^3+1/2*arctan(x/a)/a^7+1/2*arctanh(x/a)/a^7
```

3.132.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=-\frac{1}{3a^4x^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}-\frac{\log(a-x)}{4a^7}+\frac{\log(a+x)}{4a^7}$$

input
```
Integrate[1/(x^4*(a^4 - x^4)),x]
```

output
```
-1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a + x]/(
4*a^7)
```

3.132.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^4-x^4)} \, dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\int \frac{1}{a^4-x^4}dx}{a^4}-\frac{1}{3 a^4 x^3}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{\int \frac{1}{a^2-x^2}dx}{2 a^2}+\frac{\int \frac{1}{a^2+x^2}dx}{2 a^2}}{a^4}-\frac{1}{3 a^4 x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\int \frac{1}{a^2-x^2}dx}{2 a^2}+\frac{\arctan\left(\frac{x}{a}\right)}{2 a^3}}{a^4}-\frac{1}{3 a^4 x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\arctan\left(\frac{x}{a}\right)}{2 a^3}+\frac{\text{arctanh}\left(\frac{x}{a}\right)}{2 a^3}}{a^4}-\frac{1}{3 a^4 x^3}$$

input
```
Int[1/(x^4*(a^4 - x^4)),x]
```

output
```
-1/3*1/(a^4*x^3) + (ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3))/a^4
```

3.132.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

3.132.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{\ln(a-x)}{4a^7}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}-\frac{1}{3a^4x^3}+\frac{\ln(a+x)}{4a^7}$	41
parallelrisch	$-\frac{3i\ln(-ia+x)x^3-3i\ln(ia+x)x^3+3\ln(-a+x)x^3-3\ln(a+x)x^3+4a^3}{12a^7x^3}$	61
risch	$-\frac{1}{3a^4x^3}-\frac{\ln(a-x)}{4a^7}+\frac{\ln(-a-x)}{4a^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^2+1\right)}\_R\ln\left(\left(-5\_R^4a^{28}+4\right)x-a^8\_R\right)\right)}{4}$	71

input
```
int(1/x^4/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^7*ln(a-x)+1/2*arctan(x/a)/a^7-1/3/a^4/x^3+1/4*ln(a+x)/a^7
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=\frac{6\,x^3\arctan\left(\frac{x}{a}\right)+3\,x^3\log\left(a+x\right)-3\,x^3\log\left(-a+x\right)-4\,a^3}{12\,a^7x^3}$$

input
```
integrate(1/x^4/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/12*(6*x^3*arctan(x/a) + 3*x^3*log(a + x) - 3*x^3*log(-a + x) - 4*a^3)/(a
^7*x^3)
```

3.132.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=-\frac{1}{3a^4x^3}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}+\frac{i\log\left(-ia+x\right)}{4}-\frac{i\log\left(ia+x\right)}{4}}{a^7}$$

input
```
integrate(1/x**4/(a**4-x**4),x)
```

output
```
-1/(3*a**4*x**3) - (log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*l
og(I*a + x)/4)/a**7
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7}+\frac{\log\left(a+x\right)}{4\,a^7}-\frac{\log\left(-a+x\right)}{4\,a^7}-\frac{1}{3\,a^4x^3}$$

input
```
integrate(1/x^4/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/2*arctan(x/a)/a^7 + 1/4*log(a + x)/a^7 - 1/4*log(-a + x)/a^7 - 1/3/(a^4*
x^3)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (a^4 - x^4)} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\log\left(|a + x|\right)}{4\,a^7} - \frac{\log\left(|-a + x|\right)}{4\,a^7} - \frac{1}{3\,a^4 x^3}$$

input
```
integrate(1/x^4/(a^4-x^4),x, algorithm="giac")
```

output
```
1/2*arctan(x/a)/a^7 + 1/4*log(abs(a + x))/a^7 - 1/4*log(abs(-a + x))/a^7 -
 1/3/(a^4*x^3)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 (a^4 - x^4)} \, dx = \frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2\,a^7} - \frac{1}{3\,a^4\,x^3}$$

input
```
int(1/(x^4*(a^4 - x^4)),x)
```

output
```
atan(x/a)/(2*a^7) + atanh(x/a)/(2*a^7) - 1/(3*a^4*x^3)
```

3.133 $\int \frac{x^{-m}}{a^4-x^4} dx$

3.133.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{x^{-m}}{a^4-x^4} dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{4}, \frac{5-m}{4}, \frac{x^4}{a^4}\right)}{a^4(1-m)}$$

output
```
x^(1-m)*hypergeom([1, 1/4-1/4*m],[5/4-1/4*m],x^4/a^4)/a^4/(1-m)
```

3.133.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^4-x^4} dx = -\frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1}{4}-\frac{m}{4}, \frac{5}{4}-\frac{m}{4}, \frac{x^4}{a^4}\right)}{a^4(-1+m)}$$

input
```
Integrate[1/(x^m*(a^4 - x^4)),x]
```

output
```
-((x^(1 - m)*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, x^4/a^4])/(a^4*(-1
 + m)))
```

3.133.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-m}}{a^4-x^4} \, dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{1-m} \text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

input
```
Int[1/(x^m*(a^4 - x^4)),x]
```

output
```
(x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(a^4*(1 -
m))
```

3.133.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

3.133.4 Maple [F]

$$\int \frac{x^{-m}}{a^4-x^4}dx$$

input
```
int(1/(x^m)/(a^4-x^4),x)
```

output
```
int(1/(x^m)/(a^4-x^4),x)
```

3.133.5 Fricas [F]

$$\int \frac{x^{-m}}{a^4-x^4}\,dx=\int \frac{1}{(a^4-x^4)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^4-x^4),x, algorithm="fricas")
```

output
```
integral(1/((a^4 - x^4)*x^m), x)
```

3.133.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int \frac{x^{-m}}{a^4-x^4}\,dx=-\frac{mx^{1-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4},1,\frac{1}{4}-\frac{m}{4}\right)\Gamma\left(\frac{1}{4}-\frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4}-\frac{m}{4}\right)}+\frac{x^{1-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4},1,\frac{1}{4}-\frac{m}{4}\right)\Gamma\left(\frac{1}{4}-\frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4}-\frac{m}{4}\right)}$$

input
```
integrate(1/(x**m)/(a**4-x**4),x)
```

output
```
-m*x**(1 - m)*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/
4 - m/4)/(16*a**4*gamma(5/4 - m/4)) + x**(1 - m)*lerchphi(x**4*exp_polar(2
*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*gamma(5/4 - m/4))
```

3.133.7 Maxima [F]

$$\int \frac{x^{-m}}{a^4-x^4}\,dx=\int \frac{1}{(a^4-x^4)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^4-x^4),x, algorithm="maxima")
```

output
```
integrate(1/((a^4 - x^4)*x^m), x)
```

3.133.8 Giac [F]

$$\int \frac{x^{-m}}{a^4 - x^4}\,dx = \int \frac{1}{(a^4 - x^4)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^4-x^4),x, algorithm="giac")
```

output
```
integrate(1/((a^4 - x^4)*x^m), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^4 - x^4}\,dx = \int \frac{1}{x^m\,(a^4 - x^4)}\,dx$$

input
```
int(1/(x^m*(a^4 - x^4)),x)
```

output
```
int(1/(x^m*(a^4 - x^4)), x)
```

3.134 $\int \frac{x}{a^4+x^4} \, dx$

3.134.1 Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{x}{a^4+x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

output
```
1/2*arctan(x^2/a^2)/a^2
```

3.134.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^4+x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Integrate[x/(a^4 + x^4),x]
```

output
```
ArcTan[x^2/a^2]/(2*a^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^4+x^4} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{a^4+x^4}dx^2$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Int[x/(a^4 + x^4),x]
```

output
```
ArcTan[x^2/a^2]/(2*a^2)
```

3.134.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.134.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14
risch	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14
parallelrisch	$-\frac{i\ln(-ia^2+x^2)-i\ln(ia^2+x^2)}{4a^2}$	35

input
```
int(x/(a^4+x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

input
```
integrate(x/(a^4+x^4),x, algorithm="fricas")
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

3.134.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{a^4+x^4}\,dx = \frac{-\frac{i\log\left(-ia^2+x^2\right)}{4}+\frac{i\log\left(ia^2+x^2\right)}{4}}{a^2}$$

input
```
integrate(x/(a**4+x**4),x)
```

output
```
(-I*log(-I*a**2 + x**2)/4 + I*log(I*a**2 + x**2)/4)/a**2
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

input
```
integrate(x/(a^4+x^4),x, algorithm="maxima")
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

input
```
integrate(x/(a^4+x^4),x, algorithm="giac")
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\operatorname{atan}\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

input
```
int(x/(a^4 + x^4),x)
```

output
```
atan(x^2/a^2)/(2*a^2)
```

3.135 $\int \frac{x^2}{a^4+x^4} dx$

3.135.1 Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{x^2}{a^4+x^4} dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

output
```
-1/4*arctan(1-x*2^(1/2)/a)/a*2^(1/2)+1/4*arctan(1+x*2^(1/2)/a)/a*2^(1/2)+1
/8*ln(a^2+x^2-a*x*2^(1/2))/a*2^(1/2)-1/8*ln(a^2+x^2+a*x*2^(1/2))/a*2^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{a^4+x^4} dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+2\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+\log\left(a^2-\sqrt{2}ax+x^2\right)-\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

input
```
Integrate[x^2/(a^4 + x^4),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*x)/a] + 2*ArcTan[1 + (Sqrt[2]*x)/a] + Log[a^2 - Sq
rt[2]*a*x + x^2] - Log[a^2 + Sqrt[2]*a*x + x^2])/(4*Sqrt[2]*a)
```

3.135.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{a^2-\sqrt{2}xa+x^2}dx+\frac{1}{2}\int \frac{1}{a^2+\sqrt{2}xa+x^2}dx\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}x}{a}\right)^2-1}d\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}x}{a}+1\right)^2-1}d\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}+\frac{\int -\frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{a+\sqrt{2}x}{a^2+\sqrt{2}xa+x^2}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

$\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

input
```
Int[x^2/(a^4 + x^4),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*x)/a]/(Sqrt[2]*a)) + ArcTan[1 + (Sqrt[2]*x)/a]/(Sqr
t[2]*a))/2 + (Log[a^2 - Sqrt[2]*a*x + x^2]/(2*Sqrt[2]*a) - Log[a^2 + Sqrt[
2]*a*x + x^2]/(2*Sqrt[2]*a))/2
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.135.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.22

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+a^4\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$	24
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(a^4\right)^{\frac{1}{4}}}$	85

input
```
int(x^2/(a^4+x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+a^4))
```

3.135.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right) + \frac{1}{4}i\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-i\,a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-a^4\left(-\frac{1}{a^4}\right)^{\frac{3}{4}}+x\right)$$

input
```
integrate(x^2/(a^4+x^4),x, algorithm="fricas")
```

output
```
1/4*(-1/a^4)^(1/4)*log(a^4*(-1/a^4)^(3/4) + x) - 1/4*I*(-1/a^4)^(1/4)*log(
I*a^4*(-1/a^4)^(3/4) + x) + 1/4*I*(-1/a^4)^(1/4)*log(-I*a^4*(-1/a^4)^(3/4)
 + x) - 1/4*(-1/a^4)^(1/4)*log(-a^4*(-1/a^4)^(3/4) + x)
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.17

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\operatorname{RootSum}\left(256t^4+1,\left(t\mapsto t\log\left(64t^3a+x\right)\right)\right)}{a}$$

input
```
integrate(x**2/(a**4+x**4),x)
```

output
```
RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a+2\,x\right)}{2\,a}\right)}{4\,a} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a-2\,x\right)}{2\,a}\right)}{4\,a} - \frac{\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)}{8\,a} + \frac{\sqrt{2}\log\left(-\sqrt{2}ax+a^2+x^2\right)}{8\,a}$$

input
```
integrate(x^2/(a^4+x^4),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 1/4*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - 1/8*sqrt(2)*log(sqrt(2)*a*x + a^2 +
 x^2)/a + 1/8*sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2\,x\right)}{2\,|a|}\right)}{4\,a^2} + \frac{\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2\,x\right)}{2\,|a|}\right)}{4\,a^2} - \frac{\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{8\,a^2} + \frac{\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{8\,a^2}$$

input
```
integrate(x^2/(a^4+x^4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^2 +
 1/4*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a))/a^2
 - 1/8*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2 + 1/8*sqr
t(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}x}{a}\right)-(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}x}{a}\right)}{2\,a}$$

input
```
int(x^2/(a^4 + x^4),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*x)/a) - (-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(
2*a)
```

3.136 $\int \frac{1}{a^5+x^5} dx$

3.136.1 Optimal result

Integrand size = 9, antiderivative size = 201

$$\int \frac{1}{a^5+x^5} dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4} + \frac{\log(a+x)}{5a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}$$

output
```
1/5*ln(a+x)/a^4-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^4-1/2
0*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^4-1/10*arctan(1/20*(-4*x+a
*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^4-1/10*arcta
n((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a^4
```

3.136.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{1}{a^5+x^5}\,dx =$$

$$-\frac{-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log}{}$$

input
```
Integrate[(a^5 + x^5)^(-1),x]
```

output
```
-1/20*(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5
 + Sqrt[5])]*a)] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x
)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x
)/2 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - (
(1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^
2])/a^4
```

3.136.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {751, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^5+x^5}dx$$

$$\Big\downarrow \text{751}$$

$$\frac{\int \frac{1}{a+x}dx}{5a^4}+\frac{2\int \frac{4a-\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}+\frac{2\int \frac{4a-\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\int \frac{4a-\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}+\frac{2\int \frac{4a-\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a-\left(1-\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\int \frac{4a-\left(1+\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\left(5+\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)}{5a^4}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\left(5-\sqrt{5}\right)a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-\left(1+\sqrt{5}\right)a\right)}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{1103}$$

3.136. $\int \frac{1}{a^5+x^5}\,dx$

$$\frac{\log(a+x)}{5a^4}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)}{5a^4}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)}{5a^4}$$

input
```
Int[(a^5 + x^5)^(-1),x]
```

output
```
Log[a + x]/(5*a^4) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(-((1 - Sqrt[5])*a) + 4
*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] - ((1 - Sqrt[5])*Log[2*a^2 - (1 - Sqrt[5])*
a*x + 2*x^2])/4)/(5*a^4) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((1 + Sqrt[5])*
a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] - ((1 + Sqrt[5])*Log[2*a^2 - (1 + Sqr
t[5])*a*x + 2*x^2])/4)/(5*a^4)
```

3.136.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 751
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k -
1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; r/(a*n)
  Int[1/(r + s*x), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 1)/2}], x]] /; Fre
eQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.136.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\ln(a+x)}{5a^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^4+a^{12}\_Z^3+a^8\_Z^2+a^4\_Z+1\right)}\_R\ln\left(\_R\,a^5+x\right)\right)}{5}$	55
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3a^2\_R+4a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^4}+\frac{\ln(a+x)}{5a^4}$	101

input
```
int(1/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(a+x)/a^4+1/5*sum(_R*ln(_R*a^5+x),_R=RootOf(_Z^4*a^16+_Z^3*a^12+_Z^2
*a^8+_Z*a^4+1))
```

3.136.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 11094, normalized size of antiderivative = 55.19

$$\int \frac{1}{a^5+x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log(5ta+x)\right)\right)}{a^4}$$

input
```
integrate(1/(a**5+x**5),x)
```

output
```
(log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambd
a(_t, _t*log(5*_t*a + x))))/a**4
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{1}{a^5+x^5}\,dx = {} & \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{5\,a^4\sqrt{2\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5\,a^4\sqrt{-2\sqrt{5}+10}} \\ & - \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{10\,a^4\left(\sqrt{5}+1\right)} \\ & - \frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{10\,a^4\left(\sqrt{5}-1\right)} + \frac{\log(a+x)}{5\,a^4}\end{aligned}$$

input
```
integrate(1/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/(a^4*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a
*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a^4*sqrt(-2*sqrt(5) + 10
)) - 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^4*(sqrt
(5) + 1)) - 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^4
*(sqrt(5) - 1)) + 1/5*log(a + x)/a^4
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^4} + \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^4} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^4} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^4} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20a^4} + \frac{\log\left(|a+x|\right)}{5a^4}$$

input
```
integrate(1/(a^5+x^5),x, algorithm="giac")
```

output
```
1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^4 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/
(a*sqrt(-2*sqrt(5) + 10)))/a^4 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a
)*x + x^2)/a^4 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^4 -
 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^4 + 1/5*log(abs(a +
x))/a^4
```

3.136.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\ln(a+x)}{5\,a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^4} + \frac{\ln\left(x+\frac{a\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^4}$$

input
```
int(1/(a^5 + x^5),x)
```

output
```
log(a + x)/(5*a^4) - (log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4
)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^4) - (log(x - (a*((- 2*5^(
1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) +
1))/(20*a^4) + (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5
^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^4) - (log(x - (a*(5^(1/2) +
(2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(2
0*a^4)
```

3.137 $\int \frac{x}{a^5+x^5} \, dx$

3.137.1 Optimal result

Integrand size = 11, antiderivative size = 201

$$\int \frac{x}{a^5+x^5} \, dx = \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} \arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)} \arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3} - \frac{\log(a+x)}{5a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}$$

output
```
-1/5*ln(a+x)/a^3+1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^3+1/2
0*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^3+1/10*arctan((-4*x+a*(-5
^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^3-1/10*arctan(1/
20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^3
```

3.137.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x}{a^5+x^5}\,dx$$
$$= \frac{-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log(a$$

input
```
Integrate[x/(a^5 + x^5),x]
```

output
```
(-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt
[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqr
t[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 +
x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + S
qrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(2
0*a^3)
```

3.137.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^5+x^5}\,dx$$
$$\Big\downarrow \ 822$$
$$-\frac{\int \frac{1}{a+x}dx}{5a^3}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}$$
$$\Big\downarrow \ 16$$
$$\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \color{blue}{1142}$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\sqrt{5}a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}(1-\sqrt{5})\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{-\sqrt{5}a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \color{blue}{1083}$$

$$\frac{2\sqrt{5}a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-(1-\sqrt{5})a\right)-\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{-\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-2\sqrt{5}a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-(1+\sqrt{5})a\right)}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \color{blue}{217}$$

$$\frac{-\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \color{blue}{1103}$$

3.137. $\int \frac{x}{a^5+x^5}\,dx$

$$-\frac{\log(a+x)}{5a^3}+\frac{\frac{1}{4}(1+\sqrt{5})\log\left(2a^2-(1-\sqrt{5})\,ax+2x^2\right)-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\frac{1}{4}(1-\sqrt{5})\log\left(2a^2-(1+\sqrt{5})\,ax+2x^2\right)+\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{5a^3}$$

input
```
Int[x/(a^5 + x^5),x]
```

output
```
-1/5*Log[a + x]/a^3 + (-(Sqrt[10/(5 + Sqrt[5])]*ArcTan[(-((1 - Sqrt[5])*a)
 + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)]) + ((1 + Sqrt[5])*Log[2*a^2 - (1 - Sqrt
[5])*a*x + 2*x^2])/4)/(5*a^3) + (Sqrt[10/(5 - Sqrt[5])]*ArcTan[(-((1 + Sqr
t[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 - Sqrt[5])*Log[2*a^2 - (1
 + Sqrt[5])*a*x + 2*x^2])/4)/(5*a^3)
```

3.137.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.137.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.30

method	result	size
risch	$-\frac{\ln(a+x)}{5a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}\_Z^4-a^9\_Z^3+a^6\_Z^2-a^3\_Z+1\right)}\_R\ln\left(-a^{10}\_R^3+x\right)\right)}{5}$	60
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a+3a^2\_R+a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^3}-\frac{\ln(a+x)}{5a^3}$	97

input
```
int(x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(a+x)/a^3+1/5*sum(_R*ln(-_R^3*a^10+x),_R=RootOf(_Z^4*a^12-_Z^3*a^9+
_Z^2*a^6-_Z*a^3+1))
```

3.137.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 18781, normalized size of antiderivative = 93.44

$$\int \frac{x}{a^5+x^5}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int \frac{x}{a^5+x^5}\,dx = \frac{-\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1, \left(t \mapsto t\log\left(-125t^3a+x\right)\right)\right)}{a^3}$$

input
```
integrate(x/(a**5+x**5),x)
```

output
```
(-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lamb
da(_t, _t*log(-125*_t**3*a + x))))/a**3
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{x}{a^5+x^5}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^3\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^3\sqrt{-2\sqrt{5}+10}} - \frac{\log(a+x)}{5a^3} - \frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{5a^3(\sqrt{5}+1)} + \frac{\log\left(ax(\sqrt{5}-1)+2a^2+2x^2\right)}{5a^3(\sqrt{5}-1)}$$

input
```
integrate(x/(a^5+x^5),x, algorithm="maxima")
```

output
```
-2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^3
*sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) - 1/5*log(a + x)/a^3 - 1
/5*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) + 1/5*log(a
*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1))
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^5+x^5}\,dx = -\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^3} + \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^3} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^3} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^3} + \frac{\log\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20a^3} - \frac{\log\left(\left|a+x\right|\right)}{5a^3}$$

input
```
integrate(x/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(
5) + 10)))/a^3 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)
/(a*sqrt(-2*sqrt(5) + 10)))/a^3 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a +
a)*x + x^2)/a^3 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^3
+ 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^3 - 1/5*log(abs(a +
 x))/a^3
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{x}{a^5+x^5}\,dx = \frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}-\frac{\ln(a+x)}{5\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^3}-\frac{\ln\left(x+\frac{a\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}$$

input
```
int(x/(a^5 + x^5),x)
```

output
```
(log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^
(1/2) - 10)^(1/2) + 1))/(20*a^3) - log(a + x)/(5*a^3) + (log(x - (a*((- 2*
5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1
/2) + 1))/(20*a^3) - (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^
3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^3) + (log(x - (a*(5
^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1
/2) + 1))/(20*a^3)
```

3.138 $\int \frac{x^2}{a^5+x^5}\,dx$

3.138.1 Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^2} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2} + \frac{\log(a+x)}{5a^2} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}$$

output
```
1/5*ln(a+x)/a^2-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^2-1/20
*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^2+1/10*arctan((-4*x+a*(-5^
(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^2-1/10*arctan(1/2
0*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^2
```

3.138.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{a^5+x^5}\,dx =$$

$$-\frac{2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log\left(\right.}{}$$

input
```
Integrate[x^2/(a^5 + x^5),x]
```

output
```
-1/20*(2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 +
 Sqrt[5])]*a)] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)
/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)
/2 + x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((
1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2
])/a^2
```

3.138.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {822, 16, 27, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^5+x^5}\,dx$$

$$\Big\downarrow \text{822}$$

$$\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\int \frac{1}{a+x}dx}{5a^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\left(1+\sqrt{5}\right)(a+x)}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{(1-\sqrt{5})\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{(1+\sqrt{5})\int \frac{a+x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{(1-\sqrt{5})\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \; 1142$$

$$-\frac{(1+\sqrt{5})\left(\frac{1}{4}(5-\sqrt{5})\,a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{(1-\sqrt{5})\left(\frac{1}{4}(5+\sqrt{5})\,a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{(1+\sqrt{5})\left(\frac{1}{4}(5-\sqrt{5})\,a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{(1-\sqrt{5})\left(\frac{1}{4}(5+\sqrt{5})\,a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \; 1083$$

$$-\frac{(1+\sqrt{5})\left(-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{2}(5-\sqrt{5})\,a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-(1-\sqrt{5})\,a\right)\right)}{5a^2}-\frac{(1-\sqrt{5})\left(-\frac{1}{4}\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{2}(5+\sqrt{5})\,a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-(1+\sqrt{5})\,a\right)\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \; 217$$

3.138. $\int \frac{x^2}{a^5+x^5}\,dx$

$$-\frac{(1+\sqrt{5})\left(\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{(1-\sqrt{5})\left(\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5-\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{(1+\sqrt{5})\left(\frac{1}{4}\log\left(2a^2-(1-\sqrt{5})ax+2x^2\right)+\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5a^2}-\frac{(1-\sqrt{5})\left(\frac{1}{4}\log\left(2a^2-(1+\sqrt{5})ax+2x^2\right)+\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5-\sqrt{5}\right)}}\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

input
```
Int[x^2/(a^5 + x^5),x]
```

output
```
Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*(((5 - Sqrt[5])*ArcTan[(-((1 - Sqrt[5]
)*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(2*Sqrt[2*(5 + Sqrt[5])]) + Log[2*
a^2 - (1 - Sqrt[5])*a*x + 2*x^2]/4))/(5*a^2) - ((1 - Sqrt[5])*(((5 + Sqrt[
5])*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)])/(2*Sqrt[
2*(5 - Sqrt[5])]) + Log[2*a^2 - (1 + Sqrt[5])*a*x + 2*x^2]/4))/(5*a^2)
```

3.138.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.138. $\int \frac{x^2}{a^5+x^5} \, dx$

3.138.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{\ln(a+x)}{5a^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^8\_Z^4+a^6\_Z^3+a^4\_Z^2+a^2\_Z+1\right)}\_R\ln\left(x\_R^3a^5+1\right)\right)}{5}$	58
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a+2a^2\_R-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^2}+\frac{\ln(a+x)}{5a^2}$	101

input
```
int(x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(a+x)/a^2+1/5*sum(_R*ln(_R^3*a^5*x+1),_R=RootOf(_Z^4*a^8+_Z^3*a^6+_Z
^2*a^4+_Z*a^2+1))
```

3.138.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 12656, normalized size of antiderivative = 62.97

$$\int \frac{x^2}{a^5+x^5}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1, \left(t \mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^2}$$

input
```
integrate(x**2/(a**5+x**5),x)
```

output
```
(log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambd
a(_t, _t*log(25*_t**2*a + x))))/a**2
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^2\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^2\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{5a^2} + \frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{5a^2(\sqrt{5}+1)} - \frac{\log\left(ax(\sqrt{5}-1)+2a^2+2x^2\right)}{5a^2(\sqrt{5}-1)}$$

input
```
integrate(x^2/(a^5+x^5),x, algorithm="maxima")
```

output
```
-2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^2
*sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) + 1/5*log(a + x)/a^2 + 1
/5*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) - 1/5*log(a
*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1))
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^2} + \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^2} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^2} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^2} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^2} + \frac{\log\left(|a+x|\right)}{5\,a^2}$$

input
```
integrate(x^2/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(
5) + 10)))/a^2 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)
/(a*sqrt(-2*sqrt(5) + 10)))/a^2 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a +
a)*x + x^2)/a^2 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^2
- 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^2 + 1/5*log(abs(a +
 x))/a^2
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\ln(a+x)}{5\,a^2} + \frac{\ln\left(a^5+\frac{x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3a^4}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^2}$$

input
```
int(x^2/(a^5 + x^5),x)
```

output
```
log(a + x)/(5*a^2) + (log(a^5 + (a^4*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2)
 - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^2) - (log(a^5
 - (a^4*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1
/2) - 10)^(1/2) + 1))/(20*a^2) - (log(a^5 - (a^4*x*(5^(1/2) - (2*5^(1/2) -
 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^2) -
(log(a^5 - (a^4*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^
(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^2)
```

3.138. $\int \frac{x^2}{a^5+x^5}\,dx$

3.139 $\int \frac{x^3}{a^5+x^5}\,dx$

3.139.1 Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^3}{a^5+x^5}\,dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a} - \frac{\log(a+x)}{5a} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}$$

output
```
-1/5*ln(a+x)/a+1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a+1/20*l
n(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a-1/10*arctan(1/20*(-4*x+a*(5^(
1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a-1/10*arctan((-4*x
+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a
```

3.139.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{a^5+x^5}\,dx$$

$$= \frac{2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log\left(a^2\right.}{}$$

input
```
Integrate[x^3/(a^5 + x^5),x]
```

output
```
(2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt
[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt
[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x
^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sq
rt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20
*a)
```

3.139.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a^5+x^5}\,dx$$

$$\Big\downarrow \text{822}$$

$$\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\int \frac{1}{a+x}dx}{5a}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\left(5+\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)}{5a}+\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\left(5-\sqrt{5}\right)a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-\left(1+\sqrt{5}\right)a\right)}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{1103}$$

3.139. $\int \frac{x^3}{a^5+x^5}\,dx$

$$\frac{\frac{1}{4}(1-\sqrt{5})\log\left(2a^2-(1-\sqrt{5})\,ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}+$$
$$\frac{\frac{1}{4}(1+\sqrt{5})\log\left(2a^2-(1+\sqrt{5})\,ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{5a}-\frac{\log(a+x)}{5a}$$

input
```
Int[x^3/(a^5 + x^5),x]
```

output
```
-1/5*Log[a + x]/a + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(-((1 - Sqrt[5])*a) + 4*
x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + ((1 - Sqrt[5])*Log[2*a^2 - (1 - Sqrt[5])*a
*x + 2*x^2])/4)/(5*a) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((1 + Sqrt[5])*a)
+ 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 + Sqrt[5])*Log[2*a^2 - (1 + Sqrt[5
])*a*x + 2*x^2])/4)/(5*a)
```

3.139.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.139.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4\_Z^4-a^3\_Z^3+\_Z^2a^2-a\_Z+1\right)}\_R\ln\left(\_R^3a^4-\_R^2a^3+a^2\_R-a+x\right)\right)}{5}-\frac{\ln(a+x)}{5a}$	73
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3+3\_R^2a-2a^2\_R+a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a}-\frac{\ln(a+x)}{5a}$	97

input
```
int(x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum(_R*ln(_R^3*a^4-_R^2*a^3+_R*a^2-a+x),_R=RootOf(_Z^4*a^4-_Z^3*a^3+_Z
^2*a^2-_Z*a+1))-1/5*ln(a+x)/a
```

3.139.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 17865, normalized size of antiderivative = 88.88

$$\int \frac{x^3}{a^5+x^5}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{-\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 - 125t^3 + 25t^2 - 5t + 1, \left(t \mapsto t\log\left(625t^4a + x\right)\right)\right)}{a}$$

input
```
integrate(x**3/(a**5+x**5),x)
```

output
```
(-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lamb
da(_t, _t*log(625*_t**4*a + x))))/a
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a\sqrt{2\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a\sqrt{-2\sqrt{5}+10}} + \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{10a\left(\sqrt{5}+1\right)} + \frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+2a^2+2x^2\right)}{10a\left(\sqrt{5}-1\right)} - \frac{\log(a+x)}{5a}$$

input
```
integrate(x^3/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(
sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) +
 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) +
1)) + 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5
) - 1)) - 1/5*log(a + x)/a
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a} + \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a} + \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a} - \frac{\log\left(|a+x|\right)}{5\,a}$$

input
```
integrate(x^3/(a^5+x^5),x, algorithm="giac")
```

output
```
1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
+ x^2)/a - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a + 1/20*lo
g(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a - 1/5*log(abs(a + x))/a
```

3.139.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a} - \frac{\ln\left(5\,a^{10}+\frac{5\,x\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)\,a^9}{4}\right)\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a} - \frac{\ln\left(a+x\right)}{5\,a} + \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a} + \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a}$$

input
```
int(x^3/(a^5 + x^5),x)
```

output
```
(log(5*a^10 - (5*a^9*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2)
 + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) - (log(5*a^10 + (5*a^9*x*(5^(1/2) +
 (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1
))/(20*a) - log(a + x)/(5*a) + (log(5*a^10 - (5*a^9*x*(5^(1/2) - (2*5^(1/2
) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) + (l
og(5*a^10 - (5*a^9*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^
(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a)
```

3.140 $\int \frac{x^4}{a^5+x^5}\,dx$

3.140.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

output
```
1/5*ln(a^5+x^5)
```

3.140.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

input
```
Integrate[x^4/(a^5 + x^5),x]
```

output
```
Log[a^5 + x^5]/5
```

3.140.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a^5+x^5}\,dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

input
```
Int[x^4/(a^5 + x^5),x]
```

output
```
Log[a^5 + x^5]/5
```

3.140.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

3.140.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln\left(a^5+x^5\right)}{5}$	11
default	$\frac{\ln\left(a^5+x^5\right)}{5}$	11
risch	$\frac{\ln\left(a^5+x^5\right)}{5}$	11
norman	$\frac{\ln\left(a+x\right)}{5}+\frac{\ln\left(a^4-a^3x+a^2x^2-a\,x^3+x^4\right)}{5}$	37
parallelrisch	$\frac{\ln\left(a+x\right)}{5}+\frac{\ln\left(a^4-a^3x+a^2x^2-a\,x^3+x^4\right)}{5}$	37

input
```
int(x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(a^5+x^5)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

input
```
integrate(x^4/(a^5+x^5),x, algorithm="fricas")
```

output
```
1/5*log(a^5 + x^5)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{\log\left(a^5+x^5\right)}{5}$$

input
```
integrate(x**4/(a**5+x**5),x)
```

output
```
log(a**5 + x**5)/5
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

input
```
integrate(x^4/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*log(a^5 + x^5)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{a^5 + x^5} \, dx = \frac{1}{5} \log \left(\left| a^5 + x^5 \right| \right)$$

input
```
integrate(x^4/(a^5+x^5),x, algorithm="giac")
```

output
```
1/5*log(abs(a^5 + x^5))
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5 + x^5} \, dx = \frac{\ln (a^5 + x^5)}{5}$$

input
```
int(x^4/(a^5 + x^5),x)
```

output
```
log(a^5 + x^5)/5
```

3.141 $\int \frac{1}{x(a^5+x^5)}\,dx$

3.141.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

output
```
ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

3.141.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

input
```
Integrate[1/(x*(a^5 + x^5)),x]
```

output
```
Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

3.141.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^5+x^5\right)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{5}\int \frac{1}{x^5\left(a^5+x^5\right)}dx^5$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{5}\left(\frac{\int \frac{1}{x^5}dx^5}{a^5}-\frac{\int \frac{1}{a^5+x^5}dx^5}{a^5}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{5}\left(\frac{\log\left(x^5\right)}{a^5}-\frac{\int \frac{1}{a^5+x^5}dx^5}{a^5}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{5}\left(\frac{\log\left(x^5\right)}{a^5}-\frac{\log\left(a^5+x^5\right)}{a^5}\right)$$

input
```
Int[1/(x*(a^5 + x^5)),x]
```

output
```
(Log[x^5]/a^5 - Log[a^5 + x^5]/a^5)/5
```

3.141.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.141.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\ln(x)}{a^5}-\frac{\ln(a^5+x^5)}{5a^5}$	21
parallelrisch	$\frac{5\ln(x)-\ln(a+x)-\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}$	46
default	$\frac{\ln(x)}{a^5}-\frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}-\frac{\ln(a+x)}{5a^5}$	49
norman	$\frac{\ln(x)}{a^5}-\frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}-\frac{\ln(a+x)}{5a^5}$	49

input
```
int(1/x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx = -\frac{\log\left(a^5+x^5\right)-5\,\log\left(x\right)}{5\,a^5}$$

input
```
integrate(1/x/(a^5+x^5),x, algorithm="fricas")
```

output
```
-1/5*(log(a^5 + x^5) - 5*log(x))/a^5
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx = \frac{\log\left(x\right)}{a^5} - \frac{\log\left(a^5+x^5\right)}{5a^5}$$

input
```
integrate(1/x/(a**5+x**5),x)
```

output
```
log(x)/a**5 - log(a**5 + x**5)/(5*a**5)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx = -\frac{\log\left(a^5+x^5\right)}{5\,a^5} + \frac{\log\left(x^5\right)}{5\,a^5}$$

input
```
integrate(1/x/(a^5+x^5),x, algorithm="maxima")
```

output
```
-1/5*log(a^5 + x^5)/a^5 + 1/5*log(x^5)/a^5
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^5+x^5)}\,dx = -\frac{\log\left(|a^5+x^5|\right)}{5\,a^5} + \frac{\log\left(|x|\right)}{a^5}$$

input
```
integrate(1/x/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/5*log(abs(a^5 + x^5))/a^5 + log(abs(x))/a^5
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(a^5+x^5)}\,dx = -\frac{\ln\left(a^5+x^5\right) - 5\,\ln\left(x\right)}{5\,a^5}$$

input
```
int(1/(x*(a^5 + x^5)),x)
```

output
```
-(log(a^5 + x^5) - 5*log(x))/(5*a^5)
```

3.142 $\int \frac{1}{x^2(a^5+x^5)}\,dx$

3.142.1 Optimal result

Integrand size = 13, antiderivative size = 209

$$\begin{aligned}\int \frac{1}{x^2(a^5+x^5)}\,dx = &-\frac{1}{a^5x}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6}\\&+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}\\&+\frac{\log(a+x)}{5a^6}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}\\&-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}\end{aligned}$$

output
```
-1/a^5/x+1/5*ln(a+x)/a^6-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1
)/a^6-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^6+1/10*arctan(1/2
0*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^6+1
/10*arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1
/2)/a^6
```

3.142.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = -\frac{\frac{20a}{x}+2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)-}{20a^6}$$

input
```
Integrate[1/(x^2*(a^5 + x^5)),x]
```

output
```
-1/20*((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/
(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5]
)*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] - (-1 + Sqrt[5])*Log[
a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/a^6
```

3.142.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {847, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a^5+x^5)}\,dx$$

$$\Big\downarrow \ 847$$

$$-\frac{\int \frac{x^3}{a^5+x^5}dx}{a^5}-\frac{1}{a^5x}$$

$$\Big\downarrow \ 822$$

$$-\frac{\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\int \frac{1}{a+x}dx}{5a}}{a^5}-\frac{1}{a^5x}$$

$$\Big\downarrow \ 16$$

$$-\frac{\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\log(a+x)}{5a}}{a^5}-\frac{1}{a^5x}$$

$\big\downarrow$ 27

$$-\frac{\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}}{a^5}-\frac{1}{a^5x}$$

$\big\downarrow$ 1142

$$-\frac{\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1+\cdots\right.}{2a^2-\left(1+\cdots\right.}\cdots}{5a}\cdots}{a^5}$$

$$\frac{1}{a^5x}$$

$\big\downarrow$ 25

$$-\frac{\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a\cdots}{2a^2-\left(1+\sqrt{5}\right)\cdots}\cdots}{5a}\cdots}{a^5}$$

$$\frac{1}{a^5x}$$

$\big\downarrow$ 1083

$$-\frac{\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\left(5+\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)}{5a}+\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\cdots}{\cdots}}{a^5}$$

$$\frac{1}{a^5x}$$

$\big\downarrow$ 217

$$-\frac{\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a\cdots}{2a^2-\left(1+\sqrt{5}\right)\cdots}\cdots}{5a}\cdots}{a^5}$$

$$\frac{1}{a^5x}$$

$\big\downarrow$ 1103

3.142. $\int \frac{1}{x^2(a^5+x^5)}\,dx$

$$-\frac{1}{a^5x}-\frac{\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\right.}{5a}}{a^5}$$

input
```
Int[1/(x^2*(a^5 + x^5)),x]
```

output
```
-(1/(a^5*x)) - (-1/5*Log[a + x]/a + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(-((1 -
Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + ((1 - Sqrt[5])*Log[2*a^2 -
 (1 - Sqrt[5])*a*x + 2*x^2])/4)/(5*a) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((
1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 + Sqrt[5])*Log[2*a
^2 - (1 + Sqrt[5])*a*x + 2*x^2])/4)/(5*a))/a^5
```

3.142.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.142.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

method	result	size
risch	$-\frac{1}{a^5 x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{24}\_Z^4+a^{18}\_Z^3+a^{12}\_Z^2+a^6\_Z+1\right)}\_R\ln\left(\left(6\_R^5 a^{30}-5\right)x+a^{25}\_R^4\right)\right)}{5}+\frac{\ln(a+x)}{5a^6}$	76
default	$-\frac{1}{a^5 x}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3-3\_R^2a+2a^2\_R-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^6}+\frac{\ln(a+x)}{5a^6}$	109

input
```
int(1/x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/a^5/x+1/5*sum(_R*ln((6*_R^5*a^30-5)*x+a^25*_R^4),_R=RootOf(_Z^4*a^24+_Z
^3*a^18+_Z^2*a^12+_Z*a^6+1))+1/5*ln(a+x)/a^6
```

3.142.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 15275, normalized size of antiderivative = 73.09

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = -\frac{1}{a^5x} + \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a^6}$$

input
```
integrate(1/x**2/(a**5+x**5),x)
```

output
```
-1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5
*_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = -\frac{\frac{2\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a\sqrt{-2\sqrt{5}+10}} + \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a\left(\sqrt{5}+1\right)} + \frac{\left(\sqrt{5}-3\right)}{}}{10a^5} - \frac{1}{a^5x}$$

input
```
integrate(1/x^2/(a^5+x^5),x, algorithm="maxima")
```

output
```
-1/10*(2*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sq
rt(5) + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 2*sqrt(5)*(sqrt(5) - 1)*arctan(-(
a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)
) + (sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1)
) + (sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1))
 - 2*log(a + x)/a)/a^5 - 1/(a^5*x)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = -\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^6} - \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^6} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^6} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20a^6} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20a^6} + \frac{\log\left(|a+x|\right)}{5a^6} - \frac{1}{a^5x}$$

input
```
integrate(1/x^2/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^6 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)
/(a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a +
a)*x + x^2)/a^6 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^6
- 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^6 + 1/5*log(abs(a +
 x))/a^6 - 1/(a^5*x)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^5 + x^5)} \, dx = \frac{\ln(a+x)}{5a^6} - \frac{1}{a^5 x} + \frac{\ln\left(5a^{30} + \frac{5x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)a^{29}}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20a^6}$$

input
```
int(1/(x^2*(a^5 + x^5)),x)
```

output
```
log(a + x)/(5*a^6) - 1/(a^5*x) + (log(5*a^30 + (5*a^29*x*(5^(1/2) + (- 2*5
^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*
a^6) - (log(5*a^30 - (5*a^29*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*
(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x
*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1
/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*((- 2*5^(1/2) - 10)^(1/2) - 5
^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^6)
```

3.143 $\int \frac{1}{x^3(a^5+x^5)}\,dx$

3.143.1 Optimal result

Integrand size = 13, antiderivative size = 211

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = -\frac{1}{2a^5x^2} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^7} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7} - \frac{\log(a+x)}{5a^7} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}$$

output
```
-1/2/a^5/x^2-1/5*ln(a+x)/a^7+1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2
)+1)/a^7+1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^7-1/10*arctan
((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^7+1/
10*arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2)
)^(1/2)/a^7
```

3.143.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = -\frac{\frac{10a^2}{x^2}-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{(-1+\sqrt{5})a+4x}{\sqrt{2(5+\sqrt{5})}a}\right)+2\sqrt{2(5+\sqrt{5})}\arctan\left(\frac{-((1+\sqrt{5})a)+4x}{\sqrt{10-2\sqrt{5}}a}\right)+4\log(a+x)}{20a^7}$$

input
```
Integrate[1/(x^3*(a^5 + x^5)),x]
```

output
```
-1/20*((10*a^2)/x^2 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*
x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqr
t[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 4*Log[a + x] - (1 + Sqrt[5])*L
og[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (-1 + Sqrt[5])*Log[a^2 - ((1 + Sq
rt[5])*a*x)/2 + x^2])/a^7
```

3.143.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {847, 822, 16, 27, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a^5+x^5)}dx$$

$$\Big\downarrow \; 847$$

$$-\frac{\int \frac{x^2}{a^5+x^5}dx}{a^5}-\frac{1}{2a^5x^2}$$

$$\Big\downarrow \; 822$$

$$-\frac{\frac{2\int -\frac{(1+\sqrt{5})a+(1+\sqrt{5})x}{2(2a^2-(1-\sqrt{5})xa+2x^2)}dx}{5a^2}+\frac{2\int -\frac{(1-\sqrt{5})(a+x)}{2(2a^2-(1+\sqrt{5})xa+2x^2)}dx}{5a^2}+\frac{\int \frac{1}{a+x}dx}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$$\Big\downarrow \; 16$$

$$-\frac{\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\log(a+x)}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(1+\sqrt{5}\right)(a+x)}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\downarrow$ 27

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\downarrow$ 1142

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int - \ldots\right)}{5a^2}\ldots}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\ldots}{2a^2-\ldots}\right)}{5a^2}\ldots}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$\downarrow$ 1083

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(-\frac{1}{4}\int \frac{\left(1+\sqrt{5}\right)a-\ldots}{2a^2-\left(1+\sqrt{5}\right)\ldots}\right)}{\ldots}}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$\downarrow$ 217

3.143. $\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx$

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5-\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+\cdots}\cdots\right)}{5a^2}\cdots}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{1}{2a^5x^2}-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)+\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\cdots}{\sqrt{2(\cdots}}\right)}{2\sqrt{2\left(5-\sqrt{5}\cdots\right.}}\cdots\right)}{5a^2}\cdots}{a^5}$$

input
```
Int[1/(x^3*(a^5 + x^5)),x]
```

output
```
-1/2*1/(a^5*x^2) - (Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*(((5 - Sqrt[5])*Ar
cTan[(-((1 - Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(2*Sqrt[2*(5 +
 Sqrt[5])]) + Log[2*a^2 - (1 - Sqrt[5])*a*x + 2*x^2]/4))/(5*a^2) - ((1 - S
qrt[5])*(((5 + Sqrt[5])*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqr
t[5])]*a)])/(2*Sqrt[2*(5 - Sqrt[5])]) + Log[2*a^2 - (1 + Sqrt[5])*a*x + 2*
x^2]/4))/(5*a^2))/a^5
```

3.143.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.143. $\int \frac{1}{x^3(a^5+x^5)}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.143. $\int \frac{1}{x^3(a^5+x^5)} dx$

3.143.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.37

method	result	size
risch	$-\frac{1}{2a^5x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{28}\_Z^4-a^{21}\_Z^3+a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{35}-5\right)x+a^{15}\_R^2\right)\right)}{5}-\frac{\ln(a+x)}{5a^7}$	78
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a-2a^2\_R+a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^7}-\frac{1}{2a^5x^2}-\frac{\ln(a+x)}{5a^7}$	105

input
```
int(1/x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^5/x^2+1/5*sum(_R*ln((-6*_R^5*a^35-5)*x+a^15*_R^2),_R=RootOf(_Z^4*a^
28-_Z^3*a^21+_Z^2*a^14-_Z*a^7+1))-1/5*ln(a+x)/a^7
```

3.143.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 15499, normalized size of antiderivative = 73.45

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^3(a^5+x^5)}\,dx$$
$$= -\frac{1}{2a^5x^2} + \frac{-\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 - 125t^3 + 25t^2 - 5t + 1, \left(t \mapsto t\log\left(25t^2a + x\right)\right)\right)}{a^7}$$

input
```
integrate(1/x**3/(a**5+x**5),x)
```

output
```
-1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t*
*2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3(a^5+x^5)}\,dx$$
$$= \frac{\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^2\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^2\sqrt{-2\sqrt{5}+10}} - \frac{\log(a+x)}{a^2} - \frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{a^2(\sqrt{5}+1)} + \frac{\log\left(ax(\sqrt{5}-1)+2a^2+2x^2\right)}{a^2(\sqrt{5}-1)}}{5a^5} - \frac{1}{2a^5x^2}$$

input
```
integrate(1/x^3/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a
^2*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) - log(a + x)/a^2 - log(-
a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) + log(a*x*(sqrt(5)
- 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1)))/a^5 - 1/2/(a^5*x^2)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^7} - \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^7} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^7} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^7} + \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^7} - \frac{\log\left(|a+x|\right)}{5\,a^7} - \frac{1}{2\,a^5x^2}$$

input

```
integrate(1/x^3/(a^5+x^5),x, algorithm="giac")
```

output

```
1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^7 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/
(a*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a
)*x + x^2)/a^7 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^7 +
 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^7 - 1/5*log(abs(a +
x))/a^7 - 1/2/(a^5*x^2)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a^5 + x^5)}\, dx = \frac{\ln\left(a^{20} - \frac{a^{19}\, x\, \left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right) \left(\sqrt{5} + \sqrt{2\,\sqrt{5} - 10} + 1\right)}{20\, a^7} - \frac{1}{2\, a^5\, x^2} - \frac{\ln\left(a^{20} + \frac{x\, \left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)^3 a^{19}}{64}\right) \left(\sqrt{5} + \sqrt{-2\,\sqrt{5} - 10} - 1\right)}{20\, a^7} - \frac{\ln(a + x)}{5\, a^7} + \frac{\ln\left(a^{20} - \frac{a^{19}\, x\, \left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right) \left(\sqrt{5} - \sqrt{2\,\sqrt{5} - 10} + 1\right)}{20\, a^7} + \frac{\ln\left(a^{20} - \frac{a^{19}\, x\, \left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right) \left(\sqrt{-2\,\sqrt{5} - 10} - \sqrt{5} + 1\right)}{20\, a^7}$$

input
```
int(1/(x^3*(a^5 + x^5)),x)
```

output
```
(log(a^20 - (a^19*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2)
 + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^7) - 1/(2*a^5*x^2) - (log(a^20 + (a^
19*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/
2) - 10)^(1/2) - 1))/(20*a^7) - log(a + x)/(5*a^7) + (log(a^20 - (a^19*x*(
5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(
1/2) + 1))/(20*a^7) + (log(a^20 - (a^19*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1
/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^7)
```

3.144 $\int \frac{1}{x^4(a^5+x^5)}\,dx$

3.144.1 Optimal result

Integrand size = 13, antiderivative size = 211

$$\int \frac{1}{x^4(a^5+x^5)}\,dx = -\frac{1}{3a^5x^3} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8} + \frac{\log(a+x)}{5a^8} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}$$

output
```
-1/3/a^5/x^3+1/5*ln(a+x)/a^8-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2
)+1)/a^8-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^8-1/10*arctan
((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^8+1/
10*arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2)
)^(1/2)/a^8
```

3.144.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx$$
$$=\frac{-\frac{20a^3}{x^3}+6\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-6\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)+12\log(a+x)}{60a^8}$$

input
```
Integrate[1/(x^4*(a^5 + x^5)),x]
```

output
```
((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(S
qrt[2*(5 + Sqrt[5])]*a)] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])
*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a + x] - 3*(1 + Sqrt[5])*Log
[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sq
rt[5])*a*x)/2 + x^2])/(60*a^8)
```

3.144.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {847, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx$$
$$\Big\downarrow \text{847}$$
$$-\frac{\int \frac{x}{a^5+x^5}dx}{a^5}-\frac{1}{3a^5x^3}$$
$$\Big\downarrow \text{822}$$
$$-\frac{-\frac{\int \frac{1}{a+x}dx}{5a^3}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}}{a^5}-\frac{1}{3a^5x^3}$$
$$\Big\downarrow \text{16}$$

$$-\frac{-\frac{2\int \frac{(1-\sqrt{5})a+(1+\sqrt{5})x}{2(2a^2-(1-\sqrt{5})xa+2x^2)}dx}{5a^3}+\frac{2\int \frac{(1+\sqrt{5})a+(1-\sqrt{5})x}{2(2a^2-(1+\sqrt{5})xa+2x^2)}dx}{5a^3}-\frac{\log(a+x)}{5a^3}}{a^5}-\frac{1}{3a^5x^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{(1-\sqrt{5})a+(1+\sqrt{5})x}{2a^2-(1-\sqrt{5})xa+2x^2}dx}{5a^3}+\frac{\int \frac{(1+\sqrt{5})a+(1-\sqrt{5})x}{2a^2-(1+\sqrt{5})xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}}{a^5}-\frac{1}{3a^5x^3}$$

$\Big\downarrow$ 1142

$$-\frac{\frac{\frac{1}{4}(1+\sqrt{5})\int -\frac{(1-\sqrt{5})a-4x}{2a^2-(1-\sqrt{5})xa+2x^2}dx-\sqrt{5}a\int \frac{1}{2a^2-(1-\sqrt{5})xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-(1+\sqrt{5})xa+2x^2}dx+\frac{1}{4}(1-\sqrt{5})\int -\frac{(1+\sqrt{5})a-4x}{2a^2-(1+\sqrt{5})xa+2x^2}dx}{5a^3}}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\sqrt{5}a\int \frac{1}{2a^2-(1-\sqrt{5})xa+2x^2}dx-\frac{1}{4}(1+\sqrt{5})\int \frac{(1-\sqrt{5})a-4x}{2a^2-(1-\sqrt{5})xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-(1+\sqrt{5})xa+2x^2}dx-\frac{1}{4}(1-\sqrt{5})\int \frac{(1+\sqrt{5})a-4x}{2a^2-(1+\sqrt{5})xa+2x^2}dx}{5a^3}-}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$\Big\downarrow$ 1083

$$-\frac{\frac{2\sqrt{5}a\int \frac{1}{-2(5+\sqrt{5})a^2-(4x-(1-\sqrt{5})a)^2}d(4x-(1-\sqrt{5})a)-\frac{1}{4}(1+\sqrt{5})\int \frac{(1-\sqrt{5})a-4x}{2a^2-(1-\sqrt{5})xa+2x^2}dx}{5a^3}+\frac{-\frac{1}{4}(1-\sqrt{5})\int \frac{(1+\sqrt{5})a-4x}{2a^2-(1+\sqrt{5})xa+2x^2}dx-2\sqrt{5}a}{}}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$\Big\downarrow$ 217

$$-\frac{\frac{-\frac{1}{4}(1+\sqrt{5})\int \frac{(1-\sqrt{5})a-4x}{2a^2-(1-\sqrt{5})xa+2x^2}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-(1-\sqrt{5})a}{\sqrt{2(5+\sqrt{5})}a}\right)}{5a^3}+\frac{\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-(1+\sqrt{5})a}{\sqrt{2(5-\sqrt{5})}a}\right)-\frac{1}{4}(1-\sqrt{5})\int \frac{(1+\sqrt{5})a-4x}{2a^2-(1+\sqrt{5})xa+2x^2}}{5a^3}}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$\Big\downarrow$ 1103

3.144. $\int \frac{1}{x^4(a^5+x^5)}\,dx$

$$-\frac{1}{3a^5x^3}-\frac{-\frac{\log(a+x)}{5a^3}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{10}{5-\sqrt{5}}}\,\text{ar}}{5a^3}}{a^5}$$

input
```
Int[1/(x^4*(a^5 + x^5)),x]
```

output
```
-1/3*1/(a^5*x^3) - (-1/5*Log[a + x]/a^3 + (-(Sqrt[10/(5 + Sqrt[5])]*ArcTan
[(-((1 - Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)]) + ((1 + Sqrt[5])*L
og[2*a^2 - (1 - Sqrt[5])*a*x + 2*x^2])/4)/(5*a^3) + (Sqrt[10/(5 - Sqrt[5])
]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 - Sqr
t[5])*Log[2*a^2 - (1 + Sqrt[5])*a*x + 2*x^2])/4)/(5*a^3))/a^5
```

3.144.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
 - 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.144.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.38

method	result	size
risch	$-\frac{1}{3a^5x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{32}\_Z^4+a^{24}\_Z^3+a^{16}\_Z^2+a^8\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{40}+5\right)x-a^{25}\_R^3\right)\right)}{5}+\frac{\ln(-a-x)}{5a^8}$	81
default	$-\frac{1}{3a^5x^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3a^2\_R-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2a^2\_R-a^3}}{5a^8}+\frac{\ln(a+x)}{5a^8}$	109

input
```
int(1/x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^5/x^3+1/5*sum(_R*ln((-6*_R^5*a^40+5)*x-a^25*_R^3),_R=RootOf(_Z^4*a^
32+_Z^3*a^24+_Z^2*a^16+_Z*a^8+1))+1/5/a^8*ln(-a-x)
```

3.144.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 15501, normalized size of antiderivative = 73.46

$$\int \frac{1}{x^4 (a^5 + x^5)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^4 (a^5 + x^5)} \, dx = -\frac{1}{3a^5x^3} + \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t\log\left(125t^3a + x\right)\right)\right)}{a^8}$$

input
```
integrate(1/x**4/(a**5+x**5),x)
```

output
```
-1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**
2 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 (a^5 + x^5)} \, dx = \frac{\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^3\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^3\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{a^3} + \frac{\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a^3\left(\sqrt{5}+1\right)} - \frac{\log\left(ax\left(\sqrt{5}-1\right)+2a^2+2x\right.}{a^3\left(\sqrt{5}-1\right)}}{5a^5} - \frac{1}{3a^5x^3}$$

input
```
integrate(1/x^4/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a
^3*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) + log(a + x)/a^3 + log(-
a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) - log(a*x*(sqrt(5)
- 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1)))/a^5 - 1/3/(a^5*x^3)
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4(a^5+x^5)}\,dx = \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^8} - \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^8} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^8} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^8} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^8} + \frac{\log\left(|a+x|\right)}{5\,a^8} - \frac{1}{3\,a^5x^3}$$

input
```
integrate(1/x^4/(a^5+x^5),x, algorithm="giac")
```

output
```
1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^8 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/
(a*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a
)*x + x^2)/a^8 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^8 -
 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^8 + 1/5*log(abs(a +
x))/a^8 - 1/3/(a^5*x^3)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4 (a^5 + x^5)} \, dx = \frac{\ln(a+x)}{5 a^8} - \frac{1}{3 a^5 x^3} - \frac{\ln\left(a^{15} x - \frac{a^{16} \left(\sqrt{5} - \sqrt{2 \sqrt{5} - 10} + 1\right)^3}{64}\right) \left(\sqrt{5} - \sqrt{2 \sqrt{5} - 10} + 1\right)}{20 a^8} - \frac{\ln\left(a^{15} x - \frac{a^{16} \left(\sqrt{-2 \sqrt{5} - 10} - \sqrt{5} + 1\right)^3}{64}\right) \left(\sqrt{-2 \sqrt{5} - 10} - \sqrt{5} + 1\right)}{20 a^8} + \frac{\ln\left(\frac{\left(\sqrt{5} + \sqrt{-2 \sqrt{5} - 10} - 1\right)^3 a^{16}}{64} + x a^{15}\right) \left(\sqrt{5} + \sqrt{-2 \sqrt{5} - 10} - 1\right)}{20 a^8} - \frac{\ln\left(a^{15} x - \frac{a^{16} \left(\sqrt{5} + \sqrt{2 \sqrt{5} - 10} + 1\right)^3}{64}\right) \left(\sqrt{5} + \sqrt{2 \sqrt{5} - 10} + 1\right)}{20 a^8}$$

input
```
int(1/(x^4*(a^5 + x^5)),x)
```

output
```
log(a + x)/(5*a^8) - 1/(3*a^5*x^3) - (log(a^15*x - (a^16*(5^(1/2) - (2*5^(
1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a
^8) - (log(a^15*x - (a^16*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*
((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^8) + (log(a^15*x + (a^16*(
5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 1
0)^(1/2) - 1))/(20*a^8) - (log(a^15*x - (a^16*(5^(1/2) + (2*5^(1/2) - 10)^
(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^8)
```

3.145 $\int \frac{x^{-m}}{a^5+x^5} dx$

3.145.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{-m}}{a^5+x^5} dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{5}, \frac{6-m}{5}, -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

output
```
x^(1-m)*hypergeom([1, 1/5-1/5*m],[6/5-1/5*m],-x^5/a^5)/a^5/(1-m)
```

3.145.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^5+x^5} dx = -\frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1}{5}-\frac{m}{5}, \frac{6}{5}-\frac{m}{5}, -\frac{x^5}{a^5}\right)}{a^5(-1+m)}$$

input
```
Integrate[1/(x^m*(a^5 + x^5)),x]
```

output
```
-((x^(1 - m)*Hypergeometric2F1[1, 1/5 - m/5, 6/5 - m/5, -(x^5/a^5)])/(a^5*
(-1 + m)))
```

3.145.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-m}}{a^5+x^5}\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

input
```
Int[1/(x^m*(a^5 + x^5)),x]
```

output
```
(x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)])/(a^5*(1
 - m))
```

3.145.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

3.145.4 Maple [F]

$$\int \frac{x^{-m}}{a^5+x^5}dx$$

input
```
int(1/(x^m)/(a^5+x^5),x)
```

output
```
int(1/(x^m)/(a^5+x^5),x)
```

3.145.5 Fricas [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{(a^5+x^5)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^5+x^5),x, algorithm="fricas")
```

output
```
integral(1/((a^5 + x^5)*x^m), x)
```

3.145.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=-\frac{mx^{1-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}+\frac{x^{1-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}$$

input
```
integrate(1/(x**m)/(a**5+x**5),x)
```

output
```
-m*x**(1 - m)*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5
- m/5)/(25*a**5*gamma(6/5 - m/5)) + x**(1 - m)*lerchphi(x**5*exp_polar(I*p
i)/a**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*gamma(6/5 - m/5))
```

3.145.7 Maxima [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{(a^5+x^5)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^5+x^5),x, algorithm="maxima")
```

output
```
integrate(1/((a^5 + x^5)*x^m), x)
```

3.145.8 Giac [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{(a^5+x^5)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^5+x^5),x, algorithm="giac")
```

output
```
integrate(1/((a^5 + x^5)*x^m), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^5+x^5}\,dx=\int \frac{1}{x^m\,(a^5+x^5)}\,dx$$

input
```
int(1/(x^m*(a^5 + x^5)),x)
```

output
```
int(1/(x^m*(a^5 + x^5)), x)
```

3.146 $\int \frac{1+x^4}{1+x^6} dx$

3.146.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1+x^4}{1+x^6} dx = -\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(\sqrt{3}+2x\right)$$

output
```
2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))
```

3.146.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{1+x^4}{1+x^6} dx = \frac{2\arctan(x)}{3}-\frac{1}{3}\arctan\left(\frac{x}{-1+x^2}\right)$$

input
```
Integrate[(1 + x^4)/(1 + x^6),x]
```

output
```
(2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3
```

3.146.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^6+1} \, dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{1}{x^6+1}+\frac{x^4}{x^6+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)$$

input
```
Int[(1 + x^4)/(1 + x^6),x]
```

output
```
-1/3*ArcTan[Sqrt[3] - 2*x] + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.146.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

method	result
risch	$\arctan(x)+\frac{\arctan(x^3)}{3}$
default	$\frac{2\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{3}+\frac{\arctan\left(2x+\sqrt{3}\right)}{3}$
parallelrisch	$\frac{i\ln(x+i)}{3}-\frac{i\ln(x-i)}{3}+\frac{i\ln(x^2+ix-1)}{6}-\frac{i\ln(x^2-ix-1)}{6}$
meijerg	$\frac{x^5\sqrt{3}\,\ln\left(1-\sqrt{3}\,(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}}+\frac{x^5\arctan\left(\frac{(x^6)^{\frac{1}{6}}}{2-\sqrt{3}\,(x^6)^{\frac{1}{6}}}\right)}{6(x^6)^{\frac{5}{6}}}+\frac{x^5\arctan\left((x^6)^{\frac{1}{6}}\right)}{3(x^6)^{\frac{5}{6}}}-\frac{x^5\sqrt{3}\,\ln\left(1+\sqrt{3}\,(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}}$

input
```
int((x^4+1)/(x^6+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+1/3*arctan(x^3)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.26

$$\int \frac{1+x^4}{1+x^6}\,dx=\frac{1}{3}\arctan\left(x^3\right)+\arctan(x)$$

input
```
integrate((x^4+1)/(x^6+1),x, algorithm="fricas")
```

output
```
1/3*arctan(x^3) + arctan(x)
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.23

$$\int \frac{1+x^4}{1+x^6}\,dx=\operatorname{atan}(x)+\frac{\operatorname{atan}(x^3)}{3}$$

input
```
integrate((x**4+1)/(x**6+1),x)
```

output
```
atan(x) + atan(x**3)/3
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{1}{3}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2\,x-\sqrt{3}\right)+\frac{2}{3}\arctan\left(x\right)$$

input
```
integrate((x^4+1)/(x^6+1),x, algorithm="maxima")
```

output
```
1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{1}{3}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2\,x-\sqrt{3}\right)+\frac{2}{3}\arctan\left(x\right)$$

input
```
integrate((x^4+1)/(x^6+1),x, algorithm="giac")
```

output
```
1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.26

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{\mathrm{atan}(x^3)}{3}+\mathrm{atan}(x)$$

input
```
int((x^4 + 1)/(x^6 + 1),x)
```

output
```
atan(x^3)/3 + atan(x)
```

3.147 $\int \frac{1}{(5+3x+x^2)^3} dx$

3.147.1 Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{(5+3x+x^2)^3} dx = \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{12\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}$$

output
```
1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*arctan(1/11*(
3+2*x)*11^(1/2))*11^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{(5+3x+x^2)^3} dx = \frac{\frac{11(3+2x)(41+18x+6x^2)}{(5+3x+x^2)^2} + 24\sqrt{11}\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{2662}$$

input
```
Integrate[(5 + 3*x + x^2)^(-3),x]
```

output
```
((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*Sqrt[11]*ArcTan
[(3 + 2*x)/Sqrt[11]])/2662
```

3.147.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1086, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3x+5)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{11}\int \frac{1}{(x^2+3x+5)^2}dx+\frac{2x+3}{22(x^2+3x+5)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{11}\left(\frac{2}{11}\int \frac{1}{x^2+3x+5}dx+\frac{2x+3}{11(x^2+3x+5)}\right)+\frac{2x+3}{22(x^2+3x+5)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3}{11}\left(\frac{2x+3}{11(x^2+3x+5)}-\frac{4}{11}\int \frac{1}{-(2x+3)^2-11}d(2x+3)\right)+\frac{2x+3}{22(x^2+3x+5)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{11}\left(\frac{4\arctan\left(\frac{2x+3}{\sqrt{11}}\right)}{11\sqrt{11}}+\frac{2x+3}{11(x^2+3x+5)}\right)+\frac{2x+3}{22(x^2+3x+5)^2}$$

input
```
Int[(5 + 3*x + x^2)^(-3),x]
```

output
```
(3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*((3 + 2*x)/(11*(5 + 3*x + x^2)) + (4
*ArcTan[(3 + 2*x)/Sqrt[11]])/(11*Sqrt[11])))/11
```

3.147.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.147.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\frac{6}{121}x^3+\frac{27}{121}x^2+\frac{68}{121}x+\frac{123}{242}}{(x^2+3x+5)^2}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	44
default	$\frac{3+2x}{22(x^2+3x+5)^2}+\frac{\frac{9}{121}+\frac{6x}{121}}{x^2+3x+5}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	52

input
```
int(1/(x^2+3*x+5)^3,x,method=_RETURNVERBOSE)
```

output
```
(6/121*x^3+27/121*x^2+68/121*x+123/242)/(x^2+3*x+5)^2+12/1331*arctan(1/11*
(3+2*x)*11^(1/2))*11^(1/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{132\,x^3+24\,\sqrt{11}(x^4+6\,x^3+19\,x^2+30\,x+25)\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right)+594\,x^2+1496\,x+1353}{2662\,(x^4+6\,x^3+19\,x^2+30\,x+25)}$$

input
```
integrate(1/(x^2+3*x+5)^3,x, algorithm="fricas")
```

output
```
1/2662*(132*x^3 + 24*sqrt(11)*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*arctan(1/
11*sqrt(11)*(2*x + 3)) + 594*x^2 + 1496*x + 1353)/(x^4 + 6*x^3 + 19*x^2 +
30*x + 25)
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12x^3+54x^2+136x+123}{242x^4+1452x^3+4598x^2+7260x+6050} + \frac{12\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{3\sqrt{11}}{11}\right)}{1331}$$

input
```
integrate(1/(x**2+3*x+5)**3,x)
```

output
```
(12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x**2 + 7260
*x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(11)/11)/1331
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + \frac{12\,x^3+54\,x^2+136\,x+123}{242\,(x^4+6\,x^3+19\,x^2+30\,x+25)}$$

input
```
integrate(1/(x^2+3*x+5)^3,x, algorithm="maxima")
```

output
```
12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2
+ 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + \frac{12\,x^3+54\,x^2+136\,x+123}{242\,(x^2+3\,x+5)^2}$$

input
```
integrate(1/(x^2+3*x+5)^3,x, algorithm="giac")
```

output
```
12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2
+ 136*x + 123)/(x^2 + 3*x + 5)^2
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = 6\left(x+\frac{3}{2}\right)\left(\frac{1}{121\,(x^2+3\,x+5)} + \frac{1}{66\,(x^2+3\,x+5)^2}\right) + \frac{12\,\sqrt{11}\,\mathrm{atan}\left(\frac{2\,\sqrt{11}\,\left(x+\frac{3}{2}\right)}{11}\right)}{1331}$$

input
```
int(1/(3*x + x^2 + 5)^3,x)
```

output
```
6*(x + 3/2)*(1/(121*(3*x + x^2 + 5)) + 1/(66*(3*x + x^2 + 5)^2)) + (12*11^
(1/2)*atan((2*11^(1/2)*(x + 3/2))/11))/1331
```

3.148 $\int \frac{1+x^2+x^4}{(1+x^2)^4} dx$

3.148.1 Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1+x^2+x^4}{(1+x^2)^4} dx = \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7\arctan(x)}{16}$$

output
```
1/6*x/(x^2+1)^3-1/24*x/(x^2+1)^2+7/16*x/(x^2+1)+7/16*arctan(x)
```

3.148.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1+x^2+x^4}{(1+x^2)^4} dx = \frac{1}{48}\left(\frac{x(27+40x^2+21x^4)}{(1+x^2)^3} + 21\arctan(x)\right)$$

input
```
Integrate[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

output
```
((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

3.148.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1471, 25, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^2+1}{(x^2+1)^4}dx$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{x}{6(x^2+1)^3}-\frac{1}{6}\int -\frac{6x^2+5}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{6}\int \frac{6x^2+5}{(x^2+1)^3}dx+\frac{x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{1}{6}\left(\frac{21}{4}\int \frac{1}{(x^2+1)^2}dx-\frac{x}{4(x^2+1)^2}\right)+\frac{x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{1}{6}\left(\frac{21}{4}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}\right)-\frac{x}{4(x^2+1)^2}\right)+\frac{x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{6}\left(\frac{21}{4}\left(\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}\right)-\frac{x}{4(x^2+1)^2}\right)+\frac{x}{6(x^2+1)^3}$$

input
```
Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

output
```
x/(6*(1 + x^2)^3) + (-1/4*x/(1 + x^2)^2 + (21*(x/(2*(1 + x^2)) + ArcTan[x]
/2))/4)/6
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.148.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

method	result
default	$\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{(x^2+1)^3}+\frac{7\arctan(x)}{16}$
risch	$\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{(x^2+1)^3}+\frac{7\arctan(x)}{16}$
meijerg	$\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3}+\frac{7\arctan(x)}{16}-\frac{x(-15x^4+40x^2+15)}{240(x^2+1)^3}-\frac{x(-3x^4-8x^2+3)}{48(x^2+1)^3}$
parallelrisch	$-\frac{21i\ln(x-i)x^6-21i\ln(x+i)x^6+63i\ln(x-i)x^4-63i\ln(x+i)x^4-42x^5+63i\ln(x-i)x^2-63i\ln(x+i)x^2-80x^3+21i\ln(x-i)-21i}{96(x^2+1)^3}$

input
```
int((x^4+x^2+1)/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
(7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx=\frac{21\,x^5+40\,x^3+21\,(x^6+3\,x^4+3\,x^2+1)\arctan(x)+27\,x}{48\,(x^6+3\,x^4+3\,x^2+1)}$$

input
```
integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="fricas")
```

output
```
1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) + 27*x)/(x^
6 + 3*x^4 + 3*x^2 + 1)
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx=\frac{21x^5+40x^3+27x}{48x^6+144x^4+144x^2+48}+\frac{7\,\mathrm{atan}\,(x)}{16}$$

input
```
integrate((x**4+x**2+1)/(x**2+1)**4,x)
```

output
```
(21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) + 7*atan(x
)/16
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21\,x^5+40\,x^3+27\,x}{48\,(x^6+3\,x^4+3\,x^2+1)}+\frac{7}{16}\,\arctan{(x)}$$

input
```
integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="maxima")
```

output
```
1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*arctan(x)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21\,x^5+40\,x^3+27\,x}{48\,(x^2+1)^3}+\frac{7}{16}\,\arctan{(x)}$$

input
```
integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="giac")
```

output
```
1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{7\,\mathrm{atan}(x)}{16}+\frac{\frac{7\,x^5}{16}+\frac{5\,x^3}{6}+\frac{9\,x}{16}}{(x^2+1)^3}$$

input
```
int((x^2 + x^4 + 1)/(x^2 + 1)^4,x)
```

output
```
(7*atan(x))/16 + ((9*x)/16 + (5*x^3)/6 + (7*x^5)/16)/(x^2 + 1)^3
```

3.149 $\int \frac{B+Ax}{(c+2bx+ax^2)^2} dx$

3.149.1 Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2} dx = -\frac{bB-Ac-(Ab-aB)x}{2(b^2-ac)(c+2bx+ax^2)} - \frac{(Ab-aB)\text{arctanh}\left(\frac{b+ax}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

output
```
1/2*(-b*B+A*c+(A*b-B*a)*x)/(-a*c+b^2)/(a*x^2+2*b*x+c)-1/2*(A*b-B*a)*arctan
h((a*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2} dx = \frac{\frac{-bB+Ac+Abx-aBx}{c+x(2b+ax)} + \frac{(Ab-aB)\arctan\left(\frac{b+ax}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{2(b^2-ac)}$$

input
```
Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

output
```
((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B)*ArcTan[
(b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

3.149.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax+B}{(ax^2+2bx+c)^2}\,dx$$

$$\Big\downarrow \ 1159$$

$$\frac{(Ab-aB)\int \frac{1}{ax^2+2bx+c}dx}{2(b^2-ac)}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

$$\Big\downarrow \ 1083$$

$$-\frac{(Ab-aB)\int \frac{1}{4(b^2-ac)-(2b+2ax)^2}d(2b+2ax)}{b^2-ac}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

$$\Big\downarrow \ 219$$

$$-\frac{(Ab-aB)\text{arctanh}\left(\frac{2ax+2b}{2\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

input
```
Int[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

output
```
-1/2*(b*B - A*c - (A*b - a*B)*x)/((b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b
 - a*B)*ArcTanh[(2*b + 2*a*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(3/2))
```

3.149.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.149.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

method	result
default	$\frac{(-2Ab+2Ba)x+2bB-2Ac}{(4ac-4b^2)(a\,x^2+2bx+c)}+\frac{(-2Ab+2Ba)\arctan\left(\frac{2ax+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(Ab-Ba)x}{2\left(ac-b^2\right)}-\frac{Ac-bB}{2\left(ac-b^2\right)}}{a\,x^2+2bx+c}+\frac{\ln\left(\left(-a^2c+a\,b^2\right)x-\left(-ac+b^2\right)^{\frac{3}{2}}-abc+b^3\right)Ab}{4\left(-ac+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-a^2c+a\,b^2\right)x-\left(-ac+b^2\right)^{\frac{3}{2}}-abc+b^3\right)Ba}{4\left(-ac+b^2\right)^{\frac{3}{2}}}-\frac{\ln(}{}$

input
```
int((A*x+B)/(a*x^2+2*b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)+(-2*A*b+2*B*a
)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 448, normalized size of antiderivative = 4.98

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx$$
$$= \left[-\frac{2\,Bb^3+2\,Aac^2-((Ba^2-Aab)x^2+(Ba-Ab)c+2\,(Bab-Ab^2)x)\sqrt{b^2-ac}\log\left(\frac{a^2x^2+2\,abx+2\,b^2-ac+2\,\sqrt{}}{ax^2+2\,bx+c}\right.}{4\,(b^4c-2\,ab^2c^2+a^2c^3+(ab^4-2\,a^2b^2c+a^3c^2)x^2+2\,(b}\right.$$
$$-\frac{Bb^3+Aac^2-((Ba^2-Aab)x^2+(Ba-Ab)c+2\,(Bab-Ab^2)x)\sqrt{-b^2+ac}\arctan\left(-\frac{\sqrt{-b^2+ac}(ax+b)}{b^2-ac}\right)}{2\,(b^4c-2\,ab^2c^2+a^2c^3+(ab^4-2\,a^2b^2c+a^3c^2)x^2+2\,(b^5-2\,ab^3}$$

input
```
integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*b^3 + 2*A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a
*b - A*b^2)*x)*sqrt(b^2 - a*c)*log((a^2*x^2 + 2*a*b*x + 2*b^2 - a*c + 2*sq
rt(b^2 - a*c)*(a*x + b))/(a*x^2 + 2*b*x + c)) - 2*(B*a*b + A*b^2)*c + 2*(B
*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2 + a^2*c^3 + (a
*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1
/2*(B*b^3 + A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*
b^2)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(a*x + b)/(b^2 - a*c)) -
 (B*a*b + A*b^2)*c + (B*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a
*b^2*c^2 + a^2*c^3 + (a*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^
3*c + a^2*b*c^2)*x)]
```

3.149.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(75) = 150$.

Time = 0.54 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.59

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx =$$

$$-\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)-b^4\sqrt{-\frac{1}{(ac-b^2)^3}}}{-Aab+Ba^2}\right.}{4}$$

$$+\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab+a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)-2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)+b^4\sqrt{-\frac{1}{(ac-b^2)^3}}}{-Aab+Ba^2}\right.}{4}$$

$$+\frac{-Ac+Bb+x(-Ab+Ba)}{2ac^2-2b^2c+x^2\cdot(2a^2c-2ab^2)+x\,(4abc-4b^3)}$$

input
```
integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

output
```
-sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b - a**2*c**
2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)*
*3)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B
*a**2))/4 + sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b
 + a**2*c**2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a
*c - b**2)**3)*(-A*b + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/
(-A*a*b + B*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c
+ x**2*(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))
```

3.149.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = -\frac{(Ba-Ab)\arctan\left(\frac{ax+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2-ac)\sqrt{-b^2+ac}} - \frac{Bax-Abx+Bb-Ac}{2\,(ax^2+2\,bx+c)(b^2-ac)}$$

input
```
integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^2
 + a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x^2 + 2*b*x + c)*(b^2 - a*c
))
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{2\,(a\,c-b^2)\left(\frac{(4\,b^3-4\,a\,b\,c)\,(A\,b-B\,a)}{8\,(a\,c-b^2)^{5/2}}-\frac{a\,x\,(A\,b-B\,a)}{2\,(a\,c-b^2)^{3/2}}\right)}{A\,b-B\,a}\right)(A\,b-B\,a)}{2\,(a\,c-b^2)^{3/2}} - \frac{\frac{A\,c-B\,b}{2\,(a\,c-b^2)}+\frac{x\,(A\,b-B\,a)}{2\,(a\,c-b^2)}}{a\,x^2+2\,b\,x+c}$$

input
```
int((B + A*x)/(c + 2*b*x + a*x^2)^2,x)
```

output
```
(atan((2*(a*c - b^2)*(((4*b^3 - 4*a*b*c)*(A*b - B*a))/(8*(a*c - b^2)^(5/2)
) - (a*x*(A*b - B*a))/(2*(a*c - b^2)^(3/2))))/(A*b - B*a))*(A*b - B*a))/(2
*(a*c - b^2)^(3/2)) - ((A*c - B*b)/(2*(a*c - b^2)) + (x*(A*b - B*a))/(2*(a
*c - b^2)))/(c + 2*b*x + a*x^2)
```

3.149. $\int \frac{B+Ax}{(c+2bx+ax^2)^2} \, dx$

3.150 $\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx$

3.150.1 Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx=\frac{1-x}{5-4x+x^2}-2\arctan(2-x)+\frac{5}{2}\log\left(5-4x+x^2\right)$$

output
```
(1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

3.150.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx=\frac{1-x}{5-4x+x^2}-2\arctan(2-x)+\frac{5}{2}\log\left(5-4x+x^2\right)$$

input
```
Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

output
```
(1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

3.150.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^3-27x^2+55x-41}{(x^2-4x+5)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{4}\int -\frac{4(8-5x)}{x^2-4x+5}dx+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1-x}{x^2-4x+5}-\int \frac{8-5x}{x^2-4x+5}dx$$

$$\Big\downarrow \text{1142}$$

$$2\int \frac{1}{x^2-4x+5}dx+\frac{5}{2}\int -\frac{2(2-x)}{x^2-4x+5}dx+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1}{x^2-4x+5}dx-5\int \frac{2-x}{x^2-4x+5}dx+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \text{1083}$$

$$-5\int \frac{2-x}{x^2-4x+5}dx-4\int \frac{1}{-(2x-4)^2-4}d(2x-4)+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \text{217}$$

$$-5\int \frac{2-x}{x^2-4x+5}dx+2\arctan\left(\frac{1}{2}(2x-4)\right)+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \text{1103}$$

$$2\arctan\left(\frac{1}{2}(2x-4)\right)+\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)$$

input
```
Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

output
```
(1 - x)/(5 - 4*x + x^2) + 2*ArcTan[(-4 + 2*x)/2] + (5*Log[5 - 4*x + x^2])/
2
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.150. $\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2} \, dx$

3.150.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result
default	$\frac{1-x}{x^2-4x+5}+2\arctan\left(-2+x\right)+\frac{5\ln\left(x^2-4x+5\right)}{2}$
risch	$\frac{1-x}{x^2-4x+5}+2\arctan\left(-2+x\right)+\frac{5\ln\left(x^2-4x+5\right)}{2}$
parallelrisch	$-\frac{-40i\ln(x-2-i)x-10i\ln(x-2+i)x^2-50i\ln(x-2+i)-25\ln(x-2-i)x^2+10i\ln(x-2-i)x^2-25\ln(x-2+i)x^2+40i\ln(x-2+i)}{10(x^2-4x+5)}$

input
```
int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x,method=_RETURNVERBOSE)
```

output
```
(1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2}\,dx = \frac{4\left(x^2-4\,x+5\right)\arctan\left(x-2\right)+5\left(x^2-4\,x+5\right)\log\left(x^2-4\,x+5\right)-2\,x+2}{2\left(x^2-4\,x+5\right)}$$

input
```
integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="fricas")
```

output
```
1/2*(4*(x^2 - 4*x + 5)*arctan(x - 2) + 5*(x^2 - 4*x + 5)*log(x^2 - 4*x + 5
) - 2*x + 2)/(x^2 - 4*x + 5)
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2}\,dx = \frac{1-x}{x^2-4x+5}+\frac{5\log\left(x^2-4x+5\right)}{2}+2\operatorname{atan}\left(x-2\right)$$

input
```
integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
```

output
```
(1 - x)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x - 2)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -\frac{x-1}{x^2-4\,x+5} + 2\,\arctan\left(x-2\right) + \frac{5}{2}\,\log\left(x^2-4\,x+5\right)$$

input
```
integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="maxima")
```

output
```
-(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -\frac{x-1}{x^2-4\,x+5} + 2\,\arctan\left(x-2\right) + \frac{5}{2}\,\log\left(x^2-4\,x+5\right)$$

input
```
integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="giac")
```

output
```
-(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = 2\,\mathrm{atan}(x-2) + \frac{5\,\ln\left(x^2-4\,x+5\right)}{2} - \frac{x}{x^2-4\,x+5} + \frac{1}{x^2-4\,x+5}$$

input
```
int((55*x - 27*x^2 + 5*x^3 - 41)/(x^2 - 4*x + 5)^2,x)
```

output
```
2*atan(x - 2) + (5*log(x^2 - 4*x + 5))/2 - x/(x^2 - 4*x + 5) + 1/(x^2 - 4*
x + 5)
```

3.151 $\int \frac{1}{(-1+x^3)^2}\,dx$

3.151.1 Optimal result

Integrand size = 7, antiderivative size = 57

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{x}{3(1-x^3)} + \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\log(1-x) + \frac{1}{9}\log\left(1+x+x^2\right)$$

output
```
1/3*x/(-x^3+1)-2/9*ln(1-x)+1/9*ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))
*3^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{1}{9}\left(-\frac{3x}{-1+x^3} + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log(1-x) + \log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(-1 + x^3)^(-2),x]
```

output
```
((-3*x)/(-1 + x^3) + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] +
Log[1 + x + x^2])/9
```

3.151.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {749, 750, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-1)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\int \frac{1}{x^3-1}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{3}\int \frac{1}{x-1}dx\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{3}\log(1-x)\right)$$

input
```
Int[(-1 + x^3)^(-2),x]
```

output
```
x/(3*(1 - x^3)) - (2*(Log[1 - x]/3 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]])
 - Log[1 + x + x^2]/2)/3))/3
```

3.151.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.151.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{x}{3(x^3-1)}-\frac{2\ln(-1+x)}{9}+\frac{\ln(x^2+x+1)}{9}+\frac{2\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{9}$	41
default	$-\frac{1}{9(-1+x)}-\frac{2\ln(-1+x)}{9}+\frac{-1+x}{9x^2+9x+9}+\frac{\ln(x^2+x+1)}{9}+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	53
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3x(-1)^{\frac{1}{3}}}{-3x^3+3}-\frac{2x(-1)^{\frac{1}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	86

input
```
int(1/(x^3-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x/(x^3-1)-2/9*ln(-1+x)+1/9*ln(x^2+x+1)+2/9*3^(1/2)*arctan(2/3*3^(1/2)
*(x+1/2))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2\sqrt{3}(x^3-1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+(x^3-1)\log\left(x^2+x+1\right)-2\left(x^3-1\right)\log\left(x-1\right)-3x}{9\left(x^3-1\right)}$$

input
```
integrate(1/(x^3-1)^2,x, algorithm="fricas")
```

output
```
1/9*(2*sqrt(3)*(x^3 - 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + (x^3 - 1)*log(x^2
 + x + 1) - 2*(x^3 - 1)*log(x - 1) - 3*x)/(x^3 - 1)
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-1+x^3)^2}\,dx = -\frac{x}{3x^3-3} - \frac{2\log(x-1)}{9} + \frac{\log(x^2+x+1)}{9} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(1/(x**3-1)**2,x)
```

output
```
-x/(3*x**3 - 3) - 2*log(x - 1)/9 + log(x**2 + x + 1)/9 + 2*sqrt(3)*atan(2*
sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{x}{3(x^3-1)} + \frac{1}{9}\log\left(x^2+x+1\right) - \frac{2}{9}\log(x-1)$$

input
```
integrate(1/(x^3-1)^2,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2
+ x + 1) - 2/9*log(x - 1)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{x}{3(x^3-1)} + \frac{1}{9}\log\left(x^2+x+1\right) - \frac{2}{9}\log(|x-1|)$$

input
```
integrate(1/(x^3-1)^2,x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2
+ x + 1) - 2/9*log(abs(x - 1))
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-1+x^3)^2}\,dx = -\frac{2\ln(x-1)}{9} - \frac{x}{3(x^3-1)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(2x+1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

input
```
int(1/(x^3 - 1)^2,x)
```

output
```
log(2*x + 3^(1/2)*1i + 1)*((3^(1/2)*1i)/9 + 1/9) - x/(3*(x^3 - 1)) - log(x
 - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1/9) - (2*log(x - 1))/9
```

3.152 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

3.152.1 Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{4}{x}-\frac{7x}{4(1+x^2)^2}-\frac{25x}{8(1+x^2)}-\frac{57\arctan(x)}{8}$$

output
```
-4/x-7/4*x/(x^2+1)^2-25/8*x/(x^2+1)-57/8*arctan(x)
```

3.152.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{32+103x^2+57x^4}{8x(1+x^2)^2}-\frac{57\arctan(x)}{8}$$

input
```
Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
-1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

3.152.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1583, 25, 361, 25, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^4+4}{x^2(x^2+1)^3}dx$$

$$\Big\downarrow \text{1583}$$

$$-\frac{1}{4}\int -\frac{16-9x^2}{x^2(x^2+1)^2}dx-\frac{7x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{16-9x^2}{x^2(x^2+1)^2}dx-\frac{7x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{32-25x^2}{x^2(x^2+1)}dx-\frac{25x}{2(x^2+1)}\right)-\frac{7x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{32-25x^2}{x^2(x^2+1)}dx-\frac{25x}{2(x^2+1)}\right)-\frac{7x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-57\int \frac{1}{x^2+1}dx-\frac{32}{x}\right)-\frac{25x}{2(x^2+1)}\right)-\frac{7x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-57\arctan(x)-\frac{32}{x}\right)-\frac{25x}{2(x^2+1)}\right)-\frac{7x}{4(x^2+1)^2}$$

input
```
Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
(-7*x)/(4*(1 + x^2)^2) + ((-25*x)/(2*(1 + x^2)) + (-32/x - 57*ArcTan[x])/2
)/4
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1583
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1))   Int[x^
m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*(-d)^(-m/2 +
1)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e
*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && IL
tQ[q, -1] && ILtQ[m/2, 0]
```

3.152.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\frac{25}{8}x^3+\frac{39}{8}x}{(x^2+1)^2}-\frac{57\arctan(x)}{8}-\frac{4}{x}$	29
risch	$\frac{-\frac{57}{8}x^4-\frac{103}{8}x^2-4}{(x^2+1)^2 x}-\frac{57\arctan(x)}{8}$	29
meijerg	$-\frac{15x^4+25x^2+8}{2x(x^2+1)^2}-\frac{57\arctan(x)}{8}-\frac{x(-3x^2+3)}{8(x^2+1)^2}$	47
parallelrisch	$\frac{57i\ln(x-i)x^5-57i\ln(x+i)x^5-64+114i\ln(x-i)x^3-114i\ln(x+i)x^3-114x^4+57i\ln(x-i)x-57i\ln(x+i)x-206x^2}{16x(x^2+1)^2}$	87

input
```
int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,x^4+103\,x^2+57\,(x^5+2\,x^3+x)\arctan(x)+32}{8\,(x^5+2\,x^3+x)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
 + x)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = \frac{-57x^4-103x^2-32}{8x^5+16x^3+8x}-\frac{57\,\mathrm{atan}(x)}{8}$$

input
```
integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

output
```
(-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)} - \frac{57}{8}\arctan(x)$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{25\,x^3+39\,x}{8\,(x^2+1)^2} - \frac{4}{x} - \frac{57}{8}\arctan(x)$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,\mathrm{atan}(x)}{8} - \frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,(x^2+1)^2}$$

input
```
int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
```

output
```
- (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

3.153 $\int \frac{x}{1+x^6} dx$

3.153.1 Optimal result

Integrand size = 9, antiderivative size = 49

$$\int \frac{x}{1+x^6} dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)$$

output
```
1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2
)
```

3.153.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{x}{1+x^6} dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\sqrt{3}-2x\right) - 2\sqrt{3}\arctan\left(\sqrt{3}+2x\right) + 2\log\left(1+x^2\right) - \log\left(1-\sqrt{3}x+x^2\right) - \log\left(1+\sqrt{3}x+x^2\right)\right)$$

input
```
Integrate[x/(1 + x^6),x]
```

output
```
(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Lo
g[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

3.153.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {807, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^6+1}dx^2$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^2+1}dx^2+\frac{1}{3}\int \frac{2-x^2}{x^4-x^2+1}dx^2\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x^2}{x^4-x^2+1}dx^2+\frac{1}{3}\log \left(x^2+1\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{3}\log \left(x^2+1\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{3}\log \left(x^2+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-3\int \frac{1}{-x^4-3}d\left(2x^2-1\right)\right)+\frac{1}{3}\log \left(x^2+1\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\sqrt{3}\arctan \left(\frac{2x^2-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log \left(x^2+1\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan \left(\frac{2x^2-1}{\sqrt{3}}\right)-\frac{1}{2}\log \left(x^4-x^2+1\right)\right)+\frac{1}{3}\log \left(x^2+1\right)\right)$$

input
```
Int[x/(1 + x^6),x]
```

output
```
(Log[1 + x^2]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x^2)/Sqrt[3]] - Log[1 - x^2 + x^
4]/2)/3)/2
```

3.153.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.153.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\ln(x^2+1)}{6}-\frac{\ln(x^4-x^2+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	39
default	$-\frac{\ln(x^4-x^2+1)}{12}+\frac{\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2+1)}{6}$	41
meijerg	$\frac{x^2\ln\left(1+(x^6)^{\frac{1}{3}}\right)}{6(x^6)^{\frac{1}{3}}}-\frac{x^2\ln\left(1-(x^6)^{\frac{1}{3}}+(x^6)^{\frac{2}{3}}\right)}{12(x^6)^{\frac{1}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{3}}}{2-(x^6)^{\frac{1}{3}}}\right)}{6(x^6)^{\frac{1}{3}}}$	80

input
```
int(x/(x^6+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^6+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/
6*log(x^2 + 1)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{1+x^6}\,dx = \frac{\log(x^2+1)}{6} - \frac{\log(x^4-x^2+1)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x/(x**6+1),x)
```

output
```
log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3
- sqrt(3)/3)/6
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{12}\log(x^4-x^2+1) + \frac{1}{6}\log(x^2+1)$$

input
```
integrate(x/(x^6+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/
6*log(x^2 + 1)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{12}\log(x^4-x^2+1) + \frac{1}{6}\log(x^2+1)$$

input
```
integrate(x/(x^6+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/
6*log(x^2 + 1)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{1+x^6}\,dx = \frac{\ln(x^2+1)}{6} - \ln\left(x^2 - \frac{\sqrt{3}\,1\mathrm{i}}{2} - \frac{1}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x^2 + \frac{\sqrt{3}\,1\mathrm{i}}{2} - \frac{1}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x/(x^6 + 1),x)
```

output
```
log(x^2 + 1)/6 - log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12)
+ log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

3.154 $\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx$

3.154.1 Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx = \frac{\log(-nx+x^n)}{n}$$

output
```
ln(-n*x+x^n)/n
```

3.154.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx = \frac{\log(-nx+x^n)}{n}$$

input
```
Integrate[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

output
```
Log[-(n*x) + x^n]/n
```

3.154.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2027, 1016, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}-1}{x^n-nx}\,dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x^{-n}\left(x^{n-1}-1\right)}{1-nx^{1-n}}dx$$
$$\Big\downarrow \text{1016}$$
$$\int \frac{1-x^{1-n}}{x\left(1-nx^{1-n}\right)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int \frac{x^{n-1}\left(1-x^{1-n}\right)}{1-nx^{1-n}}dx^{1-n}}{1-n}$$
$$\Big\downarrow \text{86}$$
$$\frac{\int \left(x^{n-1}+\frac{1-n}{nx^{1-n}-1}\right)dx^{1-n}}{1-n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\log\left(x^{1-n}\right)+\frac{(1-n)\log\left(1-nx^{1-n}\right)}{n}}{1-n}$$

input
```
Int[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

output
```
(Log[x^(1 - n)] + ((1 - n)*Log[1 - n*x^(1 - n)])/n)/(1 - n)
```

3.154.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.154.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{\ln(-nx+x^n)}{n}$	14
norman	$\frac{\ln(nx-e^{n\ln(x)})}{n}$	17

input
```
int((-1+x^(-1+n))/(-n*x+x^n),x,method=_RETURNVERBOSE)
```

output
```
ln(-n*x+x^n)/n
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log\left(-nx+x^n\right)}{n}$$

input
```
integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="fricas")
```

output
```
log(-n*x + x^n)/n
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\begin{cases}\frac{\log\left(x-\frac{x^n}{n}\right)}{n} & \text{for } n\neq 0\\ -x+\log\left(x\right) & \text{otherwise}\end{cases}$$

input
```
integrate((-1+x**(-1+n))/(-n*x+x**n),x)
```

output
```
Piecewise((log(x - x**n/n)/n, Ne(n, 0)), (-x + log(x), True))
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log\left(nx-x^n\right)}{n}$$

input
```
integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="maxima")
```

output
```
log(n*x - x^n)/n
```

3.154.8 Giac [F]

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx = \int -\frac{x^{n-1}-1}{nx-x^n}\,dx$$

input
```
integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="giac")
```

output
```
integrate(-(x^(n - 1) - 1)/(n*x - x^n), x)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx = \frac{\ln(n\,x-x^n)}{n}$$

input
```
int(-(x^(n - 1) - 1)/(n*x - x^n),x)
```

output
```
log(n*x - x^n)/n
```

3.155 $\int \frac{x^3}{1-2x^2+3x^4}\,dx$

3.155.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = -\frac{\arctan\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{1}{12}\log\left(1-2x^2+3x^4\right)$$

output
```
1/12*ln(3*x^4-2*x^2+1)-1/12*arctan(1/2*(-3*x^2+1)*2^(1/2))*2^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\left(\sqrt{2}\arctan\left(\frac{-1+3x^2}{\sqrt{2}}\right) + \log\left(1-2x^2+3x^4\right)\right)$$

input
```
Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
```

output
```
(Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/12
```

3.155.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1434, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{3x^4-2x^2+1}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{3x^4-2x^2+1}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{3x^4-2x^2+1}dx^2+\frac{1}{6}\int -\frac{2(1-3x^2)}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{3x^4-2x^2+1}dx^2-\frac{1}{3}\int \frac{1-3x^2}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{1}{-x^4-8}d(6x^2-2)-\frac{1}{3}\int \frac{1-3x^2}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6x^2-2}{2\sqrt{2}}\right)}{3\sqrt{2}}-\frac{1}{3}\int \frac{1-3x^2}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6x^2-2}{2\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{6}\log\left(3x^4-2x^2+1\right)\right)$$

input
```
Int[x^3/(1 - 2*x^2 + 3*x^4),x]
```

output
```
(ArcTan[(-2 + 6*x^2)/(2*Sqrt[2])]/(3*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]/6)/
2
```

3.155.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.155.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(3x^4-2x^2+1)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(6x^2-2)\sqrt{2}}{4}\right)}{12}$	35
risch	$\frac{\ln(9x^4-6x^2+3)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(3x^2-1)\sqrt{2}}{2}\right)}{12}$	35

input
```
int(x^3/(3*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(3*x^4-2*x^2+1)+1/12*2^(1/2)*arctan(1/4*(6*x^2-2)*2^(1/2))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x^2-1)\right)+\frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

input
```
integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{\log\left(x^4-\frac{2x^2}{3}+\frac{1}{3}\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x^2}{2}-\frac{\sqrt{2}}{2}\right)}{12}$$

input
```
integrate(x**3/(3*x**4-2*x**2+1),x)
```

output
```
log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - sqrt(2)/2)
/12
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x^2-1)\right)+\frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

input
```
integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\big(3\,x^2-1\big)\right) + \frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

input
```
integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{\ln\left(x^4-\frac{2\,x^2}{3}+\frac{1}{3}\right)}{12} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\,\sqrt{2}\,x^2}{2}\right)}{12}$$

input
```
int(x^3/(3*x^4 - 2*x^2 + 1),x)
```

output
```
log(x^4 - (2*x^2)/3 + 1/3)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*x^2)/
2))/12
```

3.156 $\int \frac{x^5}{-4+x^2+3x^4} \, dx$

3.156.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^5}{-4+x^2+3x^4} \, dx = \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)$$

output
```
1/6*x^2+1/14*ln(-x^2+1)-8/63*ln(3*x^2+4)
```

3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{-4+x^2+3x^4} \, dx = \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)$$

input
```
Integrate[x^5/(-4 + x^2 + 3*x^4),x]
```

output
```
x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

3.156.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{3x^4+x^2-4}\,dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int -\frac{x^4}{-3x^4-x^2+4}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{x^4}{-3x^4-x^2+4}dx^2$$

$$\Big\downarrow \text{ 1141}$$

$$\frac{3}{2}\int \left(-\frac{16}{63\,(3x^2+4)}+\frac{1}{9}-\frac{1}{21\,(1-x^2)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{2}\left(\frac{x^2}{9}+\frac{1}{21}\log\left(1-x^2\right)-\frac{16}{189}\log\left(3x^2+4\right)\right)$$

input
```
Int[x^5/(-4 + x^2 + 3*x^4),x]
```

output
```
(3*(x^2/9 + Log[1 - x^2]/21 - (16*Log[4 + 3*x^2])/189))/2
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.156.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x^2}{6}+\frac{\ln(x^2-1)}{14}-\frac{8\ln(3x^2+4)}{63}$	25
risch	$\frac{x^2}{6}+\frac{\ln(x^2-1)}{14}-\frac{8\ln(3x^2+4)}{63}$	25
parallelrisch	$\frac{x^2}{6}+\frac{\ln(-1+x)}{14}+\frac{\ln(1+x)}{14}-\frac{8\ln\left(x^2+\frac{4}{3}\right)}{63}$	27
norman	$\frac{x^2}{6}+\frac{\ln(-1+x)}{14}+\frac{\ln(1+x)}{14}-\frac{8\ln(3x^2+4)}{63}$	29

input
```
int(x^5/(3*x^4+x^2-4),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2+1/14*ln(x^2-1)-8/63*ln(3*x^2+4)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(x^2-1\right)$$

input
```
integrate(x^5/(3*x^4+x^2-4),x, algorithm="fricas")
```

output
```
1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{x^2}{6} + \frac{\log\left(x^2-1\right)}{14} - \frac{8\log\left(x^2+\frac{4}{3}\right)}{63}$$

input
```
integrate(x**5/(3*x**4+x**2-4),x)
```

output
```
x**2/6 + log(x**2 - 1)/14 - 8*log(x**2 + 4/3)/63
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(x^2-1\right)$$

input
```
integrate(x^5/(3*x^4+x^2-4),x, algorithm="maxima")
```

output
```
1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^5/(3*x^4+x^2-4),x, algorithm="giac")
```

output
```
1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(abs(x^2 - 1))
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{\ln\left(x^2-1\right)}{14} - \frac{8\,\ln\left(x^2+\frac{4}{3}\right)}{63} + \frac{x^2}{6}$$

input
```
int(x^5/(x^2 + 3*x^4 - 4),x)
```

output
```
log(x^2 - 1)/14 - (8*log(x^2 + 4/3))/63 + x^2/6
```

3.157 $\int \frac{x^2}{9-10x^3+x^6} \, dx$

3.157.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^2}{9-10x^3+x^6} \, dx = -\frac{1}{24} \log \left(1-x^3\right) + \frac{1}{24} \log \left(9-x^3\right)$$

output
```
-1/24*ln(-x^3+1)+1/24*ln(-x^3+9)
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{9-10x^3+x^6} \, dx = -\frac{1}{24} \log \left(1-x^3\right) + \frac{1}{24} \log \left(9-x^3\right)$$

input
```
Integrate[x^2/(9 - 10*x^3 + x^6),x]
```

output
```
-1/24*Log[1 - x^3] + Log[9 - x^3]/24
```

3.157.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6-10x^3+9}dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{3}\int \frac{1}{x^6-10x^3+9}dx^3$$
$$\Big\downarrow \text{1081}$$
$$\frac{1}{3}\int \left(\frac{1}{8(1-x^3)}-\frac{1}{8(9-x^3)}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{1}{8}\log\left(9-x^3\right)-\frac{1}{8}\log\left(1-x^3\right)\right)$$

input
```
Int[x^2/(9 - 10*x^3 + x^6),x]
```

output
```
(-1/8*Log[1 - x^3] + Log[9 - x^3]/8)/3
```

3.157.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.157.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\ln(x^3-1)}{24}+\frac{\ln(x^3-9)}{24}$	18
risch	$-\frac{\ln(x^3-1)}{24}+\frac{\ln(x^3-9)}{24}$	18
norman	$-\frac{\ln(-1+x)}{24}+\frac{\ln(x^3-9)}{24}-\frac{\ln(x^2+x+1)}{24}$	25
parallelrisch	$-\frac{\ln(-1+x)}{24}+\frac{\ln(x^3-9)}{24}-\frac{\ln(x^2+x+1)}{24}$	25

input
```
int(x^2/(x^6-10*x^3+9),x,method=_RETURNVERBOSE)
```

output
```
-1/24*ln(x^3-1)+1/24*ln(x^3-9)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

input
```
integrate(x^2/(x^6-10*x^3+9),x, algorithm="fricas")
```

output
```
-1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = \frac{\log\left(x^3-9\right)}{24}-\frac{\log\left(x^3-1\right)}{24}$$

input
```
integrate(x**2/(x**6-10*x**3+9),x)
```

output
```
log(x**3 - 9)/24 - log(x**3 - 1)/24
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

input
```
integrate(x^2/(x^6-10*x^3+9),x, algorithm="maxima")
```

output
```
-1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\log\left(\left|x^3-1\right|\right)+\frac{1}{24}\log\left(\left|x^3-9\right|\right)$$

input
```
integrate(x^2/(x^6-10*x^3+9),x, algorithm="giac")
```

output
```
-1/24*log(abs(x^3 - 1)) + 1/24*log(abs(x^3 - 9))
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{81}{320\left(\frac{5x^3}{4}-\frac{9}{8}\right)}-\frac{41}{40}\right)}{12}$$

input
```
int(x^2/(x^6 - 10*x^3 + 9),x)
```

output
```
atanh(81/(320*((5*x^3)/4 - 9/8)) - 41/40)/12
```

3.158 $\int \frac{1-4x^2+x^3}{(-2+x)^4} dx$

3.158.1 Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1-4x^2+x^3}{(-2+x)^4} dx = -\frac{7}{3(2-x)^3}+\frac{2}{(2-x)^2}+\frac{2}{2-x}+\log(2-x)$$

output
```
-7/3/(2-x)^3+2/(2-x)^2+2/(2-x)+ln(2-x)
```

3.158.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1-4x^2+x^3}{(-2+x)^4} dx = \frac{-29+30x-6x^2}{3(-2+x)^3}+\log(-2+x)$$

input
```
Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

output
```
(-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

3.158.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-4x^2+1}{(x-2)^4} \, dx$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(\frac{1}{x-2}+\frac{2}{(x-2)^2}-\frac{4}{(x-2)^3}-\frac{7}{(x-2)^4}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

input
```
Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

output
```
-7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.158.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

method	result	size
norman	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3}+\ln(-2+x)$	22
risch	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3}+\ln(-2+x)$	22
default	$-\frac{2}{-2+x}+\frac{2}{(-2+x)^2}+\frac{7}{3(-2+x)^3}+\ln(-2+x)$	27
parallelrisch	$\frac{3\ln(-2+x)x^3-29-18\ln(-2+x)x^2+36\ln(-2+x)x-6x^2-24\ln(-2+x)+30x}{3(-2+x)^3}$	49
meijerg	$\frac{x\left(\frac{1}{4}x^2-\frac{3}{2}x+3\right)}{48\left(1-\frac{x}{2}\right)^3}+\frac{x\left(\frac{11}{2}x^2-15x+12\right)}{24\left(1-\frac{x}{2}\right)^3}+\ln\left(1-\frac{x}{2}\right)-\frac{x^3}{12\left(1-\frac{x}{2}\right)^3}$	60

input
```
int((x^3-4*x^2+1)/(-2+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(-2*x^2+10*x-29/3)/(-2+x)^3+ln(-2+x)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-3\,(x^3-6\,x^2+12\,x-8)\log(x-2)-30\,x+29}{3\,(x^3-6\,x^2+12\,x-8)}$$

input
```
integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="fricas")
```

output
```
-1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/(x^3 - 6*
x^2 + 12*x - 8)
```

3.158.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \frac{-6x^2+30x-29}{3x^3-18x^2+36x-24} + \log{(x-2)}$$

input
```
integrate((x**3-4*x**2+1)/(-2+x)**4,x)
```

output
```
(-6*x**2 + 30*x - 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-30\,x+29}{3\,(x^3-6\,x^2+12\,x-8)} + \log{(x-2)}$$

input
```
integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="maxima")
```

output
```
-1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + log(x - 2)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-30\,x+29}{3\,(x-2)^3} + \log{(|x-2|)}$$

input
```
integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="giac")
```

output
```
-1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + log(abs(x - 2))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \ln(x-2) - \frac{2\,x^2-10\,x+\frac{29}{3}}{(x-2)^3}$$

input
```
int((x^3 - 4*x^2 + 1)/(x - 2)^4,x)
```

output
```
log(x - 2) - (2*x^2 - 10*x + 29/3)/(x - 2)^3
```

3.159 $\int \frac{x^3}{(-1+x)^{12}}\,dx$

3.159.1 Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = \frac{1}{11(1-x)^{11}} - \frac{3}{10(1-x)^{10}} + \frac{1}{3(1-x)^9} - \frac{1}{8(1-x)^8}$$

output
```
1/11/(1-x)^11-3/10/(1-x)^10+1/3/(1-x)^9-1/8/(1-x)^8
```

3.159.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = \frac{1-11x+55x^2-165x^3}{1320(-1+x)^{11}}$$

input
```
Integrate[x^3/(-1 + x)^12,x]
```

output
```
(1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)
```

3.159.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x-1)^{12}} \, dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{1}{(x-1)^9}+\frac{3}{(x-1)^{10}}+\frac{3}{(x-1)^{11}}+\frac{1}{(x-1)^{12}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

input
```
Int[x^3/(-1 + x)^12,x]
```

output
```
1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)
```

3.159.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.159.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
norman	$\frac{-\frac{1}{8}x^3+\frac{1}{24}x^2-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$	22
risch	$\frac{-\frac{1}{8}x^3+\frac{1}{24}x^2-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$	22
gosper	$-\frac{165x^3-55x^2+11x-1}{1320(-1+x)^{11}}$	23
parallelrisch	$\frac{-165x^3+55x^2-11x+1}{1320(-1+x)^{11}}$	23
default	$-\frac{1}{3(-1+x)^9}-\frac{1}{11(-1+x)^{11}}-\frac{1}{8(-1+x)^8}-\frac{3}{10(-1+x)^{10}}$	30
meijerg	$\frac{x^4(-x^7+11x^6-55x^5+165x^4-330x^3+462x^2-462x+330)}{1320(1-x)^{11}}$	48

input
```
int(x^3/(-1+x)^12,x,method=_RETURNVERBOSE)
```

output
```
1/(-1+x)^11*(-1/8*x^3+1/24*x^2-1/120*x+1/1320)
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(29) = 58.

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1)}$$

input
```
integrate(x^3/(-1+x)^12,x, algorithm="fricas")
```

output
```
-1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

3.159.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = \frac{-165x^3 + 55x^2 - 11x + 1}{1320x^{11} - 14520x^{10} + 72600x^9 - 217800x^8 + 435600x^7 - 609840x^6 + 609840x^5 - 435600x^4 + 217800x^3}$$

input
```
integrate(x**3/(-1+x)**12,x)
```

output
```
(-165*x**3 + 55*x**2 - 11*x + 1)/(1320*x**11 - 14520*x**10 + 72600*x**9 -
217800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 - 435600*x**4 + 2178
00*x**3 - 72600*x**2 + 14520*x - 1320)
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3 - 55\,x^2 + 11\,x - 1}{1320\,(x^{11} - 11\,x^{10} + 55\,x^9 - 165\,x^8 + 330\,x^7 - 462\,x^6 + 462\,x^5 - 330\,x^4 + 165\,x^3 - 55\,x^2 + 11\,x - 1)}$$

input
```
integrate(x^3/(-1+x)^12,x, algorithm="maxima")
```

output
```
-1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,(x-1)^{11}}$$

input
```
integrate(x^3/(-1+x)^12,x, algorithm="giac")
```

output
```
-1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{1}{8\,(x-1)^8}-\frac{1}{3\,(x-1)^9}-\frac{3}{10\,(x-1)^{10}}-\frac{1}{11\,(x-1)^{11}}$$

input
```
int(x^3/(x - 1)^12,x)
```

output
```
- 1/(8*(x - 1)^8) - 1/(3*(x - 1)^9) - 3/(10*(x - 1)^10) - 1/(11*(x - 1)^11
)
```

3.160 $\int \frac{-3x+x^4}{(1+2x)^5} dx$

3.160.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{-3x+x^4}{(1+2x)^5} dx = -\frac{25}{128(1+2x)^4}+\frac{7}{24(1+2x)^3}-\frac{3}{32(1+2x)^2}+\frac{1}{8(1+2x)}+\frac{1}{32}\log(1+2x)$$

output
```
-25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*ln(1+2*x)
```

3.160.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{-3x+x^4}{(1+2x)^5} dx = \frac{49+368x+432x^2+384x^3+12(1+2x)^4\log(1+2x)}{384(1+2x)^4}$$

input
```
Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]
```

output
```
(49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(384*(1 + 2
*x)^4)
```

3.160.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-3x}{(2x+1)^5}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(x^3-3)}{(2x+1)^5}dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{1}{16(2x+1)}-\frac{1}{4(2x+1)^2}+\frac{3}{8(2x+1)^3}-\frac{7}{4(2x+1)^4}+\frac{25}{16(2x+1)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8(2x+1)}-\frac{3}{32(2x+1)^2}+\frac{7}{24(2x+1)^3}-\frac{25}{128(2x+1)^4}+\frac{1}{32}\log(2x+1)$$

input
```
Int[(-3*x + x^4)/(1 + 2*x)^5,x]
```

output
```
-25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1
+ 2*x)) + Log[1 + 2*x]/32
```

3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.160.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{x^3+\frac{9}{8}x^2+\frac{23}{24}x+\frac{49}{384}}{(1+2x)^4}+\frac{\ln(1+2x)}{32}$	34
norman	$\frac{-\frac{37}{12}x^3-\frac{31}{16}x^2-\frac{1}{16}x-\frac{49}{24}x^4}{(1+2x)^4}+\frac{\ln(1+2x)}{32}$	37
default	$-\frac{25}{128(1+2x)^4}+\frac{7}{24(1+2x)^3}-\frac{3}{32(1+2x)^2}+\frac{1}{8+16x}+\frac{\ln(1+2x)}{32}$	46
meijerg	$-\frac{x(1000x^3+1040x^2+420x+60)}{960(1+2x)^4}+\frac{\ln(1+2x)}{32}-\frac{x^2(4x^2+8x+6)}{4(1+2x)^4}$	57
parallelrisch	$\frac{48\ln(x+\frac{1}{2})x^4+96\ln(x+\frac{1}{2})x^3-196x^4+72\ln(x+\frac{1}{2})x^2-296x^3+24\ln(x+\frac{1}{2})x-186x^2+3\ln(x+\frac{1}{2})-6x}{96(1+2x)^4}$	69

input
```
int((x^4-3*x)/(1+2*x)^5,x,method=_RETURNVERBOSE)
```

output
```
16*(1/16*x^3+9/128*x^2+23/384*x+49/6144)/(1+2*x)^4+1/32*ln(1+2*x)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{384\,x^3+432\,x^2+12\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)\log(2\,x+1)+368\,x+49}{384\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)}$$

input
```
integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="fricas")
```

output
```
1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)*log(2*x
 + 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx=\frac{384x^3+432x^2+368x+49}{6144x^4+12288x^3+9216x^2+3072x+384}+\frac{\log{(2x+1)}}{32}$$

input
```
integrate((x**4-3*x)/(1+2*x)**5,x)
```

output
```
(384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216*x**2 + 3
072*x + 384) + log(2*x + 1)/32
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx=\frac{384\,x^3+432\,x^2+368\,x+49}{384\,(16\,x^4+32\,x^3+24\,x^2+8\,x+1)}+\frac{1}{32}\,\log\left(2\,x+1\right)$$

input
```
integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="maxima")
```

output
```
1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1
) + 1/32*log(2*x + 1)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx=\frac{1}{8\,(2\,x+1)}-\frac{3}{32\,(2\,x+1)^2}+\frac{7}{24\,(2\,x+1)^3}-\frac{25}{128\,(2\,x+1)^4}-\frac{1}{32}\,\log\left(\frac{|2\,x+1|}{2\,(2\,x+1)^2}\right)$$

input
```
integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="giac")
```

output
```
1/8/(2*x + 1) - 3/32/(2*x + 1)^2 + 7/24/(2*x + 1)^3 - 25/128/(2*x + 1)^4 -
 1/32*log(1/2*abs(2*x + 1)/(2*x + 1)^2)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx=\frac{\ln\left(x+\frac{1}{2}\right)}{32}+\frac{\frac{x^3}{16}+\frac{9\,x^2}{128}+\frac{23\,x}{384}+\frac{49}{6144}}{x^4+2\,x^3+\frac{3\,x^2}{2}+\frac{x}{2}+\frac{1}{16}}$$

input
```
int(-(3*x - x^4)/(2*x + 1)^5,x)
```

output
```
log(x + 1/2)/32 + ((23*x)/384 + (9*x^2)/128 + x^3/16 + 49/6144)/(x/2 + (3*
x^2)/2 + 2*x^3 + x^4 + 1/16)
```

3.161 $\int \frac{1}{(-1+x)^2(1+x)^3}\,dx$

3.161.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx=\frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3\text{arctanh}(x)}{8}$$

output
```
1/8/(1-x)-1/8/(1+x)^2-1/4/(1+x)+3/8*arctanh(x)
```

3.161.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx=\frac{1}{16}\left(\frac{4-6x-6x^2}{(-1+x)(1+x)^2}-3\log(-1+x)+3\log(1+x)\right)$$

input
```
Integrate[1/((-1 + x)^2*(1 + x)^3),x]
```

output
```
((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1 + x])/16
```

3.161.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)^2(x+1)^3}\,dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(-\frac{3}{8\left(x^2-1\right)}+\frac{1}{8(x-1)^2}+\frac{1}{4(x+1)^2}+\frac{1}{4(x+1)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3\text{arctanh}(x)}{8}+\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}$$

input
```
Int[1/((-1 + x)^2*(1 + x)^3),x]
```

output
```
1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8
```

3.161.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.161.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result
default	$-\frac{1}{8(-1+x)}-\frac{3\ln(-1+x)}{16}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3\ln(1+x)}{16}$
norman	$\frac{-\frac{3}{8}x-\frac{3}{8}x^2+\frac{1}{4}}{(-1+x)(1+x)^2}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$
risch	$\frac{-\frac{3}{8}x-\frac{3}{8}x^2+\frac{1}{4}}{(-1+x)(1+x)^2}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$
parallelrisch	$-\frac{3\ln(-1+x)x^3-3\ln(1+x)x^3-4+3\ln(-1+x)x^2-3\ln(1+x)x^2-3\ln(-1+x)x+3\ln(1+x)x+6x^2-3\ln(-1+x)+3\ln(1+x)+6x}{16(-1+x)(1+x)^2}$

input
```
int(1/(-1+x)^2/(1+x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/(-1+x)-3/16*ln(-1+x)-1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx$$
$$= -\frac{6\,x^2-3\,(x^3+x^2-x-1)\log{(x+1)}+3\,(x^3+x^2-x-1)\log{(x-1)}+6\,x-4}{16\,(x^3+x^2-x-1)}$$

input
```
integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*log(x + 1) + 3*(x^3 + x^2 - x - 1)*lo
g(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)
```

3.161.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{-3x^2-3x+2}{8x^3+8x^2-8x-8} - \frac{3\log(x-1)}{16} + \frac{3\log(x+1)}{16}$$

input
```
integrate(1/(-1+x)**2/(1+x)**3,x)
```

output
```
(-3*x**2 - 3*x + 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*log(x - 1)/16 + 3*log(
x + 1)/16
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{3\,x^2+3\,x-2}{8\,(x^3+x^2-x-1)} + \frac{3}{16}\,\log(x+1) - \frac{3}{16}\,\log(x-1)$$

input
```
integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*log(x + 1) - 3/16*log(x
- 1)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{8\,(x-1)} + \frac{\frac{12}{x-1}+5}{32\,\left(\frac{2}{x-1}+1\right)^2} + \frac{3}{16}\,\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

input
```
integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

output
```
-1/8/(x - 1) + 1/32*(12/(x - 1) + 5)/(2/(x - 1) + 1)^2 + 3/16*log(abs(-2/(
x - 1) - 1))
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{3\,\mathrm{atanh}(x)}{8} + \frac{\frac{3\,x^2}{8} + \frac{3\,x}{8} - \frac{1}{4}}{-x^3 - x^2 + x + 1}$$

input
```
int(1/((x - 1)^2*(x + 1)^3),x)
```

output
```
(3*atanh(x))/8 + ((3*x)/8 + (3*x^2)/8 - 1/4)/(x - x^2 - x^3 + 1)
```

3.162 $\int \frac{1}{(5-6x)^2x^2}\,dx$

3.162.1 Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{6}{25(5-6x)} - \frac{1}{25x} - \frac{12}{125}\log(5-6x) + \frac{12\log(x)}{125}$$

output
```
6/25/(5-6*x)-1/25/x-12/125*ln(5-6*x)+12/125*ln(x)
```

3.162.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{1}{125}\left(\frac{30}{5-6x} - \frac{5}{x} - 12\log(5-6x) + 12\log(x)\right)$$

input
```
Integrate[1/((5 - 6*x)^2*x^2),x]
```

output
```
(30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125
```

3.162.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5-6x)^2x^2} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{25x^2}-\frac{72}{125(6x-5)}+\frac{36}{25(6x-5)^2}+\frac{12}{125x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

input
```
Int[1/((5 - 6*x)^2*x^2),x]
```

output
```
6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125
```

3.162.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.162.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{25x}+\frac{12\ln(x)}{125}-\frac{6}{25(6x-5)}-\frac{12\ln(6x-5)}{125}$	28
risch	$\frac{-\frac{12x}{25}+\frac{1}{5}}{x(6x-5)}+\frac{12\ln(x)}{125}-\frac{12\ln(6x-5)}{125}$	31
norman	$\frac{\frac{1}{5}-\frac{72x^2}{125}}{x(6x-5)}+\frac{12\ln(x)}{125}-\frac{12\ln(6x-5)}{125}$	32
meijerg	$-\frac{1}{25x}+\frac{6}{125}+\frac{12\ln(x)}{125}+\frac{12\ln(2)}{125}+\frac{12\ln(3)}{125}-\frac{12\ln(5)}{125}+\frac{12i\pi}{125}+\frac{108x}{625(3-\frac{18x}{5})}-\frac{12\ln(1-\frac{6x}{5})}{125}$	46
parallelrisch	$\frac{72x^2\ln(x)-72\ln(x-\frac{5}{6})x^2+25-60x\ln(x)+60\ln(x-\frac{5}{6})x-72x^2}{125(6x-5)x}$	48

input
```
int(1/(-6*x+5)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/25/x+12/125*ln(x)-6/25/(6*x-5)-12/125*ln(6*x-5)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{12\,(6\,x^2-5\,x)\log(6\,x-5)-12\,(6\,x^2-5\,x)\log(x)+60\,x-25}{125\,(6\,x^2-5\,x)}$$

input
```
integrate(1/(5-6*x)^2/x^2,x, algorithm="fricas")
```

output
```
-1/125*(12*(6*x^2 - 5*x)*log(6*x - 5) - 12*(6*x^2 - 5*x)*log(x) + 60*x - 2
5)/(6*x^2 - 5*x)
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{5-12x}{150x^2-125x} + \frac{12\log(x)}{125} - \frac{12\log\left(x-\frac{5}{6}\right)}{125}$$

input
```
integrate(1/(5-6*x)**2/x**2,x)
```

output
```
(5 - 12*x)/(150*x**2 - 125*x) + 12*log(x)/125 - 12*log(x - 5/6)/125
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{12\,x-5}{25\,(6\,x^2-5\,x)} - \frac{12}{125}\log(6\,x-5) + \frac{12}{125}\log(x)$$

input
```
integrate(1/(5-6*x)^2/x^2,x, algorithm="maxima")
```

output
```
-1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log(x)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{6}{25\,(6\,x-5)} + \frac{6}{125\,\left(\frac{5}{6\,x-5}+1\right)} + \frac{12}{125}\log\left(\left|-\frac{5}{6\,x-5}-1\right|\right)$$

input
```
integrate(1/(5-6*x)^2/x^2,x, algorithm="giac")
```

output
```
-6/25/(6*x - 5) + 6/125/(5/(6*x - 5) + 1) + 12/125*log(abs(-5/(6*x - 5) -
1))
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{1}{5\,x\,\left(6\,x-5\right)}-\frac{12}{25\,\left(6\,x-5\right)}-\frac{12\,\ln\left(\frac{6\,x-5}{x}\right)}{125}$$

input
```
int(1/(x^2*(6*x - 5)^2),x)
```

output
```
1/(5*x*(6*x - 5)) - 12/(25*(6*x - 5)) - (12*log((6*x - 5)/x))/125
```

3.163 $\int \frac{1}{(-3-2x+x^2)^3}\,dx$

3.163.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{1-x}{16\,(3+2x-x^2)^2} + \frac{3(1-x)}{128\,(3+2x-x^2)} + \frac{3}{512}\log(3-x) - \frac{3}{512}\log(1+x)$$

output
```
1/16*(1-x)/(-x^2+2*x+3)^2+3/128*(1-x)/(-x^2+2*x+3)+3/512*ln(3-x)-3/512*ln(
1+x)
```

3.163.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{1}{512}\left(\frac{4(17-11x-9x^2+3x^3)}{(-3-2x+x^2)^2} + 3\log(3-x) - 3\log(1+x)\right)$$

input
```
Integrate[(-3 - 2*x + x^2)^(-3),x]
```

output
```
((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x] - 3*Log
[1 + x])/512
```

3.163.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x-3)^3} \, dx$$

$$\Big\downarrow \ 1084$$

$$\int \left(-\frac{3}{512(x+1)}-\frac{3}{256(x+1)^2}-\frac{1}{64(x+1)^3}-\frac{3}{512(3-x)}-\frac{3}{256(3-x)^2}-\frac{1}{64(3-x)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{256(3-x)}+\frac{3}{256(x+1)}-\frac{1}{128(3-x)^2}+\frac{1}{128(x+1)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

input
```
Int[(-3 - 2*x + x^2)^(-3),x]
```

output
```
-1/128*1/(3 - x)^2 - 3/(256*(3 - x)) + 1/(128*(1 + x)^2) + 3/(256*(1 + x))
 + (3*Log[3 - x])/512 - (3*Log[1 + x])/512
```

3.163.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.163.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result
norman	$\frac{-\frac{11}{128}x-\frac{9}{128}x^2+\frac{3}{128}x^3+\frac{17}{128}}{(x^2-2x-3)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$
risch	$\frac{-\frac{11}{128}x-\frac{9}{128}x^2+\frac{3}{128}x^3+\frac{17}{128}}{(x^2-2x-3)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$
default	$\frac{1}{128(1+x)^2}+\frac{3}{256(1+x)}-\frac{3\ln(1+x)}{512}-\frac{1}{128(-3+x)^2}+\frac{3}{256(-3+x)}+\frac{3\ln(-3+x)}{512}$
parallelrisch	$-\frac{3\ln(1+x)x^4-3\ln(-3+x)x^4-68-12\ln(1+x)x^3+12\ln(-3+x)x^3-6\ln(1+x)x^2+6\ln(-3+x)x^2-12x^3+36\ln(1+x)x-36\ln(}{512(x^2-2x-3)^2}$

input
```
int(1/(x^2-2*x-3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-11/128*x-9/128*x^2+3/128*x^3+17/128)/(x^2-2*x-3)^2+3/512*ln(-3+x)-3/512*
ln(1+x)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx$$
$$= \frac{12\,x^3-36\,x^2-3\,(x^4-4\,x^3-2\,x^2+12\,x+9)\log(x+1)+3\,(x^4-4\,x^3-2\,x^2+12\,x+9)\log(x-3)-}{512\,(x^4-4\,x^3-2\,x^2+12\,x+9)}$$

input
```
integrate(1/(x^2-2*x-3)^3,x, algorithm="fricas")
```

output
```
1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x + 1) + 3
*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x - 3) - 44*x + 68)/(x^4 - 4*x^3 - 2
*x^2 + 12*x + 9)
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3x^3-9x^2-11x+17}{128x^4-512x^3-256x^2+1536x+1152} + \frac{3\log{(x-3)}}{512} - \frac{3\log{(x+1)}}{512}$$

input
```
integrate(1/(x**2-2*x-3)**3,x)
```

output
```
(3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1536*x + 1
152) + 3*log(x - 3)/512 - 3*log(x + 1)/512
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3\,x^3-9\,x^2-11\,x+17}{128\,(x^4-4\,x^3-2\,x^2+12\,x+9)} - \frac{3}{512}\,\log(x+1) + \frac{3}{512}\,\log(x-3)$$

input
```
integrate(1/(x^2-2*x-3)^3,x, algorithm="maxima")
```

output
```
1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9) - 3/512
*log(x + 1) + 3/512*log(x - 3)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3\,x^3-9\,x^2-11\,x+17}{128\,(x^2-2\,x-3)^2} - \frac{3}{512}\,\log(|x+1|) + \frac{3}{512}\,\log(|x-3|)$$

input
```
integrate(1/(x^2-2*x-3)^3,x, algorithm="giac")
```

output
```
1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*log(abs(x + 1)
) + 3/512*log(abs(x - 3))
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = -\frac{3\ln\left(\frac{x+1}{x-3}\right)}{512} - 6\left(\frac{1}{256\,(-x^2+2\,x+3)} + \frac{1}{96\,(-x^2+2\,x+3)^2}\right)(x-1)$$

input
```
int(-1/(2*x - x^2 + 3)^3,x)
```

output
```
- (3*log((x + 1)/(x - 3)))/512 - 6*(1/(256*(2*x - x^2 + 3)) + 1/(96*(2*x -
 x^2 + 3)^2))*(x - 1)
```

3.164 $\int \frac{1}{(13-4x+x^2)^3} \, dx$

3.164.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{1}{(13-4x+x^2)^3} \, dx = -\frac{2-x}{36(13-4x+x^2)^2} - \frac{2-x}{216(13-4x+x^2)} + \frac{1}{648} \arctan\left(\frac{1}{3}(-2+x)\right)$$

output
```
1/36*(-2+x)/(x^2-4*x+13)^2+1/216*(-2+x)/(x^2-4*x+13)+1/648*arctan(-2/3+1/3
*x)
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{1}{(13-4x+x^2)^3} \, dx = \frac{1}{648}\left(\frac{3(-2+x)(19-4x+x^2)}{(13-4x+x^2)^2} + \arctan\left(\frac{1}{3}(-2+x)\right)\right)$$

input
```
Integrate[(13 - 4*x + x^2)^(-3),x]
```

output
```
((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 + x)/3])/64
8
```

3.164.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1086, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-4x+13)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{12}\int \frac{1}{(x^2-4x+13)^2}dx-\frac{2-x}{36(x^2-4x+13)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{12}\left(\frac{1}{18}\int \frac{1}{x^2-4x+13}dx-\frac{2-x}{18(x^2-4x+13)}\right)-\frac{2-x}{36(x^2-4x+13)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{12}\left(-\frac{1}{9}\int \frac{1}{-(2x-4)^2-36}d(2x-4)-\frac{2-x}{18(x^2-4x+13)}\right)-\frac{2-x}{36(x^2-4x+13)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{12}\left(\frac{1}{54}\arctan\left(\frac{1}{6}(2x-4)\right)-\frac{2-x}{18(x^2-4x+13)}\right)-\frac{2-x}{36(x^2-4x+13)^2}$$

input
```
Int[(13 - 4*x + x^2)^(-3),x]
```

output
```
-1/36*(2 - x)/(13 - 4*x + x^2)^2 + (-1/18*(2 - x)/(13 - 4*x + x^2) + ArcTa
n[(-4 + 2*x)/6]/54)/12
```

3.164.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.164.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{1}{216}x^3-\frac{1}{36}x^2+\frac{1}{8}x-\frac{19}{108}}{(x^2-4x+13)^2}+\frac{\arctan(-\frac{2}{3}+\frac{x}{3})}{648}$
default	$\frac{-4+2x}{72(x^2-4x+13)^2}+\frac{-4+2x}{432x^2-1728x+5616}+\frac{\arctan(-\frac{2}{3}+\frac{x}{3})}{648}$
parallelrisch	$-\frac{7098i\ln(x-2-3i)x^2-28561i\ln(x-2+3i)+17576i\ln(x-2+3i)x+169i\ln(x-2-3i)x^4+1352i\ln(x-2+3i)x^3-1352i\ln(x-2-}{219024(x^2}$

input
```
int(1/(x^2-4*x+13)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/216*x^3-1/36*x^2+1/8*x-19/108)/(x^2-4*x+13)^2+1/648*arctan(-2/3+1/3*x)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{3\,x^3-18\,x^2+(x^4-8\,x^3+42\,x^2-104\,x+169)\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)+81\,x-114}{648\,(x^4-8\,x^3+42\,x^2-104\,x+169)}$$

input
```
integrate(1/(x^2-4*x+13)^3,x, algorithm="fricas")
```

output
```
1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*arctan(1/3*x
- 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6x^2+27x-38}{216x^4-1728x^3+9072x^2-22464x+36504}+\frac{\operatorname{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648}$$

input
```
integrate(1/(x**2-4*x+13)**3,x)
```

output
```
(x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 22464*x +
36504) + atan(x/3 - 2/3)/648
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6\,x^2+27\,x-38}{216\,(x^4-8\,x^3+42\,x^2-104\,x+169)}+\frac{1}{648}\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

input
```
integrate(1/(x^2-4*x+13)^3,x, algorithm="maxima")
```

output
```
1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169) + 1/6
48*arctan(1/3*x - 2/3)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6\,x^2+27\,x-38}{216\,(x^2-4\,x+13)^2}+\frac{1}{648}\,\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

input
```
integrate(1/(x^2-4*x+13)^3,x, algorithm="giac")
```

output
```
1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan(1/3*x -
2/3)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648} + 6\,(x-2)\,\left(\frac{1}{1296\,\left(x^2-4\,x+13\right)}+\frac{1}{216\,(x^2-4\,x+13)^2}\right)$$

input
```
int(1/(x^2 - 4*x + 13)^3,x)
```

output
```
atan(x/3 - 2/3)/648 + 6*(x - 2)*(1/(1296*(x^2 - 4*x + 13)) + 1/(216*(x^2 -
 4*x + 13)^2))
```

3.165 $\int \frac{1}{(2+x)^3(3+x)^4} dx$

3.165.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

output
```
-1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

input
```
Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

3.165.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)^3(x+3)^4}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{10}{x+3}-\frac{6}{(x+3)^2}-\frac{3}{(x+3)^3}-\frac{1}{(x+3)^4}+\frac{10}{x+2}-\frac{4}{(x+2)^2}+\frac{1}{(x+2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

input
```
Int[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

3.165.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.165.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
norman	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$
risch	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$
default	$-\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\ln(2+x)-10\ln(3+x)$
parallelrisch	$\frac{60\ln(2+x)x^5-60\ln(3+x)x^5+2627+780\ln(2+x)x^4-780\ln(3+x)x^4+4020\ln(2+x)x^3-4020\ln(3+x)x^3+60x^4+10260\ln(2+x)}{6(2+x)^2(3+x)}$

input
```
int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*
ln(3+x)
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2-60\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)\log(x+3)+60\,(x^5+13\,x^4+}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 2
16*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108
)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 1
08)
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60x^4+630x^3+2450x^2+4175x+2627}{6x^5+78x^4+402x^3+1026x^2+1296x+648} + 10\log(x+2) - 10\log(x+3)$$

input
```
integrate(1/(2+x)**3/(3+x)**4,x)
```

output
```
(60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x
**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)} - 10\,\log(x+3) + 10\,\log(x+2)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
 171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x+3)^3(x+2)^2} - 10\,\log(|x+3|) + 10\,\log(|x+2|)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) -
10*log(abs(x + 3)) + 10*log(abs(x + 2))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108}-20\,\mathrm{atanh}(2\,x+5)$$

input
```
int(1/((x + 2)^3*(x + 3)^4),x)
```

output
```
((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
 67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

3.166 $\int \frac{x^6}{(-2+x^2)^2} \, dx$

3.166.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{x^6}{(-2+x^2)^2} \, dx = 4x + \frac{x^3}{3} - \frac{2x}{-2+x^2} - 5\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)$$

output
```
4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{(-2+x^2)^2} \, dx = 4x + \frac{x^3}{3} - \frac{2x}{-2+x^2} + \frac{5\log\left(\sqrt{2}-x\right)}{\sqrt{2}} - \frac{5\log\left(\sqrt{2}+x\right)}{\sqrt{2}}$$

input
```
Integrate[x^6/(-2 + x^2)^2,x]
```

output
```
4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (5*Log[Sqr
t[2] + x])/Sqrt[2]
```

3.166.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {252, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^2-2)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5}{2}\int -\frac{x^4}{2-x^2}dx+\frac{x^5}{2(2-x^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^5}{2(2-x^2)}-\frac{5}{2}\int \frac{x^4}{2-x^2}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{x^5}{2(2-x^2)}-\frac{5}{2}\int \left(-x^2+\frac{4}{2-x^2}-2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^5}{2(2-x^2)}-\frac{5}{2}\left(2\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)-\frac{x^3}{3}-2x\right)$$

input
```
Int[x^6/(-2 + x^2)^2,x]
```

output
```
x^5/(2*(2 - x^2)) - (5*(-2*x - x^3/3 + 2*Sqrt[2]*ArcTanh[x/Sqrt[2]]))/2
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.166.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$4x+\frac{x^3}{3}-\frac{2x}{x^2-2}-5\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32
risch	$\frac{x^3}{3}+4x-\frac{2x}{x^2-2}+\frac{5\sqrt{2}\,\ln\left(x-\sqrt{2}\right)}{2}-\frac{5\sqrt{2}\,\ln\left(x+\sqrt{2}\right)}{2}$	44
meijerg	$i\sqrt{2}\left(-\frac{ix\sqrt{2}\left(-\frac{7}{2}x^4-35x^2+105\right)}{42\left(-\frac{x^2}{2}+1\right)}+5i\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\right)$	46

input
```
int(x^6/(x^2-2)^2,x,method=_RETURNVERBOSE)
```

output
```
4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{(-2+x^2)^2}\,dx=\frac{2\,x^5+20\,x^3+15\,\sqrt{2}(x^2-2)\log\left(\frac{x^2-2\,\sqrt{2}x+2}{x^2-2}\right)-60\,x}{6\,(x^2-2)}$$

input
```
integrate(x^6/(x^2-2)^2,x, algorithm="fricas")
```

output
```
1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^
2 - 2)) - 60*x)/(x^2 - 2)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{x^3}{3} + 4x - \frac{2x}{x^2-2} + \frac{5\sqrt{2}\log\left(x-\sqrt{2}\right)}{2} - \frac{5\sqrt{2}\log\left(x+\sqrt{2}\right)}{2}$$

input
```
integrate(x**6/(x**2-2)**2,x)
```

output
```
x**3/3 + 4*x - 2*x/(x**2 - 2) + 5*sqrt(2)*log(x - sqrt(2))/2 - 5*sqrt(2)*l
og(x + sqrt(2))/2
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right) + 4\,x - \frac{2\,x}{x^2-2}$$

input
```
integrate(x^6/(x^2-2)^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))) + 4*x - 2*x/(x^2 -
2)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}\right|}{\left|2\,x+2\,\sqrt{2}\right|}\right) + 4\,x - \frac{2\,x}{x^2-2}$$

input
```
integrate(x^6/(x^2-2)^2,x, algorithm="giac")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2))) + 4*x
 - 2*x/(x^2 - 2)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = 4\,x - \frac{2\,x}{x^2-2} + \frac{x^3}{3} + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,1\mathrm{i}}{2}\right)\,5\mathrm{i}$$

input
```
int(x^6/(x^2 - 2)^2,x)
```

output
```
4*x + 2^(1/2)*atan((2^(1/2)*x*1i)/2)*5i - (2*x)/(x^2 - 2) + x^3/3
```

3.167 $\int \frac{x^8}{\left(4+x^2\right)^4} dx$

3.167.1 Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{x^8}{\left(4+x^2\right)^4} dx=\frac{35x}{16}-\frac{x^7}{6\left(4+x^2\right)^3}-\frac{7x^5}{24\left(4+x^2\right)^2}-\frac{35x^3}{48\left(4+x^2\right)}-\frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

output
```
35/16*x-1/6*x^7/(x^2+4)^3-7/24*x^5/(x^2+4)^2-35/48*x^3/(x^2+4)-35/8*arctan
(1/2*x)
```

3.167.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{\left(4+x^2\right)^4} dx=\frac{x(1680+1120x^2+231x^4+12x^6)}{12\left(4+x^2\right)^3}-\frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

input
```
Integrate[x^8/(4 + x^2)^4,x]
```

output
```
(x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2
])/8
```

3.167.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {252, 252, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(x^2+4)^4} dx$$

$$\Big\downarrow \ 252$$

$$\frac{7}{6}\int \frac{x^6}{(x^2+4)^3}dx-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \ 252$$

$$\frac{7}{6}\left(\frac{5}{4}\int \frac{x^4}{(x^2+4)^2}dx-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \ 252$$

$$\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\int \frac{x^2}{x^2+4}dx-\frac{x^3}{2(x^2+4)}\right)-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \ 262$$

$$\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(x-4\int \frac{1}{x^2+4}dx\right)-\frac{x^3}{2(x^2+4)}\right)-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \ 216$$

$$\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(x-2\arctan\left(\frac{x}{2}\right)\right)-\frac{x^3}{2(x^2+4)}\right)-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

input
```
Int[x^8/(4 + x^2)^4,x]
```

output
```
-1/6*x^7/(4 + x^2)^3 + (7*(-1/4*x^5/(4 + x^2)^2 + (5*(-1/2*x^3/(4 + x^2) +
 (3*(x - 2*ArcTan[x/2]))/2))/4))/6
```

3.167.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.167.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result
risch	$x+\frac{\frac{29}{4}x^{5}+\frac{136}{3}x^{3}+76x}{\left(x^{2}+4\right)^{3}}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$
default	$x-\frac{16\left(-\frac{29}{64}x^{5}-\frac{17}{6}x^{3}-\frac{19}{4}x\right)}{\left(x^{2}+4\right)^{3}}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$
meijerg	$\frac{x\left(\frac{9}{4}x^{6}+\frac{693}{16}x^{4}+210x^{2}+315\right)}{144\left(1+\frac{x^{2}}{4}\right)^{3}}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$
parallelrisch	$\frac{-420i\ln(x+2i)x^{6}-26880i\ln(x+2i)+192x^{7}-5040i\ln(x+2i)x^{4}+20160i\ln(x-2i)x^{2}+3696x^{5}+5040i\ln(x-2i)x^{4}+420i\ln(x-2i)\ldots}{192\left(x^{2}+4\right)^{3}}$

input
```
int(x^8/(x^2+4)^4,x,method=_RETURNVERBOSE)
```

output
```
x+(29/4*x^5+136/3*x^3+76*x)/(x^2+4)^3-35/8*arctan(1/2*x)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(4+x^2)^4}\,dx = \frac{24\,x^7 + 462\,x^5 + 2240\,x^3 - 105\,(x^6 + 12\,x^4 + 48\,x^2 + 64)\arctan\left(\frac{1}{2}\,x\right) + 3360\,x}{24\,(x^6 + 12\,x^4 + 48\,x^2 + 64)}$$

input
```
integrate(x^8/(x^2+4)^4,x, algorithm="fricas")
```

output
```
1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 + 64)*arcta
n(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x + \frac{87x^5 + 544x^3 + 912x}{12x^6 + 144x^4 + 576x^2 + 768} - \frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

input
```
integrate(x**8/(x**2+4)**4,x)
```

output
```
x + (87*x**5 + 544*x**3 + 912*x)/(12*x**6 + 144*x**4 + 576*x**2 + 768) - 3
5*atan(x/2)/8
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x + \frac{87\,x^5 + 544\,x^3 + 912\,x}{12\,(x^6 + 12\,x^4 + 48\,x^2 + 64)} - \frac{35}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(x^8/(x^2+4)^4,x, algorithm="maxima")
```

output
```
x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64) - 35/8*ar
ctan(1/2*x)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x + \frac{87\,x^5 + 544\,x^3 + 912\,x}{12\,(x^2+4)^3} - \frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(x^8/(x^2+4)^4,x, algorithm="giac")
```

output
```
x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*arctan(1/2*x)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x - \frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8} + \frac{\frac{29\,x^5}{4} + \frac{136\,x^3}{3} + 76\,x}{x^6 + 12\,x^4 + 48\,x^2 + 64}$$

input
```
int(x^8/(x^2 + 4)^4,x)
```

output
```
x - (35*atan(x/2))/8 + (76*x + (136*x^3)/3 + (29*x^5)/4)/(48*x^2 + 12*x^4
+ x^6 + 64)
```

3.168 $\int \frac{-4+7x}{(5+2x+3x^2)^2} dx$

3.168.1 Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{-4+7x}{(5+2x+3x^2)^2} dx = -\frac{39+19x}{28(5+2x+3x^2)} - \frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

output
```
1/28*(-39-19*x)/(3*x^2+2*x+5)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(1/2
)
```

3.168.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{-4+7x}{(5+2x+3x^2)^2} dx = \frac{-39-19x}{28(5+2x+3x^2)} - \frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

input
```
Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

output
```
(-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*
Sqrt[14])
```

3.168.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1159, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{7x-4}{(3x^2+2x+5)^2}dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{19}{28}\int \frac{1}{3x^2+2x+5}dx-\frac{19x+39}{28(3x^2+2x+5)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{19}{14}\int \frac{1}{-(6x+2)^2-56}d(6x+2)-\frac{19x+39}{28(3x^2+2x+5)}$$

$$\Big\downarrow \text{217}$$

$$-\frac{19\arctan\left(\frac{6x+2}{2\sqrt{14}}\right)}{28\sqrt{14}}-\frac{19x+39}{28(3x^2+2x+5)}$$

input
```
Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

output
```
-1/28*(39 + 19*x)/(5 + 2*x + 3*x^2) - (19*ArcTan[(2 + 6*x)/(2*Sqrt[14])])/
(28*Sqrt[14])
```

3.168.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.168.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{-\frac{19x}{84}-\frac{13}{28}}{x^2+\frac{2}{3}x+\frac{5}{3}}-\frac{19\arctan\left(\frac{(1+3x)\sqrt{14}}{14}\right)\sqrt{14}}{392}$	34
default	$\frac{-38x-78}{168x^2+112x+280}-\frac{19\sqrt{14}\,\arctan\left(\frac{(6x+2)\sqrt{14}}{28}\right)}{392}$	37

input
```
int((-4+7*x)/(3*x^2+2*x+5)^2,x,method=_RETURNVERBOSE)
```

output
```
(-19/84*x-13/28)/(x^2+2/3*x+5/3)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(
1/2)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx=-\frac{19\,\sqrt{14}(3\,x^2+2\,x+5)\arctan\left(\frac{1}{14}\,\sqrt{14}(3\,x+1)\right)+266\,x+546}{392\,(3\,x^2+2\,x+5)}$$

input
```
integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="fricas")
```

output
```
-1/392*(19*sqrt(14)*(3*x^2 + 2*x + 5)*arctan(1/14*sqrt(14)*(3*x + 1)) + 26
6*x + 546)/(3*x^2 + 2*x + 5)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = \frac{-19x-39}{84x^2+56x+140} - \frac{19\sqrt{14}\,\mathrm{atan}\left(\frac{3\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

input
```
integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

output
```
(-19*x - 39)/(84*x**2 + 56*x + 140) - 19*sqrt(14)*atan(3*sqrt(14)*x/14 + s
qrt(14)/14)/392
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right) - \frac{19x+39}{28(3x^2+2x+5)}$$

input
```
integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="maxima")
```

output
```
-19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
 + 2*x + 5)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right) - \frac{19x+39}{28(3x^2+2x+5)}$$

input
```
integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="giac")
```

output
```
-19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
 + 2*x + 5)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{\frac{19\,x}{84}+\frac{13}{28}}{x^2+\frac{2\,x}{3}+\frac{5}{3}} - \frac{19\,\sqrt{14}\,\mathrm{atan}\left(\frac{3\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

input
```
int((7*x - 4)/(2*x + 3*x^2 + 5)^2,x)
```

output
```
- ((19*x)/84 + 13/28)/((2*x)/3 + x^2 + 5/3) - (19*14^(1/2)*atan((3*14^(1/2
)*x)/14 + 14^(1/2)/14))/392
```

3.169 $\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx$

3.169.1 Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{18-7x}{20(2+4x-3x^2)}-\frac{7\operatorname{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

output
```
1/20*(-18+7*x)/(-3*x^2+4*x+2)-7/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2
)
```

3.169.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{18-7x}{20(-2-4x+3x^2)}-\frac{7\log\left(2+\sqrt{10}-3x\right)}{40\sqrt{10}}+\frac{7\log\left(-2+\sqrt{10}+3x\right)}{40\sqrt{10}}$$

input
```
Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

output
```
(18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[
10]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

3.169.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1159, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-4x}{(3x^2-4x-2)^2}\,dx$$

$$\Big\downarrow \; 1159$$

$$-\frac{7}{20}\int \frac{1}{3x^2-4x-2}dx-\frac{18-7x}{20\,(-3x^2+4x+2)}$$

$$\Big\downarrow \; 1081$$

$$-\frac{21}{20}\int \left(\frac{1}{2\sqrt{10}\,(-3x-\sqrt{10}+2)}-\frac{1}{2\sqrt{10}\,(-3x+\sqrt{10}+2)}\right)dx-\frac{18-7x}{20\,(-3x^2+4x+2)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{18-7x}{20\,(-3x^2+4x+2)}-\frac{21}{20}\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)$$

input
```
Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

output
```
-1/20*(18 - 7*x)/(2 + 4*x - 3*x^2) - (21*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqr
t[10] + Log[2 + Sqrt[10] - 3*x]/(6*Sqrt[10])))/20
```

3.169.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.169.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{14x-36}{40(3x^2-4x-2)}+\frac{7\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$	37
risch	$\frac{-\frac{7x}{60}+\frac{3}{10}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{7\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{7\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$	48

input
```
int((5-4*x)/(3*x^2-4*x-2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2)
)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{7\sqrt{10}(3\,x^2-4\,x-2)\log\left(\frac{9\,x^2+2\,\sqrt{10}(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right)-140\,x+360}{400\,(3\,x^2-4\,x-2)}$$

input
```
integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="fricas")
```

output
```
1/400*(7*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12
*x + 14)/(3*x^2 - 4*x - 2)) - 140*x + 360)/(3*x^2 - 4*x - 2)
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7x-18}{60x^2-80x-40}+\frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

input
```
integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

output
```
-(7*x - 18)/(60*x**2 - 80*x - 40) + 7*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/4
00 - 7*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7}{400}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right)-\frac{7\,x-18}{20\,(3\,x^2-4\,x-2)}$$

input
```
integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="maxima")
```

output
```
-7/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(7*x
 - 18)/(3*x^2 - 4*x - 2)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7}{400}\sqrt{10}\log\left(\frac{|6\,x-2\sqrt{10}-4|}{|6\,x+2\sqrt{10}-4|}\right)-\frac{7\,x-18}{20\,(3\,x^2-4\,x-2)}$$

input
```
integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="giac")
```

output
```
-7/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
 1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{7\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\left(\frac{3\,x}{10}-\frac{1}{5}\right)\right)}{200}+\frac{\frac{7\,x}{60}-\frac{3}{10}}{-x^2+\frac{4\,x}{3}+\frac{2}{3}}$$

input
```
int(-(4*x - 5)/(4*x - 3*x^2 + 2)^2,x)
```

output
```
(7*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + ((7*x)/60 - 3/10)/((4*
x)/3 - x^2 + 2/3)
```

3.170 $\int \frac{x^5}{(1+x^4)^3} dx$

3.170.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{x^5}{(1+x^4)^3} dx = -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{\arctan(x^2)}{16}$$

output
```
-1/8*x^2/(x^4+1)^2+1/16*x^2/(x^4+1)+1/16*arctan(x^2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(1+x^4)^3} dx = \frac{1}{16}\left(\frac{x^2(-1+x^4)}{(1+x^4)^2} + \arctan(x^2)\right)$$

input
```
Integrate[x^5/(1 + x^4)^3,x]
```

output
```
((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16
```

3.170.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {807, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^4+1)^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(x^4+1)^3}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(x^4+1)^2}dx^2-\frac{x^2}{4(x^4+1)^2}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^4+1}dx^2+\frac{x^2}{2(x^4+1)}\right)-\frac{x^2}{4(x^4+1)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\arctan(x^2)}{2}+\frac{x^2}{2(x^4+1)}\right)-\frac{x^2}{4(x^4+1)^2}\right)$$

input
```
Int[x^5/(1 + x^4)^3,x]
```

output
```
(-1/4*x^2/(1 + x^4)^2 + (x^2/(2*(1 + x^4)) + ArcTan[x^2]/2)/4)/2
```

3.170.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.170.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
meijerg	$-\frac{x^{2}\left(-3x^{4}+3\right)}{48\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$	27
risch	$\frac{\frac{1}{16}x^{6}-\frac{1}{16}x^{2}}{\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$	27
default	$\frac{\frac{1}{8}x^{6}-\frac{1}{8}x^{2}}{2\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$	28
parallelrisch	$-\frac{i\ln\left(x^{2}-i\right)x^{8}-i\ln\left(x^{2}+i\right)x^{8}+2i\ln\left(x^{2}-i\right)x^{4}-2i\ln\left(x^{2}+i\right)x^{4}-2x^{6}+i\ln\left(x^{2}-i\right)-i\ln\left(x^{2}+i\right)+2x^{2}}{32\left(x^{4}+1\right)^{2}}$	93

input
```
int(x^5/(x^4+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*x^2*(-3*x^4+3)/(x^4+1)^2+1/16*arctan(x^2)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^6 - x^2 + (x^8 + 2\,x^4 + 1)\arctan\left(x^2\right)}{16\,(x^8 + 2\,x^4 + 1)}$$

input
```
integrate(x^5/(x^4+1)^3,x, algorithm="fricas")
```

output
```
1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*arctan(x^2))/(x^8 + 2*x^4 + 1)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^6 - x^2}{16x^8 + 32x^4 + 16} + \frac{\operatorname{atan}\left(x^2\right)}{16}$$

input
```
integrate(x**5/(x**4+1)**3,x)
```

output
```
(x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + atan(x**2)/16
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^6 - x^2}{16\,(x^8 + 2\,x^4 + 1)} + \frac{1}{16}\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^4+1)^3,x, algorithm="maxima")
```

output
```
1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*arctan(x^2)
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^2-\frac{1}{x^2}}{16\left(\left(x^2-\frac{1}{x^2}\right)^2+4\right)} + \frac{1}{32}\arctan\left(\frac{x^4-1}{2\,x^2}\right)$$

input
```
integrate(x^5/(x^4+1)^3,x, algorithm="giac")
```

output
```
1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 - 1)/x^2)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{\operatorname{atan}(x^2)}{16} - \frac{\frac{x^2}{16}-\frac{x^6}{16}}{x^8+2\,x^4+1}$$

input
```
int(x^5/(x^4 + 1)^3,x)
```

output
```
atan(x^2)/16 - (x^2/16 - x^6/16)/(2*x^4 + x^8 + 1)
```

3.171 $\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx$

3.171.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx = \frac{1}{4(2+2x^2+x^4)} + \frac{1}{4}\log(2+2x^2+x^4)$$

output
```
1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx = \frac{1}{4}\left(\frac{1}{1+(1+x^2)^2} + \log\left(1+(1+x^2)^2\right)\right)$$

input
```
Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

output
```
((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4
```

3.171.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1576, 1110, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x^2+1\right)^3}{\left(x^4+2x^2+2\right)^2}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{\left(x^2+1\right)^3}{\left(x^4+2x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{1110}$$

$$\frac{1}{2}\left(\int \frac{x^2+1}{x^4+2x^2+2}dx^2-\frac{\left(x^2+1\right)^2}{2\left(x^4+2x^2+2\right)}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(x^4+2x^2+2\right)-\frac{\left(x^2+1\right)^2}{2\left(x^4+2x^2+2\right)}\right)$$

input
```
Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

output
```
(-1/2*(1 + x^2)^2/(2 + 2*x^2 + x^4) + Log[2 + 2*x^2 + x^4]/2)/2
```

3.171.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1110
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))),
x] - Simp[d*e*((m - 1)/(b*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0] && N
eQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.171.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{1}{4x^4+8x^2+8}+\frac{\ln\left(x^4+2x^2+2\right)}{4}$	29
norman	$\frac{1}{4x^4+8x^2+8}+\frac{\ln\left(x^4+2x^2+2\right)}{4}$	29
risch	$\frac{1}{4x^4+8x^2+8}+\frac{\ln\left(x^4+2x^2+2\right)}{4}$	29
parallelrisch	$\frac{\ln\left(x^4+2x^2+2\right)x^4+1+2\ln\left(x^4+2x^2+2\right)x^2+2\ln\left(x^4+2x^2+2\right)}{4x^4+8x^2+8}$	61

input
```
int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{(x^4+2\,x^2+2)\log\left(x^4+2\,x^2+2\right)+1}{4\,(x^4+2\,x^2+2)}$$

input
```
integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/4*((x^4 + 2*x^2 + 2)*log(x^4 + 2*x^2 + 2) + 1)/(x^4 + 2*x^2 + 2)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{\log\left(x^4+2x^2+2\right)}{4} + \frac{1}{4x^4+8x^2+8}$$

input
```
integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)
```

output
```
log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4\,(x^4+2\,x^2+2)} + \frac{1}{4}\,\log\left(x^4+2\,x^2+2\right)$$

input
```
integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4\,(x^4+2\,x^2+2)} - \frac{1}{4}\log\left(\frac{1}{2\,(x^4+2\,x^2+2)}\right)$$

input
```
integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="giac")
```

output
```
1/4/(x^4 + 2*x^2 + 2) - 1/4*log(1/2/(x^4 + 2*x^2 + 2))
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{\ln(x^4+2\,x^2+2)}{4} + \frac{1}{4\,(x^4+2\,x^2+2)}$$

input
```
int((x*(x^2 + 1)^3)/(2*x^2 + x^4 + 2)^2,x)
```

output
```
log(2*x^2 + x^4 + 2)/4 + 1/(4*(2*x^2 + x^4 + 2))
```

3.172 $\int \frac{x^3}{(a^4+x^4)^3} \, dx$

3.172.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8(a^4+x^4)^2}$$

output
```
-1/8/(a^4+x^4)^2
```

3.172.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8(a^4+x^4)^2}$$

input
```
Integrate[x^3/(a^4 + x^4)^3,x]
```

output
```
-1/8*1/(a^4 + x^4)^2
```

3.172.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{8(a^4+x^4)^2}$$

input
```
Int[x^3/(a^4 + x^4)^3,x]
```

output
```
-1/8*1/(a^4 + x^4)^2
```

3.172.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.172.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1}{8(a^4+x^4)^2}$	12
derivativedivides	$-\frac{1}{8(a^4+x^4)^2}$	12
default	$-\frac{1}{8(a^4+x^4)^2}$	12
norman	$-\frac{1}{8(a^4+x^4)^2}$	12
risch	$-\frac{1}{8(a^4+x^4)^2}$	12
parallelrisch	$-\frac{1}{8(a^4+x^4)^2}$	12

input
```
int(x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a^4+x^4)^2
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^8+2\,a^4x^4+x^8)}$$

input
```
integrate(x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
-1/8/(a^8 + 2*a^4*x^4 + x^8)
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8a^8+16a^4x^4+8x^8}$$

input
```
integrate(x**3/(a**4+x**4)**3,x)
```

output
```
-1/(8*a**8 + 16*a**4*x**4 + 8*x**8)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^4+x^4)^2}$$

input
```
integrate(x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
-1/8/(a^4 + x^4)^2
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^4+x^4)^2}$$

input
```
integrate(x^3/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-1/8/(a^4 + x^4)^2
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^4+x^4)^2}$$

input
```
int(x^3/(a^4 + x^4)^3,x)
```

output
```
-1/(8*(a^4 + x^4)^2)
```

3.173 $\int \frac{1}{x(a^4+x^4)^3} dx$

3.173.1 Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a^4+x^4)^3} dx = \frac{1}{8a^4(a^4+x^4)^2} + \frac{1}{4a^8(a^4+x^4)} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4+x^4)}{4a^{12}}$$

output
```
1/8/a^4/(a^4+x^4)^2+1/4/a^8/(a^4+x^4)+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

3.173.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a^4+x^4)^3} dx = \frac{\frac{3a^8+2a^4x^4}{(a^4+x^4)^2} + 8\log(x) - 2\log(a^4+x^4)}{8a^{12}}$$

input
```
Integrate[1/(x*(a^4 + x^4)^3),x]
```

output
```
((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])/(8*a^12)
```

3.173.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\,(a^4+x^4)^3}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{1}{x^4\,(a^4+x^4)^3}dx^4$$

$$\Big\downarrow \; 54$$

$$\frac{1}{4}\int \left(\frac{1}{a^{12}x^4}-\frac{1}{a^{12}\,(a^4+x^4)}-\frac{1}{a^8\,(a^4+x^4)^2}-\frac{1}{a^4\,(a^4+x^4)^3}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a^{12}}+\frac{1}{2a^4\,(a^4+x^4)^2}-\frac{\log\left(a^4+x^4\right)}{a^{12}}+\frac{1}{a^8\,(a^4+x^4)}\right)$$

input
```
Int[1/(x*(a^4 + x^4)^3),x]
```

output
```
(1/(2*a^4*(a^4 + x^4)^2) + 1/(a^8*(a^4 + x^4)) + Log[x^4]/a^12 - Log[a^4 +
 x^4]/a^12)/4
```

3.173.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.173.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{(a^4+x^4)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln(a^4+x^4)}{4a^{12}}$	45
risch	$\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{(a^4+x^4)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln(a^4+x^4)}{4a^{12}}$	45
default	$-\frac{\frac{\ln\left(a^4+x^4\right)}{2}-\frac{a^4}{2\left(a^4+x^4\right)}-\frac{a^8}{4\left(a^4+x^4\right)^2}}{2a^{12}}+\frac{\ln(x)}{a^{12}}$	52
parallelrisch	$\frac{8\ln(x)x^8+16\ln(x)x^4a^4+8\ln(x)a^8-2\ln(a^4+x^4)x^8-4\ln(a^4+x^4)x^4a^4-2\ln(a^4+x^4)a^8+2a^4x^4+3a^8}{8a^{12}(a^4+x^4)^2}$	95

input
```
int(1/x/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/8/a^4+1/4/a^8*x^4)/(a^4+x^4)^2+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{3\,a^8+2\,a^4x^4-2\left(a^8+2\,a^4x^4+x^8\right)\log\left(a^4+x^4\right)+8\left(a^8+2\,a^4x^4+x^8\right)\log\left(x\right)}{8\left(a^{20}+2\,a^{16}x^4+a^{12}x^8\right)}$$

input
```
integrate(1/x/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
1/8*(3*a^8 + 2*a^4*x^4 - 2*(a^8 + 2*a^4*x^4 + x^8)*log(a^4 + x^4) + 8*(a^8
 + 2*a^4*x^4 + x^8)*log(x))/(a^20 + 2*a^16*x^4 + a^12*x^8)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx=\frac{3a^4+2x^4}{8a^{16}+16a^{12}x^4+8a^8x^8}+\frac{\log\left(x\right)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}$$

input
```
integrate(1/x/(a**4+x**4)**3,x)
```

output
```
(3*a**4 + 2*x**4)/(8*a**16 + 16*a**12*x**4 + 8*a**8*x**8) + log(x)/a**12 -
 log(a**4 + x**4)/(4*a**12)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx=\frac{3\,a^4+2\,x^4}{8\left(a^{16}+2\,a^{12}x^4+a^8x^8\right)}-\frac{\log\left(a^4+x^4\right)}{4\,a^{12}}+\frac{\log\left(x^4\right)}{4\,a^{12}}$$

input
```
integrate(1/x/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
1/8*(3*a^4 + 2*x^4)/(a^16 + 2*a^12*x^4 + a^8*x^8) - 1/4*log(a^4 + x^4)/a^1
2 + 1/4*log(x^4)/a^12
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx=-\frac{\log\left(a^4+x^4\right)}{4\,a^{12}}+\frac{\log\left(x^4\right)}{4\,a^{12}}+\frac{6\,a^8+8\,a^4x^4+3\,x^8}{8\left(a^4+x^4\right)^2a^{12}}$$

input
```
integrate(1/x/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-1/4*log(a^4 + x^4)/a^12 + 1/4*log(x^4)/a^12 + 1/8*(6*a^8 + 8*a^4*x^4 + 3*
x^8)/((a^4 + x^4)^2*a^12)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{\ln(x)}{a^{12}} + \frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{a^8+2a^4x^4+x^8} - \frac{\ln(a^4+x^4)}{4a^{12}}$$

input
```
int(1/(x*(a^4 + x^4)^3),x)
```

output
```
log(x)/a^12 + (3/(8*a^4) + x^4/(4*a^8))/(a^8 + x^8 + 2*a^4*x^4) - log(a^4
+ x^4)/(4*a^12)
```

3.174 $\int \frac{1}{x^2(a^4+x^4)^3}\,dx$

3.174.1 Optimal result

Integrand size = 13, antiderivative size = 157

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)} + \frac{45\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}} + \frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}$$

output
```
-45/32/a^12/x+1/8/a^4/x/(a^4+x^4)^2+9/32/a^8/x/(a^4+x^4)+45/128*arctan(1-x
*2^(1/2)/a)/a^13*2^(1/2)-45/128*arctan(1+x*2^(1/2)/a)/a^13*2^(1/2)-45/256*
ln(a^2+x^2-a*x*2^(1/2))/a^13*2^(1/2)+45/256*ln(a^2+x^2+a*x*2^(1/2))/a^13*2
^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = -\frac{\frac{256a}{x}+\frac{32a^5x^3}{\left(a^4+x^4\right)^2}+\frac{104ax^3}{a^4+x^4}-90\sqrt{2}\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+90\sqrt{2}\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+45\sqrt{2}\log\left(a^2-\sqrt{2}ax+\right.}{256a^{13}}$$

input
```
Integrate[1/(x^2*(a^4 + x^4)^3),x]
```

output
```
-1/256*((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4) -
 90*Sqrt[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 + (Sqrt[2]*x)/
a] + 45*Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqrt[2]*Log[a^2 + Sqrt[2
]*a*x + x^2])/a^13
```

3.174.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {819, 819, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^4+x^4)^3} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{9 \int \frac{1}{x^2 (a^4+x^4)^2}dx}{8a^4}+\frac{1}{8a^4x (a^4+x^4)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{9 \left(\frac{5 \int \frac{1}{x^2 \left(a^4+x^4\right)}dx}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x (a^4+x^4)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{9 \left(\frac{5 \left(-\frac{\int \frac{x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x (a^4+x^4)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{a^2-\sqrt{2}xa+x^2}dx+\frac{1}{2}\int \frac{1}{a^2+\sqrt{2}xa+x^2}dx\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}x}{a}\right)^2-1}d\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}x}{a}+1\right)^2-1}d\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$\Big\downarrow$ 217

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$\Big\downarrow$ 1479

3.174. $\int \frac{1}{x^2\left(a^4+x^4\right)^3}dx$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}+\frac{\int -\frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{a+\sqrt{2}x}{a^2+\sqrt{2}xa+x^2}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \; 1103$$

3.174. $\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx$

$$\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9\left(\frac{1}{4a^4x(a^4+x^4)}+\frac{5\left(-\frac{1}{a^4x}-\frac{\frac{1}{2}\left(\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}\right)}{4a^4}\right)}{8a^4}$$

input
```
Int[1/(x^2*(a^4 + x^4)^3),x]
```

output
```
1/(8*a^4*x*(a^4 + x^4)^2) + (9*(1/(4*a^4*x*(a^4 + x^4)) + (5*(-(1/(a^4*x))
 - ((-(ArcTan[1 - (Sqrt[2]*x)/a]/(Sqrt[2]*a)) + ArcTan[1 + (Sqrt[2]*x)/a]/
(Sqrt[2]*a))/2 + (Log[a^2 - Sqrt[2]*a*x + x^2]/(2*Sqrt[2]*a) - Log[a^2 + S
qrt[2]*a*x + x^2]/(2*Sqrt[2]*a))/2)/a^4))/(4*a^4)))/(8*a^4)
```

3.174.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.174. $\int \frac{1}{x^2(a^4+x^4)^3} \, dx$

3.174.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{-\frac{45x^8}{32a^{12}}-\frac{81x^4}{32a^8}-\frac{1}{a^4}}{x(a^4+x^4)^2}+\frac{45\left(\sum_{\_R=\mathrm{RootOf}\left(a^{52}\_Z^4+1\right)}\_R\ln\left(\left(5\_R^4a^{52}+4\right)x+\_R^3a^{40}\right)\right)}{128}$	75
default	$-\frac{\frac{\frac{17}{32}a^4x^3+\frac{13}{32}x^7}{\left(a^4+x^4\right)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x^2-\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}-1\right)\right)}{256\left(a^4\right)^{\frac{1}{4}}}}{a^{12}}-\frac{1}{a^{12}x}$	124

input
```
int(1/x^2/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(-45/32/a^12*x^8-81/32/a^8*x^4-1/a^4)/x/(a^4+x^4)^2+45/128*sum(_R*ln((5*_R
^4*a^52+4)*x+_R^3*a^40),_R=RootOf(_Z^4*a^52+1))
```

3.174.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx =$$
$$-\frac{128\,a^8+324\,a^4x^4+180\,x^8+45\left(a^{20}x+2\,a^{16}x^5+a^{12}x^9\right)\left(-\frac{1}{a^{52}}\right)^{\frac{1}{4}}\log\left(a^{40}\left(-\frac{1}{a^{52}}\right)^{\frac{3}{4}}+x\right)+45\left(-i\,a^{20}x\right.}{}$$

input
```
integrate(1/x^2/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
-1/128*(128*a^8 + 324*a^4*x^4 + 180*x^8 + 45*(a^20*x + 2*a^16*x^5 + a^12*x
^9)*(-1/a^52)^(1/4)*log(a^40*(-1/a^52)^(3/4) + x) + 45*(-I*a^20*x - 2*I*a^
16*x^5 - I*a^12*x^9)*(-1/a^52)^(1/4)*log(I*a^40*(-1/a^52)^(3/4) + x) + 45*
(I*a^20*x + 2*I*a^16*x^5 + I*a^12*x^9)*(-1/a^52)^(1/4)*log(-I*a^40*(-1/a^5
2)^(3/4) + x) - 45*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(-1/a^52)^(1/4)*log(-a
^40*(-1/a^52)^(3/4) + x))/(a^20*x + 2*a^16*x^5 + a^12*x^9)
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 (a^4 + x^4)^3} \, dx = \frac{-32a^8 - 81a^4x^4 - 45x^8}{32a^{20}x + 64a^{16}x^5 + 32a^{12}x^9} + \frac{\operatorname{RootSum}\left(268435456t^4 + 4100625, \left(t \mapsto t \log\left(-\frac{2097152t^3a}{91125} + x\right)\right)\right)}{a^{13}}$$

input
```
integrate(1/x**2/(a**4+x**4)**3,x)
```

output
```
(-32*a**8 - 81*a**4*x**4 - 45*x**8)/(32*a**20*x + 64*a**16*x**5 + 32*a**12
*x**9) + RootSum(268435456*_t**4 + 4100625, Lambda(_t, _t*log(-2097152*_t*
*3*a/91125 + x)))/a**13
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a^4 + x^4)^3} \, dx = -\frac{32\,a^8 + 81\,a^4x^4 + 45\,x^8}{32\,(a^{20}x + 2\,a^{16}x^5 + a^{12}x^9)} - \frac{45\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a + 2\,x\right)}{2\,a}\right)}{a} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a - 2\,x\right)}{2\,a}\right)}{a} - \frac{\sqrt{2}\log\left(\sqrt{2}ax + a^2 + x^2\right)}{a} + \frac{\sqrt{2}\log\left(-\sqrt{2}ax + a^2 + x^2\right)}{a}\right)}{256\,a^{12}}$$

input
```
integrate(1/x^2/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
-1/32*(32*a^8 + 81*a^4*x^4 + 45*x^8)/(a^20*x + 2*a^16*x^5 + a^12*x^9) - 45
/256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - sqrt(2)*log(sqrt(2)*a*x + a^2 + x
^2)/a + sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a)/a^12
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = -\frac{45\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{128\,a^{14}} - \frac{45\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{128\,a^{14}} + \frac{45\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{256\,a^{14}} - \frac{45\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{256\,a^{14}} - \frac{17\,a^4x^3+13\,x^7}{32\,(a^4+x^4)^2a^{12}} - \frac{1}{a^{12}x}$$

input
```
integrate(1/x^2/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-45/128*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a
^14 - 45/128*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs
(a))/a^14 + 45/256*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a
^14 - 45/256*sqrt(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14 -
 1/32*(17*a^4*x^3 + 13*x^7)/((a^4 + x^4)^2*a^12) - 1/(a^12*x)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = \frac{45\,(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{64\,a^{13}} - \frac{45\,(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{64\,a^{13}} - \frac{\frac{1}{a^4}+\frac{81\,x^4}{32\,a^8}+\frac{45\,x^8}{32\,a^{12}}}{a^8\,x+2\,a^4\,x^5+x^9}$$

input
```
int(1/(x^2*(a^4 + x^4)^3),x)
```

output
```
(45*(-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(64*a^13) - (45*(-1)^(1/4)*atan(((
-1)^(1/4)*x)/a))/(64*a^13) - (1/a^4 + (81*x^4)/(32*a^8) + (45*x^8)/(32*a^1
2))/(a^8*x + x^9 + 2*a^4*x^5)
```

3.175 $\int \frac{1}{x^3(a^4+x^4)^3}\,dx$

3.175.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15\arctan\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

output
```
-15/16/a^12/x^2+1/8/a^4/x^2/(a^4+x^4)^2+5/16/a^8/x^2/(a^4+x^4)-15/16*arcta
n(x^2/a^2)/a^14
```

3.175.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=\frac{-\frac{a^2\left(8a^8+25a^4x^4+15x^8\right)}{x^2\left(a^4+x^4\right)^2}+15\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+15\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{16a^{14}}$$

input
```
Integrate[1/(x^3*(a^4 + x^4)^3),x]
```

output
```
(-((a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*ArcTan[1
- (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a])/(16*a^14)
```

3.175.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {807, 253, 253, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a^4+x^4\right)^3} \, dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a^4+x^4\right)^3}dx^2$$

$$\Big\downarrow \ 253$$

$$\frac{1}{2} \left(\frac{5 \int \frac{1}{x^4 \left(a^4+x^4\right)^2}dx^2}{4 a^4}+\frac{1}{4 a^4 x^2 \left(a^4+x^4\right)^2}\right)$$

$$\Big\downarrow \ 253$$

$$\frac{1}{2} \left(\frac{5 \left(\frac{3 \int \frac{1}{x^4 \left(a^4+x^4\right)}dx^2}{2 a^4}+\frac{1}{2 a^4 x^2 \left(a^4+x^4\right)}\right)}{4 a^4}+\frac{1}{4 a^4 x^2 \left(a^4+x^4\right)^2}\right)$$

$$\Big\downarrow \ 264$$

$$\frac{1}{2} \left(\frac{5 \left(\frac{3 \left(-\frac{\int \frac{1}{a^4+x^4}dx^2}{a^4}-\frac{1}{a^4 x^2}\right)}{2 a^4}+\frac{1}{2 a^4 x^2 \left(a^4+x^4\right)}\right)}{4 a^4}+\frac{1}{4 a^4 x^2 \left(a^4+x^4\right)^2}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(\frac{1}{4a^4x^2\left(a^4+x^4\right)^2}+\frac{5\left(\frac{1}{2a^4x^2(a^4+x^4)}+\frac{3\left(-\frac{1}{a^4x^2}-\frac{\arctan\left(\frac{x^2}{a^2}\right)}{a^6}\right)}{2a^4}\right)}{4a^4}\right)$$

input
```
Int[1/(x^3*(a^4 + x^4)^3),x]
```

output
```
(1/(4*a^4*x^2*(a^4 + x^4)^2) + (5*(1/(2*a^4*x^2*(a^4 + x^4)) + (3*(-(1/(a^
4*x^2)) - ArcTan[x^2/a^2]/a^6))/(2*a^4)))/(4*a^4))/2
```

3.175.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.175.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\frac{\frac{9}{8}a^{4}x^{2}+\frac{7}{8}x^{6}}{\left(a^{4}+x^{4}\right)^{2}}+\frac{15\arctan\left(\frac{x^{2}}{a^{2}}\right)}{8a^{2}}}{2a^{12}}-\frac{1}{2a^{12}x^{2}}$
risch	$\frac{-\frac{15x^{8}}{16a^{12}}-\frac{25x^{4}}{16a^{8}}-\frac{1}{2a^{4}}}{x^{2}(a^{4}+x^{4})^{2}}+\frac{15\left(\sum_{\_R=\mathrm{RootOf}\left(a^{28}\_Z^{2}+1\right)}\_R\ln\left(\left(-5\_R^{2}a^{28}-4\right)x^{2}-a^{16}\_R\right)\right)}{32}$
parallelrisch	$\frac{15i\ln(-ia^{2}+x^{2})x^{10}+30i\ln(-ia^{2}+x^{2})x^{6}a^{4}+15i\ln(-ia^{2}+x^{2})x^{2}a^{8}-15i\ln(ia^{2}+x^{2})x^{10}-30i\ln(ia^{2}+x^{2})x^{6}a^{4}-15i\ln(ia^{2}+x^{2}}{32a^{14}x^{2}(a^{4}+x^{4})^{2}}$

input
```
int(1/x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^12*((9/8*a^4*x^2+7/8*x^6)/(a^4+x^4)^2+15/8*arctan(x^2/a^2)/a^2)-1/2
/a^12/x^2
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{3}\left(a^{4}+x^{4}\right)^{3}}\,dx=-\frac{8\,a^{10}+25\,a^{6}x^{4}+15\,a^{2}x^{8}+15\,\left(a^{8}x^{2}+2\,a^{4}x^{6}+x^{10}\right)\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\,\left(a^{22}x^{2}+2\,a^{18}x^{6}+a^{14}x^{10}\right)}$$

input
```
integrate(1/x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
-1/16*(8*a^10 + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6 + x^10)*
arctan(x^2/a^2))/(a^22*x^2 + 2*a^18*x^6 + a^14*x^10)
```

3.175.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a^4 + x^4)^3} \, dx = \frac{-8a^8 - 25a^4x^4 - 15x^8}{16a^{20}x^2 + 32a^{16}x^6 + 16a^{12}x^{10}} + \frac{\frac{15i \log(-ia^2 + x^2)}{32} - \frac{15i \log(ia^2 + x^2)}{32}}{a^{14}}$$

input
```
integrate(1/x**3/(a**4+x**4)**3,x)
```

output
```
(-8*a**8 - 25*a**4*x**4 - 15*x**8)/(16*a**20*x**2 + 32*a**16*x**6 + 16*a**
12*x**10) + (15*I*log(-I*a**2 + x**2)/32 - 15*I*log(I*a**2 + x**2)/32)/a**
14
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a^4 + x^4)^3} \, dx = -\frac{8 a^8 + 25 a^4 x^4 + 15 x^8}{16 (a^{20} x^2 + 2 a^{16} x^6 + a^{12} x^{10})} - \frac{15 \arctan\left(\frac{x^2}{a^2}\right)}{16 a^{14}}$$

input
```
integrate(1/x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
-1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^20*x^2 + 2*a^16*x^6 + a^12*x^10) -
15/16*arctan(x^2/a^2)/a^14
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (a^4 + x^4)^3} \, dx = -\frac{9 a^4 x^2 + 7 x^6}{16 (a^4 + x^4)^2 a^{12}} - \frac{15 \arctan\left(\frac{x^2}{a^2}\right)}{16 a^{14}} - \frac{1}{2 a^{12} x^2}$$

input
```
integrate(1/x^3/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^12) - 15/16*arctan(x^2/a^2)/a^1
4 - 1/2/(a^12*x^2)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a^4+x^4)^3}\,dx = -\frac{15\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{16\,a^{14}} - \frac{\frac{a^{10}}{2}+\frac{25\,a^6\,x^4}{16}+\frac{15\,a^2\,x^8}{16}}{a^{14}\,x^2\,(a^4+x^4)^2}$$

input
```
int(1/(x^3*(a^4 + x^4)^3),x)
```

output
```
- (15*atan(x^2/a^2))/(16*a^14) - (a^10/2 + (15*a^2*x^8)/16 + (25*a^6*x^4)/
16)/(a^14*x^2*(a^4 + x^4)^2)
```

3.176 $\int \frac{x^{14}}{(3+2x^5)^3} \, dx$

3.176.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^{14}}{(3+2x^5)^3} \, dx = -\frac{9}{80(3+2x^5)^2} + \frac{3}{20(3+2x^5)} + \frac{1}{40} \log (3+2x^5)$$

output
```
-9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*ln(2*x^5+3)
```

3.176.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{x^{14}}{(3+2x^5)^3} \, dx = \frac{1}{80} \left(\frac{3(9+8x^5)}{(3+2x^5)^2} + 2 \log (3+2x^5) \right)$$

input
```
Integrate[x^14/(3 + 2*x^5)^3,x]
```

output
```
((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80
```

3.176.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(2x^5+3)^3}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{5}\int \frac{x^{10}}{(2x^5+3)^3}dx^5$$

$$\Big\downarrow \ 49$$

$$\frac{1}{5}\int \left(\frac{1}{4(2x^5+3)}-\frac{3}{2(2x^5+3)^2}+\frac{9}{4(2x^5+3)^3}\right)dx^5$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}\left(\frac{3}{4(2x^5+3)}-\frac{9}{16(2x^5+3)^2}+\frac{1}{8}\log\left(2x^5+3\right)\right)$$

input
```
Int[x^14/(3 + 2*x^5)^3,x]
```

output
```
(-9/(16*(3 + 2*x^5)^2) + 3/(4*(3 + 2*x^5)) + Log[3 + 2*x^5]/8)/5
```

3.176.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.176.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{\frac{3x^5}{10}+\frac{27}{80}}{(2x^5+3)^2}+\frac{\ln(2x^5+3)}{40}$	29
risch	$\frac{\frac{3x^5}{10}+\frac{27}{80}}{(2x^5+3)^2}+\frac{\ln(2x^5+3)}{40}$	30
meijerg	$-\frac{x^5(6x^5+6)}{360\left(1+\frac{2x^5}{3}\right)^2}+\frac{\ln\left(1+\frac{2x^5}{3}\right)}{40}$	33
default	$-\frac{9}{80(2x^5+3)^2}+\frac{3}{20(2x^5+3)}+\frac{\ln(2x^5+3)}{40}$	34
parallelrisch	$\frac{8\ln(x^5+\frac{3}{2})x^{10}+27+24\ln(x^5+\frac{3}{2})x^5+24x^5+18\ln(x^5+\frac{3}{2})}{80(2x^5+3)^2}$	49

input
```
int(x^14/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/10*x^5+27/80)/(2*x^5+3)^2+1/40*ln(2*x^5+3)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{24\,x^5+2\,(4\,x^{10}+12\,x^5+9)\log(2\,x^5+3)+27}{80\,(4\,x^{10}+12\,x^5+9)}$$

input
```
integrate(x^14/(2*x^5+3)^3,x, algorithm="fricas")
```

output
```
1/80*(24*x^5 + 2*(4*x^10 + 12*x^5 + 9)*log(2*x^5 + 3) + 27)/(4*x^10 + 12*x
^5 + 9)
```

3.176.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{24x^5+27}{320x^{10}+960x^5+720} + \frac{\log(2x^5+3)}{40}$$

input
```
integrate(x**14/(2*x**5+3)**3,x)
```

output
```
(24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{3\,(8\,x^5+9)}{80\,(4\,x^{10}+12\,x^5+9)} + \frac{1}{40}\,\log\left(2\,x^5+3\right)$$

input
```
integrate(x^14/(2*x^5+3)^3,x, algorithm="maxima")
```

output
```
3/80*(8*x^5 + 9)/(4*x^10 + 12*x^5 + 9) + 1/40*log(2*x^5 + 3)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = -\frac{3\,(x^{10}+x^5)}{20\,(2\,x^5+3)^2} + \frac{1}{40}\,\log\left(|2\,x^5+3|\right)$$

input
```
integrate(x^14/(2*x^5+3)^3,x, algorithm="giac")
```

output
```
-3/20*(x^10 + x^5)/(2*x^5 + 3)^2 + 1/40*log(abs(2*x^5 + 3))
```

3.176.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{\ln\left(x^5+\frac{3}{2}\right)}{40} + \frac{\frac{3\,x^5}{40}+\frac{27}{320}}{x^{10}+3\,x^5+\frac{9}{4}}$$

input
```
int(x^14/(2*x^5 + 3)^3,x)
```

output
```
log(x^5 + 3/2)/40 + ((3*x^5)/40 + 27/320)/(3*x^5 + x^10 + 9/4)
```

3.177 $\int \frac{x^6}{(3+2x^5)^3} dx$

3.177.1 Optimal result

Integrand size = 13, antiderivative size = 319

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = -\frac{x^2}{20(3+2x^5)^2}+\frac{x^2}{150(3+2x^5)} - \frac{\sqrt{5+\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250\,2^{9/10}3^{3/5}} - \frac{\sqrt{5-\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+\frac{2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{250\,2^{9/10}3^{3/5}} - \frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\,2^{2/5}3^{3/5}} + \frac{\left(1+\sqrt{5}\right)\log\left(3^{2/5}-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2\right)}{1000\,2^{2/5}3^{3/5}} + \frac{\left(1-\sqrt{5}\right)\log\left(3^{2/5}-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2\right)}{1000\,2^{2/5}3^{3/5}}$$

output
```
-1/20*x^2/(2*x^5+3)^2+1/150*x^2/(2*x^5+3)-1/1500*ln(3^(1/5)+2^(1/5)*x)*2^(
3/5)*3^(2/5)+1/6000*ln(2^(3/5)*3^(2/5)+2*x^2-1/2*3^(1/5)*2^(4/5)*x*(5^(1/2
)+1))*(-5^(1/2)+1)*2^(3/5)*3^(2/5)+1/6000*ln(2^(3/5)*3^(2/5)+2*x^2-1/2*3^(
1/5)*2^(4/5)*x*(-5^(1/2)+1))*(5^(1/2)+1)*2^(3/5)*3^(2/5)-1/1500*arctan(1/5
*(25-10*5^(1/2))^(1/2)+2/3*2^(7/10)*x*3^(4/5)/(5+5^(1/2))^(1/2))*(5-5^(1/2
))^(1/2)*2^(1/10)*3^(2/5)+1/1500*arctan(2/3*2^(7/10)*x*3^(4/5)/(5-5^(1/2))
^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(5+5^(1/2))^(1/2)*2^(1/10)*3^(2/5)
```

3.177.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(3+2x^5)^3}\,dx$$
$$= \frac{-\frac{300x^2}{(3+2x^5)^2}+\frac{40x^2}{3+2x^5}-4\sqrt[10]{2}3^{2/5}\sqrt{5-\sqrt{5}}\arctan\left(\frac{-3+3\sqrt{5}+4\sqrt[5]{2}3^{4/5}x}{3\sqrt{2\left(5+\sqrt{5}\right)}}\right)+4\sqrt[10]{2}3^{2/5}\sqrt{5+\sqrt{5}}\arctan\left(\frac{-3\left(1+\sqrt{\cdots}\right.}{3\cdots}\right.}{\cdots}$$

input
```
Integrate[x^6/(3 + 2*x^5)^3,x]
```

output
```
((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(2/5)*Sqrt
[5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[2*(5 +
 Sqrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt[5]]*ArcTan[(-3*(1 + Sqrt[5
]) + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 - 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Lo
g[3 + 2^(1/5)*3^(4/5)*x] + 2^(3/5)*3^(2/5)*(1 + Sqrt[5])*Log[3 + (3/2)^(4/
5)*(-1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2] - 2^(3/5)*3^(2/5)*(-1 + Sqrt[5]
)*Log[3 - (3/2)^(4/5)*(1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2])/6000
```

3.177.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 819, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(2x^5+3)^3}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{1}{10}\int \frac{x}{(2x^5+3)^2}dx-\frac{x^2}{20\,(2x^5+3)^2}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{1}{10}\left(\frac{1}{5}\int \frac{x}{2x^5+3}dx+\frac{x^2}{15\,(2x^5+3)}\right)-\frac{x^2}{20\,(2x^5+3)^2}$$

$$\Big\downarrow \ 822$$

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1+\sqrt{5}\right)x+\sqrt[5]{3}\left(1-\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}+\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1-\sqrt{5}\right)x+\sqrt[5]{3}\left(1+\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1+\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}-\frac{\int \frac{1}{\sqrt[5]{2}x+\sqrt[5]{3}}dx}{5\sqrt[5]{2}3^{3/5}}\right)+\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$$\Big\downarrow \ 16$$

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1+\sqrt{5}\right)x+\sqrt[5]{3}\left(1-\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}+\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1-\sqrt{5}\right)x+\sqrt[5]{3}\left(1+\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1+\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}-\frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{5\ 2^{2/5}3^{3/5}}\right)\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{\int \frac{\sqrt[5]{2}\left(1+\sqrt{5}\right)x+\sqrt[5]{3}\left(1-\sqrt{5}\right)}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{5\sqrt[5]{2}3^{3/5}}+\frac{\int \frac{\sqrt[5]{2}\left(1-\sqrt{5}\right)x+\sqrt[5]{3}\left(1+\sqrt{5}\right)}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1+\sqrt{5}\right)x+2\ 3^{2/5}}dx}{5\sqrt[5]{2}3^{3/5}}-\frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{5\ 2^{2/5}3^{3/5}}\right)+\frac{x^2}{15\left(2x^5+3\right)}\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{\frac{\left(1+\sqrt{5}\right)\int -\frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{4\sqrt[5]{2}}-\sqrt[5]{3}\sqrt{5}\int \frac{1}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{5\sqrt[5]{2}3^{3/5}}+\frac{\sqrt[5]{3}\sqrt{5}\int \frac{1}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1\right.}}{}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$$\Big\downarrow \ 25$$

3.177. $\int \frac{x^6}{\left(3+2x^5\right)^3}\,dx$

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{-\sqrt[5]{3}\sqrt{5}\int \frac{1}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx-\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{4\sqrt[5]{2}}}{5\sqrt[5]{2}3^{3/5}}+\frac{\sqrt[5]{3}\sqrt{5}\int \frac{1}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(\right.}\cdots\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\Big\downarrow$ 1083

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{2\sqrt[5]{3}\sqrt{5}\int \frac{1}{-\left(4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)\right)^2-2\ 6^{2/5}\left(5+\sqrt{5}\right)}d\left(4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)\right)-\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}\cdots}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+\cdots}}{4\sqrt[5]{2}}}{5\sqrt[5]{2}3^{3/5}}\cdots\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{-\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{4\sqrt[5]{2}}-2^{3/10}\sqrt{\frac{5}{5+\sqrt{5}}}\arctan\left(\frac{4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)}{2^{7/10}\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{5\sqrt[5]{2}3^{3/5}}+\frac{\frac{2^{3/10}\sqrt{5}\arctan\left(\frac{4\ 2^{2/5}\cdots}{2^{7/\cdots}}\right.}{\sqrt{5-\sqrt{\cdots}}}}{\cdots}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{\frac{\left(1+\sqrt{5}\right)\log\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}\right)}{4\sqrt[5]{2}}-2^{3/10}\sqrt{\frac{5}{5+\sqrt{5}}}\arctan\left(\frac{4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)}{2^{7/10}\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{5\sqrt[5]{2}3^{3/5}}+\frac{\frac{2^{3/10}\sqrt{5}\arctan\left(\frac{4\ 2^{2\cdots}}{2^{7\cdots}}\right.}{\sqrt{5-\cdots}}}{\cdots}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

3.177. $\int \frac{x^6}{\left(3+2x^5\right)^3}\,dx$

input
```
Int[x^6/(3 + 2*x^5)^3,x]
```

output
```
-1/20*x^2/(3 + 2*x^5)^2 + (x^2/(15*(3 + 2*x^5)) + (-1/5*Log[3^(1/5) + 2^(1
/5)*x]/(2^(2/5)*3^(3/5)) + (-(2^(3/10)*Sqrt[5/(5 + Sqrt[5])]*ArcTan[(-(6^(
1/5)*(1 - Sqrt[5])) + 4*2^(2/5)*x)/(2^(7/10)*3^(1/5)*Sqrt[5 + Sqrt[5]])])
+ ((1 + Sqrt[5])*Log[2*3^(2/5) - 6^(1/5)*(1 - Sqrt[5])*x + 2*2^(2/5)*x^2])
/(4*2^(1/5)))/(5*2^(1/5)*3^(3/5)) + ((2^(3/10)*Sqrt[5]*ArcTan[(-(6^(1/5)*(
1 + Sqrt[5])) + 4*2^(2/5)*x)/(2^(7/10)*3^(1/5)*Sqrt[5 - Sqrt[5]])])/Sqrt[5
 - Sqrt[5]] + ((1 - Sqrt[5])*Log[2*3^(2/5) - 6^(1/5)*(1 + Sqrt[5])*x + 2*2
^(2/5)*x^2])/(4*2^(1/5)))/(5*2^(1/5)*3^(3/5)))/5)/10
```

3.177.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.177.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.15

method	result
risch	$\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{(2x^5+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^5+3)}\frac{\ln(x-\_R)}{\_R^3}\right)}{500}$
meijerg	$\frac{108^{\frac{4}{5}}\left(-\frac{x^2 3^{\frac{3}{5}}2^{\frac{2}{5}}\left(-\frac{28x^5}{3}+21\right)}{105\left(1+\frac{2x^5}{3}\right)^2}+\frac{2\,108^{\frac{1}{5}}x^2\left(-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\ln\left(1+\frac{2^{\frac{1}{5}}3^{\frac{4}{5}}(x^5)^{\frac{1}{5}}}{3}\right)}{2(x^5)^{\frac{2}{5}}}-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\cos\left(\frac{2\pi}{5}\right)\ln\left(1-\frac{2\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}3^{\frac{4}{5}}(x^5)^{\frac{1}{5}}}{3}+\frac{2^{\frac{2}{5}}3^{\frac{3}{5}}(x^5)^{\frac{2}{5}}}{3}\right)}{2(x^5)^{\frac{2}{5}}}\cdots\right)}{\cdots}\right)}{\cdots}$
default	$\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{(2x^5+3)^2}+\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\,48^{\frac{1}{5}}-x48^{\frac{1}{5}}+48^{\frac{2}{5}}+4x^2\right)}{12000}-\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\,48^{\frac{1}{5}}-x48^{\frac{1}{5}}+48^{\frac{2}{5}}+4x^2\right)\sqrt{5}}{12000}+\frac{48^{\frac{3}{5}}\arctan\left(-\frac{\sqrt{5}\cdots}{\sqrt{10\,48^{\frac{2}{5}}\cdots}}\right)}{\cdots}$

input
```
int(x^6/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
```

output
```
4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/500*sum(1/_R^3*ln(x-_R),_R=RootOf(2*
_Z^5+3))
```

3.177.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.49

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(2*x^5+3)^3,x, algorithm="fricas")
```

output
```
1/216000*(2880*x^7 - 2*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5)
 + I*sqrt(-2*sqrt(5) + 10) + 1)*log(-1/6912*108^(3/5)*(-1)^(2/5)*(108^(4/5
)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1
/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/64*108^(2/5)*(-1)^(3/5)
*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 - 1/6912*108^(4/5)*(-1)^(1/5)*(
108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/
5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^
(2/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 1/16*108^(2/5)*(-1)^(3/5)
*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/4*108^(2/5)*(-1)^(3/5)*(sqr
t(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 108^(2/5)*(-1)^(3/5) + 6*x) - 2*108^
(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10)
+ 1)*log(1/384*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1
)^3 + x) + 8*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*log(-108^(2/5)*(-1
)^(3/5) + 6*x) - 6480*x^2 + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(s
qrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*
x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)*
(4*x^10 + 12*x^5 + 9) - 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/864*108^(
4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/
16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/1...
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.12

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{4x^7-9x^2}{1200x^{10}+3600x^5+2700} + \operatorname{RootSum}\left(105468750000000t^5+1, \left(t\mapsto t\log\left(-281250000t^3+x\right)\right)\right)$$

input
```
integrate(x**6/(2*x**5+3)**3,x)
```

output
```
(4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(10546875000000
0*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.05

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{3^{\frac{4}{5}}2^{\frac{4}{5}}(\sqrt{5}-5)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x+\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}-3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{2\sqrt{5}+10}} + \frac{3^{\frac{4}{5}}2^{\frac{4}{5}}(\sqrt{5}+5)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x-\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{-2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}+3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{-2\sqrt{5}+10}} - \frac{1}{1500}\cdot 3^{\frac{2}{5}}2^{\frac{3}{5}}\log\left(2^{\frac{1}{5}}x+3^{\frac{1}{5}}\right) + \frac{4x^7-9x^2}{300(4x^{10}+12x^5+9)} - \frac{\log\left(2\cdot 2^{\frac{2}{5}}x^2-x\left(\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}+3^{\frac{1}{5}}2^{\frac{1}{5}}\right)+2\cdot 3^{\frac{2}{5}}\right)}{250\left(\sqrt{5}3^{\frac{3}{5}}2^{\frac{2}{5}}+3^{\frac{3}{5}}2^{\frac{2}{5}}\right)} + \frac{\log\left(2\cdot 2^{\frac{2}{5}}x^2+x\left(\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)+2\cdot 3^{\frac{2}{5}}\right)}{250\left(\sqrt{5}3^{\frac{3}{5}}2^{\frac{2}{5}}-3^{\frac{3}{5}}2^{\frac{2}{5}}\right)}$$

input
```
integrate(x^6/(2*x^5+3)^3,x, algorithm="maxima")
```

output
```
1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*
x + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sqrt(5) + 10))/((sqr
t(5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*sqrt(5) + 10)) + 1/750*3^(4
/5)*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5
)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2
/5)*2^(1/5) + 3^(2/5)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(
3/5)*log(2^(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9
) - 1/250*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*2^(1/5)
) + 2*3^(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5)) + 1/250*log(2*2
^(2/5)*x^2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(s
qrt(5)*3^(3/5)*2^(2/5) - 3^(3/5)*2^(2/5))
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = -\frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}-\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}\right)\arctan\left(\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x\right)}{3\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}+10}+\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}+10}\right)\arctan\left(-\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x\right)}{3\sqrt{-2\sqrt{5}+10}}\right) - \frac{1}{6000}\left(\left(\frac{3}{2}\right)^{\frac{2}{5}}\left(\sqrt{5}-5\right)+\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}+3\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{1}{5}}+\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)+\left(\frac{3}{2}\right)^{\frac{2}{5}}\right) + \frac{1}{6000}\left(\left(\frac{3}{2}\right)^{\frac{2}{5}}\left(\sqrt{5}+5\right)+\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}-3\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{1}{5}}-\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)+\left(\frac{3}{2}\right)^{\frac{2}{5}}\right) - \frac{1}{750}\left(\frac{3}{2}\right)^{\frac{2}{5}}\log\left(\left|x+\left(\frac{3}{2}\right)^{\frac{1}{5}}\right|\right)+\frac{4x^7-9x^2}{300\left(2x^5+3\right)^2}$$

input
```
integrate(x^6/(2*x^5+3)^3,x, algorithm="giac")
```

output
```
-1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(2*sqrt(5) + 10) - (3/2)^(2/5)*sqrt(2*sqr
t(5) + 10))*arctan(2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5) - 1) + 4*x)/sqrt(
2*sqrt(5) + 10)) + 1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(-2*sqrt(5) + 10) + (3/
2)^(2/5)*sqrt(-2*sqrt(5) + 10))*arctan(-2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt
(5) + 1) - 4*x)/sqrt(-2*sqrt(5) + 10)) - 1/6000*((3/2)^(2/5)*(sqrt(5) - 5)
 + sqrt(5)*(3/2)^(2/5) + 3*(3/2)^(2/5))*log(x^2 - 1/2*x*(sqrt(5)*(3/2)^(1/
5) + (3/2)^(1/5)) + (3/2)^(2/5)) + 1/6000*((3/2)^(2/5)*(sqrt(5) + 5) + sqr
t(5)*(3/2)^(2/5) - 3*(3/2)^(2/5))*log(x^2 + 1/2*x*(sqrt(5)*(3/2)^(1/5) - (
3/2)^(1/5)) + (3/2)^(2/5)) - 1/750*(3/2)^(2/5)*log(abs(x + (3/2)^(1/5))) +
 1/300*(4*x^7 - 9*x^2)/(2*x^5 + 3)^2
```

3.177.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}-2^{3/5}\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}-2^{3/5}\left(\sqrt{5}-1\right)\right)}{6000} - \frac{\frac{3\,x^2}{400}-\frac{x^7}{300}}{x^{10}+3\,x^5+\frac{9}{4}} - \frac{3^{2/5}\ln\left(x+\frac{3^{1/5}\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}+2^{3/5}\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}+2^{3/5}\left(\sqrt{5}-1\right)\right)}{6000} - \frac{72^{1/5}\ln\left(x+\frac{72^{3/5}}{12}\right)}{1500} + \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2^{3/5}\left(\sqrt{5}+1\right)-2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)^3}{256}\right)\left(2^{3/5}\left(\sqrt{5}+1\right)-2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)}{6000} + \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2^{3/5}\left(\sqrt{5}+1\right)+2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)^3}{256}\right)\left(2^{3/5}\left(\sqrt{5}+1\right)+2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)}{6000}$$

input
```
int(x^6/(2*x^5 + 3)^3,x)
```

output
```
(3^(2/5)*log(x - (3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(
1/2) - 1))^3)/256)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(1/2) -
1)))/6000 - ((3*x^2)/400 - x^7/300)/(3*x^5 + x^10 + 9/4) - (3^(2/5)*log(x
+ (3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1))^3)/2
56)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1)))/6000 - (72
^(1/5)*log(x + 72^(3/5)/12))/1500 + (3^(2/5)*log(x - (3^(1/5)*(2^(3/5)*(5^
(1/2) + 1) - 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(1/2) + 1
) - 2*2^(1/10)*(5^(1/2) - 5)^(1/2)))/6000 + (3^(2/5)*log(x - (3^(1/5)*(2^(
3/5)*(5^(1/2) + 1) + 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(
1/2) + 1) + 2*2^(1/10)*(5^(1/2) - 5)^(1/2)))/6000
```

3.178 $\int \frac{9}{5x^2(3-2x^2)^3} dx$

3.178.1 Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{9}{5x^2(3-2x^2)^3} dx = -\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

output
```
-1/8/x+3/20/x/(-2*x^2+3)^2+1/8/x/(-2*x^2+3)+1/24*arctanh(1/3*x*6^(1/2))*6^
(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{9}{5x^2(3-2x^2)^3} dx = \frac{1}{240}\left(-\frac{12(12-25x^2+10x^4)}{x(3-2x^2)^2} - 5\sqrt{6}\log\left(\sqrt{6}-2x\right) + 5\sqrt{6}\log\left(\sqrt{6}+2x\right)\right)$$

input
```
Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]
```

output
```
((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[Sqrt[6] -
2*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240
```

3.178.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {27, 253, 253, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx$$

$$\Big\downarrow \ 27$$

$$\frac{9}{5} \int \frac{1}{x^2 (3-2x^2)^3} dx$$

$$\Big\downarrow \ 253$$

$$\frac{9}{5} \left(\frac{5}{12} \int \frac{1}{x^2 (3-2x^2)^2} dx + \frac{1}{12x (3-2x^2)^2}\right)$$

$$\Big\downarrow \ 253$$

$$\frac{9}{5} \left(\frac{5}{12} \left(\frac{1}{2} \int \frac{1}{x^2 (3-2x^2)} dx + \frac{1}{6x (3-2x^2)}\right) + \frac{1}{12x (3-2x^2)^2}\right)$$

$$\Big\downarrow \ 264$$

$$\frac{9}{5} \left(\frac{5}{12} \left(\frac{1}{2} \left(\frac{2}{3} \int \frac{1}{3-2x^2} dx - \frac{1}{3x}\right) + \frac{1}{6x (3-2x^2)}\right) + \frac{1}{12x (3-2x^2)^2}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{9}{5} \left(\frac{5}{12} \left(\frac{1}{2} \left(\frac{1}{3}\sqrt{\frac{2}{3}}\text{arctanh}\left(\sqrt{\frac{2}{3}}x\right) - \frac{1}{3x}\right) + \frac{1}{6x (3-2x^2)}\right) + \frac{1}{12x (3-2x^2)^2}\right)$$

input
```
Int[9/(5*x^2*(3 - 2*x^2)^3),x]
```

output
```
(9*(1/(12*x*(3 - 2*x^2)^2) + (5*(1/(6*x*(3 - 2*x^2)) + (-1/3*1/x + (Sqrt[2
/3]*ArcTanh[Sqrt[2/3]*x])/3)/2))/12))/5
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.178.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{1}{15x}-\frac{8\left(\frac{7}{16}x^{3}-\frac{27}{32}x\right)}{15\left(2x^{2}-3\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{3}\right)\sqrt{6}}{24}$	39
meijerg	$\frac{i\sqrt{6}\left(\frac{i\sqrt{6}\left(\frac{20}{3}x^{4}-\frac{50}{3}x^{2}+8\right)}{4x\left(-\frac{2x^{2}}{3}+1\right)^{2}}-\frac{15i\operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{3}}{3}\right)}{2}\right)}{180}$	51
risch	$\frac{-\frac{1}{2}x^{4}+\frac{5}{4}x^{2}-\frac{3}{5}}{\left(2x^{2}-3\right)^{2}x}+\frac{\sqrt{6}\,\ln\left(2x+\sqrt{6}\right)}{48}-\frac{\sqrt{6}\,\ln\left(2x-\sqrt{6}\right)}{48}$	56

input
```
int(9/5/x^2/(-2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15/x-8/15*(7/16*x^3-27/32*x)/(2*x^2-3)^2+1/24*arctanh(1/3*x*6^(1/2))*6^
(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx = -\frac{120 \, x^4 - 5 \, \sqrt{6}(4 \, x^5 - 12 \, x^3 + 9 \, x) \log\left(\frac{2 \, x^2 + 2 \, \sqrt{6} x + 3}{2 \, x^2 - 3}\right) - 300 \, x^2 + 144}{240 \, (4 \, x^5 - 12 \, x^3 + 9 \, x)}$$

input
```
integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/240*(120*x^4 - 5*sqrt(6)*(4*x^5 - 12*x^3 + 9*x)*log((2*x^2 + 2*sqrt(6)*
x + 3)/(2*x^2 - 3)) - 300*x^2 + 144)/(4*x^5 - 12*x^3 + 9*x)
```

3.178.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx = -\frac{9 \cdot (10x^4 - 25x^2 + 12)}{720x^5 - 2160x^3 + 1620x} - \frac{\sqrt{6} \log\left(x - \frac{\sqrt{6}}{2}\right)}{48} + \frac{\sqrt{6} \log\left(x + \frac{\sqrt{6}}{2}\right)}{48}$$

input
```
integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

output
```
-9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqrt(6)*log(
x - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx = -\frac{1}{48} \, \sqrt{6} \log\left(\frac{2 \, x - \sqrt{6}}{2 \, x + \sqrt{6}}\right) - \frac{10 \, x^4 - 25 \, x^2 + 12}{20 \, (4 \, x^5 - 12 \, x^3 + 9 \, x)}$$

input
```
integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/48*sqrt(6)*log((2*x - sqrt(6))/(2*x + sqrt(6))) - 1/20*(10*x^4 - 25*x^2
 + 12)/(4*x^5 - 12*x^3 + 9*x)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = -\frac{1}{48}\sqrt{6}\log\left(\frac{\left|4\,x-2\,\sqrt{6}\right|}{\left|4\,x+2\,\sqrt{6}\right|}\right) - \frac{14\,x^3-27\,x}{60\,\left(2\,x^2-3\right)^2} - \frac{1}{15\,x}$$

input
```
integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(6)*log(abs(4*x - 2*sqrt(6))/abs(4*x + 2*sqrt(6))) - 1/60*(14*x^
3 - 27*x)/(2*x^2 - 3)^2 - 1/15/x
```

3.178.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{3}\right)}{24} - \frac{\frac{x^4}{8}-\frac{5\,x^2}{16}+\frac{3}{20}}{x^5-3\,x^3+\frac{9\,x}{4}}$$

input
```
int(-9/(5*x^2*(2*x^2 - 3)^3),x)
```

output
```
(6^(1/2)*atanh((6^(1/2)*x)/3))/24 - (x^4/8 - (5*x^2)/16 + 3/20)/((9*x)/4 -
 3*x^3 + x^5)
```

3.179 $\int \frac{4+3x^4}{x^2(1+x^2)^3} dx$

3.179.1 Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{4+3x^4}{x^2(1+x^2)^3} dx = -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57\arctan(x)}{8}$$

output
```
-4/x-7/4*x/(x^2+1)^2-25/8*x/(x^2+1)-57/8*arctan(x)
```

3.179.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{4+3x^4}{x^2(1+x^2)^3} dx = -\frac{32+103x^2+57x^4}{8x(1+x^2)^2} - \frac{57\arctan(x)}{8}$$

input
```
Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
-1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

3.179.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1583, 25, 361, 25, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^4+4}{x^2 (x^2+1)^3} \, dx$$

$$\Big\downarrow \text{ 1583}$$

$$-\frac{1}{4} \int -\frac{16-9x^2}{x^2 (x^2+1)^2}dx - \frac{7x}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4} \int \frac{16-9x^2}{x^2 (x^2+1)^2}dx - \frac{7x}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{ 361}$$

$$\frac{1}{4} \left(-\frac{1}{2} \int -\frac{32-25x^2}{x^2 (x^2+1)}dx - \frac{25x}{2 (x^2+1)}\right) - \frac{7x}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \frac{32-25x^2}{x^2 (x^2+1)}dx - \frac{25x}{2 (x^2+1)}\right) - \frac{7x}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{1}{4} \left(\frac{1}{2} \left(-57 \int \frac{1}{x^2+1}dx - \frac{32}{x}\right) - \frac{25x}{2 (x^2+1)}\right) - \frac{7x}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4} \left(\frac{1}{2} \left(-57 \arctan(x) - \frac{32}{x}\right) - \frac{25x}{2 (x^2+1)}\right) - \frac{7x}{4 (x^2+1)^2}$$

input
```
Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
(-7*x)/(4*(1 + x^2)^2) + ((-25*x)/(2*(1 + x^2)) + (-32/x - 57*ArcTan[x])/2
)/4
```

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1583
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1))   Int[x^
m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*(-d)^(-m/2 +
1)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e
*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && IL
tQ[q, -1] && ILtQ[m/2, 0]
```

3.179. $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

3.179.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\frac{25}{8}x^{3}+\frac{39}{8}x}{\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}-\frac{4}{x}$	29
risch	$\frac{-\frac{57}{8}x^{4}-\frac{103}{8}x^{2}-4}{\left(x^{2}+1\right)^{2}x}-\frac{57\arctan\left(x\right)}{8}$	29
meijerg	$-\frac{15x^{4}+25x^{2}+8}{2x\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}-\frac{x\left(-3x^{2}+3\right)}{8\left(x^{2}+1\right)^{2}}$	47
parallelrisch	$\frac{57i\ln\left(x-i\right)x^{5}-57i\ln\left(x+i\right)x^{5}-64+114i\ln\left(x-i\right)x^{3}-114i\ln\left(x+i\right)x^{3}-114x^{4}+57i\ln\left(x-i\right)x-57i\ln\left(x+i\right)x-206x^{2}}{16x\left(x^{2}+1\right)^{2}}$	87

input
```
int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{57\,x^{4}+103\,x^{2}+57\left(x^{5}+2\,x^{3}+x\right)\arctan\left(x\right)+32}{8\left(x^{5}+2\,x^{3}+x\right)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
 + x)
```

3.179.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=\frac{-57x^{4}-103x^{2}-32}{8x^{5}+16x^{3}+8x}-\frac{57\,\mathrm{atan}\left(x\right)}{8}$$

input
```
integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

output
```
(-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx = -\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)}-\frac{57}{8}\,\arctan{(x)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx = -\frac{25\,x^3+39\,x}{8\,(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\,\arctan{(x)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx = -\frac{57\,\mathrm{atan}(x)}{8}-\frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,\left(x^2+1\right)^2}$$

input
```
int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
```

output
```
- (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

3.180 $\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} \, dx$

3.180.1 Optimal result

Integrand size = 44, antiderivative size = 38

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} \, dx = -\frac{3}{2(1-x)^2}+\frac{2}{1-x}+\frac{1}{1+x}+\log(1-x)-2\log(1+x)$$

output
```
-3/2/(1-x)^2+2/(1-x)+1/(1+x)+ln(1-x)-2*ln(1+x)
```

3.180.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} \, dx = -\frac{3}{2(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{1+x}+\log(-1+x)-2\log(1+x)$$

input
```
Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + 
x^5),x]
```

output
```
-3/(2*(-1 + x)^2) - 2/(-1 + x) + (1 + x)^(-1) + Log[-1 + x] - 2*Log[1 + x]
```

3.180.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^4+5x^3+6x^2-3x+5}{x^5-x^4-2x^3+2x^2+x-1}\,dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(-\frac{2}{x+1}-\frac{1}{(x+1)^2}+\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{3}{(x-1)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

input
```
Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x
]
```

output
```
-3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]
```

3.180.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.180.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result
default	$\ln(-1+x)-\frac{3}{2(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{1+x}-2\ln(1+x)$
norman	$\frac{-x^2-\frac{7}{2}x+\frac{3}{2}}{(-1+x)^2(1+x)}-2\ln(1+x)+\ln(-1+x)$
risch	$\frac{-x^2-\frac{7}{2}x+\frac{3}{2}}{x^3-x^2-x+1}-2\ln(1+x)+\ln(-1+x)$
parallelrisch	$\frac{2\ln(-1+x)x^3-4\ln(1+x)x^3+3-2\ln(-1+x)x^2+4\ln(1+x)x^2-2\ln(-1+x)x+4\ln(1+x)x-2x^2+2\ln(-1+x)-4\ln(1+x)-7x}{2x^3-2x^2-2x+2}$

input
```
int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x,method=_RETURNVER
BOSE)
```

output
```
ln(-1+x)-3/2/(-1+x)^2-2/(-1+x)+1/(1+x)-2*ln(1+x)
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+4\,(x^3-x^2-x+1)\log(x+1)-2\,(x^3-x^2-x+1)\log(x-1)+7\,x-3}{2\,(x^3-x^2-x+1)}$$

input
```
integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm=
"fricas")
```

output
```
-1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x + 1)*log
(x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)
```

3.180.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2x^2+7x-3}{2x^3-2x^2-2x+2} + \log(x-1) - 2\log(x+1)$$

input
```
integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-1),x)
```

output
```
-(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*log(x + 1
)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+7\,x-3}{2\,(x^3-x^2-x+1)} - 2\,\log(x+1) + \log(x-1)$$

input
```
integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm=
"maxima")
```

output
```
-1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*log(x + 1) + log(x - 1)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+7\,x-3}{2\,(x+1)(x-1)^2} - 2\,\log(|x+1|) + \log(|x-1|)$$

input
```
integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm=
"giac")
```

output
```
-1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*log(abs(x + 1)) + log(abs(x
 - 1))
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = \ln(x-1)-2\,\ln(x+1)+\frac{x^2+\frac{7\,x}{2}-\frac{3}{2}}{-x^3+x^2+x-1}$$

input
```
int((6*x^2 - 3*x + 5*x^3 - x^4 + 5)/(x + 2*x^2 - 2*x^3 - x^4 + x^5 - 1),x)
```

output
```
log(x - 1) - 2*log(x + 1) + ((7*x)/2 + x^2 - 3/2)/(x + x^2 - x^3 - 1)
```

3.181 $\int \frac{1+x^2}{x(1+x^3)^2} \, dx$

3.181.1 Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2} \, dx = \frac{x(x-x^2)}{3\left(1+x^3\right)} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{4}{9}\log(1+x) - \frac{5}{18}\log\left(1-x+x^2\right)$$

output
```
1/3*x*(-x^2+x)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)-1/9*arctan(1/3*(
1-2*x)*3^(1/2))*3^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2} \, dx = \frac{1}{18}\left(\frac{6(1+x^2)}{1+x^3} + 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 18\log(x) - 2\log(1+x) + \log\left(1-x+x^2\right) - 6\log\left(1+x^3\right)\right)$$

input
```
Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]
```

output
```
((6*(1 + x^2))/(1 + x^3) + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 18*Log[x
] - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18
```

3.181.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x\left(x^3+1\right)^2}dx$$

$$\Big\downarrow \text{ 2368}$$

$$\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}-\frac{1}{3}\int -\frac{x^2+3}{x\left(x^3+1\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\int \frac{x^2+3}{x\left(x^3+1\right)}dx+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}$$

$$\Big\downarrow \text{ 2373}$$

$$\frac{1}{3}\int \left(\frac{4-5x}{3\left(x^2-x+1\right)}+\frac{3}{x}-\frac{4}{3(x+1)}\right)dx+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{5}{6}\log\left(x^2-x+1\right)+3\log(x)-\frac{4}{3}\log(x+1)\right)+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}$$

input
```
Int[(1 + x^2)/(x*(1 + x^3)^2),x]
```

output
```
(x*(x - x^2))/(3*(1 + x^3)) + (-(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + 3*Lo
g[x] - (4*Log[1 + x])/3 - (5*Log[1 - x + x^2])/6)/3
```

3.181.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.181.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
risch	$\frac{\frac{x^2}{3}+\frac{1}{3}}{x^3+1}+\ln(x)-\frac{4\ln(1+x)}{9}-\frac{5\ln(x^2-x+1)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$
default	$-\frac{-1-x}{9(x^2-x+1)}-\frac{5\ln(x^2-x+1)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\ln(x)+\frac{2}{9(1+x)}-\frac{4\ln(1+x)}{9}$
meijerg	$\frac{x^2}{3x^3+3}-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{9(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{18(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{9(x^3)^{\frac{2}{3}}}+\frac{1}{3}+\ln(x)-\frac{2x^3}{3(2x^3+2)}-$ ln(

input
```
int((x^2+1)/x/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^2+1/3)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)+1/9*3^(1/2)*arcta
n(2/3*(x-1/2)*3^(1/2))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx$$
$$= \frac{2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 6x^2 - 5\left(x^3+1\right)\log\left(x^2-x+1\right) - 8\left(x^3+1\right)\log\left(x+1\right) + 18\left(x^3\right.}{18\left(x^3+1\right)}$$

input
```
integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="fricas")
```

output
```
1/18*(2*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x^2 - 5*(x^3 +
 1)*log(x^2 - x + 1) - 8*(x^3 + 1)*log(x + 1) + 18*(x^3 + 1)*log(x) + 6)/(
x^3 + 1)
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{x^2+1}{3x^3+3} + \log\left(x\right) - \frac{4\log\left(x+1\right)}{9} - \frac{5\log\left(x^2-x+1\right)}{18} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+1)/x/(x**3+1)**2,x)
```

output
```
(x**2 + 1)/(3*x**3 + 3) + log(x) - 4*log(x + 1)/9 - 5*log(x**2 - x + 1)/18
 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{x^2+1}{3\left(x^3+1\right)} - \frac{5}{18}\log\left(x^2-x+1\right) - \frac{4}{9}\log\left(x+1\right) + \log\left(x\right)$$

input
```
integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/(x^3 + 1) - 5/18
*log(x^2 - x + 1) - 4/9*log(x + 1) + log(x)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{x(1+x^3)^2}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{x^2+1}{3(x^2-x+1)(x+1)} - \frac{5}{18}\log\left(x^2-x+1\right) - \frac{4}{9}\log\left(|x+1|\right) + \log\left(|x|\right)$$

input
```
integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/((x^2 - x + 1)*(
x + 1)) - 5/18*log(x^2 - x + 1) - 4/9*log(abs(x + 1)) + log(abs(x))
```

3.181.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{1+x^2}{x(1+x^3)^2}\,dx = \ln(x) - \frac{4\ln(x+1)}{9} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1i}{2}\right)\left(\frac{5}{18} + \frac{\sqrt{3}\,1i}{18}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{5}{18} + \frac{\sqrt{3}\,1i}{18}\right) + \frac{\frac{x^2}{3} + \frac{1}{3}}{x^3+1}$$

input
```
int((x^2 + 1)/(x*(x^3 + 1)^2),x)
```

output
```
log(x) - (4*log(x + 1))/9 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18
 + 5/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18 - 5/18) + (x^2/3
 + 1/3)/(x^3 + 1)
```

3.182 $\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx$

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx = -\frac{2}{1+x}-\frac{7+5x}{3(1+x+x^2)}-\frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-2/(1+x)+1/3*(-7-5*x)/(x^2+x+1)-ln(1+x)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1
+2*x)*3^(1/2))*3^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx = -\frac{2}{1+x}-\frac{7+5x}{3(1+x+x^2)}-\frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

output
```
-2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/
(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

3.182.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2177, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3x-2}{(x+1)^2\left(x^2+x+1\right)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{3}\int -\frac{5x^2+19x+8}{(x+1)^2\left(x^2+x+1\right)}dx-\frac{5x+7}{3\left(x^2+x+1\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{5x^2+19x+8}{(x+1)^2\left(x^2+x+1\right)}dx-\frac{5x+7}{3\left(x^2+x+1\right)}$$

$$\Big\downarrow \text{2159}$$

$$-\frac{1}{3}\int \left(\frac{11-3x}{x^2+x+1}+\frac{3}{x+1}-\frac{6}{(x+1)^2}\right)dx-\frac{5x+7}{3\left(x^2+x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{25\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2+x+1\right)-\frac{6}{x+1}-3\log(x+1)\right)-\frac{5x+7}{3\left(x^2+x+1\right)}$$

input
```
Int[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

output
```
-1/3*(7 + 5*x)/(1 + x + x^2) + (-6/(1 + x) - (25*ArcTan[(1 + 2*x)/Sqrt[3]]
)/Sqrt[3] - 3*Log[1 + x] + (3*Log[1 + x + x^2])/2)/3
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.182.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{-\frac{5x}{3}-\frac{7}{3}}{x^2+x+1}+\frac{\ln(x^2+x+1)}{2}-\frac{25\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{2}{1+x}-\ln(1+x)$	54
risch	$\frac{-\frac{11}{3}x^2-6x-\frac{13}{3}}{(x^2+x+1)(1+x)}+\frac{\ln(4x^2+4x+4)}{2}-\frac{25\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\ln(1+x)$	61

input
```
int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-2/(1+x)-ln(1+x)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{50\sqrt{3}(x^3+2x^2+2x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+66x^2-9(x^3+2x^2+2x+1)\log\left(x^2+x+1\right)+}{18\left(x^3+2x^2+2x+1\right)}$$

input
```
integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="fricas")
```

output
```
-1/18*(50*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) +
66*x^2 - 9*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1) + 18*(x^3 + 2*x^2 + 2*
x + 1)*log(x + 1) + 108*x + 78)/(x^3 + 2*x^2 + 2*x + 1)
```

3.182.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = \frac{-11x^2-18x-13}{3x^3+6x^2+6x+3} - \log(x+1) + \frac{\log\left(x^2+x+1\right)}{2} - \frac{25\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

output
```
(-11*x**2 - 18*x - 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) + log(x**2
 + x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{25}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{11x^2+18x+13}{3\left(x^3+2x^2+2x+1\right)} + \frac{1}{2}\log\left(x^2+x+1\right) - \log(x+1)$$

input
```
integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="maxima")
```

output
```
-25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^
3 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{25}{9}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(\frac{2}{x+1}-1\right)\right) + \frac{\frac{7}{x+1}-2}{3\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}-1\right)} - \frac{2}{x+1} + \frac{1}{2}\log\left(-\frac{1}{x+1}+\frac{1}{(x+1)^2}+1\right)$$

input
```
integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="giac")
```

output
```
-25/9*sqrt(3)*arctan(-1/3*sqrt(3)*(2/(x + 1) - 1)) + 1/3*(7/(x + 1) - 2)/(
1/(x + 1) - 1/(x + 1)^2 - 1) - 2/(x + 1) + 1/2*log(-1/(x + 1) + 1/(x + 1)^
2 + 1)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\ln(x+1) - \frac{\frac{11\,x^2}{3}+6\,x+\frac{13}{3}}{x^3+2\,x^2+2\,x+1} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,25\mathrm{i}}{18}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,25\mathrm{i}}{18}\right)$$

input
```
int(-(3*x - x^2 + 2)/((x + 1)^2*(x + x^2 + 1)^2),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*25i)/18 + 1/2) - (6*x + (11*x^2)/3
 + 13/3)/(2*x + 2*x^2 + x^3 + 1) - log(x + 1) - log(x + (3^(1/2)*1i)/2 + 1
/2)*((3^(1/2)*25i)/18 - 1/2)
```

3.182. $\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2}\,dx$

3.183 $\int \frac{1}{(1-4x)^3(2-3x)}\,dx$

3.183.1 Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{1}{10(1-4x)^2} - \frac{3}{25(1-4x)} - \frac{9}{125}\log(1-4x) + \frac{9}{125}\log(2-3x)$$

output
```
1/10/(-4*x+1)^2-3/25/(-4*x+1)-9/125*ln(-4*x+1)+9/125*ln(2-3*x)
```

3.183.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{-5+120x+18(1-4x)^2\log(8-12x)-18(1-4x)^2\log(-1+4x)}{250(1-4x)^2}$$

input
```
Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

output
```
(-5 + 120*x + 18*(1 - 4*x)^2*Log[8 - 12*x] - 18*(1 - 4*x)^2*Log[-1 + 4*x])
/(250*(1 - 4*x)^2)
```

3.183.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-4x)^3(2-3x)} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{36}{125(4x-1)}-\frac{12}{25(4x-1)^2}-\frac{4}{5(4x-1)^3}+\frac{27}{125(3x-2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

input
```
Int[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

output
```
1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 -
3*x])/125
```

3.183.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.183.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{\frac{12x}{25}-\frac{1}{50}}{(-1+4x)^2}+\frac{9\ln(-2+3x)}{125}-\frac{9\ln(-1+4x)}{125}$	32
norman	$\frac{\frac{8}{25}x+\frac{8}{25}x^2}{(-1+4x)^2}+\frac{9\ln(-2+3x)}{125}-\frac{9\ln(-1+4x)}{125}$	35
default	$\frac{1}{10(-1+4x)^2}+\frac{3}{25(-1+4x)}-\frac{9\ln(-1+4x)}{125}+\frac{9\ln(-2+3x)}{125}$	36
parallelrisch	$\frac{144\ln(x-\frac{2}{3})x^2-144\ln(x-\frac{1}{4})x^2-72\ln(x-\frac{2}{3})x+72\ln(x-\frac{1}{4})x+40x^2+9\ln(x-\frac{2}{3})-9\ln(x-\frac{1}{4})+40x}{125(-1+4x)^2}$	63

input
```
int(1/(-4*x+1)^3/(2-3*x),x,method=_RETURNVERBOSE)
```

output
```
16*(3/100*x-1/800)/(-1+4*x)^2+9/125*ln(-2+3*x)-9/125*ln(-1+4*x)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = -\frac{18\,(16\,x^2-8\,x+1)\log(4\,x-1)-18\,(16\,x^2-8\,x+1)\log(3\,x-2)-120\,x+5}{250\,(16\,x^2-8\,x+1)}$$

input
```
integrate(1/(-4*x+1)^3/(2-3*x),x, algorithm="fricas")
```

output
```
-1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1)*log(3*x
 - 2) - 120*x + 5)/(16*x^2 - 8*x + 1)
```

3.183.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24x-1}{800x^2-400x+50} + \frac{9\log\left(x-\frac{2}{3}\right)}{125} - \frac{9\log\left(x-\frac{1}{4}\right)}{125}$$

input
```
integrate(1/(-4*x+1)**3/(2-3*x),x)
```

output
```
(24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x - 1/4)/1
25
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24\,x-1}{50\,(16\,x^2-8\,x+1)} - \frac{9}{125}\,\log\left(4\,x-1\right) + \frac{9}{125}\,\log\left(3\,x-2\right)$$

input
```
integrate(1/(-4*x+1)^3/(2-3*x),x, algorithm="maxima")
```

output
```
1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*log(4*x - 1) + 9/125*log(3*x -
2)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24\,x-1}{50\,(4\,x-1)^2} - \frac{9}{125}\,\log\left(|4\,x-1|\right) + \frac{9}{125}\,\log\left(|3\,x-2|\right)$$

input
```
integrate(1/(-4*x+1)^3/(2-3*x),x, algorithm="giac")
```

output
```
1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*log(abs(4*x - 1)) + 9/125*log(abs(3*x
- 2))
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{\frac{3x}{100}-\frac{1}{800}}{x^2-\frac{x}{2}+\frac{1}{16}} - \frac{18\,\mathrm{atanh}\left(\frac{24\,x}{5}-\frac{11}{5}\right)}{125}$$

input
```
int(1/((3*x - 2)*(4*x - 1)^3),x)
```

output
```
((3*x)/100 - 1/800)/(x^2 - x/2 + 1/16) - (18*atanh((24*x)/5 - 11/5))/125
```

3.184 $\int \frac{x^3}{(2-5x^2)^7} dx$

3.184.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

output
```
1/150/(-5*x^2+2)^6-1/250/(-5*x^2+2)^5
```

3.184.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{-1+15x^2}{750(2-5x^2)^6}$$

input
```
Integrate[x^3/(2 - 5*x^2)^7,x]
```

output
```
(-1 + 15*x^2)/(750*(2 - 5*x^2)^6)
```

3.184.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(2-5x^2)^7} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^2}{(2-5x^2)^7}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(-\frac{1}{5(5x^2-2)^6}-\frac{2}{5(5x^2-2)^7}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{75(2-5x^2)^6}-\frac{1}{125(2-5x^2)^5}\right)$$

input
```
Int[x^3/(2 - 5*x^2)^7,x]
```

output
```
(1/(75*(2 - 5*x^2)^6) - 1/(125*(2 - 5*x^2)^5))/2
```

3.184.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.184.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
norman	$\frac{\frac{x^2}{50}-\frac{1}{750}}{(5x^2-2)^6}$	18
gosper	$\frac{15x^2-1}{750(5x^2-2)^6}$	19
risch	$\frac{\frac{x^2}{50}-\frac{1}{750}}{(5x^2-2)^6}$	19
default	$\frac{1}{150(5x^2-2)^6}+\frac{1}{250(5x^2-2)^5}$	24
meijerg	$\frac{x^4(\frac{625}{16}x^8-\frac{375}{4}x^6+\frac{375}{4}x^4-50x^2+15)}{7680\left(1-\frac{5x^2}{2}\right)^6}$	37
parallelrisch	$\frac{125x^{12}-300x^{10}+300x^8-160x^6+48x^4}{384(5x^2-2)^6}$	38

input
```
int(x^3/(-5*x^2+2)^7,x,method=_RETURNVERBOSE)
```

output
```
(1/50*x^2-1/750)/(5*x^2-2)^6
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64)}$$

input
```
integrate(x^3/(-5*x^2+2)^7,x, algorithm="fricas")
```

output
```
1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000
*x^4 - 960*x^2 + 64)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = -\frac{1-15x^2}{11718750x^{12}-28125000x^{10}+28125000x^8-15000000x^6+4500000x^4-720000x^2+48000}$$

input
```
integrate(x**3/(-5*x**2+2)**7,x)
```

output
```
-(1 - 15*x**2)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 - 15000000
*x**6 + 4500000*x**4 - 720000*x**2 + 48000)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64)}$$

input
```
integrate(x^3/(-5*x^2+2)^7,x, algorithm="maxima")
```

output
```
1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000
*x^4 - 960*x^2 + 64)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

input
```
integrate(x^3/(-5*x^2+2)^7,x, algorithm="giac")
```

output
```
1/750*(15*x^2 - 1)/(5*x^2 - 2)^6
```

3.184.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

input
```
int(-x^3/(5*x^2 - 2)^7,x)
```

output
```
(15*x^2 - 1)/(750*(5*x^2 - 2)^6)
```

3.185 $\int \frac{x^7}{(2-5x^2)^3} dx$

3.185.1 Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^7}{(2-5x^2)^3} dx = -\frac{x^2}{250} + \frac{2}{625(2-5x^2)^2} - \frac{6}{625(2-5x^2)} - \frac{3}{625}\log(2-5x^2)$$

output
```
-1/250*x^2+2/625/(-5*x^2+2)^2-6/625/(-5*x^2+2)-3/625*ln(-5*x^2+2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(2-5x^2)^3} dx = -\frac{12-150x^4+125x^6+6(2-5x^2)^2\log(-2+5x^2)}{1250(2-5x^2)^2}$$

input
```
Integrate[x^7/(2 - 5*x^2)^3,x]
```

output
```
-1/1250*(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(2 - 5*
x^2)^2
```

3.185.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(2-5x^2)^3} \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^6}{(2-5x^2)^3}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(-\frac{1}{125}-\frac{6}{125(5x^2-2)}-\frac{12}{125(5x^2-2)^2}-\frac{8}{125(5x^2-2)^3}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{x^2}{125}-\frac{12}{625(2-5x^2)}+\frac{4}{625(2-5x^2)^2}-\frac{6}{625}\log\left(2-5x^2\right)\right)$$

input
```
Int[x^7/(2 - 5*x^2)^3,x]
```

output
```
(-1/125*x^2 + 4/(625*(2 - 5*x^2)^2) - 12/(625*(2 - 5*x^2)) - (6*Log[2 - 5*
x^2])/625)/2
```

3.185.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.185.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{\frac{12}{125}x^2-\frac{1}{10}x^6-\frac{18}{625}}{(5x^2-2)^2}-\frac{3\ln(5x^2-2)}{625}$	34
risch	$-\frac{x^2}{250}+\frac{\frac{6x^2}{125}-\frac{2}{125}}{(5x^2-2)^2}-\frac{3\ln(5x^2-2)}{625}$	35
meijerg	$-\frac{x^2(25x^4-45x^2+12)}{1000\left(1-\frac{5x^2}{2}\right)^2}-\frac{3\ln\left(1-\frac{5x^2}{2}\right)}{625}$	38
default	$-\frac{x^2}{250}+\frac{6}{625(5x^2-2)}-\frac{3\ln(5x^2-2)}{625}+\frac{2}{625(5x^2-2)^2}$	39
parallelrisch	$-\frac{250x^6+300\ln(x^2-\frac{2}{5})x^4-450x^4-240\ln(x^2-\frac{2}{5})x^2+120x^2+48\ln(x^2-\frac{2}{5})}{2500(5x^2-2)^2}$	58

input
```
int(x^7/(-5*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(12/125*x^2-1/10*x^6-18/625)/(5*x^2-2)^2-3/625*ln(5*x^2-2)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{125\,x^6-100\,x^4-40\,x^2+6\,(25\,x^4-20\,x^2+4)\log(5\,x^2-2)+20}{1250\,(25\,x^4-20\,x^2+4)}$$

input
```
integrate(x^7/(-5*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log(5*x^2 -
2) + 20)/(25*x^4 - 20*x^2 + 4)
```

3.185.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{x^2}{250} - \frac{2-6x^2}{3125x^4-2500x^2+500} - \frac{3\log(5x^2-2)}{625}$$

input
```
integrate(x**7/(-5*x**2+2)**3,x)
```

output
```
-x**2/250 - (2 - 6*x**2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*x**2 - 2)
/625
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{1}{250}\,x^2 + \frac{2\,(3\,x^2-1)}{125\,(25\,x^4-20\,x^2+4)} - \frac{3}{625}\,\log\left(5\,x^2-2\right)$$

input
```
integrate(x^7/(-5*x^2+2)^3,x, algorithm="maxima")
```

output
```
-1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(5*x^2 - 2
)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{1}{250}\,x^2 + \frac{225\,x^4-120\,x^2+16}{1250\,(5\,x^2-2)^2} - \frac{3}{625}\,\log\left(\left|5\,x^2-2\right|\right)$$

input
```
integrate(x^7/(-5*x^2+2)^3,x, algorithm="giac")
```

output
```
-1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/625*log(abs
(5*x^2 - 2))
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = \frac{\frac{6\,x^2}{3125}-\frac{2}{3125}}{x^4-\frac{4\,x^2}{5}+\frac{4}{25}} - \frac{3\,\ln\left(x^2-\frac{2}{5}\right)}{625} - \frac{x^2}{250}$$

input
```
int(-x^7/(5*x^2 - 2)^3,x)
```

output
```
((6*x^2)/3125 - 2/3125)/(x^4 - (4*x^2)/5 + 4/25) - (3*log(x^2 - 2/5))/625
- x^2/250
```

3.186 $\int \frac{1}{(-2+x)^3(1+x)^2}\,dx$

3.186.1 Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = -\frac{1}{18(-2+x)^2}+\frac{2}{27(-2+x)}+\frac{1}{27(1+x)}+\frac{1}{27}\log(-2+x)-\frac{1}{27}\log(1+x)$$

output
```
-1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

3.186.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{1}{54}\left(\frac{3(-1-5x+2x^2)}{(-2+x)^2(1+x)}+2\log(-2+x)-2\log(1+x)\right)$$

input
```
Integrate[1/((-2 + x)^3*(1 + x)^2),x]
```

output
```
((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*Log[1 + x
])/54
```

3.186.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-2)^3(x+1)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{1}{27(x+1)}-\frac{1}{27(x+1)^2}+\frac{1}{27(x-2)}-\frac{2}{27(x-2)^2}+\frac{1}{9(x-2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{27(2-x)}+\frac{1}{27(x+1)}-\frac{1}{18(2-x)^2}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(x+1)$$

input
```
Int[1/((-2 + x)^3*(1 + x)^2),x]
```

output
```
-1/18*1/(2 - x)^2 - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/27 - Log[
1 + x]/27
```

3.186.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.186.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{18(-2+x)^2}+\frac{2}{27(-2+x)}+\frac{1}{27+27x}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
norman	$\frac{\frac{1}{9}x^2-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^2(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
risch	$\frac{\frac{1}{9}x^2-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^2(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$	35
parallelrisch	$\frac{2\ln(-2+x)x^3-2\ln(1+x)x^3-3-6\ln(-2+x)x^2+6\ln(1+x)x^2+6x^2+8\ln(-2+x)-8\ln(1+x)-15x}{54(-2+x)^2(1+x)}$	71

input
```
int(1/(-2+x)^3/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{6\,x^2-2\,(x^3-3\,x^2+4)\log(x+1)+2\,(x^3-3\,x^2+4)\log(x-2)-15\,x-3}{54\,(x^3-3\,x^2+4)}$$

input
```
integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="fricas")
```

output
```
1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4)*log(x -
 2) - 15*x - 3)/(x^3 - 3*x^2 + 4)
```

3.186.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{2x^2-5x-1}{18x^3-54x^2+72} + \frac{\log(x-2)}{27} - \frac{\log(x+1)}{27}$$

input
```
integrate(1/(-2+x)**3/(1+x)**2,x)
```

output
```
(2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log(x + 1)/2
7
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{2\,x^2-5\,x-1}{18\,(x^3-3\,x^2+4)} - \frac{1}{27}\,\log(x+1) + \frac{1}{27}\,\log(x-2)$$

input
```
integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="maxima")
```

output
```
1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27*log(x -
2)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{1}{27\,(x+1)} - \frac{\frac{18}{x+1}-5}{162\,\left(\frac{3}{x+1}-1\right)^2} + \frac{1}{27}\log\left(\left|-\frac{3}{x+1}+1\right|\right)$$

input
```
integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="giac")
```

output
```
1/27/(x + 1) - 1/162*(18/(x + 1) - 5)/(3/(x + 1) - 1)^2 + 1/27*log(abs(-3/
(x + 1) + 1))
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{3}-\frac{1}{3}\right)}{27}-\frac{-\frac{x^2}{9}+\frac{5\,x}{18}+\frac{1}{18}}{x^3-3\,x^2+4}$$

input
```
int(1/((x + 1)^2*(x - 2)^3),x)
```

output
```
- (2*atanh((2*x)/3 - 1/3))/27 - ((5*x)/18 - x^2/9 + 1/18)/(x^3 - 3*x^2 + 4
)
```

3.187 $\int \frac{1}{(2+x)^3(3+x)^4} \, dx$

3.187.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)$$

output
```
-1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

3.187.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)$$

input
```
Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

3.187.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)^3(x+3)^4}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{10}{x+3}-\frac{6}{(x+3)^2}-\frac{3}{(x+3)^3}-\frac{1}{(x+3)^4}+\frac{10}{x+2}-\frac{4}{(x+2)^2}+\frac{1}{(x+2)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

input
```
Int[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

3.187.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.187.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
norman	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$
risch	$\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$
default	$-\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\ln(2+x)-10\ln(3+x)$
parallelrisch	$\frac{60\ln(2+x)x^5-60\ln(3+x)x^5+2627+780\ln(2+x)x^4-780\ln(3+x)x^4+4020\ln(2+x)x^3-4020\ln(3+x)x^3+60x^4+10260\ln(2+x)}{6(2+x)^2(3+x)}$

input
```
int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*
ln(3+x)
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx$$
$$=\frac{60\,x^4+630\,x^3+2450\,x^2-60\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)\log(x+3)+60\,(x^5+13\,x^4+}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 2
16*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108
)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 1
08)
```

3.187.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10\log(x+2) - 10\log(x+3)$$

input
```
integrate(1/(2+x)**3/(3+x)**4,x)
```

output
```
(60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x
**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x^5 + 13\,x^4 + 67\,x^3 + 171\,x^2 + 216\,x + 108)} - 10\,\log(x+3) + 10\,\log(x+2)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
 171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x+3)^3(x+2)^2} - 10\,\log(|x+3|) + 10\,\log(|x+2|)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) -
10*log(abs(x + 3)) + 10*log(abs(x + 2))
```

3.187.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+x)^3(3+x)^4} \, dx = \frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108}-20\,\mathrm{atanh}(2\,x+5)$$

input
```
int(1/((x + 2)^3*(x + 3)^4),x)
```

output
```
((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
 67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

3.188 $\int \frac{x^5}{(3+x)^2} \, dx$

3.188.1 Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{x^5}{(3+x)^2} \, dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x)$$

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

3.188.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(3+x)^2} \, dx = \frac{1}{4}\left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x}\right) + 405\log(3+x)$$

input
```
Integrate[x^5/(3 + x)^2,x]
```

output
```
(-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

3.188.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x+3)^2}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(x^3-6x^2+27x+\frac{405}{x+3}-\frac{243}{(x+3)^2}-108\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^4}{4}-2x^3+\frac{27x^2}{2}-108x+\frac{243}{x+3}+405\log(x+3)$$

input
```
Int[x^5/(3 + x)^2,x]
```

output
```
-108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

3.188.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.188.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$	33
risch	$-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$	33
norman	$\frac{-\frac{135}{2}x^2+\frac{15}{2}x^3-\frac{5}{4}x^4+\frac{1}{4}x^5+1215}{3+x}+405\ln\left(3+x\right)$	36
meijerg	$-\frac{9x\left(-\frac{1}{27}x^4+\frac{5}{27}x^3-\frac{10}{9}x^2+10x+60\right)}{4\left(1+\frac{x}{3}\right)}+405\ln\left(1+\frac{x}{3}\right)$	40
parallelrisch	$\frac{x^5-5x^4+30x^3+1620\ln(3+x)x-270x^2+4860+4860\ln(3+x)}{12+4x}$	41

input
```
int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^5-5\,x^4+30\,x^3-270\,x^2+1620\,(x+3)\log\left(x+3\right)-1296\,x+972}{4\,(x+3)}$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

output
```
1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 9
72)/(x + 3)
```

3.188.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^4}{4}-2x^3+\frac{27x^2}{2}-108x+405\log\left(x+3\right)+\frac{243}{x+3}$$

input
```
integrate(x**5/(3+x)**2,x)
```

output
```
x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log\left(x+3\right)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(3+x)^2}\,dx = -\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log\left(|x+3|\right)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="giac")
```

output
```
-1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x
+ 3) + 405*log(abs(x + 3))
```

3.188.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = 405\,\ln\left(x+3\right) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

input
```
int(x^5/(x + 3)^2,x)
```

output
```
405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

3.189 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)\,dx$

3.189.1 Optimal result

Integrand size = 17, antiderivative size = 44

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)\,dx = a\mathrm{b1}x + \frac{1}{2}(2b\mathrm{b1} + a\mathrm{c1})x^2 + \frac{1}{3}(\mathrm{b1}c + 2b\mathrm{c1})x^3 + \frac{1}{4}c\mathrm{c1}x^4$$

output
```
a*b1*x+1/2*(a*c1+2*b*b1)*x^2+1/3*(2*b*c1+b1*c)*x^3+1/4*c*c1*x^4
```

3.189.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)\,dx = \frac{1}{12}x(6a(2\mathrm{b1} + \mathrm{c1}x) + x(4b(3\mathrm{b1} + 2\mathrm{c1}x) + cx(4\mathrm{b1} + 3\mathrm{c1}x)))$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

output
```
(x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))))/12
```

3.189.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(x(a\text{c1}+2b\text{b1})+a\text{b1}+x^2(2b\text{c1}+\text{b1}c)+c\text{c1}x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2(a\text{c1}+2b\text{b1})+a\text{b1}x+\frac{1}{3}x^3(2b\text{c1}+\text{b1}c)+\frac{1}{4}c\text{c1}x^4$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

output
```
a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4
```

3.189.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.189.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{c\,\mathrm{c1}\,x^{4}}{4}+\left(\frac{2b\,\mathrm{c1}}{3}+\frac{\mathrm{b1}\,c}{3}\right)x^{3}+\left(\frac{a\,\mathrm{c1}}{2}+b\,\mathrm{b1}\right)x^{2}+a\,\mathrm{b1}\,x$	38
default	$a\,\mathrm{b1}\,x+\frac{(a\,\mathrm{c1}+2b\,\mathrm{b1})x^{2}}{2}+\frac{(2b\,\mathrm{c1}+\mathrm{b1}\,c)x^{3}}{3}+\frac{c\,\mathrm{c1}\,x^{4}}{4}$	39
gosper	$\frac{1}{4}c\,\mathrm{c1}\,x^{4}+\frac{2}{3}x^{3}b\,\mathrm{c1}+\frac{1}{3}x^{3}\,\mathrm{b1}\,c+\frac{1}{2}x^{2}a\,\mathrm{c1}+x^{2}b\,\mathrm{b1}+a\,\mathrm{b1}\,x$	40
risch	$\frac{1}{4}c\,\mathrm{c1}\,x^{4}+\frac{2}{3}x^{3}b\,\mathrm{c1}+\frac{1}{3}x^{3}\,\mathrm{b1}\,c+\frac{1}{2}x^{2}a\,\mathrm{c1}+x^{2}b\,\mathrm{b1}+a\,\mathrm{b1}\,x$	40
parallelrisch	$\frac{1}{4}c\,\mathrm{c1}\,x^{4}+\frac{2}{3}x^{3}b\,\mathrm{c1}+\frac{1}{3}x^{3}\,\mathrm{b1}\,c+\frac{1}{2}x^{2}a\,\mathrm{c1}+x^{2}b\,\mathrm{b1}+a\,\mathrm{b1}\,x$	40

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*c1*x^4+(2/3*b*c1+1/3*b1*c)*x^3+(1/2*a*c1+b*b1)*x^2+a*b1*x
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int(\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^{2}\right)\,dx=\frac{1}{4}\,cc_{1}x^{4}+\frac{1}{3}\,(b_{1}c+2\,bc_{1})x^{3}+ab_{1}x+\frac{1}{2}\,(2\,bb_{1}+ac_{1})x^{2}$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="fricas")
```

output
```
1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

3.189.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int(\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^{2}\right)\,dx=ab_{1}x+\frac{cc_{1}x^{4}}{4}+x^{3}\cdot\left(\frac{2bc_{1}}{3}+\frac{b_{1}c}{3}\right)+x^{2}\left(\frac{ac_{1}}{2}+bb_{1}\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)
```

output
```
a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 + b*b1)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)\,dx = \frac{1}{4}cc_1x^4 + \frac{1}{3}(b_1c + 2bc_1)x^3 + ab_1x + \frac{1}{2}(2bb_1 + ac_1)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="maxima")
```

output
```
1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)\,dx = \frac{1}{4}cc_1x^4 + \frac{1}{3}b_1cx^3 + \frac{2}{3}bc_1x^3 + bb_1x^2 + \frac{1}{2}ac_1x^2 + ab_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="giac")
```

output
```
1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*x^2 + a*b
1*x
```

3.189.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)\,dx = \frac{c\,c_1\,x^4}{4} + \left(\frac{2\,b\,c_1}{3} + \frac{b_1\,c}{3}\right)x^3 + \left(\frac{a\,c_1}{2} + b\,b_1\right)x^2 + a\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2),x)
```

output
```
x^2*((a*c1)/2 + b*b1) + x^3*((2*b*c1)/3 + (b1*c)/3) + a*b1*x + (c*c1*x^4)/
4
```

3.190 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^2\,dx$

3.190.1 Optimal result

Integrand size = 19, antiderivative size = 96

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx=a^2\mathrm{b1}x+\frac{1}{2}a(4b\mathrm{b1}+a\mathrm{c1})x^2+\frac{2}{3}\left(2b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3+\frac{1}{2}\left(2b\mathrm{b1}c+2b^2\mathrm{c1}+ac\mathrm{c1}\right)x^4+\frac{1}{5}c(\mathrm{b1}c+4b\mathrm{c1})x^5+\frac{1}{6}c^2\mathrm{c1}x^6$$

output
```
a^2*b1*x+1/2*a*(a*c1+4*b*b1)*x^2+2/3*(2*a*b*c1+a*b1*c+2*b^2*b1)*x^3+1/2*(a
*c*c1+2*b^2*c1+2*b*b1*c)*x^4+1/5*c*(4*b*c1+b1*c)*x^5+1/6*c^2*c1*x^6
```

3.190.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx=\frac{1}{30}x\left(15a^2(2\mathrm{b1}+\mathrm{c1}x)+5ax(4b(3\mathrm{b1}+2\mathrm{c1}x)+cx(4\mathrm{b1}+3\mathrm{c1}x))+x^2\left(10b^2(4\mathrm{b1}+3\mathrm{c1}x)+6bcx(5\mathrm{b1}+4\mathrm{c1}x)+c^2x^2(6\mathrm{b1}+5\mathrm{c1}x)\right)\right)$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

output
```
(x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x
)) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*x) + c^2*x^2*(6*b1
 + 5*c1*x))))/30
```

3.190.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^2 dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^2\text{b1}+2x^3\left(ac\text{c1}+2b^2\text{c1}+2b\text{b1}c\right)+2x^2\left(2ab\text{c1}+a\text{b1}c+2b^2\text{b1}\right)+ax(a\text{c1}+4b\text{b1})+cx^4(4b\text{c1}+\text{b1}c)+c^2\text{c1}x$$

$$\Big\downarrow \text{2009}$$

$$a^2\text{b1}x+\frac{1}{2}x^4\left(ac\text{c1}+2b^2\text{c1}+2b\text{b1}c\right)+\frac{2}{3}x^3\left(2ab\text{c1}+a\text{b1}c+2b^2\text{b1}\right)+\frac{1}{2}ax^2(a\text{c1}+4b\text{b1})+\frac{1}{5}cx^5(4b\text{c1}+\text{b1}c)+\frac{1}{6}c^2\text{c1}x^6$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

output
```
a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*c + 4*b*c1)*x^5)/5
 + (c^2*c1*x^6)/6
```

3.190.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.190.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

method	result
norman	$\frac{c^2 \,\mathrm{c1}\, x^6}{6}+\left(\frac{4}{5}\,\mathrm{c1}\, b c+\frac{1}{5}\,\mathrm{b1}\, c^2\right) x^5+\left(\frac{1}{2} a c\,\mathrm{c1}+b^2\,\mathrm{c1}+b\,\mathrm{b1}\, c\right) x^4+\left(\frac{4}{3} a b\,\mathrm{c1}+\frac{2}{3} a\,\mathrm{b1}\, c+\frac{4}{3} b^2\,\mathrm{b1}\right) x^3$
default	$\frac{c^2 \,\mathrm{c1}\, x^6}{6}+\frac{\left(4\,\mathrm{c1}\, b c+\mathrm{b1}\, c^2\right) x^5}{5}+\frac{\left(4 b\,\mathrm{b1}\, c+\mathrm{c1}\left(2 a c+4 b^2\right)\right) x^4}{4}+\frac{\left(\mathrm{b1}\left(2 a c+4 b^2\right)+4 a b\,\mathrm{c1}\right) x^3}{3}+\frac{\left(\mathrm{c1}\, a^2+4\,\mathrm{b1}\, a b\right) x^2}{2}+a^2\,\mathrm{b}$
gosper	$\frac{1}{6} c^2\,\mathrm{c1}\, x^6+\frac{4}{5} x^5\,\mathrm{c1}\, b c+\frac{1}{5} x^5\,\mathrm{b1}\, c^2+\frac{1}{2} x^4 a c\,\mathrm{c1}+x^4 b^2\,\mathrm{c1}+x^4 b\,\mathrm{b1}\, c+\frac{4}{3} x^3 a b\,\mathrm{c1}+\frac{2}{3} x^3 a\,\mathrm{b1}\, c+\frac{4}{3} x$
risch	$\frac{1}{6} c^2\,\mathrm{c1}\, x^6+\frac{4}{5} x^5\,\mathrm{c1}\, b c+\frac{1}{5} x^5\,\mathrm{b1}\, c^2+\frac{1}{2} x^4 a c\,\mathrm{c1}+x^4 b^2\,\mathrm{c1}+x^4 b\,\mathrm{b1}\, c+\frac{4}{3} x^3 a b\,\mathrm{c1}+\frac{2}{3} x^3 a\,\mathrm{b1}\, c+\frac{4}{3} x$
parallelrisch	$\frac{1}{6} c^2\,\mathrm{c1}\, x^6+\frac{4}{5} x^5\,\mathrm{c1}\, b c+\frac{1}{5} x^5\,\mathrm{b1}\, c^2+\frac{1}{2} x^4 a c\,\mathrm{c1}+x^4 b^2\,\mathrm{c1}+x^4 b\,\mathrm{b1}\, c+\frac{4}{3} x^3 a b\,\mathrm{c1}+\frac{2}{3} x^3 a\,\mathrm{b1}\, c+\frac{4}{3} x$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*c1*x^6+(4/5*c1*b*c+1/5*b1*c^2)*x^5+(1/2*a*c*c1+b^2*c1+b*b1*c)*x^4+
(4/3*a*b*c1+2/3*a*b1*c+4/3*b^2*b1)*x^3+(1/2*c1*a^2+2*b1*a*b)*x^2+a^2*b1*x
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^2c_1x^6+\frac{1}{5}\left(b_1c^2+4\,bcc_1\right)x^5+\frac{1}{2}\left(2\,bb_1c+\left(2\,b^2+ac\right)c_1\right)x^4+a^2b_1x+\frac{2}{3}\left(2\,b^2b_1+ab_1c+2\,abc_1\right)x^3+\frac{1}{2}\left(4\,abb_1+a^2c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*
c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*
b*b1 + a^2*c1)*x^2
```

3.190.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right)^2 \, dx = a^2 b_1 x + \frac{c^2 c_1 x^6}{6} + x^5 \cdot \left(\frac{4bcc_1}{5} + \frac{b_1 c^2}{5}\right) + x^4 \left(\frac{acc_1}{2} + b^2 c_1 + bb_1 c\right) + x^3 \cdot \left(\frac{4abc_1}{3} + \frac{2ab_1 c}{3} + \frac{4b^2 b_1}{3}\right) + x^2 \left(\frac{a^2 c_1}{2} + 2abb_1\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

output
```
a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4*(a*c*c1/
2 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3) + x**
2*(a**2*c1/2 + 2*a*b*b1)
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right)^2 \, dx = \frac{1}{6} c^2 c_1 x^6 + \frac{1}{5} \left(b_1 c^2 + 4 \, bcc_1\right) x^5 + \frac{1}{2} \left(2 \, bb_1 c + \left(2 \, b^2 + ac\right) c_1\right) x^4 + a^2 b_1 x + \frac{2}{3} \left(2 \, b^2 b_1 + ab_1 c + 2 \, abc_1\right) x^3 + \frac{1}{2} \left(4 \, abb_1 + a^2 c_1\right) x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*
c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*
b*b1 + a^2*c1)*x^2
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2 dx = \frac{1}{6}c^2c_1x^6 + \frac{1}{5}b_1c^2x^5 + \frac{4}{5}bcc_1x^5 + bb_1cx^4 + b^2c_1x^4 + \frac{1}{2}acc_1x^4 + \frac{4}{3}b^2b_1x^3 + \frac{2}{3}ab_1cx^3 + \frac{4}{3}abc_1x^3 + 2abb_1x^2 + \frac{1}{2}a^2c_1x^2 + a^2b_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b^2*c1*x^4
 + 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/3*a*b*c1*x^3 + 2*a
*b*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x
```

3.190.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2 dx = x^3\left(\frac{4\,b_1\,b^2}{3} + \frac{4\,a\,c_1\,b}{3} + \frac{2\,a\,b_1\,c}{3}\right) + x^4\left(c_1\,b^2 + b_1\,c\,b + \frac{a\,c\,c_1}{2}\right) + x^2\left(\frac{c_1\,a^2}{2} + 2\,b\,b_1\,a\right) + x^5\left(\frac{b_1\,c^2}{5} + \frac{4\,b\,c_1\,c}{5}\right) + \frac{c^2\,c_1\,x^6}{6} + a^2\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x)
```

output
```
x^3*((4*b^2*b1)/3 + (4*a*b*c1)/3 + (2*a*b1*c)/3) + x^4*(b^2*c1 + (a*c*c1)/
2 + b*b1*c) + x^2*((a^2*c1)/2 + 2*a*b*b1) + x^5*((b1*c^2)/5 + (4*b*c*c1)/5
) + (c^2*c1*x^6)/6 + a^2*b1*x
```

3.191 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^3 \, dx$

3.191.1 Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx &= a^3\mathrm{b1}x+\frac{1}{2}a^2(6b\mathrm{b1}+a\mathrm{c1})x^2+a\left(4b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3\\ &\quad+\frac{1}{4}\left(8b^3\mathrm{b1}+12ab\mathrm{b1}c+12ab^2\mathrm{c1}+3a^2c\mathrm{c1}\right)x^4\\ &\quad+\frac{1}{5}\left(12b^2\mathrm{b1}c+3a\mathrm{b1}c^2+8b^3\mathrm{c1}+12abc\mathrm{c1}\right)x^5\\ &\quad+\frac{1}{2}c\left(2b\mathrm{b1}c+4b^2\mathrm{c1}+ac\mathrm{c1}\right)x^6\\ &\quad+\frac{1}{7}c^2(\mathrm{b1}c+6b\mathrm{c1})x^7+\frac{1}{8}c^3\mathrm{c1}x^8\end{aligned}$$

output
```
a^3*b1*x+1/2*a^2*(a*c1+6*b*b1)*x^2+a*(2*a*b*c1+a*b1*c+4*b^2*b1)*x^3+1/4*(3
*a^2*c*c1+12*a*b^2*c1+12*a*b*b1*c+8*b^3*b1)*x^4+1/5*(12*a*b*c*c1+3*a*b1*c^
2+8*b^3*c1+12*b^2*b1*c)*x^5+1/2*c*(a*c*c1+4*b^2*c1+2*b*b1*c)*x^6+1/7*c^2*(
6*b*c1+b1*c)*x^7+1/8*c^3*c1*x^8
```

3.191.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx = a^3\mathrm{b1}x+\frac{1}{2}a^2(6b\mathrm{b1}+a\mathrm{c1})x^2+a\left(4b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3 + \frac{1}{4}\left(8b^3\mathrm{b1}+12ab\mathrm{b1}c+12ab^2\mathrm{c1}+3a^2c\mathrm{c1}\right)x^4 + \frac{1}{5}\left(12b^2\mathrm{b1}c+3a\mathrm{b1}c^2+8b^3\mathrm{c1}+12abc\mathrm{c1}\right)x^5 + \frac{1}{2}c\left(2b\mathrm{b1}c+4b^2\mathrm{c1}+ac\mathrm{c1}\right)x^6 + \frac{1}{7}c^2(\mathrm{b1}c+6b\mathrm{c1})x^7+\frac{1}{8}c^3\mathrm{c1}x^8$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

output
```
a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*
x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b
^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b
^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

3.191.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(a^3\mathrm{b1}+x^3\left(3a^2c\mathrm{c1}+12ab^2\mathrm{c1}+12ab\mathrm{b1}c+8b^3\mathrm{b1}\right)+a^2x(a\mathrm{c1}+6b\mathrm{b1})+3cx^5\left(ac\mathrm{c1}+4b^2\mathrm{c1}+2b\mathrm{b1}c\right)+3ax^2(2a$$

$$\Big\downarrow \; 2009$$

$$a^3\text{b1}x+\frac{1}{4}x^4(3a^2c\text{c1}+12ab^2\text{c1}+12ab\text{b1}c+8b^3\text{b1})+\frac{1}{2}a^2x^2(a\text{c1}+6b\text{b1})+\frac{1}{2}cx^6(ac\text{c1}+4b^2\text{c1}+2b\text{b1}c)+ax^3(2ab\text{c1}+a\text{b1}c+4b^2\text{b1})+\frac{1}{5}x^5(12abc\text{c1}+3a\text{b1}c^2+8b^3\text{c1}+12b^2\text{b1}c)+\frac{1}{7}c^2x^7(6b\text{c1}+\text{b1}c)+\frac{1}{8}c^3\text{c1}x^8$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

output
```
a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*
x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b
^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b
^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

3.191.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.191.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

method	result
norman	$\frac{c^3\,\text{c1}\,x^8}{8}+\left(\frac{6}{7}\text{c1}\,b\,c^2+\frac{1}{7}\text{b1}\,c^3\right)x^7+\left(\frac{1}{2}a\,c^2\,\text{c1}+2b^2c\,\text{c1}+\text{b1}\,b\,c^2\right)x^6+\left(\frac{12}{5}abc\,\text{c1}+\frac{3}{5}a\,\text{b1}\,c^2+\right.$
gosper	$\frac{1}{8}c^3\,\text{c1}\,x^8+\frac{6}{7}x^7\,\text{c1}\,b\,c^2+\frac{1}{7}x^7\,\text{b1}\,c^3+\frac{1}{2}x^6a\,c^2\,\text{c1}+2x^6b^2c\,\text{c1}+x^6\,\text{b1}\,b\,c^2+\frac{12}{5}x^5abc\,\text{c1}+\frac{3}{5}x^5a$
risch	$\frac{1}{8}c^3\,\text{c1}\,x^8+\frac{6}{7}x^7\,\text{c1}\,b\,c^2+\frac{1}{7}x^7\,\text{b1}\,c^3+\frac{1}{2}x^6a\,c^2\,\text{c1}+2x^6b^2c\,\text{c1}+x^6\,\text{b1}\,b\,c^2+\frac{12}{5}x^5abc\,\text{c1}+\frac{3}{5}x^5a$
parallelrisch	$\frac{1}{8}c^3\,\text{c1}\,x^8+\frac{6}{7}x^7\,\text{c1}\,b\,c^2+\frac{1}{7}x^7\,\text{b1}\,c^3+\frac{1}{2}x^6a\,c^2\,\text{c1}+2x^6b^2c\,\text{c1}+x^6\,\text{b1}\,b\,c^2+\frac{12}{5}x^5abc\,\text{c1}+\frac{3}{5}x^5a$
default	$\frac{c^3\,\text{c1}\,x^8}{8}+\frac{(6\,\text{c1}\,b\,c^2+\text{b1}\,c^3)x^7}{7}+\frac{(6\,\text{b1}\,b\,c^2+\text{c1}(c^2a+8b^2c+c(2ac+4b^2)))x^6}{6}+\frac{(\text{b1}(c^2a+8b^2c+c(2ac+4b^2))+\text{c1}(8abc+2}{5}$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*c1*x^8+(6/7*c1*b*c^2+1/7*b1*c^3)*x^7+(1/2*a*c^2*c1+2*b^2*c*c1+b1*b
*c^2)*x^6+(12/5*a*b*c*c1+3/5*a*b1*c^2+8/5*b^3*c1+12/5*b^2*b1*c)*x^5+(3/4*a
^2*c*c1+3*a*b^2*c1+3*a*b*b1*c+2*b^3*b1)*x^4+(2*a^2*b*c1+a^2*b1*c+4*a*b^2*b
1)*x^3+(1/2*c1*a^3+3*b1*a^2*b)*x^2+a^3*b1*x
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx &= \frac{1}{8}c^3c_1x^8+\frac{1}{7}\left(b_1c^3+6\,bc^2c_1\right)x^7\\ &\quad+\frac{1}{2}\left(2\,bb_1c^2+\left(4\,b^2c+ac^2\right)c_1\right)x^6\\ &\quad+\frac{1}{5}\left(12\,b^2b_1c+3\,ab_1c^2+4\left(2\,b^3+3\,abc\right)c_1\right)x^5\\ &\quad+a^3b_1x+\frac{1}{4}\left(8\,b^3b_1+12\,abb_1c+3\left(4\,ab^2+a^2c\right)c_1\right)x^4\\ &\quad+\left(4\,ab^2b_1+a^2b_1c+2\,a^2bc_1\right)x^3+\frac{1}{2}\left(6\,a^2bb_1+a^3c_1\right)x^2\end{aligned}$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*
c + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c
1)*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*
x^4 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)
*x^2
```

3.191.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^3\,dx = a^3b_1x + \frac{c^3c_1x^8}{8} + x^7\cdot\left(\frac{6bc^2c_1}{7} + \frac{b_1c^3}{7}\right) + x^6\left(\frac{ac^2c_1}{2} + 2b^2cc_1 + bb_1c^2\right) + x^5\cdot\left(\frac{12abcc_1}{5} + \frac{3ab_1c^2}{5} + \frac{8b^3c_1}{5} + \frac{12b^2b_1c}{5}\right) + x^4\cdot\left(\frac{3a^2cc_1}{4} + 3ab^2c_1 + 3abb_1c + 2b^3b_1\right) + x^3\cdot\left(2a^2bc_1 + a^2b_1c + 4ab^2b_1\right) + x^2\left(\frac{a^3c_1}{2} + 3a^2bb_1\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

output
```
a**3*b1*x + c**3*c1*x**8/8 + x**7*(6*b*c**2*c1/7 + b1*c**3/7) + x**6*(a*c*
*2*c1/2 + 2*b**2*c*c1 + b*b1*c**2) + x**5*(12*a*b*c*c1/5 + 3*a*b1*c**2/5 +
 8*b**3*c1/5 + 12*b**2*b1*c/5) + x**4*(3*a**2*c*c1/4 + 3*a*b**2*c1 + 3*a*b
*b1*c + 2*b**3*b1) + x**3*(2*a**2*b*c1 + a**2*b1*c + 4*a*b**2*b1) + x**2*(
a**3*c1/2 + 3*a**2*b*b1)
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^3\,dx = \frac{1}{8}\,c^3c_1x^8 + \frac{1}{7}\left(b_1c^3 + 6\,bc^2c_1\right)x^7 + \frac{1}{2}\left(2\,bb_1c^2 + \left(4\,b^2c + ac^2\right)c_1\right)x^6 + \frac{1}{5}\left(12\,b^2b_1c + 3\,ab_1c^2 + 4\left(2\,b^3 + 3\,abc\right)c_1\right)x^5 + a^3b_1x + \frac{1}{4}\left(8\,b^3b_1 + 12\,abb_1c + 3\left(4\,ab^2 + a^2c\right)c_1\right)x^4 + \left(4\,ab^2b_1 + a^2b_1c + 2\,a^2bc_1\right)x^3 + \frac{1}{2}\left(6\,a^2bb_1 + a^3c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*
c + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c
1)*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*
x^4 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)
*x^2
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^3 dx = {} & \frac{1}{8}c^3c_1x^8 + \frac{1}{7}b_1c^3x^7 + \frac{6}{7}bc^2c_1x^7 + bb_1c^2x^6 \\ & + 2b^2cc_1x^6 + \frac{1}{2}ac^2c_1x^6 + \frac{12}{5}b^2b_1cx^5 + \frac{3}{5}ab_1c^2x^5 \\ & + \frac{8}{5}b^3c_1x^5 + \frac{12}{5}abcc_1x^5 + 2b^3b_1x^4 + 3abb_1cx^4 \\ & + 3ab^2c_1x^4 + \frac{3}{4}a^2cc_1x^4 + 4ab^2b_1x^3 + a^2b_1cx^3 \\ & + 2a^2bc_1x^3 + 3a^2bb_1x^2 + \frac{1}{2}a^3c_1x^2 + a^3b_1x\end{aligned}$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*c^3*c1*x^8 + 1/7*b1*c^3*x^7 + 6/7*b*c^2*c1*x^7 + b*b1*c^2*x^6 + 2*b^2*
c*c1*x^6 + 1/2*a*c^2*c1*x^6 + 12/5*b^2*b1*c*x^5 + 3/5*a*b1*c^2*x^5 + 8/5*b
^3*c1*x^5 + 12/5*a*b*c*c1*x^5 + 2*b^3*b1*x^4 + 3*a*b*b1*c*x^4 + 3*a*b^2*c1
*x^4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 +
 3*a^2*b*b1*x^2 + 1/2*a^3*c1*x^2 + a^3*b1*x
```

3.191.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx = x^7\left(\frac{b_1\,c^3}{7}+\frac{6\,b\,c_1\,c^2}{7}\right)+x^3\left(2\,c_1\,a^2\,b+b_1\,c\,a^2+4\,b_1\,a\,b^2\right)+x^6\left(2\,c_1\,b^2\,c+b_1\,b\,c^2+\frac{a\,c_1\,c^2}{2}\right)+x^4\left(\frac{3\,c\,c_1\,a^2}{4}+3\,c_1\,a\,b^2+3\,b_1\,c\,a\,b+2\,b_1\,b^3\right)+x^5\left(\frac{8\,c_1\,b^3}{5}+\frac{12\,b_1\,b^2\,c}{5}+\frac{12\,a\,c_1\,b\,c}{5}+\frac{3\,a\,b_1\,c^2}{5}\right)+x^2\left(\frac{c_1\,a^3}{2}+3\,b\,b_1\,a^2\right)+\frac{c^3\,c_1\,x^8}{8}+a^3\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x)
```

output
```
x^7*((b1*c^3)/7 + (6*b*c^2*c1)/7) + x^3*(4*a*b^2*b1 + 2*a^2*b*c1 + a^2*b1*
c) + x^6*((a*c^2*c1)/2 + b*b1*c^2 + 2*b^2*c*c1) + x^4*(2*b^3*b1 + 3*a*b^2*
c1 + (3*a^2*c*c1)/4 + 3*a*b*b1*c) + x^5*((8*b^3*c1)/5 + (3*a*b1*c^2)/5 + (
12*b^2*b1*c)/5 + (12*a*b*c*c1)/5) + x^2*((a^3*c1)/2 + 3*a^2*b*b1) + (c^3*c
1*x^8)/8 + a^3*b1*x
```

3.192 $\int (\mathbf{b1} + \mathbf{c1}x) (a + 2bx + cx^2)^4 \, dx$

3.192.1 Optimal result

Integrand size = 19, antiderivative size = 263

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x) \left(a+2bx+cx^2\right)^4 \, dx = {} & a^4\mathrm{b1}x+\frac{1}{2}a^3(8b\mathrm{b1}+a\mathrm{c1})x^2+\frac{4}{3}a^2\left(6b^2\mathrm{b1}+a\mathrm{b1}c+2abc1\right)x^3 \\ & + a\left(8b^3\mathrm{b1}+6ab\mathrm{b1}c+6ab^2\mathrm{c1}+a^2c\mathrm{c1}\right)x^4+\frac{2}{5}\left(8b^4\mathrm{b1}\right. \\ & \left. +24ab^2\mathrm{b1}c+3a^2\mathrm{b1}c^2+16ab^3\mathrm{c1}+12a^2bc\mathrm{c1}\right)x^5 \\ & +\frac{1}{3}\left(16b^3\mathrm{b1}c+12ab\mathrm{b1}c^2+8b^4\mathrm{c1}+24ab^2c\mathrm{c1}+3a^2c^2\mathrm{c1}\right)x^6 \\ & +\frac{4}{7}c\left(6b^2\mathrm{b1}c+a\mathrm{b1}c^2+8b^3\mathrm{c1}+6abc\mathrm{c1}\right)x^7 \\ & +\frac{1}{2}c^2\left(2b\mathrm{b1}c+6b^2\mathrm{c1}+ac\mathrm{c1}\right)x^8 \\ & +\frac{1}{9}c^3(\mathrm{b1}c+8b\mathrm{c1})x^9+\frac{1}{10}c^4\mathrm{c1}x^{10}\end{aligned}$$

output
```
a^4*b1*x+1/2*a^3*(a*c1+8*b*b1)*x^2+4/3*a^2*(2*a*b*c1+a*b1*c+6*b^2*b1)*x^3+
a*(a^2*c*c1+6*a*b^2*c1+6*a*b*b1*c+8*b^3*b1)*x^4+2/5*(12*a^2*b*c*c1+3*a^2*b
1*c^2+16*a*b^3*c1+24*a*b^2*b1*c+8*b^4*b1)*x^5+1/3*(3*a^2*c^2*c1+24*a*b^2*c
*c1+12*a*b*b1*c^2+8*b^4*c1+16*b^3*b1*c)*x^6+4/7*c*(6*a*b*c*c1+a*b1*c^2+8*b
^3*c1+6*b^2*b1*c)*x^7+1/2*c^2*(a*c*c1+6*b^2*c1+2*b*b1*c)*x^8+1/9*c^3*(8*b*
c1+b1*c)*x^9+1/10*c^4*c1*x^10
```

3.192.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int (\text{b1}+\text{c1}x) \left(a+2bx+cx^2\right)^4 \, dx = a^4\text{b1}x+\frac{1}{2}a^3(8b\text{b1}+a\text{c1})x^2+\frac{4}{3}a^2\left(6b^2\text{b1}+a\text{b1}c+2ab\text{c1}\right)x^3 + a\left(8b^3\text{b1}+6ab\text{b1}c+6ab^2\text{c1}+a^2c\text{c1}\right)x^4+\frac{2}{5}\left(8b^4\text{b1} + 24ab^2\text{b1}c+3a^2\text{b1}c^2+16ab^3\text{c1}+12a^2bc\text{c1}\right)x^5 + \frac{1}{3}\left(16b^3\text{b1}c+12ab\text{b1}c^2+8b^4\text{c1}+24ab^2c\text{c1}+3a^2c^2\text{c1}\right)x^6 + \frac{4}{7}c\left(6b^2\text{b1}c+a\text{b1}c^2+8b^3\text{c1}+6abc\text{c1}\right)x^7 + \frac{1}{2}c^2\left(2b\text{b1}c+6b^2\text{c1}+ac\text{c1}\right)x^8 + \frac{1}{9}c^3(\text{b1}c+8b\text{c1})x^9+\frac{1}{10}c^4\text{c1}x^{10}$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

output
```
a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b
*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(
8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5
)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2
*c1)*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7
+ (c^2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9
 + (c^4*c1*x^10)/10
```

3.192.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x) \left(a+2bx+cx^2\right)^4 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^4\text{b1} + a^3x(a\text{c1} + 8b\text{b1}) + 4a^2x^2\left(2ab\text{c1} + a\text{b1}c + 6b^2\text{b1}\right) + 4ax^3\left(a^2c\text{c1} + 6ab^2\text{c1} + 6ab\text{b1}c + 8b^3\text{b1}\right) + 2x^5\left(3a\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4\text{b1}x + \frac{1}{2}a^3x^2(a\text{c1} + 8b\text{b1}) + \frac{4}{3}a^2x^3\left(2ab\text{c1} + a\text{b1}c + 6b^2\text{b1}\right) + ax^4\left(a^2c\text{c1} + 6ab^2\text{c1} + 6ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{3}x^6\left(3a^2c^2\text{c1} + 24ab^2c\text{c1} + 12ab\text{b1}c^2 + 8b^4\text{c1} + 16b^3\text{b1}c\right) + \frac{2}{5}x^5\left(12a^2bc\text{c1} + 3a^2\text{b1}c^2 + 16ab^3\text{c1} + 24ab^2\text{b1}c + 8b^4\text{b1}\right) + \frac{1}{2}c^2x^8\left(ac\text{c1} + 6b^2\text{c1} + 2b\text{b1}c\right) + \frac{4}{7}cx^7\left(6abc\text{c1} + a\text{b1}c^2 + 8b^3\text{c1} + 6b^2\text{b1}c\right) + \frac{1}{9}c^3x^9(8b\text{c1} + \text{b1}c) + \frac{1}{10}c^4\text{c1}x^{10}$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

output
```
a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b
*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(
8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5
)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2
*c1)*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7
+ (c^2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9
 + (c^4*c1*x^10)/10
```

3.192.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.192.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00

method	result
norman	$\frac{c^4 \operatorname{c1} x^{10}}{10}+\left(\frac{8}{9} \operatorname{c1} b\, c^3+\frac{1}{9} \operatorname{b1} \, c^4\right) x^9+\left(\frac{1}{2} a\, c^3 \operatorname{c1}+3 b^2 c^2 \operatorname{c1}+\operatorname{b1} b\, c^3\right) x^8+\left(\frac{24}{7} a b\, c^2 \operatorname{c1}+\frac{4}{7} a \operatorname{b1} \, c^3\right.$
gosper	$a^4 \operatorname{b1} x+\frac{8}{9} x^9 \operatorname{c1} b\, c^3+\frac{1}{2} x^2 \operatorname{c1} \, a^4+\frac{1}{9} x^9 \operatorname{b1} \, c^4+\frac{8}{3} x^6 b^4 \operatorname{c1}+\frac{16}{5} x^5 b^4 \operatorname{b1}+\frac{1}{2} x^8 a\, c^3 \operatorname{c1}+3 x^8 b^2 c^2 \operatorname{c1}$
risch	$a^4 \operatorname{b1} x+\frac{8}{9} x^9 \operatorname{c1} b\, c^3+\frac{1}{2} x^2 \operatorname{c1} \, a^4+\frac{1}{9} x^9 \operatorname{b1} \, c^4+\frac{8}{3} x^6 b^4 \operatorname{c1}+\frac{16}{5} x^5 b^4 \operatorname{b1}+\frac{1}{2} x^8 a\, c^3 \operatorname{c1}+3 x^8 b^2 c^2 \operatorname{c1}$
parallelrisch	$a^4 \operatorname{b1} x+\frac{8}{9} x^9 \operatorname{c1} b\, c^3+\frac{1}{2} x^2 \operatorname{c1} \, a^4+\frac{1}{9} x^9 \operatorname{b1} \, c^4+\frac{8}{3} x^6 b^4 \operatorname{c1}+\frac{16}{5} x^5 b^4 \operatorname{b1}+\frac{1}{2} x^8 a\, c^3 \operatorname{c1}+3 x^8 b^2 c^2 \operatorname{c1}$
default	$\frac{c^4 \operatorname{c1} \, x^{10}}{10}+\frac{\left(8 \operatorname{c1} b\, c^3+\operatorname{b1} \, c^4\right) x^9}{9}+\frac{\left(8 \operatorname{b1} b\, c^3+\operatorname{c1} \left(2\left(2 a c+4 b^2\right) c^2+16 b^2 c^2\right)\right) x^8}{8}+\frac{\left(\operatorname{b1} \left(2\left(2 a c+4 b^2\right) c^2+16 b^2 c^2\right)+\operatorname{c1} \left(8 a b\, c^2+\right.\right.}{7}$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^4*c1*x^10+(8/9*c1*b*c^3+1/9*b1*c^4)*x^9+(1/2*a*c^3*c1+3*b^2*c^2*c1+
b1*b*c^3)*x^8+(24/7*a*b*c^2*c1+4/7*a*b1*c^3+32/7*b^3*c*c1+24/7*b^2*b1*c^2)
*x^7+(a^2*c^2*c1+8*a*b^2*c*c1+4*a*b*b1*c^2+8/3*b^4*c1+16/3*b^3*b1*c)*x^6+(
24/5*a^2*b*c*c1+6/5*a^2*b1*c^2+32/5*a*b^3*c1+48/5*a*b^2*b1*c+16/5*b^4*b1)*
x^5+(a^3*c*c1+6*a^2*b^2*c1+6*a^2*b*b1*c+8*a*b^3*b1)*x^4+(8/3*c1*a^3*b+4/3*
a^3*b1*c+8*a^2*b^2*b1)*x^3+(1/2*c1*a^4+4*b1*a^3*b)*x^2+a^4*b1*x
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx \\
&= \frac{1}{10}c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8 \\
&\quad+\frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)x^7 \\
&\quad+\frac{1}{3}\left(16\,b^3b_1c+12\,abb_1c^2+\left(8\,b^4+24\,ab^2c+3\,a^2c^2\right)c_1\right)x^6+a^4b_1x \\
&\quad+\frac{2}{5}\left(8\,b^4b_1+24\,ab^2b_1c+3\,a^2b_1c^2+4\left(4\,ab^3+3\,a^2bc\right)c_1\right)x^5 \\
&\quad+\left(8\,ab^3b_1+6\,a^2bb_1c+\left(6\,a^2b^2+a^3c\right)c_1\right)x^4 \\
&\quad+\frac{4}{3}\left(6\,a^2b^2b_1+a^3b_1c+2\,a^3bc_1\right)x^3+\frac{1}{2}\left(8\,a^3bb_1+a^4c_1\right)x^2
\end{aligned}$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

output
```
1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^
2*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b
*c^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3
*a^2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^
2 + 4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*
b^2 + a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/
2*(8*a^3*b*b1 + a^4*c1)*x^2
```

3.192.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.19

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = a^4b_1x+\frac{c^4c_1x^{10}}{10}+x^9\cdot\left(\frac{8bc^3c_1}{9}+\frac{b_1c^4}{9}\right)+x^8\left(\frac{ac^3c_1}{2}+3b^2c^2c_1+bb_1c^3\right)+x^7\cdot\left(\frac{24abc^2c_1}{7}+\frac{4ab_1c^3}{7}+\frac{32b^3cc_1}{7}+\frac{24b^2b_1c^2}{7}\right)+x^6\left(a^2c^2c_1+8ab^2cc_1+4abb_1c^2+\frac{8b^4c_1}{3}+\frac{16b^3b_1c}{3}\right)+x^5\cdot\left(\frac{24a^2bcc_1}{5}+\frac{6a^2b_1c^2}{5}+\frac{32ab^3c_1}{5}+\frac{48ab^2b_1c}{5}+\frac{16b^4b_1}{5}\right)+x^4\left(a^3cc_1+6a^2b^2c_1+6a^2bb_1c+8ab^3b_1\right)+x^3\cdot\left(\frac{8a^3bc_1}{3}+\frac{4a^3b_1c}{3}+8a^2b^2b_1\right)+x^2\left(\frac{a^4c_1}{2}+4a^3bb_1\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

output
```
a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) + x**8*(a*
c**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c**2*c1/7 + 4*a*b1*
c**3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) + x**6*(a**2*c**2*c1 + 8*a*b*
*2*c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*
c*c1/5 + 6*a**2*b1*c**2/5 + 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1
/5) + x**4*(a**3*c*c1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x*
*3*(8*a**3*b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*
a**3*b*b1)
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = \frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8 + \frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)x^7 + \frac{1}{3}\left(16\,b^3b_1c+12\,abb_1c^2+\left(8\,b^4+24\,ab^2c+3\,a^2c^2\right)c_1\right)x^6+a^4b_1x + \frac{2}{5}\left(8\,b^4b_1+24\,ab^2b_1c+3\,a^2b_1c^2+4\left(4\,ab^3+3\,a^2bc\right)c_1\right)x^5 + \left(8\,ab^3b_1+6\,a^2bb_1c+\left(6\,a^2b^2+a^3c\right)c_1\right)x^4 + \frac{4}{3}\left(6\,a^2b^2b_1+a^3b_1c+2\,a^3bc_1\right)x^3+\frac{1}{2}\left(8\,a^3bb_1+a^4c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

output
```
1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^
2*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b
*c^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3
*a^2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^
2 + 4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*
b^2 + a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/
2*(8*a^3*b*b1 + a^4*c1)*x^2
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.17

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = \frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\,b_1c^4x^9+\frac{8}{9}\,bc^3c_1x^9+bb_1c^3x^8+3\,b^2c^2c_1x^8 + \frac{1}{2}\,ac^3c_1x^8+\frac{24}{7}\,b^2b_1c^2x^7+\frac{4}{7}\,ab_1c^3x^7+\frac{32}{7}\,b^3cc_1x^7 + \frac{24}{7}\,abc^2c_1x^7+\frac{16}{3}\,b^3b_1cx^6+4\,abb_1c^2x^6+\frac{8}{3}\,b^4c_1x^6 + 8\,ab^2cc_1x^6+a^2c^2c_1x^6+\frac{16}{5}\,b^4b_1x^5+\frac{48}{5}\,ab^2b_1cx^5 + \frac{6}{5}\,a^2b_1c^2x^5+\frac{32}{5}\,ab^3c_1x^5+\frac{24}{5}\,a^2bcc_1x^5+8\,ab^3b_1x^4 + 6\,a^2bb_1cx^4+6\,a^2b^2c_1x^4+a^3cc_1x^4+8\,a^2b^2b_1x^3 + \frac{4}{3}\,a^3b_1cx^3+\frac{8}{3}\,a^3bc_1x^3+4\,a^3bb_1x^2+\frac{1}{2}\,a^4c_1x^2+a^4b_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

output
```
1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x^8 + 3*b^
2*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 + 4/7*a*b1*c^3*x^7 +
 32/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3*b^3*b1*c*x^6 + 4*a*b*b1*c^
2*x^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x
^5 + 48/5*a*b^2*b1*c*x^5 + 6/5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a
^2*b*c*c1*x^5 + 8*a*b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3
*c*c1*x^4 + 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3
*b*b1*x^2 + 1/2*a^4*c1*x^2 + a^4*b1*x
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = x^9\left(\frac{b_1\,c^4}{9}+\frac{8\,b\,c_1\,c^3}{9}\right)+x^3\left(\frac{8\,c_1\,a^3\,b}{3}+\frac{4\,b_1\,c\,a^3}{3}+8\,b_1\,a^2\,b^2\right)+x^8\left(3\,c_1\,b^2\,c^2+b_1\,b\,c^3+\frac{a\,c_1\,c^3}{2}\right)+x^5\left(\frac{24\,c_1\,a^2\,b\,c}{5}+\frac{6\,b_1\,a^2\,c^2}{5}+\frac{32\,c_1\,a\,b^3}{5}+\frac{48\,b_1\,a\,b^2\,c}{5}+\frac{16\,b_1\,b^4}{5}\right)+x^6\left(c_1\,a^2\,c^2+8\,c_1\,a\,b^2\,c+4\,b_1\,a\,b\,c^2+\frac{8\,c_1\,b^4}{3}+\frac{16\,b_1\,b^3\,c}{3}\right)+x^4\left(c\,c_1\,a^3+6\,c_1\,a^2\,b^2+6\,b_1\,c\,a^2\,b+8\,b_1\,a\,b^3\right)+x^7\left(\frac{32\,c_1\,b^3\,c}{7}+\frac{24\,b_1\,b^2\,c^2}{7}+\frac{24\,a\,c_1\,b\,c^2}{7}+\frac{4\,a\,b_1\,c^3}{7}\right)+x^2\left(\frac{c_1\,a^4}{2}+4\,b\,b_1\,a^3\right)+\frac{c^4\,c_1\,x^{10}}{10}+a^4\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x)
```

output
```
x^9*((b1*c^4)/9 + (8*b*c^3*c1)/9) + x^3*(8*a^2*b^2*b1 + (8*a^3*b*c1)/3 + (
4*a^3*b1*c)/3) + x^8*(3*b^2*c^2*c1 + (a*c^3*c1)/2 + b*b1*c^3) + x^5*((16*b
^4*b1)/5 + (6*a^2*b1*c^2)/5 + (32*a*b^3*c1)/5 + (48*a*b^2*b1*c)/5 + (24*a^
2*b*c*c1)/5) + x^6*((8*b^4*c1)/3 + a^2*c^2*c1 + (16*b^3*b1*c)/3 + 4*a*b*b1
*c^2 + 8*a*b^2*c*c1) + x^4*(6*a^2*b^2*c1 + 8*a*b^3*b1 + a^3*c*c1 + 6*a^2*b
*b1*c) + x^7*((24*b^2*b1*c^2)/7 + (4*a*b1*c^3)/7 + (32*b^3*c*c1)/7 + (24*a
*b*c^2*c1)/7) + x^2*((a^4*c1)/2 + 4*a^3*b*b1) + (c^4*c1*x^10)/10 + a^4*b1*
x
```

3.192. $\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^4\,dx$

3.193 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^n \, dx$

3.193.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^n \, dx = \frac{\mathrm{c1}\left(a + 2bx + cx^2\right)^{1+n}}{2c(1+n)} - \frac{2^n(\mathrm{b1}c - b\mathrm{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}\left(a+2bx+cx^2\right)^{1+n}\text{Hypergeometric2F1}\left(-n, 1+n, 2+n, \frac{b+\sqrt{b^2-ac}}{2\sqrt{b^2-a}}\right.}{c\sqrt{b^2-ac}(1+n)}$$

output

```
1/2*c1*(c*x^2+2*b*x+a)^(1+n)/c/(1+n)-2^n*(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1+n
)*hypergeom([-n, 1+n],[2+n],1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))
*((-b-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1-n)/c/(1+n)/(-a*c+b^2)^(1
/2)
```

3.193.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.67 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.68

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^n\,dx$$
$$=\frac{1}{2}(a+x(2b+cx))^n\left(\text{c1}x^2\left(\frac{b-\sqrt{b^2-ac}+cx}{b-\sqrt{b^2-ac}}\right)^{-n}\left(\frac{b+\sqrt{b^2-ac}+cx}{b+\sqrt{b^2-ac}}\right)^{-n}\text{AppellF1}\left(2,-n,-n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right)+\frac{2^{1+n}\text{b1}\left(b-\sqrt{b^2-ac}+cx\right)\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-n}\text{Hypergeometric2F1}\left(-n,1+n,2+n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right)}{c(1+n)}\right)$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

output
```
((a + x*(2*b + c*x))^n*((c1*x^2*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b
^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])]])/(((b - Sqrt[b^2 - a*c] + c*x)/
(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]
))^n) + (2^(1 + n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*Hypergeometric2F1[-n, 1
+ n, 2 + n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*
((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n)))/2
```

3.193.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^n\,dx$$
$$\Big\downarrow \text{1160}$$

$$\frac{(\text{b1}c-b\text{c1})\int \left(cx^2+2bx+a\right)^n dx}{c}+\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}$$

$$\Big\downarrow \text{ 1096}$$

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}\text{Hypergeometric2F1}\left(-n,n+1,n+2,\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(n+1)\sqrt{b^2-ac}}$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

output
```
(c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-((b
- Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1
 + n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*S
qrt[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(1 + n))
```

3.193.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.193.4 Maple [F]

$$\int (\text{c1}\,x+\text{b1})\left(c\,x^2+2bx+a\right)^n dx$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

output
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

3.193.5 Fricas [F]

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^n dx = \int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n dx$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="fricas")
```

output
```
integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

3.193.6 Sympy [F]

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^n dx = \int (b_1 + c_1x)\left(a + 2bx + cx^2\right)^n dx$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
```

output
```
Integral((b1 + c1*x)*(a + 2*b*x + c*x**2)**n, x)
```

3.193.7 Maxima [F]

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^n dx = \int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n dx$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

3.193.8 Giac [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (c_1x + b_1)(cx^2 + 2\,bx + a)^n \, dx$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="giac")
```

output
```
integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (b_1 + c_1\,x) \left(c\,x^2 + 2\,b\,x + a\right)^n dx$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x)
```

output
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n, x)
```

3.194 $\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}\,dx$

3.194.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}\,dx=-\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}+\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

output
```
1/2*c1*ln(c*x^2+2*b*x+a)/c-(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/
c/(-a*c+b^2)^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}\,dx=\frac{(\text{b1}c-b\text{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{c\sqrt{-b^2+ac}}+\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

output
```
((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 + a*c]) +
(c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

3.194.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{(\text{b1}c-b\text{c1})\int \frac{1}{cx^2+2bx+a}dx}{c}+\frac{\text{c1}\int \frac{2(b+cx)}{cx^2+2bx+a}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{(\text{b1}c-b\text{c1})\int \frac{1}{cx^2+2bx+a}dx}{c}+\frac{\text{c1}\int \frac{b+cx}{cx^2+2bx+a}dx}{c}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\text{c1}\int \frac{b+cx}{cx^2+2bx+a}dx}{c}-\frac{2(\text{b1}c-b\text{c1})\int \frac{1}{4(b^2-ac)-(2b+2cx)^2}d(2b+2cx)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{c1}\int \frac{b+cx}{cx^2+2bx+a}dx}{c}-\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

output
```
-(((b1*c - b*c1)*ArcTanh[(2*b + 2*c*x)/(2*Sqrt[b^2 - a*c])])/(c*Sqrt[b^2 -
 a*c])) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.194.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result
default	$\frac{c1\ln\left(c\,x^{2}+2bx+a\right)}{2c}+\frac{\left(b1-\frac{c1\,b}{c}\right)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^{2}}}\right)}{\sqrt{ac-b^{2}}}$
risch	$\frac{\ln\left(-abc\,c1+a\,b1\,c^{2}+b^{3}\,c1-b^{2}\,b1\,c-\sqrt{-\left(b\,c1-b1\,c\right)^{2}\left(ac-b^{2}\right)}\,cx-\sqrt{-\left(b\,c1-b1\,c\right)^{2}\left(ac-b^{2}\right)}\,b\right)a\,c1}{2ac-2b^{2}}-\ln\left(-abc\,c1+a\,b1\,c^{2}+b^{3}\,c1\right.$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c1*ln(c*x^2+2*b*x+a)/c+(b1-c1*b/c)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2
*b)/(a*c-b^2)^(1/2))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.12

$$\int \frac{\text{b1} + \text{c1}x}{a + 2bx + cx^2}\,dx = \left[\frac{(b^2 - ac)c_1 \log\left(cx^2 + 2\,bx + a\right) - \sqrt{b^2 - ac}(b_1c - bc_1)\log\left(\frac{c^2x^2 + 2\,bcx + 2\,b^2 - ac + 2\,\sqrt{b^2 - ac}(cx + b)}{cx^2 + 2\,bx + a}\right)}{2\left(b^2c - ac^2\right)}, \frac{(b^2 - ac)c_1\,\text{l}\right.$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - sqrt(b^2 - a*c)*(b1*c - b*c1
)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x
^2 + 2*b*x + a)))/(b^2*c - a*c^2), 1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x +
 a) - 2*sqrt(-b^2 + a*c)*(b1*c - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/
(b^2 - a*c)))/(b^2*c - a*c^2)]
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(53) = 106$.

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.78

$$\int \frac{\text{b1} + \text{c1}x}{a + 2bx + cx^2}\,dx = \left(\frac{c_1}{2c} - \frac{\sqrt{-ac + b^2}(bc_1 - b_1c)}{2c\left(ac - b^2\right)}\right)\log\left(x + \frac{-2ac\left(\frac{c_1}{2c} - \frac{\sqrt{-ac+b^2}(bc_1 - b_1c)}{2c(ac-b^2)}\right) + ac_1 + 2b^2\left(\frac{c_1}{2c} - \frac{\sqrt{-ac+b^2}(bc_1 - b_1c)}{2c(ac-b^2)}\right) - bb}{bc_1 - b_1c}\right.$$
$$+ \left(\frac{c_1}{2c} + \frac{\sqrt{-ac + b^2}(bc_1 - b_1c)}{2c\left(ac - b^2\right)}\right)\log\left(x + \frac{-2ac\left(\frac{c_1}{2c} + \frac{\sqrt{-ac+b^2}(bc_1 - b_1c)}{2c(ac-b^2)}\right) + ac_1 + 2b^2\left(\frac{c_1}{2c} + \frac{\sqrt{-ac+b^2}(bc_1 - b_1c)}{2c(ac-b^2)}\right) - bb}{bc_1 - b_1c}\right.$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

output
```
(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-
2*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*
c1 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))
) - b*b1)/(b*c1 - b1*c)) + (c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*
c*(a*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*
c)/(2*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1
 - b1*c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))
```

3.194.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \frac{c_1 \log\left(cx^2 + 2\,bx + a\right)}{2\,c} + \frac{(b_1c - bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2 + ac}c}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="giac")
```

output
```
1/2*c1*log(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2
 + a*c))/(sqrt(-b^2 + a*c)*c)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.38

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \frac{b_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}} + \frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{\sqrt{a\,c-b^2}} - \frac{2\,b^2\,c_1\,\ln\left(c\,x^2+2\,b\,x+a\right)}{4\,a\,c^2-4\,b^2\,c} + \frac{2\,a\,c\,c_1\,\ln\left(c\,x^2+2\,b\,x+a\right)}{4\,a\,c^2-4\,b^2\,c} - \frac{b\,c_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}} + \frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{c\,\sqrt{a\,c-b^2}}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2),x)
```

output
```
(b1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(a*c - b^2)^(1/2)
 - (2*b^2*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) + (2*a*c*c1*log(a
 + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) - (b*c1*atan(b/(a*c - b^2)^(1/2) +
(c*x)/(a*c - b^2)^(1/2)))/(c*(a*c - b^2)^(1/2))
```

3.195 $\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^2}\,dx$

3.195.1 Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^2}\,dx=-\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{2(b^2-ac)(a+2bx+cx^2)}+\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

output
```
1/2*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)+1/2*(-b*c1+b1*c
)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^2}\,dx=\frac{\frac{-b\text{b1}+a\text{c1}-\text{b1}cx+b\text{c1}x}{a+x(2b+cx)}+\frac{(-\text{b1}c+b\text{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{2(b^2-ac)}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

output
```
((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*c) + b*c1
)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

3.195.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^2} \, dx$$

$$\Big\downarrow \text{ 1159}$$

$$-\frac{(\text{b1}c - b\text{c1}) \int \frac{1}{cx^2+2bx+a} dx}{2\left(b^2 - ac\right)} - \frac{-a\text{c1} + x(\text{b1}c - b\text{c1}) + b\text{b1}}{2\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{(\text{b1}c - b\text{c1}) \int \frac{1}{4(b^2-ac)-(2b+2cx)^2} d(2b + 2cx)}{b^2 - ac} - \frac{-a\text{c1} + x(\text{b1}c - b\text{c1}) + b\text{b1}}{2\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(\text{b1}c - b\text{c1})\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{2\left(b^2 - ac\right)^{3/2}} - \frac{-a\text{c1} + x(\text{b1}c - b\text{c1}) + b\text{b1}}{2\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

output
```
-1/2*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + (
(b1*c - b*c1)*ArcTanh[(2*b + 2*c*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(
3/2))
```

3.195.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.195.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
default	$\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{(4ac-4b^2)(c\,x^2+2bx+a)}+\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(b\,\mathrm{c1}-\mathrm{b1}\,c)x}{2\left(ac-b^2\right)}-\frac{a\,\mathrm{c1}-b\,\mathrm{b1}}{2\left(ac-b^2\right)}}{c\,x^2+2bx+a}+\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)b\,\mathrm{c1}}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)\mathrm{b1}\,c}{4(-ac+b^2)^{\frac{3}{2}}}-$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+(-2*b*c1+
2*b1*c)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/
2))
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

Time = 0.26 (sec) , antiderivative size = 447, normalized size of antiderivative = 5.02

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx$$
$$= \left[-\frac{2\,b^3b_1-2\,abb_1c-(ab_1c-abc_1+(b_1c^2-bcc_1)x^2+2\,(bb_1c-b^2c_1)x)\sqrt{b^2-ac}\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2\,\sqrt{}}{cx^2+2\,bx+a}\right.}{4\,(ab^4-2\,a^2b^2c+a^3c^2+(b^4c-2\,ab^2c^2+a^2c^3)x^2+2\,(b^5}\right.$$
$$-\frac{b^3b_1-abb_1c-(ab_1c-abc_1+(b_1c^2-bcc_1)x^2+2\,(bb_1c-b^2c_1)x)\sqrt{-b^2+ac}\arctan\left(-\frac{\sqrt{-b^2+ac}(cx+b)}{b^2-ac}\right)-}{2\,(ab^4-2\,a^2b^2c+a^3c^2+(b^4c-2\,ab^2c^2+a^2c^3)x^2+2\,(b^5-2\,ab^3c}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*b^3*b1 - 2*a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 +
2*(b*b1*c - b^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*
c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)) - 2*(a*b^2 - a^2*c)*
c1 + 2*(b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c +
a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b
*c^2)*x), -1/2*(b^3*b1 - a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x
^2 + 2*(b*b1*c - b^2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x
 + b)/(b^2 - a*c)) - (a*b^2 - a^2*c)*c1 + (b^2*b1*c - a*b1*c^2 - (b^3 - a*
b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^
3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x)]
```

3.195.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(75) = 150$.

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.63

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^2}\,dx$$
$$= \frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1 - b_1c)\log\left(x + \frac{-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1-bb}{bcc_1-b_1c^2}\right.}{4}$$
$$- \frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1 - b_1c)\log\left(x + \frac{a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1-}{bcc_1-b_1c^2}\right.}{4}$$
$$+ \frac{-ac_1 + bb_1 + x(-bc_1 + b_1c)}{2a^2c - 2ab^2 + x^2\cdot(2ac^2 - 2b^2c) + x\,(4abc - 4b^3)}$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

output
```
sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (-a**2*c**2*sqrt(-1/(a*c -
b**2)**3)*(b*c1 - b1*c) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c
) - b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c
1 - b1*c**2))/4 - sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (a**2*c**
2*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)
**3)*(b*c1 - b1*c) + b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1
 - b*b1*c)/(b*c*c1 - b1*c**2))/4 + (-a*c1 + b*b1 + x*(-b*c1 + b1*c))/(2*a*
*2*c - 2*a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))
```

3.195.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx = -\frac{(b_1c-bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2-ac)\sqrt{-b^2+ac}} - \frac{b_1cx-bc_1x+bb_1-ac_1}{2\,(cx^2+2\,bx+a)(b^2-ac)}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b
^2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2 + 2*b*x + a)*(b^2
 - a*c))
```

3.195.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.79

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{2\left(\frac{\left(4\,b^3-4\,a\,b\,c\right)\,\left(b\,c_1-b_1\,c\right)}{8\,{\left(a\,c-b^2\right)}^{5/2}}-\frac{c\,x\,\left(b\,c_1-b_1\,c\right)}{2\,{\left(a\,c-b^2\right)}^{3/2}}\right)\,\left(a\,c-b^2\right)}{b\,c_1-b_1\,c}\right)\,\left(b\,c_1-b_1\,c\right)}{2\,{\left(a\,c-b^2\right)}^{3/2}} - \frac{\frac{a\,c_1-b\,b_1}{2\,\left(a\,c-b^2\right)}+\frac{x\,\left(b\,c_1-b_1\,c\right)}{2\,\left(a\,c-b^2\right)}}{c\,x^2+2\,b\,x+a}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x)
```

output
```
(atan((2*(((4*b^3 - 4*a*b*c)*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - (c*x*(
b*c1 - b1*c))/(2*(a*c - b^2)^(3/2)))*(a*c - b^2))/(b*c1 - b1*c))*(b*c1 - b
1*c))/(2*(a*c - b^2)^(3/2)) - ((a*c1 - b*b1)/(2*(a*c - b^2)) + (x*(b*c1 -
b1*c))/(2*(a*c - b^2)))/(a + 2*b*x + c*x^2)
```

3.195. $\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^2}\,dx$

3.196 $\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^3}\,dx$

3.196.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^3}\,dx = -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{4(b^2-ac)(a+2bx+cx^2)^2}+\frac{3(\text{b1}c-b\text{c1})(b+cx)}{8(b^2-ac)^2(a+2bx+cx^2)} - \frac{3c(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8(b^2-ac)^{5/2}}$$

output
```
1/4*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^2+3/8*(-b*c1+b1
*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)-3/8*c*(-b*c1+b1*c)*arctanh((c*x+b
)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(5/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^3}\,dx = \frac{\frac{2(b^2-ac)(-b\text{b1}+a\text{c1}-\text{b1}cx+b\text{c1}x)}{(a+x(2b+cx))^2}+\frac{3(\text{b1}c-b\text{c1})(b+cx)}{a+x(2b+cx)}+\frac{3c(\text{b1}c-b\text{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{8(b^2-ac)^2}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

output
```
((2*(b^2 - a*c)*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^2
+ (3*(b1*c - b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (3*c*(b1*c - b*c1)*Arc
Tan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(8*(b^2 - a*c)^2)
```

3.196.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{3(\text{b1}c-b\text{c1})\int \frac{1}{(cx^2+2bx+a)^2}dx}{4(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4(b^2-ac)(a+2bx+cx^2)^2}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{3(\text{b1}c-b\text{c1})\left(-\frac{c\int \frac{1}{cx^2+2bx+a}dx}{2(b^2-ac)}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4(b^2-ac)(a+2bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3(\text{b1}c-b\text{c1})\left(\frac{c\int \frac{1}{4(b^2-ac)-(2b+2cx)^2}d(2b+2cx)}{b^2-ac}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4(b^2-ac)(a+2bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3(\text{b1}c-b\text{c1})\left(\frac{c\,\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4(b^2-ac)(a+2bx+cx^2)^2}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

output
```
-1/4*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^2) -
 (3*(b1*c - b*c1)*(-1/2*(b + c*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + (c*A
rcTanh[(2*b + 2*c*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(3/2))))/(4*(b^2
 - a*c))
```

3.196.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.196.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19

method	result
default	$\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{2(4ac-4b^2)(c\,x^2+2bx+a)^2}+\frac{3(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)}+\frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{\left(4ac-4b^2\right)\sqrt{ac-b^2}}\right)}{2(4ac-4b^2)}$
risch	$\frac{-\frac{3c^2(b\,\mathrm{c1}-\mathrm{b1}\,c)x^3}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{9bc(b\,\mathrm{c1}-\mathrm{b1}\,c)x^2}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{\left(5ac+4b^2\right)(b\,\mathrm{c1}-\mathrm{b1}\,c)x}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{2a^2c\,\mathrm{c1}+a\,b^2\,\mathrm{c1}-5ab\,\mathrm{b1}\,c+2b^3\,\mathrm{b1}}{8\left(a^2c^2-2a\,b^2c+b^4\right)}}{(c\,x^2+2bx+a)^2}-$ $3c\ln\Big((a^2c^3-2a\,b^2c^2+b^4c)x$ … / 16…

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^2+3/2
*(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)
+2*c/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))
)
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 1104, normalized size of antiderivative = 8.49

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx$$

$$= \left[-\frac{4\,b^5b_1 - 14\,ab^3b_1c + 10\,a^2bb_1c^2 - 6\,(b^2b_1c^3 - ab_1c^4 - (b^3c^2 - abc^3)c_1)x^3 - 18\,(b^3b_1c^2 - abb_1c^3 - (b^4c -}{16\,(a^2b^6 - 3\,a}\right.$$

$$-\frac{2\,b^5b_1 - 7\,ab^3b_1c + 5\,a^2bb_1c^2 - 3\,(b^2b_1c^3 - ab_1c^4 - (b^3c^2 - abc^3)c_1)x^3 - 9\,(b^3b_1c^2 - abb_1c^3 - (b^4c - ab^2c}{8\,(a^2b^6 - 3\,a^3b^4c + 3\,a}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*b^5*b1 - 14*a*b^3*b1*c + 10*a^2*b*b1*c^2 - 6*(b^2*b1*c^3 - a*b1*
c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 18*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c -
 a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4
 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c +
 a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(b^2 - a*c)*log((c^
2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*
x + a)) + 2*(a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - 2*(4*b^4*b1*c + a*b^2*b1*
c^2 - 5*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a
^3*b^4*c + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^
4 - a^3*c^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3
 + 2*(2*b^8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(
a*b^7 - 3*a^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x), -1/8*(2*b^5*b1 - 7*a*
b^3*b1*c + 5*a^2*b*b1*c^2 - 3*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)
*c1)*x^3 - 9*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a
^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c
1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b
*b1*c^2 - a*b^2*c*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x +
b)/(b^2 - a*c)) + (a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - (4*b^4*b1*c + a*b^2
*b1*c^2 - 5*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 -
 3*a^3*b^4*c + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2...
```

3.196.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(117) = 234$.

Time = 1.00 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.78

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1 - b_1c)\log\left(x + \frac{-3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c}{3bc^2c_1-3b_1c^3}\right.}{16}$$
$$- \frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1 - b_1c)\log\left(x + \frac{3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b}{3bc^2c_1-3b_1c^3}\right.}{16}$$
$$+ \frac{-2a^2cc_1 - ab^2c_1 + 5abb_1c - 2b^3b_1 + x^3(-3bc^2c_1 + 3b_1c^3) + x^2(-9b^2cc_1 + 9bb_1c^2) + x}{8a^4c^2 - 16a^3b^2c + 8a^2b^4 + x^4\cdot(8a^2c^4 - 16ab^2c^3 + 8b^4c^2) + x^3\cdot(32a^2bc^3 - 64ab^3c^2 + 32b^5c) + x^2\cdot(16a}$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

output
```
3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (-3*a**3*c**4*sqrt(-1/(
a*c - b**2)**5)*(b*c1 - b1*c) + 9*a**2*b**2*c**3*sqrt(-1/(a*c - b**2)**5)*
(b*c1 - b1*c) - 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b
**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/
(3*b*c**2*c1 - 3*b1*c**3))/16 - 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)
*log(x + (3*a**3*c**4*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a**2*b**2
*c**3*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a*b**4*c**2*sqrt(-1/(a*c
- b**2)**5)*(b*c1 - b1*c) - 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c
) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 + (-2*a**2*c*
c1 - a*b**2*c1 + 5*a*b*b1*c - 2*b**3*b1 + x**3*(-3*b*c**2*c1 + 3*b1*c**3)
+ x**2*(-9*b**2*c*c1 + 9*b*b1*c**2) + x*(-5*a*b*c*c1 + 5*a*b1*c**2 - 4*b**
3*c1 + 4*b**2*b1*c))/(8*a**4*c**2 - 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8
*a**2*c**4 - 16*a*b**2*c**3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b
**3*c**2 + 32*b**5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(3
2*a**3*b*c**2 - 64*a**2*b**3*c + 32*a*b**5))
```

3.196.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.49

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx = \frac{3\,(b_1c^2 - bcc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{8\,(b^4 - 2\,ab^2c + a^2c^2)\sqrt{-b^2+ac}} + \frac{3\,b_1c^3x^3 - 3\,bc^2c_1x^3 + 9\,bb_1c^2x^2 - 9\,b^2cc_1x^2 + 4\,b^2b_1cx + 5\,ab_1c^2x - 4\,b^3c_1x - 5\,abcc_1x - 2\,b^3b_1 + 5\,abb_1}{8\,(b^4 - 2\,ab^2c + a^2c^2)(cx^2 + 2\,bx + a)^2}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

output
```
3/8*(b1*c^2 - b*c*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^4 - 2*a*b^2*c
 + a^2*c^2)*sqrt(-b^2 + a*c)) + 1/8*(3*b1*c^3*x^3 - 3*b*c^2*c1*x^3 + 9*b*b
1*c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a*b1*c^2*x - 4*b^3*c1*x - 5*
a*b*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^
2*c + a^2*c^2)*(c*x^2 + 2*b*x + a)^2)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.77

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx = \frac{3\,c\,\mathrm{atan}\left(\frac{8\left(\frac{3\,c^2\,x\,(b\,c_1-b_1\,c)}{8\,{\left(a\,c-b^2\right)}^{5/2}}+\frac{3\,c\,(b\,c_1-b_1\,c)\,\left(16\,a^2\,b\,c^2-32\,a\,b^3\,c+16\,b^5\right)}{128\,{\left(a\,c-b^2\right)}^{5/2}\,\left(a^2\,c^2-2\,a\,b^2\,c+b^4\right)}\right)\,\left(a^2\,c^2-2\,a\,b^2\,c+b^4\right)}{3\,b_1\,c^2-3\,b\,c\,c_1}\right)\,(b\,c_1-b_1\,c)}{8\,{\left(a\,c-b^2\right)}^{5/2}} - \frac{\frac{2\,c\,c_1\,a^2+c_1\,a\,b^2-5\,b_1\,c\,a\,b+2\,b_1\,b^3}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{x\,(4\,b^2+5\,a\,c)\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{3\,c^2\,x^3\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{9\,b\,c\,x^2\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}}{a^2+x^2\,\left(4\,b^2+2\,a\,c\right)+c^2\,x^4+4\,a\,b\,x+4\,b\,c\,x^3}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x)
```

output
```
(3*c*atan((8*((3*c^2*x*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) + (3*c*(b*c1 -
 b1*c)*(16*b^5 + 16*a^2*b*c^2 - 32*a*b^3*c))/(128*(a*c - b^2)^(5/2)*(b^4 +
 a^2*c^2 - 2*a*b^2*c)))*(b^4 + a^2*c^2 - 2*a*b^2*c))/(3*b1*c^2 - 3*b*c*c1)
)*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - ((2*b^3*b1 + a*b^2*c1 + 2*a^2*c*c
1 - 5*a*b*b1*c)/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (x*(5*a*c + 4*b^2)*(b*c1
 - b1*c))/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (3*c^2*x^3*(b*c1 - b1*c))/(8*(
b^4 + a^2*c^2 - 2*a*b^2*c)) + (9*b*c*x^2*(b*c1 - b1*c))/(8*(b^4 + a^2*c^2
- 2*a*b^2*c)))/(a^2 + x^2*(2*a*c + 4*b^2) + c^2*x^4 + 4*a*b*x + 4*b*c*x^3)
```

3.197 $\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx$

3.197.1 Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx = -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6(b^2-ac)(a+2bx+cx^2)^3}+\frac{5(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{24(b^2-ac)^2(a+2bx+cx^2)^2}-\frac{5c(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{16(b^2-ac)^3(a+2bx+cx^2)}+\frac{5c^2(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16(b^2-ac)^{7/2}}$$

output
```
1/6*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^3+5/24*(-b*c1+b
1*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)^2-5/16*c*(-b*c1+b1*c)*(c*x+b)/(-
a*c+b^2)^3/(c*x^2+2*b*x+a)+5/16*c^2*(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2
)^(1/2))/(-a*c+b^2)^(7/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx$$

$$= \frac{\frac{8(b^2-ac)^2(-b\mathrm{b1}+a\mathrm{c1}-\mathrm{b1}cx+b\mathrm{c1}x)}{(a+x(2b+cx))^3}-\frac{10(b^2-ac)(-\mathrm{b1}c+b\mathrm{c1})(b+cx)}{(a+x(2b+cx))^2}+\frac{15c(-\mathrm{b1}c+b\mathrm{c1})(b+cx)}{a+x(2b+cx)}+\frac{15c^2(-\mathrm{b1}c+b\mathrm{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+}}\right.}{\sqrt{-b^2+ac}}}{48(b^2-ac)^3}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

output
```
((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^
3 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x))^2 + (1
5*c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (15*c^2*(-(b1*c) + b
*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)
^3)
```

3.197.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1159, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^4}dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{5(\text{b1}c-b\text{c1})\int \frac{1}{(cx^2+2bx+a)^3}dx}{6(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6(b^2-ac)(a+2bx+cx^2)^3}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\int \frac{1}{(cx^2+2bx+a)^2}dx}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6(b^2-ac)(a+2bx+cx^2)^3}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\left(-\frac{c\int \frac{1}{cx^2+2bx+a}dx}{2(b^2-ac)}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6(b^2-ac)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6(b^2-ac)(a+2bx+cx^2)^3}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\left(\frac{c\int \frac{1}{4\left(b^2-ac\right)-(2b+2cx)^2}d(2b+2cx)}{b^2-ac}-\frac{b+cx}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}\right)}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

$$\Big\downarrow \ 219$$

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{b+cx}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}\right)}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

output
```
-1/6*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^3) -
 (5*(b1*c - b*c1)*(-1/4*(b + c*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^2) - (3
*c*(-1/2*(b + c*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + (c*ArcTanh[(2*b + 2
*c*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(3/2))))/(4*(b^2 - a*c))))/(6*(
b^2 - a*c))
```

3.197.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.197.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

method	result
default	$\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{3(4ac-4b^2)(c\,x^2+2bx+a)^3}+\frac{5(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\left(\frac{2cx+2b}{2\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)^2}+\frac{3c\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)}+\frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right.}{\left(4ac-4b^2\right)\sqrt{ac-b^2}}\right.}{4ac-4b^2}\right.}{3(4ac-4b^2)}$
risch	$\frac{-\frac{5c^4(b\,\mathrm{c1}-\mathrm{b1}\,c)x^5}{16\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)}-\frac{25c^3(b\,\mathrm{c1}-\mathrm{b1}\,c)b\,x^4}{16\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)}-\frac{5\left(4ac+11b^2\right)c^2(b\,\mathrm{c1}-\mathrm{b1}\,c)x^3}{24\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)}-\frac{5b\left(4ac+b^2\right)c(b\,\mathrm{c1}-\mathrm{b1}\,c)x^2}{8\left(a^3c^3-3b^2c^2a^2+3a\,b^4c-b^6\right)}-\frac{\left(11a^2b\right.}{}}{(c\,x^2+2bx+a)^3}$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^3+5/3
*(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*(1/2*(2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*
x+a)^2+3*c/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+2*c/(4
*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))))
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(161) = 322$.

Time = 0.30 (sec) , antiderivative size = 1950, normalized size of antiderivative = 11.27

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(16*b^7*b1 - 68*a*b^5*b1*c + 118*a^2*b^3*b1*c^2 - 66*a^3*b*b1*c^3 +
 30*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - a*b*c^5)*c1)*x^5 + 150*(b^3*b1*c^4
 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*x^4 + 20*(11*b^4*b1*c^3 - 7*a*b^
2*b1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3
 + 60*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b*b1*c^4 - (b^6*c + 3*a*b^4*c^2
 - 4*a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^
5*c1)*x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^5 - (
4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^4 - (2*b^4*c^2
 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b1*c^4 - (4*a*b^3*c^2 +
a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^2*c1)*x)*sqrt(b^2 - a*c)*
log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2
 + 2*b*x + a)) + 2*(2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 -
 6*(4*b^6*b1*c - 22*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b
^7 - 22*a*b^5*c + 7*a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^
6*c + 6*a^5*b^4*c^2 - 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6
*a^2*b^4*c^5 - 4*a^3*b^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6
*a^2*b^5*c^4 - 4*a^3*b^3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2
 + 20*a^2*b^6*c^3 - 10*a^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c
+ 10*a^3*b^5*c^3 - 10*a^4*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^
2*b^8*c + 20*a^3*b^6*c^2 - 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 -...
```

3.197.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(158) = 316$.

Time = 1.53 (sec) , antiderivative size = 1027, normalized size of antiderivative = 5.94

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^4}\,dx$$

$$= \frac{5c^2\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1 - b_1c)\log\left(x + \frac{-5a^4c^6\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)+20a^3b^2c^5\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)-30a^2b^4c^4\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1}{5bc^3c_1-5b}\right.}{32}$$

$$- \frac{5c^2\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1 - b_1c)\log\left(x + \frac{5a^4c^6\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)-20a^3b^2c^5\sqrt{-\frac{1}{(ac-b^2)^7}}(bc_1-b_1c)+30a^2b^4c^4\sqrt{-\frac{1}{(ac-b^2)^7}}(bc}{5bc^3c_1-}\right.}{32}$$

$$+ \frac{-8a^3c^2c_1 - 9a^2b^2cc_1 + 33a^2bb_1c^2 + 2ab^4c_1 - 26ab^3b_1c + 8}{48a^6c^3 - 144a^5b^2c^2 + 144a^4b^4c - 48a^3b^6 + x^6\cdot(48a^3c^6 - 144a^2b^2c^5 + 144ab^4c^4 - 48b^6c^3) + x^5\cdot(288a^3}$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

output
```
5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (-5*a**4*c**6*sqrt(-
1/(a*c - b**2)**7)*(b*c1 - b1*c) + 20*a**3*b**2*c**5*sqrt(-1/(a*c - b**2)*
*7)*(b*c1 - b1*c) - 30*a**2*b**4*c**4*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*
c) + 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 5*b**8*c**2*s
qrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b
*c**3*c1 - 5*b1*c**4))/32 - 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*
log(x + (5*a**4*c**6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a**3*b**2
*c**5*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 30*a**2*b**4*c**4*sqrt(-1/(
a*c - b**2)**7)*(b*c1 - b1*c) - 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b
*c1 - b1*c) + 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*
c**2*c1 - 5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 + (-8*a**3*c**2*c1 -
9*a**2*b**2*c*c1 + 33*a**2*b*b1*c**2 + 2*a*b**4*c1 - 26*a*b**3*b1*c + 8*b*
*5*b1 + x**5*(-15*b*c**4*c1 + 15*b1*c**5) + x**4*(-75*b**2*c**3*c1 + 75*b*
b1*c**4) + x**3*(-40*a*b*c**3*c1 + 40*a*b1*c**4 - 110*b**3*c**2*c1 + 110*b
**2*b1*c**3) + x**2*(-120*a*b**2*c**2*c1 + 120*a*b*b1*c**3 - 30*b**4*c*c1
+ 30*b**3*b1*c**2) + x*(-33*a**2*b*c**2*c1 + 33*a**2*b1*c**3 - 54*a*b**3*c
*c1 + 54*a*b**2*b1*c**2 + 12*b**5*c1 - 12*b**4*b1*c))/(48*a**6*c**3 - 144*
a**5*b**2*c**2 + 144*a**4*b**4*c - 48*a**3*b**6 + x**6*(48*a**3*c**6 - 144
*a**2*b**2*c**5 + 144*a*b**4*c**4 - 48*b**6*c**3) + x**5*(288*a**3*b*c**5
- 864*a**2*b**3*c**4 + 864*a*b**5*c**3 - 288*b**7*c**2) + x**4*(144*a**...
```

3.197.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.10

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\,dx = -\frac{5\,(b_1c^3 - bc^2c_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{16\,(b^6 - 3\,ab^4c + 3\,a^2b^2c^2 - a^3c^3)\sqrt{-b^2+ac}} - \frac{15\,b_1c^5x^5 - 15\,bc^4c_1x^5 + 75\,bb_1c^4x^4 - 75\,b^2c^3c_1x^4 + 110\,b^2b_1c^3x^3 + 40\,ab_1c^4x^3 - 110\,b^3c^2c_1x^3 - 40\,abc^3c}{}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

output
```
-5/16*(b1*c^3 - b*c^2*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^6 - 3*a*b
^4*c + 3*a^2*b^2*c^2 - a^3*c^3)*sqrt(-b^2 + a*c)) - 1/48*(15*b1*c^5*x^5 -
15*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*c1*x^4 + 110*b^2*b1*c^3*x^3
 + 40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^
2*x^2 + 120*a*b*b1*c^3*x^2 - 30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b
^4*b1*c*x + 54*a*b^2*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c
*c1*x - 33*a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2
*a*b^4*c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c
^2 - a^3*c^3)*(c*x^2 + 2*b*x + a)^3)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 640, normalized size of antiderivative = 3.70

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^4}\,dx$$
$$= \frac{\frac{5\,c^4\,x^5\,(b\,c_1-b_1\,c)}{16\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)} - \frac{-8\,c_1\,a^3\,c^2-9\,c_1\,a^2\,b^2\,c+33\,b_1\,a^2\,b\,c^2+2\,c_1\,a\,b^4-26\,b_1\,a\,b^3\,c+8\,b_1\,b^5}{48\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)} + \frac{x\,(b\,c_1-b_1\,c)\,(11\,a^2\,c^2+18\,a\,b^2\,c-}{16\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+}}{x^3\,(8\,b^3+12\,a\,c\,b)+x^2\,(3\,c\,a^2+12\,a\,b^2)+x^4\,(12\,b^2\,c+}$$
$$- \frac{5\,c^2\,\mathrm{atan}\left(\frac{16\left(\frac{5\,c^3\,x\,(b\,c_1-b_1\,c)}{16\,\left(a\,c-b^2\right)^{7/2}}+\frac{5\,c^2\,(b\,c_1-b_1\,c)\,\left(-32\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-96\,a\,b^5\,c+32\,b^7\right)}{512\,\left(a\,c-b^2\right)^{7/2}\,\left(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6\right)}\right)\,\left(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6\right)}{5\,b_1\,c^3-5\,b\,c^2\,c_1}\right)\,(b\,c_1-b_1}{16\,(a\,c-b^2)^{7/2}}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x)
```

output
```
((5*c^4*x^5*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)
) - (8*b^5*b1 - 8*a^3*c^2*c1 + 2*a*b^4*c1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^
2 - 9*a^2*b^2*c*c1)/(48*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (x*
(b*c1 - b1*c)*(11*a^2*c^2 - 4*b^4 + 18*a*b^2*c))/(16*(b^6 - a^3*c^3 + 3*a^
2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^3*(4*a*c^2 + 11*b^2*c)*(b*c1 - b1*c))/(24
*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^2*(b^3 + 4*a*b*c)*(
b*c1 - b1*c))/(8*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (25*b*c^3*
x^4*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))/(x^3*
(8*b^3 + 12*a*b*c) + x^2*(12*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 12*b^2*c) +
 a^3 + c^3*x^6 + 6*b*c^2*x^5 + 6*a^2*b*x) - (5*c^2*atan((16*((5*c^3*x*(b*c
1 - b1*c))/(16*(a*c - b^2)^(7/2)) + (5*c^2*(b*c1 - b1*c)*(32*b^7 - 32*a^3*
b*c^3 + 96*a^2*b^3*c^2 - 96*a*b^5*c))/(512*(a*c - b^2)^(7/2)*(b^6 - a^3*c^
3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*
c))/(5*b1*c^3 - 5*b*c^2*c1))*(b*c1 - b1*c))/(16*(a*c - b^2)^(7/2))
```

3.198 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx$

3.198.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx = \frac{\mathrm{c1}(a + 2bx + cx^2)^{1-n}}{2c(1-n)}$$
$$- \frac{2^{-n}(\mathrm{b1}c - b\mathrm{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}(a + 2bx + cx^2)^{1-n}\,\mathrm{Hypergeometric2F1}\left(1-n, n, 2-n, \frac{b+\sqrt{b^2-ac}}{2\sqrt{b^2-a}}\right.}{c\sqrt{b^2-ac}(1-n)}$$

output
```
1/2*c1*(c*x^2+2*b*x+a)^(1-n)/c/(1-n)-(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1-n)*hy
pergeom([n, 1-n],[2-n],1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))*((-b
-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1+n)/(2^n)/c/(1-n)/(-a*c+b^2)^(
1/2)
```

3.198.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.79 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int (\text{b1}+\text{c1}x) \left(a+2bx+cx^2\right)^{-n} \, dx = \frac{1}{2}(a+x(2b+cx))^{-n}\left(\text{c1}x^2\left(\frac{b-\sqrt{b^2-ac}+cx}{b-\sqrt{b^2-ac}}\right)^n\left(\frac{b+\sqrt{b^2-ac}+cx}{b+\sqrt{b^2-ac}}\right)^n \text{AppellF1}\left(2,n,n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right)-\frac{2^{1-n}\text{b1}\left(b-\sqrt{b^2-ac}+cx\right)\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^n \text{Hypergeometric2F1}\left(1-n,n,2-n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right)}{c(-1+n)}\right)$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

output
```
(c1*x^2*((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b
^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n*AppellF1[2, n, n, 3, -((c*x)/(b
+ Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] - (2^(1 - n)*b1*(b - Sq
rt[b^2 - a*c] + c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hyper
geometric2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a
*c])])/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)
```

3.198.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x) \left(a+2bx+cx^2\right)^{-n} \, dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{(\text{b1}c-b\text{c1})\int \left(cx^2+2bx+a\right)^{-n}dx}{c}+\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}$$

$$\downarrow 1096$$

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}\text{Hypergeometric2F1}\left(1-n,n,2-n,\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(1-n)\sqrt{b^2-ac}}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

output
```
(c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-((b - Sq
rt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b*x + c*x^2)^(1 - n
)*Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b
^2 - a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))
```

3.198.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.198.4 Maple [F]

$$\int (\text{c1}\,x+\text{b1})\left(c\,x^2+2bx+a\right)^{-n}dx$$

input
```
int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

output
```
int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

3.198.5 Fricas [F]

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx = \int \frac{c_1x + b_1}{(cx^2 + 2\,bx + a)^n}\, dx$$

input
```
integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="fricas")
```

output
```
integral((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx = \text{Timed out}$$

input
```
integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
```

output
```
Timed out
```

3.198.7 Maxima [F]

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx = \int \frac{c_1x + b_1}{(cx^2 + 2\,bx + a)^n}\, dx$$

input
```
integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="maxima")
```

output
```
integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

3.198.8 Giac [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} \, dx = \int \frac{c_1 x + b_1}{(cx^2 + 2\,bx + a)^n} \, dx$$

input
```
integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="giac")
```

output
```
integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} \, dx = \int \frac{b_1 + c_1\,x}{(c\,x^2 + 2\,b\,x + a)^n} \, dx$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x)
```

output
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n, x)
```

3.199 $\int \frac{x}{3+6x+2x^2}\,dx$

3.199.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{x}{3+6x+2x^2}\,dx=\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(3-\sqrt{3}+2x\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)$$

output
```
1/4*ln(3+2*x-3^(1/2))*(1-3^(1/2))+1/4*ln(3+2*x+3^(1/2))*(1+3^(1/2))
```

3.199.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x}{3+6x+2x^2}\,dx=\frac{1}{4}\left(-\left(\left(-1+\sqrt{3}\right)\log\left(-3+\sqrt{3}-2x\right)\right)+\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)\right)$$

input
```
Integrate[x/(3 + 6*x + 2*x^2),x]
```

output
```
(-((-1 + Sqrt[3])*Log[-3 + Sqrt[3] - 2*x]) + (1 + Sqrt[3])*Log[3 + Sqrt[3]
 + 2*x])/4
```

3.199.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{2x^2+6x+3}dx$$

$$\Big\downarrow \text{1141}$$

$$2\int \left(\frac{1+\sqrt{3}}{4\left(2x+\sqrt{3}+3\right)}+\frac{1-\sqrt{3}}{4\left(2x-\sqrt{3}+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{8}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{8}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)\right)$$

input
```
Int[x/(3 + 6*x + 2*x^2),x]
```

output
```
2*(((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/8 + ((1 + Sqrt[3])*Log[3 + Sqrt[
3] + 2*x])/8)
```

3.199.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.199.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\ln(2x^2+6x+3)}{4}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{2}$	31
risch	$\frac{\ln\left(3+2x+\sqrt{3}\right)}{4}+\frac{\ln\left(3+2x+\sqrt{3}\right)\sqrt{3}}{4}+\frac{\ln\left(3+2x-\sqrt{3}\right)}{4}-\frac{\ln\left(3+2x-\sqrt{3}\right)\sqrt{3}}{4}$	56

input
```
int(x/(2*x^2+6*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(2*x^2+6*x+3)+1/2*3^(1/2)*arctanh(1/6*(4*x+6)*3^(1/2))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{3+6x+2x^2}\,dx = \frac{1}{4}\sqrt{3}\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+\frac{1}{4}\log\left(2\,x^2+6\,x+3\right)$$

input
```
integrate(x/(2*x^2+6*x+3),x, algorithm="fricas")
```

output
```
1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) +
 1/4*log(2*x^2 + 6*x + 3)
```

3.199.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{3+6x+2x^2}\,dx = \left(\frac{1}{4}-\frac{\sqrt{3}}{4}\right)\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)+\left(\frac{1}{4}+\frac{\sqrt{3}}{4}\right)\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)$$

input
```
integrate(x/(2*x**2+6*x+3),x)
```

output
```
(1/4 - sqrt(3)/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*log(x + sqr
t(3)/2 + 3/2)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x}{3+6x+2x^2}\,dx = -\frac{1}{4}\,\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right)+\frac{1}{4}\,\log\left(2\,x^2+6\,x+3\right)$$

input
```
integrate(x/(2*x^2+6*x+3),x, algorithm="maxima")
```

output
```
-1/4*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) + 1/4*log(2*x^2
+ 6*x + 3)
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{3+6x+2x^2}\,dx = -\frac{1}{4}\,\sqrt{3}\log\left(\frac{\left|4\,x-2\,\sqrt{3}+6\right|}{\left|4\,x+2\,\sqrt{3}+6\right|}\right)+\frac{1}{4}\,\log\left(\left|2\,x^2+6\,x+3\right|\right)$$

input
```
integrate(x/(2*x^2+6*x+3),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) + 1/4*
log(abs(2*x^2 + 6*x + 3))
```

3.199.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{x}{3+6x+2x^2}\,dx = \ln\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\,\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\right)-\ln\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\,\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\right)$$

input
```
int(x/(6*x + 2*x^2 + 3),x)
```

output
```
log(x + 3^(1/2)/2 + 3/2)*(3^(1/2)/4 + 1/4) - log(x - 3^(1/2)/2 + 3/2)*(3^(
1/2)/4 - 1/4)
```

3.200 $\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx$

3.200.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx=\frac{5+4x}{4(3+6x+2x^2)^2}-\frac{3+2x}{2(3+6x+2x^2)}+\frac{\operatorname{arctanh}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/4*(5+4*x)/(2*x^2+6*x+3)^2+1/2*(-3-2*x)/(2*x^2+6*x+3)+1/3*arctanh(1/3*(3+
2*x)*3^(1/2))*3^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx=\frac{1}{12}\left(-\frac{3(13+44x+36x^2+8x^3)}{(3+6x+2x^2)^2}-2\sqrt{3}\log\left(-3+\sqrt{3}-2x\right)+2\sqrt{3}\log\left(3+\sqrt{3}+2x\right)\right)$$

input
```
Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

output
```
((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3]*Log[-3
+ Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

3.200.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1159, 1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-3}{(2x^2+6x+3)^3}dx$$

$$\Big\downarrow \text{1159}$$

$$3\int \frac{1}{(2x^2+6x+3)^2}dx+\frac{4x+5}{4(2x^2+6x+3)^2}$$

$$\Big\downarrow \text{1086}$$

$$3\left(-\frac{1}{3}\int \frac{1}{2x^2+6x+3}dx-\frac{2x+3}{6(2x^2+6x+3)}\right)+\frac{4x+5}{4(2x^2+6x+3)^2}$$

$$\Big\downarrow \text{1081}$$

$$3\left(-\frac{2}{3}\int \left(\frac{1}{2\sqrt{3}(2x-\sqrt{3}+3)}-\frac{1}{2\sqrt{3}(2x+\sqrt{3}+3)}\right)dx-\frac{2x+3}{6(2x^2+6x+3)}\right)+\frac{4x+5}{4(2x^2+6x+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4x+5}{4(2x^2+6x+3)^2}+3\left(-\frac{2x+3}{6(2x^2+6x+3)}-\frac{2}{3}\left(\frac{\log(2x-\sqrt{3}+3)}{4\sqrt{3}}-\frac{\log(2x+\sqrt{3}+3)}{4\sqrt{3}}\right)\right)$$

input
```
Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

output
```
(5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) + 3*(-1/6*(3 + 2*x)/(3 + 6*x + 2*x^2) -
(2*(Log[3 - Sqrt[3] + 2*x]/(4*Sqrt[3]) - Log[3 + Sqrt[3] + 2*x]/(4*Sqrt[3]
)))/3)
```

3.200.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.200.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{-24x-30}{24(2x^2+6x+3)^2}-\frac{4x+6}{4(2x^2+6x+3)}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{3}$	56
risch	$\frac{-2x^3-9x^2-11x-\frac{13}{4}}{(2x^2+6x+3)^2}+\frac{\ln\left(3+2x+\sqrt{3}\right)\sqrt{3}}{6}-\frac{\ln\left(3+2x-\sqrt{3}\right)\sqrt{3}}{6}$	61

input
```
int((2*x-3)/(2*x^2+6*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(4*x+6)/(2*x^2+6*x+3)+1/3*3^(1/2)*arc
tanh(1/6*(4*x+6)*3^(1/2))
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3}\,dx = -\frac{24\,x^3-2\,\sqrt{3}(4\,x^4+24\,x^3+48\,x^2+36\,x+9)\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+108\,x^2+132\,x+39}{12\,(4\,x^4+24\,x^3+48\,x^2+36\,x+9)}$$

input
```
integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="fricas")
```

output
```
-1/12*(24*x^3 - 2*sqrt(3)*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*log((2*x^2
+ sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) + 108*x^2 + 132*x + 39)/
(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

3.200.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3}\,dx = \frac{-8x^3-36x^2-44x-13}{16x^4+96x^3+192x^2+144x+36} - \frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6} + \frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}$$

input
```
integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
```

output
```
(-8*x**3 - 36*x**2 - 44*x - 13)/(16*x**4 + 96*x**3 + 192*x**2 + 144*x + 36
) - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x + sqrt(3)/2 + 3/2)/
6
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right) - \frac{8\,x^3+36\,x^2+44\,x+13}{4\,(4\,x^4+24\,x^3+48\,x^2+36\,x+9)}$$

input
```
integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) - 1/4*(8*x^3 + 3
6*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{\left|4\,x-2\,\sqrt{3}+6\right|}{\left|4\,x+2\,\sqrt{3}+6\right|}\right) - \frac{8\,x^3+36\,x^2+44\,x+13}{4\,(2\,x^2+6\,x+3)^2}$$

input
```
integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) - 1/4*
(8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\left(\frac{2\,x}{3}+1\right)\right)}{3} - \frac{\frac{x^3}{2}+\frac{9\,x^2}{4}+\frac{11\,x}{4}+\frac{13}{16}}{x^4+6\,x^3+12\,x^2+9\,x+\frac{9}{4}}$$

input
```
int((2*x - 3)/(6*x + 2*x^2 + 3)^3,x)
```

output
```
(3^(1/2)*atanh(3^(1/2)*((2*x)/3 + 1)))/3 - ((11*x)/4 + (9*x^2)/4 + x^3/2 +
 13/16)/(9*x + 12*x^2 + 6*x^3 + x^4 + 9/4)
```

3.201 $\int \frac{-1+x}{(4+5x+x^2)^2}\,dx$

3.201.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{13+7x}{9(4+5x+x^2)} + \frac{7}{27}\log(1+x) - \frac{7}{27}\log(4+x)$$

output
```
1/9*(13+7*x)/(x^2+5*x+4)+7/27*ln(1+x)-7/27*ln(4+x)
```

3.201.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{1}{27}\left(\frac{39+21x}{4+5x+x^2} + 7\log(1+x) - 7\log(4+x)\right)$$

input
```
Integrate[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

output
```
((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27
```

3.201.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{(x^2+5x+4)^2}\,dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(-\frac{7}{27(x+4)}-\frac{5}{9(x+4)^2}+\frac{7}{27(x+1)}-\frac{2}{9(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9(x+1)}+\frac{5}{9(x+4)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

input
```
Int[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

output
```
2/(9*(1 + x)) + 5/(9*(4 + x)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

3.201.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.201.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{5}{9(4+x)}-\frac{7\ln(4+x)}{27}+\frac{2}{9(1+x)}+\frac{7\ln(1+x)}{27}$	28
norman	$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(4+x)}{27}$	30
risch	$\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(4+x)}{27}$	30
parallelrisch	$\frac{7\ln(1+x)x^2-7\ln(4+x)x^2+39+35\ln(1+x)x-35\ln(4+x)x+28\ln(1+x)-28\ln(4+x)+21x}{27x^2+135x+108}$	62

input
```
int((-1+x)/(x^2+5*x+4)^2,x,method=_RETURNVERBOSE)
```

output
```
5/9/(4+x)-7/27*ln(4+x)+2/9/(1+x)+7/27*ln(1+x)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = -\frac{7\,(x^2+5\,x+4)\log(x+4)-7\,(x^2+5\,x+4)\log(x+1)-21\,x-39}{27\,(x^2+5\,x+4)}$$

input
```
integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="fricas")
```

output
```
-1/27*(7*(x^2 + 5*x + 4)*log(x + 4) - 7*(x^2 + 5*x + 4)*log(x + 1) - 21*x
- 39)/(x^2 + 5*x + 4)
```

3.201.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{-1+x}{\left(4+5x+x^2\right)^2}\,dx = \frac{7x+13}{9x^2+45x+36} + \frac{7\log\left(x+1\right)}{27} - \frac{7\log\left(x+4\right)}{27}$$

input
```
integrate((-1+x)/(x**2+5*x+4)**2,x)
```

output
```
(7*x + 13)/(9*x**2 + 45*x + 36) + 7*log(x + 1)/27 - 7*log(x + 4)/27
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{-1+x}{\left(4+5x+x^2\right)^2}\,dx = \frac{7\,x+13}{9\,\left(x^2+5\,x+4\right)} - \frac{7}{27}\,\log\left(x+4\right) + \frac{7}{27}\,\log\left(x+1\right)$$

input
```
integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="maxima")
```

output
```
1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(x + 4) + 7/27*log(x + 1)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{-1+x}{\left(4+5x+x^2\right)^2}\,dx = \frac{7\,x+13}{9\,\left(x^2+5\,x+4\right)} - \frac{7}{27}\,\log\left(|x+4|\right) + \frac{7}{27}\,\log\left(|x+1|\right)$$

input
```
integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="giac")
```

output
```
1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(abs(x + 4)) + 7/27*log(abs(x + 1
))
```

3.201.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{\frac{7\,x}{9}+\frac{13}{9}}{x^2+5\,x+4} - \frac{14\,\mathrm{atanh}\left(\frac{2\,x}{3}+\frac{5}{3}\right)}{27}$$

input
```
int((x - 1)/(5*x + x^2 + 4)^2,x)
```

output
```
((7*x)/9 + 13/9)/(5*x + x^2 + 4) - (14*atanh((2*x)/3 + 5/3))/27
```

3.202 $\int \frac{1}{(2+3x+x^2)^5}\,dx$

3.202.1 Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\log(1+x) - 70\log(2+x)$$

output
```
1/4*(-3-2*x)/(x^2+3*x+2)^4+7/6*(3+2*x)/(x^2+3*x+2)^3-35/6*(3+2*x)/(x^2+3*x
+2)^2+35*(3+2*x)/(x^2+3*x+2)+70*ln(1+x)-70*ln(2+x)
```

3.202.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\log(1+x) - 70\log(2+x)$$

input
```
Integrate[(2 + 3*x + x^2)^(-5),x]
```

output
```
(-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (
35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*
Log[1 + x] - 70*Log[2 + x]
```

3.202.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3x+2)^5} \, dx$$

$$\Big\downarrow \text{1084}$$

$$\int \left(-\frac{70}{x+2}-\frac{35}{(x+2)^2}-\frac{15}{(x+2)^3}-\frac{5}{(x+2)^4}-\frac{1}{(x+2)^5}+\frac{70}{x+1}-\frac{35}{(x+1)^2}+\frac{15}{(x+1)^3}-\frac{5}{(x+1)^4}+\frac{1}{(x+1)^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{35}{x+1}+\frac{35}{x+2}-\frac{15}{2(x+1)^2}+\frac{15}{2(x+2)^2}+\frac{5}{3(x+1)^3}+\frac{5}{3(x+2)^3}-\frac{1}{4(x+1)^4}+\frac{1}{4(x+2)^4}+ 70\log(x+1)-70\log(x+2)$$

input
```
Int[(2 + 3*x + x^2)^(-5),x]
```

output
```
-1/4*1/(1 + x)^4 + 5/(3*(1 + x)^3) - 15/(2*(1 + x)^2) + 35/(1 + x) + 1/(4*
(2 + x)^4) + 5/(3*(2 + x)^3) + 15/(2*(2 + x)^2) + 35/(2 + x) + 70*Log[1 +
x] - 70*Log[2 + x]
```

3.202.3.1 Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.202.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result
norman	$\frac{70x^7+735x^6+4098x+9093x^2+\frac{9730}{3}x^5+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4}+70\ln(1+x)-70\ln(2+x)$
risch	$\frac{70x^7+735x^6+4098x+9093x^2+\frac{9730}{3}x^5+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4}+70\ln(1+x)-70\ln(2+x)$
default	$\frac{1}{4(2+x)^4}+\frac{5}{3(2+x)^3}+\frac{15}{2(2+x)^2}+\frac{35}{2+x}-70\ln(2+x)-\frac{1}{4(1+x)^4}+\frac{5}{3(1+x)^3}-\frac{15}{2(1+x)^2}+\frac{35}{1+x}+70\ln($
parallelrisch	$\frac{9315+8820x^6+49176x+38920x^5+13440\ln(1+x)-13440\ln(2+x)+93450x^4+131768x^3+109116x^2+840x^7-80640\ln(2+x)x+}{}$

input
```
int(1/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

output
```
(70*x^7+735*x^6+4098*x+9093*x^2+9730/3*x^5+15575/2*x^4+32942/3*x^3+3105/4)
/(x^2+3*x+2)^4+70*ln(1+x)-70*ln(2+x)
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(81) = 162.

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.90

$$\int \frac{1}{(2+3x+x^2)^5}\,dx$$
$$= \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2-840\,(x^8+12\,x^7+62\,x^6+180\,x^5+32}{12\,(x^8+12\,x}$$

input
```
integrate(1/(x^2+3*x+2)^5,x, algorithm="fricas")
```

output
```
1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
 - 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96
*x + 16)*log(x + 2) + 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360
*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 49176*x + 9315)/(x^8 + 12*x^7 + 6
2*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

3.202.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840x^7+8820x^6+38920x^5+93450x^4+131768x^3+109116x^2+49176x+9315}{12x^8+144x^7+744x^6+2160x^5+3852x^4+4320x^3+2976x^2+1152x+192} + 70\log(x+1) - 70\log(x+2)$$

input
```
integrate(1/(x**2+3*x+2)**5,x)
```

output
```
(840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 109116*x**
2 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4
 + 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*log(x + 1) - 70*log(x + 2)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2+49176\,x+9315}{12\,(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16)} - 70\,\log(x+2) + 70\,\log(x+1)$$

input
```
integrate(1/(x^2+3*x+2)^5,x, algorithm="maxima")
```

output
```
1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
 + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 +
248*x^2 + 96*x + 16) - 70*log(x + 2) + 70*log(x + 1)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2+49176\,x+9315}{12\,(x^2+3\,x+2)^4} - 70\,\log\left(|x+2|\right)+70\,\log\left(|x+1|\right)$$

input
```
integrate(1/(x^2+3*x+2)^5,x, algorithm="giac")
```

output
```
1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
 + 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*log(abs(x + 2)) + 70*log(abs(x +
 1))
```

3.202.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = 70\,\ln\left(\frac{x+1}{x+2}\right)+70\left(x+\frac{3}{2}\right)\left(\frac{1}{x^2+3\,x+2}-\frac{1}{6\,(x^2+3\,x+2)^2}+\frac{1}{30\,(x^2+3\,x+2)^3}-\frac{1}{140\,(x^2+3\,x+2)^4}\right)$$

input
```
int(1/(3*x + x^2 + 2)^5,x)
```

output
```
70*log((x + 1)/(x + 2)) + 70*(x + 3/2)*(1/(3*x + x^2 + 2) - 1/(6*(3*x + x^
2 + 2)^2) + 1/(30*(3*x + x^2 + 2)^3) - 1/(140*(3*x + x^2 + 2)^4))
```

3.203 $\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx$

3.203.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx=-\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}-\frac{234\arctan\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}+\frac{80\log(x)}{2401}-\frac{40\log\left(7-6x+2x^2\right)}{2401}$$

output
```
-1/490/x^2-69/1715/x+1/35*(-2+3*x)/x^2/(2*x^2-6*x+7)+80/2401*ln(x)-40/2401
*ln(2*x^2-6*x+7)-234/60025*arctan(1/5*(3-2*x)*5^(1/2))*5^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx=\frac{-\frac{1225}{x^2}-\frac{4200}{x}-\frac{140(-41+9x)}{7-6x+2x^2}+468\sqrt{5}\arctan\left(\frac{-3+2x}{\sqrt{5}}\right)+4000\log(x)-2000\log\left(7-6x+2x^2\right)}{120050}$$

input
```
Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

output
```
(-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*Sqrt[5]*Ar
cTan[(-3 + 2*x)/Sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*x + 2*x^2])/120050
```

3.203.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(2 x^2-6 x+7\right)^2} \, dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{140} \int \frac{4(9x+1)}{x^3 \left(2 x^2-6 x+7\right)}dx-\frac{2-3x}{35x^2 \left(2 x^2-6 x+7\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35} \int \frac{9x+1}{x^3 \left(2 x^2-6 x+7\right)}dx-\frac{2-3x}{35x^2 \left(2 x^2-6 x+7\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{35} \int \left(-\frac{2(400x-717)}{343 \left(2 x^2-6 x+7\right)}+\frac{400}{343x}+\frac{69}{49x^2}+\frac{1}{7x^3}\right)dx-\frac{2-3x}{35x^2 \left(2 x^2-6 x+7\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{35} \left(-\frac{234 \arctan\left(\frac{3-2x}{\sqrt{5}}\right)}{343\sqrt{5}}-\frac{1}{14x^2}-\frac{200}{343}\log\left(2x^2-6x+7\right)-\frac{69}{49x}+\frac{400\log(x)}{343}\right)-\frac{2-3x}{35x^2 \left(2 x^2-6 x+7\right)}$$

input
```
Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

output
```
-1/35*(2 - 3*x)/(x^2*(7 - 6*x + 2*x^2)) + (-1/14*1/x^2 - 69/(49*x) - (234*
ArcTan[(3 - 2*x)/Sqrt[5]])/(343*Sqrt[5]) + (400*Log[x])/343 - (200*Log[7 -
 6*x + 2*x^2])/343)/35
```

3.203.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.203.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{98x^2}-\frac{12}{343x}+\frac{80\ln(x)}{2401}-\frac{4\left(\frac{63x}{20}-\frac{287}{20}\right)}{2401\left(x^2-3x+\frac{7}{2}\right)}-\frac{40\ln(2x^2-6x+7)}{2401}+\frac{234\sqrt{5}\,\arctan\left(\frac{(4x-6)\sqrt{5}}{10}\right)}{60025}$	62
risch	$\frac{-\frac{138}{1715}x^3+\frac{407}{1715}x^2-\frac{9}{49}x-\frac{1}{14}}{x^2(2x^2-6x+7)}-\frac{40\ln(4x^2-12x+14)}{2401}+\frac{234\sqrt{5}\,\arctan\left(\frac{(2x-3)\sqrt{5}}{5}\right)}{60025}+\frac{80\ln(x)}{2401}$	67

input
```
int(1/x^3/(2*x^2-6*x+7)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/98/x^2-12/343/x+80/2401*ln(x)-4/2401*(63/20*x-287/20)/(x^2-3*x+7/2)-40/
2401*ln(2*x^2-6*x+7)+234/60025*5^(1/2)*arctan(1/10*(4*x-6)*5^(1/2))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^3 (7-6x+2x^2)^2}\,dx =$$
$$-\frac{9660\,x^3 - 468\,\sqrt{5}(2\,x^4 - 6\,x^3 + 7\,x^2)\arctan\left(\frac{1}{5}\,\sqrt{5}(2\,x-3)\right) - 28490\,x^2 + 2000\,(2\,x^4 - 6\,x^3 + 7\,x^2)\log}{120050\,(2\,x^4 - 6\,x^3 + 7\,x^2)}$$

input
```
integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="fricas")
```

output
```
-1/120050*(9660*x^3 - 468*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(
5)*(2*x - 3)) - 28490*x^2 + 2000*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2 - 6*x +
 7) - 4000*(2*x^4 - 6*x^3 + 7*x^2)*log(x) + 22050*x + 8575)/(2*x^4 - 6*x^3
 + 7*x^2)
```

3.203.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3 (7-6x+2x^2)^2}\,dx = \frac{80\log(x)}{2401} - \frac{40\log\left(x^2-3x+\frac{7}{2}\right)}{2401} + \frac{234\sqrt{5}\operatorname{atan}\left(\frac{2\sqrt{5}x}{5} - \frac{3\sqrt{5}}{5}\right)}{60025} + \frac{-276x^3+814x^2-630x-245}{6860x^4-20580x^3+24010x^2}$$

input
```
integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

output
```
80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan(2*sqrt(5
)*x/5 - 3*sqrt(5)/5)/60025 + (-276*x**3 + 814*x**2 - 630*x - 245)/(6860*x*
*4 - 20580*x**3 + 24010*x**2)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} dx = \frac{234}{60025} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}(2x-3)\right) - \frac{276x^3 - 814x^2 + 630x + 245}{3430(2x^4 - 6x^3 + 7x^2)} - \frac{40}{2401} \log(2x^2 - 6x + 7) + \frac{80}{2401} \log(x)$$

input
```
integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="maxima")
```

output
```
234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^
2 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2*x^2 - 6*x + 7) +
80/2401*log(x)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} dx = \frac{234}{60025} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}(2x-3)\right) - \frac{276x^3 - 814x^2 + 630x + 245}{3430(2x^2 - 6x + 7)x^2} - \frac{40}{2401} \log(2x^2 - 6x + 7) + \frac{80}{2401} \log(|x|)$$

input
```
integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="giac")
```

output
```
234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^
2 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*log(2*x^2 - 6*x + 7) +
80/2401*log(abs(x))
```

3.203.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} \, dx = \frac{80 \ln (x)}{2401} - \frac{\frac{69 x^3}{1715} - \frac{407 x^2}{3430} + \frac{9 x}{98} + \frac{1}{28}}{x^4 - 3 x^3 + \frac{7 x^2}{2}} - \ln \left(x - \frac{3}{2} - \frac{\sqrt{5}\, 1\mathrm{i}}{2}\right) \left(\frac{40}{2401} + \frac{\sqrt{5}\, 117\mathrm{i}}{60025}\right) + \ln \left(x - \frac{3}{2} + \frac{\sqrt{5}\, 1\mathrm{i}}{2}\right) \left(-\frac{40}{2401} + \frac{\sqrt{5}\, 117\mathrm{i}}{60025}\right)$$

input
```
int(1/(x^3*(2*x^2 - 6*x + 7)^2),x)
```

output
```
(80*log(x))/2401 - ((9*x)/98 - (407*x^2)/3430 + (69*x^3)/1715 + 1/28)/((7*
x^2)/2 - 3*x^3 + x^4) - log(x - (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/6002
5 + 40/2401) + log(x + (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025 - 40/24
01)
```

3.204 $\int \frac{x^9}{(2+3x+x^2)^5}\,dx$

3.204.1 Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = 735x+\frac{x^8(4+3x)}{4(2+3x+x^2)^4}-\frac{x^6(110+81x)}{12(2+3x+x^2)^3}+\frac{x^4(184+135x)}{2(2+3x+x^2)^2}-\frac{x^2(2206+1593x)}{2(2+3x+x^2)}-1471\log(1+x)+1472\log(2+x)$$

output
```
735*x+1/4*x^8*(4+3*x)/(x^2+3*x+2)^4-1/12*x^6*(110+81*x)/(x^2+3*x+2)^3+1/2*
x^4*(184+135*x)/(x^2+3*x+2)^2-1/2*x^2*(2206+1593*x)/(x^2+3*x+2)-1471*ln(1+
x)+1472*ln(2+x)
```

3.204.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = \frac{514+513x}{4(2+3x+x^2)^4}+\frac{415+1998x}{12(2+3x+x^2)^3}+\frac{3(451+456x)}{4(2+3x+x^2)^2}-\frac{2(1114+729x)}{2+3x+x^2}-1471\log(1+x)+1472\log(2+x)$$

input
```
Integrate[x^9/(2 + 3*x + x^2)^5,x]
```

output
```
(514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x + x^2)^3
) + (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 + 729*x))/(2 + 3*x
+ x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

3.204.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(x^2+3x+2)^5} \, dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(\frac{1472}{x+2}+\frac{1024}{(x+2)^2}+\frac{768}{(x+2)^3}+\frac{256}{(x+2)^4}+\frac{512}{(x+2)^5}-\frac{1471}{x+1}+\frac{434}{(x+1)^2}-\frac{96}{(x+1)^3}+\frac{14}{(x+1)^4}-\frac{1}{(x+1)^5}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{434}{x+1}-\frac{1024}{x+2}+\frac{48}{(x+1)^2}-\frac{384}{(x+2)^2}-\frac{14}{3(x+1)^3}-\frac{256}{3(x+2)^3}+\frac{1}{4(x+1)^4}-\frac{128}{(x+2)^4}-1471\log(x+1)+1472\log(x+2)$$

input
```
Int[x^9/(2 + 3*x + x^2)^5,x]
```

output
```
1/(4*(1 + x)^4) - 14/(3*(1 + x)^3) + 48/(1 + x)^2 - 434/(1 + x) - 128/(2 +
 x)^4 - 256/(3*(2 + x)^3) - 384/(2 + x)^2 - 1024/(2 + x) - 1471*Log[1 + x]
 + 1472*Log[2 + x]
```

3.204.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.204.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

method	result
norman	$\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{(x^2+3x+2)^4}-1471\ln(1+x)+1472\ln(2+$
risch	$\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{(x^2+3x+2)^4}-1471\ln(1+x)+1472\ln(2+$
default	$-\frac{128}{(2+x)^4}-\frac{256}{3(2+x)^3}-\frac{384}{(2+x)^2}-\frac{1024}{2+x}+1472\ln(2+x)+\frac{1}{4(1+x)^4}-\frac{14}{3(1+x)^3}+\frac{48}{(1+x)^2}-\frac{434}{1+x}-147$
parallelrisch	$-\frac{195280+184200x^6+1030560x+813888x^5+282432\ln(1+x)-282624\ln(2+x)+1955853x^4+2759400x^3+2286008x^2+17496x}{}$

input
```
int(x^9/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

output
```
(-229950*x^3-85880*x-67824*x^5-15350*x^6-1458*x^7-651951/4*x^4-571502/3*x^
2-48820/3)/(x^2+3*x+2)^4-1471*ln(1+x)+1472*ln(2+x)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx =$$
$$-\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2-17664\,(x^8+12\,x^7+62\,x}{}$$

input
```
integrate(x^9/(x^2+3*x+2)^5,x, algorithm="fricas")
```

output
```
-1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2
286008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 +
248*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 +
 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 1030560*x + 195280)
/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 1
6)
```

3.204.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = \frac{-17496x^7-184200x^6-813888x^5-1955853x^4-2759400x^3-2286008x^2-1030560x-195280}{12x^8+144x^7+744x^6+2160x^5+3852x^4+4320x^3+2976x^2+1152x+192} - 1471\log(x+1)+1472\log(x+2)$$

input
```
integrate(x**9/(x**2+3*x+2)**5,x)
```

output
```
(-17496*x**7 - 184200*x**6 - 813888*x**5 - 1955853*x**4 - 2759400*x**3 - 2
286008*x**2 - 1030560*x - 195280)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x*
*5 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) - 1471*log(x + 1) +
 1472*log(x + 2)
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2+1030560\,x+195280}{12\,(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16)} + 1472\,\log(x+2)-1471\,\log(x+1)$$

input
```
integrate(x^9/(x^2+3*x+2)^5,x, algorithm="maxima")
```

output
```
-1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2
286008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^
4 + 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x + 2) - 1471*log(x + 1)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7 + 184200\,x^6 + 813888\,x^5 + 1955853\,x^4 + 2759400\,x^3 + 2286008\,x^2 + 1030560\,x + 195280}{12\,(x+2)^4(x+1)^4} + 1472\,\log\left(|x+2|\right) - 1471\,\log\left(|x+1|\right)$$

input
```
integrate(x^9/(x^2+3*x+2)^5,x, algorithm="giac")
```

output
```
-1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2
286008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) + 1472*log(abs(x +
2)) - 1471*log(abs(x + 1))
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = 1472\,\ln(x+2) - 1471\,\ln(x+1) - \frac{1458\,x^7 + 15350\,x^6 + 67824\,x^5 + \frac{651951\,x^4}{4} + 229950\,x^3 + \frac{571502\,x^2}{3} + 85880\,x + \frac{48820}{3}}{x^8 + 12\,x^7 + 62\,x^6 + 180\,x^5 + 321\,x^4 + 360\,x^3 + 248\,x^2 + 96\,x + 16}$$

input
```
int(x^9/(3*x + x^2 + 2)^5,x)
```

output
```
1472*log(x + 2) - 1471*log(x + 1) - (85880*x + (571502*x^2)/3 + 229950*x^3
 + (651951*x^4)/4 + 67824*x^5 + 15350*x^6 + 1458*x^7 + 48820/3)/(96*x + 24
8*x^2 + 360*x^3 + 321*x^4 + 180*x^5 + 62*x^6 + 12*x^7 + x^8 + 16)
```

3.205 $\int \frac{(1+2x)^2}{(3+5x+2x^2)^5} dx$

3.205.1 Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5} dx = \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480\log(1+x) - 2480\log(3+2x)$$

output
```
1/4*(1+2*x)*(7+6*x)/(2*x^2+5*x+3)^4+1/3*(73+62*x)/(2*x^2+5*x+3)^3-155/3*(5
+4*x)/(2*x^2+5*x+3)^2+620*(5+4*x)/(2*x^2+5*x+3)+2480*ln(1+x)-2480*ln(3+2*x
)
```

3.205.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5} dx = -\frac{11+10x}{4(3+5x+2x^2)^4} + \frac{31(5+4x)}{6(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480\log(2(1+x)) - 2480\log(3+2x)$$

input
```
Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

output
```
-1/4*(11 + 10*x)/(3 + 5*x + 2*x^2)^4 + (31*(5 + 4*x))/(6*(3 + 5*x + 2*x^2)
^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5*x +
 2*x^2) + 2480*Log[2*(1 + x)] - 2480*Log[3 + 2*x]
```

3.205.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2}{(2x^2+5x+3)^5}\,dx$$

$$\Big\downarrow \text{1141}$$

$$32\int \left(-\frac{155}{2x+3}-\frac{85}{(2x+3)^2}-\frac{41}{(2x+3)^3}-\frac{16}{(2x+3)^4}-\frac{4}{(2x+3)^5}+\frac{155}{2(x+1)}-\frac{35}{2(x+1)^2}+\frac{13}{4(x+1)^3}-\frac{7}{16(x \cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$32\left(\frac{35}{2(x+1)}+\frac{85}{2(2x+3)}-\frac{13}{8(x+1)^2}+\frac{41}{4(2x+3)^2}+\frac{7}{48(x+1)^3}+\frac{8}{3(2x+3)^3}-\frac{1}{128(x+1)^4}+\frac{1}{2(2x+3)^4}+\cdots\right.$$

input
```
Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

output
```
32*(-1/128*1/(1 + x)^4 + 7/(48*(1 + x)^3) - 13/(8*(1 + x)^2) + 35/(2*(1 +
x)) + 1/(2*(3 + 2*x)^4) + 8/(3*(3 + 2*x)^3) + 41/(4*(3 + 2*x)^2) + 85/(2*(
3 + 2*x)) + (155*Log[1 + x])/2 - (155*Log[3 + 2*x])/2)
```

3.205.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.205.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
norman	$\frac{19840x^7+173600x^6+1624648x^3+\frac{1428116}{3}x+\frac{1939360}{3}x^5+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{(2x^2+5x+3)^4}+2480\ln(1+x)-2480\ln$
risch	$\frac{19840x^7+173600x^6+1624648x^3+\frac{1428116}{3}x+\frac{1939360}{3}x^5+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{(2x^2+5x+3)^4}+2480\ln(1+x)-2480\ln$
default	$\frac{16}{(3+2x)^4}+\frac{256}{3(3+2x)^3}+\frac{328}{(3+2x)^2}+\frac{1360}{3+2x}-2480\ln(3+2x)-\frac{1}{4(1+x)^4}+\frac{14}{3(1+x)^3}-\frac{52}{(1+x)^2}+\frac{560}{1+x}+2$
parallelrisch	$\frac{3909588+8332800x^6+22849856x+31029760x^5+9642240\ln(1+x)+63736000x^4+77983104x^3+56836640x^2+952320x^7+82851}{}$

input
```
int((1+2*x)^2/(2*x^2+5*x+3)^5,x,method=_RETURNVERBOSE)
```

output
```
(19840*x^7+173600*x^6+1624648*x^3+1428116/3*x+1939360/3*x^5+3552290/3*x^2+
3983500/3*x^4+325799/4)/(2*x^2+5*x+3)^4+2480*ln(1+x)-2480*ln(3+2*x)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$$
$$=\frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2-29760\,(16\,x^8+160}{}$$

input
```
integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="fricas")
```

output
```
1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
 + 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4
+ 2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3) + 29760*(16*x^8 + 160*x^7
 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(x
 + 1) + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*
x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)
```

3.205.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080x^7+2083200x^6+7757440x^5+15934000x^4+19495776x^3+14209160x^2+5712464x+977397}{192x^8+1920x^7+8352x^6+20640x^5+31692x^4+30960x^3+18792x^2+6480x+972} + 2480\log(x+1) - 2480\log\left(x+\frac{3}{2}\right)$$

input
```
integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

output
```
(238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 19495776*x**3
 + 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*x**7 + 8352*x**6 +
 20640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x**2 + 6480*x + 972) + 2480*
log(x + 1) - 2480*log(x + 3/2)
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\,(16\,x^8+160\,x^7+696\,x^6+1720\,x^5+2641\,x^4+2580\,x^3+1566\,x^2+540\,x+81)} - 2480\,\log(2\,x+3) + 2480\,\log(x+1)$$

input
```
integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="maxima")
```

output
```
1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
 + 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x
^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81) - 2480*log(2*x + 3) + 24
80*log(x + 1)
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\,(2\,x^2+5\,x+3)^4} - 2480\,\log\left(|2\,x+3|\right)+2480\,\log\left(|x+1|\right)$$

input
```
integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="giac")
```

output
```
1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
 + 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^4 - 2480*log(abs(2
*x + 3)) + 2480*log(abs(x + 1))
```

3.205.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{1240\,x^7+10850\,x^6+\frac{121210\,x^5}{3}+\frac{995875\,x^4}{12}+\frac{203081\,x^3}{2}+\frac{1776145\,x^2}{24}+\frac{357029\,x}{12}+\frac{325799}{64}}{x^8+10\,x^7+\frac{87\,x^6}{2}+\frac{215\,x^5}{2}+\frac{2641\,x^4}{16}+\frac{645\,x^3}{4}+\frac{783\,x^2}{8}+\frac{135\,x}{4}+\frac{81}{16}} - 4960\,\mathrm{atanh}(4\,x+5)$$

input
```
int((2*x + 1)^2/(5*x + 2*x^2 + 3)^5,x)
```

output
```
((357029*x)/12 + (1776145*x^2)/24 + (203081*x^3)/2 + (995875*x^4)/12 + (12
1210*x^5)/3 + 10850*x^6 + 1240*x^7 + 325799/64)/((135*x)/4 + (783*x^2)/8 +
 (645*x^3)/4 + (2641*x^4)/16 + (215*x^5)/2 + (87*x^6)/2 + 10*x^7 + x^8 + 8
1/16) - 4960*atanh(4*x + 5)
```

3.206 $\int \frac{(a-bx^2)^3}{x^7} \, dx$

3.206.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{(a-bx^2)^3}{x^7} \, dx = -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

output
```
-1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

3.206.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a-bx^2)^3}{x^7} \, dx = -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

input
```
Integrate[(a - b*x^2)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

3.206.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^3}{x^7}\,dx$$
$$\Big\downarrow \ 243$$
$$\frac{1}{2}\int \frac{(a-bx^2)^3}{x^8}dx^2$$
$$\Big\downarrow \ 49$$
$$\frac{1}{2}\int \left(\frac{a^3}{x^8}-\frac{3ba^2}{x^6}+\frac{3b^2a}{x^4}-\frac{b^3}{x^2}\right)dx^2$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\left(-\frac{a^3}{3x^6}+\frac{3a^2b}{2x^4}-\frac{3ab^2}{x^2}-b^3\log\left(x^2\right)\right)$$

input
```
Int[(a - b*x^2)^3/x^7,x]
```

output
```
(-1/3*a^3/x^6 + (3*a^2*b)/(2*x^4) - (3*a*b^2)/x^2 - b^3*Log[x^2])/2
```

3.206.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.206.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{2x^2}-b^3\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln\left(x\right)$	37
risch	$\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln\left(x\right)$	37
parallelrisch	$-\frac{12b^3\ln(x)x^6+18b^2a\,x^4-9a^2b\,x^2+2a^3}{12x^6}$	40

input
```
int((-b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^3}{x^7}\,dx=-\frac{12\,b^3x^6\log\left(x\right)+18\,ab^2x^4-9\,a^2bx^2+2\,a^3}{12\,x^6}$$

input
```
integrate((-b*x^2+a)^3/x^7,x, algorithm="fricas")
```

output
```
-1/12*(12*b^3*x^6*log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

3.206.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^3}{x^7}\,dx=-b^3\log\left(x\right)-\frac{2a^3-9a^2bx^2+18ab^2x^4}{12x^6}$$

input
```
integrate((-b*x**2+a)**3/x**7,x)
```

output
```
-b**3*log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{1}{2}\,b^3\log\left(x^2\right) - \frac{18\,ab^2x^4 - 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

input
```
integrate((-b*x^2+a)^3/x^7,x, algorithm="maxima")
```

output
```
-1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{1}{2}\,b^3\log\left(x^2\right) + \frac{11\,b^3x^6 - 18\,ab^2x^4 + 9\,a^2bx^2 - 2\,a^3}{12\,x^6}$$

input
```
integrate((-b*x^2+a)^3/x^7,x, algorithm="giac")
```

output
```
-1/2*b^3*log(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)
/x^6
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -b^3\,\ln\left(x\right) - \frac{\frac{a^3}{6} - \frac{3\,a^2\,b\,x^2}{4} + \frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

input
```
int((a - b*x^2)^3/x^7,x)
```

output
```
- b^3*log(x) - (a^3/6 - (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

3.207 $\int \frac{x^{13}}{(a^4+x^4)^5} \, dx$

3.207.1 Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{x^{13}}{(a^4+x^4)^5} \, dx = -\frac{x^{10}}{16(a^4+x^4)^4} - \frac{5x^6}{96(a^4+x^4)^3} - \frac{5x^2}{128(a^4+x^4)^2} + \frac{5x^2}{256a^4(a^4+x^4)} + \frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$$

output
```
-1/16*x^10/(a^4+x^4)^4-5/96*x^6/(a^4+x^4)^3-5/128*x^2/(a^4+x^4)^2+5/256*x^
2/a^4/(a^4+x^4)+5/256*arctan(x^2/a^2)/a^6
```

3.207.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{x^{13}}{(a^4+x^4)^5} \, dx = \frac{-\frac{a^2x^2(15a^{12}+55a^8x^4+73a^4x^8-15x^{12})}{(a^4+x^4)^4} + 15\arctan\left(\frac{x^2}{a^2}\right)}{768a^6}$$

input
```
Integrate[x^13/(a^4 + x^4)^5,x]
```

output
```
(-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 + x^4)^4)
+ 15*ArcTan[x^2/a^2])/(768*a^6)
```

3.207.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {807, 252, 252, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(a^4+x^4)^5}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{8}\int \frac{x^8}{(a^4+x^4)^4}dx^2-\frac{x^{10}}{8(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\int \frac{x^4}{(a^4+x^4)^3}dx^2-\frac{x^6}{6(a^4+x^4)^3}\right)-\frac{x^{10}}{8(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(a^4+x^4)^2}dx^2-\frac{x^2}{4(a^4+x^4)^2}\right)-\frac{x^6}{6(a^4+x^4)^3}\right)-\frac{x^{10}}{8(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \frac{1}{a^4+x^4}dx^2}{2a^4}+\frac{x^2}{2a^4(a^4+x^4)}\right)-\frac{x^2}{4(a^4+x^4)^2}\right)-\frac{x^6}{6(a^4+x^4)^3}\right)-\frac{x^{10}}{8(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{x^2}{2a^4(a^4+x^4)}+\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^6}\right)-\frac{x^2}{4(a^4+x^4)^2}\right)-\frac{x^6}{6(a^4+x^4)^3}\right)-\frac{x^{10}}{8(a^4+x^4)^4}\right)$$

input
```
Int[x^13/(a^4 + x^4)^5,x]
```

output
```
(-1/8*x^10/(a^4 + x^4)^4 + (5*(-1/6*x^6/(a^4 + x^4)^3 + (-1/4*x^2/(a^4 + x
^4)^2 + (x^2/(2*a^4*(a^4 + x^4)) + ArcTan[x^2/a^2]/(2*a^6))/4)/2))/8)/2
```

3.207.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.207.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{5a^8x^2}{256}-\frac{55a^4x^6}{768}-\frac{73x^{10}}{768}+\frac{5x^{14}}{256a^4}}{\left(a^4+x^4\right)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$
default	$\frac{\frac{5x^{14}}{128a^4}-\frac{73x^{10}}{384}-\frac{55a^4x^6}{384}-\frac{5a^8x^2}{128}}{2\left(a^4+x^4\right)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$
parallelrisch	$-\frac{-60i\ln\left(ia^2+x^2\right)x^4a^{12}+60i\ln\left(-ia^2+x^2\right)x^4a^{12}-60i\ln\left(ia^2+x^2\right)x^{12}a^4+15i\ln\left(-ia^2+x^2\right)x^{16}-90i\ln\left(ia^2+x^2\right)x^8a^8+90i\ln(}{}$

input
```
int(x^13/(a^4+x^4)^5,x,method=_RETURNVERBOSE)
```

output
```
(-5/256*a^8*x^2-55/768*a^4*x^6-73/768*x^10+5/256/a^4*x^14)/(a^4+x^4)^4+5/2
56*arctan(x^2/a^2)/a^6
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{15\,a^{14}x^2+55\,a^{10}x^6+73\,a^6x^{10}-15\,a^2x^{14}-15\,(a^{16}+4\,a^{12}x^4+6\,a^8x^8+4\,a^4x^{12}+x^{16})\arctan\left(\frac{x^2}{a^2}\right)}{768\,(a^{22}+4\,a^{18}x^4+6\,a^{14}x^8+4\,a^{10}x^{12}+a^6x^{16})}$$

input
```
integrate(x^13/(a^4+x^4)^5,x, algorithm="fricas")
```

output
```
-1/768*(15*a^14*x^2 + 55*a^10*x^6 + 73*a^6*x^10 - 15*a^2*x^14 - 15*(a^16 +
 4*a^12*x^4 + 6*a^8*x^8 + 4*a^4*x^12 + x^16)*arctan(x^2/a^2))/(a^22 + 4*a^
18*x^4 + 6*a^14*x^8 + 4*a^10*x^12 + a^6*x^16)
```

3.207.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{-15a^{12}x^2-55a^8x^6-73a^4x^{10}+15x^{14}}{768a^{20}+3072a^{16}x^4+4608a^{12}x^8+3072a^8x^{12}+768a^4x^{16}} + \frac{-\frac{5i\log(-ia^2+x^2)}{512}+\frac{5i\log(ia^2+x^2)}{512}}{a^6}$$

input
```
integrate(x**13/(a**4+x**4)**5,x)
```

output
```
(-15*a**12*x**2 - 55*a**8*x**6 - 73*a**4*x**10 + 15*x**14)/(768*a**20 + 30
72*a**16*x**4 + 4608*a**12*x**8 + 3072*a**8*x**12 + 768*a**4*x**16) + (-5*
I*log(-I*a**2 + x**2)/512 + 5*I*log(I*a**2 + x**2)/512)/a**6
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^{20}+4\,a^{16}x^4+6\,a^{12}x^8+4\,a^8x^{12}+a^4x^{16})}+\frac{5\,\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}$$

input
```
integrate(x^13/(a^4+x^4)^5,x, algorithm="maxima")
```

output
```
-1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 - 15*x^14)/(a^20 + 4*a^16*x
^4 + 6*a^12*x^8 + 4*a^8*x^12 + a^4*x^16) + 5/256*arctan(x^2/a^2)/a^6
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{5\,\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}-\frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^4+x^4)^4a^4}$$

input
```
integrate(x^13/(a^4+x^4)^5,x, algorithm="giac")
```

output
```
5/256*arctan(x^2/a^2)/a^6 - 1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10
- 15*x^14)/((a^4 + x^4)^4*a^4)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{5\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{256\,a^6}-\frac{\frac{73\,x^{10}}{768}+\frac{55\,a^4\,x^6}{768}+\frac{5\,a^8\,x^2}{256}-\frac{5\,x^{14}}{256\,a^4}}{a^{16}+4\,a^{12}\,x^4+6\,a^8\,x^8+4\,a^4\,x^{12}+x^{16}}$$

input
```
int(x^13/(a^4 + x^4)^5,x)
```

output
```
(5*atan(x^2/a^2))/(256*a^6) - ((73*x^10)/768 + (55*a^4*x^6)/768 + (5*a^8*x
^2)/256 - (5*x^14)/(256*a^4))/(a^16 + x^16 + 4*a^4*x^12 + 6*a^8*x^8 + 4*a^
12*x^4)
```

3.208 $\int \left(2\sqrt{x}-x\right)^2 x^{3/2}(1+x^2)\,dx$

3.208.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{8x^{7/2}}{7}-x^4+\frac{2x^{9/2}}{9}+\frac{8x^{11/2}}{11}-\frac{2x^6}{3}+\frac{2x^{13/2}}{13}$$

output
```
8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{10296x^{7/2}-9009x^4+2002x^{9/2}+6552x^{11/2}-6006x^6+1386x^{13/2}}{9009}$$

input
```
Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

output
```
(10296*x^(7/2) - 9009*x^4 + 2002*x^(9/2) + 6552*x^(11/2) - 6006*x^6 + 1386
*x^(13/2))/9009
```

3.208.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {10, 2035, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2} \left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{10}$$

$$\int \left(2-\sqrt{x}\right)^2 x^{5/2} \left(x^2+1\right) dx$$

$$\Big\downarrow \text{2035}$$

$$2 \int \left(2-\sqrt{x}\right)^2 x^3 \left(x^2+1\right) d\sqrt{x}$$

$$\Big\downarrow \text{2123}$$

$$2 \int \left(x^6-4x^{11/2}+4x^5+x^4-4x^{7/2}+4x^3\right) d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2 \left(\frac{x^{13/2}}{13}+\frac{4x^{11/2}}{11}+\frac{x^{9/2}}{9}+\frac{4x^{7/2}}{7}-\frac{x^6}{3}-\frac{x^4}{2}\right)$$

input
```
Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

output
```
2*((4*x^(7/2))/7 - x^4/2 + x^(9/2)/9 + (4*x^(11/2))/11 - x^6/3 + x^(13/2)/
13)
```

3.208.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.208.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{8x^{\frac{7}{2}}}{7}-x^4+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^6}{3}+\frac{2x^{\frac{13}{2}}}{13}$	32
default	$\frac{8x^{\frac{7}{2}}}{7}-x^4+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^6}{3}+\frac{2x^{\frac{13}{2}}}{13}$	32
trager	$-\frac{(2x^5+2x^4+5x^3+5x^2+5x+5)(-1+x)}{3}+\frac{2x^{\frac{7}{2}}(693x^3+3276x^2+1001x+5148)}{9009}$	52

input
```
int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=-\frac{2}{3}\,x^6-x^4+\frac{2}{9009}\left(693\,x^6+3276\,x^5+1001\,x^4+5148\,x^3\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="fricas")
```

output
```
-2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3)*sqrt(x)
```

3.208.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{2x^{\frac{13}{2}}}{13}+\frac{8x^{\frac{11}{2}}}{11}+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{7}{2}}}{7}-\frac{2x^6}{3}-x^4$$

input
```
integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
```

output
```
2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2*x**6/3 -
 x**4
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{2}{13}\,x^{\frac{13}{2}}-\frac{2}{3}\,x^6+\frac{8}{11}\,x^{\frac{11}{2}}+\frac{2}{9}\,x^{\frac{9}{2}}-x^4+\frac{8}{7}\,x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="maxima")
```

output
```
2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{2}{13}\,x^{\frac{13}{2}}-\frac{2}{3}\,x^6+\frac{8}{11}\,x^{\frac{11}{2}}+\frac{2}{9}\,x^{\frac{9}{2}}-x^4+\frac{8}{7}\,x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="giac")
```

output
```
2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx=\frac{8\,x^{7/2}}{7}-\frac{2\,x^6}{3}-x^4+\frac{2\,x^{9/2}}{9}+\frac{8\,x^{11/2}}{11}+\frac{2\,x^{13/2}}{13}$$

input
```
int(x^(3/2)*(x - 2*x^(1/2))^2*(x^2 + 1),x)
```

output
```
(8*x^(7/2))/7 - (2*x^6)/3 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 + (2*x^(
13/2))/13
```

3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) \, dx$

3.209.1 Optimal result

Integrand size = 33, antiderivative size = 55

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) \, dx=-\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}$$

output
```
-45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x
^(14/3)+8/11*x^(11/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) \, dx=-\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}$$

input
```
Integrate[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

output
```
(-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23
/5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

3.209.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2027, 10, 27, 2035, 7267, 25, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{3/2}-3x^{3/5}\right)^2 \left(4x^{3/2}-\frac{x^{2/3}}{3}\right) dx$$

$$\Big\downarrow \text{2027}$$

$$\int \left(x^{9/10}-3\right)^2 x^{6/5} \left(4x^{3/2}-\frac{x^{2/3}}{3}\right) dx$$

$$\Big\downarrow \text{10}$$

$$\int -\frac{1}{3}\left(1-12x^{5/6}\right)\left(3-x^{9/10}\right)^2 x^{28/15}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \left(1-12x^{5/6}\right)\left(3-x^{9/10}\right)^2 x^{28/15}dx$$

$$\Big\downarrow \text{2035}$$

$$-5\int \left(1-12x^{5/6}\right)\left(3-x^{9/10}\right)^2 x^{14/5}d\sqrt[15]{x}$$

$$\Big\downarrow \text{7267}$$

$$10\int -x^{17/3}\left(1-12x^{5/3}\right)\left(3-x^{9/5}\right)^2 d\sqrt[30]{x}$$

$$\Big\downarrow \text{25}$$

$$-10\int x^{17/3}\left(1-12x^{5/3}\right)\left(3-x^{9/5}\right)^2 d\sqrt[30]{x}$$

$$\Big\downarrow \text{2360}$$

$$-10\int \left(-12x^{164/15}+x^{139/15}+72x^{137/15}-6x^{112/15}-108x^{22/3}+9x^{17/3}\right) d\sqrt[30]{x}$$

$$\Big\downarrow \text{2009}$$

$$10\left(-\frac{x^{28/3}}{140}-\frac{12x^{46/5}}{23}+\frac{6x^{113/15}}{113}+\frac{36x^{37/5}}{37}-\frac{9x^{86/15}}{86}+\frac{4x^{11}}{55}\right)$$

input
```
Int[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

output
```
10*((-9*x^(86/15))/86 + (36*x^(37/5))/37 + (6*x^(113/15))/113 - (12*x^(46/
5))/23 - x^(28/3)/140 + (4*x^11)/55)
```

3.209.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.209. $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx$

3.209.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$	32
default	$-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$	32

input
```
int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x,method=_RETURNVERBOS
E)
```

output
```
-45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x
^(14/3)+8/11*x^(11/2)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8}{11}\,x^{\frac{11}{2}}-\frac{1}{14}\,x^{\frac{14}{3}}-\frac{120}{23}\,x^{\frac{23}{5}}+\frac{60}{113}\,x^{\frac{113}{30}}+\frac{360}{37}\,x^{\frac{37}{10}}-\frac{45}{43}\,x^{\frac{43}{15}}$$

input
```
integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="fr
icas")
```

output
```
8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/
37*x^(37/10) - 45/43*x^(43/15)
```

3.209.6 Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{60x^{\frac{113}{30}}}{113}-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$$

input
```
integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

output
```
60*x**(113/30)/113 - 45*x**(43/15)/43 + 360*x**(37/10)/37 - 120*x**(23/5)/
23 - x**(14/3)/14 + 8*x**(11/2)/11
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx = \frac{8}{11} x^{\frac{11}{2}} - \frac{1}{14} x^{\frac{14}{3}} - \frac{120}{23} x^{\frac{23}{5}} + \frac{60}{113} x^{\frac{113}{30}} + \frac{360}{37} x^{\frac{37}{10}} - \frac{45}{43} x^{\frac{43}{15}}$$

input
```
integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="ma
xima")
```

output
```
8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/
37*x^(37/10) - 45/43*x^(43/15)
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx = \frac{8}{11} x^{\frac{11}{2}} - \frac{1}{14} x^{\frac{14}{3}} - \frac{120}{23} x^{\frac{23}{5}} + \frac{60}{113} x^{\frac{113}{30}} + \frac{360}{37} x^{\frac{37}{10}} - \frac{45}{43} x^{\frac{43}{15}}$$

input
```
integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="gi
ac")
```

output
```
8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/
37*x^(37/10) - 45/43*x^(43/15)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8\,x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120\,x^{23/5}}{23}+\frac{360\,x^{37/10}}{37}-\frac{45\,x^{43/15}}{43}+\frac{60\,x^{113/30}}{113}$$

input
```
int(-(x^(3/2) - 3*x^(3/5))^2*(x^(2/3)/3 - 4*x^(3/2)),x)
```

output
```
(8*x^(11/2))/11 - x^(14/3)/14 - (120*x^(23/5))/23 + (360*x^(37/10))/37 - (
45*x^(43/15))/43 + (60*x^(113/30))/113
```

3.210 $\int \frac{1}{1+\sqrt{1+x}}\,dx$

3.210.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right)$$

output
```
-2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right)$$

input
```
Integrate[(1 + Sqrt[1 + x])^(-1),x]
```

output
```
2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

3.210.3 Rubi [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}+1} \, dx$$
$$\Big\downarrow \text{ 239}$$
$$\int \frac{1}{\sqrt{x+1}+1} d(x+1)$$
$$\Big\downarrow \text{ 774}$$
$$2 \int \frac{\sqrt{x+1}}{x+2} d\sqrt{x+1}$$
$$\Big\downarrow \text{ 49}$$
$$2 \int \left(1+\frac{1}{-x-2}\right) d\sqrt{x+1}$$
$$\Big\downarrow \text{ 2009}$$
$$2\left(\sqrt{x+1}-\log(x+2)\right)$$

input
```
Int[(1 + Sqrt[1 + x])^(-1),x]
```

output
```
2*(Sqrt[1 + x] - Log[2 + x])
```

3.210.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.210.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-2\ln\left(1+\sqrt{1+x}\right)+2\sqrt{1+x}$	19
trager	$2\sqrt{1+x}-\ln\left(2\sqrt{1+x}+2+x\right)$	22
default	$2\sqrt{1+x}+\ln\left(-1+\sqrt{1+x}\right)-\ln\left(1+\sqrt{1+x}\right)-\ln\left(x\right)$	31
meijerg	$\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{1+x}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{2\sqrt{\pi}}$	37

input
```
int(1/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\,\sqrt{x+1}-2\,\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

3.210.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)**(1/2)),x)
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\ln\left(\sqrt{x+1}+1\right)$$

input
```
int(1/((x + 1)^(1/2) + 1),x)
```

output
```
2*(x + 1)^(1/2) - 2*log((x + 1)^(1/2) + 1)
```

3.211 $\int \frac{x}{1+\sqrt{1+x}} \, dx$

3.211.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{1+\sqrt{1+x}} \, dx = -x + \frac{2}{3}(1+x)^{3/2}$$

output
```
-x+2/3*(1+x)^(3/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{x}{1+\sqrt{1+x}} \, dx = \frac{1}{3}(1+x)\left(-3+2\sqrt{1+x}\right)$$

input
```
Integrate[x/(1 + Sqrt[1 + x]),x]
```

output
```
((1 + x)*(-3 + 2*Sqrt[1 + x]))/3
```

3.211.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {896, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x+1}+1} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \left(\sqrt{x+1}-1\right) d(x+1)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}(x+1)^{3/2}-x-1$$

input
```
Int[x/(1 + Sqrt[1 + x]),x]
```

output
```
-1 - x + (2*(1 + x)^(3/2))/3
```

3.211.3.1 Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.211.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$	13
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$	13
trager	$-x+\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{1+x}$	16
meijerg	$\frac{-\frac{\sqrt{\pi}\,(12x+8)}{6}+\frac{\sqrt{\pi}\,(8+8x)\sqrt{1+x}}{6}}{2\sqrt{\pi}}$	32

input
```
int(x/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x)^(3/2)-1-x
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+\sqrt{1+x}}\,dx=\frac{2}{3}\,(x+1)^{\frac{3}{2}}-x$$

input
```
integrate(x/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*(x + 1)^(3/2) - x
```

3.211.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{x}{1+\sqrt{1+x}}\,dx=\frac{2x\sqrt{x+1}}{3}-x+\frac{2\sqrt{x+1}}{3}$$

input
```
integrate(x/(1+(1+x)**(1/2)),x)
```

output
```
2*x*sqrt(x + 1)/3 - x + 2*sqrt(x + 1)/3
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\left(x+1\right)^{\frac{3}{2}} - x - 1$$

input
```
integrate(x/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
2/3*(x + 1)^(3/2) - x - 1
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\left(x+1\right)^{\frac{3}{2}} - x - 1$$

input
```
integrate(x/(1+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*(x + 1)^(3/2) - x - 1
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2\left(x+1\right)^{3/2}}{3} - x$$

input
```
int(x/((x + 1)^(1/2) + 1),x)
```

output
```
(2*(x + 1)^(3/2))/3 - x
```

3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx$

3.212.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{1+x}+4\log\left(1-\sqrt{1+x}\right)$$

output
```
x+4*ln(1-(1+x)^(1/2))+4*(1+x)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = 1+x+4\sqrt{1+x}+4\log\left(-1+\sqrt{1+x}\right)$$

input
```
Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

output
```
1 + x + 4*Sqrt[1 + x] + 4*Log[-1 + Sqrt[1 + x]]
```

3.212.3 Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {938, 25, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx$$

$$\Big\downarrow \text{ 938}$$

$$\int -\frac{\sqrt{x+1}+1}{1-\sqrt{x+1}}d(x+1)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sqrt{x+1}+1}{1-\sqrt{x+1}}d(x+1)$$

$$\Big\downarrow \text{ 900}$$

$$-2\int -\frac{\sqrt{x+1}(x+2)}{x}d\sqrt{x+1}$$

$$\Big\downarrow \text{ 86}$$

$$-2\int \left(-x-3-\frac{2}{x}\right)d\sqrt{x+1}$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{1}{2}(-x-1)-2\sqrt{x+1}-2\log(-x)\right)$$

input
```
Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

output
```
-2*((-1 - x)/2 - 2*Sqrt[1 + x] - 2*Log[-x])
```

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.212.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$	21
default	$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$	21
trager	$-1+x+4\sqrt{1+x}+2\ln\left(2\sqrt{1+x}-2-x\right)$	26

input
```
int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1+x+4*(1+x)^(1/2)+4*ln(-1+(1+x)^(1/2))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

3.212.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)
```

output
```
x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)+1$$

input
```
integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + 1
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(\left|\sqrt{x+1}-1\right|\right)+1$$

input
```
integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
x + 4*sqrt(x + 1) + 4*log(abs(sqrt(x + 1) - 1)) + 1
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\ln\left(\sqrt{x+1}-1\right)+4\sqrt{x+1}$$

input
```
int(((x + 1)^(1/2) + 1)/((x + 1)^(1/2) - 1),x)
```

output
```
x + 4*log((x + 1)^(1/2) - 1) + 4*(x + 1)^(1/2)
```

3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}} \, dx$

3.213.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}} \, dx = 6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+6\log\left(1-\sqrt[6]{1+x}\right)$$

output
```
6*(1+x)^(1/6)+3*(1+x)^(1/3)+6*ln(1-(1+x)^(1/6))
```

3.213.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}} \, dx = 3\left(2\sqrt[6]{1+x}+\sqrt[3]{1+x}+2\log\left(-1+\sqrt[6]{1+x}\right)\right)$$

input
```
Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

output
```
3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[-1 + (1 + x)^(1/6)])
```

3.213.3 Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1918, 2027, 798, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{2/3}-\sqrt{x+1}}dx$$
$$\Big\downarrow \text{1918}$$
$$\int \frac{1}{(x+1)^{2/3}-\sqrt{x+1}}d(x+1)$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\sqrt{x+1}\left(\sqrt[6]{x+1}-1\right)}d(x+1)$$
$$\Big\downarrow \text{798}$$
$$6\int \frac{\sqrt[3]{x+1}}{x}d\sqrt[6]{x+1}$$
$$\Big\downarrow \text{25}$$
$$-6\int -\frac{\sqrt[3]{x+1}}{x}d\sqrt[6]{x+1}$$
$$\Big\downarrow \text{49}$$
$$-6\int \left(-x-2-\frac{1}{x}\right)d\sqrt[6]{x+1}$$
$$\Big\downarrow \text{2009}$$
$$6\left(\frac{1}{2}(x+1)^2+\sqrt[6]{x+1}+\log(-x)\right)$$

input
```
Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

output
```
6*((1 + x)^(1/6) + (1 + x)^2/2 + Log[-x])
```

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1918
```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Simp[1/Coeffic
ient[u, x, 1]   Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b,
j, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.213.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result
derivativedivides	$3(1+x)^{\frac{1}{3}}+6(1+x)^{\frac{1}{6}}+6\ln\left((1+x)^{\frac{1}{6}}-1\right)$
default	$6(1+x)^{\frac{1}{6}}+3(1+x)^{\frac{1}{3}}+\ln(x)-\ln\left((1+x)^{\frac{1}{3}}+(1+x)^{\frac{1}{6}}+1\right)+2\ln\left((1+x)^{\frac{1}{6}}-1\right)$

input
```
int(1/((1+x)^(2/3)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
3*(1+x)^(1/3)+6*(1+x)^(1/6)+6*ln((1+x)^(1/6)-1)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\,\log\left((x+1)^{\frac{1}{6}} - 1\right)$$

input
```
integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

3.213.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\sqrt[6]{x+1} + 3\sqrt[3]{x+1} + 6\log\left(\sqrt[6]{x+1} - 1\right)$$

input
```
integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
```

output
```
6*(x + 1)**(1/6) + 3*(x + 1)**(1/3) + 6*log((x + 1)**(1/6) - 1)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\,\log\left((x+1)^{\frac{1}{6}} - 1\right)$$

input
```
integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left(\left|(x+1)^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log(abs((x + 1)^(1/6) - 1))
```

3.213.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\,\ln\left((x+1)^{1/6}-1\right)+3\,(x+1)^{1/3}+6\,(x+1)^{1/6}$$

input
```
int(-1/((x + 1)^(1/2) - (x + 1)^(2/3)),x)
```

output
```
6*log((x + 1)^(1/6) - 1) + 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6)
```

3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx$

3.214.1 Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = -3\left(1+\sqrt[4]{x}\right)^{4/3}+\frac{12}{7}\left(1+\sqrt[4]{x}\right)^{7/3}$$

output
```
-3*(1+x^(1/4))^(4/3)+12/7*(1+x^(1/4))^(7/3)
```

3.214.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3}{7}\left(1+\sqrt[4]{x}\right)^{4/3}\left(-3+4\sqrt[4]{x}\right)$$

input
```
Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

output
```
(3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7
```

3.214.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sqrt[4]{x}+1}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{798}$$

$$4\int \sqrt[3]{\sqrt[4]{x}+1}\sqrt[4]{x}d\sqrt[4]{x}$$

$$\Big\downarrow \text{53}$$

$$4\int \left(\left(\sqrt[4]{x}+1\right)^{4/3}-\sqrt[3]{\sqrt[4]{x}+1}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{3}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-\frac{3}{4}\left(\sqrt[4]{x}+1\right)^{4/3}\right)$$

input
```
Int[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

output
```
4*((-3*(1 + x^(1/4))^(4/3))/4 + (3*(1 + x^(1/4))^(7/3))/7)
```

3.214.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.214.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

method	result	size
meijerg	$2\sqrt{x}\,{}_2F_1\left(-\frac{1}{3},2;3;-x^{\frac{1}{4}}\right)$	17
derivativedivides	$-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$	20
default	$-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$	20

input
```
int((1+x^(1/4))^(1/3)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*hypergeom([-1/3,2],[3],-x^(1/4))
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx=\frac{3}{7}\left(4\sqrt{x}+x^{\frac{1}{4}}-3\right)\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}$$

input
```
integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="fricas")
```

output
```
3/7*(4*sqrt(x) + x^(1/4) - 3)*(x^(1/4) + 1)^(1/3)
```

3.214.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(24) = 48.

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} - \frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} + \frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}}+7x} + \frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} - \frac{9x\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} + \frac{9x}{7x^{\frac{5}{4}}+7x}$$

input
```
integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
```

output
```
12*x**(7/4)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 6*x**(5/4)*(x**(1/4
) + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x**(5/4)/(7*x**(5/4) + 7*x) + 15*x**(
3/2)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 9*x*(x**(1/4) + 1)**(1/3)/
(7*x**(5/4) + 7*x) + 9*x/(7*x**(5/4) + 7*x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}} - 3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="maxima")
```

output
```
12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}} - 3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="giac")
```

output
```
12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3\left(x^{1/4}+1\right)^{4/3}\left(4\,x^{1/4}-3\right)}{7}$$

input
```
int((x^(1/4) + 1)^(1/3)/x^(1/2),x)
```

output
```
(3*(x^(1/4) + 1)^(4/3)*(4*x^(1/4) - 3))/7
```

3.215 $\int \frac{1}{x^3(1+x)^{3/2}}\,dx$

3.215.1 Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15}{4\sqrt{1+x}} - \frac{1}{2x^2\sqrt{1+x}} + \frac{5}{4x\sqrt{1+x}} - \frac{15}{4}\text{arctanh}\left(\sqrt{1+x}\right)$$

output
```
-15/4*arctanh((1+x)^(1/2))+15/4/(1+x)^(1/2)-1/2/x^2/(1+x)^(1/2)+5/4/x/(1+x
)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{1}{4}\left(\frac{-2+5x+15x^2}{x^2\sqrt{1+x}} - 15\text{arctanh}\left(\sqrt{1+x}\right)\right)$$

input
```
Integrate[1/(x^3*(1 + x)^(3/2)),x]
```

output
```
((-2 + 5*x + 15*x^2)/(x^2*Sqrt[1 + x]) - 15*ArcTanh[Sqrt[1 + x]])/4
```

3.215.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {52, 52, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x+1)^{3/2}}dx$$
$$\Big\downarrow \text{52}$$
$$-\frac{5}{4}\int \frac{1}{x^2(x+1)^{3/2}}dx-\frac{1}{2x^2\sqrt{x+1}}$$
$$\Big\downarrow \text{52}$$
$$-\frac{5}{4}\left(-\frac{3}{2}\int \frac{1}{x(x+1)^{3/2}}dx-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$
$$\Big\downarrow \text{61}$$
$$-\frac{5}{4}\left(-\frac{3}{2}\left(\int \frac{1}{x\sqrt{x+1}}dx+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$
$$\Big\downarrow \text{73}$$
$$-\frac{5}{4}\left(-\frac{3}{2}\left(2\int \frac{1}{x}d\sqrt{x+1}+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$
$$\Big\downarrow \text{220}$$
$$-\frac{5}{4}\left(-\frac{3}{2}\left(\frac{2}{\sqrt{x+1}}-2\text{arctanh}\left(\sqrt{x+1}\right)\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$

input
```
Int[1/(x^3*(1 + x)^(3/2)),x]
```

output
```
-1/2*1/(x^2*Sqrt[1 + x]) - (5*(-(1/(x*Sqrt[1 + x])) - (3*(2/Sqrt[1 + x] -
2*ArcTanh[Sqrt[1 + x]]))/2))/4
```

3.215.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.215.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

method	result
risch	$\frac{15x^2+5x-2}{4\sqrt{1+x}\,x^2}-\frac{15\,\mathrm{arctanh}\left(\sqrt{1+x}\right)}{4}$
trager	$\frac{15x^2+5x-2}{4\sqrt{1+x}\,x^2}+\frac{15\ln\left(\frac{2\sqrt{1+x}-2-x}{x}\right)}{8}$
derivativedivides	$-\frac{1}{8(-1+\sqrt{1+x})^2}+\frac{7}{8(-1+\sqrt{1+x})}+\frac{15\ln(-1+\sqrt{1+x})}{8}+\frac{1}{8(1+\sqrt{1+x})^2}+\frac{7}{8(1+\sqrt{1+x})}-\frac{15\ln(1+\sqrt{1+x})}{8}$ -
default	$-\frac{1}{8(-1+\sqrt{1+x})^2}+\frac{7}{8(-1+\sqrt{1+x})}+\frac{15\ln(-1+\sqrt{1+x})}{8}+\frac{1}{8(1+\sqrt{1+x})^2}+\frac{7}{8(1+\sqrt{1+x})}-\frac{15\ln(1+\sqrt{1+x})}{8}$ -
meijerg	$\frac{-\frac{\sqrt{\pi}}{2x^2}+\frac{3\sqrt{\pi}}{2x}+\frac{15\left(\frac{47}{30}-2\ln(2)+\ln(x)\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-47x^2-24x+8\right)}{16x^2}-\frac{\sqrt{\pi}\left(-60x^2-20x+8\right)}{16x^2\sqrt{1+x}}-\frac{15\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{4}}{\sqrt{\pi}}$

input
```
int(1/x^3/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(15*x^2+5*x-2)/(1+x)^(1/2)/x^2-15/4*arctanh((1+x)^(1/2))
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = -\frac{15\left(x^3+x^2\right)\log\left(\sqrt{x+1}+1\right)-15\left(x^3+x^2\right)\log\left(\sqrt{x+1}-1\right)-2\left(15\,x^2+5\,x-2\right)\sqrt{x+1}}{8\left(x^3+x^2\right)}$$

input
```
integrate(1/x^3/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(15*(x^3 + x^2)*log(sqrt(x + 1) + 1) - 15*(x^3 + x^2)*log(sqrt(x + 1)
 - 1) - 2*(15*x^2 + 5*x - 2)*sqrt(x + 1))/(x^3 + x^2)
```

3.215.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 3966, normalized size of antiderivative = 76.27

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(1+x)**(3/2),x)
```

output
```
Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*
(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)*
*(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*s
qrt(x + 1)) - 15*I*pi*(x + 1)**(17/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15
/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448
*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1))
+ 240*(x + 1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**
(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) -
448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1
)) + 120*I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 22
4*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)
**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x
 + 1)**(13/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) +
 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420
*I*pi*(x + 1)**(13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1
)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2)
+ 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**
(11/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x
 + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)*...
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15\,(x+1)^2-25\,x-17}{4\left((x+1)^{\frac{5}{2}}-2\,(x+1)^{\frac{3}{2}}+\sqrt{x+1}\right)} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\,\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate(1/x^3/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) + sqrt(x +
 1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1) - 1)
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{2}{\sqrt{x+1}} + \frac{7\,(x+1)^{\frac{3}{2}} - 9\,\sqrt{x+1}}{4\,x^2} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right) + \frac{15}{8}\,\log\left(\left|\sqrt{x+1}-1\right|\right)$$

input
```
integrate(1/x^3/(1+x)^(3/2),x, algorithm="giac")
```

output
```
2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*log(sqrt(
x + 1) + 1) + 15/8*log(abs(sqrt(x + 1) - 1))
```

3.215.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = -\frac{15\,\mathrm{atanh}\left(\sqrt{x+1}\right)}{4} - \frac{\frac{25\,x}{4} - \frac{15\,(x+1)^2}{4} + \frac{17}{4}}{\sqrt{x+1} - 2\,(x+1)^{3/2} + (x+1)^{5/2}}$$

input
```
int(1/(x^3*(x + 1)^(3/2)),x)
```

output
```
- (15*atanh((x + 1)^(1/2)))/4 - ((25*x)/4 - (15*(x + 1)^2)/4 + 17/4)/((x +
 1)^(1/2) - 2*(x + 1)^(3/2) + (x + 1)^(5/2))
```

3.216 $\int \frac{1}{(1-x)^{7/2}x^5} dx$

3.216.1 Optimal result

Integrand size = 13, antiderivative size = 118

$$\int \frac{1}{(1-x)^{7/2}x^5} dx = \frac{3003}{320(1-x)^{5/2}} + \frac{1001}{64(1-x)^{3/2}} + \frac{3003}{64\sqrt{1-x}} - \frac{1}{4(1-x)^{5/2}x^4} - \frac{13}{24(1-x)^{5/2}x^3} - \frac{143}{96(1-x)^{5/2}x^2} - \frac{429}{64(1-x)^{5/2}x} - \frac{3003}{64}\text{arctanh}\left(\sqrt{1-x}\right)$$

output
```
3003/320/(1-x)^(5/2)+1001/64/(1-x)^(3/2)-1/4/(1-x)^(5/2)/x^4-13/24/(1-x)^(
5/2)/x^3-143/96/(1-x)^(5/2)/x^2-429/64/(1-x)^(5/2)/x-3003/64*arctanh((1-x)
^(1/2))+3003/64/(1-x)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{1}{(1-x)^{7/2}x^5} dx = -\frac{240 + 520x + 1430x^2 + 6435x^3 - 69069x^4 + 105105x^5 - 45045x^6 + 45045(1-x)^{5/2}x^4\text{arctanh}\left(\sqrt{1-x}\right)}{960(1-x)^{5/2}x^4}$$

input
```
Integrate[1/((1 - x)^(7/2)*x^5),x]
```

output
```
-1/960*(240 + 520*x + 1430*x^2 + 6435*x^3 - 69069*x^4 + 105105*x^5 - 45045
*x^6 + 45045*(1 - x)^(5/2)*x^4*ArcTanh[Sqrt[1 - x]])/((1 - x)^(5/2)*x^4)
```

3.216.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {52, 52, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\int \frac{1}{(1-x)^{7/2}x^4}dx-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\left(\frac{11}{6}\int \frac{1}{(1-x)^{7/2}x^3}dx-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\int \frac{1}{(1-x)^{7/2}x^2}dx-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\int \frac{1}{(1-x)^{7/2}x}dx-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{61}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(\int \frac{1}{(1-x)^{5/2}x}dx+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{61}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(\int \frac{1}{(1-x)^{3/2}x}dx+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{61}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(\int \frac{1}{\sqrt{1-x}x}dx+\frac{2}{\sqrt{1-x}}+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-3(1$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

$\Big\downarrow$ 73

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(-2\int \frac{1}{x}d\sqrt{1-x}+\frac{2}{\sqrt{1-x}}+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

$\Big\downarrow$ 219

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(-2\text{arctanh}\left(\sqrt{1-x}\right)+\frac{2}{\sqrt{1-x}}+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

input
```
Int[1/((1 - x)^(7/2)*x^5),x]
```

output
```
-1/4*1/((1 - x)^(5/2)*x^4) + (13*(-1/3*1/((1 - x)^(5/2)*x^3) + (11*(-1/2*1
/((1 - x)^(5/2)*x^2) + (9*(-(1/((1 - x)^(5/2)*x)) + (7*(2/(5*(1 - x)^(5/2)
) + 2/(3*(1 - x)^(3/2)) + 2/Sqrt[1 - x] - 2*ArcTanh[Sqrt[1 - x]]))/2))/4))
/6))/8
```

3.216.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.216.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

method	result
risch	$\frac{45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240}{960x^4\sqrt{1-x}\,(-1+x)^2}-\frac{3003\,\operatorname{arctanh}\left(\sqrt{1-x}\right)}{64}$
trager	$-\frac{(45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240)\sqrt{1-x}}{960(-1+x)^3x^4}-\frac{3003\ln\left(-\frac{2\sqrt{1-x}+2-x}{x}\right)}{128}$
pseudoelliptic	$\frac{\frac{3003x^4\sqrt{1-x}\,(-1+x)^2\ln\left(\sqrt{1-x}-1\right)}{128}-\frac{3003x^4\sqrt{1-x}\,(-1+x)^2\ln\left(\sqrt{1-x}+1\right)}{128}+\frac{3003x^6}{64}-\frac{7007x^5}{64}+\frac{23023x^4}{320}-\frac{429x^3}{64}-\frac{143x^2}{96}-\frac{13x}{24}-}{(1-x)^{\frac{5}{2}}\left(\sqrt{1-x}-1\right)^4\left(\sqrt{1-x}+1\right)^4}$
meijerg	$\frac{-\frac{\sqrt{\pi}}{4x^4}-\frac{7\sqrt{\pi}}{6x^3}-\frac{63\sqrt{\pi}}{16x^2}-\frac{231\sqrt{\pi}}{16x}+\frac{3003\left(\frac{329177}{180180}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{128}+\frac{\sqrt{\pi}\left(-329177x^4+110880x^3+30240x^2+8960x+1920\right)}{7680x^4}-\frac{\sqrt{\pi}}{}}{\sqrt{\pi}}$
derivativedivides	$\frac{2}{5(1-x)^{\frac{5}{2}}}+\frac{10}{3(1-x)^{\frac{3}{2}}}+\frac{30}{\sqrt{1-x}}-\frac{1}{64\left(\sqrt{1-x}-1\right)^4}+\frac{17}{96\left(\sqrt{1-x}-1\right)^3}-\frac{159}{128\left(\sqrt{1-x}-1\right)^2}+\frac{1083}{128\left(\sqrt{1-x}-1\right)}+$
default	$\frac{2}{5(1-x)^{\frac{5}{2}}}+\frac{10}{3(1-x)^{\frac{3}{2}}}+\frac{30}{\sqrt{1-x}}-\frac{1}{64\left(\sqrt{1-x}-1\right)^4}+\frac{17}{96\left(\sqrt{1-x}-1\right)^3}-\frac{159}{128\left(\sqrt{1-x}-1\right)^2}+\frac{1083}{128\left(\sqrt{1-x}-1\right)}+$

input
```
int(1/(1-x)^(7/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/960*(45045*x^6-105105*x^5+69069*x^4-6435*x^3-1430*x^2-520*x-240)/x^4/(1-
x)^(1/2)/(-1+x)^2-3003/64*arctanh((1-x)^(1/2))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = -\frac{45045\left(x^7-3x^6+3x^5-x^4\right)\log\left(\sqrt{-x+1}+1\right)-45045\left(x^7-3x^6+3x^5-x^4\right)\log\left(\sqrt{-x+1}-1\right)+2\left(\right.}{1920\left(x^7-3x^6+3x^5-x^4\right.}$$

input
```
integrate(1/(1-x)^(7/2)/x^5,x, algorithm="fricas")
```

output
```
-1/1920*(45045*(x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) + 1) - 45045*(
x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) - 1) + 2*(45045*x^6 - 105105*x
^5 + 69069*x^4 - 6435*x^3 - 1430*x^2 - 520*x - 240)*sqrt(-x + 1))/(x^7 - 3
*x^6 + 3*x^5 - x^4)
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-x)**(7/2)/x**5,x)
```

output
```
Timed out
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{45045(x-1)^6+165165(x-1)^5+219219(x-1)^4+119691(x-1)^3+18304(x-1}{960\left((-x+1)^{\frac{13}{2}}-4(-x+1)^{\frac{11}{2}}+6(-x+1)^{\frac{9}{2}}-4(-x+1)^{\frac{7}{2}}+(-x+\right.} - \frac{3003}{128}\log\left(\sqrt{-x+1}+1\right)+\frac{3003}{128}\log\left(\sqrt{-x+1}-1\right)$$

input
```
integrate(1/(1-x)^(7/2)/x^5,x, algorithm="maxima")
```

output
```
1/960*(45045*(x - 1)^6 + 165165*(x - 1)^5 + 219219*(x - 1)^4 + 119691*(x -
 1)^3 + 18304*(x - 1)^2 - 1664*x + 2048)/((-x + 1)^(13/2) - 4*(-x + 1)^(11
/2) + 6*(-x + 1)^(9/2) - 4*(-x + 1)^(7/2) + (-x + 1)^(5/2)) - 3003/128*log
(sqrt(-x + 1) + 1) + 3003/128*log(sqrt(-x + 1) - 1)
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{2\left(225\,(x-1)^2-25\,x+28\right)}{15\,(x-1)^2\sqrt{-x+1}} - \frac{3249\,(x-1)^3\sqrt{-x+1}+10633\,(x-1)^2\sqrt{-x+1}-11767\,(-x+1)^{\frac{3}{2}}+4431\,\sqrt{-x+1}}{192\,x^4} - \frac{3003}{128}\,\log\left(\sqrt{-x+1}+1\right)+\frac{3003}{128}\,\log\left(\left|\sqrt{-x+1}-1\right|\right)$$

input
```
integrate(1/(1-x)^(7/2)/x^5,x, algorithm="giac")
```

output
```
2/15*(225*(x - 1)^2 - 25*x + 28)/((x - 1)^2*sqrt(-x + 1)) - 1/192*(3249*(x
 - 1)^3*sqrt(-x + 1) + 10633*(x - 1)^2*sqrt(-x + 1) - 11767*(-x + 1)^(3/2)
 + 4431*sqrt(-x + 1))/x^4 - 3003/128*log(sqrt(-x + 1) + 1) + 3003/128*log(
abs(sqrt(-x + 1) - 1))
```

3.216.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{\frac{286\,(x-1)^2}{15}-\frac{26\,x}{15}+\frac{39897\,(x-1)^3}{320}+\frac{73073\,(x-1)^4}{320}+\frac{11011\,(x-1)^5}{64}+\frac{3003\,(x-1)^6}{64}+\frac{32}{15}}{(1-x)^{5/2}-4\,(1-x)^{7/2}+6\,(1-x)^{9/2}-4\,(1-x)^{11/2}+(1-x)^{13/2}} - \frac{3003\,\mathrm{atanh}\left(\sqrt{1-x}\right)}{64}$$

input
```
int(1/(x^5*(1 - x)^(7/2)),x)
```

output
```
((286*(x - 1)^2)/15 - (26*x)/15 + (39897*(x - 1)^3)/320 + (73073*(x - 1)^4
)/320 + (11011*(x - 1)^5)/64 + (3003*(x - 1)^6)/64 + 32/15)/((1 - x)^(5/2)
 - 4*(1 - x)^(7/2) + 6*(1 - x)^(9/2) - 4*(1 - x)^(11/2) + (1 - x)^(13/2))
- (3003*atanh((1 - x)^(1/2)))/64
```

3.217 $\int \frac{1}{(-1+x)^{2/3}x^5} dx$

3.217.1 Optimal result

Integrand size = 11, antiderivative size = 104

$$\int \frac{1}{(-1+x)^{2/3}x^5} dx = \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{110\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right) - \frac{55\log(x)}{243}$$

output
```
1/4*(-1+x)^(1/3)/x^4+11/36*(-1+x)^(1/3)/x^3+11/27*(-1+x)^(1/3)/x^2+55/81*(
-1+x)^(1/3)/x+55/81*ln(1+(-1+x)^(1/3))-55/243*ln(x)-110/243*arctan(1/3*(1-
2*(-1+x)^(1/3))*3^(1/2))*3^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-1+x)^{2/3}x^5} dx = \frac{1}{972}\left(\frac{3\sqrt[3]{-1+x}(81+99x+132x^2+220x^3)}{x^4} - 440\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right) + 440\log\left(1+\sqrt[3]{-1+x}\right) - 220\log\left(1-\sqrt[3]{-1+x}+(-1+x)^{2/3}\right)\right)$$

input
```
Integrate[1/((-1 + x)^(2/3)*x^5),x]
```

output
```
((3*(-1 + x)^(1/3)*(81 + 99*x + 132*x^2 + 220*x^3))/x^4 - 440*Sqrt[3]*ArcT
an[(1 - 2*(-1 + x)^(1/3))/Sqrt[3]] + 440*Log[1 + (-1 + x)^(1/3)] - 220*Log
[1 - (-1 + x)^(1/3) + (-1 + x)^(2/3)])/972
```

3.217.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {52, 52, 52, 52, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)^{2/3}x^5}\,dx$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12}\int \frac{1}{(x-1)^{2/3}x^4}dx+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12}\left(\frac{8}{9}\int \frac{1}{(x-1)^{2/3}x^3}dx+\frac{\sqrt[3]{x-1}}{3x^3}\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\int \frac{1}{(x-1)^{2/3}x^2}dx+\frac{\sqrt[3]{x-1}}{2x^2}\right)+\frac{\sqrt[3]{x-1}}{3x^3}\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\int \frac{1}{(x-1)^{2/3}x}dx+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)+\frac{\sqrt[3]{x-1}}{3x^3}\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{70}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(\frac{3}{2}\int \frac{1}{\sqrt[3]{x-1}+1}d\sqrt[3]{x-1}+\frac{3}{2}\int \frac{1}{(x-1)^{2/3}-\sqrt[3]{x-1}+1}d\sqrt[3]{x-1}-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x}}{2x} \cdots\right.\right.$$

$$\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{16}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(\frac{3}{2}\int \frac{1}{(x-1)^{2/3}-\sqrt[3]{x-1}+1}d\sqrt[3]{x-1}+\frac{3}{2}\log\left(\sqrt[3]{x-1}+1\right)-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)\right.$$
$$\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{1083}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(-3\int \frac{1}{-(x-1)^{2/3}-3}d(2\sqrt[3]{x-1}-1)+\frac{3}{2}\log\left(\sqrt[3]{x-1}+1\right)-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)\right.$$
$$\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{217}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{x-1}-1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{x-1}+1\right)-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)+\frac{\sqrt[3]{x-1}}{3x^3}\right)$$
$$\frac{\sqrt[3]{x-1}}{4x^4}$$

input
```
Int[1/((-1 + x)^(2/3)*x^5),x]
```

output
```
(-1 + x)^(1/3)/(4*x^4) + (11*((-1 + x)^(1/3)/(3*x^3) + (8*((-1 + x)^(1/3)/
(2*x^2) + (5*((-1 + x)^(1/3)/x + (2*(Sqrt[3]*ArcTan[(-1 + 2*(-1 + x)^(1/3)
)/Sqrt[3]] + (3*Log[1 + (-1 + x)^(1/3)])/2 - Log[x]/2))/3))/6))/9))/12
```

3.217.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

3.217. $\int \frac{1}{(-1+x)^{2/3}x^5}\,dx$

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.217.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

method	result
meijerg	$\frac{(-\mathrm{signum}(-1+x))^{\frac{2}{3}}\left(-\frac{\Gamma(\frac{2}{3})}{4x^4}-\frac{2\Gamma(\frac{2}{3})}{9x^3}-\frac{5\Gamma(\frac{2}{3})}{18x^2}-\frac{40\Gamma(\frac{2}{3})}{81x}+\frac{110\left(\frac{877}{1320}+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma(\frac{2}{3})}{243}+\frac{308\Gamma(\frac{2}{3})x\,{}_3F_2(1,1}{729}\right)}{\Gamma(\frac{2}{3})\,\mathrm{signum}(-1+x)^{\frac{2}{3}}}$
risch	$\frac{220x^4-88x^3-33x^2-18x-81}{324x^4(-1+x)^{\frac{2}{3}}}+\frac{110(-\mathrm{signum}(-1+x))^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma(\frac{2}{3})+\frac{2\Gamma(\frac{2}{3})x\,{}_3F_2(1,1,\frac{5}{3};2,2;x)}{3}\right)}{243\Gamma(\frac{2}{3})\,\mathrm{signum}(-1+x)^{\frac{2}{3}}}$
derivativedivides	$-\frac{1}{324\left(1+(-1+x)^{\frac{1}{3}}\right)^4}-\frac{5}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^3}-\frac{20}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^2}-\frac{25}{81\left(1+(-1+x)^{\frac{1}{3}}\right)}+\frac{110\ln\left(1+(-1+x)\right)}{243}$
default	$-\frac{1}{324\left(1+(-1+x)^{\frac{1}{3}}\right)^4}-\frac{5}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^3}-\frac{20}{243\left(1+(-1+x)^{\frac{1}{3}}\right)^2}-\frac{25}{81\left(1+(-1+x)^{\frac{1}{3}}\right)}+\frac{110\ln\left(1+(-1+x)\right)}{243}$
trager	$\frac{(220x^3+132x^2+99x+81)(-1+x)^{\frac{1}{3}}}{324x^4}-110\ln\left(-\frac{1152\,\mathrm{RootOf}\left(2304\_Z^2+48\_Z+1\right)^2x-72\,\mathrm{RootOf}\left(2304\_Z^2+48\_Z+1\right)}{}\right)$

$$3.217. \quad \int \frac{1}{(-1+x)^{2/3}x^5}\,dx$$

input
```
int(1/(-1+x)^(2/3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/GAMMA(2/3)/signum(-1+x)^(2/3)*(-signum(-1+x))^(2/3)*(-1/4*GAMMA(2/3)/x^4
-2/9*GAMMA(2/3)/x^3-5/18*GAMMA(2/3)/x^2-40/81*GAMMA(2/3)/x+110/243*(877/13
20+1/6*Pi*3^(1/2)-3/2*ln(3)+ln(x)+I*Pi)*GAMMA(2/3)+308/729*GAMMA(2/3)*x*hy
pergeom([1,1,17/3],[2,6],x))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{440\,\sqrt{3}x^4\arctan\left(\frac{2}{3}\,\sqrt{3}(x-1)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-220\,x^4\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+}{972\,x^4}$$

input
```
integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="fricas")
```

output
```
1/972*(440*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x - 1)^(1/3) - 1/3*sqrt(3)) - 2
20*x^4*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 440*x^4*log((x - 1)^(1/3)
+ 1) + 3*(220*x^3 + 132*x^2 + 99*x + 81)*(x - 1)^(1/3))/x^4
```

3.217.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.16 (sec) , antiderivative size = 12993, normalized size of antiderivative = 124.93

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-1+x)**(2/3)/x**5,x)
```

output
```
-440*(x - 1)**(35/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)
/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(
I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 48114
0*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi
/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*
(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3
)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x -
 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma
(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 440*(x - 1)**(35/3)*
exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x -
 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gam
ma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**
(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4
/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(1
7/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3
) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*
exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 291
6*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 440*(x - 1)**(35/3)*exp(2*I*pi/
3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**
(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma...
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\,(x-1)^{\frac{1}{3}}-1\Big)\right) + \frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\left((x-1)^4+4\,(x-1)^3+6\,(x-1)^2+4\,x-3\right)} - \frac{55}{243}\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right) + \frac{110}{243}\log\left((x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="maxima")
```

output
```
110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x
- 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/
((x - 1)^4 + 4*(x - 1)^3 + 6*(x - 1)^2 + 4*x - 3) - 55/243*log((x - 1)^(2/
3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/3) + 1)
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right) + \frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\,x^4} - \frac{55}{243}\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right) + \frac{110}{243}\log\left((x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="giac")
```

output
```
110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x
- 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/
x^4 - 55/243*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^
(1/3) + 1)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110\ln\left(\frac{12100\,(x-1)^{1/3}}{6561}+\frac{12100}{6561}\right)}{243} + \frac{\frac{133\,(x-1)^{1/3}}{81}+\frac{341\,(x-1)^{4/3}}{108}+\frac{22\,(x-1)^{7/3}}{9}+\frac{55\,(x-1)^{10/3}}{81}}{4\,x+6\,(x-1)^2+4\,(x-1)^3+(x-1)^4-3} - \ln\left(\frac{55}{27}-\frac{110\,(x-1)^{1/3}}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)\left(\frac{55}{243}+\frac{\sqrt{3}\,55\mathrm{i}}{243}\right) + \ln\left(\frac{110\,(x-1)^{1/3}}{27}-\frac{55}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)\left(-\frac{55}{243}+\frac{\sqrt{3}\,55}{243}\right.$$

input
```
int(1/(x^5*(x - 1)^(2/3)),x)
```

output
```
(110*log((12100*(x - 1)^(1/3))/6561 + 12100/6561))/243 + ((133*(x - 1)^(1/
3))/81 + (341*(x - 1)^(4/3))/108 + (22*(x - 1)^(7/3))/9 + (55*(x - 1)^(10/
3))/81)/(4*x + 6*(x - 1)^2 + 4*(x - 1)^3 + (x - 1)^4 - 3) - log((3^(1/2)*5
5i)/27 - (110*(x - 1)^(1/3))/27 + 55/27)*((3^(1/2)*55i)/243 + 55/243) + lo
g((110*(x - 1)^(1/3))/27 + (3^(1/2)*55i)/27 - 55/27)*((3^(1/2)*55i)/243 -
55/243)
```

3.218 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

3.218.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{\frac{1-x}{1+x}}(1+x) - 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

output
```
-2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
(Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]
/(-1 + x)]))/Sqrt[1 - x]
```

3.218.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-x}{x+1}}dx$$

$$\Big\downarrow \text{2051}$$

$$-4\int \frac{1-x}{(x+1)\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{252}$$

$$-4\left(\frac{1}{2}\int \frac{1}{\frac{1-x}{x+1}+1}d\sqrt{\frac{1-x}{x+1}}-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
-4*(-1/2*Sqrt[(1 - x)/(1 + x)]/(1 + (1 - x)/(1 + x)) + ArcTan[Sqrt[(1 - x)
/(1 + x)]]/2)
```

3.218.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.218.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\left(\sqrt{-x^2+1}+\arcsin(x)\right)}{\sqrt{-(-1+x)(1+x)}}$
risch	$(1+x)\sqrt{-\frac{-1+x}{1+x}}-\frac{\arcsin(x)\sqrt{-\frac{-1+x}{1+x}}\,\sqrt{-(-1+x)(1+x)}}{-1+x}$
trager	$(1+x)\sqrt{-\frac{-1+x}{1+x}}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{-1+x}{1+x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)\right.$

input
```
int(((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x
))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int\sqrt{\frac{1-x}{1+x}}\,dx=(x+1)\sqrt{-\frac{x-1}{x+1}}-2\,\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

3.218.6 Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \int \sqrt{\frac{1-x}{x+1}}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2),x)
```

output
```
Integral(sqrt((1 - x)/(x + 1)), x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(
x + 1)))
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\pi\mathrm{sgn}(x+1) + \arcsin(x)\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int((-(x - 1)/(x + 1))^(1/2),x)
```

output
```
- 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)
/(x + 1) - 1)
```

3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$

3.219.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=\frac{1}{4}(a-5b)(b-x)\sqrt{\frac{-a+x}{b-x}}+\frac{1}{2}(b-x)^2\sqrt{\frac{-a+x}{b-x}}-\frac{1}{4}(a-b)(a+3b)\arctan\left(\sqrt{\frac{-a+x}{b-x}}\right)$$

output
```
-1/4*(a-b)*(a+3*b)*arctan(((-a+x)/(b-x))^(1/2))+1/4*(a-5*b)*(b-x)*((-a+x)/
(b-x))^(1/2)+1/2*(b-x)^2*((-a+x)/(b-x))^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=\frac{\sqrt{\frac{-a+x}{b-x}}\left((a-3b-2x)(b-x)\sqrt{-a+x}+(-a^2-2ab+3b^2)\sqrt{b-x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)\right)}{4\sqrt{-a+x}}$$

input
```
Integrate[x*Sqrt[(-a + x)/(b - x)],x]
```

output
```
(Sqrt[(-a + x)/(b - x)]*((a - 3*b - 2*x)*(b - x)*Sqrt[-a + x] + (-a^2 - 2*
a*b + 3*b^2)*Sqrt[b - x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]]))/(4*Sqrt[-a + x
])
```

3.219.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2052, 360, 25, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{x-a}{b-x}}dx$$

$$\Big\downarrow \text{ 2052}$$

$$-2(a-b)\int -\frac{\left(a-\frac{b(a-x)}{b-x}\right)(a-x)}{\left(1-\frac{a-x}{b-x}\right)^3(b-x)}d\sqrt{-\frac{a-x}{b-x}}$$

$$\Big\downarrow \text{ 360}$$

$$-2(a-b)\left(-\frac{1}{4}\int -\frac{a-b-\frac{4b(a-x)}{b-x}}{\left(1-\frac{a-x}{b-x}\right)^2}d\sqrt{-\frac{a-x}{b-x}}-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-2(a-b)\left(\frac{1}{4}\int \frac{a-b-\frac{4b(a-x)}{b-x}}{\left(1-\frac{a-x}{b-x}\right)^2}d\sqrt{-\frac{a-x}{b-x}}-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

$$\Big\downarrow \text{ 298}$$

$$-2(a-b)\left(\frac{1}{4}\left(\frac{1}{2}(a+3b)\int \frac{1}{1-\frac{a-x}{b-x}}d\sqrt{-\frac{a-x}{b-x}}+\frac{(a-5b)\sqrt{-\frac{a-x}{b-x}}}{2\left(1-\frac{a-x}{b-x}\right)}\right)-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-2(a-b)\left(\frac{1}{4}\left(\frac{1}{2}(a+3b)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)+\frac{(a-5b)\sqrt{-\frac{a-x}{b-x}}}{2\left(1-\frac{a-x}{b-x}\right)}\right)-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

input
```
Int[x*Sqrt[(-a + x)/(b - x)],x]
```

output
```
-2*(a - b)*(-1/4*((a - b)*Sqrt[-((a - x)/(b - x))])/(1 - (a - x)/(b - x))^
2 + (((a - 5*b)*Sqrt[-((a - x)/(b - x))])/(2*(1 - (a - x)/(b - x))) + ((a
+ 3*b)*ArcTan[Sqrt[-((a - x)/(b - x))]])/2)/4)
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

3.219. $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$

3.219.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(a-3b-2x)(b-x)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(b-x)(a-x)}}{4\sqrt{-(-b+x)(-a+x)}}+\frac{\left(\frac{1}{4}ab-\frac{3}{8}b^2+\frac{1}{8}a^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(b-x)(a-x)}}{a-x}$
default	$\frac{\sqrt{-\frac{a-x}{b-x}}\,(b-x)\left(\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)a^2+2b\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)a-3\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)b^2+2\sqrt{-ab+ax+b}\right.}{8\sqrt{-(b-x)(a-x)}}$

input
```
int(x*((-a+x)/(b-x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a-3*b-2*x)*(b-x)/(-(-b+x)*(-a+x))^(1/2)*(-(a-x)/(b-x))^(1/2)*(-(b-x)*
(a-x))^(1/2)+(1/4*a*b-3/8*b^2+1/8*a^2)*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*
x-x^2)^(1/2))*(-(a-x)/(b-x))^(1/2)*(-(b-x)*(a-x))^(1/2)/(a-x)
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=-\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)+\frac{1}{4}\left(ab-3\,b^2-(a-b)x+2\,x^2\right)\sqrt{-\frac{a-x}{b-x}}$$

input
```
integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) + 1/4*(a*b - 3*b
^2 - (a - b)*x + 2*x^2)*sqrt(-(a - x)/(b - x))
```

3.219.6 Sympy [F]

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=\int x\sqrt{\frac{-a+x}{b-x}}\,dx$$

input
```
integrate(x*((-a+x)/(b-x))**(1/2),x)
```

output
```
Integral(x*sqrt((-a + x)/(b - x)), x)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.41

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=-\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)-\frac{\left(a^2-6\,ab+5\,b^2\right)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}}-\left(a^2+2\,ab-3\,b^2\right)\sqrt{-\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1\right)}$$

input
```
integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) - 1/4*((a^2 - 6*
a*b + 5*b^2)*(-(a - x)/(b - x))^(3/2) - (a^2 + 2*a*b - 3*b^2)*sqrt(-(a - x
)/(b - x)))/((a - x)^2/(b - x)^2 - 2*(a - x)/(b - x) + 1)
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=\frac{1}{8}\left(a^2\mathrm{sgn}(-b+x)+2\,ab\mathrm{sgn}(-b+x)-3\,b^2\mathrm{sgn}(-b+x)\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}\left(a\mathrm{sgn}(-b+x)-3\,b\mathrm{sgn}(-b+x)-2\,x\mathrm{sgn}(-b+x)\right)$$

input
```
integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2*sgn(-b + x) + 2*a*b*sgn(-b + x) - 3*b^2*sgn(-b + x))*arcsin((a +
b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a*sgn(-b
 + x) - 3*b*sgn(-b + x) - 2*x*sgn(-b + x))
```

3.219.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\frac{\sqrt{-\frac{a-x}{b-x}}\left(\frac{a^2\,1\mathrm{i}}{4}+\frac{a\,b\,1\mathrm{i}}{2}-\frac{b^2\,3\mathrm{i}}{4}\right)\,1\mathrm{i}-\left(-\frac{a-x}{b-x}\right)^{3/2}\left(\frac{a^2\,1\mathrm{i}}{4}-\frac{a\,b\,3\mathrm{i}}{2}+\frac{b^2\,5\mathrm{i}}{4}\right)\,1\mathrm{i}}{\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1} - \frac{\mathrm{atan}\left(\sqrt{-\frac{a-x}{b-x}}\right)\,(a-b)\,(a+3\,b)}{4}$$

input
```
int(x*(-(a - x)/(b - x))^(1/2),x)
```

output
```
- ((-(a - x)/(b - x))^(1/2)*((a*b*1i)/2 + (a^2*1i)/4 - (b^2*3i)/4)*1i - (-
(a - x)/(b - x))^(3/2)*((a^2*1i)/4 - (a*b*3i)/2 + (b^2*5i)/4)*1i)/((a - x)
^2/(b - x)^2 - (2*(a - x))/(b - x) + 1) - (atan((-(a - x)/(b - x))^(1/2))*
(a - b)*(a + 3*b))/4
```

3.220 $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx$

3.220.1 Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx = \frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{-5+x}\sqrt{3+x}\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{5}\sqrt{3+x}}{\sqrt{-5+x}}\right)}{3\sqrt{5}}$$

output
```
1/6*arctan(1/4*(-5+x)^(1/2)*(3+x)^(1/2))+1/15*arctanh(5^(1/2)*(3+x)^(1/2)/
(-5+x)^(1/2))*5^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx = \frac{1}{15}\left(-5\arctan\left(\frac{1}{\sqrt{\frac{-5+x}{3+x}}}\right) + \sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\frac{-5+x}{3+x}}}\right)\right)$$

input
```
Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

output
```
(-5*ArcTan[1/Sqrt[(-5 + x)/(3 + x)]] + Sqrt[5]*ArcTanh[Sqrt[5]/Sqrt[(-5 +
x)/(3 + x)]])/15
```

3.220.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2003, 196, 25, 103, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-5}\sqrt{x+3}}{(x-1)\left(x^2-25\right)}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\sqrt{x+3}}{\sqrt{x-5}(x-1)(x+5)}dx$$

$$\Big\downarrow \text{196}$$

$$\frac{2}{3}\int -\frac{1}{(1-x)\sqrt{x-5}\sqrt{x+3}}dx+\frac{1}{3}\int \frac{1}{\sqrt{x-5}\sqrt{x+3}(x+5)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x-5}\sqrt{x+3}(x+5)}dx-\frac{2}{3}\int \frac{1}{(1-x)\sqrt{x-5}\sqrt{x+3}}dx$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x-5}\sqrt{x+3}(x+5)}dx+\frac{2}{3}\int \frac{1}{(x-5)(x+3)+16}d\left(\sqrt{x-5}\sqrt{x+3}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{2}{3}\int \frac{1}{2-\frac{10(x+3)}{x-5}}d\frac{\sqrt{x+3}}{\sqrt{x-5}}+\frac{2}{3}\int \frac{1}{(x-5)(x+3)+16}d\left(\sqrt{x-5}\sqrt{x+3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}\int \frac{1}{2-\frac{10(x+3)}{x-5}}d\frac{\sqrt{x+3}}{\sqrt{x-5}}+\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

input
```
Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

output
```
ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqr
t[-5 + x]]/(3*Sqrt[5])
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.220. $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx$

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

3.220.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{x-5}\,\sqrt{3+x}\,\left(\sqrt{5}\,\operatorname{arctanh}\left(\frac{(5+3x)\sqrt{5}}{5\sqrt{x^2-2x-15}}\right)-5\arctan\left(\frac{4}{\sqrt{x^2-2x-15}}\right)\right)}{30\sqrt{x^2-2x-15}}$	64

input
```
int((x-5)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x,method=_RETURNVERBOSE)
```

output
```
1/30*(x-5)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2)/(x^2-2*x
-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15)^(1/2)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \frac{1}{30}\sqrt{5}\log\left(\frac{\sqrt{x+3}\sqrt{x-5}(3\sqrt{5}+5)+\sqrt{5}(3\,x+5)+9\,x+15}{x+5}\right) + \frac{1}{3}\arctan\left(\frac{1}{4}\sqrt{x+3}\sqrt{x-5}-\frac{1}{4}\,x+\frac{1}{4}\right)$$

input
```
integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="fricas")
```

output
```
1/30*sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5)*(3*sqrt(5) + 5) + sqrt(5)*(3*x +
 5) + 9*x + 15)/(x + 5)) + 1/3*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*x
+ 1/4)
```

3.220.6 Sympy [F]

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=\int \frac{\sqrt{x+3}}{\sqrt{x-5}\,(x-1)\,(x+5)}\,dx$$

input
```
integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
```

output
```
Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

3.220.7 Maxima [F]

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=\int \frac{\sqrt{x+3}\sqrt{x-5}}{(x^2-25)(x-1)}\,dx$$

input
```
integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=-\frac{1}{30}\sqrt{5}\log\left(\frac{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2-4\sqrt{5}+12}{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2+4\sqrt{5}+12}\right)-\frac{1}{3}\arctan\left(\frac{1}{8}\left(\sqrt{x+3}-\sqrt{x-5}\right)^2\right)$$

input
```
integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="giac")
```

output
```
-1/30*sqrt(5)*log(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)/((sqrt(
x + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1/8*(sqrt(x + 3) -
 sqrt(x - 5))^2)
```

3.220.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x+3}\,\sqrt{x-5}-2\,\sqrt{2}\,\sqrt{x-5}}{x-2\,\sqrt{2}\,\sqrt{x+3}+3}\right)}{3} - \frac{\sqrt{5}\,\operatorname{atanh}\left(-\frac{\sqrt{5}\,\sqrt{x+3}\,\sqrt{x-5}-2\,\sqrt{2}\,\sqrt{5}\,\sqrt{x-5}}{5\,x-10\,\sqrt{2}\,\sqrt{x+3}+15}\right)}{15}$$

input
```
int(((x + 3)^(1/2)*(x - 5)^(1/2))/((x^2 - 25)*(x - 1)),x)
```

output
```
atan(((x + 3)^(1/2)*(x - 5)^(1/2) - 2*2^(1/2)*(x - 5)^(1/2))/(x - 2*2^(1/2
)*(x + 3)^(1/2) + 3))/3 - (5^(1/2)*atanh(-(5^(1/2)*(x + 3)^(1/2)*(x - 5)^(
1/2) - 2*2^(1/2)*5^(1/2)*(x - 5)^(1/2))/(5*x - 10*2^(1/2)*(x + 3)^(1/2) +
15)))/15
```

3.221 $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

3.221.1 Optimal result

Integrand size = 52, antiderivative size = 304

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = \frac{5}{16}(1-x)^{3/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{7(1-x^2)^{5/4}}{24\sqrt{1-x}}+\frac{x(1-x^2)^{5/4}}{6\sqrt{1-x}}+\frac{1}{6}\sqrt{1+x}(1-x^2)^{5/4}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{1+x}}\right)}{8\sqrt{2}}$$

output
```
5/16*(1-x)^(3/4)*(1+x)^(1/4)-1/16*(1-x)^(1/4)*(1+x)^(3/4)+1/24*(1-x)^(5/4)
*(1+x)^(3/4)+3/16*arctan(-1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+3/16*
arctan(1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+1/16*ln(1-(1-x)^(1/4)*2^
(1/2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)-1/16*ln(1+(1-x)^(1/4)*2
^(1/2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)+7/24*(-x^2+1)^(5/4)/(1
-x)^(1/2)+1/6*x*(-x^2+1)^(5/4)/(1-x)^(1/2)+1/6*(-x^2+1)^(5/4)*(1+x)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 11.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.52

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$
$$= -\frac{1}{48}\sqrt{1+x}\sqrt[4]{1-x^2}\left(-7+2x+8x^2-\frac{\sqrt{1-x^2}(29+22x+8x^2)}{1+x}\right)$$
$$+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{1+x}\sqrt[4]{1-x^2}}{1+x-\sqrt{1-x^2}}\right)-2\text{arctanh}\left(\frac{1+x+\sqrt{1-x^2}}{\sqrt{2}\sqrt{1+x}\sqrt[4]{1-x^2}}\right)}{8\sqrt{2}}$$

input
```
Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sq
rt[1 + x])),x]
```

output
```
-1/48*(Sqrt[1 + x]*(1 - x^2)^(1/4)*(-7 + 2*x + 8*x^2 - (Sqrt[1 - x^2]*(29
+ 22*x + 8*x^2))/(1 + x))) + (3*ArcTan[(Sqrt[2]*Sqrt[1 + x]*(1 - x^2)^(1/4
))/(1 + x - Sqrt[1 - x^2])] - 2*ArcTanh[(1 + x + Sqrt[1 - x^2])/(Sqrt[2]*S
qrt[1 + x]*(1 - x^2)^(1/4))])/(8*Sqrt[2])
```

3.221.3 Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.56, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.327$, Rules used = {2003, 2528, 90, 60, 60, 73, 770, 755, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x+1}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}\,dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{x^2(x+1)^{3/4}}{\sqrt[4]{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}dx$$
$$\Big\downarrow \text{2528}$$
$$-\frac{1}{2}\int \sqrt[4]{1-x}x(x+1)^{3/4}dx-\frac{1}{2}\int \frac{x(x+1)^{5/4}}{\sqrt[4]{1-x}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}-\frac{1}{6}\int \sqrt[4]{1-x}(x+1)^{3/4}dx\right)+ \frac{1}{2}\left(\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}-\frac{1}{2}\int \frac{(x+1)^{5/4}}{\sqrt[4]{1-x}}dx\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\int \frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}dx\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)+ \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\int \frac{\sqrt[4]{x+1}}{\sqrt[4]{1-x}}dx\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-x}(x+1)^{3/4}}dx-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)+ \frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{(1-x)^{3/4}\sqrt[4]{x+1}}dx+\sqrt[4]{1-x}(x+1)^{3/4}\right)\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int \frac{\sqrt{1-x}}{(x+1)^{3/4}}d\sqrt[4]{1-x}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)+ \frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\int \frac{1}{\sqrt[4]{x+1}}d\sqrt[4]{1-x}\right)\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)$$

$$\Big\downarrow \text{770}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\int \frac{1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)+ \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int \frac{\sqrt{1-x}}{(x+1)^{3/4}}d\sqrt[4]{1-x}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int \frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int \frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int \frac{\sqrt{1-x}}{(x+1)^{3/4}}d\sqrt[4]{1-x}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$$\Big\downarrow \text{854}$$

3.221. $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int\frac{\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$\downarrow$ 826

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}-\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)-(1-x)^{3/4}\sqrt[4]{x\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right.\right.\right.\right.$$

$\downarrow$ 1476

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{x}}}\right.\right.\right.\right.\right.$$

$\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}\right)-\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(\frac{\sqrt{2}}{}\right.}{\sqrt{2}}\right.\right.\right.\right.\right.$$

$\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{}}{2}\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\right.}{\sqrt{2}}\right.\right.\right.\right.\right.$$

3.221. $\int\frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

$$\Big\downarrow \; 1479$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{\sqrt{1-x}+\frac{\sqrt{2}\cdots}{}}}{}\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}\right)\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{x+1}}\right.}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{x+\cdots}}}}{2\sqrt{\cdots}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 27$$

3.221. $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{}{\sqrt{1-x}+}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\right.}{}\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{x+1}}\right.}{\sqrt{2}}\right.\right.\right.\right.\right.\right.$$

input
```
Int[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 +
 x])),x]
```

output
```
(((1 - x)^(3/4)*(1 + x)^(9/4))/3 + (((1 - x)^(3/4)*(1 + x)^(5/4))/2 - (5*(
-((1 - x)^(3/4)*(1 + x)^(1/4)) - 2*((-(ArcTan[1 - (Sqrt[2]*(1 - x)^(1/4))/
(1 + x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)
]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4
)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - x] + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/
4)]/(2*Sqrt[2]))/2)))/4)/2)/2 + (((1 - x)^(5/4)*(1 + x)^(7/4))/3 + (((1 -
x)^(5/4)*(1 + x)^(3/4))/2 - (3*((1 - x)^(1/4)*(1 + x)^(3/4) - 2*((-(ArcTan
[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]
*(1 - x)^(1/4))/(1 + x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - x] - (S
qrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - x] + (Sqrt
[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/(2*Sqrt[2]))/2)))/4)/6)/2
```

3.221. $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.221. $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.221. $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.221.4 Maple [F]

$$\int \frac{x^2(-x^2+1)^{\frac{1}{4}}\sqrt{1+x}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}dx$$

input
```
int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x
)
```

output
```
int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x
)
```

3.221.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.21

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$
$$= -\frac{1}{48}\left(8x^2+2x-7\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}+\frac{1}{48}\left(8x^2+22x+29\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}$$
$$+\left(\frac{1}{64}i+\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i+1)\,x+i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right)$$
$$-\left(\frac{1}{64}i-\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i-1)\,x-i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right)$$
$$+\left(\frac{1}{64}i-\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i-1)\,x+i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right)$$
$$-\left(\frac{1}{64}i+\frac{1}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i+1)\,x-i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}}{x+1}\right)$$
$$+\left(\frac{5}{64}i+\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i+1)\,x-i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right)$$
$$-\left(\frac{5}{64}i-\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i-1)\,x+i-1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right)$$
$$+\left(\frac{5}{64}i-\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}((i-1)\,x-i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right)$$
$$-\left(\frac{5}{64}i+\frac{5}{64}\right)\sqrt{2}\log\left(\frac{\sqrt{2}(-(i+1)\,x+i+1)+2\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}}{x-1}\right)$$

input
```
integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1
/2)),x, algorithm="fricas")
```

3.221. $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

output
```
-1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + 1/48*(8*x^2 + 22*x
+ 29)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + (1/64*I + 1/64)*sqrt(2)*log((sqrt(2)
*((I + 1)*x + I + 1) + 2*(-x^2 + 1)^(1/4)*sqrt(x + 1))/(x + 1)) - (1/64*I
- 1/64)*sqrt(2)*log((sqrt(2)*(-(I - 1)*x - I + 1) + 2*(-x^2 + 1)^(1/4)*sqr
t(x + 1))/(x + 1)) + (1/64*I - 1/64)*sqrt(2)*log((sqrt(2)*((I - 1)*x + I -
 1) + 2*(-x^2 + 1)^(1/4)*sqrt(x + 1))/(x + 1)) - (1/64*I + 1/64)*sqrt(2)*l
og((sqrt(2)*(-(I + 1)*x - I - 1) + 2*(-x^2 + 1)^(1/4)*sqrt(x + 1))/(x + 1)
) + (5/64*I + 5/64)*sqrt(2)*log((sqrt(2)*((I + 1)*x - I - 1) + 2*(-x^2 + 1
)^(1/4)*sqrt(-x + 1))/(x - 1)) - (5/64*I - 5/64)*sqrt(2)*log((sqrt(2)*(-(I
 - 1)*x + I - 1) + 2*(-x^2 + 1)^(1/4)*sqrt(-x + 1))/(x - 1)) + (5/64*I - 5
/64)*sqrt(2)*log((sqrt(2)*((I - 1)*x - I + 1) + 2*(-x^2 + 1)^(1/4)*sqrt(-x
 + 1))/(x - 1)) - (5/64*I + 5/64)*sqrt(2)*log((sqrt(2)*(-(I + 1)*x + I + 1
) + 2*(-x^2 + 1)^(1/4)*sqrt(-x + 1))/(x - 1))
```

3.221.6 Sympy [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = \int \frac{x^2\sqrt[4]{-(x-1)(x+1)}\sqrt{x+1}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}\,dx$$

input
```
integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-(1
+x)**(1/2)),x)
```

output
```
Integral(x**2*(-(x - 1)*(x + 1))**(1/4)*sqrt(x + 1)/(sqrt(1 - x)*(sqrt(1 -
 x) - sqrt(x + 1))), x)
```

3.221.7 Maxima [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = \int -\frac{(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

input
```
integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1
/2)),x, algorithm="maxima")
```

output
```
-integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - s
qrt(-x + 1))), x)
```

3.221.8 Giac [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=\int -\frac{(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

input
```
integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1
/2)),x, algorithm="giac")
```

output
```
integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - s
qrt(-x + 1))), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=-\int \frac{x^2\left(1-x^2\right)^{1/4}\sqrt{x+1}}{\left(\sqrt{x+1}-\sqrt{1-x}\right)\sqrt{1-x}}\,dx$$

input
```
int(-(x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*
(1 - x)^(1/2)),x)
```

output
```
-int((x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*
(1 - x)^(1/2)), x)
```

3.222 $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$

3.222.1 Optimal result

Integrand size = 56, antiderivative size = 292

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx =$$

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}\sqrt[3]{1+x}+\frac{1}{4}\sqrt{1-x}x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+\frac{1}{12}\sqrt[3]{1-x}(1+x)^{2/3}(1+3x)+\frac{1}{12}\sqrt[6]{1-x}$$

output
```
-1/12*(1-3*x)*(1-x)^(2/3)*(1+x)^(1/3)-1/4*(1-x)*(3+x)+1/12*(1-x)^(1/3)*(1+
x)^(2/3)*(1+3*x)+1/12*(1-x)^(1/6)*(1+x)^(5/6)*(2+3*x)-1/12*(1-x)^(5/6)*(1+
x)^(1/6)*(10+3*x)+1/6*arctan((1+x)^(1/6)/(1-x)^(1/6))-5/6*arctan(((1-x)^(1
/3)-(1+x)^(1/3))/(1-x)^(1/6)/(1+x)^(1/6))-4/9*arctan(1/3*((1-x)^(1/3)-2*(1
+x)^(1/3))/(1-x)^(1/3)*3^(1/2))*3^(1/2)+1/18*arctanh((1-x)^(1/6)*(1+x)^(1/
6)*3^(1/2)/((1-x)^(1/3)+(1+x)^(1/3)))*3^(1/2)+1/4*x*(1-x)^(1/2)*(1+x)^(1/2
)
```

3.222.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 36.73 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx =$$

$$-\frac{1}{12}\sqrt[3]{1+x}\left((1-3x)(1-x)^{2/3}-\frac{3\sqrt[3]{1-x}x(2+x)}{\sqrt[3]{1-x^2}}-3\sqrt[3]{1-x}x\sqrt[6]{1-x^2}-(1+3x)\sqrt[3]{1-x^2}-\frac{(2+3x)\sqrt{1-x^2}}{\sqrt[3]{1-x}}\right.$$

input
```
Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) +
(1 - x)^(2/3)*Sqrt[1 + x]),x]
```

output
```
-1/12*((1 + x)^(1/3)*((1 - 3*x)*(1 - x)^(2/3) - (3*(1 - x)^(1/3)*x*(2 + x)
)/(1 - x^2)^(1/3) - 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6) - (1 + 3*x)*(1 - x^2
)^(1/3) - ((2 + 3*x)*Sqrt[1 - x^2])/(1 - x)^(1/3) + ((10 + 3*x)*(1 - x^2)^
(5/6))/(1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x)/2])) +
 (9*ArcSin[x] + 14*ArcTan[(1 + x)^(1/3)/(1 - x^2)^(1/6)] + 7*(1 + I*Sqrt[3
])*ArcTan[((1 - I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 7*(1 - I*
Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 8*S
qrt[3]*ArcTan[(1 - (2*(1 - x^2)^(1/3))/(1 + x)^(2/3))/Sqrt[3]] - (15*2^(5/
6)*Sqrt[1 - x^2]*Hypergeometric2F1[1/6, 1/6, 7/6, (1 - x)/2])/((1 - x)^(1/
3)*Sqrt[1 + x]) - 8*Log[(1 + x)^(2/3) + (1 - x^2)^(1/3)] + 4*Log[(1 + x)^(
1/3) + x*(1 + x)^(1/3) - (1 + x)^(2/3)*(1 - x^2)^(1/3) + (1 - x^2)^(2/3)])
/36
```

3.222.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}x(x+1)^{2/3}}{(1-x)^{2/3}\sqrt{x+1}-(1-x)^{5/6}\sqrt[3]{x+1}}\,dx$$

$$\Big\downarrow \; 7292$$

$$\int \frac{x\sqrt[3]{x+1}}{\sqrt[6]{1-x}\left(\sqrt[6]{x+1}-\sqrt[6]{1-x}\right)}dx$$

$$\Big\downarrow \text{7296}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

3.222. $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}dx$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

3.222. $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}dx$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

input
```
Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x
)^(2/3)*Sqrt[1 + x]),x]
```

output
```
$Aborted
```

3.222. $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$

3.222.3.1 Defintions of rubi rules used

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

3.222.4 Maple [F]

$$\int \frac{x(1+x)^{\frac{2}{3}} \sqrt{1-x}}{-(1-x)^{\frac{5}{6}} (1+x)^{\frac{1}{3}} + (1-x)^{\frac{2}{3}} \sqrt{1+x}} dx$$

input
```
int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^
(1/2)),x)
```

output
```
int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^
(1/2)),x)
```

3.222.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x} + (1-x)^{2/3}\sqrt{1+x}} dx = \text{Too large to display}$$

input
```
integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*
(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*x^2 + 1/12*(3*x + 2)*(x + 1)^(5/6)*(-x + 1)^(1/6) + 1/12*(3*x + 1)*(x
+ 1)^(2/3)*(-x + 1)^(1/3) + 1/4*sqrt(x + 1)*x*sqrt(-x + 1) + 1/12*(3*x - 1
)*(x + 1)^(1/3)*(-x + 1)^(2/3) - 1/12*(3*x + 10)*(x + 1)^(1/6)*(-x + 1)^(5
/6) - 1/72*sqrt(2)*sqrt(25*I*sqrt(3) + 25)*log((sqrt(2)*(x + 1)*sqrt(25*I*
sqrt(3) + 25) + 10*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) + 1/72*sqrt(2)*s
qrt(25*I*sqrt(3) + 25)*log(-(sqrt(2)*(x + 1)*sqrt(25*I*sqrt(3) + 25) - 10*
(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) - 1/72*sqrt(2)*sqrt(-25*I*sqrt(3) +
 25)*log((sqrt(2)*(x + 1)*sqrt(-25*I*sqrt(3) + 25) + 10*(x + 1)^(5/6)*(-x
+ 1)^(1/6))/(x + 1)) + 1/72*sqrt(2)*sqrt(-25*I*sqrt(3) + 25)*log(-(sqrt(2)
*(x + 1)*sqrt(-25*I*sqrt(3) + 25) - 10*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x +
1)) - 1/72*sqrt(2)*sqrt(49*I*sqrt(3) + 49)*log((sqrt(2)*(x - 1)*sqrt(49*I*
sqrt(3) + 49) + 14*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) + 1/72*sqrt(2)*s
qrt(49*I*sqrt(3) + 49)*log(-(sqrt(2)*(x - 1)*sqrt(49*I*sqrt(3) + 49) - 14*
(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) - 1/72*sqrt(2)*sqrt(-49*I*sqrt(3) +
 49)*log((sqrt(2)*(x - 1)*sqrt(-49*I*sqrt(3) + 49) + 14*(x + 1)^(1/6)*(-x
+ 1)^(5/6))/(x - 1)) + 1/72*sqrt(2)*sqrt(-49*I*sqrt(3) + 49)*log(-(sqrt(2)
*(x - 1)*sqrt(-49*I*sqrt(3) + 49) - 14*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x -
1)) - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(
-x + 1)^(1/3))/(x + 1)) - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt
(3)*(x + 1)^(1/3)*(-x + 1)^(2/3))/(x - 1)) + 1/2*x - 5/18*arctan((-x + ...
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \text{Timed out}$$

input
```
integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(
2/3)*(1+x)**(1/2)),x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx=\int \frac{(x+1)^{\frac{2}{3}}x\sqrt{-x+1}}{\sqrt{x+1}(-x+1)^{\frac{2}{3}}-(x+1)^{\frac{1}{3}}(-x+1)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*
(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x +
1)^(1/3)*(-x + 1)^(5/6)), x)
```

3.222.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx=\text{Timed out}$$

input
```
integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*
(1+x)^(1/2)),x, algorithm="giac")
```

output
```
Timed out
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx=\int \frac{x\,\sqrt{1-x}\,(x+1)^{2/3}}{(1-x)^{2/3}\,\sqrt{x+1}-(1-x)^{5/6}\,(x+1)^{1/3}}\,dx$$

input
```
int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)
^(5/6)*(x + 1)^(1/3)),x)
```

output
```
int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)
^(5/6)*(x + 1)^(1/3)), x)
```

3.223 $\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx$

3.223.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

output
```
-3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

3.223.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

input
```
Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

output
```
(-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

3.223.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7270, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)^4(x+1)^2}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{(x-1)^{4/3}(x+1)^{2/3}\int \frac{1}{(x-1)^{4/3}(x+1)^{2/3}}dx}{\sqrt[3]{(1-x)^4(x+1)^2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

input
```
Int[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

output
```
(-3*(-1 + x)*(1 + x))/(2*((1 - x)^4*(1 + x)^2)^(1/3))
```

3.223.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

3.223.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{3(-1+x)(1+x)}{2\left((-1+x)^{4}(1+x)^{2}\right)^{\frac{1}{3}}}$	22
risch	$-\frac{3(-1+x)(1+x)}{2\left((-1+x)^{4}(1+x)^{2}\right)^{\frac{1}{3}}}$	22
trager	$-\frac{3\left(x^{6}-2x^{5}-x^{4}+4x^{3}-x^{2}-2x+1\right)^{\frac{2}{3}}}{2(-1+x)^{3}(1+x)}$	43

input
```
int(1/((-1+x)^4*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3\,(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1)^{\frac{2}{3}}}{2\,(x^4-2\,x^3+2\,x-1)}$$

input
```
integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x^3 + 2*x
- 1)
```

3.223.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)^4\,(x+1)^2}}\,dx$$

input
```
integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
```

output
```
Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

3.223.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

3.223.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3\left((x-1)^4\,(x+1)^2\right)^{2/3}}{2\,(x-1)^3\,(x+1)}$$

input
```
int(1/((x - 1)^4*(x + 1)^2)^(1/3),x)
```

output
```
-(3*((x - 1)^4*(x + 1)^2)^(2/3))/(2*(x - 1)^3*(x + 1))
```

3.224 $\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx$

3.224.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

output
```
4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

3.224.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

input
```
Integrate[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

output
```
(4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

3.224.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7270, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$$

$$\Big\downarrow \text{ 7270}$$

$$\frac{(x-1)^{3/4}(x+2)^{5/4}\int \frac{1}{(x-1)^{3/4}(x+2)^{5/4}}dx}{\sqrt[4]{-(1-x)^3(x+2)^5}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

input
```
Int[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

output
```
(4*(-1 + x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))
```

3.224.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.224.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{4(-1+x)(2+x)}{3\left((-1+x)^{3}(2+x)^{5}\right)^{\frac{1}{4}}}$	22
risch	$\frac{4(-1+x)(2+x)}{3\left((-1+x)^{3}(2+x)^{5}\right)^{\frac{1}{4}}}$	22
trager	$\frac{4\left(x^{8}+7x^{7}+13x^{6}-11x^{5}-50x^{4}-8x^{3}+64x^{2}+16x-32\right)^{\frac{3}{4}}}{3(-1+x)^{2}(2+x)^{4}}$	53

input
```
int(1/((-1+x)^3*(2+x)^5)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4\left(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32\right)^{\frac{3}{4}}}{3\left(x^6+6x^5+9x^4-8x^3-24x^2+16\right)}$$

input
```
integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="fricas")
```

output
```
4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16*x - 32)^
(3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)
```

3.224.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\sqrt[4]{(x-1)^3\,(x+2)^5}}\,dx$$

input
```
integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)
```

output
```
Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

3.224.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="maxima")
```

output
```
integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

3.224.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="giac")
```

output
```
integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4\left((x-1)^3(x+2)^5\right)^{3/4}}{3(x-1)^2(x+2)^4}$$

input
```
int(1/((x - 1)^3*(x + 2)^5)^(1/4),x)
```

output
```
(4*((x - 1)^3*(x + 2)^5)^(3/4))/(3*(x - 1)^2*(x + 2)^4)
```

3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$

3.225.1 Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{8\sqrt[3]{(-1+x)^7(1+x)^2}}+\frac{9(-1+x)^2(1+x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

output
```
-3/8*(-1+x)*(1+x)/((-1+x)^7*(1+x)^2)^(1/3)+9/16*(-1+x)^2*(1+x)/((-1+x)^7*(
1+x)^2)^(1/3)
```

3.225.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3(-1+x)(1+x)(-5+3x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

input
```
Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

output
```
(3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

3.225.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7270, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)^7(x+1)^2}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{(x-1)^{7/3}(x+1)^{2/3}\int \frac{1}{(x-1)^{7/3}(x+1)^{2/3}}dx}{\sqrt[3]{-(1-x)^7(x+1)^2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{(x-1)^{7/3}(x+1)^{2/3}\left(-\frac{3}{8}\int \frac{1}{(x-1)^{4/3}(x+1)^{2/3}}dx-\frac{3\sqrt[3]{x+1}}{8(x-1)^{4/3}}\right)}{\sqrt[3]{-(1-x)^7(x+1)^2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(x-1)^{7/3}(x+1)^{2/3}\left(\frac{9\sqrt[3]{x+1}}{16\sqrt[3]{x-1}}-\frac{3\sqrt[3]{x+1}}{8(x-1)^{4/3}}\right)}{\sqrt[3]{-(1-x)^7(x+1)^2}}$$

input
```
Int[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

output
```
((-1 + x)^(7/3)*(1 + x)^(2/3)*((-3*(1 + x)^(1/3))/(8*(-1 + x)^(4/3)) + (9*
(1 + x)^(1/3))/(16*(-1 + x)^(1/3))))/(-((1 - x)^7*(1 + x)^2))^(1/3)
```

3.225.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.225.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{3(-1+x)(1+x)(3x-5)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$	27
risch	$\frac{3(-1+x)\left(3x^{2}-2x-5\right)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$	29
trager	$\frac{3(3x-5)\left(x^{9}-5x^{8}+8x^{7}-14x^{5}+14x^{4}-8x^{2}+5x-1\right)^{\frac{2}{3}}}{16(-1+x)^{6}(1+x)}$	53

input
```
int(1/((-1+x)^7*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/16*(-1+x)*(1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3\left(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1\right)^{\frac{2}{3}}(3x-5)}{16\left(x^7-5x^6+9x^5-5x^4-5x^3+9x^2-5x+1\right)}$$

input
```
integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="fricas")
```

output
```
3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2/3)*(3*x
- 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x + 1)
```

3.225.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\sqrt[3]{\left(x-1\right)^{7}\left(x+1\right)^{2}}}\,dx$$

input
```
integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
```

output
```
Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

3.225.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

3.225.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3\,(3\,x-5)\,\left((x-1)^7\,(x+1)^2\right)^{2/3}}{16\,(x-1)^6\,(x+1)}$$

input
```
int(1/((x - 1)^7*(x + 1)^2)^(1/3),x)
```

output
```
(3*(3*x - 5)*((x - 1)^7*(x + 1)^2)^(2/3))/(16*(x - 1)^6*(x + 1))
```

3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$

3.226.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \sqrt{3}\arctan\left(\frac{1+\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right)$$

output
```
-1/2*ln(1+x)-3/2*ln(1+(1-x)/((-1+x)^2*(1+x))^(1/3))+arctan(1/3*(1+2*(-1+x)
/((-1+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \frac{(-1+x)^{2/3}\sqrt[3]{1+x}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x}}{2\sqrt[3]{-1+x}+\sqrt[3]{1+x}}\right) - 2\log\left(\sqrt[3]{-1+x}-\sqrt[3]{1+x}\right) + \log\left((-1+x\right.\right.}{2\sqrt[3]{(-1+x)^2(1+x)}}$$

input
```
Integrate[((-1 + x)^2*(1 + x))^(-1/3),x]
```

output
```
((-1 + x)^(2/3)*(1 + x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(
2*(-1 + x)^(1/3) + (1 + x)^(1/3))] - 2*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)]
 + Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(2
*((-1 + x)^2*(1 + x))^(1/3))
```

3.226.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2477, 474, 473, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)^2(x+1)}}dx$$

$$\Big\downarrow \text{2477}$$

$$\frac{\sqrt[3]{x-1}\sqrt[3]{x^2-1}\int \frac{1}{\sqrt[3]{x-1}\sqrt[3]{x^2-1}}dx}{\sqrt[3]{(1-x)^2(x+1)}}$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{1-x}\sqrt[3]{x^2-1}\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x^2-1}}dx}{\sqrt[3]{(1-x)^2(x+1)}}$$

$$\Big\downarrow \text{473}$$

$$\frac{(x^2-1)\int \frac{1}{\sqrt[3]{-x-1}(1-x)^{2/3}}dx}{(-x-1)^{2/3}\sqrt[3]{1-x}\sqrt[3]{(1-x)^2(x+1)}}$$

$$\Big\downarrow \text{71}$$

$$\frac{(x^2-1)\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{-x-1}}{\sqrt{3}\sqrt[3]{1-x}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\frac{\sqrt[3]{-x-1}}{\sqrt[3]{1-x}}-1\right)+\frac{1}{2}\log(1-x)\right)}{(-x-1)^{2/3}\sqrt[3]{1-x}\sqrt[3]{(1-x)^2(x+1)}}$$

input
```
Int[((-1 + x)^2*(1 + x))^(-1/3),x]
```

output
```
((-1 + x^2)*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 - x)^(1/3))/(Sqrt[3]*(1 - x
)^(1/3))] + (3*Log[-1 + (-1 - x)^(1/3)/(1 - x)^(1/3)])/2 + Log[1 - x]/2))/
((-1 - x)^(2/3)*(1 - x)^(1/3)*((1 - x)^2*(1 + x))^(1/3))
```

3.226.3.1 Defintions of rubi rules used

rule 71
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 2477
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c + d*x)^p*(b + d*
x^2)^p)   Int[(c + d*x)^p*(b + d*x^2)^p, x], x] /; EqQ[b*c - a*d, 0]] /; Fr
eeQ[p, x] && PolyQ[Px, x, 3] &&  !IntegerQ[p]
```

3.226.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.52

method	result
trager	$-\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-x^2-x+1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-x^2-x+1\right)^{\frac{1}{3}} x}{}\right.$

input
```
int(1/((-1+x)^2*(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-ln((4*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(2/3)-3*R
ootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)*x-4*RootOf(_Z^2-_Z+1)^2*x-4*RootOf(_Z
^2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)+3*(x^3-x^2-x+1)^(1/3)
*x+2*RootOf(_Z^2-_Z+1)*x+x^2-3*(x^3-x^2-x+1)^(1/3)+2*RootOf(_Z^2-_Z+1)-1)/
(-1+x))+RootOf(_Z^2-_Z+1)*ln(-(2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+
1)*(x^3-x^2-x+1)^(2/3)-2*RootOf(_Z^2-_Z+1)^2*x-5*RootOf(_Z^2-_Z+1)*x^2-3*(
x^3-x^2-x+1)^(2/3)+3*(x^3-x^2-x+1)^(1/3)*x+6*RootOf(_Z^2-_Z+1)*x+2*x^2-3*(
x^3-x^2-x+1)^(1/3)-RootOf(_Z^2-_Z+1)-4*x+2)/(-1+x))
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\begin{aligned}\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx &= -\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right)\\ &\quad+\frac{1}{2}\log\left(\frac{x^2+(x^3-x^2-x+1)^{\frac{1}{3}}(x-1)-2\,x+(x^3-x^2-x+1)^{\frac{2}{3}}+1}{x^2-2\,x+1}\right)\\ &\quad-\log\left(-\frac{x-(x^3-x^2-x+1)^{\frac{1}{3}}-1}{x-1}\right)\end{aligned}$$

input
```
integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3)
)/(x - 1)) + 1/2*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3
 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - log(-(x - (x^3 - x^2 - x + 1
)^(1/3) - 1)/(x - 1))
```

3.226.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)^2\,(x+1)}}\,dx$$

input
```
integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

output
```
Integral(((x - 1)**2*(x + 1))**(-1/3), x)
```

3.226.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

3.226.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="giac")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x-1)^2\,(x+1)\right)^{1/3}}\,dx$$

input
```
int(1/((x - 1)^2*(x + 1))^(1/3),x)
```

output
```
int(1/((x - 1)^2*(x + 1))^(1/3), x)
```

3.227 $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

3.227.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = -\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^3}}+\frac{2\sqrt{-2+x}(1+x)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{(-2+x)(1+x)^3}} - \frac{\sqrt{2}\sqrt{-2+x}(1+x)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}}$$

output
```
-4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+2*(1+x)^(3/2)*arcsinh(1/3*(-2+x)^
(1/2)*3^(1/2))*(-2+x)^(1/2)/((-2+x)*(1+x)^3)^(1/2)-(1+x)^(3/2)*arctan(2^(1
/2)*(1+x)^(1/2)/(-2+x)^(1/2))*2^(1/2)*(-2+x)^(1/2)/((-2+x)*(1+x)^3)^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = -\frac{(1+x)\left(-8+4x-3\sqrt{2}\sqrt{-2+x}\sqrt{1+x}\arctan\left(\frac{\sqrt{\frac{-2+x}{1+x}}}{\sqrt{2}}\right)-6\sqrt{-2+x}\sqrt{1+x}\operatorname{arctanh}\left(\sqrt{\frac{-2+x}{1+x}}\right)\right)}{3\sqrt{(-2+x)(1+x)^3}}$$

input
```
Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

output
```
-1/3*((1 + x)*(-8 + 4*x - 3*Sqrt[2]*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTan[Sqrt[(
-2 + x)/(1 + x)]/Sqrt[2]] - 6*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTanh[Sqrt[(-2 +
x)/(1 + x)]]))/Sqrt[(-2 + x)*(1 + x)^3]
```

3.227.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2027, 7270, 2117, 27, 140, 64, 104, 217, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\frac{1}{x}}{\sqrt{(x-2)(x+1)^3}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2+1}{x\sqrt{(x-2)(x+1)^3}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\int \frac{x^2+1}{\sqrt{x-2}x(x+1)^{3/2}}dx}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{2117}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(-\frac{2}{3}\int -\frac{3\sqrt{x+1}}{2\sqrt{x-2}x}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(\int \frac{\sqrt{x+1}}{\sqrt{x-2}x}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{140}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(\int \frac{1}{\sqrt{x-2}\sqrt{x+1}}dx+\int \frac{1}{\sqrt{x-2}x\sqrt{x+1}}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{64}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\int \frac{1}{\sqrt{x+1}}d\sqrt{x-2}+\int \frac{1}{\sqrt{x-2x\sqrt{x+1}}}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\int \frac{1}{\sqrt{x+1}}d\sqrt{x-2}+2\int \frac{1}{-\frac{2(x+1)}{x-2}-1}d\frac{\sqrt{x+1}}{\sqrt{x-2}}-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \; 217$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\int \frac{1}{\sqrt{x+1}}d\sqrt{x-2}-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\text{arcsinh}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

input
```
Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

output
```
(Sqrt[-2 + x]*(1 + x)^(3/2)*((-4*Sqrt[-2 + x])/(3*Sqrt[1 + x]) + 2*ArcSinh
[Sqrt[-2 + x]/Sqrt[3]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[1 + x])/Sqrt[-2 + x]
]))/Sqrt[-((2 - x)*(1 + x)^3)]
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

3.227. $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.227. $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.227.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^{3}}}+\frac{\left(\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)+\frac{\sqrt{2}\,\arctan\left(\frac{(-4-x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)}{2}\right)(1+x)\sqrt{(1+x)(-2+x)}}{\sqrt{(-2+x)(1+x)^{3}}}$
default	$-\frac{\left(3\sqrt{2}\,\arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)x+3\sqrt{2}\,\arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)-6\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)x-6\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)+8\sqrt{x^{2}-x-2}\right)\sqrt{(1}}{6\sqrt{(-2+x)(1+x)^{3}}}$
trager	$-\frac{4\sqrt{x^{4}+x^{3}-3x^{2}-5x-2}}{3(1+x)^{2}}-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)x^{2}+5\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\operatorname{RootOf}\left(\_Z^{2}+2\right)-4\sqrt{x^{4}+x^{3}-3}}{x(1+x)}\right.}{2}$

input
```
int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+(ln(x-1/2+(x^2-x-2)^(1/2))+1/2*2^
(1/2)*arctan(1/4*(-4-x)*2^(1/2)/(x^2-x-2)^(1/2)))/((-2+x)*(1+x)^3)^(1/2)*(
1+x)*((1+x)*(-2+x))^(1/2)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$$
$$=\frac{3\sqrt{2}(x^2+2x+1)\arctan\left(-\frac{\sqrt{2}(x^2+x)-\sqrt{2}\sqrt{x^4+x^3-3x^2-5x-2}}{2(x+1)}\right)-4x^2-3(x^2+2x+1)\log\left(-\frac{2x^2+x-2\sqrt{x^4+x}}{x-}\right.}{3(x^2+2x+1)}$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*(x^2 + x) - sqrt(2)*sq
rt(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x + 1)*log(
-(2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x + 1)) - 8*x - 4*
sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*x + 1)
```

3.227.6 Sympy [F]

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \int \frac{x^2+1}{x\sqrt{(x-2)\,(x+1)^3}}\,dx$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)
```

output
```
Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)
```

3.227.7 Maxima [F]

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3(x-2)}}\,dx$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-x-2}\right)\right)}{\operatorname{sgn}(x+1)} - \frac{\log\left(\left|-2x+2\sqrt{x^2-x-2}+1\right|\right)}{\operatorname{sgn}(x+1)} - \frac{4}{\left(x-\sqrt{x^2-x-2}+1\right)\operatorname{sgn}(x+1)}$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - x - 2)))/sgn(x + 1) - log(abs(
-2*x + 2*sqrt(x^2 - x - 2) + 1))/sgn(x + 1) - 4/((x - sqrt(x^2 - x - 2) +
1)*sgn(x + 1))
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3\,(x-2)}}\,dx$$

input
```
int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2),x)
```

output
```
int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2), x)
```

3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

3.228.1 Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = -\frac{\sqrt[3]{(-1+x)^2(1+x)}}{x} - \frac{\arctan\left(\frac{1-\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right)}{\sqrt{3}} - \sqrt{3}\arctan\left(\frac{1+\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) + \frac{\log(x)}{6} - \frac{2}{3}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right) - \frac{1}{2}\log\left(1+\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right)$$

output
```
-((-1+x)^2*(1+x))^(1/3)/x+1/6*ln(x)-2/3*ln(1+x)-3/2*ln(1+(1-x)/((-1+x)^2*(
1+x))^(1/3))-1/2*ln(1+(-1+x)/((-1+x)^2*(1+x))^(1/3))-1/3*arctan(1/3*(1-2*(
-1+x)/((-1+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)-arctan(1/3*(1+2*(-1+x)/((-1
+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx =$$

$$-\frac{(-1+x)^{4/3}(1+x)^{2/3}\left(18(-1+x)^{2/3}\sqrt[3]{1+x}-6\sqrt{3}x\arctan\left(\frac{1-\frac{2}{\sqrt[3]{\frac{-1+x}{1+x}}}}{\sqrt{3}}\right)-18\sqrt{3}x\arctan\left(\frac{1+\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

output
```
-1/18*((-1 + x)^(4/3)*(1 + x)^(2/3)*(18*(-1 + x)^(2/3)*(1 + x)^(1/3) - 6*S
qrt[3]*x*ArcTan[(1 - 2/((-1 + x)/(1 + x))^(1/3))/Sqrt[3]] - 18*Sqrt[3]*x*A
rcTan[(1 + 2/((-1 + x)/(1 + x))^(1/3))/Sqrt[3]] - 10*x*Log[2/(-1 + x)] - 3
*x*Log[1 + ((-1 + x)/(1 + x))^(-2/3) - ((-1 + x)/(1 + x))^(-1/3)] + 28*x*L
og[-1 + ((-1 + x)/(1 + x))^(-1/3)] + 6*x*Log[1 + ((-1 + x)/(1 + x))^(-1/3)
] + x*Log[1 + ((-1 + x)/(1 + x))^(-2/3) + ((-1 + x)/(1 + x))^(-1/3)]))/(x*
((-1 + x)^2*(1 + x))^(2/3))
```

3.228.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2490, 2483, 27, 108, 27, 175, 72, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{(x-1)^2(x+1)}}{x^2}dx$$

$$\Big\downarrow \; 2490$$

$$\int \frac{\sqrt[3]{\left(x-\frac{1}{3}\right)^3-\frac{4}{3}\left(x-\frac{1}{3}\right)+\frac{16}{27}}}{x^2}d\left(x-\frac{1}{3}\right)$$

$$\Big\downarrow \; 2483$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\int\frac{4\ 2^{2/3}\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}{\left(3\left(x-\frac{1}{3}\right)+1\right)^2}d\left(x-\frac{1}{3}\right)}{4\ 2^{2/3}\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\int\frac{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}{\left(3\left(x-\frac{1}{3}\right)+1\right)^2}d\left(x-\frac{1}{3}\right)}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{108}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(\frac{1}{3}\int-\frac{3\left(3\left(x-\frac{1}{3}\right)+2\right)}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+1\right)\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\frac{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{\cdots}}{3\left(3\left(x-\cdots\right.\right.}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\int\frac{3\left(x-\frac{1}{3}\right)+2}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+1\right)\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\frac{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\cdots}}{3\left(3\left(x-\frac{1}{3}\cdots\right.\right.}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{175}$$

3.228. $\int\frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\int\frac{1}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\int\frac{1}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)\right.}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$\downarrow$ 72

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\int\frac{1}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+1\right)\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{2}}{\sqrt{3}\sqrt[3]{3}}\right)}{\sqrt{}}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{}}$$

$\downarrow$ 102

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}}{\sqrt{3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}}{\sqrt{3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}-\right.}{}$$

input
```
Int[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

3.228. $\int\frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

output
```
(3*(16 - 36*(-1/3 + x) + 27*(-1/3 + x)^3)^(1/3)*(-1/3*((2 - 3*(-1/3 + x))^
(2/3)*(4 + 3*(-1/3 + x))^(1/3))/(1 + 3*(-1/3 + x)) - ArcTan[1/Sqrt[3] - (2
*(2 - 3*(-1/3 + x))^(1/3))/(Sqrt[3]*(4 + 3*(-1/3 + x))^(1/3))]/Sqrt[3] - A
rcTan[1/Sqrt[3] + (2*(2 - 3*(-1/3 + x))^(1/3))/(Sqrt[3]*(4 + 3*(-1/3 + x))
^(1/3))]/(3*Sqrt[3]) - Log[1 + (2 - 3*(-1/3 + x))^(1/3)/(4 + 3*(-1/3 + x))
^(1/3)]/2 - Log[(2 - 3*(-1/3 + x))^(1/3) - (4 + 3*(-1/3 + x))^(1/3)]/6 + L
og[1 + 3*(-1/3 + x)]/18 - Log[4 + 3*(-1/3 + x)]/6))/((2 - 3*(-1/3 + x))^(2
/3)*(4 + 3*(-1/3 + x))^(1/3))
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

3.228. $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

3.228.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.17 (sec) , antiderivative size = 1374, normalized size of antiderivative = 9.16

method	result	size
trager	Expression too large to display	1374
risch	Expression too large to display	1846

input
```
int(((-1+x)^2*(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(x^3-x^2-x+1)^(1/3)/x-1/3*ln(-(17542263127-67143384127*x+473641104429*x^5
-473641104429*x^4-315760736286*x^3+365361857286*x^2-148740584544*RootOf(4*
_Z^2-2*_Z+1)*x^3-20825761472*RootOf(4*_Z^2-2*_Z+1)*x^2+161302980208*RootOf
(4*_Z^2-2*_Z+1)*x+11734778880*RootOf(4*_Z^2-2*_Z+1)^2*x^5-11734778880*Root
Of(4*_Z^2-2*_Z+1)^2*x^4+223110876816*RootOf(4*_Z^2-2*_Z+1)*x^5-22311087681
6*RootOf(4*_Z^2-2*_Z+1)*x^4+434621440*RootOf(4*_Z^2-2*_Z+1)^2+8263365808*R
ootOf(4*_Z^2-2*_Z+1)+941414819418*(x^3-x^2-x+1)^(2/3)*RootOf(4*_Z^2-2*_Z+1
)*x^3-1170393085674*(x^3-x^2-x+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*x^4-31380493
9806*(x^3-x^2-x+1)^(2/3)*RootOf(4*_Z^2-2*_Z+1)*x^2+780262057116*(x^3-x^2-x
+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*x^3-104601646602*RootOf(4*_Z^2-2*_Z+1)*(x^
3-x^2-x+1)^(2/3)*x+520174704744*(x^3-x^2-x+1)^(1/3)*RootOf(4*_Z^2-2*_Z+1)*
x^2-86695784124*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(1/3)*x-7823185920*Roo
tOf(4*_Z^2-2*_Z+1)^2*x^3-70191362560*RootOf(4*_Z^2-2*_Z+1)^2*x^2+775799270
40*RootOf(4*_Z^2-2*_Z+1)^2*x+4240338264*(x^3-x^2-x+1)^(1/3)-21673946031*(x
^3-x^2-x+1)^(2/3)+8480676528*(x^3-x^2-x+1)^(1/3)*x-585196542837*(x^3-x^2-x
+1)^(2/3)*x^3+195065514279*(x^3-x^2-x+1)^(2/3)*x^2+65021838093*x*(x^3-x^2-
x+1)^(2/3)+114489133128*(x^3-x^2-x+1)^(1/3)*x^4-76326088752*(x^3-x^2-x+1)^
(1/3)*x^3+34867215534*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^(2/3)-5088405916
8*(x^3-x^2-x+1)^(1/3)*x^2-43347892062*RootOf(4*_Z^2-2*_Z+1)*(x^3-x^2-x+1)^
(1/3))/x/(-1+x))+2/3*RootOf(4*_Z^2-2*_Z+1)*ln((18071341751+27297150257*...
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(126) = 252$.

Time = 0.24 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$$
$$= \frac{6\sqrt{3}x\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right) - 2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right) + 3\,x\log\left(\frac{x^2+(x^3 \cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/6*(6*sqrt(3)*x*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x +
1)^(1/3))/(x - 1)) - 2*sqrt(3)*x*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*
(x^3 - x^2 - x + 1)^(1/3))/(x - 1)) + 3*x*log((x^2 + (x^3 - x^2 - x + 1)^(
1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + x*l
og((x^2 - (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2
/3) + 1)/(x^2 - 2*x + 1)) - 2*x*log((x + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x
 - 1)) - 6*x*log(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*(x^3 -
x^2 - x + 1)^(1/3))/x
```

3.228.6 Sympy [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\sqrt[3]{(x-1)^2\,(x+1)}}{x^2}\,dx$$

input
```
integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
```

output
```
Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)
```

3.228.7 Maxima [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

3.228.8 Giac [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x-1)^2\,(x+1)\right)^{1/3}}{x^2}\,dx$$

input
```
int(((x - 1)^2*(x + 1))^(1/3)/x^2,x)
```

output
```
int(((x - 1)^2*(x + 1))^(1/3)/x^2, x)
```

3.229 $\int \frac{1}{(-3-2x+x^2)^{5/2}} dx$

3.229.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(-3-2x+x^2)^{5/2}} dx = \frac{1-x}{12(-3-2x+x^2)^{3/2}} - \frac{1-x}{24\sqrt{-3-2x+x^2}}$$

output
```
1/12*(1-x)/(x^2-2*x-3)^(3/2)+1/24*(-1+x)/(x^2-2*x-3)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-3-2x+x^2)^{5/2}} dx = \frac{\sqrt{-3-2x+x^2}(5-3x-3x^2+x^3)}{24(-3+x)^2(1+x)^2}$$

input
```
Integrate[(-3 - 2*x + x^2)^(-5/2),x]
```

output
```
(Sqrt[-3 - 2*x + x^2]*(5 - 3*x - 3*x^2 + x^3))/(24*(-3 + x)^2*(1 + x)^2)
```

3.229.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x-3)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{1-x}{12\,(x^2-2x-3)^{3/2}}-\frac{1}{6}\int \frac{1}{(x^2-2x-3)^{3/2}}dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{1-x}{12\,(x^2-2x-3)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

input
```
Int[(-3 - 2*x + x^2)^(-5/2),x]
```

output
```
(1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x + x^2])
```

3.229.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p + 
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.229.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

method	result	size
trager	$\frac{x^{3}-3x^{2}-3x+5}{24(x^{2}-2x-3)^{\frac{3}{2}}}$	26
risch	$\frac{x^{3}-3x^{2}-3x+5}{24(x^{2}-2x-3)^{\frac{3}{2}}}$	26
gosper	$\frac{(1+x)(-3+x)(x^{3}-3x^{2}-3x+5)}{24(x^{2}-2x-3)^{\frac{5}{2}}}$	32
default	$-\frac{-2+2x}{24(x^{2}-2x-3)^{\frac{3}{2}}}+\frac{-2+2x}{48\sqrt{x^{2}-2x-3}}$	36

input
```
int(1/(x^2-2*x-3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^3-3*x^2-3*x+5)/(x^2-2*x-3)^(3/2)
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(31) = 62.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{x^4-4\,x^3-2\,x^2+(x^3-3\,x^2-3\,x+5)\sqrt{x^2-2\,x-3}+12\,x+9}{24\,(x^4-4\,x^3-2\,x^2+12\,x+9)}$$

input
```
integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(x^4 - 4*x^3 - 2*x^2 + (x^3 - 3*x^2 - 3*x + 5)*sqrt(x^2 - 2*x - 3) +
12*x + 9)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)
```

3.229.6 Sympy [F]

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-2x-3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x**2-2*x-3)**(5/2),x)
```

output
```
Integral((x**2 - 2*x - 3)**(-5/2), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{x}{24\sqrt{x^2-2x-3}} - \frac{1}{24\sqrt{x^2-2x-3}} - \frac{x}{12\,(x^2-2x-3)^{\frac{3}{2}}} + \frac{1}{12\,(x^2-2x-3)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="maxima")
```

output
```
1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x^2 - 2*x
- 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{((x-3)x-3)x+5}{24\,(x^2-2x-3)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="giac")
```

output
```
1/24*(((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = -\frac{(4\,x-4)\,(-8\,x^2+16\,x+40)}{768\,(x^2-2\,x-3)^{3/2}}$$

input
```
int(1/(x^2 - 2*x - 3)^(5/2),x)
```

output
```
-((4*x - 4)*(16*x - 8*x^2 + 40))/(768*(x^2 - 2*x - 3)^(3/2))
```

3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx$

3.230.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=\frac{(3-x)\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}$$

output
```
(3-x)*arctanh(1/2*(1+x)^(1/2))*(1+x)^(1/2)/(x^3-5*x^2+3*x+9)^(1/2)
```

3.230.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=-\frac{(-3+x)\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{(-3+x)^2(1+x)}}$$

input
```
Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

output
```
-(((-3 + x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[(-3 + x)^2*(1 + x)])
```

3.230.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2480, 27, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^3-5x^2+3x+9}}dx$$

$$\Big\downarrow \text{2480}$$

$$-\frac{128(3-x)\sqrt{x+1}\int -\frac{1}{128(3-x)\sqrt{x+1}}dx}{\sqrt{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-x)\sqrt{x+1}\int \frac{1}{(3-x)\sqrt{x+1}}dx}{\sqrt{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2(3-x)\sqrt{x+1}\int \frac{1}{3-x}d\sqrt{x+1}}{\sqrt{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3-x)\sqrt{x+1}\text{arctanh}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

input
```
Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

output
```
((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]
```

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

3.230.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
trager	$-\frac{\ln\left(\frac{x^2+4\sqrt{x^3-5x^2+3x+9}+2x-15}{(-3+x)^2}\right)}{2}$	35
default	$\frac{(-3+x)\sqrt{1+x}\,(\ln(\sqrt{1+x}-2)-\ln(\sqrt{1+x}+2))}{2\sqrt{x^3-5x^2+3x+9}}$	45

input
```
int(1/(x^3-5*x^2+3*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((x^2+4*(x^3-5*x^2+3*x+9)^(1/2)+2*x-15)/(-3+x)^2)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = -\frac{1}{2}\log\left(\frac{2\,x+\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right) + \frac{1}{2}\log\left(-\frac{2\,x-\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right)$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + 1/2*log(-(2*x
- sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))
```

3.230.6 Sympy [F]

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5x^2+3x+9}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
```

output
```
Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)
```

3.230.7 Maxima [F]

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5\,x^2+3\,x+9}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9), x)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = -\frac{\log\left(\sqrt{x+1}+2\right)}{2\,\mathrm{sgn}\,(x-3)} + \frac{\log\left(\left|\sqrt{x+1}-2\right|\right)}{2\,\mathrm{sgn}\,(x-3)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(sqrt(x + 1) + 2)/sgn(x - 3) + 1/2*log(abs(sqrt(x + 1) - 2))/sgn(x
 - 3)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5\,x^2+3\,x+9}}\,dx$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2),x)
```

output
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2), x)
```

3.231 $\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx$

3.231.1 Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \frac{(3-x)(1+x)}{8(9+3x-5x^2+x^3)^{3/2}} + \frac{5(3-x)^2(1+x)}{64(9+3x-5x^2+x^3)^{3/2}} - \frac{15(3-x)^3(1+x)}{256(9+3x-5x^2+x^3)^{3/2}} + \frac{15(3-x)^3(1+x)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512(9+3x-5x^2+x^3)^{3/2}}$$

output
```
1/8*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+5/64*(3-x)^2*(1+x)/(x^3-5*x^2+3*x+
9)^(3/2)-15/256*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+15/512*(3-x)^3*(1+x)
^(3/2)*arctanh(1/2*(1+x)^(1/2))/(x^3-5*x^2+3*x+9)^(3/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \frac{86-140x+30x^2-15(-3+x)^2\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512(-3+x)\sqrt{(-3+x)^2(1+x)}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

output
```
(86 - 140*x + 30*x^2 - 15*(-3 + x)^2*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/(
512*(-3 + x)*Sqrt[(-3 + x)^2*(1 + x)])
```

3.231.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2480, 27, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{3/2}}dx$$

$$\Big\downarrow \text{2480}$$

$$-\frac{2097152(3-x)^3(x+1)^{3/2}\int -\frac{1}{2097152(3-x)^3(x+1)^{3/2}}dx}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-x)^3(x+1)^{3/2}\int \frac{1}{(3-x)^3(x+1)^{3/2}}dx}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\int \frac{1}{(3-x)^2(x+1)^{3/2}}dx+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\int \frac{1}{(3-x)(x+1)^{3/2}}dx+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\left(\frac{1}{4}\int \frac{1}{(3-x)\sqrt{x+1}}dx-\frac{1}{2\sqrt{x+1}}\right)+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\left(\frac{1}{2}\int \frac{1}{3-x}d\sqrt{x+1}-\frac{1}{2\sqrt{x+1}}\right)+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\left(\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x+1}}{2}\right)-\frac{1}{2\sqrt{x+1}}\right)+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

output
```
((3 - x)^3*(1 + x)^(3/2)*(1/(8*(3 - x)^2*Sqrt[1 + x]) + (5*(1/(4*(3 - x)*S
qrt[1 + x]) + (3*(-1/2*1/Sqrt[1 + x] + ArcTanh[Sqrt[1 + x]/2]/4))/8))/16))
/(9 + 3*x - 5*x^2 + x^3)^(3/2)
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

3.231.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

method	result
risch	$\frac{15x^2-70x+43}{256(-3+x)\sqrt{(1+x)(-3+x)^2}}+\frac{\left(\frac{15\ln(\sqrt{1+x}-2)}{1024}-\frac{15\ln(\sqrt{1+x}+2)}{1024}\right)\sqrt{1+x}\,(-3+x)}{\sqrt{(1+x)(-3+x)^2}}$
trager	$\frac{(15x^2-70x+43)\sqrt{x^3-5x^2+3x+9}}{256(-3+x)^3(1+x)}+\frac{15\ln\left(\frac{-x^2+4\sqrt{x^3-5x^2+3x+9}-2x+15}{(-3+x)^2}\right)}{1024}$
default	$\frac{(-3+x)^3(1+x)\left(15(1+x)^{\frac{5}{2}}\ln(\sqrt{1+x}-2)-15(1+x)^{\frac{5}{2}}\ln(\sqrt{1+x}+2)-120(1+x)^{\frac{3}{2}}\ln(\sqrt{1+x}-2)+120(1+x)^{\frac{3}{2}}\ln(\sqrt{1+x}+2)+240\ln(\sqrt{\right.}}{1024(\sqrt{1+x}+2)^2(\sqrt{1+x}-2)^2(x^3-5x^2+3x+9)^{\frac{3}{2}}}$

input
```
int(1/(x^3-5*x^2+3*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/256*(15*x^2-70*x+43)/(-3+x)/((1+x)*(-3+x)^2)^(1/2)+(15/1024*ln((1+x)^(1/
2)-2)-15/1024*ln((1+x)^(1/2)+2))/((1+x)*(-3+x)^2)^(1/2)*(1+x)^(1/2)*(-3+x)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx =$$
$$-\frac{15\left(x^4-8x^3+18x^2-27\right)\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)-15\left(x^4-8x^3+18x^2-27\right)\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+}}{x-3}\right.}{1024\left(x^4-8x^3+18x^2-27\right)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="fricas")
```

output
```
-1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x^2 + 3*x
+ 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(-(2*x - sqrt(x^3 -
 5*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 - 5*x^2 + 3*x + 9)*(15*x^2 -
70*x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27)
```

3.231.6 Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)
```

3.231.7 Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2), x)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = -\frac{15\log\left(\sqrt{x+1}+2\right)}{1024\,\mathrm{sgn}(x-3)} + \frac{15\log\left(\left|\sqrt{x+1}-2\right|\right)}{1024\,\mathrm{sgn}(x-3)} + \frac{1}{32\sqrt{x+1}\mathrm{sgn}(x-3)} + \frac{7\,(x+1)^{\frac{3}{2}}-36\sqrt{x+1}}{256\,(x-3)^2\mathrm{sgn}(x-3)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="giac")
```

output
```
-15/1024*log(sqrt(x + 1) + 2)/sgn(x - 3) + 15/1024*log(abs(sqrt(x + 1) - 2
))/sgn(x - 3) + 1/32/(sqrt(x + 1)*sgn(x - 3)) + 1/256*(7*(x + 1)^(3/2) - 3
6*sqrt(x + 1))/((x - 3)^2*sgn(x - 3))
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{3/2}}\,dx$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2),x)
```

output
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2), x)
```

3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$

3.232.1 Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \sqrt{3}\arctan\left(\frac{1+\frac{2(-3+x)}{\sqrt[3]{9+3x-5x^2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-3+x}{\sqrt[3]{9+3x-5x^2+x^3}}\right)$$

output
```
-1/2*ln(1+x)-3/2*ln(1+(3-x)/(x^3-5*x^2+3*x+9)^(1/3))+arctan(1/3*(1+2*(-3+x
)/(x^3-5*x^2+3*x+9)^(1/3))*3^(1/2))*3^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \frac{(-3+x)^{2/3}\sqrt[3]{1+x}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x}}{2\sqrt[3]{-3+x}+\sqrt[3]{1+x}}\right)-2\log\left(\sqrt[3]{-3+x}-\sqrt[3]{1+x}\right)+\log\left((-3+x\right.\right.}{2\sqrt[3]{(-3+x)^2(1+x)}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

output
```
((-3 + x)^(2/3)*(1 + x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(
2*(-3 + x)^(1/3) + (1 + x)^(1/3))] - 2*Log[(-3 + x)^(1/3) - (1 + x)^(1/3)]
 + Log[(-3 + x)^(2/3) + (-3 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(2
*((-3 + x)^2*(1 + x))^(1/3))
```

3.232.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2480, 27, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}dx$$

$$\Big\downarrow \text{ 2480}$$

$$\frac{16\ 2^{2/3}(x-3)^{2/3}\sqrt[3]{x+1}\int \frac{1}{16\ 2^{2/3}(x-3)^{2/3}\sqrt[3]{x+1}}dx}{\sqrt[3]{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(x-3)^{2/3}\sqrt[3]{x+1}\int \frac{1}{(x-3)^{2/3}\sqrt[3]{x+1}}dx}{\sqrt[3]{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{ 71}$$

$$\frac{(x-3)^{2/3}\sqrt[3]{x+1}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-3}}+\frac{1}{\sqrt{3}}\right)-\frac{1}{2}\log(x-3)-\frac{3}{2}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-3}}-1\right)\right)}{\sqrt[3]{x^3-5x^2+3x+9}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

output
```
((-3 + x)^(2/3)*(1 + x)^(1/3)*(-(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/
3))/(Sqrt[3]*(-3 + x)^(1/3))]) - Log[-3 + x]/2 - (3*Log[-1 + (1 + x)^(1/3)
/(-3 + x)^(1/3)])/2))/(9 + 3*x - 5*x^2 + x^3)^(1/3)
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 71
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

3.232.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.95

method	result
trager	$-\ln\left(-\frac{-16\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2 x^2+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}+45\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3\right.}{}\right.$

input
```
int(1/(x^3-5*x^2+3*x+9)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-ln(-(-16*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+
9)^(2/3)+45*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)*x+48*RootOf(_Z^2-3
*_Z+9)^2*x-24*RootOf(_Z^2-3*_Z+9)*x^2-216*(x^3-5*x^2+3*x+9)^(2/3)-135*Root
Of(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)+81*(x^3-5*x^2+3*x+9)^(1/3)*x+156*R
ootOf(_Z^2-3*_Z+9)*x-9*x^2-243*(x^3-5*x^2+3*x+9)^(1/3)-252*RootOf(_Z^2-3*_
Z+9)+90*x-189)/(-3+x))+1/3*RootOf(_Z^2-3*_Z+9)*ln((4*RootOf(_Z^2-3*_Z+9)^2
*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3)-72*RootOf(_Z^2-3*_Z+9)
*(x^3-5*x^2+3*x+9)^(1/3)*x-12*RootOf(_Z^2-3*_Z+9)^2*x+33*RootOf(_Z^2-3*_Z+
9)*x^2+135*(x^3-5*x^2+3*x+9)^(2/3)+216*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+
9)^(1/3)+81*(x^3-5*x^2+3*x+9)^(1/3)*x-78*RootOf(_Z^2-3*_Z+9)*x-180*x^2-243
*(x^3-5*x^2+3*x+9)^(1/3)-63*RootOf(_Z^2-3*_Z+9)+792*x-756)/(-3+x))
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$$
$$= -\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-3)+2\sqrt{3}(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}{3\,(x-3)}\right)$$
$$+\frac{1}{2}\log\left(\frac{x^2+(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}(x-3)-6\,x+(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}+9}{x^2-6\,x+9}\right)$$
$$-\log\left(-\frac{x-(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}-3}{x-3}\right)$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(1/3*(sqrt(3)*(x - 3) + 2*sqrt(3)*(x^3 - 5*x^2 + 3*x + 9)^(
1/3))/(x - 3)) + 1/2*log((x^2 + (x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x - 3) - 6*
x + (x^3 - 5*x^2 + 3*x + 9)^(2/3) + 9)/(x^2 - 6*x + 9)) - log(-(x - (x^3 -
 5*x^2 + 3*x + 9)^(1/3) - 3)/(x - 3))
```

3.232.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)
```

3.232.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

3.232.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx=\int \frac{1}{\left(x^3-5\,x^2+3\,x+9\right)^{1/3}}\,dx$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3),x)
```

output
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3), x)
```

3.233 $\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} \, dx$

3.233.1 Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} \, dx = \frac{3(3-x)(1+x)}{4(9+3x-5x^2+x^3)^{2/3}}$$

output
```
3/4*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(2/3)
```

3.233.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} \, dx = -\frac{3(-3+x)(1+x)}{4((-3+x)^2(1+x))^{2/3}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

output
```
(-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))
```

3.233.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2480, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{2/3}}\,dx$$

$$\Big\downarrow \; 2480$$

$$\frac{512\sqrt[3]{2}(x-3)^{4/3}(x+1)^{2/3}\int \frac{1}{512\sqrt[3]{2}(x-3)^{4/3}(x+1)^{2/3}}dx}{(x^3-5x^2+3x+9)^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{(x-3)^{4/3}(x+1)^{2/3}\int \frac{1}{(x-3)^{4/3}(x+1)^{2/3}}dx}{(x^3-5x^2+3x+9)^{2/3}}$$

$$\Big\downarrow \; 48$$

$$-\frac{3(x-3)(x+1)}{4\,(x^3-5x^2+3x+9)^{2/3}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

output
```
(-3*(-3 + x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

3.233.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{3(-3+x)(1+x)}{4\left((1+x)(-3+x)^{2}\right)^{\frac{2}{3}}}$	20
trager	$-\frac{3\left(x^{3}-5x^{2}+3x+9\right)^{\frac{1}{3}}}{4(-3+x)}$	23
gosper	$-\frac{3(1+x)(-3+x)}{4\left(x^{3}-5x^{2}+3x+9\right)^{\frac{2}{3}}}$	24

input
```
int(1/(x^3-5*x^2+3*x+9)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4/((1+x)*(-3+x)^2)^(2/3)*(-3+x)*(1+x)
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = -\frac{3\left(x^3-5\,x^2+3\,x+9\right)^{\frac{1}{3}}}{4\,(x-3)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="fricas")
```

output
```
-3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)
```

3.233.6 Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)
```

3.233.7 Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

3.233.8 Giac [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

3.233.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx=-\frac{3\,{\left(x^3-5\,x^2+3\,x+9\right)}^{1/3}}{4\,\left(x-3\right)}$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(2/3),x)
```

output
```
-(3*(3*x - 5*x^2 + x^3 + 9)^(1/3))/(4*(x - 3))
```

3.234 $\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx$

3.234.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3(3-x)(1+x)}{20\,(9+3x-5x^2+x^3)^{4/3}} + \frac{9(3-x)^2(1+x)}{80\,(9+3x-5x^2+x^3)^{4/3}} - \frac{27(3-x)^3(1+x)}{320\,(9+3x-5x^2+x^3)^{4/3}}$$

output
```
3/20*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(4/3)+9/80*(3-x)^2*(1+x)/(x^3-5*x^2+3*x
+9)^(4/3)-27/320*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(4/3)
```

3.234.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.36

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3(-3+x)(1+x)\,(29-42x+9x^2)}{320\,((-3+x)^2(1+x))^{4/3}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

output
```
(3*(-3 + x)*(1 + x)*(29 - 42*x + 9*x^2))/(320*((-3 + x)^2*(1 + x))^(4/3))
```

3.234.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2480, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 2480}$$

$$\frac{262144\ 2^{2/3}(x-3)^{8/3}(x+1)^{4/3}\int \frac{1}{262144\ 2^{2/3}(x-3)^{8/3}(x+1)^{4/3}}dx}{\left(x^3-5x^2+3x+9\right)^{4/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\int \frac{1}{(x-3)^{8/3}(x+1)^{4/3}}dx}{\left(x^3-5x^2+3x+9\right)^{4/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\left(-\frac{3}{10}\int \frac{1}{(x-3)^{5/3}(x+1)^{4/3}}dx-\frac{3}{20(x-3)^{5/3}\sqrt[3]{x+1}}\right)}{\left(x^3-5x^2+3x+9\right)^{4/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\left(-\frac{3}{10}\left(-\frac{3}{8}\int \frac{1}{(x-3)^{2/3}(x+1)^{4/3}}dx-\frac{3}{8(x-3)^{2/3}\sqrt[3]{x+1}}\right)-\frac{3}{20(x-3)^{5/3}\sqrt[3]{x+1}}\right)}{\left(x^3-5x^2+3x+9\right)^{4/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\left(-\frac{3}{10}\left(-\frac{9\sqrt[3]{x-3}}{32\sqrt[3]{x+1}}-\frac{3}{8\sqrt[3]{x+1}(x-3)^{2/3}}\right)-\frac{3}{20(x-3)^{5/3}\sqrt[3]{x+1}}\right)}{\left(x^3-5x^2+3x+9\right)^{4/3}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

output
```
((-3 + x)^(8/3)*(1 + x)^(4/3)*(-3/(20*(-3 + x)^(5/3)*(1 + x)^(1/3)) - (3*(
-3/(8*(-3 + x)^(2/3)*(1 + x)^(1/3)) - (9*(-3 + x)^(1/3))/(32*(1 + x)^(1/3)
)))/10))/(9 + 3*x - 5*x^2 + x^3)^(4/3)
```

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

3.234.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{\frac{27}{320} x^{2}-\frac{63}{160} x+\frac{87}{320}}{\left(-3+x \right) \left(\left(1+x \right) \left(-3+x \right)^{2}\right)^{\frac{1}{3}}}$	29
gosper	$\frac{3 \left(1+x \right) \left(-3+x \right) \left(9 x^{2}-42 x +29\right)}{320 \left(x^{3}-5 x^{2}+3 x +9\right)^{\frac{4}{3}}}$	34
trager	$\frac{3 \left(9 x^{2}-42 x +29\right) \left(x^{3}-5 x^{2}+3 x +9\right)^{\frac{2}{3}}}{320 \left(-3+x \right)^{3} \left(1+x \right)}$	38

input
```
int(1/(x^3-5*x^2+3*x+9)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/320*(9*x^2-42*x+29)/(-3+x)/((1+x)*(-3+x)^2)^(1/3)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3\,(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}(9\,x^2-42\,x+29)}{320\,(x^4-8\,x^3+18\,x^2-27)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="fricas")
```

output
```
3/320*(x^3 - 5*x^2 + 3*x + 9)^(2/3)*(9*x^2 - 42*x + 29)/(x^4 - 8*x^3 + 18*
x^2 - 27)
```

3.234.6 Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)
```

3.234.7 Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

3.234.8 Giac [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3\,(9\,x^2-42\,x+29)\,(x^3-5\,x^2+3\,x+9)^{2/3}}{320\,(x+1)\,(x-3)^3}$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(4/3),x)
```

output
```
(3*(9*x^2 - 42*x + 29)*(3*x - 5*x^2 + x^3 + 9)^(2/3))/(320*(x + 1)*(x - 3)
^3)
```

3.235 $\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx$

3.235.1 Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

output
```
-1/2*arcsin(1/41*(3-4*x)*41^(1/2))*2^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}x}{-2+\sqrt{4+3x-2x^2}}\right)$$

input
```
Integrate[1/Sqrt[4 + 3*x - 2*x^2],x]
```

output
```
Sqrt[2]*ArcTan[(Sqrt[2]*x)/(-2 + Sqrt[4 + 3*x - 2*x^2])]
```

3.235.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^2+3x+4}}dx$$
$$\Big\downarrow \; 1090$$
$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{41}(3-4x)^2}}d(3-4x)}{\sqrt{82}}$$
$$\Big\downarrow \; 223$$
$$-\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[4 + 3*x - 2*x^2],x]
```

output
```
-(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

3.235.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.235.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\sqrt{2}\,\arcsin\left(\frac{4\sqrt{41}\,\left(x-\frac{3}{4}\right)}{41}\right)}{2}$	15
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2+2\right)x+4\sqrt{-2x^2+3x+4}-3\,\mathrm{RootOf}\left(\_Z^2+2\right)\right)}{2}$	42

input
```
int(1/(-2*x^2+3*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x^2+3\,x+4}-2\,\sqrt{2}}{2\,x}\right)$$

input
```
integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(1/2*(sqrt(2)*sqrt(-2*x^2 + 3*x + 4) - 2*sqrt(2))/x)
```

3.235.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{asin}\left(\frac{4\sqrt{41}\left(x-\frac{3}{4}\right)}{41}\right)}{2}$$

input
```
integrate(1/(-2*x**2+3*x+4)**(1/2),x)
```

output
```
sqrt(2)*asin(4*sqrt(41)*(x - 3/4)/41)/2
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\frac{1}{2}\sqrt{2}\arcsin\left(-\frac{1}{41}\sqrt{41}(4x-3)\right)$$

input
```
integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arcsin(-1/41*sqrt(41)*(4*x - 3))
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{1}{8}\sqrt{-2x^2+3x+4}(4x-3)+\frac{41}{32}\sqrt{2}\arcsin\left(\frac{1}{41}\sqrt{41}(4x-3)\right)$$

input
```
integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(-2*x^2 + 3*x + 4)*(4*x - 3) + 41/32*sqrt(2)*arcsin(1/41*sqrt(41)*
(4*x - 3))
```

3.235.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{41}\,(4x-3)}{41}\right)}{2}$$

input
```
int(1/(3*x - 2*x^2 + 4)^(1/2),x)
```

output
```
(2^(1/2)*asin((41^(1/2)*(4*x - 3))/41))/2
```

3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}} \, dx$

3.236.1 Optimal result

Integrand size = 14, antiderivative size = 8

$$\int \frac{1}{\sqrt{-3+4x-x^2}} \, dx = -\arcsin(2-x)$$

output
```
arcsin(-2+x)
```

3.236.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt{-3+4x-x^2}} \, dx = -2\arctan\left(\frac{\sqrt{-3+4x-x^2}}{-1+x}\right)$$

input
```
Integrate[1/Sqrt[-3 + 4*x - x^2],x]
```

output
```
-2*ArcTan[Sqrt[-3 + 4*x - x^2]/(-1 + x)]
```

3.236.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2+4x-3}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{4}(4-2x)^2}}d(4-2x)$$

$$\Big\downarrow \text{223}$$

$$-\arcsin\left(\frac{1}{2}(4-2x)\right)$$

input
```
Int[1/Sqrt[-3 + 4*x - x^2],x]
```

output
```
-ArcSin[(4 - 2*x)/2]
```

3.236.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.236.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

method	result	size
default	$\arcsin\left(-2+x\right)$	5
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)x+\sqrt{-x^{2}+4x-3}+2\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)$	39

input
```
int(1/(-x^2+4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-2+x)
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2+4\,x-3}(x-2)}{x^2-4\,x+3}\right)$$

input
```
integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^2 + 4*x - 3)*(x - 2)/(x^2 - 4*x + 3))
```

3.236.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \operatorname{asin}\left(x-2\right)$$

input
```
integrate(1/(-x**2+4*x-3)**(1/2),x)
```

output
```
asin(x - 2)
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arcsin\left(-x+2\right)$$

input
```
integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-x + 2)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2+4\,x-3}(x-2)+\frac{1}{2}\,\arcsin\left(x-2\right)$$

input
```
integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 4*x - 3)*(x - 2) + 1/2*arcsin(x - 2)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \mathrm{asin}(x-2)$$

input
```
int(1/(4*x - x^2 - 3)^(1/2),x)
```

output
```
asin(x - 2)
```

3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx$

3.237.1 Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx=\frac{\arcsin(5+6x)}{\sqrt{3}}$$

output
```
1/3*arcsin(5+6*x)*3^(1/2)
```

3.237.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-2-5x-3x^2}}{\sqrt{3}(1+x)}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[-2 - 5*x - 3*x^2]/(Sqrt[3]*(1 + x))])/Sqrt[3]
```

3.237.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2-5x-2}}dx$$
$$\Big\downarrow \ 1090$$
$$-\frac{\int \frac{1}{\sqrt{1-(-6x-5)^2}}d(-6x-5)}{\sqrt{3}}$$
$$\Big\downarrow \ 223$$
$$\frac{\arcsin(6x+5)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

output
```
ArcSin[5 + 6*x]/Sqrt[3]
```

3.237.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.237.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arcsin(6x+5)\sqrt{3}}{3}$	12
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^2+3\right)x+6\sqrt{-3x^2-5x-2}-5\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	42

input
```
int(1/(-3*x^2-5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(6*x+5)*3^(1/2)
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2-5x-2}(6x+5)}{6(3x^2+5x+2)}\right)$$

input
```
integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5)/(3*x^2 +
5*x + 2))
```

3.237.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asin}(6x+5)}{3}$$

input
```
integrate(1/(-3*x**2-5*x-2)**(1/2),x)
```

output
```
sqrt(3)*asin(6*x + 5)/3
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{1}{3}\sqrt{3}\arcsin\left(6\,x+5\right)$$

input
```
integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsin(6*x + 5)
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{1}{12}\sqrt{-3\,x^2-5\,x-2}(6\,x+5)+\frac{1}{72}\sqrt{3}\arcsin\left(6\,x+5\right)$$

input
```
integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5) + 1/72*sqrt(3)*arcsin(6*x + 5)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x+5)}{3}$$

input
```
int(1/(- 5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin(6*x + 5))/3
```

3.238 $\int \frac{1}{\sqrt{1-x^2}(4+x^2)}\,dx$

3.238.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

output
```
1/10*arctan(1/2*x*5^(1/2)/(-x^2+1)^(1/2))*5^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx=-\frac{\arctan\left(\frac{x\sqrt{5-5x^2}}{2(-1+x^2)}\right)}{2\sqrt{5}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

output
```
-1/2*ArcTan[(x*Sqrt[5 - 5*x^2])/(2*(-1 + x^2))]/Sqrt[5]
```

3.238.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\,(x^2+4)}\,dx$$

$$\Big\downarrow \text{ 291}$$

$$\int \frac{1}{\frac{5x^2}{1-x^2}+4}d\frac{x}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

input
```
Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

output
```
ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

3.238.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.238.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{\sqrt{5}\,\arctan\left(\frac{2\sqrt{5}\,\sqrt{-x^{2}+1}}{5x}\right)}{10}$	24
default	$-\frac{\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^{2}+1}\,\sqrt{5}\,x}{2x^{2}-2}\right)}{10}$	29
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+5\right)\ln\left(\frac{-9\operatorname{RootOf}\left(\_Z^{2}+5\right)x^{2}+20x\sqrt{-x^{2}+1}+4\operatorname{RootOf}\left(\_Z^{2}+5\right)}{x^{2}+4}\right)}{20}$	50

input
```
int(1/(x^2+4)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*5^(1/2)*arctan(2/5/x*5^(1/2)*(-x^2+1)^(1/2))
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = -\frac{1}{10}\,\sqrt{5}\arctan\left(\frac{2\,\sqrt{5}\sqrt{-x^2+1}}{5\,x}\right)$$

input
```
integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/10*sqrt(5)*arctan(2/5*sqrt(5)*sqrt(-x^2 + 1)/x)
```

3.238.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)}\,(x^2+4)}\,dx$$

input
```
integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)
```

3.238.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \int \frac{1}{(x^2+4)\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{1}{20}\sqrt{5}\left(\pi\text{sgn}(x) + 2\arctan\left(-\frac{\sqrt{5}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{5\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

input
```
integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*(pi*sgn(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 + 1) - 1)^2/
x^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

3.238.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\sqrt{5}\ln\left(\frac{\frac{\sqrt{5}\,(-1+x\,2\mathrm{i})\,1\mathrm{i}}{5}-\sqrt{1-x^2}\,1\mathrm{i}}{x-2\mathrm{i}}\right)1\mathrm{i}}{20} - \frac{\sqrt{5}\ln\left(\frac{\frac{\sqrt{5}\,(1+x\,2\mathrm{i})\,1\mathrm{i}}{5}+\sqrt{1-x^2}\,1\mathrm{i}}{x+2\mathrm{i}}\right)1\mathrm{i}}{20}$$

input
```
int(1/((1 - x^2)^(1/2)*(x^2 + 4)),x)
```

output
```
(5^(1/2)*log(((5^(1/2)*(x*2i - 1)*1i)/5 - (1 - x^2)^(1/2)*1i)/(x - 2i))*1i
)/20 - (5^(1/2)*log(((5^(1/2)*(x*2i + 1)*1i)/5 + (1 - x^2)^(1/2)*1i)/(x +
2i))*1i)/20
```

3.239 $\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$

3.239.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}$$

output
```
1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{8+2x^2-x\sqrt{1+4x^2}}{2\sqrt{15}}\right)}{2\sqrt{15}}$$

input
```
Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

output
```
ArcTanh[(8 + 2*x^2 - x*Sqrt[1 + 4*x^2])/(2*Sqrt[15])]/(2*Sqrt[15])
```

3.239.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+4)\sqrt{4x^2+1}}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{4-\frac{15x^2}{4x^2+1}}d\frac{x}{\sqrt{4x^2+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

input
```
Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

output
```
ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

3.239.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.239.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{15}}{2\sqrt{4x^2+1}}\right)\sqrt{15}}{30}$	22
pseudoelliptic	$\frac{\sqrt{15}\,\operatorname{arctanh}\left(\frac{2\sqrt{4x^2+1}\,\sqrt{15}}{15x}\right)}{30}$	24
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-15\right)\ln\left(\frac{31\operatorname{RootOf}\left(\_Z^2-15\right)x^2+60\sqrt{4x^2+1}\,x+4\operatorname{RootOf}\left(\_Z^2-15\right)}{x^2+4}\right)}{60}$	50

input

```
int(1/(x^2+4)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$$
$$= \frac{1}{60}\sqrt{15}\log\left(\frac{961\,x^2+8\,\sqrt{15}(31\,x^2+4)+4\,\sqrt{4\,x^2+1}\left(31\,\sqrt{15}x+120\,x\right)+124}{x^2+4}\right)$$

input

```
integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
1/60*sqrt(15)*log((961*x^2 + 8*sqrt(15)*(31*x^2 + 4) + 4*sqrt(4*x^2 + 1)*(
31*sqrt(15)*x + 120*x) + 124)/(x^2 + 4))
```

3.239.6 Sympy [F]

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \int \frac{1}{(x^2+4)\sqrt{4x^2+1}}\,dx$$

input
```
integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)
```

output
```
Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)
```

3.239.7 Maxima [F]

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+1}(x^2+4)}\,dx$$

input
```
integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)
```

3.239.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = -\frac{1}{60}\sqrt{15}\log\left(\frac{\left(2\,x-\sqrt{4\,x^2+1}\right)^2-8\sqrt{15}+31}{\left(2\,x-\sqrt{4\,x^2+1}\right)^2+8\sqrt{15}+31}\right)$$

input
```
integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(15)*log(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/((2*x - s
qrt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))
```

3.239.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = -\frac{\sqrt{15}\left(\ln\left(x-2\mathrm{i}\right)-\ln\left(x+\frac{\sqrt{15}\sqrt{x^2+\frac{1}{4}}}{4}-\frac{1}{8}\mathrm{i}\right)\right)}{60} + \frac{\sqrt{15}\left(\ln\left(x+2\mathrm{i}\right)-\ln\left(x-\frac{\sqrt{15}\sqrt{x^2+\frac{1}{4}}}{4}+\frac{1}{8}\mathrm{i}\right)\right)}{60}$$

input
```
int(1/((x^2 + 4)*(4*x^2 + 1)^(1/2)),x)
```

output
```
(15^(1/2)*(log(x + 2i) - log(x - (15^(1/2)*(x^2 + 1/4)^(1/2))/4 + 1i/8)))/
60 - (15^(1/2)*(log(x - 2i) - log(x + (15^(1/2)*(x^2 + 1/4)^(1/2))/4 - 1i/
8)))/60
```

3.240 $\int \frac{x}{(3-x^2)\sqrt{5-x^2}} \, dx$

3.240.1 Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

output
```
ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

3.240.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {353, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(3-x^2)\sqrt{5-x^2}}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{x^4-2}d\sqrt{5-x^2}$$

$$\Big\downarrow \text{220}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

output
```
ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

3.240.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.240.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-x^2+5}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$	21
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x^2-7\operatorname{RootOf}\left(\_Z^2-2\right)-4\sqrt{-x^2+5}}{x^2-3}\right)}{4}$	48
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4+2\sqrt{3}\,\left(x+\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{3}\right)^2+2\sqrt{3}\,\left(x+\sqrt{3}\right)+2}}\right)}{4}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4-2\sqrt{3}\,\left(x-\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{3}\right)^2-2\sqrt{3}\,\left(x-\sqrt{3}\right)+2}}\right)}{4}$	100

input
```
int(x/(-x^2+3)/(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)
```

3.240.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{x^4-4\sqrt{2}(x^2-7)\sqrt{-x^2+5}-22\,x^2+89}{x^4-6\,x^2+9}\right)$$

input
```
integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log((x^4 - 4*sqrt(2)*(x^2 - 7)*sqrt(-x^2 + 5) - 22*x^2 + 89)/(
x^4 - 6*x^2 + 9))
```

3.240.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{5-x^2}-\sqrt{2}\right)-\log\left(\sqrt{5-x^2}+\sqrt{2}\right)\right)}{4}$$

input
```
integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
```

output
```
-sqrt(2)*(log(sqrt(5 - x**2) - sqrt(2)) - log(sqrt(5 - x**2) + sqrt(2)))/4
```

3.240.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.67

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx$$
$$= \frac{1}{12}\sqrt{3}\left(\sqrt{3}\sqrt{2}\log\left(\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x+2\sqrt{3}|}+\frac{4}{|2x+2\sqrt{3}|}\right)+\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x-2\sqrt{3}|}+\frac{4}{|2x\right.\right.$$

input
```
integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*(sqrt(3)*sqrt(2)*log(sqrt(3) + 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2
*x + 2*sqrt(3)) + 4/abs(2*x + 2*sqrt(3))) + sqrt(3)*sqrt(2)*log(-sqrt(3) +
 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2*x - 2*sqrt(3)) + 4/abs(2*x - 2*sqrt(3))))
```

3.240.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x^2+5}\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x^2+5}\right|\right)$$

input
```
integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*log(abs(-sqrt(2) +
 sqrt(-x^2 + 5)))
```

3.240.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\sqrt{2}\left(\ln\left(\frac{\frac{\sqrt{2}\left(\sqrt{3}\,x+5\right)\,1\mathrm{i}}{2}+\sqrt{5-x^2}\,1\mathrm{i}}{x+\sqrt{3}}\right)+\ln\left(\frac{\frac{\sqrt{2}\left(\sqrt{3}\,x-5\right)\,1\mathrm{i}}{2}-\sqrt{5-x^2}\,1\mathrm{i}}{x-\sqrt{3}}\right)\right)}{4}$$

input
```
int(-x/((x^2 - 3)*(5 - x^2)^(1/2)),x)
```

output
```
(2^(1/2)*(log(((2^(1/2)*(3^(1/2)*x + 5)*1i)/2 + (5 - x^2)^(1/2)*1i)/(x + 3
^(1/2))) + log(((2^(1/2)*(3^(1/2)*x - 5)*1i)/2 - (5 - x^2)^(1/2)*1i)/(x -
3^(1/2)))))/4
```

3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)} dx$

3.241.1 Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)} dx = -\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(1/2*(-x^2+3)^(1/2)*2^(1/2))*2^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)} dx = -\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

output
```
-(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

3.241.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {353, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{3-x^2}\,(5-x^2)}dx^2$$

$$\Big\downarrow \text{ 73}$$

$$-\int \frac{1}{x^4+2}d\sqrt{3-x^2}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

output
```
-(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

3.241.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.241.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-x^2+3}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$	21
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^2-\operatorname{RootOf}\left(\_Z^2+2\right)-4\sqrt{-x^2+3}}{x^2-5}\right)}{4}$	48
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4+2\sqrt{5}\,\left(x+\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{5}\right)^2+2\sqrt{5}\,\left(x+\sqrt{5}\right)-2}}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4-2\sqrt{5}\,\left(x-\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{5}\right)^2-2\sqrt{5}\,\left(x-\sqrt{5}\right)-2}}\right)}{4}$	100

input
```
int(x/(-x^2+5)/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2*(-x^2+3)^(1/2)*2^(1/2))*2^(1/2)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx=-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{-x^2+3}}{4\,(x^2-3)}\right)$$

input
```
integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 1)*sqrt(-x^2 + 3)/(x^2 - 3))
```

3.241.6 Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3-x^2}}{2}\right)}{2}$$

input
```
integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)
```

output
```
-sqrt(2)*atan(sqrt(2)*sqrt(3 - x**2)/2)/2
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(20) = 40.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\,|2\,x+2\sqrt{5}|}+\frac{2\sqrt{3}}{|2\,x+2\sqrt{5}|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\,|2\,x-2\sqrt{5}|}-\frac{2\sqrt{3}}{|2\,x-2\sqrt{5}|}\right.\right.$$

input
```
integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(5)*(sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2*x + 2*sq
rt(5)) + 2*sqrt(3)/abs(2*x + 2*sqrt(5))) - sqrt(5)*sqrt(2)*arcsin(2/3*sqrt
(5)*sqrt(3)*x/abs(2*x - 2*sqrt(5)) - 2*sqrt(3)/abs(2*x - 2*sqrt(5))))
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x^2+3}\right)$$

input
```
integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))
```

3.241.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(\sqrt{5}\,x+3)}{2}+\sqrt{3-x^2}\,1\mathrm{i}}{x+\sqrt{5}}\right)\,1\mathrm{i}}{4} - \frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(\sqrt{5}\,x-3)}{2}-\sqrt{3-x^2}\,1\mathrm{i}}{x-\sqrt{5}}\right)\,1\mathrm{i}}{4}$$

input
```
int(-x/((3 - x^2)^(1/2)*(x^2 - 5)),x)
```

output
```
- (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x + 3))/2 + (3 - x^2)^(1/2)*1i)/(x + 5^(
1/2)))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x - 3))/2 - (3 - x^2)^(1/2)
*1i)/(x - 5^(1/2)))*1i)/4
```

3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)} dx$

3.242.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = -\frac{1}{2}\arctan\left(\frac{x}{\sqrt{2+x^2}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}}$$

output
```
-1/2*arctan(x/(x^2+2)^(1/2))-1/6*arctanh(x*3^(1/2)/(x^2+2)^(1/2))*3^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{1}{6}\left(3\arctan\left(1+x^2-x\sqrt{2+x^2}\right)-\sqrt{3}\operatorname{arctanh}\left(\frac{1-x^2+x\sqrt{2+x^2}}{\sqrt{3}}\right)\right)$$

input
```
Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

output
```
(3*ArcTan[1 + x^2 - x*Sqrt[2 + x^2]] - Sqrt[3]*ArcTanh[(1 - x^2 + x*Sqrt[2
 + x^2])/Sqrt[3]])/6
```

3.242.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1489, 291, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2}\,(x^4-1)}dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{1}{2}\int \frac{1}{(1-x^2)\sqrt{x^2+2}}dx-\frac{1}{2}\int \frac{1}{(x^2+1)\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{291}$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{3x^2}{x^2+2}}d\frac{x}{\sqrt{x^2+2}}-\frac{1}{2}\int \frac{1}{\frac{x^2}{x^2+2}+1}d\frac{x}{\sqrt{x^2+2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{3x^2}{x^2+2}}d\frac{x}{\sqrt{x^2+2}}-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

input
```
Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

output
```
-1/2*ArcTan[x/Sqrt[2 + x^2]] - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(2*Sqrt[
3])
```

3.242.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

3.242.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,\sqrt{x^2+2}}{3x}\right)}{6}+\frac{\arctan\left(\frac{\sqrt{x^2+2}}{x}\right)}{2}$
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(2x+4)\sqrt{3}}{6\sqrt{(-1+x)^2+1+2x}}\right)}{12}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(-2x+4)\sqrt{3}}{6\sqrt{(1+x)^2+1-2x}}\right)}{12}-\frac{\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)}{2}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{x^2+2}\,x+\operatorname{RootOf}\left(\_Z^2+1\right)}{x^2+1}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-2\operatorname{RootOf}\left(\_Z^2-3\right)x^2+3\sqrt{x^2+2}\,x-}{(-1+x)(1+x)}\right.}{12}$

input
```
int(1/(x^4-1)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*3^(1/2)*arctanh(1/3*3^(1/2)*(x^2+2)^(1/2)/x)+1/2*arctan((x^2+2)^(1/2)
/x)
```

3.242.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{4\,x^2-\sqrt{3}(2\,x^2+1)-\sqrt{x^2+2}(2\,\sqrt{3}x-3\,x)+2}{x^2-1}\right) - \frac{1}{2}\arctan\left(-x^2+\sqrt{x^2+2}x-1\right)$$

input
```
integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((4*x^2 - sqrt(3)*(2*x^2 + 1) - sqrt(x^2 + 2)*(2*sqrt(3)*x
 - 3*x) + 2)/(x^2 - 1)) - 1/2*arctan(-x^2 + sqrt(x^2 + 2)*x - 1)
```

3.242.6 Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \int \frac{1}{(x-1)\,(x+1)\,(x^2+1)\,\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(x**4-1)/(x**2+2)**(1/2),x)
```

output
```
Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)
```

3.242.7 Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \int \frac{1}{(x^4-1)\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)
```

3.242.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = -\frac{1}{12}\,\sqrt{3}\log\left(\frac{\left|2\left(x-\sqrt{x^2+2}\right)^2-4\sqrt{3}-8\right|}{\left|2\left(x-\sqrt{x^2+2}\right)^2+4\sqrt{3}-8\right|}\right) + \frac{1}{2}\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2}\right)^2\right)$$

input
```
integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs(2*(x -
sqrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x - sqrt(x^2 + 2))^2)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{\sqrt{3}\left(\ln(x-1)-\ln\left(x+\sqrt{3}\sqrt{x^2+2}+2\right)\right)}{12} - \frac{\sqrt{3}\left(\ln(x+1)-\ln\left(\sqrt{3}\sqrt{x^2+2}-x+2\right)\right)}{12} + \frac{\ln\left(\sqrt{x^2+2}+2-x\,1i\right)\,1i}{4} - \frac{\ln\left(\sqrt{x^2+2}+2+x\,1i\right)\,1i}{4} + \frac{\ln(x-i)\,1i}{4} - \frac{\ln(x+1i)\,1i}{4}$$

input
```
int(1/((x^2 + 2)^(1/2)*(x^4 - 1)),x)
```

output
```
(log((x^2 + 2)^(1/2) - x*1i + 2)*1i)/4 - (log(x*1i + (x^2 + 2)^(1/2) + 2)*
1i)/4 + (log(x - 1i)*1i)/4 - (log(x + 1i)*1i)/4 + (3^(1/2)*(log(x - 1) - l
og(x + 3^(1/2)*(x^2 + 2)^(1/2) + 2)))/12 - (3^(1/2)*(log(x + 1) - log(3^(1
/2)*(x^2 + 2)^(1/2) - x + 2)))/12
```

3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$

3.243.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*
x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

output
```
ArcTanh[(1 + x - Sqrt[4 + 2*x + x^2])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x +
Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt[7]
```

3.243.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1366, 25, 1112, 220, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2-1)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{1366}$$

$$\frac{1}{2}\int -\frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+2x+4}}dx-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{1112}$$

$$2\int \frac{1}{4(x^2+2x+4)-12}d\sqrt{x^2+2x+4}-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{220}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{1}{28-\frac{4(2x+5)^2}{x^2+2x+4}}d\left(-\frac{2(2x+5)}{\sqrt{x^2+2x+4}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

output
```
-1/2*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/Sqrt[7] - ArcTanh[Sq
rt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

3.243.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.243. $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$

3.243.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}+5\operatorname{RootOf}\left(\_Z^2-7\right)}{-1+x}\right)}{14}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{x^2+2x+4}-\operatorname{RootOf}}{1+x}\right.}{6}$

input
```
int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^
(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x-10}{x-1}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right)$$

input
```
integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7)
- 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x
 + 1))
```

3.243.6 Sympy [F]

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)
```

output
```
Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = -\frac{1}{14}\sqrt{7}\,\text{arsinh}\left(\frac{4\sqrt{3}x}{3\,|2\,x-2|}+\frac{10\sqrt{3}}{3\,|2\,x-2|}\right) - \frac{1}{6}\sqrt{3}\,\text{arsinh}\left(\frac{2\sqrt{3}}{|2\,x+2|}\right)$$

input
```
integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
-1/14*sqrt(7)*arcsinh(4/3*sqrt(3)*x/abs(2*x - 2) + 10/3*sqrt(3)/abs(2*x -
2)) - 1/6*sqrt(3)*arcsinh(2*sqrt(3)/abs(2*x + 2))
```

3.243.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(48) = 96.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2\,x-2\sqrt{7}+2\sqrt{x^2+2\,x+4}+2\right|}{\left|-2\,x+2\sqrt{7}+2\sqrt{x^2+2\,x+4}+2\right|}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2\,x-2\sqrt{3}+2\sqrt{x^2+2\,x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2\,x+4}+1\right)}\right)$$

input
```
integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*
x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*
x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x +
 4) + 1))
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \int \frac{x}{(x^2-1)\,\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)),x)
```

output
```
int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)), x)
```

3.244 $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx$

3.244.1 Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx = -\frac{\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12}\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

output
```
1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^
(1/2))*3^(1/2)-1/156*arctanh(1/13*(7+3*x)*13^(1/2)/(x^2+2*x+5)^(1/2))*13^(
1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx = \frac{1}{156}\left(13\sqrt{3}\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right) + 13\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right) - 2\sqrt{13}\operatorname{arctanh}\left(\frac{2-x+\sqrt{5+2x+x^2}}{\sqrt{13}}\right)\right)$$

input
```
Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

output
```
(13*Sqrt[3]*ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]
+ 13*ArcTanh[Sqrt[5 + 2*x + x^2]] - 2*Sqrt[13]*ArcTanh[(2 - x + Sqrt[5 + 2
*x + x^2])/Sqrt[13]])/156
```

3.244.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {2535, 1154, 219, 1358, 27, 1313, 217, 1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2x+5}\,(x^3-8)}dx$$

$$\Big\downarrow \text{ 2535}$$

$$-\frac{1}{12}\int \frac{1}{(2-x)\sqrt{x^2+2x+5}}dx-\frac{1}{12}\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{6}\int \frac{1}{52-\frac{4(3x+7)^2}{x^2+2x+5}}d\left(-\frac{2(3x+7)}{\sqrt{x^2+2x+5}}\right)-\frac{1}{12}\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{1}{12}\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{ 1358}$$

$$\frac{1}{12}\left(-3\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\frac{1}{2}\int \frac{2(x+1)}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{12}\left(-3\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1313}$$

$$\frac{1}{12}\left(12\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+5}-24}d\frac{2(x+1)}{\sqrt{x^2+2x+5}}-\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{12}\left(-\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1357}$$

$$\frac{1}{12}\left(2\int \frac{1}{2-2(x^2+2x+5)}d\sqrt{x^2+2x+5}-\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{12}\left(\text{arctanh}\left(\sqrt{x^2+2x+5}\right)-\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

input
```
Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

output
```
-1/12*ArcTanh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/Sqrt[13] + (-(Sqrt
[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]) + ArcTanh[Sqrt[5 + 2*x
+ x^2]])/12
```

3.244. $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)}\,dx$

3.244.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

3.244. $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)}\,dx$

rule 2535
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]*((a_) + (b_.)*(x_)^3)), x_Sy
mbol] :> With[{r = Numerator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, S
imp[r/(3*a)   Int[1/((r - s*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[r/(3*a
)   Int[(2*r + s*x)/((r^2 + r*s*x + s^2*x^2)*Sqrt[d + e*x + f*x^2]), x], x]
] /; FreeQ[{a, b, d, e, f}, x] && NegQ[a/b]
```

3.244.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(14+6x)\sqrt{13}}{26\sqrt{(-2+x)^{2}+1+6x}}\right)}{156}+\frac{\operatorname{arctanh}\left(\sqrt{x^{2}+2x+5}\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^{2}+2x+5}}\right)}{12}$
trager	$\operatorname{RootOf}\left(144\_Z^{2}-12\_Z+1\right)\ln\left(\frac{2880\operatorname{RootOf}\left(144\_Z^{2}-12\_Z+1\right)^{2}x-126\operatorname{RootOf}\left(144\_Z^{2}-12\_Z+1\right)\sqrt{x^{2}+2}}{12\operatorname{RootOf}}\right.$

input
```
int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/156*13^(1/2)*arctanh(1/26*(14+6*x)*13^(1/2)/((-2+x)^2+1+6*x)^(1/2))+1/1
2*arctanh((x^2+2*x+5)^(1/2))-1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(
1/2)*(2*x+2))
```

3.244.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx$$
$$= \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)$$
$$-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)$$
$$+\frac{1}{156}\sqrt{13}\log\left(\frac{\sqrt{13}(3x+7)+\sqrt{x^2+2x+5}(3\sqrt{13}-13)-9x-21}{x-2}\right)$$
$$-\frac{1}{24}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right)+\frac{1}{24}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)$$

input
```
integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)
) - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5))
+ 1/156*sqrt(13)*log((sqrt(13)*(3*x + 7) + sqrt(x^2 + 2*x + 5)*(3*sqrt(13)
 - 13) - 9*x - 21)/(x - 2)) - 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) +
 3*x + 6) + 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)
```

3.244.6 Sympy [F]

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x-2)\,(x^2+2x+4)\,\sqrt{x^2+2x+5}}\,dx$$

input
```
integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
```

output
```
Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.244.7 Maxima [F]

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x^3-8)\sqrt{x^2+2\,x+5}}\,dx$$

input
```
integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(64) = 128$.

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

$$\begin{aligned}\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx =& \frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\Big(x-\sqrt{x^2+2\,x+5}+2\Big)\right) \\ &-\frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\Big(x-\sqrt{x^2+2\,x+5}\Big)\right) \\ &+\frac{1}{156}\,\sqrt{13}\log\left(\frac{\left|-2\,x-2\,\sqrt{13}+2\,\sqrt{x^2+2\,x+5}+4\right|}{\left|-2\,x+2\,\sqrt{13}+2\,\sqrt{x^2+2\,x+5}+4\right|}\right) \\ &-\frac{1}{24}\,\log\left(\Big(x-\sqrt{x^2+2\,x+5}\Big)^2+4\,x-4\,\sqrt{x^2+2\,x+5} \right. \\ &\left. +7\right)+\frac{1}{24}\,\log\left(\Big(x-\sqrt{x^2+2\,x+5}\Big)^2+3\right)\end{aligned}$$

input
```
integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) - 1/12*sqr
t(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/156*sqrt(13)*log(a
bs(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)/abs(-2*x + 2*sqrt(13) +
2*sqrt(x^2 + 2*x + 5) + 4)) - 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x -
 4*sqrt(x^2 + 2*x + 5) + 7) + 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x^3-8)\,\sqrt{x^2+2\,x+5}}\,dx$$

input
```
int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)),x)
```

output
```
int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)), x)
```

3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

3.245.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}$$

output
```
1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*arctanh(1/15
*(1+2*x)*165^(1/2)/(4*x^2+4*x+5)^(1/2))*165^(1/2)
```

3.245.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx=\frac{1}{2}\text{RootSum}\left[69-108\#1+58\#1^2-4\#1^3+\#1^4\&,\frac{-5\log\left(-2x+\sqrt{5+4x+4x^2}-\#1\right)+\log\left(-2x+\sqrt{5+4x+4x^2}-\#1\right)\#1^2}{-27+29\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

output
```
RootSum[69 - 108*#1 + 58*#1^2 - 4*#1^3 + #1^4 & , (-5*Log[-2*x + Sqrt[5 +
4*x + 4*x^2] - #1] + Log[-2*x + Sqrt[5 + 4*x + 4*x^2] - #1]*#1^2)/(-27 + 2
9*#1 - 3*#1^2 + #1^3) & ]/2
```

3.245.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1358, 27, 1313, 220, 1357, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx$$

$$\Big\downarrow \text{ 1358}$$

$$\frac{1}{8}\int \frac{4(2x+1)}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx-\frac{1}{2}\int \frac{1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx-\frac{1}{2}\int \frac{1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx$$

$$\Big\downarrow \text{ 1313}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx+4\int \frac{1}{\frac{176(2x+1)^2}{4x^2+4x+5}-240}d\frac{4(2x+1)}{\sqrt{4x^2+4x+5}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

$$\Big\downarrow \text{ 1357}$$

$$-\int \frac{1}{-4x^2-4x-16}d\sqrt{4x^2+4x+5}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

input
```
Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

output
```
ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1
+ 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

3.245. $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

3.245.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result
default	$\frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}\,\sqrt{11}}{11}\right)\sqrt{11}}{11}-\frac{\sqrt{165}\,\operatorname{arctanh}\left(\frac{\sqrt{165}\,(8x+4)}{60\sqrt{4x^2+4x+5}}\right)}{165}$
trager	$-\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)\ln\left(\frac{3524400\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)^5 x+111270\operatorname{RootOf}\left(27\right.}{}\right.$

input
```
int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165^(1/2)*ar
ctanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

3.245.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.76

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$$
$$= \frac{1}{330}\sqrt{165}\sqrt{2i\sqrt{15}-14}\log\left(\sqrt{165}\sqrt{2i\sqrt{15}-14}\left(i\sqrt{15}+15\right)-480x-240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$
$$-\frac{1}{330}\sqrt{165}\sqrt{2i\sqrt{15}-14}\log\left(\sqrt{165}\sqrt{2i\sqrt{15}-14}\left(-i\sqrt{15}-15\right)-480x-240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$
$$-\frac{1}{330}\sqrt{165}\sqrt{-2i\sqrt{15}-14}\log\left(\sqrt{165}\left(i\sqrt{15}-15\right)\sqrt{-2i\sqrt{15}-14}-480x+240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$
$$+\frac{1}{330}\sqrt{165}\sqrt{-2i\sqrt{15}-14}\log\left(\sqrt{165}\left(-i\sqrt{15}+15\right)\sqrt{-2i\sqrt{15}-14}-480x+240i\sqrt{15}+240\sqrt{4x^2+4x+5}-240\right)$$

input
```
integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/330*sqrt(165)*sqrt(2*I*sqrt(15) - 14)*log(sqrt(165)*sqrt(2*I*sqrt(15) -
14)*(I*sqrt(15) + 15) - 480*x - 240*I*sqrt(15) + 240*sqrt(4*x^2 + 4*x + 5)
 - 240) - 1/330*sqrt(165)*sqrt(2*I*sqrt(15) - 14)*log(sqrt(165)*sqrt(2*I*s
qrt(15) - 14)*(-I*sqrt(15) - 15) - 480*x - 240*I*sqrt(15) + 240*sqrt(4*x^2
 + 4*x + 5) - 240) - 1/330*sqrt(165)*sqrt(-2*I*sqrt(15) - 14)*log(sqrt(165
)*(I*sqrt(15) - 15)*sqrt(-2*I*sqrt(15) - 14) - 480*x + 240*I*sqrt(15) + 24
0*sqrt(4*x^2 + 4*x + 5) - 240) + 1/330*sqrt(165)*sqrt(-2*I*sqrt(15) - 14)*
log(sqrt(165)*(-I*sqrt(15) + 15)*sqrt(-2*I*sqrt(15) - 14) - 480*x + 240*I*
sqrt(15) + 240*sqrt(4*x^2 + 4*x + 5) - 240)
```

3.245. $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

3.245.6 Sympy [F]

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{(x^2+x+4)\sqrt{4x^2+4x+5}}\,dx$$

input
```
integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
```

output
```
Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)
```

3.245.7 Maxima [F]

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{\sqrt{4\,x^2+4\,x+5}(x^2+x+4)}\,dx$$

input
```
integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(52) = 104$.

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\begin{aligned}\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx &= \frac{1}{165}\,\sqrt{165}\sqrt{15}\arctan\left(-\frac{2\,x-\sqrt{4\,x^2+4\,x+5}+1}{\sqrt{15}+\sqrt{11}}\right) \\ &\quad -\frac{1}{165}\,\sqrt{165}\sqrt{15}\arctan\left(-\frac{2\,x-\sqrt{4\,x^2+4\,x+5}+1}{\sqrt{15}-\sqrt{11}}\right) \\ &\quad -\frac{1}{330}\,\sqrt{165}\log\left(90000\left(2\,x-\sqrt{4\,x^2+4\,x+5}+1\right)^2 \right. \\ &\qquad \left. +90000\left(\sqrt{15}+\sqrt{11}\right)^2\right) \\ &\quad +\frac{1}{330}\,\sqrt{165}\log\left(90000\left(2\,x-\sqrt{4\,x^2+4\,x+5}+1\right)^2 \right. \\ &\qquad \left. +90000\left(\sqrt{15}-\sqrt{11}\right)^2\right)\end{aligned}$$

input
```
integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x + 5) + 1)/(sqrt(1
5) + sqrt(11))) - 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x
 + 5) + 1)/(sqrt(15) - sqrt(11))) - 1/330*sqrt(165)*log(90000*(2*x - sqrt(
4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) + sqrt(11))^2) + 1/330*sqrt(165)
*log(90000*(2*x - sqrt(4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) - sqrt(11
))^2)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{\sqrt{4\,x^2+4\,x+5}\,(x^2+x+4)}\,dx$$

input
```
int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)),x)
```

output
```
int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)), x)
```

3.246 $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$

3.246.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = -2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)$$

output
```
-2*arctan(1/2*(1-x)*2^(1/2)/(x^2+x+1)^(1/2))*2^(1/2)+arctanh(1/2*(1+x)*2^(
1/2)/(x^2+x+1)^(1/2))*2^(1/2)
```

3.246.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.84

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[2-4\#1+2\#1^2 +\#1^4\&, \frac{2\log\left(-x+\sqrt{1+x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1+x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x+x\right.}{-1+\#1+\#1^3}$$

input
```
Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

output
```
RootSum[2 - 4*#1 + 2*#1^2 + #1^4 & , (2*Log[-x + Sqrt[1 + x + x^2] - #1] -
 6*Log[-x + Sqrt[1 + x + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x + x^2] - #1]*
#1^2)/(-1 + #1 + #1^3) & ]/2
```

3.246.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1369, 27, 1363, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1369}$$

$$\frac{1}{2}\int \frac{2(1-x)}{(x^2+1)\sqrt{x^2+x+1}}dx-\frac{1}{2}\int -\frac{4(x+1)}{(x^2+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x}{(x^2+1)\sqrt{x^2+x+1}}dx+2\int \frac{x+1}{(x^2+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1363}$$

$$2\int \frac{1}{\frac{(x+1)^2}{x^2+x+1}-2}d\left(-\frac{x+1}{\sqrt{x^2+x+1}}\right)-4\int \frac{1}{\frac{(1-x)^2}{x^2+x+1}+2}d\frac{1-x}{\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{216}$$

$$2\int \frac{1}{\frac{(x+1)^2}{x^2+x+1}-2}d\left(-\frac{x+1}{\sqrt{x^2+x+1}}\right)-2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

$$\Big\downarrow \text{220}$$

$$\sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

input
```
Int[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

output
```
-2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*ArcTanh[(
1 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

3.246. $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$

3.246.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(46) = 92$.

Time = 1.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.29

method	result
default	$\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}}{2}\right)-2\arctan\left(\frac{\sqrt{2}\,(-1+x)}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,(-1-x)}\right)\right)}{\sqrt{\frac{\frac{(-1+x)^2}{(-1-x)^2}+3}{\left(\frac{-1+x}{-1-x}+1\right)^2}}\left(\frac{-1+x}{-1-x}+1\right)}$
trager	$\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)\ln\left(-\frac{12\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^4 x+92x\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^2+64\operatorname{R}}{2x\operatorname{RootOf}\left(4\_Z\right.}\right.$

input
```
int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-1+x)^2/(-1-x)^2+3)^(1/2)*2^(1/2)*(arctanh(1/2*((-1+x)^2/(-1-x)^2+3)^(1/
2)*2^(1/2))-2*arctan(2^(1/2)/((-1+x)^2/(-1-x)^2+3)^(1/2)*(-1+x)/(-1-x)))/(
((-1+x)^2/(-1-x)^2+3)/((-1+x)/(-1-x)+1)^2)^(1/2)/((-1+x)/(-1-x)+1)
```

3.246.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.88

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \frac{1}{2}\sqrt{8i-6}\log\left(-10\,x-(i-3)\,\sqrt{8i-6}+10\,\sqrt{x^2+x+1}+10i\right) - \frac{1}{2}\sqrt{8i-6}\log\left(-10\,x+(i-3)\,\sqrt{8i-6}+10\,\sqrt{x^2+x+1}+10i\right) + \frac{1}{2}\sqrt{-8i-6}\log\left(-10\,x+(i+3)\,\sqrt{-8i-6}+10\,\sqrt{x^2+x+1}-10i\right) - \frac{1}{2}\sqrt{-8i-6}\log\left(-10\,x-(i+3)\,\sqrt{-8i-6}+10\,\sqrt{x^2+x+1}-10i\right)$$

input
```
integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(8*I - 6)*log(-10*x - (I - 3)*sqrt(8*I - 6) + 10*sqrt(x^2 + x + 1)
 + 10*I) - 1/2*sqrt(8*I - 6)*log(-10*x + (I - 3)*sqrt(8*I - 6) + 10*sqrt(x
^2 + x + 1) + 10*I) + 1/2*sqrt(-8*I - 6)*log(-10*x + (I + 3)*sqrt(-8*I - 6
) + 10*sqrt(x^2 + x + 1) - 10*I) - 1/2*sqrt(-8*I - 6)*log(-10*x - (I + 3)*
sqrt(-8*I - 6) + 10*sqrt(x^2 + x + 1) - 10*I)
```

3.246.6 Sympy [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}\,dx$$

input
```
integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

output
```
Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)
```

3.246.7 Maxima [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{\sqrt{x^2+x+1}(x^2+1)}\,dx$$

input
```
integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

3.246.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.71

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$$
$$= -\frac{1}{2}\sqrt{2}\Big(\pi+4\arctan\Big(-\Big(x-\sqrt{x^2+x+1}\Big)\Big(\sqrt{2}+2\Big)-\sqrt{2}-1\Big)\Big)$$
$$+\frac{1}{2}\sqrt{2}\Big(\pi+4\arctan\Big(\Big(x-\sqrt{x^2+x+1}\Big)\Big(\sqrt{2}-2\Big)+\sqrt{2}-1\Big)\Big)$$
$$-\frac{1}{2}\sqrt{2}\log\left(\left(x+\sqrt{2}-\sqrt{x^2+x+1}-1\right)^2+\left(x-\sqrt{x^2+x+1}+1\right)^2\right)$$
$$+\frac{1}{2}\sqrt{2}\log\left(\left(x-\sqrt{2}-\sqrt{x^2+x+1}-1\right)^2+\left(x-\sqrt{x^2+x+1}+1\right)^2\right)$$

input
```
integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(pi + 4*arctan(-(x - sqrt(x^2 + x + 1))*(sqrt(2) + 2) - sqrt(
2) - 1)) + 1/2*sqrt(2)*(pi + 4*arctan((x - sqrt(x^2 + x + 1))*(sqrt(2) - 2
) + sqrt(2) - 1)) - 1/2*sqrt(2)*log((x + sqrt(2) - sqrt(x^2 + x + 1) - 1)^
2 + (x - sqrt(x^2 + x + 1) + 1)^2) + 1/2*sqrt(2)*log((x - sqrt(2) - sqrt(x
^2 + x + 1) - 1)^2 + (x - sqrt(x^2 + x + 1) + 1)^2)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{(x^2+1)\,\sqrt{x^2+x+1}}\,dx$$

input
```
int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)),x)
```

output
```
int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)), x)
```

3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)}\,dx$

3.247.1 Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = -\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right)}{6\sqrt{14}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right)}{3\sqrt{7}}$$

output
```
-1/21*arctanh((1+x)*7^(1/2)/(x^2+6*x-1)^(1/2))*7^(1/2)-5/84*arctan(1/4*(2-
x)*7^(1/2)*2^(1/2)/(x^2+6*x-1)^(1/2))*14^(1/2)
```

3.247.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \text{RootSum}\Big[171-104\#1+46\#1^2-8\#1^3 +3\#1^4\&, \frac{4\log\left(-x+\sqrt{-1+6x+x^2}-\#1\right)-\log\left(-x+\sqrt{-1+6x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{-1\right.}}{-26+23\#1-6\#1^2+3\#1^3}$$

input
```
Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

output
```
RootSum[171 - 104*#1 + 46*#1^2 - 8*#1^3 + 3*#1^4 & , (4*Log[-x + Sqrt[-1 +
 6*x + x^2] - #1] - Log[-x + Sqrt[-1 + 6*x + x^2] - #1]*#1 + Log[-x + Sqrt
[-1 + 6*x + x^2] - #1]*#1^2)/(-26 + 23*#1 - 6*#1^2 + 3*#1^3) & ]
```

3.247.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1368, 27, 1362, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{42}\int -\frac{14(2-x)}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx-\frac{1}{42}\int -\frac{70(x+1)}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{3}\int \frac{x+1}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx-\frac{1}{3}\int \frac{2-x}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx$$

$$\Big\downarrow \text{1362}$$

$$\frac{40}{3}\int \frac{1}{\frac{112(2-x)^2}{x^2+6x-1}+128}d\left(-\frac{2(2-x)}{\sqrt{x^2+6x-1}}\right)+\frac{64}{3}\int \frac{1}{\frac{7168(x+1)^2}{x^2+6x-1}-1024}d\frac{16(x+1)}{\sqrt{x^2+6x-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{64}{3}\int \frac{1}{\frac{7168(x+1)^2}{x^2+6x-1}-1024}d\frac{16(x+1)}{\sqrt{x^2+6x-1}}-\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\text{arctanh}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

input
```
Int[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

output
```
(-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt[14]) - A
rcTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7])
```

3.247.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.247.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(53) = 106$.

Time = 1.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

method	result
default	$\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\left(5\sqrt{14}\,\arctan\left(\frac{\sqrt{14}\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,(-2+x)}{4\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)(-1-x)}\right)-4\sqrt{7}\,\text{arctanh}\left(\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,\sqrt{7}}{21}\right)\right)}{84\sqrt{-\frac{3\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)}{\left(\frac{-2+x}{-1-x}+1\right)^2}}\left(\frac{-2+x}{-1-x}+1\right)}$
trager	$-\frac{672\ln\left(\frac{159860736\,\text{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)^5 x+2221632\,\text{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)^3 x-4044096\,\text{RootOf}\left(451584\_\right.}{}\right.}{}$

input
```
int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(5*14^(1/2)*arctan(1/4*14^(1/2)*(-6*(
-2+x)^2/(-1-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x)/(-1-x))-4*7^(1/2
)*arctanh(1/21*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*7^(1/2)))/(-3*(2*(-2+x)^2/(
-1-x)^2-5)/((-2+x)/(-1-x)+1)^2)^(1/2)/((-2+x)/(-1-x)+1)
```

3.247.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.27

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx$$
$$= \frac{1}{168}\sqrt{14}\sqrt{20i\sqrt{2}-17}\log\left(\sqrt{14}\sqrt{20i\sqrt{2}-17}\left(13i\sqrt{2}+16\right)-198x-132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$
$$-\frac{1}{168}\sqrt{14}\sqrt{20i\sqrt{2}-17}\log\left(\sqrt{14}\sqrt{20i\sqrt{2}-17}\left(-13i\sqrt{2}-16\right)-198x-132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$
$$-\frac{1}{168}\sqrt{14}\sqrt{-20i\sqrt{2}-17}\log\left(\sqrt{14}\left(13i\sqrt{2}-16\right)\sqrt{-20i\sqrt{2}-17}-198x+132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$
$$+\frac{1}{168}\sqrt{14}\sqrt{-20i\sqrt{2}-17}\log\left(\sqrt{14}\left(-13i\sqrt{2}+16\right)\sqrt{-20i\sqrt{2}-17}-198x+132i\sqrt{2}+198\sqrt{x^2+6x-1}-132\right)$$

input
```
integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="fricas")
```

output
```
1/168*sqrt(14)*sqrt(20*I*sqrt(2) - 17)*log(sqrt(14)*sqrt(20*I*sqrt(2) - 17
)*(13*I*sqrt(2) + 16) - 198*x - 132*I*sqrt(2) + 198*sqrt(x^2 + 6*x - 1) -
132) - 1/168*sqrt(14)*sqrt(20*I*sqrt(2) - 17)*log(sqrt(14)*sqrt(20*I*sqrt(
2) - 17)*(-13*I*sqrt(2) - 16) - 198*x - 132*I*sqrt(2) + 198*sqrt(x^2 + 6*x
 - 1) - 132) - 1/168*sqrt(14)*sqrt(-20*I*sqrt(2) - 17)*log(sqrt(14)*(13*I*
sqrt(2) - 16)*sqrt(-20*I*sqrt(2) - 17) - 198*x + 132*I*sqrt(2) + 198*sqrt(
x^2 + 6*x - 1) - 132) + 1/168*sqrt(14)*sqrt(-20*I*sqrt(2) - 17)*log(sqrt(1
4)*(-13*I*sqrt(2) + 16)*sqrt(-20*I*sqrt(2) - 17) - 198*x + 132*I*sqrt(2) +
 198*sqrt(x^2 + 6*x - 1) - 132)
```

3.247. $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)}\,dx$

3.247.6 Sympy [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx=\int \frac{2x+1}{\sqrt{x^2+6x-1}\cdot(3x^2+4x+4)}\,dx$$

input
```
integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
```

output
```
Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)
```

3.247.7 Maxima [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx=\int \frac{2\,x+1}{(3\,x^2+4\,x+4)\sqrt{x^2+6\,x-1}}\,dx$$

input
```
integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

3.247.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(51) = 102$.

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\begin{aligned}\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = &-\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan(2)+\arctan\left(\frac{1}{8}\left(x-\sqrt{x^2+6\,x-1}\right)\left(\sqrt{14}+\sqrt{2}\right)+\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right)\\&+\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan\left(\frac{1}{2}\right)+\arctan\left(-\frac{1}{8}\left(x-\sqrt{x^2+6\,x-1}\right)\left(\sqrt{14}-\sqrt{2}\right)-\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right)\\&+\frac{1}{42}\sqrt{7}\log\left(4\left(4\sqrt{7}\sqrt{2}+3\,x+\sqrt{7}-4\sqrt{2}-3\sqrt{x^2+6\,x-1}+2\right)^2\right.\\&\left.+16\left(\sqrt{7}\sqrt{2}-3\,x-\sqrt{7}-\sqrt{2}+3\sqrt{x^2+6\,x-1}-2\right)^2\right)\\&-\frac{1}{42}\sqrt{7}\log\left(4\left(4\sqrt{7}\sqrt{2}+3\,x-\sqrt{7}+4\sqrt{2}-3\sqrt{x^2+6\,x-1}+2\right)^2\right.\\&\left.+16\left(\sqrt{7}\sqrt{2}-3\,x+\sqrt{7}+\sqrt{2}+3\sqrt{x^2+6\,x-1}-2\right)^2\right)\end{aligned}$$

input
```
integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="giac")
```

output
```
-5/84*sqrt(7)*sqrt(2)*(arctan(2) + arctan(1/8*(x - sqrt(x^2 + 6*x - 1))*(s
qrt(14) + sqrt(2)) + 1/8*sqrt(14) + 3/8*sqrt(2))) + 5/84*sqrt(7)*sqrt(2)*(
arctan(1/2) + arctan(-1/8*(x - sqrt(x^2 + 6*x - 1))*(sqrt(14) - sqrt(2)) -
 1/8*sqrt(14) + 3/8*sqrt(2))) + 1/42*sqrt(7)*log(4*(4*sqrt(7)*sqrt(2) + 3*
x + sqrt(7) - 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) + 2)^2 + 16*(sqrt(7)*sqrt(
2) - 3*x - sqrt(7) - sqrt(2) + 3*sqrt(x^2 + 6*x - 1) - 2)^2) - 1/42*sqrt(7
)*log(4*(4*sqrt(7)*sqrt(2) + 3*x - sqrt(7) + 4*sqrt(2) - 3*sqrt(x^2 + 6*x
- 1) + 2)^2 + 16*(sqrt(7)*sqrt(2) - 3*x + sqrt(7) + sqrt(2) + 3*sqrt(x^2 +
 6*x - 1) - 2)^2)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+6\,x-1}\,(3\,x^2+4\,x+4)}\,dx$$

input
```
int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)),x)
```

output
```
int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)), x)
```

3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

3.248.1 Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = -\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right)}{\sqrt{35}} - \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

output
```
-1/35*(2*A+B)*arctan((2-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)-1/70*
(A+B)*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)
```

3.248.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$= \frac{((1+4i)A+(1+2i)B)\text{arctanh}\left(\frac{(4-i)\sqrt{10}-(2-i)\sqrt{10}x+(2-i)\sqrt{13-22x+10x^2}}{\sqrt{35}}\right)+((1-4i)A+(1-2i)B)\text{arctanh}}{2\sqrt{35}}$$

input
```
Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
(((1 + 4*I)*A + (1 + 2*I)*B)*ArcTanh[((4 - I)*Sqrt[10] - (2 - I)*Sqrt[10]*
x + (2 - I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]] + ((1 - 4*I)*A + (1 - 2*I)
*B)*ArcTanh[((4 + I)*Sqrt[10] - (2 + I)*Sqrt[10]*x + (2 + I)*Sqrt[13 - 22*
x + 10*x^2])/Sqrt[35]])/(2*Sqrt[35])
```

3.248.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax+B}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{70}\int \frac{70(A+B)(2-x)}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx-\frac{1}{70}\int \frac{70(2A+B-(2A+B)x)}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{27}$$

$$(A+B)\int \frac{2-x}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx-\int \frac{2A+B-(2A+B)x}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{1362}$$

$$32(2A+B)^2\int \frac{1}{-\frac{8960(2-x)^2(2A+B)^2}{10x^2-22x+13}-256(2A+B)^2}d\frac{8(2A+B)(2-x)}{\sqrt{10x^2-22x+13}}+8(A+B)\int \frac{1}{64-\frac{560(1-x)^2}{10x^2-22x+13}}d\left(-\frac{2(1-x)}{\sqrt{10x^2-22x+13}}\right)$$

$$\Big\downarrow \text{217}$$

$$8(A+B)\int \frac{1}{64-\frac{560(1-x)^2}{10x^2-22x+13}}d\left(-\frac{2(1-x)}{\sqrt{10x^2-22x+13}}\right)-\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

input
```
Int[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
-(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])/Sqrt[35]
) - ((A + B)*ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])])/(2*
Sqrt[35])
```

3.248.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.248. $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

3.248.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(64) = 128$.

Time = 0.94 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.40

method	result
default	$\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\left(\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)A-4\arctan\left(\frac{\sqrt{35}\,(-2+x)}{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,(1-x)}\right)A+\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)B-2\operatorname{arcta}}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(\frac{-2+x}{1-x}+1\right)^2}}\left(\frac{-2+x}{1-x}+1\right)}$

input
```
int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/(1-x)^2+9
)^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1
-x))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*B-2*arctan(35^(1/
2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*B)/(((-2+x)^2/(1-x)^2+9)/((-2+
x)/(1-x)+1)^2)^(1/2)/((-2+x)/(1-x)+1)
```

3.248.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(61) = 122$.

Time = 0.34 (sec) , antiderivative size = 1339, normalized size of antiderivative = 16.74

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fri
cas")
```

output
```
1/280*sqrt(35)*sqrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B -
13*A^2*B^2 - 6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^2*B^2 +
2310*A*B^3 + 350*B^4 + (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A
*B^3 - B^4)*(15*A + 7*B) - 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*B^2 + 3*B^3
))*sqrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 -
 6*A*B^3 - B^4))*sqrt(10*x^2 - 22*x + 13) - 71*(34*A^4 + 87*A^3*B + 81*A^2
*B^2 + 33*A*B^3 + 5*B^4)*x + 2*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6*A*B
^3 - B^4)*(221*A^2 + 234*A*B + 65*B^2 - 11*(17*A^2 + 18*A*B + 5*B^2)*x))/x
) - 1/280*sqrt(35)*sqrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*
B - 13*A^2*B^2 - 6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^2*B^
2 + 2310*A*B^3 + 350*B^4 - (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 -
 6*A*B^3 - B^4)*(15*A + 7*B) - 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*B^2 + 3
*B^3))*sqrt(-15*A^2 - 14*A*B - 3*B^2 + 4*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B
^2 - 6*A*B^3 - B^4))*sqrt(10*x^2 - 22*x + 13) - 71*(34*A^4 + 87*A^3*B + 81
*A^2*B^2 + 33*A*B^3 + 5*B^4)*x + 2*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B^2 - 6
*A*B^3 - B^4)*(221*A^2 + 234*A*B + 65*B^2 - 11*(17*A^2 + 18*A*B + 5*B^2)*x
))/x) - 1/280*sqrt(35)*sqrt(-15*A^2 - 14*A*B - 3*B^2 - 4*sqrt(-4*A^4 - 12*
A^3*B - 13*A^2*B^2 - 6*A*B^3 - B^4))*log(-(2380*A^4 + 6090*A^3*B + 5670*A^
2*B^2 + 2310*A*B^3 + 350*B^4 + (sqrt(35)*sqrt(-4*A^4 - 12*A^3*B - 13*A^2*B
^2 - 6*A*B^3 - B^4)*(15*A + 7*B) + 2*sqrt(35)*(8*A^3 + 18*A^2*B + 13*A*...
```

3.248.6 Sympy [F]

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{Ax+B}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

input
```
integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

output
```
Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

3.248.7 Maxima [F]

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{Ax+B}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

input
```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

3.248.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(61) = 122$.

Time = 0.42 (sec) , antiderivative size = 629, normalized size of antiderivative = 7.86

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$= \frac{2\sqrt{35}(2\,A^2+3\,AB+B^2)\sqrt{A^2+2\,AB+B^2}\left(\arctan(3)+\arctan\left(-\frac{5\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)\left(300\sqrt{14}-1129\right)-}{2329\sqrt{35}-4358\sqrt{10}}\right.\right.}{35\left(15\,A^2+14\,AB+3\,B^2-\sqrt{289\,A^4+612\,A^3B+494\,A^2B^2+180\,AB^3+25\,B^4}\right)}$$
$$-\frac{2\sqrt{35}(2\,A^2+3\,AB+B^2)\sqrt{A^2+2\,AB+B^2}\left(\arctan\left(\frac{1}{7}\right)+\arctan\left(-\frac{5\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)\left(62556\sqrt{14}+2\right.}{496201\sqrt{35}+92}\right.\right.}{35\left(15\,A^2+14\,AB+3\,B^2-\sqrt{289\,A^4+612\,A^3B+494\,A^2B^2+180\,AB^3+2\right.}$$
$$+\frac{1}{140}\sqrt{35}\sqrt{A^2+2\,AB+B^2}\log\left(25\left(546\sqrt{14}\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)+2807\sqrt{10}x-234\sqrt{35}\right.\right.$$
$$+25\left(78\sqrt{14}\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)+401\sqrt{10}x+48\sqrt{35}\sqrt{14}+208\sqrt{14}\sqrt{10}+141\sqrt{35}+611\right.$$
$$-\frac{1}{140}\sqrt{35}\sqrt{A^2+2\,AB+B^2}\log\left(625\left(18\sqrt{14}\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)-75\sqrt{10}x+8\sqrt{35}\sqrt{14}-\right.\right.$$
$$+625\left(6\sqrt{14}\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)-25\sqrt{10}x+6\sqrt{35}\sqrt{14}-18\sqrt{14}\sqrt{10}-25\sqrt{35}+75\sqrt{10}-\right.$$

input
```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="gia
c")
```

output
```
2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arctan(3) + a
rctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(300*sqrt(14) - 1129) -
7658*sqrt(35) + 14361*sqrt(10))/(2329*sqrt(35) - 4358*sqrt(10))))/(15*A^2
+ 14*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*A^2*B^2 + 180*A*B^3 + 25
*B^4)) - 2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arct
an(1/7) + arctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(62556*sqrt(1
4) + 245977) - 1617962*sqrt(35) - 3089577*sqrt(10))/(496201*sqrt(35) + 929
846*sqrt(10))))/(15*A^2 + 14*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*
A^2*B^2 + 180*A*B^3 + 25*B^4)) + 1/140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*lo
g(25*(546*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 2807*sqrt(10)
*x - 234*sqrt(35)*sqrt(14) - 1014*sqrt(14)*sqrt(10) - 1203*sqrt(35) - 5213
*sqrt(10) - 2807*sqrt(10*x^2 - 22*x + 13))^2 + 25*(78*sqrt(14)*(sqrt(10)*x
 - sqrt(10*x^2 - 22*x + 13)) + 401*sqrt(10)*x + 48*sqrt(35)*sqrt(14) + 208
*sqrt(14)*sqrt(10) + 141*sqrt(35) + 611*sqrt(10) - 401*sqrt(10*x^2 - 22*x
+ 13))^2) - 1/140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*log(625*(18*sqrt(14)*(s
qrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 75*sqrt(10)*x + 8*sqrt(35)*sqrt(14
) - 24*sqrt(14)*sqrt(10) - 37*sqrt(35) + 111*sqrt(10) + 75*sqrt(10*x^2 - 2
2*x + 13))^2 + 625*(6*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 2
5*sqrt(10)*x + 6*sqrt(35)*sqrt(14) - 18*sqrt(14)*sqrt(10) - 25*sqrt(35) +
75*sqrt(10) + 25*sqrt(10*x^2 - 22*x + 13))^2)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx=\int \frac{B+A\,x}{\left(5\,x^2-18\,x+17\right)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

input
```
int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

output
```
int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

3.249.1 Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

output
```
1/70*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)
```

3.249.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. $2(38) = 76$.

Time = 0.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$=-\frac{\operatorname{arctanh}\left(\frac{-135+145x-50x^2+\sqrt{10}(-9+5x)\sqrt{13-22x+10x^2}}{-20\sqrt{14}+10\sqrt{14}x-2\sqrt{35}\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

input
```
Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
-1/2*ArcTanh[(-135 + 145*x - 50*x^2 + Sqrt[10]*(-9 + 5*x)*Sqrt[13 - 22*x +
 10*x^2])/(-20*Sqrt[14] + 10*Sqrt[14]*x - 2*Sqrt[35]*Sqrt[13 - 22*x + 10*x
^2])]/Sqrt[35]
```

3.249.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1362, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{1362}$$

$$8\int \frac{1}{64-\frac{560(1-x)^2}{10x^2-22x+13}}d\frac{2(1-x)}{\sqrt{10x^2-22x+13}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

input
```
Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])
```

3.249.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

3.249.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{75\operatorname{RootOf}\left(\_Z^2-35\right)x^2-158\operatorname{RootOf}\left(\_Z^2-35\right)x+140\sqrt{10x^2-22x+13}\,x+87\operatorname{RootOf}\left(\_Z^2-35\right)-140\sqrt{10x^2}}{5x^2-18x+17}\right.}{140}$
default	$-\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\,\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(\frac{-2+x}{1-x}+1\right)^2}}\left(\frac{-2+x}{1-x}+1\right)}$

input
```
int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/140*RootOf(_Z^2-35)*ln(-(75*RootOf(_Z^2-35)*x^2-158*RootOf(_Z^2-35)*x+1
40*(10*x^2-22*x+13)^(1/2)*x+87*RootOf(_Z^2-35)-140*(10*x^2-22*x+13)^(1/2))
/(5*x^2-18*x+17))
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(26) = 52.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$=\frac{1}{280}\sqrt{35}\log\left(\frac{11225\,x^4-47220\,x^3-8\sqrt{35}(75\,x^3-233\,x^2+245\,x-87)\sqrt{10\,x^2-22\,x+13}+75534\,x^2}{25\,x^4-180\,x^3+494\,x^2-612\,x+289}\right.$$

input
```
integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fric
as")
```

output
```
1/280*sqrt(35)*log((11225*x^4 - 47220*x^3 - 8*sqrt(35)*(75*x^3 - 233*x^2 +
 245*x - 87)*sqrt(10*x^2 - 22*x + 13) + 75534*x^2 - 54372*x + 14849)/(25*x
^4 - 180*x^3 + 494*x^2 - 612*x + 289))
```

3.249.6 Sympy [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

input
```
integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

output
```
Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

3.249.7 Maxima [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

input
```
integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

3.249.8 Giac [F(-1)]

Timed out.

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \text{Timed out}$$

input
```
integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{(5\,x^2-18\,x+17)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

input
```
int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

output
```
int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

3.250 $\int x^4\sqrt{5-x^2}\,dx$

3.250.1 Optimal result

Integrand size = 15, antiderivative size = 65

$$\int x^4\sqrt{5-x^2}\,dx=-\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\arcsin\left(\frac{x}{\sqrt{5}}\right)$$

output
```
125/16*arcsin(1/5*x*5^(1/2))-25/16*x*(-x^2+5)^(1/2)-5/24*x^3*(-x^2+5)^(1/2
)+1/6*x^5*(-x^2+5)^(1/2)
```

3.250.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^4\sqrt{5-x^2}\,dx=\frac{1}{48}x\sqrt{5-x^2}(-75-10x^2+8x^4)-\frac{125}{8}\arctan\left(\frac{x}{\sqrt{5}-\sqrt{5-x^2}}\right)$$

input
```
Integrate[x^4*Sqrt[5 - x^2],x]
```

output
```
(x*Sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4))/48 - (125*ArcTan[x/(Sqrt[5] - Sqr
t[5 - x^2])])/8
```

3.250.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{5-x^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{6}\int \frac{x^4}{\sqrt{5-x^2}}dx+\frac{1}{6}\sqrt{5-x^2}x^5$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{6}\left(\frac{15}{4}\int \frac{x^2}{\sqrt{5-x^2}}dx-\frac{1}{4}x^3\sqrt{5-x^2}\right)+\frac{1}{6}\sqrt{5-x^2}x^5$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{6}\left(\frac{15}{4}\left(\frac{5}{2}\int \frac{1}{\sqrt{5-x^2}}dx-\frac{1}{2}x\sqrt{5-x^2}\right)-\frac{1}{4}x^3\sqrt{5-x^2}\right)+\frac{1}{6}\sqrt{5-x^2}x^5$$

$$\Big\downarrow \text{223}$$

$$\frac{5}{6}\left(\frac{15}{4}\left(\frac{5}{2}\arcsin\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}\right)-\frac{1}{4}x^3\sqrt{5-x^2}\right)+\frac{1}{6}\sqrt{5-x^2}x^5$$

input
```
Int[x^4*Sqrt[5 - x^2],x]
```

output
```
(x^5*Sqrt[5 - x^2])/6 + (5*(-1/4*(x^3*Sqrt[5 - x^2]) + (15*(-1/2*(x*Sqrt[5
 - x^2]) + (5*ArcSin[x/Sqrt[5]])/2))/4))/6
```

3.250.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.250.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{x(8x^4-10x^2-75)(x^2-5)}{48\sqrt{-x^2+5}}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	40
pseudoelliptic	$-\frac{125\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)}{16}+\frac{(8x^5-10x^3-75x)\sqrt{-x^2+5}}{48}$	43
default	$-\frac{x^3(-x^2+5)^{\frac{3}{2}}}{6}-\frac{5x(-x^2+5)^{\frac{3}{2}}}{8}+\frac{25x\sqrt{-x^2+5}}{16}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	49
meijerg	$\frac{125i\left(\frac{i\sqrt{\pi}\,x\sqrt{5}\left(-\frac{8}{5}x^4+2x^2+15\right)\sqrt{-\frac{x^2}{5}+1}}{300}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{4}\right)}{4\sqrt{\pi}}$	52
trager	$\frac{x(8x^4-10x^2-75)\sqrt{-x^2+5}}{48}+\frac{125\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+5}+x\right)}{16}$	53

input
```
int(x^4*(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(8*x^4-10*x^2-75)*(x^2-5)/(-x^2+5)^(1/2)+125/16*arcsin(1/5*x*5^(1/
2))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int x^4\sqrt{5-x^2}\,dx = \frac{1}{48}\left(8\,x^5-10\,x^3-75\,x\right)\sqrt{-x^2+5}-\frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

input
```
integrate(x^4*(-x^2+5)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2 + 5) - 125/16*arctan(sqrt(-x^2 + 5)
/x)
```

3.250.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.35

$$\int x^4\sqrt{5-x^2}\,dx = \begin{cases} \frac{ix^7}{6\sqrt{x^2-5}}-\frac{25ix^5}{24\sqrt{x^2-5}}-\frac{25ix^3}{48\sqrt{x^2-5}}+\frac{125ix}{16\sqrt{x^2-5}}-\frac{125i\operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{for } |x^2|>5 \\ -\frac{x^7}{6\sqrt{5-x^2}}+\frac{25x^5}{24\sqrt{5-x^2}}+\frac{25x^3}{48\sqrt{5-x^2}}-\frac{125x}{16\sqrt{5-x^2}}+\frac{125\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(-x**2+5)**(1/2),x)
```

output
```
Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 - 5)) - 25*
I*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5)) - 125*I*acosh(sqr
t(5)*x/5)/16, Abs(x**2) > 5), (-x**7/(6*sqrt(5 - x**2)) + 25*x**5/(24*sqrt
(5 - x**2)) + 25*x**3/(48*sqrt(5 - x**2)) - 125*x/(16*sqrt(5 - x**2)) + 12
5*asin(sqrt(5)*x/5)/16, True))
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int x^4\sqrt{5-x^2}\,dx = -\frac{1}{6}\left(-x^2+5\right)^{\frac{3}{2}}x^3 - \frac{5}{8}\left(-x^2+5\right)^{\frac{3}{2}}x + \frac{25}{16}\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

input
```
integrate(x^4*(-x^2+5)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(-x^2 + 5)^(3/2)*x^3 - 5/8*(-x^2 + 5)^(3/2)*x + 25/16*sqrt(-x^2 + 5)*
x + 125/16*arcsin(1/5*sqrt(5)*x)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int x^4\sqrt{5-x^2}\,dx = \frac{1}{48}\left(2\left(4x^2-5\right)x^2-75\right)\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

input
```
integrate(x^4*(-x^2+5)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)
*x)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int x^4\sqrt{5-x^2}\,dx = \frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{16} - \sqrt{5-x^2}\left(-\frac{x^5}{6}+\frac{5\,x^3}{24}+\frac{25\,x}{16}\right)$$

input
```
int(x^4*(5 - x^2)^(1/2),x)
```

output
```
(125*asin((5^(1/2)*x)/5))/16 - (5 - x^2)^(1/2)*((25*x)/16 + (5*x^3)/24 - x
^5/6)
```

3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$

3.251.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}-\frac{\sqrt{2+x^2}}{15x}$$

output
```
-1/10*(x^2+2)^(1/2)/x^5+1/15*(x^2+2)^(1/2)/x^3-1/15*(x^2+2)^(1/2)/x
```

3.251.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}(-3+2x^2-2x^4)}{30x^5}$$

input
```
Integrate[1/(x^6*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[2 + x^2]*(-3 + 2*x^2 - 2*x^4))/(30*x^5)
```

3.251.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^2+2}}dx$$

$$\Big\downarrow \ 245$$

$$-\frac{2}{5}\int \frac{1}{x^4\sqrt{x^2+2}}dx-\frac{\sqrt{x^2+2}}{10x^5}$$

$$\Big\downarrow \ 245$$

$$-\frac{2}{5}\left(-\frac{1}{3}\int \frac{1}{x^2\sqrt{x^2+2}}dx-\frac{\sqrt{x^2+2}}{6x^3}\right)-\frac{\sqrt{x^2+2}}{10x^5}$$

$$\Big\downarrow \ 242$$

$$-\frac{\sqrt{x^2+2}}{10x^5}-\frac{2}{5}\left(\frac{\sqrt{x^2+2}}{6x}-\frac{\sqrt{x^2+2}}{6x^3}\right)$$

input
```
Int[1/(x^6*Sqrt[2 + x^2]),x]
```

output
```
-1/10*Sqrt[2 + x^2]/x^5 - (2*(-1/6*Sqrt[2 + x^2]/x^3 + Sqrt[2 + x^2]/(6*x)
))/5
```

3.251.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

3.251.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\sqrt{x^{2}+2}\,(2x^{4}-2x^{2}+3)}{30x^{5}}$	25
trager	$-\frac{\sqrt{x^{2}+2}\,(2x^{4}-2x^{2}+3)}{30x^{5}}$	25
pseudoelliptic	$-\frac{\sqrt{x^{2}+2}\,(2x^{4}-2x^{2}+3)}{30x^{5}}$	25
meijerg	$-\frac{\sqrt{2}\,\left(\frac{2}{3}x^{4}-\frac{2}{3}x^{2}+1\right)\sqrt{1+\frac{x^{2}}{2}}}{10x^{5}}$	30
risch	$-\frac{2x^{6}+2x^{4}-x^{2}+6}{30x^{5}\sqrt{x^{2}+2}}$	30
default	$-\frac{\sqrt{x^{2}+2}}{10x^{5}}+\frac{\sqrt{x^{2}+2}}{15x^{3}}-\frac{\sqrt{x^{2}+2}}{15x}$	38

input
```
int(1/x^6/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(x^2+2)^(1/2)*(2*x^4-2*x^2+3)/x^5
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx=-\frac{2\,x^5+(2\,x^4-2\,x^2+3)\sqrt{x^2+2}}{30\,x^5}$$

input
```
integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(2*x^5 + (2*x^4 - 2*x^2 + 3)*sqrt(x^2 + 2))/x^5
```

3.251.6 Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{1+\frac{2}{x^2}}}{15} + \frac{\sqrt{1+\frac{2}{x^2}}}{15x^2} - \frac{\sqrt{1+\frac{2}{x^2}}}{10x^4}$$

input
```
integrate(1/x**6/(x**2+2)**(1/2),x)
```

output
```
-sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x**2)/(10*x
**4)
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{x^2+2}}{15\,x} + \frac{\sqrt{x^2+2}}{15\,x^3} - \frac{\sqrt{x^2+2}}{10\,x^5}$$

input
```
integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 + 2)/x^5
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = \frac{32\left(5\left(x-\sqrt{x^2+2}\right)^4 - 5\left(x-\sqrt{x^2+2}\right)^2 + 2\right)}{15\left(\left(x-\sqrt{x^2+2}\right)^2 - 2\right)^5}$$

input
```
integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x - sqrt(x
^2 + 2))^2 - 2)^5
```

3.251.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\sqrt{x^2+2}\left(\frac{1}{15\,x}-\frac{1}{15\,x^3}+\frac{1}{10\,x^5}\right)$$

input
```
int(1/(x^6*(x^2 + 2)^(1/2)),x)
```

output
```
-(x^2 + 2)^(1/2)*(1/(15*x) - 1/(15*x^3) + 1/(10*x^5))
```

3.252 $\int \frac{1}{(3+2x^2)^{7/2}} \, dx$

3.252.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(3+2x^2)^{7/2}} \, dx = \frac{x}{15(3+2x^2)^{5/2}} + \frac{4x}{135(3+2x^2)^{3/2}} + \frac{8x}{405\sqrt{3+2x^2}}$$

output
```
1/15*x/(2*x^2+3)^(5/2)+4/135*x/(2*x^2+3)^(3/2)+8/405*x/(2*x^2+3)^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{(3+2x^2)^{7/2}} \, dx = \frac{x(135+120x^2+32x^4)}{405(3+2x^2)^{5/2}}$$

input
```
Integrate[(3 + 2*x^2)^(-7/2),x]
```

output
```
(x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

3.252.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+3)^{7/2}}dx$$
$$\Big\downarrow \text{209}$$
$$\frac{4}{15}\int \frac{1}{(2x^2+3)^{5/2}}dx+\frac{x}{15(2x^2+3)^{5/2}}$$
$$\Big\downarrow \text{209}$$
$$\frac{4}{15}\left(\frac{2}{9}\int \frac{1}{(2x^2+3)^{3/2}}dx+\frac{x}{9(2x^2+3)^{3/2}}\right)+\frac{x}{15(2x^2+3)^{5/2}}$$
$$\Big\downarrow \text{208}$$
$$\frac{x}{15(2x^2+3)^{5/2}}+\frac{4}{15}\left(\frac{2x}{27\sqrt{2x^2+3}}+\frac{x}{9(2x^2+3)^{3/2}}\right)$$

input
```
Int[(3 + 2*x^2)^(-7/2),x]
```

output
```
x/(15*(3 + 2*x^2)^(5/2)) + (4*(x/(9*(3 + 2*x^2)^(3/2)) + (2*x)/(27*Sqrt[3
+ 2*x^2])))/15
```

3.252.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.252.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
trager	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
risch	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
pseudoelliptic	$\frac{32x^5+120x^3+135x}{405(2x^2+3)^{\frac{5}{2}}}$	26
meijerg	$\frac{\sqrt{3}\,x\left(\frac{32}{9}x^4+\frac{40}{3}x^2+15\right)}{1215\left(1+\frac{2x^2}{3}\right)^{\frac{5}{2}}}$	28
default	$\frac{x}{15(2x^2+3)^{\frac{5}{2}}}+\frac{4x}{135(2x^2+3)^{\frac{3}{2}}}+\frac{8x}{405\sqrt{2x^2+3}}$	38

input
```
int(1/(2*x^2+3)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/405*x*(32*x^4+120*x^2+135)/(2*x^2+3)^(5/2)
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{(32\,x^5+120\,x^3+135\,x)\sqrt{2\,x^2+3}}{405\,(8\,x^6+36\,x^4+54\,x^2+27)}$$

input
```
integrate(1/(2*x^2+3)^(7/2),x, algorithm="fricas")
```

output
```
1/405*(32*x^5 + 120*x^3 + 135*x)*sqrt(2*x^2 + 3)/(8*x^6 + 36*x^4 + 54*x^2
+ 27)
```

3.252.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(42) = 84$.

Time = 2.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{32x^5}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}} + \frac{120x^3}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}} + \frac{135x}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}$$

input
```
integrate(1/(2*x**2+3)**(7/2),x)
```

output
```
32*x**5/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x**2 + 3) + 3645*sq
rt(2*x**2 + 3)) + 120*x**3/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*
x**2 + 3) + 3645*sqrt(2*x**2 + 3)) + 135*x/(1620*x**4*sqrt(2*x**2 + 3) + 4
860*x**2*sqrt(2*x**2 + 3) + 3645*sqrt(2*x**2 + 3))
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{8\,x}{405\,\sqrt{2\,x^2+3}} + \frac{4\,x}{135\,(2\,x^2+3)^{\frac{3}{2}}} + \frac{x}{15\,(2\,x^2+3)^{\frac{5}{2}}}$$

input
```
integrate(1/(2*x^2+3)^(7/2),x, algorithm="maxima")
```

output
```
8/405*x/sqrt(2*x^2 + 3) + 4/135*x/(2*x^2 + 3)^(3/2) + 1/15*x/(2*x^2 + 3)^(
5/2)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{(8\,(4\,x^2+15)x^2+135)x}{405\,(2\,x^2+3)^{\frac{5}{2}}}$$

input
```
integrate(1/(2*x^2+3)^(7/2),x, algorithm="giac")
```

output
```
1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^(5/2)
```

3.252.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.82

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{2\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{405\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)} + \frac{2\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{405\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)} + \frac{\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{1440\left(-x^3+\frac{3\mathrm{i}\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}-\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)} + \frac{\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{1440\left(-x^3-\frac{3\mathrm{i}\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}+\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)} + \frac{\sqrt{2}\sqrt{6}\sqrt{x^2+\frac{3}{2}}\,19\mathrm{i}}{25920\left(x^2+1\mathrm{i}\sqrt{6}\,x-\frac{3}{2}\right)} + \frac{\sqrt{2}\sqrt{6}\sqrt{x^2+\frac{3}{2}}\,19\mathrm{i}}{25920\left(-x^2+1\mathrm{i}\sqrt{6}\,x+\frac{3}{2}\right)}$$

input
```
int(1/(2*x^2 + 3)^(7/2),x)
```

output
```
(2*2^(1/2)*(x^2 + 3/2)^(1/2))/(405*(x - (6^(1/2)*1i)/2)) + (2*2^(1/2)*(x^2
 + 3/2)^(1/2))/(405*(x + (6^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 3/2)^(1/2))/(1
440*((9*x)/2 - (6^(1/2)*3i)/4 + (6^(1/2)*x^2*3i)/2 - x^3)) + (2^(1/2)*(x^2
 + 3/2)^(1/2))/(1440*((9*x)/2 + (6^(1/2)*3i)/4 - (6^(1/2)*x^2*3i)/2 - x^3)
) + (2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i + x^2 - 3
/2)) + (2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i - x^2
+ 3/2))
```

3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}} \, dx$

3.253.1 Optimal result

Integrand size = 20, antiderivative size = 12

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}} \, dx = \log \left(a+\sqrt{1+x^2}\right)$$

output
```
ln(a+(x^2+1)^(1/2))
```

3.253.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}} \, dx = \log \left(a+\sqrt{1+x^2}\right)$$

input
```
Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

output
```
Log[a + Sqrt[1 + x^2]]
```

3.253.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a\sqrt{x^2+1}+x^2+1}dx$$
$$\Big\downarrow \text{2586}$$
$$\frac{1}{2}\int \frac{1}{x^2+a\sqrt{x^2+1}+1}dx^2$$
$$\Big\downarrow \text{7267}$$
$$\int \frac{1}{a+\sqrt{x^2+1}}d\sqrt{x^2+1}$$
$$\Big\downarrow \text{16}$$
$$\log\left(a+\sqrt{x^2+1}\right)$$

input
```
Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

output
```
Log[a + Sqrt[1 + x^2]]
```

3.253.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.253.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 354, normalized size of antiderivative = 29.50

method	result
default	$\frac{\sqrt{x^2+1}}{a} - \frac{\sqrt{\left(x-\sqrt{(1+a)(a-1)}\right)^2+2\sqrt{(1+a)(a-1)}\left(x-\sqrt{(1+a)(a-1)}\right)+a^2}}{2a} + \frac{a\ln\left(\frac{2a^2+2\sqrt{(1+a)(a-1)}\left(x-\sqrt{(1+a)(a-1)}\right)+2\sqrt{a^2}}{}\right.}{}$

input
```
int(x/(1+x^2+a*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(x^2+1)^(1/2)-1/2/a*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1))^(1/2)*(
x-((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2+2*((1+a)*(a-
1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x-((1+a)*(a-1))^(1/2))^2
+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x-((1+a)*(a-1)
)^(1/2)))-1/2/a*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)
*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2-2*((1+a)*(a-1))^(1/2
)*(x+((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a
)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x+((1+a)*(a-1))^(1/2))
)+1/2/a^2*ln(-a^2+x^2+1)-1/2*(-a^2+1)/a^2*ln(-a^2+x^2+1)
```

3.253.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 5.17

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \frac{1}{2}\log\left(-a^2+x^2+1\right) - \frac{1}{2}\log\left(ax+x^2-\sqrt{x^2+1}(a+x)+1\right) + \frac{1}{2}\log\left(-ax+x^2+\sqrt{x^2+1}(a-x)+1\right)$$

input
```
integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a + x) + 1) +
 1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)
```

3.253.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = -a\left(-\frac{\log\left(2a+2\sqrt{x^2+1}\right)}{2a}+\frac{\log\left(-2\sqrt{x^2+1}\right)}{2a}\right) + \frac{\log\left(2a\sqrt{x^2+1}+2x^2+2\right)}{2}$$

input
```
integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)
```

output
```
-a*(-log(2*a + 2*sqrt(x**2 + 1))/(2*a) + log(-2*sqrt(x**2 + 1))/(2*a)) + l
og(2*a*sqrt(x**2 + 1) + 2*x**2 + 2)/2
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(a+\sqrt{x^2+1}\right)$$

input
```
integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
log(a + sqrt(x^2 + 1))
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(\left|a+\sqrt{x^2+1}\right|\right)$$

input
```
integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
log(abs(a + sqrt(x^2 + 1)))
```

3.253.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 12.83

$$\begin{aligned}\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx &= \frac{\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)}{2}+\frac{\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)}{2}\\ &\quad-\frac{a\left(\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}-x\,\sqrt{a-1}\,\sqrt{a+1}+1\right)\right)}{2\,\sqrt{(a-1)\,(a+1)+1}}\\ &\quad-\frac{a\left(\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}+x\,\sqrt{a-1}\,\sqrt{a+1}+1\right)\right)}{2\,\sqrt{(a-1)\,(a+1)+1}}\end{aligned}$$

input
```
int(x/(a*(x^2 + 1)^(1/2) + x^2 + 1),x)
```

output
```
log(x + (a - 1)^(1/2)*(a + 1)^(1/2))/2 + log(x - (a - 1)^(1/2)*(a + 1)^(1/
2))/2 - (a*(log(x + (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^
2)^(1/2) - x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(
1/2)) - (a*(log(x - (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^
2)^(1/2) + x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(
1/2))
```

3.254 $\int \frac{1-x+x^2}{(1+x^2)^{3/2}} dx$

3.254.1 Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}} dx = \frac{1}{\sqrt{1+x^2}} + \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)+1/(x^2+1)^(1/2)
```

3.254.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}} dx = \frac{1}{\sqrt{1+x^2}} - \log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

output
```
1/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

3.254.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2345, 25, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-x+1}{(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{\sqrt{x^2+1}}-\int -\frac{1}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{\sqrt{x^2+1}}dx+\frac{1}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{222}$$

$$\text{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$$

input
```
Int[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

output
```
1/Sqrt[1 + x^2] + ArcSinh[x]
```

3.254.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.254.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\operatorname{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$	11
risch	$\operatorname{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$	11
trager	$\frac{1}{\sqrt{x^2+1}}-\ln\left(-\sqrt{x^2+1}+x\right)$	23
meijerg	$\frac{x}{\sqrt{x^2+1}}+\frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^2+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^2+1}}}{\sqrt{\pi}}$	56

input
```
int((x^2-x+1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)+1/(x^2+1)^(1/2)
```

3.254.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx=-\frac{(x^2+1)\log\left(-x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}}{x^2+1}$$

input
```
integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-((x^2 + 1)*log(-x + sqrt(x^2 + 1)) - sqrt(x^2 + 1))/(x^2 + 1)
```

3.254.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 4.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{x^2\,\mathrm{asinh}\,(x)}{x^2+1} + \frac{\mathrm{asinh}\,(x)}{x^2+1} + \frac{1}{\sqrt{x^2+1}}$$

input
```
integrate((x**2-x+1)/(x**2+1)**(3/2),x)
```

output
```
x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{x^2+1}} + \mathrm{arsinh}\,(x)$$

input
```
integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
1/sqrt(x^2 + 1) + arcsinh(x)
```

3.254.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{x^2+1}} - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
1/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

3.254.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{\operatorname{asinh}(x)+x^2\operatorname{asinh}(x)+\sqrt{x^2+1}}{x^2+1}$$

input
```
int((x^2 - x + 1)/(x^2 + 1)^(3/2),x)
```

output
```
(asinh(x) + x^2*asinh(x) + (x^2 + 1)^(1/2))/(x^2 + 1)
```

3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx$

3.255.1 Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \operatorname{arcsinh}(x) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}$$

output
```
arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}} - \log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/(2 + x^2),x]
```

output
```
-(ArcTanh[(2 + x^2 - x*Sqrt[1 + x^2])/Sqrt[2]]/Sqrt[2]) - Log[-x + Sqrt[1
+ x^2]]
```

3.255.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {301, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

$$\Big\downarrow \ 301$$

$$\int \frac{1}{\sqrt{x^2+1}}dx-\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)}dx$$

$$\Big\downarrow \ 222$$

$$\text{arcsinh}(x)-\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)}dx$$

$$\Big\downarrow \ 291$$

$$\text{arcsinh}(x)-\int \frac{1}{2-\frac{x^2}{x^2+1}}d\frac{x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \ 219$$

$$\text{arcsinh}(x)-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[1 + x^2]/(2 + x^2),x]
```

output
```
ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

3.255.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.255.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\operatorname{arcsinh}(x)-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{2}$	23
pseudoelliptic	$\frac{\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+1}}{x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x^2+1}-x}{x}\right)}{2}$	56
trager	$-\ln\left(-\sqrt{x^2+1}+x\right)-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-2\right)x^2+4x\sqrt{x^2+1}+2\operatorname{RootOf}\left(\_Z^2-2\right)}{x^2+2}\right)}{4}$	63

input
```
int((x^2+1)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

3.255.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{9\,x^2 - 2\sqrt{2}(3\,x^2+2) - 2\sqrt{x^2+1}\left(3\sqrt{2}x - 4\,x\right) + 6}{x^2+2}\right) - \log\left(-x + \sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 2) - 2*sqrt(x^2 + 1)*(3*sqrt(2
)*x - 4*x) + 6)/(x^2 + 2)) - log(-x + sqrt(x^2 + 1))
```

3.255.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(x**2+2),x)
```

output
```
Integral(sqrt(x**2 + 1)/(x**2 + 2), x)
```

3.255.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*x^4 + 8*x
^2 + 4), x)
```

3.255.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1
))^2 + 2*sqrt(2) + 3)) - log(-x + sqrt(x^2 + 1))
```

3.255.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \operatorname{asinh}(x)+\frac{\sqrt{2}\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4} - \frac{\sqrt{2}\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4}$$

input
```
int((x^2 + 1)^(1/2)/(x^2 + 2),x)
```

output
```
asinh(x) + (2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1
/2)*1i + 1)))/4 - (2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i -
 2^(1/2)*x*1i + 1)))/4
```

3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx$

3.256.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{x\sqrt{1+x^2}}{4\,(2+x^2)} + \frac{3\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}$$

output
```
3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+
2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \frac{1}{8}\left(-\frac{2x\sqrt{1+x^2}}{2+x^2} + 3\sqrt{2}\operatorname{arctanh}\left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}}\right)\right)$$

input
```
Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

output
```
((-2*x*Sqrt[1 + x^2])/(2 + x^2) + 3*Sqrt[2]*ArcTanh[(2 + x^2 - x*Sqrt[1 +
x^2])/Sqrt[2]])/8
```

3.256.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}dx$$

$$\Big\downarrow \ 296$$

$$\frac{3}{4}\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)}dx-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

$$\Big\downarrow \ 291$$

$$\frac{3}{4}\int \frac{1}{2-\frac{x^2}{x^2+1}}d\frac{x}{\sqrt{x^2+1}}-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

$$\Big\downarrow \ 219$$

$$\frac{3\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

input
```
Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

output
```
-1/4*(x*Sqrt[1 + x^2])/(2 + x^2) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/
(4*Sqrt[2])
```

3.256.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.256.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^{2}+1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^{2}+1}}{4(x^{2}+2)}$	38
default	$\frac{x}{4\sqrt{x^{2}+1}\left(\frac{x^{2}}{x^{2}+1}-2\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^{2}+1}}\right)\sqrt{2}}{8}$	46
pseudoelliptic	$\frac{(3x^{2}+6)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^{2}+1}}{x}\right)-2x\sqrt{x^{2}+1}}{8x^{2}+16}$	48
trager	$-\frac{x\sqrt{x^{2}+1}}{4(x^{2}+2)}+\frac{3\,\mathrm{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^{2}-2\right)x^{2}+4x\sqrt{x^{2}+1}+2\,\mathrm{RootOf}\left(\_Z^{2}-2\right)}{x^{2}+2}\right)}{16}$	66

input
```
int(1/(x^2+2)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+
2)
```

3.256.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx$$
$$=\frac{3\,\sqrt{2}(x^2+2)\log\left(\frac{9\,x^2+2\,\sqrt{2}(3\,x^2+2)+2\,\sqrt{x^2+1}\left(3\,\sqrt{2}x+4\,x\right)+6}{x^2+2}\right)-4\,x^2-4\,\sqrt{x^2+1}x-8}{16\,(x^2+2)}$$

input
```
integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(3*sqrt(2)*(x^2 + 2)*log((9*x^2 + 2*sqrt(2)*(3*x^2 + 2) + 2*sqrt(x^2
+ 1)*(3*sqrt(2)*x + 4*x) + 6)/(x^2 + 2)) - 4*x^2 - 4*sqrt(x^2 + 1)*x - 8)/
(x^2 + 2)
```

3.256.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}\,dx$$

input
```
integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)
```

3.256.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \int \frac{1}{(x^2+2)^2\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)
```

3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right) - \frac{3\left(x-\sqrt{x^2+1}\right)^2+1}{2\left(\left(x-\sqrt{x^2+1}\right)^4+6\left(x-\sqrt{x^2+1}\right)^2+1\right)}$$

input
```
integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-3/16*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 +
 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 + 1)/((x - sqrt(x^
2 + 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{3\sqrt{2}\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16} + \frac{3\sqrt{2}\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16} - \frac{\sqrt{x^2+1}}{8\,\left(x-\sqrt{2}\,1\mathrm{i}\right)} - \frac{\sqrt{x^2+1}}{8\,\left(x+\sqrt{2}\,1\mathrm{i}\right)}$$

input
```
int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^2),x)
```

output
```
(3*2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2^(1/2)*x*1i +
1)))/16 - (3*2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(
1/2)*1i + 1)))/16 - (x^2 + 1)^(1/2)/(8*(x - 2^(1/2)*1i)) - (x^2 + 1)^(1/2)
/(8*(x + 2^(1/2)*1i))
```

3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$

3.257.1 Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-2+x^2}}\right) - \sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{-2+x^2}}\right)$$

output
```
arctanh(x/(x^2-2)^(1/2))-1/2*arctanh(1/3*x*6^(1/2)/(x^2-2)^(1/2))*6^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{6-x^2+x\sqrt{-2+x^2}}{2\sqrt{6}}\right) - \log\left(-x+\sqrt{-2+x^2}\right)$$

input
```
Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

output
```
-(Sqrt[3/2]*ArcTanh[(6 - x^2 + x*Sqrt[-2 + x^2])/(2*Sqrt[6])]) - Log[-x +
Sqrt[-2 + x^2]]
```

3.257.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {385, 25, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2-6)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{385}$$

$$\int \frac{1}{\sqrt{x^2-2}}dx+6\int -\frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{\sqrt{x^2-2}}dx-6\int \frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{x^2}{x^2-2}}d\frac{x}{\sqrt{x^2-2}}-6\int \frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-6\int \frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{291}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-6\int \frac{1}{6-\frac{4x^2}{x^2-2}}d\frac{x}{\sqrt{x^2-2}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

input
```
Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

output
```
ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]
]
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

3.257.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\ln\left(\frac{x+\sqrt{x^{2}-2}}{x}\right)}{2}-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{\sqrt{x^{2}-2}\,\sqrt{6}}{2x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x^{2}-2}-x}{x}\right)}{2}$
trager	$-\ln\left(x-\sqrt{x^{2}-2}\right)+\frac{\operatorname{RootOf}\left(\_Z^{2}-6\right)\ln\left(\frac{5\operatorname{RootOf}\left(\_Z^{2}-6\right)x^{2}-12\sqrt{x^{2}-2}\,x-6\operatorname{RootOf}\left(\_Z^{2}-6\right)}{x^{2}-6}\right)}{4}$
default	$\ln\left(x+\sqrt{x^{2}-2}\right)-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{8+2\left(x-\sqrt{6}\right)\sqrt{6}}{4\sqrt{\left(x-\sqrt{6}\right)^{2}+2\left(x-\sqrt{6}\right)\sqrt{6}+4}}\right)}{4}+\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{8-2\left(x+\sqrt{6}\right)\sqrt{6}}{4\sqrt{\left(x+\sqrt{6}\right)^{2}-2\left(x+\sqrt{6}\right)\sqrt{6}+4}}\right)}{4}$

input
```
int(x^2/(x^2-6)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln((x+(x^2-2)^(1/2))/x)-1/2*6^(1/2)*arctanh(1/2*(x^2-2)^(1/2)/x*6^(1/2
))-1/2*ln(((x^2-2)^(1/2)-x)/x)
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(30) = 60$.

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \frac{1}{4}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(5x^2-6)-25x^2+2(5\sqrt{3}\sqrt{2}x-12x)\sqrt{x^2-2}+30}{x^2-6}\right) - \log\left(-x+\sqrt{x^2-2}\right)$$

input
```
integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(5*x^2 - 6) - 25*x^2 + 2*(5*sq
rt(3)*sqrt(2)*x - 12*x)*sqrt(x^2 - 2) + 30)/(x^2 - 6)) - log(-x + sqrt(x^2
 - 2))
```

3.257.6 Sympy [F]

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \int \frac{x^2}{(x^2-6)\sqrt{x^2-2}}\,dx$$

input
```
integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
```

output
```
Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)
```

3.257.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$$
$$= \frac{1}{12}\sqrt{6}\left(2\sqrt{6}\log\left(x+\sqrt{x^2-2}\right)-3\log\left(\sqrt{6}+\frac{4\sqrt{x^2-2}}{|2x-2\sqrt{6}|}+\frac{8}{|2x-2\sqrt{6}|}\right)+3\log\left(-\sqrt{6}+\frac{4\sqrt{x^2}}{|2x+2}\right.\right.$$

input
```
integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) + 4*sqrt(x^
2 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) + 3*log(-sqrt(6) + 4
*sqrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x + 2*sqrt(6))))
```

3.257.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(30) = 60.

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\frac{1}{4}\sqrt{6}\log\left(\frac{\left|2\left(x-\sqrt{x^2-2}\right)^2-8\sqrt{6}-20\right|}{\left|2\left(x-\sqrt{x^2-2}\right)^2+8\sqrt{6}-20\right|}\right) - \frac{1}{2}\log\left(\left(x-\sqrt{x^2-2}\right)^2\right)$$

input
```
integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(6)*log(abs(2*(x - sqrt(x^2 - 2))^2 - 8*sqrt(6) - 20)/abs(2*(x -
sqrt(x^2 - 2))^2 + 8*sqrt(6) - 20)) - 1/2*log((x - sqrt(x^2 - 2))^2)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2-2}\,(x^2-6)}\,dx$$

input
```
int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)),x)
```

output
```
int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)), x)
```

3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2} dx$

3.258.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

output
```
2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)+x*(-x^2+1)^(1/2)/(x^2+1)
```

3.258.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

input
```
Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

output
```
(x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

3.258.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5}{\sqrt{1-x^2}\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x\sqrt{1-x^2}}{x^2+1}-\frac{1}{4}\int -\frac{16}{\sqrt{1-x^2}\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{1}{\sqrt{1-x^2}\left(x^2+1\right)}dx+\frac{\sqrt{1-x^2}x}{x^2+1}$$

$$\Big\downarrow \text{291}$$

$$4\int \frac{1}{\frac{2x^2}{1-x^2}+1}d\frac{x}{\sqrt{1-x^2}}+\frac{\sqrt{1-x^2}x}{x^2+1}$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)+\frac{\sqrt{1-x^2}x}{x^2+1}$$

input
```
Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

output
```
(x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.258.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

method	result	size
pseudoelliptic	$\frac{\left(-2x^{2}-2\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}}{2x}\right)+x\sqrt{-x^{2}+1}}{x^{2}+1}$	50
risch	$-\frac{x\left(x^{2}-1\right)}{\left(x^{2}+1\right)\sqrt{-x^{2}+1}}-2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}\,x}{x^{2}-1}\right)$	53
trager	$\frac{x\sqrt{-x^{2}+1}}{x^{2}+1}-\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}+2\right)x^{2}+4x\sqrt{-x^{2}+1}-\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x^{2}+1}\right)$	69
default	$-2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}\,x}{x^{2}-1}\right)-\frac{\sqrt{-x^{2}+1}\,x}{2\left(x^{2}-1\right)\left(\frac{\left(-x^{2}+1\right)x^{2}}{\left(x^{2}-1\right)^{2}}+\frac{1}{2}\right)}$	70

input
```
int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-2*x^2-2)*2^(1/2)*arctan(1/2/x*2^(1/2)*(-x^2+1)^(1/2))+x*(-x^2+1)^(1/2))
/(x^2+1)
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = -\frac{2\sqrt{2}(x^2+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right) - \sqrt{-x^2+1}x}{x^2+1}$$

input
```
integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) - sqrt(-x^2 + 1
)*x)/(x^2 + 1)
```

3.258.6 Sympy [F]

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \int \frac{x^2+5}{\sqrt{-(x-1)(x+1)}\,(x^2+1)^2}\,dx$$

input
```
integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
```

output
```
Integral((x**2 + 5)/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

3.258.7 Maxima [F]

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \int \frac{x^2+5}{(x^2+1)^2\sqrt{-x^2+1}}\,dx$$

input
```
integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)
```

3.258.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(39) = 78.

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.62

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \sqrt{2}\left(\pi\text{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{2\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2+8}$$

input
```
integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
 1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) -
 1)/x)/((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 8)
```

3.258.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)1\mathrm{i} - \sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)1\mathrm{i} + \frac{\sqrt{1-x^2}}{2\,(x-\mathrm{i})} + \frac{\sqrt{1-x^2}}{2\,(x+1\mathrm{i})}$$

input
```
int((x^2 + 5)/((1 - x^2)^(1/2)*(x^2 + 1)^2),x)
```

output
```
2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i
- 2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i))*1
i + (1 - x^2)^(1/2)/(2*(x - 1i)) + (1 - x^2)^(1/2)/(2*(x + 1i))
```

3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}} \, dx$

3.259.1 Optimal result

Integrand size = 33, antiderivative size = 88

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}} \, dx = -x-4\sqrt{1-x^2}+5\arcsin(x)+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}} - \frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)$$

output
```
-x+5*arcsin(x)+20*ln(5+(-x^2+1)^(1/2))+25/12*arctan(1/12*x*6^(1/2))*6^(1/2
)-25/12*arctan(5/12*x*6^(1/2)/(-x^2+1)^(1/2))*6^(1/2)-4*(-x^2+1)^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}} \, dx = -x-4\sqrt{1-x^2}+10\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right) - \frac{25\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{-1+\sqrt{1-x^2}}\right)}{\sqrt{6}}-20\log\left(-1+\sqrt{1-x^2}\right) +20\log\left(-4-x^2+4\sqrt{1-x^2}\right)$$

input
```
Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

output
```
-x - 4*Sqrt[1 - x^2] + 10*ArcTan[x/(-1 + Sqrt[1 - x^2])] - (25*ArcTan[(Sqr
t[3/2]*x)/(-1 + Sqrt[1 - x^2])])/Sqrt[6] - 20*Log[-1 + Sqrt[1 - x^2]] + 20
*Log[-4 - x^2 + 4*Sqrt[1 - x^2]]
```

3.259.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x-\sqrt{1-x^2}}{\sqrt{1-x^2}+5}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{4x}{\sqrt{1-x^2}+5}-\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$5\arcsin(x)-\frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-x$$

input
```
Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

output
```
-x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6]
) - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt[6]) + 20*Log[5 +
Sqrt[1 - x^2]]
```

3.259.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.259.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

method	result
default	$\frac{25 \arctan\left(\frac{x\sqrt{6}}{12}\right)\sqrt{6}}{12}+10\ln\left(x^{2}+24\right)-x+5\arcsin\left(x\right)+\frac{25\sqrt{6}\,\arctan\left(\frac{5\sqrt{6}\,\sqrt{-x^{2}+1}\,x}{12\left(x^{2}-1\right)}\right)}{12}-4\sqrt{-x^{2}+1}+20$
trager	Expression too large to display

input
```
int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+25/12*6^(
1/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x^2+1)^(1/2)+20*arc
tanh(1/5*(-x^2+1)^(1/2))
```

3.259.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2\,x}\right) + \frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3\,x}\right)-x - 4\sqrt{-x^2+1}-10\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 10\log\left(x^2+24\right)-10\log\left(-\frac{x^2+6\sqrt{-x^2+1}-6}{x^2}\right) + 10\log\left(\frac{x^2-4\sqrt{-x^2+1}+4}{x^2}\right)$$

input
```
integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) + 25/12*sqrt(6)*arctan(1/2*(sqrt(6)*s
qrt(-x^2 + 1) - sqrt(6))/x) + 25/12*sqrt(6)*arctan(1/3*(sqrt(6)*sqrt(-x^2
+ 1) - sqrt(6))/x) - x - 4*sqrt(-x^2 + 1) - 10*arctan((sqrt(-x^2 + 1) - 1)
/x) + 10*log(x^2 + 24) - 10*log(-(x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2) + 10*lo
g((x^2 - 4*sqrt(-x^2 + 1) + 4)/x^2)
```

3.259.6 Sympy [F]

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \int \frac{4x-\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\,dx$$

input
```
integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

output
```
Integral((4*x - sqrt(1 - x**2))/(sqrt(1 - x**2) + 5), x)
```

3.259.7 Maxima [F]

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx=\int \frac{4\,x-\sqrt{-x^2+1}}{\sqrt{-x^2+1}+5}\,dx$$

input
```
integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
-x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) + 5), x) +
 20*log(sqrt(-x^2 + 1) + 5)
```

3.259.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(66) = 132.

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx &= \frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)-\frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{3\,x}\right)\\&\quad-\frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{2\,x}\right)-x-4\sqrt{-x^2+1}\\&\quad+5\,\arcsin\left(x\right)+10\,\log\left(x^2+24\right)-10\,\log\left(\frac{3\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\right)\\&\quad+10\,\log\left(\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+3\right)\end{aligned}$$

input
```
integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*sqrt(6)*(
sqrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(6)*(sqrt(-x^2 + 1)
 - 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) + 10*log(x^2 + 24) - 10*log(
3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*log(2*(sqrt(-x^2 + 1) - 1)^2/x^2 +
3)
```

3.259.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.81

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = 5\,\mathrm{asin}(x)-x-4\sqrt{1-x^2} - \frac{\sqrt{24}\,\ln\left(\frac{\frac{2\sqrt{6}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x-\sqrt{6}\,2\mathrm{i}}\right)\left(125+\sqrt{24}\,100\mathrm{i}\right)\,1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(\frac{-\frac{\sqrt{24}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x+\sqrt{24}\,1\mathrm{i}}\right)\left(-125+\sqrt{24}\,100\mathrm{i}\right)\,1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(x-\sqrt{6}\,2\mathrm{i}\right)\left(25+\sqrt{24}\,20\mathrm{i}\right)\,1\mathrm{i}}{48} - \frac{\sqrt{24}\,\ln\left(x+\sqrt{24}\,1\mathrm{i}\right)\left(-25+\sqrt{24}\,20\mathrm{i}\right)\,1\mathrm{i}}{48}$$

input
```
int((4*x - (1 - x^2)^(1/2))/((1 - x^2)^(1/2) + 5),x)
```

output
```
5*asin(x) - x - 4*(1 - x^2)^(1/2) - (24^(1/2)*log(((2*6^(1/2)*x)/5 + (1 -
x^2)^(1/2)*1i + 1i/5)/(x - 6^(1/2)*2i))*(24^(1/2)*100i + 125)*1i)/240 - (2
4^(1/2)*log(((1 - x^2)^(1/2)*1i - (24^(1/2)*x)/5 + 1i/5)/(x + 24^(1/2)*1i)
)*(24^(1/2)*100i - 125)*1i)/240 - (24^(1/2)*log(x - 6^(1/2)*2i)*(24^(1/2)*
20i + 25)*1i)/48 - (24^(1/2)*log(x + 24^(1/2)*1i)*(24^(1/2)*20i - 25)*1i)/
48
```

3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx$

3.260.1 Optimal result

Integrand size = 44, antiderivative size = 136

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx=\frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41\operatorname{arcsinh}(x)}{54}+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)+\frac{7}{27}\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right)-\frac{7}{54}\log\left(3+2x+3x^2\right)$$

output
```
8/9*x-1/6*x^2-41/54*arcsinh(x)+7/27*arctanh(1/2*(1-x)/(x^2+1)^(1/2))-7/54*
ln(3*x^2+2*x+3)+4/27*arctan(1/4*(1+3*x)*2^(1/2))*2^(1/2)+4/27*arctan(1/2*(
1+x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)+8/9*(x^2+1)^(1/2)-1/6*x*(x^2+1)^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{1}{54}\left(48x-9x^2+48\sqrt{1+x^2}-9x\sqrt{1+x^2}+16\sqrt{2}\arctan\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)+55\log\left(-x+\sqrt{1+x^2}\right)-14\log\left(-2-x-x^2+(1+x)\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3
/2))),x]
```

output
```
(48*x - 9*x^2 + 48*Sqrt[1 + x^2] - 9*x*Sqrt[1 + x^2] + 16*Sqrt[2]*ArcTan[(
1 + x - Sqrt[1 + x^2])/Sqrt[2]] + 55*Log[-x + Sqrt[1 + x^2]] - 14*Log[-2 -
 x - x^2 + (1 + x)*Sqrt[1 + x^2]])/54
```

3.260.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(2-\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\left(-x^3+(x^2+1)^{3/2}+1\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{x^2}{-x^3+\sqrt{x^2+1}x^2+\sqrt{x^2+1}+1}-\frac{2x^2}{\sqrt{x^2+1}\left(x^3-(x^2+1)^{3/2}-1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{41\operatorname{arcsinh}(x)}{54}+\frac{4}{27}\sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{3x+1}{2\sqrt{2}}\right)+\frac{7}{27}\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{8x}{9}$$

input
```
Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),
x]
```

output
```
(8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[
x])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[
(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2]
)])/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

3.260.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.260.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(99) = 198$.

Time = 0.04 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.81

$$-\frac{x^2}{6}+\frac{8x}{9}-\frac{7\ln\left(3x^2+2x+3\right)}{54}+\frac{4\sqrt{2}\arctan\left(\frac{(6x+2)\sqrt{2}}{8}\right)}{27}-\frac{41\operatorname{arcsinh}(x)}{54}-\frac{\sqrt{2}\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\left(-\sqrt{2}\arctan\right.}{12}$$

input
```
int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

output
```
-1/6*x^2+8/9*x-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+2)*2^(1/2
))-41/54*arcsinh(x)-1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arc
tan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x
))+5*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-
x)+1)^2)^(1/2)/((1+x)/(1-x)+1)+3/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2
^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*
(1+x)/(1-x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((
1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/6*x*(x^2+1)^(1/2)+8/9*(x^2+1)^(1/
2)+1/216*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*arctan(1/2*2^(1/2
)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+43*arctanh(
(2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2
)/((1+x)/(1-x)+1)-1/36*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-11*2^(1/2)*ar
ctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-
x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x
)+1)^2)^(1/2)/((1+x)/(1-x)+1)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.25

$$\begin{aligned}
\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx &= -\frac{1}{6}\,x^2-\frac{1}{18}\,\sqrt{x^2+1}(3\,x-16) \\
&+\frac{4}{27}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}(3\,x+1)\right)+\frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}(3\,x-1)+\frac{3}{2}\,\sqrt{2}\sqrt{x^2+1}\right) \\
&-\frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}(x+1)+\frac{1}{2}\,\sqrt{2}\sqrt{x^2+1}\right)+\frac{8}{9}\,x \\
&+\frac{7}{54}\,\log\left(3\,x^2-\sqrt{x^2+1}(3\,x-1)-x+2\right)-\frac{7}{54}\,\log\left(3\,x^2+2\,x+3\right) \\
&-\frac{7}{54}\,\log\left(x^2-\sqrt{x^2+1}(x+1)+x+2\right)+\frac{41}{54}\,\log\left(-x+\sqrt{x^2+1}\right)
\end{aligned}$$

input
```
integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, alg
orithm="fricas")
```

output
```
-1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)
*(3*x + 1)) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(3*x - 1) + 3/2*sqrt(2)*sqr
t(x^2 + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x + 1) + 1/2*sqrt(2)*sqrt(
x^2 + 1)) + 8/9*x + 7/54*log(3*x^2 - sqrt(x^2 + 1)*(3*x - 1) - x + 2) - 7/
54*log(3*x^2 + 2*x + 3) - 7/54*log(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 2) +
41/54*log(-x + sqrt(x^2 + 1))
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2
),x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = \int \frac{x^2\left(\sqrt{x^2+1}-2\right)}{\left(x^3-\left(x^2+1\right)^{\frac{3}{2}}-1\right)\sqrt{x^2+1}}\,dx$$

input
```
integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, alg
orithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9 + 5*x^8
- 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x^10 + 11*x^9 - 11*
x^8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3 - 7*x^2 - 2*(x^12 + 3*x^10
 - 2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt
(x^2 + 1) - 2), x) + 1/6*log(x^2 + x + 1) + 1/6*log(x - 1)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = -\frac{1}{6}\,x^2-\frac{1}{18}\,\sqrt{x^2+1}(3\,x-16) + \frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(3\,x-3\,\sqrt{x^2+1}-1\right)\right) + \frac{4}{27}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}(3\,x+1)\right)-\frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(x-\sqrt{x^2+1}+1\right)\right) + \frac{8}{9}\,x+\frac{7}{54}\,\log\left(3\left(x-\sqrt{x^2+1}\right)^2-2\,x+2\,\sqrt{x^2+1}+1\right) - \frac{7}{54}\,\log\left(\left(x-\sqrt{x^2+1}\right)^2+2\,x-2\,\sqrt{x^2+1}+3\right) - \frac{7}{54}\,\log\left(3\,x^2+2\,x+3\right)+\frac{41}{54}\,\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, alg
orithm="giac")
```

output
```
-1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2
)*(3*x - 3*sqrt(x^2 + 1) - 1)) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 1)
) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 1) + 1)) + 8/9*x + 7/
54*log(3*(x - sqrt(x^2 + 1))^2 - 2*x + 2*sqrt(x^2 + 1) + 1) - 7/54*log((x
- sqrt(x^2 + 1))^2 + 2*x - 2*sqrt(x^2 + 1) + 3) - 7/54*log(3*x^2 + 2*x + 3
) + 41/54*log(-x + sqrt(x^2 + 1))
```

3.260.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{8x}{9} - \frac{41\,\mathrm{asinh}(x)}{54} - \left(\frac{x}{6}-\frac{8}{9}\right)\sqrt{x^2+1} - \frac{x^2}{6} + \frac{\sqrt{2}\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(-\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)1\mathrm{i}}{8} + \frac{\sqrt{2}\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)1\mathrm{i}}{8} + \frac{\sqrt{2}\left(\frac{4}{81}+\frac{\sqrt{2}\,44\mathrm{i}}{81}\right)\left(\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)-\ln\left(1+\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)\sqrt{x^2+1}-\frac{x}{3}-\frac{\sqrt{2}\,x\,2\mathrm{i}}{3}\right)\right)1\mathrm{i}}{8\sqrt{\left(\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)^2+1}} + \frac{\sqrt{2}\left(-\frac{4}{81}+\frac{\sqrt{2}\,44\mathrm{i}}{81}\right)\left(\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)-\ln\left(1-\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)\sqrt{x^2+1}-\frac{x}{3}+\frac{\sqrt{2}\,x\,2\mathrm{i}}{3}\right)\right)1\mathrm{i}}{8\sqrt{\left(-\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)^2+1}}$$

input
```
int(-(x^2*((x^2 + 1)^(1/2) - 2))/((x^2 + 1)^(1/2)*((x^2 + 1)^(3/2) - x^3 +
 1)),x)
```

output
```
(8*x)/9 - (41*asinh(x))/54 - (x/6 - 8/9)*(x^2 + 1)^(1/2) - x^2/6 + (2^(1/2
)*log(x - (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 - 16/27)*1i)/8 + (2^(1/2
)*log(x + (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 + 16/27)*1i)/8 + (2^(1/2
)*((2^(1/2)*44i)/81 + 4/81)*(log(x + (2^(1/2)*2i)/3 + 1/3) - log(((2^(1/2)
*1i)/3 + 2/3)*(x^2 + 1)^(1/2) - x/3 - (2^(1/2)*x*2i)/3 + 1))*1i)/(8*(((2^(
1/2)*2i)/3 + 1/3)^2 + 1)^(1/2)) + (2^(1/2)*((2^(1/2)*44i)/81 - 4/81)*(log(
x - (2^(1/2)*2i)/3 + 1/3) - log((2^(1/2)*x*2i)/3 - ((2^(1/2)*1i)/3 - 2/3)*
(x^2 + 1)^(1/2) - x/3 + 1))*1i)/(8*(((2^(1/2)*2i)/3 - 1/3)^2 + 1)^(1/2))
```

3.261 $\int x\sqrt{2rx-x^2}\,dx$

3.261.1 Optimal result

Integrand size = 16, antiderivative size = 64

$$\int x\sqrt{2rx-x^2}\,dx=-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

output
```
-1/3*(2*r*x-x^2)^(3/2)+r^3*arctan(x/(2*r*x-x^2)^(1/2))-1/2*r*(r-x)*(2*r*x-
x^2)^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int x\sqrt{2rx-x^2}\,dx=\frac{1}{6}\sqrt{-x(-2r+x)}\left(-3r^2-rx+2x^2+\frac{6r^3\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

input
```
Integrate[x*Sqrt[2*r*x - x^2],x]
```

output
```
(Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 + (6*r^3*Log[-Sqrt[x] + Sqrt[
-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/6
```

3.261.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1160, 1087, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{2rx-x^2}\,dx$$

$$\Big\downarrow \text{ 1160}$$

$$r\int \sqrt{2rx-x^2}dx-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$r\left(\frac{1}{2}r^2\int \frac{1}{\sqrt{2rx-x^2}}dx-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 1091}$$

$$r\left(r^2\int \frac{1}{\frac{x^2}{2rx-x^2}+1}d\frac{x}{\sqrt{2rx-x^2}}-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 216}$$

$$r\left(r^2\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

input
```
Int[x*Sqrt[2*r*x - x^2],x]
```

output
```
-1/3*(2*r*x - x^2)^(3/2) + r*(-1/2*((r - x)*Sqrt[2*r*x - x^2]) + r^2*ArcTa
n[x/Sqrt[2*r*x - x^2]])
```

3.261.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.261.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right)r^{3}-\frac{\sqrt{x(2r-x)}\,(r+x)\left(r-\frac{2x}{3}\right)}{2}$	44
risch	$-\frac{(3r^{2}+rx-2x^{2})x(2r-x)}{6\sqrt{-x(-2r+x)}}+\frac{r^{3}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{2}$	60
default	$-\frac{(2rx-x^{2})^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^{2}}}{4}+\frac{r^{2}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{2}\right)$	64

input
```
int(x*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(1/x*(x*(2*r-x))^(1/2))*r^3-1/2*(x*(2*r-x))^(1/2)*(r+x)*(r-2/3*x)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int x\sqrt{2rx-x^2}\,dx = -r^3\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{6}\left(3\,r^2+rx-2\,x^2\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-r^3*arctan(sqrt(2*r*x - x^2)/x) - 1/6*(3*r^2 + r*x - 2*x^2)*sqrt(2*r*x -
x^2)
```

3.261.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int x\sqrt{2rx-x^2}\,dx = \frac{r^3\left(\begin{cases}-i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0\\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{2}+\sqrt{2rx-x^2}\left(-\frac{r^2}{2}-\frac{rx}{6}+\frac{x^2}{3}\right)$$

input
```
integrate(x*(2*r*x-x**2)**(1/2),x)
```

output
```
r**3*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0)),
((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/2 + sqrt(2*r*x - x**2)*(-
r**2/2 - r*x/6 + x**2/3)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)-\frac{1}{2}\,\sqrt{2\,rx-x^2}r^2+\frac{1}{2}\,\sqrt{2\,rx-x^2}rx-\frac{1}{3}\,\left(2\,rx-x^2\right)^{\frac{3}{2}}$$

input
```
integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*r^3*arcsin((r - x)/r) - 1/2*sqrt(2*r*x - x^2)*r^2 + 1/2*sqrt(2*r*x -
x^2)*r*x - 1/3*(2*r*x - x^2)^(3/2)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r)-\frac{1}{6}\left(3\,r^2+(r-2\,x)x\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*r^3*arcsin((r - x)/r)*sgn(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sqrt(2*r*x -
 x^2)
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}-\frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}$$

input
```
int(x*(2*r*x - x^2)^(1/2),x)
```

output
```
- ((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 - (r^3*log(x - r - (x*
(2*r - x))^(1/2)*1i)*1i)/2
```

3.262 $\int x^2\sqrt{2rx-x^2}\,dx$

3.262.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int x^2\sqrt{2rx-x^2}\,dx=-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{5}{4}r^4\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

output
```
-5/12*r*(2*r*x-x^2)^(3/2)-1/4*x*(2*r*x-x^2)^(3/2)+5/4*r^4*arctan(x/(2*r*x-
x^2)^(1/2))-5/8*r^2*(r-x)*(2*r*x-x^2)^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{2rx-x^2}\,dx=\frac{1}{24}\sqrt{-x(-2r+x)}\left(-15r^3-5r^2x-2rx^2+6x^3+\frac{30r^4\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

input
```
Integrate[x^2*Sqrt[2*r*x - x^2],x]
```

output
```
(Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 + (30*r^4*Log[
-Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/24
```

3.262.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1134, 1160, 1087, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{2rx-x^2}\,dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{5}{4}r\int x\sqrt{2rx-x^2}dx-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{5}{4}r\left(r\int \sqrt{2rx-x^2}dx-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{4}r\left(r\left(\frac{1}{2}r^2\int \frac{1}{\sqrt{2rx-x^2}}dx-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{5}{4}r\left(r\left(r^2\int \frac{1}{\frac{x^2}{2rx-x^2}+1}d\frac{x}{\sqrt{2rx-x^2}}-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4}r\left(r\left(r^2\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

input
```
Int[x^2*Sqrt[2*r*x - x^2],x]
```

output
```
-1/4*(x*(2*r*x - x^2)^(3/2)) + (5*r*(-1/3*(2*r*x - x^2)^(3/2) + r*(-1/2*((
r - x)*Sqrt[2*r*x - x^2]) + r^2*ArcTan[x/Sqrt[2*r*x - x^2]])))/4
```

3.262.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.262.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{5 \arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right) r^4}{4}-\frac{5 \sqrt{x(2r-x)}\,\left(r^3+\frac{1}{3}r^2 x+\frac{2}{15}r\,x^2-\frac{2}{5}x^3\right)}{8}$	57
risch	$-\frac{\left(15r^3+5r^2 x+2r\,x^2-6x^3\right)x(2r-x)}{24\sqrt{-x(-2r+x)}}+\frac{5r^4 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{8}$	69
default	$-\frac{x\left(2rx-x^2\right)^{\frac{3}{2}}}{4}+\frac{5r\left(-\frac{\left(2rx-x^2\right)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4}+\frac{r^2 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}\right)\right)}{4}$	83

input
```
int(x^2*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4*arctan(1/x*(x*(2*r-x))^(1/2))*r^4-5/8*(x*(2*r-x))^(1/2)*(r^3+1/3*r^2*
x+2/15*r*x^2-2/5*x^3)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int x^2\sqrt{2rx-x^2}\,dx=-\frac{5}{4}\,r^4\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{24}\,\left(15\,r^3+5\,r^2x+2\,rx^2-6\,x^3\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-5/4*r^4*arctan(sqrt(2*r*x - x^2)/x) - 1/24*(15*r^3 + 5*r^2*x + 2*r*x^2 -
6*x^3)*sqrt(2*r*x - x^2)
```

3.262.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^2\sqrt{2rx-x^2}\,dx = \frac{5r^4\left(\begin{cases}-i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0\\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{8} + \sqrt{2rx-x^2}\left(-\frac{5r^3}{8}-\frac{5r^2x}{24}-\frac{rx^2}{12}+\frac{x^3}{4}\right)$$

input
```
integrate(x**2*(2*r*x-x**2)**(1/2),x)
```

output
```
5*r**4*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0))
, ((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/8 + sqrt(2*r*x - x**2)*
(-5*r**3/8 - 5*r**2*x/24 - r*x**2/12 + x**3/4)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}r^4\arcsin\left(\frac{r-x}{r}\right)-\frac{5}{8}\sqrt{2\,rx-x^2}r^3 + \frac{5}{8}\sqrt{2\,rx-x^2}r^2x-\frac{5}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r-\frac{1}{4}\left(2\,rx-x^2\right)^{\frac{3}{2}}x$$

input
```
integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-5/8*r^4*arcsin((r - x)/r) - 5/8*sqrt(2*r*x - x^2)*r^3 + 5/8*sqrt(2*r*x -
x^2)*r^2*x - 5/12*(2*r*x - x^2)^(3/2)*r - 1/4*(2*r*x - x^2)^(3/2)*x
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}\,r^4\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r) - \frac{1}{24}\left(15\,r^3+\left(5\,r^2+2\,(r-3\,x)x\right)x\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

output
```
-5/8*r^4*arcsin((r - x)/r)*sgn(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r - 3*x)*x)
*x)*sqrt(2*r*x - x^2)
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{x\,\left(2\,r\,x-x^2\right)^{3/2}}{4} - \frac{5\,r\,\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}\right)}{4}$$

input
```
int(x^2*(2*r*x - x^2)^(1/2),x)
```

output
```
- (x*(2*r*x - x^2)^(3/2))/4 - (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 -
 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4
```

3.263 $\int x^3\sqrt{2rx-x^2}\,dx$

3.263.1 Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2} -\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

output
```
-7/12*r^2*(2*r*x-x^2)^(3/2)-7/20*r*x*(2*r*x-x^2)^(3/2)-1/5*x^2*(2*r*x-x^2)
^(3/2)+7/4*r^5*arctan(x/(2*r*x-x^2)^(1/2))-7/8*r^3*(r-x)*(2*r*x-x^2)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{2rx-x^2}\,dx = \frac{1}{120}\sqrt{-x(-2r+x)}\left(-105r^4-35r^3x-14r^2x^2-6rx^3+24x^4 +\frac{210r^5\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

input
```
Integrate[x^3*Sqrt[2*r*x - x^2],x]
```

output
```
(Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^
4 + (210*r^5*Log[-Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/12
0
```

3.263.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1134, 1134, 1160, 1087, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{2rx-x^2}\,dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{7}{5}r\int x^2\sqrt{2rx-x^2}dx-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\int x\sqrt{2rx-x^2}dx-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\int\sqrt{2rx-x^2}dx-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\left(\frac{1}{2}r^2\int\frac{1}{\sqrt{2rx-x^2}}dx-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\left(r^2\int\frac{1}{\frac{x^2}{2rx-x^2}+1}d\frac{x}{\sqrt{2rx-x^2}}-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\left(r^2\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

input
```
Int[x^3*Sqrt[2*r*x - x^2],x]
```

output
```
-1/5*(x^2*(2*r*x - x^2)^(3/2)) + (7*r*(-1/4*(x*(2*r*x - x^2)^(3/2)) + (5*r
*(-1/3*(2*r*x - x^2)^(3/2) + r*(-1/2*((r - x)*Sqrt[2*r*x - x^2]) + r^2*Arc
Tan[x/Sqrt[2*r*x - x^2]])))/4))/5
```

3.263.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.263.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$-\frac{7 \arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right) r^5}{4}-\frac{7 \sqrt{x(2r-x)}\,\left(r^4+\frac{1}{3} r^3 x+\frac{2}{15} r^2 x^2+\frac{2}{35} r\, x^3-\frac{8}{35} x^4\right)}{8}$	65
risch	$-\frac{\left(105 r^4+35 r^3 x+14 r^2 x^2+6 r\, x^3-24 x^4\right) x(2r-x)}{120 \sqrt{-x(-2r+x)}}+\frac{7 r^5 \arctan\left(\frac{x-r}{\sqrt{2 r x-x^2}}\right)}{8}$	77
default	$-\frac{x^2\left(2 r x-x^2\right)^{\frac{3}{2}}}{5}+\frac{7 r\left(-\frac{x\left(2 r x-x^2\right)^{\frac{3}{2}}}{4}+\frac{5 r\left(-\frac{\left(2 r x-x^2\right)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2 r x-x^2}}{4}+\frac{r^2 \arctan\left(\frac{x-r}{\sqrt{2 r x-x^2}}\right)}{2}\right)\right)}{4}\right)}{5}$	104

input
```
int(x^3*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-7/4*arctan(1/x*(x*(2*r-x))^(1/2))*r^5-7/8*(x*(2*r-x))^(1/2)*(r^4+1/3*r^3*
x+2/15*r^2*x^2+2/35*r*x^3-8/35*x^4)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{4}\,r^5\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right) - \frac{1}{120}\left(105\,r^4+35\,r^3x+14\,r^2x^2+6\,rx^3-24\,x^4\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-7/4*r^5*arctan(sqrt(2*r*x - x^2)/x) - 1/120*(105*r^4 + 35*r^3*x + 14*r^2*
x^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)
```

3.263.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{2rx-x^2}\,dx = \frac{7r^5\left(\begin{cases}-i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0\\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{8} + \sqrt{2rx-x^2}\left(-\frac{7r^4}{8}-\frac{7r^3x}{24}-\frac{7r^2x^2}{60}-\frac{rx^3}{20}+\frac{x^4}{5}\right)$$

input
```
integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

output
```
7*r**5*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0))
, ((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/8 + sqrt(2*r*x - x**2)*
(-7*r**4/8 - 7*r**3*x/24 - 7*r**2*x**2/60 - r*x**3/20 + x**4/5)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}r^5\arcsin\left(\frac{r-x}{r}\right)-\frac{7}{8}\sqrt{2\,rx-x^2}r^4+\frac{7}{8}\sqrt{2\,rx-x^2}r^3x - \frac{7}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r^2-\frac{7}{20}\left(2\,rx-x^2\right)^{\frac{3}{2}}rx-\frac{1}{5}\left(2\,rx-x^2\right)^{\frac{3}{2}}x^2$$

input
```
integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-7/8*r^5*arcsin((r - x)/r) - 7/8*sqrt(2*r*x - x^2)*r^4 + 7/8*sqrt(2*r*x -
x^2)*r^3*x - 7/12*(2*r*x - x^2)^(3/2)*r^2 - 7/20*(2*r*x - x^2)^(3/2)*r*x -
 1/5*(2*r*x - x^2)^(3/2)*x^2
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}\,r^5\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r) - \frac{1}{120}\left(105\,r^4+\left(35\,r^3+2\left(7\,r^2+3\,(r-4\,x)x\right)x\right)x\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

output
```
-7/8*r^5*arcsin((r - x)/r)*sgn(r) - 1/120*(105*r^4 + (35*r^3 + 2*(7*r^2 +
3*(r - 4*x)*x)*x)*x)*sqrt(2*r*x - x^2)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7\,r\left(\frac{x\,{\left(2\,r\,x-x^2\right)}^{3/2}}{4}+\frac{5\,r\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}\right)}{4}\right)}{5} - \frac{x^2\,{\left(2\,r\,x-x^2\right)}^{3/2}}{5}$$

input
```
int(x^3*(2*r*x - x^2)^(1/2),x)
```

output
```
- (7*r*((x*(2*r*x - x^2)^(3/2))/4 + (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12
*r^2 - 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4))/5
 - (x^2*(2*r*x - x^2)^(3/2))/5
```

3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$

3.264.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{2}\arctan\left(\sqrt{2x+x^2}\right) - \frac{\operatorname{arctanh}\left(\frac{1+2x}{\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}$$

output
```
-1/2*arctan((x^2+2*x)^(1/2))-1/6*arctanh(1/3*(1+2*x)*3^(1/2)/(x^2+2*x)^(1/
2))*3^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{\sqrt{x}\sqrt{2+x}\left(3\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)-\sqrt{3}\operatorname{arctanh}\left(\frac{1-x+\sqrt{x}\sqrt{2+x}}{\sqrt{3}}\right)\right)}{3\sqrt{x(2+x)}}$$

input
```
Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[2 + x]*(3*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]] - Sqrt[3]*ArcT
anh[(1 - x + Sqrt[x]*Sqrt[2 + x])/Sqrt[3]]))/(3*Sqrt[x*(2 + x)])
```

3.264.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1316, 25, 1112, 216, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{1316}$$

$$\frac{1}{2}\int -\frac{1}{(1-x)\sqrt{x^2+2x}}dx+\frac{1}{2}\int -\frac{1}{(x+1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x}}dx-\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{1112}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x}}dx-2\int \frac{1}{4(x^2+2x)+4}d\sqrt{x^2+2x}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x}}dx-\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right)$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{1}{12-\frac{4(2x+1)^2}{x^2+2x}}d\left(-\frac{2(2x+1)}{\sqrt{x^2+2x}}\right)-\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right)-\frac{\text{arctanh}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

input
```
Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

output
```
-1/2*ArcTan[Sqrt[2*x + x^2]] - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])
]/(2*Sqrt[3])
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1316
```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Simp[1/2   Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x]
, x] + Simp[1/2   Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.264.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{3}\, \sqrt{x (2+x)}}{3 x}\right)}{3}+\arctan\left(\frac{\sqrt{x (2+x)}}{x}\right)$
default	$-\frac{\sqrt{3}\, \operatorname{arctanh}\left(\frac{(2+4 x) \sqrt{3}}{6 \sqrt{(-1+x)^{2}-1+4 x}}\right)}{6}+\frac{\arctan\left(\frac{1}{\sqrt{(1+x)^{2}-1}}\right)}{2}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2 x}}{1+x}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(\frac{-2 \operatorname{RootOf}\left(\_Z^{2}-3\right) x+3 \sqrt{x^{2}+2 x}-\operatorname{Root}}{-1+x}\right.}{6}$

input
```
int(1/(x^2-1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*3^(1/2)*arctanh(1/3*3^(1/2)*(x*(2+x))^(1/2)/x)+arctan((x*(2+x))^(1/2)
/x)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}(2\,x+1)+\sqrt{x^2+2\,x}(2\,\sqrt{3}-3)-4\,x-2}{x-1}\right) - \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(sqrt(3)*(2*x + 1) + sqrt(x^2 + 2*x)*(2*sqrt(3) - 3) - 4*
x - 2)/(x - 1)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.264.6 Sympy [F]

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+2)}\,(x-1)\,(x+1)}\,dx$$

input
```
integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^2+2\,x}}{|2\,x-2|}+\frac{6}{|2\,x-2|}+2\right) + \frac{1}{2}\arcsin\left(\frac{2}{|2\,x+2|}\right)$$

input
```
integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2 + 2*x)/abs(2*x - 2) + 6/abs(2*x - 2) +
 2) + 1/2*arcsin(2/abs(2*x + 2))
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\,x-2\sqrt{3}+2\sqrt{x^2+2\,x}+2\right|}{\left|-2\,x+2\sqrt{3}+2\sqrt{x^2+2\,x}+2\right|}\right) - \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(-2*x + 2
*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}\,(x^2-1)}\,dx$$

input
```
int(1/((2*x + x^2)^(1/2)*(x^2 - 1)),x)
```

output
```
int(1/((2*x + x^2)^(1/2)*(x^2 - 1)), x)
```

3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

3.265.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(1/3*(1-2*x)*3^(1/2)/(-x^2+2*x)^(1/2))*3^(1/2)-5/6*(-x^2+2*x)^(1
/2)/(1+x)^2-2/3*(-x^2+2*x)^(1/2)/(1+x)
```

3.265.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{x(-18+x+4x^2)-2\sqrt{3}\sqrt{-2+x}\sqrt{x}(1+x)^2\text{arctanh}\left(\frac{1-\sqrt{-2+x}\sqrt{x}+x}{\sqrt{3}}\right)}{6\sqrt{-((-2+x)x)}(1+x)^2}$$

input
```
Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

output
```
(x*(-18 + x + 4*x^2) - 2*Sqrt[3]*Sqrt[-2 + x]*Sqrt[x]*(1 + x)^2*ArcTanh[(1
 - Sqrt[-2 + x]*Sqrt[x] + x)/Sqrt[3]])/(6*Sqrt[-((-2 + x)*x)]*(1 + x)^2)
```

3.265.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1237, 25, 1228, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x-2}{(x+1)^3\sqrt{2x-x^2}}dx$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{6}\int -\frac{7-5x}{(x+1)^2\sqrt{2x-x^2}}dx-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}\int \frac{7-5x}{(x+1)^2\sqrt{2x-x^2}}dx-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{6}\left(-3\int \frac{1}{(x+1)\sqrt{2x-x^2}}dx-\frac{4\sqrt{2x-x^2}}{x+1}\right)-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{6}\left(6\int \frac{1}{-\frac{4(1-2x)^2}{2x-x^2}-12}d\left(-\frac{2(1-2x)}{\sqrt{2x-x^2}}\right)-\frac{4\sqrt{2x-x^2}}{x+1}\right)-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)-\frac{4\sqrt{2x-x^2}}{x+1}\right)-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

input
```
Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

output
```
(-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) + ((-4*Sqrt[2*x - x^2])/(1 + x) + Sqrt[
3]*ArcTan[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])])/6
```

3.265.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.265. $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

3.265.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,(1+x)^{2}\arctan\left(\frac{\sqrt{3}\,\sqrt{-x(-2+x)}}{3x}\right)+(-4x-9)\sqrt{-x(-2+x)}}{6(1+x)^{2}}$	50
risch	$\frac{x(-2+x)(4x+9)}{6(1+x)^{2}\sqrt{-x(-2+x)}}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^{2}+1+4x}}\right)}{6}$	56
trager	$-\frac{(4x+9)\sqrt{-x^{2}+2x}}{6(1+x)^{2}}-\frac{\operatorname{RootOf}\left(\_Z^{2}+3\right)\ln\left(\frac{-2\operatorname{RootOf}\left(\_Z^{2}+3\right)x+3\sqrt{-x^{2}+2x}+\operatorname{RootOf}\left(\_Z^{2}+3\right)}{1+x}\right)}{6}$	69
default	$-\frac{5\sqrt{-(1+x)^{2}+1+4x}}{6(1+x)^{2}}-\frac{2\sqrt{-(1+x)^{2}+1+4x}}{3(1+x)}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^{2}+1+4x}}\right)}{6}$	74

input
```
int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*(1+x)^2*arctan(1/3*3^(1/2)*(-x*(-2+x))^(1/2)/x)+(-4*x-9)*(-
x*(-2+x))^(1/2))/(1+x)^2
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{2\sqrt{3}(x^2+2x+1)\arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{3x}\right)-\sqrt{-x^2+2x}(4x+9)}{6(x^2+2x+1)}$$

input
```
integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*sqrt(-x^2 + 2*x)/x) - sq
rt(-x^2 + 2*x)*(4*x + 9))/(x^2 + 2*x + 1)
```

3.265.6 Sympy [F]

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \int \frac{3x-2}{\sqrt{-x\,(x-2)}\,(x+1)^3}\,dx$$

input
```
integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
```

output
```
Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{1}{6}\sqrt{3}\arcsin\left(\frac{2\,x}{|x+1|}-\frac{1}{|x+1|}\right) - \frac{5\sqrt{-x^2+2\,x}}{6\,(x^2+2\,x+1)} - \frac{2\sqrt{-x^2+2\,x}}{3\,(x+1)}$$

input
```
integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arcsin(2*x/abs(x + 1) - 1/abs(x + 1)) - 5/6*sqrt(-x^2 + 2*x)/
(x^2 + 2*x + 1) - 2/3*sqrt(-x^2 + 2*x)/(x + 1)
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.86

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{x-1}-1\right)\right) + \frac{\frac{34\left(\sqrt{-x^2+2\,x}-1\right)}{x-1} - \frac{39\left(\sqrt{-x^2+2\,x}-1\right)^2}{(x-1)^2} + \frac{18\left(\sqrt{-x^2+2\,x}-1\right)^3}{(x-1)^3} - 26}{24\left(\frac{\sqrt{-x^2+2\,x}-1}{x-1} - \frac{\left(\sqrt{-x^2+2\,x}-1\right)^2}{(x-1)^2} - 1\right)^2}$$

input
```
integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 1)) + 1
/24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 + 2*x) - 1)^2/(x -
1)^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 - 26)/((sqrt(-x^2 + 2*x) - 1)
/(x - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 - 1)^2
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \int \frac{3\,x-2}{\sqrt{2\,x-x^2}\,{(x+1)}^3}\,dx$$

input
```
int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3),x)
```

output
```
int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3), x)
```

3.266 $\int \frac{1}{\sqrt{1+x+x^2}}\,dx$

3.266.1 Optimal result

Integrand size = 10, antiderivative size = 12

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
arcsinh(1/3*(1+2*x)*3^(1/2))
```

3.266.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = -\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/Sqrt[1 + x + x^2],x]
```

output
```
-Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]
```

3.266.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \ 1090$$

$$\frac{\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)}{\sqrt{3}}$$

$$\Big\downarrow \ 222$$

$$\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)$$

input
```
Int[1/Sqrt[1 + x + x^2],x]
```

output
```
ArcSinh[(1 + 2*x)/Sqrt[3]]
```

3.266.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.266.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x+\frac{1}{2})}{3}\right)$	10
trager	$-\ln\left(2\sqrt{x^2+x+1}-1-2x\right)$	19

input
```
int(1/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(2/3*3^(1/2)*(x+1/2))
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(1/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.266.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)$$

input
```
integrate(1/(x**2+x+1)**(1/2),x)
```

output
```
asinh(2*sqrt(3)*(x + 1/2)/3)
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(1/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.266.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(11) = 22.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+1}(2x+1) - \frac{3}{8}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(1/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 + x + 1)*(2*x + 1) - 3/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)$$

input
```
int(1/(x + x^2 + 1)^(1/2),x)
```

output
```
log(x + (x + x^2 + 1)^(1/2) + 1/2)
```

3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}} dx$

3.267.1 Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{x^3}{\sqrt{1+x+x^2}} dx = \frac{1}{3}x^2\sqrt{1+x+x^2} - \frac{1}{24}(1+10x)\sqrt{1+x+x^2} + \frac{7}{16}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
7/16*arcsinh(1/3*(1+2*x)*3^(1/2))+1/3*x^2*(x^2+x+1)^(1/2)-1/24*(1+10*x)*(x
^2+x+1)^(1/2)
```

3.267.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{1+x+x^2}} dx = \frac{1}{24}\sqrt{1+x+x^2}(-1-10x+8x^2) - \frac{7}{16}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^3/Sqrt[1 + x + x^2],x]
```

output
```
(Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2))/24 - (7*Log[-1 - 2*x + 2*Sqrt[1 +
x + x^2]])/16
```

3.267.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1166, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\int -\frac{x(5x+4)}{2\sqrt{x^2+x+1}}dx+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^2\sqrt{x^2+x+1}-\frac{1}{6}\int \frac{x(5x+4)}{\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{6}\left(\frac{21}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx-\frac{1}{4}(10x+1)\sqrt{x^2+x+1}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{6}\left(\frac{7}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)-\frac{1}{4}(10x+1)\sqrt{x^2+x+1}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{6}\left(\frac{21}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{4}(10x+1)\sqrt{x^2+x+1}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

input
```
Int[x^3/Sqrt[1 + x + x^2],x]
```

output
```
(x^2*Sqrt[1 + x + x^2])/3 + (-1/4*((1 + 10*x)*Sqrt[1 + x + x^2]) + (21*Arc
Sinh[(1 + 2*x)/Sqrt[3]])/8)/6
```

3.267.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.267. $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$

3.267.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{(8x^2-10x-1)\sqrt{x^2+x+1}}{24}+\frac{7\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{16}$	33
trager	$\left(\frac{1}{3}x^2-\frac{5}{12}x-\frac{1}{24}\right)\sqrt{x^2+x+1}-\frac{7\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{16}$	39
default	$\frac{x^2\sqrt{x^2+x+1}}{3}-\frac{5x\sqrt{x^2+x+1}}{12}-\frac{\sqrt{x^2+x+1}}{24}+\frac{7\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{16}$	47

input
```
int(x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*x^2-10*x-1)*(x^2+x+1)^(1/2)+7/16*arcsinh(2/3*3^(1/2)*(x+1/2))
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx=\frac{1}{24}\left(8\,x^2-10\,x-1\right)\sqrt{x^2+x+1}-\frac{7}{16}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(8*x^2 - 10*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
 + 1) - 1)
```

3.267.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx=\left(\frac{x^2}{3}-\frac{5x}{12}-\frac{1}{24}\right)\sqrt{x^2+x+1}+\frac{7\,\mathrm{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{16}$$

input
```
integrate(x**3/(x**2+x+1)**(1/2),x)
```

output
```
(x**2/3 - 5*x/12 - 1/24)*sqrt(x**2 + x + 1) + 7*asinh(2*sqrt(3)*(x + 1/2)/
3)/16
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{3}\sqrt{x^2+x+1}x^2 - \frac{5}{12}\sqrt{x^2+x+1}x - \frac{1}{24}\sqrt{x^2+x+1} + \frac{7}{16}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(x^2 + x + 1)*x^2 - 5/12*sqrt(x^2 + x + 1)*x - 1/24*sqrt(x^2 + x +
 1) + 7/16*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{24}\left(2\left(4x-5\right)x-1\right)\sqrt{x^2+x+1} - \frac{7}{16}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 +
x + 1) - 1)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2+x+1}}\,dx$$

input
```
int(x^3/(x + x^2 + 1)^(1/2),x)
```

output
```
int(x^3/(x + x^2 + 1)^(1/2), x)
```

3.268 $\int \frac{1}{(1+x+x^2)^{3/2}} \, dx$

3.268.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{(1+x+x^2)^{3/2}} \, dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

output
```
2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x+x^2)^{3/2}} \, dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

input
```
Integrate[(1 + x + x^2)^(-3/2),x]
```

output
```
(2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

3.268.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+x+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

input
```
Int[(1 + x + x^2)^(-3/2),x]
```

output
```
(2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

3.268.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

3.268.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16
default	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16
trager	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16
risch	$\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$	16

input
```
int(1/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{2\left(2\,x^2+\sqrt{x^2+x+1}(2\,x+1)+2\,x+2\right)}{3\,(x^2+x+1)}$$

input
```
integrate(1/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(2*x^2 + sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)/(x^2 + x + 1)
```

3.268.6 Sympy [F]

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**2+x+1)**(3/2),x)
```

output
```
Integral((x**2 + x + 1)**(-3/2), x)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\,x}{3\,\sqrt{x^2+x+1}} + \frac{2}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(1/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(1/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
2/3*(2*x + 1)/sqrt(x^2 + x + 1)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\,\left(x+\frac{1}{2}\right)}{3\,\sqrt{x^2+x+1}}$$

input
```
int(1/(x + x^2 + 1)^(3/2),x)
```

output
```
(4*(x + 1/2))/(3*(x + x^2 + 1)^(1/2))
```

3.269 $\int \frac{x}{(1+x+x^2)^{3/2}} \, dx$

3.269.1 Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{(1+x+x^2)^{3/2}} \, dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

output
```
-2/3*(2+x)/(x^2+x+1)^(1/2)
```

3.269.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x+x^2)^{3/2}} \, dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

input
```
Integrate[x/(1 + x + x^2)^(3/2),x]
```

output
```
(-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

3.269.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+x+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 1158$$

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

input
```
Int[x/(1 + x + x^2)^(3/2),x]
```

output
```
(-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

3.269.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

3.269.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
trager	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
risch	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
default	$-\frac{1}{\sqrt{x^2+x+1}}-\frac{1+2x}{3\sqrt{x^2+x+1}}$	27

input
```
int(x/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2+x)/(x^2+x+1)^(1/2)
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\left(x^2+\sqrt{x^2+x+1}(x+2)+x+1\right)}{3\left(x^2+x+1\right)}$$

input
```
integrate(x/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(x^2 + sqrt(x^2 + x + 1)*(x + 2) + x + 1)/(x^2 + x + 1)
```

3.269.6 Sympy [F]

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(x**2+x+1)**(3/2),x)
```

output
```
Integral(x/(x**2 + x + 1)**(3/2), x)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,x}{3\,\sqrt{x^2+x+1}} - \frac{4}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(x/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(x/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(x + 2)/sqrt(x^2 + x + 1)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,x+4}{3\,\sqrt{x^2+x+1}}$$

input
```
int(x/(x + x^2 + 1)^(3/2),x)
```

output
```
-(2*x + 4)/(3*(x + x^2 + 1)^(1/2))
```

3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}} \, dx$

3.270.1 Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{x^3}{(1+x+x^2)^{3/2}} \, dx = -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-3/2*arcsinh(1/3*(1+2*x)*3^(1/2))-2/3*x^2*(2+x)/(x^2+x+1)^(1/2)+1/3*(5+2*x
)*(x^2+x+1)^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(1+x+x^2)^{3/2}} \, dx = \frac{5+7x+3x^2}{3\sqrt{1+x+x^2}} + \frac{3}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^3/(1 + x + x^2)^(3/2),x]
```

output
```
(5 + 7*x + 3*x^2)/(3*Sqrt[1 + x + x^2]) + (3*Log[-1 - 2*x + 2*Sqrt[1 + x +
 x^2]])/2
```

3.270.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1164, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^2+x+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{2}{3}\int \frac{2x(x+2)}{\sqrt{x^2+x+1}}dx-\frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{3}\int \frac{x(x+2)}{\sqrt{x^2+x+1}}dx-\frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1225}$$

$$\frac{4}{3}\left(\frac{1}{4}(2x+5)\sqrt{x^2+x+1}-\frac{9}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx\right)-\frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{4}{3}\left(\frac{1}{4}(2x+5)\sqrt{x^2+x+1}-\frac{3}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)\right)-\frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{4}{3}\left(\frac{1}{4}(2x+5)\sqrt{x^2+x+1}-\frac{9}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

input
```
Int[x^3/(1 + x + x^2)^(3/2),x]
```

output
```
(-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + (4*(((5 + 2*x)*Sqrt[1 + x + x^2])
/4 - (9*ArcSinh[(1 + 2*x)/Sqrt[3]])/8))/3
```

3.270.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.270. $\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx$

3.270.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	33
trager	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}}+\frac{3\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{2}$	40
default	$\frac{x^2}{\sqrt{x^2+x+1}}+\frac{3x}{2\sqrt{x^2+x+1}}+\frac{5}{4\sqrt{x^2+x+1}}+\frac{\frac{5}{12}+\frac{5x}{6}}{\sqrt{x^2+x+1}}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	61

input
```
int(x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^2+7*x+5)/(x^2+x+1)^(1/2)-3/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{19\,x^2+18\,(x^2+x+1)\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)+4\,(3\,x^2+7\,x+5)\sqrt{x^2+x}}{12\,(x^2+x+1)}$$

input
```
integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(19*x^2 + 18*(x^2 + x + 1)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + 4*(3
*x^2 + 7*x + 5)*sqrt(x^2 + x + 1) + 19*x + 19)/(x^2 + x + 1)
```

3.270.6 Sympy [F]

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(x**2+x+1)**(3/2),x)
```

output
```
Integral(x**3/(x**2 + x + 1)**(3/2), x)
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{x^2+x+1}} + \frac{7\,x}{3\,\sqrt{x^2+x+1}} + \frac{5}{3\,\sqrt{x^2+x+1}} - \frac{3}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
x^2/sqrt(x^2 + x + 1) + 7/3*x/sqrt(x^2 + x + 1) + 5/3/sqrt(x^2 + x + 1) -
3/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{(3\,x+7)x+5}{3\,\sqrt{x^2+x+1}} + \frac{3}{2}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*log(-2*x + 2*sqrt(x^2 + x +
1) - 1)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x^2+x+1)^{3/2}}\,dx$$

input
```
int(x^3/(x + x^2 + 1)^(3/2),x)
```

output
```
int(x^3/(x + x^2 + 1)^(3/2), x)
```

3.271 $\int x^2\sqrt{1+x+x^2}\,dx$

3.271.1 Optimal result

Integrand size = 14, antiderivative size = 65

$$\int x^2\sqrt{1+x+x^2}\,dx=\frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-5/24*(x^2+x+1)^(3/2)+1/4*x*(x^2+x+1)^(3/2)+3/128*arcsinh(1/3*(1+2*x)*3^(1
/2))+1/64*(1+2*x)*(x^2+x+1)^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{1+x+x^2}\,dx=\frac{1}{192}\sqrt{1+x+x^2}\left(-37+14x+8x^2+48x^3\right)-\frac{3}{128}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^2*Sqrt[1 + x + x^2],x]
```

output
```
(Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/192 - (3*Log[-1 - 2*x +
2*Sqrt[1 + x + x^2]])/128
```

3.271.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1166, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{x^2+x+1}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{4}\int -\frac{1}{2}(5x+2)\sqrt{x^2+x+1}dx+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{1}{8}\int (5x+2)\sqrt{x^2+x+1}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \sqrt{x^2+x+1}dx-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{3}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{3}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

input
```
Int[x^2*Sqrt[1 + x + x^2],x]
```

output
```
(x*(1 + x + x^2)^(3/2))/4 + ((-5*(1 + x + x^2)^(3/2))/3 + (((1 + 2*x)*Sqrt
[1 + x + x^2])/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/8)/2)/8
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

3.271.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{(48x^3+8x^2+14x-37)\sqrt{x^2+x+1}}{192}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	38
trager	$\left(\frac{1}{4}x^3+\frac{1}{24}x^2+\frac{7}{96}x-\frac{37}{192}\right)\sqrt{x^2+x+1}-\frac{3\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{128}$	44
default	$\frac{x(x^2+x+1)^{\frac{3}{2}}}{4}-\frac{5(x^2+x+1)^{\frac{3}{2}}}{24}+\frac{(1+2x)\sqrt{x^2+x+1}}{64}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	49

input
```
int(x^2*(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*x^3+8*x^2+14*x-37)*(x^2+x+1)^(1/2)+3/128*arcsinh(2/3*3^(1/2)*(x+
1/2))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{1+x+x^2}\,dx=\frac{1}{192}\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/192*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2*
sqrt(x^2 + x + 1) - 1)
```

3.271.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{1+x+x^2}\,dx = \sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right)+\frac{3\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$$

input
```
integrate(x**2*(x**2+x+1)**(1/2),x)
```

output
```
sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 3*asinh(2*sqrt(3
)*(x + 1/2)/3)/128
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x-\frac{5}{24}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{32}\sqrt{x^2+x+1}x + \frac{1}{64}\sqrt{x^2+x+1}+\frac{3}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 + x + 1)^(3/2)*x - 5/24*(x^2 + x + 1)^(3/2) + 1/32*sqrt(x^2 + x +
 1)*x + 1/64*sqrt(x^2 + x + 1) + 3/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{192}\left(2\left(4\left(6x+1\right)x+7\right)x-37\right)\sqrt{x^2+x+1} - \frac{3}{128}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x +
2*sqrt(x^2 + x + 1) - 1)
```

3.271.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{3\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128} - \frac{\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{4} - \frac{5\,(8\,x^2+2\,x+5)\,\sqrt{x^2+x+1}}{192} + \frac{x\,\left(x^2+x+1\right)^{3/2}}{4}$$

input
```
int(x^2*(x + x^2 + 1)^(1/2),x)
```

output
```
(3*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 - ((x/2 + 1/4)*(x + x^2 + 1)^(1
/2))/4 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/192 + (x*(x + x^2 + 1)^
(3/2))/4
```

3.272 $\int (1+x+x^2)^{3/2} \, dx$

3.272.1 Optimal result

Integrand size = 10, antiderivative size = 55

$$\int (1+x+x^2)^{3/2} \, dx = \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{27}{128}\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
1/8*(1+2*x)*(x^2+x+1)^(3/2)+27/128*arcsinh(1/3*(1+2*x)*3^(1/2))+9/64*(1+2*
x)*(x^2+x+1)^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (1+x+x^2)^{3/2} \, dx = \frac{1}{64}\sqrt{1+x+x^2}(17+42x+24x^2+16x^3) - \frac{27}{128}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[(1 + x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3))/64 - (27*Log[-1 - 2*x +
2*Sqrt[1 + x + x^2]])/128
```

3.272.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+x+1\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{16} \int \sqrt{x^2+x+1}dx+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{16} \left(\frac{3}{8} \int \frac{1}{\sqrt{x^2+x+1}}dx+\frac{1}{4} \sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{9}{16} \left(\frac{1}{8} \sqrt{3} \int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)+\frac{1}{4} \sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{9}{16} \left(\frac{3}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{4} \sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

input
```
Int[(1 + x + x^2)^(3/2),x]
```

output
```
((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (9*(((1 + 2*x)*Sqrt[1 + x + x^2])/4 +
(3*ArcSinh[(1 + 2*x)/Sqrt[3]])/8))/16
```

3.272.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.272.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{(16x^3+24x^2+42x+17)\sqrt{x^2+x+1}}{64}+\frac{27\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	38
default	$\frac{(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{8}+\frac{9(1+2x)\sqrt{x^2+x+1}}{64}+\frac{27\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	43
trager	$\left(\frac{1}{4}x^3+\frac{3}{8}x^2+\frac{21}{32}x+\frac{17}{64}\right)\sqrt{x^2+x+1}+\frac{27\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{128}$	44

input
```
int((x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*x^3+24*x^2+42*x+17)*(x^2+x+1)^(1/2)+27/128*arcsinh(2/3*3^(1/2)*(x
+1/2))
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \frac{1}{64}\left(16\,x^3+24\,x^2+42\,x+17\right)\sqrt{x^2+x+1} - \frac{27}{128}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(16*x^3 + 24*x^2 + 42*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x + 2
*sqrt(x^2 + x + 1) - 1)
```

3.272.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1} + \left(\frac{x^2}{3}+\frac{x}{12}+\frac{5}{24}\right)\sqrt{x^2+x+1} + \sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right) + \frac{27\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$$

input
```
integrate((x**2+x+1)**(3/2),x)
```

output
```
(x/2 + 1/4)*sqrt(x**2 + x + 1) + (x**2/3 + x/12 + 5/24)*sqrt(x**2 + x + 1)
 + sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 27*asinh(2*sq
rt(3)*(x + 1/2)/3)/128
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \left(1+x+x^2\right)^{3/2}\,dx = \frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x + \frac{1}{8}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{9}{32}\sqrt{x^2+x+1}x + \frac{9}{64}\sqrt{x^2+x+1} + \frac{27}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$

input
```
integrate((x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 + x + 1)^(3/2)*x + 1/8*(x^2 + x + 1)^(3/2) + 9/32*sqrt(x^2 + x +
1)*x + 9/64*sqrt(x^2 + x + 1) + 27/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int (1+x+x^2)^{3/2}\,dx = \frac{1}{64}(2(4(2x+3)x+21)x+17)\sqrt{x^2+x+1} - \frac{27}{128}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x +
 2*sqrt(x^2 + x + 1) - 1)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int (1+x+x^2)^{3/2}\,dx = \frac{27\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128} + \frac{\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{3/2}}{4} + \frac{9\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{16}$$

input
```
int((x + x^2 + 1)^(3/2),x)
```

output
```
(27*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 + ((x + 1/2)*(x + x^2 + 1)^(3/
2))/4 + (9*(x/2 + 1/4)*(x + x^2 + 1)^(1/2))/16
```

3.273 $\int (1+x+x^2)^{5/2}\,dx$

3.273.1 Optimal result

Integrand size = 10, antiderivative size = 74

$$\int (1+x+x^2)^{5/2}\,dx = \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{135\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)}{1024}$$

output
```
5/64*(1+2*x)*(x^2+x+1)^(3/2)+1/12*(1+2*x)*(x^2+x+1)^(5/2)+135/1024*arcsinh
(1/3*(1+2*x)*3^(1/2))+45/512*(1+2*x)*(x^2+x+1)^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int (1+x+x^2)^{5/2}\,dx = \frac{\sqrt{1+x+x^2}(383+1142x+1256x^2+1264x^3+640x^4+256x^5)}{1536} - \frac{135\log\left(-1-2x+2\sqrt{1+x+x^2}\right)}{1024}$$

input
```
Integrate[(1 + x + x^2)^(5/2),x]
```

output
```
(Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^5
))/1536 - (135*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/1024
```

3.273.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+x+1\right)^{5/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}\int \left(x^2+x+1\right)^{3/2}dx+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}\left(\frac{9}{16}\int \sqrt{x^2+x+1}dx+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}\left(\frac{9}{16}\left(\frac{3}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{5}{8}\left(\frac{9}{16}\left(\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{5}{8}\left(\frac{9}{16}\left(\frac{3}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

input
```
Int[(1 + x + x^2)^(5/2),x]
```

output
```
((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (5*(((1 + 2*x)*(1 + x + x^2)^(3/2))/8
 + (9*(((1 + 2*x)*Sqrt[1 + x + x^2])/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/8)
)/16))/8
```

3.273.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.273.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{(256x^5+640x^4+1264x^3+1256x^2+1142x+383)\sqrt{x^2+x+1}}{1536}+\frac{135\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$	48
trager	$\left(\frac{1}{6}x^5+\frac{5}{12}x^4+\frac{79}{96}x^3+\frac{157}{192}x^2+\frac{571}{768}x+\frac{383}{1536}\right)\sqrt{x^2+x+1}+\frac{135\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{1024}$	54
default	$\frac{(1+2x)\left(x^2+x+1\right)^{\frac{5}{2}}}{12}+\frac{5(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{64}+\frac{45(1+2x)\sqrt{x^2+x+1}}{512}+\frac{135\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$	58

input
```
int((x^2+x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(256*x^5+640*x^4+1264*x^3+1256*x^2+1142*x+383)*(x^2+x+1)^(1/2)+135/
1024*arcsinh(2/3*3^(1/2)*(x+1/2))
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{1536}\left(256\,x^5+640\,x^4+1264\,x^3+1256\,x^2+1142\,x+383\right)\sqrt{x^2+x+1} - \frac{135}{1024}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(5/2),x, algorithm="fricas")
```

output
```
1/1536*(256*x^5 + 640*x^4 + 1264*x^3 + 1256*x^2 + 1142*x + 383)*sqrt(x^2 +
 x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(71) = 142.

Time = 0.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.27

$$\int (1+x+x^2)^{5/2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}+2\left(\frac{x^2}{3}+\frac{x}{12}+\frac{5}{24}\right)\sqrt{x^2+x+1} +3\sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right)+2\sqrt{x^2+x+1}\left(\frac{x^4}{5}+\frac{x^3}{40}+\frac{3x^2}{80}-\frac{27x}{320}+\frac{33}{640}\right) +\sqrt{x^2+x+1}\left(\frac{x^5}{6}+\frac{x^4}{60}+\frac{11x^3}{480}-\frac{47x^2}{960}+\frac{103x}{3840}+\frac{443}{7680}\right)+\frac{135\,\mathrm{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$$

input
```
integrate((x**2+x+1)**(5/2),x)
```

output
```
(x/2 + 1/4)*sqrt(x**2 + x + 1) + 2*(x**2/3 + x/12 + 5/24)*sqrt(x**2 + x +
1) + 3*sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 2*sqrt(x*
*2 + x + 1)*(x**4/5 + x**3/40 + 3*x**2/80 - 27*x/320 + 33/640) + sqrt(x**2
 + x + 1)*(x**5/6 + x**4/60 + 11*x**3/480 - 47*x**2/960 + 103*x/3840 + 443
/7680) + 135*asinh(2*sqrt(3)*(x + 1/2)/3)/1024
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{6}\left(x^2+x+1\right)^{\frac{5}{2}}x + \frac{1}{12}\left(x^2+x+1\right)^{\frac{5}{2}} + \frac{5}{32}\left(x^2+x+1\right)^{\frac{3}{2}}x + \frac{5}{64}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{45}{256}\sqrt{x^2+x+1}x + \frac{45}{512}\sqrt{x^2+x+1} + \frac{135}{1024}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate((x^2+x+1)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(x^2 + x + 1)^(5/2)*x + 1/12*(x^2 + x + 1)^(5/2) + 5/32*(x^2 + x + 1)^
(3/2)*x + 5/64*(x^2 + x + 1)^(3/2) + 45/256*sqrt(x^2 + x + 1)*x + 45/512*s
qrt(x^2 + x + 1) + 135/1024*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{1536}(2(4(2(8(2x+5)x+79)x+157)x+571)x+383)\sqrt{x^2+x+1} - \frac{135}{1024}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(5/2),x, algorithm="giac")
```

output
```
1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*sqrt(x^2 +
 x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int (1+x+x^2)^{5/2}\,dx = \frac{135\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{1024} + \frac{5\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{3/2}}{32} + \frac{\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{5/2}}{6} + \frac{45\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{128}$$

input
```
int((x + x^2 + 1)^(5/2),x)
```

output
```
(135*log(x + (x + x^2 + 1)^(1/2) + 1/2))/1024 + (5*(x + 1/2)*(x + x^2 + 1)
^(3/2))/32 + ((x + 1/2)*(x + x^2 + 1)^(5/2))/6 + (45*(x/2 + 1/4)*(x + x^2
+ 1)^(1/2))/128
```

3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$

3.274.1 Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{x} + \frac{1}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

3.274.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{x} - \text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]
```

output
```
-(Sqrt[1 + x + x^2]/x) - ArcTanh[x - Sqrt[1 + x + x^2]]
```

3.274.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^2+x+1}}dx$$
$$\Big\downarrow \text{1157}$$
$$-\frac{1}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{x}$$
$$\Big\downarrow \text{1154}$$
$$\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}-\frac{\sqrt{x^2+x+1}}{x}$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

input
```
Int[1/(x^2*Sqrt[1 + x + x^2]),x]
```

output
```
-(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

3.274.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

3.274.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}-\frac{\sqrt{x^2+x+1}}{x}$	31
risch	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}-\frac{\sqrt{x^2+x+1}}{x}$	31
trager	$-\frac{\sqrt{x^2+x+1}}{x}+\frac{\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{2}$	35

input
```
int(1/x^2/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{x\log\left(-x+\sqrt{x^2+x+1}+1\right)-x\log\left(-x+\sqrt{x^2+x+1}-1\right)-2\,x-2\sqrt{x^2+x+1}}{2\,x}$$

input
```
integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x*log(-x + sqrt(x^2 + x + 1) + 1) - x*log(-x + sqrt(x^2 + x + 1) - 1)
 - 2*x - 2*sqrt(x^2 + x + 1))/x
```

3.274.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^2\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/x**2/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2 + x + 1)), x)
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{x^2+x+1}}{x} + \frac{1}{2}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x^2 + x + 1)/x + 1/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(
x))
```

3.274.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1} + \frac{1}{2}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{2}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
(x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2 + x + 1))^2 - 1) + 1/2*log(abs(
-x + sqrt(x^2 + x + 1) + 1)) - 1/2*log(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.274.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\frac{x}{2}+1}{\sqrt{x^2+x+1}}\right)}{2} - \frac{\sqrt{x^2+x+1}}{x}$$

input
```
int(1/(x^2*(x + x^2 + 1)^(1/2)),x)
```

output
```
atanh((x/2 + 1)/(x + x^2 + 1)^(1/2))/2 - (x + x^2 + 1)^(1/2)/x
```

3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

3.275.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx=-\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+
1)^(1/2)/x
```

3.275.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx=\frac{(-2+3x)\sqrt{1+x+x^2}}{4x^2}-\frac{1}{4}\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

output
```
((-2 + 3*x)*Sqrt[1 + x + x^2])/(4*x^2) - ArcTanh[x - Sqrt[1 + x + x^2]]/4
```

3.275.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1167}$$

$$-\frac{1}{2}\int \frac{2x+3}{2x^2\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{2x+3}{x^2\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{4}\left(\frac{3\sqrt{x^2+x+1}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{4}\left(\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}+\frac{3\sqrt{x^2+x+1}}{x}\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+\frac{3\sqrt{x^2+x+1}}{x}\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

output
```
-1/2*Sqrt[1 + x + x^2]/x^2 + ((3*Sqrt[1 + x + x^2])/x + ArcTanh[(2 + x)/(2
*Sqrt[1 + x + x^2])]/2)/4
```

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.275.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result	size
trager	$\frac{(-2+3x)\sqrt{x^2+x+1}}{4x^2}-\frac{\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{8}$	42
risch	$\frac{3x^3+x^2+x-2}{4x^2\sqrt{x^2+x+1}}+\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	42
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{3\sqrt{x^2+x+1}}{4x}$	44

input
```
int(1/x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2+3*x)/x^2*(x^2+x+1)^(1/2)-1/8*ln((-2-x+2*(x^2+x+1)^(1/2))/x)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{x^2\log\left(-x+\sqrt{x^2+x+1}+1\right)-x^2\log\left(-x+\sqrt{x^2+x+1}-1\right)+6\,x^2+2\,\sqrt{x^2+x+1}(3\,x-2)}{8\,x^2}$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1)
- 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

3.275.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{3\sqrt{x^2+x+1}}{4\,x} - \frac{\sqrt{x^2+x+1}}{2\,x^2} + \frac{1}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt
(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{\left(x-\sqrt{x^2+x+1}\right)^3 + 9\,x - 9\sqrt{x^2+x+1} + 8}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2} + \frac{1}{8}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{8}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt
(x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*l
og(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x^2+x+1}}\,dx$$

input
```
int(1/(x^3*(x + x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^3*(x + x^2 + 1)^(1/2)), x)
```

3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$

3.276.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx=\frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}+\frac{3}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x/(x^2+x+1)^(1/2)-5/3*(x^
2+x+1)^(1/2)/x
```

3.276.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx=\frac{-3-7x-5x^2}{3x\sqrt{1+x+x^2}}-3\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

output
```
(-3 - 7*x - 5*x^2)/(3*x*Sqrt[1 + x + x^2]) - 3*ArcTanh[x - Sqrt[1 + x + x^
2]]
```

3.276.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^2+x+1)^{3/2}} \, dx$$

$$\Big\downarrow \ 1165$$

$$\frac{2}{3} \int \frac{5-2x}{2x^2\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3} \int \frac{5-2x}{x^2\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 1228$$

$$\frac{1}{3} \left(-\frac{9}{2} \int \frac{1}{x\sqrt{x^2+x+1}}dx-\frac{5\sqrt{x^2+x+1}}{x}\right)+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{3} \left(9 \int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{x}\right)+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3} \left(\frac{9}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{5\sqrt{x^2+x+1}}{x}\right)+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

input
```
Int[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

output
```
(2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) + ((-5*Sqrt[1 + x + x^2])/x + (9*ArcTa
nh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2)/3
```

3.276.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.276.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}+\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	41
trager	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}-\frac{3\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{2}$	47
default	$-\frac{1}{x\sqrt{x^2+x+1}}-\frac{3}{2\sqrt{x^2+x+1}}-\frac{5(1+2x)}{6\sqrt{x^2+x+1}}+\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	56

input
```
int(1/x^2/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*x^2+7*x+3)/x/(x^2+x+1)^(1/2)+3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2)
)
```

3.276.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx = -\frac{10\,x^3+10\,x^2-9\,(x^3+x^2+x)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\,(x^3+x^2+x)\log\left(-x+\sqrt{x^2+x+1}-1\right)}{6\,(x^3+x^2+x)}$$

input
```
integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(10*x^3 + 10*x^2 - 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) + 1)
+ 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(5*x^2 + 7*x + 3)*
sqrt(x^2 + x + 1) + 10*x)/(x^3 + x^2 + x)
```

3.276.6 Sympy [F]

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx = \int \frac{1}{x^2(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(x**2+x+1)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx = -\frac{5\,x}{3\,\sqrt{x^2+x+1}} - \frac{7}{3\,\sqrt{x^2+x+1}} - \frac{1}{\sqrt{x^2+x+1}x} + \frac{3}{2}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
-5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 + x + 1)*x)
 + 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx = -\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}} + \frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1} + \frac{3}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{3}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(x + 2)/sqrt(x^2 + x + 1) + (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^
2 + x + 1))^2 - 1) + 3/2*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 3/2*log(ab
s(-x + sqrt(x^2 + x + 1) - 1))
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1 + x + x^2)^{3/2}} \, dx = \int \frac{1}{x^2 (x^2 + x + 1)^{3/2}} \, dx$$

input
```
int(1/(x^2*(x + x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(x + x^2 + 1)^(3/2)), x)
```

3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx$

3.277.1 Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx = \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
-3/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x^2/(x^2+x+1)^(1/2)-7/6*
(x^2+x+1)^(1/2)/x^2+37/12*(x^2+x+1)^(1/2)/x
```

3.277.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx = \frac{-6+15x+23x^2+37x^3}{12x^2\sqrt{1+x+x^2}} + \frac{3}{4}\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

output
```
(-6 + 15*x + 23*x^2 + 37*x^3)/(12*x^2*Sqrt[1 + x + x^2]) + (3*ArcTanh[x -
Sqrt[1 + x + x^2]])/4
```

3.277.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1165, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(x^2+x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{2}{3}\int \frac{7-4x}{2x^3\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{7-4x}{x^3\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{14x+37}{2x^2\sqrt{x^2+x+1}}dx-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-\frac{1}{4}\int \frac{14x+37}{x^2\sqrt{x^2+x+1}}dx-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{3}\left(\frac{1}{4}\left(\frac{9}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx+\frac{37\sqrt{x^2+x+1}}{x}\right)-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(\frac{1}{4}\left(\frac{37\sqrt{x^2+x+1}}{x}-9\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}\right)-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{1}{4}\left(\frac{37\sqrt{x^2+x+1}}{x}-\frac{9}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)\right)-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

input
```
Int[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

output
```
(2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) + ((-7*Sqrt[1 + x + x^2])/(2*x^2) +
((37*Sqrt[1 + x + x^2])/x - (9*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2)/
4)/3
```

3.277.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.277.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\,x^2}-\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	46
trager	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\,x^2}-\frac{3\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{8}$	50
default	$-\frac{1}{2x^2\sqrt{x^2+x+1}}+\frac{5}{4x\sqrt{x^2+x+1}}+\frac{3}{8\sqrt{x^2+x+1}}+\frac{\frac{37}{24}+\frac{37x}{12}}{\sqrt{x^2+x+1}}-\frac{3\,\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$	69

input
```
int(1/x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(37*x^3+23*x^2+15*x-6)/(x^2+x+1)^(1/2)/x^2-3/8*arctanh(1/2*(2+x)/(x^2
+x+1)^(1/2))
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}}\,dx=\frac{74\,x^4+74\,x^3+74\,x^2-9\,(x^4+x^3+x^2)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\,(x^4+x}{24\,(x^4+}$$

input
```
integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(74*x^4 + 74*x^3 + 74*x^2 - 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x
 + 1) + 1) + 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(37*x
^3 + 23*x^2 + 15*x - 6)*sqrt(x^2 + x + 1))/(x^4 + x^3 + x^2)
```

3.277.6 Sympy [F]

$$\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx = \int \frac{1}{x^3(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

output
```
Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx &= \frac{37\,x}{12\sqrt{x^2+x+1}} + \frac{23}{12\sqrt{x^2+x+1}} \\ &+ \frac{5}{4\sqrt{x^2+x+1}x} - \frac{1}{2\sqrt{x^2+x+1}x^2} - \frac{3}{8}\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\sqrt{3}}{3\,|x|}\right)\end{aligned}$$

input
```
integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
37/12*x/sqrt(x^2 + x + 1) + 23/12/sqrt(x^2 + x + 1) + 5/4/(sqrt(x^2 + x +
1)*x) - 1/2/(sqrt(x^2 + x + 1)*x^2) - 3/8*arcsinh(1/3*sqrt(3)*x/abs(x) + 2
/3*sqrt(3)/abs(x))
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx &= \frac{2\,(2\,x+1)}{3\sqrt{x^2+x+1}} \\ &- \frac{3\left(x-\sqrt{x^2+x+1}\right)^3 + 8\left(x-\sqrt{x^2+x+1}\right)^2 - 13\,x + 13\sqrt{x^2+x+1} - 16}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2 - 1\right)^2} \\ &- \frac{3}{8}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) + \frac{3}{8}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
2/3*(2*x + 1)/sqrt(x^2 + x + 1) - 1/4*(3*(x - sqrt(x^2 + x + 1))^3 + 8*(x
- sqrt(x^2 + x + 1))^2 - 13*x + 13*sqrt(x^2 + x + 1) - 16)/((x - sqrt(x^2
+ x + 1))^2 - 1)^2 - 3/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 3/8*log(ab
s(-x + sqrt(x^2 + x + 1) - 1))
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}} \, dx = \int \frac{1}{x^3 (x^2+x+1)^{3/2}} \, dx$$

input
```
int(1/(x^3*(x + x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x^3*(x + x^2 + 1)^(3/2)), x)
```

3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$

3.278.1 Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)$$

output
```
-arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))
```

3.278.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = 2\operatorname{arctanh}\left(1+x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

output
```
2*ArcTanh[1 + x - Sqrt[1 + x + x^2]]
```

3.278.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{4-\frac{(1-x)^2}{x^2+x+1}}d\frac{1-x}{\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

input
```
Int[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

output
```
-ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

3.278.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.278.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2-x}}\right)$	22
trager	$-\ln\left(\frac{2\sqrt{x^2+x+1}+1-x}{1+x}\right)$	25

input
```
int(1/(1+x)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)$$

input
```
integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)
```

3.278.6 Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/(1+x)/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x+1|} - \frac{\sqrt{3}}{3\,|x+1|}\right)$$

input
```
integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right) + \log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

input
```
integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \int \frac{1}{(x+1)\,\sqrt{x^2+x+1}}\,dx$$

input
```
int(1/((x + 1)*(x + x^2 + 1)^(1/2)),x)
```

output
```
int(1/((x + 1)*(x + x^2 + 1)^(1/2)), x)
```

3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$

3.279.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) - \frac{\text{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\text{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/2*arctanh(1/2*(4+x)/(x^2+2*x+4)^(1/2))-1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)
*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1
/2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = -\text{arctanh}\left(\frac{1}{2}\left(x-\sqrt{4+2x+x^2}\right)\right) + \frac{\text{arctanh}\left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\text{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

output
```
-ArcTanh[(x - Sqrt[4 + 2*x + x^2])/2] + ArcTanh[(1 + x - Sqrt[4 + 2*x + x^
2])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt
[7]
```

3.279.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2x+4}\,(x^3-x)}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x\,(x^2-1)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x}{(x^2-1)\sqrt{x^2+2x+4}}-\frac{1}{x\sqrt{x^2+2x+4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\text{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

output
```
ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sq
rt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*
Sqrt[3])
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.279.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{8+2x}{4\sqrt{x^2+2x+4}}\right)}{2}-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{x^2+2x+4}-\operatorname{RootOf}\left(\_Z^2-3\right)}{1+x}\right)}{6}-\frac{\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}+5\operatorname{RootOf}\left(\_\right.}{-1+x}\right.}{14}$

input
```
int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/4*(8+2*x)/(x^2+2*x+4)^(1/2))-1/14*7^(1/2)*arctanh(1/14*(10+4
*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3
)^(1/2))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx$$
$$= \frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x-10}{x-1}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right) + \frac{1}{2}\log\left(-x+\sqrt{x^2+2x+4}+2\right) - \frac{1}{2}\log\left(-x+\sqrt{x^2+2x+4}-2\right)$$

input
```
integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7)
- 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x
 + 1)) + 1/2*log(-x + sqrt(x^2 + 2*x + 4) + 2) - 1/2*log(-x + sqrt(x^2 + 2
*x + 4) - 2)
```

3.279.6 Sympy [F]

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \int \frac{1}{x\,(x-1)\,(x+1)\,\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
```

output
```
Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

3.279.7 Maxima [F]

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \int \frac{1}{(x^3-x)\sqrt{x^2+2\,x+4}}\,dx$$

input
```
integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)
```

3.279.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(66) = 132.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx &= \frac{1}{14}\,\sqrt{7}\log\left(\frac{\left|-2\,x-2\,\sqrt{7}+2\,\sqrt{x^2+2\,x+4}+2\right|}{\left|-2\,x+2\,\sqrt{7}+2\,\sqrt{x^2+2\,x+4}+2\right|}\right)\\ &+\frac{1}{6}\,\sqrt{3}\log\left(-\frac{\left|-2\,x-2\,\sqrt{3}+2\,\sqrt{x^2+2\,x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2\,x+4}+1\right)}\right)\\ &+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+2\,x+4}+2\right|\right)\\ &-\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+2\,x+4}-2\right|\right)\end{aligned}$$

input
```
integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*
x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*
x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x +
 4) + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) + 2)) - 1/2*log(abs(-x +
sqrt(x^2 + 2*x + 4) - 2))
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = -\int \frac{1}{(x-x^3)\,\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int(-1/((x - x^3)*(2*x + x^2 + 4)^(1/2)),x)
```

output
```
-int(1/((x - x^3)*(2*x + x^2 + 4)^(1/2)), x)
```

3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2} dx$

3.280.1 Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2} dx = \frac{\sqrt{4+2x+x^2}}{1-x} + \operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right) - \frac{2\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{\sqrt{7}}$$

output
```
arcsinh(1/3*(1+x)*3^(1/2))-2/7*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/
2))*7^(1/2)+(x^2+2*x+4)^(1/2)/(1-x)
```

3.280.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2} dx = -\frac{\sqrt{4+2x+x^2}}{-1+x} - \frac{4\operatorname{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}} - \log\left(-1-x+\sqrt{4+2x+x^2}\right)$$

input
```
Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

output
```
-(Sqrt[4 + 2*x + x^2]/(-1 + x)) - (4*ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])
/Sqrt[7]])/Sqrt[7] - Log[-1 - x + Sqrt[4 + 2*x + x^2]]
```

3.280.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1161, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{1}{2}\int -\frac{2(x+1)}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\int \frac{x+1}{(1-x)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{ 1269}$$

$$\int \frac{1}{\sqrt{x^2+2x+4}}dx-2\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 1090}$$

$$-2\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{\int \frac{1}{\sqrt{\frac{1}{12}(2x+2)^2+1}}d(2x+2)}{2\sqrt{3}}+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 222}$$

$$-2\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\text{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 1154}$$

$$4\int \frac{1}{28-\frac{4(2x+5)^2}{x^2+2x+4}}d\left(-\frac{2(2x+5)}{\sqrt{x^2+2x+4}}\right)+\text{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)-\frac{2\text{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\frac{\sqrt{x^2+2x+4}}{1-x}$$

input
```
Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

output
```
Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(2 + 2*x)/(2*Sqrt[3])] - (2*ArcTanh[
(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.280.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{x^2+2x+4}}{-1+x}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}$
trager	$-\frac{\sqrt{x^2+2x+4}}{-1+x}+\ln\left(1+x+\sqrt{x^2+2x+4}\right)+\frac{2\,\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}-5\,\mathrm{RootOf}}{-1+x}\right.}{7}$
default	$-\frac{\left((-1+x)^2+3+4x\right)^{\frac{3}{2}}}{7(-1+x)}+\frac{2\sqrt{(-1+x)^2+3+4x}}{7}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}+\frac{(2x+2)\sqrt{}}{}$

input
```
int((x^2+2*x+4)^(1/2)/(-1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(-1+x)*(x^2+2*x+4)^(1/2)+arcsinh(1/3*(1+x)*3^(1/2))-2/7*7^(1/2)*arctanh
(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$$
$$= \frac{2\sqrt{7}(x-1)\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)-7(x-1)\log\left(-x+\sqrt{x^2+2x+4}-1\right)-7x-7}{7(x-1)}$$

input
```
integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="fricas")
```

output
```
1/7*(2*sqrt(7)*(x - 1)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqr
t(7) - 7) - 4*x - 10)/(x - 1)) - 7*(x - 1)*log(-x + sqrt(x^2 + 2*x + 4) -
1) - 7*x - 7*sqrt(x^2 + 2*x + 4) + 7)/(x - 1)
```

3.280.6 Sympy [F]

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

input
```
integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

output
```
Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{2}{7}\sqrt{7}\,\mathrm{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right) - \frac{\sqrt{x^2+2x+4}}{x-1} + \mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

input
```
integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="maxima")
```

output
```
-2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x - 1)) -
sqrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/3*sqrt(3))
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(53) = 106$.

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{2}{7}\sqrt{7}\log\left(\sqrt{7}\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}+2\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) + \log\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}+1\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - \log\left(\left|\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - \sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

input
```
integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="giac")
```

output
```
-2/7*sqrt(7)*log(sqrt(7)*(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)/(x -
 1)) + 2)*sgn(1/(x - 1)) + log(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)
/(x - 1) + 1)*sgn(1/(x - 1)) - log(abs(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) +
 sqrt(7)/(x - 1) - 1))*sgn(1/(x - 1)) - sqrt(4/(x - 1) + 7/(x - 1)^2 + 1)*
sgn(1/(x - 1))
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \int \frac{\sqrt{x^2+2\,x+4}}{(x-1)^2}\,dx$$

input
```
int((2*x + x^2 + 4)^(1/2)/(x - 1)^2,x)
```

output
```
int((2*x + x^2 + 4)^(1/2)/(x - 1)^2, x)
```

3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$

3.281.1 Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \operatorname{arctanh}\left(\sqrt{4+2x+x^2}\right)$$

output
```
arctanh((x^2+2*x+4)^(1/2))-1/8*arctan(1/2*(1+x)*2^(1/2)/(x^2+2*x+4)^(1/2))
*2^(1/2)-1/4*(3-x)*(x^2+2*x+4)^(1/2)/(x^2+2*x+3)
```

3.281.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\left(\frac{2(-3+x)\sqrt{4+2x+x^2}}{3+2x+x^2} + \sqrt{2}\arctan\left(\frac{3+2x+x^2-(1+x)\sqrt{4+2x+x^2}}{\sqrt{2}}\right)\right) + \operatorname{arctanh}\left(\sqrt{4+2x+x^2}\right)$$

input
```
Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

output
```
((2*(-3 + x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) + Sqrt[2]*ArcTan[(3 + 2*
x + x^2 - (1 + x)*Sqrt[4 + 2*x + x^2])/Sqrt[2]])/8 + ArcTanh[Sqrt[4 + 2*x
+ x^2]]
```

3.281.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1349, 27, 1358, 27, 1313, 217, 1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(x^2+2x+3)^2\sqrt{x^2+2x+4}}\,dx$$

$$\Big\downarrow \text{1349}$$

$$\frac{1}{8}\int -\frac{2(4x+5)}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{4x+5}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-\frac{\sqrt{x^2+2x+4}(3-x)}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{1358}$$

$$\frac{1}{4}\left(-\int \frac{1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-2\int \frac{2(x+1)}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\int \frac{1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-4\int \frac{x+1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{1313}$$

$$\frac{1}{4}\left(4\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+4}-16}d\frac{2(x+1)}{\sqrt{x^2+2x+4}}-4\int \frac{x+1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(-4\int \frac{x+1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{\sqrt{2}}\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{1357}$$

$$\frac{1}{4}\left(8\int \frac{1}{2-2(x^2+2x+4)}d\sqrt{x^2+2x+4}-\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{\sqrt{2}}\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(4\text{arctanh}\left(\sqrt{x^2+2x+4}\right)-\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{\sqrt{2}}\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

input
```
Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

output
```
-1/4*((3 - x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) + (-(ArcTan[(1 + x)/(Sq
rt[2]*Sqrt[4 + 2*x + x^2])]/Sqrt[2]) + 4*ArcTanh[Sqrt[4 + 2*x + x^2]])/4
```

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.281. $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

3.281. $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$

3.281.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12}+\operatorname{arctanh}\left(\sqrt{x^2+2x+4}\right)-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(2x+2)}{4\sqrt{x^2+2x+4}}\right)}{8}$
default	$-\frac{1}{2\left(\sqrt{x^2+2x+4}+1\right)}+\frac{\ln\left(\sqrt{x^2+2x+4}+1\right)}{2}-\frac{1}{2\left(\sqrt{x^2+2x+4}-1\right)}-\frac{\ln\left(\sqrt{x^2+2x+4}-1\right)}{2}+\frac{\frac{3}{4}+\frac{3x}{4}}{\sqrt{x^2+2x+4}\left(\frac{(1+x)^2}{x^2+2x+4}+2\right)}-$
trager	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12}-3\ln\left(-\frac{48384\,\mathrm{RootOf}\left(384\_Z^2+128\_Z+11\right)^2 x+960\sqrt{x^2+2x+4}\,\mathrm{RootOf}\left(384\_Z^2+128\_Z+11\right)+1}{48\,\mathrm{RootOf}\left(384\right.}\right.$

input
```
int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-3+x)/(x^2+2*x+3)*(x^2+2*x+4)^(1/2)+arctanh((x^2+2*x+4)^(1/2))-1/8*2^
(1/2)*arctan(1/4*2^(1/2)/(x^2+2*x+4)^(1/2)*(2*x+2))
```

3.281.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(61) = 122.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.29

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$$
$$=\frac{\sqrt{2}(x^2+2\,x+3)\arctan\left(-\frac{1}{2}\sqrt{2}(x+2)+\frac{1}{2}\sqrt{2}\sqrt{x^2+2\,x+4}\right)-\sqrt{2}(x^2+2\,x+3)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\right.}{}$$

input
```
integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*(x + 2) + 1/2*sqrt(2)*sqr
t(x^2 + 2*x + 4)) - sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*x + 1/2*sq
rt(2)*sqrt(x^2 + 2*x + 4)) + 2*x^2 - 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2
+ 2*x + 4)*(x + 2) + 3*x + 5) + 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2*x
 + 4)*x + x + 3) + 2*sqrt(x^2 + 2*x + 4)*(x - 3) + 4*x + 6)/(x^2 + 2*x + 3
)
```

3.281.6 Sympy [F]

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2x+3}{(x^2+2x+3)^2\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
```

output
```
Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)
```

3.281.7 Maxima [F]

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2\,x+3}{\sqrt{x^2+2\,x+4}(x^2+2\,x+3)^2}\,dx$$

input
```
integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)
```

3.281.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(61) = 122$.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.09

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2\,x+4}+2\right)\right)$$
$$-\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2\,x+4}\right)\right)$$
$$+\frac{4\left(x-\sqrt{x^2+2\,x+4}\right)^3+13\left(x-\sqrt{x^2+2\,x+4}\right)^2+26\,x-26\sqrt{x^2+2\,x+4}+26}{2\left(\left(x-\sqrt{x^2+2\,x+4}\right)^4+4\left(x-\sqrt{x^2+2\,x+4}\right)^3+8\left(x-\sqrt{x^2+2\,x+4}\right)^2+8\,x-8\sqrt{x^2+2\,x+4}+\cdots\right.}$$
$$-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+4}\right)^2+4\,x-4\sqrt{x^2+2\,x+4}+6\right)$$
$$+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+4}\right)^2+2\right)$$

input
```
integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) - 1/8*sqrt(
2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*(4*(x - sqrt(x^2 +
 2*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 + 26*x - 26*sqrt(x^2 + 2*x +
 4) + 26)/((x - sqrt(x^2 + 2*x + 4))^4 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8
*(x - sqrt(x^2 + 2*x + 4))^2 + 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*log
((x - sqrt(x^2 + 2*x + 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*log(
(x - sqrt(x^2 + 2*x + 4))^2 + 2)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx=\int \frac{2\,x+3}{{\left(x^2+2\,x+3\right)}^2\,\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)),x)
```

output
```
int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)), x)
```

3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)}\,dx$

3.282.1 Optimal result

Integrand size = 34, antiderivative size = 36

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{-3+2x+x^2}+\frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

output
```
(x^2+2*x-3)^(1/2)+1/2*(x^2+2*x-3)^(1/2)/(1-x)
```

3.282.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{(-3+2x)\sqrt{-3+2x+x^2}}{2(-1+x)}$$

input
```
Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

output
```
((-3 + 2*x)*Sqrt[-3 + 2*x + x^2])/(2*(-1 + x))
```

3.282.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2027, 2004, 1213, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2}{\sqrt{x^2+2x-3}\,(2x^2+x-3)}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(2x+3)}{\sqrt{x^2+2x-3}\,(2x^2+x-3)}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{x^2}{(x-1)\sqrt{x^2+2x-3}}dx$$

$$\Big\downarrow \text{1213}$$

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}-\int -\frac{x+1}{\sqrt{x^2+2x-3}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x+1}{\sqrt{x^2+2x-3}}dx+\frac{\sqrt{x^2+2x-3}}{2(1-x)}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

input
```
Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

output
```
Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.282.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{(2x-3)(3+x)}{2\sqrt{x^2+2x-3}}$	21
trager	$\frac{(2x-3)\sqrt{x^2+2x-3}}{-2+2x}$	23
risch	$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$	23
default	$\sqrt{x^2+2x-3}-\frac{\sqrt{(-1+x)^2-4+4x}}{2(-1+x)}$	31

input
```
int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x-3)*(3+x)/(x^2+2*x-3)^(1/2)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\sqrt{x^2+2\,x-3}(2\,x-3)}{2\,(x-1)}$$

input
```
integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="fricas
")
```

output
```
1/2*sqrt(x^2 + 2*x - 3)*(2*x - 3)/(x - 1)
```

3.282.6 Sympy [F]

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+3)}\,(x-1)}\,dx$$

input
```
integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{x^2+2\,x-3} - \frac{\sqrt{x^2+2\,x-3}}{2\,(x-1)}$$

input
```
integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="maxima
")
```

output
```
sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{x^2+2\,x-3}+\frac{2}{x-\sqrt{x^2+2\,x-3}-1}$$

input
```
integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\left(x-\frac{3}{2}\right)\sqrt{x^2+2\,x-3}}{x-1}$$

input
```
int((3*x^2 + 2*x^3)/((x + 2*x^2 - 3)*(2*x + x^2 - 3)^(1/2)),x)
```

output
```
((x - 3/2)*(2*x + x^2 - 3)^(1/2))/(x - 1)
```

3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$

3.283.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}-\frac{1}{8}\text{arcsinh}\left(\frac{1+2x}{\sqrt{7}}\right)+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}}-\text{arctanh}\left(\sqrt{2+x+x^2}\right)$$

output
```
-1/8*arcsinh(1/7*(1+2*x)*7^(1/2))-arctanh((x^2+x+2)^(1/2))+1/3*arctan(1/3*
(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)-7/4*(x^2+x+2)^(1/2)+1/2*x*(x^2+x+
2)^(1/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = -\frac{\arctan\left(\frac{2+2x+2x^2-(1+2x)\sqrt{2+x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}-\text{arctanh}\left(\sqrt{2+x+x^2}\right)+\frac{1}{8}\left(2(-7+2x)\sqrt{2+x+x^2}+\log\left(-1-2x+2\sqrt{2+x+x^2}\right)\right)$$

input
```
Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

output
```
-(ArcTan[(2 + 2*x + 2*x^2 - (1 + 2*x)*Sqrt[2 + x + x^2])/Sqrt[3]]/Sqrt[3])
 - ArcTanh[Sqrt[2 + x + x^2]] + (2*(-7 + 2*x)*Sqrt[2 + x + x^2] + Log[-1 -
 2*x + 2*Sqrt[2 + x + x^2]])/8
```

3.283.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{x^2}{\sqrt{x^2+x+2}}-\frac{x}{\sqrt{x^2+x+2}}+\frac{x+1}{(x^2+x+1)\sqrt{x^2+x+2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{7}}\right)+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\text{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{1}{2}\sqrt{x^2+x+2}x-\frac{7}{4}\sqrt{x^2+x+2}$$

input
```
Int[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

output
```
(-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 + 2*x)/Sqr
t[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/Sqrt[3] - ArcTanh[
Sqrt[2 + x + x^2]]
```

3.283.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.283.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{(2x-7)\sqrt{x^2+x+2}}{4}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{7}\left(x+\frac{1}{2}\right)}{7}\right)}{8}-\operatorname{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$	63
default	$\frac{x\sqrt{x^2+x+2}}{2}-\frac{7\sqrt{x^2+x+2}}{4}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{7}\left(x+\frac{1}{2}\right)}{7}\right)}{8}-\operatorname{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$	69
trager	Expression too large to display	1101

input
```
int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x-7)*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*7^(1/2)*(x+1/2))-arctanh((x^2+
x+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(70) = 140$.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+2}(2x-7) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3)+\frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1)+\frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{2}\log\left(2x^2-\sqrt{x^2+x+2}(2x+3)+4x+5\right) - \frac{1}{2}\log\left(2x^2-\sqrt{x^2+x+2}(2x-1)+3\right) + \frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)$$

input
```
integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x + 3
) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x
- 1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*
(2*x + 3) + 4*x + 5) - 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x - 1) + 3) +
1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

3.283.6 Sympy [F]

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}\,dx$$

input
```
integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

output
```
Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)
```

3.283.7 Maxima [F]

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx=\int \frac{x^4+1}{\sqrt{x^2+x+2}(x^2+x+1)}\,dx$$

input
```
integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx &= \frac{1}{4}\sqrt{x^2+x+2}(2x-7)\\ &\quad-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}+3\right)\right)\\ &\quad+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}-1\right)\right)\\ &\quad+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+x+2}\right)^2+3x-3\sqrt{x^2+x+2}+3\right)\\ &\quad-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+x+2}\right)^2-x+\sqrt{x^2+x+2}+1\right)\\ &\quad+\frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)\end{aligned}$$

input
```
integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2
*sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x
^2 + x + 2) - 1)) + 1/2*log((x - sqrt(x^2 + x + 2))^2 + 3*x - 3*sqrt(x^2 +
 x + 2) + 3) - 1/2*log((x - sqrt(x^2 + x + 2))^2 - x + sqrt(x^2 + x + 2) +
 1) + 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}\,dx$$

input
```
int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)),x)
```

output
```
int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)), x)
```

3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx$

3.284.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx=\frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8(1+x)}{405\sqrt{4+2x+x^2}}$$

output
```
1/15*(1+x)/(x^2+2*x+4)^(5/2)+4/135*(1+x)/(x^2+2*x+4)^(3/2)+8/405*(1+x)/(x^
2+2*x+4)^(1/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx=\frac{(1+x)\left(203+152x+108x^2+32x^3+8x^4\right)}{405\left(4+2x+x^2\right)^{5/2}}$$

input
```
Integrate[(4 + 2*x + x^2)^(-7/2),x]
```

output
```
((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5
/2))
```

3.284.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+2x+4)^{7/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4}{15}\int \frac{1}{(x^2+2x+4)^{5/2}}dx+\frac{x+1}{15\,(x^2+2x+4)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{4}{15}\left(\frac{2}{9}\int \frac{1}{(x^2+2x+4)^{3/2}}dx+\frac{x+1}{9\,(x^2+2x+4)^{3/2}}\right)+\frac{x+1}{15\,(x^2+2x+4)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{x+1}{15\,(x^2+2x+4)^{5/2}}+\frac{4}{15}\left(\frac{2(x+1)}{27\sqrt{x^2+2x+4}}+\frac{x+1}{9\,(x^2+2x+4)^{3/2}}\right)$$

input
```
Int[(4 + 2*x + x^2)^(-7/2),x]
```

output
```
(1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*((1 + x)/(9*(4 + 2*x + x^2)^(3/2))
 + (2*(1 + x))/(27*Sqrt[4 + 2*x + x^2])))/15
```

3.284.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.284.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$	38
trager	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$	38
risch	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$	38
default	$\frac{2x+2}{30(x^2+2x+4)^{\frac{5}{2}}}+\frac{\frac{4}{135}+\frac{4x}{135}}{(x^2+2x+4)^{\frac{3}{2}}}+\frac{\frac{8}{405}+\frac{8x}{405}}{\sqrt{x^2+2x+4}}$	53

input
```
int(1/(x^2+2*x+4)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/405*(8*x^5+40*x^4+140*x^3+260*x^2+355*x+203)/(x^2+2*x+4)^(5/2)
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(46) = 92.

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\,x^6+48\,x^5+192\,x^4+448\,x^3+768\,x^2+(8\,x^5+40\,x^4+140\,x^3+260\,x^2+355\,x}{405\,(x^6+6\,x^5+24\,x^4+56\,x^3+96\,x^2+96\,x+6}$$

input
```
integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="fricas")
```

output
```
1/405*(8*x^6 + 48*x^5 + 192*x^4 + 448*x^3 + 768*x^2 + (8*x^5 + 40*x^4 + 14
0*x^3 + 260*x^2 + 355*x + 203)*sqrt(x^2 + 2*x + 4) + 768*x + 512)/(x^6 + 6
*x^5 + 24*x^4 + 56*x^3 + 96*x^2 + 96*x + 64)
```

3.284.6 Sympy [F]

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \int \frac{1}{(x^2+2x+4)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(x**2+2*x+4)**(7/2),x)
```

output
```
Integral((x**2 + 2*x + 4)**(-7/2), x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\,x}{405\,\sqrt{x^2+2\,x+4}} + \frac{8}{405\,\sqrt{x^2+2\,x+4}} + \frac{4\,x}{135\,(x^2+2\,x+4)^{\frac{3}{2}}} + \frac{4}{135\,(x^2+2\,x+4)^{\frac{3}{2}}} + \frac{x}{15\,(x^2+2\,x+4)^{\frac{5}{2}}} + \frac{1}{15\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="maxima")
```

output
```
8/405*x/sqrt(x^2 + 2*x + 4) + 8/405/sqrt(x^2 + 2*x + 4) + 4/135*x/(x^2 + 2
*x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^2 + 2*x + 4)^(5/2)
 + 1/15/(x^2 + 2*x + 4)^(5/2)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{(4\,((2\,(x+5)x+35)x+65)x+355)x+203}{405\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="giac")
```

output
```
1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*x + 4)^(5
/2)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{51\,x+8\,x\,{\left(x^2+2\,x+4\right)}^2+8\,{\left(x^2+2\,x+4\right)}^2+12\,x^2+12\,x\,\left(x^2+2\,x+4\right)+75}{{\left(x^2+2\,x+4\right)}^{3/2}\,\left(405\,x^2+810\,x+1620\right)}$$

input
```
int(1/(2*x + x^2 + 4)^(7/2),x)
```

output
```
(51*x + 8*x*(2*x + x^2 + 4)^2 + 8*(2*x + x^2 + 4)^2 + 12*x^2 + 12*x*(2*x +
 x^2 + 4) + 75)/((2*x + x^2 + 4)^(3/2)*(810*x + 405*x^2 + 1620))
```

3.285 $\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx$

3.285.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = -\frac{4+3x}{39\,(1+8x+3x^2)^{3/2}} + \frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}$$

output
```
1/39*(-4-3*x)/(3*x^2+8*x+1)^(3/2)+2/169*(4+3*x)/(3*x^2+8*x+1)^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{(4+3x)\,(-7+48x+18x^2)}{507\,(1+8x+3x^2)^{3/2}}$$

input
```
Integrate[(1 + 8*x + 3*x^2)^(-5/2),x]
```

output
```
((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))
```

3.285.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+8x+1)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1089}$$

$$-\frac{2}{13}\int \frac{1}{(3x^2+8x+1)^{3/2}}dx-\frac{3x+4}{39\,(3x^2+8x+1)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\,(3x^2+8x+1)^{3/2}}$$

input
```
Int[(1 + 8*x + 3*x^2)^(-5/2),x]
```

output
```
-1/39*(4 + 3*x)/(1 + 8*x + 3*x^2)^(3/2) + (2*(4 + 3*x))/(169*Sqrt[1 + 8*x
+ 3*x^2])
```

3.285.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.285.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
trager	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
risch	$\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$	30
default	$-\frac{6x+8}{78(3x^2+8x+1)^{\frac{3}{2}}}+\frac{6x+8}{169\sqrt{3x^2+8x+1}}$	40

input
```
int(1/(3*x^2+8*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/507*(54*x^3+216*x^2+171*x-28)/(3*x^2+8*x+1)^(3/2)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = -\frac{252\,x^4+1344\,x^3+1960\,x^2-(54\,x^3+216\,x^2+171\,x-28)\sqrt{3\,x^2+8\,x+1}+448\,x+28}{507\,(9\,x^4+48\,x^3+70\,x^2+16\,x+1)}$$

input
```
integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/507*(252*x^4 + 1344*x^3 + 1960*x^2 - (54*x^3 + 216*x^2 + 171*x - 28)*sq
rt(3*x^2 + 8*x + 1) + 448*x + 28)/(9*x^4 + 48*x^3 + 70*x^2 + 16*x + 1)
```

3.285.6 Sympy [F]

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \int \frac{1}{(3x^2+8x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(3*x**2+8*x+1)**(5/2),x)
```

output
```
Integral((3*x**2 + 8*x + 1)**(-5/2), x)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{6\,x}{169\,\sqrt{3\,x^2+8\,x+1}} + \frac{8}{169\,\sqrt{3\,x^2+8\,x+1}} - \frac{x}{13\,(3\,x^2+8\,x+1)^{\frac{3}{2}}} - \frac{4}{39\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="maxima")
```

output
```
6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/13*x/(3*x^
2 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{9\,(6\,(x+4)x+19)x-28}{507\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="giac")
```

output
```
1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+8x+3x^2)^{5/2}} \, dx = \frac{(12\,x+16)\,\left(72\,x^2+192\,x-28\right)}{8112\,\left(3\,x^2+8\,x+1\right)^{3/2}}$$

input
```
int(1/(8*x + 3*x^2 + 1)^(5/2),x)
```

output
```
((12*x + 16)*(192*x + 72*x^2 - 28))/(8112*(8*x + 3*x^2 + 1)^(3/2))
```

3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$

3.286.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{2-3x}{57\,(5+4x-3x^2)^{3/2}} - \frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}$$

output
```
1/57*(-2+3*x)/(-3*x^2+4*x+5)^(3/2)-2/361*(2-3*x)/(-3*x^2+4*x+5)^(1/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{-98+99x+108x^2-54x^3}{1083\,(5+4x-3x^2)^{3/2}}$$

input
```
Integrate[(5 + 4*x - 3*x^2)^(-5/2),x]
```

output
```
(-98 + 99*x + 108*x^2 - 54*x^3)/(1083*(5 + 4*x - 3*x^2)^(3/2))
```

3.286.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2+4x+5)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2}{19}\int \frac{1}{(-3x^2+4x+5)^{3/2}}dx-\frac{2-3x}{57(-3x^2+4x+5)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57(-3x^2+4x+5)^{3/2}}$$

input
```
Int[(5 + 4*x - 3*x^2)^(-5/2),x]
```

output
```
-1/57*(2 - 3*x)/(5 + 4*x - 3*x^2)^(3/2) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x
- 3*x^2])
```

3.286.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.286.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{54x^{3}-108x^{2}-99x+98}{1083\left(-3x^{2}+4x+5\right)^{\frac{3}{2}}}$	30
default	$-\frac{-6x+4}{114\left(-3x^{2}+4x+5\right)^{\frac{3}{2}}}-\frac{-6x+4}{361\sqrt{-3x^{2}+4x+5}}$	40
trager	$-\frac{\left(54x^{3}-108x^{2}-99x+98\right)\sqrt{-3x^{2}+4x+5}}{1083\left(3x^{2}-4x-5\right)^{2}}$	42
risch	$\frac{54x^{3}-108x^{2}-99x+98}{1083\left(3x^{2}-4x-5\right)\sqrt{-3x^{2}+4x+5}}$	42

input
```
int(1/(-3*x^2+4*x+5)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1083/(-3*x^2+4*x+5)^(3/2)*(54*x^3-108*x^2-99*x+98)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(5+4x-3x^2\right)^{5/2}}\,dx = -\frac{\left(54\,x^{3}-108\,x^{2}-99\,x+98\right)\sqrt{-3\,x^{2}+4\,x+5}}{1083\,\left(9\,x^{4}-24\,x^{3}-14\,x^{2}+40\,x+25\right)}$$

input
```
integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="fricas")
```

output
```
-1/1083*(54*x^3 - 108*x^2 - 99*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(9*x^4 - 24*
x^3 - 14*x^2 + 40*x + 25)
```

3.286.6 Sympy [F]

$$\int \frac{1}{\left(5+4x-3x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-3x^{2}+4x+5\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

output
```
Integral((-3*x**2 + 4*x + 5)**(-5/2), x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{6\,x}{361\,\sqrt{-3\,x^2+4\,x+5}} - \frac{4}{361\,\sqrt{-3\,x^2+4\,x+5}} + \frac{x}{19\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}} - \frac{2}{57\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}}$$

input
```
integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="maxima")
```

output
```
6/361*x/sqrt(-3*x^2 + 4*x + 5) - 4/361/sqrt(-3*x^2 + 4*x + 5) + 1/19*x/(-3
*x^2 + 4*x + 5)^(3/2) - 2/57/(-3*x^2 + 4*x + 5)^(3/2)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{(9\,(6\,(x-2)x-11)x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\,(3\,x^2-4\,x-5)^2}$$

input
```
integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="giac")
```

output
```
-1/1083*(9*(6*(x - 2)*x - 11)*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(3*x^2 - 4*x
- 5)^2
```

3.286.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{(12\,x-8)\,\left(-72\,x^2+96\,x+196\right)}{17328\,(-3\,x^2+4\,x+5)^{3/2}}$$

input
```
int(1/(4*x - 3*x^2 + 5)^(5/2),x)
```

output
```
((12*x - 8)*(96*x - 72*x^2 + 196))/(17328*(4*x - 3*x^2 + 5)^(3/2))
```

3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$

3.287.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\operatorname{arcsinh}(1+x)$$

output
```
1/(1+x)+arcsinh(1+x)-(x^2+2*x+2)^(1/2)/(1+x)
```

3.287.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=-\frac{-1+\sqrt{2+2x+x^2}+(1+x)\log\left(-1-x+\sqrt{2+2x+x^2}\right)}{1+x}$$

input
```
Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

output
```
-((-1 + Sqrt[2 + 2*x + x^2] + (1 + x)*Log[-1 - x + Sqrt[2 + 2*x + x^2]])/(
1 + x))
```

3.287.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2x+2}+1}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{x^2+2x+2}}{(x+1)^2}-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\text{arcsinh}(x+1)-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}$$

input
```
Int[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

output
```
(1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

3.287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.287.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\left((1+x)^2+1\right)^{\frac{3}{2}}}{1+x}+(1+x)\sqrt{(1+x)^2+1}+\operatorname{arcsinh}(1+x)+\frac{1}{1+x}$	40
trager	$-\frac{x}{1+x}-\frac{\sqrt{x^2+2x+2}}{1+x}-\ln\left(\sqrt{x^2+2x+2}-1-x\right)$	45

input
```
int(1/(1+(x^2+2*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1/(1+x)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=-\frac{(x+1)\log\left(-x+\sqrt{x^2+2\,x+2}-1\right)+x+\sqrt{x^2+2\,x+2}}{x+1}$$

input
```
integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
-((x + 1)*log(-x + sqrt(x^2 + 2*x + 2) - 1) + x + sqrt(x^2 + 2*x + 2))/(x
+ 1)
```

3.287.6 Sympy [F]

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

input
```
integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
```

output
```
Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)
```

3.287.7 Maxima [F]

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x+2}+1}\,dx$$

input
```
integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)
```

3.287.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \frac{2}{\left(x-\sqrt{x^2+2\,x+2}\right)^2+2\,x-2\,\sqrt{x^2+2\,x+2}} + \frac{1}{x+1} - \log\left(-x+\sqrt{x^2+2\,x+2}-1\right)$$

input
```
integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="giac")
```

output
```
2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1/(x + 1)
- log(-x + sqrt(x^2 + 2*x + 2) - 1)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \frac{1}{x+1} + \int \frac{\sqrt{x^2+2\,x+2}}{(x+1)^2}\,dx$$

input
```
int(1/((2*x + x^2 + 2)^(1/2) + 1),x)
```

output
```
1/(x + 1) + int((2*x + x^2 + 2)^(1/2)/(x + 1)^2, x)
```

3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}} dx$

3.288.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx=-x+\sqrt{1+x+x^2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log\left(x+\sqrt{1+x+x^2}\right)$$

output
```
-x-3/2*arcsinh(1/3*(1+2*x)*3^(1/2))+2*ln(x+(x^2+x+1)^(1/2))+(x^2+x+1)^(1/2
)
```

3.288.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx=-x+\sqrt{1+x+x^2}+2\log\left(-2-x+\sqrt{1+x+x^2}\right)-\frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[(x + Sqrt[1 + x + x^2])^(-1),x]
```

output
```
-x + Sqrt[1 + x + x^2] + 2*Log[-2 - x + Sqrt[1 + x + x^2]] - Log[-1 - 2*x
+ 2*Sqrt[1 + x + x^2]]/2
```

3.288.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+x+1}+x}dx$$

$$\Big\downarrow \text{ 2541}$$

$$2\int \frac{\left(x+\sqrt{x^2+x+1}\right)^2+x+\sqrt{x^2+x+1}+1}{\left(x+\sqrt{x^2+x+1}\right)\left(2\left(x+\sqrt{x^2+x+1}\right)+1\right)^2}d\left(x+\sqrt{x^2+x+1}\right)$$

$$\Big\downarrow \text{ 1195}$$

$$2\int \left(-\frac{3}{2\left(2\left(x+\sqrt{x^2+x+1}\right)+1\right)}-\frac{3}{2\left(2\left(x+\sqrt{x^2+x+1}\right)+1\right)^2}+\frac{1}{x+\sqrt{x^2+x+1}}\right)d\left(x+\sqrt{x^2+x+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{3}{4\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{4}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)\right)$$

input
```
Int[(x + Sqrt[1 + x + x^2])^(-1),x]
```

output
```
2*(3/(4*(1 + 2*(x + Sqrt[1 + x + x^2]))) + Log[x + Sqrt[1 + x + x^2]] - (3
*Log[1 + 2*(x + Sqrt[1 + x + x^2])])/4)
```

3.288.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.288.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

method	result	size
default	$\sqrt{(1+x)^{2}-x}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^{2}-x}}\right)-x+\ln\left(1+x\right)$	52
trager	$\sqrt{x^{2}+x+1}-x-\frac{\ln\left(\frac{2x^{2}\sqrt{x^{2}+x+1}+2x^{3}+8x\sqrt{x^{2}+x+1}+9x^{2}+14\sqrt{x^{2}+x+1}+12x+13}{(1+x)^{4}}\right)}{2}$	71

input
```
int(1/(x+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
((1+x)^2-x)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(1/2*(1-x)/((1+x
)^2-x)^(1/2))-x+ln(1+x)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{x^2+x+1}+\log(x+1)-\log\left(-x+\sqrt{x^2+x+1}\right) +\log\left(-x+\sqrt{x^2+x+1}-2\right)+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

output
```
-x + sqrt(x^2 + x + 1) + log(x + 1) - log(-x + sqrt(x^2 + x + 1)) + log(-x
 + sqrt(x^2 + x + 1) - 2) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.288.6 Sympy [F]

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/(x+(x**2+x+1)**(1/2)),x)
```

output
```
Integral(1/(x + sqrt(x**2 + x + 1)), x)
```

3.288.7 Maxima [F]

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x + sqrt(x^2 + x + 1)), x)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)+\log\left(|x+1|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

input
```
integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

output
```
-x + sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + log(abs
(x + 1)) - log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x +
1) - 2))
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \ln\left(x+1\right)-x+\int \frac{\sqrt{x^2+x+1}}{x+1}\,dx$$

input
```
int(1/(x + (x + x^2 + 1)^(1/2)),x)
```

output
```
log(x + 1) - x + int((x + x^2 + 1)^(1/2)/(x + 1), x)
```

3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$

3.289.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{64}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-1/9*x^3-1/6*x^4-5/36*(x^2+x+1)^(3/2)+1/6*x*(x^2+x+1)^(3/2)+1/64*arcsinh(1
/3*(1+2*x)*3^(1/2))+1/96*(1+2*x)*(x^2+x+1)^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{18}x^3(2+3x)+\frac{1}{288}\sqrt{1+x+x^2}\left(-37+14x+8x^2+48x^3\right)-\frac{1}{64}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

output
```
-1/18*(x^3*(2 + 3*x)) + (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/
288 - Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/64
```

3.289.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2\sqrt{x^2+x+1}+2x+1}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{2x^3}{3}+\frac{2}{3}\sqrt{x^2+x+1}x^2-\frac{x^2}{3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{64}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}$$

input
```
Int[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

output
```
-1/9*x^3 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x^2)^(3/
2))/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqrt[3]]/64
```

3.289.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.289.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

method	result	size
trager	$-\frac{(2+3x)x^3}{18}+\frac{\left(\frac{1}{2}x^3+\frac{1}{12}x^2+\frac{7}{48}x-\frac{37}{96}\right)\sqrt{x^2+x+1}}{3}+\frac{\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{64}$	55
default	$-\frac{x^3}{9}-\frac{x^4}{6}+\frac{x(x^2+x+1)^{\frac{3}{2}}}{6}-\frac{5(x^2+x+1)^{\frac{3}{2}}}{36}+\frac{(1+2x)\sqrt{x^2+x+1}}{96}+\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{64}$	59

input
```
int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(2+3*x)*x^3+1/3*(1/2*x^3+1/12*x^2+7/48*x-37/96)*(x^2+x+1)^(1/2)+1/64
*ln(1+2*x+2*(x^2+x+1)^(1/2))
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{6}x^4-\frac{1}{9}x^3+\frac{1}{288}\left(48x^3+8x^2+14x-37\right)\sqrt{x^2+x+1} - \frac{1}{64}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

output
```
-1/6*x^4 - 1/9*x^3 + 1/288*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1)
- 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.289.6 Sympy [F]

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \int \frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

input
```
integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

output
```
Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)
```

3.289.7 Maxima [F]

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \int \frac{x^2}{2\,x+2\,\sqrt{x^2+x+1}+1}\,dx$$

input
```
integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{6}\,x^4 - \frac{1}{9}\,x^3 + \frac{1}{288}\,(2\,(4\,(6\,x+1)x+7)x-37)\sqrt{x^2+x+1} - \frac{1}{64}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1
) - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{64} - \frac{\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{6} - \frac{x^3}{9} - \frac{x^4}{6} - \frac{5\,(8\,x^2+2\,x+5)\,\sqrt{x^2+x+1}}{288} + \frac{x\,(x^2+x+1)^{3/2}}{6}$$

input
```
int(x^2/(2*x + 2*(x + x^2 + 1)^(1/2) + 1),x)
```

output
```
log(x + (x + x^2 + 1)^(1/2) + 1/2)/64 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/2))/
6 - x^3/9 - x^4/6 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/288 + (x*(x
+ x^2 + 1)^(3/2))/6
```

3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$

3.290.1 Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{1+x+x^2}+\frac{5}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)-\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\log(1+x)$$

output
```
x+5/2*arcsinh(1/3*(1+2*x)*3^(1/2))+4*arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))-ar
ctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+ln(x)-4*ln(1+x)-3*(x^2+x+1)^(1/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{1+x+x^2}-8\log\left(-2-x+\sqrt{1+x+x^2}\right)+2\log\left(-1-x+\sqrt{1+x+x^2}\right)+\frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

output
```
x - 3*Sqrt[1 + x + x^2] - 8*Log[-2 - x + Sqrt[1 + x + x^2]] + 2*Log[-1 - x
 + Sqrt[1 + x + x^2]] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

3.290.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+x+1}-3x}{\sqrt{x^2+x+1}-1}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}-\frac{3x}{\sqrt{x^2+x+1}-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5}{2}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-3\sqrt{x^2+x+1}+x+\log(x)-4\log(x+1)$$

input
```
Int[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

output
```
x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1
- x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log
[x] - 4*Log[1 + x]
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.290.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result
default	$\ln(x) - 4\ln(1+x) + x + \sqrt{x^2+x+1} + \frac{5\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2} - \operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right) - 4\sqrt{(1+x}$
trager	$-1 + x - 3\sqrt{x^2+x+1} + \frac{\ln\left(\frac{-8+3865870x^6-96x+1790544x^5+8\sqrt{x^2+x+1}+445596x^4-1526x^3-507x^2+2392341x^{10}+308624x^{12}+}{}\right.}{}$

input
```
int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-4*ln(1+x)+x+(x^2+x+1)^(1/2)+5/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh
(1/2*(2+x)/(x^2+x+1)^(1/2))-4*((1+x)^2-x)^(1/2)+4*arctanh(1/2*(1-x)/((1+x)
^2-x)^(1/2))
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx &= x - 3\sqrt{x^2+x+1} - 4\log(x+1) + \log(x)\\ &\quad - \log\left(-x+\sqrt{x^2+x+1}+1\right) + 4\log\left(-x+\sqrt{x^2+x+1}\right)\\ &\quad + \log\left(-x+\sqrt{x^2+x+1}-1\right) - 4\log\left(-x+\sqrt{x^2+x+1}-2\right)\\ &\quad - \frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)\end{aligned}$$

input
```
integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="fricas
")
```

output
```
x - 3*sqrt(x^2 + x + 1) - 4*log(x + 1) + log(x) - log(-x + sqrt(x^2 + x +
1) + 1) + 4*log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 1)
- 4*log(-x + sqrt(x^2 + x + 1) - 2) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) -
 1)
```

3.290.6 Sympy [F]

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = -\int \frac{3x}{\sqrt{x^2+x+1}-1}\,dx - \int \left(-\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\right)dx$$

input
```
integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

output
```
-Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2 + x + 1)/
(sqrt(x**2 + x + 1) - 1), x)
```

3.290.7 Maxima [F]

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = \int -\frac{3\,x-\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\,dx$$

input
```
integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="maxima
")
```

output
```
3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2*sqrt(x^2
 + x + 1) + 2), x)
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{x^2+x+1}-\frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right) -4\log\left(|x+1|\right)+\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) +4\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right) +\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right) -4\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

input
```
integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

output
```
x - 3*sqrt(x^2 + x + 1) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*log(
abs(x + 1)) + log(abs(x)) - log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 4*log(a
bs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 1)) - 4*log
(abs(-x + sqrt(x^2 + x + 1) - 2))
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-4\ln(x+1)+\ln(x)-\int \frac{(3x-1)\sqrt{x^2+x+1}}{x\,(x+1)}\,dx$$

input
```
int(-(3*x - (x + x^2 + 1)^(1/2))/((x + x^2 + 1)^(1/2) - 1),x)
```

output
```
x - 4*log(x + 1) + log(x) - int(((3*x - 1)*(x + x^2 + 1)^(1/2))/(x*(x + 1)
), x)
```

3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$

3.291.1 Optimal result

Integrand size = 31, antiderivative size = 158

$$\begin{aligned}\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx &= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2} \\ &\quad -2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+x^2} \\ &\quad +\frac{11}{2}\operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)+\frac{43}{8}\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right) \\ &\quad -2\sqrt{7}\operatorname{arctanh}\left(\frac{1+5x}{2\sqrt{7}\sqrt{1+x+x^2}}\right) \\ &\quad +2\sqrt{7}\operatorname{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)\end{aligned}$$

output
```
11/2*arcsinh(1/3*(1+x)*3^(1/2))+43/8*arcsinh(1/3*(1+2*x)*3^(1/2))-2*arctan
h(1/14*(1+5*x)*7^(1/2)/(x^2+x+1)^(1/2))*7^(1/2)+2*arctanh(1/7*(1-2*x)*7^(1
/2)/(x^2+2*x+4)^(1/2))*7^(1/2)-2*(x^2+x+1)^(1/2)+1/4*(1+2*x)*(x^2+x+1)^(1/
2)-2*(x^2+2*x+4)^(1/2)+1/2*(1+x)*(x^2+2*x+4)^(1/2)
```

3.291.2 Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\Bigg(-14\sqrt{1+x+x^2}+4x\sqrt{1+x+x^2} - 12\sqrt{4+2x+x^2}+4x\sqrt{4+2x+x^2} - 32\sqrt{7}\text{arctanh}\left(\frac{3+x-\sqrt{1+x+x^2}}{\sqrt{7}}\right) - 32\sqrt{7}\text{arctanh}\left(\frac{3+x-\sqrt{4+2x+x^2}}{\sqrt{7}}\right) - 43\log\left(-1-2x+2\sqrt{1+x+x^2}\right) - 44\log\left(-1-x+\sqrt{4+2x+x^2}\right)\Bigg)$$

input
```
Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

output
```
(-14*Sqrt[1 + x + x^2] + 4*x*Sqrt[1 + x + x^2] - 12*Sqrt[4 + 2*x + x^2] +
4*x*Sqrt[4 + 2*x + x^2] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[1 + x + x^2])/S
qrt[7]] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[4 + 2*x + x^2])/Sqrt[7]] - 43*L
og[-1 - 2*x + 2*Sqrt[1 + x + x^2]] - 44*Log[-1 - x + Sqrt[4 + 2*x + x^2]])
/8
```

3.291.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt{x^2+2x+4}-\sqrt{x^2+x+1}}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{x}{\sqrt{x^2+x+1}-\sqrt{x^2+2x+4}}-\frac{1}{\sqrt{x^2+x+1}-\sqrt{x^2+2x+4}}\right)dx$$

$$\downarrow 2009$$

$$\frac{11}{2}\text{arcsinh}\left(\frac{x+1}{\sqrt{3}}\right)+\frac{43}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{7}\text{arctanh}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2\sqrt{7}\text{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{x^2+2x+4}}\right)+\frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}$$

input
```
Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

output
```
-2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x +
x^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 +
 (43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7
]*Sqrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x
+ x^2])]
```

3.291.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.291.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

method	result
default	$-2\sqrt{(3+x)^2-5x-8}+\frac{43\,\text{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{8}+2\sqrt{7}\,\text{arctanh}\left(\frac{(-1-5x)\sqrt{7}}{14\sqrt{(3+x)^2-5x-8}}\right)-2\sqrt{(3+x)^2}$

input
```
int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*3^(1/2)*(x+1/2))+2*7^(1/2)*arcta
nh(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)^2-4*x-5)^(1/2)+11
/2*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh(1/14*(2-4*x)*7^(1/2)/((3+x
)^2-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1)^(1/2)+1/4*(2*x+2)*(x^2+2*x+4)^(1/2
)
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx &= \frac{1}{4}\sqrt{x^2+x+1}(2x-7)+\frac{1}{2}\sqrt{x^2+2x+4}(x-3)\\ &\quad+2\sqrt{7}\log\left(\frac{2\sqrt{7}(5x+1)+2\sqrt{x^2+x+1}(5\sqrt{7}-14)-25x-5}{x+3}\right)\\ &\quad+2\sqrt{7}\log\left(\frac{\sqrt{7}(2x-1)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x+2}{x+3}\right)\\ &\quad-\frac{11}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)-\frac{43}{8}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)\end{aligned}$$

input
```
integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/4*sqrt(x^2 + x + 1)*(2*x - 7) + 1/2*sqrt(x^2 + 2*x + 4)*(x - 3) + 2*sqrt
(7)*log((2*sqrt(7)*(5*x + 1) + 2*sqrt(x^2 + x + 1)*(5*sqrt(7) - 14) - 25*x
 - 5)/(x + 3)) + 2*sqrt(7)*log((sqrt(7)*(2*x - 1) + sqrt(x^2 + 2*x + 4)*(2
*sqrt(7) - 7) - 4*x + 2)/(x + 3)) - 11/2*log(-x + sqrt(x^2 + 2*x + 4) - 1)
 - 43/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.291.6 Sympy [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx=\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
```

output
```
Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

3.291.7 Maxima [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx=\int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

input
```
integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="maxima"
)
```

output
```
integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

3.291.8 Giac [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx=\int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

input
```
integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int -\frac{x+1}{\sqrt{x^2+x+1}-\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)),x)
```

output
```
int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)), x)
```

3.292 $\int \frac{1}{\sqrt{-1+x}x^3} \, dx$

3.292.1 Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{1}{\sqrt{-1+x}x^3} \, dx = \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4}\arctan\left(\sqrt{-1+x}\right)$$

output
```
3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

3.292.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+x}x^3} \, dx = \frac{1}{4}\left(\frac{\sqrt{-1+x}(2+3x)}{x^2} + 3\arctan\left(\sqrt{-1+x}\right)\right)$$

input
```
Integrate[1/(Sqrt[-1 + x]*x^3),x]
```

output
```
((Sqrt[-1 + x]*(2 + 3*x))/x^2 + 3*ArcTan[Sqrt[-1 + x]])/4
```

3.292.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}x^3}dx$$
$$\Big\downarrow \ 52$$
$$\frac{3}{4}\int \frac{1}{\sqrt{x-1}x^2}dx+\frac{\sqrt{x-1}}{2x^2}$$
$$\Big\downarrow \ 52$$
$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-1}x}dx+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}$$
$$\Big\downarrow \ 73$$
$$\frac{3}{4}\left(\int \frac{1}{x}d\sqrt{x-1}+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}$$
$$\Big\downarrow \ 216$$
$$\frac{3}{4}\left(\arctan\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}$$

input
```
Int[1/(Sqrt[-1 + x]*x^3),x]
```

output
```
Sqrt[-1 + x]/(2*x^2) + (3*(Sqrt[-1 + x]/x + ArcTan[Sqrt[-1 + x]]))/4
```

3.292.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.292.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}$
default	$\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^2}+\frac{3\sqrt{-1+x}}{4x}$
risch	$\frac{3x^2-x-2}{4x^2\sqrt{-1+x}}+\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}$
trager	$\frac{(2+3x)\sqrt{-1+x}}{4x^2}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-1+x}-x+2}{x}\right)}{8}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(-1+x)}\left(-\frac{\sqrt{\pi}}{2x^2}-\frac{\sqrt{\pi}}{2x}+\frac{3\left(\frac{7}{6}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^2+8x+8\right)}{16x^2}-\frac{\sqrt{\pi}\,(12x+8)\sqrt{1-x}}{16x^2}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-x}}{2}\right)}{4}\right.}{\sqrt{\pi}\,\sqrt{\operatorname{signum}(-1+x)}}$

input
```
int(1/x^3/(-1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,x^2\arctan\left(\sqrt{x-1}\right)+(3\,x+2)\sqrt{x-1}}{4\,x^2}$$

input
```
integrate(1/x^3/(-1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(3*x^2*arctan(sqrt(x - 1)) + (3*x + 2)*sqrt(x - 1))/x^2
```

3.292.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \begin{cases} \frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-1+x)**(1/2),x)
```

output
```
Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*
x**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (
-3*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1
- 1/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\,\left((x-1)^2+2\,x-1\right)} + \frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

input
```
integrate(1/x^3/(-1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/((x - 1)^2 + 2*x - 1) + 3/4*arctan(s
qrt(x - 1))
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\,x^2}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

input
```
integrate(1/x^3/(-1+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1))
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,\mathrm{atan}\left(\sqrt{x-1}\right)}{4}+\frac{3\,\sqrt{x-1}}{4\,x}+\frac{\sqrt{x-1}}{2\,x^2}$$

input
```
int(1/(x^3*(x - 1)^(1/2)),x)
```

output
```
(3*atan((x - 1)^(1/2)))/4 + (3*(x - 1)^(1/2))/(4*x) + (x - 1)^(1/2)/(2*x^2
)
```

3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$

3.293.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

output
```
-1/(1-3/x)^(1/3)
```

3.293.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{\frac{-3+x}{x}}}$$

input
```
Integrate[1/((1 - 3/x)^(4/3)*x^2),x]
```

output
```
-((-3 + x)/x)^(-1/3)
```

3.293.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$$

$$\Big\downarrow \; 793$$

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

input
```
Int[1/((1 - 3/x)^(4/3)*x^2),x]
```

output
```
-(1 - 3/x)^(-1/3)
```

3.293.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.293.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
default	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
risch	$-\frac{1}{\left(\frac{-3+x}{x}\right)^{\frac{1}{3}}}$	12
gosper	$-\frac{-3+x}{x\left(\frac{-3+x}{x}\right)^{\frac{4}{3}}}$	18
trager	$-\frac{x\left(-\frac{3-x}{x}\right)^{\frac{2}{3}}}{-3+x}$	21

input
```
int(1/(1-3/x)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(1-3/x)^(1/3)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2}\,dx = -\frac{x\left(\frac{x-3}{x}\right)^{\frac{2}{3}}}{x-3}$$

input
```
integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="fricas")
```

output
```
-x*((x - 3)/x)^(2/3)/(x - 3)
```

3.293.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3}x^2}\,dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

input
```
integrate(1/(1-3/x)**(4/3)/x**2,x)
```

output
```
-1/(1 - 3/x)**(1/3)
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3}x^2}\,dx = -\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="maxima")
```

output
```
-1/(-3/x + 1)^(1/3)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3}x^2}\,dx = -\frac{1}{\left(\frac{x-3}{x}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="giac")
```

output
```
-1/((x - 3)/x)^(1/3)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\left(1-\frac{3}{x}\right)^{1/3}}$$

input
```
int(1/(x^2*(1 - 3/x)^(4/3)),x)
```

output
```
-1/(1 - 3/x)^(1/3)
```

3.294 $\int \frac{(-1+3x)^{4/3}}{x^2} dx$

3.294.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx = 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) + 2\log(x) - 6\log\left(1+\sqrt[3]{-1+3x}\right)$$

output
```
12*(-1+3*x)^(1/3)-(-1+3*x)^(4/3)/x+2*ln(x)-6*ln(1+(-1+3*x)^(1/3))+4*arctan
(1/3*(1-2*(-1+3*x)^(1/3))*3^(1/2))*3^(1/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx = \frac{\sqrt[3]{-1+3x}(1+9x)}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) - 4\log\left(1+\sqrt[3]{-1+3x}\right) + 2\log\left(1-\sqrt[3]{-1+3x}+(-1+3x)^{2/3}\right)$$

input
```
Integrate[(-1 + 3*x)^(4/3)/x^2,x]
```

output
```
((-1 + 3*x)^(1/3)*(1 + 9*x))/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*x)^(1/3))
/Sqrt[3]] - 4*Log[1 + (-1 + 3*x)^(1/3)] + 2*Log[1 - (-1 + 3*x)^(1/3) + (-1
 + 3*x)^(2/3)]
```

3.294.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {51, 60, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x-1)^{4/3}}{x^2}\,dx$$

$$\Big\downarrow \text{51}$$

$$4\int \frac{\sqrt[3]{3x-1}}{x}dx-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{60}$$

$$4\left(3\sqrt[3]{3x-1}-\int \frac{1}{x(3x-1)^{2/3}}dx\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{70}$$

$$4\left(-\frac{3}{2}\int \frac{1}{\sqrt[3]{3x-1}+1}d\sqrt[3]{3x-1}-\frac{3}{2}\int \frac{1}{(3x-1)^{2/3}-\sqrt[3]{3x-1}+1}d\sqrt[3]{3x-1}+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{16}$$

$$4\left(-\frac{3}{2}\int \frac{1}{(3x-1)^{2/3}-\sqrt[3]{3x-1}+1}d\sqrt[3]{3x-1}+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}-\frac{3}{2}\log\left(\sqrt[3]{3x-1}+1\right)\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{1083}$$

$$4\left(3\int \frac{1}{-(3x-1)^{2/3}-3}d\left(2\sqrt[3]{3x-1}-1\right)+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}-\frac{3}{2}\log\left(\sqrt[3]{3x-1}+1\right)\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{217}$$

$$4\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{3x-1}-1}{\sqrt{3}}\right)+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}-\frac{3}{2}\log\left(\sqrt[3]{3x-1}+1\right)\right)-\frac{(3x-1)^{4/3}}{x}$$

input
```
Int[(-1 + 3*x)^(4/3)/x^2,x]
```

output
```
-((-1 + 3*x)^(4/3)/x) + 4*(3*(-1 + 3*x)^(1/3) - Sqrt[3]*ArcTan[(-1 + 2*(-1
 + 3*x)^(1/3))/Sqrt[3]] + Log[x]/2 - (3*Log[1 + (-1 + 3*x)^(1/3)])/2)
```

3.294.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.294.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
meijerg	$-\frac{4\,\mathrm{signum}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}\left(\frac{3\Gamma\left(\frac{2}{3}\right)}{4x}+3\left(2+\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)-\frac{3\Gamma\left(\frac{2}{3}\right)x\,{}_3F_2\left(\frac{2}{3},1,1;2,3;3x\right)}{2}\right)}{3\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x-\frac{1}{3}\right)\right)^{\frac{4}{3}}}$
pseudoelliptic	$\frac{(27x+3)(-1+3x)^{\frac{1}{3}}-6x\left(2\sqrt{3}\arctan\left(\frac{\left(2(-1+3x)^{\frac{1}{3}}-1\right)\sqrt{3}}{3}\right)-\ln\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)+2\ln\left(1+(-1+3x)^{\frac{1}{3}}\right)\right)}{\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)\left(1+(-1+3x)^{\frac{1}{3}}\right)}$
derivativedivides	$9(-1+3x)^{\frac{1}{3}}+\frac{1+(-1+3x)^{\frac{1}{3}}}{(-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1}+2\ln\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)-4\sqrt{3}\,\mathrm{arc}$
default	$9(-1+3x)^{\frac{1}{3}}+\frac{1+(-1+3x)^{\frac{1}{3}}}{(-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1}+2\ln\left((-1+3x)^{\frac{2}{3}}-(-1+3x)^{\frac{1}{3}}+1\right)-4\sqrt{3}\,\mathrm{arc}$
risch	$\frac{(-1+3x)^{\frac{1}{3}}}{x}+\frac{\left(-\frac{4(-1+3x)^{\frac{2}{3}}\left(-\mathrm{signum}\left(x-\frac{1}{3}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+2\Gamma\left(\frac{2}{3}\right)x\,{}_3F_2\left(1,1,\frac{5}{3};2,2;3x\right)\right)}{\left((-1+3x)^2\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x-\frac{1}{3}\right)^{\frac{2}{3}}}+\frac{9(-1+3x)}{}\right.}{(-1+3x)^{\frac{2}{3}}}$
trager	$\frac{(1+9x)(-1+3x)^{\frac{1}{3}}}{x}-4\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)(-1+3x)^{\frac{2}{3}}-\mathrm{RootOf}\left(\_Z^2-\_\right.}{x}\right.$

input
```
int((-1+3*x)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3/GAMMA(2/3)*signum(x-1/3)^(4/3)/(-signum(x-1/3))^(4/3)*(3/4*GAMMA(2/3)
/x+3*(2+1/6*Pi*3^(1/2)-1/2*ln(3)+ln(x)+I*Pi)*GAMMA(2/3)-3/2*GAMMA(2/3)*x*h
ypergeom([2/3,1,1],[2,3],3*x))
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int\frac{(-1+3x)^{4/3}}{x^2}\,dx=$$
$$-\frac{4\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}(3x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2x\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)+4x\log\left((3x-1)^{\frac{1}{3}}+1\right.}{x}$$

input
```
integrate((-1+3*x)^(4/3)/x^2,x, algorithm="fricas")
```

output
```
-(4*sqrt(3)*x*arctan(2/3*sqrt(3)*(3*x - 1)^(1/3) - 1/3*sqrt(3)) - 2*x*log(
(3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/3) + 1) - (9
*x + 1)*(3*x - 1)^(1/3))/x
```

3.294.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 7.62

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = \frac{189\cdot\sqrt[3]{3}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}e^{\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{84\cdot\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{84\left(x-\frac{1}{3}\right)\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} - \frac{84\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{i\pi}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{84\left(x-\frac{1}{3}\right)e^{\frac{2i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{28\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} - \frac{28e^{\frac{i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{i\pi}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)} + \frac{28e^{\frac{2i\pi}{3}}\log\left(-\sqrt[3]{3}\sqrt[3]{x-\frac{1}{3}}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{7}{3}\right)}{9\left(x-\frac{1}{3}\right)e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)+3e^{\frac{i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-1+3*x)**(4/3)/x**2,x)
```

output
```
189*3**(1/3)*(x - 1/3)**(4/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi
/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3
)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi
/3)*gamma(10/3)) + 84*(x - 1/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I
*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3
)*gamma(10/3)) - 84*(x - 1/3)*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*ex
p(I*pi/3)*gamma(10/3)) + 84*(x - 1/3)*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3
)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamm
a(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 28*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*
exp(I*pi/3)*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*ex
p(I*pi/3)*gamma(10/3)) + 28*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) +
3*exp(I*pi/3)*gamma(10/3))
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(3\,x-1)^{\frac{1}{3}}-1\right)\right)+9\,(3\,x-1)^{\frac{1}{3}} + \frac{(3\,x-1)^{\frac{1}{3}}}{x}+2\,\log\left((3\,x-1)^{\frac{2}{3}}-(3\,x-1)^{\frac{1}{3}}+1\right)-4\,\log\left((3\,x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate((-1+3*x)^(4/3)/x^2,x, algorithm="maxima")
```

output
```
-4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
 + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*lo
g((3*x - 1)^(1/3) + 1)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(3\,x-1)^{\frac{1}{3}}-1\right)\right)+9\,(3\,x-1)^{\frac{1}{3}} + \frac{(3\,x-1)^{\frac{1}{3}}}{x}+2\,\log\left((3\,x-1)^{\frac{2}{3}}-(3\,x-1)^{\frac{1}{3}}+1\right)-4\,\log\left((3\,x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate((-1+3*x)^(4/3)/x^2,x, algorithm="giac")
```

output
```
-4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
 + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*lo
g((3*x - 1)^(1/3) + 1)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = 9\,(3\,x-1)^{1/3} -4\,\ln\left(144\,(3\,x-1)^{1/3}+144\right)+\frac{(3\,x-1)^{1/3}}{x}+\ln\left(18-36\,(3\,x-1)^{1/3}+\sqrt{3}\,18\mathrm{i}\right)\,\left(2+\sqrt{3}\,2\mathrm{i}\right)-\ln\left(36\,(3\,x-\right.$$

input
```
int((3*x - 1)^(4/3)/x^2,x)
```

output
```
9*(3*x - 1)^(1/3) - 4*log(144*(3*x - 1)^(1/3) + 144) + (3*x - 1)^(1/3)/x +
 log(3^(1/2)*18i - 36*(3*x - 1)^(1/3) + 18)*(3^(1/2)*2i + 2) - log(3^(1/2)
*18i + 36*(3*x - 1)^(1/3) - 18)*(3^(1/2)*2i - 2)
```

3.295 $\int (4-3x)^{4/3}x^2\,dx$

3.295.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int (4-3x)^{4/3}x^2\,dx=-\frac{16}{63}(4-3x)^{7/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{1}{117}(4-3x)^{13/3}$$

output
```
-16/63*(4-3*x)^(7/3)+4/45*(4-3*x)^(10/3)-1/117*(4-3*x)^(13/3)
```

3.295.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int (4-3x)^{4/3}x^2\,dx=-\frac{1}{455}(4-3x)^{7/3}\left(16+28x+35x^2\right)$$

input
```
Integrate[(4 - 3*x)^(4/3)*x^2,x]
```

output
```
-1/455*((4 - 3*x)^(7/3)*(16 + 28*x + 35*x^2))
```

3.295.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4-3x)^{4/3}x^2\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{1}{9}(4-3x)^{10/3}-\frac{8}{9}(4-3x)^{7/3}+\frac{16}{9}(4-3x)^{4/3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

input
```
Int[(4 - 3*x)^(4/3)*x^2,x]
```

output
```
(-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(13/3)/117
```

3.295.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.295.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.45

method	result	size
meijerg	$\frac{4\,2^{\frac{2}{3}} x^{3} {}_{2}F_{1}\left(-\frac{4}{3},3;4;\frac{3x}{4}\right)}{3}$	18
gosper	$-\frac{(4-3x)^{\frac{7}{3}}\left(35x^{2}+28x+16\right)}{455}$	20
pseudoelliptic	$-\frac{\left(35x^{2}+28x+16\right)(-4+3x)^{2}(4-3x)^{\frac{1}{3}}}{455}$	27
derivativedivides	$-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$	29
default	$-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$	29
trager	$\left(-\frac{9}{13}x^{4}+\frac{84}{65}x^{3}-\frac{32}{455}x^{2}-\frac{64}{455}x-\frac{256}{455}\right)(4-3x)^{\frac{1}{3}}$	29
risch	$\frac{\left(315x^{4}-588x^{3}+32x^{2}+64x+256\right)(-4+3x)}{455(4-3x)^{\frac{2}{3}}}$	35

input
```
int((4-3*x)^(4/3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*2^(2/3)*x^3*hypergeom([-4/3,3],[4],3/4*x)
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{455}\left(315\,x^4-588\,x^3+32\,x^2+64\,x+256\right)(-3\,x+4)^{\frac{1}{3}}$$

input
```
integrate((4-3*x)^(4/3)*x^2,x, algorithm="fricas")
```

output
```
-1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^(1/3)
```

3.295.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.45

$$\int (4-3x)^{4/3}x^2\,dx = \begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{13} + \frac{84x^3\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{65} - \frac{32x^2\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{64x\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{256\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} \\ -\frac{9x^4\sqrt[3]{4-3x}}{13} + \frac{84x^3\sqrt[3]{4-3x}}{65} - \frac{32x^2\sqrt[3]{4-3x}}{455} - \frac{64x\sqrt[3]{4-3x}}{455} - \frac{256\sqrt[3]{4-3x}}{455} \end{cases}$$

input
```
integrate((4-3*x)**(4/3)*x**2,x)
```

output
```
Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(I*pi/3)/13 + 84*x**3*(3*x - 4)**(1
/3)*exp(I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 64*x*(3*x
- 4)**(1/3)*exp(I*pi/3)/455 - 256*(3*x - 4)**(1/3)*exp(I*pi/3)/455, Abs(x)
 > 4/3), (-9*x**4*(4 - 3*x)**(1/3)/13 + 84*x**3*(4 - 3*x)**(1/3)/65 - 32*x
**2*(4 - 3*x)**(1/3)/455 - 64*x*(4 - 3*x)**(1/3)/455 - 256*(4 - 3*x)**(1/3
)/455, True))
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{117}\,(-3\,x+4)^{\frac{13}{3}} + \frac{4}{45}\,(-3\,x+4)^{\frac{10}{3}} - \frac{16}{63}\,(-3\,x+4)^{\frac{7}{3}}$$

input
```
integrate((4-3*x)^(4/3)*x^2,x, algorithm="maxima")
```

output
```
-1/117*(-3*x + 4)^(13/3) + 4/45*(-3*x + 4)^(10/3) - 16/63*(-3*x + 4)^(7/3)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{117}\,(3\,x-4)^4(-3\,x+4)^{\frac{1}{3}} - \frac{4}{45}\,(3\,x-4)^3(-3\,x+4)^{\frac{1}{3}} - \frac{16}{63}\,(3\,x-4)^2(-3\,x+4)^{\frac{1}{3}}$$

input
```
integrate((4-3*x)^(4/3)*x^2,x, algorithm="giac")
```

output
```
-1/117*(3*x - 4)^4*(-3*x + 4)^(1/3) - 4/45*(3*x - 4)^3*(-3*x + 4)^(1/3) -
16/63*(3*x - 4)^2*(-3*x + 4)^(1/3)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{(4-3\,x)^{7/3}\,\left(1092\,x+35\,(3\,x-4)^2-416\right)}{4095}$$

input
```
int(x^2*(4 - 3*x)^(4/3),x)
```

output
```
-((4 - 3*x)^(7/3)*(1092*x + 35*(3*x - 4)^2 - 416))/4095
```

3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

3.296.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

output
```
4*(1-2*x^(1/3))^(3/4)+6*arctan((1-2*x^(1/3))^(1/4))-6*arctanh((1-2*x^(1/3)
)^(1/4))
```

3.296.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

input
```
Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]
```

output
```
4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
 2*x^(1/3))^(1/4)]
```

3.296.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {798, 60, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-2 \sqrt[3]{x}\right)^{3/4}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \frac{\left(1-2 \sqrt[3]{x}\right)^{3/4}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{60}$$

$$3 \left(\int \frac{1}{\sqrt[4]{1-2 \sqrt[3]{x}} \sqrt[3]{x}}d\sqrt[3]{x}+\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}\right)$$

$$\Big\downarrow \text{73}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-2 \int \frac{2 x^{2/3}}{1-x^{4/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \int \frac{x^{2/3}}{1-x^{4/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{827}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \left(\frac{1}{2} \int \frac{1}{1-x^{2/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}-\frac{1}{2} \int \frac{1}{x^{2/3}+1}d\sqrt[4]{1-2 \sqrt[3]{x}}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \left(\frac{1}{2} \int \frac{1}{1-x^{2/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}-\frac{1}{2} \arctan \left(\sqrt[4]{1-2 \sqrt[3]{x}}\right)\right)\right)$$

$$\Big\downarrow \text{219}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \left(\frac{1}{2} \text{arctanh}\left(\sqrt[4]{1-2 \sqrt[3]{x}}\right)-\frac{1}{2} \arctan \left(\sqrt[4]{1-2 \sqrt[3]{x}}\right)\right)\right)$$

input
```
Int[(1 - 2*x^(1/3))^(3/4)/x,x]
```

3.296. $\int \frac{\left(1-2 \sqrt[3]{x}\right)^{3/4}}{x} \, dx$

output
```
3*((4*(1 - 2*x^(1/3))^(3/4))/3 - 4*(-1/2*ArcTan[(1 - 2*x^(1/3))^(1/4)] + A
rcTanh[(1 - 2*x^(1/3))^(1/4)]/2))
```

3.296.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.296. $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

3.296.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result
derivativedivides	$4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-2\right.\right.$
default	$4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-2\right.\right.$
meijerg	$-\frac{9\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{4\left(\frac{4}{3}-2\ln(2)-\frac{\pi}{2}+\frac{\ln(x)}{3}+i\pi\right)\pi\sqrt{2}}{3\Gamma\left(\frac{3}{4}\right)}+\frac{2\pi\sqrt{2}\,x^{\frac{1}{3}}\,{}_{3}F_{2}\left(\frac{1}{4},1,1;2,2;2x^{\frac{1}{3}}\right)}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}$

input
```
int((1-2*x^(1/3))^(3/4)/x,x,method=_RETURNVERBOSE)
```

output
```
4*(1-2*x^(1/3))^(3/4)+3*ln((1-2*x^(1/3))^(1/4)-1)-3*ln((1-2*x^(1/3))^(1/4)
+1)+6*arctan((1-2*x^(1/3))^(1/4))
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="fricas")
```

output
```
4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^
(1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

3.296.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = -\frac{3\cdot 2^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((1-2*x**(1/3))**(3/4)/x,x)
```

output
```
-3*2**(3/4)*x**(1/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,),
 1/(2*x**(1/3)))/gamma(1/4)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="maxima")
```

output
```
4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^
(1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

3.296.8 Giac [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\log\left(\left|\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="giac")
```

output
```
4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^
(1/3) + 1)^(1/4) + 1) + 3*log(abs((-2*x^(1/3) + 1)^(1/4) - 1))
```

3.296.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 6\,\mathrm{atan}\Big(\left(1-2\,x^{1/3}\right)^{1/4}\Big)-6\,\mathrm{atanh}\Big(\left(1-2\,x^{1/3}\right)^{1/4}\Big)+4\left(1-2\,x^{1/3}\right)^{3/4}$$

input
```
int((1 - 2*x^(1/3))^(3/4)/x,x)
```

output
```
6*atan((1 - 2*x^(1/3))^(1/4)) - 6*atanh((1 - 2*x^(1/3))^(1/4)) + 4*(1 - 2*
x^(1/3))^(3/4)
```

3.297 $\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx$

3.297.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}$$

output
```
-27/2*(3-2*x^(1/2))^(1/4)+27/10*(3-2*x^(1/2))^(5/4)-1/2*(3-2*x^(1/2))^(9/4
)+1/26*(3-2*x^(1/2))^(13/4)
```

3.297.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{4}{65}\sqrt[4]{3-2\sqrt{x}}\left(144+24\sqrt{x}+10x+5x^{3/2}\right)$$

input
```
Integrate[x/(3 - 2*Sqrt[x])^(3/4),x]
```

output
```
(-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/65
```

3.297.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{3/2}}{\left(3-2\sqrt{x}\right)^{3/4}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{1}{8}\left(3-2\sqrt{x}\right)^{9/4}+\frac{9}{8}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{8}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{8\left(3-2\sqrt{x}\right)^{3/4}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{52}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{4}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{20}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{4}\sqrt[4]{3-2\sqrt{x}}\right)$$

input
```
Int[x/(3 - 2*Sqrt[x])^(3/4),x]
```

output
```
2*((-27*(3 - 2*Sqrt[x])^(1/4))/4 + (27*(3 - 2*Sqrt[x])^(5/4))/20 - (3 - 2*
Sqrt[x])^(9/4)/4 + (3 - 2*Sqrt[x])^(13/4)/52)
```

3.297.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.297.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

method	result	size
meijerg	$\frac{3^{\frac{1}{4}} x^{2} {}_{2}F_{1}\left(\frac{3}{4},4;5;\frac{2\sqrt{x}}{3}\right)}{6}$	20
derivativedivides	$-\frac{27\left(3-2\sqrt{x}\right)^{\frac{1}{4}}}{2}+\frac{27\left(3-2\sqrt{x}\right)^{\frac{5}{4}}}{10}-\frac{\left(3-2\sqrt{x}\right)^{\frac{9}{4}}}{2}+\frac{\left(3-2\sqrt{x}\right)^{\frac{13}{4}}}{26}$	46
default	$-\frac{27\left(3-2\sqrt{x}\right)^{\frac{1}{4}}}{2}+\frac{27\left(3-2\sqrt{x}\right)^{\frac{5}{4}}}{10}-\frac{\left(3-2\sqrt{x}\right)^{\frac{9}{4}}}{2}+\frac{\left(3-2\sqrt{x}\right)^{\frac{13}{4}}}{26}$	46

input
```
int(x/(3-2*x^(1/2))^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/4)*x^2*hypergeom([3/4,4],[5],2/3*x^(1/2))
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{4}{65}\left(\left(5\,x+24\right)\sqrt{x}+10\,x+144\right)\left(-2\,\sqrt{x}+3\right)^{\frac{1}{4}}$$

input
```
integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="fricas")
```

output
```
-4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

3.297.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 3303, normalized size of antiderivative = 47.87

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(3-2*x**(1/2))**(3/4),x)
```

output
```
Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x*
*(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) + 26304*3**(1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4
)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2
*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3*
*(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14040
0*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*sq
rt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2
*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3*
*(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14040
0*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*s
qrt(3)*x**(19/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(
2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 28080...
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{1}{26}\left(-2\sqrt{x}+3\right)^{\frac{13}{4}} - \frac{1}{2}\left(-2\sqrt{x}+3\right)^{\frac{9}{4}} + \frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}} - \frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

input
```
integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="maxima")
```

output
```
1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10*(-2*sqrt
(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{1}{26}\left(2\sqrt{x}-3\right)^{3}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}} - \frac{1}{2}\left(2\sqrt{x}-3\right)^{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}} + \frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}} - \frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

input
```
integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="giac")
```

output
```
-1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) - 3)^2*(-2
*sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)
^(1/4)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{27\left(3-2\sqrt{x}\right)^{5/4}}{10} - \frac{27\left(3-2\sqrt{x}\right)^{1/4}}{2} - \frac{\left(3-2\sqrt{x}\right)^{9/4}}{2} + \frac{\left(3-2\sqrt{x}\right)^{13/4}}{26}$$

input
```
int(x/(3 - 2*x^(1/2))^(3/4),x)
```

output
```
(27*(3 - 2*x^(1/2))^(5/4))/10 - (27*(3 - 2*x^(1/2))^(1/4))/2 - (3 - 2*x^(1
/2))^(9/4)/2 + (3 - 2*x^(1/2))^(13/4)/26
```

3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

3.298.1 Optimal result

Integrand size = 17, antiderivative size = 193

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5\arctan\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} - \frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}}$$

output
```
5/4*arctan(-1+2^(1/2)*(-1+2*x^(1/2))^(1/4))*2^(1/2)+5/4*arctan(1+2^(1/2)*(
-1+2*x^(1/2))^(1/4))*2^(1/2)-5/8*ln(1-2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x
^(1/2))^(1/2))*2^(1/2)+5/8*ln(1+2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2)
)^(1/2))*2^(1/2)-5/2*(-1+2*x^(1/2))^(1/4)/x^(1/2)-(-1+2*x^(1/2))^(5/4)/x
```

3.298.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = \frac{2\left(2-9\sqrt{x}\right)\sqrt[4]{-1+2\sqrt{x}}+5\sqrt{2}x\arctan\left(\frac{-1+\sqrt{-1+2\sqrt{x}}}{\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}}\right)+5\sqrt{2}x\operatorname{arctanh}\left(\frac{\sqrt{2}\ldots}{\ldots}\right)}{4x}$$

input
```
Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

output
```
(2*(2 - 9*Sqrt[x])*(-1 + 2*Sqrt[x])^(1/4) + 5*Sqrt[2]*x*ArcTan[(-1 + Sqrt[
-1 + 2*Sqrt[x]])/(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))] + 5*Sqrt[2]*x*ArcTanh[(
Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))/(1 + Sqrt[-1 + 2*Sqrt[x]])])/(4*x)
```

3.298.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {798, 51, 51, 73, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2\sqrt{x}-1\right)^{5/4}}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(2\sqrt{x}-1\right)^{5/4}}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{51}$$

$$2\left(\frac{5}{4}\int \frac{\sqrt[4]{2\sqrt{x}-1}}{x}d\sqrt{x}-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{51}$$

$$2\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\left(2\sqrt{x}-1\right)^{3/4}\sqrt{x}}d\sqrt{x}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{73}$$

$$2\left(\frac{5}{4}\left(\int \frac{1}{\frac{x^2}{2}+\frac{1}{2}}d\sqrt[4]{2\sqrt{x}-1}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{2(1-x)}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{1}{2}\int \frac{2(x+1)}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\int \frac{x+1}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\right.$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{(}{}\right.$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{5}{4}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt[4]{2\sqrt{x}-1}}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-\right.}{\sqrt{2}}\right.\right.$$

3.298. $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{5}{4}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{2\sqrt{x}-1}}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}+\right.\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{5}{4}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{2\sqrt{x}-1}}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right.}{\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{5}{4}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+\right.}{2\sqrt{2}}\right.\right.$$

input
```
Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

output
```
2*(-1/2*(-1 + 2*Sqrt[x])^(5/4)/x + (5*(-((-1 + 2*Sqrt[x])^(1/4)/Sqrt[x]) -
 ArcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)]/Sqrt[2] + ArcTan[1 + Sqrt[2]*(
-1 + 2*Sqrt[x])^(1/4)]/Sqrt[2] - Log[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) +
x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + x]/(2*Sqrt[2])))
/4)
```

3.298. $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

3.298.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.298. $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.298.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result
meijerg	$\frac{5\,\mathrm{signum}\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}\left(-\frac{2\Gamma\left(\frac{3}{4}\right)}{5x}+\frac{2\Gamma\left(\frac{3}{4}\right)}{\sqrt{x}}+\frac{\left(-2\ln\left(2\right)+\frac{\pi}{2}-\frac{3}{2}+\frac{\ln\left(x\right)}{2}+i\pi\right)\Gamma\left(\frac{3}{4}\right)}{2}+\frac{\Gamma\left(\frac{3}{4}\right)\sqrt{x}\,{}_{3}F_{2}\left(1,1,\frac{7}{4};2,4;2\sqrt{x}\right)}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(-1+2\sqrt{x}\right)\right)^{\frac{5}{4}}}$
derivativedivides	$\frac{-\frac{9\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}}{4}-\frac{5\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}\right)+2\arctan\cdots\right.}{8}$
default	$\frac{-\frac{9\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}}{4}-\frac{5\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\left(-1+2\sqrt{x}\right)^{\frac{1}{4}}\right)+2\arctan\cdots\right.}{8}$

input
```
int((-1+2*x^(1/2))^(5/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
5/2/GAMMA(3/4)*signum(-1+2*x^(1/2))^(5/4)/(-signum(-1+2*x^(1/2)))^(5/4)*(-
2/5*GAMMA(3/4)/x+2*GAMMA(3/4)/x^(1/2)+1/2*(-2*ln(2)+1/2*Pi-3/2+1/2*ln(x)+I
*Pi)*GAMMA(3/4)+1/4*GAMMA(3/4)*x^(1/2)*hypergeom([1,1,7/4],[2,4],2*x^(1/2)
))
```

3.298.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = \frac{(5i+5)\,\sqrt{2}x\log\left((i+1)\,\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)-(5i-5)\,\sqrt{2}x\log\left(-(i-1)\,\sqrt{2}+\right.}{}$$

input
```
integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="fricas")
```

output
```
1/8*((5*I + 5)*sqrt(2)*x*log((I + 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) -
(5*I - 5)*sqrt(2)*x*log(-(I - 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) + (5*I
 - 5)*sqrt(2)*x*log((I - 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) - (5*I + 5)
*sqrt(2)*x*log(-(I + 1)*sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4)) - 4*(9*sqrt(x)
- 2)*(2*sqrt(x) - 1)^(1/4))/x
```

3.298.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = -\frac{4\cdot\sqrt[4]{2}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

output
```
-4*2**(1/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), exp_polar(2*I*pi)/(2*sqr
t(x)))/(x**(3/8)*gamma(7/4))
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = \frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) + \frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) + \frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{\left(2\sqrt{x}-1\right)^2+4\sqrt{x}-1}$$

input
```
integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="maxima")
```

output
```
5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqr
t(2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sq
rt(2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - (9*(
2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/((2*sqrt(x) - 1)^2 + 4*sqr
t(x) - 1)
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = \frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) + \frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right) + \frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right) - \frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{4x}$$

input
```
integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="giac")
```

output
```
5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqr
t(2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sq
rt(2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 1/4*
(9*(2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/x
```

3.298.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.40

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = -\frac{5\left(2\sqrt{x}-1\right)^{1/4}}{4\,x} - \frac{9\left(2\sqrt{x}-1\right)^{5/4}}{4\,x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{4}+\frac{5}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{4}-\frac{5}{4}\mathrm{i}\right)$$

input
```
int((2*x^(1/2) - 1)^(5/4)/x^2,x)
```

output
```
2^(1/2)*atan(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 - 1i/2))*(5/4 + 5i/4) - (9
*(2*x^(1/2) - 1)^(5/4))/(4*x) - (5*(2*x^(1/2) - 1)^(1/4))/(4*x) + 2^(1/2)*
atan(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 + 1i/2))*(5/4 - 5i/4)
```

3.299 $\int x^6 \sqrt[3]{1+x^7}\,dx$

3.299.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

output
```
3/28*(x^7+1)^(4/3)
```

3.299.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

input
```
Integrate[x^6*(1 + x^7)^(1/3),x]
```

output
```
(3*(1 + x^7)^(4/3))/28
```

3.299.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[3]{x^7+1} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

input
```
Int[x^6*(1 + x^7)^(1/3),x]
```

output
```
(3*(1 + x^7)^(4/3))/28
```

3.299.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.299.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
default	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
risch	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
pseudoelliptic	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
trager	$\left(\frac{3}{28}+\frac{3x^7}{28}\right)(x^7+1)^{\frac{1}{3}}$	16
meijerg	$\frac{x^7 {}_2F_1\left(-\frac{1}{3},1;2;-x^7\right)}{7}$	17
gosper	$\frac{3(1+x)(x^6-x^5+x^4-x^3+x^2-x+1)(x^7+1)^{\frac{1}{3}}}{28}$	37

input
```
int(x^6*(x^7+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/28*(x^7+1)^(4/3)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6\sqrt[3]{1+x^7}\,dx=\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^6*(x^7+1)^(1/3),x, algorithm="fricas")
```

output
```
3/28*(x^7 + 1)^(4/3)
```

3.299.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int x^6\sqrt[3]{1+x^7}\,dx=\frac{3x^7\sqrt[3]{x^7+1}}{28}+\frac{3\sqrt[3]{x^7+1}}{28}$$

input
```
integrate(x**6*(x**7+1)**(1/3),x)
```

output
```
3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6\sqrt[3]{1+x^7}\,dx=\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^6*(x^7+1)^(1/3),x, algorithm="maxima")
```

output
```
3/28*(x^7 + 1)^(4/3)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6\sqrt[3]{1+x^7}\,dx=\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^6*(x^7+1)^(1/3),x, algorithm="giac")
```

output
```
3/28*(x^7 + 1)^(4/3)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3\,(x^7+1)^{4/3}}{28}$$

input
```
int(x^6*(x^7 + 1)^(1/3),x)
```

output
```
(3*(x^7 + 1)^(4/3))/28
```

3.300 $\int \frac{x^6}{(1+x^7)^{5/3}}\,dx$

3.300.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14(1+x^7)^{2/3}}$$

output
```
-3/14/(x^7+1)^(2/3)
```

3.300.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14(1+x^7)^{2/3}}$$

input
```
Integrate[x^6/(1 + x^7)^(5/3),x]
```

output
```
-3/(14*(1 + x^7)^(2/3))
```

3.300.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^7+1)^{5/3}}\,dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{3}{14\,(x^7+1)^{2/3}}$$

input
```
Int[x^6/(1 + x^7)^(5/3),x]
```

output
```
-3/(14*(1 + x^7)^(2/3))
```

3.300.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.300.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
default	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
trager	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
risch	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
pseudoelliptic	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
meijerg	$\frac{x^7 {}_2F_1(1,\frac{5}{3};2;-x^7)}{7}$	17
gosper	$-\frac{3(1+x)(x^6-x^5+x^4-x^3+x^2-x+1)}{14(x^7+1)^{\frac{5}{3}}}$	37

input
```
int(x^6/(x^7+1)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3/14/(x^7+1)^(2/3)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(x^7+1)^(5/3),x, algorithm="fricas")
```

output
```
-3/14/(x^7 + 1)^(2/3)
```

3.300.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x**6/(x**7+1)**(5/3),x)
```

output
```
-3/(14*(x**7 + 1)**(2/3))
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(x^7+1)^(5/3),x, algorithm="maxima")
```

output
```
-3/14/(x^7 + 1)^(2/3)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(x^7+1)^(5/3),x, algorithm="giac")
```

output
```
-3/14/(x^7 + 1)^(2/3)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{2/3}}$$

input
```
int(x^6/(x^7 + 1)^(5/3),x)
```

output
```
-3/(14*(x^7 + 1)^(2/3))
```

3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$

3.301.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx=-\frac{\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}+\frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)$$

output
```
-1/18*ln(x)+1/42*ln(3+(2*x^7-27)^(1/3))-1/63*arctan(1/9*(3-2*(2*x^7-27)^(1
/3))*3^(1/2))*3^(1/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx=\frac{1}{126}\left(-2\sqrt{3}\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)+2\log\left(3+\sqrt[3]{-27+2x^7}\right)-\log\left(9-3\sqrt[3]{-27+2x^7}+\left(-27+2x^7\right)^{2/3}\right)\right)$$

input
```
Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])] + 2*Log[3 + (-
27 + 2*x^7)^(1/3)] - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7)^(2/3)])
/126
```

3.301.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(2x^7-27\right)^{2/3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{7}\int \frac{1}{x^7\left(2x^7-27\right)^{2/3}}dx^7$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{7}\left(\frac{1}{6}\int \frac{1}{\sqrt[3]{2x^7-27}+3}d\sqrt[3]{2x^7-27}+\frac{1}{2}\int \frac{1}{x^{14}-3\sqrt[3]{2x^7-27}+9}d\sqrt[3]{2x^7-27}-\frac{1}{18}\log\left(x^7\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{7}\left(\frac{1}{2}\int \frac{1}{x^{14}-3\sqrt[3]{2x^7-27}+9}d\sqrt[3]{2x^7-27}-\frac{1}{18}\log\left(x^7\right)+\frac{1}{6}\log\left(\sqrt[3]{2x^7-27}+3\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{7}\left(-\int \frac{1}{-x^{14}-27}d\left(2\sqrt[3]{2x^7-27}-3\right)-\frac{1}{18}\log\left(x^7\right)+\frac{1}{6}\log\left(\sqrt[3]{2x^7-27}+3\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{7}\left(\frac{\arctan\left(\frac{2\sqrt[3]{2x^7-27}-3}{3\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log\left(x^7\right)}{18}+\frac{1}{6}\log\left(\sqrt[3]{2x^7-27}+3\right)\right)$$

input
```
Int[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

output
```
(ArcTan[(-3 + 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(3*Sqrt[3]) - Log[x^7]/1
8 + Log[3 + (-27 + 2*x^7)^(1/3)]/6)/7
```

3.301.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.301.4 Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{\ln\left(3+\left(2x^{7}-27\right)^{\frac{1}{3}}\right)}{63}-\frac{\ln\left(\left(2x^{7}-27\right)^{\frac{2}{3}}-3\left(2x^{7}-27\right)^{\frac{1}{3}}+9\right)}{126}+\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(2x^{7}-27\right)^{\frac{1}{3}}}{9}-\frac{\sqrt{3}}{3}\right)}{63}$
meijerg	$\frac{\left(-\operatorname{signum}\left(-1+\frac{2x^{7}}{27}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{9\ln(3)}{2}+7\ln(x)+\ln(2)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{4\Gamma\left(\frac{2}{3}\right)x^{7}{}_{3}F_{2}\left(1,1,\frac{5}{3};2,2;\frac{2x^{7}}{27}\right)}{81}\right)}{63\operatorname{signum}\left(-1+\frac{2x^{7}}{27}\right)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$
trager	$-\frac{\ln\left(-\frac{757355840490254039191854\operatorname{RootOf}\left(81\_Z^{2}+9\_Z+1\right)^{2}x^{7}+1193513320861007233414\operatorname{RootOf}\left(81\_Z^{2}+9\_Z+1\right)x^{7}-20}{}\right.}{}$

input
```
int(1/x/(2*x^7-27)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/63*ln(3+(2*x^7-27)^(1/3))-1/126*ln((2*x^7-27)^(2/3)-3*(2*x^7-27)^(1/3)+9
)+1/63*3^(1/2)*arctan(2/9*3^(1/2)*(2*x^7-27)^(1/3)-1/3*3^(1/2))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{63}\sqrt{3}\arctan\left(\frac{2}{9}\sqrt{3}\left(2\,x^{7}-27\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right) - \frac{1}{126}\log\left(\left(2\,x^{7}-27\right)^{\frac{2}{3}}-3\left(2\,x^{7}-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2\,x^{7}-27\right)^{\frac{1}{3}}+3\right)$$

input
```
integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="fricas")
```

output
```
1/63*sqrt(3)*arctan(2/9*sqrt(3)*(2*x^7 - 27)^(1/3) - 1/3*sqrt(3)) - 1/126*
log((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)
^(1/3) + 3)
```

3.301.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = -\frac{\sqrt[3]{2}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{27e^{2i\pi}}{2x^7}\right)}{14x^{\frac{14}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(2*x**7-27)**(2/3),x)
```

output
```
-2**(1/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), 27*exp_polar(2*I*pi)/(2*x**
7))/(14*x**(14/3)*gamma(5/3))
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2x^7-27\right)^{\frac{1}{3}}+3\right)$$

input
```
integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="maxima")
```

output
```
1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2
*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(1/3)
 + 3)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx = \frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right) + \frac{1}{63}\log\left(\left|\left(2x^7-27\right)^{\frac{1}{3}}+3\right|\right)$$

input
```
integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="giac")
```

output
```
1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2
*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log(abs((2*x^7 - 27)^(
1/3) + 3))
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx = \frac{\ln\left(\frac{(2x^7-27)^{1/3}}{49}+\frac{3}{49}\right)}{63} - \ln\left(\frac{27}{14}-\frac{9\left(2x^7-27\right)^{1/3}}{7}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(\frac{1}{126}+\frac{\sqrt{3}\,1\mathrm{i}}{126}\right)+\ln\left(\frac{9\left(2x^7-27\right)^{1/3}}{7}-\frac{27}{14}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(-\frac{1}{126}+\frac{\sqrt{3}}{12\ldots}\right.$$

input
```
int(1/(x*(2*x^7 - 27)^(2/3)),x)
```

output
```
log((2*x^7 - 27)^(1/3)/49 + 3/49)/63 - log((3^(1/2)*27i)/14 - (9*(2*x^7 -
27)^(1/3))/7 + 27/14)*((3^(1/2)*1i)/126 + 1/126) + log((3^(1/2)*27i)/14 +
(9*(2*x^7 - 27)^(1/3))/7 - 27/14)*((3^(1/2)*1i)/126 - 1/126)
```

3.302 $\int \frac{(1+x^7)^{2/3}}{x^8} dx$

3.302.1 Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{(1+x^7)^{2/3}}{x^8} dx = -\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)$$

output
```
-1/7*(x^7+1)^(2/3)/x^7-1/3*ln(x)+1/7*ln(1-(x^7+1)^(1/3))+2/21*arctan(1/3*(
1+2*(x^7+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.302.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{(1+x^7)^{2/3}}{x^8} dx = \frac{1}{21}\left(-\frac{3(1+x^7)^{2/3}}{x^7} + 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1+x^7}\right) - \log\left(1+\sqrt[3]{1+x^7}+(1+x^7)^{2/3}\right)\right)$$

input
```
Integrate[(1 + x^7)^(2/3)/x^8,x]
```

output
```
((-3*(1 + x^7)^(2/3))/x^7 + 2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[
3]] + 2*Log[-1 + (1 + x^7)^(1/3)] - Log[1 + (1 + x^7)^(1/3) + (1 + x^7)^(2
/3)])/21
```

3.302.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {798, 51, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^7+1)^{2/3}}{x^8}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{7}\int \frac{(x^7+1)^{2/3}}{x^{14}}dx^7$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{7}\left(\frac{2}{3}\int \frac{1}{x^7\sqrt[3]{x^7+1}}dx^7-\frac{(x^7+1)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{x^7+1}}d\sqrt[3]{x^7+1}+\frac{3}{2}\int \frac{1}{x^{14}+\sqrt[3]{x^7+1}+1}d\sqrt[3]{x^7+1}-\frac{1}{2}\log\left(x^7\right)\right)-\frac{(x^7+1)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(\frac{3}{2}\int \frac{1}{x^{14}+\sqrt[3]{x^7+1}+1}d\sqrt[3]{x^7+1}-\frac{1}{2}\log\left(x^7\right)+\frac{3}{2}\log\left(1-\sqrt[3]{x^7+1}\right)\right)-\frac{(x^7+1)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(-3\int \frac{1}{-x^{14}-3}d\left(2\sqrt[3]{x^7+1}+1\right)-\frac{1}{2}\log\left(x^7\right)+\frac{3}{2}\log\left(1-\sqrt[3]{x^7+1}\right)\right)-\frac{(x^7+1)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)-\frac{\log\left(x^7\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{x^7+1}\right)\right)-\frac{(x^7+1)^{2/3}}{x^7}\right)$$

input
```
Int[(1 + x^7)^(2/3)/x^8,x]
```

output
```
(-((1 + x^7)^(2/3)/x^7) + (2*(Sqrt[3]*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[
3]] - Log[x^7]/2 + (3*Log[1 - (1 + x^7)^(1/3)])/2))/3)/7
```

3.302.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.302. $\int \frac{(1+x^7)^{2/3}}{x^8}\,dx$

3.302.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 6.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

method	result
meijerg	$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)x^{7}}-\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}-1+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{\pi\sqrt{3}\,x^{7}\,{}_{3}F_{2}\left(1,1,\frac{4}{3};2,3;-x^{7}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$
risch	$-\frac{\left(x^{7}+1\right)^{\frac{2}{3}}}{7x^{7}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}-\frac{2\pi\sqrt{3}\,x^{7}\,{}_{3}F_{2}\left(1,1,\frac{4}{3};2,2;-x^{7}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(1+2\left(x^{7}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,x^{7}-\ln\left(\left(x^{7}+1\right)^{\frac{2}{3}}+\left(x^{7}+1\right)^{\frac{1}{3}}+1\right)x^{7}+2\ln\left(\left(x^{7}+1\right)^{\frac{1}{3}}-1\right)x^{7}-3\left(x^{7}+1\right)^{\frac{2}{3}}}{21\left(\left(x^{7}+1\right)^{\frac{2}{3}}+\left(x^{7}+1\right)^{\frac{1}{3}}+1\right)\left(\left(x^{7}+1\right)^{\frac{1}{3}}-1\right)}$
trager	$-\frac{\left(x^{7}+1\right)^{\frac{2}{3}}}{7x^{7}}-\frac{2\ln\left(\frac{3593313\,\mathrm{RootOf}\left(9\_Z^{2}+3\_Z+1\right)^{2}x^{7}+3486414\,\mathrm{RootOf}\left(9\_Z^{2}+3\_Z+1\right)x^{7}-106899x^{7}+6095754\left(x^{7}+1\right)^{\frac{2}{3}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int((x^7+1)^(2/3)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/Pi*3^(1/2)*GAMMA(2/3)*(Pi*3^(1/2)/GAMMA(2/3)/x^7-2/3*(-1/6*Pi*3^(1/2
)-3/2*ln(3)-1+7*ln(x))*Pi*3^(1/2)/GAMMA(2/3)+1/9*Pi*3^(1/2)/GAMMA(2/3)*x^7
*hypergeom([1,1,4/3],[2,3],-x^7))
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{\left(1+x^{7}\right)^{2/3}}{x^{8}}\,dx=\frac{2\sqrt{3}x^{7}\arctan\left(\frac{2}{3}\sqrt{3}(x^{7}+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-x^{7}\log\left((x^{7}+1)^{\frac{2}{3}}+(x^{7}+1)^{\frac{1}{3}}+1\right)+2\,x^{7}\log\cdots}{21\,x^{7}}$$

input
```
integrate((x^7+1)^(2/3)/x^8,x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(3)*x^7*arctan(2/3*sqrt(3)*(x^7 + 1)^(1/3) + 1/3*sqrt(3)) - x^
7*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2*x^7*log((x^7 + 1)^(1/3) -
 1) - 3*(x^7 + 1)^(2/3))/x^7
```

3.302.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = -\frac{\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^7}\right)}{7x^{\frac{7}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((x**7+1)**(2/3)/x**8,x)
```

output
```
-gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**7)/(7*x**(7/3)*g
amma(4/3))
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7} - \frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right) + \frac{2}{21}\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate((x^7+1)^(2/3)/x^8,x, algorithm="maxima")
```

output
```
2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(
2/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log((x^7
 + 1)^(1/3) - 1)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\,(x^7+1)^{\frac{1}{3}}+1\Big)\right) - \frac{(x^7+1)^{\frac{2}{3}}}{7\,x^7} - \frac{1}{21}\log\Big((x^7+1)^{\frac{2}{3}} + (x^7+1)^{\frac{1}{3}} + 1\Big) + \frac{2}{21}\log\Big(\Big|(x^7+1)^{\frac{1}{3}} - 1\Big|\Big)$$

input
```
integrate((x^7+1)^(2/3)/x^8,x, algorithm="giac")
```

output
```
2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(
2/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log(abs(
(x^7 + 1)^(1/3) - 1))
```

3.302.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2\,\ln\left(\frac{4\,(x^7+1)^{1/3}}{49} - \frac{4}{49}\right)}{21} + \ln\left(\frac{4\,(x^7+1)^{1/3}}{49} - 9\left(-\frac{1}{21} + \frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(-\frac{1}{21} + \frac{\sqrt{3}\,1\mathrm{i}}{21}\right) - \ln\left(\frac{4\,(x^7+1)^{1/3}}{49} - 9\left(\frac{1}{21} + \frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(\frac{1}{21}\right.$$

input
```
int((x^7 + 1)^(2/3)/x^8,x)
```

output
```
(2*log((4*(x^7 + 1)^(1/3))/49 - 4/49))/21 + log((4*(x^7 + 1)^(1/3))/49 - 9
*((3^(1/2)*1i)/21 - 1/21)^2)*((3^(1/2)*1i)/21 - 1/21) - log((4*(x^7 + 1)^(
1/3))/49 - 9*((3^(1/2)*1i)/21 + 1/21)^2)*((3^(1/2)*1i)/21 + 1/21) - (x^7 +
 1)^(2/3)/(7*x^7)
```

3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2} dx$

3.303.1 Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2} dx = -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}$$

output
```
-(4*x^4+3)^(1/4)/x-1/2*arctan(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)+1/2*arcta
nh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2} dx = -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[(3 + 4*x^4)^(1/4)/x^2,x]
```

output
```
-((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + A
rcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

3.303.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {809, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{4x^4+3}}{x^2}dx$$

$$\Big\downarrow \text{809}$$

$$4\int \frac{x^2}{(4x^4+3)^{3/4}}dx-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{854}$$

$$4\int \frac{x^2}{\sqrt{4x^4+3}\left(1-\frac{4x^4}{4x^4+3}\right)}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{827}$$

$$4\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{1}{4}\int \frac{1}{\frac{2x^2}{\sqrt{4x^4+3}}+1}d\frac{x}{\sqrt[4]{4x^4+3}}\right)-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)-\frac{\sqrt[4]{4x^4+3}}{x}$$

input
```
Int[(3 + 4*x^4)^(1/4)/x^2,x]
```

output
```
-((3 + 4*x^4)^(1/4)/x) + 4*(-1/4*ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqr
t[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/(4*Sqrt[2]))
```

3.303.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.303. $\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx$

3.303.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

method	result
meijerg	$-\frac{3^{\frac{1}{4}} {}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{4x^4}{3}\right)}{x}$
risch	$-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{4\,3^{\frac{1}{4}} x^3 {}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)}{9}$
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4x^4+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)x+\sqrt{2}\,\arctan\left(\frac{\left(4x^4+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)x-2\left(4x^4+3\right)^{\frac{1}{4}}}{2x}$
trager	$-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2+2\right)\sqrt{4x^4+3}\,x^2+8\operatorname{RootOf}\left(\_Z^2+2\right)x^4-4\left(4x^4+3\right)^{\frac{3}{4}}x+8x^3\left(4\right.\right.}{4}$

input
```
int((4*x^4+3)^(1/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3^(1/4)/x*hypergeom([-1/4,-1/4],[3/4],-4/3*x^4)
```

3.303.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(55) = 110.

Time = 2.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{2\sqrt{2}x\arctan\left(\frac{4}{3}\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3+\frac{2}{3}\sqrt{2}(4x^4+3)^{\frac{3}{4}}x\right)-\sqrt{2}x\log\left(-256x^8-192x^4-4\sqrt{2}(16x^5+3\right.}{8x}$$

input
```
integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*x*arctan(4/3*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 2/3*sqrt(2)*(
4*x^4 + 3)^(3/4)*x) - sqrt(2)*x*log(-256*x^8 - 192*x^4 - 4*sqrt(2)*(16*x^5
 + 3*x)*(4*x^4 + 3)^(3/4) - 8*sqrt(2)*(16*x^7 + 9*x^3)*(4*x^4 + 3)^(1/4) -
 16*(8*x^6 + 3*x^2)*sqrt(4*x^4 + 3) - 9) + 8*(4*x^4 + 3)^(1/4))/x
```

3.303.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{\sqrt[4]{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((4*x**4+3)**(1/4)/x**2,x)
```

output
```
3**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), 4*x**4*exp_polar(I*pi)/3)
/(4*x*gamma(3/4))
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4x^4+3)^{\frac{1}{4}}}{x}$$

input
```
integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sq
rt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3
)^(1/4)/x
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) - \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4\,x^4+3)^{\frac{1}{4}}}{x}$$

input
```
integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sq
rt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3
)^(1/4)/x
```

3.303.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{3^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{4\,x^4}{3}\right)}{x}$$

input
```
int((4*x^4 + 3)^(1/4)/x^2,x)
```

output
```
-(3^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(4*x^4)/3))/x
```

3.304 $\int x^2(3+4x^4)^{5/4}\,dx$

3.304.1 Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

output
```
15/32*x^3*(4*x^4+3)^(1/4)+1/8*x^3*(4*x^4+3)^(5/4)-45/256*arctan(x*2^(1/2)/
(4*x^4+3)^(1/4))*2^(1/2)+45/256*arctanh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=\frac{1}{32}x^3\sqrt[4]{3+4x^4}\left(27+16x^4\right)-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

input
```
Integrate[x^2*(3 + 4*x^4)^(5/4),x]
```

output
```
(x^3*(3 + 4*x^4)^(1/4)*(27 + 16*x^4))/32 - (45*ArcTan[(Sqrt[2]*x)/(3 + 4*x
^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(1
28*Sqrt[2])
```

3.304.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {811, 811, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(4x^4+3\right)^{5/4} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{15}{8}\int x^2 \sqrt[4]{4x^4+3}dx+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

$$\Big\downarrow \text{811}$$

$$\frac{15}{8}\left(\frac{3}{4}\int \frac{x^2}{\left(4x^4+3\right)^{3/4}}dx+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

$$\Big\downarrow \text{854}$$

$$\frac{15}{8}\left(\frac{3}{4}\int \frac{x^2}{\sqrt{4x^4+3}\left(1-\frac{4x^4}{4x^4+3}\right)}d\frac{x}{\sqrt[4]{4x^4+3}}+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

$$\Big\downarrow \text{827}$$

$$\frac{15}{8}\left(\frac{3}{4}\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{1}{4}\int \frac{1}{\frac{2x^2}{\sqrt{4x^4+3}}+1}d\frac{x}{\sqrt[4]{4x^4+3}}\right)+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

$$\Big\downarrow \text{216}$$

$$\frac{15}{8}\left(\frac{3}{4}\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

$$\Big\downarrow \text{219}$$

$$\frac{15}{8}\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

input
```
Int[x^2*(3 + 4*x^4)^(5/4),x]
```

output
```
(x^3*(3 + 4*x^4)^(5/4))/8 + (15*((x^3*(3 + 4*x^4)^(1/4))/4 + (3*(-1/4*ArcT
an[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4
)^(1/4)]/(4*Sqrt[2])))/4))/8
```

3.304.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.304.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.20

method	result
meijerg	$3^{\frac{1}{4}} x^{3} {}_{2}F_{1}\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^{4}}{3}\right)$
risch	$\frac{x^{3}\left(16x^{4}+27\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{32}+\frac{5\,3^{\frac{1}{4}} x^{3} {}_{2}F_{1}\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^{4}}{3}\right)}{32}$
pseudoelliptic	$\frac{\frac{9x^{7}\left(4x^{4}+3\right)^{\frac{1}{4}}}{2}+\frac{243x^{3}\left(4x^{4}+3\right)^{\frac{1}{4}}}{32}+\frac{405\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{256}+\frac{405\sqrt{2}\,\arctan\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{256}}{\left(-2x^{2}+\sqrt{4x^{4}+3}\right)^{2}\left(2x^{2}+\sqrt{4x^{4}+3}\right)^{2}}$
trager	$\frac{x^{3}\left(16x^{4}+27\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{32}-\frac{45\,\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-4\sqrt{4x^{4}+3}\,\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{2}-8\,\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{4}+4(4x^{4}+\right.}{512}$

input
```
int(x^2*(4*x^4+3)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
3^(1/4)*x^3*hypergeom([-5/4,3/4],[7/4],-4/3*x^4)
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=\frac{45}{256}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right)$$
$$+\frac{45}{512}\,\sqrt{2}\log\left(8\,x^4+4\,\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}x^3+4\,\sqrt{4\,x^4+3}x^2+2\,\sqrt{2}(4\,x^4+3)^{\frac{3}{4}}x+3\right)$$
$$+\frac{1}{32}\left(16\,x^7+27\,x^3\right)\left(4\,x^4+3\right)^{\frac{1}{4}}$$

input
```
integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="fricas")
```

output
```
45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 45/512*sqrt(2)*lo
g(8*x^4 + 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 + 2*sqrt
(2)*(4*x^4 + 3)^(3/4)*x + 3) + 1/32*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4)
```

3.304.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int x^2 (3+4x^4)^{5/4}\,dx = \frac{3\cdot\sqrt[4]{3}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(4*x**4+3)**(5/4),x)
```

output
```
3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_polar(I*p
i)/3)/(4*gamma(7/4))
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int x^2 (3+4x^4)^{5/4}\,dx = \frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) + \frac{9\left(\frac{20(4x^4+3)^{\frac{1}{4}}}{x}-\frac{9(4x^4+3)^{\frac{5}{4}}}{x^5}\right)}{32\left(\frac{8(4x^4+3)}{x^4}-\frac{(4x^4+3)^2}{x^8}-16\right)}$$

input
```
integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="maxima")
```

output
```
45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*lo
g(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) + 9/32
*(20*(4*x^4 + 3)^(1/4)/x - 9*(4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (
4*x^4 + 3)^2/x^8 - 16)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int x^2(3+4x^4)^{5/4}\,dx = \frac{1}{32}x^8\left(\frac{9\,(4\,x^4+3)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}-\frac{20\,(4\,x^4+3)^{\frac{1}{4}}}{x}\right)+\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right)-\frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right)$$

input
```
integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="giac")
```

output
```
1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4)/x) + 45
/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log(
-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int x^2(3+4x^4)^{5/4}\,dx = \int x^2\,\left(4\,x^4+3\right)^{5/4}\,dx$$

input
```
int(x^2*(4*x^4 + 3)^(5/4),x)
```

output
```
int(x^2*(4*x^4 + 3)^(5/4), x)
```

3.305 $\int x^6 \sqrt[4]{3+4x^4}\,dx$

3.305.1 Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4} + \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

output
```
3/128*x^3*(4*x^4+3)^(1/4)+1/8*x^7*(4*x^4+3)^(1/4)+27/1024*arctan(x*2^(1/2)
/(4*x^4+3)^(1/4))*2^(1/2)-27/1024*arctanh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/
2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{1}{128}x^3\sqrt[4]{3+4x^4}(3+16x^4) + \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

input
```
Integrate[x^6*(3 + 4*x^4)^(1/4),x]
```

output
```
(x^3*(3 + 4*x^4)^(1/4)*(3 + 16*x^4))/128 + (27*ArcTan[(Sqrt[2]*x)/(3 + 4*x
^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(5
12*Sqrt[2])
```

3.305.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {811, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[4]{4x^4+3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{8}\int \frac{x^6}{(4x^4+3)^{3/4}}dx+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\int \frac{x^2}{(4x^4+3)^{3/4}}dx\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

$$\Big\downarrow \text{854}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\int \frac{x^2}{\sqrt{4x^4+3}\left(1-\frac{4x^4}{4x^4+3}\right)}d\frac{x}{\sqrt[4]{4x^4+3}}\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

$$\Big\downarrow \text{827}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{1}{4}\int \frac{1}{\frac{2x^2}{\sqrt{4x^4+3}}+1}d\frac{x}{\sqrt[4]{4x^4+3}}\right)\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

input
```
Int[x^6*(3 + 4*x^4)^(1/4),x]
```

output
```
(x^7*(3 + 4*x^4)^(1/4))/8 + (3*((x^3*(3 + 4*x^4)^(1/4))/16 - (9*(-1/4*ArcT
an[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4
)^(1/4)]/(4*Sqrt[2])))/16))/8
```

3.305.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.305.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.22

method	result
meijerg	$\frac{3^{\frac{1}{4}} x^{7} {}_{2}F_{1}\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};-\frac{4x^{4}}{3}\right)}{7}$
risch	$\frac{x^{3}\left(16x^{4}+3\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{128}-\frac{3\,3^{\frac{1}{4}} x^{3} {}_{2}F_{1}\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^{4}}{3}\right)}{128}$
pseudoelliptic	$\frac{\frac{9x^{7}\left(4x^{4}+3\right)^{\frac{1}{4}}}{8}+\frac{27x^{3}\left(4x^{4}+3\right)^{\frac{1}{4}}}{128}-\frac{243\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{1024}-\frac{243\sqrt{2}\,\arctan\left(\frac{\left(4x^{4}+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)}{1024}}{\left(-2x^{2}+\sqrt{4x^{4}+3}\right)^{2}\left(2x^{2}+\sqrt{4x^{4}+3}\right)^{2}}$
trager	$\frac{x^{3}\left(16x^{4}+3\right)\left(4x^{4}+3\right)^{\frac{1}{4}}}{128}+\frac{27\,\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-4\sqrt{4x^{4}+3}\,\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{2}-8\,\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{4}+4(4x^{4}+3\right.}{2048}$

input
```
int(x^6*(4*x^4+3)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/7*3^(1/4)*x^7*hypergeom([-1/4,7/4],[11/4],-4/3*x^4)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = -\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{27}{2048}\sqrt{2}\log\left(8x^4 - 4\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3 + 4\sqrt{4x^4+3}x^2 - 2\sqrt{2}(4x^4+3)^{\frac{3}{4}}x + 3\right) + \frac{1}{128}(16x^7+3x^3)(4x^4+3)^{\frac{1}{4}}$$

input
```
integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="fricas")
```

output
```
-27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)
*log(8*x^4 - 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 - 2*s
qrt(2)*(4*x^4 + 3)^(3/4)*x + 3) + 1/128*(16*x^7 + 3*x^3)*(4*x^4 + 3)^(1/4)
```

3.305.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{\sqrt[4]{3}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6*(4*x**4+3)**(1/4),x)
```

output
```
3**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), 4*x**4*exp_polar(I*pi
)/3)/(4*gamma(11/4))
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$\int x^6\sqrt[4]{3+4x^4}\,dx = -\frac{27}{1024}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) + \frac{27}{2048}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{9\left(\frac{12\,(4\,x^4+3)^{\frac{1}{4}}}{x}+\frac{(4\,x^4+3)^{\frac{5}{4}}}{x^5}\right)}{128\left(\frac{8\,(4\,x^4+3)}{x^4}-\frac{(4\,x^4+3)^2}{x^8}-16\right)}$$

input
```
integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="maxima")
```

output
```
-27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)
*log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - 9
/128*(12*(4*x^4 + 3)^(1/4)/x + (4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 -
 (4*x^4 + 3)^2/x^8 - 16)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{1}{128}\,x^8\left(\frac{(4\,x^4+3)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}+\frac{12\,(4\,x^4+3)^{\frac{1}{4}}}{x}\right) - \frac{27}{1024}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) + \frac{27}{2048}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right)$$

input
```
integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="giac")
```

output
```
1/128*x^8*((4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^(1/4)/x) - 27/
1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*log
(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \int x^6\,\left(4\,x^4+3\right)^{1/4}\,dx$$

input
```
int(x^6*(4*x^4 + 3)^(1/4),x)
```

output
```
int(x^6*(4*x^4 + 3)^(1/4), x)
```

3.306 $\int \sqrt[3]{x(1-x^2)}\,dx$

3.306.1 Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \sqrt[3]{x(1-x^2)}\,dx = \frac{1}{2}x\sqrt[3]{x(1-x^2)} + \frac{\arctan\left(\frac{2x-\sqrt[3]{x(1-x^2)}}{\sqrt{3}\sqrt[3]{x(1-x^2)}}\right)}{2\sqrt{3}} + \frac{\log(x)}{12} - \frac{1}{4}\log\left(x+\sqrt[3]{x(1-x^2)}\right)$$

output
```
1/2*x*(x*(-x^2+1))^(1/3)+1/12*ln(x)-1/4*ln(x+(x*(-x^2+1))^(1/3))+1/6*arcta
n(1/3*(2*x-(x*(-x^2+1))^(1/3))/(x*(-x^2+1))^(1/3)*3^(1/2))*3^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.47

$$\int \sqrt[3]{x(1-x^2)}\,dx = \frac{\sqrt[3]{x-x^3}\left(6x^{4/3}\sqrt[3]{-1+x^2}+2\sqrt{3}\arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{-1+x^2}}\right)+2\log\left(-x^{2/3}+\sqrt[3]{-1+x^2}\right)-\log\left(x^{4/3}+\right.\right.}{12\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

input
```
Integrate[(x*(1 - x^2))^(1/3),x]
```

output
```
((x - x^3)^(1/3)*(6*x^(4/3)*(-1 + x^2)^(1/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*x
^(2/3))/(x^(2/3) + 2*(-1 + x^2)^(1/3))] + 2*Log[-x^(2/3) + (-1 + x^2)^(1/3
)] - Log[x^(4/3) + x^(2/3)*(-1 + x^2)^(1/3) + (-1 + x^2)^(2/3)]))/(12*x^(1
/3)*(-1 + x^2)^(1/3))
```

3.306.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2078, 1910, 1938, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x\left(1-x^2\right)}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt[3]{x-x^3}dx$$

$$\Big\downarrow \text{ 1910}$$

$$\frac{1}{3}\int \frac{x}{\left(x-x^3\right)^{2/3}}dx+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{\left(1-x^2\right)^{2/3}x^{2/3}\int \frac{\sqrt[3]{x}}{\left(1-x^2\right)^{2/3}}dx}{3\left(x-x^3\right)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\left(1-x^2\right)^{2/3}x^{2/3}\int \frac{x}{\left(1-x^2\right)^{2/3}}d\sqrt[3]{x}}{\left(x-x^3\right)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 807}$$

$$\frac{\left(1-x^2\right)^{2/3}x^{2/3}\int \frac{x^{2/3}}{(1-x)^{2/3}}dx^{2/3}}{2\left(x-x^3\right)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 853}$$

$$\frac{\left(1-x^2\right)^{2/3}x^{2/3}\left(-\frac{\arctan\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x^{2/3}-\sqrt[3]{1-x}\right)\right)}{2\left(x-x^3\right)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

input
```
Int[(x*(1 - x^2))^(1/3),x]
```

output
```
(x*(x - x^3)^(1/3))/2 + (x^(2/3)*(1 - x^2)^(2/3)*(-(ArcTan[(1 - (2*x^(2/3)
)/(1 - x)^(1/3))/Sqrt[3]]/Sqrt[3]) - Log[-(1 - x)^(1/3) - x^(2/3)]/2))/(2*
(x - x^3)^(2/3))
```

3.306.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

3.306. $\int \sqrt[3]{x\left(1-x^2\right)}\,dx$

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.306.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.16

method	result
meijerg	$\frac{3x^{\frac{4}{3}}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)}{4}$
pseudoelliptic	$\frac{x\left(2\sqrt{3}\,\arctan\left(\frac{\left(-2\left(-x^3+x\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)+6\left(-x^3+x\right)^{\frac{1}{3}}x-2\ln\left(\frac{\left(-x^3+x\right)^{\frac{1}{3}}+x}{x}\right)+\ln\left(\frac{\left(-x^3+x\right)^{\frac{2}{3}}-\left(-x^3+x\right)^{\frac{1}{3}}x+x^2}{x^2}\right)\right)}{12\left(\left(-x^3+x\right)^{\frac{1}{3}}+x\right)\left(\left(-x^3+x\right)^{\frac{2}{3}}-\left(-x^3+x\right)^{\frac{1}{3}}x+x^2\right)}$
trager	$\frac{\left(-x^3+x\right)^{\frac{1}{3}}x}{2}-\frac{\ln\left(4959\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^2+6768\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+x\right)^{\frac{2}{3}}+22833\,\mathrm{RootOf}\left(9\_\right.\right.}{}$
risch	Expression too large to display

input
```
int((x*(-x^2+1))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],x^2)
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \sqrt[3]{x(1-x^2)}\,dx =$$
$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{44032959556\sqrt{3}(-x^3+x)^{\frac{1}{3}}x-\sqrt{3}(16754327161\,x^2-2707204793)+10524305234\sqrt{3}(}{81835897185\,x^2-1102302937}\right.$$
$$+\frac{1}{2}\left(-x^3+x\right)^{\frac{1}{3}}x-\frac{1}{12}\log\left(3\left(-x^3+x\right)^{\frac{1}{3}}x+3\left(-x^3+x\right)^{\frac{2}{3}}+1\right)$$

input
```
integrate((x*(-x^2+1))^(1/3),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan((44032959556*sqrt(3)*(-x^3 + x)^(1/3)*x - sqrt(3)*(167
54327161*x^2 - 2707204793) + 10524305234*sqrt(3)*(-x^3 + x)^(2/3))/(818358
97185*x^2 - 1102302937)) + 1/2*(-x^3 + x)^(1/3)*x - 1/12*log(3*(-x^3 + x)^
(1/3)*x + 3*(-x^3 + x)^(2/3) + 1)
```

3.306.6 Sympy [F]

$$\int \sqrt[3]{x(1-x^2)}\,dx = \int \sqrt[3]{x(1-x^2)}\,dx$$

input
```
integrate((x*(-x**2+1))**(1/3),x)
```

output
```
Integral((x*(1 - x**2))**(1/3), x)
```

3.306.7 Maxima [F]

$$\int \sqrt[3]{x(1-x^2)}\,dx = \int \left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((x*(-x^2+1))^(1/3),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 1)*x)^(1/3), x)
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \sqrt[3]{x(1-x^2)}\,dx = \frac{1}{2}x^2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}} - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}-1\right)\right) + \frac{1}{12}\log\left(\left(\frac{1}{x^2}-1\right)^{\frac{2}{3}} - \left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}+1\right) - \frac{1}{6}\log\left(\left|\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}+1\right|\right)$$

input
```
integrate((x*(-x^2+1))^(1/3),x, algorithm="giac")
```

output
```
1/2*x^2*(1/x^2 - 1)^(1/3) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 - 1)^
(1/3) - 1)) + 1/12*log((1/x^2 - 1)^(2/3) - (1/x^2 - 1)^(1/3) + 1) - 1/6*lo
g(abs((1/x^2 - 1)^(1/3) + 1))
```

3.306.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.31

$$\int \sqrt[3]{x(1-x^2)}\,dx = \frac{3\,x\,(x-x^3)^{1/3}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ x^2\right)}{4\,(1-x^2)^{1/3}}$$

input
```
int((-x*(x^2 - 1))^(1/3),x)
```

output
```
(3*x*(x - x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, x^2))/(4*(1 - x^2)^(1/3))
```

3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$

3.307.1 Optimal result

Integrand size = 13, antiderivative size = 126

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx=\frac{7}{64}\sqrt{\left(1+\sqrt[3]{x}\right)x}-\frac{21\sqrt{\left(1+\sqrt[3]{x}\right)x}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{\left(1+\sqrt[3]{x}\right)x}+\frac{3}{40}x^{2/3}\sqrt{\left(1+\sqrt[3]{x}\right)x}+\frac{3}{5}x\sqrt{\left(1+\sqrt[3]{x}\right)x}+\frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{\left(1+\sqrt[3]{x}\right)x}}\right)$$

output
```
21/128*arctanh(x^(2/3)/((1+x^(1/3))*x)^(1/2))+7/64*((1+x^(1/3))*x)^(1/2)-2
1/128*((1+x^(1/3))*x)^(1/2)/x^(1/3)-7/80*x^(1/3)*((1+x^(1/3))*x)^(1/2)+3/4
0*x^(2/3)*((1+x^(1/3))*x)^(1/2)+3/5*x*((1+x^(1/3))*x)^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx=\frac{\sqrt{x+x^{4/3}}\left(-105+70\sqrt[3]{x}-56x^{2/3}+48x+384x^{4/3}\right)}{640\sqrt[3]{x}}+\frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\right)$$

input
```
Integrate[Sqrt[(1 + x^(1/3))*x],x]
```

output
```
(Sqrt[x + x^(4/3)]*(-105 + 70*x^(1/3) - 56*x^(2/3) + 48*x + 384*x^(4/3)))/
(640*x^(1/3)) + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

3.307.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2078, 1910, 1930, 1930, 1930, 1916, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\left(\sqrt[3]{x}+1\right)x}\,dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{x^{4/3}+x}dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{1}{10}\int \frac{x}{\sqrt{x^{4/3}+x}}dx+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\int \frac{x^{2/3}}{\sqrt{x^{4/3}+x}}dx\right)+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\int \frac{\sqrt[3]{x}}{\sqrt{x^{4/3}+x}}dx\right)\right)+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\int \frac{1}{\sqrt{x^{4/3}+x}}dx\right)\right)\right)+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1916}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\left(\frac{3\sqrt{x^{4/3}+x}}{\sqrt[3]{x}}-\frac{1}{2}\int\frac{1}{\sqrt[3]{x}\sqrt{x^{4/3}+x}}dx\right)\right)\right)\right)+$$
$$\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$\downarrow$ 1935

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\left(\frac{3\sqrt{x^{4/3}+x}}{\sqrt[3]{x}}-3\int\frac{1}{1-\frac{x^{4/3}}{x^{4/3}+x}}d\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)\right.\right.\right.$$
$$\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$\downarrow$ 219

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\left(\frac{3\sqrt{x^{4/3}+x}}{\sqrt[3]{x}}-3\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)\right)\right)\right)\right)$$
$$\frac{3}{5}\sqrt{x^{4/3}+x}x$$

input
```
Int[Sqrt[(1 + x^(1/3))*x],x]
```

output
```
(3*x*Sqrt[x + x^(4/3)])/5 + ((3*x^(2/3)*Sqrt[x + x^(4/3)])/4 - (7*(x^(1/3)
*Sqrt[x + x^(4/3)] - (5*((3*Sqrt[x + x^(4/3)])/2 - (3*((3*Sqrt[x + x^(4/3)
])/x^(1/3) - 3*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]]))/4))/6))/8)/10
```

3.307.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

3.307. $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.307.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

method	result
meijerg	$-\frac{3\left(\frac{\sqrt{\pi}\, x^{\frac{1}{6}}\left(-1152 x^{\frac{4}{3}}-144 x+168 x^{\frac{2}{3}}-210 x^{\frac{1}{3}}+315\right)\sqrt{x^{\frac{1}{3}}+1}}{2880}-\frac{7\sqrt{\pi}\, \operatorname{arcsinh}\left(x^{\frac{1}{6}}\right)}{64}\right)}{2\sqrt{\pi}}$
derivativedivides	$\frac{\sqrt{\left(x^{\frac{1}{3}}+1\right)x}\left(768 x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672 x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\, x^{\frac{1}{3}}-210\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}+105\ln(\ldots}{1280 x^{\frac{1}{3}}\sqrt{\left(x^{\frac{1}{3}}+1\right)x^{\frac{1}{3}}}}$
default	$\frac{\sqrt{\left(x^{\frac{1}{3}}+1\right)x}\left(768 x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672 x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\, x^{\frac{1}{3}}-210\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}+105\ln(\ldots}{1280 x^{\frac{1}{3}}\sqrt{\left(x^{\frac{1}{3}}+1\right)x^{\frac{1}{3}}}}$

input
```
int(((x^(1/3)+1)*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2/Pi^(1/2)*(1/2880*Pi^(1/2)*x^(1/6)*(-1152*x^(4/3)-144*x+168*x^(2/3)-21
0*x^(1/3)+315)*(x^(1/3)+1)^(1/2)-7/64*Pi^(1/2)*arcsinh(x^(1/6)))
```

3.307.5 Fricas [A] (verification not implemented)

Time = 42.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$$
$$= \frac{35\,x\log\left(\frac{32\,x^2+48\,x^{\frac{5}{3}}+2\left(16\,x^{\frac{4}{3}}+16\,x+3\,x^{\frac{2}{3}}\right)\sqrt{x^{\frac{4}{3}}+x}+18\,x^{\frac{4}{3}}+x}{x}\right)+2\left(384\,x^2+3\,(16\,x-35)x^{\frac{2}{3}}-56\,x^{\frac{4}{3}}+70\,x\right)\sqrt{x}}{1280\,x}$$

input
```
integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="fricas")
```

output
```
1/1280*(35*x*log((32*x^2 + 48*x^(5/3) + 2*(16*x^(4/3) + 16*x + 3*x^(2/3))*
sqrt(x^(4/3) + x) + 18*x^(4/3) + x)/x) + 2*(384*x^2 + 3*(16*x - 35)*x^(2/3
) - 56*x^(4/3) + 70*x)*sqrt(x^(4/3) + x))/x
```

3.307.6 Sympy [F]

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = \int \sqrt{x\left(\sqrt[3]{x}+1\right)}\,dx$$

input
```
integrate(((1+x**(1/3))*x)**(1/2),x)
```

output
```
Integral(sqrt(x*(x**(1/3) + 1)), x)
```

3.307.7 Maxima [F]

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx = \int \sqrt{x\left(x^{\frac{1}{3}}+1\right)}\,dx$$

input
```
integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x*(x^(1/3) + 1)), x)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx$$
$$= \frac{1}{1280}\left(2\left(2\left(4\left(6\,x^{\frac{1}{3}}\left(8\,x^{\frac{1}{3}}+1\right)-7\right)x^{\frac{1}{3}}+35\right)x^{\frac{1}{3}}-105\right)\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-105\,\log\left(\left|2\,\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-2\,x^{\frac{1}{3}}-1\right.\right.\right.$$

input
```
integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="giac")
```

output
```
1/1280*(2*(2*(4*(6*x^(1/3)*(8*x^(1/3) + 1) - 7)*x^(1/3) + 35)*x^(1/3) - 10
5)*sqrt(x^(2/3) + x^(1/3)) - 105*log(abs(2*sqrt(x^(2/3) + x^(1/3)) - 2*x^(
1/3) - 1)))*sgn(x)
```

3.307.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \sqrt{(1+\sqrt[3]{x})\,x}\,dx = \frac{2\,x\,\sqrt{x+x^{4/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{9}{2};\ \frac{11}{2};\ -x^{1/3}\right)}{3\,\sqrt{x^{1/3}+1}}$$

input
```
int((x*(x^(1/3) + 1))^(1/2),x)
```

output
```
(2*x*(x + x^(4/3))^(1/2)*hypergeom([-1/2, 9/2], 11/2, -x^(1/3)))/(3*(x^(1/
3) + 1)^(1/2))
```

3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$

3.308.1 Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}$$

output
```
-1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1}{3}\left(\sqrt{6}-\sqrt{6}x^4+\sqrt{3+6x^8}\right)\right)}{2\sqrt{3}}$$

input
```
Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[6] - Sqrt[6]*x^4 + Sqrt[3 + 6*x^8])/3]/Sqrt[3]
```

3.308.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1799, 25, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^4-1)\sqrt{2x^8+1}}dx$$

$$\Big\downarrow \text{ 1799}$$

$$\frac{1}{4}\int -\frac{1}{(1-x^4)\sqrt{2x^8+1}}dx^4$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{4}\int \frac{1}{(1-x^4)\sqrt{2x^8+1}}dx^4$$

$$\Big\downarrow \text{ 488}$$

$$\frac{1}{4}\int \frac{1}{3-x^8}d\frac{-2x^4-1}{\sqrt{2x^8+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{-2x^4-1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

input
```
Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

output
```
ArcTanh[(-1 - 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

3.308.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

3.308.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3\sqrt{2x^8+1}}\right)\sqrt{3}}{12}$	28
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2-3\right)x^4+3\sqrt{2x^8+1}+\operatorname{RootOf}\left(\_Z^2-3\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{12}$	58

input
```
int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{2\,x^4-\sqrt{3}(2\,x^4+1)-\sqrt{2\,x^8+1}\left(\sqrt{3}-3\right)+1}{x^4-1}\right)$$

input
```
integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((2*x^4 - sqrt(3)*(2*x^4 + 1) - sqrt(2*x^8 + 1)*(sqrt(3) -
 3) + 1)/(x^4 - 1))
```

3.308.6 Sympy [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{x^3}{(x-1)(x+1)(x^2+1)\sqrt{2x^8+1}}\,dx$$

input
```
integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)
```

output
```
Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)
```

3.308.7 Maxima [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{x^3}{\sqrt{2\,x^8+1}(x^4-1)}\,dx$$

input
```
integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)), x)
```

3.308.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{\left|-2\sqrt{2}x^4-2\sqrt{3}+2\sqrt{2}+2\sqrt{2\,x^8+1}\right|}{2\left(\sqrt{2}x^4-\sqrt{3}-\sqrt{2}-\sqrt{2\,x^8+1}\right)}\right)$$

input
```
integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) + 2*sqrt(
2*x^8 + 1))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8 + 1)))
```

3.308.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\sqrt{3}\left(\ln\left(x^4+\frac{\sqrt{2}\sqrt{3}\sqrt{x^8+\frac{1}{2}}}{2}+\frac{1}{2}\right)-\ln\left(x^4-1\right)\right)}{12}$$

input
```
int(x^3/((x^4 - 1)*(2*x^8 + 1)^(1/2)),x)
```

output
```
-(3^(1/2)*(log(x^4 + (2^(1/2)*3^(1/2)*(x^8 + 1/2)^(1/2))/2 + 1/2) - log(x^
4 - 1)))/12
```

3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$

3.309.1 Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = -\frac{1}{40}(1+2x^5)\sqrt{1+x^5+x^{10}} + \frac{1}{15}(1+x^5+x^{10})^{3/2} - \frac{3}{80}\operatorname{arcsinh}\left(\frac{1+2x^5}{\sqrt{3}}\right)$$

output
```
1/15*(x^10+x^5+1)^(3/2)-3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))-1/40*(2*x^5+1)
*(x^10+x^5+1)^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{1}{120}\sqrt{1+x^5+x^{10}}(5+2x^5+8x^{10}) + \frac{3}{80}\log\left(-1-2x^5+2\sqrt{1+x^5+x^{10}}\right)$$

input
```
Integrate[x^9*Sqrt[1 + x^5 + x^10],x]
```

output
```
(Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10))/120 + (3*Log[-1 - 2*x^5 + 2*Sq
rt[1 + x^5 + x^10]])/80
```

3.309.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1693, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \sqrt{x^{10}+x^5+1}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int x^5\sqrt{x^{10}+x^5+1}dx^5$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{2}\int \sqrt{x^{10}+x^5+1}dx^5\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(-\frac{3}{8}\int \frac{1}{\sqrt{x^{10}+x^5+1}}dx^5-\frac{1}{4}\sqrt{x^{10}+x^5+1}\left(2x^5+1\right)\right)+\frac{1}{3}\left(x^{10}+x^5+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(-\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{x^{10}}{3}+1}}d\left(2x^5+1\right)-\frac{1}{4}\sqrt{x^{10}+x^5+1}\left(2x^5+1\right)\right)+\frac{1}{3}\left(x^{10}+x^5+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(-\frac{3}{8}\text{arcsinh}\left(\frac{2x^5+1}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{x^{10}+x^5+1}\left(2x^5+1\right)\right)+\frac{1}{3}\left(x^{10}+x^5+1\right)^{3/2}\right)$$

input
```
Int[x^9*Sqrt[1 + x^5 + x^10],x]
```

output
```
((1 + x^5 + x^10)^(3/2)/3 + (-1/4*((1 + 2*x^5)*Sqrt[1 + x^5 + x^10]) - (3*
ArcSinh[(1 + 2*x^5)/Sqrt[3]])/8)/2)/5
```

3.309.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.309.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{3\,\operatorname{arcsinh}\left(\frac{\left(2x^{5}+1\right)\sqrt{3}}{3}\right)}{80}+\frac{\left(8x^{10}+2x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}}{120}$	41
trager	$\left(\frac{1}{15}x^{10}+\frac{1}{60}x^{5}+\frac{1}{24}\right)\sqrt{x^{10}+x^{5}+1}+\frac{3\ln\left(-2x^{5}+2\sqrt{x^{10}+x^{5}+1}-1\right)}{80}$	47
risch	$\frac{\left(8x^{10}+2x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}}{120}-\frac{3\ln\left(2x^{5}+2\sqrt{x^{10}+x^{5}+1}+1\right)}{80}$	48

input
```
int(x^9*(x^10+x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))+1/120*(8*x^10+2*x^5+5)*(x^10+x^5+1)^(
1/2)
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{1}{120}\left(8\,x^{10}+2\,x^5+5\right)\sqrt{x^{10}+x^5+1} + \frac{3}{80}\log\left(-2\,x^5+2\,\sqrt{x^{10}+x^5+1}-1\right)$$

input
```
integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(8*x^10 + 2*x^5 + 5)*sqrt(x^10 + x^5 + 1) + 3/80*log(-2*x^5 + 2*sqrt
(x^10 + x^5 + 1) - 1)
```

3.309.6 Sympy [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \int x^9\sqrt{(x^2+x+1)\,(x^8-x^7+x^5-x^4+x^3-x+1)}\,dx$$

input
```
integrate(x**9*(x**10+x**5+1)**(1/2),x)
```

output
```
Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x**3 - x +
1)), x)
```

3.309.7 Maxima [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\int \sqrt{x^{10}+x^5+1}x^9\,dx$$

input
```
integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^10 + x^5 + 1)*x^9, x)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\frac{1}{120}\sqrt{x^{10}+x^5+1}\left(2\left(4x^5+1\right)x^5+5\right)+\frac{3}{80}\log\left(-2x^5+2\sqrt{x^{10}+x^5+1}-1\right)$$

input
```
integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*log(-2*x^5 + 2*s
qrt(x^10 + x^5 + 1) - 1)
```

3.309.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\frac{\sqrt{x^{10}+x^5+1}\left(8x^{10}+2x^5+5\right)}{120}-\frac{3\ln\left(\sqrt{x^{10}+x^5+1}+x^5+\frac{1}{2}\right)}{80}$$

input
```
int(x^9*(x^5 + x^10 + 1)^(1/2),x)
```

output
```
((x^5 + x^10 + 1)^(1/2)*(2*x^5 + 8*x^10 + 5))/120 - (3*log((x^5 + x^10 + 1
)^(1/2) + x^5 + 1/2))/80
```

3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$

3.310.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = -\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{128}\text{arctanh}\left(\frac{4+x^2}{2\sqrt{4+2x^2+x^4}}\right)$$

output
```
1/128*arctanh(1/2*(x^2+4)/(x^4+2*x^2+4)^(1/2))-1/16*(x^4+2*x^2+4)^(1/2)/x^
4+3/64*(x^4+2*x^2+4)^(1/2)/x^2
```

3.310.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{1}{64}\left(\frac{(-4+3x^2)\sqrt{4+2x^2+x^4}}{x^4}-\text{arctanh}\left(\frac{1}{2}\left(x^2-\sqrt{4+2x^2+x^4}\right)\right)\right)$$

input
```
Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

output
```
(((-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/x^4 - ArcTanh[(x^2 - Sqrt[4 + 2*x^2
+ x^4])/2])/64
```

3.310.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1167, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^4+2x^2+4}}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{x^4+2x^2+4}}dx^2$$

$$\Big\downarrow \text{ 1167}$$

$$\frac{1}{2}\left(-\frac{1}{8}\int \frac{x^2+3}{x^4\sqrt{x^4+2x^2+4}}dx^2-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{3\sqrt{x^4+2x^2+4}}{4x^2}-\frac{1}{4}\int \frac{1}{x^2\sqrt{x^4+2x^2+4}}dx^2\right)-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{1}{16-x^4}d\frac{2(x^2+4)}{\sqrt{x^4+2x^2+4}}+\frac{3\sqrt{x^4+2x^2+4}}{4x^2}\right)-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{8}\text{arctanh}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)+\frac{3\sqrt{x^4+2x^2+4}}{4x^2}\right)-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

output
```
(-1/8*Sqrt[4 + 2*x^2 + x^4]/x^4 + ((3*Sqrt[4 + 2*x^2 + x^4])/(4*x^2) + Arc
Tanh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/8)/8)/2
```

3.310.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.310.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
trager	$\frac{(3x^2-4)\sqrt{x^4+2x^2+4}}{64x^4}+\frac{\ln\left(\frac{x^2+2\sqrt{x^4+2x^2+4}+4}{x^2}\right)}{128}$	52
default	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
risch	$\frac{3x^6+2x^4+4x^2-16}{64x^4\sqrt{x^4+2x^2+4}}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
elliptic	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)x^4+6x^2\sqrt{x^4+2x^2+4}-8\sqrt{x^4+2x^2+4}}{128x^4}$	62

input
```
int(1/x^5/(x^4+2*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(3*x^2-4)/x^4*(x^4+2*x^2+4)^(1/2)+1/128*ln((x^2+2*(x^4+2*x^2+4)^(1/2)
+4)/x^2)
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$$
$$=\frac{x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}+2\right)-x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}-2\right)+6\,x^4+2\,\sqrt{x^4+2\,x^2+4}(3\,x^2-}{128\,x^4}$$

input
```
integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(x^4*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2) - x^4*log(-x^2 + sqrt(x^4
 + 2*x^2 + 4) - 2) + 6*x^4 + 2*sqrt(x^4 + 2*x^2 + 4)*(3*x^2 - 4))/x^4
```

3.310.6 Sympy [F]

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{x^5\sqrt{x^4+2x^2+4}}\,dx$$

input
```
integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{3\sqrt{x^4+2x^2+4}}{64x^2} - \frac{\sqrt{x^4+2x^2+4}}{16x^4} + \frac{1}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3} + \frac{4\sqrt{3}}{3x^2}\right)$$

input
```
integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
3/64*sqrt(x^4 + 2*x^2 + 4)/x^2 - 1/16*sqrt(x^4 + 2*x^2 + 4)/x^4 + 1/128*ar
csinh(1/3*sqrt(3) + 4/3*sqrt(3)/x^2)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{\left(x^2-\sqrt{x^4+2x^2+4}\right)^3 + 36x^2 - 36\sqrt{x^4+2x^2+4} + 64}{32\left(\left(x^2-\sqrt{x^4+2x^2+4}\right)^2-4\right)^2} - \frac{1}{128}\log\left(x^2-\sqrt{x^4+2x^2+4}+2\right) + \frac{1}{128}\log\left(-x^2+\sqrt{x^4+2x^2+4}+2\right)$$

input
```
integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
1/32*((x^2 - sqrt(x^4 + 2*x^2 + 4))^3 + 36*x^2 - 36*sqrt(x^4 + 2*x^2 + 4)
+ 64)/((x^2 - sqrt(x^4 + 2*x^2 + 4))^2 - 4)^2 - 1/128*log(x^2 - sqrt(x^4 +
 2*x^2 + 4) + 2) + 1/128*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{x^5\,\sqrt{x^4+2\,x^2+4}}\,dx$$

input
```
int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)),x)
```

output
```
int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)), x)
```

3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$

3.311.1 Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \operatorname{arctanh}\left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}}\right)$$

output
```
arctanh((x^2+1)/(x^4+3*x^2+1)^(1/2))
```

3.311.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\operatorname{arctanh}\left(x^2-\sqrt{1+3x^2+x^4}\right) - \frac{1}{2}\log\left(-3-2x^2+2\sqrt{1+3x^2+x^4}\right)$$

input
```
Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

output
```
-ArcTanh[x^2 - Sqrt[1 + 3*x^2 + x^4]] - Log[-3 - 2*x^2 + 2*Sqrt[1 + 3*x^2
+ x^4]]/2
```

3.311.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1578, 25, 1239, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x\sqrt{x^4+3x^2+1}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int -\frac{1-x^2}{x^2\sqrt{x^4+3x^2+1}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^2\sqrt{x^4+3x^2+1}}dx^2$$

$$\Big\downarrow \text{1239}$$

$$2\int \frac{1}{4-x^4}d\frac{2(x^2+1)}{\sqrt{x^4+3x^2+1}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

input
```
Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

output
```
ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

3.311.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1239
```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Simp[4*f*((a - d)/(b*d - a*e))   Subst[Int[1/(4*(
a - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
 FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[
e*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.311.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
trager	$\ln\left(\frac{x^{2}+\sqrt{x^{4}+3x^{2}+1}+1}{x}\right)$	23
default	$\frac{\ln\left(x^{2}+\frac{3}{2}+\sqrt{x^{4}+3x^{2}+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^{2}+2}{2\sqrt{x^{4}+3x^{2}+1}}\right)}{2}$	46
elliptic	$\frac{\ln\left(x^{2}+\frac{3}{2}+\sqrt{x^{4}+3x^{2}+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^{2}+2}{2\sqrt{x^{4}+3x^{2}+1}}\right)}{2}$	46
pseudoelliptic	$\frac{\ln\left(2x^{2}+3+2\sqrt{x^{4}+3x^{2}+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^{2}+2}{2\sqrt{x^{4}+3x^{2}+1}}\right)}{2}$	50

input
```
int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((x^2+(x^4+3*x^2+1)^(1/2)+1)/x)
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\frac{1}{2}\log\left(4\,x^4+11\,x^2-\sqrt{x^4+3\,x^2+1}(4\,x^2+5)+5\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}+1\right)$$

input
```
integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(4*x^4 + 11*x^2 - sqrt(x^4 + 3*x^2 + 1)*(4*x^2 + 5) + 5) + 1/2*log
(-x^2 + sqrt(x^4 + 3*x^2 + 1) + 1)
```

3.311.6 Sympy [F]

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \int \frac{(x-1)\,(x+1)}{x\sqrt{x^4+3x^2+1}}\,dx$$

input
```
integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)
```

3.311.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \frac{1}{2}\log\left(2\,x^2+2\,\sqrt{x^4+3\,x^2+1}+3\right) + \frac{1}{2}\log\left(\frac{2\,\sqrt{x^4+3\,x^2+1}}{x^2}+\frac{2}{x^2}+3\right)$$

input
```
integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(2*x^2 + 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(2*sqrt(x^4 + 3*x^2
+ 1)/x^2 + 2/x^2 + 3)
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(19) = 38$.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\frac{1}{2}\log\left(2\,x^2 - 2\,\sqrt{x^4+3\,x^2+1}+3\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}-1\right)$$

input
```
integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(2*x^2 - 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(-x^2 + sqrt(x^4 +
3*x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 3*x^2 + 1) - 1)
```

3.311.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \frac{\ln\left(\frac{1}{x^2}\right)}{2} + \frac{\ln\left(\sqrt{x^4+3\,x^2+1}+x^2+\frac{3}{2}\right)}{2} + \frac{\ln\left(\frac{2\,\sqrt{x^4+3\,x^2+1}}{3}+x^2+\frac{2}{3}\right)}{2}$$

input
```
int((x^2 - 1)/(x*(3*x^2 + x^4 + 1)^(1/2)),x)
```

output
```
log(1/x^2)/2 + log((3*x^2 + x^4 + 1)^(1/2) + x^2 + 3/2)/2 + log((2*(3*x^2
+ x^4 + 1)^(1/2))/3 + x^2 + 2/3)/2
```

3.312 $\int (-3x+2x^3)(-3x^2+x^4)^{3/5}\,dx$

3.312.1 Optimal result

Integrand size = 23, antiderivative size = 17

$$\int (-3x+2x^3)(-3x^2+x^4)^{3/5}\,dx = \frac{5}{16}(-3x^2+x^4)^{8/5}$$

output
```
5/16*(x^4-3*x^2)^(8/5)
```

3.312.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 10.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int (-3x+2x^3)(-3x^2+x^4)^{3/5}\,dx = \frac{5(x^2(-3+x^2))^{3/5}\left(-39x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{5},\frac{8}{5},\frac{13}{5},\frac{x^2}{3}\right)+16x^4\,\text{Hypergeometric2F1}\left(-\right.\right.}{208\left(1-\frac{x^2}{3}\right)^{3/5}}$$

input
```
Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

output
```
(5*(x^2*(-3 + x^2))^(3/5)*(-39*x^2*Hypergeometric2F1[-3/5, 8/5, 13/5, x^2/
3] + 16*x^4*Hypergeometric2F1[-3/5, 13/5, 18/5, x^2/3]))/(208*(1 - x^2/3)^
(3/5))
```

3.312.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^3-3x\right)\left(x^4-3x^2\right)^{3/5}\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

input
```
Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

output
```
(5*(-3*x^2 + x^4)^(8/5))/16
```

3.312.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.312.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{5(x^4-3x^2)^{\frac{8}{5}}}{16}$	14
gosper	$\frac{5(x^4-3x^2)^{\frac{3}{5}}x^2(x^2-3)}{16}$	22
trager	$\frac{5(x^4-3x^2)^{\frac{3}{5}}x^2(x^2-3)}{16}$	22
risch	$\frac{5x^2(x^2(x^2-3))^{\frac{3}{5}}(x^2-3)}{16}$	22
pseudoelliptic	$\frac{5x^2(x^2(x^2-3))^{\frac{3}{5}}(x^2-3)}{16}$	22
meijerg	$\frac{5\,3^{\frac{3}{5}}\operatorname{signum}\left(-1+\frac{x^2}{3}\right)^{\frac{3}{5}}x^{\frac{26}{5}}\,{}_2F_1\left(-\frac{3}{5},\frac{13}{5};\frac{18}{5};\frac{x^2}{3}\right)}{13\left(-\operatorname{signum}\left(-1+\frac{x^2}{3}\right)\right)^{\frac{3}{5}}}-\frac{15\,3^{\frac{3}{5}}\operatorname{signum}\left(-1+\frac{x^2}{3}\right)^{\frac{3}{5}}x^{\frac{16}{5}}\,{}_2F_1\left(-\frac{3}{5},\frac{8}{5};\frac{13}{5};\frac{x^2}{3}\right)}{16\left(-\operatorname{signum}\left(-1+\frac{x^2}{3}\right)\right)^{\frac{3}{5}}}$	84

input
```
int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x,method=_RETURNVERBOSE)
```

output
```
5/16*(x^4-3*x^2)^(8/5)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

input
```
integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="fricas")
```

output
```
5/16*(x^4 - 3*x^2)^(8/5)
```

3.312.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int\left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5x^4(x^4-3x^2)^{\frac{3}{5}}}{16}-\frac{15x^2(x^4-3x^2)^{\frac{3}{5}}}{16}$$

input
```
integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
```

output
```
5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)/16
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int\left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

input
```
integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="maxima")
```

output
```
5/16*(x^4 - 3*x^2)^(8/5)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int\left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

input
```
integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="giac")
```

output
```
5/16*(x^4 - 3*x^2)^(8/5)
```

3.312.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5\,x^2\,(x^2-3)\,\left(x^4-3\,x^2\right)^{3/5}}{16}$$

input
```
int(-(3*x - 2*x^3)*(x^4 - 3*x^2)^(3/5),x)
```

output
```
(5*x^2*(x^2 - 3)*(x^4 - 3*x^2)^(3/5))/16
```

3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx$

3.313.1 Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\frac{4}{27}\sqrt[4]{-1+3x^3} - \frac{4}{33}\left(-1+3x^3\right)^{11/12} + \frac{4}{243}\left(-1+3x^3\right)^{9/4}$$

output
```
-4/27*(3*x^3-1)^(1/4)-4/33*(3*x^3-1)^(11/12)+4/243*(3*x^3-1)^(9/4)
```

3.313.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\frac{4\sqrt[4]{-1+3x^3}\left(88+66x^3-99x^6+81(-1+3x^3)^{2/3}\right)}{2673}$$

input
```
Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

output
```
(-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2/3)))/267
3
```

3.313.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^8-2x^5-\left(3x^3-1\right)^{2/3}x^2}{\left(3x^3-1\right)^{3/4}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{3x^8}{\left(3x^3-1\right)^{3/4}}-\frac{2x^5}{\left(3x^3-1\right)^{3/4}}-\frac{x^2}{\sqrt[12]{3x^3-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

input
```
Int[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

output
```
(-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)
^(9/4))/243
```

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.313.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

method	result
meijerg	$-\frac{(-\operatorname{signum}(3x^3-1))^{\frac{3}{4}} x^6 {}_2F_1(\frac{3}{4},2;3;3x^3)}{3\operatorname{signum}(3x^3-1)^{\frac{3}{4}}}+\frac{(-\operatorname{signum}(3x^3-1))^{\frac{3}{4}} x^9 {}_2F_1(\frac{3}{4},3;4;3x^3)}{3\operatorname{signum}(3x^3-1)^{\frac{3}{4}}}-\frac{(-\operatorname{signum}(3x^3-1))^{\frac{1}{12}} x^3 {}_2F_1(\frac{1}{12},1;2;3x^3)}{3\operatorname{signum}(3x^3-1)^{\frac{1}{12}}}$

input
```
int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x,method=_RETURNVER
BOSE)
```

output
```
-1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^6*hypergeom([3/4,2],
[3],3*x^3)+1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^9*hypergeo
m([3/4,3],[4],3*x^3)-1/3/signum(3*x^3-1)^(1/12)*(-signum(3*x^3-1))^(1/12)*
x^3*hypergeom([1/12,1],[2],3*x^3)
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(9\,x^6-6\,x^3-8\right)\left(3\,x^3-1\right)^{\frac{1}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}$$

input
```
integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm=
"fricas")
```

output
```
4/243*(9*x^6 - 6*x^3 - 8)*(3*x^3 - 1)^(1/4) - 4/33*(3*x^3 - 1)^(11/12)
```

3.313.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.67 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.80

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\frac{4(3x^3-1)^{\frac{11}{12}}}{33} - 2\left(\begin{cases} \frac{4x^3\sqrt[4]{3x^3-1}}{45}+\frac{16\sqrt[4]{3x^3-1}}{135} & \text{for } |x^3|>\frac{1}{3} \\ -\frac{4x^3\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{45}-\frac{16\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{135} & \text{otherwise} \end{cases}\right) + 3\left(\begin{cases} \frac{4x^6\sqrt[4]{3x^3-1}}{81}+\frac{32x^3\sqrt[4]{3x^3-1}}{1215}+\frac{128\sqrt[4]{3x^3-1}}{3645} & \text{for } |x^3|>\frac{1}{3} \\ \frac{4x^6\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{81}+\frac{32x^3\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{1215}+\frac{128\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{3645} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

output
```
-4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(1/4)/45 +
 16*(3*x**3 - 1)**(1/4)/135, Abs(x**3) > 1/3), (-4*x**3*(1 - 3*x**3)**(1/4
)*exp(-3*I*pi/4)/45 - 16*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/135, True)) +
3*Piecewise((4*x**6*(3*x**3 - 1)**(1/4)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1
215 + 128*(3*x**3 - 1)**(1/4)/3645, Abs(x**3) > 1/3), (4*x**6*(1 - 3*x**3)
**(1/4)*exp(I*pi/4)/81 + 32*x**3*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/1215 + 12
8*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/3645, True))
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(3x^3-1\right)^{\frac{9}{4}} - \frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}} - \frac{4}{27}\left(3x^3-1\right)^{\frac{1}{4}}$$

input
```
integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm=
"maxima")
```

output
```
4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4
)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

input
```
integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm=
"giac")
```

output
```
4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4
)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\left(3\,x^3-1\right)^{1/4}\left(\frac{8\,x^3}{81}-\frac{4\,x^6}{27}+\frac{4\left(3\,x^3-1\right)^{2/3}}{33}+\frac{32}{243}\right)$$

input
```
int(-(x^2*(3*x^3 - 1)^(2/3) + 2*x^5 - 3*x^8)/(3*x^3 - 1)^(3/4),x)
```

output
```
-(3*x^3 - 1)^(1/4)*((8*x^3)/81 - (4*x^6)/27 + (4*(3*x^3 - 1)^(2/3))/33 + 3
2/243)
```

3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$

3.314.1 Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log(-1+x^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}$$

output
```
-1/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/6)-1/18*ln(x^3
-1)*3^(2/3)+1/6*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

3.314.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \frac{-6\arctan\left(\frac{3^{5/6}x}{\sqrt[3]{3}x+2\sqrt[3]{2+x^3}}\right)+\sqrt{3}\left(2\log\left(-3x+3^{2/3}\sqrt[3]{2+x^3}\right)-\log\left(3x^2+3^{2/3}x\sqrt[3]{2+x^3}+\sqrt[3]{3}(2+x\right.\right.}{6\ 3^{5/6}}$$

input
```
Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

output
```
(-6*ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*(2 + x^3)^(1/3))] + Sqrt[3]*(2*Log[-
3*x + 3^(2/3)*(2 + x^3)^(1/3)] - Log[3*x^2 + 3^(2/3)*x*(2 + x^3)^(1/3) + 3
^(1/3)*(2 + x^3)^(2/3)]))/(6*3^(5/6))
```

3.314.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-1)\sqrt[3]{x^3+2}}dx$$
$$\Big\downarrow \text{ 901}$$
$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(x^3-1\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}$$

input
```
Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

output
```
-(ArcTan[(1 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]]/3^(5/6)) - Log[-1 +
x^3]/(6*3^(1/3)) + Log[3^(1/3)*x - (2 + x^3)^(1/3)]/(2*3^(1/3))
```

3.314.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.314.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

method	result	size
pseudoelliptic	$\frac{3^{\frac{2}{3}} \ln\left(\frac{-3^{\frac{1}{3}} x+\left(x^{3}+2\right)^{\frac{1}{3}}}{x}\right)}{9}-\frac{3^{\frac{2}{3}} \ln\left(\frac{3^{\frac{2}{3}} x^{2}+3^{\frac{1}{3}} \left(x^{3}+2\right)^{\frac{1}{3}} x+\left(x^{3}+2\right)^{\frac{2}{3}}}{x^{2}}\right)}{18}+\frac{3^{\frac{1}{6}} \arctan\left(\frac{\sqrt{3}\left(2\,3^{\frac{2}{3}} \left(x^{3}+2\right)^{\frac{1}{3}}+3 x\right)}{9 x}\right)}{3}$	93
trager	Expression too large to display	818

input
```
int(1/(x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(2/3)*ln((-3^(1/3)*x+(x^3+2)^(1/3))/x)-1/18*3^(2/3)*ln((3^(2/3)*x^2+
3^(1/3)*(x^3+2)^(1/3)*x+(x^3+2)^(2/3))/x^2)+1/3*3^(1/6)*arctan(1/9*3^(1/2)
*(2*3^(2/3)*(x^3+2)^(1/3)+3*x)/x)
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 1.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{1}{27}\cdot 3^{\frac{2}{3}}\log\left(\frac{9\cdot 3^{\frac{1}{3}}(x^3+2)^{\frac{1}{3}}x^2-2\cdot 3^{\frac{2}{3}}(x^3-1)-9\,(x^3+2)^{\frac{2}{3}}x}{x^3-1}\right)-\frac{1}{54}$$
$$\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}(7\,x^4+2\,x)(x^3+2)^{\frac{2}{3}}+3^{\frac{1}{3}}(31\,x^6+46\,x^3+4)+9\,(5\,x^5+4\,x^2)(x^3+2)^{\frac{1}{3}}}{x^6-2\,x^3+1}\right)$$
$$-\frac{1}{9}$$
$$\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7\,x^7-5\,x^4-2\,x)(x^3+2)^{\frac{2}{3}}-3^{\frac{1}{3}}(127\,x^9+402\,x^6+192\,x^3+8)-18\,(31\,x^8+46\right.}{3\,(251\,x^9+462\,x^6+24\,x^3-8)}\right.$$

input
```
integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
1/27*3^(2/3)*log((9*3^(1/3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(2/3)*(x^3 - 1) - 9*
(x^3 + 2)^(2/3)*x)/(x^3 - 1)) - 1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 2*x)*
(x^3 + 2)^(2/3) + 3^(1/3)*(31*x^6 + 46*x^3 + 4) + 9*(5*x^5 + 4*x^2)*(x^3 +
 2)^(1/3))/(x^6 - 2*x^3 + 1)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)
*(7*x^7 - 5*x^4 - 2*x)*(x^3 + 2)^(2/3) - 3^(1/3)*(127*x^9 + 402*x^6 + 192*
x^3 + 8) - 18*(31*x^8 + 46*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(251*x^9 + 462*x^
6 + 24*x^3 - 8))
```

3.314.6 Sympy [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx=\int \frac{1}{(x-1)\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

input
```
integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
```

output
```
Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

3.314.7 Maxima [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx=\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

input
```
integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

3.314.8 Giac [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx=\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

input
```
integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3-1)\,(x^3+2)^{1/3}}\,dx$$

input
```
int(1/((x^3 - 1)*(x^3 + 2)^(1/3)),x)
```

output
```
int(1/((x^3 - 1)*(x^3 + 2)^(1/3)), x)
```

3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

3.315.1 Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} -\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}+\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1+x*2^(1/2)/(x^4+2)^(1/4))*2^(1/2)+1/4*arctan(1+x*2^(1/2)/(x^4
+2)^(1/4))*2^(1/2)-1/8*ln(1-x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(
1/2)+1/8*ln(1+x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(1/2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{-x^2+\sqrt{2+x^4}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{x^2+\sqrt{2+x^4}}\right)}{2\sqrt{2}}$$

input
```
Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2]*x*(2 + x^4)^(1/4))/(-x^2 + Sqrt[2 + x^4])] + ArcTanh[(Sqr
t[2]*x*(2 + x^4)^(1/4))/(x^2 + Sqrt[2 + x^4])])/(2*Sqrt[2])
```

3.315.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {902, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+1)\sqrt[4]{x^4+2}}dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\int \frac{\frac{x^2}{\sqrt{x^4+2}}+1}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\int \frac{1}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{x^2}{\sqrt{x^4+2}}-1}d\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{x^4+2}}-1}d\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\downarrow$ 1479

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2x}{\sqrt[4]{x^4+2}}}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2x}{\sqrt[4]{x^4+2}}}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2x}{\sqrt[4]{x^4+2}}}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

3.315. $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*
x)/(2 + x^4)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2
]*x)/(2 + x^4)^(1/4)]/Sqrt[2] + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2
 + x^4)^(1/4)]/(2*Sqrt[2]))/2
```

3.315.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.315. $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.315.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{-\left(x^4+2\right)^{\frac{1}{4}}\sqrt{2}\,x+x^2+\sqrt{x^4+2}}{\left(x^4+2\right)^{\frac{1}{4}}\sqrt{2}\,x+x^2+\sqrt{x^4+2}}\right)+2\arctan\left(\frac{\left(x^4+2\right)^{\frac{1}{4}}\sqrt{2}+x}{x}\right)+2\arctan\left(\frac{\left(x^4+2\right)^{\frac{1}{4}}\sqrt{2}-x}{x}\right)\right)}{8}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\left(x^4+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3-\sqrt{x^4+2}\,\mathrm{RootOf}\left(\_Z^4+1\right)x^2+\left(x^4+2\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4+1\right)^3}{x^4+1}\right)}{4}+$

input
```
int(1/(x^4+1)/(x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

3.315. $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

output
```
-1/8*2^(1/2)*(ln((-(x^4+2)^(1/4)*2^(1/2)*x+x^2+(x^4+2)^(1/2))/((x^4+2)^(1/
4)*2^(1/2)*x+x^2+(x^4+2)^(1/2)))+2*arctan(((x^4+2)^(1/4)*2^(1/2)+x)/x)+2*a
rctan(((x^4+2)^(1/4)*2^(1/2)-x)/x))
```

3.315.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.60

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$$
$$= -\left(\frac{1}{16}i + \frac{1}{16}\right)\sqrt{2}\log\left(\frac{(i+1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 - 2i\sqrt{x^4+2}x^2 + (i-1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right)$$
$$+ \left(\frac{1}{16}i - \frac{1}{16}\right)\sqrt{2}\log\left(\frac{-(i-1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 + 2i\sqrt{x^4+2}x^2 - (i+1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right)$$
$$- \left(\frac{1}{16}i - \frac{1}{16}\right)\sqrt{2}\log\left(\frac{(i-1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 + 2i\sqrt{x^4+2}x^2 + (i+1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right)$$
$$+ \left(\frac{1}{16}i + \frac{1}{16}\right)\sqrt{2}\log\left(\frac{-(i+1)\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3 - 2i\sqrt{x^4+2}x^2 - (i-1)\sqrt{2}(x^4+2)^{\frac{3}{4}}x + 2}{x^4+1}\right)$$

input
```
integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="fricas")
```

output
```
-(1/16*I + 1/16)*sqrt(2)*log(((I + 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 - 2*I*sq
rt(x^4 + 2)*x^2 + (I - 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1)) + (1/1
6*I - 1/16)*sqrt(2)*log((-(I - 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*I*sqrt(x
^4 + 2)*x^2 - (I + 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1)) - (1/16*I
- 1/16)*sqrt(2)*log(((I - 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*I*sqrt(x^4 +
2)*x^2 + (I + 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1)) + (1/16*I + 1/1
6)*sqrt(2)*log((-(I + 1)*sqrt(2)*(x^4 + 2)^(1/4)*x^3 - 2*I*sqrt(x^4 + 2)*x
^2 - (I - 1)*sqrt(2)*(x^4 + 2)^(3/4)*x + 2)/(x^4 + 1))
```

3.315.6 Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+1)\sqrt[4]{x^4+2}}\,dx$$

input
```
integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

output
```
Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)
```

3.315.7 Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

input
```
integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

3.315.8 Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

input
```
integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+1)\,(x^4+2)^{1/4}}\,dx$$

input
```
int(1/((x^4 + 1)*(x^4 + 2)^(1/4)),x)
```

output
```
int(1/((x^4 + 1)*(x^4 + 2)^(1/4)), x)
```

3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx$

3.316.1 Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=\frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)$$

output
```
1/3*x*(x^3+2)^(2/3)+5/6*ln(-x+(x^3+2)^(1/3))-5/9*arctan(1/3*(1+2*x/(x^3+2)
^(1/3))*3^(1/2))*3^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=\frac{1}{18}\left(6x\left(2+x^3\right)^{2/3}-10\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x+2\sqrt[3]{2+x^3}}\right)+10\log\left(-x+\sqrt[3]{2+x^3}\right)-5\log\left(x^2+x\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)\right)$$

input
```
Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

output
```
(6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x^3)^(1/3
))] + 10*Log[-x + (2 + x^3)^(1/3)] - 5*Log[x^2 + x*(2 + x^3)^(1/3) + (2 +
x^3)^(2/3)])/18
```

3.316.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{\sqrt[3]{x^3+2}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{1}{3}x\left(x^3+2\right)^{2/3}-\frac{5}{3}\int \frac{1}{\sqrt[3]{x^3+2}}dx$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{3}x\left(x^3+2\right)^{2/3}-\frac{5}{3}\left(\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+2}-x\right)\right)$$

input
```
Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

output
```
(x*(2 + x^3)^(2/3))/3 - (5*(ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/Sq
rt[3] - Log[-x + (2 + x^3)^(1/3)]/2))/3
```

3.316.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

3.316.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.46

method	result
risch	$\frac{x\left(x^3+2\right)^{\frac{2}{3}}}{3}-\frac{5\,2^{\frac{2}{3}} x\, {}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x^3}{2}\right)}{6}$
meijerg	$-\frac{2^{\frac{2}{3}} x\, {}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x^3}{2}\right)}{2}+\frac{2^{\frac{2}{3}} x^4\, {}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{x^3}{2}\right)}{8}$
pseudoelliptic	$\frac{6x\left(x^3+2\right)^{\frac{2}{3}}+10\sqrt{3}\,\arctan\left(\frac{\left(x+2\left(x^3+2\right)^{\frac{1}{3}}\right)\sqrt{3}}{3x}\right)-5\ln\left(\frac{\left(x^3+2\right)^{\frac{2}{3}}+x\left(x^3+2\right)^{\frac{1}{3}}+x^2}{x^2}\right)+10\ln\left(\frac{-x+\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)}{9\left(\left(x^3+2\right)^{\frac{2}{3}}+x\left(x^3+2\right)^{\frac{1}{3}}+x^2\right)\left(-x+\left(x^3+2\right)^{\frac{1}{3}}\right)}$
trager	$\frac{x\left(x^3+2\right)^{\frac{2}{3}}}{3}+\frac{5\ln\left(-8\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2 x^3+6\,\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(x^3+2\right)^{\frac{2}{3}} x-10\,\mathrm{RootOf}\left(4\_Z^2+2\_\right.\right.}{9}$

input
```
int((x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx=\frac{1}{3}\left(x^3+2\right)^{\frac{2}{3}}x+\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}(x^3+2)^{\frac{1}{3}}}{3\,x}\right)+\frac{5}{9}\log\left(-\frac{x-(x^3+2)^{\frac{1}{3}}}{x}\right)-\frac{5}{18}\log\left(\frac{x^2+(x^3+2)^{\frac{1}{3}}x+(x^3+2)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 2)^(2/3)*x + 5/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
 + 2)^(1/3))/x) + 5/9*log(-(x - (x^3 + 2)^(1/3))/x) - 5/18*log((x^2 + (x^3
 + 2)^(1/3)*x + (x^3 + 2)^(2/3))/x^2)
```

3.316.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)} - \frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((x**3-1)/(x**3+2)**(1/3),x)
```

output
```
2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi)/2)
/(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp
_polar(I*pi)/2)/(6*gamma(4/3))
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+2\right)^{\frac{1}{3}}}{x}+1\right)\right) + \frac{2\left(x^3+2\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+2}{x^3}-1\right)} - \frac{5}{18}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+2\right)^{\frac{2}{3}}}{x^2}+1\right) + \frac{5}{9}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}-1\right)$$

input
```
integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(x^3 + 2)^
(2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/3)/x + (x^3 + 2)^(
2/3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)
```

3.316.8 Giac [F]

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3-1}{(x^3+2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3-1}{(x^3+2)^{1/3}}\,dx$$

input
```
int((x^3 - 1)/(x^3 + 2)^(1/3),x)
```

output
```
int((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx$

3.317.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx = \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}$$

output
```
1/8*x*(x^4+1)^(3/4)/(x^4+2)+3/32*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/
4)+3/32*arctanh(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)
```

3.317.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx = \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}$$

input
```
Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

output
```
(x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

3.317.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {903, 902, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+1)^{3/4}}{(x^4+2)^2}dx$$

$$\Big\downarrow \; 903$$

$$\frac{3}{8}\int \frac{1}{\sqrt[4]{x^4+1}(x^4+2)}dx+\frac{(x^4+1)^{3/4}x}{8(x^4+2)}$$

$$\Big\downarrow \; 902$$

$$\frac{3}{8}\int \frac{1}{2-\frac{x^4}{x^4+1}}d\frac{x}{\sqrt[4]{x^4+1}}+\frac{(x^4+1)^{3/4}x}{8(x^4+2)}$$

$$\Big\downarrow \; 756$$

$$\frac{3}{8}\left(\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}}+\sqrt{2}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}\right)+\frac{(x^4+1)^{3/4}x}{8(x^4+2)}$$

$$\Big\downarrow \; 216$$

$$\frac{3}{8}\left(\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}\right)+\frac{(x^4+1)^{3/4}x}{8(x^4+2)}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{8}\left(\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}\right)+\frac{(x^4+1)^{3/4}x}{8(x^4+2)}$$

input
```
Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

output
```
(x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*(ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))
]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4))))/8
```

3.317.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.317.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{-3(x^4+2)\left(2\arctan\left(\frac{2^{\frac{1}{4}}(x^4+1)^{\frac{1}{4}}}{x}\right)-\ln\left(\frac{-x2^{\frac{3}{4}}-2(x^4+1)^{\frac{1}{4}}}{x2^{\frac{3}{4}}-2(x^4+1)^{\frac{1}{4}}}\right)\right)2^{\frac{1}{4}}+8(x^4+1)^{\frac{3}{4}}x}{64x^4+128}$
trager	$\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{-2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4\right.}{}\right.}{}$
risch	$\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-\right.}{}\right.}{}$

input
```
int((x^4+1)^(3/4)/(x^4+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-3*(x^4+2)*(2*arctan(1/x*2^(1/4)*(x^4+1)^(1/4))-ln((-x*2^(3/4)-2*(x^4+1)^
(1/4))/(x*2^(3/4)-2*(x^4+1)^(1/4))))*2^(1/4)+8*(x^4+1)^(3/4)*x)/(64*x^4+12
8)
```

3.317.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.23

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \frac{3\cdot 8^{\frac{3}{4}}(x^4+2)\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2+8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)-3\cdot 8^{\frac{3}{4}}(-i\,x^4-2i)}{}$$

input
```
integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="fricas")
```

output
```
1/512*(3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*
sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
 - 3*8^(3/4)*(-I*x^4 - 2*I)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*I*8^(1
/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*I*x^4 + 2*I) - 16*(x^4 + 1)^(3/4)*x)/(x
^4 + 2)) - 3*8^(3/4)*(I*x^4 + 2*I)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8
*I*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(-3*I*x^4 - 2*I) - 16*(x^4 + 1)^(3/
4)*x)/(x^4 + 2)) - 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3
- 8*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x
)/(x^4 + 2)) + 64*(x^4 + 1)^(3/4)*x)/(x^4 + 2)
```

3.317.6 Sympy [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

input
```
integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
```

output
```
Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)
```

3.317.7 Maxima [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

input
```
integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

3.317.8 Giac [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

input
```
integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="giac")
```

output
```
integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{3/4}}{(x^4+2)^2}\,dx$$

input
```
int((x^4 + 1)^(3/4)/(x^4 + 2)^2,x)
```

output
```
int((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$

3.318.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx=-\frac{5x(-2+x^5)}{33\left(3+x^5\right)^{11/5}}+\frac{5x}{297\left(3+x^5\right)^{6/5}}+\frac{97x}{891\sqrt[5]{3+x^5}}$$

output
```
-5/33*x*(x^5-2)/(x^5+3)^(11/5)+5/297*x/(x^5+3)^(6/5)+97/891*x/(x^5+3)^(1/5
)
```

3.318.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.54

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx=\frac{x(1188+462x^5+97x^{10})}{891\left(3+x^5\right)^{11/5}}$$

input
```
Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

output
```
(x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))
```

3.318.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {903, 25, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^5-2)^2}{(x^5+3)^{16/5}}dx$$

$$\Big\downarrow \text{ 903}$$

$$\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}-\frac{20}{33}\int -\frac{2-x^5}{(x^5+3)^{11/5}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{20}{33}\int \frac{2-x^5}{(x^5+3)^{11/5}}dx+\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}$$

$$\Big\downarrow \text{ 903}$$

$$\frac{20}{33}\left(\frac{5}{9}\int \frac{1}{(x^5+3)^{6/5}}dx+\frac{x(2-x^5)}{18(x^5+3)^{6/5}}\right)+\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}+\frac{20}{33}\left(\frac{5x}{27\sqrt[5]{x^5+3}}+\frac{(2-x^5)x}{18(x^5+3)^{6/5}}\right)$$

input
```
Int[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

output
```
(x*(2 - x^5)^2)/(33*(3 + x^5)^(11/5)) + (20*((x*(2 - x^5))/(18*(3 + x^5)^(
6/5)) + (5*x)/(27*(3 + x^5)^(1/5))))/33
```

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

3.318.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	23
trager	$\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	23
risch	$\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	23
pseudoelliptic	$\frac{97x^{11}+462x^{6}+1188x}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$	24
meijerg	$\frac{4\,3^{\frac{4}{5}}x\left(\frac{25}{9}x^{10}+\frac{55}{3}x^{5}+33\right)}{2673\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}+\frac{3^{\frac{4}{5}}x^{11}}{891\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}-\frac{2\,3^{\frac{4}{5}}x^{6}\left(11+\frac{5x^{5}}{3}\right)}{2673\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}$	70

input
```
int((x^5-2)^2/(x^5+3)^(16/5),x,method=_RETURNVERBOSE)
```

output
```
1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{(97\,x^{11}+462\,x^6+1188\,x)(x^5+3)^{\frac{4}{5}}}{891\,(x^{15}+9\,x^{10}+27\,x^5+27)}$$

input
```
integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="fricas")
```

output
```
1/891*(97*x^11 + 462*x^6 + 1188*x)*(x^5 + 3)^(4/5)/(x^15 + 9*x^10 + 27*x^5
 + 27)
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \text{Timed out}$$

input
```
integrate((x**5-2)**2/(x**5+3)**(16/5),x)
```

output
```
Timed out
```

3.318.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = -\frac{4\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-\frac{33\,(x^5+3)^2}{x^{10}}-3\right)}{891\,(x^5+3)^{\frac{11}{5}}}-\frac{2\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-6\right)}{297\,(x^5+3)^{\frac{11}{5}}}+\frac{x^{11}}{33\,(x^5+3)^{\frac{11}{5}}}$$

input
```
integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="maxima")
```

output
```
-4/891*x^11*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^10 - 3)/(x^5 + 3)^(11/5)
- 2/297*x^11*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^(11/5) + 1/33*x^11/(x^5 + 3)
^(11/5)
```

3.318.8 Giac [F]

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \int \frac{(x^5-2)^2}{(x^5+3)^{\frac{16}{5}}}\,dx$$

input
```
integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="giac")
```

output
```
integrate((x^5 - 2)^2/(x^5 + 3)^(16/5), x)
```

3.318.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{97\,x^{11}+462\,x^6+1188\,x}{891\,(x^5+3)^{11/5}}$$

input
```
int((x^5 - 2)^2/(x^5 + 3)^(16/5),x)
```

output
```
(1188*x + 462*x^6 + 97*x^11)/(891*(x^5 + 3)^(11/5))
```

3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

3.319.1 Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

output
```
-1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/1
8*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

3.319.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{3+3x+3x^2+x^3}}{2\sqrt[3]{3}+2\sqrt[3]{3}x+\sqrt[3]{3+3x+3x^2+x^3}}\right)}{3^{5/6}} + \frac{2\log\left(\sqrt[3]{3}+\sqrt[3]{3}x-\sqrt[3]{3+3x+3x^2+x^3}\right)-\log\left(3^{2/3}+2\,3^{2/3}x+3^{2/3}x^2+\sqrt[3]{3}(1+x)\sqrt[3]{3+3x+3x^2+}\right.}{6\sqrt[3]{3}}$$

input
```
Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

output
```
ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x +
(3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3
+ 3*x + 3*x^2 + x^3)^(1/3)] - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + 3^
(1/3)*(1 + x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3
)])/(6*3^(1/3))
```

3.319.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {939, 938, 25, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3+3x^2+3x)\sqrt[3]{x^3+3x^2+3x+3}}dx$$

$$\Big\downarrow \text{939}$$

$$\int \frac{1}{((x+1)^3-1)\sqrt[3]{(x+1)^3+2}}dx$$

$$\Big\downarrow \text{938}$$

$$\int -\frac{1}{(1-(x+1)^3)\sqrt[3]{(x+1)^3+2}}d(x+1)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(1-(x+1)^3\right)\sqrt[3]{(x+1)^3+2}}d(x+1)$$

$$\Big\downarrow \text{901}$$

$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}$$

input
```
Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

output
```
-(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
 - Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^
(1/3)]/(2*3^(1/3))
```

3.319.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 939
```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[NormalizePseudoBinomial[u, x]^p
*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x] && PseudoBinomialP
airQ[u, v, x]
```

3.319. $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

3.319.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.17 (sec) , antiderivative size = 2517, normalized size of antiderivative = 27.97

method	result	size
trager	Expression too large to display	2517

input
```
int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/9*ln(-(6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x+2269837425*(x^3+3*x^2+3*x+3
)^(1/3)*RootOf(_Z^3-9)^2+19831665822*RootOf(_Z^3-9)*x+275418203481*RootOf(
RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+275418203481*RootOf(Root
Of(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+15322002984*(x^3+3*x^2+3*x+3)^
(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^
2*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootO
f(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2+2269837425*(x^3+3*x^2+3*x+3)^(1/3)*Roo
tOf(_Z^3-9)^2*x^2+4539674850*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+45
966008952*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3
-9)+81*_Z^2)*RootOf(_Z^3-9)+91806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootO
f(_Z^3-9)+81*_Z^2)*x^3+6610555274*RootOf(_Z^3-9)*x^3-15589709631*RootOf(Ro
otOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-1122547
122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*
x^2-15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*Roo
tOf(_Z^3-9)^2*x-1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*
_Z^2)*RootOf(_Z^3-9)^3*x+150700526433*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(
_Z^3-9)+81*_Z^2)+10851288846*RootOf(_Z^3-9)+19831665822*RootOf(_Z^3-9)*x^2
+6809512275*(x^3+3*x^2+3*x+3)^(2/3)+2619276618*RootOf(_Z^3-9)^3*RootOf(Roo
tOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+36375989139*RootOf(RootOf(_Z^3-
9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-374182374*RootOf(R...
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.

Time = 4.91 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.09

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{1}{54}$$
$$\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}(7\,x^4+28\,x^3+42\,x^2+30\,x+9)(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}+3^{\frac{1}{3}}(31\,x^6+186\,x^5+465\,x^4+66}{x^6+6\,x^5+15\,x^4}\right.$$
$$+\frac{1}{27}$$
$$\cdot 3^{\frac{2}{3}}\log\left(\frac{2\cdot 3^{\frac{2}{3}}(x^3+3\,x^2+3\,x)-9\cdot 3^{\frac{1}{3}}(x^3+3\,x^2+3\,x+3)^{\frac{1}{3}}(x^2+2\,x+1)+9\,(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}(x}{x^3+3\,x^2+3\,x}\right.$$
$$-\frac{1}{9}$$
$$\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7\,x^7+49\,x^6+147\,x^5+240\,x^4+225\,x^3+117\,x^2+27\,x)(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}-\right.}{}\right.$$

input
```
integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

output
```
-1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3
*x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 60
3*x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3
 + 3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27
*3^(2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x
 + 3)^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^
3 + 3*x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49
*x^6 + 147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x +
3)^(2/3) - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5
+ 22032*x^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7
 + 868*x^6 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3
+ 3*x^2 + 3*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 343
98*x^5 + 38556*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

3.319.6 Sympy [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

input
```
integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

output
```
Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

3.319.7 Maxima [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^3+3\,x^2+3\,x\right)}\,dx$$

input
```
integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

3.319.8 Giac [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^3+3\,x^2+3\,x\right)}\,dx$$

input
```
integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{(x^3+3\,x^2+3\,x)\,\left(x^3+3\,x^2+3\,x+3\right)^{1/3}}\,dx$$

input
```
int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
```

output
```
int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

3.320.1 Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

3.320.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2213}$$

$$\int \frac{1}{\frac{2x^2}{x^4+1}+1}d\frac{x}{\sqrt{x^4+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

input
```
Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

3.320.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.320.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$	19
pseudoelliptic	$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$	19
elliptic	$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$	22
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x-\sqrt{x^4+1}}{x^2+1}\right)}{2}$	37

input
```
int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

3.320.6 Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\right)dx$$

input
```
integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x
**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

3.320.7 Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.320.8 Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx=-\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
int(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

output
```
-int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$

3.321.1 Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

3.321.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2213, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{2213}$$
$$\int \frac{1}{1-\frac{2x^2}{x^4+1}}d\frac{x}{\sqrt{x^4+1}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

input
```
Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

3.321.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.321.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^{4}+1}}{2x}\right)\sqrt{2}}{2}$	22
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x+\sqrt{x^{4}+1}}{(-1+x)(1+x)}\right)}{2}$	38
default	$\frac{\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\left(x^{2}-x+1\right)\sqrt{2}}{\sqrt{x^{4}+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^{2}+x+1\right)\sqrt{2}}{\sqrt{x^{4}+1}}\right)\right)}{4}$	47
pseudoelliptic	$\frac{\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\left(x^{2}-x+1\right)\sqrt{2}}{\sqrt{x^{4}+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^{2}+x+1\right)\sqrt{2}}{\sqrt{x^{4}+1}}\right)\right)}{4}$	47

input
```
int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2/x*2^(1/2)*(x^4+1)^(1/2))*2^(1/2)
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
 + 1))
```

3.321.6 Sympy [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx - \int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x*
*2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

3.321.7 Maxima [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.321.8 Giac [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{(x^2-1)\sqrt{x^4+1}}\,dx$$

input
```
int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

output
```
int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$

3.322.1 Optimal result

Integrand size = 18, antiderivative size = 16

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(\frac{-1+x^2}{\sqrt{1+x^4}}\right)$$

output
```
arctanh((x^2-1)/(x^4+1)^(1/2))
```

3.322.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(1+2x^2-2\sqrt{1+x^4}\right)-\frac{1}{2}\log\left(1-x^2+\sqrt{1+x^4}\right)$$

input
```
Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[1 + 2*x^2 - 2*Sqrt[1 + x^4]] - Log[1 - x^2 + Sqrt[1 + x^4]]/2
```

3.322.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1579, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^2+1}{x^2\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+1}}dx^2+\int \frac{1}{x^2\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2\sqrt{x^4+1}}dx^2+\operatorname{arcsinh}\left(x^2\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+1}}dx^4+\operatorname{arcsinh}\left(x^2\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+1}-1}d\sqrt{x^4+1}+\operatorname{arcsinh}\left(x^2\right)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}\left(x^2\right)-\operatorname{arctanh}\left(\sqrt{x^4+1}\right)\right)$$

input
```
Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
(ArcSinh[x^2] - ArcTanh[Sqrt[1 + x^4]])/2
```

3.322.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.322. $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$

3.322.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	18
trager	$\ln\left(\frac{x^{2}+\sqrt{x^{4}+1}-1}{x}\right)$	18
elliptic	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	18
pseudoelliptic	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	18
meijerg	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}+\frac{\left(-2\ln\left(2\right)+4\ln\left(x\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{4}+1}}{2}\right)}{4\sqrt{\pi}}$	44

input
```
int((x^2+1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(x^2)-1/2*arctanh(1/(x^4+1)^(1/2))
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(2\,x^{4}-x^{2}-\sqrt{x^{4}+1}\left(2\,x^{2}-1\right)+1\right)+\frac{1}{2}\log\left(-x^{2}+\sqrt{x^{4}+1}-1\right)$$

input
```
integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(2*x^4 - x^2 - sqrt(x^4 + 1)*(2*x^2 - 1) + 1) + 1/2*log(-x^2 + sqr
t(x^4 + 1) - 1)
```

3.322.6 Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}\left(x^2\right)}{2}$$

input
```
integrate((x**2+1)/x/(x**4+1)**(1/2),x)
```

output
```
-asinh(x**(-2))/2 + asinh(x**2)/2
```

3.322.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right) + \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^
4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

3.322.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(x^2 - sqrt(x^4 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2
*log(-x^2 + sqrt(x^4 + 1))
```

3.322.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2} - \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int((x^2 + 1)/(x*(x^4 + 1)^(1/2)),x)
```

output
```
asinh(x^2)/2 - atanh((x^4 + 1)^(1/2))/2
```

3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}} \, dx$

3.323.1 Optimal result

Integrand size = 18, antiderivative size = 16

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}} \, dx = \operatorname{arctanh}\left(\frac{1+x^2}{\sqrt{1+x^4}}\right)$$

output
```
arctanh((x^2+1)/(x^4+1)^(1/2))
```

3.323.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}} \, dx = -\operatorname{arctanh}\left(x^2-\sqrt{1+x^4}\right)-\frac{1}{2}\log\left(-x^2+\sqrt{1+x^4}\right)$$

input
```
Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
-ArcTanh[x^2 - Sqrt[1 + x^4]] - Log[-x^2 + Sqrt[1 + x^4]]/2
```

3.323.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1579, 25, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \textbf{1579}$$

$$\frac{1}{2}\int -\frac{1-x^2}{x^2\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^2\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \textbf{538}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+1}}dx^2-\int \frac{1}{x^2\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \textbf{222}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)-\int \frac{1}{x^2\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \textbf{243}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)-\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+1}}dx^4\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)-\int \frac{1}{\sqrt{x^4+1}-1}d\sqrt{x^4+1}\right)$$

$$\Big\downarrow \textbf{220}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)+\operatorname{arctanh}\left(\sqrt{x^4+1}\right)\right)$$

input
```
Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
(ArcSinh[x^2] + ArcTanh[Sqrt[1 + x^4]])/2
```

3.323. $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$

3.323.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.323. $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$

3.323.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	18
trager	$\ln\left(\frac{x^{2}+\sqrt{x^{4}+1}+1}{x}\right)$	18
elliptic	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	18
pseudoelliptic	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	18
meijerg	$\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}-\frac{\left(-2\ln\left(2\right)+4\ln\left(x\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{4}+1}}{2}\right)}{4\sqrt{\pi}}$	44

input
```
int((x^2-1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(x^2)+1/2*arctanh(1/(x^4+1)^(1/2))
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\,\log\left(2\,x^{4}+x^{2}-\sqrt{x^{4}+1}\left(2\,x^{2}+1\right)+1\right)+\frac{1}{2}\,\log\left(-x^{2}+\sqrt{x^{4}+1}+1\right)$$

input
```
integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) + 1/2*log(-x^2 + sqr
t(x^4 + 1) + 1)
```

3.323.6 Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx=\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

input
```
integrate((x**2-1)/x/(x**4+1)**(1/2),x)
```

output
```
asinh(x**(-2))/2 + asinh(x**2)/2
```

3.323.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx=\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)-\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

3.323.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx=-\frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(x^2 - sqrt(x^4 + 1) + 1) + 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/
2*log(-x^2 + sqrt(x^4 + 1))
```

3.323.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2} + \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int((x^2 - 1)/(x*(x^4 + 1)^(1/2)),x)
```

output
```
asinh(x^2)/2 + atanh((x^4 + 1)^(1/2))/2
```

3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$

3.324.1 Optimal result

Integrand size = 27, antiderivative size = 26

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(x*3^(1/2)/(x^4+x^2+1)^(1/2))*3^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

3.324.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2212, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2212}$$

$$\int \frac{1}{1-\frac{3x^2}{x^4+x^2+1}}d\frac{x}{\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

3.324.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

3.324.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+x^2+1}\,\sqrt{2}\,\sqrt{6}}{6x}\right)\sqrt{6}\,\sqrt{2}}{6}$	31
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^2-3\right)x+\sqrt{x^4+x^2+1}}{(-1+x)(1+x)}\right)}{3}$	42
default	$\frac{\sqrt{3}\left(\operatorname{arctanh}\left(\frac{\left(2x^2-x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)-\operatorname{arctanh}\left(\frac{\left(2x^2+x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)\right)}{6}$	59
pseudoelliptic	$\frac{\sqrt{3}\left(\operatorname{arctanh}\left(\frac{\left(2x^2-x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)-\operatorname{arctanh}\left(\frac{\left(2x^2+x+2\right)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)\right)}{6}$	59

input
```
int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(1/6*(x^4+x^2+1)^(1/2)*2^(1/2)/x*6^(1/2))*6^(1/2)*2^(1/2)
```

3.324.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x^4+2\sqrt{3}\sqrt{x^4+x^2+1}x+4x^2+1}{x^4-2x^2+1}\right)$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((x^4 + 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*x + 4*x^2 + 1)/(x^4 -
 2*x^2 + 1))
```

3.324.6 Sympy [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx - \int \frac{1}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x) -
Integral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x)
```

3.324.7 Maxima [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

3.324.8 Giac [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{(x^2-1)\sqrt{x^4+x^2+1}}\,dx$$

input
```
int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

3.325.1 Optimal result

Integrand size = 27, antiderivative size = 15

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx=\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

output
```
arctan(x/(x^4+x^2+1)^(1/2))
```

3.325.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx=\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

input
```
Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

3.325.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \ 2212$$

$$\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \ 216$$

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

3.325.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

3.325.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$	14
pseudoelliptic	$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$	14
elliptic	$-\arctan\left(\frac{\sqrt{x^4+x^2+1}}{x}\right)$	18
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{x^4+x^2+1}}{x^2+1}\right)$	37

input
```
int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/(x^4+x^2+1)^(1/2))
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
arctan(x/sqrt(x^4 + x^2 + 1))
```

3.325.6 Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\right)dx$$

input
```
integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) -
Integral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x)
```

3.325.7 Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.325.8 Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$

input
```
int(-(x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
-int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$

3.326.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{1+x^2+x^4}}{x}$$

output
```
1/x*(x^4+x^2+1)^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{1+x^2+x^4}}{x}$$

input
```
Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
Sqrt[1 + x^2 + x^4]/x
```

3.326.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{2023}$$
$$\frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
Int[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
Sqrt[1 + x^2 + x^4]/x
```

3.326.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.326.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{x^{4}+x^{2}+1}}{x}$	15
trager	$\frac{\sqrt{x^{4}+x^{2}+1}}{x}$	15
risch	$\frac{\sqrt{x^{4}+x^{2}+1}}{x}$	15
elliptic	$\frac{\sqrt{x^{4}+x^{2}+1}}{x}$	15
gosper	$\frac{\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{\sqrt{x^{4}+x^{2}+1}\, x}$	29
pseudoelliptic	$\frac{\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{\sqrt{x^{4}+x^{2}+1}\, x}$	29

input
```
int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/x*(x^4+x^2+1)^(1/2)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx=\frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^4 + x^2 + 1)/x
```

3.326.6 Sympy [F]

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx=\int \frac{(x-1)\,(x+1)\,(x^2+1)}{x^2\sqrt{(x^2-x+1)\,(x^2+x+1)}}\,dx$$

input
```
integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x +
1))), x)
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^2+x+1}\sqrt{x^2-x+1}}{x}$$

input
```
integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x
```

3.326.8 Giac [F]

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^4-1}{\sqrt{x^4+x^2+1}x^2}\,dx$$

input
```
integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2), x)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
int((x^4 - 1)/(x^2*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
(x^2 + x^4 + 1)^(1/2)/x
```

3.327 $\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$

3.327.1 Optimal result

Integrand size = 44, antiderivative size = 74

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \frac{\arctan\left(\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right)}{\sqrt{2}\sqrt{1-b}}$$

output
```
1/2*arctan(1/2*(a+2*(a^2-b+1)*x+a*x^2)*2^(1/2)/(1-b)^(1/2)/(2*a*x^3+x^4+2*
b*x^2+2*a*x+1)^(1/2))*2^(1/2)/(1-b)^(1/2)
```

3.327.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{-1+b}x}{1+2ax+x^2-\sqrt{1+2bx^2+x^4+2a(x+x^3)}}\right)}{\sqrt{-1+b}}$$

input
```
Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3
+ x^4]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-1 + b]*x)/(1 + 2*a*x + x^2 - Sqrt[1 + 2*
b*x^2 + x^4 + 2*a*(x + x^3)])])/Sqrt[-1 + b])
```

3.327.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2507}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(2ax+x^2+1)\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\,dx$$

$$\Big\downarrow \text{2507}$$

$$\frac{\arctan\left(\frac{2x(a^2-b+1)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

input
```
Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]
),x]
```

output
```
ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x
 + 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

3.327.3.1 Defintions of rubi rules used

rule 2507
```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

3.327.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{\ln(2)+\ln\left(\frac{\sqrt{2b-2}\,\sqrt{2a\,x^3+x^4+2x^2b+2ax+1}-a\,x^2+\left(-2a^2+2b-2\right)x-a}{2ax+x^2+1}\right)}{\sqrt{2b-2}}$	78
pseudoelliptic	$\frac{\ln(2)+\ln\left(\frac{\sqrt{2b-2}\,\sqrt{2a\,x^3+x^4+2x^2b+2ax+1}-a\,x^2+\left(-2a^2+2b-2\right)x-a}{2ax+x^2+1}\right)}{\sqrt{2b-2}}$	78
elliptic	Expression too large to display	258804

input
```
int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/(2*b-2)^(1/2)*(ln(2)+ln(((2*b-2)^(1/2)*(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/
2)-a*x^2+(-2*a^2+2*b-2)*x-a)/(2*a*x+x^2+1)))
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.41

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$$
$$= \left[\frac{\sqrt{2}\log\left(\frac{4\,a^3x^3+(a^2+2\,b-2)x^4+4\,a^3x+2\,(2\,a^4+5\,a^2-2\,(2\,a^2+3)b+4\,b^2+2)x^2+a^2-\frac{2\,\sqrt{2}\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}\left((ab-a)x^2+ab-2\,\left(a^2-\left(a^2\right.\right.\right.}{\sqrt{b-1}}}{4\,ax^3+x^4+2\,(2\,a^2+1)x^2+4\,ax+1}\right.}{4\,\sqrt{b-1}}\right.$$

input
```
integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, al
gorithm="fricas")
```

output
```
[1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(2*a^4 + 5
*a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2)*sqrt(2*a*x^3 + x
^4 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b - 2*(a^2 - (a^2 + 2)*b + b^
2 + 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 +
4*a*x + 1))/sqrt(b - 1), 1/2*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(
2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(b - 1)*sqrt(-1/(b - 1))/(a*x^2 + 2*(
a^2 - b + 1)*x + a))]
```

3.327.6 Sympy [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\int \frac{x^2}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+x^4 \cdots}} - \int \left(-\frac{1}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+ \cdots}}\right.$$

input
```
integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2)
,x)
```

output
```
-Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*
sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b
*x**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x*
*2 + x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(
2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x)
```

3.327.7 Maxima [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, al
gorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x
^2 + 1)), x)
```

3.327.8 Giac [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, al
gorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x
^2 + 1)), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+2\,a\,x+1)\,\sqrt{x^4+2\,a\,x^3+2\,b\,x^2+2\,a\,x+1}}\,dx$$

input
```
int(-(x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1
/2)),x)
```

output
```
-int((x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1
/2)), x)
```

3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

3.328.1 Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

output
```
arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

3.328.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = i\text{arctanh}\left(\sqrt{2}+\sqrt{2}x^4-ix^3\sqrt{-x^2+\sqrt{1+x^4}} + \frac{\sqrt{1+x^4}\left(-2x^2+i\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}\right)$$

input
```
Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

output
```
I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt
[1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

3.328.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2553, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+1)\sqrt{\sqrt{x^4+1}-x^2}}dx$$

$$\Big\downarrow \text{ 2553}$$

$$\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}-x^2}+1}d\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

input
```
Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

output
```
ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

3.328.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

3.328.4 Maple [F]

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+(7\,x^5-x)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqr
t(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

3.328.6 Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,(x^4+1)}\,dx$$

input
```
integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

3.328.7 Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.328.8 Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,(x^4+1)}\,dx$$

input
```
int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
```

output
```
int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

3.329 $\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx$

3.329.1 Optimal result

Integrand size = 31, antiderivative size = 24

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\right)$$

output
```
arctan(x/(-x^2+(1+x^(2*n))^(1/n))^(1/2))
```

3.329.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \cot^{-1}\left(\frac{\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}{x}\right)$$

input
```
Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

output
```
ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]
```

3.329.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2553, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^{2n}+1)\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}dx$$

$$\Big\downarrow \text{2553}$$

$$\int \frac{1}{\frac{x^2}{(x^{2n}+1)^{\frac{1}{n}}-x^2}+1}d\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}\right)$$

input
```
Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

output
```
ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

3.329.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

3.329.4 Maple [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}dx$$

input
```
int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

output
```
int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

3.329.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.329.6 Sympy [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\frac{1}{n}}}\,(x^{2n}+1)}\,dx$$

input
```
integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)), x)
```

3.329.7 Maxima [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\left(\frac{1}{n}\right)}}\,(x^{2n}+1)}\,dx$$

input
```
integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

3.329.8 Giac [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\left(\frac{1}{n}\right)}}\,(x^{2n}+1)}\,dx$$

input
```
integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{(x^{2n}+1)\,\sqrt{(x^{2n}+1)^{1/n}-x^2}}\,dx$$

input
```
int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)),x)
```

output
```
int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)), x)
```

3.330 $\int \cos^2(x) \, dx$

3.330.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x) \, dx = \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.330.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x) \, dx = \frac{x}{2} + \frac{1}{4} \sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

3.330.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \; 24$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.330.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.330.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

input
```
integrate(cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

3.330.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(cos(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.330.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

3.331 $\int \cos^3(x)\,dx$

3.331.1 Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

output
```
sin(x)-1/3*sin(x)^3
```

3.331.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

input
```
Integrate[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

3.331.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{3113}$$

$$-\int \left(1-\sin^2(x)\right) d(-\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\sin(x)-\frac{\sin^3(x)}{3}$$

input
```
Int[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

3.331.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.331.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12
parallelrisch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12

input
```
int(cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2+cos(x)^2)*sin(x)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x)\,dx = \frac{1}{3}\left(\cos\left(x\right)^2+2\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*sin(x)
```

3.331.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x)\,dx = -\frac{\sin^3\left(x\right)}{3}+\sin\left(x\right)$$

input
```
integrate(cos(x)**3,x)
```

output
```
-sin(x)**3/3 + sin(x)
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="maxima")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="giac")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

3.331.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin(x)^3}{3}$$

input
```
int(cos(x)^3,x)
```

output
```
sin(x) - sin(x)^3/3
```

3.332 $\int \sin^4(x)\,dx$

3.332.1 Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)$$

output
```
3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

3.332.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

input
```
Integrate[Sin[x]^4,x]
```

output
```
(3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

3.332.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)$$

input
```
Int[Sin[x]^4,x]
```

output
```
-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4
```

3.332.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.332.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	siz
risch	$\frac{3x}{8}+\frac{\sin\left(4x\right)}{32}-\frac{\sin\left(2x\right)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin\left(4x\right)}{32}-\frac{\sin\left(2x\right)}{4}$	17
default	$-\frac{\left(\sin^{3}\left(x\right)+\frac{3\sin\left(x\right)}{2}\right)\cos\left(x\right)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{4}}$	82

input
```
int(sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \sin^4(x)\,dx = \frac{1}{8}\left(2\,\cos\left(x\right)^3 - 5\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{8}\,x$$

input
```
integrate(sin(x)^4,x, algorithm="fricas")
```

output
```
1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

3.332.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{\sin^3\left(x\right)\cos\left(x\right)}{4} - \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

input
```
integrate(sin(x)**4,x)
```

output
```
3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.332.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3\,x}{8} - \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

input
```
int(sin(x)^4,x)
```

output
```
(3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

3.333 $\int \cos^6(x)\,dx$

3.333.1 Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)$$

output
```
5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

3.333.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

input
```
Integrate[Cos[x]^6,x]
```

output
```
(5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

3.333.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)$$

input
```
Int[Cos[x]^6,x]
```

output
```
(Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/
2))/4))/6
```

3.333.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.333.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
default	$\frac{\left(\cos^{5}(x)+\frac{5\left(\cos^{3}(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{5\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{24}+\frac{15\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{4}-\frac{15\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{4}+\frac{5\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{24}-\frac{11\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{6}}$

input
```
int(cos(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/16*x+1/192*sin(6*x)+3/64*sin(4*x)+15/64*sin(2*x)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^6(x)\,dx = \frac{1}{48}\left(8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{5}{16}x$$

input
```
integrate(cos(x)^6,x, algorithm="fricas")
```

output
```
1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

3.333.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

input
```
integrate(cos(x)**6,x)
```

output
```
5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \cos^6(x)\,dx = -\frac{1}{48}\sin(2x)^3 + \frac{5}{16}x + \frac{3}{64}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)^6,x, algorithm="maxima")
```

output
```
-1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5}{16}\,x + \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) + \frac{15}{64}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^6,x, algorithm="giac")
```

output
```
5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

3.333.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5\,x}{16} + \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} + \frac{\sin\left(6\,x\right)}{192}$$

input
```
int(cos(x)^6,x)
```

output
```
(5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

3.334 $\int \sin^8(x)\,dx$

3.334.1 Optimal result

Integrand size = 4, antiderivative size = 44

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{35}{128}\cos(x)\sin(x) - \frac{35}{192}\cos(x)\sin^3(x) - \frac{7}{48}\cos(x)\sin^5(x) - \frac{1}{8}\cos(x)\sin^7(x)$$

output
```
35/128*x-35/128*cos(x)*sin(x)-35/192*cos(x)*sin(x)^3-7/48*cos(x)*sin(x)^5-
1/8*cos(x)*sin(x)^7
```

3.334.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{7}{32}\sin(2x) + \frac{7}{128}\sin(4x) - \frac{1}{96}\sin(6x) + \frac{\sin(8x)}{1024}$$

input
```
Integrate[Sin[x]^8,x]
```

output
```
(35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + Sin[8*x]/1
024
```

3.334.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 2.250$, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^8dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\int \sin^6(x)dx-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\int \sin(x)^6dx-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

input
```
Int[Sin[x]^8,x]
```

output
```
-1/8*(Cos[x]*Sin[x]^7) + (7*(-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin
[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4))/6))/8
```

3.334.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.334.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result
risch	$\frac{35x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(6x)}{96}+\frac{7\sin(4x)}{128}-\frac{7\sin(2x)}{32}$
parallelrisch	$\frac{35x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(6x)}{96}+\frac{7\sin(4x)}{128}-\frac{7\sin(2x)}{32}$
default	$-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}$
norman	$\frac{35x}{128}-\frac{805\left(\tan^3\left(\frac{x}{2}\right)\right)}{192}-\frac{2681\left(\tan^5\left(\frac{x}{2}\right)\right)}{192}-\frac{5053\left(\tan^7\left(\frac{x}{2}\right)\right)}{192}+\frac{5053\left(\tan^9\left(\frac{x}{2}\right)\right)}{192}+\frac{2681\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{192}+\frac{805\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{192}+\frac{35\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{64}+3$

input
```
int(sin(x)^8,x,method=_RETURNVERBOSE)
```

output
```
35/128*x+1/1024*sin(8*x)-1/96*sin(6*x)+7/128*sin(4*x)-7/32*sin(2*x)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \sin^8(x)\,dx = \frac{1}{384}\left(48\,\cos(x)^7 - 200\,\cos(x)^5 + 326\,\cos(x)^3 - 279\,\cos(x)\right)\sin(x) + \frac{35}{128}\,x$$

input
```
integrate(sin(x)^8,x, algorithm="fricas")
```

output
```
1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*sin(x) + 35
/128*x
```

3.334.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{\sin^7(x)\cos(x)}{8} - \frac{7\sin^5(x)\cos(x)}{48} - \frac{35\sin^3(x)\cos(x)}{192} - \frac{35\sin(x)\cos(x)}{128}$$

input
```
integrate(sin(x)**8,x)
```

output
```
35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)**3*cos(x
)/192 - 35*sin(x)*cos(x)/128
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \sin^8(x)\,dx = \frac{1}{24}\,\sin(2\,x)^3 + \frac{35}{128}\,x + \frac{1}{1024}\,\sin(8\,x) + \frac{7}{128}\,\sin(4\,x) - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^8,x, algorithm="maxima")
```

output
```
1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1/4*sin(2*
x)
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \sin^8(x)\,dx = \frac{35}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{96}\,\sin\left(6\,x\right) + \frac{7}{128}\,\sin\left(4\,x\right) - \frac{7}{32}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^8,x, algorithm="giac")
```

output
```
35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/32*sin(2*x
)
```

3.334.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \sin^8(x)\,dx = \frac{35\,x}{128} - \frac{7\,\sin\left(2\,x\right)}{32} + \frac{7\,\sin\left(4\,x\right)}{128} - \frac{\sin\left(6\,x\right)}{96} + \frac{\sin\left(8\,x\right)}{1024}$$

input
```
int(sin(x)^8,x)
```

output
```
(35*x)/128 - (7*sin(2*x))/32 + (7*sin(4*x))/128 - sin(6*x)/96 + sin(8*x)/1
024
```

3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

3.335.1 Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{1}{8}\cos(x)\sin(x)$$

output
```
3/8*x+1/2*cos(x)-1/8*cos(x)*sin(x)
```

3.335.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{1}{16}(3\pi+6x+8\cos(x)-2\cos(x)\sin(x))$$

input
```
Integrate[Cos[Pi/4 + x/2]^4,x]
```

output
```
(3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16
```

3.335.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3115, 3042, 3114}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\frac{x}{2}+\frac{3\pi}{4}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)dx+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sin\left(\frac{x}{2}+\frac{3\pi}{4}\right)^2dx+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3114}$$

$$\frac{3}{4}\left(\frac{x}{2}+\frac{\cos(x)}{2}\right)+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

input
```
Int[Cos[Pi/4 + x/2]^4,x]
```

output
```
(3*(x/2 + Cos[x]/2))/4 + (Cos[Pi/4 + x/2]^3*Sin[Pi/4 + x/2])/2
```

3.335.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.335.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result
risch	$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$
parallelrisch	$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$
derivativedivides	$\frac{\left(\cos^{3}\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$
default	$\frac{\left(\cos^{3}\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^{3}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3\left(\tan^{5}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}-\frac{5\left(\tan^{7}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{3x\left(\tan^{2}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+\frac{9x\left(\tan^{4}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{4}+\frac{3x\left(\tan^{6}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)}{2}+3x(\tan}{\left(1+\tan^{2}\left(\frac{\pi}{8}+\frac{x}{4}\right)\right)^{4}}$

input
```
int(cos(1/4*Pi+1/2*x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*x+1/2*cos(x)-1/16*sin(2*x)
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{1}{4}\left(2\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^3+3\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+\frac{3}{8}x$$

input
```
integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="fricas")
```

output
```
1/4*(2*cos(1/4*pi + 1/2*x)^3 + 3*cos(1/4*pi + 1/2*x))*sin(1/4*pi + 1/2*x)
+ 3/8*x
```

3.335.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(17) = 34.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3x\sin^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3x\sin^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{3x\cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3\sin^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{5\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}$$

input
```
integrate(cos(1/4*pi+1/2*x)**4,x)
```

output
```
3*x*sin(x/2 + pi/4)**4/8 + 3*x*sin(x/2 + pi/4)**2*cos(x/2 + pi/4)**2/4 + 3
*x*cos(x/2 + pi/4)**4/8 + 3*sin(x/2 + pi/4)**3*cos(x/2 + pi/4)/4 + 5*sin(x
/2 + pi/4)*cos(x/2 + pi/4)**3/4
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3}{16}\pi+\frac{3}{8}x+\frac{1}{16}\sin\left(\pi+2x\right)+\frac{1}{2}\sin\left(\frac{1}{2}\pi+x\right)$$

input
```
integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="maxima")
```

output
```
3/16*pi + 3/8*x + 1/16*sin(pi + 2*x) + 1/2*sin(1/2*pi + x)
```

3.335.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3}{8}\,x+\frac{1}{2}\,\cos\left(x\right)-\frac{1}{16}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/2*cos(x) - 1/16*sin(2*x)
```

3.335.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3\,x}{8}+\frac{\sin\left(\Pi+2\,x\right)}{16}+\frac{\sin\left(\frac{\Pi}{2}+x\right)}{2}$$

input
```
int(cos(Pi/4 + x/2)^4,x)
```

output
```
(3*x)/8 + sin(Pi + 2*x)/16 + sin(Pi/2 + x)/2
```

3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$

3.336.1 Optimal result

Integrand size = 14, antiderivative size = 31

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)$$

output
```
-1/3*sin(5/12*Pi+3*x)+1/9*sin(5/12*Pi+3*x)^3
```

3.336.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{4}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{36}\cos\left(3\left(\frac{\pi}{12}-3x\right)\right)$$

input
```
Integrate[-Sin[Pi/12 - 3*x]^3,x]
```

output
```
-1/4*Cos[Pi/12 - 3*x] + Cos[3*(Pi/12 - 3*x)]/36
```

3.336.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {25, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin^3\left(\frac{\pi}{12}-3x\right)dx$$

$$\Big\downarrow \; 3042$$

$$-\int \sin\left(\frac{\pi}{12}-3x\right)^3dx$$

$$\Big\downarrow \; 3113$$

$$-\frac{1}{3}\int \left(1-\cos^2\left(\frac{\pi}{12}-3x\right)\right)d\cos\left(\frac{\pi}{12}-3x\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{1}{3}\cos^3\left(\frac{\pi}{12}-3x\right)-\cos\left(\frac{\pi}{12}-3x\right)\right)$$

input
```
Int[-Sin[Pi/12 - 3*x]^3,x]
```

output
```
(-Cos[Pi/12 - 3*x] + Cos[Pi/12 - 3*x]^3/3)/3
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.336.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$	22
parallelrisch	$\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$	22
derivativedivides	$-\frac{\left(2+\cos^2\left(\frac{5\pi}{12}+3x\right)\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
default	$-\frac{\left(2+\cos^2\left(\frac{5\pi}{12}+3x\right)\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
norman	$\frac{-\frac{4\left(\tan^3\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{9}-\frac{2\left(\tan^5\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{3}-\frac{2\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)^3}$	51

input
```
int(-cos(5/12*Pi+3*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*sin(1/4*Pi+9*x)-1/4*sin(5/12*Pi+3*x)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{9}\left(\cos\left(\frac{5}{12}\,\pi+3\,x\right)^2+2\right)\sin\left(\frac{5}{12}\,\pi+3\,x\right)$$

input
```
integrate(-cos(5/12*pi+3*x)^3,x, algorithm="fricas")
```

output
```
-1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)
```

3.336.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = -\frac{2\sin^3\left(3x+\frac{5\pi}{12}\right)}{9} - \frac{\sin\left(3x+\frac{5\pi}{12}\right)\cos^2\left(3x+\frac{5\pi}{12}\right)}{3}$$

input
```
integrate(-cos(5/12*pi+3*x)**3,x)
```

output
```
-2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12)**2/3
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = \frac{1}{9}\sin\left(\frac{5}{12}\pi+3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi+3x\right)$$

input
```
integrate(-cos(5/12*pi+3*x)^3,x, algorithm="maxima")
```

output
```
1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = \frac{1}{9}\sin\left(\frac{5}{12}\pi+3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi+3x\right)$$

input
```
integrate(-cos(5/12*pi+3*x)^3,x, algorithm="giac")
```

output
```
1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

3.336.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=\frac{\sin\left(\frac{5\,\Pi}{12}+3\,x\right)\,\left(\sin\left(\frac{5\,\Pi}{12}+3\,x\right)^2-3\right)}{9}$$

input
```
int(-cos((5*Pi)/12 + 3*x)^3,x)
```

output
```
(sin((5*Pi)/12 + 3*x)*(sin((5*Pi)/12 + 3*x)^2 - 3))/9
```

3.337 $\int \csc^6(x)\,dx$

3.337.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \csc^6(x)\,dx = -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}$$

output
```
-cot(x)-2/3*cot(x)^3-1/5*cot(x)^5
```

3.337.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \csc^6(x)\,dx = -\frac{8\cot(x)}{15} - \frac{4}{15}\cot(x)\csc^2(x) - \frac{1}{5}\cot(x)\csc^4(x)$$

input
```
Integrate[Csc[x]^6,x]
```

output
```
(-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

3.337.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \csc(x)^6 dx$$
$$\Big\downarrow \ 4254$$
$$-\int \left(\cot^4(x)+2\cot^2(x)+1\right)d\cot(x)$$
$$\Big\downarrow \ 2009$$
$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

input
```
Int[Csc[x]^6,x]
```

output
```
-Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

3.337.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.337.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\left(-\frac{8}{15}-\frac{\left(\csc^{4}(x)\right)}{5}-\frac{4\left(\csc^{2}(x)\right)}{15}\right)\cot\left(x\right)$	18
parallelrisch	$-\frac{\cot(x)\left(\csc^{4}(x)\right)(8+\cos(4x)-6\cos(2x))}{15}$	21
risch	$-\frac{16i\left(10\,\mathrm{e}^{4ix}-5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}-1\right)^{5}}$	29
norman	$\frac{-\frac{1}{160}-\frac{5\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{96}-\frac{5\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{96}+\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{160}}{\tan\left(\frac{x}{2}\right)^{5}}$	50

input
```
int(1/sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
(-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)
```

3.337.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \csc^{6}(x)\,dx=-\frac{8\cos\left(x\right)^{5}-20\cos\left(x\right)^{3}+15\cos\left(x\right)}{15\left(\cos\left(x\right)^{4}-2\cos\left(x\right)^{2}+1\right)\sin\left(x\right)}$$

input
```
integrate(1/sin(x)^6,x, algorithm="fricas")
```

output
```
-1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*
sin(x))
```

3.337.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \csc^6(x)\,dx = -\frac{8\cos(x)}{15\sin(x)} - \frac{4\cos(x)}{15\sin^3(x)} - \frac{\cos(x)}{5\sin^5(x)}$$

input
```
integrate(1/sin(x)**6,x)
```

output
```
-8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^6(x)\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

input
```
integrate(1/sin(x)^6,x, algorithm="maxima")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

3.337.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^6(x)\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

input
```
integrate(1/sin(x)^6,x, algorithm="giac")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

3.337.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \csc^6(x)\,dx = -\frac{8\,\cos(x)\,\sin(x)^4 + 4\,\cos(x)\,\sin(x)^2 + 3\,\cos(x)}{15\sin(x)^5}$$

input
```
int(1/sin(x)^6,x)
```

output
```
-(3*cos(x) + 4*cos(x)*sin(x)^2 + 8*cos(x)*sin(x)^4)/(15*sin(x)^5)
```

3.338 $\int \csc^7(x)\,dx$

3.338.1 Optimal result

Integrand size = 4, antiderivative size = 36

$$\int \csc^7(x)\,dx = -\frac{5}{16}\text{arctanh}(\cos(x)) - \frac{5}{16}\cot(x)\csc(x) - \frac{5}{24}\cot(x)\csc^3(x) - \frac{1}{6}\cot(x)\csc^5(x)$$

output
```
-5/16*arctanh(cos(x))-5/16*cot(x)*csc(x)-5/24*cot(x)*csc(x)^3-1/6*cot(x)*c
sc(x)^5
```

3.338.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \csc^7(x)\,dx = -\frac{5}{64}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{1}{384}\csc^6\left(\frac{x}{2}\right) - \frac{5}{16}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{5}{16}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{5}{64}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right) + \frac{1}{384}\sec^6\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]^7,x]
```

output
```
(-5*Csc[x/2]^2)/64 - Csc[x/2]^4/64 - Csc[x/2]^6/384 - (5*Log[Cos[x/2]])/16
 + (5*Log[Sin[x/2]])/16 + (5*Sec[x/2]^2)/64 + Sec[x/2]^4/64 + Sec[x/2]^6/3
84
```

3.338.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 2.000$, Rules used = {3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^7dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\int \csc^5(x)dx-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \csc(x)^5dx-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

input
```
Int[Csc[x]^7,x]
```

output
```
-1/6*(Cot[x]*Csc[x]^5) + (5*(-1/4*(Cot[x]*Csc[x]^3) + (3*(-1/2*ArcTanh[Cos
[x]] - (Cot[x]*Csc[x])/2))/4))/6
```

3.338.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.338.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\left(-\frac{\left(\csc^{5}\left(x\right)\right)}{6}-\frac{5\left(\csc^{3}\left(x\right)\right)}{24}-\frac{5\csc\left(x\right)}{16}\right)\cot\left(x\right)+\frac{5\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)}{16}$	32
parallelrisch	$-\frac{75\left(\csc^{6}\left(x\right)\right)\left(\left(\cos\left(2x\right)-\frac{2\cos\left(4x\right)}{5}+\frac{\cos\left(6x\right)}{15}-\frac{2}{3}\right)\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)+\frac{44\cos\left(x\right)}{25}-\frac{34\cos\left(3x\right)}{45}+\frac{2\cos\left(5x\right)}{15}\right)}{512}$	51
norman	$\frac{-\frac{1}{384}-\frac{3\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{128}-\frac{15\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{128}+\frac{15\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{128}+\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^{6}}+\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16}$	58
risch	$\frac{15\,{\rm e}^{11ix}-85\,{\rm e}^{9ix}+198\,{\rm e}^{7ix}+198\,{\rm e}^{5ix}-85\,{\rm e}^{3ix}+15\,{\rm e}^{ix}}{24\left({\rm e}^{2ix}-1\right)^{6}}-\frac{5\ln\left({\rm e}^{ix}+1\right)}{16}+\frac{5\ln\left({\rm e}^{ix}-1\right)}{16}$	76

input
```
int(csc(x)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*csc(x)^5-5/24*csc(x)^3-5/16*csc(x))*cot(x)+5/16*ln(csc(x)-cot(x))
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

$$\int \csc^7(x)\,dx$$
$$= \frac{30\,\cos\left(x\right)^{5} - 80\,\cos\left(x\right)^{3} - 15\,\left(\cos\left(x\right)^{6} - 3\,\cos\left(x\right)^{4} + 3\,\cos\left(x\right)^{2} - 1\right)\log\left(\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right) + 15\,\left(\cos\left(x\right)^{6} -\right.}{96\,\left(\cos\left(x\right)^{6} - 3\,\cos\left(x\right)^{4} + 3\,\cos\left(x\right)^{2} - 1\right)}$$

input
```
integrate(csc(x)^7,x, algorithm="fricas")
```

output
```
1/96*(30*cos(x)^5 - 80*cos(x)^3 - 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 -
 1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*lo
g(-1/2*cos(x) + 1/2) + 66*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

3.338.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \csc^7(x)\,dx = -\frac{-15\cos^5(x) + 40\cos^3(x) - 33\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{5\log(\cos(x) - 1)}{32} - \frac{5\log(\cos(x) + 1)}{32}$$

input
```
integrate(csc(x)**7,x)
```

output
```
-(-15*cos(x)**5 + 40*cos(x)**3 - 33*cos(x))/(48*cos(x)**6 - 144*cos(x)**4
+ 144*cos(x)**2 - 48) + 5*log(cos(x) - 1)/32 - 5*log(cos(x) + 1)/32
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \csc^7(x)\,dx = \frac{15\,\cos(x)^5 - 40\,\cos(x)^3 + 33\,\cos(x)}{48\,(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1)} - \frac{5}{32}\,\log(\cos(x)+1) + \frac{5}{32}\,\log(\cos(x)-1)$$

input
```
integrate(csc(x)^7,x, algorithm="maxima")
```

output
```
1/48*(15*cos(x)^5 - 40*cos(x)^3 + 33*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*co
s(x)^2 - 1) - 5/32*log(cos(x) + 1) + 5/32*log(cos(x) - 1)
```

3.338.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(28) = 56.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \csc^7(x)\,dx = -\frac{\left(\frac{9\,(\cos(x)-1)}{\cos(x)+1} - \frac{45\,(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{110\,(\cos(x)-1)^3}{(\cos(x)+1)^3} - 1\right)(\cos(x)+1)^3}{384\,(\cos(x)-1)^3} - \frac{15\,(\cos(x)-1)}{128\,(\cos(x)+1)} + \frac{3\,(\cos(x)-1)^2}{128\,(\cos(x)+1)^2} - \frac{(\cos(x)-1)^3}{384\,(\cos(x)+1)^3} + \frac{5}{32}\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)^7,x, algorithm="giac")
```

output
```
-1/384*(9*(cos(x) - 1)/(cos(x) + 1) - 45*(cos(x) - 1)^2/(cos(x) + 1)^2 + 1
10*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1)*(cos(x) + 1)^3/(cos(x) - 1)^3 - 15/1
28*(cos(x) - 1)/(cos(x) + 1) + 3/128*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/384
*(cos(x) - 1)^3/(cos(x) + 1)^3 + 5/32*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.338.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \csc^7(x)\,dx = \frac{\frac{5\cos(x)^5}{16} - \frac{5\cos(x)^3}{6} + \frac{11\cos(x)}{16}}{\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1} - \frac{5\,\mathrm{atanh}(\cos(x))}{16}$$

input
```
int(1/sin(x)^7,x)
```

output
```
((11*cos(x))/16 - (5*cos(x)^3)/6 + (5*cos(x)^5)/16)/(3*cos(x)^2 - 3*cos(x)
^4 + cos(x)^6 - 1) - (5*atanh(cos(x)))/16
```

3.339 $\int \sec^{12}(x)\,dx$

3.339.1 Optimal result

Integrand size = 4, antiderivative size = 41

$$\int \sec^{12}(x)\,dx = \tan(x) + \frac{5\tan^3(x)}{3} + 2\tan^5(x) + \frac{10\tan^7(x)}{7} + \frac{5\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

output
```
tan(x)+5/3*tan(x)^3+2*tan(x)^5+10/7*tan(x)^7+5/9*tan(x)^9+1/11*tan(x)^11
```

3.339.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sec^{12}(x)\,dx = \tan(x) + \frac{5\tan^3(x)}{3} + 2\tan^5(x) + \frac{10\tan^7(x)}{7} + \frac{5\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

input
```
Integrate[Sec[x]^12,x]
```

output
```
Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 +
Tan[x]^11/11
```

3.339.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{12}(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(x+\frac{\pi}{2}\right)^{12}dx$$

$$\Big\downarrow \ 4254$$

$$-\int \left(\tan^{10}(x)+5\tan^8(x)+10\tan^6(x)+10\tan^4(x)+5\tan^2(x)+1\right)d(-\tan(x))$$

$$\Big\downarrow \ 2009$$

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

input
```
Int[Sec[x]^12,x]
```

output
```
Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 +
Tan[x]^11/11
```

3.339.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.339.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

method	result	size
default	$-\left(-\frac{256}{693}-\frac{\left(\sec^{10}(x)\right)}{11}-\frac{10\left(\sec^{8}(x)\right)}{99}-\frac{80\left(\sec^{6}(x)\right)}{693}-\frac{32\left(\sec^{4}(x)\right)}{231}-\frac{128\left(\sec^{2}(x)\right)}{693}\right)\tan\left(x\right)$	37
parallelrisch	$\frac{\tan(x)\left(\sec^{10}(x)\right)\left(512+\cos(10x)+12\cos(8x)+67\cos(6x)+232\cos(4x)+562\cos(2x)\right)}{1386}$	39
risch	$\frac{512i\left(462\,{\rm e}^{10ix}+330\,{\rm e}^{8ix}+165\,{\rm e}^{6ix}+55\,{\rm e}^{4ix}+11\,{\rm e}^{2ix}+1\right)}{693\left({\rm e}^{2ix}+1\right)^{11}}$	50

input
```
int(1/cos(x)^12,x,method=_RETURNVERBOSE)
```

output
```
-(-256/693-1/11*sec(x)^10-10/99*sec(x)^8-80/693*sec(x)^6-32/231*sec(x)^4-1
28/693*sec(x)^2)*tan(x)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \sec^{12}(x)\,dx = \frac{\left(256\,\cos\left(x\right)^{10}+128\,\cos\left(x\right)^{8}+96\,\cos\left(x\right)^{6}+80\,\cos\left(x\right)^{4}+70\,\cos\left(x\right)^{2}+63\right)\sin\left(x\right)}{693\,\cos\left(x\right)^{11}}$$

input
```
integrate(1/cos(x)^12,x, algorithm="fricas")
```

output
```
1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 + 70*cos(x
)^2 + 63)*sin(x)/cos(x)^11
```

3.339.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \sec^{12}(x)\,dx = \frac{256\sin(x)}{693\cos(x)} + \frac{128\sin(x)}{693\cos^3(x)} + \frac{32\sin(x)}{231\cos^5(x)} + \frac{80\sin(x)}{693\cos^7(x)} + \frac{10\sin(x)}{99\cos^9(x)} + \frac{\sin(x)}{11\cos^{11}(x)}$$

input
```
integrate(1/cos(x)**12,x)
```

output
```
256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/(231*cos(
x)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x)**9) + sin(x)/(11
*cos(x)**11)
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec^{12}(x)\,dx = \frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

input
```
integrate(1/cos(x)^12,x, algorithm="maxima")
```

output
```
1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3
+ tan(x)
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec^{12}(x)\,dx = \frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

input
```
integrate(1/cos(x)^12,x, algorithm="giac")
```

output
```
1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3
+ tan(x)
```

3.339.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sec^{12}(x)\,dx$$

$$= \frac{256\sin(x)\cos(x)^{10} + 128\sin(x)\cos(x)^{8} + 96\sin(x)\cos(x)^{6} + 80\sin(x)\cos(x)^{4} + 70\sin(x)\cos(x)^{2}}{693\cos(x)^{11}}$$

input
```
int(1/cos(x)^12,x)
```

output
```
(63*sin(x) + 70*cos(x)^2*sin(x) + 80*cos(x)^4*sin(x) + 96*cos(x)^6*sin(x)
+ 128*cos(x)^8*sin(x) + 256*cos(x)^10*sin(x))/(693*cos(x)^11)
```

3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$

3.340.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)$$

output
```
1/6*arctanh(sin(1/4*Pi+3*x))+1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)
```

3.340.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)$$

input
```
Integrate[Sec[Pi/4 + 3*x]^3,x]
```

output
```
ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

3.340.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3\left(3x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(3x+\frac{3\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\int \sec\left(3x+\frac{\pi}{4}\right)dx+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(3x+\frac{3\pi}{4}\right)dx+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{6}\text{arctanh}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

input
```
Int[Sec[Pi/4 + 3*x]^3,x]
```

output
```
ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

3.340.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.340.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$	40
default	$\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$	40
parallelrisch	$\frac{(1-\sin(6x))\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)+(\sin(6x)-1)\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)-2\sin\left(\frac{\pi}{4}+3x\right)}{6\sin(6x)-6}$	61
norman	$\frac{\left(\frac{\tan^3\left(\frac{\pi}{8}+\frac{3x}{2}\right)}{3}+\frac{\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)}{3}\right)}{\left(\tan^2\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)^2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)}{6}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)}{6}$	66
risch	$-\frac{i\left((-1)^{\frac{3}{4}}\mathrm{e}^{9ix}-(-1)^{\frac{1}{4}}\mathrm{e}^{3ix}\right)}{3(i\mathrm{e}^{6ix}+1)^2}-\frac{\ln\left((-1)^{\frac{1}{4}}\mathrm{e}^{3ix}-i\right)}{6}+\frac{\ln\left((-1)^{\frac{1}{4}}\mathrm{e}^{3ix}+i\right)}{6}$	67

input
```
int(1/cos(1/4*Pi+3*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/4*Pi+3*x)
)
```

3.340.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(30) = 60.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$$
$$=\frac{\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)+2\sin\left(\frac{1}{4}\pi+3x\right)}{12\cos\left(\frac{1}{4}\pi+3x\right)^2}$$

input
```
integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="fricas")
```

output
```
1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi + 3*x)^2
*log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(1/4*pi + 3*x)^2
```

3.340.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(29) = 58.

Time = 0.53 (sec) , antiderivative size = 388, normalized size of antiderivative = 9.70

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} + \frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} - \frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} + \frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} - \frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} + \frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} + \frac{2\tan^3\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6} + \frac{2\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}$$

input
```
integrate(1/cos(1/4*pi+3*x)**3,x)
```

output
```
-log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
 12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + p
i/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - log(tan(3
*x/2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) +
 log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
 12*tan(3*x/2 + pi/8)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + p
i/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3
*x/2 + pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) +
 2*tan(3*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2
+ 6) + 2*tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)*
*2 + 6)
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)-1\right)$$

input
```
integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="maxima")
```

output
```
-1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3
*x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)$$

input
```
integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="giac")
```

output
```
-1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3
*x) + 1) - 1/12*log(-sin(1/4*pi + 3*x) + 1)
```

3.340.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = \frac{\ln\left(\tan\left(\frac{\Pi}{8}+\frac{3x}{2}+\frac{\pi}{4}\right)\right)}{6}+\frac{\tan\left(\frac{\Pi}{4}+3x\right)}{6\cos\left(\frac{\Pi}{4}+3x\right)}$$

input
```
int(1/cos(Pi/4 + 3*x)^3,x)
```

output
```
log(tan(Pi/8 + (3*x)/2 + pi/4))/6 + tan(Pi/4 + 3*x)/(6*cos(Pi/4 + 3*x))
```

3.341 $\int \tan^6(x)\,dx$

3.341.1 Optimal result

Integrand size = 4, antiderivative size = 22

$$\int \tan^6(x)\,dx = -x+\tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}$$

output
```
-x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

3.341.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \tan^6(x)\,dx = -\arctan(\tan(x))+\tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}$$

input
```
Integrate[Tan[x]^6,x]
```

output
```
-ArcTan[Tan[x]] + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

3.341.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^6dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^5(x)}{5}-\int \tan^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^5(x)}{5}-\int \tan(x)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^2(x)dx+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2dx+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

$$\Big\downarrow \text{24}$$

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

input
```
Int[Tan[x]^6,x]
```

output
```
-x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

3.341.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.341.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
norman	$-x+\tan(x)-\frac{(\tan^3(x))}{3}+\frac{(\tan^5(x))}{5}$	19
parallelrisch	$-x+\tan(x)-\frac{(\tan^3(x))}{3}+\frac{(\tan^5(x))}{5}$	19
derivativedivides	$\frac{(\tan^5(x))}{5}-\frac{(\tan^3(x))}{3}+\tan(x)-\arctan(\tan(x))$	21
default	$\frac{(\tan^5(x))}{5}-\frac{(\tan^3(x))}{3}+\tan(x)-\arctan(\tan(x))$	21
risch	$-x+\frac{2i(45\,e^{8ix}+90\,e^{6ix}+140\,e^{4ix}+70\,e^{2ix}+23)}{15(e^{2ix}+1)^5}$	47

input
```
int(tan(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^5 - \frac{1}{3}\,\tan\left(x\right)^3 - x + \tan\left(x\right)$$

input
```
integrate(tan(x)^6,x, algorithm="fricas")
```

output
```
1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.341.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \tan^6(x)\,dx = -x + \frac{\sin^5\left(x\right)}{5\cos^5\left(x\right)} - \frac{\sin^3\left(x\right)}{3\cos^3\left(x\right)} + \frac{\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
integrate(tan(x)**6,x)
```

output
```
-x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^5 - \frac{1}{3}\,\tan\left(x\right)^3 - x + \tan\left(x\right)$$

input
```
integrate(tan(x)^6,x, algorithm="maxima")
```

output
```
1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\, dx = \frac{1}{5}\tan{(x)}^5 - \frac{1}{3}\tan{(x)}^3 - x + \tan{(x)}$$

input
```
integrate(tan(x)^6,x, algorithm="giac")
```

output
```
1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\, dx = \frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

input
```
int(tan(x)^6,x)
```

output
```
tan(x) - x - tan(x)^3/3 + tan(x)^5/5
```

3.342 $\int \cot^5(x)\,dx$

3.342.1 Optimal result

Integrand size = 4, antiderivative size = 20

$$\int \cot^5(x)\,dx = \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\sin(x))$$

output
```
1/2*cot(x)^2-1/4*cot(x)^4+ln(sin(x))
```

3.342.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \cot^5(x)\,dx = \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\cos(x)) + \log(\tan(x))$$

input
```
Integrate[Cot[x]^5,x]
```

output
```
Cot[x]^2/2 - Cot[x]^4/4 + Log[Cos[x]] + Log[Tan[x]]
```

3.342.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 2.750$, Rules used = {3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\int -\cot^3(x)dx-\frac{\cot^4(x)}{4}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot^3(x)dx-\frac{1}{4}\cot^4(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int -\tan\left(x+\frac{\pi}{2}\right)^3dx-\frac{1}{4}\cot^4(x)$$

$$\Big\downarrow \text{25}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^3dx-\frac{\cot^4(x)}{4}$$

$$\Big\downarrow \text{3954}$$

$$-\int -\cot(x)dx-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(x)dx-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}+\log(\sin(x))$$

input
```
Int[Cot[x]^5,x]
```

output
```
Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.342.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}-\frac{\ln(1+\tan^2(x))}{2}$	26
default	$-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}-\frac{\ln(1+\tan^2(x))}{2}$	26
norman	$\frac{-\frac{1}{4}+\frac{(\tan^2(x))}{2}}{\tan(x)^4}-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	27
parallelrisch	$\frac{4\ln(\tan(x))(\tan^4(x))-2\ln(1+\tan^2(x))(\tan^4(x))-1+2(\tan^2(x))}{4\tan(x)^4}$	37
risch	$-ix-\frac{4(e^{6ix}-e^{4ix}+e^{2ix})}{(e^{2ix}-1)^4}+\ln(e^{2ix}-1)$	43

input
```
int(1/tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2-1/2*ln(1+tan(x)^2)
```

3.342.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \cot^5(x)\,dx = \frac{2\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^4+3\tan(x)^4+2\tan(x)^2-1}{4\tan(x)^4}$$

input
```
integrate(1/tan(x)^5,x, algorithm="fricas")
```

output
```
1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan(x)^2 - 1
)/tan(x)^4
```

3.342.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \cot^5(x)\,dx = \frac{4\sin^2(x) - 1}{4\sin^4(x)} + \log(\sin(x))$$

input
```
integrate(1/tan(x)**5,x)
```

output
```
(4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \cot^5(x)\,dx = \frac{4\sin(x)^2 - 1}{4\sin(x)^4} + \frac{1}{2}\log\left(\sin(x)^2\right)$$

input
```
integrate(1/tan(x)^5,x, algorithm="maxima")
```

output
```
1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)
```

3.342.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cot^5(x)\,dx = -\frac{3\tan(x)^4 - 2\tan(x)^2 + 1}{4\tan(x)^4} - \frac{1}{2}\log\left(\tan(x)^2 + 1\right) + \frac{1}{2}\log\left(\tan(x)^2\right)$$

input
```
integrate(1/tan(x)^5,x, algorithm="giac")
```

output
```
-1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*log(tan(x)^2 + 1) + 1/2*
log(tan(x)^2)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \cot^5(x)\,dx = \ln(\tan(x)) - \frac{\ln\left(\tan(x)^2+1\right)}{2} + \frac{\frac{\tan(x)^2}{2} - \frac{1}{4}}{\tan(x)^4}$$

input
```
int(1/tan(x)^5,x)
```

output
```
log(tan(x)) - log(tan(x)^2 + 1)/2 + (tan(x)^2/2 - 1/4)/tan(x)^4
```

3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$

3.343.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)$$

output
```
x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3
```

3.343.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = -\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$$

input
```
Integrate[Cot[Pi/4 + x/3]^4,x]
```

output
```
-(Cot[Pi/4 + x/3]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[Pi/4 + x/3]^2])
```

3.343.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4\left(\frac{x}{3}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(\frac{x}{3}+\frac{3\pi}{4}\right)^4dx$$

$$\Big\downarrow \text{3954}$$

$$-\int \cot^2\left(\frac{x}{3}+\frac{\pi}{4}\right)dx-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan\left(\frac{x}{3}+\frac{3\pi}{4}\right)^2dx-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{24}$$

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

input
```
Int[Cot[Pi/4 + x/3]^4,x]
```

output
```
x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

3.343.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.343.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\left(\cot^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	25
derivativedivides	$-\left(\cot^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\,\operatorname{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	38
default	$-\left(\cot^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\,\operatorname{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$	38
norman	$\frac{-1+x\left(\tan^{3}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)+3\left(\tan^{2}\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}}$	38
risch	$x+\frac{4i\left(-3\,{\mathrm e}^{\frac{4ix}{3}}-3i\,{\mathrm e}^{\frac{2ix}{3}}+2\right)}{\left({\mathrm e}^{\frac{i(3\pi+4x)}{6}}-1\right)^{3}}$	38

input
```
int(cot(1/4*Pi+1/3*x)^4,x,method=_RETURNVERBOSE)
```

output
```
x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{4\,\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)^2+\left(x\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-x\right)\sin\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)+2\,\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-2}{\left(\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-1\right)\sin\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)}$$

input
```
integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="fricas")
```

output
```
(4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi + 2/3*x)
 + 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)*sin(1/2*pi + 2/3*
x))
```

3.343.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

input
```
integrate(cot(1/4*pi+1/3*x)**4,x)
```

output
```
x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{3}{4}\pi+x+\frac{3\,\tan\left(\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)^2-1}{\tan\left(\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)^3}$$

input
```
integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="maxima")
```

output
```
3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3
```

3.343.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{3}{4}\,\pi + \frac{1}{8}\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^3 + x + \frac{15\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^2 - 1}{8\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^3} - \frac{15}{8}\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)$$

input
```
integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="giac")
```

output
```
3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6*x)^2 - 1
)/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = -\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)^3 + 3\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right) + x$$

input
```
int(cot(Pi/4 + x/3)^4,x)
```

output
```
x + 3*cot(Pi/4 + x/3) - cot(Pi/4 + x/3)^3
```

3.344 $\int \cos^6(x)\sin^4(x)\,dx$

3.344.1 Optimal result

Integrand size = 9, antiderivative size = 56

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256}+\frac{3}{256}\cos(x)\sin(x)+\frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)$$

output
```
3/256*x+3/256*cos(x)*sin(x)+1/128*cos(x)^3*sin(x)+1/160*cos(x)^5*sin(x)-3/
80*cos(x)^7*sin(x)-1/10*cos(x)^7*sin(x)^3
```

3.344.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256}+\frac{1}{512}\sin(2x)-\frac{1}{256}\sin(4x)-\frac{\sin(6x)}{1024}+\frac{\sin(8x)}{2048}+\frac{\sin(10x)}{5120}$$

input
```
Integrate[Cos[x]^6*Sin[x]^4,x]
```

output
```
(3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x]/2048 +
Sin[10*x]/5120
```

3.344.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^4\cos(x)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\int \cos^6(x)\sin^2(x)dx-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \cos(x)^6\sin(x)^2dx-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \cos^6(x)dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \; 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \; 24$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

input
```
Int[Cos[x]^6*Sin[x]^4,x]
```

output
```
-1/10*(Cos[x]^7*Sin[x]^3) + (3*(-1/8*(Cos[x]^7*Sin[x]) + ((Cos[x]^5*Sin[x]
)/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4))/6)/8))/1
0
```

3.344.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.344.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$	35
parallelrisch	$\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$	35
default	$-\frac{(\cos^7(x))(\sin^3(x))}{10}-\frac{3(\cos^7(x))\sin(x)}{80}+\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160}+\frac{3x}{256}$	42

input
```
int(cos(x)^6*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/256*x+1/5120*sin(10*x)+1/2048*sin(8*x)-1/1024*sin(6*x)-1/256*sin(4*x)+1/
512*sin(2*x)
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \cos^6(x)\sin^4(x)\,dx$$
$$=\frac{1}{1280}\left(128\cos(x)^9-176\cos(x)^7+8\cos(x)^5+10\cos(x)^3+15\cos(x)\right)\sin(x)+\frac{3}{256}x$$

input
```
integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")
```

output
```
1/1280*(128*cos(x)^9 - 176*cos(x)^7 + 8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x)
)*sin(x) + 3/256*x
```

3.344.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} + \frac{\sin(x)\cos^9(x)}{10} - \frac{11\sin(x)\cos^7(x)}{80} + \frac{\sin(x)\cos^5(x)}{160} + \frac{\sin(x)\cos^3(x)}{128} + \frac{3\sin(x)\cos(x)}{256}$$

input
```
integrate(cos(x)**6*sin(x)**4,x)
```

output
```
3*x/256 + sin(x)*cos(x)**9/10 - 11*sin(x)*cos(x)**7/80 + sin(x)*cos(x)**5/
160 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{1}{320}\sin(2x)^5 + \frac{3}{256}x + \frac{1}{2048}\sin(8x) - \frac{1}{256}\sin(4x)$$

input
```
integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")
```

output
```
1/320*sin(2*x)^5 + 3/256*x + 1/2048*sin(8*x) - 1/256*sin(4*x)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3}{256}x + \frac{1}{5120}\sin(10x) + \frac{1}{2048}\sin(8x) - \frac{1}{1024}\sin(6x) - \frac{1}{256}\sin(4x) + \frac{1}{512}\sin(2x)$$

input
```
integrate(cos(x)^6*sin(x)^4,x, algorithm="giac")
```

output
```
3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) - 1/256*sin
(4*x) + 1/512*sin(2*x)
```

3.344.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \cos^6(x)\sin^4(x)\,dx = \left(\frac{\cos(x)^5}{10}+\frac{\cos(x)^3}{16}+\frac{\cos(x)}{32}\right)\sin(x)^5+\frac{3x}{256}-\frac{\sin(2x)}{128}+\frac{\sin(4x)}{1024}$$

input
```
int(cos(x)^6*sin(x)^4,x)
```

output
```
(3*x)/256 - sin(2*x)/128 + sin(4*x)/1024 + sin(x)^5*(cos(x)/32 + cos(x)^3/
16 + cos(x)^5/10)
```

3.345 $\int \cos^6(x) \sin^7(x) \, dx$

3.345.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos^6(x) \sin^7(x) \, dx = -\frac{1}{7} \cos^7(x) + \frac{\cos^9(x)}{3} - \frac{3 \cos^{11}(x)}{11} + \frac{\cos^{13}(x)}{13}$$

output
```
-1/7*cos(x)^7+1/3*cos(x)^9-3/11*cos(x)^11+1/13*cos(x)^13
```

3.345.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \cos^6(x) \sin^7(x) \, dx = -\frac{5 \cos(x)}{1024} - \frac{5 \cos(3x)}{4096} + \frac{3 \cos(5x)}{4096} + \frac{3 \cos(7x)}{14336} - \frac{\cos(9x)}{6144} - \frac{\cos(11x)}{45056} + \frac{\cos(13x)}{53248}$$

input
```
Integrate[Cos[x]^6*Sin[x]^7,x]
```

output
```
(-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos[7*x])/14
336 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

3.345.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(x)\cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^7\cos(x)^6dx$$

$$\Big\downarrow \text{3045}$$

$$-\int \cos^6(x)\left(1-\cos^2(x)\right)^3d\cos(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(-\cos^{12}(x)+3\cos^{10}(x)-3\cos^8(x)+\cos^6(x)\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

input
```
Int[Cos[x]^6*Sin[x]^7,x]
```

output
```
-1/7*Cos[x]^7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

3.345.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.345.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{(\cos^7(x))}{7}+\frac{(\cos^9(x))}{3}-\frac{3(\cos^{11}(x))}{11}+\frac{(\cos^{13}(x))}{13}$	26
default	$-\frac{(\cos^7(x))}{7}+\frac{(\cos^9(x))}{3}-\frac{3(\cos^{11}(x))}{11}+\frac{(\cos^{13}(x))}{13}$	26
risch	$-\frac{5\cos(x)}{1024}+\frac{\cos(13x)}{53248}-\frac{\cos(11x)}{45056}-\frac{\cos(9x)}{6144}+\frac{3\cos(7x)}{14336}+\frac{3\cos(5x)}{4096}-\frac{5\cos(3x)}{4096}$	42
parallelrisch	$\frac{320}{3003}-\frac{5\cos(x)}{1024}+\frac{\cos(13x)}{53248}-\frac{\cos(11x)}{45056}-\frac{\cos(9x)}{6144}+\frac{3\cos(7x)}{14336}+\frac{3\cos(5x)}{4096}-\frac{5\cos(3x)}{4096}$	43

input
```
int(cos(x)^6*sin(x)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7*cos(x)^7+1/3*cos(x)^9-3/11*cos(x)^11+1/13*cos(x)^13
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

input
```
integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")
```

output
```
1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos^{13}(x)}{13} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^9(x)}{3} - \frac{\cos^7(x)}{7}$$

input
```
integrate(cos(x)**6*sin(x)**7,x)
```

output
```
cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

input
```
integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")
```

output
```
1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

input
```
integrate(cos(x)^6*sin(x)^7,x, algorithm="giac")
```

output
```
1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos(x)^{13}}{13} - \frac{3\cos(x)^{11}}{11} + \frac{\cos(x)^9}{3} - \frac{\cos(x)^7}{7}$$

input
```
int(cos(x)^6*sin(x)^7,x)
```

output
```
cos(x)^9/3 - cos(x)^7/7 - (3*cos(x)^11)/11 + cos(x)^13/13
```

3.346 $\int \sin^{10}(x)\tan(x)\,dx$

3.346.1 Optimal result

Integrand size = 7, antiderivative size = 46

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{5\cos^2(x)}{2} - \frac{5\cos^4(x)}{2} + \frac{5\cos^6(x)}{3} - \frac{5\cos^8(x)}{8} + \frac{\cos^{10}(x)}{10} - \log(\cos(x))$$

output
```
5/2*cos(x)^2-5/2*cos(x)^4+5/3*cos(x)^6-5/8*cos(x)^8+1/10*cos(x)^10-ln(cos(
x))
```

3.346.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{5\cos^2(x)}{2} - \frac{5\cos^4(x)}{2} + \frac{5\cos^6(x)}{3} - \frac{5\cos^8(x)}{8} + \frac{\cos^{10}(x)}{10} - \log(\cos(x))$$

input
```
Integrate[Sin[x]^10*Tan[x],x]
```

output
```
(5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]
^10/10 - Log[Cos[x]]
```

3.346.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(x)\tan(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)^{10}\tan(x)dx$$

$$\Big\downarrow \ 3070$$

$$-\int \left(1-\cos^2(x)\right)^5\sec(x)d\cos(x)$$

$$\Big\downarrow \ 243$$

$$-\frac{1}{2}\int \left(1-\cos^2(x)\right)^5\sec(x)d\cos^2(x)$$

$$\Big\downarrow \ 49$$

$$-\frac{1}{2}\int \left(-\cos^8(x)+5\cos^6(x)-10\cos^4(x)+10\cos^2(x)+\sec(x)-5\right)d\cos^2(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\cos^{10}(x)}{5}-\frac{5\cos^8(x)}{4}+\frac{10\cos^6(x)}{3}-5\cos^4(x)+5\cos^2(x)-\log\left(\cos^2(x)\right)\right)$$

input
```
Int[Sin[x]^10*Tan[x],x]
```

output
```
(5*Cos[x]^2 - 5*Cos[x]^4 + (10*Cos[x]^6)/3 - (5*Cos[x]^8)/4 + Cos[x]^10/5
- Log[Cos[x]^2])/2
```

3.346.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.346.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\left(\sin^{10}\left(x\right)\right)}{10}-\frac{\left(\sin^{8}\left(x\right)\right)}{8}-\frac{\left(\sin^{6}\left(x\right)\right)}{6}-\frac{\left(\sin^{4}\left(x\right)\right)}{4}-\frac{\left(\sin^{2}\left(x\right)\right)}{2}-\ln\left(\cos\left(x\right)\right)$
risch	$ix+\frac{281\,\mathrm{e}^{2ix}}{1024}+\frac{281\,\mathrm{e}^{-2ix}}{1024}-\ln\left(\mathrm{e}^{2ix}+1\right)+\frac{\cos(10x)}{5120}-\frac{3\cos(8x)}{1024}+\frac{67\cos(6x)}{3072}-\frac{29\cos(4x)}{256}$
parallelrisch	$-\frac{469}{46080}+\ln\left(\frac{2}{\cos(x)+1}\right)-\ln\left(-\cot\left(x\right)+1+\csc\left(x\right)\right)-\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)+\frac{\cos(10x)}{5120}$

input
```
int(sin(x)^11/cos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/10*sin(x)^10-1/8*sin(x)^8-1/6*sin(x)^6-1/4*sin(x)^4-1/2*sin(x)^2-ln(cos
(x))
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{1}{10}\cos(x)^{10} - \frac{5}{8}\cos(x)^{8} + \frac{5}{3}\cos(x)^{6} - \frac{5}{2}\cos(x)^{4} + \frac{5}{2}\cos(x)^{2} - \log(-\cos(x))$$

input
```
integrate(sin(x)^11/cos(x),x, algorithm="fricas")
```

output
```
1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2*cos(x)^2
 - log(-cos(x))
```

3.346.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \sin^{10}(x)\tan(x)\,dx = -\log(\cos(x)) + \frac{\cos^{10}(x)}{10} - \frac{5\cos^{8}(x)}{8} + \frac{5\cos^{6}(x)}{3} - \frac{5\cos^{4}(x)}{2} + \frac{5\cos^{2}(x)}{2}$$

input
```
integrate(sin(x)**11/cos(x),x)
```

output
```
-log(cos(x)) + cos(x)**10/10 - 5*cos(x)**8/8 + 5*cos(x)**6/3 - 5*cos(x)**4
/2 + 5*cos(x)**2/2
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(\sin(x)^{2} - 1\right)$$

input
```
integrate(sin(x)^11/cos(x),x, algorithm="maxima")
```

output
```
-1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^
2 - 1/2*log(sin(x)^2 - 1)
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(-\sin(x)^{2}+1\right)$$

input
```
integrate(sin(x)^11/cos(x),x, algorithm="giac")
```

output
```
-1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^
2 - 1/2*log(-sin(x)^2 + 1)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{\sin(x)^{10}}{10} - \frac{\sin(x)^{8}}{8} - \frac{\sin(x)^{6}}{6} - \frac{\sin(x)^{4}}{4} - \frac{\sin(x)^{2}}{2} - \ln(\cos(x))$$

input
```
int(sin(x)^11/cos(x),x)
```

output
```
- log(cos(x)) - sin(x)^2/2 - sin(x)^4/4 - sin(x)^6/6 - sin(x)^8/8 - sin(x)
^10/10
```

3.347 $\int \csc^6(x) \sec^6(x) \, dx$

3.347.1 Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \csc^6(x) \sec^6(x) \, dx = -10 \cot(x) - \frac{5 \cot^3(x)}{3} - \frac{\cot^5(x)}{5} + 10 \tan(x) + \frac{5 \tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

output
```
-10*cot(x)-5/3*cot(x)^3-1/5*cot(x)^5+10*tan(x)+5/3*tan(x)^3+1/5*tan(x)^5
```

3.347.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \csc^6(x) \sec^6(x) \, dx = -\frac{128 \cot(x)}{15} - \frac{19}{15} \cot(x) \csc^2(x) - \frac{1}{5} \cot(x) \csc^4(x) + \frac{128 \tan(x)}{15} + \frac{19}{15} \sec^2(x) \tan(x) + \frac{1}{5} \sec^4(x) \tan(x)$$

input
```
Integrate[Csc[x]^6*Sec[x]^6,x]
```

output
```
(-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5 + (128*Ta
n[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

3.347.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(x)\sec^6(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(x)^6\sec(x)^6dx$$

$$\Big\downarrow \; 3100$$

$$\int \left(\tan^2(x)+1\right)^5\cot^6(x)d\tan(x)$$

$$\Big\downarrow \; 244$$

$$\int \left(\tan^4(x)+5\tan^2(x)+\cot^6(x)+5\cot^4(x)+10\cot^2(x)+10\right)d\tan(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

input
```
Int[Csc[x]^6*Sec[x]^6,x]
```

output
```
-10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Ta
n[x]^5/5
```

3.347.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.347.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$-\frac{(\sec^5(x))(\csc^5(x))(\cos(10x)-5\cos(6x)+10\cos(2x))}{30}$	28
risch	$-\frac{512i(10\,e^{8ix}-5\,e^{4ix}+1)}{15(e^{2ix}-1)^5(e^{2ix}+1)^5}$	38
default	$\frac{1}{5\sin(x)^5\cos(x)^5}-\frac{2}{5\sin(x)^5\cos(x)^3}+\frac{16}{15\sin(x)^3\cos(x)^3}-\frac{32}{15\sin(x)^3\cos(x)}+\frac{128}{15\cos(x)\sin(x)}-\frac{256\cot(x)}{15}$	56

input
```
int(1/cos(x)^6/sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/30*sec(x)^5*csc(x)^5*(cos(10*x)-5*cos(6*x)+10*cos(2*x))
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{256\cos(x)^{10}-640\cos(x)^8+480\cos(x)^6-80\cos(x)^4-10\cos(x)^2-3}{15\left(\cos(x)^9-2\cos(x)^7+\cos(x)^5\right)\sin(x)}$$

input
```
integrate(1/cos(x)^6/sin(x)^6,x, algorithm="fricas")
```

output
```
-1/15*(256*cos(x)^10 - 640*cos(x)^8 + 480*cos(x)^6 - 80*cos(x)^4 - 10*cos(
x)^2 - 3)/((cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x))
```

3.347.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{256\cos(2x)}{15\sin(2x)} - \frac{128\cos(2x)}{15\sin^3(2x)} - \frac{32\cos(2x)}{5\sin^5(2x)}$$

input
```
integrate(1/cos(x)**6/sin(x)**6,x)
```

output
```
-256*cos(2*x)/(15*sin(2*x)) - 128*cos(2*x)/(15*sin(2*x)**3) - 32*cos(2*x)/
(5*sin(2*x)**5)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \csc^6(x)\sec^6(x)\,dx = \frac{1}{5}\tan(x)^5 + \frac{5}{3}\tan(x)^3 - \frac{150\tan(x)^4 + 25\tan(x)^2 + 3}{15\tan(x)^5} + 10\tan(x)$$

input
```
integrate(1/cos(x)^6/sin(x)^6,x, algorithm="maxima")
```

output
```
1/5*tan(x)^5 + 5/3*tan(x)^3 - 1/15*(150*tan(x)^4 + 25*tan(x)^2 + 3)/tan(x)
^5 + 10*tan(x)
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{32\left(15\tan(2x)^4 + 10\tan(2x)^2 + 3\right)}{15\tan(2x)^5}$$

input
```
integrate(1/cos(x)^6/sin(x)^6,x, algorithm="giac")
```

output
```
-32/15*(15*tan(2*x)^4 + 10*tan(2*x)^2 + 3)/tan(2*x)^5
```

3.347.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{32\left(\frac{\cos(2x)}{3}-\frac{\cos(6x)}{6}+\frac{\cos(10x)}{30}\right)}{\sin(2x)^5}$$

input
```
int(1/(cos(x)^6*sin(x)^6),x)
```

output
```
-(32*(cos(2*x)/3 - cos(6*x)/6 + cos(10*x)/30))/sin(2*x)^5
```

3.348 $\int \cos^2(x) \sin^2(x) \, dx$

3.348.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x) \sin^2(x) \, dx = \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)$$

output
```
1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

3.348.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x) \sin^2(x) \, dx = \frac{x}{8} - \frac{1}{32} \sin(4x)$$

input
```
Integrate[Cos[x]^2*Sin[x]^2,x]
```

output
```
x/8 - Sin[4*x]/32
```

3.348.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2\cos(x)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

input
```
Int[Cos[x]^2*Sin[x]^2,x]
```

output
```
-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4
```

3.348.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.348.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos^3(x))\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

input
```
int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x-1/32*sin(4*x)
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\,\cos\left(x\right)^3-\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{8}\,x$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

3.348.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8}-\frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

input
```
integrate(cos(x)**2*sin(x)**2,x)
```

output
```
x/8 - sin(2*x)*cos(2*x)/16
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
1/8*x - 1/32*sin(4*x)
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
1/8*x - 1/32*sin(4*x)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos\left(x\right)\,\sin\left(x\right)^3}{4} - \frac{\cos\left(x\right)\,\sin\left(x\right)}{8} + \frac{x}{8}$$

input
```
int(cos(x)^2*sin(x)^2,x)
```

output
```
x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

3.349 $\int \cos^4(x)\sin^4(x)\,dx$

3.349.1 Optimal result

Integrand size = 9, antiderivative size = 46

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x) - \frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)$$

output
```
3/128*x+3/128*cos(x)*sin(x)+1/64*cos(x)^3*sin(x)-1/16*cos(x)^5*sin(x)-1/8*
cos(x)^5*sin(x)^3
```

3.349.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

input
```
Integrate[Cos[x]^4*Sin[x]^4,x]
```

output
```
(3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

3.349.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\cos^4(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)^4\cos(x)^4dx$$

$$\Big\downarrow \text{ 3048}$$

$$\frac{3}{8}\int \cos^4(x)\sin^2(x)dx-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{8}\int \cos(x)^4\sin(x)^2dx-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 3048}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \cos^4(x)dx-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

input
```
Int[Cos[x]^4*Sin[x]^4,x]
```

output
```
-1/8*(Cos[x]^5*Sin[x]^3) + (3*(-1/6*(Cos[x]^5*Sin[x]) + ((Cos[x]^3*Sin[x])
/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)/6))/8
```

3.349.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.349.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

method	result
risch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
parallelrisch	$\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$
default	$-\frac{(\cos^5(x))(\sin^3(x))}{8}-\frac{(\cos^5(x))\sin(x)}{16}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{64}+\frac{3x}{128}$
norman	$\frac{\frac{3x}{128}-\frac{23\left(\tan^3\left(\frac{x}{2}\right)\right)}{64}+\frac{333\left(\tan^5\left(\frac{x}{2}\right)\right)}{64}-\frac{671\left(\tan^7\left(\frac{x}{2}\right)\right)}{64}+\frac{671\left(\tan^9\left(\frac{x}{2}\right)\right)}{64}-\frac{333\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{64}+\frac{23\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{64}+\frac{3\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{64}+\frac{3x\left(\tan^2\left(\right.\right.}{16}}{(1+}$

input
```
int(cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/128*x+1/1024*sin(8*x)-1/128*sin(4*x)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{1}{128}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{128}x$$

input
```
integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

output
```
1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x
```

3.349.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128}-\frac{\sin^3(2x)\cos(2x)}{128}-\frac{3\sin(2x)\cos(2x)}{256}$$

input
```
integrate(cos(x)**4*sin(x)**4,x)
```

output
```
3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

input
```
integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

output
```
3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

input
```
integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
```

output
```
3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cos^4(x)\sin^4(x)\,dx = \left(\frac{\cos\left(x\right)^3}{8} + \frac{\cos\left(x\right)}{16}\right)\,\sin\left(x\right)^5 + \frac{3\,x}{128} - \frac{\sin\left(2\,x\right)}{64} + \frac{\sin\left(4\,x\right)}{512}$$

input
```
int(cos(x)^4*sin(x)^4,x)
```

output
```
(3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

3.350 $\int \cos^6(x)\sin^6(x)\,dx$

3.350.1 Optimal result

Integrand size = 9, antiderivative size = 68

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024} + \frac{5\cos(x)\sin(x)}{1024} + \frac{5\cos^3(x)\sin(x)}{1536} + \frac{1}{384}\cos^5(x)\sin(x) - \frac{1}{64}\cos^7(x)\sin(x) - \frac{1}{24}\cos^7(x)\sin^3(x) - \frac{1}{12}\cos^7(x)\sin^5(x)$$

output
```
5/1024*x+5/1024*cos(x)*sin(x)+5/1536*cos(x)^3*sin(x)+1/384*cos(x)^5*sin(x)
-1/64*cos(x)^7*sin(x)-1/24*cos(x)^7*sin(x)^3-1/12*cos(x)^7*sin(x)^5
```

3.350.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024} - \frac{15\sin(4x)}{8192} + \frac{3\sin(8x)}{8192} - \frac{\sin(12x)}{24576}$$

input
```
Integrate[Cos[x]^6*Sin[x]^6,x]
```

output
```
(5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/24576
```

3.350.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.444$, Rules used = {3042, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(x)\cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^6\cos(x)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{5}{12}\int \cos^6(x)\sin^4(x)dx-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{12}\int \cos(x)^6\sin(x)^4dx-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{5}{12}\left(\frac{3}{10}\int \cos^6(x)\sin^2(x)dx-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{12}\left(\frac{3}{10}\int \cos(x)^6\sin(x)^2dx-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \cos^6(x)dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\downarrow 3042$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\downarrow 3115$$

$$\frac{5}{12}\Bigg(\frac{3}{10}\Bigg(\frac{1}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\Bigg)+\frac{1}{6}\sin(x)\cos^5(x)\Bigg)-\frac{1}{8}\sin(x)\cos^7(x)\Bigg)-\frac{1}{10}\sin^3(x)\cos^7(x)$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\downarrow 3042$$

$$\frac{5}{12}\Bigg(\frac{3}{10}\Bigg(\frac{1}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\Bigg)+\frac{1}{6}\sin(x)\cos^5(x)\Bigg)-\frac{1}{8}\sin(x)\cos^7(x)\Bigg)-\frac{1}{10}\sin^3(x)$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\downarrow 3115$$

$$\frac{5}{12}\Bigg(\frac{3}{10}\Bigg(\frac{1}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\Bigg)+\frac{1}{6}\sin(x)\cos^5(x)\Bigg)-\frac{1}{8}\sin(x)\cos^7(x)\Bigg)-\frac{1}{10}$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\downarrow 24$$

$$\frac{5}{12}\Bigg(\frac{3}{10}\Bigg(\frac{1}{8}\Bigg(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\Bigg)-\frac{1}{8}\sin(x)\cos^7(x)\Bigg)-\frac{1}{10}$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

input
```
Int[Cos[x]^6*Sin[x]^6,x]
```

output
```
-1/12*(Cos[x]^7*Sin[x]^5) + (5*(-1/10*(Cos[x]^7*Sin[x]^3) + (3*(-1/8*(Cos[
x]^7*Sin[x]) + ((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 +
(Cos[x]*Sin[x])/2))/4))/6)/8))/10))/12
```

3.350. $\int \cos^6(x)\sin^6(x)\,dx$

3.350.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.350.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

method	result	s
risch	$\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$	2
parallelrisch	$\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$	2
default	$-\frac{(\sin^5(x))(\cos^7(x))}{12}-\frac{(\cos^7(x))(\sin^3(x))}{24}-\frac{(\cos^7(x))\sin(x)}{64}+\frac{\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{384}+\frac{5x}{1024}$	5

input
```
int(sin(x)^6*cos(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/1024*x-1/24576*sin(12*x)+3/8192*sin(8*x)-15/8192*sin(4*x)
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \cos^6(x) \sin^6(x) \, dx = -\frac{1}{3072} \left(256 \cos(x)^{11} - 640 \cos(x)^9 + 432 \cos(x)^7 - 8 \cos(x)^5 - 10 \cos(x)^3 - 15 \cos(x)\right) \sin(x) + \frac{5}{1024} x$$

input
```
integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")
```

output
```
-1/3072*(256*cos(x)^11 - 640*cos(x)^9 + 432*cos(x)^7 - 8*cos(x)^5 - 10*cos
(x)^3 - 15*cos(x))*sin(x) + 5/1024*x
```

3.350.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \cos^6(x) \sin^6(x) \, dx = \frac{5x}{1024} - \frac{\sin^5(2x) \cos(2x)}{768} - \frac{5 \sin^3(2x) \cos(2x)}{3072} - \frac{5 \sin(2x) \cos(2x)}{2048}$$

input
```
integrate(cos(x)**6*sin(x)**6,x)
```

output
```
5*x/1024 - sin(2*x)**5*cos(2*x)/768 - 5*sin(2*x)**3*cos(2*x)/3072 - 5*sin(
2*x)*cos(2*x)/2048
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \cos^6(x) \sin^6(x) \, dx = \frac{1}{6144} \sin(4x)^3 + \frac{5}{1024} x + \frac{3}{8192} \sin(8x) - \frac{1}{512} \sin(4x)$$

input
```
integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")
```

output
```
1/6144*sin(4*x)^3 + 5/1024*x + 3/8192*sin(8*x) - 1/512*sin(4*x)
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5}{1024}\,x - \frac{1}{24576}\,\sin(12\,x) + \frac{3}{8192}\,\sin(8\,x) - \frac{15}{8192}\,\sin(4\,x)$$

input
```
integrate(cos(x)^6*sin(x)^6,x, algorithm="giac")
```

output
```
5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)
```

3.350.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\sin^6(x)\,dx = \left(\frac{\cos(x)^5}{12} + \frac{\cos(x)^3}{24} + \frac{\cos(x)}{64}\right)\sin(x)^7 + \frac{5\,x}{1024} - \frac{15\,\sin(2\,x)}{4096} + \frac{3\,\sin(4\,x)}{4096} - \frac{\sin(6\,x)}{12288}$$

input
```
int(cos(x)^6*sin(x)^6,x)
```

output
```
(5*x)/1024 - (15*sin(2*x))/4096 + (3*sin(4*x))/4096 - sin(6*x)/12288 + sin
(x)^7*(cos(x)/64 + cos(x)^3/24 + cos(x)^5/12)
```

3.351 $\int \cos^8(x)\sin^8(x)\,dx$

3.351.1 Optimal result

Integrand size = 9, antiderivative size = 90

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768} + \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x)$$

output
```
35/32768*x+35/32768*cos(x)*sin(x)+35/49152*cos(x)^3*sin(x)+7/12288*cos(x)^
5*sin(x)+1/2048*cos(x)^7*sin(x)-1/256*cos(x)^9*sin(x)-5/384*cos(x)^9*sin(x
)^3-1/32*cos(x)^9*sin(x)^5-1/16*cos(x)^9*sin(x)^7
```

3.351.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.42

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768} - \frac{7\sin(4x)}{16384} + \frac{7\sin(8x)}{65536} - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}$$

input
```
Integrate[Cos[x]^8*Sin[x]^8,x]
```

output
```
(35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x]/49152 +
 Sin[16*x]/524288
```

3.351.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.889$, Rules used = {3042, 3048, 3042, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(x)\cos^8(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^8\cos(x)^8dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\int \cos^8(x)\sin^6(x)dx-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\int \cos(x)^8\sin(x)^6dx-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\left(\frac{5}{14}\int \cos^8(x)\sin^4(x)dx-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\int \cos(x)^8\sin(x)^4dx-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\int \cos^8(x)\sin^2(x)dx-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\int \cos(x)^8\sin(x)^2dx-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \cos^8(x)dx-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3042

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \sin\left(x+\frac{\pi}{2}\right)^8dx-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3115

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \cos^6(x)dx+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3042

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3115

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\,\mathrm{co}\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3042

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3115

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\,\mathrm{c}\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\Big\downarrow$ 3042

3.351. $\int \cos^8(x)\sin^8(x)\,dx$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\text{s}\right.\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)\right.\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1}{8}\sin(x)\cos^7(x)+\frac{7}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)\right)\right)-\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

input
```
Int[Cos[x]^8*Sin[x]^8,x]
```

output
```
-1/16*(Cos[x]^9*Sin[x]^7) + (7*(-1/14*(Cos[x]^9*Sin[x]^5) + (5*(-1/12*(Cos
[x]^9*Sin[x]^3) + (-1/10*(Cos[x]^9*Sin[x]) + ((Cos[x]^7*Sin[x])/8 + (7*((C
os[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2)
)/4))/6))/8)/10)/4))/14))/16
```

3.351.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.351.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

method	result
risch	$\frac{35x}{32768}+\frac{\sin(16x)}{524288}-\frac{\sin(12x)}{49152}+\frac{7\sin(8x)}{65536}-\frac{7\sin(4x)}{16384}$
parallelrisch	$\frac{35x}{32768}+\frac{\sin(16x)}{524288}-\frac{\sin(12x)}{49152}+\frac{7\sin(8x)}{65536}-\frac{7\sin(4x)}{16384}$
default	$-\frac{(\cos^9(x))(\sin^7(x))}{16}-\frac{(\cos^9(x))(\sin^5(x))}{32}-\frac{5(\sin^3(x))(\cos^9(x))}{384}-\frac{(\cos^9(x))\sin(x)}{256}+\frac{\left(\cos^7(x)+\frac{7(\cos^5(x))}{6}+\frac{35(\cos}{24}\right.}{2048}$

input
```
int(cos(x)^8*sin(x)^8,x,method=_RETURNVERBOSE)
```

output
```
35/32768*x+1/524288*sin(16*x)-1/49152*sin(12*x)+7/65536*sin(8*x)-7/16384*s
in(4*x)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \cos^8(x)\sin^8(x)\,dx$$
$$=\frac{1}{98304}\left(6144\cos(x)^{15}-21504\cos(x)^{13}+25856\cos(x)^{11}-10880\cos(x)^9+48\cos(x)^7+56\cos(x)^5+\right.$$
$$+\frac{35}{32768}x$$

input
```
integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")
```

output
```
1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 10880*cos(x)
^9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x))*sin(x) + 35/327
68*x
```

3.351.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768} - \frac{\sin^7(2x)\cos(2x)}{4096} - \frac{7\sin^5(2x)\cos(2x)}{24576} - \frac{35\sin^3(2x)\cos(2x)}{98304} - \frac{35\sin(2x)\cos(2x)}{65536}$$

input
```
integrate(cos(x)**8*sin(x)**8,x)
```

output
```
35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/24576 - 35
*sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.33

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{1}{12288}\sin(4x)^3 + \frac{35}{32768}x + \frac{1}{524288}\sin(16x) + \frac{7}{65536}\sin(8x) - \frac{1}{2048}\sin(4x)$$

input
```
integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")
```

output
```
1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*sin(8*x) -
1/2048*sin(4*x)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.31

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35}{32768}x + \frac{1}{524288}\sin(16x) - \frac{1}{49152}\sin(12x) + \frac{7}{65536}\sin(8x) - \frac{7}{16384}\sin(4x)$$

input
```
integrate(cos(x)^8*sin(x)^8,x, algorithm="giac")
```

output
```
35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin(8*x) - 7
/16384*sin(4*x)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \cos^8(x)\sin^8(x)\,dx = \left(\frac{\cos(x)^7}{16}+\frac{\cos(x)^5}{32}+\frac{5\cos(x)^3}{384}+\frac{\cos(x)}{256}\right)\sin(x)^9 + \frac{35\,x}{32768}-\frac{7\,\sin(2\,x)}{8192}+\frac{7\,\sin(4\,x)}{32768}-\frac{\sin(6\,x)}{24576}+\frac{\sin(8\,x)}{262144}$$

input
```
int(cos(x)^8*sin(x)^8,x)
```

output
```
(35*x)/32768 - (7*sin(2*x))/8192 + (7*sin(4*x))/32768 - sin(6*x)/24576 + s
in(8*x)/262144 + sin(x)^9*(cos(x)/256 + (5*cos(x)^3)/384 + cos(x)^5/32 + c
os(x)^7/16)
```

3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$

3.352.1 Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

output
```
cos(x)^(-1+2*m)*(cos(x)^2)^(1/2-m)*hypergeom([1/2-m, 1/2+m],[3/2+m],sin(x)
^2)*sin(x)^(1+2*m)/(1+2*m)
```

3.352.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}+m,\frac{3}{2}+m,\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

input
```
Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

output
```
(Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[1/2 - m, 1/2 + m
, 3/2 + m, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

3.352.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{2m}(x)\cos^{2m}(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)^{2m}\cos(x)^{2m}dx$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1),\frac{1}{2}(2m+3),\sin^2(x)\right)}{2m+1}$$

input
```
Int[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

output
```
(Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*m)/2, (1
+ 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

3.352.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.352.4 Maple [F]

$$\int \left(\cos^{2m}\left(x\right)\right) \left(\sin^{2m}\left(x\right)\right) dx$$

input
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

output
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

3.352.5 Fricas [F]

$$\int \cos^{2m}(x) \sin^{2m}(x) \, dx = \int \cos\left(x\right)^{2\,m} \sin\left(x\right)^{2\,m} \, dx$$

input
```
integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")
```

output
```
integral(cos(x)^(2*m)*sin(x)^(2*m), x)
```

3.352.6 Sympy [F]

$$\int \cos^{2m}(x) \sin^{2m}(x) \, dx = \int \sin^{2m}\left(x\right) \cos^{2m}\left(x\right) dx$$

input
```
integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
```

output
```
Integral(sin(x)**(2*m)*cos(x)**(2*m), x)
```

3.352.7 Maxima [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

input
```
integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")
```

output
```
integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

3.352.8 Giac [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

input
```
integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="giac")
```

output
```
integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = -\frac{\cos(x)^{2m+1}\sin(x)^{2m+1}\,{}_2F_1\left(\frac{1}{2}-m,m+\frac{1}{2};\ m+\frac{3}{2};\ \cos(x)^2\right)}{(2m+1)\ \left(\sin(x)^2\right)^{m+\frac{1}{2}}}$$

input
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

output
```
-(cos(x)^(2*m + 1)*sin(x)^(2*m + 1)*hypergeom([1/2 - m, m + 1/2], m + 3/2,
 cos(x)^2))/((2*m + 1)*(sin(x)^2)^(m + 1/2))
```

3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$

3.353.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\cot^2\left(\frac{\pi}{4}+2x\right)+\frac{1}{2}\log\left(\tan\left(\frac{\pi}{4}+2x\right)\right)$$

output
```
-1/4*cot(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

3.353.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\csc^2\left(\frac{\pi}{4}+2x\right)-\frac{1}{2}\log\left(\cos\left(\frac{1}{4}(\pi+8x)\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

input
```
Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

output
```
-1/4*Csc[Pi/4 + 2*x]^2 - Log[Cos[(Pi + 8*x)/4]]/2 + Log[Sin[Pi/4 + 2*x]]/2
```

3.353.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3\left(2x+\frac{\pi}{4}\right)\sec\left(2x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2x+\frac{\pi}{4}\right)^3\sec\left(2x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{1}{2}\int \cot^3\left(2x+\frac{\pi}{4}\right)\left(\tan^2\left(2x+\frac{\pi}{4}\right)+1\right)d\tan\left(2x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}\int \left(\cot^3\left(2x+\frac{\pi}{4}\right)+\cot\left(2x+\frac{\pi}{4}\right)\right)d\tan\left(2x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{2}\cot^2\left(2x+\frac{\pi}{4}\right)\right)$$

input
```
Int[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

output
```
(-1/2*Cot[Pi/4 + 2*x]^2 + Log[Tan[Pi/4 + 2*x]])/2
```

3.353.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

3.353.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{1}{4\sin\left(\frac{\pi}{4}+2x\right)^{2}}+\frac{\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$
default	$-\frac{1}{4\sin\left(\frac{\pi}{4}+2x\right)^{2}}+\frac{\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$
risch	$\frac{ie^{4ix}}{\left(ie^{4ix}-1\right)^{2}}+\frac{\ln\left(ie^{4ix}-1\right)}{2}-\frac{\ln\left(ie^{4ix}+1\right)}{2}$
parallelrisch	$\ln\left(\sqrt{\tan}\left(\frac{\pi}{8}+x\right)\right)+\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)-1}}\right)+\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)+1}}\right)-\frac{\left(\tan^{2}\left(\frac{\pi}{8}+x\right)\right)}{16}-\frac{\left(\cot^{2}\left(\frac{\pi}{8}+x\right)\right)}{16}$
norman	$\frac{-\frac{1}{16}-\frac{\left(\tan^{4}\left(\frac{\pi}{8}+x\right)\right)}{16}}{\tan\left(\frac{\pi}{8}+x\right)^{2}}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$

input
```
int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

3.353.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \csc^{3}\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)^{2}-1\right)\log\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)^{2}\right)-\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)^{2}-1\right)\log\left(-\frac{1}{4}\,\cos\left(\frac{1}{4}\,\pi+2\,x\right)^{2}+\frac{1}{4}\right)+}{4\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)^{2}-1\right)}$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="fricas")
```

output
```
-1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1/4*pi + 2
*x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos(1/4*pi + 2*x)^2 -
 1)
```

3.353.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(22) = 44$.

Time = 0.57 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2} - \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2} + \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)\right)}{2} - \frac{\tan^2\left(x+\frac{\pi}{8}\right)}{16} - \frac{1}{16\tan^2\left(x+\frac{\pi}{8}\right)}$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
```

output
```
-log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x + pi/8))/
2 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4\sin\left(\frac{1}{4}\pi+2x\right)^2} - \frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^2-1\right) + \frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^2\right)$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="maxima")
```

output
```
-1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4*log(sin(
1/4*pi + 2*x)^2)
```

3.353.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(24) = 48.

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\left(\frac{4\,(\cos(\frac{1}{4}\,\pi+2\,x)-1)}{\cos(\frac{1}{4}\,\pi+2\,x)+1}-1\right)\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1\right)}{16\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1\right)} + \frac{\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1}{16\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1\right)} + \frac{1}{4}\log\left(-\frac{\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1}{\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1}\right) - \frac{1}{2}\log\left(\left|-\frac{\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1}{\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1}-1\right|\right)$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="giac")
```

output
```
-1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(cos(1/4*pi
+ 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi + 2*x) - 1)/(cos(1/
4*pi + 2*x) + 1) + 1/4*log(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1
)) - 1/2*log(abs(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1))
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = \frac{\ln\left(\tan\left(\frac{\Pi}{4}+2\,x\right)\right)}{2} - \frac{1}{4\sin\left(\frac{\Pi}{4}+2\,x\right)^2}$$

input
```
int(1/(cos(Pi/4 + 2*x)*sin(Pi/4 + 2*x)^3),x)
```

output
```
log(tan(Pi/4 + 2*x))/2 - 1/(4*sin(Pi/4 + 2*x)^2)
```

3.354 $\int \sec^2(x)\tan^2(x)\,dx$

3.354.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

output
```
1/3*tan(x)^3
```

3.354.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

input
```
Integrate[Sec[x]^2*Tan[x]^2,x]
```

output
```
Tan[x]^3/3
```

3.354.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^2\sec(x)^2dx$$

$$\Big\downarrow \text{ 3087}$$

$$\int \tan^2(x)d\tan(x)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\tan^3(x)}{3}$$

input
```
Int[Sec[x]^2*Tan[x]^2,x]
```

output
```
Tan[x]^3/3
```

3.354.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.354.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\tan^3(x))}{3}$	7
default	$\frac{(\tan^3(x))}{3}$	7
risch	$-\frac{2i(3\,\mathrm{e}^{4ix}+1)}{3(\mathrm{e}^{2ix}+1)^3}$	22

input
```
int(tan(x)^2*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(x)^3
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{(\cos(x)^2-1)\sin(x)}{3\cos(x)^3}$$

input
```
integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

3.354.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

input
```
integrate(sec(x)**2*tan(x)**2,x)
```

output
```
-sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{1}{3}\,\tan{(x)}^3$$

input
```
integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{1}{3}\,\tan{(x)}^3$$

input
```
integrate(sec(x)^2*tan(x)^2,x, algorithm="giac")
```

output
```
1/3*tan(x)^3
```

3.354.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan(x)^3}{3}$$

input
```
int(tan(x)^2/cos(x)^2,x)
```

output
```
tan(x)^3/3
```

3.355 $\int \cot^3(x)\csc(x)\,dx$

3.355.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

output
```
csc(x)-1/3*csc(x)^3
```

3.355.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \csc(x) - \frac{\csc^3(x)}{3}$$

input
```
Integrate[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

3.355.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \left(\csc^2(x)-1\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\csc(x)-\frac{\csc^3(x)}{3}$$

input
```
Int[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

3.355.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.355.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\csc(x) - \frac{(\csc^3(x))}{3}$	10
default	$\csc(x) - \frac{(\csc^3(x))}{3}$	10
risch	$\frac{2i(3\,e^{5ix}-2\,e^{3ix}+3\,e^{ix})}{3(e^{2ix}-1)^3}$	35

input
```
int(cot(x)^3*csc(x),x,method=_RETURNVERBOSE)
```

output
```
csc(x)-1/3*csc(x)^3
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\cos{(x)}^2-2}{3\left(\cos{(x)}^2-1\right)\sin{(x)}}$$

input
```
integrate(cot(x)^3*csc(x),x, algorithm="fricas")
```

output
```
1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

3.355.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\csc(x)\,dx = -\frac{1-3\sin^2(x)}{3\sin^3(x)}$$

input
```
integrate(cot(x)**3*csc(x),x)
```

output
```
-(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\sin(x)^2-1}{3\sin(x)^3}$$

input
```
integrate(cot(x)^3*csc(x),x, algorithm="maxima")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.355.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\sin(x)^2-1}{3\sin(x)^3}$$

input
```
integrate(cot(x)^3*csc(x),x, algorithm="giac")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.355.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

input
```
int(cot(x)^3/sin(x),x)
```

output
```
(sin(x)^2 - 1/3)/sin(x)^3
```

3.356 $\int \sec^3(x)\tan(x)\,dx$

3.356.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^3(x)\tan(x)\,dx=\frac{\sec^3(x)}{3}$$

output
```
1/3*sec(x)^3
```

3.356.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan(x)\,dx=\frac{\sec^3(x)}{3}$$

input
```
Integrate[Sec[x]^3*Tan[x],x]
```

output
```
Sec[x]^3/3
```

3.356.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\sec(x)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec^2(x)d\sec(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\sec^3(x)}{3}$$

input
```
Int[Sec[x]^3*Tan[x],x]
```

output
```
Sec[x]^3/3
```

3.356.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.356.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^3(x))}{3}$	7
default	$\frac{(\sec^3(x))}{3}$	7
risch	$\frac{8\,\mathrm{e}^{3ix}}{3(\mathrm{e}^{2ix}+1)^3}$	17

input
```
int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*sec(x)^3
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos(x)^3}$$

input
```
integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

output
```
1/3/cos(x)^3
```

3.356.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos^3(x)}$$

input
```
integrate(sec(x)**3*tan(x),x)
```

output
```
1/(3*cos(x)**3)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

input
```
integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

output
```
1/3/cos(x)^3
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

input
```
integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

output
```
1/3/cos(x)^3
```

3.356.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos(x)^3}$$

input
```
int(tan(x)/cos(x)^3,x)
```

output
```
1/(3*cos(x)^3)
```

3.357 $\int \cot^2(x)\csc^3(x)\,dx$

3.357.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{1}{8}\text{arctanh}(\cos(x)) + \frac{1}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)$$

output
```
1/8*arctanh(cos(x))+1/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3
```

3.357.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{1}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) + \frac{1}{8}\log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{1}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Cot[x]^2*Csc[x]^3,x]
```

output
```
Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8 - Sec[x/
2]^2/32 + Sec[x/2]^4/64
```

3.357.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\csc^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^2\sec\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\frac{1}{2}\cot(x)\csc(x)-\frac{\int \csc(x)dx}{2}\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\cot(x)\csc(x)-\frac{\int \csc(x)dx}{2}\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{2}\text{arctanh}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

input
```
Int[Cot[x]^2*Csc[x]^3,x]
```

output
```
-1/4*(Cot[x]*Csc[x]^3) + (ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2)/4
```

3.357.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.357.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\cos^{3}(x)}{4\sin(x)^{4}}-\frac{\cos^{3}(x)}{8\sin(x)^{2}}-\frac{\cos(x)}{8}-\frac{\ln(\csc(x)-\cot(x))}{8}$	36
risch	$-\frac{e^{7ix}+7\,e^{5ix}+7\,e^{3ix}+e^{ix}}{4(e^{2ix}-1)^{4}}-\frac{\ln(e^{ix}-1)}{8}+\frac{\ln(e^{ix}+1)}{8}$	58

input
```
int(cot(x)^2*csc(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x)^3/sin(x)^4-1/8*cos(x)^3/sin(x)^2-1/8*cos(x)-1/8*ln(csc(x)-cot(
x))
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{2\cos(x)^3 - \left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(x\right.}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

input
```
integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*cos(x)^3 - (cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + (c
os(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) + 2*cos(x))/(cos(x)^4 - 2
*cos(x)^2 + 1)
```

3.357.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{-\cos^3(x) - \cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} - \frac{\log(\cos(x) - 1)}{16} + \frac{\log(\cos(x) + 1)}{16}$$

input
```
integrate(cot(x)**2*csc(x)**3,x)
```

output
```
(-cos(x)**3 - cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) - log(cos(x) - 1)/1
6 + log(cos(x) + 1)/16
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{\cos(x)^3 + \cos(x)}{8\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)} + \frac{1}{16}\log(\cos(x) + 1) - \frac{1}{16}\log(\cos(x) - 1)$$

input
```
integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")
```

output
```
-1/8*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/16*log(cos(x) + 1
) - 1/16*log(cos(x) - 1)
```

3.357.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{\frac{1}{\cos(x)} + \cos(x)}{8\left(\left(\frac{1}{\cos(x)} + \cos(x)\right)^2 - 4\right)} + \frac{1}{32}\log\left(\left|\frac{1}{\cos(x)} + \cos(x) + 2\right|\right) - \frac{1}{32}\log\left(\left|\frac{1}{\cos(x)} + \cos(x) - 2\right|\right)$$

input
```
integrate(cot(x)^2*csc(x)^3,x, algorithm="giac")
```

output
```
-1/8*(1/cos(x) + cos(x))/((1/cos(x) + cos(x))^2 - 4) + 1/32*log(abs(1/cos(
x) + cos(x) + 2)) - 1/32*log(abs(1/cos(x) + cos(x) - 2))
```

3.357.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64\tan\left(\frac{x}{2}\right)^4} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$$

input
```
int(cot(x)^2/sin(x)^3,x)
```

output
```
tan(x/2)^4/64 - 1/(64*tan(x/2)^4) - log(tan(x/2))/8
```

3.358 $\int \cot^3(x) \csc^4(x) \, dx$

3.358.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^3(x) \csc^4(x) \, dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

output
```
1/4*csc(x)^4-1/6*csc(x)^6
```

3.358.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x) \csc^4(x) \, dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

input
```
Integrate[Cot[x]^3*Csc[x]^4,x]
```

output
```
Csc[x]^4/4 - Csc[x]^6/6
```

3.358.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc^4(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)^4\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^4\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \ 3086$$

$$-\int -\csc^3(x)\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \ 25$$

$$\int \csc^3(x)\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \ 244$$

$$\int \left(\csc^3(x)-\csc^5(x)\right)d\csc(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

input
```
Int[Cot[x]^3*Csc[x]^4,x]
```

output
```
Csc[x]^4/4 - Csc[x]^6/6
```

3.358.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.358.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\cos^4(x)}{6\sin(x)^6}-\frac{\cos^4(x)}{12\sin(x)^4}$	22
norman	$\frac{-\frac{1}{384}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^8\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^6}$	34
risch	$\frac{4\,\mathrm{e}^{8ix}+\frac{8\,\mathrm{e}^{6ix}}{3}+4\,\mathrm{e}^{4ix}}{(\mathrm{e}^{2ix}-1)^6}$	34
parallelrisch	$\frac{-(\tan^{12}(\frac{x}{2}))+9(\tan^8(\frac{x}{2}))+9(\tan^4(\frac{x}{2}))-1}{384\tan(\frac{x}{2})^6}$	35

input
```
int(cos(x)^3/sin(x)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

3.358.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\cos(x)^2 - 1}{12\,(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1)}$$

input
```
integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
```

output
```
1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

3.358.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

input
```
integrate(cos(x)**3/sin(x)**7,x)
```

output
```
-(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\sin(x)^2 - 2}{12\,\sin(x)^6}$$

input
```
integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
```

output
```
1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\sin{(x)}^2-2}{12\,\sin{(x)}^6}$$

input
```
integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
```

output
```
1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

3.358.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\frac{\sin(x)^2}{4}-\frac{1}{6}}{\sin{(x)}^6}$$

input
```
int(cos(x)^3/sin(x)^7,x)
```

output
```
(sin(x)^2/4 - 1/6)/sin(x)^6
```

3.359 $\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx$

3.359.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2}{3}\sec^{\frac{3}{2}}(x) - \frac{4}{7}\sec^{\frac{7}{2}}(x) + \frac{2}{11}\sec^{\frac{11}{2}}(x)$$

output
```
2/3*sec(x)^(3/2)-4/7*sec(x)^(7/2)+2/11*sec(x)^(11/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{1}{924}(135 + 44\cos(2x) + 77\cos(4x))\sec^{\frac{11}{2}}(x)$$

input
```
Integrate[Sec[x]^(13/2)*Sin[x]^5,x]
```

output
```
((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924
```

3.359.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3102, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(x)\sec^{\frac{13}{2}}(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)^{13/2}}{\csc(x)^5}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\int \sqrt{\sec(x)}\left(1-\sec^2(x)\right)^2d\sec(x)$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\sec^{\frac{9}{2}}(x)-2\sec^{\frac{5}{2}}(x)+\sqrt{\sec(x)}\right)d\sec(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{11}\sec^{\frac{11}{2}}(x)-\frac{4}{7}\sec^{\frac{7}{2}}(x)+\frac{2}{3}\sec^{\frac{3}{2}}(x)$$

input
```
Int[Sec[x]^(13/2)*Sin[x]^5,x]
```

output
```
(2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

3.359.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.359.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{2\left(\sec^{\frac{3}{2}}(x)\right)}{3}-\frac{4\left(\sec^{\frac{7}{2}}(x)\right)}{7}+\frac{2\left(\sec^{\frac{11}{2}}(x)\right)}{11}$	20
default	$\frac{2\left(\sec^{\frac{3}{2}}(x)\right)}{3}-\frac{4\left(\sec^{\frac{7}{2}}(x)\right)}{7}+\frac{2\left(\sec^{\frac{11}{2}}(x)\right)}{11}$	20

input
```
int(sec(x)^(3/2)*tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
2/3*sec(x)^(3/2)-4/7*sec(x)^(7/2)+2/11*sec(x)^(11/2)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx=\frac{2\left(77\cos(x)^4-66\cos(x)^2+21\right)}{231\cos(x)^{\frac{11}{2}}}$$

input
```
integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")
```

output
```
2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

3.359.6 Sympy [A] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\tan^4(x)\sec^{\frac{3}{2}}(x)}{11} - \frac{16\tan^2(x)\sec^{\frac{3}{2}}(x)}{77} + \frac{64\sec^{\frac{3}{2}}(x)}{231}$$

input
```
integrate(sec(x)**(3/2)*tan(x)**5,x)
```

output
```
2*tan(x)**4*sec(x)**(3/2)/11 - 16*tan(x)**2*sec(x)**(3/2)/77 + 64*sec(x)**
(3/2)/231
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2}{3\cos(x)^{\frac{3}{2}}} - \frac{4}{7\cos(x)^{\frac{7}{2}}} + \frac{2}{11\cos(x)^{\frac{11}{2}}}$$

input
```
integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")
```

output
```
2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(77\cos(x)^4 - 66\cos(x)^2 + 21\right)}{231\cos(x)^{\frac{11}{2}}}$$

input
```
integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="giac")
```

output
```
2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(\frac{1}{\cos(x)}\right)^{11/2}\left(77\cos(x)^4-66\cos(x)^2+21\right)}{231}$$

input
```
int(tan(x)^5*(1/cos(x))^(3/2),x)
```

output
```
(2*(1/cos(x))^(11/2)*(77*cos(x)^4 - 66*cos(x)^2 + 21))/231
```

3.360 $\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$

3.360.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{5}\tan^{\frac{5}{2}}(x) + \frac{2}{9}\tan^{\frac{9}{2}}(x)$$

output
```
2/5*tan(x)^(5/2)+2/9*tan(x)^(9/2)
```

3.360.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{45}(7+2\cos(2x))\sec^2(x)\tan^{\frac{5}{2}}(x)$$

input
```
Integrate[Sec[x]^4*Tan[x]^(3/2),x]
```

output
```
(2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45
```

3.360.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(x)\sec^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^{3/2}\sec(x)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\int \tan^{\frac{3}{2}}(x)\left(\tan^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^{\frac{7}{2}}(x)+\tan^{\frac{3}{2}}(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

input
```
Int[Sec[x]^4*Tan[x]^(3/2),x]
```

output
```
(2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

3.360.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.360.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\left(\tan^{\frac{5}{2}}(x)\right)}{5}+\frac{2\left(\tan^{\frac{9}{2}}(x)\right)}{9}$	14
default	$\frac{2\left(\tan^{\frac{5}{2}}(x)\right)}{5}+\frac{2\left(\tan^{\frac{9}{2}}(x)\right)}{9}$	14

input
```
int(sec(x)^4*tan(x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*tan(x)^(5/2)+2/9*tan(x)^(9/2)
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = -\frac{2\left(4\cos(x)^4+\cos(x)^2-5\right)\sqrt{\frac{\sin(x)}{\cos(x)}}}{45\cos(x)^4}$$

input
```
integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4
```

3.360.6 Sympy [F]

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \int \tan^{\frac{3}{2}}(x) \sec^4(x) \, dx$$

input
```
integrate(sec(x)**4*tan(x)**(3/2),x)
```

output
```
Integral(tan(x)**(3/2)*sec(x)**4, x)
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \frac{2}{9} \tan(x)^{\frac{9}{2}} + \frac{2}{5} \tan(x)^{\frac{5}{2}}$$

input
```
integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")
```

output
```
2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \frac{2}{9} \tan(x)^{\frac{9}{2}} + \frac{2}{5} \tan(x)^{\frac{5}{2}}$$

input
```
integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="giac")
```

output
```
2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = -\frac{4\sqrt{\sin(2x)}\,\left(2\cos(2x)^2+5\,\cos(2x)-7\right)}{45\left(\cos(2x)+1\right)^{5/2}}$$

input
```
int(tan(x)^(3/2)/cos(x)^4,x)
```

output
```
-(4*sin(2*x)^(1/2)*(5*cos(2*x) + 2*cos(2*x)^2 - 7))/(45*(cos(2*x) + 1)^(5/
2))
```

3.361 $\int \cot^4(x)\csc^3(x)\,dx$

3.361.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \cot^4(x)\csc^3(x)\,dx = -\frac{1}{16}\text{arctanh}(\cos(x)) - \frac{1}{16}\cot(x)\csc(x) + \frac{1}{8}\cot(x)\csc^3(x) - \frac{1}{6}\cot^3(x)\csc^3(x)$$

output
```
-1/16*arctanh(cos(x))-1/16*cot(x)*csc(x)+1/8*cot(x)*csc(x)^3-1/6*cot(x)^3*
csc(x)^3
```

3.361.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(38) = 76.

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \cot^4(x)\csc^3(x)\,dx = -\frac{1}{64}\csc^2\left(\frac{x}{2}\right) + \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{1}{384}\csc^6\left(\frac{x}{2}\right) - \frac{1}{16}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{16}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{64}\sec^2\left(\frac{x}{2}\right) - \frac{1}{64}\sec^4\left(\frac{x}{2}\right) + \frac{1}{384}\sec^6\left(\frac{x}{2}\right)$$

input
```
Integrate[Cot[x]^4*Csc[x]^3,x]
```

output
```
-1/64*Csc[x/2]^2 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/16 + Log
[Sin[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6/384
```

3.361.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 3091, 3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(x)\csc^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^4 \sec\left(x-\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{2}\int \cot^2(x)\csc^3(x)dx-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sec\left(x-\frac{\pi}{2}\right)^3 \tan\left(x-\frac{\pi}{2}\right)^2 dx-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \csc^3(x)dx+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \csc(x)^3dx+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

input
```
Int[Cot[x]^4*Csc[x]^3,x]
```

output
```
-1/6*(Cot[x]^3*Csc[x]^3) + ((Cot[x]*Csc[x]^3)/4 + (-1/2*ArcTanh[Cos[x]] -
(Cot[x]*Csc[x])/2)/4)/2
```

3.361.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.361.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
default	$-\frac{\cos^{5}(x)}{6\sin(x)^{6}}-\frac{\cos^{5}(x)}{24\sin(x)^{4}}+\frac{\cos^{5}(x)}{48\sin(x)^{2}}+\frac{(\cos^{3}(x))}{48}+\frac{\cos(x)}{16}+\frac{\ln(\csc(x)-\cot(x))}{16}$	52
risch	$\frac{3\,e^{11ix}+47\,e^{9ix}+78\,e^{7ix}+78\,e^{5ix}+47\,e^{3ix}+3\,e^{ix}}{24(e^{2ix}-1)^{6}}+\frac{\ln(e^{ix}-1)}{16}-\frac{\ln(e^{ix}+1)}{16}$	76

input
```
int(cot(x)^4*csc(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/sin(x)^6*cos(x)^5-1/24/sin(x)^4*cos(x)^5+1/48/sin(x)^2*cos(x)^5+1/48*
cos(x)^3+1/16*cos(x)+1/16*ln(csc(x)-cot(x))
```

3.361.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \cot^4(x)\csc^3(x)\,dx$$
$$= \frac{6\cos(x)^5 + 16\cos(x)^3 - 3\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^6 - 3\right.}{96\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

input
```
integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")
```

output
```
1/96*(6*cos(x)^5 + 16*cos(x)^3 - 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1
)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(-
1/2*cos(x) + 1/2) - 6*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

3.361.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \cot^4(x)\csc^3(x)\,dx = -\frac{-3\cos^5(x) - 8\cos^3(x) + 3\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{\log(\cos(x) - 1)}{32} - \frac{\log(\cos(x) + 1)}{32}$$

input
```
integrate(cot(x)**4*csc(x)**3,x)
```

output
```
-(-3*cos(x)**5 - 8*cos(x)**3 + 3*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 + 1
44*cos(x)**2 - 48) + log(cos(x) - 1)/32 - log(cos(x) + 1)/32
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)} - \frac{1}{32}\log(\cos(x) + 1) + \frac{1}{32}\log(\cos(x) - 1)$$

input
```
integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")
```

output
```
1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x
)^2 - 1) - 1/32*log(cos(x) + 1) + 1/32*log(cos(x) - 1)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^2 - 1\right)^3} - \frac{1}{32}\log(\cos(x) + 1) + \frac{1}{32}\log(-\cos(x) + 1)$$

input
```
integrate(cot(x)^4*csc(x)^3,x, algorithm="giac")
```

output
```
1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32*log(cos(
x) + 1) + 1/32*log(-cos(x) + 1)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16} + \frac{\frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{1}{384}}{\tan\left(\frac{x}{2}\right)^6} - \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^6}{384}$$

input
```
int(cot(x)^4/sin(x)^3,x)
```

output
```
log(tan(x/2))/16 + (tan(x/2)^2/128 + tan(x/2)^4/128 - 1/384)/tan(x/2)^6 -
tan(x/2)^2/128 - tan(x/2)^4/128 + tan(x/2)^6/384
```

3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

3.362.1 Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=-\frac{1}{4}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)-\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

output
```
-1/4*arctanh(sin(1/4*Pi+1/2*x))-1/4*sec(1/4*Pi+1/2*x)*tan(1/4*Pi+1/2*x)+1/
2*sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)
```

3.362.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=-\frac{1}{4}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)-\frac{1}{4}\sec^2\left(\frac{1}{4}(\pi+2x)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^4\left(\frac{1}{4}(\pi+2x)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

input
```
Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

output
```
-1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[(Pi + 2*x)/4]^2*Sin[Pi/4 + x/2])/4 +
(Sec[(Pi + 2*x)/4]^4*Sin[Pi/4 + x/2])/2
```

3.362.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(\frac{x}{2}+\frac{\pi}{4}\right)^2\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\int \sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\int \csc\left(\frac{x}{2}+\frac{3\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\left(-\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\frac{1}{2}\int \sec\left(\frac{x}{2}+\frac{\pi}{4}\right)dx\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\left(-\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\frac{1}{2}\int \csc\left(\frac{x}{2}+\frac{3\pi}{4}\right)dx\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(-\text{arctanh}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

input
```
Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

output
```
(Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2])/2 + (-ArcTanh[Sin[Pi/4 + x/2]] - Sec[P
i/4 + x/2]*Tan[Pi/4 + x/2])/4
```

3.362.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.362.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4}+\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^2}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}-\frac{\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}{4}$	76
default	$\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4}+\frac{\sin^3\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^2}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}-\frac{\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}{4}$	76
risch	$\frac{i\left(-(-1)^{\frac{3}{4}}e^{\frac{7ix}{2}}+7(-1)^{\frac{1}{4}}e^{\frac{5ix}{2}}+7(-1)^{\frac{3}{4}}e^{\frac{3ix}{2}}-(-1)^{\frac{1}{4}}e^{\frac{ix}{2}}\right)}{2\left(ie^{ix}+1\right)^4}-\frac{\ln\left(e^{\frac{i(\pi+2x)}{4}}+i\right)}{4}+\frac{\ln\left(e^{\frac{i(\pi+2x)}{4}}-i\right)}{4}$	88

input
```
int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^4+1/4*sin(1/4*Pi+1/2*x)^3/cos(1/
4*Pi+1/2*x)^2+1/4*sin(1/4*Pi+1/2*x)-1/4*ln(sec(1/4*Pi+1/2*x)+tan(1/4*Pi+1/
2*x))
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx =$$
$$-\frac{\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)-\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(-\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+2\left(\cos\left(\frac{1}{4}\pi+\right.\right.}{8\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4}$$

input
```
integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="fricas")
```

output
```
-1/8*(cos(1/4*pi + 1/2*x)^4*log(sin(1/4*pi + 1/2*x) + 1) - cos(1/4*pi + 1/
2*x)^4*log(-sin(1/4*pi + 1/2*x) + 1) + 2*(cos(1/4*pi + 1/2*x)^2 - 2)*sin(1
/4*pi + 1/2*x))/cos(1/4*pi + 1/2*x)^4
```

3.362.6 Sympy [F]

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \int \tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\,dx$$

input
```
integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)
```

output
```
Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^3+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4-2\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^2+1\right)} - \frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right) + \frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-1\right)$$

input
```
integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="maxima")
```

output
```
1/4*(sin(1/4*pi + 1/2*x)^3 + sin(1/4*pi + 1/2*x))/(sin(1/4*pi + 1/2*x)^4 -
 2*sin(1/4*pi + 1/2*x)^2 + 1) - 1/8*log(sin(1/4*pi + 1/2*x) + 1) + 1/8*log
(sin(1/4*pi + 1/2*x) - 1)
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\left(\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)^2-4\right)} - \frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+2\right|\right) + \frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-2\right|\right)$$

input
```
integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="giac")
```

output
```
1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi + 1/2*x)
+ sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1
/4*pi + 1/2*x) + 2)) + 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1
/2*x) - 2))
```

3.362.9 Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{2\left(\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^7}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^5}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^3}{4}+\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)}{4}\right)}{\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^2-1\right)^4} - \frac{\operatorname{atanh}\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)\right)}{2}$$

input
```
int(tan(Pi/4 + x/2)^2/cos(Pi/4 + x/2)^3,x)
```

output
```
(2*(tan(Pi/8 + x/4)/4 + (7*tan(Pi/8 + x/4)^3)/4 + (7*tan(Pi/8 + x/4)^5)/4
+ tan(Pi/8 + x/4)^7/4))/(tan(Pi/8 + x/4)^2 - 1)^4 - atanh(tan(Pi/8 + x/4))
/2
```

3.362. $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

3.363 $\int (1+\cot^3(x)) (a\sec^2(x)-\sin(2x))^2 \, dx$

3.363.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int (1+\cot^3(x)) (a\sec^2(x)-\sin(2x))^2 \, dx = \frac{x}{2}+4ax+2\cos^2(x)+\cos^4(x)+4a\cot(x) - \frac{1}{2}a^2\cot^2(x)+(4-a)a\log(\cos(x)) + (4+a^2)\log(\sin(x))+\frac{1}{2}\cos(x)\sin(x) - \cos^3(x)\sin(x)+a^2\tan(x)+\frac{1}{3}a^2\tan^3(x)$$

output
```
1/2*x+4*a*x+2*cos(x)^2+cos(x)^4+4*a*cot(x)-1/2*a^2*cot(x)^2+(4-a)*a*ln(cos
(x))+(a^2+4)*ln(sin(x))+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)+a^2*tan(x)+1/3*a
^2*tan(x)^3
```

3.363.2 Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int (1+\cot^3(x)) (a\sec^2(x)-\sin(2x))^2 \, dx = -\frac{2\cos^3(x)\sin(x)(-a\sec^2(x)+\sin(2x))^2(-96a\cot^2(x)-8a^2(2+\cos(2x))\sec^2(x)-3\cot(x)(4x+32a}{3(-4a}$$

input
```
Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

output
```
(-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2 - 8*a^2*(
2 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos[2*x] + Cos[4*x] -
 4*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[Cos[x]] + 32*Log[Sin[x]] +
8*a^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4*a + 2*Sin[2*x] + Sin[4*x])^2)
```

3.363.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4889, 2336, 27, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cot^3(x)+1\right)\left(a\sec^2(x)-\sin(2x)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\cot(x)^3+1\right)\left(a\sec(x)^2-\sin(2x)\right)^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(\tan^3(x)+1\right)\cot^3(x)\left(a\tan^4(x)+2a\tan^2(x)+a-2\tan(x)\right)^2}{\left(\tan^2(x)+1\right)^3}d\tan(x)$$

$$\Big\downarrow \text{2336}$$

$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}-\frac{1}{4}\int -\frac{4\cot^3(x)\left(a^2\tan^9(x)+3a^2\tan^7(x)-(4-a)a\tan^6(x)+3a^2\tan^5(x)-(4-3a)a\tan^4(x)+\left(a^2-4a+1\right)t\right.}{\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\cot^3(x)\left(a^2\tan^9(x)+3a^2\tan^7(x)-(4-a)a\tan^6(x)+3a^2\tan^5(x)-(4-3a)a\tan^4(x)+\left(a^2-4a+1\right)\tan^3(a\right.}{\left(\tan^2(x)+1\right)^2}$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{1}{2}\int -\frac{\cot^3(x)\left(2a^2\tan^7(x)+4a^2\tan^5(x)-2(4-a)a\tan^4(x)+\left(2a^2+1\right)\tan^3(x)+4\left(a^2+2\right)\tan^2(x)-8a\tan}{\tan^2(x)+1}$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{\cot^3(x)\left(2a^2\tan^7(x)+4a^2\tan^5(x)-2(4-a)a\tan^4(x)+\left(2a^2+1\right)\tan^3(x)+4\left(a^2+2\right)\tan^2(x)-8a\tan(x)\right.}{\tan^2(x)+1}$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{2}\int\left(2a^2\cot^3(x)-8a\cot^2(x)+2\left(a^2+4\right)\cot(x)+2a^2+2a^2\tan^2(x)+\frac{8a-8(a+1)\tan(x)+1}{\tan^2(x)+1}\right)d\tan(x)+$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}a^2\tan^3(x)+2a^2\tan(x)-a^2\cot^2(x)+2\left(a^2+4\right)\log(\tan(x))+(8a+1)\arctan(\tan(x))+8a\cot(x)-4(a+\right.$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

input
```
Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

output
```
(1 - Tan[x])/(1 + Tan[x]^2)^2 + (4 + Tan[x])/(2*(1 + Tan[x]^2)) + ((1 + 8*
a)*ArcTan[Tan[x]] + 8*a*Cot[x] - a^2*Cot[x]^2 + 2*(4 + a^2)*Log[Tan[x]] -
4*(1 + a)*Log[1 + Tan[x]^2] + 2*a^2*Tan[x] + (2*a^2*Tan[x]^3)/3)/2
```

3.363.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.363.4 Maple [A] (verified)

Time = 59.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
parts	$-\frac{\sin(2x)\cos(2x)}{4}+\frac{x}{2}+\cos^4(x)+2(\cos^2(x))+4\ln(\sin(x))-4a(-x-\cot(x))+a^2\left(-\frac{1}{2\sin(x)^2}+\mathrm{l}\right.$
default	$\cos^4(x)+2(\cos^2(x))+4\ln(\sin(x))-4a(-x-\cot(x))+a^2\left(-\frac{1}{2\sin(x)^2}+\ln(\tan(x))\right)+\frac{x}{2}+$
risch	$\frac{x}{2}-\frac{i e^{-4ix}}{16}+4ax-4ix+\frac{e^{4ix}}{16}-4iax+\frac{3e^{2ix}}{4}+\frac{3e^{-2ix}}{4}+\frac{e^{-4ix}}{16}+\frac{ie^{4ix}}{16}+\frac{2a(12ie^{8ix}+3a e^{8ix}+6ia e^{6ix}+24ie}{}$

input
```
int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*sin(2*x)*cos(2*x)+1/2*x+cos(x)^4+2*cos(x)^2+4*ln(sin(x))-4*a*(-x-cot(
x))+a^2*(-1/2/sin(x)^2+ln(tan(x)))-a^2*(-2/3-1/3*sec(x)^2)*tan(x)-4*a*ln(s
ec(x))
```

3.363.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$$
$$= \frac{24\cos(x)^9+24\cos(x)^7+3\,(4\,(8\,a+1)x-27)\cos(x)^5+3\,(4\,a^2-4\,(8\,a+1)x+11)\cos(x)^3-12\,((a^2\,\cdots}{}$$

input
```
integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="fricas")
```

output
```
1/24*(24*cos(x)^9 + 24*cos(x)^7 + 3*(4*(8*a + 1)*x - 27)*cos(x)^5 + 3*(4*a
^2 - 4*(8*a + 1)*x + 11)*cos(x)^3 - 12*((a^2 - 4*a)*cos(x)^5 - (a^2 - 4*a)
*cos(x)^3)*log(cos(x)^2) + 12*((a^2 + 4)*cos(x)^5 - (a^2 + 4)*cos(x)^3)*lo
g(-1/4*cos(x)^2 + 1/4) - 4*(6*cos(x)^8 - 9*cos(x)^6 - (4*a^2 - 24*a - 3)*c
os(x)^4 + 2*a^2*cos(x)^2 + 2*a^2)*sin(x))/(cos(x)^5 - cos(x)^3)
```

3.363.6 Sympy [A] (verification not implemented)

Time = 158.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx = -\frac{a^2\log\left(\sin^2(x)-1\right)}{2}+a^2\log\left(\sin(x)\right)+\frac{a^2\tan^3(x)}{3}+a^2\tan(x)-\frac{a^2}{2\sin^2(x)}+4ax+4a\log\left(\cos(x)\right)+\frac{4a\cos(x)}{\sin(x)}+\frac{x}{2}+4\log\left(\sin(x)\right)+\sin^4(x)-4\sin^2(x)-\frac{\sin(4x)}{8}$$

input
```
integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

output
```
-a**2*log(sin(x)**2 - 1)/2 + a**2*log(sin(x)) + a**2*tan(x)**3/3 + a**2*ta
n(x) - a**2/(2*sin(x)**2) + 4*a*x + 4*a*log(cos(x)) + 4*a*cos(x)/sin(x) +
x/2 + 4*log(sin(x)) + sin(x)**4 - 4*sin(x)**2 - sin(4*x)/8
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx = \frac{1}{3}\left(\tan(x)^3+3\,\tan(x)\right)a^2-\frac{1}{2}\,a^2\left(\frac{1}{\sin(x)^2}+\log\left(\sin(x)^2-1\right)-\log\left(\sin(x)^2\right)\right) + 4\,a\left(x+\frac{1}{\tan(x)}\right)+2\,a\log\left(-\sin(x)^2+1\right)+\frac{1}{2}\,x+\frac{1}{8}\,\cos(4\,x) + \frac{3}{2}\,\cos(2\,x)+2\,\log\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right) + 2\,\log\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)-\frac{1}{8}\,\sin(4\,x)$$

input
```
integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="maxima")
```

output
```
1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^2 - 1) -
log(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2 + 1) + 1/2*x + 1/8
*cos(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/8*sin(4*x)
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.69

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx = \frac{1}{3}\,a^2\tan(x)^3+a^2\tan(x) + \frac{1}{2}\,(8\,a+1)x-2\,(a+1)\log\left(\tan(x)^2+1\right)+\left(a^2+4\right)\log\left(|\tan(x)|\right) - \frac{a^2\tan(x)^6-4\,a\tan(x)^6+3\,a^2\tan(x)^4-8\,a\tan(x)^5-8\,a\tan(x)^4-\tan(x)^5+3\,a^2\tan(x)^2-16\,a\,\mathrm{ta}}{2\left(\tan(x)^3+\tan(x)\right)^2}$$

input
```
integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="giac")
```

output
```
1/3*a^2*tan(x)^3 + a^2*tan(x) + 1/2*(8*a + 1)*x - 2*(a + 1)*log(tan(x)^2 +
 1) + (a^2 + 4)*log(abs(tan(x))) - 1/2*(a^2*tan(x)^6 - 4*a*tan(x)^6 + 3*a^
2*tan(x)^4 - 8*a*tan(x)^5 - 8*a*tan(x)^4 - tan(x)^5 + 3*a^2*tan(x)^2 - 16*
a*tan(x)^3 - 4*tan(x)^4 - 4*a*tan(x)^2 + tan(x)^3 + a^2 - 8*a*tan(x) - 6*t
an(x)^2)/(tan(x)^3 + tan(x))^2
```

3.363.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

$$\int \left(1+\cot^3(x)\right) \left(a\sec^2(x)-\sin(2x)\right)^2 \,dx = a^2\tan(x) - \frac{\tan(x)^4\left(\frac{a^2}{2}-2\right)-4\,a\tan(x)+\frac{a^2}{2}-\tan(x)^5\left(4\,a+\frac{1}{2}\right)-\tan(x)^3\left(8\,a-\frac{1}{2}\right)+\tan(x)^2\left(a^2-3\right)}{\tan\left(x\right)^6+2\tan\left(x\right)^4+\tan\left(x\right)^2} - \ln\left(\tan(x)-\mathrm{i}\right)\left(a\left(2+2\mathrm{i}\right)+2+\frac{1}{4}\mathrm{i}\right) - \ln\left(\tan(x)+1\mathrm{i}\right)\left(a\left(2-2\mathrm{i}\right)+2-\frac{1}{4}\mathrm{i}\right)+\frac{a^2\tan(x)^3}{3}+\ln\left(\tan(x)\right)\left(a^2+4\right)$$

input
```
int((cot(x)^3 + 1)*(sin(2*x) - a/cos(x)^2)^2,x)
```

output
```
a^2*tan(x) - (tan(x)^4*(a^2/2 - 2) - 4*a*tan(x) + a^2/2 - tan(x)^5*(4*a +
1/2) - tan(x)^3*(8*a - 1/2) + tan(x)^2*(a^2 - 3))/(tan(x)^2 + 2*tan(x)^4 +
 tan(x)^6) - log(tan(x) - 1i)*(a*(2 + 2i) + (2 + 1i/4)) - log(tan(x) + 1i)
*(a*(2 - 2i) + (2 - 1i/4)) + (a^2*tan(x)^3)/3 + log(tan(x))*(a^2 + 4)
```

3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx$

3.364.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227x}{32}+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x)-\frac{3\sin^3(x)}{2}-\frac{1}{16}\cos(x)\sin^3(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^5(x)}{80}$$

output
```
227/32*x+10*cos(x)-3*cos(x)^2-2/3*cos(x)^3-3*sin(x)-99/32*cos(x)*sin(x)-3/
2*sin(x)^3-1/16*cos(x)*sin(x)^3+3/8*sin(x)^4-3/80*sin(x)^5
```

3.364.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{27}{16}\cos(2x)-\frac{1}{6}\cos(3x)+\frac{3}{64}\cos(4x)-\frac{531\sin(x)}{128}-\frac{25}{16}\sin(2x)+\frac{99}{256}\sin(3x)+\frac{1}{128}\sin(4x)-\frac{3\sin(5x)}{1280}$$

input
```
Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

output
```
(227*x)/32 + (19*Cos[x])/2 - (27*Cos[2*x])/16 - Cos[3*x]/6 + (3*Cos[4*x])/
64 - (531*Sin[x])/128 - (25*Sin[2*x])/16 + (99*Sin[3*x])/256 + Sin[4*x]/12
8 - (3*Sin[5*x])/1280
```

3.364.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\frac{\sin(x)}{2}\right)^4 (4-3\cos(x))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(1-\frac{\sin(x)}{2}\right)^4 (4-3\cos(x))dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(-3\cos(x)-\frac{1}{16}\sin^4(x)(3\cos(x)-4)+\frac{1}{2}\sin^3(x)(3\cos(x)-4)-\frac{3}{2}\sin^2(x)(3\cos(x)-4)+2\sin(x)(3\cos(x)-\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-\frac{1}{3}(4-3\cos(x))^2+2\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin(x)\cos(x)$$

input
```
Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

output
```
(227*x)/32 - (4 - 3*Cos[x])^2/3 + 2*Cos[x] - (2*Cos[x]^3)/3 - 3*Sin[x] - (
99*Cos[x]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4
)/8 - (3*Sin[x]^5)/80
```

3.364.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.364.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

method	result
parts	$\frac{227x}{32}-\frac{3(\sin(x)-2)^5}{80}-3\cos(x)\sin(x)+\frac{2(2+\sin^2(x))\cos(x)}{3}-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{16}+8\cos(x)$
risch	$\frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{531\sin(x)}{128}-\frac{3\sin(5x)}{1280}+\frac{3\cos(4x)}{64}+\frac{\sin(4x)}{128}-\frac{\cos(3x)}{6}+\frac{99\sin(3x)}{256}-\frac{27\cos(2x)}{16}-\frac{25\sin(}{16}$
parallelrisch	$-\frac{409}{960}+\frac{227x}{32}-\frac{25\sin(2x)}{16}-\frac{3\sin(5x)}{1280}+\frac{99\sin(3x)}{256}+\frac{\sin(4x)}{128}-\frac{531\sin(x)}{128}+\frac{3\cos(4x)}{64}-\frac{27\cos(2x)}{16}-\frac{\cos(3x)}{6}$
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{16}+\frac{227x}{32}+\frac{2(2+\sin^2(x))\cos(x)}{3}-3\cos(x)\sin(x)+8\cos(x)-\frac{3(\sin^5(x))}{80}+3($
norman	$\frac{28\left(\tan^8\left(\frac{x}{2}\right)\right)+114\left(\tan^6\left(\frac{x}{2}\right)\right)+\frac{268\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{470\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}+\frac{227x}{32}-\frac{391\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}-\frac{306\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}-\frac{185\left(\tan^7\left(\frac{x}{2}\right)\right)}{8}+\frac{3(\tan^9(}{16}}{(1+\tan^2(\frac{x}{2}))}$

input
```
int((4-3*cos(x))*(1-1/2*sin(x))^4,x,method=_RETURNVERBOSE)
```

output
```
227/32*x-3/80*(sin(x)-2)^5-3*cos(x)*sin(x)+2/3*(2+sin(x)^2)*cos(x)-1/16*(s
in(x)^3+3/2*sin(x))*cos(x)+8*cos(x)
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{3}{8}\cos(x)^4 - \frac{2}{3}\cos(x)^3 - \frac{15}{4}\cos(x)^2 - \frac{1}{160}\left(6\cos(x)^4 - 10\cos(x)^3 - 252\cos(x)^2 + 505\cos(x) + 726\right)\sin(x) + \frac{227}{32}x + 10\cos(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="fricas")
```

output
```
3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 - 10*cos(x
)^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x + 10*cos(x)
```

3.364.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{3x\sin^4(x)}{32} + \frac{3x\sin^2(x)\cos^2(x)}{16} + 3x\sin^2(x) + \frac{3x\cos^4(x)}{32} + 3x\cos^2(x) + 4x - \frac{3\sin^5(x)}{80} + \frac{3\sin^4(x)}{8} - \frac{5\sin^3(x)\cos(x)}{32} - \frac{3\sin^3(x)}{2} + 2\sin^2(x)\cos(x) - \frac{3\sin(x)\cos^3(x)}{32} - 3\sin(x)\cos(x) - 3\sin(x) + \frac{4\cos^3(x)}{3} - 3\cos^2(x) + 8\cos(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

output
```
3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3*x*cos(x)
**4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 + 3*sin(x)**4/8 - 5*sin(x)**
3*cos(x)/32 - 3*sin(x)**3/2 + 2*sin(x)**2*cos(x) - 3*sin(x)*cos(x)**3/32 -
 3*sin(x)*cos(x) - 3*sin(x) + 4*cos(x)**3/3 - 3*cos(x)**2 + 8*cos(x)
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = -\frac{3}{80}\sin(x)^5+\frac{3}{8}\sin(x)^4-\frac{2}{3}\cos(x)^3-\frac{3}{2}\sin(x)^3-3\cos(x)^2+\frac{227}{32}x+10\cos(x)+\frac{1}{128}\sin(4x)-\frac{25}{16}\sin(2x)-3\sin(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="maxima")
```

output
```
-3/80*sin(x)^5 + 3/8*sin(x)^4 - 2/3*cos(x)^3 - 3/2*sin(x)^3 - 3*cos(x)^2 +
 227/32*x + 10*cos(x) + 1/128*sin(4*x) - 25/16*sin(2*x) - 3*sin(x)
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227}{32}x+\frac{3}{64}\cos(4x)-\frac{1}{6}\cos(3x)-\frac{27}{16}\cos(2x)+\frac{19}{2}\cos(x)-\frac{3}{1280}\sin(5x)+\frac{1}{128}\sin(4x)+\frac{99}{256}\sin(3x)-\frac{25}{16}\sin(2x)-\frac{531}{128}\sin(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="giac")
```

output
```
227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*cos(x) - 3
/1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/16*sin(2*x) - 531/1
28*sin(x)
```

3.364.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = -\frac{6\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^9}{5}+6\cos\left(\frac{x}{2}\right)^8+\frac{17\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^7}{5} - \frac{52\cos\left(\frac{x}{2}\right)^6}{3}+\frac{93\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^5}{10} + 2\cos\left(\frac{x}{2}\right)^4-\frac{191\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)^3}{8} + 28\cos\left(\frac{x}{2}\right)^2+\frac{3\,\sin\left(\frac{x}{2}\right)\,\cos\left(\frac{x}{2}\right)}{16}+\frac{227\,x}{32}$$

input
```
int(-(3*cos(x) - 4)*(sin(x)/2 - 1)^4,x)
```

output
```
(227*x)/32 - (191*cos(x/2)^3*sin(x/2))/8 + (93*cos(x/2)^5*sin(x/2))/10 + (
17*cos(x/2)^7*sin(x/2))/5 - (6*cos(x/2)^9*sin(x/2))/5 + 28*cos(x/2)^2 + 2*
cos(x/2)^4 - (52*cos(x/2)^6)/3 + 6*cos(x/2)^8 + (3*cos(x/2)*sin(x/2))/16
```

3.365 $\int \left(\frac{1}{2}-3 \cot(x)\right) (3-2 \cot(x))^3 \, dx$

3.365.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \left(\frac{1}{2}-3 \cot(x)\right) (3-2 \cot(x))^3 \, dx = -\frac{285x}{2}+5(3-2 \cot(x))^2+(3-2 \cot(x))^3 - 42 \cot(x)+4 \log(\sin(x))$$

output
```
-285/2*x+5*(3-2*cot(x))^2+(3-2*cot(x))^3-42*cot(x)+4*ln(sin(x))
```

3.365.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \left(\frac{1}{2}-3 \cot(x)\right) (3-2 \cot(x))^3 \, dx = \frac{27x}{2}+56 \cot^2(x) - 8 \cot^3(x) \text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2}, -\tan^2(x)\right) - 180 \cot(x) \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}, -\tan^2(x)\right)+4 \log(\cos(x))+4 \log(\tan(x))$$

input
```
Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

output
```
(27*x)/2 + 56*Cot[x]^2 - 8*Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[
x]^2] - 180*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2] + 4*Log[Cos[
x]] + 4*Log[Tan[x]]
```

3.365.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 4011, 3042, 4011, 3042, 4008, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\tan\left(x+\frac{\pi}{2}\right)+3\right)^3\left(3\tan\left(x+\frac{\pi}{2}\right)+\frac{1}{2}\right)dx$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-10\cot(x)-\frac{9}{2}\right)(3-2\cot(x))^2dx+(3-2\cot(x))^3$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\tan\left(x+\frac{\pi}{2}\right)+3\right)^2\left(10\tan\left(x+\frac{\pi}{2}\right)-\frac{9}{2}\right)dx+(3-2\cot(x))^3$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-21\cot(x)-\frac{67}{2}\right)(3-2\cot(x))dx+(3-2\cot(x))^3+5(3-2\cot(x))^2$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\tan\left(x+\frac{\pi}{2}\right)+3\right)\left(21\tan\left(x+\frac{\pi}{2}\right)-\frac{67}{2}\right)dx+(3-2\cot(x))^3+5(3-2\cot(x))^2$$

$$\Big\downarrow \text{4008}$$

$$-4\int -\cot(x)dx-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)$$

$$\Big\downarrow \; 25$$

$$4\int \cot(x)dx-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)$$

$$\Big\downarrow \; 3042$$

$$4\int -\tan\left(x+\frac{\pi}{2}\right)dx-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)$$

$$\Big\downarrow \; 25$$

$$-4\int \tan\left(x+\frac{\pi}{2}\right)dx-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)$$

$$\Big\downarrow \; 3956$$

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

input
```
Int[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

output
```
(-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] + 4*Log[Sin
[x]]
```

3.365.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

3.365. $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.365.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

method	result
parallelrisch	$4\ln\left(\tan\left(x\right)\right)-2\ln\left(\sec^{2}\left(x\right)\right)-\frac{285x}{2}-8(\cot^{3}\left(x\right))-156\cot\left(x\right)+56(\cot^{2}\left(x\right))$
derivativedivides	$-8(\cot^{3}\left(x\right))+56(\cot^{2}\left(x\right))-156\cot\left(x\right)-2\ln\left(\cot^{2}\left(x\right)+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$
default	$-8(\cot^{3}\left(x\right))+56(\cot^{2}\left(x\right))-156\cot\left(x\right)-2\ln\left(\cot^{2}\left(x\right)+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$
norman	$\frac{-8-156(\tan^{2}(x))-\frac{285x\left(\tan^{3}(x)\right)}{2}+56\tan(x)}{\tan(x)^{3}}+4\ln\left(\tan\left(x\right)\right)-2\ln\left(1+\tan^{2}\left(x\right)\right)$
parts	$\frac{27x}{2}-156\cot\left(x\right)+78\pi-156\,\mathrm{arccot}\left(\cot\left(x\right)\right)+56(\cot^{2}\left(x\right))-56\ln\left(\cot^{2}\left(x\right)+1\right)-8$
risch	$-\frac{285x}{2}-4ix+\frac{\left(-\frac{224}{1873}-\frac{264i}{1873}\right)\left(1873\,\mathrm{e}^{4ix}-1260i\mathrm{e}^{2ix}-3358\,\mathrm{e}^{2ix}+1221+1036i\right)}{\left(\mathrm{e}^{2ix}-1\right)^{3}}+4\ln\left(\mathrm{e}^{2ix}-1\right)$

input
```
int((1/2-3*cot(x))*(3-2*cot(x))^3,x,method=_RETURNVERBOSE)
```

output
```
4*ln(tan(x))-2*ln(sec(x)^2)-285/2*x-8*cot(x)^3-156*cot(x)+56*cot(x)^2
```

3.365.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int\left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$$
$$=\frac{4\left(\cos\left(2\,x\right)-1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)\sin\left(2\,x\right)-296\,\cos\left(2\,x\right)^{2}-\left(285\,x\cos\left(2\,x\right)-285\,x+224\right)\sin\left(2\,x\right)}{2\left(\cos\left(2\,x\right)-1\right)\sin\left(2\,x\right)}$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="fricas")
```

output
```
1/2*(4*(cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2)*sin(2*x) - 296*cos(2*x)^2 -
 (285*x*cos(2*x) - 285*x + 224)*sin(2*x) + 32*cos(2*x) + 328)/((cos(2*x) -
 1)*sin(2*x))
```

3.365.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = -\frac{285x}{2} - 2\log\left(\tan^2(x)+1\right) + 4\log(\tan(x)) - \frac{156}{\tan(x)} + \frac{56}{\tan^2(x)} - \frac{8}{\tan^3(x)}$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
```

output
```
-285*x/2 - 2*log(tan(x)**2 + 1) + 4*log(tan(x)) - 156/tan(x) + 56/tan(x)**
2 - 8/tan(x)**3
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = -\frac{285}{2}\,x - \frac{4\left(39\,\tan(x)^2-14\,\tan(x)+2\right)}{\tan(x)^3} - 2\,\log\left(\tan(x)^2+1\right) + 4\,\log(\tan(x))$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="maxima")
```

output
```
-285/2*x - 4*(39*tan(x)^2 - 14*tan(x) + 2)/tan(x)^3 - 2*log(tan(x)^2 + 1)
+ 4*log(tan(x))
```

3.365.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = \tan\left(\frac{1}{2}x\right)^3 + 14\tan\left(\frac{1}{2}x\right)^2 - \frac{285}{2}x - \frac{22\tan\left(\frac{1}{2}x\right)^3 + 225\tan\left(\frac{1}{2}x\right)^2 - 42\tan\left(\frac{1}{2}x\right) + 3}{3\tan\left(\frac{1}{2}x\right)^3} - 4\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 4\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right) + 75\tan\left(\frac{1}{2}x\right)$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="giac")
```

output
```
tan(1/2*x)^3 + 14*tan(1/2*x)^2 - 285/2*x - 1/3*(22*tan(1/2*x)^3 + 225*tan(
1/2*x)^2 - 42*tan(1/2*x) + 3)/tan(1/2*x)^3 - 4*log(tan(1/2*x)^2 + 1) + 4*l
og(abs(tan(1/2*x))) + 75*tan(1/2*x)
```

3.365.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = x\left(-\frac{285}{2} - 4i\right) + 4\ln\left(e^{x\,2i} - 1\right) + \frac{64i}{3\,e^{x\,2i} - 3\,e^{x\,4i} + e^{x\,6i} - 1} + \frac{-224 + 96i}{1 + e^{x\,4i} - 2\,e^{x\,2i}} + \frac{-224 - 264i}{e^{x\,2i} - 1}$$

input
```
int((2*cot(x) - 3)^3*(3*cot(x) - 1/2),x)
```

output
```
4*log(exp(x*2i) - 1) - x*(285/2 + 4i) + 64i/(3*exp(x*2i) - 3*exp(x*4i) + e
xp(x*6i) - 1) - (224 - 96i)/(exp(x*4i) - 2*exp(x*2i) + 1) - (224 + 264i)/(
exp(x*2i) - 1)
```

3.366 $\int \cos(5x)\sec^5(x)\,dx$

3.366.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \cos(5x)\sec^5(x)\,dx = 16x - 15\tan(x) + \frac{5\tan^3(x)}{3}$$

output
```
16*x-15*tan(x)+5/3*tan(x)^3
```

3.366.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \cos(5x)\sec^5(x)\,dx = 16x - \frac{50\tan(x)}{3} + \frac{5}{3}\sec^2(x)\tan(x)$$

input
```
Integrate[Cos[5*x]*Sec[x]^5,x]
```

output
```
16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3
```

3.366.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4889, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(5x)\sec^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(5x)}{\cos(x)^5}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{5\tan^4(x)-10\tan^2(x)+1}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(5\tan^2(x)+\frac{16}{\tan^2(x)+1}-15\right)d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$16\arctan(\tan(x))+\frac{5\tan^3(x)}{3}-15\tan(x)$$

input
```
Int[Cos[5*x]*Sec[x]^5,x]
```

output
```
16*ArcTan[Tan[x]] - 15*Tan[x] + (5*Tan[x]^3)/3
```

3.366.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.366.4 Maple [A] (verified)

Time = 33.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$16x - 5\left(-\frac{2}{3} - \frac{(\sec^2(x))}{3}\right)\tan(x) - 20\tan(x)$	21
risch	$16x - \frac{20i(6\,e^{4ix}+9\,e^{2ix}+5)}{3(e^{2ix}+1)^3}$	33

input
```
int(cos(5*x)/cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cos(5x)\sec^5(x)\,dx = \frac{48\,x\cos(x)^3 - 5\left(10\,\cos(x)^2 - 1\right)\sin(x)}{3\,\cos(x)^3}$$

input
```
integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")
```

output
```
1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3
```

3.366.6 Sympy [A] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \cos(5x)\sec^5(x)\,dx = 16x - \frac{20\sin(x)}{\cos(x)} + \frac{5\tan^3(x)}{3} + 5\tan(x)$$

input
```
integrate(cos(5*x)/cos(x)**5,x)
```

output
```
16*x - 20*sin(x)/cos(x) + 5*tan(x)**3/3 + 5*tan(x)
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos(5x)\sec^5(x)\,dx = \frac{5}{3}\tan(x)^3 + 16x - 15\tan(x)$$

input
```
integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")
```

output
```
5/3*tan(x)^3 + 16*x - 15*tan(x)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos(5x)\sec^5(x)\,dx = \frac{5}{3}\tan(x)^3 + 16x - 15\tan(x)$$

input
```
integrate(cos(5*x)/cos(x)^5,x, algorithm="giac")
```

output
```
5/3*tan(x)^3 + 16*x - 15*tan(x)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cos(5x)\sec^5(x)\,dx = \frac{48\,x\cos(x)^3 - 50\,\sin(x)\,\cos(x)^2 + 5\,\sin(x)}{3\cos(x)^3}$$

input
```
int(cos(5*x)/cos(x)^5,x)
```

output
```
(5*sin(x) + 48*x*cos(x)^3 - 50*cos(x)^2*sin(x))/(3*cos(x)^3)
```

3.367 $\int \cos(4x)\sec(x)\,dx$

3.367.1 Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

output
```
arctanh(sin(x))-8/3*sin(x)^3
```

3.367.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

input
```
Integrate[Cos[4*x]*Sec[x],x]
```

output
```
ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

3.367.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4864, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(4x)\sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(4x)}{\cos(x)}dx$$

$$\Big\downarrow \text{ 4864}$$

$$\int \frac{8\sin^4(x)-8\sin^2(x)+1}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(\frac{1}{1-\sin^2(x)}-8\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\text{arctanh}(\sin(x))-\frac{8\sin^3(x)}{3}$$

input
```
Int[Cos[4*x]*Sec[x],x]
```

output
```
ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

3.367.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.367.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

method	result	size
default	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+\frac{8\left(2+\cos^{2}\left(x\right)\right)\sin\left(x\right)}{3}-8\sin\left(x\right)$	22
risch	$i\,\mathrm{e}^{i x}-i\,\mathrm{e}^{-i x}+\ln\left(i+\mathrm{e}^{i x}\right)-\ln\left(\mathrm{e}^{i x}-i\right)+\frac{2\sin\left(3 x\right)}{3}$	44

input
```
int(cos(4*x)/cos(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x)+tan(x))+8/3*(2+cos(x)^2)*sin(x)-8*sin(x)
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \cos(4x)\sec(x)\,dx = \frac{8}{3}\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)+\frac{1}{2}\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(cos(4*x)/cos(x),x, algorithm="fricas")
```

output
```
8/3*(cos(x)^2 - 1)*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.367.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \cos(4x)\sec(x)\,dx = -\frac{\log(\sin(x)-1)}{2} + \frac{\log(\sin(x)+1)}{2} - \frac{8\sin^3(x)}{3}$$

input
```
integrate(cos(4*x)/cos(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - 8*sin(x)**3/3
```

3.367.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \cos(4x)\sec(x)\,dx = -\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(\sin(x)-1)$$

input
```
integrate(cos(4*x)/cos(x),x, algorithm="maxima")
```

output
```
-8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \cos(4x)\sec(x)\,dx = -\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)$$

input
```
integrate(cos(4*x)/cos(x),x, algorithm="giac")
```

output
```
-8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.367.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cos(4x)\sec(x)\,dx = \operatorname{atanh}(\sin(x)) - \frac{8\sin(x)^3}{3}$$

input
```
int(cos(4*x)/cos(x),x)
```

output
```
atanh(sin(x)) - (8*sin(x)^3)/3
```

3.368 $\int \cos(x)\cos(4x)\,dx$

3.368.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

3.368.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

input
```
Integrate[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

3.368.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(x)\cos(4x)dx$$

$$\Big\downarrow \text{ 4771}$$

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

input
```
Int[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

3.368.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.368.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
parallelrisch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{2\left(\tan^2(2x)\right)\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

input
```
int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

3.368.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{15}\left(24\,\cos\left(x\right)^4-8\,\cos\left(x\right)^2-1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

output
```
1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

3.368.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(x)\cos(4x)\,dx = -\frac{\sin\left(x\right)\cos\left(4x\right)}{15}+\frac{4\sin\left(4x\right)\cos\left(x\right)}{15}$$

input
```
integrate(cos(x)*cos(4*x),x)
```

output
```
-sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

3.368.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

input
```
int(cos(4*x)*cos(x),x)
```

output
```
sin(3*x)/6 + sin(5*x)/10
```

3.369 $\int \cos(4x)\sec^5(x)\,dx$

3.369.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \cos(4x)\sec^5(x)\,dx = \frac{35}{8}\operatorname{arctanh}(\sin(x)) - \frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)$$

output
```
35/8*arctanh(sin(x))-29/8*sec(x)*tan(x)+1/4*sec(x)^3*tan(x)
```

3.369.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec^5(x)\,dx = \frac{1}{8}\left(35\operatorname{arctanh}(\sin(x)) - 27\sec^3(x)\tan(x) + 29\sec(x)\tan^3(x)\right)$$

input
```
Integrate[Cos[4*x]*Sec[x]^5,x]
```

output
```
(35*ArcTanh[Sin[x]] - 27*Sec[x]^3*Tan[x] + 29*Sec[x]*Tan[x]^3)/8
```

3.369.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4864, 1471, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(4x)\sec^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(4x)}{\cos(x)^5}dx$$

$$\Big\downarrow \text{4864}$$

$$\int \frac{8\sin^4(x)-8\sin^2(x)+1}{\left(1-\sin^2(x)\right)^3}d\sin(x)$$

$$\Big\downarrow \text{1471}$$

$$\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}-\frac{1}{4}\int -\frac{3-32\sin^2(x)}{\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{3-32\sin^2(x)}{\left(1-\sin^2(x)\right)^2}d\sin(x)+\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{1}{4}\left(\frac{35}{2}\int \frac{1}{1-\sin^2(x)}d\sin(x)-\frac{29\sin(x)}{2\left(1-\sin^2(x)\right)}\right)+\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{35}{2}\text{arctanh}(\sin(x))-\frac{29\sin(x)}{2\left(1-\sin^2(x)\right)}\right)+\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}$$

input
```
Int[Cos[4*x]*Sec[x]^5,x]
```

output
```
Sin[x]/(4*(1 - Sin[x]^2)^2) + ((35*ArcTanh[Sin[x]])/2 - (29*Sin[x])/(2*(1
- Sin[x]^2)))/4
```

3.369.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.369. $\int \cos(4x)\sec^5(x)\,dx$

3.369.4 Maple [A] (verified)

Time = 38.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
default	$-\left(-\frac{\left(\sec^{3}\left(x\right)\right)}{4}-\frac{3\sec\left(x\right)}{8}\right)\tan\left(x\right)+\frac{35\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)}{8}-4\sec\left(x\right)\tan\left(x\right)$	31
risch	$\frac{i\left(29\,{\mathrm e}^{7 i x}+21\,{\mathrm e}^{5 i x}-21\,{\mathrm e}^{3 i x}-29\,{\mathrm e}^{i x}\right)}{4\left({\mathrm e}^{2 i x}+1\right)^{4}}+\frac{35\ln\left(i+{\mathrm e}^{i x}\right)}{8}-\frac{35\ln\left({\mathrm e}^{i x}-i\right)}{8}$	65

input
```
int(cos(4*x)/cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+35/8*ln(sec(x)+tan(x))-4*sec(x)*tan(x)
```

3.369.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int\cos(4x)\sec^5(x)\,dx = \frac{35\,\cos\left(x\right)^{4}\log\left(\sin\left(x\right)+1\right)-35\,\cos\left(x\right)^{4}\log\left(-\sin\left(x\right)+1\right)-2\,\left(29\,\cos\left(x\right)^{2}-2\right)\sin\left(x\right)}{16\,\cos\left(x\right)^{4}}$$

input
```
integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")
```

output
```
1/16*(35*cos(x)^4*log(sin(x) + 1) - 35*cos(x)^4*log(-sin(x) + 1) - 2*(29*c
os(x)^2 - 2)*sin(x))/cos(x)^4
```

3.369.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(27) = 54.

Time = 6.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.88

$$\int\cos(4x)\sec^5(x)\,dx = -\frac{35\log\left(\sin\left(x\right)-1\right)}{16}+\frac{35\log\left(\sin\left(x\right)+1\right)}{16}-\frac{3\sin^{3}\left(x\right)}{8\sin^{4}\left(x\right)-16\sin^{2}\left(x\right)+8}+\frac{5\sin\left(x\right)}{8\sin^{4}\left(x\right)-16\sin^{2}\left(x\right)+8}+\frac{8\sin\left(x\right)}{2\sin^{2}\left(x\right)-2}$$

input
```
integrate(cos(4*x)/cos(x)**5,x)
```

output
```
-35*log(sin(x) - 1)/16 + 35*log(sin(x) + 1)/16 - 3*sin(x)**3/(8*sin(x)**4
- 16*sin(x)**2 + 8) + 5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 8*sin(x)
/(2*sin(x)**2 - 2)
```

3.369.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cos(4x)\sec^5(x)\,dx = \frac{5\,\sin(x)^3 - 3\,\sin(x)}{8\left(\sin(x)^4 - 2\,\sin(x)^2 + 1\right)} + \frac{3\,\sin(x)}{\sin(x)^2 - 1} + \frac{35}{16}\,\log(\sin(x) + 1) - \frac{35}{16}\,\log(\sin(x) - 1)$$

input
```
integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")
```

output
```
1/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*sin(x)/(sin(x)
^2 - 1) + 35/16*log(sin(x) + 1) - 35/16*log(sin(x) - 1)
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cos(4x)\sec^5(x)\,dx = \frac{29\,\sin(x)^3 - 27\,\sin(x)}{8\left(\sin(x)^2 - 1\right)^2} + \frac{35}{16}\,\log(\sin(x) + 1) - \frac{35}{16}\,\log(-\sin(x) + 1)$$

input
```
integrate(cos(4*x)/cos(x)^5,x, algorithm="giac")
```

output
```
1/8*(29*sin(x)^3 - 27*sin(x))/(sin(x)^2 - 1)^2 + 35/16*log(sin(x) + 1) - 3
5/16*log(-sin(x) + 1)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cos(4x)\sec^5(x)\,dx = \frac{35\,\mathrm{atanh}(\sin(x))}{8} - \frac{\frac{27\,\sin(x)}{8} - \frac{29\sin(x)^3}{8}}{\sin(x)^4 - 2\sin(x)^2 + 1}$$

input
```
int(cos(4*x)/cos(x)^5,x)
```

output
```
(35*atanh(sin(x)))/8 - ((27*sin(x))/8 - (29*sin(x)^3)/8)/(sin(x)^4 - 2*sin
(x)^2 + 1)
```

3.370 $\int \cos^4(x)\cos(4x)\,dx$

3.370.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

output
```
1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

3.370.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

input
```
Integrate[Cos[x]^4*Cos[4*x],x]
```

output
```
x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

3.370.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(x)\cos(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)^4\cos(4x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{1}{4}\cos(2x)+\frac{3}{8}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{16}\cos(8x)+\frac{1}{16}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

input
```
Int[Cos[x]^4*Cos[4*x],x]
```

output
```
x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

3.370.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.370.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29
risch	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29
parallelrisch	$\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$	29

input
```
int(cos(x)^4*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^4(x)\cos(4x)\,dx = \frac{1}{48}\left(48\cos(x)^7-8\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{16}x$$

input
```
integrate(cos(x)^4*cos(4*x),x, algorithm="fricas")
```

output
```
1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

3.370.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(31) = 62.

Time = 1.59 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\begin{aligned}\int \cos^4(x)\cos(4x)\,dx = {} & \frac{x\sin^4(x)\cos(4x)}{16}-\frac{x\sin^3(x)\sin(4x)\cos(x)}{4}\\ & -\frac{3x\sin^2(x)\cos^2(x)\cos(4x)}{8}+\frac{x\sin(x)\sin(4x)\cos^3(x)}{4}\\ & +\frac{x\cos^4(x)\cos(4x)}{16}-\frac{\sin^4(x)\sin(4x)}{24}-\frac{5\sin^3(x)\cos(x)\cos(4x)}{48}\\ & -\frac{11\sin(x)\cos^3(x)\cos(4x)}{48}+\frac{7\sin(4x)\cos^4(x)}{24}\end{aligned}$$

input
```
integrate(cos(x)**4*cos(4*x),x)
```

output
```
x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin(x)**2*co
s(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*cos(x)**4*cos(4*x)/
16 - sin(x)**4*sin(4*x)/24 - 5*sin(x)**3*cos(x)*cos(4*x)/48 - 11*sin(x)*co
s(x)**3*cos(4*x)/48 + 7*sin(4*x)*cos(x)**4/24
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \cos^4(x)\cos(4x)\,dx = -\frac{1}{6}\sin(2x)^3 + \frac{1}{16}x + \frac{1}{128}\sin(8x) + \frac{3}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)^4*cos(4*x),x, algorithm="maxima")
```

output
```
-1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*sin(2*x)
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\cos(4x)\,dx = \frac{1}{16}x + \frac{1}{128}\sin(8x) + \frac{1}{24}\sin(6x) + \frac{3}{32}\sin(4x) + \frac{1}{8}\sin(2x)$$

input
```
integrate(cos(x)^4*cos(4*x),x, algorithm="giac")
```

output
```
1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin(2*x)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16} + \frac{\frac{\tan(x)^7}{16} + \frac{11\tan(x)^5}{48} + \frac{5\tan(x)^3}{48} + \frac{15\tan(x)}{16}}{\left(\tan(x)^2+1\right)^4}$$

input
```
int(cos(4*x)*cos(x)^4,x)
```

output
```
x/16 + ((15*tan(x))/16 + (5*tan(x)^3)/48 + (11*tan(x)^5)/48 + tan(x)^7/16)
/(tan(x)^2 + 1)^4
```

3.371 $\int \cos(5x)\csc^5(x)\,dx$

3.371.1 Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \cos(5x)\csc^5(x)\,dx = 6\csc^2(x) - \frac{\csc^4(x)}{4} + 16\log(\sin(x))$$

output
```
6*csc(x)^2-1/4*csc(x)^4+16*ln(sin(x))
```

3.371.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos(5x)\csc^5(x)\,dx = 6\csc^2(x) - \frac{\csc^4(x)}{4} + 16\log(\sin(x))$$

input
```
Integrate[Cos[5*x]*Csc[x]^5,x]
```

output
```
6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

3.371.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4866, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(5x)\csc^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(5x)}{\sin(x)^5}dx$$

$$\Big\downarrow \text{4866}$$

$$-\int \frac{\cos(x)\left(16\cos^4(x)-20\cos^2(x)+5\right)}{\left(1-\cos^2(x)\right)^3}d\cos(x)$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{2}\int \frac{16\cos^4(x)-20\cos^2(x)+5}{\left(1-\cos^2(x)\right)^3}d\cos^2(x)$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{2}\int \left(-\frac{16}{\cos^2(x)-1}-\frac{12}{\left(\cos^2(x)-1\right)^2}-\frac{1}{\left(\cos^2(x)-1\right)^3}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{12}{1-\cos^2(x)}-\frac{1}{2\left(1-\cos^2(x)\right)^2}+16\log\left(1-\cos^2(x)\right)\right)$$

input
```
Int[Cos[5*x]*Csc[x]^5,x]
```

output
```
(-1/2*1/(1 - Cos[x]^2)^2 + 12/(1 - Cos[x]^2) + 16*Log[1 - Cos[x]^2])/2
```

3.371.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.371.4 Maple [A] (verified)

Time = 25.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

method	result	size
default	$-\frac{5}{4\sin(x)^4}+\frac{5(\cos^4(x))}{\sin(x)^4}-4(\cot^4(x))+8(\cot^2(x))+16\ln(\sin(x))$	35
risch	$-16ix-\frac{4(6\,e^{6ix}-11\,e^{4ix}+6\,e^{2ix})}{(e^{2ix}-1)^4}+16\ln(e^{2ix}-1)$	49

input
```
int(cos(5*x)/sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/4/sin(x)^4+5/sin(x)^4*cos(x)^4-4*cot(x)^4+8*cot(x)^2+16*ln(sin(x))
```

3.371.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \cos(5x)\csc^5(x)\,dx = -\frac{24\cos(x)^2 - 64\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 23}{4\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

input
```
integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")
```

output
```
-1/4*(24*cos(x)^2 - 64*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*sin(x)) - 23)/(
cos(x)^4 - 2*cos(x)^2 + 1)
```

3.371.6 Sympy [A] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \cos(5x)\csc^5(x)\,dx = 8\log\left(\sin^2(x)\right) + \frac{6}{\sin^2(x)} - \frac{1}{4\sin^4(x)}$$

input
```
integrate(cos(5*x)/sin(x)**5,x)
```

output
```
8*log(sin(x)**2) + 6/sin(x)**2 - 1/(4*sin(x)**4)
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \cos(5x)\csc^5(x)\,dx = \frac{5}{\sin(x)^2} + \frac{4\sin(x)^2 - 1}{4\sin(x)^4} + \frac{11}{2}\log\left(\sin(x)^2\right) + 5\log\left(\sin(x)\right)$$

input
```
integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")
```

output
```
5/sin(x)^2 + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 11/2*log(sin(x)^2) + 5*log(si
n(x))
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos(5x)\csc^5(x)\,dx = \frac{24\sin(x)^2 - 1}{4\sin(x)^4} + 16\log(|\sin(x)|)$$

input
```
integrate(cos(5*x)/sin(x)^5,x, algorithm="giac")
```

output
```
1/4*(24*sin(x)^2 - 1)/sin(x)^4 + 16*log(abs(sin(x)))
```

3.371.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos(5x)\csc^5(x)\,dx = 8\ln\left(\sin(x)^2\right) + \frac{6\sin(x)^2 - \frac{1}{4}}{\sin(x)^4}$$

input
```
int(cos(5*x)/sin(x)^5,x)
```

output
```
8*log(sin(x)^2) + (6*sin(x)^2 - 1/4)/sin(x)^4
```

3.372 $\int \csc^4(x)\sin(4x)\,dx$

3.372.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \csc^4(x)\sin(4x)\,dx = -2\csc^2(x) - 8\log(\sin(x))$$

output
```
-2*csc(x)^2-8*ln(sin(x))
```

3.372.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \csc^4(x)\sin(4x)\,dx = -2\csc^2(x) - 8\log(\sin(x))$$

input
```
Integrate[Csc[x]^4*Sin[4*x],x]
```

output
```
-2*Csc[x]^2 - 8*Log[Sin[x]]
```

3.372.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4878, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(4x)\csc^4(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(4x)}{\sin(x)^4}dx$$
$$\Big\downarrow \text{4878}$$
$$\int 4\left(1-2\sin^2(x)\right)\csc^3(x)d\sin(x)$$
$$\Big\downarrow \text{27}$$
$$4\int \csc^3(x)\left(1-2\sin^2(x)\right)d\sin(x)$$
$$\Big\downarrow \text{244}$$
$$4\int \left(\csc^3(x)-2\csc(x)\right)d\sin(x)$$
$$\Big\downarrow \text{2009}$$
$$4\left(-\frac{1}{2}\csc^2(x)-2\log(\sin(x))\right)$$

input
```
Int[Csc[x]^4*Sin[4*x],x]
```

output
```
4*(-1/2*Csc[x]^2 - 2*Log[Sin[x]])
```

3.372.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.372.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{2}{\sin(x)^2} - 4(\cot^2(x)) - 8\ln(\sin(x))$	19
risch	$8ix + \frac{8\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}-1)^2} - 8\ln(\mathrm{e}^{2ix} - 1)$	32

input
```
int(sin(4*x)/sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2\left(4\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\sin(x)\right)-1\right)}{\cos(x)^2-1}$$

input
```
integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")
```

output
```
-2*(4*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)
```

3.372.6 Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \csc^4(x)\sin(4x)\,dx = -8\log(\sin(x)) - \frac{2}{\sin^2(x)}$$

input
```
integrate(sin(4*x)/sin(x)**4,x)
```

output
```
-8*log(sin(x)) - 2/sin(x)**2
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2}{\sin(x)^2} - 2\log\left(\sin(x)^2\right) - 4\log(\sin(x))$$

input
```
integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")
```

output
```
-2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2}{\sin{(x)}^2} - 8\,\log{(|\sin{(x)}|)}$$

input
```
integrate(sin(4*x)/sin(x)^4,x, algorithm="giac")
```

output
```
-2/sin(x)^2 - 8*log(abs(sin(x)))
```

3.372.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \csc^4(x)\sin(4x)\,dx = 8\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - 8\,\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{2\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2}{2}$$

input
```
int(sin(4*x)/sin(x)^4,x)
```

output
```
8*log(tan(x/2)^2 + 1) - 8*log(tan(x/2)) - 1/(2*tan(x/2)^2) - tan(x/2)^2/2
```

3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

3.373.1 Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{x}{2\sqrt{3}} + \frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}} + \frac{1}{2}\log(\sin(x)) - \frac{1}{4}\log(1+\cos(x)\sin(x))$$

output
```
1/2*ln(sin(x))-1/4*ln(1+cos(x)*sin(x))-1/6*x*3^(1/2)+1/6*arctan((1-2*cos(x
)^2)/(2+2*cos(x)*sin(x)+3^(1/2)))*3^(1/2)
```

3.373.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{1+2\tan(x)}{\sqrt{3}}\right) + 6\log(\sin(x)) - 3\log(2+\sin(2x))\right)$$

input
```
Integrate[Cot[x]/(2 + Sin[2*x]),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Log[2 + Sin
[2*x]])/12
```

3.373.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4889, 27, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sin(2x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(2x)+2)\tan(x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\cot(x)}{2\left(\tan^2(x)+\tan(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\cot(x)}{\tan^2(x)+\tan(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\int -\frac{\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)+\log(\tan(x))\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\log(\tan(x))-\int \frac{\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\tan^2(x)+\tan(x)+1}d\tan(x)-\frac{1}{2}\int \frac{2\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)+\log(\tan(x))\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)+\int \frac{1}{-(2\tan(x)+1)^2-3}d(2\tan(x)+1)+\log(\tan(x))\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)-\frac{\arctan\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(\tan(x))\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\tan^2(x)+\tan(x)+1\right)+\log(\tan(x))\right)$$

input
```
Int[Cot[x]/(2 + Sin[2*x]),x]
```

output
```
(-(ArcTan[(1 + 2*Tan[x])/Sqrt[3]]/Sqrt[3]) + Log[Tan[x]] - Log[1 + Tan[x]
+ Tan[x]^2]/2)/2
```

3.373.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.373. $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.373.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\ln(\tan(x))}{2}-\frac{\ln(\tan^2(x)+\tan(x)+1)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}$	35
risch	$-\frac{\ln\left(e^{2ix}-i\sqrt{3}+2i\right)}{4}+\frac{i\ln\left(e^{2ix}-i\sqrt{3}+2i\right)\sqrt{3}}{12}-\frac{\ln\left(e^{2ix}+i\sqrt{3}+2i\right)}{4}-\frac{i\ln\left(e^{2ix}+i\sqrt{3}+2i\right)\sqrt{3}}{12}+\frac{\ln(e^{2ix}-1)}{2}$	88

input
```
int(cos(x)/sin(x)/(2+sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tan(x))-1/4*ln(tan(x)^2+tan(x)+1)-1/6*3^(1/2)*arctan(1/3*(2*tan(x)+
1)*3^(1/2))
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) - \frac{1}{8}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right) + \frac{1}{4}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
 1)) - 1/8*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/4*log(-1/4*
cos(x)^2 + 1/4)
```

3.373.6 Sympy [F]

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \int \frac{\cos(x)}{(\sin(2x)+2)\sin(x)}\,dx$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
```

output
```
Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)
```

3.373.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.25

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx =$$

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(-2\left(4\sin(2x)+1\right)\cos(4x)+\cos(4x)^2+16\cos(2x)^2+8\cos(2x)\sin(4x)+\sin(4\right.\right.$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="maxima")
```

output
```
-1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 16*
cos(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2*x)^2 + 8*sin(2*x)
 + 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 2*sqrt(3)*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*
x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2
 + sin(2*x)^2 + 4*sin(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + s
in(2*x)^2 - 4*sqrt(3) + 7)))
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{\cot(x)}{2+\sin(2x)}\,dx &= -\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right)\\ &\quad -\frac{1}{4}\log\left(\tan(x)^2+\tan(x)+1\right)+\frac{1}{2}\log\left(|\tan(x)|\right)\end{aligned}$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(
sqrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) - 1/4*log(tan(x)
^2 + tan(x) + 1) + 1/2*log(abs(tan(x)))
```

3.373.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \frac{\cot(x)}{2+\sin(2x)}\,dx &= \frac{\ln(\tan(x))}{2}+\ln\left(\tan(x)+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)\\ &\quad -\ln\left(\tan(x)+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)\end{aligned}$$

input
```
int(cos(x)/(sin(x)*(sin(2*x) + 2)),x)
```

output
```
log(tan(x))/2 + log(tan(x) - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/4)
 - log(tan(x) + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 + 1/4)
```

3.373. $\int \frac{\cot(x)}{2+\sin(2x)}\, dx$

3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$

3.374.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{2}\log\left(-4+\csc^2(x)\right)$$

output
```
-1/2*ln(-4+csc(x)^2)
```

3.374.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

input
```
Integrate[Cos[x]*Cot[x]*Sec[3*x],x]
```

output
```
Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

3.374.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4856, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(x)\sec(3x)dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{\csc(x)}{1-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\csc(x)}{1-4\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(4\int \frac{1}{1-4\sin^2(x)}d\sin^2(x)+\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(4\int \frac{1}{1-4\sin^2(x)}d\sin^2(x)+\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(\sin^2(x)\right)-\log\left(1-4\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]*Cot[x]*Sec[3*x],x]
```

output
```
(Log[Sin[x]^2] - Log[1 - 4*Sin[x]^2])/2
```

3.374.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.374.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
default	$-\frac{\ln(4(\cos^2(x))-3)}{2}+\frac{\ln(-1+\cos(x))}{2}+\frac{\ln(\cos(x)+1)}{2}$	27
risch	$\ln\left(e^{2ix}-1\right)-\frac{\ln(e^{4ix}-e^{2ix}+1)}{2}$	27

input
```
int(cos(x)^2/cos(3*x)/sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(4*cos(x)^2-3)+1/2*ln(-1+cos(x))+1/2*ln(cos(x)+1)
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{2}\log\left(4\cos(x)^2-3\right)+\log\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="fricas")
```

output
```
-1/2*log(4*cos(x)^2 - 3) + log(1/2*sin(x))
```

3.374.6 Sympy [F]

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \int \frac{\cos^2(x)}{\sin(x)\cos(3x)}\,dx$$

input
```
integrate(cos(x)**2/cos(3*x)/sin(x),x)
```

output
```
Integral(cos(x)**2/(sin(x)*cos(3*x)), x)
```

3.374.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(9) = 18.

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 8.36

$$\begin{aligned}\int \cos(x)\cot(x)\sec(3x)\,dx = &-\frac{1}{4}\log\left(-2\left(\cos(2x)-1\right)\cos(4x)+\cos(4x)^2+\cos(2x)^2\right.\\&\left.+\sin(4x)^2-2\sin(4x)\sin(2x)+\sin(2x)^2-2\cos(2x)+1\right)\\&+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\\&+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\end{aligned}$$

input
```
integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="maxima")
```

output
```
-1/4*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/2*log(cos(x)^2 +
 sin(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.374.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \frac{1}{2}\log\left(-\cos(x)^2+1\right) - \frac{1}{2}\log\left(\left|4\cos(x)^2-3\right|\right)$$

input
```
integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="giac")
```

output
```
1/2*log(-cos(x)^2 + 1) - 1/2*log(abs(4*cos(x)^2 - 3))
```

3.374.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right)^4 - 14\tan\left(\frac{x}{2}\right)^2 + 1\right)}{2}$$

input
```
int(cos(x)^2/(cos(3*x)*sin(x)),x)
```

output
```
log(tan(x/2)) - log(tan(x/2)^4 - 14*tan(x/2)^2 + 1)/2
```

3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$

3.375.1 Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx=-\arctan(\cos(2x))$$

output
```
-arctan(cos(2*x))
```

3.375.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx=-\arctan(\cos(2x))$$

input
```
Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

output
```
-ArcTan[Cos[2*x]]
```

3.375.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4878, 27, 1432, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\sin^4(x)+\cos^4(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin(2x)}{\sin(x)^4+\cos(x)^4}dx$$
$$\Big\downarrow \ 4878$$
$$\int \frac{2\sin(x)}{2\sin^4(x)-2\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \ 27$$
$$2\int \frac{\sin(x)}{2\sin^4(x)-2\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \ 1432$$
$$\int \frac{1}{2\sin^4(x)-2\sin^2(x)+1}d\sin^2(x)$$
$$\Big\downarrow \ 1082$$
$$\int \frac{1}{-\sin^4(x)-1}d(1-2\sin^2(x))$$
$$\Big\downarrow \ 217$$
$$-\arctan\left(1-2\sin^2(x)\right)$$

input
```
Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

output
```
-ArcTan[1 - 2*Sin[x]^2]
```

3.375.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.375.4 Maple [A] (verified)

Time = 12.97 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

method	result	size
derivativedivides	$-\arctan\left(2(\cos^2(x))-1\right)$	12
default	$-\arctan\left(2(\cos^2(x))-1\right)$	12
risch	$-\frac{i\ln(\mathrm{e}^{4ix}+2i\mathrm{e}^{2ix}+1)}{2}+\frac{i\ln(\mathrm{e}^{4ix}-2i\mathrm{e}^{2ix}+1)}{2}$	40

input
```
int(sin(2*x)/(cos(x)^4+sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
-arctan(2*cos(x)^2-1)
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan\left(2\,\cos\left(x\right)^2-1\right)$$

input
```
integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="fricas")
```

output
```
-arctan(2*cos(x)^2 - 1)
```

3.375.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)
```

output
```
Timed out
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = \arctan\left(2\,\sin\left(x\right)^2-1\right)$$

input
```
integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="maxima")
```

output
```
arctan(2*sin(x)^2 - 1)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx = \arctan\left(2\,\sin\left(x\right)^2 - 1\right)$$

input
```
integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="giac")
```

output
```
arctan(2*sin(x)^2 - 1)
```

3.375.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx = \mathrm{atan}\left(\tan(x)^2\right)$$

input
```
int(sin(2*x)/(cos(x)^4 + sin(x)^4),x)
```

output
```
atan(tan(x)^2)
```

3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$

3.376.1 Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{\cos(x)-\sqrt{3}\sin(x)}{2\left(2+\sqrt{3}\right)+\sqrt{3}\cos(x)+\sin(x)}\right)}{\sqrt{3}}$$

output
```
1/6*x*3^(1/2)+1/3*arctan((cos(x)-sin(x)*3^(1/2))/(sin(x)+cos(x)*3^(1/2)+4+
2*3^(1/2)))*3^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=-\frac{\arctan\left(\frac{-1+\left(-4+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

output
```
-(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])
```

3.376.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3603, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\sqrt{3}\cos(x)+4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)+\sqrt{3}\cos(x)+4}dx$$

$$\Big\downarrow \text{ 3603}$$

$$2\int \frac{1}{\left(4-\sqrt{3}\right)\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+\sqrt{3}+4}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$-2\int \frac{1}{-\left(\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+1\right)^2-12}d\left(\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+1\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+1}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

output
```
ArcTan[(1 + (4 - Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3]
```

3.376.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.376.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{52 \arctan\left(\frac{26 \tan\left(\frac{x}{2}\right)+2\sqrt{3}+8}{16\sqrt{3}+12}\right)}{\left(\sqrt{3}-4\right)\left(16\sqrt{3}+12\right)}$	43
risch	$\frac{i\sqrt{3}\,\ln\left(\frac{i\sqrt{3}}{2}+\sqrt{3}+\frac{3}{2}+i+e^{ix}\right)}{6}-\frac{i\sqrt{3}\,\ln\left(e^{ix}+\sqrt{3}-\frac{3}{2}+i-\frac{i\sqrt{3}}{2}\right)}{6}$	52
parallelrisch	$-\frac{i\left(\ln\left(13\tan\left(\frac{x}{2}\right)+4-6i+(1-8i)\sqrt{3}\right)-\ln\left(13\tan\left(\frac{x}{2}\right)+4+6i+(1+8i)\sqrt{3}\right)\right)\left(4+\sqrt{3}\right)}{8\sqrt{3}+6}$	57

input
```
int(1/(4+sin(x)+cos(x)*3^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8)/(16*3^(
1/2)+12))
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{2\left(\left(4\sqrt{3}\cos(x)+3\right)\sin(x)+\sqrt{3}\cos(x)+3\right)}{3\left(4\cos(x)^2-3\right)}\right)$$

input
```
integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(2/3*((4*sqrt(3)*cos(x) + 3)*sin(x) + sqrt(3)*cos(x) + 3
)/(4*cos(x)^2 - 3))
```

3.376.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(48) = 96.

Time = 4.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{13906891405206808\sqrt{3}\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-41720674215620424+24087442555831531\sqrt{3}} + \frac{24087442555831531\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-41720674215620424+24087442555831531\sqrt{3}}$$

input
```
integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

output
```
-13906891405206808*sqrt(3)*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt
(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 2408744255583153
1*sqrt(3)) + 24087442555831531*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 +
sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 240874425558
31531*sqrt(3))
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{(\sqrt{3}-4)\sin(x)}{\cos(x)+1}-1\right)\right)$$

input
```
integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3) - 4)*sin(x)/(cos(x) + 1) - 1))
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = \frac{\left(x+2\arctan\left(\frac{\sqrt{3}\cos(x)-8\sqrt{3}\sin(x)+\sqrt{3}+4\cos(x)+7\sin(x)+4}{8\sqrt{3}\cos(x)+\sqrt{3}\sin(x)+8\sqrt{3}-7\cos(x)+4\sin(x)+19}\right)\right)(\sqrt{3}+4)}{2(4\sqrt{3}+3)}$$

input
```
integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="giac")
```

output
```
1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) + 4*cos(x)
+ 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*sqrt(3) - 7*cos(x)
+ 4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.43

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{\sqrt{12}\,\mathrm{atan}\left(\frac{\sqrt{12}\left(\tan\left(\frac{x}{2}\right)\left(\sqrt{3}-4\right)-1\right)}{12}\right)}{6}$$

input
```
int(1/(sin(x) + 3^(1/2)*cos(x) + 4),x)
```

output
```
-(12^(1/2)*atan((12^(1/2)*(tan(x/2)*(3^(1/2) - 4) - 1))/12))/6
```

3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$

3.377.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{8+3\cos(x)+3\sin(x)}\right)}{\sqrt{23}}$$

output
```
-1/23*arctanh((cos(x)-sin(x))*23^(1/2)/(8+3*cos(x)+3*sin(x)))*23^(1/2)
```

3.377.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{-4+\tan\left(\frac{x}{2}\right)}{\sqrt{23}}\right)}{\sqrt{23}}$$

input
```
Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

output
```
(2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]
```

3.377.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3603, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\sin(x)+4\cos(x)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{4\sin(x)+4\cos(x)+3}dx$$

$$\Big\downarrow \text{3603}$$

$$2\int \frac{1}{-\tan^2\left(\frac{x}{2}\right)+8\tan\left(\frac{x}{2}\right)+7}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{1081}$$

$$-2\int \left(\frac{1}{2\sqrt{23}\left(-\tan\left(\frac{x}{2}\right)-\sqrt{23}+4\right)}-\frac{1}{2\sqrt{23}\left(-\tan\left(\frac{x}{2}\right)+\sqrt{23}+4\right)}\right)d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{23}+4\right)}{2\sqrt{23}}-\frac{\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{23}+4\right)}{2\sqrt{23}}\right)$$

input
```
Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

output
```
2*(Log[4 - Sqrt[23] - Tan[x/2]]/(2*Sqrt[23]) - Log[4 + Sqrt[23] - Tan[x/2]
]/(2*Sqrt[23]))
```

3.377.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.377.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2\sqrt{23}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-8\right)\sqrt{23}}{46}\right)}{23}$	20
risch	$\frac{\sqrt{23}\,\ln\left(e^{ix}+\frac{3}{8}+\frac{3i}{8}-\frac{\sqrt{23}}{8}+\frac{i\sqrt{23}}{8}\right)}{23}-\frac{\sqrt{23}\,\ln\left(e^{ix}+\frac{3}{8}+\frac{3i}{8}+\frac{\sqrt{23}}{8}-\frac{i\sqrt{23}}{8}\right)}{23}$	54

input
```
int(1/(3+4*cos(x)+4*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/23*23^(1/2)*arctanh(1/46*(2*tan(1/2*x)-8)*23^(1/2))
```

3.377.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$$
$$= \frac{1}{46}\sqrt{23}\log\left(-\frac{6\sqrt{23}\cos(x)^2+8\left(\sqrt{23}-3\right)\cos(x)-2\left(4\sqrt{23}-7\cos(x)+12\right)\sin(x)-3\sqrt{23}-48}{8\left(4\cos(x)+3\right)\sin(x)+24\cos(x)+25}\right)$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="fricas")
```

output
```
1/46*sqrt(23)*log(-(6*sqrt(23)*cos(x)^2 + 8*(sqrt(23) - 3)*cos(x) - 2*(4*s
qrt(23) - 7*cos(x) + 12)*sin(x) - 3*sqrt(23) - 48)/(8*(4*cos(x) + 3)*sin(x
) + 24*cos(x) + 25))
```

3.377.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23} - \frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x)
```

output
```
sqrt(23)*log(tan(x/2) - 4 + sqrt(23))/23 - sqrt(23)*log(tan(x/2) - sqrt(23
) - 4)/23
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{1}{23}\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="maxima")
```

output
```
-1/23*sqrt(23)*log(-(sqrt(23) - sin(x)/(cos(x) + 1) + 4)/(sqrt(23) + sin(x
)/(cos(x) + 1) - 4))
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{1}{23}\sqrt{23}\log\left(\frac{\left|-2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}{\left|2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}\right)$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="giac")
```

output
```
-1/23*sqrt(23)*log(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(23) + 2*
tan(1/2*x) - 8))
```

3.377.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{2\sqrt{23}\,\mathrm{atanh}\left(\frac{\sqrt{23}\,\left(\tan\left(\frac{x}{2}\right)-4\right)}{23}\right)}{23}$$

input
```
int(1/(4*cos(x) + 4*sin(x) + 3),x)
```

output
```
(2*23^(1/2)*atanh((23^(1/2)*(tan(x/2) - 4))/23))/23
```

3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$

3.378.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{x}{3}+\frac{1}{3}\arctan\left(\frac{2\cos(x)\sin(x)}{1+2\sin^2(x)}\right)$$

output
```
1/3*x+1/3*arctan(2*cos(x)*sin(x)/(1+2*sin(x)^2))
```

3.378.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\arctan(3\tan(x))$$

input
```
Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[3*Tan[x]]/3
```

3.378.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sin^2(x)-3\cos^2(x)+4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{5\sin(x)^2-3\cos(x)^2+4}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{9\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\arctan(3\tan(x))$$

input
```
Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[3*Tan[x]]/3
```

3.378.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.378.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\arctan(3\tan(x))}{3}$	8
risch	$\frac{i\ln(\mathrm{e}^{2ix}-2)}{6}-\frac{i\ln(\mathrm{e}^{2ix}-\frac{1}{2})}{6}$	24
parallelrisch	$-\frac{i\left(\ln\left(\frac{-3i\sin(x)-\cos(x)}{\cos(x)+1}\right)-\ln\left(\frac{3i\sin(x)-\cos(x)}{\cos(x)+1}\right)\right)}{6}$	43

input
```
int(1/(4-3*cos(x)^2+5*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(3*tan(x))
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-9}{6\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))
```

3.378.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(22) = 44$.

Time = 4.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 8.11

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{4478554083\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{3166815962\sqrt{2}\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{131836323\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{93222358\sqrt{2}\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}$$

input
```
integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)
```

output
```
4478554083*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + p
i*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 3166815962*s
qrt(2)*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi*fl
oor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 131836323*sqrt(1
2*sqrt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi
/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 93222358*sqrt(2)*sqrt(12*sqrt
(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/pi
))/(2305195203 + 1630019160*sqrt(2))
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\arctan\left(3\tan\left(x\right)\right)$$

input
```
integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="maxima")
```

output
```
1/3*arctan(3*tan(x))
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\,x - \frac{1}{3}\,\arctan\left(\frac{\sin(2\,x)}{\cos(2\,x)-2}\right)$$

input
```
integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="giac")
```

output
```
1/3*x - 1/3*arctan(sin(2*x)/(cos(2*x) - 2))
```

3.378.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{x}{3} - \frac{\mathrm{atan}(\tan(x))}{3} + \frac{\mathrm{atan}(3\tan(x))}{3}$$

input
```
int(1/(5*sin(x)^2 - 3*cos(x)^2 + 4),x)
```

output
```
x/3 - atan(tan(x))/3 + atan(3*tan(x))/3
```

3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

3.379.1 Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx=\frac{4x}{25}-\frac{3}{25}\log(2\cos(x)+\sin(x))+\frac{2}{5(2+\tan(x))}$$

output
```
4/25*x-3/25*ln(2*cos(x)+sin(x))+2/5/(2+tan(x))
```

3.379.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx =\frac{-5+4x+\cot(x)(8x-6\log(2\cos(x)+\sin(x)))-3\log(2\cos(x)+\sin(x))}{25+50\cot(x)}$$

input
```
Integrate[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

output
```
(-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos[x] + Sin
[x]])/(25 + 50*Cot[x])
```

3.379.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4853, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan(x)+4\cot(x)+4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)+4\cot(x)+4}dx$$

$$\Big\downarrow \text{4853}$$

$$\int \frac{\tan(x)}{(\tan(x)+2)^2\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{594}$$

$$\frac{2}{5(\tan(x)+2)}-\frac{1}{5}\int -\frac{2\tan(x)+1}{(\tan(x)+2)\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\int \frac{2\tan(x)+1}{(\tan(x)+2)\left(\tan^2(x)+1\right)}d\tan(x)+\frac{2}{5(\tan(x)+2)}$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{5}\int \left(\frac{3\tan(x)+4}{5\left(\tan^2(x)+1\right)}-\frac{3}{5(\tan(x)+2)}\right)d\tan(x)+\frac{2}{5(\tan(x)+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{4}{5}\arctan(\tan(x))+\frac{3}{10}\log\left(\tan^2(x)+1\right)-\frac{3}{5}\log(\tan(x)+2)\right)+\frac{2}{5(\tan(x)+2)}$$

input
```
Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

output
```
((4*ArcTan[Tan[x]])/5 - (3*Log[2 + Tan[x]])/5 + (3*Log[1 + Tan[x]^2])/10)/
5 + 2/(5*(2 + Tan[x]))
```

3.379.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

3.379. $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

3.379.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{3\ln\left(1+\tan^2(x)\right)}{50}+\frac{4\arctan(\tan(x))}{25}+\frac{2}{5(2+\tan(x))}-\frac{3\ln(2+\tan(x))}{25}$	31
default	$\frac{3\ln\left(1+\tan^2(x)\right)}{50}+\frac{4\arctan(\tan(x))}{25}+\frac{2}{5(2+\tan(x))}-\frac{3\ln(2+\tan(x))}{25}$	31
norman	$\frac{\frac{8x}{25}+\frac{4x\tan(x)}{25}+\frac{2}{5}}{2+\tan(x)}-\frac{3\ln(2+\tan(x))}{25}+\frac{3\ln\left(1+\tan^2(x)\right)}{50}$	35
parallelrisch	$\frac{(3\tan(x)+6)\ln\left(\sec^2(x)\right)+(-6\tan(x)-12)\ln(2+\tan(x))+8x\tan(x)+16x+20}{100+50\tan(x)}$	44
risch	$\frac{4x}{25}+\frac{3ix}{25}+\frac{16}{25(5\,e^{2ix}+3+4i)}-\frac{12i}{25(5\,e^{2ix}+3+4i)}-\frac{3\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{25}$	52

input
```
int(1/(4+4*cot(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
3/50*ln(1+tan(x)^2)+4/25*arctan(tan(x))+2/5/(2+tan(x))-3/25*ln(2+tan(x))
```

3.379.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = -\frac{3\left(\tan\left(x\right)+2\right)\log\left(\frac{\tan\left(x\right)^2+4\tan\left(x\right)+4}{\tan\left(x\right)^2+1}\right)-8\left(x-1\right)\tan\left(x\right)-16\,x-4}{50\left(\tan\left(x\right)+2\right)}$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="fricas")
```

output
```
-1/50*(3*(tan(x) + 2)*log((tan(x)^2 + 4*tan(x) + 4)/(tan(x)^2 + 1)) - 8*(x
 - 1)*tan(x) - 16*x - 4)/(tan(x) + 2)
```

3.379.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(26) = 52$.

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.64

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{8x\tan(x)}{50\tan(x)+100} + \frac{16x}{50\tan(x)+100} - \frac{6\log(\tan(x)+2)\tan(x)}{50\tan(x)+100} - \frac{12\log(\tan(x)+2)}{50\tan(x)+100} + \frac{3\log(\tan^2(x)+1)\tan(x)}{50\tan(x)+100} + \frac{6\log(\tan^2(x)+1)}{50\tan(x)+100} + \frac{20}{50\tan(x)+100}$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x)
```

output
```
8*x*tan(x)/(50*tan(x) + 100) + 16*x/(50*tan(x) + 100) - 6*log(tan(x) + 2)*
tan(x)/(50*tan(x) + 100) - 12*log(tan(x) + 2)/(50*tan(x) + 100) + 3*log(ta
n(x)**2 + 1)*tan(x)/(50*tan(x) + 100) + 6*log(tan(x)**2 + 1)/(50*tan(x) +
100) + 20/(50*tan(x) + 100)
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4}{25}x + \frac{2}{5(\tan(x)+2)} + \frac{3}{50}\log\left(\tan(x)^2+1\right) - \frac{3}{25}\log(\tan(x)+2)$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="maxima")
```

output
```
4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(tan(x) + 2)
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4}{25}\,x + \frac{2}{5\,(\tan{(x)}+2)} + \frac{3}{50}\,\log\left(\tan{(x)}^2+1\right) - \frac{3}{25}\,\log\left(|\tan{(x)}+2|\right)$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="giac")
```

output
```
4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(abs(tan(x) +
 2))
```

3.379.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{2}{5\,\left(\tan{(x)}+2\right)} - \frac{3\,\ln\left(\tan(x)+2\right)}{25} + \ln\left(\tan(x)-\mathrm{i}\right)\,\left(\frac{3}{50}-\frac{2}{25}\mathrm{i}\right) + \ln\left(\tan(x)+1\mathrm{i}\right)\,\left(\frac{3}{50}+\frac{2}{25}\mathrm{i}\right)$$

input
```
int(1/(4*cot(x) + tan(x) + 4),x)
```

output
```
log(tan(x) - 1i)*(3/50 - 2i/25) - (3*log(tan(x) + 2))/25 + log(tan(x) + 1i
)*(3/50 + 2i/25) + 2/(5*(tan(x) + 2))
```

3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$

3.380.1 Optimal result

Integrand size = 9, antiderivative size = 67

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8x}{15\sqrt{15}} - \frac{8\arctan\left(\frac{1-2\cos^2(x)}{4+\sqrt{15}+2\cos(x)\sin(x)}\right)}{15\sqrt{15}} + \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}$$

output
```
8/225*x*15^(1/2)-8/225*arctan((1-2*cos(x)^2)/(4+2*cos(x)*sin(x)+15^(1/2)))
*15^(1/2)+1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)
```

3.380.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{\sec^2(x)(4+\sin(2x))\left(15(-15+\cos(2x))+8\sqrt{15}\arctan\left(\frac{1+4\tan(x)}{\sqrt{15}}\right)(4+\sin(2x))\right)}{900(2\sec(x)+\sin(x))^2}$$

input
```
Integrate[(2*Sec[x] + Sin[x])^(-2),x]
```

output
```
(Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan[(1 + 4*T
an[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x])^2)
```

3.380.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4889, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+2\sec(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(x)+2\sec(x))^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{\left(2\tan^2(x)+\tan(x)+2\right)^2}d\tan(x)$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{4}{15}\int \frac{1}{2\tan^2(x)+\tan(x)+2}d\tan(x)+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8}{15}\int \frac{1}{-(4\tan(x)+1)^2-15}d(4\tan(x)+1)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{8\arctan\left(\frac{4\tan(x)+1}{\sqrt{15}}\right)}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}$$

input
```
Int[(2*Sec[x] + Sin[x])^(-2),x]
```

output
```
(8*ArcTan[(1 + 4*Tan[x])/Sqrt[15]])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2
+ Tan[x] + 2*Tan[x]^2))
```

3.380.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.380.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{1+4\tan(x)}{30+15\tan(x)+30(\tan^2(x))}+\frac{8\sqrt{15}\,\arctan\left(\frac{(1+4\tan(x))\sqrt{15}}{15}\right)}{225}$	39
risch	$\frac{\left(\frac{8}{3615}-\frac{2i}{241}\right)\left(241\,e^{2ix}-15+4i\right)}{e^{4ix}+8ie^{2ix}-1}+\frac{4i\sqrt{15}\,\ln\left(e^{2ix}+i\sqrt{15}+4i\right)}{225}-\frac{4i\sqrt{15}\,\ln\left(e^{2ix}-i\sqrt{15}+4i\right)}{225}$	76

input
```
int(1/(2*sec(x)+sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)+8/225*15^(1/2)*arctan(1/15*(1+4*ta
n(x))*15^(1/2))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{4\left(\sqrt{15}\cos(x)\sin(x)+2\sqrt{15}\right)\arctan\left(\frac{8\sqrt{15}\cos(x)\sin(x)+\sqrt{15}}{15\left(2\cos(x)^2-1\right)}\right)+15\cos(x)^2-120}{225\left(\cos(x)\sin(x)+2\right)}$$

input
```
integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="fricas")
```

output
```
1/225*(4*(sqrt(15)*cos(x)*sin(x) + 2*sqrt(15))*arctan(1/15*(8*sqrt(15)*cos
(x)*sin(x) + sqrt(15))/(2*cos(x)^2 - 1)) + 15*cos(x)^2 - 120)/(cos(x)*sin(
x) + 2)
```

3.380.6 Sympy [F]

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \int \frac{1}{(\sin(x)+2\sec(x))^2}\,dx$$

input
```
integrate(1/(2*sec(x)+sin(x))**2,x)
```

output
```
Integral((sin(x) + 2*sec(x))**(-2), x)
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8}{225}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,\tan{(x)}+1)\right) + \frac{4\,\tan{(x)}+1}{15\,\left(2\,\tan{(x)}^2+\tan{(x)}+2\right)}$$

input
```
integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="maxima")
```

output
```
8/225*sqrt(15)*arctan(1/15*sqrt(15)*(4*tan(x) + 1)) + 1/15*(4*tan(x) + 1)/
(2*tan(x)^2 + tan(x) + 2)
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8}{225}\,\sqrt{15}\left(x+\arctan\left(-\frac{\sqrt{15}\sin{(2\,x)}-\cos{(2\,x)}-4\,\sin{(2\,x)}-1}{\sqrt{15}\cos{(2\,x)}+\sqrt{15}-4\,\cos{(2\,x)}+\sin{(2\,x)}+4}\right)\right) + \frac{4\,\tan{(x)}+1}{15\,\left(2\,\tan{(x)}^2+\tan{(x)}+2\right)}$$

input
```
integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="giac")
```

output
```
8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin(2*x) - 1
)/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x) + 4))) + 1/15*(4*t
an(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

3.380.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.79

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$$
$$= \frac{4\sqrt{15}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^3}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^2}{15}+\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{15}}{15}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{15}}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{15}\right)\right)}{225}$$
$$-\frac{\frac{7\tan\left(\frac{x}{2}\right)^3}{30}+\frac{2\tan\left(\frac{x}{2}\right)^2}{15}-\frac{7\tan\left(\frac{x}{2}\right)}{30}}{\tan\left(\frac{x}{2}\right)^4-\tan\left(\frac{x}{2}\right)^3+2\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(sin(x) + 2/cos(x))^2,x)
```

output
```
(4*15^(1/2)*(2*atan((2*15^(1/2)*tan(x/2))/5 + 15^(1/2)/15 - (2*15^(1/2)*ta
n(x/2)^2)/15 + (2*15^(1/2)*tan(x/2)^3)/15) - 2*atan(15^(1/2)/15 - (2*15^(1
/2)*tan(x/2))/15)))/225 - ((2*tan(x/2)^2)/15 - (7*tan(x/2))/30 + (7*tan(x/
2)^3)/30)/(tan(x/2) + 2*tan(x/2)^2 - tan(x/2)^3 + tan(x/2)^4 + 1)
```

3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2} dx$

3.381.1 Optimal result

Integrand size = 9, antiderivative size = 55

$$\int \frac{1}{(\cos(x)+2\sec(x))^2} dx = \frac{x}{6\sqrt{6}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}} + \frac{\tan(x)}{6(3+2\tan^2(x))}$$

output
```
1/36*x*6^(1/2)-1/36*arctan(cos(x)*sin(x)/(2+cos(x)^2+6^(1/2)))*6^(1/2)+1/6
*tan(x)/(3+2*tan(x)^2)
```

3.381.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{1}{(\cos(x)+2\sec(x))^2} dx = \frac{(5+\cos(2x))\sec^4(x)\left(\sqrt{6}\arctan\left(\sqrt{\frac{2}{3}}\tan(x)\right)(5+\cos(2x))+6\sin(2x)\right)}{144(1+2\sec^2(x))^2}$$

input
```
Integrate[(Cos[x] + 2*Sec[x])^(-2),x]
```

output
```
((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + Cos[2*x])
+ 6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

3.381.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4889, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{(\cos(x)+2\sec(x))^2}dx$$
$$\Big\downarrow \ 4889$$
$$\int \frac{1}{\left(2\tan^2(x)+3\right)^2}d\tan(x)$$
$$\Big\downarrow \ 215$$
$$\frac{1}{6}\int \frac{1}{2\tan^2(x)+3}d\tan(x)+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}$$
$$\Big\downarrow \ 216$$
$$\frac{\arctan\left(\sqrt{\frac{2}{3}}\tan(x)\right)}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}$$

input
```
Int[(Cos[x] + 2*Sec[x])^(-2),x]
```

output
```
ArcTan[Sqrt[2/3]*Tan[x]]/(6*Sqrt[6]) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

3.381.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.381.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\tan(x)}{18+12(\tan^2(x))}+\frac{\sqrt{6}\,\arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)}{36}$	29
risch	$\frac{i(5\,\mathrm{e}^{2ix}+1)}{3\,\mathrm{e}^{4ix}+30\,\mathrm{e}^{2ix}+3}+\frac{i\sqrt{6}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{6}+5\right)}{72}-\frac{i\sqrt{6}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{6}+5\right)}{72}$	68

input
```
int(1/(cos(x)+2*sec(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(x)/(3+2*tan(x)^2)+1/36*6^(1/2)*arctan(1/3*tan(x)*6^(1/2))
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = -\frac{\left(\sqrt{6}\cos\left(x\right)^2+2\sqrt{6}\right)\arctan\left(\frac{5\sqrt{6}\cos(x)^2-2\sqrt{6}}{12\cos(x)\sin(x)}\right)-12\cos\left(x\right)\sin\left(x\right)}{72\left(\cos\left(x\right)^2+2\right)}$$

input
```
integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="fricas")
```

output
```
-1/72*((sqrt(6)*cos(x)^2 + 2*sqrt(6))*arctan(1/12*(5*sqrt(6)*cos(x)^2 - 2*
sqrt(6))/(cos(x)*sin(x))) - 12*cos(x)*sin(x))/(cos(x)^2 + 2)
```

3.381.6 Sympy [F]

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx=\int \frac{1}{(\cos{(x)}+2\sec{(x)})^2}\,dx$$

input
```
integrate(1/(cos(x)+2*sec(x))**2,x)
```

output
```
Integral((cos(x) + 2*sec(x))**(-2), x)
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx=\frac{1}{36}\,\sqrt{6}\arctan\left(\frac{1}{3}\,\sqrt{6}\tan{(x)}\right)+\frac{\tan{(x)}}{6\,(2\,\tan{(x)}^2+3)}$$

input
```
integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="maxima")
```

output
```
1/36*sqrt(6)*arctan(1/3*sqrt(6)*tan(x)) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx=\frac{1}{36}\,\sqrt{6}\left(x+\arctan\left(-\frac{\sqrt{6}\sin{(2\,x)}-2\,\sin{(2\,x)}}{\sqrt{6}\cos{(2\,x)}+\sqrt{6}-2\,\cos{(2\,x)}+2}\right)\right)+\frac{\tan{(x)}}{6\,(2\,\tan{(x)}^2+3)}$$

input
```
integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="giac")
```

output
```
1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6)*cos(2*x
) + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

3.381.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = \frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\tan\left(\frac{x}{2}\right)^3}{4}+\frac{5\sqrt{6}\tan\left(\frac{x}{2}\right)}{12}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{6}\tan\left(\frac{x}{2}\right)}{4}\right)\right)}{72} + \frac{\frac{\tan\left(\frac{x}{2}\right)}{9}-\frac{\tan\left(\frac{x}{2}\right)^3}{9}}{\tan\left(\frac{x}{2}\right)^4+\frac{2\tan\left(\frac{x}{2}\right)^2}{3}+1}$$

input
```
int(1/(cos(x) + 2/cos(x))^2,x)
```

output
```
(6^(1/2)*(2*atan((5*6^(1/2)*tan(x/2))/12 + (6^(1/2)*tan(x/2)^3)/4) + 2*ata
n((6^(1/2)*tan(x/2))/4)))/72 + (tan(x/2)/9 - tan(x/2)^3/9)/((2*tan(x/2)^2)
/3 + tan(x/2)^4 + 1)
```

3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

3.382.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx=-\frac{67x}{250}-\frac{28}{125}\log(\cos(x)+3\sin(x))-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}$$

output
```
-67/250*x-28/125*ln(cos(x)+3*sin(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x)
)
```

3.382.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx=\frac{1}{500}\left((56+67i)\log(i-\tan(x))+(56-67i)\log(i+\tan(x))-112\log(1+3\tan(x))-\frac{350}{(1+3\tan(x))^2}-\frac{290}{1+3\tan(x)}\right)$$

input
```
Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

output
```
((56 + 67*I)*Log[I - Tan[x]] + (56 - 67*I)*Log[I + Tan[x]] - 112*Log[1 + 3
*Tan[x]] - 350/(1 + 3*Tan[x])^2 - 290/(1 + 3*Tan[x]))/500
```

3.382.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4111, 27, 3042, 4012, 25, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-6\tan^2(x)-\tan(x)+5}{(3\tan(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-6\tan(x)^2-\tan(x)+5}{(3\tan(x)+1)^3}dx$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{10}\int \frac{2(4-17\tan(x))}{(3\tan(x)+1)^2}dx-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{4-17\tan(x)}{(3\tan(x)+1)^2}dx-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{4-17\tan(x)}{(3\tan(x)+1)^2}dx-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{5}\left(\frac{1}{10}\int -\frac{29\tan(x)+47}{3\tan(x)+1}dx-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(-\frac{1}{10}\int \frac{29\tan(x)+47}{3\tan(x)+1}dx-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{1}{10}\int \frac{29\tan(x)+47}{3\tan(x)+1}dx-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{1}{5}\left(\frac{1}{10}\left(-\frac{56}{5}\int \frac{3-\tan(x)}{3\tan(x)+1}dx-\frac{67x}{5}\right)-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{10}\left(-\frac{56}{5}\int\frac{3-\tan(x)}{3\tan(x)+1}dx-\frac{67x}{5}\right)-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{1}{5}\left(\frac{1}{10}\left(-\frac{67x}{5}-\frac{56}{5}\log(3\sin(x)+\cos(x))\right)-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

input
```
Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

output
```
-7/(10*(1 + 3*Tan[x])^2) + (((-67*x)/5 - (56*Log[Cos[x] + 3*Sin[x]])/5)/10
 - 29/(10*(1 + 3*Tan[x])))/5
```

3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.382.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{14\ln\left(1+\tan^2(x)\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$
default	$\frac{14\ln\left(1+\tan^2(x)\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$
risch	$-\frac{67x}{250}+\frac{28ix}{125}+\frac{\left(-\frac{36}{24125}-\frac{621i}{48250}\right)\left(965\,e^{2ix}-324+768i\right)}{\left(5\,e^{2ix}-4+3i\right)^2}-\frac{28\ln\left(e^{2ix}-\frac{4}{5}+\frac{3i}{5}\right)}{125}$
norman	$\frac{-\frac{87\tan(x)}{50}-\frac{67x}{250}-\frac{201x\tan(x)}{125}-\frac{603x\left(\tan^2(x)\right)}{250}-\frac{32}{25}}{(1+3\tan(x))^2}-\frac{28\ln(1+3\tan(x))}{125}+\frac{14\ln\left(1+\tan^2(x)\right)}{125}$
parallelrisch	$-\frac{4536\ln\left(\frac{1}{3}+\tan(x)\right)\left(\tan^2(x)\right)-2268\ln\left(1+\tan^2(x)\right)\left(\tan^2(x)\right)+5427x\left(\tan^2(x)\right)+2880+3024\ln\left(\frac{1}{3}+\tan(x)\right)\tan(x)-1}{2250(1+3\tan(x)}$

input
```
int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
14/125*ln(1+tan(x)^2)-67/250*arctan(tan(x))-7/10/(1+3*tan(x))^2-29/50/(1+3
*tan(x))-28/125*ln(1+3*tan(x))
```

3.382.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{9\,(134\,x-1)\tan\left(x\right)^2+56\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)\log\left(\frac{9\,\tan(x)^2+6\,\tan(x)+1}{\tan(x)^2+1}\right)+12\,(67\,x+72)\tan\left(x\right.}{500\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)}$$

input
```
integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="fricas")
```

output
```
-1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*log((9*tan
(x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*tan(x) + 134*x + 63
9)/(9*tan(x)^2 + 6*tan(x) + 1)
```

3.382.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(39) = 78$.

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 6.00

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{603x\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{402x\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{67x}{2250\tan^2(x)+1500\tan(x)+250} - \frac{504\log(3\tan(x)+1)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{336\log(3\tan(x)+1)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{56\log(3\tan(x)+1)}{2250\tan^2(x)+1500\tan(x)+250} + \frac{252\log(\tan^2(x)+1)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} + \frac{168\log(\tan^2(x)+1)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} + \frac{28\log(\tan^2(x)+1)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{435\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{320}{2250\tan^2(x)+1500\tan(x)+250}$$

input
```
integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

output
```
-603*x*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 402*x*tan(x)/(2250
*tan(x)**2 + 1500*tan(x) + 250) - 67*x/(2250*tan(x)**2 + 1500*tan(x) + 250
) - 504*log(3*tan(x) + 1)*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) -
 336*log(3*tan(x) + 1)*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) - 56*lo
g(3*tan(x) + 1)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 252*log(tan(x)**2 +
 1)*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) + 168*log(tan(x)**2 + 1
)*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 28*log(tan(x)**2 + 1)/(225
0*tan(x)**2 + 1500*tan(x) + 250) - 435*tan(x)/(2250*tan(x)**2 + 1500*tan(x
) + 250) - 320/(2250*tan(x)**2 + 1500*tan(x) + 250)
```

3.382. $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

3.382.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67}{250}\,x - \frac{87\,\tan(x)+64}{50\,\left(9\,\tan(x)^2+6\,\tan(x)+1\right)} + \frac{14}{125}\,\log\left(\tan(x)^2+1\right) - \frac{28}{125}\,\log\left(3\,\tan(x)+1\right)$$

input
```
integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="maxima")
```

output
```
-67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 14/125*log
(tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67}{250}\,x - \frac{87\,\tan(x)+64}{50\,(3\,\tan(x)+1)^2} + \frac{14}{125}\,\log\left(\tan(x)^2+1\right) - \frac{28}{125}\,\log\left(|3\,\tan(x)+1|\right)$$

input
```
integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="giac")
```

output
```
-67/250*x - 1/50*(87*tan(x) + 64)/(3*tan(x) + 1)^2 + 14/125*log(tan(x)^2 +
 1) - 28/125*log(abs(3*tan(x) + 1))
```

3.382.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{28\,\ln\left(\tan(x)+\frac{1}{3}\right)}{125} - \frac{\frac{29\tan(x)}{150}+\frac{32}{225}}{\tan(x)^2+\frac{2\tan(x)}{3}+\frac{1}{9}} + \ln\left(\tan(x)-\mathrm{i}\right)\left(\frac{14}{125}+\frac{67}{500}\mathrm{i}\right) + \ln\left(\tan(x)+1\mathrm{i}\right)\left(\frac{14}{125}-\frac{67}{500}\mathrm{i}\right)$$

input
```
int(-(tan(x) + 6*tan(x)^2 - 5)/(3*tan(x) + 1)^3,x)
```

output
```
log(tan(x) - 1i)*(14/125 + 67i/500) - (28*log(tan(x) + 1/3))/125 + log(tan
(x) + 1i)*(14/125 - 67i/500) - ((29*tan(x))/150 + 32/225)/((2*tan(x))/3 +
tan(x)^2 + 1/9)
```

3.382. $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

3.383 $\int \cos^2(x) \sec(3x) \, dx$

3.383.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{2} \operatorname{arctanh}(2 \sin(x))$$

output
```
1/2*arctanh(2*sin(x))
```

3.383.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{2} \operatorname{arctanh}(2 \sin(x))$$

input
```
Integrate[Cos[x]^2*Sec[3*x],x]
```

output
```
ArcTanh[2*Sin[x]]/2
```

3.383.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4878, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\sec(3x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(x)^2}{\cos(3x)}dx$$
$$\Big\downarrow \ 4878$$
$$\int \frac{1}{1-4\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \ 219$$
$$\frac{1}{2}\text{arctanh}(2\sin(x))$$

input
```
Int[Cos[x]^2*Sec[3*x],x]
```

output
```
ArcTanh[2*Sin[x]]/2
```

3.383.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.383.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

method	result	size
default	$\frac{\ln(1+2\sin(x))}{4} - \frac{\ln(2\sin(x)-1)}{4}$	20
risch	$-\frac{\ln(-i e^{ix}+e^{2ix}-1)}{4} + \frac{\ln(i e^{ix}+e^{2ix}-1)}{4}$	38

input
```
int(cos(x)^2/cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(1+2*sin(x))-1/4*ln(2*sin(x)-1)
```

3.383.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \cos^2(x)\sec(3x)\,dx = \frac{1}{4}\log\left(2\sin\left(x\right)+1\right) - \frac{1}{4}\log\left(-2\sin\left(x\right)+1\right)$$

input
```
integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
```

output
```
1/4*log(2*sin(x) + 1) - 1/4*log(-2*sin(x) + 1)
```

3.383.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(7) = 14$.

Time = 1.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 8.44

$$\int \cos^2(x)\sec(3x)\,dx = -\frac{\log(\sin(3x)-1)}{12} + \frac{\log(\sin(3x)+1)}{12} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}{12} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}{12}$$

input
```
integrate(cos(x)**2/cos(3*x),x)
```

output
```
-log(sin(3*x) - 1)/12 + log(sin(3*x) + 1)/12 - log(tan(x/2) - 1)/6 + log(t
an(x/2) + 1)/6 - log(tan(x/2)**2 - 4*tan(x/2) + 1)/12 + log(tan(x/2)**2 +
4*tan(x/2) + 1)/12
```

3.383.7 Maxima [F]

$$\int \cos^2(x)\sec(3x)\,dx = \int \frac{\cos(x)^2}{\cos(3x)}\,dx$$

input
```
integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)^2/cos(3*x), x)
```

3.383.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \cos^2(x)\sec(3x)\,dx = \frac{1}{4}\log(|2\sin(x)+1|) - \frac{1}{4}\log(|2\sin(x)-1|)$$

input
```
integrate(cos(x)^2/cos(3*x),x, algorithm="giac")
```

output
```
1/4*log(abs(2*sin(x) + 1)) - 1/4*log(abs(2*sin(x) - 1))
```

3.383.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos^2(x) \sec(3x) \, dx = \frac{\operatorname{atanh}(2 \sin(x))}{2}$$

input
```
int(cos(x)^2/cos(3*x),x)
```

output
```
atanh(2*sin(x))/2
```

3.384 $\int \sec(2x)\sin(x)\,dx$

3.384.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.384.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

input
```
Integrate[Sec[2*x]*Sin[x],x]
```

output
```
(ArcTanh[Sqrt[2] - Tan[x/2]] + ArcTanh[Sqrt[2] + Tan[x/2]])/Sqrt[2]
```

3.384.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4857, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{2\cos^2(x)-1}d\cos(x)$$

$$\Big\downarrow \text{220}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

input
```
Int[Sec[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

3.384.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.384.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$	47

input
```
int(sin(x)/cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.384.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)$$

input
```
integrate(sin(x)/cos(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

3.384.6 Sympy [F]

$$\int \sec(2x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(2x)}\,dx$$

input
```
integrate(sin(x)/cos(2*x),x)
```

output
```
Integral(sin(x)/cos(2*x), x)
```

3.384.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 8.60

$$\begin{aligned}\int \sec(2x)\sin(x)\,dx = &\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x)\right.\\&\left.+\cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right)\\&-\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x)\right.\\&\left.+\cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right)\end{aligned}$$

input
```
integrate(sin(x)/cos(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1)
```

3.384.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}{\left|4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}\right)$$

input
```
integrate(sin(x)/cos(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sq
rt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

3.384.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sec(2x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

input
```
int(sin(x)/cos(2*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

3.385 $\int \sec(2x)\sin^2(x)\,dx$

3.385.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} + \frac{1}{4}\text{arctanh}(2\cos(x)\sin(x))$$

output
```
-1/2*x+1/4*arctanh(2*cos(x)*sin(x))
```

3.385.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} - \frac{1}{4}\log(\cos(x)-\sin(x)) + \frac{1}{4}\log(\cos(x)+\sin(x))$$

input
```
Integrate[Sec[2*x]*Sin[x]^2,x]
```

output
```
-1/2*x - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4
```

3.385.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4889, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^2}{\cos(2x)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{\tan^2(x)}{1-\tan^4(x)}d\tan(x)$$

$$\Big\downarrow \text{ 827}$$

$$\frac{1}{2}\int \frac{1}{1-\tan^2(x)}d\tan(x)-\frac{1}{2}\int \frac{1}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\int \frac{1}{1-\tan^2(x)}d\tan(x)-\frac{1}{2}\arctan(\tan(x))$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\text{arctanh}(\tan(x))-\frac{1}{2}\arctan(\tan(x))$$

input
```
Int[Sec[2*x]*Sin[x]^2,x]
```

output
```
-1/2*ArcTan[Tan[x]] + ArcTanh[Tan[x]]/2
```

3.385.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.385.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\ln(\tan(x)-1)}{4}-\frac{\arctan(\tan(x))}{2}+\frac{\ln(\tan(x)+1)}{4}$	21
risch	$-\frac{x}{2}-\frac{\ln(\mathrm{e}^{2ix}-i)}{4}+\frac{\ln(\mathrm{e}^{2ix}+i)}{4}$	27

input
```
int(sin(x)^2/cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(tan(x)-1)-1/2*arctan(tan(x))+1/4*ln(tan(x)+1)
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right) + 1\right) - \frac{1}{8}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right) + 1\right)$$

input
```
integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")
```

output
```
-1/2*x + 1/8*log(2*cos(x)*sin(x) + 1) - 1/8*log(-2*cos(x)*sin(x) + 1)
```

3.385.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} - \frac{\log\left(\sin\left(2x\right) - 1\right)}{8} + \frac{\log\left(\sin\left(2x\right) + 1\right)}{8}$$

input
```
integrate(sin(x)**2/cos(2*x),x)
```

output
```
-x/2 - log(sin(2*x) - 1)/8 + log(sin(2*x) + 1)/8
```

3.385.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\begin{aligned}\int \sec(2x)\sin^2(x)\,dx = & -\frac{1}{2}\,x - \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right) \\ & + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right) \\ & + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right) \\ & - \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right)\end{aligned}$$

input
```
integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")
```

output
```
-1/2*x - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)
*sin(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt
(2)*sin(x) + 2) - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*s
qrt(2)*sin(x) + 2)
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x + \frac{1}{4}\,\log\left(|\tan\left(x\right) + 1|\right) - \frac{1}{4}\,\log\left(|\tan\left(x\right) - 1|\right)$$

input
```
integrate(sin(x)^2/cos(2*x),x, algorithm="giac")
```

output
```
-1/2*x + 1/4*log(abs(tan(x) + 1)) - 1/4*log(abs(tan(x) - 1))
```

3.385.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \sec(2x)\sin^2(x)\,dx = \frac{\operatorname{atanh}(\tan(x))}{2} - \frac{x}{2}$$

input
```
int(sin(x)^2/cos(2*x),x)
```

output
```
atanh(tan(x))/2 - x/2
```

3.386 $\int \sec(3x)\sin^3(x)\,dx$

3.386.1 Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

output
```
1/3*ln(cos(x))-1/24*ln(3-4*cos(x)^2)
```

3.386.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(1-4\sin^2(x)\right)$$

input
```
Integrate[Sec[3*x]*Sin[x]^3,x]
```

output
```
Log[Cos[x]]/3 - Log[1 - 4*Sin[x]^2]/24
```

3.386.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4866, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^3}{\cos(3x)}dx$$

$$\Big\downarrow \text{ 4866}$$

$$-\int -\frac{(1-\cos^2(x))\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{(1-\cos^2(x))\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(1-\cos^2(x))\sec(x)}{3-4\cos^2(x)}d\cos^2(x)$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(\frac{\sec(x)}{3}-\frac{1}{3(4\cos^2(x)-3)}\right)d\cos^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\cos^2(x)\right)-\frac{1}{12}\log\left(3-4\cos^2(x)\right)\right)$$

input
```
Int[Sec[3*x]*Sin[x]^3,x]
```

output
```
(Log[Cos[x]^2]/3 - Log[3 - 4*Cos[x]^2]/12)/2
```

3.386.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.386.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(\cos(x))}{3}-\frac{\ln(4(\cos^2(x))-3)}{24}$	18
risch	$-\frac{ix}{4}+\frac{\ln(e^{2ix}+1)}{3}-\frac{\ln(e^{4ix}-e^{2ix}+1)}{24}$	33

input
```
int(sin(x)^3/cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(cos(x))-1/24*ln(4*cos(x)^2-3)
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sec(3x)\sin^3(x)\,dx = -\frac{1}{24}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log\left(-\cos(x)\right)$$

input
```
integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")
```

output
```
-1/24*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

3.386.6 Sympy [F]

$$\int \sec(3x)\sin^3(x)\,dx = \int \frac{\sin^3(x)}{\cos(3x)}\,dx$$

input
```
integrate(sin(x)**3/cos(3*x),x)
```

output
```
Integral(sin(x)**3/cos(3*x), x)
```

3.386.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.86

$$\begin{aligned}\int \sec(3x)\sin^3(x)\,dx = &-\frac{1}{48}\log\left(-2\left(\cos(2x)-1\right)\cos(4x)+\cos(4x)^2+\cos(2x)^2\right.\\ &\left.+\sin(4x)^2-2\sin(4x)\sin(2x)+\sin(2x)^2-2\cos(2x)+1\right)\\ &+\frac{1}{6}\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\end{aligned}$$

input
```
integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")
```

output
```
-1/48*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^
2 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^
2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{6}\log\left(-\sin(x)^2+1\right) - \frac{1}{24}\log\left(\left|4\sin(x)^2-1\right|\right)$$

input
```
integrate(sin(x)^3/cos(3*x),x, algorithm="giac")
```

output
```
1/6*log(-sin(x)^2 + 1) - 1/24*log(abs(4*sin(x)^2 - 1))
```

3.386.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sec(3x)\sin^3(x)\,dx = \frac{\ln(\cos(x))}{3} - \frac{\ln\left(\cos(x)^2-\frac{3}{4}\right)}{24}$$

input
```
int(sin(x)^3/cos(3*x),x)
```

output
```
log(cos(x))/3 - log(cos(x)^2 - 3/4)/24
```

3.387 $\int \cos(x)\csc(3x)\,dx$

3.387.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

output
```
1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

3.387.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

input
```
Integrate[Cos[x]*Csc[3*x],x]
```

output
```
Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

3.387.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4856, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(3x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\sin^2(x)\right)-\frac{1}{3}\log\left(3-4\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]*Csc[3*x],x]
```

output
```
(Log[Sin[x]^2]/3 - Log[3 - 4*Sin[x]^2]/3)/2
```

3.387.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.387.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{\ln\left(\mathrm{e}^{2 i x}-1\right)}{3}-\frac{\ln\left(\mathrm{e}^{4 i x}+\mathrm{e}^{2 i x}+1\right)}{6}$	27
default	$\frac{\ln\left(\cos\left(x\right)+1\right)}{6}-\frac{\ln\left(2\cos\left(x\right)-1\right)}{6}+\frac{\ln\left(-1+\cos\left(x\right)\right)}{6}-\frac{\ln\left(1+2\cos\left(x\right)\right)}{6}$	34

input
```
int(cos(x)/sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(exp(2*I*x)-1)-1/6*ln(exp(4*I*x)+exp(2*I*x)+1)
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{6}\log\left(4\cos\left(x\right)^{2}-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/sin(3*x),x, algorithm="fricas")
```

output
```
-1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

3.387.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = -\frac{\log\left(4\sin^{2}\left(x\right)-3\right)}{6}+\frac{\log\left(\sin\left(x\right)\right)}{3}$$

input
```
integrate(cos(x)/sin(3*x),x)
```

output
```
-log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

3.387.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\begin{aligned}\int \cos(x)\csc(3x)\,dx = &-\frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}\right.\\&\left.+2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)\\&-\frac{1}{12}\log\left(-2\left(\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}\right.\\&\left.-2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)\\&+\frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)\\&+\frac{1}{6}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)\end{aligned}$$

input
```
integrate(cos(x)/sin(3*x),x, algorithm="maxima")
```

output
```
-1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2
*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2
- 2*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{6}\,\log\left(-\cos\left(x\right)^2+1\right)-\frac{1}{6}\,\log\left(\left|4\,\cos\left(x\right)^2-1\right|\right)$$

input
```
integrate(cos(x)/sin(3*x),x, algorithm="giac")
```

output
```
1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

3.387.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = \frac{\ln\left(\sin\left(x\right)\right)}{3}-\frac{\ln\left(\frac{1}{4}-\cos\left(x\right)^2\right)}{6}$$

input
```
int(cos(x)/sin(3*x),x)
```

output
```
log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

3.388 $\int \csc(4x)\sin(x)\,dx$

3.388.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

3.388.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4878, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\sin(4x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{1}{8\sin^4(x)-12\sin^2(x)+4}d\sin(x)$$

$$\Big\downarrow \text{ 1406}$$

$$2\int \frac{1}{8\sin^2(x)-8}d\sin(x)-2\int \frac{1}{8\sin^2(x)-4}d\sin(x)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\operatorname{arctanh}(\sin(x))$$

input
```
Int[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

3.388.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.388.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(\sin(x)+1)}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}$	28
risch	$\frac{\ln(\mathrm{e}^{ix}-i)}{4}-\frac{\ln(i+\mathrm{e}^{ix})}{4}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{8}$	72

input
```
int(sin(x)/sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(sin(x)-1)-1/8*ln(sin(x)+1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.388.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right) - \frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)$$

input
```
integrate(sin(x)/sin(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.388.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 3.24 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx = &\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} \\ &- \frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} - \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} \\ &+ \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} \\ &+ \frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} \\ &- \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} \\ &- \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}\end{aligned}$$

input
```
integrate(sin(x)/sin(4*x),x)
```

output
```
27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(
x/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt
(2) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808)
+ 27720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt
(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(
x/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2)
 + 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - s
qrt(2) - 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/
(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110
880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2)
+ 156808)
```

3.388.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

input
```
integrate(sin(x)/sin(4*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^
2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.388.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

input
```
integrate(sin(x)/sin(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.388.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{4} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

input
```
int(sin(x)/sin(4*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

3.389 $\int \csc(4x)\sin^3(x)\,dx$

3.389.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \csc(4x)\sin^3(x)\,dx=-\frac{1}{4}\text{arctanh}(\sin(x))+\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}$$

output
```
-1/4*arctanh(sin(x))+1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.389.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int \csc(4x)\sin^3(x)\,dx=\frac{1}{16}\left(4\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-4\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}\left(-\log\left(\sqrt{2}-2\sin(x)\right)+\log\left(\sqrt{2}+2\sin(x)\right)\right)\right)$$

input
```
Integrate[Csc[4*x]*Sin[x]^3,x]
```

output
```
(4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[Cos[x/2] + Sin[x/2]] + Sqrt[2]*(-Log[S
qrt[2] - 2*Sin[x]] + Log[Sqrt[2] + 2*Sin[x]]))/16
```

3.389.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4878, 27, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\csc(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{\sin(4x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{\sin^2(x)}{4\left(2\sin^4(x)-3\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\sin^2(x)}{2\sin^4(x)-3\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{4}\left(2\int \frac{1}{2\sin^2(x)-2}d\sin(x)-\int \frac{1}{2\sin^2(x)-1}d\sin(x)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-\text{arctanh}(\sin(x))\right)$$

input
```
Int[Csc[4*x]*Sin[x]^3,x]
```

output
```
(-ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2])/4
```

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.389.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{8}+\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(\sin(x)+1)}{8}$	28
risch	$-\frac{\ln(i+e^{ix})}{4}+\frac{\ln(e^{ix}-i)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{16}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{16}$	72

input
```
int(sin(x)^3/sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(sin(x)-1)-1/8*ln(sin(x)+1)
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin^3(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

input
```
integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) -
1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.389.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 7.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\int \csc(4x)\sin^3(x)\,dx = \frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{2894292447518688\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{2894292447518688\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{16372590531019792+11577169790074752\sqrt{2}}$$

input
```
integrate(sin(x)**3/sin(4*x),x)
```

3.389. $\int \csc(4x)\sin^3(x)\,dx$

output
```
4093147632754948*log(tan(x/2) - 1)/(16372590531019792 + 11577169790074752*
sqrt(2)) + 2894292447518688*sqrt(2)*log(tan(x/2) - 1)/(16372590531019792 +
 11577169790074752*sqrt(2)) - 4093147632754948*log(tan(x/2) + 1)/(16372590
531019792 + 11577169790074752*sqrt(2)) - 2894292447518688*sqrt(2)*log(tan(
x/2) + 1)/(16372590531019792 + 11577169790074752*sqrt(2)) + 14471462237593
44*log(tan(x/2) - 1 + sqrt(2))/(16372590531019792 + 11577169790074752*sqrt
(2)) + 1023286908188737*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(1637259053101
9792 + 11577169790074752*sqrt(2)) + 1447146223759344*log(tan(x/2) + 1 + sq
rt(2))/(16372590531019792 + 11577169790074752*sqrt(2)) + 1023286908188737*
sqrt(2)*log(tan(x/2) + 1 + sqrt(2))/(16372590531019792 + 11577169790074752
*sqrt(2)) - 1447146223759344*log(tan(x/2) - sqrt(2) - 1)/(1637259053101979
2 + 11577169790074752*sqrt(2)) - 1023286908188737*sqrt(2)*log(tan(x/2) - s
qrt(2) - 1)/(16372590531019792 + 11577169790074752*sqrt(2)) - 144714622375
9344*log(tan(x/2) - sqrt(2) + 1)/(16372590531019792 + 11577169790074752*sq
rt(2)) - 1023286908188737*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(16372590531
019792 + 11577169790074752*sqrt(2))
```

3.389.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$
\begin{aligned}
\int \csc(4x)\sin^3(x)\,dx =& \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\
&- \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\
&+ \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\
&- \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\
&- \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) \\
&+ \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)
\end{aligned}
$$

input
```
integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")
```

output
```
1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^
2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.389.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin^3(x)\,dx = -\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1)$$

input
```
integrate(sin(x)^3/sin(4*x),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) -
1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.389.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin^3(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sin(x)\right)}{8} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

input
```
int(sin(x)^3/sin(4*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/8 - atanh(sin(x/2)/cos(x/2))/2
```

3.390 $\int \sqrt{1+\sin(2x)}\,dx$

3.390.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{\cos(2x)}{\sqrt{1+\sin(2x)}}$$

output
```
-cos(2*x)/(1+sin(2*x))^(1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \sqrt{1+\sin(2x)}\,dx = \frac{(-\cos(x)+\sin(x))\sqrt{1+\sin(2x)}}{\cos(x)+\sin(x)}$$

input
```
Integrate[Sqrt[1 + Sin[2*x]],x]
```

output
```
((-Cos[x] + Sin[x])*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

3.390.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(2x)+1}dx$$

$$\Big\downarrow \text{3125}$$

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

input
```
Int[Sqrt[1 + Sin[2*x]],x]
```

output
```
-(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

3.390.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.390.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{(-1+\sin(2x))\sqrt{1+\sin(2x)}}{\cos(2x)}$	22

input
```
int((1+sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+sin(2*x))*(1+sin(2*x))^(1/2)/cos(2*x)
```

3.390.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{(\cos(2x) - \sin(2x) + 1)\sqrt{\sin(2x) + 1}}{\cos(2x) + \sin(2x) + 1}$$

input
```
integrate((1+sin(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x) + 1)
```

3.390.6 Sympy [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin(2x) + 1}\,dx$$

input
```
integrate((1+sin(2*x))**(1/2),x)
```

output
```
Integral(sqrt(sin(2*x) + 1), x)
```

3.390.7 Maxima [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin(2x)+1}\,dx$$

input
```
integrate((1+sin(2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*x) + 1), x)
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \sqrt{1+\sin(2x)}\,dx = \sqrt{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+x\right)\right)\sin\left(-\frac{1}{4}\pi+x\right)$$

input
```
integrate((1+sin(2*x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*sgn(cos(-1/4*pi + x))*sin(-1/4*pi + x)
```

3.390.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \sqrt{1+\sin(2x)}\,dx = \frac{(\sin(2x)-1)\,\sqrt{\sin(2x)+1}}{\cos(2x)}$$

input
```
int((sin(2*x) + 1)^(1/2),x)
```

output
```
((sin(2*x) - 1)*(sin(2*x) + 1)^(1/2))/cos(2*x)
```

3.391 $\int \sqrt{1-\sin(2x)}\,dx$

3.391.1 Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

output
```
cos(2*x)/(1-sin(2*x))^(1/2)
```

3.391.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{(\cos(x)+\sin(x))\sqrt{1-\sin(2x)}}{\cos(x)-\sin(x)}$$

input
```
Integrate[Sqrt[1 - Sin[2*x]],x]
```

output
```
((Cos[x] + Sin[x])*Sqrt[1 - Sin[2*x]])/(Cos[x] - Sin[x])
```

3.391.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\sin(2x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{1-\sin(2x)}dx$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

input
```
Int[Sqrt[1 - Sin[2*x]],x]
```

output
```
Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

3.391.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.391.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

method	result	size
default	$-\frac{(-1+\sin(2x))(1+\sin(2x))}{\cos(2x)\sqrt{1-\sin(2x)}}$	31

input
```
int((1-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1+sin(2*x))*(1+sin(2*x))/cos(2*x)/(1-sin(2*x))^(1/2)
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{(\cos(2x)+\sin(2x)+1)\sqrt{-\sin(2x)+1}}{\cos(2x)-\sin(2x)+1}$$

input
```
integrate((1-sin(2*x))^(1/2),x, algorithm="fricas")
```

output
```
(cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x) + 1)
```

3.391.6 Sympy [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{1-\sin(2x)}\,dx$$

input
```
integrate((1-sin(2*x))**(1/2),x)
```

output
```
Integral(sqrt(1 - sin(2*x)), x)
```

3.391.7 Maxima [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{-\sin(2x)+1}\,dx$$

input
```
integrate((1-sin(2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-sin(2*x) + 1), x)
```

3.391.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \sqrt{1-\sin(2x)}\,dx = -\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+x\right)\right) - \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+x\right)\right)\right)$$

input
```
integrate((1-sin(2*x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*(cos(-1/4*pi + x)*sgn(sin(-1/4*pi + x)) - sgn(sin(-1/4*pi + x)))
```

3.391.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\sqrt{1-\sin(2x)}\,(\sin(2x)+1)}{\cos(2x)}$$

input
```
int((1 - sin(2*x))^(1/2),x)
```

output
```
((1 - sin(2*x))^(1/2)*(sin(2*x) + 1))/cos(2*x)
```

3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}} \, dx$

3.392.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{1}{\sqrt{1+\cos(2x)}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1+cos(2*x))^(1/2))*2^(1/2)
```

3.392.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{1+\cos(2x)}} \, dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{1+\cos(2x)}}$$

input
```
Integrate[1/Sqrt[1 + Cos[2*x]],x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/Sqrt[1 + Cos[2*x]]
```

3.392.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(2x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(2x+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\int \frac{1}{2-\frac{\sin^2(2x)}{\cos(2x)+1}}d\left(-\frac{\sin(2x)}{\sqrt{\cos(2x)+1}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 + Cos[2*x]],x]
```

output
```
ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

3.392.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.392.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\sqrt{2}\,\operatorname{am}^{-1}(x\mid 1)}{2}$	9
risch	$-\frac{\sqrt{2}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)}{\sqrt{\left(e^{2ix}+1\right)^{2} e^{-2ix}}}+\frac{\sqrt{2}\,\ln\left(i+e^{ix}\right)\cos\left(x\right)}{\sqrt{\left(e^{2ix}+1\right)^{2} e^{-2ix}}}$	67

input
```
int(1/(1+cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*InverseJacobiAM(x,1)
```

3.392.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(-\frac{\cos\left(2\,x\right)^{2}-2\,\sqrt{2}\sqrt{\cos\left(2\,x\right)+1}\sin\left(2\,x\right)-2\,\cos\left(2\,x\right)-3}{\cos\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}\right)$$

input
```
integrate(1/(1+cos(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(cos(2*x)^2 - 2*sqrt(2)*sqrt(cos(2*x) + 1)*sin(2*x) - 2*c
os(2*x) - 3)/(cos(2*x)^2 + 2*cos(2*x) + 1))
```

3.392.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \int \frac{1}{\sqrt{\cos(2x)+1}}\,dx$$

input
```
integrate(1/(1+cos(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(cos(2*x) + 1), x)
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right) - \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

input
```
integrate(1/(1+cos(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*sqrt(2)*log(cos(
x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)}{8\,\mathrm{sgn}(\cos(x))} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)}{8\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate(1/(1+cos(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/8*sqrt(2)*log(
abs(1/sin(x) + sin(x) - 2))/sgn(cos(x))
```

3.392.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\sqrt{2}\,\mathrm{asinh}\left(\frac{\sin(x)}{\cos(x)}\right)}{2}$$

input
```
int(1/(cos(2*x) + 1)^(1/2),x)
```

output
```
(2^(1/2)*asinh(sin(x)/cos(x)))/2
```

3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$

3.393.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1-cos(2*x))^(1/2))*2^(1/2)
```

3.393.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{1-\cos(2x)}}$$

input
```
Integrate[1/Sqrt[1 - Cos[2*x]],x]
```

output
```
-(((Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])/Sqrt[1 - Cos[2*x]])
```

3.393.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(2x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\int \frac{1}{2-\frac{\sin^2(2x)}{1-\cos(2x)}}d\frac{\sin(2x)}{\sqrt{1-\cos(2x)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 - Cos[2*x]],x]
```

output
```
-(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

3.393.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.393.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sin(x)\,\operatorname{arctanh}(\cos(x))\sqrt{2}}{\sqrt{2-2\cos(2x)}}$	17
risch	$-\frac{\sqrt{2}\,\ln(e^{ix}+1)\,\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}+\frac{\sqrt{2}\,\ln(e^{ix}-1)\,\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}$	67

input
```
int(1/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)
```

3.393.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{(\cos(2x)+3)\sin(2x)-2\left(\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\cos(2x)+1}}{(\cos(2x)-1)\sin(2x)}\right)$$

input
```
integrate(1/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-((cos(2*x) + 3)*sin(2*x) - 2*(sqrt(2)*cos(2*x) + sqrt(2))
*sqrt(-cos(2*x) + 1))/((cos(2*x) - 1)*sin(2*x)))
```

3.393.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$$

input
```
integrate(1/(1-cos(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(1 - cos(2*x)), x)
```

3.393.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(25) = 50.

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.37

$$\begin{aligned}\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = &-\frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 + \sin\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 + 2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right) + 1\right)\\ &+\frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 + \sin\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 - 2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right) + 1\right)\end{aligned}$$

input
```
integrate(1/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(
sin(2*x), cos(2*x)))^2 + 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1) + 1/4
*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(sin(
2*x), cos(2*x)))^2 - 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)
```

3.393.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \frac{\sqrt{2}\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(1/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(abs(tan(1/2*x)))/sgn(sin(x))
```

3.393.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\sqrt{2}\,\sin\left(2\,x\right)\,\mathrm{atanh}\left(\sqrt{{\cos\left(x\right)}^2}\right)}{2\,\sqrt{1-{\cos\left(2\,x\right)}^2}}$$

input
```
int(1/(1 - cos(2*x))^(1/2),x)
```

output
```
-(2^(1/2)*sin(2*x)*atanh((cos(x)^2)^(1/2)))/(2*(1 - cos(2*x)^2)^(1/2))
```

3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$

3.394.1 Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}} - \frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

output
```
-1/6*sin(3*x)/(1-cos(3*x))^(3/2)-1/12*arctanh(1/2*sin(3*x)*2^(1/2)/(1-cos(
3*x))^(1/2))*2^(1/2)
```

3.394.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\left(\csc^2\left(\frac{3x}{4}\right)+4\log\left(\cos\left(\frac{3x}{4}\right)\right)-4\log\left(\sin\left(\frac{3x}{4}\right)\right)-\sec^2\left(\frac{3x}{4}\right)\right)\sin^3\left(\frac{3x}{2}\right)}{12(1-\cos(3x))^{3/2}}$$

input
```
Integrate[(1 - Cos[3*x])^(-3/2),x]
```

output
```
-1/12*((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - Sec[(
3*x)/4]^2)*Sin[(3*x)/2]^3)/(1 - Cos[3*x])^(3/2)
```

3.394.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cos(3x))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(1-\sin\left(3x+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3129$$

$$\frac{1}{4}\int \frac{1}{\sqrt{1-\cos(3x)}}dx-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \frac{1}{\sqrt{1-\sin\left(3x+\frac{\pi}{2}\right)}}dx-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

$$\Big\downarrow \; 3128$$

$$-\frac{1}{6}\int \frac{1}{2-\frac{\sin^2(3x)}{1-\cos(3x)}}d\frac{\sin(3x)}{\sqrt{1-\cos(3x)}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

input
```
Int[(1 - Cos[3*x])^(-3/2),x]
```

output
```
-1/6*ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/Sqrt[2] - Sin[3*x]/(6*
(1 - Cos[3*x])^(3/2))
```

3.394.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.394.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\left(\frac{\cos\left(\frac{3x}{2}\right)}{2}+\frac{\left(\ln\left(\cos\left(\frac{3x}{2}\right)+1\right)-\ln\left(\cos\left(\frac{3x}{2}\right)-1\right)\right)\left(\sin^{2}\left(\frac{3x}{2}\right)\right)}{4}\right)\sqrt{2}}{3\sin\left(\frac{3x}{2}\right)\sqrt{2-2\cos\left(3x\right)}}$	52

input
```
int(1/(1-cos(3*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(1/2*cos(3/2*x)+1/4*(ln(cos(3/2*x)+1)-ln(cos(3/2*x)-1))*sin(3/2*x)^2)
/sin(3/2*x)*2^(1/2)/(sin(3/2*x)^2)^(1/2)
```

3.394.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \frac{\left(\sqrt{2}\cos(3x) - \sqrt{2}\right)\log\left(-\frac{(\cos(3x)+3)\sin(3x)-2\left(\sqrt{2}\cos(3x)+\sqrt{2}\right)\sqrt{-\cos(3x)+1}}{(\cos(3x)-1)\sin(3x)}\right)\sin(3x)}{24(\cos(3x)-1)\sin(3x)}$$

input
```
integrate(1/(1-cos(3*x))^(3/2),x, algorithm="fricas")
```

output
```
1/24*((sqrt(2)*cos(3*x) - sqrt(2))*log(-((cos(3*x) + 3)*sin(3*x) - 2*(sqrt
(2)*cos(3*x) + sqrt(2))*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x)))*si
n(3*x) + 4*(cos(3*x) + 1)*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x))
```

3.394.6 Sympy [F]

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{1}{(1-\cos(3x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1-cos(3*x))**(3/2),x)
```

output
```
Integral((1 - cos(3*x))**(-3/2), x)
```

3.394.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(42) = 84$.

Time = 0.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 8.17

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \frac{4(\sin(6x) - 2\sin(3x))\cos\left(\frac{3}{2}\pi + \frac{3}{2}\arctan(\sin(3x), \cos(3x))\right) - 4(\sin(6x) -}{}$$

input
```
integrate(1/(1-cos(3*x))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(sin(6*x) - 2*sin(3*x))*cos(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x
))) - 4*(sin(6*x) - 2*sin(3*x))*cos(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x
))) + (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^
2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos(3*x) - 1)*log(cos(1/2*arcta
n2(sin(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin(3*x), cos(3*x)))^2 + 2*cos
(1/2*arctan2(sin(3*x), cos(3*x))) + 1) - (2*(2*cos(3*x) - 1)*cos(6*x) - co
s(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2
+ 4*cos(3*x) - 1)*log(cos(1/2*arctan2(sin(3*x), cos(3*x)))^2 + sin(1/2*arc
tan2(sin(3*x), cos(3*x)))^2 - 2*cos(1/2*arctan2(sin(3*x), cos(3*x))) + 1)
- 4*(cos(6*x) - 2*cos(3*x) + 1)*sin(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x
))) + 4*(cos(6*x) - 2*cos(3*x) + 1)*sin(1/2*pi + 1/2*arctan2(sin(3*x), cos
(3*x))))/(sqrt(2)*cos(6*x)^2 + 4*sqrt(2)*cos(3*x)^2 + sqrt(2)*sin(6*x)^2 -
 4*sqrt(2)*sin(6*x)*sin(3*x) + 4*sqrt(2)*sin(3*x)^2 - 2*(2*sqrt(2)*cos(3*x
) - sqrt(2))*cos(6*x) - 4*sqrt(2)*cos(3*x) + sqrt(2))
```

3.394.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(\cos\left(\frac{3}{2}x\right)-1\right)}{\cos\left(\frac{3}{2}x\right)+1}-1\right)\left(\cos\left(\frac{3}{2}x\right)+1\right)}{48\left(\cos\left(\frac{3}{2}x\right)-1\right)\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)} + \frac{\sqrt{2}\log\left(-\frac{\cos\left(\frac{3}{2}x\right)-1}{\cos\left(\frac{3}{2}x\right)+1}\right)}{24\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)} - \frac{\sqrt{2}\left(\cos\left(\frac{3}{2}x\right)-1\right)}{48\left(\cos\left(\frac{3}{2}x\right)+1\right)\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)}$$

input
```
integrate(1/(1-cos(3*x))^(3/2),x, algorithm="giac")
```

output
```
-1/48*sqrt(2)*(2*(cos(3/2*x) - 1)/(cos(3/2*x) + 1) - 1)*(cos(3/2*x) + 1)/(
(cos(3/2*x) - 1)*sgn(sin(3/2*x))) + 1/24*sqrt(2)*log(-(cos(3/2*x) - 1)/(co
s(3/2*x) + 1))/sgn(sin(3/2*x)) - 1/48*sqrt(2)*(cos(3/2*x) - 1)/((cos(3/2*x
) + 1)*sgn(sin(3/2*x)))
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{1}{\left(1-\cos\left(3\,x\right)\right)^{3/2}} \,dx$$

input
```
int(1/(1 - cos(3*x))^(3/2),x)
```

output
```
int(1/(1 - cos(3*x))^(3/2), x)
```

3.395 $\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} \, dx$

3.395.1 Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} \, dx = \frac{32 \cos \left(\frac{2x}{3}\right)}{5 \sqrt{1-\sin \left(\frac{2x}{3}\right)}} + \frac{8}{5} \cos \left(\frac{2x}{3}\right) \sqrt{1-\sin \left(\frac{2x}{3}\right)} + \frac{3}{5} \cos \left(\frac{2x}{3}\right) \left(1-\sin \left(\frac{2x}{3}\right)\right)^{3/2}$$

output
```
3/5*cos(2/3*x)*(1-sin(2/3*x))^(3/2)+32/5*cos(2/3*x)/(1-sin(2/3*x))^(1/2)+8
/5*cos(2/3*x)*(1-sin(2/3*x))^(1/2)
```

3.395.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} \, dx = \frac{\left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} \left(150 \cos \left(\frac{x}{3}\right)+25 \cos (x)-3 \cos \left(\frac{5x}{3}\right)+150 \sin \left(\frac{x}{3}\right)-25 \sin (x)-3 \sin \left(\frac{5x}{3}\right)\right)}{20 \left(\cos \left(\frac{x}{3}\right)-\sin \left(\frac{x}{3}\right)\right)^5}$$

input
```
Integrate[(1 - Sin[(2*x)/3])^(5/2),x]
```

output
```
((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)/3] + 150
*Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] - Sin[x/3])^5)
```

3.395.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{5/2} dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5} \int \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} dx+\frac{3}{5} \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} \cos \left(\frac{2 x}{3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5} \int \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} dx+\frac{3}{5} \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} \cos \left(\frac{2 x}{3}\right)$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5} \left(\frac{4}{3} \int \sqrt{1-\sin \left(\frac{2 x}{3}\right)}dx+\sqrt{1-\sin \left(\frac{2 x}{3}\right)} \cos \left(\frac{2 x}{3}\right)\right)+\frac{3}{5} \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} \cos \left(\frac{2 x}{3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5} \left(\frac{4}{3} \int \sqrt{1-\sin \left(\frac{2 x}{3}\right)}dx+\sqrt{1-\sin \left(\frac{2 x}{3}\right)} \cos \left(\frac{2 x}{3}\right)\right)+\frac{3}{5} \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} \cos \left(\frac{2 x}{3}\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{3}{5} \left(1-\sin \left(\frac{2 x}{3}\right)\right)^{3/2} \cos \left(\frac{2 x}{3}\right)+\frac{8}{5} \left(\sqrt{1-\sin \left(\frac{2 x}{3}\right)} \cos \left(\frac{2 x}{3}\right)+\frac{4 \cos \left(\frac{2 x}{3}\right)}{\sqrt{1-\sin \left(\frac{2 x}{3}\right)}}\right)$$

input
```
Int[(1 - Sin[(2*x)/3])^(5/2),x]
```

output
```
(8*((4*Cos[(2*x)/3])/Sqrt[1 - Sin[(2*x)/3]] + Cos[(2*x)/3]*Sqrt[1 - Sin[(2
*x)/3]]))/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5
```

3.395.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.395.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{\left(-1+\sin\left(\frac{2x}{3}\right)\right)\left(\sin\left(\frac{2x}{3}\right)+1\right)\left(3\left(\sin^{2}\left(\frac{2x}{3}\right)\right)-14\sin\left(\frac{2x}{3}\right)+43\right)}{5\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$	47

input
```
int((1-sin(2/3*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-1+sin(2/3*x))*(sin(2/3*x)+1)*(3*sin(2/3*x)^2-14*sin(2/3*x)+43)/cos(
2/3*x)/(1-sin(2/3*x))^(1/2)
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx =$$
$$-\frac{\left(3\cos\left(\frac{2}{3}x\right)^{3}-11\cos\left(\frac{2}{3}x\right)^{2}+\left(3\cos\left(\frac{2}{3}x\right)^{2}+14\cos\left(\frac{2}{3}x\right)-32\right)\sin\left(\frac{2}{3}x\right)-46\cos\left(\frac{2}{3}x\right)-32\right)\sqrt{-\sin}}{5\left(\cos\left(\frac{2}{3}x\right)-\sin\left(\frac{2}{3}x\right)+1\right)}$$

input
```
integrate((1-sin(2/3*x))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*cos(2/3*x)^3 - 11*cos(2/3*x)^2 + (3*cos(2/3*x)^2 + 14*cos(2/3*x) -
 32)*sin(2/3*x) - 46*cos(2/3*x) - 32)*sqrt(-sin(2/3*x) + 1)/(cos(2/3*x) -
sin(2/3*x) + 1)
```

3.395.6 Sympy [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx = \int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{\frac{5}{2}} dx$$

input
```
integrate((1-sin(2/3*x))**(5/2),x)
```

output
```
Integral((1 - sin(2*x/3))**(5/2), x)
```

3.395.7 Maxima [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx = \int \left(-\sin\left(\frac{2}{3}x\right)+1\right)^{\frac{5}{2}} dx$$

input
```
integrate((1-sin(2/3*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-sin(2/3*x) + 1)^(5/2), x)
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx = -\frac{1}{20}\sqrt{2}\left(150\cos\left(-\frac{1}{4}\pi+\frac{1}{3}x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{3}x\right)\right)-25\cos\left(-\frac{3}{4}\pi+x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{3}x\right)\right)\right)$$

input
```
integrate((1-sin(2/3*x))^(5/2),x, algorithm="giac")
```

output
```
-1/20*sqrt(2)*(150*cos(-1/4*pi + 1/3*x)*sgn(sin(-1/4*pi + 1/3*x)) - 25*cos
(-3/4*pi + x)*sgn(sin(-1/4*pi + 1/3*x)) + 3*cos(-5/4*pi + 5/3*x)*sgn(sin(-
1/4*pi + 1/3*x)) - 128*sgn(sin(-1/4*pi + 1/3*x)))
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx = \int \left(1-\sin\left(\frac{2\,x}{3}\right)\right)^{5/2} dx$$

input
```
int((1 - sin((2*x)/3))^(5/2),x)
```

output
```
int((1 - sin((2*x)/3))^(5/2), x)
```

3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$

3.396.1 Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{3}{4\sqrt{1+2\sin(x)}} - \frac{4}{\sqrt[4]{1+2\sin(x)}} - \frac{1}{2}\sqrt{1+2\sin(x)} + \frac{1}{12}(1+2\sin(x))^{3/2}$$

output
```
-4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(
1+2*sin(x))^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\frac{-3+\cos(2x)+4\sin(x)+24\sqrt[4]{1+2\sin(x)}}{6\sqrt{1+2\sin(x)}}$$

input
```
Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/
2),x]
```

output
```
-1/6*(-3 + Cos[2*x] + 4*Sin[x] + 24*(1 + 2*Sin[x])^(1/4))/Sqrt[1 + 2*Sin[x
]]
```

3.396.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4856, 25, 7267, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)\left(2\sqrt[4]{2\sin(x)+1}-\cos^2(x)\right)}{(2\sin(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)\left(2\sqrt[4]{2\sin(x)+1}-\cos(x)^2\right)}{(2\sin(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{4856}$$

$$\int -\frac{-\sin^2(x)-2\sqrt[4]{2\sin(x)+1}+1}{(2\sin(x)+1)^{3/2}}d\sin(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{-\sin^2(x)-2\sqrt[4]{2\sin(x)+1}+1}{(2\sin(x)+1)^{3/2}}d\sin(x)$$

$$\Big\downarrow \text{7267}$$

$$\frac{1}{2}\int -\csc^3(x)\left(-(2\sin(x)+1)^2+2(2\sin(x)+1)-8\sqrt[4]{2\sin(x)+1}+3\right)d\sqrt[4]{2\sin(x)+1}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \csc^3(x)\left(-(2\sin(x)+1)^2+2(2\sin(x)+1)-8\sqrt[4]{2\sin(x)+1}+3\right)d\sqrt[4]{2\sin(x)+1}$$

$$\Big\downarrow \text{2010}$$

$$-\frac{1}{2}\int \left(3\csc^3(x)-8\csc^2(x)-(2\sin(x)+1)^{5/4}+2\sqrt[4]{2\sin(x)+1}\right)d\sqrt[4]{2\sin(x)+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{6}(2\sin(x)+1)^{3/2}-\sqrt{2\sin(x)+1}+\frac{3\csc^2(x)}{2}-8\csc(x)\right)$$

input
```
Int[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

3.396. $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}dx$

output
```
(-8*Csc[x] + (3*Csc[x]^2)/2 - Sqrt[1 + 2*Sin[x]] + (1 + 2*Sin[x])^(3/2)/6)
/2
```

3.396.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.396. $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$

3.396.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$	42
default	$-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$	42
parts	$-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$	42

input
```
int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(
1+2*sin(x))^(1/2)
```

3.396.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\frac{\left(\cos(x)^2+2\sin(x)-2\right)\sqrt{2\sin(x)+1}+12\,(2\sin(x)+1)^{\frac{3}{4}}}{3\,(2\sin(x)+1)}$$

input
```
integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/3*((cos(x)^2 + 2*sin(x) - 2)*sqrt(2*sin(x) + 1) + 12*(2*sin(x) + 1)^(3/
4))/(2*sin(x) + 1)
```

3.396.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(48) = 96$.

Time = 37.51 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.18

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{4(2\sin(x)+1)^{\frac{3}{4}}\sin^2(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{3}{4}}\sin(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} + \frac{3(2\sin(x)+1)^{\frac{3}{4}}\cos^2(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{3}{4}}}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{24\sin(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{12}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}}$$

input
```
integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)
```

output
```
4*(2*sin(x) + 1)**(3/4)*sin(x)**2/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*s
in(x) + 1)**(1/4)) - 2*(2*sin(x) + 1)**(3/4)*sin(x)/(6*(2*sin(x) + 1)**(1/
4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) + 3*(2*sin(x) + 1)**(3/4)*cos(x)**2/(
6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 2*(2*sin(x) +
1)**(3/4)/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 24*
sin(x)/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 12/(6*
(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4))
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}(2\sin(x)+1)^{\frac{3}{2}} - \frac{16(2\sin(x)+1)^{\frac{1}{4}}-3}{4\sqrt{2\sin(x)+1}} - \frac{1}{2}\sqrt{2\sin(x)+1}$$

input
```
integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, al
gorithm="maxima")
```

output
```
1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x
) + 1) - 1/2*sqrt(2*sin(x) + 1)
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}\,(2\,\sin\left(x\right)+1)^{\frac{3}{2}} - \frac{16\,(2\,\sin\left(x\right)+1)^{\frac{1}{4}}-3}{4\,\sqrt{2\,\sin\left(x\right)+1}} - \frac{1}{2}\,\sqrt{2\,\sin\left(x\right)+1}$$

input
```
integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, al
gorithm="giac")
```

output
```
1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x
) + 1) - 1/2*sqrt(2*sin(x) + 1)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\int -\frac{\cos\left(x\right)\,\left(2\,(2\,\sin\left(x\right)+1)^{1/4}-\cos\left(x\right)^2\right)}{(2\,\sin\left(x\right)+1)^{3/2}}\,dx$$

input
```
int((cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2),x)
```

output
```
-int(-(cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2), x
)
```

3.397 $\int \sqrt{\tan(x)}\,dx$

3.397.1 Optimal result

Integrand size = 6, antiderivative size = 98

$$\int \sqrt{\tan(x)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}$$

output
```
1/2*arctan(-1+2^(1/2)*tan(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*tan(x)^(1
/2))*2^(1/2)+1/4*ln(1-2^(1/2)*tan(x)^(1/2)+tan(x))*2^(1/2)-1/4*ln(1+2^(1/2
)*tan(x)^(1/2)+tan(x))*2^(1/2)
```

3.397.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \sqrt{\tan(x)}\,dx = \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(x)}\right) - \operatorname{arctanh}\left(\sqrt[4]{-\tan^2(x)}\right)\right)\sqrt[4]{-\tan(x)}}{\sqrt[4]{\tan(x)}}$$

input
```
Integrate[Sqrt[Tan[x]],x]
```

output
```
((ArcTan[(-Tan[x]^2)^(1/4)] - ArcTanh[(-Tan[x]^2)^(1/4)])*(-Tan[x])^(1/4))
/Tan[x]^(1/4)
```

3.397.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.833, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)}dx$$

$$\Big\downarrow \text{3957}$$

$$\int \frac{\sqrt{\tan(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{\tan(x)+1}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}+\frac{1}{2}\int \frac{1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(x)-1}d\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(x)-1}d\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\right.\right.$$

$\downarrow$ 25

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{a}{}\right.\right.$$

$\downarrow$ 27

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(x)}+1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}\right.}{\sqrt{2}}\right.\right.$$

$\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\mathrm{ta}\right.}{}\right.\right.$$

input
```
Int[Sqrt[Tan[x]],x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[
Tan[x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
- Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]))/2)
```

3.397.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.397. $\int \sqrt{\tan(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.397.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.50

method	result	size
lookup	$\frac{(\sqrt{\tan}(x))\cos(x)\sqrt{2}\,\operatorname{arccos}(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,(\sqrt{\tan}(x))\cos(x)+\sin(x)\right)}{2}$	49
default	$\frac{(\sqrt{\tan}(x))\cos(x)\sqrt{2}\,\operatorname{arccos}(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,(\sqrt{\tan}(x))\cos(x)+\sin(x)\right)}{2}$	49
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}{1+\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,(\sqrt{\tan}(x))\right)+2\arctan\left(-1+\sqrt{2}\,(\sqrt{\tan}(x))\right)\right)}{4}$	62

input
```
int(tan(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x)
)-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

3.397.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\begin{aligned}\int\sqrt{\tan(x)}\,dx=&\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\sqrt{\tan(x)}\right)\\&-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\sqrt{\tan(x)}\right)\\&+\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\sqrt{\tan(x)}\right)\\&-\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\sqrt{\tan(x)}\right)\end{aligned}$$

input
```
integrate(tan(x)^(1/2),x, algorithm="fricas")
```

output
```
(1/4*I - 1/4)*sqrt(2)*log((I + 1)*sqrt(2) + 2*sqrt(tan(x))) - (1/4*I + 1/4
)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*sqrt(tan(x))) + (1/4*I + 1/4)*sqrt(2)*l
og((I - 1)*sqrt(2) + 2*sqrt(tan(x))) - (1/4*I - 1/4)*sqrt(2)*log(-(I + 1)*
sqrt(2) + 2*sqrt(tan(x)))
```

3.397.6 Sympy [F]

$$\int \sqrt{\tan(x)}\,dx = \int \sqrt{\tan(x)}\,dx$$

input
```
integrate(tan(x)**(1/2),x)
```

output
```
Integral(sqrt(tan(x)), x)
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \sqrt{\tan(x)}\,dx &= \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)\\&\quad+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)\\&\quad-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)\\&\quad+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)\end{aligned}$$

input
```
integrate(tan(x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*s
qrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x)
 + 1)
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*s
qrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x)
 + 1)
```

3.397.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \sqrt{\tan(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\sqrt{\tan(x)}-\tan(x)-1\right)-\ln\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{4} + \frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\sqrt{\tan(x)}-1\right)+\mathrm{atan}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{2}$$

input
```
int(tan(x)^(1/2),x)
```

output
```
(2^(1/2)*(log(2^(1/2)*tan(x)^(1/2) - tan(x) - 1) - log(tan(x) + 2^(1/2)*ta
n(x)^(1/2) + 1)))/4 + (2^(1/2)*(atan(2^(1/2)*tan(x)^(1/2) - 1) + atan(2^(1
/2)*tan(x)^(1/2) + 1)))/2
```

3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

3.398.1 Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = -\frac{1}{10}\sqrt{3}\arctan\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{3}{20}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1+\tan^2(5x)\right)$$

output
```
3/20*ln(1+tan(5*x)^(2/3))-1/20*ln(1+tan(5*x)^2)-1/10*arctan(1/3*(1-2*tan(5
*x)^(2/3))*3^(1/2))*3^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{-1+2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)$$

input
```
Integrate[Tan[5*x]^(-1/3),x]
```

output
```
(Sqrt[3]*ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2
/3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

3.398.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {3042, 3957, 266, 807, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{1}{5}\int \frac{1}{\sqrt[3]{\tan(5x)}\left(\tan^2(5x)+1\right)}d\tan(5x)$$

$$\Big\downarrow \text{266}$$

$$\frac{3}{5}\int \frac{\sqrt[3]{\tan(5x)}}{\tan^2(5x)+1}d\sqrt[3]{\tan(5x)}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{10}\int \frac{1}{\tan(5x)+1}d\tan^{\frac{2}{3}}(5x)$$

$$\Big\downarrow \text{750}$$

$$\frac{3}{10}\left(\frac{1}{3}\int \left(2-\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)+\frac{1}{3}\int \frac{1}{\tan^{\frac{2}{3}}(5x)+1}d\tan^{\frac{2}{3}}(5x)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{10}\left(\frac{1}{3}\int \left(2-\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\tan^{\frac{2}{3}}(5x)-\frac{1}{2}\int \left(2\tan^{\frac{2}{3}}(5x)-1\right)d\tan^{\frac{2}{3}}(5x)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\tan^{\frac{2}{3}}(5x)+\frac{1}{2}\int \left(1-2\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)-3\int\frac{1}{-2\tan^{\frac{2}{3}}(5x)-2}d\left(2\tan^{\frac{2}{3}}(5x)-1\right)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)+\sqrt{3}\arctan\left(\frac{2\tan^{\frac{2}{3}}(5x)-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{10}\left(\frac{\arctan\left(\frac{2\tan^{\frac{2}{3}}(5x)-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

input
```
Int[Tan[5*x]^(-1/3),x]
```

output
```
(3*(ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]]/Sqrt[3] + Log[1 + Tan[5*x]^(2/
3)]/3))/10
```

3.398.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.398. $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

3.398.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan^{\frac{2}{3}}(5x)\right)}{10}-\frac{\ln\left(\tan^{\frac{4}{3}}(5x)-\left(\tan^{\frac{2}{3}}(5x)\right)+1\right)}{20}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2\left(\tan^{\frac{2}{3}}(5x)\right)-1\right)\sqrt{3}}{3}\right)}{10}$	53
default	$\frac{\ln\left(1+\tan^{\frac{2}{3}}(5x)\right)}{10}-\frac{\ln\left(\tan^{\frac{4}{3}}(5x)-\left(\tan^{\frac{2}{3}}(5x)\right)+1\right)}{20}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2\left(\tan^{\frac{2}{3}}(5x)\right)-1\right)\sqrt{3}}{3}\right)}{10}$	53

input
```
int(1/tan(5*x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/10*ln(1+tan(5*x)^(2/3))-1/20*ln(tan(5*x)^(4/3)-tan(5*x)^(2/3)+1)+1/10*3^
(1/2)*arctan(1/3*(2*tan(5*x)^(2/3)-1)*3^(1/2))
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan\left(5\,x\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right) - \frac{1}{20}\log\left(\tan\left(5\,x\right)^{\frac{4}{3}}-\tan\left(5\,x\right)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan\left(5\,x\right)^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/tan(5*x)^(1/3),x, algorithm="fricas")
```

output
```
1/10*sqrt(3)*arctan(2/3*sqrt(3)*tan(5*x)^(2/3) - 1/3*sqrt(3)) - 1/20*log(t
an(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

3.398.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \int \frac{1}{\sqrt[3]{\tan{(5x)}}}\,dx$$

input
```
integrate(1/tan(5*x)**(1/3),x)
```

output
```
Integral(tan(5*x)**(-1/3), x)
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\tan{(5x)}^{\frac{2}{3}} - 1\Big)\right) - \frac{1}{20}\log\left(\tan{(5x)}^{\frac{4}{3}} - \tan{(5x)}^{\frac{2}{3}} + 1\right) + \frac{1}{10}\log\left(\tan{(5x)}^{\frac{2}{3}} + 1\right)$$

input
```
integrate(1/tan(5*x)^(1/3),x, algorithm="maxima")
```

output
```
1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x
)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\tan{(5x)}^{\frac{2}{3}} - 1\Big)\right) - \frac{1}{20}\log\left(\tan{(5x)}^{\frac{4}{3}} - \tan{(5x)}^{\frac{2}{3}} + 1\right) + \frac{1}{10}\log\left(\tan{(5x)}^{\frac{2}{3}} + 1\right)$$

input
```
integrate(1/tan(5*x)^(1/3),x, algorithm="giac")
```

output
```
1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x
)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

3.398.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{\ln\left(81\tan(5\,x)^{2/3}+81\right)}{10} - \ln\left(81-162\tan(5\,x)^{2/3}+\sqrt{3}\,81\mathrm{i}\right)\left(\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right) + \ln\left(162\tan(5\,x)^{2/3}-81+\sqrt{3}\,81\mathrm{i}\right)\left(-\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right)$$

input
```
int(1/tan(5*x)^(1/3),x)
```

output
```
log(81*tan(5*x)^(2/3) + 81)/10 - log(3^(1/2)*81i - 162*tan(5*x)^(2/3) + 81
)*((3^(1/2)*1i)/20 + 1/20) + log(3^(1/2)*81i + 162*tan(5*x)^(2/3) - 81)*((
3^(1/2)*1i)/20 - 1/20)
```

3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

3.399.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} + \frac{13\text{arctanh}\left(\frac{3+\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{4+3\tan(2x)}}$$

output
```
-9/500*arctan(1/2*(1-3*tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)+13/
500*arctanh(1/2*(3+tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)-3/25/(4
+3*tan(2*x))^(1/2)
```

3.399.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{(3+4i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\left(\frac{4}{25}-\frac{3i}{25}\right)(4+3\tan(2x))\right)+(3-4i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\ldots\right)}{50\sqrt{4+3\tan(2x)}}$$

input
```
Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

output
```
-1/50*((3 + 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 - (3*I)/25)*(4 + 3*
Tan[2*x])] + (3 - 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 + (3*I)/25)*(
4 + 3*Tan[2*x])])/Sqrt[4 + 3*Tan[2*x]]
```

3.399.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3964, 3042, 4019, 27, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\tan(2x)+4)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\tan(2x)+4)^{3/2}}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{1}{25}\int \frac{4-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{25}\int \frac{4-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{4019}$$

$$\frac{1}{25}\left(\frac{1}{10}\int \frac{9(\tan(2x)+3)}{\sqrt{3\tan(2x)+4}}dx-\frac{1}{10}\int -\frac{13(1-3\tan(2x))}{\sqrt{3\tan(2x)+4}}dx\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{25}\left(\frac{13}{10}\int \frac{1-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx+\frac{9}{10}\int \frac{\tan(2x)+3}{\sqrt{3\tan(2x)+4}}dx\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{25}\left(\frac{13}{10}\int \frac{1-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx+\frac{9}{10}\int \frac{\tan(2x)+3}{\sqrt{3\tan(2x)+4}}dx\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{4018}$$

$$\frac{1}{25}\left(-\frac{9}{10}\int \frac{1}{\frac{(1-3\tan(2x))^2}{3\tan(2x)+4}+2}d\frac{1-3\tan(2x)}{\sqrt{3\tan(2x)+4}}-\frac{117}{10}\int \frac{1}{\frac{81(\tan(2x)+3)^2}{3\tan(2x)+4}-162}d\frac{9(\tan(2x)+3)}{\sqrt{3\tan(2x)+4}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{25}\left(-\frac{117}{10}\int \frac{1}{\frac{81(\tan(2x)+3)^2}{3\tan(2x)+4}-162}d\frac{9(\tan(2x)+3)}{\sqrt{3\tan(2x)+4}}-\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{10\sqrt{2}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{25}\left(\frac{13\text{arctanh}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{10\sqrt{2}}-\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{10\sqrt{2}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

input
```
Int[(4 + 3*Tan[2*x])^(-3/2),x]
```

output
```
((-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(10*Sqrt[2])
 + (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(10*Sqrt[2]
))/25 - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

3.399.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

3.399.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

method	result
derivativedivides	$-\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)+3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}-$
default	$-\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)+3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}-$

3.399. $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

input
```
int(1/(4+3*tan(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/25/(4+3*tan(2*x))^(1/2)+13/1000*2^(1/2)*ln(9+3*tan(2*x)+3*(4+3*tan(2*x)
)^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)+3*2^(1/2
))*2^(1/2))-13/1000*2^(1/2)*ln(9+3*tan(2*x)-3*(4+3*tan(2*x))^(1/2)*2^(1/2)
)+9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/2))
```

3.399.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \frac{\sqrt{117i+44}(3\,\tan{(2\,x)}+4)\log\left(-(7i-24)\,\sqrt{117i+44}+125\,\sqrt{3\,\tan{(2\,x)}+4}\right.}{}$$

input
```
integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="fricas")
```

output
```
1/500*(sqrt(117*I + 44)*(3*tan(2*x) + 4)*log(-(7*I - 24)*sqrt(117*I + 44)
+ 125*sqrt(3*tan(2*x) + 4)) - sqrt(117*I + 44)*(3*tan(2*x) + 4)*log((7*I -
 24)*sqrt(117*I + 44) + 125*sqrt(3*tan(2*x) + 4)) + sqrt(-117*I + 44)*(3*t
an(2*x) + 4)*log((7*I + 24)*sqrt(-117*I + 44) + 125*sqrt(3*tan(2*x) + 4))
- sqrt(-117*I + 44)*(3*tan(2*x) + 4)*log(-(7*I + 24)*sqrt(-117*I + 44) + 1
25*sqrt(3*tan(2*x) + 4)) - 60*sqrt(3*tan(2*x) + 4))/(3*tan(2*x) + 4)
```

3.399.6 Sympy [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \int \frac{1}{(3\tan{(2x)}+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4+3*tan(2*x))**(3/2),x)
```

output
```
Integral((3*tan(2*x) + 4)**(-3/2), x)
```

3.399.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3213 vs. 2(69) = 138.

Time = 0.52 (sec) , antiderivative size = 3213, normalized size of antiderivative = 36.93

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="maxima")
```

output
```
-1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 + 200
0*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin
(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))*sin(1/2*arctan2(-3*c
os(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x
) + 4))^2 - 2000*(cos(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) +
 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3
 - 80*(48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) - 80*(2
5*(cos(4*x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8
*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 25*cos(4*x)
+ 48*sin(4*x) + 75)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x)
+ 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*a
rctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x)
+ 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arctan2(1/3*
25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2
 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(
4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan
2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8...
```

3.399.8 Giac [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \int \frac{1}{(3\,\tan(2\,x)+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((3*tan(2*x) + 4)^(-3/2), x)
```

3.399. $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

3.399.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{3}{25\sqrt{3\tan(2x)+4}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{3\tan(2x)+4}\left(\frac{1}{10}-\frac{3}{10}\mathrm{i}\right)\right)\left(\frac{9}{500}+\frac{13}{500}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{3\tan(2x)+4}\left(\frac{1}{10}+\frac{3}{10}\mathrm{i}\right)\right)\left(\frac{9}{500}-\frac{13}{500}\mathrm{i}\right)$$

input

```
int(1/(3*tan(2*x) + 4)^(3/2),x)
```

output

```
2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 - 3i/10))*(9/500 + 13i/5
00) + 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 + 3i/10))*(9/500 -
 13i/500) - 3/(25*(3*tan(2*x) + 4)^(1/2))
```

3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$

3.400.1 Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{1}{3}\log(4-3\tan(x)) + \frac{8}{3\sqrt{4-3\tan(x)}} + \frac{2}{3}\sqrt{4-3\tan(x)}$$

output
```
1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)
```

3.400.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{1}{3}\left(\log(4-3\tan(x)) + \frac{2(8-3\tan(x))}{\sqrt{4-3\tan(x)}}\right)$$

input
```
Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2)
,x]
```

output
```
(Log[4 - 3*Tan[x]] + (2*(8 - 3*Tan[x]))/Sqrt[4 - 3*Tan[x]])/3
```

3.400.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4842, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(3\tan(x)-\sqrt{4-3\tan(x)}\right)\sec^2(x)}{(4-3\tan(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(3\tan(x)-\sqrt{4-3\tan(x)}\right)\sec(x)^2}{(4-3\tan(x))^{3/2}}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \left(\frac{3\tan(x)}{(4-3\tan(x))^{3/2}}+\frac{1}{3\tan(x)-4}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

input
```
Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
```

output
```
Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3
```

3.400.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.400.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\ln(4-3\tan(x))}{3}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2\sqrt{4-3\tan(x)}}{3}$	31
default	$\frac{\ln(4-3\tan(x))}{3}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2\sqrt{4-3\tan(x)}}{3}$	31

input
```
int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)
```

3.400.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{(4\cos(x)-3\sin(x))\log\left(\frac{7}{4}\cos(x)^2-6\cos(x)\sin(x)+\frac{9}{4}\right)}{6}$$

input
```
integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, al
gorithm="fricas")
```

output
```
1/6*((4*cos(x) - 3*sin(x))*log(7/4*cos(x)^2 - 6*cos(x)*sin(x) + 9/4) - (4*
cos(x) - 3*sin(x))*log(cos(x)^2) + 4*sqrt((4*cos(x) - 3*sin(x))/cos(x))*(8
*cos(x) - 3*sin(x)))/(4*cos(x) - 3*sin(x))
```

3.400.6 Sympy [F]

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = -\int \frac{\sqrt{4-3\tan(x)}}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\,dx - \int \left(-\frac{3\tan(x)}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\right)dx$$

input
```
integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)
```

output
```
-Integral(sqrt(4 - 3*tan(x))/(-3*sqrt(4 - 3*tan(x))*cos(x)**2*tan(x) + 4*s
qrt(4 - 3*tan(x))*cos(x)**2), x) - Integral(-3*tan(x)/(-3*sqrt(4 - 3*tan(x
))*cos(x)**2*tan(x) + 4*sqrt(4 - 3*tan(x))*cos(x)**2), x)
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{-3\tan(x)+4} + \frac{8}{3\sqrt{-3\tan(x)+4}} + \frac{1}{3}\log(-3\tan(x)+4)$$

input
```
integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, al
gorithm="maxima")
```

output
```
2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(-3*tan(x) + 4)
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{-3\tan(x)+4} + \frac{8}{3\sqrt{-3\tan(x)+4}} + \frac{1}{3}\log(|-3\tan(x)+4|)$$

input
```
integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, al
gorithm="giac")
```

output
```
2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(abs(-3*tan(x)
+ 4))
```

3.400.9 Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{\ln\left(e^{x\,2i}\left(-\frac{16}{3}-4i\right)-\frac{16}{3}+4i\right)}{3} - \frac{\ln\left(e^{x\,2i}\left(\frac{16}{3}-4i\right)+\frac{16}{3}-4i\right)}{3} + \frac{2\,e^{x\,1i}\cos(x)\left(\frac{32\,e^{x\,1i}\cos(x)}{3}-4\,e^{x\,1i}\sin(x)\right)\sqrt{4-\frac{3\sin(x)}{\cos(x)}}}{8\,e^{x\,2i}+8\cos(2x)\,e^{x\,2i}-6\sin(2x)\,e^{x\,2i}}$$

input
```
int((3*tan(x) - (4 - 3*tan(x))^(1/2))/(cos(x)^2*(4 - 3*tan(x))^(3/2)),x)
```

output
```
log(- exp(x*2i)*(16/3 + 4i) - (16/3 - 4i))/3 - log(exp(x*2i)*(16/3 - 4i) +
 (16/3 - 4i))/3 + (2*exp(x*1i)*cos(x)*((32*exp(x*1i)*cos(x))/3 - 4*exp(x*1
i)*sin(x))*(4 - (3*sin(x))/cos(x))^(1/2))/(8*exp(x*2i) + 8*cos(2*x)*exp(x*
2i) - 6*sin(2*x)*exp(x*2i))
```

3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$

3.401.1 Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{x}{2}+\frac{\arctan\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{1+\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}} + \frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}$$

output
```
-1/2*x+1/2*ln(cos(x))+ln(1-tan(x)^(1/2))+1/2*arctan(1/2*(1-tan(x))*2^(1/2)
/tan(x)^(1/2))*2^(1/2)+1/2*arctanh(1/2*(1+tan(x))*2^(1/2)/tan(x)^(1/2))*2^
(1/2)+1/(1-tan(x)^(1/2))
```

3.401.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{1}{2}\arctan(\tan(x))+\frac{1}{2}\log(\cos(x)) +\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}} -\frac{2}{3}\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\tan^2(x)\right)\tan^{\frac{3}{2}}(x)$$

input
```
Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

output
```
-1/2*ArcTan[Tan[x]] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Ta
n[x]])^(-1) - (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3
```

3.401.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4153, 7267, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{\left(1-\sqrt{\tan(x)}\right)^2(\tan^2(x)+1)}d\tan(x)$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\tan^{\frac{3}{2}}(x)}{\left(1-\sqrt{\tan(x)}\right)^2(\tan^2(x)+1)}d\sqrt{\tan(x)}$$

$$\Big\downarrow \text{7276}$$

$$2\int \left(-\frac{\sqrt{\tan(x)}\left(\sqrt{\tan(x)}+1\right)^2}{2(\tan^2(x)+1)}+\frac{1}{2\left(\sqrt{\tan(x)}-1\right)}+\frac{1}{2\left(\sqrt{\tan(x)}-1\right)^2}\right)d\sqrt{\tan(x)}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{2\sqrt{2}}-\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{1}{4}\arctan(\tan(x))+\frac{1}{2\left(1-\sqrt{\tan(x)}\right)}-\frac{1}{8}\log\left(\tan^2\right.\right.$$

input
```
Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

output
```
2*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/(2*Sqrt[2]) - ArcTan[1 + Sqrt[2]*Sqrt[
Tan[x]]]/(2*Sqrt[2]) - ArcTan[Tan[x]]/4 + Log[1 - Sqrt[Tan[x]]]/2 - Log[1
- Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]
] + Tan[x]]/(4*Sqrt[2]) - Log[1 + Tan[x]^2]/8 + 1/(2*(1 - Sqrt[Tan[x]])))
```

3.401.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.401. $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2} dx$

3.401.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}{1+\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,(\sqrt{\tan}(x))\right)+2\arctan\left(-1+\sqrt{2}\,(\sqrt{\tan}(x))\right)\right)}{4}$
default	$-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}{1+\sqrt{2}\,(\sqrt{\tan}(x))+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,(\sqrt{\tan}(x))\right)+2\arctan\left(-1+\sqrt{2}\,(\sqrt{\tan}(x))\right)\right)}{4}$

input
```
int(tan(x)/(-1+tan(x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(tan(x))-1/4*2^(1/2)*(ln((1-2^(1/2)*tan(x)^(1/2)+tan(x))/(1+2^(
1/2)*tan(x)^(1/2)+tan(x)))+2*arctan(1+2^(1/2)*tan(x)^(1/2))+2*arctan(-1+2^
(1/2)*tan(x)^(1/2)))-1/4*ln(1+tan(x)^2)-1/(-1+tan(x)^(1/2))+ln(-1+tan(x)^(
1/2))
```

3.401.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 603, normalized size of antiderivative = 7.18

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/8*(2*(2*sqrt(-I) - I + 1)*(tan(x) - 1)*log(-1/2*(2*sqrt(-I) - I + 1)^2*
(4*(-1)^(1/4) + 2*I + 1) - (2*(-1)^(1/4) + I + 1)^3 - ((2*(-1)^(1/4) + I +
 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) + 4*(2*(-1)^(1/4) + I
 + 1)^2 + 6*sqrt(tan(x)) - 16*(-1)^(1/4) - 8*I - 9) + 2*(2*(-1)^(1/4) + I
+ 1)*(tan(x) - 1)*log((2*(-1)^(1/4) + I + 1)^3 - 7/2*(2*(-1)^(1/4) + I + 1
)^2 + 6*sqrt(tan(x)) + 14*(-1)^(1/4) + 7*I + 14) - ((2*sqrt(-I) - I + 1)*(
tan(x) - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) - 4*sqrt(-3/16*(2*sqrt(-
I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(
2*(-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x)
+ 4)*log(1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1
)^(1/4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2
*(-1)^(1/4) + I + 1)^2 + sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^
(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^
(1/4) + 1/2*I - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-
1)^(1/4) - I + 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - ((2*sqrt(
-I) - I + 1)*(tan(x) - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) + 4*sqrt(-
3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(
-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) -
1) - 4*tan(x) + 4)*log(1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1)
 + 1/2*((2*(-1)^(1/4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) ...
```

3.401.6 Sympy [F]

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \int \frac{\tan\left(x\right)}{\left(\sqrt{\tan\left(x\right)}-1\right)^2}\,dx$$

input
```
integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
```

output
```
Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2}-2\right)\log\left(\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2}+2\right)\log\left(-\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) - \frac{1}{\sqrt{\tan\left(x\right)}-1} + \log\left(\sqrt{\tan\left(x\right)}-1\right)$$

input
```
integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) -
 1/4*sqrt(2)*(sqrt(2) + 2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x))))
 - 1/8*sqrt(2)*(sqrt(2) - 2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/8*
sqrt(2)*(sqrt(2) + 2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/(sqrt(ta
n(x)) - 1) + log(sqrt(tan(x)) - 1)
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(x\right)}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) - \frac{1}{\sqrt{\tan\left(x\right)}-1} - \frac{1}{4}\log\left(\tan\left(x\right)^2+1\right) + \log\left(\left|\sqrt{\tan\left(x\right)}-1\right|\right)$$

input
```
integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) - 1/2*(s
qrt(2) + 1)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + 1/4*sqrt(2)*
log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan
(x)) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1) - 1/4*log(tan(x)^2 + 1) + log(ab
s(sqrt(tan(x)) - 1))
```

3.401.9 Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.71

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \ln\left(612\sqrt{\tan(x)}-612\right)-\frac{1}{\sqrt{\tan(x)}-1}+\left(\sum_{k=1}^{4}\ln\left(4\sqrt{\tan(x)}+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2\sqrt{\tan(x)}\,80+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^3\sqrt{\tan(x)}\,448+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^4\sqrt{\tan(x)}\,128+32\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2-384\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^3-256\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^4-\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)\sqrt{\tan(x)}\,48-4\right)\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)\right)$$

input
```
int(tan(x)/(tan(x)^(1/2) - 1)^2,x)
```

output
```
log(612*tan(x)^(1/2) - 612) - 1/(tan(x)^(1/2) - 1) + symsum(log(4*tan(x)^(
1/2) + 80*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^2*tan(x)^(1/2) + 448*
root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3*tan(x)^(1/2) + 128*root(z^4 +
 z^3 + z^2/2 - z/8 + 1/64, z, k)^4*tan(x)^(1/2) + 32*root(z^4 + z^3 + z^2/
2 - z/8 + 1/64, z, k)^2 - 384*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3
 - 256*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^4 - 48*root(z^4 + z^3 +
z^2/2 - z/8 + 1/64, z, k)*tan(x)^(1/2) - 4)*root(z^4 + z^3 + z^2/2 - z/8 +
 1/64, z, k), k, 1, 4)
```

3.401. $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$

3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$

3.402.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\arcsin(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-1/2*arcsin(cos(x)-sin(x))-1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

3.402.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx=\frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x))-\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

input
```
Integrate[Sin[x]/Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

3.402.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \; 4794$$

$$-\frac{1}{2}\arcsin(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

input
```
Int[Sin[x]/Sqrt[Sin[2*x]],x]
```

output
```
-1/2*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

3.402.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.402.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,E\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)\right)-\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\,\mathrm{ta}}}{}$

input
```
int(sin(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*(1+tan(1/2*x
))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*
x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)
^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan
(1/2*x)^2+2*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/
2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+tan(1/2*x))^(1/2)*(-2*ta
n(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2
^(1/2))+2*tan(1/2*x)^4-2*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)
/(1+tan(1/2*x)^2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

3.402.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) + \frac{1}{8}\log\Big(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2+16\cos(x)\sin(x)+1\Big)$$

input
```
integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) + 1/8*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

3.402.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.402.7 Maxima [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(x)/sqrt(sin(2*x)), x)
```

3.402.8 Giac [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(x)/sqrt(sin(2*x)), x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2\,x)}}\,dx$$

input
```
int(sin(x)/sin(2*x)^(1/2),x)
```

output
```
int(sin(x)/sin(2*x)^(1/2), x)
```

3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

3.403.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) + \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

3.403.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

input
```
Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

3.403.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))$$

input
```
Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

3.403.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.403.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

method	result	size
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}}\,\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,F\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^{3}\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$	98

input
```
int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*
2^(1/2))
```

3.403.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^{2}+2\cos(x)\sin(x)-1}\right) - \frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{8}\log\left(-32\cos(x)^{4} + 4\sqrt{2}\left(4\cos(x)^{3}-\left(4\cos(x)^{2}+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^{2}+16\cos(x)\sin(x)+1\right)$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

3.403.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.403.7 Maxima [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.403.8 Giac [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin(2\,x)}}\,dx$$

input
```
int(cos(x)/sin(2*x)^(1/2),x)
```

output
```
int(cos(x)/sin(2*x)^(1/2), x)
```

3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$

3.404.1 Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = -\frac{1}{4}\arcsin(\cos(x)-\sin(x)) + \frac{1}{4}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{1}{2}\cos(x)\sqrt{\sin(2x)}$$

output
```
-1/4*arcsin(cos(x)-sin(x))+1/4*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/2*cos(x)
*sin(2*x)^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \frac{1}{4}\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - 2\cos(x)\sqrt{\sin(2x)}\right)$$

input
```
Integrate[Sin[x]*Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] - 2*Cos[
x]*Sqrt[Sin[2*x]])/4
```

3.404.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sqrt{\sin(2x)}dx$$

$$\Big\downarrow \text{4790}$$

$$\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx - \frac{1}{2}\sqrt{\sin(2x)}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx - \frac{1}{2}\sqrt{\sin(2x)}\cos(x)$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))\right)-\frac{1}{2}\sqrt{\sin(2x)}\cos(x)$$

input
```
Int[Sin[x]*Sqrt[Sin[2*x]],x]
```

output
```
(-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)/2
 - (Cos[x]*Sqrt[Sin[2*x]])/2
```

3.404.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

3.404.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

method	result
default	$\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}}\,(\tan^2(\frac{x}{2})-1)\left(\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2})+2}\,\sqrt{-\tan(\frac{x}{2})}\,F\left(\sqrt{1+\tan(\frac{x}{2})},\frac{\sqrt{2}}{2}\right)(\tan^2(\frac{x}{2}))+\sqrt{1+\tan(\frac{x}{2})}\,\sqrt{-2\tan(\frac{x}{2}\cdots\right.}{\sqrt{\tan(\frac{x}{2})\,(\tan^2(\frac{x}{2})-1)}\,\sqrt{\tan^3(\frac{x}{2})-\tan(\frac{x}{2})}\,(1+\tan^2(\frac{x}{2}))}$

input
```
int(sin(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+tan(1/2*x))^(1/2
)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/
2),1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*
(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))+2*tan(1/2*
x)^3-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1
/2*x))^(1/2)/(1+tan(1/2*x)^2)
```

3.404.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\int \sin(x)\sqrt{\sin(2x)}\,dx$$
$$= -\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) + \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{8}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{16}\log\Big(-32\cos(x)^4 + 4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x)+1\Big)$$

input
```
integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)
*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) -
1)) - 1/8*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x
) - sin(x))) - 1/16*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2
 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin
(x) + 1)
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)*sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.404.7 Maxima [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\sin(x)\,dx$$

input
```
integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*x))*sin(x), x)
```

3.404.8 Giac [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\sin(x)\,dx$$

input
```
integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(2*x))*sin(x), x)
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\,\sin(x)\,dx$$

input
```
int(sin(2*x)^(1/2)*sin(x),x)
```

output
```
int(sin(2*x)^(1/2)*sin(x), x)
```

3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$

3.405.1 Optimal result

Integrand size = 16, antiderivative size = 47

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = -\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\sin(x)\sqrt{\sin(2x)}$$

output
```
-1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/2*cos(x)*sin(2*x)^(1/2)+1/2*sin(x)
*sin(2*x)^(1/2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \frac{1}{2}\left(-\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \cos(x)\sqrt{\sin(2x)} + \sin(x)\sqrt{\sin(2x)}\right)$$

input
```
Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

output
```
(-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] + Sin[x]*S
qrt[Sin[2*x]])/2
```

3.405.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2x)}(\cos(x)-\sin(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(2x)}(\cos(x)-\sin(x))dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\sqrt{\sin(2x)}\cos(x)-\sin(x)\sqrt{\sin(2x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

input
```
Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

output
```
-1/2*Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + (Cos[x]*Sqrt[Sin[2*x]])/2 + (
Sin[x]*Sqrt[Sin[2*x]])/2
```

3.405.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.405.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 5.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 8.43

method	result
parts	$\frac{2\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{\left(1+\tan\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}\,\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,E\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-t}}$
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(4\sqrt{\left(1+\tan\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}\,\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,E\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)}{}$

input
```
int((cos(x)-sin(x))*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*((1+tan(1/2*x))
*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(
1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+ta
n(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+t
an(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(
1/2)+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/((1+tan(1/2*x))*(tan(1/2*x)
-1)*tan(1/2*x))^(1/2)-(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1
)*((1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipt
icF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-
2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1
/2*2^(1/2))+2*tan(1/2*x)^3-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(1+tan(1/2*x)^2)
```

3.405.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(35) = 70$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x)) + \frac{1}{8}\log\Big(-32\cos(x)^4 + 4\sqrt{2}\big(4\cos(x)^3 - \big(4\cos(x)^2+1\big)\sin(x) - 5\cos(x)\big)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1\Big)$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1/8*log(-32*cos(x)^4 +
 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*s
in(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx = \text{Timed out}$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.405.7 Maxima [F]

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx = \int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

3.405.8 Giac [F]

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx = \int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx=\int \sqrt{\sin\left(2\,x\right)}\,\left(\cos\left(x\right)-\sin\left(x\right)\right)\,dx$$

input
```
int(sin(2*x)^(1/2)*(cos(x) - sin(x)),x)
```

output
```
int(sin(2*x)^(1/2)*(cos(x) - sin(x)), x)
```

3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx$

3.406.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx = -\frac{1}{16} \arcsin(\cos(x)-\sin(x)) + \frac{1}{16} \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}}$$

output
```
-1/16*arcsin(cos(x)-sin(x))+1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/5*sin(
x)^5/sin(2*x)^(5/2)-1/4*sin(x)/sin(2*x)^(1/2)
```

3.406.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx = \frac{1}{80}\left(5\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right) + 2\sec(x)\left(-6+\sec^2(x)\right)\sqrt{\sin(2x)}\right)$$

input
```
Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]
```

output
```
(5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]) + 2*
Sec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80
```

3.406.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 4782, 3042, 4782, 3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^7}{\sin(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4782}$$

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{4}\int \frac{\sin^3(x)}{\sin^{\frac{3}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{4}\int \frac{\sin(x)^3}{\sin(2x)^{3/2}}dx$$

$$\Big\downarrow \text{4782}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \csc(x)\sqrt{\sin(2x)}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{\sqrt{\sin(2x)}}{\sin(x)}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4796}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))\right)-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

input
```
Int[Sin[x]^7/Sin[2*x]^(7/2),x]
```

output
```
((-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)/
2 - Sin[x]/Sqrt[Sin[2*x]])/4 + Sin[x]^5/(5*Sin[2*x]^(5/2))
```

3.406.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4782
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e
*Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c,
d, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m,
 2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.406. $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

3.406.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 6.09 (sec) , antiderivative size = 510, normalized size of antiderivative = 8.36

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(5\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,F\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\left(\tan^{14}\left(\frac{x}{2}\right)\right)+35\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan \cdots}\right.}{}$

input
```
int(sin(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2688*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(1+tan(1/2
*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/
2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^14+35*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2
*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2
))*tan(1/2*x)^12+10*tan(1/2*x)^15+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+
2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*t
an(1/2*x)^10+66*tan(1/2*x)^13+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(
1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1
/2*x)^8-1014*tan(1/2*x)^11+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2
)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*
x)^6+2002*tan(1/2*x)^9+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^4
-2002*tan(1/2*x)^7+35*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1
/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+1014
*tan(1/2*x)^5+5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-66*tan(1/2*x)^3-10*tan(
1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(1/2*x)^2)^7/(tan(1/2*x)
^3-tan(1/2*x))^(1/2)
```

3.406.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(47) = 94.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{10 \arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\cos(x)^3 - 10 \arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)\cos\cdots}{}$$

input
```
integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

output
```
1/320*(10*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*
sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*cos(x)^3 - 10*arctan(-(2*sqrt(2)
*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x)))*cos(x)^3 - 5*co
s(x)^3*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x)
- 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1) - 48
*cos(x)^3 - 8*sqrt(2)*(6*cos(x)^2 - 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**7/sin(2*x)**(7/2),x)
```

output
```
Timed out
```

3.406.7 Maxima [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2\,x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

3.406.8 Giac [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2x)^{7/2}}\,dx$$

input
```
int(sin(x)^7/sin(2*x)^(7/2),x)
```

output
```
int(sin(x)^7/sin(2*x)^(7/2), x)
```

3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx$

3.407.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) - \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}}$$

output
```
-1/16*arcsin(cos(x)-sin(x))-1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/5*cos(
x)^5/sin(2*x)^(5/2)+1/4*cos(x)/sin(2*x)^(1/2)
```

3.407.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)} \, dx = \frac{1}{16}\left(-\arcsin(\cos(x)-\sin(x)) - \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right) + \left(\frac{3\csc(x)}{20} - \frac{\csc^3(x)}{40}\right)\sqrt{\sin(2x)}$$

input
```
Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/16 + ((
3*Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]
```

3.407.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 4781, 3042, 4781, 3042, 4795, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^7}{\sin(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4781}$$

$$-\frac{1}{4}\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(2x)}dx-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \frac{\cos(x)^3}{\sin(2x)^{3/2}}dx-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4781}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \sec(x)\sqrt{\sin(2x)}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{\sqrt{\sin(2x)}}{\cos(x)}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4795}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4794}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\arcsin(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

input
```
Int[Cos[x]^7/Sin[2*x]^(7/2),x]
```

output
```
((-1/2*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)/
2 + Cos[x]/Sqrt[Sin[2*x]])/4 - Cos[x]^5/(5*Sin[2*x]^(5/2))
```

3.407.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4781
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e*Co
s[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4795
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

3.407.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 1108, normalized size of antiderivative = 18.16

method	result	size
default	Expression too large to display	1108

input
```
int(cos(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/160*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(192*(tan(1/2*x)*(
tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(
1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*
(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6-96*(tan(1/2*x)*(tan(1/2*x)^2
-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2
)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-
1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+
tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^10+96*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^8-384*(
tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^
(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+
tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4+192*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan
(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))
*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(tan(1/2*x)^2
-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^8+4
8*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x
))^(1/2)*tan(1/2*x)^8-192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipti...
```

3.407.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.36

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$$

$$= \frac{10\left(\cos(x)^2-1\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\sin(x)-10\left(\cos(x)^2-1\right)\arctan\left(-\frac{2\sqrt{\ldots}}{\ldots}\right.}{\ldots}$$

input
```
integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

output
```
1/320*(10*(cos(x)^2 - 1)*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - si
n(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*sin(x) - 10*(cos(
x)^2 - 1)*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x
) - sin(x)))*sin(x) + 5*(cos(x)^2 - 1)*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos
(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x
)^2 + 16*cos(x)*sin(x) + 1)*sin(x) + 8*sqrt(2)*(6*cos(x)^2 - 5)*sqrt(cos(x
)*sin(x)) + 48*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

3.407.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**7/sin(2*x)**(7/2),x)
```

output
```
Timed out
```

3.407.7 Maxima [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

3.407.8 Giac [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos\left(x\right)^{7}}{\sin\left(2\,x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos\left(x\right)^{7}}{\sin\left(2\,x\right)^{7/2}}\,dx$$

input
```
int(cos(x)^7/sin(2*x)^(7/2),x)
```

output
```
int(cos(x)^7/sin(2*x)^(7/2), x)
```

3.408 $\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx$

3.408.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = -\frac{1}{5} \csc^5(x) \sin^{\frac{5}{2}}(2x)$$

output
```
-1/5*csc(x)^5*sin(2*x)^(5/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = -\frac{1}{5} \csc^5(x) \sin^{\frac{5}{2}}(2x)$$

input
```
Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]
```

output
```
-1/5*(Csc[x]^5*Sin[2*x]^(5/2))
```

3.408.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2x)\csc^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2x)^{3/2}}{\sin(x)^5}dx$$

$$\Big\downarrow \text{4780}$$

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

input
```
Int[Csc[x]^5*Sin[2*x]^(3/2),x]
```

output
```
-1/5*(Csc[x]^5*Sin[2*x]^(5/2))
```

3.408.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

3.408.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 508, normalized size of antiderivative = 31.75

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(96\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}E\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\sqrt{\left(1+\tan\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-\right.}\right.}{}$

input
```
int(sin(2*x)^(3/2)/sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(96*(tan(1/2*x)*(tan
(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2
*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(ta
n(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)*(tan(1/2*x)^2-1)
)^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*E
llipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*
tan(1/2*x))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan
(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+40*tan(1/2*x)^4*(ta
n(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+tan(1/2*x
)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan
(1/2*x))^(1/2)+28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2
*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4+(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*
((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-28*(tan(1/2*
x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*ta
n(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)
-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/((1+tan(1/2*x))*(ta
n(1/2*x)-1)*tan(1/2*x))^(1/2)
```

3.408.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \frac{4\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)^2 + \left(\cos(x)^2-1\right)\sin(x)\right)}{5\left(\cos(x)^2-1\right)\sin(x)}$$

input
```
integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")
```

output
```
4/5*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2 + (cos(x)^2 - 1)*sin(x))/((cos(x
)^2 - 1)*sin(x))
```

3.408.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = \text{Timed out}$$

input
```
integrate(sin(2*x)**(3/2)/sin(x)**5,x)
```

output
```
Timed out
```

3.408.7 Maxima [F]

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = \int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5} \, dx$$

input
```
integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")
```

output
```
integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

3.408.8 Giac [F]

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = \int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5} \, dx$$

input
```
integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="giac")
```

output
```
integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

3.408.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \frac{4\sqrt{\sin(2x)}\,\left(\sin(x)^2-1\right)}{5\sin(x)^3}$$

input
```
int(sin(2*x)^(3/2)/sin(x)^5,x)
```

output
```
(4*sin(2*x)^(1/2)*(sin(x)^2 - 1))/(5*sin(x)^3)
```

3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$

3.409.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{4}{5}\sec(x)\sqrt{\sin(2x)}+\frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}$$

output
```
4/5*sec(x)*sin(2*x)^(1/2)+1/5*sec(x)^3*sin(2*x)^(1/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{5}\sec(x)\left(4+\sec^2(x)\right)\sqrt{\sin(2x)}$$

input
```
Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

output
```
(Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5
```

3.409.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4787, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin(2x)}\cos(x)^3}dx$$

$$\Big\downarrow \text{4787}$$

$$\frac{4}{5}\int \frac{\sec(x)}{\sqrt{\sin(2x)}}dx+\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{1}{\cos(x)\sqrt{\sin(2x)}}dx+\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)$$

$$\Big\downarrow \text{4779}$$

$$\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

input
```
Int[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

output
```
(4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

3.409.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4787
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Cos[a +
 b*x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ
[m + 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.409.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 4.94 (sec) , antiderivative size = 2946, normalized size of antiderivative = 95.03

method	result	size
default	Expression too large to display	2946

input
```
int(1/cos(x)^3/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(-56*tan(1/2*x)-96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan
(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+8*
tan(1/2*x)^3-8*tan(1/2*x)^5+56*tan(1/2*x)^7-32*(1+tan(1/2*x))^(1/2)*(-2*ta
n(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2
^(1/2))+6*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)
*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*(1+tan(1/2*x))^(
1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))
^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*
x)^3-tan(1/2*x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)-24
*arctan(((tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*
x)^3-tan(1/2*x))^(3/2)-12*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-
tan(1/2*x))^(1/2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(3/2)-24*arct
an(((tan(1/2*x)^3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-
tan(1/2*x))^(3/2)-3*ln(1/tan(1/2*x)*(tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*
x))^(1/2)+2*tan(1/2*x)-1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+6*arctan(((tan(
1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x
))^(1/2)+3*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/
2)-2*tan(1/2*x)+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+6*arctan(((tan(1/2*x)^
3-tan(1/2*x))^(1/2)-tan(1/2*x))/tan(1/2*x))*(tan(1/2*x)^3-tan(1/2*x))^(1/2
)+6*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*...
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{4\,\cos{(x)}^3 + \sqrt{2}\left(4\,\cos{(x)}^2 + 1\right)\sqrt{\cos{(x)}\sin{(x)}}}{5\,\cos{(x)}^3}$$

input
```
integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(x)**3/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

3.409.7 Maxima [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{1}{\cos\left(x\right)^3\sqrt{\sin\left(2\,x\right)}}\,dx$$

input
```
integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

3.409.8 Giac [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{1}{\cos\left(x\right)^3\sqrt{\sin\left(2\,x\right)}}\,dx$$

input
```
integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

3.409.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{\sqrt{\sin\left(2\,x\right)}\,\left(2\,\cos\left(2\,x\right)+3\right)}{5\,{\cos\left(x\right)}^3}$$

input
```
int(1/(sin(2*x)^(1/2)*cos(x)^3),x)
```

output
```
(sin(2*x)^(1/2)*(2*cos(2*x) + 3))/(5*cos(x)^3)
```

3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$

3.410.1 Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4\sin(x)}{3\sqrt{\sin(2x)}}$$

output
```
-2/3*cos(x)/sin(2*x)^(3/2)+4/3*sin(x)/sin(2*x)^(1/2)
```

3.410.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \left(-\frac{1}{6}\cot(x)\csc(x) + \frac{\sec(x)}{2}\right)\sqrt{\sin(2x)}$$

input
```
Integrate[Csc[x]/Sin[2*x]^(3/2),x]
```

output
```
(-1/6*(Cot[x]*Csc[x]) + Sec[x]/2)*Sqrt[Sin[2*x]]
```

3.410.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4796, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)\sin(2x)^{3/2}}dx$$
$$\Big\downarrow \text{4796}$$
$$2\int \frac{\cos(x)}{\sin^{\frac{5}{2}}(2x)}dx$$
$$\Big\downarrow \text{3042}$$
$$2\int \frac{\cos(x)}{\sin(2x)^{5/2}}dx$$
$$\Big\downarrow \text{4791}$$
$$2\left(\frac{2}{3}\int \frac{\sin(x)}{\sin^{\frac{3}{2}}(2x)}dx-\frac{\cos(x)}{3\sin^{\frac{3}{2}}(2x)}\right)$$
$$\Big\downarrow \text{3042}$$
$$2\left(\frac{2}{3}\int \frac{\sin(x)}{\sin(2x)^{3/2}}dx-\frac{\cos(x)}{3\sin^{\frac{3}{2}}(2x)}\right)$$
$$\Big\downarrow \text{4780}$$
$$2\left(\frac{2\sin(x)}{3\sqrt{\sin(2x)}}-\frac{\cos(x)}{3\sin^{\frac{3}{2}}(2x)}\right)$$

input
```
Int[Csc[x]/Sin[2*x]^(3/2),x]
```

output
```
2*(-1/3*Cos[x]/Sin[2*x]^(3/2) + (2*Sin[x])/(3*Sqrt[Sin[2*x]]))
```

3.410.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
&& IntegerQ[2*p]
```

3.410.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.17

method	result	size
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,F\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)-\left(\tan^4\left(\frac{x}{2}\right)\right)+1\right)}{12\tan\left(\frac{x}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$	121

input
```
int(1/sin(x)/sin(2*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(2*
(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF
((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)-tan(1/2*x)^4+1)/(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

3.410.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \frac{4\,\cos{(x)}^{3}+\sqrt{2}\left(4\,\cos{(x)}^{2}-3\right)\sqrt{\cos{(x)}\sin{(x)}}-4\,\cos{(x)}}{6\,\left(\cos{(x)}^{3}-\cos{(x)}\right)}$$

input
```
integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 - 3)*sqrt(cos(x)*sin(x)) - 4*cos(x))
/(cos(x)^3 - cos(x))
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

output
```
Timed out
```

3.410.7 Maxima [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \int \frac{1}{\sin{(2\,x)}^{\frac{3}{2}}\sin{(x)}}\,dx$$

input
```
integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

3.410.8 Giac [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \int \frac{1}{\sin{(2\,x)}^{\frac{3}{2}}\sin{(x)}}\,dx$$

input
```
integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

3.410.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = -\frac{\sqrt{\sin{(2\,x)}}\,(2\,\cos{(2\,x)}-1)}{6\,\left(\cos{(x)}-\cos{(x)}^3\right)}$$

input
```
int(1/(sin(2*x)^(3/2)*sin(x)),x)
```

output
```
-(sin(2*x)^(1/2)*(2*cos(2*x) - 1))/(6*(cos(x) - cos(x)^3))
```

3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

3.411.1 Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx=\frac{33}{32}\text{arctanh}\left(\frac{1}{2}\sec(x)\sqrt{\sin(2x)}\right)-\frac{9\cos(x)}{16\sqrt{\sin(2x)}}-\frac{5\cos(x)\cot(x)}{24\sqrt{\sin(2x)}}+\frac{\cos(x)\cot^2(x)}{20\sqrt{\sin(2x)}}$$

output
```
33/32*arctanh(1/2*sin(2*x)^(1/2)/cos(x))-9/16*cos(x)/sin(2*x)^(1/2)-5/24*c
os(x)*cot(x)/sin(2*x)^(1/2)+1/20*cos(x)*cot(x)^2/sin(2*x)^(1/2)
```

3.411.2 Mathematica [A] (verified)

Time = 7.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx=\frac{\cos(x)\sqrt{\sin(2x)}\left(\frac{1}{15}\csc(x)\left(-147-50\cot(x)+12\csc^2(x)\right)+\frac{33\arctan\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{\cos(x)}{2+2\cos(x)}}\sec(x)}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)(\cos}{16(\cos(x)+\cos(3x)-6\sin(x))}$$

input
```
Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]
^(5/2)),x]
```

output
```
(Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))/15 + (33
*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]]*Sqrt[-(Cos[x]/(2 + 2*Cos[x])
)]*Sec[x])/Sqrt[Tan[x/2]])*(Cos[2*x] - 3*Tan[x]))/(16*(Cos[x] + Cos[3*x] -
 6*Sin[x]))
```

3.411.3 Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4890, 4889, 25, 2035, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^3(\cos(2x)-3\tan(x))}{\left(\sin(x)^2-\sin(2x)\right)\sin(2x)^{5/2}}dx$$

$$\Big\downarrow \text{ 4890}$$

$$\frac{\sin^5(x)\int \frac{\cos(x)^3\csc^5(x)(\cos(2x)-3\tan(x))\tan^{\frac{5}{2}}(x)}{\sin(x)^2-\sin(2x)}dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{\sin^5(x)\int -\frac{-3\tan^3(x)-\tan^2(x)-3\tan(x)+1}{(2-\tan(x))\tan^{\frac{7}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sin^5(x)\int \frac{-3\tan^3(x)-\tan^2(x)-3\tan(x)+1}{(2-\tan(x))\tan^{\frac{7}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{ 2035}$$

$$-\frac{2\sin^5(x)\int \frac{\cot^6(x)\left(-3\tan^3(x)-\tan^2(x)-3\tan(x)+1\right)}{2-\tan(x)}d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{2333}$$

$$-\frac{2\sin^5(x)\int \left(\frac{\cot^6(x)}{2}-\frac{5\cot^4(x)}{4}-\frac{9\cot^2(x)}{8}+\frac{33}{8(\tan(x)-2)}\right)d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sin^5(x)\left(-\frac{33\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{1}{10}\cot^5(x)+\frac{5\cot^3(x)}{12}+\frac{9\cot(x)}{8}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

input
```
Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)
),x]
```

output
```
(-2*((-33*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]])/(8*Sqrt[2]) + (9*Cot[x])/8 + (5*C
ot[x]^3)/12 - Cot[x]^5/10)*Sin[x]^5)/(Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

3.411.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.411. $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

3.411.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.31 (sec) , antiderivative size = 761, normalized size of antiderivative = 11.19

method	result	size
default	Expression too large to display	761

input
```
int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(-3024*(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-t
an(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x
))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+932*(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^
(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2
*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+3*2^(1/2)
*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+t
an(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*sum((34*_alpha^3+13*_alpha^2+3
4*_alpha-21)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x)+1)^(1
/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((1+
tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(
_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)^2+200*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^5-1920*tan(
1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)-24*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(ta
n(1/2*x)-1)*tan(1/2*x))^(1/2)-552*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(
1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4-24*(tan(1/2*x)*(tan(
1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan...
```

3.411.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx =$$
$$-\frac{495\left(\cos\left(x\right)^2 - 1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos\left(x\right)\sin\left(x\right)}(4\cos\left(x\right) + 3\sin\left(x\right)) + \frac{1}{2}\cos\left(x\right)^2 + \frac{7}{2}\cos\left(x\right)\sin\left(x\right) + \right.}{}$$

input
```
integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),
x, algorithm="fricas")
```

output
```
-1/1920*(495*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
 + 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 495*(cos(x
)^2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*c
os(x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) -
135)*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(
x))
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/
2),x)
```

output
```
Timed out
```

3.411.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),
x, algorithm="maxima")
```

output
```
Timed out
```

3.411.8 Giac [F]

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int \frac{(\cos(2x) - 3\tan(x))\cos(x)^3}{(\sin(x)^2 - \sin(2x))\sin(2x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),
x, algorithm="giac")
```

output
```
integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(
5/2)), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int -\frac{\cos(x)^3(\cos(2x) - 3\tan(x))}{\sin(2x)^{5/2}(\sin(2x) - \sin(x)^2)}\,dx$$

input
```
int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2
)),x)
```

output
```
int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2
)), x)
```

3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

3.412.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\cos(x)\sin(x)\sqrt{\sec^4(x)\tan(x)}$$

output
```
2/3*cos(x)*sin(x)*(sec(x)^4*tan(x))^(1/2)
```

3.412.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\cos(x)\sin(x)\sqrt{\sec^4(x)\tan(x)}$$

input
```
Integrate[Sqrt[Sec[x]^4*Tan[x]],x]
```

output
```
(2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

3.412.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4889, 2096, 2001, 1383, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)\sec^4(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\tan(x)\sec(x)^4}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{\tan(x)\left(\tan^2(x)+1\right)}{\sqrt{\tan(x)\left(\tan^2(x)+1\right)^2}}d\tan(x)$$
$$\Big\downarrow \text{ 2096}$$
$$\int \frac{\tan(x)\left(\tan^2(x)+1\right)}{\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}d\tan(x)$$
$$\Big\downarrow \text{ 2001}$$
$$\frac{\sqrt{\tan(x)}\sqrt{\tan^4(x)+2\tan^2(x)+1}\int \frac{\sqrt{\tan(x)}(\tan^2(x)+1)}{\sqrt{\tan^4(x)+2\tan^2(x)+1}}d\tan(x)}{\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$
$$\Big\downarrow \text{ 1383}$$
$$\frac{\sqrt{\tan(x)}(\tan^2(x)+1)\int \sqrt{\tan(x)}d\tan(x)}{\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$
$$\Big\downarrow \text{ 15}$$
$$\frac{2\tan^2(x)\left(\tan^2(x)+1\right)}{3\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

input
```
Int[Sqrt[Sec[x]^4*Tan[x]],x]
```

output
```
(2*Tan[x]^2*(1 + Tan[x]^2))/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])
```

3.412.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 1383
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^(2
*p)   Int[u*(d + e*x^n)^(q + 2*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q}
, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &&  !Int
egerQ[p]
```

rule 2001
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Simp[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p)   Int[x^(m + j*p)*(A + B*x^(k
- j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B
, j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && P
osQ[k - j]
```

rule 2096
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_), x_Symbol] :> Int[(f*x)^m*ExpandToSu
m[z, x]*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p}, x] && BinomialQ[z, x] &
& GeneralizedTrinomialQ[u, x] && EqQ[BinomialDegree[z, x] - GeneralizedTrin
omialDegree[u, x], 0] &&  !(BinomialMatchQ[z, x] && GeneralizedTrinomialMat
chQ[u, x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.412. $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

3.412.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{2\cos(x)\sin(x)\sqrt{(\sec^4(x))\tan(x)}}{3}$	16

input
```
int((sin(x)/cos(x)^5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*cos(x)*sin(x)*(sec(x)^4*tan(x))^(1/2)
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\sqrt{\frac{\sin(x)}{\cos(x)^5}}\cos(x)\sin(x)$$

input
```
integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \text{Timed out}$$

input
```
integrate((sin(x)/cos(x)**5)**(1/2),x)
```

output
```
Timed out
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.32

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\tan{(x)}^{\frac{3}{2}}$$

input
```
integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="maxima")
```

output
```
2/3*tan(x)^(3/2)
```

3.412.8 Giac [F]

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin{(x)}}{\cos{(x)}^5}}\,dx$$

input
```
integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(x)/cos(x)^5), x)
```

3.412.9 Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{\sin{(2x)}\sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$$

input
```
int((sin(x)/cos(x)^5)^(1/2),x)
```

output
```
(sin(2*x)*(sin(x)/cos(x)^5)^(1/2))/3
```

3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

3.413.1 Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \frac{3\arctan\left(\frac{(1-\cot(x))\csc^2(x)\sqrt{\sin^4(x)\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{3\log\left(\cos(x)+\sin(x)-\sqrt{2}\cot(x)\csc(x)\sqrt{\sin^4(x)\tan(x)}\right)}{4\sqrt{2}} - \frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}$$

output
```
3/8*arctan(1/2*(1-cot(x))*csc(x)^2*(sin(x)^4*tan(x))^(1/2)*2^(1/2))*2^(1/2
)+3/8*ln(cos(x)+sin(x)-cot(x)*csc(x)*2^(1/2)*(sin(x)^4*tan(x))^(1/2))*2^(1
/2)-1/2*cot(x)*(sin(x)^4*tan(x))^(1/2)
```

3.413.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = -\frac{1}{8}\csc^3(x)\left(3\arcsin(\cos(x)-\sin(x)) + 3\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + 2\sin(x)\sqrt{\sin(2x)}\right)\sqrt{\sin(2x)}\sqrt{\sin^4(x)\tan(x)}$$

input
```
Integrate[Sqrt[Sin[x]^4*Tan[x]],x]
```

output
```
-1/8*(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + Sqrt[S
in[2*x]]] + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin[x]^4*Tan[x]])
```

3.413.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 4889, 7270, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\sin(x)^4\tan(x)}dx$$

$$\Big\downarrow \ 4889$$

$$\int \frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \ 7270$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\int \frac{\tan^{\frac{5}{2}}(x)}{(\tan^2(x)+1)^2}d\tan(x)}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \ 252$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{4}\int \frac{\sqrt{\tan(x)}}{\tan^2(x)+1}d\tan(x)-\frac{\tan^{\frac{3}{2}}(x)}{2(\tan^2(x)+1)}\right)}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \ 266$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\int \frac{\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{\tan^{\frac{3}{2}}(x)}{2(\tan^2(x)+1)}\right)}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \ 826$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\tan(x)+1}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{1}{2}\int\frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)-\frac{\tan^{\frac{3}{2}}(x)}{2(\tan^2(x)+1)}\right)}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 1476

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}+\frac{1}{2}\int\frac{1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)-\frac{1}{2}\int\frac{1-}{\tan}\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 1082

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(x)-1}d\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(x)-1}d\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan}\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 217

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)-\frac{t}{2(ta}\right.}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 1479

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{}\right.}{\sqrt{2}}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 25

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{ta}\right.}{\sqrt{2}}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 27

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(x)}+1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\downarrow$ 1103

3.413. $\int\sqrt{\sin^4(x)\tan(x)}\,dx$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\mathrm{lo}}{}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

input
```
Int[Sqrt[Sin[x]^4*Tan[x]],x]
```

output
```
(Sqrt[Tan[x]^5/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2)*((3*((-(ArcTan[1 - Sqrt[2]
*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2])/2 + (L
og[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[T
an[x]] + Tan[x]]/(2*Sqrt[2]))/2))/2 - Tan[x]^(3/2)/(2*(1 + Tan[x]^2))))/Ta
n[x]^(5/2)
```

3.413.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.413. $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.413.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(73) = 146$.

Time = 7.83 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.25

method	result
default	$\frac{\left(4\cos(x)\sin(x)\sqrt{2}\sqrt{-\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}+4\sqrt{2}\sqrt{-\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}\,\sin(x)-3\ln\left(-\frac{\cos(x)\cot(x)-2\cot(x)-2\sin(x)\sqrt{-\left(\cot^3(x)\right)+3\csc(x)\left(\cot^2(x)\right)}}{}\right.\right.}{}$

input
```
int((sin(x)^5/cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(4*cos(x)*sin(x)*2^(1/2)*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+4*2^(1/2
)*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)-3*ln(-(cos(x)*cot(x)-2*cot(x)
-2*sin(x)*(-cot(x)^3+3*csc(x)*cot(x)^2-3*cot(x)*csc(x)^2+csc(x)^3-csc(x)+c
ot(x))^(1/2)-2*cos(x)-sin(x)+csc(x)+2)/(-1+cos(x)))+3*ln(-(cos(x)*cot(x)-2
*cot(x)+2*sin(x)*(-cot(x)^3+3*csc(x)*cot(x)^2-3*cot(x)*csc(x)^2+csc(x)^3-c
sc(x)+cot(x))^(1/2)-2*cos(x)-sin(x)+csc(x)+2)/(-1+cos(x)))+6*arctan((-2^(1
/2)*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos(x)))-6*ar
ctan((2^(1/2)*(-cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos
(x))))*(sin(x)^4*tan(x))^(1/2)*cos(x)/(-1+cos(x))/(cos(x)+1)^2/(-cos(x)*si
n(x)/(cos(x)+1)^2)^(1/2)*32^(1/2)
```

3.413.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 628, normalized size of antiderivative = 6.83

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \text{Too large to display}$$

input
```
integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="fricas")
```

output
```
1/64*(-(3*I - 3)*sqrt(2)*log((2*I*cos(x)^4 - 3*I*cos(x)^2 + 2*(cos(x)^3 -
cos(x))*sin(x) + ((I + 1)*sqrt(2)*cos(x)^2 - (I - 1)*sqrt(2)*cos(x)*sin(x)
)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + I)/(cos(x)^2 - 1))*sin
(x) + (3*I - 3)*sqrt(2)*log((2*I*cos(x)^4 - 3*I*cos(x)^2 + 2*(cos(x)^3 - c
os(x))*sin(x) + (-(I + 1)*sqrt(2)*cos(x)^2 + (I - 1)*sqrt(2)*cos(x)*sin(x)
)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + I)/(cos(x)^2 - 1))*sin
(x) + (3*I + 3)*sqrt(2)*log((-2*I*cos(x)^4 + 3*I*cos(x)^2 + 2*(cos(x)^3 -
cos(x))*sin(x) + (-(I - 1)*sqrt(2)*cos(x)^2 + (I + 1)*sqrt(2)*cos(x)*sin(x
))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - I)/(cos(x)^2 - 1))*si
n(x) - (3*I + 3)*sqrt(2)*log((-2*I*cos(x)^4 + 3*I*cos(x)^2 + 2*(cos(x)^3 -
 cos(x))*sin(x) + ((I - 1)*sqrt(2)*cos(x)^2 - (I + 1)*sqrt(2)*cos(x)*sin(x
))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - I)/(cos(x)^2 - 1))*si
n(x) + (3*I + 3)*sqrt(2)*log((cos(x)^2 + ((I + 1)*sqrt(2)*cos(x)^2 - (I -
1)*sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))
 - 1)/(cos(x)^2 - 1))*sin(x) - (3*I - 3)*sqrt(2)*log((cos(x)^2 + (-(I - 1)*
sqrt(2)*cos(x)^2 + (I + 1)*sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x
)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1))*sin(x) + (3*I - 3)*sqrt(2)*lo
g((cos(x)^2 + ((I - 1)*sqrt(2)*cos(x)^2 - (I + 1)*sqrt(2)*cos(x)*sin(x))*s
qrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1))*sin(x)
 - (3*I + 3)*sqrt(2)*log((cos(x)^2 + (-(I + 1)*sqrt(2)*cos(x)^2 + (I - ...
```

3.413.6 Sympy [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin^5(x)}{\cos(x)}}\,dx$$

input
```
integrate((sin(x)**5/cos(x))**(1/2),x)
```

output
```
Integral(sqrt(sin(x)**5/cos(x)), x)
```

3.413.7 Maxima [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

input
```
integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(x)^5/cos(x)), x)
```

3.413.8 Giac [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

input
```
integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(x)^5/cos(x)), x)
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

input
```
int((sin(x)^5/cos(x))^(1/2),x)
```

output
```
int((sin(x)^5/cos(x))^(1/2), x)
```

3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$

3.414.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{5}\cos^3(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)} + \frac{3}{11}\cos(x)\sin^3(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}$$

output
```
3/5*cos(x)^3*sin(x)*(sec(x)^12*tan(x)^2)^(1/3)+3/11*cos(x)*sin(x)^3*(sec(x
)^12*tan(x)^2)^(1/3)
```

3.414.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3\cos(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\left(-3+8(-\tan^2(x))^{5/6}+3\cos(2x)\left(-1+(-\tan^2(x))^{5/6}\right)\right)}{55\left(-\tan^2(x)\right)^{5/6}}$$

input
```
Integrate[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

output
```
(3*Cos[x]*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3)*(-3 + 8*(-Tan[x]^2)^(5/6) + 3*
Cos[2*x]*(-1 + (-Tan[x]^2)^(5/6))))/(55*(-Tan[x]^2)^(5/6))
```

3.414.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 4889, 2058, 34, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\tan^2(x)\sec^{12}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\tan(x)^2\sec(x)^{12}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\int \sqrt[3]{\tan^2(x)}\left(\tan^2(x)+1\right)d\tan(x)}{\sqrt[3]{\tan^2(x)}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\int \tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)d\tan(x)}{\tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\int \left(\tan^{\frac{8}{3}}(x)+\tan^{\frac{2}{3}}(x)\right)d\tan(x)}{\tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\left(\frac{3}{11}\tan^{\frac{11}{3}}(x)+\frac{3}{5}\tan^{\frac{5}{3}}(x)\right)}{\tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)^2}$$

input
```
Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

output
```
((Tan[x]^2*(1 + Tan[x]^2)^6)^(1/3)*((3*Tan[x]^(5/3))/5 + (3*Tan[x]^(11/3))
/11))/(Tan[x]^(2/3)*(1 + Tan[x]^2)^2)
```

3.414.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.414.4 Maple [F]

$$\int \left(\frac{\sin^2(x)}{\cos(x)^{14}}\right)^{\frac{1}{3}} dx$$

input
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

output
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\, dx = \frac{3}{55}\left(6\cos(x)^3 + 5\cos(x)\right)\left(-\frac{\cos(x)^2 - 1}{\cos(x)^{14}}\right)^{\frac{1}{3}} \sin(x)$$

input
```
integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")
```

output
```
3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*sin(x)
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\, dx = \text{Timed out}$$

input
```
integrate((sin(x)**2/cos(x)**14)**(1/3),x)
```

output
```
Timed out
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.28

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{11}\tan(x)^{\frac{11}{3}} + \frac{3}{5}\tan(x)^{\frac{5}{3}}$$

input
```
integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")
```

output
```
3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)
```

3.414.8 Giac [F]

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \int \left(\frac{\sin(x)^2}{\cos(x)^{14}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="giac")
```

output
```
integrate((sin(x)^2/cos(x)^14)^(1/3), x)
```

3.414.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{6\sin(2x)\,(1-\cos(2x))^{1/3}\,(3\cos(2x)+8)}{55\,(\cos(2x)+1)^{7/3}}$$

input
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

output
```
(6*sin(2*x)*(1 - cos(2*x))^(1/3)*(3*cos(2*x) + 8))/(55*(cos(2*x) + 1)^(7/3
))
```

3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$

3.415.1 Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{4\cos^5(x)\sin(x)}{9\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} - \frac{8\cos^3(x)\sin^3(x)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} + \frac{4\cos(x)\sin^5(x)}{7\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

output
```
-4/9*cos(x)^5*sin(x)/(cos(x)^11*sin(x)^13)^(1/4)-8*cos(x)^3*sin(x)^3/(cos(
x)^11*sin(x)^13)^(1/4)+4/7*cos(x)*sin(x)^5/(cos(x)^11*sin(x)^13)^(1/4)
```

3.415.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{4\cos(x)(15+8\cos(2x)-16\cos(4x))\sin(x)}{63\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

input
```
Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

output
```
(-4*Cos[x]*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[x])/(63*(Cos[x]^11*Sin[x]^1
3)^(1/4))
```

3.415.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4889, 7270, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[4]{\sin(x)^{13}\cos(x)^{11}}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{\sqrt[4]{\frac{\tan^{13}(x)}{(\tan^2(x)+1)^{12}}}(\tan^2(x)+1)}d\tan(x)$$

$$\Big\downarrow \text{7270}$$

$$\frac{\tan^{\frac{13}{4}}(x)\int \frac{(\tan^2(x)+1)^2}{\tan^{\frac{13}{4}}(x)}d\tan(x)}{\sqrt[4]{\frac{\tan^{13}(x)}{(\tan^2(x)+1)^{12}}}(\tan^2(x)+1)^3}$$

$$\Big\downarrow \text{244}$$

$$\frac{\tan^{\frac{13}{4}}(x)\int \left(\tan^{\frac{3}{4}}(x)+\frac{2}{\tan^{\frac{5}{4}}(x)}+\frac{1}{\tan^{\frac{13}{4}}(x)}\right)d\tan(x)}{\sqrt[4]{\frac{\tan^{13}(x)}{(\tan^2(x)+1)^{12}}}(\tan^2(x)+1)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^{\frac{13}{4}}(x)\left(\frac{4}{7}\tan^{\frac{7}{4}}(x)-\frac{4}{9\tan^{\frac{9}{4}}(x)}-\frac{8}{\sqrt[4]{\tan(x)}}\right)}{\sqrt[4]{\frac{\tan^{13}(x)}{(\tan^2(x)+1)^{12}}}(\tan^2(x)+1)^3}$$

input
```
Int[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

3.415. $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}dx$

output
```
(Tan[x]^(13/4)*(-4/(9*Tan[x]^(9/4)) - 8/Tan[x]^(1/4) + (4*Tan[x]^(7/4))/7)
)/((Tan[x]^13/(1 + Tan[x]^2)^12)^(1/4)*(1 + Tan[x]^2)^3)
```

3.415.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.415.4 Maple [F]

$$\int \frac{1}{\left(\left(\cos^{11}\left(x\right)\right)\left(\sin^{13}\left(x\right)\right)\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

output
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$$
$$= \frac{4\left(128\cos(x)^4 - 144\cos(x)^2 + 9\right)\left(\left(\cos(x)^{23} - 6\cos(x)^{21} + 15\cos(x)^{19} - 20\cos(x)^{17} + 15\cos(x)^{15} - \right.\right.}{63\left(\cos(x)^{22} - 6\cos(x)^{20} + 15\cos(x)^{18} - 20\cos(x)^{16} + 15\cos(x)^{14} - 6\cos(x)^{12}\right.}$$

input
```
integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="fricas")
```

output
```
4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21 + 15*cos(
x)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(x)^11)*sin(x))^(3/
4)/(cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20*cos(x)^16 + 15*cos(x)^14 -
 6*cos(x)^12 + cos(x)^10)
```

3.415.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

output
```
Timed out
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \frac{4}{23}\tan(x)^{\frac{23}{4}} + \frac{8}{15}\tan(x)^{\frac{15}{4}} + \frac{4}{7}\tan(x)^{\frac{7}{4}} - \frac{4\left(35\tan(x)^7 + 161\tan(x)^5 + 345\tan(x)^3 - 805\tan(x)\right)}{805\tan(x)^{\frac{5}{4}}} + \frac{4\left(21\tan(x)^7 + 135\tan(x)^5 - 945\tan(x)^3 - 35\tan(x)\right)}{315\tan(x)^{\frac{13}{4}}}$$

input
```
integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="maxima")
```

output
```
4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/805*(35*tan
(x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)^(5/4) + 4/315*(21
*tan(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*tan(x))/tan(x)^(13/4)
```

3.415.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \int \frac{1}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="giac")
```

output
```
integrate((cos(x)^11*sin(x)^13)^(-1/4), x)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{2^{3/4}\left(-32\cos(2x)^2+8\cos(2x)+31\right)\left(924\sin(2x)-132\sin(4x)-660\sin(6x)+165\sin(8x)+3\right.}{2016\,(\mathrm{co}}$$

input
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

output
```
-(2^(3/4)*(8*cos(2*x) - 32*cos(2*x)^2 + 31)*(924*sin(2*x) - 132*sin(4*x) -
 660*sin(6*x) + 165*sin(8*x) + 330*sin(10*x) - 110*sin(12*x) - 110*sin(14*
x) + 44*sin(16*x) + 22*sin(18*x) - 10*sin(20*x) - 2*sin(22*x) + sin(24*x))
^(3/4))/(2016*(cos(2*x) - 1)^6*(cos(2*x) + 1)^5)
```

3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$

3.416.1 Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = -\sqrt{2}\log\left(\cos(x)+\sin(x)-\sqrt{2}\sec(x)\sqrt{\cos^3(x)\sin(x)}\right) - \frac{\arcsin(\cos(x)-\sin(x))\cos(x)\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}} - \frac{\operatorname{arctanh}(\sin(x))\cos(x)\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}} - \frac{\sin(2x)}{\sqrt{\cos^3(x)\sin(x)}}$$

output
```
-ln(cos(x)+sin(x)-sec(x)*2^(1/2)*(cos(x)^3*sin(x))^(1/2))*2^(1/2)-sin(2*x)
/(cos(x)^3*sin(x))^(1/2)-arcsin(cos(x)-sin(x))*cos(x)*sin(2*x)^(1/2)/(cos(
x)^3*sin(x))^(1/2)-arctanh(sin(x))*cos(x)*sin(2*x)^(1/2)/(cos(x)^3*sin(x))
^(1/2)
```

3.416.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \frac{-4\cos^3(x)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\cos^2(x)\right)\sin(x)-3\cos(x)\sqrt[4]{\sin^2(x)}\left(2\sin(x)+\operatorname{arctanh}(\sin(x))\sqrt{}\right.}{3\sqrt{\cos^3(x)\sin(x)}\sqrt[4]{\sin^2(x)}}$$

input
```
Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

output
```
(-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2]*Sin[x] - 3*Cos[x]*
(Sin[x]^2)^(1/4)*(2*Sin[x] + ArcTanh[Sin[x]]*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[
x]^3*Sin[x]]*(Sin[x]^2)^(1/4))
```

3.416.3 Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.59, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4889, 7270, 2035, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\sin(x)\cos^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\sin(x)\cos(x)^3}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}}\left(-\tan^2(x)-\frac{\sqrt{2}\tan(x)}{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}}+1\right)\cot(x)d\tan(x)$$

$$\Big\downarrow \text{7270}$$

$$\frac{\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\int \frac{-\tan^2(x)-\frac{\sqrt{2}\tan(x)}{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}}+1}{\sqrt{\tan(x)}(\tan^2(x)+1)}d\tan(x)}{\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{2035}$$

$$\frac{2\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\int \frac{-\tan^2(x)-\frac{\sqrt{2}\tan(x)}{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}}+1}{\tan^2(x)+1}d\sqrt{\tan(x)}}{\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{7276}$$

$$\frac{2\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\int\left(-\frac{\tan^2(x)}{\tan^2(x)+1}-\sqrt{2}\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}+\frac{1}{\tan^2(x)+1}\right)d\sqrt{\tan(x)}}{\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(-\frac{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}\sqrt{\tan^2(x)+1}\cot(x)\text{arcsinh}(\tan(x))}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}\right.}{\sqrt{2}}\right.}{\sqrt{\tan(x)}}$$

input
```
Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

output
```
(2*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2)*(-(ArcTan[1 - Sqrt[2]*Sqrt
[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] - Log[1 - Sq
rt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]] +
Tan[x]]/(2*Sqrt[2]) - Sqrt[Tan[x]] - (ArcSinh[Tan[x]]*Cot[x]*Sqrt[Tan[x]/(
1 + Tan[x]^2)]*Sqrt[1 + Tan[x]^2])/Sqrt[2]))/Sqrt[Tan[x]]
```

3.416.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.416. $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.416.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(92) = 184.

Time = 3.68 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.29

method	result
default	$-\frac{2\sin(x)\cos(x)}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{2\sqrt{2}\,\cos(x)\sqrt{\cos(x)\sin(x)}\,\operatorname{arctanh}(-\csc(x)+\cot(x))}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}\left(\ln\left(2\cot(x)\sqrt{2}\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}+2\right.\right.}{}$
parts	$-\frac{2\sin(x)\cos(x)}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{2\sqrt{2}\,\cos(x)\sqrt{\cos(x)\sin(x)}\,\operatorname{arctanh}(-\csc(x)+\cot(x))}{\sqrt{(\cos^3(x))\sin(x)}}+\frac{\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}\left(\ln\left(2\cot(x)\sqrt{2}\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^2}}+2\right.\right.}{}$

input
```
int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2*sin(x)*cos(x)/(cos(x)^3*sin(x))^(1/2)+2*2^(1/2)*cos(x)*(cos(x)*sin(x))^
(1/2)*arctanh(-csc(x)+cot(x))/(cos(x)^3*sin(x))^(1/2)+1/2*(cos(x)*sin(x)/(
cos(x)+1)^2)^(1/2)*(ln(2*cot(x)*2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)
+2*csc(x)*2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+2+2*cot(x))+2*arctan(
(2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)-cos(x)+1)/(-1+cos(x)))-
ln(-2*cot(x)*2^(1/2)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)-2*csc(x)*2^(1/2)*(
cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+2+2*cot(x))+2*arctan((2^(1/2)*(cos(x)*si
n(x)/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos(x))))/(cos(x)^3*sin(x))^
(1/2)*(cos(x)^2+cos(x))*2^(1/2)
```

3.416.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.44

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$$
$$= \frac{-(i-1)\,\sqrt{2}\cos(x)^2\log\left(\frac{2\cos(x)^3+2i\cos(x)^2\sin(x)+\sqrt{\cos(x)^3\sin(x)}\left((i+1)\sqrt{2}\cos(x)+(i-1)\sqrt{2}\sin(x)\right)-\cos(x)}{\cos(x)}\right)+(i-1)}{}$$

input
```
integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="
fricas")
```

output
```
1/8*(-(I - 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 + 2*I*cos(x)^2*sin(x) + sqr
t(cos(x)^3*sin(x))*((I + 1)*sqrt(2)*cos(x) + (I - 1)*sqrt(2)*sin(x)) - cos
(x))/cos(x)) + (I - 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 + 2*I*cos(x)^2*sin
(x) + sqrt(cos(x)^3*sin(x))*(-(I + 1)*sqrt(2)*cos(x) - (I - 1)*sqrt(2)*sin
(x)) - cos(x))/cos(x)) + (I + 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^3 - 2*I*co
s(x)^2*sin(x) + sqrt(cos(x)^3*sin(x))*(-(I - 1)*sqrt(2)*cos(x) - (I + 1)*s
qrt(2)*sin(x)) - cos(x))/cos(x)) - (I + 1)*sqrt(2)*cos(x)^2*log((2*cos(x)^
3 - 2*I*cos(x)^2*sin(x) + sqrt(cos(x)^3*sin(x))*((I - 1)*sqrt(2)*cos(x) +
(I + 1)*sqrt(2)*sin(x)) - cos(x))/cos(x)) + (I - 1)*sqrt(2)*cos(x)^2*log((
sqrt(cos(x)^3*sin(x))*((I + 1)*sqrt(2)*cos(x) - (I - 1)*sqrt(2)*sin(x)) -
cos(x))/cos(x)) - (I + 1)*sqrt(2)*cos(x)^2*log((sqrt(cos(x)^3*sin(x))*(-(I
 - 1)*sqrt(2)*cos(x) + (I + 1)*sqrt(2)*sin(x)) - cos(x))/cos(x)) + (I + 1)
*sqrt(2)*cos(x)^2*log((sqrt(cos(x)^3*sin(x))*((I - 1)*sqrt(2)*cos(x) - (I
+ 1)*sqrt(2)*sin(x)) - cos(x))/cos(x)) - (I - 1)*sqrt(2)*cos(x)^2*log((sqr
t(cos(x)^3*sin(x))*(-(I + 1)*sqrt(2)*cos(x) + (I - 1)*sqrt(2)*sin(x)) - co
s(x))/cos(x)) + 4*sqrt(2)*cos(x)^2*log(-(cos(x)^4 - 2*cos(x)^2 + 2*sqrt(co
s(x)^3*sin(x))*sqrt(cos(x)*sin(x)))/cos(x)^4) - 16*sqrt(cos(x)^3*sin(x)))/
cos(x)^2
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \text{Timed out}$$

input
```
integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)
```

output
```
Timed out
```

3.416.7 Maxima [F]

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int -\frac{\sqrt{\sin(2x)} - \cos(2x)}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

input
```
integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="
maxima")
```

output
```
1/2*sqrt(2)*integrate(2*((((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x)
+ 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(si
n(x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (co
s(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x),
 cos(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((cos(1/2*arctan
2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x),
-cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(
1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(
x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x),
cos(2*x) + 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4)), x) - 1/2*sqrt(2)*integrate(-2*(((cos(1/2*arctan2(sin(x), -cos(x) + 1
))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/
2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -
cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arc
tan2(sin(x), cos(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) - (((c
os(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arct
an2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) + (cos(1/2
*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(s
in(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arc...
```

3.416.8 Giac [F]

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx=\int -\frac{\sqrt{\sin(2x)}-\cos(2x)}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

input
```
integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="
giac")
```

output
```
integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)), x)
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx=\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

input
```
int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
```

output
```
int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2), x)
```

3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

3.417.1 Optimal result

Integrand size = 41, antiderivative size = 364

$$\begin{aligned}&\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx\\
&= -2\sqrt{2}\coth^{-1}\left(\frac{\cos(x)(\cos(x)+\sin(x))}{\sqrt{2}\sqrt{\cos^3(x)\sin(x)}}\right)+\sqrt[4]{2}\coth^{-1}\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)+\sin(x)\right)}{2^{3/4}\sqrt{\cos^3(x)\sin(x)}}\right)\\
&\quad-\sqrt[4]{2}\coth^{-1}\left(\frac{\sqrt{2}+\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)-2\sqrt{2}\arctan\left(\frac{\cos(x)(\cos(x)-\sin(x))}{\sqrt{2}\sqrt{\cos^3(x)\sin(x)}}\right)\\
&\quad+\sqrt[4]{2}\arctan\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)-\sin(x)\right)}{2^{3/4}\sqrt{\cos^3(x)\sin(x)}}\right)-\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)+4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(}\end{aligned}$$

output
```
2^(1/4)*arccoth(1/2*cos(x)*(sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x
))^(1/2))-2^(1/4)*arccoth(1/2*(2^(1/2)+tan(x))*2^(1/4)/tan(x)^(1/2))+2^(1/
4)*arctan(1/2*cos(x)*(-sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x))^(1
/2))-2^(1/4)*arctan(1/2*(2^(1/2)-tan(x))*2^(1/4)/tan(x)^(1/2))-2*arccoth(1
/2*cos(x)*(cos(x)+sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)-2*arcta
n(1/2*cos(x)*(cos(x)-sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)+4*cs
c(x)*sec(x)*(cos(x)^3*sin(x))^(1/2)+1/4*csc(x)^2*ln(1+cos(x)^2)*sec(x)^2*(
cos(x)^3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+1/2*csc(x)^2*ln(sin(x))*sec
(x)^2*(cos(x)^3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+4/tan(x)^(1/2)-1/4*c
sc(x)^2*ln(1+cos(x)^2)*(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)+1/2*csc(x)^2*l
n(sin(x))*(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)
```

3.417.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 19.66 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)}$$
$$+ \frac{\cot(x)\left(-2\log\left(\sec^2(x)\right) + 2\log(\tan(x)) + \log\left(2+\tan^2(x)\right)\right)\sqrt{\cos(x)\sin^3(x)}}{4\sqrt{\cos^3(x)\sin(x)}}$$
$$+ \frac{4}{\sqrt{\tan(x)}}$$
$$+ \frac{\left(2\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right) - 2\arctan\left(1+\sqrt[4]{2}\sqrt{\tan(x)}\right) - 4\sqrt[4]{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right) + 4\sqrt[4]{2}\,\mathrm{a}\right.}{}$$
$$+ \frac{1}{4}\csc^2(x)\left(2\log(\tan(x)) - \log\left(2+\tan^2(x)\right)\right)\sqrt{\cos(x)\sin^3(x)}\sqrt{\tan(x)}$$
$$+ \frac{4\sqrt{2}\cos^2(x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{2\sin^2(x)}{3+\cos(2x)}\right)\tan^{\frac{3}{2}}(x)}{3(3+\cos(2x))^{3/4}}$$

input
```
Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + S
qrt[Tan[x]]),x]
```

output
```
4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] + (Cot[x]*(-2*Log[Sec[x]^2] + 2*Log[
Tan[x]] + Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3])/(4*Sqrt[Cos[x]^3*Sin[x
]]) + 4/Sqrt[Tan[x]] + ((2*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]] - 2*ArcTan[1 +
 2^(1/4)*Sqrt[Tan[x]]] - 4*2^(1/4)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]] + 4*2^
(1/4)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]] + 2*2^(1/4)*Log[1 - Sqrt[2]*Sqrt[Ta
n[x]] + Tan[x]] - 2*2^(1/4)*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]] - Log[2
 - 2*2^(1/4)*Sqrt[Tan[x]] + Sqrt[2]*Tan[x]] + Log[2 + 2*2^(1/4)*Sqrt[Tan[x
]] + Sqrt[2]*Tan[x]])*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]
]) + (Csc[x]^2*(2*Log[Tan[x]] - Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3]*S
qrt[Tan[x]])/4 + (4*Sqrt[2]*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7
/4, (2*Sin[x]^2)/(3 + Cos[2*x])]*Tan[x]^(3/2))/(3*(3 + Cos[2*x])^(3/4))
```

3.417.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 781 vs. 2(364) = 728.

Time = 3.30 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3042, 4889, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin^3(x)\cos(x)}-2\sin(2x)}{\sqrt{\tan(x)}-\sqrt{\sin(x)\cos^3(x)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\sin(x)^3\cos(x)}-2\sin(2x)}{\sqrt{\tan(x)}-\sqrt{\sin(x)\cos(x)^3}}dx$$

$$\Big\downarrow \ 4889$$

$$\int \frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}-\frac{4\tan(x)}{\tan^2(x)+1}}{(\tan^2(x)+1)\left(\sqrt{\tan(x)}-\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\right)}d\tan(x)$$

$$\Big\downarrow \ 7276$$

$$\int \left(\frac{4\tan(x)}{(\tan^2(x)+1)^2\left(\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}-\sqrt{\tan(x)}\right)}-\frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}}{(\tan^2(x)+1)\left(\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}-\sqrt{\tan(x)}\right)}\right)d\tan(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[4]{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}} - \frac{\sqrt[4]{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\arctan\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}} - \frac{2\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}} + \frac{2\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}} - \sqrt[4]{2}\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\arctan\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right) + \frac{4}{\sqrt{\tan(x)}} + \frac{\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}} - \frac{\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}} - \frac{\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}\sqrt{\tan(x)}} + \frac{\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\log\left(\tan(x)+2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}\sqrt{\tan(x)}} + \frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\log(\tan(x))}{2\tan^{\frac{3}{2}}(x)} - \frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\log\left(\tan^2(x)+2\right)}{4\tan^{\frac{3}{2}}(x)} + \frac{\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}} - \frac{\log\left(\tan(x)+2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}} + 4\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\cot(x) + \sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)^2\cot^2(x)\log\left(\sqrt{\tan(x)}\right) - \frac{1}{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)^2\cot^2(x)\log\left(\tan^2(x)+1\right) + \frac{1}{4}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)^2\cot^2(x)\log\left(\tan^2(x)+2\right)$$

3.417. $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

input
```
Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Ta
n[x]]),x]
```

output
```
-(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 + 2^(1/4)*S
qrt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) - Log[
Sqrt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) + 4/Sqrt[Tan[x]] + 4*Cot[
x]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2) + (2^(1/4)*ArcTan[1 - 2^(1
/4)*Sqrt[Tan[x]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/Sqrt[Tan[x
]] - (2^(1/4)*ArcTan[1 + 2^(1/4)*Sqrt[Tan[x]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^
2]*(1 + Tan[x]^2))/Sqrt[Tan[x]] - (2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x
]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/Sqrt[Tan[x]] + (2*Sqrt[2
]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[
x]^2))/Sqrt[Tan[x]] + (Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sqrt
[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/Sqrt[Tan[x]] - (Sqrt[2]*Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2
))/Sqrt[Tan[x]] - (Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sqrt[Tan[x
]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/(2^(3/4)*Sqrt[Tan[x]]) + (Log[Sqrt[2]
+ 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]
^2))/(2^(3/4)*Sqrt[Tan[x]]) + (Log[Tan[x]]*Sqrt[Tan[x]^3/(1 + Tan[x]^2)^2]
*(1 + Tan[x]^2))/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sqrt[Tan[x]^3/(1 +
Tan[x]^2)^2]*(1 + Tan[x]^2))/(4*Tan[x]^(3/2)) + Cot[x]^2*Log[Sqrt[Tan[x]]]
*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*Sqrt[Tan[x]^3/(1 + Tan[x]^2)^2]*(1 + Tan[x]
^2)^2 - (Cot[x]^2*Log[1 + Tan[x]^2]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*Sqrt[...
```

3.417.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.417. $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.417.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 57.05 (sec) , antiderivative size = 133928, normalized size of antiderivative = 367.93

method	result	size
default	Expression too large to display	133928
parts	Expression too large to display	149826

input
```
int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)
^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.417.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+
tan(x)^(1/2)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   not
invertible
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = \text{Timed out}$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1
/2)+tan(x)**(1/2)),x)
```

output
```
Timed out
```

3.417.7 Maxima [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = \int -\frac{\sqrt{\cos(x)\sin(x)^3}-2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)}-\sqrt{\tan(x)}}\,dx$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+
tan(x)^(1/2)),x, algorithm="maxima")
```

output
```
-2*integrate(-1/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sq
rt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) +
2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt
(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sq
rt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*c
os(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*ar
ctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sq
rt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos
(4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*
sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x
) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arc
tan2(sin(x), cos(x) + 1)) + (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqr
t(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(
4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*s
in(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x)
 - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(
2*x) + sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*c
os(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(
2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*...
```

3.417.8 Giac [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \int -\frac{\sqrt{\cos(x)\sin(x)^3} - 2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)} - \sqrt{\tan(x)}}\,dx$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+
tan(x)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = -\int \frac{2\,\sin\left(2\,x\right)-\sqrt{\cos\left(x\right)\,\sin\left(x\right)^3}}{\sqrt{\tan\left(x\right)}-\sqrt{\cos\left(x\right)^3\,\sin\left(x\right)}}\,dx$$

input
```
int(-(2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(
x))^(1/2)),x)
```

output
```
-int((2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(
x))^(1/2)), x)
```

3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx$

3.418.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\begin{aligned}\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx &= -\frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}}\\ &-\frac{9}{4}\sec^8(x)\left(\cos^5(x)\sin(x)\right)^{4/3}+\frac{3}{2}\sqrt[3]{\cos^5(x)\sin(x)}\sqrt[3]{\sec^6(x)\tan(x)}\\ &+\frac{3}{4}\sqrt[3]{\cos^5(x)\sin(x)}\tan^2(x)\sqrt[3]{\sec^6(x)\tan(x)}+\frac{3}{14}\sqrt[3]{\cos^5(x)\sin(x)}\tan^4(x)\sqrt[3]{\sec^6(x)\tan(x)}\end{aligned}$$

output
```
-9/10*sin(x)^4/(cos(x)^5*sin(x))^(2/3)-9/4*sec(x)^8*(cos(x)^5*sin(x))^(4/3
)+3/2*(cos(x)^5*sin(x))^(1/3)*(sec(x)^6*tan(x))^(1/3)+3/4*(cos(x)^5*sin(x)
)^(1/3)*tan(x)^2*(sec(x)^6*tan(x))^(1/3)+3/14*(cos(x)^5*sin(x))^(1/3)*tan(
x)^4*(sec(x)^6*tan(x))^(1/3)
```

3.418.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx = -\frac{3\sin(x)\left(924\sin(x)+252\sin(3x)-5(158\cos(x)+57\cos(3x)+9\cos(5x))\sqrt[3]{\sec^6(x)\tan(x)}\right)}{2240\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

input
```
Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

output
```
(-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*x] + 9*Co
s[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x])^(2/3))
```

3.418.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4889, 25, 7270, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\tan(x)\sec^6(x)}-3\tan(x)}{\left(\sin(x)\cos^5(x)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\tan(x)\sec(x)^6}-3\tan(x)}{\left(\sin(x)\cos(x)^5\right)^{2/3}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int -\frac{3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}}{\left(\frac{\tan(x)}{\left(\tan^2(x)+1\right)^3}\right)^{2/3}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}}{\left(\frac{\tan(x)}{\left(\tan^2(x)+1\right)^3}\right)^{2/3}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{\tan^{\frac{2}{3}}(x)\int \frac{\left(\tan^2(x)+1\right)\left(3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\right)}{\tan^{\frac{2}{3}}(x)}d\tan(x)}{\left(\frac{\tan(x)}{\left(\tan^2(x)+1\right)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2035}$$

$$-\frac{3\tan^{\frac{2}{3}}(x)\int\left(\tan^2(x)+1\right)\left(3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\right)d\sqrt[3]{\tan(x)}}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{3\tan^{\frac{2}{3}}(x)\int\left(\left(3\tan(x)-\sqrt[3]{\left(\tan^{\frac{7}{3}}(x)+\sqrt[3]{\tan(x)}\right)^3}\right)\tan^2(x)+3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\right)d\sqrt[3]{\tan(}}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\tan^{\frac{2}{3}}(x)\left(\frac{3}{10}\tan^{\frac{10}{3}}(x)+\frac{3}{4}\tan^{\frac{4}{3}}(x)-\frac{\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\sqrt[3]{\tan(x)}}{2(\tan^2(x)+1)}-\frac{\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\tan^{\frac{13}{3}}(x)}{14(\tan^2(x)+1)}-\right.}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

input
```
Int[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

output
```
(-3*Tan[x]^(2/3)*((3*Tan[x]^(4/3))/4 + (3*Tan[x]^(10/3))/10 - (Tan[x]^(1/3
)*(Tan[x]*(1 + Tan[x]^2)^3)^(1/3))/(2*(1 + Tan[x]^2)) - (Tan[x]^(7/3)*(Tan
[x]*(1 + Tan[x]^2)^3)^(1/3))/(4*(1 + Tan[x]^2)) - (Tan[x]^(13/3)*(Tan[x]*(
1 + Tan[x]^2)^3)^(1/3))/(14*(1 + Tan[x]^2))))/((Tan[x]/(1 + Tan[x]^2)^3)^(
2/3)*(1 + Tan[x]^2)^2)
```

3.418.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

3.418. $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.418.4 Maple [F]

$$\int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}} - 3\tan(x)}{\left((\cos^5(x))\sin(x)\right)^{\frac{2}{3}}}dx$$

input
```
int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

output
```
int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \frac{-3\tan(x) + \sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3\left(\cos(x)^5\sin(x)\right)^{\frac{1}{3}}\left(21\left(3\cos(x)^2+2\right)\sin(x) - 5\left(9\cos(x)^5+3\cos(x)^3+2\cos(x)\right)\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}\right)}{140\cos(x)^5}$$

input
```
integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, al
gorithm="fricas")
```

output
```
-3/140*(cos(x)^5*sin(x))^(1/3)*(21*(3*cos(x)^2 + 2)*sin(x) - 5*(9*cos(x)^5
 + 3*cos(x)^3 + 2*cos(x))*(sin(x)/cos(x)^7)^(1/3))/cos(x)^5
```

3.418.6 Sympy [F(-1)]

Timed out.

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x
)
```

output
```
Timed out
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3}{20}\tan\left(x\right)^{\frac{20}{3}} - \frac{3}{7}\tan\left(x\right)^{\frac{14}{3}} - \frac{9}{10}\tan\left(x\right)^{\frac{10}{3}} - \frac{3}{8}\tan\left(x\right)^{\frac{8}{3}} - \frac{9}{4}\tan\left(x\right)^{\frac{4}{3}} + \frac{3\left(14\tan\left(x\right)^{7}+60\tan\left(x\right)^{5}+105\tan\left(x\right)^{3}+140\tan\left(x\right)\right)}{280\tan\left(x\right)^{\frac{1}{3}}}$$

input
```
integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, al
gorithm="maxima")
```

output
```
-3/20*tan(x)^(20/3) - 3/7*tan(x)^(14/3) - 9/10*tan(x)^(10/3) - 3/8*tan(x)^
(8/3) - 9/4*tan(x)^(4/3) + 3/280*(14*tan(x)^7 + 60*tan(x)^5 + 105*tan(x)^3
 + 140*tan(x))/tan(x)^(1/3)
```

3.418.8 Giac [F]

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx=\int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}-3\tan(x)}{(\cos(x)^5\sin(x))^{\frac{2}{3}}}\,dx$$

input
```
integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, al
gorithm="giac")
```

output
```
integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx=\int -\frac{3\tan(x)-\left(\frac{\sin(x)}{\cos(x)^7}\right)^{1/3}}{(\cos(x)^5\sin(x))^{2/3}}\,dx$$

input
```
int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3),x)
```

output
```
int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3), x)
```

3.419 $\int (1 + 2\cos^2(x))^{5/2} \sin(x)\, dx$

3.419.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int (1 + 2\cos^2(x))^{5/2} \sin(x)\, dx = -\frac{5\operatorname{arcsinh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}} - \frac{5}{16}\cos(x)\sqrt{1 + 2\cos^2(x)} - \frac{5}{24}\cos(x)\left(1 + 2\cos^2(x)\right)^{3/2} - \frac{1}{6}\cos(x)\left(1 + 2\cos^2(x)\right)^{5/2}$$

output
```
-5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arc
sinh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (1 + 2\cos^2(x))^{5/2} \sin(x)\, dx = \frac{1}{96}\left(-2\sqrt{2 + \cos(2x)}(92\cos(x) + 23\cos(3x) + 2\cos(5x)) - 15\sqrt{2}\log\left(\sqrt{2}\cos(x) + \sqrt{2 + \cos(2x)}\right)\right)$$

input
```
Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

output
```
(-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 15*Sqrt[2]
*Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

3.419.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 25, 3669, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\left(2\cos^2(x)+1\right)^{5/2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(2\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{5/2}\left(-\cos\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \cos\left(x+\frac{\pi}{2}\right)\left(2\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{5/2}dx$$

$$\Big\downarrow \ 3669$$

$$-\int \left(2\cos^2(x)+1\right)^{5/2}d\cos(x)$$

$$\Big\downarrow \ 211$$

$$-\frac{5}{6}\int \left(2\cos^2(x)+1\right)^{3/2}d\cos(x)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \ 211$$

$$-\frac{5}{6}\left(\frac{3}{4}\int \sqrt{2\cos^2(x)+1}d\cos(x)+\frac{1}{4}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}\right)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \ 211$$

$$-\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{2\cos^2(x)+1}}d\cos(x)+\frac{1}{2}\sqrt{2\cos^2(x)+1}\cos(x)\right)+\frac{1}{4}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}\right)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \ 222$$

$$-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\operatorname{arcsinh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}+\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)+1}\right)+\frac{1}{4}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}\right)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

input
```
Int[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

output
```
-1/6*(Cos[x]*(1 + 2*Cos[x]^2)^(5/2)) - (5*((Cos[x]*(1 + 2*Cos[x]^2)^(3/2))
/4 + (3*(ArcSinh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) + (Cos[x]*Sqrt[1 + 2*Cos[x]^2
])/2))/4))/6
```

3.419.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.419.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{5\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}{6}-\frac{5\,\operatorname{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\left(\cos^2(x)\right)}}{16}$
default	$-\frac{5\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\left(\cos^2(x)\right)\right)^{\frac{5}{2}}}{6}-\frac{5\,\operatorname{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\left(\cos^2(x)\right)}}{16}$

input
```
int((1+2*cos(x)^2)^(5/2)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arc
sinh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{48}\left(32\cos(x)^5+52\cos(x)^3+33\cos(x)\right)\sqrt{2\cos(x)^2+1} + \frac{5}{256}\sqrt{2}\log\Big(2048\cos(x)^8+2048\cos(x)^6+640\cos(x)^4+64\cos(x)^2 - 8\left(128\sqrt{2}\cos(x)^7+96\sqrt{2}\cos(x)^5+20\sqrt{2}\cos(x)^3+\sqrt{2}\cos(x)\right)\sqrt{2\cos(x)^2+1} + 1\Big)$$

input
```
integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="fricas")
```

output
```
-1/48*(32*cos(x)^5 + 52*cos(x)^3 + 33*cos(x))*sqrt(2*cos(x)^2 + 1) + 5/256
*sqrt(2)*log(2048*cos(x)^8 + 2048*cos(x)^6 + 640*cos(x)^4 + 64*cos(x)^2 -
8*(128*sqrt(2)*cos(x)^7 + 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 + sqrt
(2)*cos(x))*sqrt(2*cos(x)^2 + 1) + 1)
```

3.419.6 Sympy [F(-1)]

Timed out.

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = \text{Timed out}$$

input
```
integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
```

output
```
Timed out
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{6}\left(2\cos(x)^2+1\right)^{\frac{5}{2}}\cos(x) - \frac{5}{24}\left(2\cos(x)^2+1\right)^{\frac{3}{2}}\cos(x) - \frac{5}{32}\sqrt{2}\,\mathrm{arsinh}\left(\sqrt{2}\cos(x)\right) - \frac{5}{16}\sqrt{2\cos(x)^2+1}\cos(x)$$

input
```
integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="maxima")
```

output
```
-1/6*(2*cos(x)^2 + 1)^(5/2)*cos(x) - 5/24*(2*cos(x)^2 + 1)^(3/2)*cos(x) -
5/32*sqrt(2)*arcsinh(sqrt(2)*cos(x)) - 5/16*sqrt(2*cos(x)^2 + 1)*cos(x)
```

3.419.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{48}\left(4\left(8\cos(x)^2+13\right)\cos(x)^2+33\right)\sqrt{2\cos(x)^2+1}\cos(x) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2+1}\right)$$

input
```
integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="giac")
```

output
```
-1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*cos(x) + 5/
32*sqrt(2)*log(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))
```

3.419.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{5\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\cos(x)\right)}{32} - \frac{\sqrt{2}\sqrt{\cos(x)^2+\frac{1}{2}}\left(\frac{4\cos(x)^5}{3}+\frac{13\cos(x)^3}{6}+\frac{11\cos(x)}{8}\right)}{2}$$

input
```
int(sin(x)*(2*cos(x)^2 + 1)^(5/2),x)
```

output
```
- (5*2^(1/2)*asinh(2^(1/2)*cos(x)))/32 - (2^(1/2)*(cos(x)^2 + 1/2)^(1/2)*(
(11*cos(x))/8 + (13*cos(x)^3)/6 + (4*cos(x)^5)/3))/2
```

3.419. $\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx$

3.420 $\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} dx$

3.420.1 Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} dx = \frac{625}{32}\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right) + \frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)} + \frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2} + \frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}$$

output
```
625/32*arcsin(2/5*sin(x)*5^(1/2))+25/24*sin(x)*(5-4*sin(x)^2)^(3/2)+1/6*si
n(x)*(5-4*sin(x)^2)^(5/2)+125/16*sin(x)*(5-4*sin(x)^2)^(1/2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} dx = \frac{1}{96}\left(1875\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right) + 2\sqrt{3+2\cos(2x)}(515\sin(x)+90\sin(3x)+8\sin(5x))\right)$$

input
```
Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

output
```
(1875*ArcSin[(2*Sin[x])/Sqrt[5]] + 2*Sqrt[3 + 2*Cos[2*x]]*(515*Sin[x] + 90
*Sin[3*x] + 8*Sin[5*x]))/96
```

3.420.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4856, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \left(\sin^2(x)+5\cos^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x) \left(\sin(x)^2+5\cos(x)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4856}$$

$$\int \left(5-4\sin^2(x)\right)^{5/2} d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{25}{6} \int \left(5-4\sin^2(x)\right)^{3/2} d\sin(x)+\frac{1}{6}\sin(x) \left(5-4\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{25}{6} \left(\frac{15}{4} \int \sqrt{5-4\sin^2(x)}d\sin(x)+\frac{1}{4}\sin(x) \left(5-4\sin^2(x)\right)^{3/2}\right)+\frac{1}{6}\sin(x) \left(5-4\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{25}{6} \left(\frac{15}{4} \left(\frac{5}{2} \int \frac{1}{\sqrt{5-4\sin^2(x)}}d\sin(x)+\frac{1}{2}\sqrt{5-4\sin^2(x)}\sin(x)\right)+\frac{1}{4}\sin(x) \left(5-4\sin^2(x)\right)^{3/2}\right)+ \frac{1}{6}\sin(x) \left(5-4\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{25}{6} \left(\frac{15}{4} \left(\frac{5}{4} \arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{1}{2}\sin(x)\sqrt{5-4\sin^2(x)}\right)+\frac{1}{4}\sin(x) \left(5-4\sin^2(x)\right)^{3/2}\right)+ \frac{1}{6}\sin(x) \left(5-4\sin^2(x)\right)^{5/2}$$

input
```
Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

output
```
(Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6 + (25*((Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/4
 + (15*((5*ArcSin[(2*Sin[x])/Sqrt[5]])/4 + (Sin[x]*Sqrt[5 - 4*Sin[x]^2])/2
))/4))/6
```

3.420.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.420.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	si
derivativedivides	$\frac{625 \arcsin\left(\frac{2 \sin(x) \sqrt{5}}{5}\right)}{32}+\frac{25 \sin(x) \left(5-4 \left(\sin^{2}(x)\right)\right)^{\frac{3}{2}}}{24}+\frac{\sin(x) \left(5-4 \left(\sin^{2}(x)\right)\right)^{\frac{5}{2}}}{6}+\frac{125 \sin(x) \sqrt{5-4 \left(\sin^{2}(x)\right)}}{16}$	54
default	$\frac{625 \arcsin\left(\frac{2 \sin(x) \sqrt{5}}{5}\right)}{32}+\frac{25 \sin(x) \left(5-4 \left(\sin^{2}(x)\right)\right)^{\frac{3}{2}}}{24}+\frac{\sin(x) \left(5-4 \left(\sin^{2}(x)\right)\right)^{\frac{5}{2}}}{6}+\frac{125 \sin(x) \sqrt{5-4 \left(\sin^{2}(x)\right)}}{16}$	54

input
```
int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
625/32*arcsin(2/5*sin(x)*5^(1/2))+25/24*sin(x)*(5-4*sin(x)^2)^(3/2)+1/6*si
n(x)*(5-4*sin(x)^2)^(5/2)+125/16*sin(x)*(5-4*sin(x)^2)^(1/2)
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cos(x) \left(5\cos^2(x) + \sin^2(x)\right)^{5/2} dx = \frac{1}{48}\left(128 \cos(x)^4 + 264 \cos(x)^2 + 433\right)\sqrt{4 \cos(x)^2 + 1}\sin(x) + \frac{625}{64}\arctan\left(\frac{4\left(8 \cos(x)^2 - 3\right)\sqrt{4 \cos(x)^2 + 1}\sin(x) - 25 \cos(x)\sin(x)}{64 \cos(x)^4 - 23 \cos(x)^2 - 16}\right) + \frac{625}{64}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/48*(128*cos(x)^4 + 264*cos(x)^2 + 433)*sqrt(4*cos(x)^2 + 1)*sin(x) + 625
/64*arctan((4*(8*cos(x)^2 - 3)*sqrt(4*cos(x)^2 + 1)*sin(x) - 25*cos(x)*sin
(x))/(64*cos(x)^4 - 23*cos(x)^2 - 16)) + 625/64*arctan(sin(x)/cos(x))
```

3.420.6 Sympy [F(-1)]

Timed out.

$$\int \cos(x) \left(5\cos^2(x) + \sin^2(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx = \frac{1}{6}\left(-4\,\sin\left(x\right)^2+5\right)^{\frac{5}{2}}\sin\left(x\right) + \frac{25}{24}\left(-4\,\sin\left(x\right)^2+5\right)^{\frac{3}{2}}\sin\left(x\right) + \frac{125}{16}\sqrt{-4\,\sin\left(x\right)^2+5}\sin\left(x\right) + \frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-4*sin(x)^2 + 5)^(5/2)*sin(x) + 25/24*(-4*sin(x)^2 + 5)^(3/2)*sin(x)
+ 125/16*sqrt(-4*sin(x)^2 + 5)*sin(x) + 625/32*arcsin(2/5*sqrt(5)*sin(x))
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx = \frac{1}{48}\left(8\left(16\,\sin\left(x\right)^2-65\right)\sin\left(x\right)^2+825\right)\sqrt{-4\,\sin\left(x\right)^2+5}\sin\left(x\right) + \frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/48*(8*(16*sin(x)^2 - 65)*sin(x)^2 + 825)*sqrt(-4*sin(x)^2 + 5)*sin(x) +
625/32*arcsin(2/5*sqrt(5)*sin(x))
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} dx = \int \cos(x) \left(5\cos(x)^2+\sin(x)^2\right)^{5/2} dx$$

input
```
int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2),x)
```

output
```
int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2), x)
```

3.421 $\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx$

3.421.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \frac{3}{16} \arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right) - \frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}$$

output
```
3/16*arctan(2*sin(x)/(-1-4*sin(x)^2)^(1/2))+1/4*sin(x)*(-1-4*sin(x)^2)^(3/
2)-3/8*sin(x)*(-1-4*sin(x)^2)^(1/2)
```

3.421.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \frac{\sqrt{-3+2\cos(2x)}\left(-3\text{arcsinh}(2\sin(x)) + 2\sqrt{3-2\cos(2x)}(-11\sin(x) + 2\sin(3x))\right)}{16\sqrt{1+4\sin^2(x)}}$$

input
```
Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(Sqrt[-3 + 2*Cos[2*x]]*(-3*ArcSinh[2*Sin[x]] + 2*Sqrt[3 - 2*Cos[2*x]]*(-11
*Sin[x] + 2*Sin[3*x])))/(16*Sqrt[1 + 4*Sin[x]^2])
```

3.421.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4856, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \left(-5\sin^2(x)-\cos^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x) \left(-5\sin(x)^2-\cos(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4856}$$

$$\int \left(-4\sin^2(x)-1\right)^{3/2} d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\int \sqrt{-4\sin^2(x)-1}d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\left(\frac{1}{2}\sin(x)\sqrt{-4\sin^2(x)-1}-\frac{1}{2}\int \frac{1}{\sqrt{-4\sin^2(x)-1}}d\sin(x)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\left(\frac{1}{2}\sin(x)\sqrt{-4\sin^2(x)-1}-\frac{1}{2}\int \frac{1}{\frac{4\sin^2(x)}{-4\sin^2(x)-1}+1}d\frac{\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\left(\frac{1}{2}\sin(x)\sqrt{-4\sin^2(x)-1}-\frac{1}{4}\arctan\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)\right)$$

input
```
Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4 - (3*(-1/4*ArcTan[(2*Sin[x])/Sqrt[-1 -
4*Sin[x]^2]] + (Sin[x]*Sqrt[-1 - 4*Sin[x]^2])/2))/4
```

3.421.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.421.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{3\arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\left(\sin^2(x)\right)}}\right)}{16}+\frac{\sin(x)\left(-1-4\left(\sin^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3\sin(x)\sqrt{-1-4\left(\sin^2(x)\right)}}{8}$	47
default	$\frac{3\arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\left(\sin^2(x)\right)}}\right)}{16}+\frac{\sin(x)\left(-1-4\left(\sin^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3\sin(x)\sqrt{-1-4\left(\sin^2(x)\right)}}{8}$	47

input
```
int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/16*arctan(2*sin(x)/(-1-4*sin(x)^2)^(1/2))+1/4*sin(x)*(-1-4*sin(x)^2)^(3/
2)-3/8*sin(x)*(-1-4*sin(x)^2)^(1/2)
```

3.421.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2} dx = \frac{1}{128}\left(12i\, e^{(4i\,x)}\log\left(-\frac{1}{2}\sqrt{e^{(4i\,x)}-3\,e^{(2i\,x)}+1}\left(4\,e^{(2i\,x)}-5\right)+2\,e^{(4i\,x)}-\frac{11}{2}\,e^{(2i\,x)}+\frac{5}{2}\right)\right.$$

input
```
integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/128*(12*I*e^(4*I*x)*log(-1/2*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1)*(4*e^(2*I
*x) - 5) + 2*e^(4*I*x) - 11/2*e^(2*I*x) + 5/2) - 12*I*e^(4*I*x)*log(sqrt(e
^(4*I*x) - 3*e^(2*I*x) + 1) - e^(2*I*x) - 1) - 8*(2*I*e^(6*I*x) - 11*I*e^(
4*I*x) + 11*I*e^(2*I*x) - 2*I)*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - 145*I*e
^(4*I*x))*e^(-4*I*x)
```

3.421.6 Sympy [F(-1)]

Timed out.

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.421.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \cos(x) \left(-\cos^2(x)-5\sin^2(x)\right)^{3/2} \, dx = \frac{1}{4} \left(-4 \sin{(x)}^2 - 1\right)^{\frac{3}{2}} \sin{(x)} - \frac{3}{8} \sqrt{-4 \sin{(x)}^2 - 1} \sin{(x)} - \frac{3}{16} i \operatorname{arsinh}{(2 \sin{(x)})}$$

input
```
integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-4*sin(x)^2 - 1)^(3/2)*sin(x) - 3/8*sqrt(-4*sin(x)^2 - 1)*sin(x) - 3/
16*I*arcsinh(2*sin(x))
```

3.421.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \cos(x) \left(-\cos^2(x)-5\sin^2(x)\right)^{3/2} \, dx = -\frac{1}{8} i \left(8 \sin{(x)}^2 + 5\right) \sqrt{4 \sin{(x)}^2 + 1} \sin{(x)} + \frac{3}{16} i \log\left(\sqrt{4 \sin{(x)}^2 + 1} - 2 \sin{(x)}\right)$$

input
```
integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*I*(8*sin(x)^2 + 5)*sqrt(4*sin(x)^2 + 1)*sin(x) + 3/16*I*log(sqrt(4*si
n(x)^2 + 1) - 2*sin(x))
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \int \cos(x) \left(-\cos(x)^2 - 5\sin(x)^2\right)^{3/2} dx$$

input
```
int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2),x)
```

output
```
int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2), x)
```

3.422 $\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx$

3.422.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}} - \frac{\cos(x)}{15\left(-2+7\cos^2(x)\right)^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7\cos^2(x)}}$$

output
```
1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*c
os(x)/(-2+7*cos(x)^2)^(1/2)
```

3.422.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)(67+56\cos(2x)+49\cos(4x))}{15\sqrt{2}(3+7\cos(2x))^{5/2}}$$

input
```
Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

output
```
(Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[2*x])^(5/
2))
```

3.422.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4857, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\left(5\cos(x)^2-2\sin(x)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{\left(7\cos^2(x)-2\right)^{7/2}}d\cos(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{2}{5}\int \frac{1}{\left(7\cos^2(x)-2\right)^{5/2}}d\cos(x)+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2}{5}\left(-\frac{1}{3}\int \frac{1}{\left(7\cos^2(x)-2\right)^{3/2}}d\cos(x)-\frac{\cos(x)}{6\left(7\cos^2(x)-2\right)^{3/2}}\right)+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}+\frac{2}{5}\left(\frac{\cos(x)}{6\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{6\left(7\cos^2(x)-2\right)^{3/2}}\right)$$

input
```
Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

output
```
Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) + (2*(-1/6*Cos[x]/(-2 + 7*Cos[x]^2)^(3
/2) + Cos[x]/(6*Sqrt[-2 + 7*Cos[x]^2])))/5
```

3.422.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.422.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\cos(x)}{10(-2+7(\cos^2(x)))^{\frac{5}{2}}}-\frac{\cos(x)}{15(-2+7(\cos^2(x)))^{\frac{3}{2}}}+\frac{\cos(x)}{15\sqrt{-2+7(\cos^2(x))}}$	44
default	$\frac{\cos(x)}{10(-2+7(\cos^2(x)))^{\frac{5}{2}}}-\frac{\cos(x)}{15(-2+7(\cos^2(x)))^{\frac{3}{2}}}+\frac{\cos(x)}{15\sqrt{-2+7(\cos^2(x))}}$	44

input
```
int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*c
os(x)/(-2+7*cos(x)^2)^(1/2)
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\frac{\left(98\,\cos\left(x\right)^{5}-70\,\cos\left(x\right)^{3}+15\,\cos\left(x\right)\right)\sqrt{7\,\cos\left(x\right)^{2}-2}}{30\,\left(343\,\cos\left(x\right)^{6}-294\,\cos\left(x\right)^{4}+84\,\cos\left(x\right)^{2}-8\right)}$$

input
```
integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/30*(98*cos(x)^5 - 70*cos(x)^3 + 15*cos(x))*sqrt(7*cos(x)^2 - 2)/(343*cos
(x)^6 - 294*cos(x)^4 + 84*cos(x)^2 - 8)
```

3.422.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\frac{\cos\left(x\right)}{15\,\sqrt{7\,\cos\left(x\right)^{2}-2}}-\frac{\cos\left(x\right)}{15\,\left(7\,\cos\left(x\right)^{2}-2\right)^{\frac{3}{2}}}+\frac{\cos\left(x\right)}{10\,\left(7\,\cos\left(x\right)^{2}-2\right)^{\frac{5}{2}}}$$

input
```
integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*cos(x)/sqrt(7*cos(x)^2 - 2) - 1/15*cos(x)/(7*cos(x)^2 - 2)^(3/2) + 1/
10*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

3.422.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\frac{\left(14\left(7\cos\left(x\right)^2-5\right)\cos\left(x\right)^2+15\right)\cos\left(x\right)}{30\left(7\cos\left(x\right)^2-2\right)^{\frac{5}{2}}}$$

input
```
integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
1/30*(14*(7*cos(x)^2 - 5)*cos(x)^2 + 15)*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\frac{\cos\left(x\right)\,\left(98\cos\left(x\right)^4-70\cos\left(x\right)^2+15\right)}{30\left(7\cos\left(x\right)^2-2\right)^{5/2}}$$

input
```
int(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x)
```

output
```
(cos(x)*(98*cos(x)^4 - 70*cos(x)^2 + 15))/(30*(7*cos(x)^2 - 2)^(5/2))
```

3.423 $\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx$

3.423.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

output
```
2/25*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2)+1/10*sin(x)/(2-5*sin(x)^2)^(1/2)
```

3.423.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{1}{50}\left(4\sqrt{5}\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right) + \frac{5\sin(x)}{\sqrt{2-5\sin^2(x)}}\right)$$

input
```
Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(4*Sqrt[5]*ArcSin[Sqrt[5/2]*Sin[x]] + (5*Sin[x])/Sqrt[2 - 5*Sin[x]^2])/50
```

3.423.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4856, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1-2\sin^2(x)}{(2-5\sin^2(x))^{3/2}}d\sin(x)$$

$$\Big\downarrow \text{298}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{2-5\sin^2(x)}}d\sin(x)+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

input
```
Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2]
)
```

3.423.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.423.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{20\sqrt{5}\,\arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right)\left(\cos^2(x)\right)+5\sin(x)\sqrt{5(\cos^2(x))-3}-12\arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right)\sqrt{5}}{250(\cos^2(x))-150}$	58

input
```
int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/50/(5*cos(x)^2-3)*(20*5^(1/2)*arcsin(1/2*sin(x)*10^(1/2))*cos(x)^2+5*sin
(x)*(5*cos(x)^2-3)^(1/2)-12*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2))
```

3.423.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(28) = 56.

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = -\frac{\left(5\sqrt{5}\cos(x)^2-3\sqrt{5}\right)\arctan\left(\frac{\left(50\sqrt{5}\cos(x)^4-80\sqrt{5}\cos(x)^2+31\sqrt{5}\right)\sqrt{5\cos(x)^2-3}}{10\left(25\cos(x)^4-35\cos(x)^2+12\right)\sin(x)}\right)-5\sqrt{5\cos(x)^2-3}\sin(x)}{50\left(5\cos(x)^2-3\right)}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*cos(x)^4 -
 80*sqrt(5)*cos(x)^2 + 31*sqrt(5))*sqrt(5*cos(x)^2 - 3)/((25*cos(x)^4 - 35
*cos(x)^2 + 12)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*cos(x)^2 - 3)
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.423.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(28) = 56.

Time = 0.35 (sec) , antiderivative size = 716, normalized size of antiderivative = 18.36

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/50*(5*cos(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*cos(4*x) - 2*cos(2*x) +
 5))*sin(2*x) - 5*(cos(2*x) - 1)*sin(1/2*arctan2(5*sin(4*x) - 2*sin(2*x),
5*cos(4*x) - 2*cos(2*x) + 5)) + 2*(-10*(2*cos(2*x) - 5)*cos(4*x) + 25*cos(
4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^
2 - 20*cos(2*x) + 25)^(1/4)*(sqrt(5)*arctan2(1/12*sqrt(6)*(sqrt(6)*(25/36)
^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 -
 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*si
n(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin
(2*x)^2 - 5/12*cos(2*x) + 25/24)) + 5*sin(2*x)), 5/12*sqrt(6)*cos(2*x) + 1
/2*(25/36)^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*co
s(2*x)^2 - 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25
)^(1/4)*cos(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 -
 25/24*sin(2*x)^2 - 5/12*cos(2*x) + 25/24)) - 1/12*sqrt(6)) + sqrt(5)*arct
an2(1/12*sqrt(6)*(sqrt(6)*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2
*x)^3 + 2*(cos(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980
*cos(2*x) + 2401)^(1/4)*sin(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24
*cos(2*x)^2 - 1/24*sin(2*x)^2 - 5/12*cos(2*x) + 49/24)) + sin(2*x)), 1/12*
sqrt(6)*cos(2*x) + 1/2*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)
^3 + 2*(cos(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*co
s(2*x) + 2401)^(1/4)*cos(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24...
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2}{25}\sqrt{5}\arcsin\left(\frac{1}{2}\sqrt{10}\sin(x)\right) - \frac{\sqrt{-5\sin(x)^2+2}\sin(x)}{10\left(5\sin(x)^2-2\right)}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2 + 2)*sin(
x)/(5*sin(x)^2 - 2)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx = \int \frac{\cos\left(2\,x\right)\,\cos\left(x\right)}{{\left(2-5\,{\sin\left(x\right)}^2\right)}^{3/2}}\,dx$$

input
```
int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2),x)
```

output
```
int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2), x)
```

3.424 $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

3.424.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = -\frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}+\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}$$

output
```
-1/2*arcsin(2/3*cos(x))-55/27*cos(x)/(9-4*cos(x)^2)^(3/2)+295/243*cos(x)/(
9-4*cos(x)^2)^(1/2)
```

3.424.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = \frac{2550\cos(x)-590\cos(3x)+243i(7-2\cos(2x))^{3/2}\log\left(2i\cos(x)+\sqrt{7-\right.}{486(7-2\cos(2x))^{3/2}}$$

input
```
Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

output
```
(2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[(2*I)*Cos
[x] + Sqrt[7 - 2*Cos[2*x]]])/(486*(7 - 2*Cos[2*x])^(3/2))
```

3.424.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4879, 1471, 27, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(5x)}{(9\sin^2(x)+5\cos^2(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(5x)}{(9\sin(x)^2+5\cos(x)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{16\cos^4(x)-12\cos^2(x)+1}{(9-4\cos^2(x))^{5/2}}d\cos(x)$$

$$\Big\downarrow \text{1471}$$

$$\frac{1}{27}\int \frac{4(27\cos^2(x)+13)}{(9-4\cos^2(x))^{3/2}}d\cos(x)-\frac{55\cos(x)}{27(9-4\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{27}\int \frac{27\cos^2(x)+13}{(9-4\cos^2(x))^{3/2}}d\cos(x)-\frac{55\cos(x)}{27(9-4\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{4}{27}\left(\frac{295\cos(x)}{36\sqrt{9-4\cos^2(x)}}-\frac{27}{4}\int \frac{1}{\sqrt{9-4\cos^2(x)}}d\cos(x)\right)-\frac{55\cos(x)}{27(9-4\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{4}{27}\left(\frac{295\cos(x)}{36\sqrt{9-4\cos^2(x)}}-\frac{27}{8}\arcsin\left(\frac{2\cos(x)}{3}\right)\right)-\frac{55\cos(x)}{27(9-4\cos^2(x))^{3/2}}$$

input
```
Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

output
```
(-55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2)) + (4*((-27*ArcSin[(2*Cos[x])/3])/
8 + (295*Cos[x])/(36*Sqrt[9 - 4*Cos[x]^2])))/27
```

3.424.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.424. $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

3.424.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(36) = 72.

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.19

method	result
derivativedivides	$-\frac{\arcsin\left(\frac{2\cos(x)}{3}\right)}{2}-\frac{55\sqrt{-4\left(\cos(x)-\frac{3}{2}\right)^{2}-12\cos(x)+18}}{2592\left(\cos(x)-\frac{3}{2}\right)^{2}}-\frac{295\sqrt{-\left(\cos(x)-\frac{3}{2}\right)^{2}-3\cos(x)+\frac{9}{2}}}{972\left(\cos(x)-\frac{3}{2}\right)}+\frac{55\sqrt{-4(\cos(x)-}}{2592(c}$
default	$-\frac{\arcsin\left(\frac{2\cos(x)}{3}\right)}{2}-\frac{55\sqrt{-4\left(\cos(x)-\frac{3}{2}\right)^{2}-12\cos(x)+18}}{2592\left(\cos(x)-\frac{3}{2}\right)^{2}}-\frac{295\sqrt{-\left(\cos(x)-\frac{3}{2}\right)^{2}-3\cos(x)+\frac{9}{2}}}{972\left(\cos(x)-\frac{3}{2}\right)}+\frac{55\sqrt{-4(\cos(x)-}}{2592(c}$

input
```
int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsin(2/3*cos(x))-55/2592/(cos(x)-3/2)^2*(-4*(cos(x)-3/2)^2-12*cos(x
)+18)^(1/2)-295/972/(cos(x)-3/2)*(-(cos(x)-3/2)^2-3*cos(x)+9/2)^(1/2)+55/2
592/(cos(x)+3/2)^2*(-4*(cos(x)+3/2)^2+12*cos(x)+18)^(1/2)-295/972/(cos(x)+
3/2)*(-(cos(x)+3/2)^2+3*cos(x)+9/2)^(1/2)
```

3.424.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.73

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = \frac{243\left(16\cos(x)^4-72\cos(x)^2+81\right)\arctan\left(-\frac{81\cos(x)\sin(x)-4\left(8\cos(x)^3-9\cos\right.}{64\cos(x)^4-225\cos(}\right.}{}$$

input
```
integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/972*(243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(-(81*cos(x)*sin(x) - 4*
(8*cos(x)^3 - 9*cos(x))*sqrt(-4*cos(x)^2 + 9))/(64*cos(x)^4 - 225*cos(x)^2
 + 81)) - 243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(sin(x)/cos(x)) - 80*
(59*cos(x)^3 - 108*cos(x))*sqrt(-4*cos(x)^2 + 9))/(16*cos(x)^4 - 72*cos(x)
^2 + 81)
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = -2\left(\frac{2\cos(x)^2}{(-4\cos(x)^2+9)^{\frac{3}{2}}} - \frac{3}{(-4\cos(x)^2+9)^{\frac{3}{2}}}\right)\cos(x) + \frac{52\cos(x)}{243\sqrt{-4\cos(x)^2+9}} + \frac{26\cos(x)}{27(-4\cos(x)^2+9)^{\frac{3}{2}}} - \frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

input
```
integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-2*(2*cos(x)^2/(-4*cos(x)^2 + 9)^(3/2) - 3/(-4*cos(x)^2 + 9)^(3/2))*cos(x)
 + 52/243*cos(x)/sqrt(-4*cos(x)^2 + 9) + 26/27*cos(x)/(-4*cos(x)^2 + 9)^(3
/2) - 1/2*arcsin(2/3*cos(x))
```

3.424.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = -\frac{20(59\cos(x)^2-108)\sqrt{-4\cos(x)^2+9}\cos(x)}{243(4\cos(x)^2-9)^2} - \frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

input
```
integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x)^2 - 9)^
2 - 1/2*arcsin(2/3*cos(x))
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(5x)}{(5\cos^2(x) + 9\sin^2(x))^{5/2}}\,dx = \int \frac{\sin(5\,x)}{\left(5\cos(x)^2 + 9\sin(x)^2\right)^{5/2}}\,dx$$

input
```
int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x)
```

output
```
int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2), x)
```

3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx$

3.425.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = -\frac{1}{4(-5+4\sin^2(x))^{3/2}} - \frac{5}{8\sqrt{-5+4\sin^2(x)}} + \frac{1}{8}\sqrt{-5+4\sin^2(x)}$$

output
```
-1/4/(-5+4*sin(x)^2)^(3/2)-5/8/(-5+4*sin(x)^2)^(1/2)+1/8*(-5+4*sin(x)^2)^(
1/2)
```

3.425.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \frac{12+11\cos(2x)+\cos(4x)}{4(-5+4\sin^2(x))^{3/2}}$$

input
```
Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

output
```
(12 + 11*Cos[2*x] + Cos[4*x])/(4*(-5 + 4*Sin[x]^2)^(3/2))
```

3.425.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4856, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\cos(x)\cos(2x)}{(4\sin^2(x)-5)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(3x)\cos(x)\cos(2x)}{(4\sin(x)^2-5)^{5/2}}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{\sin(x)\left(8\sin^4(x)-10\sin^2(x)+3\right)}{(4\sin^2(x)-5)^{5/2}}d\sin(x)$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{8\sin^4(x)-10\sin^2(x)+3}{(4\sin^2(x)-5)^{5/2}}d\sin^2(x)$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{1}{2}\int \left(\frac{1}{2\sqrt{4\sin^2(x)-5}}+\frac{5}{2\left(4\sin^2(x)-5\right)^{3/2}}+\frac{3}{(4\sin^2(x)-5)^{5/2}}\right)d\sin^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\sqrt{4\sin^2(x)-5}-\frac{5}{4\sqrt{4\sin^2(x)-5}}-\frac{1}{2\left(4\sin^2(x)-5\right)^{3/2}}\right)$$

input
```
Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

output
```
(-1/2*1/(-5 + 4*Sin[x]^2)^(3/2) - 5/(4*Sqrt[-5 + 4*Sin[x]^2]) + Sqrt[-5 +
4*Sin[x]^2]/4)/2
```

3.425.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.425.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\sqrt{-4(\cos^2(x))-1}}{8}-\frac{1}{4(-4(\cos^2(x))-1)^{\frac{3}{2}}}-\frac{5}{8\sqrt{-4(\cos^2(x))-1}}$	38
default	$\frac{\sqrt{-4(\cos^2(x))-1}}{8}-\frac{1}{4(-4(\cos^2(x))-1)^{\frac{3}{2}}}-\frac{5}{8\sqrt{-4(\cos^2(x))-1}}$	38

input
```
int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/8*(-4*cos(x)^2-1)^(1/2)-1/4/(-4*cos(x)^2-1)^(3/2)-5/8/(-4*cos(x)^2-1)^(1
/2)
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \frac{(4\cos(x)^4+7\cos(x)^2+1)\sqrt{-4\cos(x)^2-1}}{2(16\cos(x)^4+8\cos(x)^2+1)}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="fri
cas")
```

output
```
1/2*(4*cos(x)^4 + 7*cos(x)^2 + 1)*sqrt(-4*cos(x)^2 - 1)/(16*cos(x)^4 + 8*c
os(x)^2 + 1)
```

3.425.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.425.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(37) = 74.

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.92

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx =$$

$$-\frac{(\cos(11x)+14\cos(9x)+58\cos(7x)+94\cos(5x)+58\cos(3x)+15\cos(x))\cos\left(\frac{5}{2}\arctan(\sin(4x)\right.}{8(2(3\cos(2x)-}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="max
ima")
```

output
```
-1/8*((cos(11*x) + 14*cos(9*x) + 58*cos(7*x) + 94*cos(5*x) + 58*cos(3*x) +
 15*cos(x))*cos(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x)
- 1)) - (sin(11*x) + 14*sin(9*x) + 58*sin(7*x) + 94*sin(5*x) + 58*sin(3*x)
 + 13*sin(x))*sin(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x
) - 1)))/(2*(3*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + sin(4*
x)^2 + 6*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)^(5/4)
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \frac{1}{8}\sqrt{4\,\sin\left(x\right)^2-5}-\frac{20\,\sin\left(x\right)^2-23}{8\left(4\,\sin\left(x\right)^2-5\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="gia
c")
```

output
```
1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5)^(3/2)
```

3.425.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \frac{2\cos\left(2\,x\right)^2+11\,\cos\left(2\,x\right)+11}{4\left(-2\,\cos\left(2\,x\right)-3\right)^{3/2}}$$

input
```
int((cos(2*x)*sin(3*x)*cos(x))/(4*sin(x)^2 - 5)^(5/2),x)
```

output
```
(11*cos(2*x) + 2*cos(2*x)^2 + 11)/(4*(- 2*cos(2*x) - 3)^(3/2))
```

3.426 $\int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}}\,dx$

3.426.1 Optimal result

Integrand size = 33, antiderivative size = 111

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = 2\arctan\left(\frac{\cos(x)}{\sqrt{-5+\sin^2(x)}}\right)-\frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-5+\sin^2(x)}}\right)}{\sqrt{5}}-\frac{2\arctan\left(\frac{\sqrt{-5+\sin^2(x)}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)+2\sqrt{-5+\sin^2(x)}+\frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}$$

output
```
2*arctan(cos(x)/(-5+sin(x)^2)^(1/2))-2*arctanh(sin(x)/(-5+sin(x)^2)^(1/2))
-1/5*arctan(cos(x)*5^(1/2)/(-5+sin(x)^2)^(1/2))*5^(1/2)-2/5*arctan(1/5*(-5
+sin(x)^2)^(1/2)*5^(1/2))*5^(1/2)+2*(-5+sin(x)^2)^(1/2)+2/5*(-5+sin(x)^2)^
(1/2)/sin(x)
```

3.426.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.00 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.66

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = \frac{2\sqrt{2}(-2\cos^3(x)+\cos(2x)+2\cos^2(x)\cot(x))\left(18+2\cos(2x)+20\sqrt{2}\text{arctanh}\left(\frac{2\sqrt{2}\tan\left(\frac{x}{2}\right)}{\sqrt{-((9+\cos(2x))\sec^4\left(\frac{x}{2}\right))}}\right)\text{co}\right.}{}$$

input
```
Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5
 + Sin[x]^2],x]
```

output
```
(2*Sqrt[2]*(-2*Cos[x]^3 + Cos[2*x] + 2*Cos[x]^2*Cot[x])*(18 + 2*Cos[2*x] +
 20*Sqrt[2]*ArcTanh[(2*Sqrt[2]*Tan[x/2])/Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)
]]*Cos[x/2]^3*Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)]*Sin[x/2] + 85*Sin[x] + Sq
rt[10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[-9 - Cos[2*x]]]*Sqrt[-9 - Cos[2*x]]*S
in[x] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[2*x]]/Sqrt[10]]*Sqrt[-9 - Cos[2*x]
]*Sin[x] + (10*I)*Sqrt[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sqrt[
-9 - Cos[2*x]]]*Sin[x] + 5*Sin[3*x]))/(5*Sqrt[-9 - Cos[2*x]]*(-6*Cos[x] -
2*Cos[3*x] + 2*Sin[x] + 2*Sin[2*x] - 2*Sin[3*x] + Sin[4*x]))
```

3.426.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x) \left(\sin(x)\cos(2x) - 2(\sin(x)-1)\cos^3(x)\right)}{\sqrt{\sin^2(x)-5}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)\cos(2x) - 2(\sin(x)-1)\cos(x)^3}{\sin(x)^2\sqrt{\sin(x)^2-5}}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{(\cos(2x)-2\cos^3(x))\csc(x)}{\sqrt{\sin^2(x)-5}} + \frac{2\cos(x)\cot^2(x)}{\sqrt{\sin^2(x)-5}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2\arctan\left(\frac{\cos(x)}{\sqrt{-\cos^2(x)-4}}\right) - \frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)}{\sqrt{5}} - \frac{2\arctan\left(\frac{\sqrt{-\cos^2(x)-4}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right) + 2\sqrt{-\cos^2(x)-4} + \frac{2}{5}\sqrt{\sin^2(x)-5}\csc(x)$$

input
```
Int[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5 + Sin
[x]^2],x]
```

output
```
2*ArcTan[Cos[x]/Sqrt[-4 - Cos[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-4 - C
os[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-4 - Cos[x]^2]/Sqrt[5]])/Sqrt[5] - 2*Ar
cTanh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-4 - Cos[x]^2] + (2*Csc[x]*Sqrt
[-5 + Sin[x]^2])/5
```

3.426.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.426.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18

method	result
default	$-2\ln\left(\sin\left(x\right)+\sqrt{-5+\sin^{2}\left(x\right)}\right)+2\sqrt{-5+\sin^{2}\left(x\right)}+\frac{2\sqrt{-5+\sin^{2}(x)}}{5\sin(x)}+\frac{2\sqrt{5}\,\arctan\left(\frac{\sqrt{5}}{\sqrt{-5+\sin^{2}(x)}}\right)}{5}-$
parts	$-2\ln\left(\sin\left(x\right)+\sqrt{-5+\sin^{2}\left(x\right)}\right)+2\sqrt{-5+\sin^{2}\left(x\right)}+\frac{2\sqrt{-5+\sin^{2}(x)}}{5\sin(x)}+\frac{2\sqrt{5}\,\arctan\left(\frac{\sqrt{5}}{\sqrt{-5+\sin^{2}(x)}}\right)}{5}-$

input
```
int((-2*cos(x)^3*(sin(x)-1)+sin(x)*cos(2*x))/sin(x)^2/(-5+sin(x)^2)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
-2*ln(sin(x)+(-5+sin(x)^2)^(1/2))+2*(-5+sin(x)^2)^(1/2)+2/5*(-5+sin(x)^2)^
(1/2)/sin(x)+2/5*5^(1/2)*arctan(5^(1/2)/(-5+sin(x)^2)^(1/2))-1/10*((-5+sin
(x)^2)*cos(x)^2)^(1/2)*(-5^(1/2)*arctan(1/5*(3*sin(x)^2-5)*5^(1/2)/(-cos(x
)^4-4*cos(x)^2)^(1/2))-10*arcsin(1+1/2*cos(x)^2))/cos(x)/(-5+sin(x)^2)^(1/
2)
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx =$$

$$-\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-\cos(x)^2-4}}{\cos(x)+4}\right)\sin(x)-3\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-\cos(x)^2-4}}{\cos(x)-4}\right)\sin(x)+20\arctan\left(\frac{\sqrt{-\cos(x)^2-4}}{\cos(x)}\right)\mathrm{s}}{10\,\mathrm{si}}$$

input
```
integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)
^(1/2),x, algorithm="fricas")
```

output
```
-1/10*(sqrt(5)*arctan(sqrt(5)*sqrt(-cos(x)^2 - 4)/(cos(x) + 4))*sin(x) - 3
*sqrt(5)*arctan(sqrt(5)*sqrt(-cos(x)^2 - 4)/(cos(x) - 4))*sin(x) + 20*arct
an(sqrt(-cos(x)^2 - 4)/cos(x))*sin(x) - 10*log(2*cos(x)^2 + 2*sqrt(-cos(x)
^2 - 4)*sin(x) + 3)*sin(x) - 4*sqrt(-cos(x)^2 - 4)*(5*sin(x) + 1))/sin(x)
```

3.426.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-5+sin(x)*
*2)**(1/2),x)
```

output
```
Timed out
```

3.426.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$= \frac{2}{5}\sqrt{5}\operatorname{arcsin}\left(\frac{\sqrt{5}}{|\sin(x)|}\right) - \frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(\frac{\cos(x)}{2(\cos(x)+1)} - \frac{2}{\cos(x)+1}\right) - \frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(-\frac{\cos(x)}{2(\cos(x)-1)} - \frac{2}{\cos(x)-1}\right) + 2\sqrt{\sin(x)^2-5} + \frac{2\sqrt{\sin(x)^2-5}}{5\sin(x)} - 2i\operatorname{arsinh}\left(\frac{1}{2}\cos(x)\right) - 2\log\left(2\sqrt{\sin(x)^2-5} + 2\sin(x)\right)$$

input
```
integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)
^(1/2),x, algorithm="maxima")
```

output
```
2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(1/2*cos(x
)/(cos(x) + 1) - 2/(cos(x) + 1)) - 1/10*I*sqrt(5)*arcsinh(-1/2*cos(x)/(cos
(x) - 1) - 2/(cos(x) - 1)) + 2*sqrt(sin(x)^2 - 5) + 2/5*sqrt(sin(x)^2 - 5)
/sin(x) - 2*I*arcsinh(1/2*cos(x)) - 2*log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))
```

3.426.8 Giac [F]

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = \int -\frac{2(\sin(x)-1)\cos(x)^3 - \cos(2x)\sin(x)}{\sqrt{\sin(x)^2-5}\sin(x)^2}\,dx$$

input
```
integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)
^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)
*sin(x)^2), x)
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$=\int \frac{\cos\left(2\,x\right)\,\sin\left(x\right)-2\cos\left(x\right)^3\left(\sin\left(x\right)-1\right)}{\sin\left(x\right)^2\,\sqrt{\sin\left(x\right)^2-5}}\,dx$$

input
```
int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(
1/2)),x)
```

output
```
int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(
1/2)), x)
```

3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

3.427.1 Optimal result

Integrand size = 39, antiderivative size = 112

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{5\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{-1+4\cos^2(x)}}\right) -\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{-1+8\cos^2(x)}}\right)-\frac{1}{2}\sqrt{-1+4\cos^2(x)}\sin(x)-\frac{1}{2}\sqrt{-1+8\cos^2(x)}\sin(x)$$

output
```
3/4*arcsin(2/3*sin(x)*3^(1/2))-3/4*arctan(sin(x)/(-1+4*cos(x)^2)^(1/2))-3/
4*arctan(sin(x)/(-1+8*cos(x)^2)^(1/2))+5/8*arcsin(2/7*sin(x)*14^(1/2))*2^(
1/2)-1/2*sin(x)*(-1+4*cos(x)^2)^(1/2)-1/2*sin(x)*(-1+8*cos(x)^2)^(1/2)
```

3.427.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{1}{8}\left(5\sqrt{2}\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)+6\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)+3\arctan\left(\frac{7-8\sin(x)}{\sqrt{3+4\cos(2x)}}\right)+3\arctan\left(\frac{3-4\sin(x)}{\sqrt{1+2\cos(2x)}}\right)-3\arctan\left(\frac{3+4\sin(x)}{\sqrt{1+2\cos(2x)}}\right)-3\arctan\left(\frac{7+8\sin(x)}{\sqrt{3+4\cos(2x)}}\right)-4\sqrt{1+2\cos(2x)}\sin(x)-4\sqrt{3+4\cos(2x)}\sin(x)\right)$$

input
```
Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),
x]
```

output
```
(5*Sqrt[2]*ArcSin[2*Sqrt[2/7]*Sin[x]] + 6*ArcSin[(2*Sin[x])/Sqrt[3]] + 3*A
rcTan[(7 - 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] + 3*ArcTan[(3 - 4*Sin[x])/Sqrt[
1 + 2*Cos[2*x]]] - 3*ArcTan[(3 + 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTa
n[(7 + 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] - 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4
*Sqrt[3 + 4*Cos[2*x]]*Sin[x])/8
```

3.427.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4878, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(3x)}{\sqrt{3\cos^2(x)-\sin^2(x)}-\sqrt{8\cos^2(x)-1}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(3x)}{\sqrt{3\cos(x)^2-\sin(x)^2}-\sqrt{8\cos(x)^2-1}}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int -\frac{1-4\sin^2(x)}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-4\sin^2(x)}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{1}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}-\frac{4\sin^2(x)}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)-\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}$$

input
```
Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

output
```
(5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]]
)/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[3
- 4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2])/2 - (Sin[x]*Sqrt[3 - 4*S
in[x]^2])/2
```

3.427.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.427. $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.427.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(84) = 168.

Time = 1.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

method	result
default	$\frac{3\sqrt{-8(\sin(x)+1)^2+16\sin(x)+15}}{8}+\frac{5\arcsin\left(\frac{2\sin(x)\sqrt{14}}{7}\right)\sqrt{2}}{8}-\frac{3\arctan\left(\frac{14+16\sin(x)}{2\sqrt{-8(\sin(x)+1)^2+16\sin(x)+15}}\right)}{8}-\frac{3\sqrt{-8(\sin(x)-1)^2}}{8}$

input
```
int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,method
=_RETURNVERBOSE)
```

output
```
3/8*(-8*(sin(x)+1)^2+16*sin(x)+15)^(1/2)+5/8*arcsin(2/7*sin(x)*14^(1/2))*2
^(1/2)-3/8*arctan(1/2*(14+16*sin(x))/(-8*(sin(x)+1)^2+16*sin(x)+15)^(1/2))
-3/8*(-8*(sin(x)-1)^2-16*sin(x)+15)^(1/2)+3/8*arctan(1/2*(14-16*sin(x))/(-
8*(sin(x)-1)^2-16*sin(x)+15)^(1/2))+3/8*(-4*(sin(x)+1)^2+8*sin(x)+7)^(1/2)
+3/4*arcsin(2/3*sin(x)*3^(1/2))-3/8*arctan(1/2*(6+8*sin(x))/(-4*(sin(x)+1)
^2+8*sin(x)+7)^(1/2))-3/8*(-4*(sin(x)-1)^2-8*sin(x)+7)^(1/2)+3/8*arctan(1/
2*(6-8*sin(x))/(-4*(sin(x)-1)^2-8*sin(x)+7)^(1/2))-1/2*sin(x)*(-8*sin(x)^2
+7)^(1/2)-1/2*sin(x)*(3-4*sin(x)^2)^(1/2)
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(84) = 168$.

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.74

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$= -\frac{5}{32}\sqrt{2}\arctan\left(\frac{\left(512\sqrt{2}\cos(x)^4-576\sqrt{2}\cos(x)^2+113\sqrt{2}\right)\sqrt{8\cos(x)^2-1}}{16\left(128\cos(x)^4-88\cos(x)^2+9\right)\sin(x)}\right)$$
$$-\frac{1}{2}\sqrt{8\cos(x)^2-1}\sin(x)-\frac{1}{2}\sqrt{4\cos(x)^2-1}\sin(x)$$
$$+\frac{3}{8}\arctan\left(\frac{4\left(8\cos(x)^2-5\right)\sqrt{4\cos(x)^2-1}\sin(x)-9\cos(x)\sin(x)}{64\cos(x)^4-71\cos(x)^2+16}\right)$$
$$+\frac{3}{8}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{3}{8}\arctan\left(\frac{9\cos(x)^2-2}{2\sqrt{8\cos(x)^2-1}\sin(x)}\right)$$
$$+\frac{3}{4}\arctan\left(\frac{\sqrt{4\cos(x)^2-1}}{\sin(x)}\right)$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
 algorithm="fricas")
```

output
```
-5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos(x)^2 + 1
13*sqrt(2))*sqrt(8*cos(x)^2 - 1)/((128*cos(x)^4 - 88*cos(x)^2 + 9)*sin(x))
) - 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)*sin(x) + 3/
8*arctan((4*(8*cos(x)^2 - 5)*sqrt(4*cos(x)^2 - 1)*sin(x) - 9*cos(x)*sin(x)
)/(64*cos(x)^4 - 71*cos(x)^2 + 16)) + 3/8*arctan(sin(x)/cos(x)) + 3/8*arct
an(1/2*(9*cos(x)^2 - 2)/(sqrt(8*cos(x)^2 - 1)*sin(x))) + 3/4*arctan(sqrt(4
*cos(x)^2 - 1)/sin(x))
```

3.427.6 Sympy [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$=\int \frac{\cos(3x)}{\sqrt{-\sin^2(x)+3\cos^2(x)}-\sqrt{8\cos^2(x)-1}}\,dx$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2
)),x)
```

output
```
Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)**2 - 1))
, x)
```

3.427.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
 algorithm="maxima")
```

output
```
Timed out
```

3.427.8 Giac [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$=\int -\frac{\cos(3x)}{\sqrt{8\cos(x)^2-1}-\sqrt{3\cos(x)^2-\sin(x)^2}}\,dx$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
 algorithm="giac")
```

output
```
integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)),
x)
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$= -\int -\frac{\cos(3x)}{\sqrt{3\cos(x)^2-\sin(x)^2}-\sqrt{8\cos(x)^2-1}}\,dx$$

input
```
int(cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)),x)
```

output
```
-int(-cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)), x
)
```

3.428 $\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx$

3.428.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx=\frac{5}{36}(2-3\sin^2(x))^{8/5}-\frac{20}{117}(2-3\sin^2(x))^{13/5}$$

output
```
5/36*(2-3*sin(x)^2)^(8/5)-20/117*(2-3*sin(x)^2)^(13/5)
```

3.428.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx=-\frac{5(1+3\cos(2x))^{8/5}(-5+24\cos(2x))}{936\ 2^{3/5}}$$

input
```
Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

output
```
(-5*(1 + 3*Cos[2*x])^(8/5)*(-5 + 24*Cos[2*x]))/(936*2^(3/5))
```

3.428.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4878, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-3 \sin ^2(x)\right)^{3/5} \sin (4 x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2-3 \sin (x)^2\right)^{3/5} \sin (4 x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int 4 \sin (x) \left(2-3 \sin ^2(x)\right)^{3/5} \left(1-2 \sin ^2(x)\right)d\sin (x)$$

$$\Big\downarrow \text{27}$$

$$4 \int \sin (x) \left(2-3 \sin ^2(x)\right)^{3/5} \left(1-2 \sin ^2(x)\right)d\sin (x)$$

$$\Big\downarrow \text{353}$$

$$2 \int \left(2-3 \sin ^2(x)\right)^{3/5} \left(1-2 \sin ^2(x)\right)d\sin ^2(x)$$

$$\Big\downarrow \text{53}$$

$$2 \int \left(\frac{2}{3} \left(2-3 \sin ^2(x)\right)^{8/5}-\frac{1}{3} \left(2-3 \sin ^2(x)\right)^{3/5}\right)d\sin ^2(x)$$

$$\Big\downarrow \text{2009}$$

$$2 \left(\frac{5}{72} \left(2-3 \sin ^2(x)\right)^{8/5}-\frac{10}{117} \left(2-3 \sin ^2(x)\right)^{13/5}\right)$$

input
```
Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

output
```
2*((5*(2 - 3*Sin[x]^2)^(8/5))/72 - (10*(2 - 3*Sin[x]^2)^(13/5))/117)
```

3.428.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.428.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{5\left(2-3\left(\sin^{2}\left(x\right)\right)\right)^{\frac{8}{5}}}{12}-\frac{5\left(16+48\cos\left(2x\right)\right)^{\frac{13}{5}}}{239616}-\frac{5\left(16+48\cos\left(2x\right)\right)^{\frac{8}{5}}}{4608}$	38

input
```
int((2-3*sin(x)^2)^(3/5)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
5/12*(2-3*sin(x)^2)^(8/5)-5/239616*(16+48*cos(2*x))^(13/5)-5/4608*(16+48*c
os(2*x))^(8/5)
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{5}{468}\left(144\cos\left(x\right)^4-135\cos\left(x\right)^2+29\right)\left(3\cos\left(x\right)^2-1\right)^{\frac{3}{5}}$$

input
```
integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="fricas")
```

output
```
-5/468*(144*cos(x)^4 - 135*cos(x)^2 + 29)*(3*cos(x)^2 - 1)^(3/5)
```

3.428.6 Sympy [F(-1)]

Timed out.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = \text{Timed out}$$

input
```
integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)
```

output
```
Timed out
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{20}{117}\left(-3\sin\left(x\right)^2+2\right)^{\frac{13}{5}}+\frac{5}{36}\left(-3\sin\left(x\right)^2+2\right)^{\frac{8}{5}}$$

input
```
integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="maxima")
```

output
```
-20/117*(-3*sin(x)^2 + 2)^(13/5) + 5/36*(-3*sin(x)^2 + 2)^(8/5)
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{20}{117}\left(3\sin\left(x\right)^2-2\right)^2\left(-3\sin\left(x\right)^2+2\right)^{\frac{3}{5}}+\frac{5}{36}\left(-3\sin\left(x\right)^2+2\right)^{\frac{8}{5}}$$

input
```
integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="giac")
```

output
```
-20/117*(3*sin(x)^2 - 2)^2*(-3*sin(x)^2 + 2)^(3/5) + 5/36*(-3*sin(x)^2 + 2
)^(8/5)
```

3.428.9 Mupad [F(-1)]

Timed out.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = \int \sin\left(4\,x\right)\,\left(2-3\sin\left(x\right)^2\right)^{3/5}\,dx$$

input
```
int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5),x)
```

output
```
int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5), x)
```

3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$

3.429.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \frac{1}{2}\sqrt{\cos(2x)}\sin(x)$$

output
```
1/4*arcsin(sin(x)*2^(1/2))*2^(1/2)+1/2*sin(x)*cos(2*x)^(1/2)
```

3.429.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{1}{4}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) + 2\sqrt{\cos(2x)}\sin(x)\right)$$

input
```
Integrate[Cos[x]*Sqrt[Cos[2*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] + 2*Sqrt[Cos[2*x]]*Sin[x])/4
```

3.429.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4856, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)\sqrt{\cos(2x)}dx$$

$$\Big\downarrow \; 4856$$

$$\int \sqrt{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-2\sin^2(x)}}d\sin(x)+\frac{1}{2}\sqrt{1-2\sin^2(x)}\sin(x)$$

$$\Big\downarrow \; 223$$

$$\frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{1-2\sin^2(x)}$$

input
```
Int[Cos[x]*Sqrt[Cos[2*x]],x]
```

output
```
ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sin[x]*Sqrt[1 - 2*Sin[x]^2])/2
```

3.429.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.429.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(23) = 46.

Time = 0.47 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
default	$-\frac{\sqrt{(2(\cos^2(x))-1)(\sin^2(x))}\left(-\sqrt{2}\,\arcsin(4(\sin^2(x))-1)-4\sqrt{-2(\sin^4(x))+\sin^2(x)}\right)}{8\sin(x)\sqrt{2(\cos^2(x))-1}}$	62

input
```
int(cos(x)*cos(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(-2^(1/2)*arcsin(4*sin(x)^2-1)-4*(-2*
sin(x)^4+sin(x)^2)^(1/2))/sin(x)/(2*cos(x)^2-1)^(1/2)
```

3.429.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$
$$= -\frac{1}{16}\sqrt{2}\arctan\left(\frac{(32\sqrt{2}\cos(x)^4 - 48\sqrt{2}\cos(x)^2 + 17\sqrt{2})\sqrt{2\cos(x)^2-1}}{8\,(8\cos(x)^4 - 10\cos(x)^2 + 3)\sin(x)}\right)$$
$$+ \frac{1}{2}\sqrt{2\cos(x)^2-1}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*s
qrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x))) + 1/
2*sqrt(2*cos(x)^2 - 1)*sin(x)
```

3.429.6 Sympy [F]

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \cos{(x)}\sqrt{\cos{(2x)}}\,dx$$

input
```
integrate(cos(x)*cos(2*x)**(1/2),x)
```

output
```
Integral(cos(x)*sqrt(cos(2*x)), x)
```

3.429.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(23) = 46$.

Time = 0.32 (sec) , antiderivative size = 488, normalized size of antiderivative = 14.79

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*(2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*
arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - (cos(2*x) - 1)*sin(1/2*arctan2
(sin(4*x), cos(4*x) + 1))) + arctan2(-(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x
) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - cos(2*x)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*c
os(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + si
n(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 1) - arctan2(-(cos(4*x)
^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1))*sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (co
s(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))
) - 1) - arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*
arctan2(sin(4*x), cos(4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + arctan2((cos(4*x)
^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos(4*x)
 + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(s
in(4*x), cos(4*x) + 1)) - 1))
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{1}{4}\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(x\right)\right) + \frac{1}{2}\sqrt{-2\sin\left(x\right)^2+1}\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*sin(x)
```

3.429.9 Mupad [F(-1)]

Timed out.

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \sqrt{\cos\left(2\,x\right)}\,\cos\left(x\right)\,dx$$

input
```
int(cos(2*x)^(1/2)*cos(x),x)
```

output
```
int(cos(2*x)^(1/2)*cos(x), x)
```

3.430 $\int \cos^{\frac{3}{2}}(2x) \sin(x)\,dx$

3.430.1 Optimal result

Integrand size = 11, antiderivative size = 55

$$\int \cos^{\frac{3}{2}}(2x) \sin(x)\,dx = -\frac{3\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}} + \frac{3}{8}\cos(x)\sqrt{\cos(2x)} - \frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)$$

output
```
-1/4*cos(x)*cos(2*x)^(3/2)-3/16*arctanh(cos(x)*2^(1/2)/cos(2*x)^(1/2))*2^(
1/2)+3/8*cos(x)*cos(2*x)^(1/2)
```

3.430.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \cos^{\frac{3}{2}}(2x) \sin(x)\,dx = -\frac{1}{8}\sqrt{\cos(2x)}(-2\cos(x) + \cos(3x)) - \frac{3\log\left(\sqrt{2}\cos(x) + \sqrt{\cos(2x)}\right)}{8\sqrt{2}}$$

input
```
Integrate[Cos[2*x]^(3/2)*Sin[x],x]
```

output
```
-1/8*(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x])) - (3*Log[Sqrt[2]*Cos[x] + Sqr
t[Cos[2*x]]])/(8*Sqrt[2])
```

3.430.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4857, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^{\frac{3}{2}}(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)\cos(2x)^{3/2}dx$$

$$\Big\downarrow \; 4857$$

$$-\int \left(2\cos^2(x)-1\right)^{3/2}d\cos(x)$$

$$\Big\downarrow \; 211$$

$$\frac{3}{4}\int \sqrt{2\cos^2(x)-1}d\cos(x)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

$$\Big\downarrow \; 211$$

$$\frac{3}{4}\left(\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)-1}-\frac{1}{2}\int \frac{1}{\sqrt{2\cos^2(x)-1}}d\cos(x)\right)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

$$\Big\downarrow \; 224$$

$$\frac{3}{4}\left(\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)-1}-\frac{1}{2}\int \frac{1}{1-\frac{2\cos^2(x)}{2\cos^2(x)-1}}d\frac{\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{4}\left(\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)-1}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)}{2\sqrt{2}}\right)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

input
```
Int[Cos[2*x]^(3/2)*Sin[x],x]
```

output
```
-1/4*(Cos[x]*(-1 + 2*Cos[x]^2)^(3/2)) + (3*(-1/2*ArcTanh[(Sqrt[2]*Cos[x])/
Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2] + (Cos[x]*Sqrt[-1 + 2*Cos[x]^2])/2))/4
```

3.430.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.430.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{3 \ln\left(\cos(x)\sqrt{2}+\sqrt{2(\cos^2(x))-1}\right)\sqrt{2}}{16}-\frac{(\cos^3(x))\sqrt{2(\cos^2(x))-1}}{2}+\frac{5\cos(x)\sqrt{2(\cos^2(x))-1}}{8}$	55

input
```
int(cos(2*x)^(3/2)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-3/16*ln(cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)-1/2*cos(x)^3*(2*cos(
x)^2-1)^(1/2)+5/8*cos(x)*(2*cos(x)^2-1)^(1/2)
```

3.430.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$$
$$= -\frac{1}{8}\left(4\cos(x)^{3} - 5\cos(x)\right)\sqrt{2\cos(x)^{2} - 1} + \frac{3}{128}\sqrt{2}\log\Big(2048\cos(x)^{8} - 2048\cos(x)^{6} + 640\cos(x)^{4} - 64\cos(x)^{2} - 8\left(128\sqrt{2}\cos(x)^{7} - 96\sqrt{2}\cos(x)^{5} + 20\sqrt{2}\cos(x)^{3} - \sqrt{2}\cos(x)\right)\sqrt{2\cos(x)^{2} - 1} + 1\Big)$$

input
```
integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")
```

output
```
-1/8*(4*cos(x)^3 - 5*cos(x))*sqrt(2*cos(x)^2 - 1) + 3/128*sqrt(2)*log(2048
*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8*(128*sqrt(2)*co
s(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(
2*cos(x)^2 - 1) + 1)
```

3.430.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \int \sin(x)\cos^{\frac{3}{2}}(2x)\,dx$$

input
```
integrate(cos(2*x)**(3/2)*sin(x),x)
```

output
```
Integral(sin(x)*cos(2*x)**(3/2), x)
```

3.430.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(39) = 78.

Time = 0.34 (sec) , antiderivative size = 790, normalized size of antiderivative = 14.36

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \text{Too large to display}$$

input
```
integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
```

output
```
-1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(((cos(
4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x)*sin(1/2*arctan2(
sin(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x) +
1)) - (cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - (cos(4*x) - 2)*sin(
1/2*arctan2(sin(4*x), cos(4*x))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos
(4*x) + 1))) + 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/
2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*co
s(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + s
in(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 + 1) - 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arcta
n2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) +
 3*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x)
, cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arct
an2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1
/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*co
s(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*
arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + si
n(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*...
```

3.430.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{1}{8}\left(4\cos(x)^2-5\right)\sqrt{2\cos(x)^2-1}\cos(x) + \frac{3}{16}\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)$$

input
```
integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="giac")
```

output
```
-1/8*(4*cos(x)^2 - 5)*sqrt(2*cos(x)^2 - 1)*cos(x) + 3/16*sqrt(2)*log(abs(-
sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1)))
```

3.430.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{\cos\left(2\,x\right)^{3/2}\,\cos\left(x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \cos\left(2\,x\right)+1\right)}{{\left(-\cos\left(2\,x\right)\right)}^{3/2}}$$

input
```
int(cos(2*x)^(3/2)*sin(x),x)
```

output
```
-(cos(2*x)^(3/2)*cos(x)*hypergeom([-3/2, 1/2], 3/2, cos(2*x) + 1))/(-cos(2
*x))^(3/2)
```

3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$

3.431.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

output
```
-1/3*cos(3*x)/cos(2*x)^(3/2)
```

3.431.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

input
```
Integrate[Sin[x]/Cos[2*x]^(5/2),x]
```

output
```
-1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

3.431.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4819}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)}{\cos(2x)^{5/2}}dx$$
$$\Big\downarrow \text{4819}$$
$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

input
```
Int[Sin[x]/Cos[2*x]^(5/2),x]
```

output
```
-1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

3.431.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4819
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> Simp[(-(m + 2))*(e*Cos[a + b*x])^(m + 1)*(Cos[(m + 1)*(a + b*x)]/(d*e*
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

3.431.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

method	result	size
default	$\frac{\sqrt{1-2\left(\sin^{2}\left(x\right)\right)}\,\cos\left(x\right)\left(4\left(\sin^{2}\left(x\right)\right)-1\right)}{12\left(\sin^{4}\left(x\right)\right)-12\left(\sin^{2}\left(x\right)\right)+3}$	39

input
```
int(sin(x)/cos(2*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(4*sin(x)^4-4*sin(x)^2+1)*(1-2*sin(x)^2)^(1/2)*cos(x)*(4*sin(x)^2-1)
```

3.431.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\left(4\,\cos\left(x\right)^{3}-3\,\cos\left(x\right)\right)\sqrt{2\,\cos\left(x\right)^{2}-1}}{3\,\left(4\,\cos\left(x\right)^{4}-4\,\cos\left(x\right)^{2}+1\right)}$$

input
```
integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(4*cos(x)^3 - 3*cos(x))*sqrt(2*cos(x)^2 - 1)/(4*cos(x)^4 - 4*cos(x)^2
 + 1)
```

3.431.6 Sympy [F]

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = \int \frac{\sin\left(x\right)}{\cos^{\frac{5}{2}}\left(2x\right)}\,dx$$

input
```
integrate(sin(x)/cos(2*x)**(5/2),x)
```

output
```
Integral(sin(x)/cos(2*x)**(5/2), x)
```

3.431.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.62

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx =$$
$$-\frac{\sqrt{2}\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)+1\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)\right)\right)+\left(\sqrt{2}\cos\left(\frac{3}{2}\arctan\left(\sin\right.\right.\right.}{3\left(\cos\left(4x\right)^{2}+\sin\left(4x\right)^{2}+2\cos\left(4x\right)+1\right)}$$

input
```
integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(2)*sin(3/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(3/2*arctan2(sin
(4*x), cos(4*x))) + (sqrt(2)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + sqrt(2
))*cos(3/2*arctan2(sin(4*x), cos(4*x) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*
cos(4*x) + 1)^(3/4)
```

3.431.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\left(4\cos\left(x\right)^{2}-3\right)\cos\left(x\right)}{3\left(2\cos\left(x\right)^{2}-1\right)^{\frac{3}{2}}}$$

input
```
integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(4*cos(x)^2 - 3)*cos(x)/(2*cos(x)^2 - 1)^(3/2)
```

3.431.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos\left(3x\right)}{3\cos\left(2x\right)^{3/2}}$$

input
```
int(sin(x)/cos(2*x)^(5/2),x)
```

output
```
-cos(3*x)/(3*cos(2*x)^(3/2))
```

3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$

3.432.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = 2\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x)$$

output
```
-5/2*arctan(sin(x)/cos(2*x)^(1/2))+2*arcsin(sin(x)*2^(1/2))*2^(1/2)-1/2*se
c(x)*cos(2*x)^(1/2)*tan(x)
```

3.432.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - 5\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \sqrt{\cos(2x)}\sec(x)\tan(x)\right)$$

input
```
Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

output
```
(4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]] - Sqrt
[Cos[2*x]]*Sec[x]*Tan[x])/2
```

3.432.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 4864, 315, 25, 398, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(2x)^{3/2}}{\cos(x)^3}dx$$

$$\Big\downarrow \; 4864$$

$$\int \frac{\left(1-2\sin^2(x)\right)^{3/2}}{\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \; 315$$

$$-\frac{1}{2}\int -\frac{3-8\sin^2(x)}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)-\frac{\sqrt{1-2\sin^2(x)}\sin(x)}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\int \frac{3-8\sin^2(x)}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \; 398$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-2\sin^2(x)}}d\sin(x)-5\int \frac{1}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \; 223$$

$$\frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)-5\int \frac{1}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)-5\int \frac{1}{\frac{\sin^2(x)}{1-2\sin^2(x)}+1}d\frac{\sin(x)}{\sqrt{1-2\sin^2(x)}}\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

$$\downarrow 216$$

$$\frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)-5\arctan\left(\frac{\sin(x)}{\sqrt{1-2\sin^2(x)}}\right)\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

input
```
Int[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

output
```
(4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[1 - 2*Sin[x]^2]])
/2 - (Sin[x]*Sqrt[1 - 2*Sin[x]^2])/(2*(1 - Sin[x]^2))
```

3.432.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.432.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(37) = 74$.

Time = 0.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{(2(\cos^2(x))-1)(\sin^2(x))}\left(4\sqrt{2}\arcsin\left(4(\cos^2(x))-3\right)(\cos^2(x))-5\arctan\left(\frac{3(\cos^2(x))-2}{2\sqrt{-2(\sin^4(x))+\sin^2(x)}}\right)(\cos^2(x))+2\sqrt{-2(\sin^4(x)}\right.}{4\cos(x)^2\sin(x)\sqrt{2(\cos^2(x))-1}}$

input
```
int(cos(2*x)^(3/2)/cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2-3)*cos(x
)^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(1/2))*cos(x)^2+2*(
-2*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2*cos(x)^2-1)^(1/2)
```

3.432.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.41

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)\cos(x)^2-5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{2\cos(x)^2-1}\sin(x)}\right)\cos(x}{4\cos(x)^2}$$

input
```
integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17
*sqrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x)))*co
s(x)^2 - 5*arctan(1/2*(3*cos(x)^2 - 2)/(sqrt(2*cos(x)^2 - 1)*sin(x)))*cos(
x)^2 + 2*sqrt(2*cos(x)^2 - 1)*sin(x))/cos(x)^2
```

3.432.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \text{Timed out}$$

input
```
integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

output
```
Timed out
```

3.432.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

input
```
integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")
```

output
```
integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

3.432.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

input
```
integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="giac")
```

output
```
integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{3/2}}{\cos(x)^3}\,dx$$

input
```
int(cos(2*x)^(3/2)/cos(x)^3,x)
```

output
```
int(cos(2*x)^(3/2)/cos(x)^3, x)
```

3.433 $\int \frac{\sin^2(x)(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx$

3.433.1 Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{\sin^2(x)\,(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}} - \frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)} - \frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{63\cos(x)}{20\sqrt{\cos(2x)}} + \frac{3\cos(x)\sin^2(x)}{10\cos^{\frac{5}{2}}(2x)}$$

output
```
-11/20*cos(x)/cos(2*x)^(3/2)-2/3*cos(x)^3/cos(2*x)^(3/2)+3/10*cos(x)*sin(x
)^2/cos(2*x)^(5/2)-1/2*arctanh(cos(x)*2^(1/2)/cos(2*x)^(1/2))*2^(1/2)+63/2
0*cos(x)/cos(2*x)^(1/2)
```

3.433.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)\,(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx = \frac{250\cos(x)+45\cos(3x)+169\cos(5x)-120\sqrt{2}\cos^{\frac{5}{2}}(2x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{240\cos^{\frac{5}{2}}(2x)}$$

input
```
Integrate[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

output
```
(250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(5/2)*Log[
Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(240*Cos[2*x]^(5/2))
```

3.433.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.80, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4877, 25, 3042, 4866, 292, 292, 208, 4879, 27, 357, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\sin(4x)\cos(x)\right)}{\cos^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2\left(3\sin(x)^3-\sin(4x)\cos(x)\right)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4877}$$

$$3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}dx+\int -\frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{25}$$

$$3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}dx-\int \frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$3\int \frac{\sin(x)^5}{\cos(2x)^{7/2}}dx-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4866}$$

$$-3\int \frac{\left(1-\cos^2(x)\right)^2}{\left(2\cos^2(x)-1\right)^{7/2}}d\cos(x)-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \text{292}$$

$$-3\left(-\frac{4}{5}\int \frac{1-\cos^2(x)}{\left(2\cos^2(x)-1\right)^{5/2}}d\cos(x)-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}\right)-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \text{292}$$

$$-3\left(-\frac{4}{5}\left(-\frac{2}{3}\int \frac{1}{\left(2\cos^2(x)-1\right)^{3/2}}d\cos(x)-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}\right)-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \text{208}$$

$$-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{4879}$$

$$\int \frac{4\cos^2(x)\left(1-\cos^2(x)\right)}{\left(2\cos^2(x)-1\right)^{5/2}}d\cos(x)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\cos^2(x)\left(1-\cos^2(x)\right)}{\left(2\cos^2(x)-1\right)^{5/2}}d\cos(x)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{357}$$

$$4\left(-\frac{1}{2}\int \frac{\cos^2(x)}{\left(2\cos^2(x)-1\right)^{3/2}}d\cos(x)-\frac{\cos^3(x)}{6\left(2\cos^2(x)-1\right)^{3/2}}\right)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{252}$$

$$4\left(\frac{1}{2}\left(\frac{\cos(x)}{2\sqrt{2\cos^2(x)-1}}-\frac{1}{2}\int \frac{1}{\sqrt{2\cos^2(x)-1}}d\cos(x)\right)-\frac{\cos^3(x)}{6\left(2\cos^2(x)-1\right)^{3/2}}\right)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{224}$$

3.433. $\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}dx$

$$4\left(\frac{1}{2}\left(\frac{\cos(x)}{2\sqrt{2\cos^2(x)-1}}-\frac{1}{2}\int\frac{1}{1-\frac{2\cos^2(x)}{2\cos^2(x)-1}}d\frac{\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)-\frac{\cos^3(x)}{6\left(2\cos^2(x)-1\right)^{3/2}}\right)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{1}{2}\left(\frac{\cos(x)}{2\sqrt{2\cos^2(x)-1}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)}{2\sqrt{2}}\right)-\frac{\cos^3(x)}{6\left(2\cos^2(x)-1\right)^{3/2}}\right)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

input
```
Int[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

output
```
4*(-1/6*Cos[x]^3/(-1 + 2*Cos[x]^2)^(3/2) + (-1/2*ArcTanh[(Sqrt[2]*Cos[x])/
Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2] + Cos[x]/(2*Sqrt[-1 + 2*Cos[x]^2]))/2) - 3*
(-1/5*(Cos[x]*(1 - Cos[x]^2)^2)/(-1 + 2*Cos[x]^2)^(5/2) - (4*(-1/3*(Cos[x]
*(1 - Cos[x]^2))/(-1 + 2*Cos[x]^2)^(3/2) + (2*Cos[x])/(3*Sqrt[-1 + 2*Cos[x
]^2])))/5)
```

3.433.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.433. $\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.433. $\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(65) = 130$.

Time = 2.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.07

method	result
default	$-\frac{120\sqrt{2}\,\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2(\sin^2(x))}\right)(\sin^6(x))+338\sqrt{1-2(\sin^2(x))}\,\cos(x)(\sin^4(x))-180\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2(\sin^2(x))}\right)\sqrt{2}\,(\mathrm{s}}{30(8(}$
parts	$-\frac{\sqrt{1-2(\sin^2(x))}\,\cos(x)(43(\sin^4(x))-36(\sin^2(x))+8)}{5(8(\sin^6(x))-12(\sin^4(x))+6(\sin^2(x))-1)}-\frac{12\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2(\sin^2(x))}\right)\sqrt{2}\,(\sin^4(x))+8\sqrt{1-2(\sin^2(x))}\,(\sin}{}$

input
```
int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/30/(8*sin(x)^6-12*sin(x)^4+6*sin(x)^2-1)*(120*2^(1/2)*ln(cos(x)*2^(1/2)
+(1-2*sin(x)^2)^(1/2))*sin(x)^6+338*(1-2*sin(x)^2)^(1/2)*cos(x)*sin(x)^4-1
80*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)^(1/2))*2^(1/2)*sin(x)^4-276*(1-2*sin(x
)^2)^(1/2)*sin(x)^2*cos(x)+90*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)^(1/2))*2^(1
/2)*sin(x)^2+58*(1-2*sin(x)^2)^(1/2)*cos(x)-15*ln(cos(x)*2^(1/2)+(1-2*sin(
x)^2)^(1/2))*2^(1/2))
```

3.433.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$$

$$=\frac{15\left(8\sqrt{2}\cos(x)^6-12\sqrt{2}\cos(x)^4+6\sqrt{2}\cos(x)^2-\sqrt{2}\right)\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(x\right.}{}$$

input
```
integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorith
m="fricas")
```

output
```
1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*cos(x)^2 -
 sqrt(2))*log(2048*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 -
 8*(128*sqrt(2)*cos(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqr
t(2)*cos(x))*sqrt(2*cos(x)^2 - 1) + 1) + 16*(169*cos(x)^5 - 200*cos(x)^3 +
 60*cos(x))*sqrt(2*cos(x)^2 - 1))/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 -
 1)
```

3.433.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
```

output
```
Timed out
```

3.433.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(65) = 130$.

Time = 0.41 (sec) , antiderivative size = 1359, normalized size of antiderivative = 15.62

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \text{Too large to display}$$

input
```
integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorith
m="maxima")
```

output
```
1/48*(4*(4*sqrt(2)*sin(4*x)*sin(5/2*arctan2(sin(4*x), cos(4*x))) + 4*(sqrt
(2)*cos(4*x) + sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 3*sqrt(2)*c
os(8*x) + 7*sqrt(2)*cos(4*x) + 4*sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*
x) + 1)) + 12*sqrt(2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3
/2*arctan2(sin(4*x), cos(4*x) + 1)) - 12*(sqrt(2)*cos(4*x)^2 + sqrt(2)*sin
(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*cos(1/2*arctan2(sin(4*x), cos(4*x)
 + 1)) - 4*(4*sqrt(2)*cos(5/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - 4*(s
qrt(2)*cos(4*x) + sqrt(2))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 3*sqrt(2
)*sin(8*x) - 7*sqrt(2)*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1))
- 3*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*((sqrt(2)*cos(4*x)^2
+ sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 +
 sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 +
 sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x),
cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(
1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*
sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(
4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) +
 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan
2(sin(4*x), cos(4*x) + 1)) + 1) + (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x...
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}\cos\left(x\right)+\sqrt{2\cos\left(x\right)^2-1}\right|\right) + \frac{\left(\left(169\cos\left(x\right)^2-200\right)\cos\left(x\right)^2+60\right)\cos\left(x\right)}{15\left(2\cos\left(x\right)^2-1\right)^{\frac{5}{2}}}$$

input
```
integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorith
m="giac")
```

output
```
1/2*sqrt(2)*log(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/15*((169*
cos(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5/2)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)\,(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx=\int \frac{\sin\left(x\right)^2\,\left(3\,\sin\left(x\right)^3-\sin\left(4\,x\right)\,\cos\left(x\right)\right)}{\cos\left(2\,x\right)^{7/2}}\,dx$$

input
```
int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2),x)
```

output
```
int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2), x)
```

3.434 $\int (4-5\sec^2(x))^{3/2}\,dx$

3.434.1 Optimal result

Integrand size = 12, antiderivative size = 68

$$\int (4-5\sec^2(x))^{3/2}\,dx = 8\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{7}{2}\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)}$$

output
```
8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-7/2*arctan(5^(1/2)*tan(x)/(-1-5*t
an(x)^2)^(1/2))*5^(1/2)-5/2*(-1-5*tan(x)^2)^(1/2)*tan(x)
```

3.434.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.69

$$\int (4-5\sec^2(x))^{3/2}\,dx = -\frac{(-5+4\cos^2(x))\sec(x)\sqrt{4-5\sec^2(x)}\left(7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sin(x)}{\sqrt{-3+2\cos(2x)}}\right)\cos^2(x)+16i\cos^2(x)\log\left(\sqrt{-3+2\,\mathrm{co}\right.\right.}{2(-3+2\cos(2x))^{3/2}}$$

input
```
Integrate[(4 - 5*Sec[x]^2)^(3/2),x]
```

output
```
-1/2*((-5 + 4*Cos[x]^2)*Sec[x]*Sqrt[4 - 5*Sec[x]^2]*(7*Sqrt[5]*ArcTan[(Sqr
t[5]*Sin[x])/Sqrt[-3 + 2*Cos[2*x]]]*Cos[x]^2 + (16*I)*Cos[x]^2*Log[Sqrt[-3
 + 2*Cos[2*x]] + (2*I)*Sin[x]] + 5*Sqrt[-3 + 2*Cos[2*x]]*Sin[x]))/(-3 + 2*
Cos[2*x])^(3/2)
```

3.434.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4616, 318, 25, 398, 224, 216, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sec^2(x)\right)^{3/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(4-5 \sec(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4616}$$

$$\int \frac{\left(-5 \tan^2(x)-1\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{318}$$

$$\frac{1}{2} \int -\frac{35 \tan^2(x)+3}{\sqrt{-5 \tan^2(x)-1} \left(\tan^2(x)+1\right)}d\tan(x)-\frac{5}{2} \tan(x) \sqrt{-5 \tan^2(x)-1}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2} \int \frac{35 \tan^2(x)+3}{\sqrt{-5 \tan^2(x)-1} \left(\tan^2(x)+1\right)}d\tan(x)-\frac{5}{2} \sqrt{-5 \tan^2(x)-1} \tan(x)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2} \left(32 \int \frac{1}{\sqrt{-5 \tan^2(x)-1} \left(\tan^2(x)+1\right)}d\tan(x)-35 \int \frac{1}{\sqrt{-5 \tan^2(x)-1}}d\tan(x)\right)-\frac{5}{2} \tan(x) \sqrt{-5 \tan^2(x)-1}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(32\int \frac{1}{\sqrt{-5\tan^2(x)-1}\,(\tan^2(x)+1)}d\tan(x)-35\int \frac{1}{\frac{5\tan^2(x)}{-5\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(32\int \frac{1}{\sqrt{-5\tan^2(x)-1}\,(\tan^2(x)+1)}d\tan(x)-7\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(32\int \frac{1}{\frac{4\tan^2(x)}{-5\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-5\tan^2(x)-1}}-7\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(16\arctan\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-7\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

input
```
Int[(4 - 5*Sec[x]^2)^(3/2),x]
```

output
```
(16*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - 7*Sqrt[5]*ArcTan[(Sqrt[5]*T
an[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2
```

3.434.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.434.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(54) = 108$.

Time = 7.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.19

method	result
default	$-\frac{(4-5(\sec^2(x)))^{\frac{3}{2}}\left(7(\cos^3(x))\sqrt{5}\arctan\left(\frac{(4\sin(x)-1)\sqrt{5}}{5(\cos(x)+1)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)+7(\cos^3(x))\sqrt{5}\arctan\left(\frac{(4\sin(x)+1)\sqrt{5}}{5(\cos(x)+1)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)-32\right.}{4(4(\cos^2(x))-5)\sqrt{\frac{4(\cos^2(x)}{(\cos(x)+}}}$

input
```
int((4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4-5*sec(x)^2)^(3/2)/(4*cos(x)^2-5)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/
2)/(cos(x)+1)*(7*cos(x)^3*5^(1/2)*arctan(1/5*(4*sin(x)-1)/(cos(x)+1)/((4*c
os(x)^2-5)/(cos(x)+1)^2)^(1/2)*5^(1/2))+7*cos(x)^3*5^(1/2)*arctan(1/5*(4*s
in(x)+1)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)*5^(1/2))-32*cos(x)
^3*arctan(2*sin(x)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))+10*cos(
x)^2*sin(x)*((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)+10*cos(x)*sin(x)*((4*cos(x
)^2-5)/(cos(x)+1)^2)^(1/2))
```

3.434.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(54) = 108$.

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.91

$$\int (4-5\sec^2(x))^{3/2}\,dx = \frac{7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)}{5\sin(x)}\right)\cos(x)+8\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x}{64\cos(x)^4-143\cos(x)^2+80}\right.}{2\cos(x)}$$

input
```
integrate((4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(7*sqrt(5)*arctan(1/5*sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)/s
in(x))*cos(x) + 8*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/
cos(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80))*cos(x
) - 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin
(x))/cos(x)
```

3.434.6 Sympy [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(4-5\sec^2{(x)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sec(x)**2)**(3/2),x)
```

output
```
Integral((4 - 5*sec(x)**2)**(3/2), x)
```

3.434.7 Maxima [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sec\left(x\right)^{2}+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

3.434.8 Giac [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sec\left(x\right)^{2}+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(4-\frac{5}{\cos{(x)}^2}\right)^{3/2}\,dx$$

input
```
int((4 - 5/cos(x)^2)^(3/2),x)
```

output
```
int((4 - 5/cos(x)^2)^(3/2), x)
```

3.435 $\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx$

3.435.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}}$$

output
```
1/8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-5/4*tan(x)/(-1-5*tan(x)^2)^(1/2
)
```

3.435.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = -\frac{(-3+2\cos(2x))^{3/2}\sec^3(x)\left(\text{arcsinh}(2\sin(x))(-3+2\cos(2x))+10\sqrt{3-2\cos(2x)}\sin(x)\right)}{8(4-5\sec^2(x))^{3/2}\sqrt{-(1+4\sin^2(x))^2}}$$

input
```
Integrate[(4 - 5*Sec[x]^2)^(-3/2),x]
```

output
```
-1/8*((-3 + 2*Cos[2*x])^(3/2)*Sec[x]^3*(ArcSinh[2*Sin[x]]*(-3 + 2*Cos[2*x]
) + 10*Sqrt[3 - 2*Cos[2*x]]*Sin[x]))/((4 - 5*Sec[x]^2)^(3/2)*Sqrt[-(1 + 4*
Sin[x]^2)^2])
```

3.435.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(4-5\sec(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\int \frac{1}{(-5\tan^2(x)-1)^{3/2}(\tan^2(x)+1)}d\tan(x)$$

$$\Big\downarrow \text{ 296}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{-5\tan^2(x)-1}(\tan^2(x)+1)}d\tan(x)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{4}\int \frac{1}{\frac{4\tan^2(x)}{-5\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-5\tan^2(x)-1}}-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

input
```
Int[(4 - 5*Sec[x]^2)^(-3/2),x]
```

output
```
ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan
[x]^2])
```

3.435.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.435.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(32) = 64$.

Time = 1.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.10

method	result
default	$\frac{(\sec^3(x))(4(\cos^2(x))-5)\left(\arctan\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}\cos(x)+\arctan\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\sqrt{\frac{4(\cos}{(\cos}}\right.}{8(4-5(\sec^2(x)))^{\frac{3}{2}}}$

input
```
int(1/(4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*sec(x)^3*(4*cos(x)^2-5)*(arctan(2*sin(x)/(cos(x)+1)/((4*cos(x)^2-5)/(c
os(x)+1)^2)^(1/2))*((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)*cos(x)+arctan(2*sin
(x)/(cos(x)+1)/((4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*((4*cos(x)^2-5)/(cos(x
)+1)^2)^(1/2)-10*sin(x))/(4-5*sec(x)^2)^(3/2)
```

3.435.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx =$$
$$-\frac{20\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)\sin(x)-\left(4\cos(x)^2-5\right)\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+\cos(x)\sin(x)}{64\cos(x)^4-143\cos(x)^2+80}\right)+}{16\left(4\cos(x)^2-5\right)}$$

input
```
integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(x)^2 - 5)
*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x)
+ cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80)) + (4*cos(x)^2 - 5)*arc
tan(sin(x)/cos(x)))/(4*cos(x)^2 - 5)
```

3.435.6 Sympy [F]

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{(4-5\sec^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sec(x)**2)**(3/2),x)
```

output
```
Integral((4 - 5*sec(x)**2)**(-3/2), x)
```

3.435.7 Maxima [F]

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{(-5\,\sec(x)^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

3.435.8 Giac [F]

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{(-5\,\sec(x)^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{\left(4-\frac{5}{\cos(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(4 - 5/cos(x)^2)^(3/2),x)
```

output
```
int(1/(4 - 5/cos(x)^2)^(3/2), x)
```

3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$

3.436.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{1}{4}\text{arctanh}\left(\frac{2\tan(x)}{\sqrt{1+5\tan^2(x)}}\right)-\frac{\cos(x)}{4\sqrt{1+5\tan^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}}-\frac{1}{8}\cos(x)\sqrt{1+5\tan^2(x)}+\frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)}$$

output
```
-1/4*arctanh(2*tan(x)/(1+5*tan(x)^2)^(1/2))-1/4*cos(x)/(1+5*tan(x)^2)^(1/2
)-5/2*cot(x)/(1+5*tan(x)^2)^(1/2)-1/8*cos(x)*(1+5*tan(x)^2)^(1/2)+9/2*cot(
x)*(1+5*tan(x)^2)^(1/2)
```

3.436.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{(-3+2\cos(2x))^{3/2}\left(-1+2\cot^2(x)\csc(x)\right)\sin^2(x)\left(-2\text{arcsinh}(2\sin(x))\left(4+\csc^2(x)\right)+(-2+164\csc(x)\right.}{2\sqrt{-(3-2\cos(2x))^2}\left(5+\cot^2(x)\right)(4+4\cos(2x)-3\sin(x)+\sin(3x))}$$

input
```
Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

output
```
-1/2*((-3 + 2*Cos[2*x])^(3/2)*(-1 + 2*Cot[x]^2*Csc[x])*Sin[x]^2*(-2*ArcSin
h[2*Sin[x]]*(4 + Csc[x]^2) + (-2 + 164*Csc[x] - 3*Csc[x]^2 + 16*Csc[x]^3)*
Sqrt[1 + 4*Sin[x]^2])*Tan[x])/(Sqrt[-(3 - 2*Cos[2*x])^2]*(5 + Cot[x]^2)*(4
 + 4*Cos[2*x] - 3*Sin[x] + Sin[3*x])*Sqrt[1 + 5*Tan[x]^2])
```

3.436.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4877, 27, 3042, 4147, 245, 208, 4153, 374, 25, 445, 25, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)-2\cot^2(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)-2\cot(x)^2}{\left(5\tan(x)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{\sin(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx+\int -\frac{2\cot^2(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx-2\int \frac{\cot^2(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\left(5\tan(x)^2+1\right)^{3/2}}dx-2\int \frac{1}{\tan(x)^2\left(5\tan(x)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\int \frac{\cos^2(x)}{\left(5\sec^2(x)-4\right)^{3/2}}d\sec(x)-2\int \frac{1}{\tan(x)^2\left(5\tan(x)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{245}$$

$$-2\int \frac{1}{\tan(x)^2\left(5\tan(x)^2+1\right)^{3/2}}dx+\frac{5}{2}\int \frac{1}{\left(5\sec^2(x)-4\right)^{3/2}}d\sec(x)+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{208}$$

$$-2\int \frac{1}{\tan(x)^2\left(5\tan(x)^2+1\right)^{3/2}}dx-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{4153}$$

$$-2\int \frac{\cot^2(x)}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{3/2}}d\tan(x)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{374}$$

$$-2\left(\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}-\frac{1}{4}\int -\frac{\cot^2(x)\left(10\tan^2(x)+9\right)}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan(x)\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{25}$$

$$-2\left(\frac{1}{4}\int \frac{\cot^2(x)\left(10\tan^2(x)+9\right)}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan(x)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{445}$$

$$-2\left(\frac{1}{4}\left(-\int -\frac{1}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan(x)-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{25}$$

$$-2\left(\frac{1}{4}\left(\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan(x)-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{291}$$

$$-2\left(\frac{1}{4}\left(\int \frac{1}{1-\frac{4\tan^2(x)}{5\tan^2(x)+1}}d\frac{\tan(x)}{\sqrt{5\tan^2(x)+1}}-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{219}$$

3.436. $\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$

$$-2\left(\frac{1}{4}\left(\frac{1}{2}\operatorname{arctanh}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

input
```
Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

output
```
Cos[x]/(4*Sqrt[-4 + 5*Sec[x]^2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) -
2*((5*Cot[x])/(4*Sqrt[1 + 5*Tan[x]^2]) + (ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Ta
n[x]^2]]/2 - 9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/4)
```

3.436.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.436. $\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.436. $\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx$

3.436.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(74) = 148$.

Time = 4.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.61

method	result
default	$-\frac{(\sec^3(x))\csc(x)(4(\cos^2(x))-5)\left(-2\cos(x)\sin(x)\operatorname{arctanh}\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{-\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right)\sqrt{-\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}+2(\cos^2(x))\sin(x)-2\right.}{8(5(\sec^2(x))-4)^{\frac{3}{2}}}$
parts	$-\frac{8\cos(x)-30\sec(x)+25(\sec^3(x))}{8(5(\sec^2(x))-4)^{\frac{3}{2}}\left(-2+\sqrt{5}\right)^2\left(2+\sqrt{5}\right)^2}+\frac{(\sec^3(x))\csc(x)(4(\cos^2(x))-5)\left(\cos(x)\sin(x)\operatorname{arctanh}\left(\frac{2\sin(x)}{(\cos(x)+1)\sqrt{-\frac{4(\cos^2(x))-5}{(\cos(x)+1)^2}}}\right.\right.}{}$

input
```
int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*sec(x)^3*csc(x)*(4*cos(x)^2-5)*(-2*cos(x)*sin(x)*arctanh(2*sin(x)/(co
s(x)+1)/(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^
2)^(1/2)+2*cos(x)^2*sin(x)-2*sin(x)*arctanh(2*sin(x)/(cos(x)+1)/(-(4*cos(x
)^2-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)-164*cos(x
)^2-5*sin(x)+180)/(5*sec(x)^2-4)^(3/2)
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \frac{2\left(4\cos(x)^2-5\right)\log\left(\sqrt{-\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)-2\sin(x)\right)\sin(x)+\left(164\cos(x)\right.}{8\left(4\cos(x)^2-5\right)\sin}$$

input
```
integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(x) - 2*si
n(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*sin(x) - 180*cos(x)
)*sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^2 - 5)*sin(x))
```

3.436.6 Sympy [F]

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = -\int \left(-\frac{\sin(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\right)dx - \int \frac{2\cot^2(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\,dx$$

input
```
integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

output
```
-Integral(-sin(x)/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 +
1)), x) - Integral(2*cot(x)**2/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5
*tan(x)**2 + 1)), x)
```

3.436.7 Maxima [F(-1)]

Timed out.

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.436.8 Giac [F]

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \int -\frac{2\cot(x)^2-\sin(x)}{(5\tan(x)^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2), x)
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx=\int \frac{\sin\left(x\right)-2\cot(x)^2}{\left(5\tan\left(x\right)^2+1\right)^{3/2}}\,dx$$

input
```
int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2),x)
```

output
```
int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2), x)
```

3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$

3.437.1 Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{2}{3}\sqrt{4-\cot^2(x)}\tan(x)-\frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x)$$

output
```
-2/3*(4-cot(x)^2)^(1/2)*tan(x)-1/3*(4-cot(x)^2)^(1/2)*tan(x)^3
```

3.437.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \frac{(3+\cos(2x))(-3+5\cos(2x))\csc(x)\sec^3(x)}{12\sqrt{4-\cot^2(x)}}$$

input
```
Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

output
```
((3 + Cos[2*x])*(-3 + 5*Cos[2*x])*Csc[x]*Sec[x]^3)/(12*Sqrt[4 - Cot[x]^2])
```

3.437.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4889, 27, 941, 955, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\cos(2x)-3)\sec^4(x)}{\sqrt{4-\cot^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2x)-3}{\cos(x)^4\sqrt{4-\cot(x)^2}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int -\frac{2(2\tan^2(x)+1)}{\sqrt{4-\cot^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{2\tan^2(x)+1}{\sqrt{4-\cot^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{941}$$

$$-2\int \frac{(\cot^2(x)+2)\tan^2(x)}{\sqrt{4-\cot^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{955}$$

$$-2\left(\frac{4}{3}\int \frac{1}{\sqrt{4-\cot^2(x)}}d\tan(x)+\frac{1}{6}\tan^3(x)\sqrt{4-\cot^2(x)}\right)$$

$$\Big\downarrow \text{746}$$

$$-2\left(\frac{1}{6}\tan^3(x)\sqrt{4-\cot^2(x)}+\frac{1}{3}\tan(x)\sqrt{4-\cot^2(x)}\right)$$

input
```
Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

output
```
-2*((Sqrt[4 - Cot[x]^2]*Tan[x])/3 + (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/6)
```

3.437.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.437. $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$

3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 2.73 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

method	result	size
default	$\frac{(\sec^3(x))\csc(x)(25(\cos^4(x))-10(\cos^2(x))-8)}{6\sqrt{-5(\cot^2(x))+4(\csc^2(x))}}-\frac{5\cot(x)-4\sec(x)\csc(x)}{2\sqrt{-5(\cot^2(x))+4(\csc^2(x))}}$	67

input
```
int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*sec(x)^3*csc(x)*(25*cos(x)^4-10*cos(x)^2-8)/(-5*cot(x)^2+4*csc(x)^2)^(
1/2)-1/2/(-5*cot(x)^2+4*csc(x)^2)^(1/2)*(5*cot(x)-4*sec(x)*csc(x))
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{\left(\cos(x)^2+1\right)\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2-1}}\sin(x)}{3\cos(x)^3}$$

input
```
integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 + 1)*sqrt((5*cos(x)^2 - 4)/(cos(x)^2 - 1))*sin(x)/cos(x)^3
```

3.437.6 Sympy [F]

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \int \frac{\cos(2x)-3}{\sqrt{-(\cot(x)-2)(\cot(x)+2)}\cos^4(x)}\,dx$$

input
```
integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)
```

output
```
Integral((cos(2*x) - 3)/(sqrt(-(cot(x) - 2)*(cot(x) + 2))*cos(x)**4), x)
```

3.437.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{1}{48}\left(-\frac{1}{\tan(x)^2}+4\right)^{\frac{3}{2}}\tan(x)^3 + \frac{3}{16}\sqrt{-\frac{1}{\tan(x)^2}+4}\tan(x) - \frac{8\tan(x)^4+26\tan(x)^2-7}{8\sqrt{2\tan(x)+1}\sqrt{2\tan(x)-1}}$$

input
```
integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(-1/tan(x)^2 + 4)^(3/2)*tan(x)^3 + 3/16*sqrt(-1/tan(x)^2 + 4)*tan(x)
 - 1/8*(8*tan(x)^4 + 26*tan(x)^2 - 7)/(sqrt(2*tan(x) + 1)*sqrt(2*tan(x) -
1))
```

3.437.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.46

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \frac{\frac{125\sqrt{5}\left(\frac{21\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^2}{\cos(x)^2}+125\right)\cos(x)^3}{\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^3} - \frac{\sqrt{5}\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^3}{\cos(x)^3} - \frac{105\sqrt{5}\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)}{\cos(x)}}{2400\,\mathrm{sgn}(\sin(x))} + \frac{2}{3}i\,\mathrm{sgn}(\sin(x))$$

input
```
integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2400*(125*sqrt(5)*(21*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^2/cos(
x)^2 + 125)*cos(x)^3/(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3 - sqrt(
5)*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3/cos(x)^3 - 105*sqrt(5)*(s
qrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))/cos(x))/sgn(sin(x)) + 2/3*I*sgn(
sin(x))
```

3.437.9 Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{\tan(x)\,\left(\tan(x)^2+2\right)\,\sqrt{4-\frac{1}{\tan(x)^2}}}{3}$$

input
```
int((cos(2*x) - 3)/(cos(x)^4*(4 - cot(x)^2)^(1/2)),x)
```

output
```
-(tan(x)*(tan(x)^2 + 2)*(4 - 1/tan(x)^2)^(1/2))/3
```

3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx$

3.438.1 Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\text{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

output
```
-1/18*arctanh(1/3*(5-4*sec(x)^2)^(1/2)*3^(1/2))*3^(1/2)-1/25*arctanh(1/5*(
5-4*sec(x)^2)^(1/2)*5^(1/2))*5^(1/2)-2/15/(5-4*sec(x)^2)^(1/2)
```

3.438.2 Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\left(5-4\sec^2(x)\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{-\cos^2(x)}\left(60\sqrt{-\cos^2(x)}+9\text{arcsinh}\left(\frac{1}{2}\sqrt{5}\sqrt{-\cos^2(x)}\right)\sqrt{30-50\cos(2x)}+25\text{arctanh}\left(\frac{\sqrt{3}\sqrt{-\cos^2(x)}}{\sqrt{-1+5\sin^2(x)}}\right)\right.}{450\left(-1+5\sin^2(x)\right)}$$

input
```
Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2
)),x]
```

output
```
-1/450*(Sqrt[-Cos[x]^2]*(60*Sqrt[-Cos[x]^2] + 9*ArcSinh[(Sqrt[5]*Sqrt[-Cos
[x]^2])/2]*Sqrt[30 - 50*Cos[2*x]] + 25*ArcTanh[(Sqrt[3]*Sqrt[-Cos[x]^2])/S
qrt[-1 + 5*Sin[x]^2]]*Sqrt[-3 + 15*Sin[x]^2])*Sqrt[Sec[x]^2 - 5*Tan[x]^2])
/(-1 + 5*Sin[x]^2)
```

3.438.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4873, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\sin^2(x)+3)\tan^3(x)}{(\cos^2(x)-2)(5-4\sec^2(x))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(\sin(x)^2+3)\tan(x)^3}{(\cos(x)^2-2)(5-4\sec(x)^2)^{3/2}}dx$$
$$\Big\downarrow \text{4873}$$
$$-\int -\frac{(1-\cos^2(x))(4-\cos^2(x))\sec^3(x)}{(2-\cos^2(x))(5-4\sec^2(x))^{3/2}}d\cos(x)$$
$$\Big\downarrow \text{25}$$
$$\int \frac{(1-\cos^2(x))(4-\cos^2(x))\sec^3(x)}{(2-\cos^2(x))(5-4\sec^2(x))^{3/2}}d\cos(x)$$
$$\Big\downarrow \text{7276}$$
$$\int \left(-\frac{3\sec(x)}{2(5-4\sec^2(x))^{3/2}}+\frac{2\sec^3(x)}{(5-4\sec^2(x))^{3/2}}+\frac{\cos(x)}{2(\cos^2(x)-2)(5-4\sec^2(x))^{3/2}}\right)d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}-\frac{2}{15\sqrt{5-4\sec^2(x)}}$$

input
```
Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

output
```
-1/6*ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/Sqrt[3] - ArcTanh[Sqrt[5 - 4*Se
c[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])
```

3.438.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4873
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-(b*c*d^(n - 1))^(-1)   Subst[Int[Subst
For[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*
(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
 c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan]
)
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.438.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1497 vs. $2(55) = 110$.

Time = 2.00 (sec) , antiderivative size = 1498, normalized size of antiderivative = 20.52

method	result	size
default	Expression too large to display	1498

input
```
int((3+sin(x)^2)*tan(x)^3/(cos(x)^2-2)/(5-4*sec(x)^2)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
3/5*sec(x)^3*(5*cos(x)^2-4)*(50*3^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)
^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*
cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*cos(x)+50*3^(1/2)*2^(
1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/
2)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-
6^(1/2)))*cos(x)-25*6^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*ar
ctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*cos(x)^2-4)/
(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*cos(x)+25*6^(1/2)*2^(1/2)*((5*cos
(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)
-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*co
s(x)+100*3^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(
1/2)+4*2^(1/2)+10*cos(x)+4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)
/(2*3^(1/2)+6^(1/2)))*cos(x)-100*3^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/
2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x)^
2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)-6^(1/2)))*cos(x)-50*6^(1/2)*((5*cos(x)
^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)
/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*3^(1/2)+6^(1/2)))*cos(x
)-50*6^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)
+4*2^(1/2)-10*cos(x)-4)/(cos(x)+1)/((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)/(2*
3^(1/2)-6^(1/2)))*cos(x)+72*5^(1/2)*((5*cos(x)^2-4)/(cos(x)+1)^2)^(1/2)...
```

3.438.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(55) = 110$.

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.52

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx =$$

$$-\frac{480\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}\cos(x)^2 - 18\left(5\sqrt{5}\cos(x)^2-4\sqrt{5}\right)\log\left(625\cos(x)^8-1000\cos(x)^6+500\cos(x)^4-8\right.}{}$$

input
```
integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqrt(5)*cos(
x)^2 - 4*sqrt(5))*log(625*cos(x)^8 - 1000*cos(x)^6 + 500*cos(x)^4 - 80*cos
(x)^2 - (125*sqrt(5)*cos(x)^8 - 150*sqrt(5)*cos(x)^6 + 50*sqrt(5)*cos(x)^4
 - 4*sqrt(5)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 2) - 25*(5*sqrt(3
)*cos(x)^2 - 4*sqrt(3))*log((1921*cos(x)^8 - 3464*cos(x)^6 + 2040*cos(x)^4
 - 416*cos(x)^2 - 8*(62*sqrt(3)*cos(x)^8 - 87*sqrt(3)*cos(x)^6 + 36*sqrt(3
)*cos(x)^4 - 4*sqrt(3)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 16)/(co
s(x)^8 - 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2 + 16)))/(5*cos(x)^2 - 4)
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.438.7 Maxima [F]

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = \int \frac{(\sin(x)^2+3)\tan(x)^3}{(\cos(x)^2-2)\,(-4\sec(x)^2+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2))
, x)
```

3.438.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(55) = 110$.

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx =$$

$$-\frac{5\sqrt{15}\sqrt{5}\log\left(-\frac{2\left(\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2-4\sqrt{15}-16\right)}{\left|2\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2+8\sqrt{15}-32\right|}\right)-18\sqrt{5}\log\left(\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2\right)}{900\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algo
rithm="giac")
```

output
```
-1/900*(5*sqrt(15)*sqrt(5)*log(-2*((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))
^2 - 4*sqrt(15) - 16)/abs(2*(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*
sqrt(15) - 32)) - 18*sqrt(5)*log((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2
) + 120*cos(x)/sqrt(5*cos(x)^2 - 4))/sgn(cos(x))
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = \int \frac{\tan(x)^3\,\left(\sin(x)^2+3\right)}{\left(\cos(x)^2-2\right)\,\left(5-\frac{4}{\cos(x)^2}\right)^{3/2}}\,dx$$

input
```
int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)),x)
```

output
```
int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)), x)
```

3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx$

3.439.1 Optimal result

Integrand size = 48, antiderivative size = 57

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{3}{4}\log(\tan(x))+\frac{3}{8}\log\left(4+9\tan^2(x)\right)-\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}}$$

output
```
-3/4*ln(tan(x))+3/8*ln(4+9*tan(x)^2)-1/4*cot(x)/(4+9*tan(x)^2)^(1/2)-7/8*t
an(x)/(4+9*tan(x)^2)^(1/2)
```

3.439.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

Time = 5.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \frac{5\cot(x)+6\sqrt{\frac{13-5\cos(2x)}{1+\cos(2x)}}\log\left(1+7\tan^2\left(\frac{x}{2}\right)+\right.}{}$$

input
```
Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(
4*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]
```

output
```
(5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Tan[x/2]^2
+ Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*Log[Tan[x/2]]*Sqrt[
-5 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]
)
```

3.439.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 4889, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{5\tan^2(x)+4\sec^2(x)}\right)}{\left(5\tan^2(x)+4\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(x)^2\left(\sec(x)^2-3\tan(x)\sqrt{5\tan(x)^2+4\sec(x)^2}\right)}{\left(5\tan(x)^2+4\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(\tan^2(x)-3\sqrt{9\tan^2(x)+4}\tan(x)+1\right)\cot^2(x)}{\left(9\tan^2(x)+4\right)^{3/2}}d\tan(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{1}{\left(9\tan^2(x)+4\right)^{3/2}}+\frac{\cot^2(x)}{\left(9\tan^2(x)+4\right)^{3/2}}-\frac{3\cot(x)}{9\tan^2(x)+4}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

input
```
Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[
x]^2 + 5*Tan[x]^2)^(3/2),x]
```

output
```
(-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4 + 9*Tan[
x]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

3.439. $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}dx$

3.439.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.439.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result
parts	$-\frac{(\sec^3(x))\csc(x)(25(\cos^4(x))-80(\cos^2(x))+63)\sqrt{4}}{16(-5+9(\sec^2(x)))^{\frac{3}{2}}}-\frac{3\ln(\cos(x)+1)}{8}+\frac{3\ln(5(\cos^2(x))-9)}{8}-\frac{3\ln(-1+\cos(x))}{8}$
default	$-\frac{6(-5+9(\sec^2(x)))^{\frac{3}{2}}\ln(\csc(x)-\cot(x))-3(-5+9(\sec^2(x)))^{\frac{3}{2}}\ln\left(-\frac{5(\cos^2(x))-9}{(\cos(x)+1)^2}\right)+25\cot(x)-80\sec(x)\csc(x)+63(\sec^3(x))\csc(}{8(-5+9(\sec^2(x)))^{\frac{3}{2}}}$

input
```
int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2
+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*sec(x)^3*csc(x)*(25*cos(x)^4-80*cos(x)^2+63)/(-5+9*sec(x)^2)^(3/2)*4
^(1/2)-3/8*ln(cos(x)+1)+3/8*ln(5*cos(x)^2-9)-3/8*ln(-1+cos(x))
```

3.439. $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}}\,dx$

3.439.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \frac{3\left(5\cos(x)^2-9\right)\log\left(-\frac{5}{4}\cos(x)^2+\frac{9}{4}\right)\sin(x)}{}$$

input
```
integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*se
c(x)^2+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*cos(x)^2 -
9)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt(-(5*cos(x)^2 - 9)
/cos(x)^2))/((5*cos(x)^2 - 9)*sin(x))
```

3.439.6 Sympy [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \int \frac{-3\sqrt{5\tan^2(x)+4\sec^2(x)}\tan(x)+\sec^2(x)}{\left(5\tan^2(x)+4\sec^2(x)\right)^{\frac{3}{2}}\sin^2(x)}$$

input
```
integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/
(4*sec(x)**2+5*tan(x)**2)**(3/2),x)
```

output
```
Integral((-3*sqrt(5*tan(x)**2 + 4*sec(x)**2)*tan(x) + sec(x)**2)/((5*tan(x
)**2 + 4*sec(x)**2)**(3/2)*sin(x)**2), x)
```

3.439.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{7\tan(x)}{8\sqrt{9\tan(x)^2+4}} - \frac{1}{4\sqrt{9\tan(x)^2+4}\tan(x)} + \frac{3}{8}\log\left(9\tan(x)^2+4\right) - \frac{3}{4}\log\left(\tan(x)\right)$$

input
```
integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*se
c(x)^2+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(x)) + 3/8
*log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

3.439.8 Giac [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx=\int \frac{\sec(x)^2-3\sqrt{4\sec(x)^2+5\tan(x)^2}\tan(x}{\left(4\sec(x)^2+5\tan(x)^2\right)^{\frac{3}{2}}\sin(x)^2}$$

input
```
integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*se
c(x)^2+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2
 + 5*tan(x)^2)^(3/2)*sin(x)^2), x)
```

3.439.9 Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.98

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx=\frac{3\ln\left(\left(\cos(2x)+\sin(2x)\,1i\right)\left(5\cos(2x)-13\right.\right.}{8}$$
$$-\frac{3\ln\left(\cos(2x)\,852930i-852930\sin(2x)-852930i\right)}{4}$$
$$-\frac{\frac{18\sin(2x)\sqrt{13-5\cos(2x)}}{\sqrt{\cos(2x)+1}}-\frac{5\sin(4x)\sqrt{13-5\cos(2x)}}{\sqrt{\cos(2x)+1}}}{80\cos(2x)^2-288\cos(2x)+208}$$

input
```
int((1/cos(x)^2 - 3*tan(x)*(4/cos(x)^2 + 5*tan(x)^2)^(1/2))/(sin(x)^2*(4/c
os(x)^2 + 5*tan(x)^2)^(3/2)),x)
```

output
```
(3*log((cos(2*x) + sin(2*x)*1i)*(5*cos(2*x) - 13)))/8 - (3*log(cos(2*x)*85
2930i - 852930*sin(2*x) - 852930i))/4 - ((18*sin(2*x)*(13 - 5*cos(2*x))^(1
/2))/(cos(2*x) + 1)^(1/2) - (5*sin(4*x)*(13 - 5*cos(2*x))^(1/2))/(cos(2*x)
 + 1)^(1/2))/(80*cos(2*x)^2 - 288*cos(2*x) + 208)
```

3.440 $\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx$

3.440.1 Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = -32\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) + 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2}$$

output
```
-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+16*(1+5*tan(x)^2)^(1/2)-4/3*(1+5*tan(
x)^2)^(3/2)+1/5*(1+5*tan(x)^2)^(5/2)
```

3.440.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \frac{5\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},\frac{4\cos^2(x)}{5}\right)\left(1+5\tan^2(x)\right)^{5/2}}{(3-2\cos(2x))^{5/2}}$$

input
```
Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
(5*Sqrt[5]*Hypergeometric2F1[-5/2, -5/2, -3/2, (4*Cos[x]^2)/5]*(1 + 5*Tan[
x]^2)^(5/2))/(3 - 2*Cos[2*x])^(5/2)
```

3.440.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4153, 353, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\left(5\tan^2(x)+1\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\left(5\tan(x)^2+1\right)^{5/2}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)\left(5\tan^2(x)+1\right)^{5/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(5\tan^2(x)+1\right)^{5/2}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\int \frac{\left(5\tan^2(x)+1\right)^{3/2}}{\tan^2(x)+1}d\tan^2(x)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\int \frac{\sqrt{5\tan^2(x)+1}}{\tan^2(x)+1}d\tan^2(x)\right)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\left(2\sqrt{5\tan^2(x)+1}-4\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\left(2\sqrt{5\tan^2(x)+1}-\frac{8}{5}\int \frac{1}{\frac{\tan^4(x)}{5}+\frac{4}{5}}d\sqrt{5\tan^2(x)+1}\right)\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{2}{5}(5\tan^2(x)+1)^{5/2}-4\left(\frac{2}{3}(5\tan^2(x)+1)^{3/2}-4\left(2\sqrt{5\tan^2(x)+1}-4\arctan\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)\right)\right)\right)$$

input
```
Int[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
((2*(1 + 5*Tan[x]^2)^(5/2))/5 - 4*((2*(1 + 5*Tan[x]^2)^(3/2))/3 - 4*(-4*Ar
cTan[Sqrt[1 + 5*Tan[x]^2]/2] + 2*Sqrt[1 + 5*Tan[x]^2])))/2
```

3.440.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.440.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{223\sqrt{1+5(\tan^2(x))}}{15}+5(\tan^4(x))\sqrt{1+5(\tan^2(x))}-\frac{14(\tan^2(x))\sqrt{1+5(\tan^2(x))}}{3}-32\arctan\left(\frac{\sqrt{}}{}\right.$
default	$\frac{223\sqrt{1+5(\tan^2(x))}}{15}+5(\tan^4(x))\sqrt{1+5(\tan^2(x))}-\frac{14(\tan^2(x))\sqrt{1+5(\tan^2(x))}}{3}-32\arctan\left(\frac{\sqrt{}}{}\right.$

input
```
int(tan(x)*(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
223/15*(1+5*tan(x)^2)^(1/2)+5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*tan(x)^2*
(1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx=\frac{1}{15}\left(75\,\tan\left(x\right)^4-70\,\tan\left(x\right)^2+223\right)\sqrt{5\,\tan\left(x\right)^2+1}-16\,\arctan\left(\frac{5\,\tan\left(x\right)^2-3}{4\,\sqrt{5\,\tan\left(x\right)^2+1}}\right)$$

input
```
integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*arctan(1/
4*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))
```

3.440.6 Sympy [F]

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2}\,dx = \int \left(5\tan^2(x)+1\right)^{\frac{5}{2}}\tan(x)\,dx$$

input
```
integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
```

output
```
Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)
```

3.440.7 Maxima [F]

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2}\,dx = \int \left(5\tan(x)^2+1\right)^{\frac{5}{2}}\tan(x)\,dx$$

input
```
integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

3.440.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2}\,dx = \frac{1}{5}\left(5\tan(x)^2+1\right)^{\frac{5}{2}} - \frac{4}{3}\left(5\tan(x)^2+1\right)^{\frac{3}{2}} + 16\sqrt{5\tan(x)^2+1} - 32\arctan\left(\frac{1}{2}\sqrt{5\tan(x)^2+1}\right)$$

input
```
integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt(5*tan(x)
^2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))
```

3.440.9 Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \frac{\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}\left(25\tan(x)^4-\frac{70\tan(x)^2}{3}+\frac{223}{3}\right)}{5}$$
$$-\ln\left(\tan(x)-\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)16\mathrm{i}$$
$$-\ln\left(\tan(x)+\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right)16\mathrm{i}+\ln(\tan(x)-\mathrm{i})\,16\mathrm{i}+\ln(\tan(x)+1\mathrm{i})\,16\mathrm{i}$$

input
```
int(tan(x)*(5*tan(x)^2 + 1)^(5/2),x)
```

output
```
log(tan(x) - 1i)*16i - log(tan(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 -
 1i/5)*16i - log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*16i
 + log(tan(x) + 1i)*16i + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2)*(25*tan(x)^4 - (
70*tan(x)^2)/3 + 223/3))/5
```

3.441 $\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx$

3.441.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx = \frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) - \frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}$$

output
```
1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*
tan(x)^2)^(3/2)
```

3.441.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx = \frac{(-3+2\cos(2x))\left(-6\cos(x)+8\cos(3x)-3(-3+2\cos(2x))^{3/2}\log\left(2\cos(x)+\right.\right.}{96\left(1+5\tan^2(x)\right)^{5/2}}$$

input
```
Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^(3/2)*Log
[2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])*Sec[x]^5)/(96*(1 + 5*Tan[x]^2)^(5/2))
```

3.441.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 4153, 353, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(5\tan^2(x)+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(5\tan(x)^2+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{5/2}}d\tan^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{1}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{3/2}}d\tan^2(x)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan^2(x)+\frac{1}{2\sqrt{5\tan^2(x)+1}}\right)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{10}\int \frac{1}{\frac{\tan^4(x)}{5}+\frac{4}{5}}d\sqrt{5\tan^2(x)+1}+\frac{1}{2\sqrt{5\tan^2(x)+1}}\right)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{4}\arctan\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)+\frac{1}{2\sqrt{5\tan^2(x)+1}}\right)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

input
```
Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
(-1/6*1/(1 + 5*Tan[x]^2)^(3/2) + (ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/4 + 1/(2*
Sqrt[1 + 5*Tan[x]^2]))/4)/2
```

3.441.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.441. $\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx$

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.441.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{1+5\left(\tan^2(x)\right)}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5(\tan^2(x))}}-\frac{1}{12(1+5(\tan^2(x)))^{\frac{3}{2}}}$	41
default	$\frac{\arctan\left(\frac{\sqrt{1+5\left(\tan^2(x)\right)}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5(\tan^2(x))}}-\frac{1}{12(1+5(\tan^2(x)))^{\frac{3}{2}}}$	41

input
```
int(tan(x)/(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*
tan(x)^2)^(3/2)
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx=\frac{3\left(25\tan(x)^4+10\tan(x)^2+1\right)\arctan\left(\frac{5\tan(x)^2-3}{4\sqrt{5\tan(x)^2+1}}\right)+4\left(15\tan(x)^2-1\right)\sqrt{}}{192\left(25\tan(x)^4+10\tan(x)^2+1\right)}$$

input
```
integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(
5*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 + 1))/(25*tan(x)^4
+ 10*tan(x)^2 + 1)
```

3.441.6 Sympy [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{5\tan^2(x)+1}}{2}\right)}{32} + \frac{1}{16\sqrt{5\tan^2(x)+1}} - \frac{1}{12\left(5\tan^2(x)+1\right)^{\frac{3}{2}}}$$

input
```
integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
```

output
```
atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) - 1/(12*(5
*tan(x)**2 + 1)**(3/2))
```

3.441.7 Maxima [F]

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \int \frac{\tan(x)}{\left(5\tan(x)^2+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)
```

3.441.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{15\tan(x)^2-1}{48\left(5\tan(x)^2+1\right)^{\frac{3}{2}}} + \frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{5\tan(x)^2+1}\right)$$

input
```
integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*sqrt(5*tan
(x)^2 + 1))
```

3.441.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.19

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{\ln\left(\tan(x)-\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)\,1\mathrm{i}}{64} + \frac{\ln\left(\tan(x)+\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right)\,1\mathrm{i}}{64} - \frac{\ln(\tan(x)-\mathrm{i})\,1\mathrm{i}}{64} - \frac{\ln(\tan(x)+1\mathrm{i})\,1\mathrm{i}}{64} - \frac{\sqrt{\tan(x)^2+\frac{1}{5}}\,1\mathrm{i}}{96\left(\tan(x)-\frac{\sqrt{5}\,1\mathrm{i}}{5}\right)} + \frac{\sqrt{\tan(x)^2+\frac{1}{5}}\,1\mathrm{i}}{96\left(\tan(x)+\frac{\sqrt{5}\,1\mathrm{i}}{5}\right)} + \frac{\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{240\left(\tan(x)^2+\frac{2\mathrm{i}\sqrt{5}\tan(x)}{5}-\frac{1}{5}\right)} - \frac{\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{240\left(-\tan(x)^2+\frac{2\mathrm{i}\sqrt{5}\tan(x)}{5}+\frac{1}{5}\right)}$$

input
```
int(tan(x)/(5*tan(x)^2 + 1)^(5/2),x)
```

output
```
(log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*1i)/64 + (log(t
an(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 - 1i/5)*1i)/64 - (log(tan(x)
- 1i)*1i)/64 - (log(tan(x) + 1i)*1i)/64 - ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*
(tan(x) - (5^(1/2)*1i)/5)) + ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*(tan(x) + (5^
(1/2)*1i)/5)) + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*(tan(x)^2 + (5^(1/2)
*tan(x)*2i)/5 - 1/5)) - (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*((5^(1/2)*ta
n(x)*2i)/5 - tan(x)^2 + 1/5))
```

3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

3.442.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

output
```
1/2*ln(cos(x))/(a^3-b^3)^(1/3)+3/4*ln((a^3-b^3)^(1/3)-(a^3+b^3*tan(x)^2)^(
1/3))/(a^3-b^3)^(1/3)+1/2*arctan(1/3*(1+2*(a^3+b^3*tan(x)^2)^(1/3)/(a^3-b^
3)^(1/3))*3^(1/2))*3^(1/2)/(a^3-b^3)^(1/3)
```

3.442.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)+2\log(\cos(x))+3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

input
```
Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/S
qrt[3]] + 2*Log[Cos[x]] + 3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(
1/3)])/(4*(a^3 - b^3)^(1/3))
```

3.442.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4153, 353, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan(x)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{(\tan^2(x)+1)\sqrt[3]{a^3+b^3\tan^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt[3]{a^3+b^3\tan^2(x)}}d\tan^2(x)$$

$\Big\downarrow$ 67

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}}d\sqrt[3]{a^3+b^3\tan^2(x)}}{2\sqrt[3]{a^3-b^3}}+\frac{3}{2}\int \frac{1}{\tan^4(x)+\left(a^3-b^3\right)^{2/3}+\sqrt[3]{a^3-b^3}\sqrt[3]{a^3+b^3\tan}}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{\tan^4(x)+\left(a^3-b^3\right)^{2/3}+\sqrt[3]{a^3-b^3}\sqrt[3]{a^3+b^3\tan^2(x)}}d\sqrt[3]{a^3+b^3\tan^2(x)}-\frac{\log\left(\tan^2(x)+1\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{\cdots}\right.}{\cdots}\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{-\tan^4(x)-3}d\left(\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1\right)}{\sqrt[3]{a^3-b^3}}-\frac{\log\left(\tan^2(x)+1\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{2\sqrt[3]{a^3-b^3}}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a^3-b^3}}-\frac{\log\left(\tan^2(x)+1\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{2\sqrt[3]{a^3-b^3}}\right)$$

input
```
Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

3.442. $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

output
```
((Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sq
rt[3]])/(a^3 - b^3)^(1/3) - Log[1 + Tan[x]^2]/(2*(a^3 - b^3)^(1/3)) + (3*L
og[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(2*(a^3 - b^3)^(1/3)))
/2
```

3.442.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.442. $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.442.4 Maple [F]

$$\int \frac{\tan(x)}{(a^3 + b^3 (\tan^2(x)))^{\frac{1}{3}}} dx$$

input
```
int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

output
```
int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

3.442.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} dx = \text{Timed out}$$

input
```
integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.442.6 Sympy [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} dx = \int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} dx$$

input
```
integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
```

output
```
Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)
```

3.442.7 Maxima [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx=\int \frac{\tan(x)}{\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3), x)
```

3.442.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx &= \frac{3\left(a^3-b^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}+\left(a^3-b^3\right)^{\frac{1}{3}}\right)}{3\left(a^3-b^3\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}a^3-\sqrt{3}b^3\right)} \\ &\quad -\frac{\log\left(\left(b^3\tan(x)^2+a^3\right)^{\frac{2}{3}}+\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}\left(a^3-b^3\right)^{\frac{1}{3}}+\left(a^3-b^3\right)^{\frac{2}{3}}\right)}{4\left(a^3-b^3\right)^{\frac{1}{3}}} \\ &\quad +\frac{\log\left(\left|\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}-\left(a^3-b^3\right)^{\frac{1}{3}}\right|\right)}{2\left(a^3-b^3\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="giac")
```

output
```
3/2*(a^3 - b^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b^3*tan(x)^2 + a^3)^(1/3) + (
a^3 - b^3)^(1/3))/(a^3 - b^3)^(1/3))/(sqrt(3)*a^3 - sqrt(3)*b^3) - 1/4*log
((b^3*tan(x)^2 + a^3)^(2/3) + (b^3*tan(x)^2 + a^3)^(1/3)*(a^3 - b^3)^(1/3)
 + (a^3 - b^3)^(2/3))/(a^3 - b^3)^(1/3) + 1/2*log(abs((b^3*tan(x)^2 + a^3)
^(1/3) - (a^3 - b^3)^(1/3)))/(a^3 - b^3)^(1/3)
```

3.442.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{9\,a^3-9\,b^3}{4\,(a-b)^{2/3}\,(a^2+a\,b+b^2)^{2/3}}\right)}{2\,(a-b)^{1/3}\,(a^2+a\,b+b^2)^{1/3}} + \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a^3-9\,b^3)}{16\,(a-b)^{2/3}\,(a^2+a\,b+b^2)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(a-b)^{1/3}\,(a^2+a\,b+b^2)^{1/3}} - \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a^3-9\,b^3)}{16\,(a-b)^{2/3}\,(a^2+a\,b+b^2)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(a-b)^{1/3}\,(a^2+a\,b+b^2)^{1/3}}$$

input
```
int(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x)
```

output
```
log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - (9*a^3 - 9*b^3)/(4*(a - b)^(2/3)*(a
*b + a^2 + b^2)^(2/3)))/(2*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3)) + (log((
9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(1/2)*1i - 1)^2*(9*a^3 - 9*b^3))/(16
*(a - b)^(2/3)*(a*b + a^2 + b^2)^(2/3)))*(3^(1/2)*1i - 1))/(4*(a - b)^(1/3
)*(a*b + a^2 + b^2)^(1/3)) - (log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(
1/2)*1i + 1)^2*(9*a^3 - 9*b^3))/(16*(a - b)^(2/3)*(a*b + a^2 + b^2)^(2/3))
)*(3^(1/2)*1i + 1))/(4*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3))
```

3.443 $\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx$

3.443.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}$$

output
```
2*ln(cos(x))+3*ln(2-(1-7*tan(x)^2)^(1/3))+2*arctan(1/3*(1+(1-7*tan(x)^2)^(
1/3))*3^(1/2))*3^(1/2)+3/4*(1-7*tan(x)^2)^(2/3)
```

3.443.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}$$

input
```
Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

output
```
2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3
*Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

3.443.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 4153, 353, 60, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\left(1-7\tan(x)^2\right)^{2/3}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)\left(1-7\tan^2(x)\right)^{2/3}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(1-7\tan^2(x)\right)^{2/3}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt[3]{1-7\tan^2(x)}\left(\tan^2(x)+1\right)}d\tan^2(x)+\frac{3}{2}\left(1-7\tan^2(x)\right)^{2/3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(8\left(-\frac{3}{4}\int \frac{1}{2-\sqrt[3]{1-7\tan^2(x)}}d\sqrt[3]{1-7\tan^2(x)}+\frac{3}{2}\int \frac{1}{\tan^4(x)+2\sqrt[3]{1-7\tan^2(x)}+4}d\sqrt[3]{1-7\tan^2(x)}-\frac{1}{4}\text{lo}\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(8\left(\frac{3}{2}\int \frac{1}{\tan^4(x)+2\sqrt[3]{1-7\tan^2(x)}+4}d\sqrt[3]{1-7\tan^2(x)}-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{3}{4}\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right.\right.\right.$$

$$\downarrow 1083$$

$$\frac{1}{2}\left(8\left(-3\int \frac{1}{-\tan^4(x)-12}d\left(2\sqrt[3]{1-7\tan^2(x)}+2\right)-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{3}{4}\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)\right)+\frac{3}{2}\right.$$

$$\downarrow 217$$

$$\frac{1}{2}\left(8\left(\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-7\tan^2(x)}+2}{2\sqrt{3}}\right)-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{3}{4}\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)\right)+\frac{3}{2}(1-7\text{ta}\right.$$

input
```
Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

output
```
(8*((Sqrt[3]*ArcTan[(2 + 2*(1 - 7*Tan[x]^2)^(1/3))/(2*Sqrt[3])])/2 - Log[1
 + Tan[x]^2]/4 + (3*Log[2 - (1 - 7*Tan[x]^2)^(1/3)])/4) + (3*(1 - 7*Tan[x]
^2)^(2/3))/2)/2
```

3.443.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

3.443. $\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.443.4 Maple [F]

$$\int \tan (x) \left(1-7\left(\tan ^{2}(x)\right)\right)^{\frac{2}{3}} d x$$

input
```
int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

output
```
int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

3.443.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(58) = 116$.

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int \tan(x) \left(1 -7\tan^2(x)\right)^{2/3} dx = 2\sqrt{3}\arctan\left(\frac{7\sqrt{3}\tan(x)^2 + 4\sqrt{3}\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} - 16\sqrt{3}\left(-7\tan(x)^2 + 1\right)^{\frac{1}{3}} - \sqrt{3}}{7\tan(x)^2 - 65}\right) + \frac{3}{4}\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} + \log\left(\frac{7\tan(x)^2 + 6\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} - 12\left(-7\tan(x)^2 + 1\right)^{\frac{1}{3}} + 7}{\tan(x)^2 + 1}\right)$$

input
```
integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="fricas")
```

output
```
2*sqrt(3)*arctan((7*sqrt(3)*tan(x)^2 + 4*sqrt(3)*(-7*tan(x)^2 + 1)^(2/3) -
 16*sqrt(3)*(-7*tan(x)^2 + 1)^(1/3) - sqrt(3))/(7*tan(x)^2 - 65)) + 3/4*(-
7*tan(x)^2 + 1)^(2/3) + log((7*tan(x)^2 + 6*(-7*tan(x)^2 + 1)^(2/3) - 12*(
-7*tan(x)^2 + 1)^(1/3) + 7)/(tan(x)^2 + 1))
```

3.443.6 Sympy [F]

$$\int \tan(x) \left(1 - 7\tan^2(x)\right)^{2/3} dx = \int \left(1 - 7\tan^2(x)\right)^{\frac{2}{3}} \tan(x)\, dx$$

input
```
integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
```

output
```
Integral((1 - 7*tan(x)**2)**(2/3)*tan(x), x)
```

3.443.7 Maxima [F]

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = \int \left(-7 \tan(x)^2+1\right)^{\frac{2}{3}} \tan(x)\, dx$$

input
```
integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x), x)
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\left(-7 \tan(x)^2+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{3}{4}\left(-7 \tan(x)^2+1\right)^{\frac{2}{3}} - \log\left(\left(-7 \tan(x)^2+1\right)^{\frac{2}{3}}+2\left(-7 \tan(x)^2+1\right)^{\frac{1}{3}}+4\right) + 2\log\left(\left|\left(-7 \tan(x)^2+1\right)^{\frac{1}{3}}-2\right|\right)$$

input
```
integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="giac")
```

output
```
2*sqrt(3)*arctan(1/3*sqrt(3)*((-7*tan(x)^2 + 1)^(1/3) + 1)) + 3/4*(-7*tan(
x)^2 + 1)^(2/3) - log((-7*tan(x)^2 + 1)^(2/3) + 2*(-7*tan(x)^2 + 1)^(1/3)
+ 4) + 2*log(abs((-7*tan(x)^2 + 1)^(1/3) - 2))
```

3.443.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = 2\ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-288\right) + \frac{3\left(1-7\tan(x)^2\right)^{2/3}}{4} + \ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-72\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right) - \ln\left(144\left(1-7\tan(x)^2\right)^{1/3}-72\left(1+\sqrt{3}\,1\mathrm{i}\right.\right.$$

input
```
int(tan(x)*(1 - 7*tan(x)^2)^(2/3),x)
```

output
```
2*log(144*(1 - 7*tan(x)^2)^(1/3) - 288) + (3*(1 - 7*tan(x)^2)^(2/3))/4 + l
og(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i - 1)^2)*(3^(1/2)*1i - 1) -
log(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i + 1)^2)*(3^(1/2)*1i + 1)
```

3.443. $\int \tan(x) \left(1 - 7\tan^2(x)\right)^{2/3} dx$

3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$

3.444.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

output
```
-arctan((a^4+b^4*csc(x)^2)^(1/4)/a)/a+arctanh((a^4+b^4*csc(x)^2)^(1/4)/a)/
a
```

3.444.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(52) = 104.

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.92

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$$
$$=\frac{\sqrt[4]{-a^4-2b^4+a^4\cos(2x)}\left(-2\arctan\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{-b^4-a^4\sin^2(x)}}\right)+2\arctan\left(1+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{-b^4-a^4\sin^2(x)}}\right)\right.}{2\,2^{3/4}a\sqrt[4]{a^4+b^4}}$$

input
```
Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

output
```
((-a^4 - 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]
])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(-
b^4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[-b^4 - a^4*Sin[x]^2
] - (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*S
in[x])/Sqrt[-b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Si
n[x]^2)^(1/4)]))/(2*2^(3/4)*a*(a^4 + b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

3.444.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 25, 4627, 243, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4+b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4+b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \; 4627$$

$$-\int \frac{\sin(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}d\csc(x)$$

$$\Big\downarrow \; 243$$

$$-\frac{1}{2}\int \frac{\sin(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}d\csc^2(x)$$

$$\Big\downarrow \; 73$$

$$-\frac{2\int -\frac{b^4\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}}{b^4}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{b^4\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}}{b^4}$$

$$\Big\downarrow \ 27$$

$$2\int \frac{\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}$$

$$\Big\downarrow \ 827$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}-\frac{1}{2}\int \frac{1}{\csc^4(x)+a^2}d\sqrt[4]{a^4+b^4\csc^2(x)}\right)$$

$$\Big\downarrow \ 216$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

$$\Big\downarrow \ 219$$

$$2\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{2a}-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

input
```
Int[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

output
```
2*(-1/2*ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a + ArcTanh[(a^4 + b^4*Csc[x]
^2)^(1/4)/a]/(2*a))
```

3.444.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.444. $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$

3.444.4 Maple [F]

$$\int \frac{\cot(x)}{(a^4 + b^4 (\csc^2(x)))^{\frac{1}{4}}} dx$$

input
```
int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

3.444.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx = \text{Timed out}$$

input
```
integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.444.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx = \int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx$$

input
```
integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)
```

output
```
Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a} - \frac{\log\left(-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

output
```
-arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 + b^4/sin(x)^2)
^(1/4))/a - 1/2*log(-a + (a^4 + b^4/sin(x)^2)^(1/4))/a
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(\left|a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a} - \frac{\log\left(\left|-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

output
```
-arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 + b^4/sin(x
)^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 + b^4/sin(x)^2)^(1/4)))/a
```

3.444.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx = -\frac{\operatorname{atan}\left(\frac{\left(\frac{b^4}{\sin(x)^2}+a^4\right)^{1/4}}{a}\right)-\operatorname{atanh}\left(\frac{\left(\frac{b^4}{\sin(x)^2}+a^4\right)^{1/4}}{a}\right)}{a}$$

input
```
int(cot(x)/(b^4/sin(x)^2 + a^4)^(1/4),x)
```

output
```
-(atan((b^4/sin(x)^2 + a^4)^(1/4)/a) - atanh((b^4/sin(x)^2 + a^4)^(1/4)/a)
)/a
```

3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$

3.445.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

output
```
-arctan((a^4-b^4*csc(x)^2)^(1/4)/a)/a+arctanh((a^4-b^4*csc(x)^2)^(1/4)/a)/
a
```

3.445.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(54) = 108.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx = \frac{\sqrt[4]{-a^4+2b^4+a^4\cos(2x)}\left(-2\arctan\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)+2\arctan\left(1+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)-\right.}{2\,2^{3/4}a\sqrt[4]{a^4-b^4\csc^2(}}$$

input
```
Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

output
```
((-a^4 + 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]
])/(b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^
4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] -
 (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x
])/Sqrt[b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2
)^(1/4)]))/(2*2^(3/4)*a*(a^4 - b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

3.445.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 25, 4627, 243, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4-b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4-b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\sin(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}d\csc(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\sin(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}d\csc^2(x)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{b^4\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}}{b^4}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}-\frac{1}{2}\int \frac{1}{\csc^4(x)+a^2}d\sqrt[4]{a^4-b^4\csc^2(x)}\right)$$

$$\Big\downarrow \text{216}$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{2a}-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

input
```
Int[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

output
```
2*(-1/2*ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a + ArcTanh[(a^4 - b^4*Csc[x]
^2)^(1/4)/a]/(2*a))
```

3.445.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.445. $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.445.4 Maple [F]

$$\int \frac{\cot(x)}{(a^4 - b^4(\csc^2(x)))^{\frac{1}{4}}} dx$$

input
```
int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

3.445.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.445.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = \int \frac{\cot(x)}{\sqrt[4]{(a^2 - b^2 \csc(x))\,(a^2 + b^2 \csc(x))}}\,dx$$

input
```
integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
```

output
```
Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4), x)
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a} - \frac{\log\left(-a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

output
```
-arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/sin(x)^2)
^(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(\left|a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}-\frac{\log\left(\left|-a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

output
```
-arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 - b^4/sin(x
)^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 - b^4/sin(x)^2)^(1/4)))/a
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx = \int \frac{\cot(x)}{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{1/4}}\,dx$$

input
```
int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4), x)
```

3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$

3.446.1 Optimal result

Integrand size = 61, antiderivative size = 133

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=\sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{1-3\sec^2(x)}}{\sqrt{3}}\right)+\frac{1}{4}\log\left(\sec^2(x)\right)-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{3}\log\left(1-\sqrt{1-3\sec^2(x)}\right)-\sqrt[6]{1-3\sec^2(x)}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}+\frac{1}{2\left(1-\sqrt{1-3\sec^2(x)}\right)}$$

output
```
1/4*ln(sec(x)^2)-3/2*ln(1-(1-3*sec(x)^2)^(1/6))+1/3*ln(1-(1-3*sec(x)^2)^(1
/2))-(1-3*sec(x)^2)^(1/6)-1/4*(1-3*sec(x)^2)^(2/3)+arctan(1/3*(1+2*(1-3*se
c(x)^2)^(1/6))*3^(1/2))*3^(1/2)+1/2/(1-(1-3*sec(x)^2)^(1/2))
```

3.446.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 25.30 (sec) , antiderivative size = 1447, normalized size of antiderivative = 10.88

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))
/((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

output
```
-1/3*((6 + ((-5 + Cos[2*x])/(1 + Cos[2*x]))^(1/3) + Cos[2*x]*((-5 + Cos[2*
x])/(1 + Cos[2*x]))^(1/3))*(3*Sec[x]^2 + (1 - 3*Sec[x]^2)^(1/3))*Tan[x]*(-
2 - 3*Tan[x]^2)^(5/6)*(2 + 3*Tan[x]^2)*Sqrt[-(2 + 3*Tan[x]^2)^2]*(-1 + (-2
 - 3*Tan[x]^2)^(1/3))*(1 + (-2 - 3*Tan[x]^2)^(1/3) + (-2 - 3*Tan[x]^2)^(2/
3))*(6*ArcTan[Sqrt[2 + 3*Tan[x]^2]]*Sqrt[-2 - 3*Tan[x]^2] - 5*Sqrt[2 + 3*T
an[x]^2] - 4*ArcTanh[Sqrt[-2 - 3*Tan[x]^2]]*Sqrt[2 + 3*Tan[x]^2] + Cos[2*x
]*Sqrt[2 + 3*Tan[x]^2] + 5*Log[Sec[x]^2]*Sqrt[2 + 3*Tan[x]^2] - 9*Log[1 -
(-2 - 3*Tan[x]^2)^(1/3)]*Sqrt[2 + 3*Tan[x]^2] - 12*(-2 - 3*Tan[x]^2)^(1/6)
*Sqrt[2 + 3*Tan[x]^2] + 36*Hypergeometric2F1[1/6, 1, 7/6, -2 - 3*Tan[x]^2]
*(-2 - 3*Tan[x]^2)^(1/6)*Sqrt[2 + 3*Tan[x]^2] - 3*(-2 - 3*Tan[x]^2)^(2/3)*
Sqrt[2 + 3*Tan[x]^2] + Sqrt[-(2 + 3*Tan[x]^2)^2] + Cos[2*x]*Sqrt[-(2 + 3*T
an[x]^2)^2] - 6*ArcTan[(1 + 2*(-2 - 3*Tan[x]^2)^(1/3))/Sqrt[3]]*Sqrt[6 + 9
*Tan[x]^2]))/((-1 + Sqrt[(-5 + Cos[2*x])/(1 + Cos[2*x])])*(1 - 3*Sec[x]^2)
^(5/6)*(6 + (1 - 3*Sec[x]^2)^(1/3) + Cos[2*x]*(1 - 3*Sec[x]^2)^(1/3))*(12*
Csc[x]*Sec[x]*(-2 - 3*Tan[x]^2)^(5/6) + 12*Cos[2*x]*Csc[x]*Sec[x]*(-2 - 3*
Tan[x]^2)^(5/6) - 88*Sin[2*x]*(-2 - 3*Tan[x]^2)^(5/6) - 16*Cot[x]^2*Sin[2*
x]*(-2 - 3*Tan[x]^2)^(5/6) + 48*Sec[x]^2*Tan[x]*(-2 - 3*Tan[x]^2)^(5/6) +
48*Cos[2*x]*Sec[x]^2*Tan[x]*(-2 - 3*Tan[x]^2)^(5/6) - 180*Sin[2*x]*Tan[x]^
2*(-2 - 3*Tan[x]^2)^(5/6) + 63*Sec[x]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5/6) +
 63*Cos[2*x]*Sec[x]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5/6) - 162*Sin[2*x]*T...
```

3.446. $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$

3.446.3 Rubi [A] (verified)

Time = 4.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.066$, Rules used = {3042, 4861, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec^2(x)\left(3\tan^2(x)+\sin^2(x)\sqrt[3]{1-3\sec^2(x)}\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)\sec(x)^2\left(3\tan(x)^2+\sin(x)^2\sqrt[3]{1-3\sec(x)^2}\right)}{(1-3\sec(x)^2)^{5/6}\left(1-\sqrt{1-3\sec(x)^2}\right)}dx$$

$$\Big\downarrow \text{4861}$$

$$-\int \frac{(1-\cos^2(x))\sec^5(x)\left(\sqrt[3]{1-3\sec^2(x)}\cos^2(x)+3\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}d\cos(x)$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{\left(-\sqrt[3]{(\cos^2(x)-3)\sec^2(x)}\cos^2(x)-3\right)\sec^5(x)}{(1-3\sec^2(x))^{5/6}\left(\sqrt{(\cos^2(x)-3)\sec^2(x)}-1\right)}+\frac{\left(\sqrt[3]{(\cos^2(x)-3)\sec^2(x)}\cos^2(x)+3\right)\sec^3(x)}{(1-3\sec^2(x))^{5/6}\left(\sqrt{(\cos^2(x)-3)\sec^2(x)}-1\right)}\right)d\cos$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3-\cos^2(x)}\sec(x)\arcsin\left(\frac{\cos(x)}{\sqrt{3}}\right)}{2\sqrt{1-3\sec^2(x)}}+\sqrt{3}\arctan\left(\frac{2\sqrt[6]{1-3\sec^2(x)}+1}{\sqrt{3}}\right)+\frac{\cos^2(x)}{6}-\frac{1}{4}(1-3\sec^2(x))^{2/3}-\sqrt[6]{1-3\sec^2(x)}-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{2}\log\left(1-\sqrt{1-3\sec^2(x)}\right)-\frac{3-\cos^2(x)}{6\sqrt{1-3\sec^2(x)}}+\frac{1}{3}\log\left(1-\sqrt{-((3-\cos^2(x))\sec^2(x))}\right)$$

input
```
Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 -
 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

output
```
Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + Cos[x]^2/6 + Log[
1 - Sqrt[-((3 - Cos[x]^2)*Sec[x]^2)]]/3 - (3*Log[1 - (1 - 3*Sec[x]^2)^(1/6
)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/2 - (3 - Cos[x]^2)/(6*Sqrt[1 - 3*Sec
[x]^2]) + (ArcSin[Cos[x]/Sqrt[3]]*Sqrt[3 - Cos[x]^2]*Sec[x])/(2*Sqrt[1 - 3
*Sec[x]^2]) - (1 - 3*Sec[x]^2)^(1/6) - (1 - 3*Sec[x]^2)^(2/3)/4
```

3.446.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.446.4 Maple [F]

$$\int \frac{\tan(x)\left((1-3(\sec^2(x)))^{\frac{1}{3}}(\sin^2(x))+3(\tan^2(x))\right)}{\cos(x)^2(1-3(\sec^2(x)))^{\frac{5}{6}}\left(1-\sqrt{1-3(\sec^2(x))}\right)}dx$$

input
```
int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)
^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

output
```
int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)
^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

3.446.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*
sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Curv
e not irreducible after change of variable 0 -> infinity
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Timed out}$$

input
```
integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/
(1-3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

output
```
Timed out
```

3.446.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Timed out}$$

input
```
integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*
sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="maxima")
```

output
```
Timed out
```

3.446.8 Giac [F]

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=\int -\frac{\left(\left(-3\sec(x)^2+1\right)^{\frac{1}{3}}\sin(x)^2+3\tan(x)\right.}{\left(-3\sec(x)^2+1\right)^{\frac{5}{6}}\left(\sqrt{-3\sec(x)^2+1}-\right.}$$

input
```
integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*
sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=$$
$$-\int \frac{\tan(x)\left(\sin(x)^2\left(1-\frac{3}{\cos(x)^2}\right)^{1/3}+3\tan(x)^2\right)}{\cos(x)^2\left(\sqrt{1-\frac{3}{\cos(x)^2}}-1\right)\left(1-\frac{3}{\cos(x)^2}\right)^{5/6}}\,dx$$

input
```
int(-(tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
 - 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)),x)
```

output
```
-int((tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
 - 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)), x)
```

3.447 $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}}\,dx$

3.447.1 Optimal result

Integrand size = 29, antiderivative size = 100

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = 2\text{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right) - \frac{11\text{arctanh}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)}{4\sqrt{2}} + \frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}} + \frac{2\tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}} + \frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}$$

output
```
2*arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))-11/8*arctanh(2^(1/2)*tan(x)/(tan
(x)*tan(2*x))^(1/2))*2^(1/2)+3/4*tan(x)/(tan(x)*tan(2*x))^(1/2)+1/2*tan(x)
/(tan(x)*tan(2*x))^(3/2)+2/3*tan(x)^3/(tan(x)*tan(2*x))^(3/2)
```

3.447.2 Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.69

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \frac{\left(-\cos(2x)+2\tan^2(x)\right)\left(\frac{4\sqrt{2}\left(-2\text{arctanh}\left(\sqrt{\frac{\cos(2x)}{1+\cos(2x)}}\right)+\sqrt{2}\text{arctanh}\left(\sqrt{1-\right.\right.}{\sqrt{1-\tan^2(x)}}\right.}{}$$

input
```
Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

output
```
((-Cos[2*x] + 2*Tan[x]^2)*((4*Sqrt[2]*(-2*ArcTanh[Sqrt[Cos[2*x]/(1 + Cos[2
*x])]] + Sqrt[2]*ArcTanh[Sqrt[1 - Tan[x]^2]])*Cos[2*x]*Tan[x])/Sqrt[1 - Ta
n[x]^2] - 3*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Sin[x]*Sqrt[-1 + Tan[x]^2]
+ (-3*Cot[x] - 4*Cos[x]*Sin[x] + (5 + 9*Cos[2*x])*Tan[x]^3)/3)*Tan[2*x]^2)
/(2*(-3 + 6*Cos[2*x] + Cos[4*x])*(Tan[x]*Tan[2*x])^(3/2))
```

3.447.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 4897, 3042, 4889, 27, 2058, 34, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)\left(2\tan^2(x)-\cos(2x)\right)}{(\tan(x)\tan(2x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2\left(2\tan(x)^2-\cos(2x)\right)}{(\tan(x)\tan(2x))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec^2(x)\left(2\tan^2(x)-\cos(2x)\right)}{(\sec(2x)-1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2\left(2\tan(x)^2-\cos(2x)\right)}{(\sec(2x)-1)^{3/2}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int -\frac{-2\tan^4(x)-3\tan^2(x)+1}{2\sqrt{2}\left(\frac{\tan^2(x)}{1-\tan^2(x)}\right)^{3/2}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-2\tan^4(x)-3\tan^2(x)+1}{\left(\frac{\tan^2(x)}{1-\tan^2(x)}\right)^{3/2}(\tan^2(x)+1)}d\tan(x)}{2\sqrt{2}}$$

$$\Big\downarrow \text{2058}$$

$$-\frac{\sqrt{\tan^2(x)}\int \frac{(1-\tan^2(x))^{3/2}(-2\tan^4(x)-3\tan^2(x)+1)}{\tan^2(x)^{3/2}(\tan^2(x)+1)}d\tan(x)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$\Big\downarrow \text{34}$$

$$-\frac{\tan(x)\int \frac{\cot^3(x)(1-\tan^2(x))^{3/2}(-2\tan^4(x)-3\tan^2(x)+1)}{\tan^2(x)+1}d\tan(x)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{\tan(x)\int \left((1-\tan^2(x))^{3/2}\cot^3(x)-4(1-\tan^2(x))^{3/2}\cot(x)+\frac{2\tan(x)(1-\tan^2(x))^{3/2}}{\tan^2(x)+1}\right)d\tan(x)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(x)\left(\frac{11}{2}\text{arctanh}\left(\sqrt{1-\tan^2(x)}\right)-4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\tan^2(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1-\tan^2(x))^{3/2}-\frac{3}{2}\sqrt{1-\tan^2(x)}-\frac{1}{2}(1-\right.}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

input
```
Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

output
```
-1/2*(Tan[x]*((11*ArcTanh[Sqrt[1 - Tan[x]^2]])/2 - 4*Sqrt[2]*ArcTanh[Sqrt[
1 - Tan[x]^2]/Sqrt[2]] - (3*Sqrt[1 - Tan[x]^2])/2 - (2*(1 - Tan[x]^2)^(3/2
))/3 - (Cot[x]^2*(1 - Tan[x]^2)^(3/2))/2))/(Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan
[x]^2)]*Sqrt[1 - Tan[x]^2])
```

3.447.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.447. $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}}dx$

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.447.4 Maple [F]

$$\int \frac{-\cos(2x) + 2(\tan^2(x))}{\cos(x)^2 (\tan(x)\tan(2x))^{\frac{3}{2}}} dx$$

input
```
int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
```

output
```
int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
```

3.447.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(78) = 156$.

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.71

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx =$$

$$-\frac{24\left(\cos(x)^5-\cos(x)^3\right)\log\left(-\frac{4\sqrt{2}\left(8\cos(x)^5-6\cos(x)^3+\cos(x)\right)\sqrt{-\frac{\cos(x)^2-1}{2\cos(x)^2-1}}-\left(32\cos(x)^4-16\cos(x)^2+1\right)\sin(x)}{\sin(x)}\right)\sin(x)}{}$$

input
```
integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algor
ithm="fricas")
```

output
```
-1/48*(24*(cos(x)^5 - cos(x)^3)*log(-(4*sqrt(2)*(8*cos(x)^5 - 6*cos(x)^3 +
 cos(x))*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - (32*cos(x)^4 - 16*cos(x)
^2 + 1)*sin(x))/sin(x))*sin(x) - 33*(sqrt(2)*cos(x)^5 - sqrt(2)*cos(x)^3)*
log(4*(sqrt(2)*(2*(3*sqrt(2) - 4)*cos(x)^3 - (3*sqrt(2) - 4)*cos(x))*sqrt(
-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) + (3*(2*sqrt(2) - 3)*cos(x)^2 - 2*sqrt(2
) + 3)*sin(x))/((cos(x)^2 - 1)*sin(x)))*sin(x) - 2*sqrt(2)*(22*cos(x)^6 -
47*cos(x)^4 + 26*cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - 44
*(cos(x)^5 - cos(x)^3)*sin(x))/((cos(x)^5 - cos(x)^3)*sin(x))
```

3.447.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

output
```
Timed out
```

3.447.7 Maxima [F]

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx=\int \frac{2\,\tan\left(x\right)^{2}-\cos\left(2\,x\right)}{\left(\tan\left(2\,x\right)\tan\left(x\right)\right)^{\frac{3}{2}}\cos\left(x\right)^{2}}\,dx$$

input
```
integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algor
ithm="maxima")
```

output
```
integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2), x)
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(78) = 156$.

Time = 0.40 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.93

$$\begin{aligned}&\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx=\\&-\frac{\sqrt{2}\left(2\left(-\tan\left(x\right)^{2}+1\right)^{\frac{3}{2}}+3\sqrt{-\tan\left(x\right)^{2}+1}\right)}{12\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}\\&+\frac{11\sqrt{2}\log\left(\sqrt{-\tan\left(x\right)^{2}+1}+1\right)}{16\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}-\frac{11\sqrt{2}\log\left(-\sqrt{-\tan\left(x\right)^{2}+1}+1\right)}{16\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}\\&+\frac{\log\left(\frac{\sqrt{2}-\sqrt{-\tan(x)^{2}+1}}{\sqrt{2}+\sqrt{-\tan(x)^{2}+1}}\right)}{\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}-\frac{\sqrt{2}\sqrt{-\tan\left(x\right)^{2}+1}}{8\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)\tan\left(x\right)^{2}}\end{aligned}$$

input
```
integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algor
ithm="giac")
```

output
```
-1/12*sqrt(2)*(2*(-tan(x)^2 + 1)^(3/2) + 3*sqrt(-tan(x)^2 + 1))/(sgn(tan(x
)^2 - 1)*sgn(tan(x))) + 11/16*sqrt(2)*log(sqrt(-tan(x)^2 + 1) + 1)/(sgn(ta
n(x)^2 - 1)*sgn(tan(x))) - 11/16*sqrt(2)*log(-sqrt(-tan(x)^2 + 1) + 1)/(sg
n(tan(x)^2 - 1)*sgn(tan(x))) + log((sqrt(2) - sqrt(-tan(x)^2 + 1))/(sqrt(2
) + sqrt(-tan(x)^2 + 1)))/(sgn(tan(x)^2 - 1)*sgn(tan(x))) - 1/8*sqrt(2)*sq
rt(-tan(x)^2 + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x))*tan(x)^2)
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = -\int \frac{\cos\left(2\,x\right)-2\tan(x)^2}{\cos\left(x\right)^2\left(\tan\left(2\,x\right)\,\tan\left(x\right)\right)^{3/2}}\,dx$$

input
```
int(-(cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)),x)
```

output
```
-int((cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)), x)
```

3.448 $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$

3.448.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3\cos^n(x)}}{\sqrt{3}a}\right)}{a^4n} - \frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} - \frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}$$

output
```
-3/a^3/n/(a^3-b^3*cos(x)^n)^(1/3)+1/2*ln(cos(x))/a^4-3/2*ln(a-(a^3-b^3*cos
(x)^n)^(1/3))/a^4/n-arctan(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2))*3
^(1/2)/a^4/n
```

3.448.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},1-\frac{b^3\cos^n(x)}{a^3}\right)}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}$$

input
```
Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

output
```
(-3*Hypergeometric2F1[-1/3, 1, 2/3, 1 - (b^3*Cos[x]^n)/a^3])/(a^3*n*(a^3 -
 b^3*Cos[x]^n)^(1/3))
```

3.448.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 25, 3709, 798, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\left(a^3-b^3\sin\left(x+\frac{\pi}{2}\right)^n\right)^{4/3}}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\left(a^3-b^3\sin\left(x+\frac{\pi}{2}\right)^n\right)^{4/3}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3709$$

$$-\int \frac{\sec(x)}{(a^3-b^3\cos^n(x))^{4/3}}d\cos(x)$$

$$\Big\downarrow \ 798$$

$$-\frac{\int \frac{\sec(x)}{(a^3-b^3\cos^n(x))^{4/3}}d\cos^n(x)}{n}$$

$$\Big\downarrow \ 61$$

$$-\frac{\frac{\int \frac{\sec(x)}{\sqrt[3]{a^3-b^3\cos^n(x)}}d\cos^n(x)}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x)}}}{n}$$

$$\Big\downarrow \ 67$$

$$-\frac{\frac{-\frac{3\int \frac{1}{a-\sqrt[3]{a^3-b^3\cos^n(x)}}d\sqrt[3]{a^3-b^3\cos^n(x)}}{2a}+\frac{3}{2}\int \frac{1}{\cos^{2n}(x)+a^2+a\sqrt[3]{a^3-b^3\cos^n(x)}}d\sqrt[3]{a^3-b^3\cos^n(x)}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{}{a^3}}{n}$$

$\downarrow$ 16

$$-\frac{\frac{\frac{3}{2}\int \frac{1}{\cos^{2n}(x)+a^2+a\sqrt[3]{a^3-b^3\cos^n(x)}}d\sqrt[3]{a^3-b^3\cos^n(x)}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x}}{n}$$

$\downarrow$ 1082

$$-\frac{\frac{-\frac{3\int \frac{1}{-\cos^{2n}(x)-3}d\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}+1\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x)}}}{n}$$

$\downarrow$ 217

$$-\frac{\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}+1}{\sqrt{3}}\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x)}}}{n}$$

input
```
Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

output
```
-((3/(a^3*(a^3 - b^3*Cos[x]^n)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a^3 - b^
3*Cos[x]^n)^(1/3))/a)/Sqrt[3]])/a - Log[Cos[x]^n]/(2*a) + (3*Log[a - (a^3
- b^3*Cos[x]^n)^(1/3)])/(2*a))/a^3)/n)
```

3.448. $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$

3.448.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.448. $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.448.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-\frac{\frac{\ln\left(a-\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right)}{a^4}+\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}+\left(a^3-b^3(\cos^n(x))\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right.}{3a}\right.}{a^4}}{n}$
default	$-\frac{\frac{\ln\left(a-\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right)}{a^4}+\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}+\left(a^3-b^3(\cos^n(x))\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(a^3-b^3(\cos^n(x))\right)^{\frac{1}{3}}\right.}{3a}\right.}{a^4}}{n}$

input
```
int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/n*(1/a^4*ln(a-(a^3-b^3*cos(x)^n)^(1/3))+1/a^4*(-1/2*ln(a^2+a*(a^3-b^3*c
os(x)^n)^(1/3)+(a^3-b^3*cos(x)^n)^(2/3))+3^(1/2)*arctan(1/3*(a+2*(a^3-b^3*
cos(x)^n)^(1/3))/a*3^(1/2)))+3/a^3/(a^3-b^3*cos(x)^n)^(1/3))
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.65

$$\int\frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx=$$
$$-\frac{2\left(\sqrt{3}b^3\cos(x)^n-\sqrt{3}a^3\right)\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}}{3a}\right)-\left(b^3\cos(x)^n-a^3\right)\log\left(a^2+\left(-b^3\cos(x)^n+\right.\right.}{2\left(a^4b^3n\right.}$$

input
```
integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="fricas")
```

output
```
-1/2*(2*(sqrt(3)*b^3*cos(x)^n - sqrt(3)*a^3)*arctan(1/3*(sqrt(3)*a + 2*sqr
t(3)*(-b^3*cos(x)^n + a^3)^(1/3))/a) - (b^3*cos(x)^n - a^3)*log(a^2 + (-b^
3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)) + 2*(b^3*cos(x)^n
 - a^3)*log(-a + (-b^3*cos(x)^n + a^3)^(1/3)) - 6*(-b^3*cos(x)^n + a^3)^(2
/3)*a)/(a^4*b^3*n*cos(x)^n - a^7*n)
```

3.448.6 Sympy [F]

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}}\, dx = \int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{\frac{4}{3}}}\, dx$$

input
```
integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
```

output
```
Integral(tan(x)/(a**3 - b**3*cos(x)**n)**(4/3), x)
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}}\, dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^4 n} + \frac{\log\left(a^2 + \left(-b^3\cos(x)^n + a^3\right)^{\frac{1}{3}}a + \left(-b^3\cos(x)^n + a^3\right)^{\frac{2}{3}}\right)}{2\,a^4 n} - \frac{\log\left(-a + \left(-b^3\cos(x)^n + a^3\right)^{\frac{1}{3}}\right)}{a^4 n} - \frac{3}{\left(-b^3\cos(x)^n + a^3\right)^{\frac{1}{3}} a^3 n}$$

input
```
integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a)/(a^4*n)
 + 1/2*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/
3))/(a^4*n) - log(-a + (-b^3*cos(x)^n + a^3)^(1/3))/(a^4*n) - 3/((-b^3*cos
(x)^n + a^3)^(1/3)*a^3*n)
```

3.448.8 Giac [F]

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{(-b^3 \cos(x)^n + a^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="giac")
```

output
```
integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3), x)
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{(a^3 - b^3 \cos(x)^n)^{4/3}}\,dx$$

input
```
int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3),x)
```

output
```
int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3), x)
```

3.449 $\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx$

3.449.1 Optimal result

Integrand size = 15, antiderivative size = 95

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx=\frac{\arctan\left(\frac{1-\sqrt[3]{1+2\cos^9(x)}}{\sqrt{3}\sqrt[6]{1+2\cos^9(x)}}\right)}{3\sqrt{3}}+\frac{1}{3}\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right)-\frac{1}{9}\text{arctanh}\left(\sqrt{1+2\cos^9(x)}\right)-\frac{2}{15}(1+2\cos^9(x))^{5/6}$$

output
```
1/3*arctanh((1+2*cos(x)^9)^(1/6))-1/9*arctanh((1+2*cos(x)^9)^(1/2))-2/15*(
1+2*cos(x)^9)^(5/6)+1/9*arctan(1/3*(1-(1+2*cos(x)^9)^(1/3))/(1+2*cos(x)^9)
^(1/6)*3^(1/2))*3^(1/2)
```

3.449.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.62

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx=\frac{1}{90}\left(10\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)-10\sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)+20\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right)-12(1+2\cos^9(x))^{5/6}-5\log\left(1-\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)+5\log\left(1+\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)\right)$$

input
```
Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

output
```
(10*Sqrt[3]*ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] - 10*Sqrt[3]*Ar
cTan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] + 20*ArcTanh[(1 + 2*Cos[x]^9)
^(1/6)] - 12*(1 + 2*Cos[x]^9)^(5/6) - 5*Log[1 - (1 + 2*Cos[x]^9)^(1/6) + (
1 + 2*Cos[x]^9)^(1/3)] + 5*Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^
9)^(1/3)])/90
```

3.449.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 3709, 798, 60, 73, 27, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2 \cos^9(x)+1\right)^{5/6} \tan(x) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(2 \sin\left(x+\frac{\pi}{2}\right)^9+1\right)^{5/6}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(2 \sin\left(x+\frac{\pi}{2}\right)^9+1\right)^{5/6}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3709}$$

$$-\int \left(2 \cos^9(x)+1\right)^{5/6} \sec(x)d\cos(x)$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{1}{9} \int \left(2 \cos^9(x)+1\right)^{5/6} \sec(x)d\cos^9(x)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{9} \left(-\int \frac{\sec(x)}{\sqrt[6]{2 \cos^9(x)+1}}d\cos^9(x)-\frac{6}{5} \left(2 \cos^9(x)+1\right)^{5/6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{9}\left(-3\int -\frac{2\cos^{36}(x)}{1-\cos^{54}(x)}d\sqrt[6]{2\cos^9(x)+1}-\frac{6}{5}\left(2\cos^9(x)+1\right)^{5/6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(6\int \frac{\cos^{36}(x)}{1-\cos^{54}(x)}d\sqrt[6]{2\cos^9(x)+1}-\frac{6}{5}\left(2\cos^9(x)+1\right)^{5/6}\right)$$

$$\Big\downarrow \text{825}$$

$$\frac{1}{9}\left(6\left(\frac{1}{3}\int \frac{1}{1-\cos^{18}(x)}d\sqrt[6]{2\cos^9(x)+1}+\frac{1}{3}\int -\frac{\sqrt[6]{2\cos^9(x)+1}+1}{2\left(\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1\right)}d\sqrt[6]{2\cos^9(x)+1}+\frac{1}{3}\int -\frac{}{2}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(6\left(\frac{1}{3}\int \frac{1}{1-\cos^{18}(x)}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{6}\int \frac{\sqrt[6]{2\cos^9(x)+1}+1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{6}\int \frac{1-}{\cos^{18}(x)}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{9}\left(6\left(-\frac{1}{6}\int \frac{\sqrt[6]{2\cos^9(x)+1}+1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{6}\int \frac{1-\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)+\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)}\right.\right.$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{2}\int -\frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{3}{2}\int \frac{1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)}\right.\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(3\int \frac{1}{-\cos^{18}(x)-3}d\left(2\sqrt[6]{2\cos^9(x)+1}-1\right)+\frac{1}{2}\int \frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}\right)+\right.\right.$$

$$\Big\downarrow \text{217}$$

3.449. $\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\sqrt{3}\arctan\left(\frac{2\sqrt[6]{2\cos^9(x)+1}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[6]{2\cos^9(x)+1}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\cos^{18}(x)\right.\right.\right.\right.$$

input
```
Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

output
```
((-6*(1 + 2*Cos[x]^9)^(5/6))/5 + 6*(ArcTanh[(1 + 2*Cos[x]^9)^(1/6)]/3 + (-
(Sqrt[3]*ArcTan[(-1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]) - Log[1 + Cos[x]
^18 - (1 + 2*Cos[x]^9)^(1/6)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*(1 + 2*Cos[x
]^9)^(1/6))/Sqrt[3]]) + Log[1 + Cos[x]^18 + (1 + 2*Cos[x]^9)^(1/6)]/2)/6))
/9
```

3.449.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.449. $\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx$

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.449.4 Maple [F]

$$\int \left(1+2\left(\cos^{9}\left(x\right)\right)\right)^{\frac{5}{6}} \tan\left(x\right) dx$$

input
```
int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

output
```
int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

3.449.5 Fricas [F(-1)]

Timed out.

$$\int \left(1+2\cos^9(x)\right)^{5/6} \tan(x)\, dx = \text{Timed out}$$

input
```
integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="fricas")
```

output
```
Timed out
```

3.449.6 Sympy [F(-1)]

Timed out.

$$\int \left(1+2\cos^9(x)\right)^{5/6} \tan(x)\, dx = \text{Timed out}$$

input
```
integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

output
```
Timed out
```

3.449.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(72) = 144$.

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right) - \frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}} + \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}} + \left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}} - \left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) + \frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)$$

input
```
integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9
+ 1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1)
 - 1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log
((2*cos(x)^9 + 1)^(1/6) + 1) - 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(72) = 144$.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.54

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right) - \frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}} + \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}} + \left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}} - \left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) + \frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{9}\log\left(\left|\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9
+ 1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1)
 - 1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log
((2*cos(x)^9 + 1)^(1/6) + 1) - 1/9*log(abs((2*cos(x)^9 + 1)^(1/6) - 1))
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx = \int \tan(x)\ \left(2\cos(x)^9+1\right)^{5/6}\,dx$$

input
```
int(tan(x)*(2*cos(x)^9 + 1)^(5/6),x)
```

output
```
int(tan(x)*(2*cos(x)^9 + 1)^(5/6), x)
```

3.450 $\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx$

3.450.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{4}{125\sqrt[3]{2-5\sin^3(x)}}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}$$

output
```
4/125/(2-5*sin(x)^3)^(1/3)+2/125*(2-5*sin(x)^3)^(2/3)-1/625*(2-5*sin(x)^3)
^(5/3)
```

3.450.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{36-30\sin^3(x)-25\sin^6(x)}{625\sqrt[3]{2-5\sin^3(x)}}$$

input
```
Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

output
```
(36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))
```

3.450.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4834, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(x)\cos(x)}{\left(2-5\sin^3(x)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^8\cos(x)}{(2-5\sin(x)^3)^{4/3}}dx$$

$$\Big\downarrow \text{4834}$$

$$\int \frac{\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}d\sin(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sin^6(x)}{\left(2-5\sin^3(x)\right)^{4/3}}d\sin^3(x)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{1}{25}\left(2-5\sin^3(x)\right)^{2/3}-\frac{4}{25\sqrt[3]{2-5\sin^3(x)}}+\frac{4}{25\left(2-5\sin^3(x)\right)^{4/3}}\right)d\sin^3(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{6}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{12}{125\sqrt[3]{2-5\sin^3(x)}}\right)$$

input
```
Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

output
```
(12/(125*(2 - 5*Sin[x]^3)^(1/3)) + (6*(2 - 5*Sin[x]^3)^(2/3))/125 - (3*(2
- 5*Sin[x]^3)^(5/3))/625)/3
```

3.450. $\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx$

3.450.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.450.4 Maple [F]

$$\int \frac{\cot(x)\left(\sin^9(x)\right)}{\left(2-5\left(\sin^3(x)\right)\right)^{\frac{4}{3}}}dx$$

input
```
int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

output
```
int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{25\cos{(x)}^6-75\cos{(x)}^4+75\cos{(x)}^2+30\left(\cos{(x)}^2-1\right)\sin{(x)}+11}{625\left(5\left(\cos{(x)}^2-1\right)\sin{(x)}+2\right)^{\frac{1}{3}}}$$

input
```
integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="fricas")
```

output
```
1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1)*sin(x)
+ 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)
```

3.450.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)
```

output
```
Timed out
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\frac{1}{625}\left(-5\sin{(x)}^3+2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\sin{(x)}^3+2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\sin{(x)}^3+2\right)^{\frac{1}{3}}}$$

input
```
integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="maxima")
```

output
```
-1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5
*sin(x)^3 + 2)^(1/3)
```

3.450.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\frac{1}{625}\left(-5\,\sin\left(x\right)^3+2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\,\sin\left(x\right)^3+2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\,\sin\left(x\right)^3+2\right)^{\frac{1}{3}}}$$

input
```
integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="giac")
```

output
```
-1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5
*sin(x)^3 + 2)^(1/3)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \int \frac{\cot(x)\,\sin\left(x\right)^9}{\left(2-5\sin\left(x\right)^3\right)^{4/3}}\,dx$$

input
```
int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3),x)
```

output
```
int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3), x)
```

3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$

3.451.1 Optimal result

Integrand size = 33, antiderivative size = 20

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2$$

output
```
-3/32*(1+(1-8*tan(x)^2)^(1/3))^2
```

3.451.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2$$

input
```
Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^
(2/3),x]
```

output
```
(-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

3.451.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4842, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\sec^2(x)}{\left(1-8\tan^2(x)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)\left(\sqrt[3]{1-8\tan(x)^2}+1\right)\sec(x)^2}{\left(1-8\tan(x)^2\right)^{2/3}}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{\tan(x)\left(\sqrt[3]{1-8\tan^2(x)}+1\right)}{\left(1-8\tan^2(x)\right)^{2/3}}d\tan(x)$$

$$\Big\downarrow \text{7237}$$

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

input
```
Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),
x]
```

output
```
(-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

3.451.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.451.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\frac{3(1-8(\tan^2(x)))^{\frac{1}{3}}}{16}-\frac{3(1-8(\tan^2(x)))^{\frac{2}{3}}}{32}$	26
default	$-\frac{3(1-8(\tan^2(x)))^{\frac{1}{3}}}{16}-\frac{3(1-8(\tan^2(x)))^{\frac{2}{3}}}{32}$	26

input
```
int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,method
=_RETURNVERBOSE)
```

output
```
-3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)
```

3.451.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}-\frac{3}{16}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{1}{3}}$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="fricas")
```

output
```
-3/32*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) - 3/16*((9*cos(x)^2 - 8)/cos(x)^2)
^(1/3)
```

3.451.6 Sympy [F]

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\tan(x)}{\left(1-8\tan^2(x)\right)^{\frac{2}{3}}\cos^2(x)}\,dx$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/
3),x)
```

output
```
Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*tan(x)/((1 - 8*tan(x)**2)**(2/3)*c
os(x)**2), x)
```

3.451.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.30

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{3\left(\frac{\left(9\sin(x)^2-1\right)(3\sin(x)-1)^{\frac{1}{3}}(\sin(x)+1)^{\frac{1}{3}}(\sin(x)-1)^{\frac{1}{3}}}{(3\sin(x)+1)^{\frac{1}{3}}}+\frac{2\left(9\sin(x)^2-1\right)(\sin(x)+1)^{\frac{2}{3}}(\sin(x)-1)^{\frac{2}{3}}}{(3\sin(x)+1)^{\frac{2}{3}}}\right)}{32\left(\sin(x)^2-1\right)(3\sin(x)-1)^{\frac{2}{3}}}$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="maxima")
```

output
```
-3/32*((9*sin(x)^2 - 1)*(3*sin(x) - 1)^(1/3)*(sin(x) + 1)^(1/3)*(sin(x) -
1)^(1/3)/(3*sin(x) + 1)^(1/3) + 2*(9*sin(x)^2 - 1)*(sin(x) + 1)^(2/3)*(sin
(x) - 1)^(2/3)/(3*sin(x) + 1)^(2/3))/((sin(x)^2 - 1)*(3*sin(x) - 1)^(2/3))
```

3.451.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{3}{32}\left(-8\,\tan\left(x\right)^2+1\right)^{\frac{2}{3}}-\frac{3}{16}\left(-8\,\tan\left(x\right)^2+1\right)^{\frac{1}{3}}$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="giac")
```

output
```
-3/32*(-8*tan(x)^2 + 1)^(2/3) - 3/16*(-8*tan(x)^2 + 1)^(1/3)
```

3.451.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{3\left(\left(18\cos\left(x\right)^2-16\right)^{1/3}+2\left(2\cos\left(x\right)^2\right)^{1/3}\right)\left(18\cos\left(x\right)^2-16\right)^{1/3}}{32\left(2\cos\left(x\right)^2\right)^{2/3}}$$

input
```
int((tan(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)
),x)
```

output
```
-(3*((18*cos(x)^2 - 16)^(1/3) + 2*(2*cos(x)^2)^(1/3))*(18*cos(x)^2 - 16)^(
1/3))/(32*(2*cos(x)^2)^(2/3))
```

3.451. $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$

3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$

3.452.1 Optimal result

Integrand size = 31, antiderivative size = 27

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\log(\tan(x))+\frac{3}{2}\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)$$

output
```
-ln(tan(x))+3/2*ln(1-(1-8*tan(x)^2)^(1/3))
```

3.452.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \frac{3\cos^2(x)\sqrt[3]{\sec^2(x)-9\tan^2(x)}\left(\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\right.\right.}{}$$

input
```
Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2
/3),x]
```

output
```
(3*Cos[x]^2*(Sec[x]^2 - 9*Tan[x]^2)^(1/3)*(Hypergeometric2F1[2/3, 1, 5/3,
(2*Cos[x]^2)/(-7 + 9*Cos[2*x])] + 2*Hypergeometric2F1[1/3, 1, 4/3, (2*Cos[
x]^2)/(-7 + 9*Cos[2*x])]*(Sec[x]^2 - 9*Tan[x]^2)^(1/3)))/(-4 + 36*Sin[x]^2
)
```

3.452.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3042, 4866, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\csc(x)\sec(x)}{\left(1-8\tan^2(x)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sqrt[3]{1-8\tan(x)^2}+1\right)\sec(x)}{\sin(x)\left(1-8\tan(x)^2\right)^{2/3}}dx$$

$$\Big\downarrow \text{4866}$$

$$-\int \frac{\sec(x)\left(\sqrt[3]{9-8\sec^2(x)}+1\right)}{\left(1-\cos^2(x)\right)\left(9-8\sec^2(x)\right)^{2/3}}d\cos(x)$$

$$\Big\downarrow \text{7276}$$

$$-\int \left(-\frac{\sec(x)}{\left(\cos^2(x)-1\right)\sqrt[3]{9-8\sec^2(x)}}-\frac{\sec(x)}{\left(\cos^2(x)-1\right)\left(9-8\sec^2(x)\right)^{2/3}}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\frac{1}{2}\log\left(1-\sec^2(x)\right)$$

input
```
Int[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
```

output
```
-1/2*Log[1 - Sec[x]^2] + (3*Log[1 - (9 - 8*Sec[x]^2)^(1/3)])/2
```

3.452. $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}dx$

3.452.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.452.4 Maple [F]

$$\int \frac{\cot(x)\left(1+(1-8(\tan^2(x)))^{\frac{1}{3}}\right)}{\cos(x)^2(1-8(\tan^2(x)))^{\frac{2}{3}}}dx$$

input
```
int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

output
```
int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

3.452.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(23) = 46.

Time = 0.96 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{1}{2}\log\left(\frac{16\left(145\cos(x)^4-200\cos(x)^2+3\left(11\cos(x)^4-8\cos(x)^2\right)\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}+3\left(19\cos(x)^4-16\right.\right.}{\cos(x)^4-2\cos(x)^2+1}\right.$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="fricas")
```

output
```
-1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos(x)^2)*((
9*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*cos(x)^2)*((9*cos(x)
^2 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos(x)^2 + 1))
```

3.452.6 Sympy [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\cot(x)}{(1-8\tan^2(x))^{\frac{2}{3}}\cos^2(x)}\,dx$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/
3),x)
```

output
```
Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*cot(x)/((1 - 8*tan(x)**2)**(2/3)*c
os(x)**2), x)
```

3.452.7 Maxima [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\left(\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}+1\right)\cot(x)}{\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}\cos(x)^2}\,dx$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="maxima")
```

output
```
integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*co
s(x)^2), x)
```

3.452.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{1}{2}\log\left(\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}+\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}+1\right) + \log\left(\left|\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="giac")
```

output
```
-1/2*log((-8*tan(x)^2 + 1)^(2/3) + (-8*tan(x)^2 + 1)^(1/3) + 1) + log(abs(
(-8*tan(x)^2 + 1)^(1/3) - 1))
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx=\int \frac{\cot(x)\,\left(\left(1-8\tan(x)^2\right)^{1/3}+1\right)}{\cos\left(x\right)^2\,\left(1-8\tan\left(x\right)^2\right)^{2/3}}\,dx$$

input
```
int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)
),x)
```

output
```
int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)
), x)
```

3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

3.453.1 Optimal result

Integrand size = 52, antiderivative size = 101

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = -\frac{3\arctan\left(\frac{\sqrt[4]{-1+5\sin^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+5\sin^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+2\sqrt[4]{-1+5\sin^2(x)}-\frac{\sqrt[4]{-1+5\sin^2(x)}}{2\left(2+\sqrt{-1+5\sin^2(x)}\right)}$$

output
```
2*(-1+5*sin(x)^2)^(1/4)-3/2*arctan(1/2*(-1+5*sin(x)^2)^(1/4)*2^(1/2))*2^(1
/2)-1/4*arctanh(1/2*(-1+5*sin(x)^2)^(1/4)*2^(1/2))*2^(1/2)-1/2*(-1+5*sin(x
)^2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/2))
```

3.453.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=\frac{1}{4}\left(-6\sqrt{2}\arctan\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)-2\sqrt[4]{4-5\cos^2(x)}\left(-4+\frac{1}{2+\sqrt{4-5\cos^2(x)}}\right)\right)$$

input
```
Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)
^(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

output
```
(-6*Sqrt[2]*ArcTan[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - Sqrt[2]*ArcTanh[(3 -
5*Cos[2*x])^(1/4)/2^(3/4)] - 2*(4 - 5*Cos[x]^2)^(1/4)*(-4 + (2 + Sqrt[4 -
5*Cos[x]^2])^(-1)))/4
```

3.453.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3042, 4861, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\left(5\cos^2(x)-\sqrt{5\sin^2(x)-1}\right)}{\sqrt[4]{5\sin^2(x)-1}\left(\sqrt{5\sin^2(x)-1}+2\right)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(x)\left(5\cos(x)^2-\sqrt{5\sin(x)^2-1}\right)}{\sqrt[4]{5\sin(x)^2-1}\left(\sqrt{5\sin(x)^2-1}+2\right)}dx$$
$$\Big\downarrow \text{4861}$$

$$-\int \frac{5\cos^2(x)-\sqrt{4-5\cos^2(x)}}{\sqrt[4]{4-5\cos^2(x)}\left(\sqrt{4-5\cos^2(x)}\cos(x)+2\cos(x)\right)}d\cos(x)$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{5\cos(x)}{\sqrt[4]{4-5\cos^2(x)}\left(\sqrt{4-5\cos^2(x)}+2\right)}-\frac{\sqrt[4]{4-5\cos^2(x)}\sec(x)}{\sqrt{4-5\cos^2(x)}+2}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-2\sqrt{2}\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)+\frac{\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(\sqrt{4-5\cos^2(x)}+2\right)}$$

input
```
Int[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)
*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

output
```
ArcTan[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/Sqrt[2] - 2*Sqrt[2]*ArcTan[(4 - 5*C
os[x]^2)^(1/4)/Sqrt[2]] - ArcTanh[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[
2]) + 2*(4 - 5*Cos[x]^2)^(1/4) - (4 - 5*Cos[x]^2)^(1/4)/(2*(2 + Sqrt[4 - 5
*Cos[x]^2]))
```

3.453.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

3.453. $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.453.4 Maple [F]

$$\int \frac{\left(5(\cos^2(x)) - \sqrt{-1+5(\sin^2(x))}\right)\tan(x)}{\left(-1+5(\sin^2(x))\right)^{\frac{1}{4}}\left(2+\sqrt{-1+5(\sin^2(x))}\right)}dx$$

input
```
int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1
+5*sin(x)^2)^(1/2)),x)
```

output
```
int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1
+5*sin(x)^2)^(1/2)),x)
```

3.453.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(81) = 162$.

Time = 39.14 (sec) , antiderivative size = 461, normalized size of antiderivative = 4.56

$$\int \frac{\left(5\cos^2(x) - \sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$= \frac{70\left(5\sqrt{2}\cos(x)^4 - 4\sqrt{2}\cos(x)^2\right)\arctan\left(-\frac{2\left(\left(5\sqrt{2}\cos(x)^2-4\sqrt{2}\right)\left(-5\cos(x)^2+4\right)^{\frac{3}{4}}-2\sqrt{2}\left(-5\cos(x)^2+4\right)^{\frac{5}{4}}\right)}{5\left(5\cos(x)^4-4\cos(x)^2\right)}\right) - 50}{}$$

input
```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/
(2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="fricas")
```

output
```
1/160*(70*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*arctan(-2/5*((5*sqrt(2
)*cos(x)^2 - 4*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 2*sqrt(2)*(-5*cos(x)^2 +
 4)^(5/4))/(5*cos(x)^4 - 4*cos(x)^2)) - 50*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)
*cos(x)^2)*arctan(2/5*(sqrt(2)*(-5*cos(x)^2 + 4)^(3/4) + 2*sqrt(2)*(-5*cos
(x)^2 + 4)^(1/4))/cos(x)^2) + 35*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)
*log(-(125*cos(x)^6 - 1700*cos(x)^4 - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2))
*(-5*cos(x)^2 + 4)^(5/4) + 2560*cos(x)^2 + 4*(25*sqrt(2)*cos(x)^4 - 100*sq
rt(2)*cos(x)^2 + 64*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(25*cos(x)^4 - 6
0*cos(x)^2 + 32)*sqrt(-5*cos(x)^2 + 4) - 1024)/(5*cos(x)^6 - 4*cos(x)^4))
+ 25*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(-(25*cos(x)^4 - 320*cos
(x)^2 - 4*(5*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(
5*cos(x)^2 - 8)*sqrt(-5*cos(x)^2 + 4) - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2
))*(-5*cos(x)^2 + 4)^(1/4) + 256)/cos(x)^4) + 16*(5*cos(x)^2 - 2*(10*cos(x
)^2 - 1)*sqrt(-5*cos(x)^2 + 4) - 4)*(-5*cos(x)^2 + 4)^(3/4))/(5*cos(x)^4 -
 4*cos(x)^2)
```

3.453.6 Sympy [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=\int \frac{\left(-\sqrt{5\sin^2\left(x\right)-1}+5\cos^2\left(x\right)\right)\tan\left(x\right)}{\left(\sqrt{5\sin^2\left(x\right)-1}+2\right)\sqrt[4]{5\sin^2\left(x\right)-1}}\,dx$$

input
```
integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(
1/4)/(2+(-1+5*sin(x)**2)**(1/2)),x)
```

output
```
Integral((-sqrt(5*sin(x)**2 - 1) + 5*cos(x)**2)*tan(x)/((sqrt(5*sin(x)**2
- 1) + 2)*(5*sin(x)**2 - 1)**(1/4)), x)
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$= -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(5\sin\left(x\right)^2-1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\left(5\sin\left(x\right)^2-1\right)^{\frac{1}{4}}}{\sqrt{2}+\left(5\sin\left(x\right)^2-1\right)^{\frac{1}{4}}}\right)+2\left(5\sin\left(x\right)^2-1\right)^{\frac{1}{4}}-\frac{\left(5\sin\left(x\right)^2-1\right)^{\frac{1}{4}}}{2\left(\sqrt{5\sin\left(x\right)^2-1}+2\right)}$$

input
```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/
(2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="maxima")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqrt(2)*log(
-(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/(sqrt(2) + (5*sin(x)^2 - 1)^(1/4))) +
2*(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1
) + 2)
```

3.453.8 Giac [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = \int \frac{\left(5\cos\left(x\right)^2-\sqrt{5\sin\left(x\right)^2-1}\right)\tan\left(x\right)}{\left(5\sin\left(x\right)^2-1\right)^{\frac{1}{4}}\left(\sqrt{5\sin\left(x\right)^2-1}+2\right)}\,dx$$

input
```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/
(2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="giac")
```

output
```
integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/
4)*(sqrt(5*sin(x)^2 - 1) + 2)), x)
```

3.453. $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=\int \frac{\tan(x)\left(5\cos(x)^2-\sqrt{5\sin(x)^2-1}\right)}{\left(5\sin(x)^2-1\right)^{1/4}\left(\sqrt{5\sin(x)^2-1}+2\right)}\,dx$$

input
```
int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)
*((5*sin(x)^2 - 1)^(1/2) + 2)),x)
```

output
```
int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)
*((5*sin(x)^2 - 1)^(1/2) + 2)), x)
```

3.454 $\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx$

3.454.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx=-\frac{3}{40}\cos^{\frac{5}{3}}(2x)-\frac{3}{64}\cos^{\frac{8}{3}}(2x)$$

output
```
-3/40*cos(2*x)^(5/3)-3/64*cos(2*x)^(8/3)
```

3.454.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx=-\frac{3}{320}\cos^{\frac{5}{3}}(2x)(8+5\cos(2x))$$

input
```
Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

output
```
(-3*Cos[2*x]^(5/3)*(8 + 5*Cos[2*x]))/320
```

3.454.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4857, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(x)\cos^{\frac{2}{3}}(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)\cos(x)^3\cos(2x)^{2/3}dx$$

$$\Big\downarrow \; 4857$$

$$-\int \cos^3(x)\left(2\cos^2(x)-1\right)^{2/3}d\cos(x)$$

$$\Big\downarrow \; 243$$

$$-\frac{1}{2}\int \cos^2(x)\left(2\cos^2(x)-1\right)^{2/3}d\cos^2(x)$$

$$\Big\downarrow \; 53$$

$$-\frac{1}{2}\int \left(\frac{1}{2}\left(2\cos^2(x)-1\right)^{5/3}+\frac{1}{2}\left(2\cos^2(x)-1\right)^{2/3}\right)d\cos^2(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{3}{32}\left(2\cos^2(x)-1\right)^{8/3}-\frac{3}{20}\left(2\cos^2(x)-1\right)^{5/3}\right)$$

input
```
Int[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

output
```
((-3*(-1 + 2*Cos[x]^2)^(5/3))/20 - (3*(-1 + 2*Cos[x]^2)^(8/3))/32)/2
```

3.454.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.454.4 Maple [F]

$$\int \left(\cos^4(x)\right)\left(\cos^{\frac{2}{3}}(2x)\right)\tan(x)\,dx$$

input
```
int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

output
```
int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{320} \left(20 \cos(x)^4 - 4 \cos(x)^2 - 3\right) \left(2 \cos(x)^2 - 1\right)^{\frac{2}{3}}$$

input
```
integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="fricas")
```

output
```
-3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)
```

3.454.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = \text{Timed out}$$

input
```
integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)
```

output
```
Timed out
```

3.454.7 Maxima [F]

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = \int \cos(2x)^{\frac{2}{3}} \cos(x)^4 \tan(x) \, dx$$

input
```
integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="maxima")
```

output
```
integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

3.454.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx = -\frac{3}{64}\left(2\cos(x)^2-1\right)^{\frac{8}{3}} - \frac{3}{40}\left(2\cos(x)^2-1\right)^{\frac{5}{3}}$$

input
```
integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="giac")
```

output
```
-3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx = \int \cos(2x)^{2/3}\cos(x)^4\tan(x)\,dx$$

input
```
int(cos(2*x)^(2/3)*cos(x)^4*tan(x),x)
```

output
```
int(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$

3.455.1 Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \frac{\arctan\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{1+\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{\cos(2x)} - \frac{1}{5}\cos^{\frac{5}{4}}(2x) + \frac{1}{36}\cos^{\frac{9}{4}}(2x)$$

output
```
7/4*cos(2*x)^(1/4)-1/5*cos(2*x)^(5/4)+1/36*cos(2*x)^(9/4)+1/2*arctan(1/2*(
1-cos(2*x)^(1/2))/cos(2*x)^(1/4)*2^(1/2))*2^(1/2)-1/2*arctanh(1/2*(1+cos(2
*x)^(1/2))/cos(2*x)^(1/4)*2^(1/2))*2^(1/2)
```

3.455.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \frac{1}{360}\left(180\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}\right) - 180\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}\right) + 635\sqrt[4]{\cos(2x)} - 72\cos^{\frac{5}{4}}(2x) + 5\sqrt[4]{\cos(2x)}\cos(4x) + 90\sqrt{2}\log\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right) - 90\sqrt{2}\log\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right)\right)$$

input
```
Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

output
```
(180*Sqrt[2]*ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)] - 180*Sqrt[2]*ArcTan[1 + S
qrt[2]*Cos[2*x]^(1/4)] + 635*Cos[2*x]^(1/4) - 72*Cos[2*x]^(5/4) + 5*Cos[2*
x]^(1/4)*Cos[4*x] + 90*Sqrt[2]*Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2
*x]]] - 90*Sqrt[2]*Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]])/360
```

3.455.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4861, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{3/4}}dx$$

$$\Big\downarrow \text{4861}$$

$$-\int \frac{(1-\cos^2(x))^3\sec(x)}{(2\cos^2(x)-1)^{3/4}}d\cos(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{(1-\cos^2(x))^3\sec(x)}{(2\cos^2(x)-1)^{3/4}}d\cos^2(x)$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(-\frac{1}{4}(2\cos^2(x)-1)^{5/4}+\sqrt[4]{2\cos^2(x)-1}+\frac{\sec(x)}{(2\cos^2(x)-1)^{3/4}}-\frac{7}{4(2\cos^2(x)-1)^{3/4}}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt[4]{2\cos^2(x)-1}\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt[4]{2\cos^2(x)-1}+1\right)+\frac{1}{18}(2\cos^2(x)-1)^{9/4}-\frac{2}{5}(2\cos^2\right.$$

input
```
Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

output
```
(Sqrt[2]*ArcTan[1 - Sqrt[2]*(-1 + 2*Cos[x]^2)^(1/4)] - Sqrt[2]*ArcTan[1 +
Sqrt[2]*(-1 + 2*Cos[x]^2)^(1/4)] + (7*(-1 + 2*Cos[x]^2)^(1/4))/2 - (2*(-1
+ 2*Cos[x]^2)^(5/4))/5 + (-1 + 2*Cos[x]^2)^(9/4)/18 + Log[1 - Sqrt[2]*(-1
+ 2*Cos[x]^2)^(1/4) + Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2] - Log[1 + Sqrt[2]*(-1
 + 2*Cos[x]^2)^(1/4) + Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2])/2
```

3.455.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

3.455. $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$

3.455.4 Maple [F]

$$\int \frac{(\sin^6(x))\tan(x)}{\cos(2x)^{\frac{3}{4}}}dx$$

input
```
int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

output
```
int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

3.455.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.455.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
```

output
```
Timed out
```

3.455.7 Maxima [F]

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4), x)
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx =& \frac{1}{36}\cos(2x)^{\frac{9}{4}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\cos(2x)^{\frac{1}{4}}\right)\right) \\ &- \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\cos(2x)^{\frac{1}{4}}\right)\right) \\ &- \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\cos(2x)^{\frac{1}{4}}+\sqrt{\cos(2x)}+1\right) \\ &+ \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\cos(2x)^{\frac{1}{4}}+\sqrt{\cos(2x)}+1\right) \\ &- \frac{1}{5}\cos(2x)^{\frac{5}{4}} + \frac{7}{4}\cos(2x)^{\frac{1}{4}}\end{aligned}$$

input
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="giac")
```

output
```
1/36*cos(2*x)^(9/4) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*cos(2*x)
^(1/4))) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*cos(2*x)^(1/4))) -
 1/4*sqrt(2)*log(sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) + 1/4*sqrt(2
)*log(-sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) - 1/5*cos(2*x)^(5/4) +
 7/4*cos(2*x)^(1/4)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin\left(x\right)^6\tan(x)}{\cos\left(2\,x\right)^{3/4}}\,dx$$

input
```
int((sin(x)^6*tan(x))/cos(2*x)^(3/4),x)
```

output
```
int((sin(x)^6*tan(x))/cos(2*x)^(3/4), x)
```

3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$

3.456.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\text{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

output
```
-arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))
```

3.456.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\csc(x)\sqrt{\tan(x)\tan(2x)}}{\sqrt{2}}$$

input
```
Integrate[Sqrt[Tan[x]*Tan[2*x]],x]
```

output
```
-((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Csc[x]*Sqrt[Tan
[x]*Tan[2*x]])/Sqrt[2])
```

3.456.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4897, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\tan(x)\tan(2x)}dx$$

$$\Big\downarrow \; 4897$$

$$\int \sqrt{\sec(2x)-1}dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(2x+\frac{\pi}{2}\right)-1}dx$$

$$\Big\downarrow \; 4261$$

$$-\int \frac{1}{\frac{\tan^2(2x)}{\sec(2x)-1}-1}d\left(-\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

$$\Big\downarrow \; 220$$

$$-\text{arctanh}\left(\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

input
```
Int[Sqrt[Tan[x]*Tan[2*x]],x]
```

output
```
-ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]
```

3.456.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.456.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.69 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

method	result	size
default	$\frac{\sqrt{\frac{\sin^2(x)}{2\left(\cos^2(x)\right)-1}}\,\sin(x)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}\,\operatorname{arctanh}\left(\frac{\cos(x)\sqrt{2}}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right)\sqrt{4}}{-2+2\cos(x)}$	78

input
```
int((tan(x)*tan(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(sin(x)^2/(2*cos(x)^2-1))^(1/2)*sin(x)*((2*cos(x)^2-1)/(cos(x)+1)^2)^(
1/2)*arctanh(cos(x)/(cos(x)+1)/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)*2^(1/2)
)/(-1+cos(x))*4^(1/2)
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.94

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \frac{1}{2}\log\left(-\frac{\tan(x)^3 - 2\sqrt{2}(\tan(x)^2-1)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2-1}} - 3\tan(x)}{\tan(x)^3+\tan(x)}\right)$$

input
```
integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(-(tan(x)^3 - 2*sqrt(2)*(tan(x)^2 - 1)*sqrt(-tan(x)^2/(tan(x)^2 - 1
)) - 3*tan(x))/(tan(x)^3 + tan(x)))
```

3.456.6 Sympy [F]

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan(x)\tan(2x)}\,dx$$

input
```
integrate((tan(x)*tan(2*x))**(1/2),x)
```

output
```
Integral(sqrt(tan(x)*tan(2*x)), x)
```

3.456.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(15) = 30.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 15.24

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$
$$= \frac{1}{4}\log\left(4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + 4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + 8\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right) + 4\right) - \frac{1}{4}\log\left(\cos(2x)^2+\sin(2x)^2 + \sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + \sin\left(\frac{1}{2}\arctan(\mathrm{s}\right.\right.\right.$$
$$+2\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\left(\cos(2x)\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right) + \sin(2\right.$$

input
```
integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(s
in(4*x), cos(4*x) + 1))^2 + 4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 8*(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 4) - 1
/4*log(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
 + 1)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sin(1/2*arctan2(sin(4*
x), cos(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)
*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arc
tan2(sin(4*x), cos(4*x) + 1))))
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{-\tan(x)^2+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-\tan(x)^2+1}\right)\right)\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan$$

input
```
integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((sqrt(2)*log(sqrt(2) + sqrt(-tan(x)^2 + 1)) - sqrt(2)*log(sqr
t(2) - sqrt(-tan(x)^2 + 1)))*sgn(tan(x)^2 - 1)*sgn(tan(x)) + (sqrt(2)*log(
sqrt(2) + 1) - sqrt(2)*log(sqrt(2) - 1))*sgn(tan(x)))
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan(2x)\,\tan(x)}\,dx$$

input
```
int((tan(2*x)*tan(x))^(1/2),x)
```

output
```
int((tan(2*x)*tan(x))^(1/2), x)
```

3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$

3.457.1 Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = -\frac{\arcsin(\tan(x))}{\sqrt{2}} + \arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$

output
```
arctan(2^(1/2)*tan(x)/(1-tan(x)^2)^(1/2))-1/2*arcsin(tan(x))*2^(1/2)
```

3.457.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \frac{\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\right)\cos(x)\sqrt{\cot(2x)\tan(x)}}{\sqrt{\cos(2x)}}$$

input
```
Integrate[Sqrt[Cot[2*x]*Tan[x]],x]
```

output
```
((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])*Cos[x]*S
qrt[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

3.457.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4889, 27, 301, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)\cot(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)\cot(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt{1-\tan^2(x)}}{\sqrt{2}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{1-\tan^2(x)}}{\tan^2(x)+1}d\tan(x)}{\sqrt{2}}$$

$$\Big\downarrow \text{301}$$

$$\frac{2\int \frac{1}{\sqrt{1-\tan^2(x)}(\tan^2(x)+1)}d\tan(x)-\int \frac{1}{\sqrt{1-\tan^2(x)}}d\tan(x)}{\sqrt{2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\int \frac{1}{\sqrt{1-\tan^2(x)}(\tan^2(x)+1)}d\tan(x)-\arcsin(\tan(x))}{\sqrt{2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{2\int \frac{1}{\frac{2\tan^2(x)}{1-\tan^2(x)}+1}d\frac{\tan(x)}{\sqrt{1-\tan^2(x)}}-\arcsin(\tan(x))}{\sqrt{2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\arcsin(\tan(x))}{\sqrt{2}}$$

input
```
Int[Sqrt[Cot[2*x]*Tan[x]],x]
```

output
```
(-ArcSin[Tan[x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]])/Sq
rt[2]
```

3.457.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.457. $\int \sqrt{\cot(2x)\tan(x)}\, dx$

3.457.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(26) = 52$.

Time = 6.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.47

method	result
default	$\frac{\sqrt{2}\,\sqrt{2-(\sec^2(x))}\left(2\sqrt{2}\,\arctan\left(\frac{\sin(x)\sqrt{2}}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right)-\arctan\left(\frac{1+2\sin(x)}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right)-\arctan\left(\frac{2\sin(x)-1}{(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}\right.\right.}{4(\cos(x)+1)\sqrt{\frac{2\left(\cos^2(x)\right)-1}{(\cos(x)+1)^2}}}$

input
```
int((cot(2*x)/cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*(2-sec(x)^2)^(1/2)*(2*2^(1/2)*arctan(sin(x)/(cos(x)+1)/((2*cos
(x)^2-1)/(cos(x)+1)^2)^(1/2)*2^(1/2))-arctan((1+2*sin(x))/(cos(x)+1)/((2*c
os(x)^2-1)/(cos(x)+1)^2)^(1/2))-arctan((2*sin(x)-1)/(cos(x)+1)/((2*cos(x)^
2-1)/(cos(x)+1)^2)^(1/2)))*cos(x)/(cos(x)+1)/((2*cos(x)^2-1)/(cos(x)+1)^2)
^(1/2)
```

3.457.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.59

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(3\cos(2x)^2+2\cos(2x)-1\right)\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}}{4\cos(2x)\sin(2x)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{2}\cos(2x)^2+\sqrt{2}\cos(2x)-\sqrt{2}\right)\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}}{4\cos(2x)\sin(2x)}\right)$$

input
```
integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/4*sqrt(2)*(3*cos(2*x)^2 + 2*cos(2*x) - 1)*sqrt(cos(2*
x)/(cos(2*x) + 1))/(cos(2*x)*sin(2*x))) - 1/2*arctan(1/4*sqrt(2)*(2*sqrt(2
)*cos(2*x)^2 + sqrt(2)*cos(2*x) - sqrt(2))*sqrt(cos(2*x)/(cos(2*x) + 1))/(
cos(2*x)*sin(2*x)))
```

3.457.6 Sympy [F]

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \int \sqrt{\frac{\cot{(2x)}}{\cot{(x)}}}\,dx$$

input
```
integrate((cot(2*x)/cot(x))**(1/2),x)
```

output
```
Integral(sqrt(cot(2*x)/cot(x)), x)
```

3.457.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.84

$$\int \sqrt{\cot(2x)\tan(x)}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\sqrt{2}\arctan\left(\left(\cos{(4x)}^2 + \sin{(4x)}^2 + 2\cos{(4x)} + 1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan{(\sin{(4x)}, \cos{(4x)} + 1)}\right) + \right.\right.$$

input
```
integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1
/4)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin
(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) +
 cos(2*x)) - 2*arctan2(((abs(2*e^(2*I*x) + 2)^4 + 16*cos(2*x)^4 + 16*sin(2
*x)^4 + 8*(cos(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^
2 - 64*cos(2*x)^3 + 32*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2
*x)^2 - 64*cos(2*x) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(2*x) - 1)*sin(2*x)/
abs(2*e^(2*I*x) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x
)^2 - 8*cos(2*x) + 4)/abs(2*e^(2*I*x) + 2)^2)) + 2*sin(2*x))/abs(2*e^(2*I*
x) + 2), ((abs(2*e^(2*I*x) + 2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^4 + 8*(cos
(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 64*cos(2*x
)^3 + 32*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2 - 64*cos
(2*x) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(2*x) - 1)*sin(2*x)/abs(2*e^(2*I*x
) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x)^2 - 8*cos(2*
x) + 4)/abs(2*e^(2*I*x) + 2)^2)) + 2*cos(2*x) - 2)/abs(2*e^(2*I*x) + 2)))
```

3.457.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.31

$$\int \sqrt{\cot(2x)\tan(x)}\,dx$$
$$= \frac{1}{2}\left(\pi - \sqrt{2}\arctan(-i) - \sqrt{2}\arctan\left(\sqrt{2}\right) - i\log\left(2\sqrt{2}+3\right)\right)\operatorname{sgn}(\sin(2x))$$
$$- \frac{\sqrt{2}(-i\sqrt{2}\log(2i\sqrt{2}+3i) - 2\arctan(-i))\operatorname{sgn}(\cos(x)) + 2\left(\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{2}\sqrt{-2\cos(x)^4+3\cos}\right.}{4\cos(x)^2-3}\right.\right.\right.}{4\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(2x))}$$

input
```
integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="giac")
```

output
```
1/2*(pi - sqrt(2)*arctan(-I) - sqrt(2)*arctan(sqrt(2)) - I*log(2*sqrt(2) +
 3))*sgn(sin(2*x)) - 1/4*(sqrt(2)*(-I*sqrt(2)*log(2*I*sqrt(2) + 3*I) - 2*a
rctan(-I))*sgn(cos(x)) + 2*(sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(2)*sqrt(
-2*cos(x)^4 + 3*cos(x)^2 - 1) - 1)/(4*cos(x)^2 - 3) - 1)) + arcsin(4*cos(x
)^2 - 3))*sgn(cos(x)))/(sgn(cos(x))*sgn(sin(2*x)))
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \int \sqrt{\frac{\cot(2x)}{\cot(x)}}\,dx$$

input
```
int((cot(2*x)/cot(x))^(1/2),x)
```

output
```
int((cot(2*x)/cot(x))^(1/2), x)
```

3.458 $\int \frac{1}{x^5(5+x^2)}\,dx$

3.458.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{x^5(5+x^2)}\,dx=-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\log(x)}{125}-\frac{1}{250}\log\left(5+x^2\right)$$

output
```
-1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

3.458.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(5+x^2)}\,dx=-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\log(x)}{125}-\frac{1}{250}\log\left(5+x^2\right)$$

input
```
Integrate[1/(x^5*(5 + x^2)),x]
```

output
```
-1/20*1/x^4 + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

3.458.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^2+5)} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^6 (x^2+5)} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{1}{125x^2} - \frac{1}{25x^4} + \frac{1}{5x^6} - \frac{1}{125(x^2+5)}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{10x^4} + \frac{1}{25x^2} + \frac{\log(x^2)}{125} - \frac{1}{125}\log(x^2+5)\right)$$

input
```
Int[1/(x^5*(5 + x^2)),x]
```

output
```
(-1/10*1/x^4 + 1/(25*x^2) + Log[x^2]/125 - Log[5 + x^2]/125)/2
```

3.458.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.458.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$	24
norman	$\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$	25
risch	$\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$	25
meijerg	$-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln(5)}{250}-\frac{\ln\left(1+\frac{x^2}{5}\right)}{250}$	30
parallelrisch	$\frac{4x^4\ln(x)-2\ln(x^2+5)x^4-25+10x^2}{500x^4}$	31

input
```
int(1/x^5/(x^2+5),x,method=_RETURNVERBOSE)
```

output
```
-1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5(5+x^2)}\,dx = -\frac{2\,x^4\log\left(x^2+5\right)-4\,x^4\log\left(x\right)-10\,x^2+25}{500\,x^4}$$

input
```
integrate(1/x^5/(x^2+5),x, algorithm="fricas")
```

output
```
-1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4
```

3.458.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5(5+x^2)}\,dx = \frac{\log\left(x\right)}{125}-\frac{\log\left(x^2+5\right)}{250}+\frac{2x^2-5}{100x^4}$$

input
```
integrate(1/x**5/(x**2+5),x)
```

output
```
log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)
```

3.458.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5 (5 + x^2)} \, dx = \frac{2 \, x^2 - 5}{100 \, x^4} - \frac{1}{250} \log \left(x^2 + 5\right) + \frac{1}{250} \log \left(x^2\right)$$

input
```
integrate(1/x^5/(x^2+5),x, algorithm="maxima")
```

output
```
1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (5 + x^2)} \, dx = -\frac{3 \, x^4 - 10 \, x^2 + 25}{500 \, x^4} - \frac{1}{250} \log \left(x^2 + 5\right) + \frac{1}{250} \log \left(x^2\right)$$

input
```
integrate(1/x^5/(x^2+5),x, algorithm="giac")
```

output
```
-1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

3.458.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5 (5 + x^2)} \, dx = \frac{\ln (x)}{125} - \frac{\ln \left(x^2 + 5\right)}{250} + \frac{\frac{x^2}{50} - \frac{1}{20}}{x^4}$$

input
```
int(1/(x^5*(x^2 + 5)),x)
```

output
```
log(x)/125 - log(x^2 + 5)/250 + (x^2/50 - 1/20)/x^4
```

3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$

3.459.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{x^6(5+x^2)}\,dx=-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

output
```
-1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

3.459.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(5+x^2)}\,dx=-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

input
```
Integrate[1/(x^6*(5 + x^2)),x]
```

output
```
-1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

3.459.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^2+5)} \, dx$$
$$\Big\downarrow \text{264}$$
$$-\frac{1}{5} \int \frac{1}{x^4 (x^2+5)}dx - \frac{1}{25x^5}$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{5} \left(\frac{1}{5} \int \frac{1}{x^2 (x^2+5)}dx + \frac{1}{15x^3}\right) - \frac{1}{25x^5}$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{5} \left(\frac{1}{5} \left(-\frac{1}{5} \int \frac{1}{x^2+5}dx - \frac{1}{5x}\right) + \frac{1}{15x^3}\right) - \frac{1}{25x^5}$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{5} \left(\frac{1}{5} \left(-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{1}{5x}\right) + \frac{1}{15x^3}\right) - \frac{1}{25x^5}$$

input
```
Int[1/(x^6*(5 + x^2)),x]
```

output
```
-1/25*1/x^5 + (1/(15*x^3) + (-1/5*1/x - ArcTan[x/Sqrt[5]]/(5*Sqrt[5]))/5)/
5
```

3.459.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.459.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	29
risch	$\frac{-\frac{1}{125}x^4+\frac{1}{75}x^2-\frac{1}{25}}{x^5}-\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	30
meijerg	$\frac{\sqrt{5}\left(-\frac{2\sqrt{5}}{x}+\frac{10\sqrt{5}}{3x^3}-\frac{10\sqrt{5}}{x^5}-2\arctan\left(\frac{x\sqrt{5}}{5}\right)\right)}{1250}$	40

input
```
int(1/x^6/(x^2+5),x,method=_RETURNVERBOSE)
```

output
```
-1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{3\sqrt{5}x^5\arctan\left(\frac{1}{5}\sqrt{5}x\right)+15\,x^4-25\,x^2+75}{1875\,x^5}$$

input
```
integrate(1/x^6/(x^2+5),x, algorithm="fricas")
```

output
```
-1/1875*(3*sqrt(5)*x^5*arctan(1/5*sqrt(5)*x) + 15*x^4 - 25*x^2 + 75)/x^5
```

3.459.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6 (5 + x^2)} \, dx = -\frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625} + \frac{-3x^4 + 5x^2 - 15}{375x^5}$$

input
```
integrate(1/x**6/(x**2+5),x)
```

output
```
-sqrt(5)*atan(sqrt(5)*x/5)/625 + (-3*x**4 + 5*x**2 - 15)/(375*x**5)
```

3.459.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6 (5 + x^2)} \, dx = -\frac{1}{625} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) - \frac{3\,x^4 - 5\,x^2 + 15}{375\,x^5}$$

input
```
integrate(1/x^6/(x^2+5),x, algorithm="maxima")
```

output
```
-1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

3.459.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6 (5 + x^2)} \, dx = -\frac{1}{625} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) - \frac{3\,x^4 - 5\,x^2 + 15}{375\,x^5}$$

input
```
integrate(1/x^6/(x^2+5),x, algorithm="giac")
```

output
```
-1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

3.459.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{\frac{x^4}{125}-\frac{x^2}{75}+\frac{1}{25}}{x^5}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)}{625}$$

input
```
int(1/(x^6*(x^2 + 5)),x)
```

output
```
- (x^4/125 - x^2/75 + 1/25)/x^5 - (5^(1/2)*atan((5^(1/2)*x)/5))/625
```

3.460 $\int \frac{1}{x(-4+x^2)^4} dx$

3.460.1 Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{x(-4+x^2)^4} dx = \frac{1}{24(4-x^2)^3} + \frac{1}{64(4-x^2)^2} + \frac{1}{128(4-x^2)} + \frac{\log(x)}{256} - \frac{1}{512}\log(4-x^2)$$

output
```
1/24/(-x^2+4)^3+1/64/(-x^2+4)^2+1/128/(-x^2+4)+1/256*ln(x)-1/512*ln(-x^2+4
)
```

3.460.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{x(-4+x^2)^4} dx = \frac{-\frac{4(88-30x^2+3x^4)}{(-4+x^2)^3} + 6\log(x) - 3\log(4-x^2)}{1536}$$

input
```
Integrate[1/(x*(-4 + x^2)^4),x]
```

output
```
((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x^2])/1536
```

3.460.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^2-4)^4}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(4-x^2)^4}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{1}{256x^2}-\frac{1}{256(x^2-4)}+\frac{1}{64(x^2-4)^2}-\frac{1}{16(x^2-4)^3}+\frac{1}{4(x^2-4)^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{64(4-x^2)}+\frac{1}{32(4-x^2)^2}+\frac{1}{12(4-x^2)^3}+\frac{\log(x^2)}{256}-\frac{1}{256}\log(4-x^2)\right)$$

input
```
Int[1/(x*(-4 + x^2)^4),x]
```

output
```
(1/(12*(4 - x^2)^3) + 1/(32*(4 - x^2)^2) + 1/(64*(4 - x^2)) + Log[x^2]/256
 - Log[4 - x^2]/256)/2
```

3.460.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.460.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

method	result
risch	$\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{(x^2-4)^3}+\frac{\ln(x)}{256}-\frac{\ln(x^2-4)}{512}$
norman	$\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{(x^2-4)^3}+\frac{\ln(x)}{256}-\frac{\ln(-2+x)}{512}-\frac{\ln(2+x)}{512}$
meijerg	$\frac{11}{3072}+\frac{\ln(x)}{256}-\frac{\ln(2)}{256}+\frac{i\pi}{512}+\frac{x^2\left(\frac{11}{16}x^4-\frac{27}{4}x^2+18\right)}{12288\left(1-\frac{x^2}{4}\right)^3}-\frac{\ln\left(1-\frac{x^2}{4}\right)}{512}$
default	$\frac{1}{1536(2+x)^3}+\frac{3}{2048(2+x)^2}+\frac{11}{4096(2+x)}-\frac{\ln(2+x)}{512}+\frac{\ln(x)}{256}-\frac{1}{1536(-2+x)^3}+\frac{3}{2048(-2+x)^2}-\frac{11}{4096(-2+x)}-$
parallelrisch	$\frac{6\ln(x)x^6-3\ln(-2+x)x^6-3\ln(2+x)x^6-352-72x^4\ln(x)+36\ln(-2+x)x^4+36\ln(2+x)x^4-12x^4+288x^2\ln(x)-144\ln(-2+x)x}{1536(x^2-4)^3}$

input
```
int(1/x/(x^2-4)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/128*x^4+5/64*x^2-11/48)/(x^2-4)^3+1/256*ln(x)-1/512*ln(x^2-4)
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx = -\frac{12\,x^4-120\,x^2+3\,(x^6-12\,x^4+48\,x^2-64)\log\left(x^2-4\right)-6\,(x^6-12\,x^4+48\,x^2-64)\log\left(x\right)+352}{1536\,(x^6-12\,x^4+48\,x^2-64)}$$

input
```
integrate(1/x/(x^2-4)^4,x, algorithm="fricas")
```

output
```
-1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x^2 - 4) -
6*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x) + 352)/(x^6 - 12*x^4 + 48*x^2 - 64)
```

3.460.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(-4+x^{2}\right)^{4}}\,dx=\frac{-3x^{4}+30x^{2}-88}{384x^{6}-4608x^{4}+18432x^{2}-24576}+\frac{\log\left(x\right)}{256}-\frac{\log\left(x^{2}-4\right)}{512}$$

input
```
integrate(1/x/(x**2-4)**4,x)
```

output
```
(-3*x**4 + 30*x**2 - 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 24576) + log
(x)/256 - log(x**2 - 4)/512
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(-4+x^{2}\right)^{4}}\,dx=-\frac{3\,x^{4}-30\,x^{2}+88}{384\,\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)}-\frac{1}{512}\,\log\left(x^{2}-4\right)+\frac{1}{512}\,\log\left(x^{2}\right)$$

input
```
integrate(1/x/(x^2-4)^4,x, algorithm="maxima")
```

output
```
-1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512*log(x^2
- 4) + 1/512*log(x^2)
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\left(-4+x^{2}\right)^{4}}\,dx=\frac{11\,x^{6}-156\,x^{4}+768\,x^{2}-1408}{3072\,\left(x^{2}-4\right)^{3}}+\frac{1}{512}\,\log\left(x^{2}\right)-\frac{1}{512}\,\log\left(\left|x^{2}-4\right|\right)$$

input
```
integrate(1/x/(x^2-4)^4,x, algorithm="giac")
```

output
```
1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*log(x^2) -
1/512*log(abs(x^2 - 4))
```

3.460.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx = \frac{\ln\left(x\right)}{256} - \frac{\ln\left(x^2-4\right)}{512} - \frac{\frac{x^4}{128}-\frac{5\,x^2}{64}+\frac{11}{48}}{x^6-12\,x^4+48\,x^2-64}$$

input
```
int(1/(x*(x^2 - 4)^4),x)
```

output
```
log(x)/256 - log(x^2 - 4)/512 - (x^4/128 - (5*x^2)/64 + 11/48)/(48*x^2 - 1
2*x^4 + x^6 - 64)
```

3.461 $\int \frac{1}{x(-2+x^2)^{5/2}}\,dx$

3.461.1 Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{1}{x(-2+x^2)^{5/2}}\,dx=-\frac{1}{6(-2+x^2)^{3/2}}+\frac{1}{4\sqrt{-2+x^2}}+\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/6/(x^2-2)^(3/2)+1/8*arctan(1/2*(x^2-2)^(1/2)*2^(1/2))*2^(1/2)+1/4/(x^2-
2)^(1/2)
```

3.461.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(-2+x^2)^{5/2}}\,dx=\frac{-8+3x^2}{12(-2+x^2)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[1/(x*(-2 + x^2)^(5/2)),x]
```

output
```
(-8 + 3*x^2)/(12*(-2 + x^2)^(3/2)) + ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/(4*Sqr
t[2])
```

3.461.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {243, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^2-2\right)^{5/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(x^2-2\right)^{5/2}}dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2\left(x^2-2\right)^{3/2}}dx^2-\frac{1}{3\left(x^2-2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{x^2-2}}dx^2+\frac{1}{\sqrt{x^2-2}}\right)-\frac{1}{3\left(x^2-2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{x^4+2}d\sqrt{x^2-2}+\frac{1}{\sqrt{x^2-2}}\right)-\frac{1}{3\left(x^2-2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{x^2-2}}\right)-\frac{1}{3\left(x^2-2\right)^{3/2}}\right)$$

input
```
Int[1/(x*(-2 + x^2)^(5/2)),x]
```

output
```
(-1/3*1/(-2 + x^2)^(3/2) + (1/Sqrt[-2 + x^2] + ArcTan[Sqrt[-2 + x^2]/Sqrt[
2]]/Sqrt[2])/2)/2
```

3.461.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.461.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	35
default	$-\frac{1}{6(x^2-2)^{\frac{3}{2}}}+\frac{1}{4\sqrt{x^2-2}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	37
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{x^2-2}\,\sqrt{2}}{2}\right)\sqrt{2}\,(x^2-2)^{\frac{3}{2}}+2x^2-\frac{16}{3}}{8(x^2-2)^{\frac{3}{2}}}$	41
trager	$\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}}-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\sqrt{x^2-2}-\operatorname{RootOf}\left(\_Z^2+2\right)}{x}\right)}{8}$	47
meijerg	$\frac{\sqrt{2}\left(-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)\right)^{\frac{5}{2}}\left(\frac{3\left(\frac{8}{3}-3\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{4}-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(-6x^2+16\right)}{8\left(-\frac{x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^2}{2}+1}}{2}\right)}{2}\right)}{12\sqrt{\pi}\,\operatorname{signum}\left(-1+\frac{x^2}{2}\right)^{\frac{5}{2}}}$	96

input
```
int(1/x/(x^2-2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^2-8)/(x^2-2)^(3/2)-1/8*2^(1/2)*arctan(1/(x^2-2)^(1/2)*2^(1/2))
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=\frac{3\,\sqrt{2}(x^4-4\,x^2+4)\arctan\left(-\frac{1}{2}\,\sqrt{2}x+\frac{1}{2}\,\sqrt{2}\sqrt{x^2-2}\right)+(3\,x^2-8)\sqrt{x^2-2}}{12\,(x^4-4\,x^2+4)}$$

input
```
integrate(1/x/(x^2-2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(2)*(x^4 - 4*x^2 + 4)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt
(x^2 - 2)) + (3*x^2 - 8)*sqrt(x^2 - 2))/(x^4 - 4*x^2 + 4)
```

3.461.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 984, normalized size of antiderivative = 18.92

$$\int \frac{1}{x(-2+x^2)^{5/2}}\,dx = \begin{cases} \frac{6ix^4\log(x)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} - \frac{3ix^4\log(x^2)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} - \frac{6x^4\operatorname{asin}\left(\frac{\sqrt{2}}{x}\right)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} + \frac{6\sqrt{2}x^2\sqrt{x^2-2}}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} \\ -\frac{3ix^4\log(x^2)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} + \frac{6ix^4\log\left(\sqrt{1-\frac{x^2}{2}}+1\right)}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} - \frac{3\pi x^4}{24\sqrt{2}x^4-96\sqrt{2}x^2+96\sqrt{2}} + \frac{3ix^4\log(2)}{24\sqrt{2}x^4-96\sqrt{2}x^2+9} \end{cases}$$

input
```
integrate(1/x/(x**2-2)**(5/2),x)
```

output
```
Piecewise((6*I*x**4*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)
) - 3*I*x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) -
6*x**4*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) +
6*sqrt(2)*x**2*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt
(2)) - 24*I*x**2*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) +
 12*I*x**2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24
*x**2*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 1
6*sqrt(2)*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
+ 24*I*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(
x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*asin(sqrt(2)/x
)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)), Abs(x**2) > 2), (-3*I*
x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*I*x**4
*log(sqrt(1 - x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)
) - 3*pi*x**4/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 3*I*x**4*
log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2)*I*x**2
*sqrt(2 - x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I*x*
*2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*I*x**2*
log(sqrt(1 - x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
 + 12*pi*x**2/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*x**2
*log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(2)*I...
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = -\frac{1}{8}\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right) + \frac{1}{4\sqrt{x^2-2}} - \frac{1}{6\left(x^2-2\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(x^2-2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*arcsin(sqrt(2)/abs(x)) + 1/4/sqrt(x^2 - 2) - 1/6/(x^2 - 2)^(3
/2)
```

3.461.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right) + \frac{3x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(x^2-2)^(5/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/(x^2 - 2)
^(3/2)
```

3.461.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{x^2-2}}{2}\right)}{8} + \frac{\frac{x^2}{4}-\frac{2}{3}}{\left(x^2-2\right)^{3/2}}$$

input
```
int(1/(x*(x^2 - 2)^(5/2)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/8 + (x^2/4 - 2/3)/(x^2 - 2)^(3
/2)
```

3.462 $\int \frac{\left(-10+x^2\right)^{5/2}}{x} \, dx$

3.462.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\left(-10+x^2\right)^{5/2}}{x} \, dx = 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

output
```
-10/3*(x^2-10)^(3/2)+1/5*(x^2-10)^(5/2)-100*arctan(1/10*(x^2-10)^(1/2)*10^
(1/2))*10^(1/2)+100*(x^2-10)^(1/2)
```

3.462.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{\left(-10+x^2\right)^{5/2}}{x} \, dx = \frac{1}{15}\sqrt{-10+x^2}\left(2300-110x^2+3x^4\right) - 100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

input
```
Integrate[(-10 + x^2)^(5/2)/x,x]
```

output
```
(Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-
10 + x^2]/Sqrt[10]]
```

3.462.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {243, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-10)^{5/2}}{x} \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(x^2-10)^{5/2}}{x^2}dx^2$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\int \frac{(x^2-10)^{3/2}}{x^2}dx^2\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\int \frac{\sqrt{x^2-10}}{x^2}dx^2\right)\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\left(2\sqrt{x^2-10}-10\int \frac{1}{x^2\sqrt{x^2-10}}dx^2\right)\right)\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\left(2\sqrt{x^2-10}-20\int \frac{1}{x^4+10}d\sqrt{x^2-10}\right)\right)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\left(2\sqrt{x^2-10}-2\sqrt{10}\arctan\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)\right)\right)\right)$$

input
```
Int[(-10 + x^2)^(5/2)/x,x]
```

output
```
((2*(-10 + x^2)^(5/2))/5 - 10*((2*(-10 + x^2)^(3/2))/3 - 10*(2*Sqrt[-10 +
x^2] - 2*Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]])))/2
```

3.462. $\int \frac{(-10+x^2)^{5/2}}{x} \, dx$

3.462.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.462.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-100\arctan\left(\frac{\sqrt{x^2-10}\,\sqrt{10}}{10}\right)\sqrt{10}+\frac{\sqrt{x^2-10}\,(3x^4-110x^2+2300)}{15}$
default	$\frac{(x^2-10)^{\frac{5}{2}}}{5}-\frac{10(x^2-10)^{\frac{3}{2}}}{3}+100\sqrt{x^2-10}+100\sqrt{10}\,\arctan\left(\frac{\sqrt{10}}{\sqrt{x^2-10}}\right)$
trager	$\left(\frac{1}{5}x^4-\frac{22}{3}x^2+\frac{460}{3}\right)\sqrt{x^2-10}-100\,\mathrm{RootOf}\left(\_Z^2+10\right)\ln\left(\frac{\sqrt{x^2-10}+\mathrm{RootOf}\left(\_Z^2+10\right)}{x}\right)$
meijerg	$-\frac{375\sqrt{2}\,\sqrt{5}\,\mathrm{signum}\left(-1+\frac{x^2}{10}\right)^{\frac{5}{2}}\left(-\frac{8\left(\frac{46}{15}-3\ln(2)+2\ln(x)-\ln(5)+i\pi\right)\sqrt{\pi}}{15}+\frac{368\sqrt{\pi}}{225}-\frac{4\sqrt{\pi}\left(\frac{3}{25}x^4-\frac{22}{5}x^2+92\right)\sqrt{1-\frac{x^2}{10}}}{225}+\frac{16\sqrt{\pi}\,\ln\left(\cdots\right.}{\cdots}\right.}{4\sqrt{\pi}\left(-\mathrm{signum}\left(-1+\frac{x^2}{10}\right)\right)^{\frac{5}{2}}}$

input
```
int((x^2-10)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-100*arctan(1/10*(x^2-10)^(1/2)*10^(1/2))*10^(1/2)+1/15*(x^2-10)^(1/2)*(3*
x^4-110*x^2+2300)
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{15}\left(3\,x^4-110\,x^2+2300\right)\sqrt{x^2-10} - 200\,\sqrt{10}\arctan\left(-\frac{1}{10}\,\sqrt{10}x+\frac{1}{10}\,\sqrt{10}\sqrt{x^2-10}\right)$$

input
```
integrate((x^2-10)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/15*(3*x^4 - 110*x^2 + 2300)*sqrt(x^2 - 10) - 200*sqrt(10)*arctan(-1/10*s
qrt(10)*x + 1/10*sqrt(10)*sqrt(x^2 - 10))
```

3.462.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.70

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \begin{cases} \frac{x^4\sqrt{x^2-10}}{5} - \frac{22x^2\sqrt{x^2-10}}{3} + \frac{460\sqrt{x^2-10}}{3} - 100\sqrt{10}i\log(x) + 50\sqrt{10}i\log(x^2) + 100\sqrt{10} \\ \frac{ix^4\sqrt{10-x^2}}{5} - \frac{22ix^2\sqrt{10-x^2}}{3} + \frac{460i\sqrt{10-x^2}}{3} + 50\sqrt{10}i\log(x^2) - 100\sqrt{10}i\log\left(\sqrt{1-\frac{x^2}{10}} + \right. \end{cases}$$

input
```
integrate((x**2-10)**(5/2)/x,x)
```

output
```
Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 460*sqrt(x
**2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(x**2) + 100*sqrt(1
0)*asin(sqrt(10)/x), Abs(x**2) > 10), (I*x**4*sqrt(10 - x**2)/5 - 22*I*x**
2*sqrt(10 - x**2)/3 + 460*I*sqrt(10 - x**2)/3 + 50*sqrt(10)*I*log(x**2) -
100*sqrt(10)*I*log(sqrt(1 - x**2/10) + 1), True))
```

3.462.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} + 100\sqrt{10}\arcsin\left(\frac{\sqrt{10}}{|x|}\right) + 100\sqrt{x^2-10}$$

input
```
integrate((x^2-10)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsin(sqrt(10
)/abs(x)) + 100*sqrt(x^2 - 10)
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} - 100\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\sqrt{x^2-10}\right) + 100\sqrt{x^2-10}$$

input
```
integrate((x^2-10)^(5/2)/x,x, algorithm="giac")
```

output
```
1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arctan(1/10*sq
rt(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)
```

3.462.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = 100\sqrt{x^2-10} - 100\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\sqrt{x^2-10}}{10}\right) - \frac{10\left(x^2-10\right)^{3/2}}{3} + \frac{\left(x^2-10\right)^{5/2}}{5}$$

input
```
int((x^2 - 10)^(5/2)/x,x)
```

output
```
100*(x^2 - 10)^(1/2) - 100*10^(1/2)*atan((10^(1/2)*(x^2 - 10)^(1/2))/10) -
 (10*(x^2 - 10)^(3/2))/3 + (x^2 - 10)^(5/2)/5
```

3.463 $\int x^{1+2n}\,dx$

3.463.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int x^{1+2n}\,dx=\frac{x^{2(1+n)}}{2(1+n)}$$

output
```
1/2*x^(2+2*n)/(1+n)
```

3.463.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx=\frac{x^{2+2n}}{2+2n}$$

input
```
Integrate[x^(1 + 2*n),x]
```

output
```
x^(2 + 2*n)/(2 + 2*n)
```

3.463.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n+1}\,dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{2(n+1)}}{2(n+1)}$$

input
```
Int[x^(1 + 2*n),x]
```

output
```
x^(2*(1 + n))/(2*(1 + n))
```

3.463.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.463.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{2+2n}}{2+2n}$	15
default	$\frac{x^{2+2n}}{2+2n}$	16
risch	$\frac{x\,x^{1+2n}}{2+2n}$	16
parallelrisch	$\frac{x\,x^{1+2n}}{2+2n}$	16
norman	$\frac{x\,\mathrm{e}^{(1+2n)\ln(x)}}{2+2n}$	18

input
```
int(x^(1+2*n),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2+2*n)/(1+n)
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx = \frac{x x^{2\,n+1}}{2\,(n+1)}$$

input
```
integrate(x^(1+2*n),x, algorithm="fricas")
```

output
```
1/2*x*x^(2*n + 1)/(n + 1)
```

3.463.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx = \begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for } n \neq -1 \\ \log{(x)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1+2*n),x)
```

output
```
Piecewise((x**(2*n + 2)/(2*n + 2), Ne(n, -1)), (log(x), True))
```

3.463.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{1+2n}\,dx = \frac{x^{2\,n+2}}{2\,(n+1)}$$

input
```
integrate(x^(1+2*n),x, algorithm="maxima")
```

output
```
1/2*x^(2*n + 2)/(n + 1)
```

3.463.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{1+2n}\,dx = \frac{x^{2\,n+2}}{2\,(n+1)}$$

input
```
integrate(x^(1+2*n),x, algorithm="giac")
```

output
```
1/2*x^(2*n + 2)/(n + 1)
```

3.463.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^{1+2n}\,dx = \begin{cases} \ln(x) & \text{if } n=-1 \\ \frac{x^{2\,n+2}}{2\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int(x^(2*n + 1),x)
```

output
```
piecewise(n == -1, log(x), n ~= -1, x^(2*n + 2)/(2*(n + 1)))
```

3.464 $\int \frac{x^7}{(-5+x^2)^3} dx$

3.464.1 Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{x^7}{(-5+x^2)^3} dx = \frac{x^2}{2} - \frac{125}{4(5-x^2)^2} + \frac{75}{2(5-x^2)} + \frac{15}{2}\log(5-x^2)$$

output
```
1/2*x^2-125/4/(-x^2+5)^2+75/2/(-x^2+5)+15/2*ln(-x^2+5)
```

3.464.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(-5+x^2)^3} dx = \frac{1}{4}\left(2x^2 - \frac{125}{(-5+x^2)^2} - \frac{150}{-5+x^2} + 30\log(-5+x^2)\right)$$

input
```
Integrate[x^7/(-5 + x^2)^3,x]
```

output
```
(2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4
```

3.464.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {243, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(x^2-5)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int -\frac{x^6}{(5-x^2)^3}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{x^6}{(5-x^2)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{1}{2}\int \left(-1-\frac{15}{x^2-5}-\frac{75}{(x^2-5)^2}-\frac{125}{(x^2-5)^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(x^2+\frac{75}{5-x^2}-\frac{125}{2(5-x^2)^2}+15\log\left(5-x^2\right)\right)$$

input
```
Int[x^7/(-5 + x^2)^3,x]
```

output
```
(x^2 - 125/(2*(5 - x^2)^2) + 75/(5 - x^2) + 15*Log[5 - x^2])/2
```

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.464.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
norman	$\frac{-75x^2+\frac{1}{2}x^6+\frac{1125}{4}}{(x^2-5)^2}+\frac{15\ln(x^2-5)}{2}$	30
risch	$\frac{x^2}{2}+\frac{-\frac{75x^2}{2}+\frac{625}{4}}{(x^2-5)^2}+\frac{15\ln(x^2-5)}{2}$	30
default	$\frac{x^2}{2}+\frac{15\ln(x^2-5)}{2}-\frac{125}{4(x^2-5)^2}-\frac{75}{2(x^2-5)}$	33
meijerg	$\frac{x^2\left(\frac{4}{25}x^4-\frac{18}{5}x^2+12\right)}{8\left(-\frac{x^2}{5}+1\right)^2}+\frac{15\ln\left(-\frac{x^2}{5}+1\right)}{2}$	38
parallelrisch	$\frac{2x^6+30\ln(x^2-5)x^4+1125-300\ln(x^2-5)x^2-300x^2+750\ln(x^2-5)}{4(x^2-5)^2}$	52

input
```
int(x^7/(x^2-5)^3,x,method=_RETURNVERBOSE)
```

output
```
(-75*x^2+1/2*x^6+1125/4)/(x^2-5)^2+15/2*ln(x^2-5)
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{2\,x^6-20\,x^4-100\,x^2+30\,(x^4-10\,x^2+25)\log(x^2-5)+625}{4\,(x^4-10\,x^2+25)}$$

input
```
integrate(x^7/(x^2-5)^3,x, algorithm="fricas")
```

output
```
1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 - 5) + 625)
/(x^4 - 10*x^2 + 25)
```

3.464.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{x^2}{2} + \frac{625-150x^2}{4x^4-40x^2+100} + \frac{15\log(x^2-5)}{2}$$

input
```
integrate(x**7/(x**2-5)**3,x)
```

output
```
x**2/2 + (625 - 150*x**2)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2 - 5)/2
```

3.464.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{2}\,x^2 - \frac{25\,(6\,x^2-25)}{4\,(x^4-10\,x^2+25)} + \frac{15}{2}\,\log\left(x^2-5\right)$$

input
```
integrate(x^7/(x^2-5)^3,x, algorithm="maxima")
```

output
```
1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 - 5)
```

3.464.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{2}\,x^2 - \frac{5\,(9\,x^4-60\,x^2+100)}{4\,(x^2-5)^2} + \frac{15}{2}\,\log\left(\left|x^2-5\right|\right)$$

input
```
integrate(x^7/(x^2-5)^3,x, algorithm="giac")
```

output
```
1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*log(abs(x^2 - 5))
```

3.464.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{15\,\ln\left(x^2-5\right)}{2} - \frac{\frac{75\,x^2}{2}-\frac{625}{4}}{x^4-10\,x^2+25} + \frac{x^2}{2}$$

input
```
int(x^7/(x^2 - 5)^3,x)
```

output
```
(15*log(x^2 - 5))/2 - ((75*x^2)/2 - 625/4)/(x^4 - 10*x^2 + 25) + x^2/2
```

3.465 $\int \frac{-4x^3+3x^5}{(-1+x^2)^5} \, dx$

3.465.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5} \, dx = \frac{1}{8(1-x^2)^4} + \frac{1}{3(1-x^2)^3} - \frac{3}{4(1-x^2)^2}$$

output
```
1/8/(-x^2+1)^4+1/3/(-x^2+1)^3-3/4/(-x^2+1)^2
```

3.465.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5} \, dx = \frac{-7+28x^2-18x^4}{24(-1+x^2)^4}$$

input
```
Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

output
```
(-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

3.465.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2027, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^5-4x^3}{\left(x^2-1\right)^5}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x^3\left(3x^2-4\right)}{\left(x^2-1\right)^5}dx$$
$$\Big\downarrow \text{ 354}$$
$$\frac{1}{2}\int \frac{x^2\left(4-3x^2\right)}{\left(1-x^2\right)^5}dx^2$$
$$\Big\downarrow \text{ 86}$$
$$\frac{1}{2}\int \left(\frac{3}{\left(x^2-1\right)^3}+\frac{2}{\left(x^2-1\right)^4}-\frac{1}{\left(x^2-1\right)^5}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{3}{2\left(1-x^2\right)^2}+\frac{2}{3\left(1-x^2\right)^3}+\frac{1}{4\left(1-x^2\right)^4}\right)$$

input
```
Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

output
```
(1/(4*(1 - x^2)^4) + 2/(3*(1 - x^2)^3) - 3/(2*(1 - x^2)^2))/2
```

3.465.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.465.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result
norman	$\frac{-\frac{3}{4}x^4+\frac{7}{6}x^2-\frac{7}{24}}{(x^2-1)^4}$
risch	$\frac{-\frac{3}{4}x^4+\frac{7}{6}x^2-\frac{7}{24}}{(x^2-1)^4}$
gosper	$-\frac{18x^4-28x^2+7}{24(x^2-1)^4}$
parallelrisch	$\frac{-18x^4+28x^2-7}{24(x^2-1)^4}$
meijerg	$-\frac{x^6(-x^2+4)}{8(-x^2+1)^4}+\frac{x^4(x^4-4x^2+6)}{6(-x^2+1)^4}$
default	$\frac{1}{128(-1+x)^4}-\frac{11}{192(-1+x)^3}-\frac{27}{256(-1+x)^2}+\frac{27}{256(-1+x)}+\frac{1}{128(1+x)^4}+\frac{11}{192(1+x)^3}-\frac{27}{256(1+x)^2}-\frac{27}{256(1+x)}$

input
```
int((3*x^5-4*x^3)/(x^2-1)^5,x,method=_RETURNVERBOSE)
```

output
```
(-3/4*x^4+7/6*x^2-7/24)/(x^2-1)^4
```

3.465.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

input
```
integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="fricas")
```

output
```
-1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

3.465.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{-18x^4+28x^2-7}{24x^8-96x^6+144x^4-96x^2+24}$$

input
```
integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
```

output
```
(-18*x**4 + 28*x**2 - 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2 + 24)
```

3.465.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

input
```
integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="maxima")
```

output
```
-1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

3.465.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^2-1)^4}$$

input
```
integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="giac")
```

output
```
-1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4
```

3.465.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{\frac{3\,x^4}{4}-\frac{7\,x^2}{6}+\frac{7}{24}}{x^8-4\,x^6+6\,x^4-4\,x^2+1}$$

input
```
int(-(4*x^3 - 3*x^5)/(x^2 - 1)^5,x)
```

output
```
-((3*x^4)/4 - (7*x^2)/6 + 7/24)/(6*x^4 - 4*x^2 - 4*x^6 + x^8 + 1)
```

3.466 $\int x^3(1+x^2)^{9/14}\,dx$

3.466.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3\left(1+x^2\right)^{9/14}\,dx=-\frac{7}{23}\left(1+x^2\right)^{23/14}+\frac{7}{37}\left(1+x^2\right)^{37/14}$$

output
```
-7/23*(x^2+1)^(23/14)+7/37*(x^2+1)^(37/14)
```

3.466.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^3\left(1+x^2\right)^{9/14}\,dx=\frac{7}{851}\left(1+x^2\right)^{9/14}\left(-14+9x^2+23x^4\right)$$

input
```
Integrate[x^3*(1 + x^2)^(9/14),x]
```

output
```
(7*(1 + x^2)^(9/14)*(-14 + 9*x^2 + 23*x^4))/851
```

3.466.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (x^2+1)^{9/14} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^2 (x^2+1)^{9/14} dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left((x^2+1)^{23/14} - (x^2+1)^{9/14} \right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{14}{37} (x^2+1)^{37/14} - \frac{14}{23} (x^2+1)^{23/14} \right)$$

input
```
Int[x^3*(1 + x^2)^(9/14),x]
```

output
```
((-14*(1 + x^2)^(23/14))/23 + (14*(1 + x^2)^(37/14))/37)/2
```

3.466.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.466.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{7(x^2+1)^{\frac{23}{14}}(23x^2-14)}{851}$	17
meijerg	$\frac{x^4 {}_2F_1(-\frac{9}{14},2;3;-x^2)}{4}$	17
pseudoelliptic	$\frac{7(x^2+1)^{\frac{23}{14}}(23x^2-14)}{851}$	17
trager	$\left(\frac{7}{37}x^4+\frac{63}{851}x^2-\frac{98}{851}\right)(x^2+1)^{\frac{9}{14}}$	21
risch	$\frac{7(x^2+1)^{\frac{9}{14}}(23x^4+9x^2-14)}{851}$	22

input
```
int(x^3*(x^2+1)^(9/14),x,method=_RETURNVERBOSE)
```

output
```
7/851*(x^2+1)^(23/14)*(23*x^2-14)
```

3.466.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7}{851}\left(23\,x^4+9\,x^2-14\right)\left(x^2+1\right)^{\frac{9}{14}}$$

input
```
integrate(x^3*(x^2+1)^(9/14),x, algorithm="fricas")
```

output
```
7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)
```

3.466.6 Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7x^4(x^2+1)^{\frac{9}{14}}}{37}+\frac{63x^2(x^2+1)^{\frac{9}{14}}}{851}-\frac{98(x^2+1)^{\frac{9}{14}}}{851}$$

input
```
integrate(x**3*(x**2+1)**(9/14),x)
```

output
```
7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 98*(x**2 +
 1)**(9/14)/851
```

3.466.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}} - \frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

input
```
integrate(x^3*(x^2+1)^(9/14),x, algorithm="maxima")
```

output
```
7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

3.466.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}} - \frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

input
```
integrate(x^3*(x^2+1)^(9/14),x, algorithm="giac")
```

output
```
7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

3.466.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3(1+x^2)^{9/14}\,dx = \left(x^2+1\right)^{9/14}\left(\frac{7\,x^4}{37} + \frac{63\,x^2}{851} - \frac{98}{851}\right)$$

input
```
int(x^3*(x^2 + 1)^(9/14),x)
```

output
```
(x^2 + 1)^(9/14)*((63*x^2)/851 + (7*x^4)/37 - 98/851)
```

3.467 $\int \frac{x^5}{(-4+x^2)^{13/6}} \, dx$

3.467.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x^5}{(-4+x^2)^{13/6}} \, dx = -\frac{48}{7\,(-4+x^2)^{7/6}} - \frac{24}{\sqrt[6]{-4+x^2}} + \frac{3}{5}\left(-4+x^2\right)^{5/6}$$

output
```
-48/7/(x^2-4)^(7/6)-24/(x^2-4)^(1/6)+3/5*(x^2-4)^(5/6)
```

3.467.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{(-4+x^2)^{13/6}} \, dx = \frac{3(1152-336x^2+7x^4)}{35\,(-4+x^2)^{7/6}}$$

input
```
Integrate[x^5/(-4 + x^2)^(13/6),x]
```

output
```
(3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

3.467.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^2-4)^{13/6}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(x^2-4)^{13/6}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{\sqrt[6]{x^2-4}}+\frac{8}{(x^2-4)^{7/6}}+\frac{16}{(x^2-4)^{13/6}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{6}{5}(x^2-4)^{5/6}-\frac{48}{\sqrt[6]{x^2-4}}-\frac{96}{7(x^2-4)^{7/6}}\right)$$

input
```
Int[x^5/(-4 + x^2)^(13/6),x]
```

output
```
(-96/(7*(-4 + x^2)^(7/6)) - 48/(-4 + x^2)^(1/6) + (6*(-4 + x^2)^(5/6))/5)/
2
```

3.467.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.467.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
pseudoelliptic	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$	20
trager	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$	22
risch	$\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$	22
gosper	$\frac{3(-2+x)(2+x)(7x^4-336x^2+1152)}{35(x^2-4)^{\frac{13}{6}}}$	28
meijerg	$\frac{2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^2}{4}\right)\right)^{\frac{13}{6}} x^6 {}_2F_1\left(\frac{13}{6},3;4;\frac{x^2}{4}\right)}{192\operatorname{signum}\left(-1+\frac{x^2}{4}\right)^{\frac{13}{6}}}$	42

input
```
int(x^5/(x^2-4)^(13/6),x,method=_RETURNVERBOSE)
```

output
```
3/5*(x^4-48*x^2+1152/7)/(x^2-4)^(7/6)
```

3.467.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3\,(7\,x^4-336\,x^2+1152)(x^2-4)^{\frac{5}{6}}}{35\,(x^4-8\,x^2+16)}$$

input
```
integrate(x^5/(x^2-4)^(13/6),x, algorithm="fricas")
```

output
```
3/35*(7*x^4 - 336*x^2 + 1152)*(x^2 - 4)^(5/6)/(x^4 - 8*x^2 + 16)
```

3.467.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(32) = 64.

Time = 0.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{21x^4}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}} - \frac{1008x^2}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}} + \frac{3456}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}$$

input
```
integrate(x**5/(x**2-4)**(13/6),x)
```

output
```
21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 1008*x**2/(3
5*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456/(35*x**2*(x**2 -
4)**(1/6) - 140*(x**2 - 4)**(1/6))
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3}{5}\,(x^2-4)^{\frac{5}{6}} - \frac{24}{(x^2-4)^{\frac{1}{6}}} - \frac{48}{7\,(x^2-4)^{\frac{7}{6}}}$$

input
```
integrate(x^5/(x^2-4)^(13/6),x, algorithm="maxima")
```

output
```
3/5*(x^2 - 4)^(5/6) - 24/(x^2 - 4)^(1/6) - 48/7/(x^2 - 4)^(7/6)
```

3.467.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3}{5}\,(x^2-4)^{\frac{5}{6}} - \frac{24\,(7\,x^2-26)}{7\,(x^2-4)^{\frac{7}{6}}}$$

input
```
integrate(x^5/(x^2-4)^(13/6),x, algorithm="giac")
```

output
```
3/5*(x^2 - 4)^(5/6) - 24/7*(7*x^2 - 26)/(x^2 - 4)^(7/6)
```

3.467.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3\,(7\,x^4-336\,x^2+1152)}{35\,(x^2-4)^{7/6}}$$

input
```
int(x^5/(x^2 - 4)^(13/6),x)
```

output
```
(3*(7*x^4 - 336*x^2 + 1152))/(35*(x^2 - 4)^(7/6))
```

3.468 $\int \frac{1}{(1+2x^2)^{5/2}} dx$

3.468.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{(1+2x^2)^{5/2}} dx = \frac{x}{3(1+2x^2)^{3/2}} + \frac{2x}{3\sqrt{1+2x^2}}$$

output
```
1/3*x/(2*x^2+1)^(3/2)+2/3*x/(2*x^2+1)^(1/2)
```

3.468.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+2x^2)^{5/2}} dx = \frac{3x+4x^3}{3(1+2x^2)^{3/2}}$$

input
```
Integrate[(1 + 2*x^2)^(-5/2),x]
```

output
```
(3*x + 4*x^3)/(3*(1 + 2*x^2)^(3/2))
```

3.468.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+1)^{5/2}}dx$$

$$\Big\downarrow \; 209$$

$$\frac{2}{3}\int \frac{1}{(2x^2+1)^{3/2}}dx+\frac{x}{3(2x^2+1)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3(2x^2+1)^{3/2}}$$

input
```
Int[(1 + 2*x^2)^(-5/2),x]
```

output
```
x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

3.468.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.468.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
trager	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
meijerg	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
risch	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
pseudoelliptic	$\frac{4x^3+3x}{3(2x^2+1)^{\frac{3}{2}}}$	21
default	$\frac{x}{3(2x^2+1)^{\frac{3}{2}}}+\frac{2x}{3\sqrt{2x^2+1}}$	26

input
```
int(1/(2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(4*x^2+3)/(2*x^2+1)^(3/2)
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{(4\,x^3+3\,x)\sqrt{2\,x^2+1}}{3\,(4\,x^4+4\,x^2+1)}$$

input
```
integrate(1/(2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(4*x^3 + 3*x)*sqrt(2*x^2 + 1)/(4*x^4 + 4*x^2 + 1)
```

3.468.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(27) = 54.

Time = 0.86 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{4x^3}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}} + \frac{3x}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}$$

input
```
integrate(1/(2*x**2+1)**(5/2),x)
```

output
```
4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x**2*sqrt(2
*x**2 + 1) + 3*sqrt(2*x**2 + 1))
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{2\,x^2+1}} + \frac{x}{3\,(2\,x^2+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)
```

3.468.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{(4\,x^2+3)x}{3\,(2\,x^2+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)
```

3.468.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\,\left(-x^2+1\mathrm{i}\,\sqrt{2}\,x+\frac{1}{2}\right)} + \frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\,\left(x^2+1\mathrm{i}\,\sqrt{2}\,x-\frac{1}{2}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\,\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\,\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}$$

input

```
int(1/(2*x^2 + 1)^(5/2),x)
```

output

```
((x^2 + 1/2)^(1/2)*1i)/(24*(2^(1/2)*x*1i - x^2 + 1/2)) + ((x^2 + 1/2)^(1/2
)*1i)/(24*(2^(1/2)*x*1i + x^2 - 1/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x
- (2^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x + (2^(1/2)*1i)/2))
```

3.469 $\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx$

3.469.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx=\frac{1-x}{6(-1-2x+x^2)^{3/2}}-\frac{1-x}{6\sqrt{-1-2x+x^2}}$$

output
```
1/6*(1-x)/(x^2-2*x-1)^(3/2)+1/6*(-1+x)/(x^2-2*x-1)^(1/2)
```

3.469.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx=\frac{2-3x^2+x^3}{6(-1-2x+x^2)^{3/2}}$$

input
```
Integrate[(-1 - 2*x + x^2)^(-5/2),x]
```

output
```
(2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))
```

3.469.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x-1)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{1-x}{6(x^2-2x-1)^{3/2}}-\frac{1}{3}\int \frac{1}{(x^2-2x-1)^{3/2}}dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{1-x}{6(x^2-2x-1)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

input
```
Int[(-1 - 2*x + x^2)^(-5/2),x]
```

output
```
(1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])
```

3.469.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.469.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x^3-3x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$	23
trager	$\frac{x^3-3x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$	23
risch	$\frac{x^3-3x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$	23
default	$-\frac{-2+2x}{12(x^2-2x-1)^{\frac{3}{2}}}+\frac{-2+2x}{12\sqrt{x^2-2x-1}}$	36

input
```
int(1/(x^2-2*x-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^3-3*x^2+2)/(x^2-2*x-1)^(3/2)
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x^4-4\,x^3+2\,x^2+(x^3-3\,x^2+2)\sqrt{x^2-2\,x-1}+4\,x+1}{6\,(x^4-4\,x^3+2\,x^2+4\,x+1)}$$

input
```
integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(x^4 - 4*x^3 + 2*x^2 + (x^3 - 3*x^2 + 2)*sqrt(x^2 - 2*x - 1) + 4*x + 1
)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)
```

3.469.6 Sympy [F]

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-2x-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x**2-2*x-1)**(5/2),x)
```

output
```
Integral((x**2 - 2*x - 1)**(-5/2), x)
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x}{6\sqrt{x^2-2x-1}} - \frac{1}{6\sqrt{x^2-2x-1}} - \frac{x}{6(x^2-2x-1)^{\frac{3}{2}}} + \frac{1}{6(x^2-2x-1)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="maxima")
```

output
```
1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2 - 2*x - 1
)^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)
```

3.469.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.49

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{(x-3)x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="giac")
```

output
```
1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)
```

3.469.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x^3-3x^2+2}{6(x^2-2x-1)^{3/2}}$$

input
```
int(1/(x^2 - 2*x - 1)^(5/2),x)
```

output
```
(x^3 - 3*x^2 + 2)/(6*(x^2 - 2*x - 1)^(3/2))
```

3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$

3.470.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{x^4\,(-8+x^2)^{3/2}}\,dx = \frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{48x\sqrt{-8+x^2}} - \frac{x}{192\sqrt{-8+x^2}}$$

output
```
1/24/x^3/(x^2-8)^(1/2)+1/48/x/(x^2-8)^(1/2)-1/192*x/(x^2-8)^(1/2)
```

3.470.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\,(-8+x^2)^{3/2}}\,dx = \frac{8+4x^2-x^4}{192x^3\sqrt{-8+x^2}}$$

input
```
Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]
```

output
```
(8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

3.470.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {245, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^2-8)^{3/2}} \, dx$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{6} \int \frac{1}{x^2 (x^2-8)^{3/2}}dx+\frac{1}{24 x^3 \sqrt{x^2-8}}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{6} \left(\frac{1}{4} \int \frac{1}{(x^2-8)^{3/2}}dx+\frac{1}{8 x \sqrt{x^2-8}}\right)+\frac{1}{24 x^3 \sqrt{x^2-8}}$$

$$\Big\downarrow \text{208}$$

$$\frac{1}{6} \left(\frac{1}{8 x \sqrt{x^2-8}}-\frac{x}{32 \sqrt{x^2-8}}\right)+\frac{1}{24 x^3 \sqrt{x^2-8}}$$

input
```
Int[1/(x^4*(-8 + x^2)^(3/2)),x]
```

output
```
1/(24*x^3*Sqrt[-8 + x^2]) + (1/(8*x*Sqrt[-8 + x^2]) - x/(32*Sqrt[-8 + x^2]
))/6
```

3.470.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

3.470.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$	23
trager	$-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$	23
risch	$-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$	23
pseudoelliptic	$-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$	23
default	$\frac{1}{24x^{3}\sqrt{x^{2}-8}}+\frac{1}{48x\sqrt{x^{2}-8}}-\frac{x}{192\sqrt{x^{2}-8}}$	36
meijerg	$-\frac{\sqrt{2}\,\left(-\operatorname{signum}\left(-1+\frac{x^{2}}{8}\right)\right)^{\frac{3}{2}}\left(-\frac{1}{8}x^{4}+\frac{1}{2}x^{2}+1\right)}{96\operatorname{signum}\left(-1+\frac{x^{2}}{8}\right)^{\frac{3}{2}}x^{3}\sqrt{1-\frac{x^{2}}{8}}}$	52

input
```
int(1/x^4/(x^2-8)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(x^4-4*x^2-8)/x^3/(x^2-8)^(1/2)
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\left(-8+x^2\right)^{3/2}}\,dx = -\frac{x^5-8\,x^3+\left(x^4-4\,x^2-8\right)\sqrt{x^2-8}}{192\,\left(x^5-8\,x^3\right)}$$

input
```
integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="fricas")
```

output
```
-1/192*(x^5 - 8*x^3 + (x^4 - 4*x^2 - 8)*sqrt(x^2 - 8))/(x^5 - 8*x^3)
```

3.470.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.26

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = \begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for } \frac{1}{|x^2|} > \frac{1}{8} \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(x**2-8)**(3/2),x)
```

output
```
Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I*x**2*sqr
t(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/x**2)/(192*x**4 -
1536*x**2), 1/Abs(x**2) > 1/8), (-x**4*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x
**2) + 4*x**2*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)
/(192*x**4 - 1536*x**2), True))
```

3.470.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = -\frac{x}{192\sqrt{x^2-8}} + \frac{1}{48\sqrt{x^2-8}x} + \frac{1}{24\sqrt{x^2-8}x^3}$$

input
```
integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="maxima")
```

output
```
-1/192*x/sqrt(x^2 - 8) + 1/48/(sqrt(x^2 - 8)*x) + 1/24/(sqrt(x^2 - 8)*x^3)
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = -\frac{x}{512\sqrt{x^2-8}} - \frac{3\left(x-\sqrt{x^2-8}\right)^4 + 96\left(x-\sqrt{x^2-8}\right)^2 + 320}{96\left(\left(x-\sqrt{x^2-8}\right)^2+8\right)^3}$$

input
```
integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="giac")
```

output
```
-1/512*x/sqrt(x^2 - 8) - 1/96*(3*(x - sqrt(x^2 - 8))^4 + 96*(x - sqrt(x^2
- 8))^2 + 320)/((x - sqrt(x^2 - 8))^2 + 8)^3
```

3.470.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} \, dx = \frac{-x^4 + 4\, x^2 + 8}{192\, x^3 \sqrt{x^2 - 8}}$$

input
```
int(1/(x^4*(x^2 - 8)^(3/2)),x)
```

output
```
(4*x^2 - x^4 + 8)/(192*x^3*(x^2 - 8)^(1/2))
```

3.471 $\int \frac{(5+x^2)^2}{x^{13/3}} \, dx$

3.471.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(5+x^2)^2}{x^{13/3}} \, dx = -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2}$$

output
```
-15/2/x^(10/3)-15/2/x^(4/3)+3/2*x^(2/3)
```

3.471.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{(5+x^2)^2}{x^{13/3}} \, dx = \frac{3(-5-5x^2+x^4)}{2x^{10/3}}$$

input
```
Integrate[(5 + x^2)^2/x^(13/3),x]
```

output
```
(3*(-5 - 5*x^2 + x^4))/(2*x^(10/3))
```

3.471.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5)^2}{x^{13/3}}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{10}{x^{7/3}}+\frac{25}{x^{13/3}}+\frac{1}{\sqrt[3]{x}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

input
```
Int[(5 + x^2)^2/x^(13/3),x]
```

output
```
-15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

3.471.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.471.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{\frac{3}{2}x^4-\frac{15}{2}-\frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
trager	$\frac{\frac{3}{2}x^4-\frac{15}{2}-\frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
risch	$\frac{\frac{3}{2}x^4-\frac{15}{2}-\frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
derivativedivides	$-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$	17
default	$-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$	17

input
```
int((x^2+5)^2/x^(13/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(x^4-5*x^2-5)/x^(10/3)
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3\,(x^4-5\,x^2-5)}{2\,x^{\frac{10}{3}}}$$

input
```
integrate((x^2+5)^2/x^(13/3),x, algorithm="fricas")
```

output
```
3/2*(x^4 - 5*x^2 - 5)/x^(10/3)
```

3.471.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3x^{\frac{2}{3}}}{2}-\frac{15}{2x^{\frac{4}{3}}}-\frac{15}{2x^{\frac{10}{3}}}$$

input
```
integrate((x**2+5)**2/x**(13/3),x)
```

output
```
3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

3.471.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,(x^2+1)}{2\,x^{\frac{10}{3}}}$$

input
```
integrate((x^2+5)^2/x^(13/3),x, algorithm="maxima")
```

output
```
3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,(x^2+1)}{2\,x^{\frac{10}{3}}}$$

input
```
integrate((x^2+5)^2/x^(13/3),x, algorithm="giac")
```

output
```
3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

3.471.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = -\frac{-3\,x^4+15\,x^2+15}{2\,x^{10/3}}$$

input
```
int((x^2 + 5)^2/x^(13/3),x)
```

output
```
-(15*x^2 - 3*x^4 + 15)/(2*x^(10/3))
```

3.471. $\int \frac{(5+x^2)^2}{x^{13/3}}\,dx$

3.472 $\int \frac{1}{x^7 (1+x^2)^3} \, dx$

3.472.1 Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{3}{x^2} - \frac{1}{4(1+x^2)^2} - \frac{2}{1+x^2} - 10\log(x) + 5\log(1+x^2)$$

output
```
-1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

3.472.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{2-5x^2+20x^4+90x^6+60x^8}{12x^6 (1+x^2)^2} - 10\log(x) + 5\log(1+x^2)$$

input
```
Integrate[1/(x^7*(1 + x^2)^3),x]
```

output
```
-1/12*(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(x^6*(1 + x^2)^2) - 10*Log[x]
 + 5*Log[1 + x^2]
```

3.472.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^2+1)^3} \, dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int \frac{1}{x^8 (x^2+1)^3} dx^2$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{2} \int \left(-\frac{10}{x^2}+\frac{6}{x^4}-\frac{3}{x^6}+\frac{1}{x^8}+\frac{10}{x^2+1}+\frac{4}{(x^2+1)^2}+\frac{1}{(x^2+1)^3}\right) dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\frac{1}{3x^6}+\frac{3}{2x^4}-\frac{4}{x^2+1}-\frac{6}{x^2}-\frac{1}{2(x^2+1)^2}-10\log\left(x^2\right)+10\log\left(x^2+1\right)\right)$$

input
```
Int[1/(x^7*(1 + x^2)^3),x]
```

output
```
(-1/3*1/x^6 + 3/(2*x^4) - 6/x^2 - 1/(2*(1 + x^2)^2) - 4/(1 + x^2) - 10*Log
[x^2] + 10*Log[1 + x^2])/2
```

3.472.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.472.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4(x^2+1)^2}-\frac{2}{x^2+1}-10\ln(x)+5\ln(x^2+1)$
norman	$\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6(x^2+1)^2}-10\ln(x)+5\ln(x^2+1)$
risch	$\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6(x^2+1)^2}-10\ln(x)+5\ln(x^2+1)$
meijerg	$-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{9}{4}-10\ln(x)+\frac{x^2(9x^2+10)}{4(x^2+1)^2}+5\ln(x^2+1)$
parallelrisch	$-\frac{120x^{10}\ln(x)-60\ln(x^2+1)x^{10}+2+240\ln(x)x^8-120\ln(x^2+1)x^8+60x^8+120\ln(x)x^6-60x^6\ln(x^2+1)+90x^6+20x^4-5x^2}{12x^6(x^2+1)^2}$

input
```
int(1/x^7/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^7(1+x^2)^3}\,dx = -\frac{60\,x^8+90\,x^6+20\,x^4-5\,x^2-60\,(x^{10}+2\,x^8+x^6)\log(x^2+1)+120\,(x^{10}+2\,x^8+x^6)\log(x)+2}{12\,(x^{10}+2\,x^8+x^6)}$$

input
```
integrate(1/x^7/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^10 + 2*x^8 + x^6)*log(x^2
+ 1) + 120*(x^10 + 2*x^8 + x^6)*log(x) + 2)/(x^10 + 2*x^8 + x^6)
```

3.472.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -10 \log{(x)} + 5 \log{(x^2+1)} + \frac{-60x^8 - 90x^6 - 20x^4 + 5x^2 - 2}{12x^{10} + 24x^8 + 12x^6}$$

input
```
integrate(1/x**7/(x**2+1)**3,x)
```

output
```
-10*log(x) + 5*log(x**2 + 1) + (-60*x**8 - 90*x**6 - 20*x**4 + 5*x**2 - 2)
/(12*x**10 + 24*x**8 + 12*x**6)
```

3.472.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{60 x^8 + 90 x^6 + 20 x^4 - 5 x^2 + 2}{12 (x^{10} + 2 x^8 + x^6)} + 5 \log{(x^2+1)} - 5 \log{(x^2)}$$

input
```
integrate(1/x^7/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^10 + 2*x^8 + x^6) + 5*log(
x^2 + 1) - 5*log(x^2)
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{30 x^4 + 68 x^2 + 39}{4 (x^2+1)^2} + \frac{110 x^6 - 36 x^4 + 9 x^2 - 2}{12 x^6} + 5 \log{(x^2+1)} - 5 \log{(x^2)}$$

input
```
integrate(1/x^7/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4 + 9*x^2 -
 2)/x^6 + 5*log(x^2 + 1) - 5*log(x^2)
```

3.472.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (1 + x^2)^3} \, dx = 5 \ln \left(x^2 + 1\right) - 10 \ln (x) - \frac{5 x^8 + \frac{15 x^6}{2} + \frac{5 x^4}{3} - \frac{5 x^2}{12} + \frac{1}{6}}{x^{10} + 2 x^8 + x^6}$$

input
```
int(1/(x^7*(x^2 + 1)^3),x)
```

output
```
5*log(x^2 + 1) - 10*log(x) - ((5*x^4)/3 - (5*x^2)/12 + (15*x^6)/2 + 5*x^8
+ 1/6)/(x^6 + 2*x^8 + x^10)
```

3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx$

3.473.1 Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}$$

output
```
-9/10*(1+2/x^2)^(7/9)*x/(x^2+2)^(1/2)
```

3.473.2 Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}$$

input
```
Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

output
```
(-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

3.473.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2088, 942, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{7/9}}{(x^2+2)^{3/2}}\,dx$$
$$\Big\downarrow \text{2088}$$
$$\int \frac{\left(\frac{2}{x^2}+1\right)^{7/9}}{(x^2+2)^{3/2}}dx$$
$$\Big\downarrow \text{942}$$
$$\frac{\left(\frac{2}{x^2}+1\right)^{7/9}x^{14/9}\int \frac{1}{x^{14/9}(x^2+2)^{13/18}}dx}{(x^2+2)^{7/9}}$$
$$\Big\downarrow \text{242}$$
$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

input
```
Int[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

output
```
(-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

3.473.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

rule 2088
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && BinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && BinomialMatchQ[v, x])
```

3.473.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$	22
risch	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$	22

input
```
int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-9/10*x/(x^2+2)^(1/2)*((x^2+2)/x^2)^(7/9)
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\,x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\,\sqrt{x^2+2}}$$

input
```
integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-9/10*x*((x^2 + 2)/x^2)^(7/9)/sqrt(x^2 + 2)
```

3.473.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)
```

output
```
Timed out
```

3.473.7 Maxima [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

3.473.8 Giac [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

3.473.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\,x\,(x^2+2)^{5/18}\,\left(\frac{1}{x^2}\right)^{7/9}}{10}$$

input
```
int(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x)
```

output
```
-(9*x*(x^2 + 2)^(5/18)*(1/x^2)^(7/9))/10
```

3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$

3.474.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=\frac{x^5}{7\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}+\frac{x^5}{175\left(\sqrt{10}-x^2\right)^{5/2}}$$

output
```
1/70*x^5*10^(1/2)/(-x^2+10^(1/2))^(7/2)+1/175*x^5/(-x^2+10^(1/2))^(5/2)
```

3.474.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=-\frac{x^5\left(-7\sqrt{10}+2x^2\right)}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

input
```
Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

output
```
-1/350*(x^5*(-7*Sqrt[10] + 2*x^2))/(Sqrt[10] - x^2)^(7/2)
```

3.474.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}} \, dx$$

$$\Big\downarrow \text{245}$$

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{1}{5}\sqrt{\frac{2}{5}}\int \frac{x^6}{\left(\sqrt{10}-x^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{242}$$

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}$$

input
```
Int[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

output
```
x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^2)^(7/2))
```

3.474.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

3.474.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

method	result
gosper	$\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$
pseudoelliptic	$\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$
meijerg	$\frac{10^{\frac{3}{4}}x^{5}\left(-\frac{\sqrt{2}\,\sqrt{5}\,x^{2}}{5}+7\right)}{35000\left(1-\frac{\sqrt{10}\,x^{2}}{10}\right)^{\frac{7}{2}}}$
risch	$\frac{2x^{7}-7\sqrt{10}\,x^{5}}{350\left(x^{2}-\sqrt{10}\right)^{3}\sqrt{-x^{2}+\sqrt{10}}}$
trager	$-\frac{2\sqrt{10}\,\left(\sqrt{10}\,x^{2}-35\right)x^{5}\sqrt{-x^{2}+\sqrt{10}}}{35\left(\sqrt{10}\,x^{2}-10\right)^{4}}$
default	$\frac{x^{3}}{4\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{3\sqrt{10}\left(\frac{x}{6\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{\sqrt{10}\left(\frac{x\sqrt{10}}{70\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\sqrt{10}\left(\frac{x\sqrt{10}}{50\left(-x^{2}+\sqrt{10}\right)^{\frac{5}{2}}}+\frac{2\sqrt{10}\left(\frac{x\sqrt{10}}{30\left(-x^{2}+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{\cdots}{15\sqrt{\cdots}}\cdots\right)}{25}\cdots\right)}{35}\cdots\right)}{6}\cdots\right)}{4}\cdots$

input
```
int(x^4/(-x^2+10^(1/2))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/350*x^5*(-2*x^2+7*10^(1/2))/(-x^2+10^(1/2))^(7/2)
```

3.474. $\int \frac{x^{4}}{\left(\sqrt{10}-x^{2}\right)^{9/2}}\,dx$

3.474.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = -\frac{\left(2\,x^{15}-160\,x^{11}-2600\,x^7+\sqrt{10}(x^{13}-340\,x^9-700\,x^5)\right)\sqrt{-x^2+\sqrt{10}}}{350\,(x^{16}-40\,x^{12}+600\,x^8-4000\,x^4+10000)}$$

input
```
integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="fricas")
```

output
```
-1/350*(2*x^15 - 160*x^11 - 2600*x^7 + sqrt(10)*(x^13 - 340*x^9 - 700*x^5)
)*sqrt(-x^2 + sqrt(10))/(x^16 - 40*x^12 + 600*x^8 - 4000*x^4 + 10000)
```

3.474.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
```

output
```
Timed out
```

3.474.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \frac{x}{175\,\sqrt{-x^2+\sqrt{10}}}+\frac{\sqrt{10}x}{350\,\left(-x^2+\sqrt{10}\right)^{\frac{3}{2}}} + \frac{x^3}{4\,\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\,x}{140\,\left(-x^2+\sqrt{10}\right)^{\frac{5}{2}}}-\frac{3\,\sqrt{10}x}{28\,\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}$$

input
```
integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="maxima")
```

output
```
1/175*x/sqrt(-x^2 + sqrt(10)) + 1/350*sqrt(10)*x/(-x^2 + sqrt(10))^(3/2) +
 1/4*x^3/(-x^2 + sqrt(10))^(7/2) + 3/140*x/(-x^2 + sqrt(10))^(5/2) - 3/28*
sqrt(10)*x/(-x^2 + sqrt(10))^(7/2)
```

3.474.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = -\frac{16\left(7\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^2+20\right)}{175\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^7}$$

input
```
integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="giac")
```

output
```
-16/175*(7*(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 + sqrt(10))
- 10^(1/4))/x)^2 + 20)/(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2
+ sqrt(10)) - 10^(1/4))/x)^7
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$$

input
```
int(x^4/(10^(1/2) - x^2)^(9/2),x)
```

output
```
int(x^4/(10^(1/2) - x^2)^(9/2), x)
```

3.475 $\int \frac{x^2}{(3-x^2)^{3/2}}\,dx$

3.475.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{3-x^2}} - \arcsin\left(\frac{x}{\sqrt{3}}\right)$$

output
```
-arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

3.475.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{3-x^2}} + 2\arctan\left(\frac{x}{\sqrt{3}-\sqrt{3-x^2}}\right)$$

input
```
Integrate[x^2/(3 - x^2)^(3/2),x]
```

output
```
x/Sqrt[3 - x^2] + 2*ArcTan[x/(Sqrt[3] - Sqrt[3 - x^2])]
```

3.475.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3-x^2)^{3/2}} dx$$
$$\Big\downarrow \text{ 252}$$
$$\frac{x}{\sqrt{3-x^2}} - \int \frac{1}{\sqrt{3-x^2}}dx$$
$$\Big\downarrow \text{ 223}$$
$$\frac{x}{\sqrt{3-x^2}} - \arcsin\left(\frac{x}{\sqrt{3}}\right)$$

input
```
Int[x^2/(3 - x^2)^(3/2),x]
```

output
```
x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

3.475.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.475.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
default	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$	22
risch	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$	22
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)\sqrt{-x^2+3}+x}{\sqrt{-x^2+3}}$	37
meijerg	$\frac{i\left(-\frac{i\sqrt{\pi}\,x\sqrt{3}}{3\sqrt{-\frac{x^2}{3}+1}}+i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{\sqrt{\pi}}$	40
trager	$-\frac{x\sqrt{-x^2+3}}{x^2-3}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+3}+x\right)$	48

input
```
int(x^2/(-x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx=\frac{(x^2-3)\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)-\sqrt{-x^2+3}x}{x^2-3}$$

input
```
integrate(x^2/(-x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
((x^2 - 3)*arctan(sqrt(-x^2 + 3)/x) - sqrt(-x^2 + 3)*x)/(x^2 - 3)
```

3.475.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\frac{x^2 \operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3} - \frac{x\sqrt{3-x^2}}{x^2-3} + \frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}$$

input
```
integrate(x**2/(-x**2+3)**(3/2),x)
```

output
```
-x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(3 - x**2)/(x**2 - 3) + 3*asin(
sqrt(3)*x/3)/(x**2 - 3)
```

3.475.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{-x^2+3}} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

input
```
integrate(x^2/(-x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)
```

3.475.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+3}x}{x^2-3} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

input
```
integrate(x^2/(-x^2+3)^(3/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)
```

3.475.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right) - \frac{\sqrt{3-x^2}}{2\,\left(x-\sqrt{3}\right)} - \frac{\sqrt{3-x^2}}{2\,\left(x+\sqrt{3}\right)}$$

input
```
int(x^2/(3 - x^2)^(3/2),x)
```

output
```
- asin((3^(1/2)*x)/3) - (3 - x^2)^(1/2)/(2*(x - 3^(1/2))) - (3 - x^2)^(1/2
)/(2*(x + 3^(1/2)))
```

3.476 $\int \frac{(25-x^2)^{3/2}}{x^4} dx$

3.476.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \arcsin\left(\frac{x}{5}\right)$$

output
```
-1/3*(-x^2+25)^(3/2)/x^3+arcsin(1/5*x)+(-x^2+25)^(1/2)/x
```

3.476.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \frac{\sqrt{25-x^2}(-25+4x^2)}{3x^3} - 2\arctan\left(\frac{\sqrt{25-x^2}}{5+x}\right)$$

input
```
Integrate[(25 - x^2)^(3/2)/x^4,x]
```

output
```
(Sqrt[25 - x^2]*(-25 + 4*x^2))/(3*x^3) - 2*ArcTan[Sqrt[25 - x^2]/(5 + x)]
```

3.476.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(25-x^2)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \; 247$$

$$-\int \frac{\sqrt{25-x^2}}{x^2}dx-\frac{(25-x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 247$$

$$\int \frac{1}{\sqrt{25-x^2}}dx+\frac{\sqrt{25-x^2}}{x}-\frac{(25-x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 223$$

$$\arcsin\left(\frac{x}{5}\right)+\frac{\sqrt{25-x^2}}{x}-\frac{(25-x^2)^{3/2}}{3x^3}$$

input
```
Int[(25 - x^2)^(3/2)/x^4,x]
```

output
```
Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

3.476.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

3.476.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{4x^4-125x^2+625}{3x^3\sqrt{-x^2+25}}+\arcsin\left(\frac{x}{5}\right)$	32
meijerg	$-\frac{3i\left(-\frac{1000i\sqrt{\pi}\left(1-\frac{4x^2}{25}\right)\sqrt{-\frac{x^2}{25}+1}}{9x^3}+\frac{8i\sqrt{\pi}\arcsin\left(\frac{x}{5}\right)}{3}\right)}{8\sqrt{\pi}}$	43
trager	$\frac{(4x^2-25)\sqrt{-x^2+25}}{3x^3}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+25}\right)$	50
pseudoelliptic	$\frac{-3\arctan\left(\frac{\sqrt{-x^2+25}}{x}\right)x^3+4\sqrt{-x^2+25}\,x^2-25\sqrt{-x^2+25}}{3x^3}$	51
default	$-\frac{(-x^2+25)^{\frac{5}{2}}}{75x^3}+\frac{2(-x^2+25)^{\frac{5}{2}}}{1875x}+\frac{2x(-x^2+25)^{\frac{3}{2}}}{1875}+\frac{\sqrt{-x^2+25}\,x}{25}+\arcsin\left(\frac{x}{5}\right)$	58

input
```
int((-x^2+25)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*x^4-125*x^2+625)/x^3/(-x^2+25)^(1/2)+arcsin(1/5*x)
```

3.476.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = -\frac{6\,x^3\arctan\left(\frac{\sqrt{-x^2+25}-5}{x}\right)-(4\,x^2-25)\sqrt{-x^2+25}}{3\,x^3}$$

input
```
integrate((-x^2+25)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(6*x^3*arctan((sqrt(-x^2 + 25) - 5)/x) - (4*x^2 - 25)*sqrt(-x^2 + 25)
)/x^3
```

3.476.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \operatorname{asin}\left(\frac{x}{5}\right) + \frac{4\sqrt{25-x^2}}{3x} - \frac{25\sqrt{25-x^2}}{3x^3}$$

input
```
integrate((-x**2+25)**(3/2)/x**4,x)
```

output
```
asin(x/5) + 4*sqrt(25 - x**2)/(3*x) - 25*sqrt(25 - x**2)/(3*x**3)
```

3.476.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \frac{1}{25}\sqrt{-x^2+25}x + \frac{2\,(-x^2+25)^{\frac{3}{2}}}{75\,x} - \frac{(-x^2+25)^{\frac{5}{2}}}{75\,x^3} + \arcsin\left(\frac{1}{5}\,x\right)$$

input
```
integrate((-x^2+25)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 + 25)^(5/2)
/x^3 + arcsin(1/5*x)
```

3.476.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = -\frac{x^3\left(\frac{15\left(\sqrt{-x^2+25}-5\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+25}-5\right)^3} + \frac{5\left(\sqrt{-x^2+25}-5\right)}{8\,x} - \frac{\left(\sqrt{-x^2+25}-5\right)^3}{24\,x^3} + \arcsin\left(\frac{1}{5}\,x\right)$$

input
```
integrate((-x^2+25)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/24*x^3*(15*(sqrt(-x^2 + 25) - 5)^2/x^2 - 1)/(sqrt(-x^2 + 25) - 5)^3 + 5
/8*(sqrt(-x^2 + 25) - 5)/x - 1/24*(sqrt(-x^2 + 25) - 5)^3/x^3 + arcsin(1/5
*x)
```

3.476.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \operatorname{asin}\left(\frac{x}{5}\right)+\frac{4\sqrt{25-x^2}}{3\,x}-\frac{25\sqrt{25-x^2}}{3\,x^3}$$

input
```
int((25 - x^2)^(3/2)/x^4,x)
```

output
```
asin(x/5) + (4*(25 - x^2)^(1/2))/(3*x) - (25*(25 - x^2)^(1/2))/(3*x^3)
```

3.477 $\int \frac{1}{(1-2x^2)^{7/2}}\,dx$

3.477.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx=\frac{x}{5\,(1-2x^2)^{5/2}}+\frac{4x}{15\,(1-2x^2)^{3/2}}+\frac{8x}{15\sqrt{1-2x^2}}$$

output
```
1/5*x/(-2*x^2+1)^(5/2)+4/15*x/(-2*x^2+1)^(3/2)+8/15*x/(-2*x^2+1)^(1/2)
```

3.477.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx=\frac{x(15-40x^2+32x^4)}{15\,(1-2x^2)^{5/2}}$$

input
```
Integrate[(1 - 2*x^2)^(-7/2),x]
```

output
```
(x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))
```

3.477.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-2x^2)^{7/2}} \, dx$$
$$\Big\downarrow \text{209}$$
$$\frac{4}{5}\int \frac{1}{(1-2x^2)^{5/2}}dx+\frac{x}{5(1-2x^2)^{5/2}}$$
$$\Big\downarrow \text{209}$$
$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-2x^2)^{3/2}}dx+\frac{x}{3(1-2x^2)^{3/2}}\right)+\frac{x}{5(1-2x^2)^{5/2}}$$
$$\Big\downarrow \text{208}$$
$$\frac{x}{5(1-2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-2x^2}}+\frac{x}{3(1-2x^2)^{3/2}}\right)$$

input
```
Int[(1 - 2*x^2)^(-7/2),x]
```

output
```
x/(5*(1 - 2*x^2)^(5/2)) + (4*(x/(3*(1 - 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 -
2*x^2])))/5
```

3.477.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

3.477.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{x(32x^4-40x^2+15)}{15(-2x^2+1)^{\frac{5}{2}}}$	25
meijerg	$\frac{x(32x^4-40x^2+15)}{15(-2x^2+1)^{\frac{5}{2}}}$	25
pseudoelliptic	$\frac{32x^5-40x^3+15x}{15(-2x^2+1)^{\frac{5}{2}}}$	26
trager	$-\frac{(32x^4-40x^2+15)x\sqrt{-2x^2+1}}{15(2x^2-1)^3}$	34
risch	$\frac{x(32x^4-40x^2+15)}{15(2x^2-1)^2\sqrt{-2x^2+1}}$	34
default	$\frac{x}{5(-2x^2+1)^{\frac{5}{2}}}+\frac{4x}{15(-2x^2+1)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{-2x^2+1}}$	38

input
```
int(1/(-2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x/(-2*x^2+1)^(5/2)*(32*x^4-40*x^2+15)
```

3.477.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = -\frac{(32\,x^5-40\,x^3+15\,x)\sqrt{-2\,x^2+1}}{15\,(8\,x^6-12\,x^4+6\,x^2-1)}$$

input
```
integrate(1/(-2*x^2+1)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(32*x^5 - 40*x^3 + 15*x)*sqrt(-2*x^2 + 1)/(8*x^6 - 12*x^4 + 6*x^2 -
1)
```

3.477.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 291, normalized size of antiderivative = 5.94

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \begin{cases} -\frac{32ix^5}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} + \frac{40ix^3}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} - \frac{15}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{}} \\ \frac{32x^5}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} - \frac{40x^3}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} + \frac{15x}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1}} \end{cases}$$

input
```
integrate(1/(-2*x**2+1)**(7/2),x)
```

output
```
Piecewise((-32*I*x**5/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt(2*x**2 - 1)
 + 15*sqrt(2*x**2 - 1)) + 40*I*x**3/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sq
rt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)) - 15*I*x/(60*x**4*sqrt(2*x**2 - 1) -
 60*x**2*sqrt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)), Abs(x**2) > 1/2), (32*x*
*5/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1 - 2*x*
*2)) - 40*x**3/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*s
qrt(1 - 2*x**2)) + 15*x/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**
2) + 15*sqrt(1 - 2*x**2)), True))
```

3.477.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \frac{8\,x}{15\,\sqrt{-2\,x^2+1}} + \frac{4\,x}{15\,(-2\,x^2+1)^{\frac{3}{2}}} + \frac{x}{5\,(-2\,x^2+1)^{\frac{5}{2}}}$$

input
```
integrate(1/(-2*x^2+1)^(7/2),x, algorithm="maxima")
```

output
```
8/15*x/sqrt(-2*x^2 + 1) + 4/15*x/(-2*x^2 + 1)^(3/2) + 1/5*x/(-2*x^2 + 1)^(
5/2)
```

3.477.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = -\frac{(8\,(4\,x^2-5)x^2+15)\sqrt{-2\,x^2+1}x}{15\,(2\,x^2-1)^3}$$

input
```
integrate(1/(-2*x^2+1)^(7/2),x, algorithm="giac")
```

output
```
-1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3
```

3.477.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.65

$$\begin{aligned}\int \frac{1}{(1-2x^2)^{7/2}}\,dx = {} & \frac{19\sqrt{\frac{1}{2}-x^2}}{480\,\left(x^2-\sqrt{2}\,x+\frac{1}{2}\right)} \\ & -\frac{19\sqrt{\frac{1}{2}-x^2}}{480\,\left(x^2+\sqrt{2}\,x+\frac{1}{2}\right)}-\frac{\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{160\,\left(x^3-\frac{3\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}-\frac{\sqrt{2}}{4}\right)} \\ & -\frac{\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{160\,\left(x^3+\frac{3\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}+\frac{\sqrt{2}}{4}\right)}-\frac{2\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{15\,\left(x-\frac{\sqrt{2}}{2}\right)}-\frac{2\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{15\,\left(x+\frac{\sqrt{2}}{2}\right)}\end{aligned}$$

input
```
int(1/(1 - 2*x^2)^(7/2),x)
```

output
```
(19*(1/2 - x^2)^(1/2))/(480*(x^2 - 2^(1/2)*x + 1/2)) - (19*(1/2 - x^2)^(1/
2))/(480*(2^(1/2)*x + x^2 + 1/2)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(160*((3*x
)/2 - 2^(1/2)/4 - (3*2^(1/2)*x^2)/2 + x^3)) - (2^(1/2)*(1/2 - x^2)^(1/2))/
(160*((3*x)/2 + 2^(1/2)/4 + (3*2^(1/2)*x^2)/2 + x^3)) - (2*2^(1/2)*(1/2 -
x^2)^(1/2))/(15*(x - 2^(1/2)/2)) - (2*2^(1/2)*(1/2 - x^2)^(1/2))/(15*(x +
2^(1/2)/2))
```

3.478 $\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx$

3.478.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx=-\frac{3-x}{6(-7+6x-x^2)^{3/2}}-\frac{3-x}{6\sqrt{-7+6x-x^2}}$$

output
```
1/6*(-3+x)/(-x^2+6*x-7)^(3/2)+1/6*(-3+x)/(-x^2+6*x-7)^(1/2)
```

3.478.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx=-\frac{-18+24x-9x^2+x^3}{6(-7+6x-x^2)^{3/2}}$$

input
```
Integrate[(-7 + 6*x - x^2)^(-5/2),x]
```

output
```
-1/6*(-18 + 24*x - 9*x^2 + x^3)/(-7 + 6*x - x^2)^(3/2)
```

3.478.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^2+6x-7)^{5/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{1}{3}\int \frac{1}{(-x^2+6x-7)^{3/2}}dx-\frac{3-x}{6\,(-x^2+6x-7)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\,(-x^2+6x-7)^{3/2}}$$

input
```
Int[(-7 + 6*x - x^2)^(-5/2),x]
```

output
```
-1/6*(3 - x)/(-7 + 6*x - x^2)^(3/2) - (3 - x)/(6*Sqrt[-7 + 6*x - x^2])
```

3.478.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.478.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{x^{3}-9x^{2}+24x-18}{6(-x^{2}+6x-7)^{\frac{3}{2}}}$	28
trager	$-\frac{(x^{3}-9x^{2}+24x-18)\sqrt{-x^{2}+6x-7}}{6(x^{2}-6x+7)^{2}}$	38
risch	$\frac{x^{3}-9x^{2}+24x-18}{6(x^{2}-6x+7)\sqrt{-x^{2}+6x-7}}$	38
default	$-\frac{-2x+6}{12(-x^{2}+6x-7)^{\frac{3}{2}}}-\frac{-2x+6}{12\sqrt{-x^{2}+6x-7}}$	40

input
```
int(1/(-x^2+6*x-7)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(-x^2+6*x-7)^(3/2)*(x^3-9*x^2+24*x-18)
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(x^3-9\,x^2+24\,x-18)\sqrt{-x^2+6\,x-7}}{6\,(x^4-12\,x^3+50\,x^2-84\,x+49)}$$

input
```
integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(x^3 - 9*x^2 + 24*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^4 - 12*x^3 + 50*x^2
 - 84*x + 49)
```

3.478.6 Sympy [F]

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \int \frac{1}{(-x^2+6x-7)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**2+6*x-7)**(5/2),x)
```

output
```
Integral((-x**2 + 6*x - 7)**(-5/2), x)
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \frac{x}{6\sqrt{-x^2+6x-7}} - \frac{1}{2\sqrt{-x^2+6x-7}} + \frac{x}{6(-x^2+6x-7)^{\frac{3}{2}}} - \frac{1}{2(-x^2+6x-7)^{\frac{3}{2}}}$$

input
```
integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="maxima")
```

output
```
1/6*x/sqrt(-x^2 + 6*x - 7) - 1/2/sqrt(-x^2 + 6*x - 7) + 1/6*x/(-x^2 + 6*x
- 7)^(3/2) - 1/2/(-x^2 + 6*x - 7)^(3/2)
```

3.478.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(((x-9)x+24)x-18)\sqrt{-x^2+6x-7}}{6(x^2-6x+7)^2}$$

input
```
integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(((x - 9)*x + 24)*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^2 - 6*x + 7)^2
```

3.478.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(4x-12)(8x^2-48x+48)}{192(-x^2+6x-7)^{3/2}}$$

input
```
int(1/(6*x - x^2 - 7)^(5/2),x)
```

output
```
-((4*x - 12)*(8*x^2 - 48*x + 48))/(192*(6*x - x^2 - 7)^(3/2))
```

3.479 $\int (1-2x-2x^2)^3 \, dx$

3.479.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int (1-2x-2x^2)^3 \, dx = x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}$$

output
```
x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

3.479.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (1-2x-2x^2)^3 \, dx = x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}$$

input
```
Integrate[(1 - 2*x - 2*x^2)^3,x]
```

output
```
x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

3.479.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2-2x+1\right)^3 \, dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(-8x^6-24x^5-12x^4+16x^3+6x^2-6x+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

input
```
Int[(1 - 2*x - 2*x^2)^3,x]
```

output
```
x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

3.479.3.1 Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.479.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
norman	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
risch	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
parallelrisch	$x-3x^2+2x^3+4x^4-\frac{12}{5}x^5-4x^6-\frac{8}{7}x^7$	33
gosper	$-\frac{x(40x^6+140x^5+84x^4-140x^3-70x^2+105x-35)}{35}$	34

input
```
int((-2*x^2-2*x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8}{7}\,x^7-4\,x^6-\frac{12}{5}\,x^5+4\,x^4+2\,x^3-3\,x^2+x$$

input
```
integrate((-2*x^2-2*x+1)^3,x, algorithm="fricas")
```

output
```
-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

3.479.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

input
```
integrate((-2*x**2-2*x+1)**3,x)
```

output
```
-8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3 \, dx = -\frac{8}{7}\,x^7 - 4\,x^6 - \frac{12}{5}\,x^5 + 4\,x^4 + 2\,x^3 - 3\,x^2 + x$$

input
```
integrate((-2*x^2-2*x+1)^3,x, algorithm="maxima")
```

output
```
-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

3.479.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3 \, dx = -\frac{8}{7}\,x^7 - 4\,x^6 - \frac{12}{5}\,x^5 + 4\,x^4 + 2\,x^3 - 3\,x^2 + x$$

input
```
integrate((-2*x^2-2*x+1)^3,x, algorithm="giac")
```

output
```
-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

3.479.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3 \, dx = -\frac{8\,x^7}{7} - 4\,x^6 - \frac{12\,x^5}{5} + 4\,x^4 + 2\,x^3 - 3\,x^2 + x$$

input
```
int(-(2*x + 2*x^2 - 1)^3,x)
```

output
```
x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

3.480 $\int (-1+5x)(-1-x+x^2)^2\,dx$

3.480.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=-x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}$$

output
```
-x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

3.480.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=-x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}$$

input
```
Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

output
```
-x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

3.480.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x-1)\left(x^2-x-1\right)^2 dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(5x^5-11x^4-3x^3+11x^2+3x-1\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

input
```
Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

output
```
-x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

3.480.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.480.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
gosper	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
default	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
norman	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
risch	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30
parallelrisch	$-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$	30

input
```
int((-1+5*x)*(x^2-x-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int(-1+5x)\left(-1-x+x^2\right)^2\,dx=\frac{5}{6}\,x^6-\frac{11}{5}\,x^5-\frac{3}{4}\,x^4+\frac{11}{3}\,x^3+\frac{3}{2}\,x^2-x$$

input
```
integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="fricas")
```

output
```
5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

3.480.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int(-1+5x)\left(-1-x+x^2\right)^2\,dx=\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

input
```
integrate((-1+5*x)*(x**2-x-1)**2,x)
```

output
```
5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5}{6}\,x^6 - \frac{11}{5}\,x^5 - \frac{3}{4}\,x^4 + \frac{11}{3}\,x^3 + \frac{3}{2}\,x^2 - x$$

input
```
integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="maxima")
```

output
```
5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5}{6}\,x^6 - \frac{11}{5}\,x^5 - \frac{3}{4}\,x^4 + \frac{11}{3}\,x^3 + \frac{3}{2}\,x^2 - x$$

input
```
integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="giac")
```

output
```
5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

3.480.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx = \frac{5\,x^6}{6} - \frac{11\,x^5}{5} - \frac{3\,x^4}{4} + \frac{11\,x^3}{3} + \frac{3\,x^2}{2} - x$$

input
```
int((5*x - 1)*(x - x^2 + 1)^2,x)
```

output
```
(3*x^2)/2 - x + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$

3.481.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{1-2x}{6(1-8x+2x^2)^{3/2}} - \frac{2(2-x)}{21\sqrt{1-8x+2x^2}}$$

output
```
1/6*(1-2*x)/(2*x^2-8*x+1)^(3/2)-2/21*(2-x)/(2*x^2-8*x+1)^(1/2)
```

3.481.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{-1+54x-48x^2+8x^3}{42(1-8x+2x^2)^{3/2}}$$

input
```
Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

output
```
(-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

3.481.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+1}{(2x^2-8x+1)^{5/2}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{1-2x}{6\left(2x^2-8x+1\right)^{3/2}}-\frac{2}{3}\int \frac{1}{(2x^2-8x+1)^{3/2}}dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{1-2x}{6\left(2x^2-8x+1\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

input
```
Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

output
```
(1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x
^2])
```

3.481.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.481.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$	30
trager	$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$	30
risch	$\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$	30
default	$-\frac{4x-8}{12(2x^2-8x+1)^{\frac{3}{2}}}+\frac{4x-8}{42\sqrt{2x^2-8x+1}}-\frac{1}{2(2x^2-8x+1)^{\frac{3}{2}}}$	54

input
```
int((1+3*x)/(2*x^2-8*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/42*(8*x^3-48*x^2+54*x-1)/(2*x^2-8*x+1)^(3/2)
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = -\frac{4\,x^4-32\,x^3+68\,x^2-(8\,x^3-48\,x^2+54\,x-1)\sqrt{2\,x^2-8\,x+1}-16\,x+1}{42\,(4\,x^4-32\,x^3+68\,x^2-16\,x+1)}$$

input
```
integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/42*(4*x^4 - 32*x^3 + 68*x^2 - (8*x^3 - 48*x^2 + 54*x - 1)*sqrt(2*x^2 -
8*x + 1) - 16*x + 1)/(4*x^4 - 32*x^3 + 68*x^2 - 16*x + 1)
```

3.481.6 Sympy [F]

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \int \frac{3x+1}{(2x^2-8x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
```

output
```
Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{2\,x}{21\,\sqrt{2\,x^2-8\,x+1}} - \frac{4}{21\,\sqrt{2\,x^2-8\,x+1}} - \frac{x}{3\,(2\,x^2-8\,x+1)^{\frac{3}{2}}} + \frac{1}{6\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="maxima")
```

output
```
2/21*x/sqrt(2*x^2 - 8*x + 1) - 4/21/sqrt(2*x^2 - 8*x + 1) - 1/3*x/(2*x^2 -
 8*x + 1)^(3/2) + 1/6/(2*x^2 - 8*x + 1)^(3/2)
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{2\,(4\,(x-6)x+27)x-1}{42\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="giac")
```

output
```
1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^(3/2)
```

3.481.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{8\,x^3-48\,x^2+54\,x-1}{42\,(2\,x^2-8\,x+1)^{3/2}}$$

input
```
int((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x)
```

output
```
(54*x - 48*x^2 + 8*x^3 - 1)/(42*(2*x^2 - 8*x + 1)^(3/2))
```

3.482 $\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}} \, dx$

3.482.1 Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}} \, dx = -\frac{4(1+x)}{15\,(1+2x-4x^2)^{3/2}} - \frac{7+122x}{75\sqrt{1+2x-4x^2}}$$

output
```
-4/15*(1+x)/(-4*x^2+2*x+1)^(3/2)+1/75*(-7-122*x)/(-4*x^2+2*x+1)^(1/2)
```

3.482.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}} \, dx = -\frac{27+156x+216x^2-488x^3}{75\,(1+2x-4x^2)^{3/2}}$$

input
```
Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

output
```
-1/75*(27 + 156*x + 216*x^2 - 488*x^3)/(1 + 2*x - 4*x^2)^(3/2)
```

3.482.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2191, 27, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3-8x-1}{(-4x^2+2x+1)^{5/2}} \, dx$$

$$\Big\downarrow \ 2191$$

$$-\frac{1}{30}\int \frac{2(30x+23)}{(-4x^2+2x+1)^{3/2}}dx - \frac{4(x+1)}{15(-4x^2+2x+1)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{15}\int \frac{30x+23}{(-4x^2+2x+1)^{3/2}}dx - \frac{4(x+1)}{15(-4x^2+2x+1)^{3/2}}$$

$$\Big\downarrow \ 1158$$

$$-\frac{4(x+1)}{15(-4x^2+2x+1)^{3/2}} - \frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

input
```
Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

output
```
(-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[1 + 2*x -
 4*x^2])
```

3.482.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.482.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{488x^3-216x^2-156x-27}{75(-4x^2+2x+1)^{\frac{3}{2}}}$	30
trager	$\frac{(488x^3-216x^2-156x-27)\sqrt{-4x^2+2x+1}}{75(4x^2-2x-1)^2}$	42
risch	$-\frac{488x^3-216x^2-156x-27}{75(4x^2-2x-1)\sqrt{-4x^2+2x+1}}$	42
default	$\frac{\frac{61}{240}-\frac{61x}{60}}{(-4x^2+2x+1)^{\frac{3}{2}}}+\frac{\frac{61}{150}-\frac{122x}{75}}{\sqrt{-4x^2+2x+1}}-\frac{49}{48(-4x^2+2x+1)^{\frac{3}{2}}}+\frac{2x^2}{(-4x^2+2x+1)^{\frac{3}{2}}}-\frac{x}{4(-4x^2+2x+1)^{\frac{3}{2}}}$	86

input
```
int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/75/(-4*x^2+2*x+1)^(3/2)*(488*x^3-216*x^2-156*x-27)
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{432\,x^4-432\,x^3-108\,x^2-(488\,x^3-216\,x^2-156\,x-27)\sqrt{-4\,x^2+2\,x+1}+108\,x+27}{75\,(16\,x^4-16\,x^3-4\,x^2+4\,x+1)}$$

input
```
integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/75*(432*x^4 - 432*x^3 - 108*x^2 - (488*x^3 - 216*x^2 - 156*x - 27)*sqrt
(-4*x^2 + 2*x + 1) + 108*x + 27)/(16*x^4 - 16*x^3 - 4*x^2 + 4*x + 1)
```

3.482.6 Sympy [F]

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = \int \frac{8x^3-8x-1}{(-4x^2+2x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
```

output
```
Integral((8*x**3 - 8*x - 1)/(-4*x**2 + 2*x + 1)**(5/2), x)
```

3.482.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{122\,x}{75\,\sqrt{-4\,x^2+2\,x+1}} + \frac{2\,x^2}{(-4\,x^2+2\,x+1)^{\frac{3}{2}}} + \frac{61}{150\,\sqrt{-4\,x^2+2\,x+1}} - \frac{19\,x}{15\,(-4\,x^2+2\,x+1)^{\frac{3}{2}}} - \frac{23}{30\,(-4\,x^2+2\,x+1)^{\frac{3}{2}}}$$

input
```
integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="maxima")
```

output
```
-122/75*x/sqrt(-4*x^2 + 2*x + 1) + 2*x^2/(-4*x^2 + 2*x + 1)^(3/2) + 61/150
/sqrt(-4*x^2 + 2*x + 1) - 19/15*x/(-4*x^2 + 2*x + 1)^(3/2) - 23/30/(-4*x^2
 + 2*x + 1)^(3/2)
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = \frac{(4\,(2\,(61\,x-27)x-39)x-27)\sqrt{-4\,x^2+2\,x+1}}{75\,(4\,x^2-2\,x-1)^2}$$

input
```
integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="giac")
```

output
```
1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*x^2 - 2*x
 - 1)^2
```

3.482.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{-488\,x^3+216\,x^2+156\,x+27}{75\,(-4\,x^2+2\,x+1)^{3/2}}$$

input
```
int(-(8*x - 8*x^3 + 1)/(2*x - 4*x^2 + 1)^(5/2),x)
```

output
```
-(156*x + 216*x^2 - 488*x^3 + 27)/(75*(2*x - 4*x^2 + 1)^(3/2))
```

3.483 $\int x^2 \cos^5(x)\,dx$

3.483.1 Optimal result

Integrand size = 8, antiderivative size = 83

$$\int x^2 \cos^5(x)\,dx = \frac{16}{15}x\cos(x) + \frac{8}{45}x\cos^3(x) + \frac{2}{25}x\cos^5(x) - \frac{298\sin(x)}{225} + \frac{8}{15}x^2\sin(x) + \frac{4}{15}x^2\cos^2(x)\sin(x) + \frac{1}{5}x^2\cos^4(x)\sin(x) + \frac{76\sin^3(x)}{675} - \frac{2\sin^5(x)}{125}$$

output
```
16/15*x*cos(x)+8/45*x*cos(x)^3+2/25*x*cos(x)^5-298/225*sin(x)+8/15*x^2*sin
(x)+4/15*x^2*cos(x)^2*sin(x)+1/5*x^2*cos(x)^4*sin(x)+76/675*sin(x)^3-2/125
*sin(x)^5
```

3.483.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int x^2 \cos^5(x)\,dx = \frac{5}{4}x\cos(x) + \frac{5}{72}x\cos(3x) + \frac{1}{200}x\cos(5x) + \frac{5}{8}(-2+x^2)\sin(x) + \frac{5}{432}(-2+9x^2)\sin(3x) + \frac{(-2+25x^2)\sin(5x)}{2000}$$

input
```
Integrate[x^2*Cos[x]^5,x]
```

output
```
(5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 + x^2)*Sin[
x])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*Sin[5*x])/2000
```

3.483.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {3042, 3792, 3042, 3113, 2009, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^5(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin\left(x+\frac{\pi}{2}\right)^5 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{4}{5}\int x^2\cos^3(x)dx-\frac{2}{25}\int \cos^5(x)dx+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int x^2\sin\left(x+\frac{\pi}{2}\right)^3dx-\frac{2}{25}\int \sin\left(x+\frac{\pi}{2}\right)^5dx+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3113}$$

$$\frac{4}{5}\int x^2\sin\left(x+\frac{\pi}{2}\right)^3dx+\frac{2}{25}\int \left(\sin^4(x)-2\sin^2(x)+1\right)d(-\sin(x))+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}\int x^2\sin\left(x+\frac{\pi}{2}\right)^3dx+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3792}$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\cos(x)dx-\frac{2}{9}\int \cos^3(x)dx+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(x+\frac{\pi}{2}\right)dx-\frac{2}{9}\int \sin\left(x+\frac{\pi}{2}\right)^3dx+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 3113$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(x+\frac{\pi}{2}\right)dx+\frac{2}{9}\int\left(1-\sin^2(x)\right)d(-\sin(x))+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 3777$$

$$\frac{4}{5}\left(\frac{2}{3}\left(2\int -x\sin(x)dx+x^2\sin(x)\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 25$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2\int x\sin(x)dx\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2\int x\sin(x)dx\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 3777$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2(\int\cos(x)dx-x\cos(x))\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2\left(\int\sin\left(x+\frac{\pi}{2}\right)dx-x\cos(x)\right)\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right.$$
$$\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

3.483. $\int x^2\cos^5(x)\,dx$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{5}x^2\sin(x)\cos^4(x)+$$
$$\frac{4}{5}\left(\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{3}(x^2\sin(x)-2(\sin(x)-x\cos(x)))+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+$$
$$\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

input
```
Int[x^2*Cos[x]^5,x]
```

output
```
(2*x*Cos[x]^5)/25 + (x^2*Cos[x]^4*Sin[x])/5 + (2*(-Sin[x] + (2*Sin[x]^3)/3
 - Sin[x]^5/5))/25 + (4*((2*x*Cos[x]^3)/9 + (x^2*Cos[x]^2*Sin[x])/3 + (2*(
-Sin[x] + Sin[x]^3/3))/9 + (2*(x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x])))/3))
/5
```

3.483.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.483. $\int x^2\cos^5(x)\,dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.483.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result
risch	$\frac{5x\cos(x)}{4}+\frac{5(x^2-2)\sin(x)}{8}+\frac{x\cos(5x)}{200}+\frac{(25x^2-2)\sin(5x)}{2000}+\frac{5x\cos(3x)}{72}+\frac{5(9x^2-2)\sin(3x)}{432}$
parallelrisch	$\frac{(5625x^2-1250)\sin(3x)}{54000}+\frac{(675x^2-54)\sin(5x)}{54000}+\frac{5x^2\sin(x)}{8}+\frac{5x\cos(x)}{4}+\frac{5x\cos(3x)}{72}+\frac{x\cos(5x)}{200}-\frac{5\sin(x)}{4}$
default	$\frac{x^2\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{5}+\frac{2\left(\cos^5(x)\right)x}{25}-\frac{2\left(\frac{8}{3}+\cos^4(x)+\frac{4\left(\cos^2(x)\right)}{3}\right)\sin(x)}{125}+\frac{8x\left(\cos^3(x)\right)}{45}-\frac{8(2+\cos^2(x$

input
```
int(x^2*cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
5/4*x*cos(x)+5/8*(x^2-2)*sin(x)+1/200*x*cos(5*x)+1/2000*(25*x^2-2)*sin(5*x
)+5/72*x*cos(3*x)+5/432*(9*x^2-2)*sin(3*x)
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int x^2\cos^5(x)\,dx$$
$$=\frac{2}{25}\,x\cos\left(x\right)^5+\frac{8}{45}\,x\cos\left(x\right)^3+\frac{16}{15}\,x\cos\left(x\right)$$
$$+\frac{1}{3375}\left(27\left(25\,x^2-2\right)\cos\left(x\right)^4+4\left(225\,x^2-68\right)\cos\left(x\right)^2+1800\,x^2-4144\right)\sin\left(x\right)$$

input
```
integrate(x^2*cos(x)^5,x, algorithm="fricas")
```

output
```
2/25*x*cos(x)^5 + 8/45*x*cos(x)^3 + 16/15*x*cos(x) + 1/3375*(27*(25*x^2 -
2)*cos(x)^4 + 4*(225*x^2 - 68)*cos(x)^2 + 1800*x^2 - 4144)*sin(x)
```

3.483.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int x^2\cos^5(x)\,dx = \frac{8x^2\sin^5(x)}{15} + \frac{4x^2\sin^3(x)\cos^2(x)}{3} + x^2\sin(x)\cos^4(x) + \frac{16x\sin^4(x)\cos(x)}{15} + \frac{104x\sin^2(x)\cos^3(x)}{45} + \frac{298x\cos^5(x)}{225} - \frac{4144\sin^5(x)}{3375} - \frac{1712\sin^3(x)\cos^2(x)}{675} - \frac{298\sin(x)\cos^4(x)}{225}$$

input
```
integrate(x**2*cos(x)**5,x)
```

output
```
8*x**2*sin(x)**5/15 + 4*x**2*sin(x)**3*cos(x)**2/3 + x**2*sin(x)*cos(x)**4
 + 16*x*sin(x)**4*cos(x)/15 + 104*x*sin(x)**2*cos(x)**3/45 + 298*x*cos(x)*
*5/225 - 4144*sin(x)**5/3375 - 1712*sin(x)**3*cos(x)**2/675 - 298*sin(x)*c
os(x)**4/225
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int x^2\cos^5(x)\,dx = \frac{1}{200}\,x\cos(5\,x) + \frac{5}{72}\,x\cos(3\,x) + \frac{5}{4}\,x\cos(x) + \frac{1}{2000}\,\left(25\,x^2-2\right)\sin(5\,x) + \frac{5}{432}\,\left(9\,x^2-2\right)\sin(3\,x) + \frac{5}{8}\,\left(x^2-2\right)\sin(x)$$

input
```
integrate(x^2*cos(x)^5,x, algorithm="maxima")
```

output
```
1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*si
n(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int x^2 \cos^5(x)\,dx = \frac{1}{200}x\cos(5x) + \frac{5}{72}x\cos(3x) + \frac{5}{4}x\cos(x) + \frac{1}{2000}\left(25x^2-2\right)\sin(5x) + \frac{5}{432}\left(9x^2-2\right)\sin(3x) + \frac{5}{8}\left(x^2-2\right)\sin(x)$$

input
```
integrate(x^2*cos(x)^5,x, algorithm="giac")
```

output
```
1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*si
n(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

3.483.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int x^2 \cos^5(x)\,dx = \frac{8x\cos(x)^3}{45} - \frac{4144\sin(x)}{3375} + \frac{2x\cos(x)^5}{25} + \frac{8x^2\sin(x)}{15} - \frac{272\cos(x)^2\sin(x)}{3375} - \frac{2\cos(x)^4\sin(x)}{125} + \frac{16x\cos(x)}{15} + \frac{4x^2\cos(x)^2\sin(x)}{15} + \frac{x^2\cos(x)^4\sin(x)}{5}$$

input
```
int(x^2*cos(x)^5,x)
```

output
```
(8*x*cos(x)^3)/45 - (4144*sin(x))/3375 + (2*x*cos(x)^5)/25 + (8*x^2*sin(x)
)/15 - (272*cos(x)^2*sin(x))/3375 - (2*cos(x)^4*sin(x))/125 + (16*x*cos(x)
)/15 + (4*x^2*cos(x)^2*sin(x))/15 + (x^2*cos(x)^4*sin(x))/5
```

3.484 $\int x^3 \sin^3(x) \, dx$

3.484.1 Optimal result

Integrand size = 8, antiderivative size = 73

$$\int x^3 \sin^3(x) \, dx = \frac{40}{9} x \cos(x) - \frac{2}{3} x^3 \cos(x) - \frac{40 \sin(x)}{9} + 2x^2 \sin(x) + \frac{2}{9} x \cos(x) \sin^2(x) - \frac{1}{3} x^3 \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3} x^2 \sin^3(x)$$

output
```
40/9*x*cos(x)-2/3*x^3*cos(x)-40/9*sin(x)+2*x^2*sin(x)+2/9*x*cos(x)*sin(x)^
2-1/3*x^3*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)^3
```

3.484.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int x^3 \sin^3(x) \, dx = \frac{1}{108} \left(-81x\left(-6+x^2\right)\cos(x) + 3x\left(-2+3x^2\right)\cos(3x) + 243\left(-2+x^2\right)\sin(x) - \left(-2+9x^2\right)\sin(3x)\right)$$

input
```
Integrate[x^3*Sin[x]^3,x]
```

output
```
(-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 + x^2)*Sin[
x] - (-2 + 9*x^2)*Sin[3*x])/108
```

3.484.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin^3(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sin(x)^3 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{2}{3} \int x^3 \sin(x) dx - \frac{2}{3} \int x \sin^3(x) dx - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3} \int x^3 \sin(x) dx - \frac{2}{3} \int x \sin(x)^3 dx - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3} \left(3 \int x^2 \cos(x) dx - x^3 \cos(x)\right) - \frac{2}{3} \int x \sin(x)^3 dx - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3} \left(3 \int x^2 \sin\left(x + \frac{\pi}{2}\right) dx - x^3 \cos(x)\right) - \frac{2}{3} \int x \sin(x)^3 dx - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3} \left(3 \left(2 \int -x \sin(x) dx + x^2 \sin(x)\right) - x^3 \cos(x)\right) - \frac{2}{3} \int x \sin(x)^3 dx - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3} \left(3 \left(x^2 \sin(x) - 2 \int x \sin(x) dx\right) - x^3 \cos(x)\right) - \frac{2}{3} \int x \sin(x)^3 dx - \frac{1}{3} x^3 \sin^2(x) \cos(x) + \frac{1}{3} x^2 \sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2\int x\sin(x)dx\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2(\int\cos(x)dx-x\cos(x))\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2\left(\int\sin\left(x+\frac{\pi}{2}\right)dx-x\cos(x)\right)\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3117}$$

$$-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3791}$$

$$-\frac{2}{3}\left(\frac{2}{3}\int x\sin(x)dx+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\left(\frac{2}{3}\int x\sin(x)dx+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2}{3}\left(\frac{2}{3}(\int\cos(x)dx-x\cos(x))+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\left(\frac{2}{3}\left(\int\sin\left(x+\frac{\pi}{2}\right)dx-x\cos(x)\right)+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3117}$$

3.484. $\int x^3\sin^3(x)\,dx$

$$-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))-\frac{2}{3}\left(\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)+\frac{2}{3}(\sin(x)-x\cos(x))\right)$$

input
```
Int[x^3*Sin[x]^3,x]
```

output
```
-1/3*(x^3*Cos[x]*Sin[x]^2) + (x^2*Sin[x]^3)/3 - (2*(-1/3*(x*Cos[x]*Sin[x]^
2) + Sin[x]^3/9 + (2*(-(x*Cos[x]) + Sin[x]))/3))/3 + (2*(-(x^3*Cos[x]) + 3
*(x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x]))))/3
```

3.484.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.484.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

method	result
risch	$\left(-\frac{3}{4}x^{3}+\frac{9}{2}x\right)\cos\left(x\right)+\frac{9\left(x^{2}-2\right)\sin\left(x\right)}{4}+\left(\frac{1}{12}x^{3}-\frac{1}{18}x\right)\cos\left(3x\right)-\frac{\left(9x^{2}-2\right)\sin\left(3x\right)}{108}$
default	$-\frac{x^{3}\left(2+\sin^{2}\left(x\right)\right)\cos\left(x\right)}{3}+2x^{2}\sin\left(x\right)-\frac{40\sin\left(x\right)}{9}+4x\cos\left(x\right)+\frac{x^{2}\left(\sin^{3}\left(x\right)\right)}{3}+\frac{2x\left(2+\sin^{2}\left(x\right)\right)\cos\left(x\right)}{9}-\frac{2\left(\sin^{3}\left(x\right)\right)}{27}$
norman	$\frac{\frac{40x}{9}-\frac{2x^{3}}{3}-\frac{496\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{27}-\frac{80\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{9}+\frac{16x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}-\frac{16x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{3}-\frac{40x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{9}+4x^{2}\tan\left(\frac{x}{2}\right)-2x^{3}\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2x^{3}\left(\tan^{4}\left(\frac{x}{2}\right)\right.}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}$

input
```
int(x^3*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/4*x^3+9/2*x)*cos(x)+9/4*(x^2-2)*sin(x)+(1/12*x^3-1/18*x)*cos(3*x)-1/10
8*(9*x^2-2)*sin(3*x)
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^3\sin^3(x)\,dx = \frac{1}{9}\left(3\,x^3-2\,x\right)\cos\left(x\right)^3-\frac{1}{3}\left(3\,x^3-14\,x\right)\cos\left(x\right) - \frac{1}{27}\left(\left(9\,x^2-2\right)\cos\left(x\right)^2-63\,x^2+122\right)\sin\left(x\right)$$

input
```
integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

output
```
1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9*x^2 - 2)
*cos(x)^2 - 63*x^2 + 122)*sin(x)
```

3.484.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int x^3 \sin^3(x)\,dx = -x^3 \sin^2(x)\cos(x) - \frac{2x^3\cos^3(x)}{3} + \frac{7x^2\sin^3(x)}{3} + 2x^2\sin(x)\cos^2(x) + \frac{14x\sin^2(x)\cos(x)}{3} + \frac{40x\cos^3(x)}{9} - \frac{122\sin^3(x)}{27} - \frac{40\sin(x)\cos^2(x)}{9}$$

input
```
integrate(x**3*sin(x)**3,x)
```

output
```
-x**3*sin(x)**2*cos(x) - 2*x**3*cos(x)**3/3 + 7*x**2*sin(x)**3/3 + 2*x**2*
sin(x)*cos(x)**2 + 14*x*sin(x)**2*cos(x)/3 + 40*x*cos(x)**3/9 - 122*sin(x)
**3/27 - 40*sin(x)*cos(x)**2/9
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int x^3 \sin^3(x)\,dx = \frac{1}{36}\left(3x^3 - 2x\right)\cos(3x) - \frac{3}{4}\left(x^3 - 6x\right)\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{9}{4}\left(x^2 - 2\right)\sin(x)$$

input
```
integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

output
```
1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*s
in(3*x) + 9/4*(x^2 - 2)*sin(x)
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int x^3 \sin^3(x)\,dx = \frac{1}{36}\left(3x^3 - 2x\right)\cos(3x) - \frac{3}{4}\left(x^3 - 6x\right)\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{9}{4}\left(x^2 - 2\right)\sin(x)$$

input
```
integrate(x^3*sin(x)^3,x, algorithm="giac")
```

output
```
1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*s
in(3*x) + 9/4*(x^2 - 2)*sin(x)
```

3.484.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int x^3 \sin^3(x)\, dx = \frac{7\,x^2\ \sin(x)}{3} - \frac{2\,x\cos(x)^3}{9} - x^3\ \cos(x) - \frac{122\ \sin(x)}{27} + \frac{x^3\cos(x)^3}{3} + \frac{2\cos(x)^2\ \sin(x)}{27} + \frac{14\,x\ \cos(x)}{3} - \frac{x^2\cos(x)^2\ \sin(x)}{3}$$

input
```
int(x^3*sin(x)^3,x)
```

output
```
(7*x^2*sin(x))/3 - (2*x*cos(x)^3)/9 - x^3*cos(x) - (122*sin(x))/27 + (x^3*
cos(x)^3)/3 + (2*cos(x)^2*sin(x))/27 + (14*x*cos(x))/3 - (x^2*cos(x)^2*sin
(x))/3
```

3.485 $\int x^2 \sin^6(x) \, dx$

3.485.1 Optimal result

Integrand size = 8, antiderivative size = 105

$$\int x^2 \sin^6(x) \, dx = -\frac{245x}{1152} + \frac{5x^3}{48} + \frac{245\cos(x)\sin(x)}{1152} - \frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{24}x^2\cos(x)\sin^3(x) + \frac{5}{48}x\sin^4(x) + \frac{1}{108}\cos(x)\sin^5(x) - \frac{1}{6}x^2\cos(x)\sin^5(x) + \frac{1}{18}x\sin^6(x)$$

output
```
-245/1152*x+5/48*x^3+245/1152*cos(x)*sin(x)-5/16*x^2*cos(x)*sin(x)+5/16*x*
sin(x)^2+65/1728*cos(x)*sin(x)^3-5/24*x^2*cos(x)*sin(x)^3+5/48*x*sin(x)^4+
1/108*cos(x)*sin(x)^5-1/6*x^2*cos(x)*sin(x)^5+1/18*x*sin(x)^6
```

3.485.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int x^2 \sin^6(x) \, dx = \frac{1440x^3 - 3240x\cos(2x) + 324x\cos(4x) - 24x\cos(6x) - 1620(-1+2x^2)\sin(2x) + 81(-1+8x^2)\sin(4x)}{13824}$$

input
```
Integrate[x^2*Sin[x]^6,x]
```

output
```
(1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 1620*(-1 +
2*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*x^2)*Sin[6*x])/138
24
```

3.485.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.72, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.500, Rules used = {3042, 3792, 3042, 3115, 3042, 3115, 3042, 3115, 24, 3792, 3042, 3115, 3042, 3115, 24, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^6(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin(x)^6 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{5}{6} \int x^2 \sin^4(x) dx - \frac{1}{18} \int \sin^6(x) dx - \frac{1}{6} x^2 \sin^5(x) \cos(x) + \frac{1}{18} x \sin^6(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6} \int x^2 \sin(x)^4 dx - \frac{1}{18} \int \sin(x)^6 dx - \frac{1}{6} x^2 \sin^5(x) \cos(x) + \frac{1}{18} x \sin^6(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6} \int x^2 \sin(x)^4 dx + \frac{1}{18} \left(\frac{1}{6} \sin^5(x) \cos(x) - \frac{5}{6} \int \sin^4(x) dx \right) - \frac{1}{6} x^2 \sin^5(x) \cos(x) + \frac{1}{18} x \sin^6(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6} \int x^2 \sin(x)^4 dx + \frac{1}{18} \left(\frac{1}{6} \sin^5(x) \cos(x) - \frac{5}{6} \int \sin(x)^4 dx \right) - \frac{1}{6} x^2 \sin^5(x) \cos(x) + \frac{1}{18} x \sin^6(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6} \int x^2 \sin(x)^4 dx + \frac{1}{18} \left(\frac{1}{6} \sin^5(x) \cos(x) - \frac{5}{6} \left(\frac{3}{4} \int \sin^2(x) dx - \frac{1}{4} \sin^3(x) \cos(x) \right) \right) - \frac{1}{6} x^2 \sin^5(x) \cos(x) + \frac{1}{18} x \sin^6(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\int\sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3792}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin^2(x)dx-\frac{1}{8}\int\sin^4(x)dx-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx-\frac{1}{8}\int\sin(x)^4dx-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\int\sin^2(x)dx\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\int\sin(x)^2dx\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

3.485. $\int x^2\sin^6(x)\,dx$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)-$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3792}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int x^2dx}{2}-\frac{1}{2}\int\sin^2(x)dx-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3\right.\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int\sin^2(x)dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3(x\right.\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int\sin(x)^2dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3(x\right.\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3115}$$

3.485. $\int x^2\sin^6(x)\,dx$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \; 24$$

$$-\frac{1}{6}x^2\sin^5(x)\cos(x)+$$
$$\frac{5}{6}\left(-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{3}{4}\left(\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)\right)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\right.$$
$$\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

input
```
Int[x^2*Sin[x]^6,x]
```

output
```
-1/6*(x^2*Cos[x]*Sin[x]^5) + (x*Sin[x]^6)/18 + ((Cos[x]*Sin[x]^5)/6 - (5*(
-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4))/6)/18 + (5*(-1/
4*(x^2*Cos[x]*Sin[x]^3) + (x*Sin[x]^4)/8 + ((Cos[x]*Sin[x]^3)/4 - (3*(x/2
- (Cos[x]*Sin[x])/2))/4)/8 + (3*(x^3/6 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]
^2)/2 + (-1/2*x + (Cos[x]*Sin[x])/2)/2))/4))/6
```

3.485.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.485.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

method	result
risch	$\frac{5x^3}{48}-\frac{x\cos(6x)}{576}-\frac{(18x^2-1)\sin(6x)}{3456}+\frac{3x\cos(4x)}{128}+\frac{3(8x^2-1)\sin(4x)}{512}-\frac{15x\cos(2x)}{64}-\frac{15(2x^2-1)\sin(2x)}{128}$
default	$x^2\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+\frac{x\left(\sin^6(x)\right)}{18}+\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{108}+\frac{115x}{1152}+$

input
```
int(x^2*sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/48*x^3-1/576*x*cos(6*x)-1/3456*(18*x^2-1)*sin(6*x)+3/128*x*cos(4*x)+3/51
2*(8*x^2-1)*sin(4*x)-15/64*x*cos(2*x)-15/128*(2*x^2-1)*sin(2*x)
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int x^2\sin^6(x)\,dx = -\frac{1}{18}\,x\cos(x)^6+\frac{13}{48}\,x\cos(x)^4+\frac{5}{48}\,x^3-\frac{11}{16}\,x\cos(x)^2 - \frac{1}{3456}\left(32\left(18\,x^2-1\right)\cos(x)^5-2\left(936\,x^2-97\right)\cos(x)^3+3\left(792\,x^2-299\right)\cos(x)\right)\sin(x) + \frac{299}{1152}\,x$$

input
```
integrate(x^2*sin(x)^6,x, algorithm="fricas")
```

output
```
-1/18*x*cos(x)^6 + 13/48*x*cos(x)^4 + 5/48*x^3 - 11/16*x*cos(x)^2 - 1/3456
*(32*(18*x^2 - 1)*cos(x)^5 - 2*(936*x^2 - 97)*cos(x)^3 + 3*(792*x^2 - 299)
*cos(x))*sin(x) + 299/1152*x
```

3.485.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.83

$$\int x^2\sin^6(x)\,dx = \frac{5x^3\sin^6(x)}{48} + \frac{5x^3\sin^4(x)\cos^2(x)}{16} + \frac{5x^3\sin^2(x)\cos^4(x)}{16} + \frac{5x^3\cos^6(x)}{48} - \frac{11x^2\sin^5(x)\cos(x)}{16} - \frac{5x^2\sin^3(x)\cos^3(x)}{6} - \frac{5x^2\sin(x)\cos^5(x)}{16} + \frac{299x\sin^6(x)}{1152} + \frac{35x\sin^4(x)\cos^2(x)}{384} - \frac{125x\sin^2(x)\cos^4(x)}{384} - \frac{245x\cos^6(x)}{1152} + \frac{299\sin^5(x)\cos(x)}{1152} + \frac{25\sin^3(x)\cos^3(x)}{54} + \frac{245\sin(x)\cos^5(x)}{1152}$$

input
```
integrate(x**2*sin(x)**6,x)
```

output
```
5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(x)**2*cos
(x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(x)/16 - 5*x**2*sin
(x)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16 + 299*x*sin(x)**6/1152 + 3
5*x*sin(x)**4*cos(x)**2/384 - 125*x*sin(x)**2*cos(x)**4/384 - 245*x*cos(x)
**6/1152 + 299*sin(x)**5*cos(x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin
(x)*cos(x)**5/1152
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^2\sin^6(x)\,dx = \frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\left(18\,x^2-1\right)\sin(6\,x) + \frac{3}{512}\left(8\,x^2-1\right)\sin(4\,x) - \frac{15}{128}\left(2\,x^2-1\right)\sin(2\,x)$$

input
```
integrate(x^2*sin(x)^6,x, algorithm="maxima")
```

output
```
5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456
*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*s
in(2*x)
```

3.485.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^2 \sin^6(x)\,dx = \frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\left(18\,x^2 - 1\right)\sin(6\,x) + \frac{3}{512}\left(8\,x^2 - 1\right)\sin(4\,x) - \frac{15}{128}\left(2\,x^2 - 1\right)\sin(2\,x)$$

input
```
integrate(x^2*sin(x)^6,x, algorithm="giac")
```

output
```
5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456
*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*s
in(2*x)
```

3.485.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int x^2 \sin^6(x)\,dx = \frac{15\,\sin(2\,x)}{128} - \frac{3\,\sin(4\,x)}{512} + \frac{\sin(6\,x)}{3456} - \frac{3\,x\left(2\sin(2\,x)^2 - 1\right)}{128} + \frac{x\left(2\sin(3\,x)^2 - 1\right)}{576} - \frac{15\,x^2\,\sin(2\,x)}{64} + \frac{3\,x^2\,\sin(4\,x)}{64} - \frac{x^2\,\sin(6\,x)}{192} + \frac{5\,x^3}{48} + \frac{15\,x\left(2\sin(x)^2 - 1\right)}{64}$$

input
```
int(x^2*sin(x)^6,x)
```

output
```
(15*sin(2*x))/128 - (3*sin(4*x))/512 + sin(6*x)/3456 - (3*x*(2*sin(2*x)^2
- 1))/128 + (x*(2*sin(3*x)^2 - 1))/576 - (15*x^2*sin(2*x))/64 + (3*x^2*sin
(4*x))/64 - (x^2*sin(6*x))/192 + (5*x^3)/48 + (15*x*(2*sin(x)^2 - 1))/64
```

3.486 $\int x^2 \cos(x) \sin^2(x) \, dx$

3.486.1 Optimal result

Integrand size = 10, antiderivative size = 44

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{4}{9} x \cos(x) - \frac{4 \sin(x)}{9} + \frac{2}{9} x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3} x^2 \sin^3(x)$$

output
```
4/9*x*cos(x)-4/9*sin(x)+2/9*x*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)
^3
```

3.486.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{1}{54} \left(27x \cos(x) - 3x \cos(3x) + \left(-26 + 9x^2 + \left(2 - 9x^2\right) \cos(2x)\right) \sin(x)\right)$$

input
```
Integrate[x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x])*Sin[x])
/54
```

3.486.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3924, 3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2(x) \cos(x) \, dx$$

$$\Big\downarrow \text{ 3924}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\int x \sin^3(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\int x \sin(x)^3dx$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\left(\frac{2}{3}\int x \sin(x)dx + \frac{\sin^3(x)}{9} - \frac{1}{3}x \sin^2(x) \cos(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\left(\frac{2}{3}\int x \sin(x)dx + \frac{\sin^3(x)}{9} - \frac{1}{3}x \sin^2(x) \cos(x)\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\left(\frac{2}{3}(\int \cos(x)dx - x \cos(x)) + \frac{\sin^3(x)}{9} - \frac{1}{3}x \sin^2(x) \cos(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\left(\frac{2}{3}\left(\int \sin\left(x + \frac{\pi}{2}\right) dx - x \cos(x)\right) + \frac{\sin^3(x)}{9} - \frac{1}{3}x \sin^2(x) \cos(x)\right)$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2}{3}\left(\frac{\sin^3(x)}{9} - \frac{1}{3}x \sin^2(x) \cos(x) + \frac{2}{3}(\sin(x) - x \cos(x))\right)$$

input
```
Int[x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(x^2*Sin[x]^3)/3 - (2*(-1/3*(x*Cos[x]*Sin[x]^2) + Sin[x]^3/9 + (2*(-(x*Cos
[x]) + Sin[x]))/3))/3
```

3.486.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.486.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^2(\sin^3(x))}{3}+\frac{2x(2+\sin^2(x))\cos(x)}{9}-\frac{2(\sin^3(x))}{27}-\frac{4\sin(x)}{9}$	32
risch	$\frac{x\cos(x)}{2}+\frac{(x^2-2)\sin(x)}{4}-\frac{x\cos(3x)}{18}-\frac{(9x^2-2)\sin(3x)}{108}$	36
parallelrisch	$\frac{x\cos(x)}{2}+\frac{x^2\sin(x)}{4}-\frac{\sin(x)}{2}-\frac{x\cos(3x)}{18}-\frac{x^2\sin(3x)}{12}+\frac{\sin(3x)}{54}$	40
norman	$\frac{\frac{4x}{9}-\frac{64\left(\tan^3\left(\frac{x}{2}\right)\right)}{27}-\frac{8\left(\tan^5\left(\frac{x}{2}\right)\right)}{9}+\frac{4x\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{4x\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{4x\left(\tan^6\left(\frac{x}{2}\right)\right)}{9}+\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)x^2}{3}-\frac{8\tan\left(\frac{x}{2}\right)}{9}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	76

input
```
int(x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3-4/9*sin(x)
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2\cos(x)\sin^2(x)\,dx = -\frac{2}{9}\,x\cos{(x)}^3+\frac{2}{3}\,x\cos{(x)} - \frac{1}{27}\left(\left(9\,x^2-2\right)\cos{(x)}^2-9\,x^2+14\right)\sin{(x)}$$

input
```
integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

output
```
-2/9*x*cos(x)^3 + 2/3*x*cos(x) - 1/27*((9*x^2 - 2)*cos(x)^2 - 9*x^2 + 14)*
sin(x)
```

3.486.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int x^2\cos(x)\sin^2(x)\,dx = \frac{x^2\sin^3(x)}{3} + \frac{2x\sin^2(x)\cos(x)}{3} + \frac{4x\cos^3(x)}{9} - \frac{14\sin^3(x)}{27} - \frac{4\sin(x)\cos^2(x)}{9}$$

input
```
integrate(x**2*cos(x)*sin(x)**2,x)
```

output
```
x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/3 + 4*x*cos(x)**3/9 - 14*sin(x)**3
/27 - 4*sin(x)*cos(x)**2/9
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{18}x\cos(3x) + \frac{1}{2}x\cos(x) - \frac{1}{108}(9x^2-2)\sin(3x) + \frac{1}{4}(x^2-2)\sin(x)$$

input
```
integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 -
2)*sin(x)
```

3.486.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{18}x\cos(3x) + \frac{1}{2}x\cos(x) - \frac{1}{108}(9x^2-2)\sin(3x) + \frac{1}{4}(x^2-2)\sin(x)$$

input
```
integrate(x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

output
```
-1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 -
2)*sin(x)
```

3.486.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^2 \cos(x) \sin^2(x)\,dx = \frac{x^2 \sin(x)^3}{3} + \frac{4\,x\cos(x)^3}{9} + \frac{2\,x\,\cos(x)\,\sin(x)^2}{3} - \frac{4\cos(x)^2\,\sin(x)}{9} - \frac{14\sin(x)^3}{27}$$

input
```
int(x^2*cos(x)*sin(x)^2,x)
```

output
```
(4*x*cos(x)^3)/9 - (14*sin(x)^3)/27 + (x^2*sin(x)^3)/3 - (4*cos(x)^2*sin(x
))/9 + (2*x*cos(x)*sin(x)^2)/3
```

3.487 $\int x \cos^2(x) \cot^2(x) \, dx$

3.487.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x \cos^2(x) \cot^2(x) \, dx = -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2} x \cos(x) \sin(x)$$

output
```
-3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)
```

3.487.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x \cos^2(x) \cot^2(x) \, dx = -\frac{3x^2}{4} - \frac{1}{8} \cos(2x) - x \cot(x) + \log(\sin(x)) - \frac{1}{4} x \sin(2x)$$

input
```
Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

3.487.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4908, 3042, 3791, 15, 4203, 15, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(x)\cot^2(x)\,dx$$

$$\Big\downarrow \text{ 4908}$$

$$\int x\cot^2(x)dx-\int x\cos^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\int x\sin\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3791}$$

$$-\frac{\int xdx}{2}+\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\cos^2(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 15}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{x^2}{4}-\frac{\cos^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 4203}$$

$$-\int xdx-\int -\cot(x)dx-\frac{x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 15}$$

$$-\int -\cot(x)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 25}$$

$$\int \cot(x)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \tan \left(x+\frac{\pi}{2}\right) dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{3956}$$

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

3.487.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.487.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

method	result
parallelrisch	$-\frac{3x^2}{4}-\frac{3}{8}+\ln\left(\frac{\csc(x)}{2}-\frac{\cot(x)}{2}\right)-\ln\left(\frac{1}{\cos(x)+1}\right)+\frac{x\cot(x)\cos(2x)}{4}-\frac{5x\cot(x)}{4}-\frac{\cos(2x)}{8}$
risch	$-\frac{3x^2}{4}+\frac{i(i+2x)e^{2ix}}{16}-\frac{i(-i+2x)e^{-2ix}}{16}-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$
norman	$\frac{\tan^3\left(\frac{x}{2}\right)-\frac{x}{2}-\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)x^2}{2}-\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)x^2}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan^2\left(\right.\right.$

input
```
int(x*cos(x)^4/sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*x^2-3/8+ln(1/2*csc(x)-1/2*cot(x))-ln(1/(cos(x)+1))+1/4*x*cot(x)*cos(2
*x)-5/4*x*cot(x)-1/8*cos(2*x)
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\cos^2(x)\cot^2(x)\,dx = \frac{4\,x\cos\left(x\right)^3-12\,x\cos\left(x\right)-\left(6\,x^2+2\,\cos\left(x\right)^2-1\right)\sin\left(x\right)+8\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)\sin\left(x\right)}{8\,\sin\left(x\right)}$$

input
```
integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")
```

output
```
1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(
1/2*sin(x))*sin(x))/sin(x)
```

3.487.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(32) = 64.

Time = 0.68 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.36

$$\int x\cos^2(x)\cot^2(x)\,dx = -\frac{3x^2\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{6x^2\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{3x^2\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{2x\tan^6\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{6x\tan^4\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{6x\tan^2\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{2x}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{4\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{8\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} - \frac{4\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{4\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^5\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{8\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{4\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)} + \frac{4\tan^3\left(\frac{x}{2}\right)}{4\tan^5\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(x*cos(x)**4/sin(x)**2,x)
```

output
```
-3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x**2*
tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 3*x**2*tan(x/2)
/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 2*x*tan(x/2)**6/(4*tan(x/2
)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*ta
n(x/2)**3 + 4*tan(x/2)) - 6*x*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
 4*tan(x/2)) - 2*x/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(ta
n(x/2)**2 + 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) -
8*log(tan(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(
x/2)) - 4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4
*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
4*tan(x/2)) + 8*log(tan(x/2))*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
 4*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4
*tan(x/2)) + 4*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

3.487.7 Maxima [F(-2)]

Exception generated.

$$\int x\cos^2(x)\cot^2(x)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.487.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 6.24

$$\int x\cos^2(x)\cot^2(x)\,dx =$$

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\tan}{}$$

input
```
integrate(x*cos(x)^4/sin(x)^2,x, algorithm="giac")
```

output
```
-1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*t
an(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1
/2*x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(
1/2*x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(
1/2*x) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

3.487.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\cos^2(x)\cot^2(x)\,dx = \ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right) - \mathrm{e}^{-x\,2\mathrm{i}}\left(\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right) + \mathrm{e}^{x\,2\mathrm{i}}\left(-\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right) - \frac{3\,x^2}{4} - x\,2\mathrm{i} - \frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

input
```
int((x*cos(x)^4)/sin(x)^2,x)
```

output
```
log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1
i)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

3.488 $\int x \sec(x) \tan^3(x) \, dx$

3.488.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int x \sec(x) \tan^3(x) \, dx = \frac{5}{6} \text{arctanh}(\sin(x)) - x \sec(x) + \frac{1}{3} x \sec^3(x) - \frac{1}{6} \sec(x) \tan(x)$$

output
```
5/6*arctanh(sin(x))-x*sec(x)+1/3*x*sec(x)^3-1/6*sec(x)*tan(x)
```

3.488.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(30) = 60.

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.47

$$\begin{aligned}\int x \sec(x) \tan^3(x) \, dx = & -\frac{1}{24} \sec^3(x) \left(4x + 12x \cos(2x) + 5 \cos(3x) \log \left(\cos \left(\frac{x}{2}\right) - \sin \left(\frac{x}{2}\right)\right)\right. \\ & + 15 \cos(x) \left(\log \left(\cos \left(\frac{x}{2}\right) - \sin \left(\frac{x}{2}\right)\right) - \log \left(\cos \left(\frac{x}{2}\right) + \sin \left(\frac{x}{2}\right)\right)\right) \\ & \left. - 5 \cos(3x) \log \left(\cos \left(\frac{x}{2}\right) + \sin \left(\frac{x}{2}\right)\right) + 2 \sin(2x)\right)\end{aligned}$$

input
```
Integrate[x*Sec[x]*Tan[x]^3,x]
```

output
```
-1/24*(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x/2]]
 + 15*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) - 5*Cos
[3*x]*Log[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))
```

3.488.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4917, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan^3(x)\sec(x)\,dx$$

$$\Big\downarrow \ 4917$$

$$-\int \left(\frac{\sec^3(x)}{3}-\sec(x)\right)dx+\frac{1}{3}x\sec^3(x)-x\sec(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{5}{6}\text{arctanh}(\sin(x))+\frac{1}{3}x\sec^3(x)-x\sec(x)-\frac{1}{6}\tan(x)\sec(x)$$

input
```
Int[x*Sec[x]*Tan[x]^3,x]
```

output
```
(5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6
```

3.488.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4917
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Sec[a + b*x]^n*Tan[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m, 0] && (IntegerQ[n/2] || I
ntegerQ[(p - 1)/2])
```

3.488.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	si
parallelrisch	$\ln\left(\left(-\cot(x)+1+\csc(x)\right)^{\frac{5}{6}}\right)+\ln\left(\frac{1}{\left(-\cot(x)+\csc(x)-1\right)^{\frac{5}{6}}}\right)+\frac{x\left(\sec^3(x)\right)}{3}+\frac{(-6x-\tan(x))\sec(x)}{6}$	43
norman	$\frac{\frac{2x}{3}-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)-2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{\tan\left(\frac{x}{2}\right)}{3}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{6}+\frac{5\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{6}$	76
risch	$-\frac{e^{ix}\left(6x\,e^{4ix}+4x\,e^{2ix}-i e^{4ix}+6x+i\right)}{3\left(e^{2ix}+1\right)^3}+\frac{5\ln\left(i+e^{ix}\right)}{6}-\frac{5\ln\left(e^{ix}-i\right)}{6}$	78

input
```
int(x*sin(x)^3/cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
ln((-cot(x)+1+csc(x))^(5/6))+ln(1/(-cot(x)+csc(x)-1)^(5/6))+1/3*x*sec(x)^3
+1/6*(-6*x-tan(x))*sec(x)
```

3.488.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int x\sec(x)\tan^3(x)\,dx$$
$$=\frac{5\cos(x)^3\log(\sin(x)+1)-5\cos(x)^3\log(-\sin(x)+1)-12\,x\cos(x)^2-2\cos(x)\sin(x)+4\,x}{12\cos(x)^3}$$

input
```
integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")
```

output
```
1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) - 12*x*cos(
x)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3
```

3.488.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(29) = 58$.

Time = 0.64 (sec) , antiderivative size = 551, normalized size of antiderivative = 18.37

$$\int x\sec(x)\tan^3(x)\,dx = \frac{4x\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{12x\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{12x\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{4x}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{5\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{15\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{15\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{5\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{5\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^6\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{15\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^4\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{15\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{5\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} - \frac{2\tan^5\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6} + \frac{2\tan\left(\frac{x}{2}\right)}{6\tan^6\left(\frac{x}{2}\right)-18\tan^4\left(\frac{x}{2}\right)+18\tan^2\left(\frac{x}{2}\right)-6}$$

input
```
integrate(x*sin(x)**3/cos(x)**4,x)
```

output
```
4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12
*x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*
x*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 4*x/
(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) - 1
)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*l
og(tan(x/2) - 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)
**2 - 6) - 15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**
4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*tan(x/2)
**4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**6/(6*tan(x/2)**6
 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) + 1)*tan(x/2)**4
/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) +
 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*
log(tan(x/2) + 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) -
2*tan(x/2)**5/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*ta
n(x/2)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)
```

3.488.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 619, normalized size of antiderivative = 20.63

$$\int x \sec(x) \tan^3(x) \, dx = \text{Too large to display}$$

input
```
integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")
```

output
```
-1/12*(48*x*sin(3*x)*sin(2*x) + 4*(6*x*cos(5*x) + 4*x*cos(3*x) + 6*x*cos(x
) + sin(5*x) - sin(x))*cos(6*x) + 12*(6*x*cos(4*x) + 6*x*cos(2*x) + 2*x -
sin(4*x) - sin(2*x))*cos(5*x) + 12*(4*x*cos(3*x) + 6*x*cos(x) - sin(x))*co
s(4*x) + 16*(3*x*cos(2*x) + x)*cos(3*x) + 12*(6*x*cos(x) - sin(x))*cos(2*x
) + 24*x*cos(x) - 5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2
 + 6*(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x)
 + sin(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) +
 9*sin(2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) +
5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) +
 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6
*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*
cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*(6*x*sin(5*x) +
4*x*sin(3*x) + 6*x*sin(x) - cos(5*x) + cos(x))*sin(6*x) + 4*(18*x*sin(4*x)
 + 18*x*sin(2*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*sin(5*x) + 12*(4*x*sin(3*x
) + 6*x*sin(x) + cos(x))*sin(4*x) + 12*(6*x*sin(x) + cos(x))*sin(2*x) - 4*
sin(x))/(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(
2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))
*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^
2 + 6*cos(2*x) + 1)
```

3.488.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(24) = 48$.

Time = 0.43 (sec) , antiderivative size = 341, normalized size of antiderivative = 11.37

$$\int x\sec(x)\tan^3(x)\,dx$$
$$= \frac{8\,x\tan\left(\frac{1}{2}\,x\right)^6 + 5\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^6 - 5\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^6 -}{}$$

input
```
integrate(x*sin(x)^3/cos(x)^4,x, algorithm="giac")
```

output
```
1/12*(8*x*tan(1/2*x)^6 + 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/
2*x)^2 + 1))*tan(1/2*x)^6 - 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4 - 15*log(2*(tan(1/2*x)^2
+ 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 15*log(2*(tan(1/2*x
)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5
- 24*x*tan(1/2*x)^2 + 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 - 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(
1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*x - 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x)^4 + 3*tan(1/2*x
)^2 - 1)
```

3.488.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int x\sec(x)\tan^3(x)\,dx = -\frac{x\cos(x)^2 - \frac{x}{3} + \frac{\sin(2x)}{12}}{\cos(x)^3} - \frac{\operatorname{atan}(\cos(x) + \sin(x)\ 1\mathrm{i})\ 5\mathrm{i}}{3}$$

input
```
int((x*sin(x)^3)/cos(x)^4,x)
```

output
```
- (atan(cos(x) + sin(x)*1i)*5i)/3 - (sin(2*x)/12 - x/3 + x*cos(x)^2)/cos(x
)^3
```

3.489 $\int x \sec^2(x) \tan(x) \, dx$

3.489.1 Optimal result

Integrand size = 8, antiderivative size = 16

$$\int x \sec^2(x) \tan(x) \, dx = \frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2}$$

output
```
1/2*x*sec(x)^2-1/2*tan(x)
```

3.489.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x \sec^2(x) \tan(x) \, dx = \frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2}$$

input
```
Integrate[x*Sec[x]^2*Tan[x],x]
```

output
```
(x*Sec[x]^2)/2 - Tan[x]/2
```

3.489.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4244, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan(x)\sec^2(x)\,dx$$
$$\Big\downarrow \text{4244}$$
$$\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \sec^2(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{4254}$$
$$\frac{1}{2}\int 1d(-\tan(x))+\frac{1}{2}x\sec^2(x)$$
$$\Big\downarrow \text{24}$$
$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

input
```
Int[x*Sec[x]^2*Tan[x],x]
```

output
```
(x*Sec[x]^2)/2 - Tan[x]/2
```

3.489.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.489.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x}{2\cos(x)^2}-\frac{\tan(x)}{2}$	13
parallelrisch	$\frac{x(\sec^2(x))}{2}-\frac{\tan(x)}{2}$	13
risch	$\frac{2x\,e^{2ix}-ie^{2ix}-i}{(e^{2ix}+1)^2}$	30
norman	$\frac{\tan^3(\frac{x}{2})+x(\tan^2(\frac{x}{2}))+\frac{x}{2}+\frac{x(\tan^4(\frac{x}{2}))}{2}-\tan(\frac{x}{2})}{(\tan^2(\frac{x}{2})-1)^2}$	45

input
```
int(x*sin(x)/cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/cos(x)^2-1/2*tan(x)
```

3.489.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x\sec^2(x)\tan(x)\,dx = -\frac{\cos(x)\sin(x)-x}{2\cos(x)^2}$$

input
```
integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")
```

output
```
-1/2*(cos(x)*sin(x) - x)/cos(x)^2
```

3.489.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(12) = 24.

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 8.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{x\tan^4\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{x}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}+\frac{2\tan^3\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}-\frac{2\tan\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}$$

input
```
integrate(x*sin(x)/cos(x)**3,x)
```

output
```
x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)**2/(2*tan
(x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*
tan(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) - 2*tan(x/2)/(2*tan(x/2)**
4 - 4*tan(x/2)**2 + 2)
```

3.489.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 8.25

$$\int x\sec^2(x)\tan(x)\,dx = \frac{4\,x\cos(2\,x)^2+4\,x\sin(2\,x)^2+(2\,x\cos(2\,x)+\sin(2\,x))\cos(4\,x)+2\,x\cos(2\,x)+(2\,x\sin(2\,x)-\cos(2\,x)}{2\,(2\,\cos(2\,x)+1)\cos(4\,x)+\cos(4\,x)^2+4\,\cos(2\,x)^2+\sin(4\,x)^2+4\,\sin(4\,x)\sin(2\,x)+4\,\sin(2\,x)}$$

input
```
integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")
```

output
```
(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*
x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*s
in(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

3.489.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\sec^2(x)\tan(x)\,dx = \frac{x\tan\left(\frac{1}{2}\,x\right)^4 + 2\,x\tan\left(\frac{1}{2}\,x\right)^2 + 2\,\tan\left(\frac{1}{2}\,x\right)^3 + x - 2\,\tan\left(\frac{1}{2}\,x\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}$$

input
```
integrate(x*sin(x)/cos(x)^3,x, algorithm="giac")
```

output
```
1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x)
)/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

3.489.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{2\,x - \sin\left(2\,x\right)}{4\cos\left(x\right)^2}$$

input
```
int((x*sin(x))/cos(x)^3,x)
```

output
```
(2*x - sin(2*x))/(4*cos(x)^2)
```

3.490 $\int x \sin^2(x) \tan(x) \, dx$

3.490.1 Optimal result

Integrand size = 8, antiderivative size = 62

$$\int x \sin^2(x) \tan(x) \, dx = \frac{x}{4} + \frac{ix^2}{2} - x \log\left(1 + e^{2ix}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2ix}\right) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2} x \sin^2(x)$$

output
```
1/4*x+1/2*I*x^2-x*ln(1+exp(2*I*x))+1/2*I*polylog(2,-exp(2*I*x))-1/4*cos(x)
*sin(x)-1/2*x*sin(x)^2
```

3.490.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x \sin^2(x) \tan(x) \, dx = \frac{ix^2}{2} + \frac{1}{4} x \cos(2x) - x \log\left(1 + e^{2ix}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2ix}\right) - \frac{1}{8} \sin(2x)$$

input
```
Integrate[x*Sin[x]^2*Tan[x],x]
```

output
```
(I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^
((2*I)*x)] - Sin[2*x]/8
```

3.490.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {4907, 3042, 3924, 3042, 3115, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^2(x)\tan(x)\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int x\tan(x)dx-\int x\cos(x)\sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan(x)dx-\int x\cos(x)\sin(x)dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{1}{2}\int \sin^2(x)dx+\int x\tan(x)dx-\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(x)^2dx+\int x\tan(x)dx-\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{3115}$$

$$\int x\tan(x)dx+\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{24}$$

$$\int x\tan(x)dx-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{2ix}x}{1+e^{2ix}}dx+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{1}{2}i\int \log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(\frac{1}{4}\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[x*Sin[x]^2*Tan[x],x]
```

output
```
(I/2)*x^2 - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)] - PolyLog[2, -E^((2*I)*
x)]/4) - (x*Sin[x]^2)/2 + (x/2 - (Cos[x]*Sin[x])/2)/2
```

3.490.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.490.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{i x^{2}}{2}+\frac{\left(i+2 x \right) {\mathrm e}^{2 i x}}{16}+\frac{\left(-i+2 x \right) {\mathrm e}^{-2 i x}}{16}-x \ln \left({\mathrm e}^{2 i x}+1\right)+\frac{i \operatorname{Li}_{2}\left(-{\mathrm e}^{2 i x}\right)}{2}$	57

input
```
int(x*sin(x)^3/cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x^2+1/16*(I+2*x)*exp(2*I*x)+1/16*(-I+2*x)*exp(-2*I*x)-x*ln(exp(2*I*x
)+1)+1/2*I*polylog(2,-exp(2*I*x))
```

3.490.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\begin{aligned}\int x\sin^2(x)\tan(x)\,dx &= \frac{1}{2}x\cos(x)^2 - \frac{1}{2}x\log(i\cos(x) + \sin(x) + 1) \\ &- \frac{1}{2}x\log(i\cos(x) - \sin(x) + 1) - \frac{1}{2}x\log(-i\cos(x) + \sin(x) + 1) \\ &- \frac{1}{2}x\log(-i\cos(x) - \sin(x) + 1) - \frac{1}{4}\cos(x)\sin(x) - \frac{1}{4}x \\ &- \frac{1}{2}i\,\mathrm{Li}_2(i\cos(x) + \sin(x)) + \frac{1}{2}i\,\mathrm{Li}_2(i\cos(x) - \sin(x)) \\ &+ \frac{1}{2}i\,\mathrm{Li}_2(-i\cos(x) + \sin(x)) - \frac{1}{2}i\,\mathrm{Li}_2(-i\cos(x) - \sin(x))\end{aligned}$$

input
```
integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")
```

output
```
1/2*x*cos(x)^2 - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - s
in(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin
(x) + 1) - 1/4*cos(x)*sin(x) - 1/4*x - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/
2*I*dilog(I*cos(x) - sin(x)) + 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dil
og(-I*cos(x) - sin(x))
```

3.490.6 Sympy [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin^3(x)}{\cos(x)}\,dx$$

input
```
integrate(x*sin(x)**3/cos(x),x)
```

output
```
Integral(x*sin(x)**3/cos(x), x)
```

3.490.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int x\sin^2(x)\tan(x)\,dx = \frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right) - \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")
```

output
```
1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x) - 1/2*x*l
og(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dilog(-e^(2*I*x)) - 1
/8*sin(2*x)
```

3.490.8 Giac [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin\left(x\right)^3}{\cos\left(x\right)}\,dx$$

input
```
integrate(x*sin(x)^3/cos(x),x, algorithm="giac")
```

output
```
integrate(x*sin(x)^3/cos(x), x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin\left(x\right)^3}{\cos\left(x\right)}\,dx$$

input
```
int((x*sin(x)^3)/cos(x),x)
```

output
```
int((x*sin(x)^3)/cos(x), x)
```

3.491 $\int x \tan^3(x)\,dx$

3.491.1 Optimal result

Integrand size = 6, antiderivative size = 59

$$\int x\tan^3(x)\,dx=\frac{x}{2}-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)$$

output
```
1/2*x-1/2*I*x^2+x*ln(1+exp(2*I*x))-1/2*I*polylog(2,-exp(2*I*x))-1/2*tan(x)
+1/2*x*tan(x)^2
```

3.491.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x\tan^3(x)\,dx=-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

input
```
Integrate[x*Tan[x]^3,x]
```

output
```
(-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] + (
x*Sec[x]^2)/2 - Tan[x]/2
```

3.491.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4203, 3042, 3954, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan(x)^3dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{1}{2}\int \tan^2(x)dx-\int x\tan(x)dx+\frac{1}{2}x\tan^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int x\tan(x)dx-\frac{1}{2}\int \tan(x)^2dx+\frac{1}{2}x\tan^2(x)$$

$$\Big\downarrow \text{3954}$$

$$\frac{1}{2}(\int 1dx-\tan(x))-\int x\tan(x)dx+\frac{1}{2}x\tan^2(x)$$

$$\Big\downarrow \text{24}$$

$$-\int x\tan(x)dx+\frac{1}{2}x\tan^2(x)+\frac{1}{2}(x-\tan(x))$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2ix}x}{1+e^{2ix}}dx-\frac{ix^2}{2}+\frac{1}{2}x\tan^2(x)+\frac{1}{2}(x-\tan(x))$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\int \log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)-\frac{ix^2}{2}+\frac{1}{2}x\tan^2(x)+\frac{1}{2}(x-\tan(x))$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{4}\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)-\frac{ix^2}{2}+\frac{1}{2}x\tan^2(x)+\frac{1}{2}(x-\tan(x))$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)-\frac{ix^2}{2}+\frac{1}{2}x\tan^2(x)+\frac{1}{2}(x-\tan(x))$$

input
```
Int[x*Tan[x]^3,x]
```

output
```
(-1/2*I)*x^2 + (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)] - PolyLog[2, -E^((2*
I)*x)]/4) + (x - Tan[x])/2 + (x*Tan[x]^2)/2
```

3.491.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.491. $\int x\tan^3(x)\,dx$

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.491.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{i x^2}{2}+\frac{2x\,\mathrm{e}^{2ix}-i\mathrm{e}^{2ix}-i}{\left(\mathrm{e}^{2ix}+1\right)^2}+x\ln\left(\mathrm{e}^{2ix}+1\right)-\frac{i\,\mathrm{Li}_2\left(-\mathrm{e}^{2ix}\right)}{2}$	59

input
```
int(x*sin(x)^3/cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x^2+(2*x*exp(2*I*x)-I*exp(2*I*x)-I)/(exp(2*I*x)+1)^2+x*ln(exp(2*I*x
)+1)-1/2*I*polylog(2,-exp(2*I*x))
```

3.491.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int x\tan^3(x)\,dx$$
$$= \frac{x\cos(x)^2\log(i\cos(x)+\sin(x)+1)+x\cos(x)^2\log(i\cos(x)-\sin(x)+1)+x\cos(x)^2\log(-i\cos(x)\cdots}{}$$

input
```
integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")
```

output
```
1/2*(x*cos(x)^2*log(I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(I*cos(x) - sin
(x) + 1) + x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(-I*cos(
x) - sin(x) + 1) + I*cos(x)^2*dilog(I*cos(x) + sin(x)) - I*cos(x)^2*dilog(
I*cos(x) - sin(x)) - I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + I*cos(x)^2*dil
og(-I*cos(x) - sin(x)) - cos(x)*sin(x) + x)/cos(x)^2
```

3.491.6 Sympy [F]

$$\int x\tan^3(x)\,dx = \int \frac{x\sin^3(x)}{\cos^3(x)}\,dx$$

input
```
integrate(x*sin(x)**3/cos(x)**3,x)
```

output
```
Integral(x*sin(x)**3/cos(x)**3, x)
```

3.491.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.56

$$\int x\tan^3(x)\,dx =$$

$$-\frac{x^2\cos(4x) + i\,x^2\sin(4x) + x^2 - 2\,(x\cos(4x) + 2\,x\cos(2x) + i\,x\sin(4x) + 2i\,x\sin(2x) + x)\,\mathrm{arctan}$$

input
```
integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")
```

output
```
-(x^2*cos(4*x) + I*x^2*sin(4*x) + x^2 - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x
*sin(4*x) + 2*I*x*sin(2*x) + x)*arctan2(sin(2*x), cos(2*x) + 1) + 2*(x^2 +
 2*I*x + 1)*cos(2*x) + (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x)
+ 1)*dilog(-e^(2*I*x)) - (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*
x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 2*(I*x^2
 - 2*x + I)*sin(2*x) + 2)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*s
in(2*x) - 2*I)
```

3.491.8 Giac [F]

$$\int x\tan^3(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

input
```
integrate(x*sin(x)^3/cos(x)^3,x, algorithm="giac")
```

output
```
integrate(x*sin(x)^3/cos(x)^3, x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int x\tan^3(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

input
```
int((x*sin(x)^3)/cos(x)^3,x)
```

output
```
int((x*sin(x)^3)/cos(x)^3, x)
```

3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2} \, dx$

3.492.1 Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2} \, dx = \frac{2}{1+\frac{\cot(x)}{x}}$$

output
```
2/(1+cot(x)/x)
```

3.492.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2} \, dx = \frac{2x\sin(x)}{\cos(x)+x\sin(x)}$$

input
```
Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

output
```
(2*x*Sin[x])/(Cos[x] + x*Sin[x])
```

3.492.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7262, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+\sin(2x)}{(x\sin(x)+\cos(x))^2}dx$$

$$\Big\downarrow \text{7262}$$

$$-2\int \frac{1}{\left(\frac{\cot(x)}{x}+1\right)^2}d\frac{\cot(x)}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

input
```
Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

output
```
2/(1 + Cot[x]/x)
```

3.492.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 7262
```
Int[(u_)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol] :> With[{c
= Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Simp[c*p   Subst[Int[(b + a*x^p
)^m, x], x, v*w^(m*q + 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q}, x]
 && EqQ[p + q*(m*p + 1), 0] && IntegerQ[p] && IntegerQ[m]
```

3.492.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

method	result	size
risch	$-\frac{2i}{x+i}-\frac{4ix}{(x+i)(x\,\mathrm{e}^{2ix}-x+i\mathrm{e}^{2ix}+i)}$	44

input
```
int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)
```

3.492.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx=-\frac{2\cos(x)}{x\sin(x)+\cos(x)}$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="fricas")
```

output
```
-2*cos(x)/(x*sin(x) + cos(x))
```

3.492.6 Sympy [F]

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx=\int \frac{2x+\sin(2x)}{(x\sin(x)+\cos(x))^2}\,dx$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

output
```
Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)
```

3.492.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 6.50

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2\left(\cos\left(2\,x\right)^2+2\,x\sin\left(2\,x\right)+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)}{\left(x^2+1\right)\cos\left(2\,x\right)^2+\left(x^2+1\right)\sin\left(2\,x\right)^2+x^2-2\left(x^2-1\right)\cos\left(2\,x\right)+4\,x\sin\left(2\,x\right)+1}$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="maxima")
```

output
```
-2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^2 + 1)*co
s(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(2*x) + 4*x*sin(2*x
) + 1)
```

3.492.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2}{x\tan\left(x\right)+1}$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="giac")
```

output
```
-2/(x*tan(x) + 1)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = \int \frac{2\,x+\sin\left(2\,x\right)}{\left(\cos\left(x\right)+x\,\sin\left(x\right)\right)^2}\,dx$$

input
```
int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2,x)
```

output
```
int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2, x)
```

3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$

3.493.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\cot(x)+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

output
```
-cot(x)+x*csc(x)/(x*cos(x)-sin(x))
```

3.493.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{\cos(x)+x\sin(x)}{x\cos(x)-\sin(x)}$$

input
```
Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]
```

output
```
(Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])
```

3.493.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5105, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}dx$$

$$\Big\downarrow \text{5105}$$

$$\int \csc^2(x)dx+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^2dx+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\int 1d\cot(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

input
```
Int[x^2/(x*Cos[x] - Sin[x])^2,x]
```

output
```
-Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

3.493.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5105
```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Simp[1/d^2   I
nt[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

3.493.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\cos(x)+x\sin(x)}{x\cos(x)-\sin(x)}$	20
risch	$\frac{2i(x-i)}{i\,\mathrm{e}^{2ix}+x\,\mathrm{e}^{2ix}-i+x}$	29
norman	$\frac{\tan^2\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)-1}{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}$	37

input
```
int(x^2/(x*cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
(cos(x)+x*sin(x))/(x*cos(x)-sin(x))
```

3.493.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx=\frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

input
```
integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="fricas")
```

output
```
(x*sin(x) + cos(x))/(x*cos(x) - sin(x))
```

3.493.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(15) = 30.

Time = 0.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\frac{2x\tan\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}+\frac{\tan^2\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)} - \frac{1}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(x**2/(x*cos(x)-sin(x))**2,x)
```

output
```
-2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*tan(x/2)**
2 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))
```

3.493.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(20) = 40.

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{2\,(2\,x\cos(2\,x)+(x^2-1)\sin(2\,x))}{(x^2+1)\cos(2\,x)^2+(x^2+1)\sin(2\,x)^2+x^2+2\,(x^2-1)\cos(2\,x)-4\,x\sin(2\,x)+1}$$

input
```
integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="maxima")
```

output
```
2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*si
n(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1)
```

3.493.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\frac{2\,x\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)^2+1}{x\tan\left(\frac{1}{2}\,x\right)^2-x+2\,\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="giac")
```

output
```
-(2*x*tan(1/2*x) - tan(1/2*x)^2 + 1)/(x*tan(1/2*x)^2 - x + 2*tan(1/2*x))
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \int \frac{x^2}{\left(\sin\left(x\right)-x\,\cos\left(x\right)\right)^2}\,dx$$

input
```
int(x^2/(sin(x) - x*cos(x))^2,x)
```

output
```
int(x^2/(sin(x) - x*cos(x))^2, x)
```

3.494 $\int a^{mx} b^{nx}\, dx$

3.494.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int a^{mx} b^{nx}\, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

output
```
a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

3.494.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx} b^{nx}\, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

input
```
Integrate[a^(m*x)*b^(n*x),x]
```

output
```
(a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

3.494.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^{mx}b^{nx}\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int e^{x(m\log(a)+n\log(b))}dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

input
```
Int[a^(m*x)*b^(n*x),x]
```

output
```
(a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

3.494.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

3.494.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{a^{mx} b^{nx}}{m \ln(a)+n \ln(b)}$	23
risch	$\frac{a^{mx} b^{nx}}{m \ln(a)+n \ln(b)}$	23
parallelrisch	$\frac{a^{mx} b^{nx}}{m \ln(a)+n \ln(b)}$	23
norman	$\frac{{\rm e}^{m x \ln(a)} {\rm e}^{n x \ln(b)}}{m \ln(a)+n \ln(b)}$	25
meijerg	$-\frac{1-{\rm e}^{x n \ln(b) \left(1+\frac{m \ln(a)}{n \ln(b)}\right)}}{n \ln(b) \left(1+\frac{m \ln(a)}{n \ln(b)}\right)}$	48

input
```
int(a^(m*x)*b^(n*x),x,method=_RETURNVERBOSE)
```

output
```
a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

3.494.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx} b^{nx}\, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

input
```
integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")
```

output
```
a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))
```

3.494.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int a^{mx} b^{nx}\, dx = \begin{cases} \frac{a^{mx} b^{nx}}{m \log(a)+n \log(b)} & \text{for}\ m \neq -\frac{n \log(b)}{\log(a)} \\ b^{nx} x e^{-nx \log(b)} & \text{otherwise} \end{cases}$$

input
```
integrate(a**(m*x)*b**(n*x),x)
```

output
```
Piecewise((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(b)/log(a)
)), (b**(n*x)*x*exp(-n*x*log(b)), True))
```

3.494.7 Maxima [F(-2)]

Exception generated.

$$\int a^{mx} b^{nx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((log(b)*n)/(log(a)*m)>0)', see `
assume?` f
```

3.494.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 14.77

$$\int a^{mx} b^{nx}\,dx$$
$$= 2\left(\frac{2\,(m\log(|a|)+n\log(|b|))\cos\left(-\frac{1}{2}\,\pi mx\text{sgn}(a)-\frac{1}{2}\,\pi nx\text{sgn}(b)+\frac{1}{2}\,\pi mx+\frac{1}{2}\,\pi nx\right)}{(\pi m\text{sgn}\,(a)+\pi n\text{sgn}\,(b)-\pi m-\pi n)^2+4\,(m\log(|a|)+n\log(|b|))^2} - \frac{(\pi m\text{sgn}(a)+\pi n\text{s}}{(\pi m\text{sg}}\right.$$
$$\left.+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi mx\text{sgn}(a)+\frac{1}{2}i\,\pi nx\text{sgn}(b)-\frac{1}{2}i\,\pi mx-\frac{1}{2}i\,\pi nx\right)}}{i\,\pi m\text{sgn}\,(a)+i\,\pi n\text{sgn}\,(b)-i\,\pi m-i\,\pi n+2\,m\log(|a|)+2\,n\log(|b|)} - \frac{i\,e^{\left(-\frac{1}{2}i\,\pi mx\text{sgn}\right.}}{-i\,\pi m\text{sgn}\,(a)-i\,\pi n\text{sgn}\,(b)}\right.\right.$$

input
```
integrate(a^(m*x)*b^(n*x),x, algorithm="giac")
```

output
```
2*(2*(m*log(abs(a)) + n*log(abs(b)))*cos(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*s
gn(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n
)^2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - (pi*m*sgn(a) + pi*n*sgn(b) -
pi*m - pi*n)*sin(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2
*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) +
 n*log(abs(b)))^2))*e^((m*log(abs(a)) + n*log(abs(b)))*x) + I*(I*e^(1/2*I*
pi*m*x*sgn(a) + 1/2*I*pi*n*x*sgn(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x)/(I*pi*m
*sgn(a) + I*pi*n*sgn(b) - I*pi*m - I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(
b))) - I*e^(-1/2*I*pi*m*x*sgn(a) - 1/2*I*pi*n*x*sgn(b) + 1/2*I*pi*m*x + 1/
2*I*pi*n*x)/(-I*pi*m*sgn(a) - I*pi*n*sgn(b) + I*pi*m + I*pi*n + 2*m*log(ab
s(a)) + 2*n*log(abs(b))))*e^((m*log(abs(a)) + n*log(abs(b)))*x)
```

3.494.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx} b^{nx} \, dx = \frac{a^{m\,x}\,b^{n\,x}}{m\,\ln(a) + n\,\ln(b)}$$

input
```
int(a^(m*x)*b^(n*x),x)
```

output
```
(a^(m*x)*b^(n*x))/(m*log(a) + n*log(b))
```

3.495 $\int a^{-x}b^{-x}(a^x-b^x)^2\,dx$

3.495.1 Optimal result

Integrand size = 22, antiderivative size = 34

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-2x+\frac{a^xb^{-x}-a^{-x}b^x}{\log(a)-\log(b)}$$

output
```
-2*x+(a^x/(b^x)-b^x/(a^x))/(ln(a)-ln(b))
```

3.495.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-2x+\frac{e^{x(\log(a)-\log(b))}}{\log(a)-\log(b)}+\frac{e^{x(-\log(a)+\log(b))}}{-\log(a)+\log(b)}$$

input
```
Integrate[(a^x - b^x)^2/(a^x*b^x),x]
```

output
```
-2*x + E^(x*(Log[a] - Log[b]))/(Log[a] - Log[b]) + E^(x*(-Log[a] + Log[b])
)/(-Log[a] + Log[b])
```

3.495.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2725, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx$$
$$\Big\downarrow \text{2725}$$
$$\int (a^x-b^x)^2e^{-x(\log(a)+\log(b))}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(a^{2x}e^{-x(\log(a)+\log(b))}-2a^xb^xe^{-x(\log(a)+\log(b))}+b^{2x}e^{-x(\log(a)+\log(b))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^{-x}b^x}{\log(a)-\log(b)}+\frac{a^xb^{-x}}{\log(a)-\log(b)}-2x$$

input
```
Int[(a^x - b^x)^2/(a^x*b^x),x]
```

output
```
-2*x + a^x/(b^x*(Log[a] - Log[b])) - b^x/(a^x*(Log[a] - Log[b]))
```

3.495.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.495.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result	size
risch	$-2x-\frac{a^{-x} b^{x}}{\ln(a)-\ln(b)}+\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$	42
parallelrisch	$-\frac{\left(2x\,a^{x} b^{x} \ln(a)-2x\,a^{x} b^{x} \ln(b)-a^{2x}+b^{2x}\right) a^{-x} b^{-x}}{\ln(a)-\ln(b)}$	57
norman	$\left(\frac{{\mathrm e}^{2x \ln(a)}}{\ln(a)-\ln(b)}-\frac{{\mathrm e}^{2x \ln(b)}}{\ln(a)-\ln(b)}-2x\,{\mathrm e}^{x \ln(a)} {\mathrm e}^{x \ln(b)}\right) {\mathrm e}^{-x \ln(a)} {\mathrm e}^{-x \ln(b)}$	65

input
```
int((a^x-b^x)^2/(a^x)/(b^x),x,method=_RETURNVERBOSE)
```

output
```
-2*x-1/(ln(a)-ln(b))/(a^x)*b^x+a^x/(b^x)/(ln(a)-ln(b))
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-\frac{2\left(x\log\left(a\right)-x\log\left(b\right)\right)a^xb^x-a^{2\,x}+b^{2\,x}}{a^xb^x(\log\left(a\right)-\log\left(b\right))}$$

input
```
integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="fricas")
```

output
```
-(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(log(a) -
log(b)))
```

3.495.6 Sympy [F(-2)]

Exception generated.

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a**x-b**x)**2/(a**x)/(b**x),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

3.495.7 Maxima [F(-2)]

Exception generated.

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?
` for more
```

3.495.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 436, normalized size of antiderivative = 12.82

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/(
(pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a)
- pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(
b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))
) + I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn
(b) + 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*
x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^
(x*(log(abs(a)) - log(abs(b)))) - 2*(2*(log(abs(a)) - log(abs(b)))*cos(1/2
*pi*x*sgn(a) - 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)
) - log(abs(b)))^2) - (pi*sgn(a) - pi*sgn(b))*sin(1/2*pi*x*sgn(a) - 1/2*pi
*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*
e^(-x*(log(abs(a)) - log(abs(b)))) + I*(-I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi
*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) - 2*log(abs(a)) + 2*log(abs(b))) + I
*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) -
2*log(abs(a)) + 2*log(abs(b))))*e^(-x*(log(abs(a)) - log(abs(b)))) - 2*x
```

3.495.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=\frac{\frac{a^x}{b^x}-\frac{b^x}{a^x}}{\ln(a)-\ln(b)}-2\,x$$

input
```
int((a^x - b^x)^2/(a^x*b^x),x)
```

output
```
(a^x/b^x - b^x/a^x)/(log(a) - log(b)) - 2*x
```

3.496 $\int (-e^{-x} + e^x) \, dx$

3.496.1 Optimal result

Integrand size = 11, antiderivative size = 9

$$\int (-e^{-x} + e^x) \, dx = e^{-x} + e^x$$

output
```
exp(-x)+exp(x)
```

3.496.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (-e^{-x} + e^x) \, dx = e^{-x} + e^x$$

input
```
Integrate[-E^(-x) + E^x,x]
```

output
```
E^(-x) + E^x
```

3.496.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e^x - e^{-x}) \, dx$$

$$\Big\downarrow \text{2009}$$

$$e^{-x} + e^x$$

input
```
Int[-E^(-x) + E^x,x]
```

output
```
E^(-x) + E^x
```

3.496.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.496.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$e^{-x} + e^{x}$	8
default	$e^{-x} + e^{x}$	8
risch	$e^{-x} + e^{x}$	8
parts	$e^{-x} + e^{x}$	8
meijerg	$-2 + e^{-x} + e^{x}$	9
norman	$(1 + e^{2x}) e^{-x}$	12
parallelrisch	$(1 + e^{2x}) e^{-x}$	12

input
```
int(-1/exp(x)+exp(x),x,method=_RETURNVERBOSE)
```

output
```
1/exp(x)+exp(x)
```

3.496.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \left(-e^{-x}+e^x\right)\,dx=\left(e^{(2x)}+1\right)e^{(-x)}$$

input
```
integrate(-1/exp(x)+exp(x),x, algorithm="fricas")
```

output
```
(e^(2*x) + 1)*e^(-x)
```

3.496.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)\,dx=e^x+e^{-x}$$

input
```
integrate(-1/exp(x)+exp(x),x)
```

output
```
exp(x) + exp(-x)
```

3.496.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)\,dx=e^{(-x)}+e^x$$

input
```
integrate(-1/exp(x)+exp(x),x, algorithm="maxima")
```

output
```
e^(-x) + e^x
```

3.496.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)\,dx=e^{(-x)}+e^x$$

input
```
integrate(-1/exp(x)+exp(x),x, algorithm="giac")
```

output
```
e^(-x) + e^x
```

3.496.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \left(-e^{-x}+e^x\right)\,dx=2\cosh(x)$$

input
```
int(exp(x) - exp(-x),x)
```

output
```
2*cosh(x)
```

3.497 $\int \left(-e^{-x}+e^x\right)^2 \, dx$

3.497.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \left(-e^{-x}+e^x\right)^2 \, dx=-\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x$$

output
```
-1/2/exp(2*x)+1/2*exp(2*x)-2*x
```

3.497.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^x\right)^2 \, dx=-\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x$$

input
```
Integrate[(-E^(-x) + E^x)^2,x]
```

output
```
-1/2*1/E^(2*x) + E^(2*x)/2 - 2*x
```

3.497.3 Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^2 dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-3x}\left(1-e^{2x}\right)^2 de^x$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int e^{-2x}\left(1-e^{2x}\right)^2 de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(1+e^{-2x}-2e^{-x}\right) de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-e^{-x}+e^{2x}-2\log\left(e^{2x}\right)\right)$$

input
```
Int[(-E^(-x) + E^x)^2,x]
```

output
```
(-E^(-x) + E^(2*x) - 2*Log[E^(2*x)])/2
```

3.497.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.497.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$-2x+\frac{{\mathrm e}^{2x}}{2}-\frac{{\mathrm e}^{-2x}}{2}$	17
parts	$-2x+\frac{{\mathrm e}^{2x}}{2}-\frac{{\mathrm e}^{-2x}}{2}$	17
derivativedivides	$\frac{{\mathrm e}^{2x}}{2}-\frac{{\mathrm e}^{-2x}}{2}-2\ln\left({\mathrm e}^{x}\right)$	19
default	$\frac{{\mathrm e}^{2x}}{2}-\frac{{\mathrm e}^{-2x}}{2}-2\ln\left({\mathrm e}^{x}\right)$	19
parallelrisch	$\frac{\left(-1+{\mathrm e}^{4x}-4\,{\mathrm e}^{2x}x\right){\mathrm e}^{-2x}}{2}$	20
norman	$\left(-\frac{1}{2}+\frac{{\mathrm e}^{4x}}{2}-2\,{\mathrm e}^{2x}x\right){\mathrm e}^{-2x}$	21

input
```
int((-1/exp(x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/2*exp(2*x)-1/2*exp(-2*x)
```

3.497.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \left(-e^{-x}+e^{x}\right)^{2}\,dx=-\frac{1}{2}\,\left(4\,xe^{(2\,x)}-e^{(4\,x)}+1\right)e^{(-2\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)
```

3.497.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^2\,dx=-2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$$

input
```
integrate((-1/exp(x)+exp(x))**2,x)
```

output
```
-2*x + exp(2*x)/2 - exp(-2*x)/2
```

3.497.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(-e^{-x}+e^x\right)^2\,dx=-2\,x+\frac{1}{2}\,e^{(2\,x)}-\frac{1}{2}\,e^{(-2\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^2,x, algorithm="maxima")
```

output
```
-2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

3.497.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(-e^{-x}+e^x\right)^2\,dx=\frac{1}{2}\left(2\,e^{(2\,x)}-1\right)e^{(-2\,x)}-2\,x+\frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^2,x, algorithm="giac")
```

output
```
1/2*(2*e^(2*x) - 1)*e^(-2*x) - 2*x + 1/2*e^(2*x)
```

3.497.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.36

$$\int \left(-e^{-x}+e^x\right)^2 \, dx = \sinh(2\,x) - 2\,x$$

input
```
int((exp(-x) - exp(x))^2,x)
```

output
```
sinh(2*x) - 2*x
```

3.498 $\int (-e^{-x}+e^x)^3 \, dx$

3.498.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int (-e^{-x}+e^x)^3 \, dx = \frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}$$

output
```
1/3/exp(3*x)-3/exp(x)-3*exp(x)+1/3*exp(3*x)
```

3.498.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int (-e^{-x}+e^x)^3 \, dx = \frac{1}{3}e^{-3x}(1 - 9e^{2x} - 9e^{4x} + e^{6x})$$

input
```
Integrate[(-E^(-x) + E^x)^3,x]
```

output
```
(1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))
```

3.498.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2720, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^3 \, dx$$
$$\Big\downarrow \ 2720$$
$$\int -e^{-4x}\left(1-e^{2x}\right)^3 de^x$$
$$\Big\downarrow \ 25$$
$$-\int e^{-4x}\left(1-e^{2x}\right)^3 de^x$$
$$\Big\downarrow \ 244$$
$$-\int \left(3+e^{-4x}-3e^{-2x}-e^{2x}\right) de^x$$
$$\Big\downarrow \ 2009$$
$$\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

input
```
Int[(-E^(-x) + E^x)^3,x]
```

output
```
1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

3.498.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.498.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
default	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
risch	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
parts	$\frac{e^{3x}}{3}-3\,e^{x}-3\,e^{-x}+\frac{e^{-3x}}{3}$	24
meijerg	$\frac{16}{3}+\frac{e^{-3x}}{3}-3\,e^{-x}-3\,e^{x}+\frac{e^{3x}}{3}$	25
norman	$\left(\frac{1}{3}-3\,e^{2x}-3\,e^{4x}+\frac{e^{6x}}{3}\right)e^{-3x}$	26
parallelrisch	$-\frac{\left(-e^{6x}-1+9\,e^{4x}+9\,e^{2x}\right)e^{-3x}}{3}$	27

input
```
int((-1/exp(x)+exp(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(x)^3-3*exp(x)-3/exp(x)+1/3/exp(x)^3
```

3.498.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^{x}\right)^{3}\,dx=\frac{1}{3}\,\left(e^{(6\,x)}-9\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)e^{(-3\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^3,x, algorithm="fricas")
```

output
```
1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)
```

3.498.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^3 dx = \frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3}$$

input
```
integrate((-1/exp(x)+exp(x))**3,x)
```

output
```
exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

3.498.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \left(-e^{-x}+e^x\right)^3 dx = \frac{1}{3}\,e^{(3\,x)} - 3\,e^{(-x)} + \frac{1}{3}\,e^{(-3\,x)} - 3\,e^x$$

input
```
integrate((-1/exp(x)+exp(x))^3,x, algorithm="maxima")
```

output
```
1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x
```

3.498.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \left(-e^{-x}+e^x\right)^3 dx = -\frac{1}{3}\left(9\,e^{(2\,x)} - 1\right)e^{(-3\,x)} + \frac{1}{3}\,e^{(3\,x)} - 3\,e^x$$

input
```
integrate((-1/exp(x)+exp(x))^3,x, algorithm="giac")
```

output
```
-1/3*(9*e^(2*x) - 1)*e^(-3*x) + 1/3*e^(3*x) - 3*e^x
```

3.498.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{\mathrm{e}^{-3\,x}}{3}-3\,\mathrm{e}^{-x}+\frac{\mathrm{e}^{3\,x}}{3}-3\,\mathrm{e}^{x}$$

input
```
int(-(exp(-x) - exp(x))^3,x)
```

output
```
exp(-3*x)/3 - 3*exp(-x) + exp(3*x)/3 - 3*exp(x)
```

3.499 $\int (-e^{-x}+e^x)^4\,dx$

3.499.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int (-e^{-x}+e^x)^4\,dx=-\frac{1}{4}e^{-4x}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}+6x$$

output
```
-1/4/exp(4*x)+2/exp(2*x)-2*exp(2*x)+1/4*exp(4*x)+6*x
```

3.499.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int (-e^{-x}+e^x)^4\,dx=\frac{1}{4}e^{-4x}\left(-1+8e^{2x}-8e^{6x}+e^{8x}\right)+6\log\left(e^x\right)$$

input
```
Integrate[(-E^(-x) + E^x)^4,x]
```

output
```
(-1 + 8*E^(2*x) - 8*E^(6*x) + E^(8*x))/(4*E^(4*x)) + 6*Log[E^x]
```

3.499.3 Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x - e^{-x}\right)^4 dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^{-5x}\left(1 - e^{2x}\right)^4 de^x$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int e^{-3x}\left(1 - e^{2x}\right)^4 de^{2x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(-4 + e^{-3x} - 4e^{-2x} + 6e^{-x} + e^{2x}\right) de^{2x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{e^{-2x}}{2} + 4e^{-x} - \frac{7e^{2x}}{2} + 6\log\left(e^{2x}\right)\right)$$

input
```
Int[(-E^(-x) + E^x)^4,x]
```

output
```
(-1/2*1/E^(2*x) + 4/E^x - (7*E^(2*x))/2 + 6*Log[E^(2*x)])/2
```

3.499.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.499.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
risch	$6x+\frac{{\mathrm e}^{4x}}{4}-2\,{\mathrm e}^{2x}+2\,{\mathrm e}^{-2x}-\frac{{\mathrm e}^{-4x}}{4}$	29
parts	$6x+\frac{{\mathrm e}^{4x}}{4}-2\,{\mathrm e}^{2x}+2\,{\mathrm e}^{-2x}-\frac{{\mathrm e}^{-4x}}{4}$	29
derivativedivides	$\frac{{\mathrm e}^{4x}}{4}-2\,{\mathrm e}^{2x}+2\,{\mathrm e}^{-2x}+6\ln\left({\mathrm e}^{x}\right)-\frac{{\mathrm e}^{-4x}}{4}$	31
default	$\frac{{\mathrm e}^{4x}}{4}-2\,{\mathrm e}^{2x}+2\,{\mathrm e}^{-2x}+6\ln\left({\mathrm e}^{x}\right)-\frac{{\mathrm e}^{-4x}}{4}$	31
norman	$\left(-\frac{1}{4}+2\,{\mathrm e}^{2x}-2\,{\mathrm e}^{6x}+\frac{{\mathrm e}^{8x}}{4}+6x\,{\mathrm e}^{4x}\right){\mathrm e}^{-4x}$	33
parallelrisch	$-\frac{\left(-{\mathrm e}^{8x}+1+8\,{\mathrm e}^{6x}-24\ln\left({\mathrm e}^{x}\right){\mathrm e}^{4x}-8\,{\mathrm e}^{2x}\right){\mathrm e}^{-4x}}{4}$	36

input
```
int((-1/exp(x)+exp(x))^4,x,method=_RETURNVERBOSE)
```

output
```
6*x+1/4*exp(4*x)-2*exp(2*x)+2*exp(-2*x)-1/4*exp(-4*x)
```

3.499.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(-e^{-x}+e^x\right)^4\,dx=\frac{1}{4}\,\left(24\,xe^{(4\,x)}+e^{(8\,x)}-8\,e^{(6\,x)}+8\,e^{(2\,x)}-1\right)e^{(-4\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^4,x, algorithm="fricas")
```

output
```
1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)
```

3.499.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(-e^{-x}+e^x\right)^4\,dx=6x+\frac{e^{4x}}{4}-2e^{2x}+2e^{-2x}-\frac{e^{-4x}}{4}$$

input
```
integrate((-1/exp(x)+exp(x))**4,x)
```

output
```
6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4
```

3.499.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)^4\,dx=6\,x+\frac{1}{4}\,e^{(4\,x)}-2\,e^{(2\,x)}+2\,e^{(-2\,x)}-\frac{1}{4}\,e^{(-4\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^4,x, algorithm="maxima")
```

output
```
6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)
```

3.499.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^x\right)^4\,dx=-\frac{1}{4}\left(18\,e^{(4\,x)}-8\,e^{(2\,x)}+1\right)e^{(-4\,x)}+6\,x+\frac{1}{4}\,e^{(4\,x)}-2\,e^{(2\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^4,x, algorithm="giac")
```

output
```
-1/4*(18*e^(4*x) - 8*e^(2*x) + 1)*e^(-4*x) + 6*x + 1/4*e^(4*x) - 2*e^(2*x)
```

3.499.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx=6\,x+2\,\mathrm{e}^{-2\,x}-2\,\mathrm{e}^{2\,x}-\frac{\mathrm{e}^{-4\,x}}{4}+\frac{\mathrm{e}^{4\,x}}{4}$$

input
```
int((exp(-x) - exp(x))^4,x)
```

output
```
6*x + 2*exp(-2*x) - 2*exp(2*x) - exp(-4*x)/4 + exp(4*x)/4
```

3.500 $\int (-e^{-x} + e^x)^n \, dx$

3.500.1 Optimal result

Integrand size = 13, antiderivative size = 48

$$\int (-e^{-x} + e^x)^n \, dx = -\frac{(-e^{-x} + e^x)^n (1 - e^{2x}) \text{Hypergeometric2F1}\left(1, \frac{2+n}{2}, 1 - \frac{n}{2}, e^{2x}\right)}{n}$$

output
```
-(-1/exp(x)+exp(x))^n*(1-exp(2*x))*hypergeom([1, 1+1/2*n],[1-1/2*n],exp(2*
x))/n
```

3.500.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (-e^{-x} + e^x)^n \, dx = \frac{(-e^{-x} + e^x)^n (-1 + e^{2x}) \text{Hypergeometric2F1}\left(1, 1 + \frac{n}{2}, 1 - \frac{n}{2}, e^{2x}\right)}{n}$$

input
```
Integrate[(-E^(-x) + E^x)^n,x]
```

output
```
((-E^(-x) + E^x)^n*(-1 + E^(2*x))*Hypergeometric2F1[1, 1 + n/2, 1 - n/2, E
^(2*x)])/n
```

3.500.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 1938, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^n dx$$

$$\Big\downarrow \ 2720$$

$$\int e^{-x}\left(e^x-e^{-x}\right)^n de^x$$

$$\Big\downarrow \ 1938$$

$$\left(e^x\right)^n\left(e^x-e^{-x}\right)^n\left(e^{2x}-1\right)^{-n}\int \left(e^x\right)^{-n-1}\left(-1+e^{2x}\right)^n de^x$$

$$\Big\downarrow \ 279$$

$$\left(e^x\right)^n\left(e^x-e^{-x}\right)^n\left(1-e^{2x}\right)^{-n}\int \left(e^x\right)^{-n-1}\left(1-e^{2x}\right)^n de^x$$

$$\Big\downarrow \ 278$$

$$-\frac{\left(e^x-e^{-x}\right)^n\left(1-e^{2x}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

input
```
Int[(-E^(-x) + E^x)^n,x]
```

output
```
-(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^(2*x)])/((1
- E^(2*x))^n*n))
```

3.500.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.500.4 Maple [F]

$$\int \left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)^{n} dx$$

input
```
int((-1/exp(x)+exp(x))^n,x)
```

output
```
int((-1/exp(x)+exp(x))^n,x)
```

3.500.5 Fricas [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\, dx=\int \left(-e^{(-x)}+e^{x}\right)^{n} dx$$

input
```
integrate((-1/exp(x)+exp(x))^n,x, algorithm="fricas")
```

output
```
integral((-e^(-x) + e^x)^n, x)
```

3.500.6 Sympy [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(e^{x}-e^{-x}\right)^{n}\,dx$$

input
```
integrate((-1/exp(x)+exp(x))**n,x)
```

output
```
Integral((exp(x) - exp(-x))**n, x)
```

3.500.7 Maxima [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}\,dx$$

input
```
integrate((-1/exp(x)+exp(x))^n,x, algorithm="maxima")
```

output
```
integrate((-e^(-x) + e^x)^n, x)
```

3.500.8 Giac [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}\,dx$$

input
```
integrate((-1/exp(x)+exp(x))^n,x, algorithm="giac")
```

output
```
integrate((-e^(-x) + e^x)^n, x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \left(-e^{-x}+e^x\right)^n \,dx=\int \left({\mathrm{e}}^x-{\mathrm{e}}^{-x}\right)^n \,dx$$

input
```
int((exp(x) - exp(-x))^n,x)
```

output
```
int((exp(x) - exp(-x))^n, x)
```

3.501 $\int (a^{-4x}-a^{2x})^3\,dx$

3.501.1 Optimal result

Integrand size = 15, antiderivative size = 43

$$\int (a^{-4x}-a^{2x})^3\,dx = 3x-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}$$

output
```
3*x-1/12/(a^(12*x))/ln(a)+1/2/(a^(6*x))/ln(a)-1/6*a^(6*x)/ln(a)
```

3.501.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int (a^{-4x}-a^{2x})^3\,dx = -\frac{a^{-12x}-6a^{-6x}+2a^{6x}-36\log(a^x)}{12\log(a)}$$

input
```
Integrate[(a^(-4*x) - a^(2*x))^3,x]
```

output
```
-1/12*(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*Log[a^x])/Log[a]
```

3.501.3 Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2720, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{-4x}-a^{2x}\right)^3 dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-14x}\left(1-a^{6x}\right)^3 da^{2x}}{2\log(a)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int a^{-6x}\left(1-a^{6x}\right)^3 da^{6x}}{6\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^{-6x}-3a^{-4x}+3a^{-2x}-1\right) da^{6x}}{6\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^{-4x}+3a^{-2x}-a^{6x}+3\log\left(a^{6x}\right)}{6\log(a)}$$

input
```
Int[(a^(-4*x) - a^(2*x))^3,x]
```

output
```
(-1/2*1/a^(4*x) + 3/a^(2*x) - a^(6*x) + 3*Log[a^(6*x)])/(6*Log[a])
```

3.501.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.501.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
risch	$3x-\frac{a^{6x}}{6\ln(a)}+\frac{a^{-6x}}{2\ln(a)}-\frac{a^{-12x}}{12\ln(a)}$	44
norman	$\left(-\frac{1}{12\ln(a)}+3x\,\mathrm{e}^{12x\ln(a)}+\frac{\mathrm{e}^{6x\ln(a)}}{2\ln(a)}-\frac{\mathrm{e}^{18x\ln(a)}}{6\ln(a)}\right)\mathrm{e}^{-12x\ln(a)}$	56
parallelrisch	$-\frac{\left(1-36a^{8x}a^{4x}x\ln(a)+2a^{12x}a^{6x}-6a^{2x}a^{4x}\right)a^{-12x}}{12\ln(a)}$	63

input
```
int((1/(a^(4*x))-a^(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
3*x-1/6/ln(a)*(a^(2*x))^3+1/2/ln(a)/(a^(2*x))^3-1/12/ln(a)/(a^(2*x))^6
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=\frac{36\,a^{12\,x}x\log\left(a\right)-2\,a^{18\,x}+6\,a^{6\,x}-1}{12\,a^{12\,x}\log\left(a\right)}$$

input
```
integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="fricas")
```

output
```
1/12*(36*a^(12*x)*x*log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x)*log(a))
```

3.501.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=3x+\begin{cases}\frac{-24a^{6x}\log\left(a\right)^2+72a^{-6x}\log\left(a\right)^2-12a^{-12x}\log\left(a\right)^2}{144\log\left(a\right)^3} & \text{for }\log\left(a\right)^3\neq 0\\ -3x & \text{otherwise}\end{cases}$$

input
```
integrate((1/(a**(4*x))-a**(2*x))**3,x)
```

output
```
3*x + Piecewise(((-24*a**(6*x)*log(a)**2 + 72*log(a)**2/a**(6*x) - 12*log(
a)**2/a**(12*x))/(144*log(a)**3), Ne(log(a)**3, 0)), (-3*x, True))
```

3.501.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=3\,x-\frac{a^{6\,x}}{6\,\log\left(a\right)}-\frac{1}{12\,a^{12\,x}\log\left(a\right)}+\frac{1}{2\,a^{6\,x}\log\left(a\right)}$$

input
```
integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="maxima")
```

output
```
3*x - 1/6*a^(6*x)/log(a) - 1/12/(a^(12*x)*log(a)) + 1/2/(a^(6*x)*log(a))
```

3.501.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=-\frac{2\,a^{6\,x}+\frac{9\,a^{12\,x}-6\,a^{6\,x}+1}{a^{12\,x}}-6\,\log\left(a^{6\,x}\right)}{12\,\log\left(a\right)}$$

input
```
integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="giac")
```

output
```
-1/12*(2*a^(6*x) + (9*a^(12*x) - 6*a^(6*x) + 1)/a^(12*x) - 6*log(a^(6*x)))
/log(a)
```

3.501.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=3\,x+\frac{1}{2\,a^{6\,x}\,\ln\left(a\right)}-\frac{a^{6\,x}}{6\,\ln\left(a\right)}-\frac{1}{12\,a^{12\,x}\,\ln\left(a\right)}$$

input
```
int(-(a^(2*x) - 1/a^(4*x))^3,x)
```

output
```
3*x + 1/(2*a^(6*x)*log(a)) - a^(6*x)/(6*log(a)) - 1/(12*a^(12*x)*log(a))
```

3.502 $\int \left(a^{kx}+a^{lx}\right) \, dx$

3.502.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \left(a^{kx}+a^{lx}\right) \, dx=\frac{a^{kx}}{k \log (a)}+\frac{a^{lx}}{l \log (a)}$$

output
```
a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)
```

3.502.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right) \, dx=\frac{a^{kx}}{k \log (a)}+\frac{a^{lx}}{l \log (a)}$$

input
```
Integrate[a^(k*x) + a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

3.502.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

input
```
Int[a^(k*x) + a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

3.502.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.502.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a^{kx} l+a^{lx} k}{k \ln(a) l}$	27
default	$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$	28
risch	$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$	28
parts	$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$	28
norman	$\frac{{\mathrm e}^{k x \ln(a)}}{k\ln(a)}+\frac{{\mathrm e}^{l x \ln(a)}}{l\ln(a)}$	30
meijerg	$-\frac{1-{\mathrm e}^{k x \ln(a)}}{k\ln(a)}-\frac{1-{\mathrm e}^{l x \ln(a)}}{l\ln(a)}$	40

input
```
int(a^(k*x)+a^(l*x),x,method=_RETURNVERBOSE)
```

output
```
(a^(k*x)*l+a^(l*x)*k)/k/ln(a)/l
```

3.502.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{lx}k+a^{kx}l}{kl\log\left(a\right)}$$

input
```
integrate(a^(k*x)+a^(l*x),x, algorithm="fricas")
```

output
```
(a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))
```

3.502.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\begin{cases}\frac{a^{kx}}{k\log\left(a\right)} & \text{for}\ k\log\left(a\right)\neq 0\\ x & \text{otherwise}\end{cases}+\begin{cases}\frac{a^{lx}}{l\log\left(a\right)} & \text{for}\ l\log\left(a\right)\neq 0\\ x & \text{otherwise}\end{cases}$$

input
```
integrate(a**(k*x)+a**(l*x),x)
```

output
```
Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Piecewise((
a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

3.502.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{kx}}{k\log\left(a\right)}+\frac{a^{lx}}{l\log\left(a\right)}$$

input
```
integrate(a^(k*x)+a^(l*x),x, algorithm="maxima")
```

output
```
a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

3.502.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

input
```
integrate(a^(k*x)+a^(l*x),x, algorithm="giac")
```

output
```
a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

3.502.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{k\,x}\,l+a^{l\,x}\,k}{k\,l\,\ln(a)}$$

input
```
int(a^(k*x) + a^(l*x),x)
```

output
```
(a^(k*x)*l + a^(l*x)*k)/(k*l*log(a))
```

3.503 $\int (a^{kx} + a^{lx})^2 \, dx$

3.503.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int (a^{kx} + a^{lx})^2 \, dx = \frac{a^{2kx}}{2k\log(a)} + \frac{a^{2lx}}{2l\log(a)} + \frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

output
```
1/2*a^(2*k*x)/k/ln(a)+1/2*a^(2*l*x)/l/ln(a)+2*a^((k+l)*x)/(k+l)/ln(a)
```

3.503.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int (a^{kx} + a^{lx})^2 \, dx = \frac{a^{2kx}}{2k\log(a)} + \frac{a^{2lx}}{2l\log(a)} + \frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^2,x]
```

output
```
a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k +
l)*Log[a])
```

3.503.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^2 dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int \left(a^{kx}+a^{lx}\right)^2 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(a^{2kx}+a^{2lx}+2a^{(k+l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2a^{x(k+l)}}{k+l}+\frac{a^{2kx}}{2k}+\frac{a^{2lx}}{2l}}{\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^2,x]
```

output
```
(a^(2*k*x)/(2*k) + a^(2*l*x)/(2*l) + (2*a^((k + l)*x))/(k + l))/Log[a]
```

3.503.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.503.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k\ln(a)}+\frac{a^{2lx}}{2l\ln(a)}+\frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{e^{2kx\ln(a)}}{2k\ln(a)}+\frac{e^{2lx\ln(a)}}{2l\ln(a)}+\frac{2\,e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$	59
parallelrisch	$\frac{a^{2kx}lk+a^{2kx}l^2+4a^{kx}a^{lx}kl+a^{2lx}k^2+a^{2lx}kl}{2\ln(a)kl(k+l)}$	75
meijerg	$-\frac{1-e^{2kx\ln(a)}}{2k\ln(a)}-\frac{2\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{1-e^{2lx\ln(a)}}{2l\ln(a)}$	77

input
```
int((a^(k*x)+a^(l*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2+2/ln(a)/(k+l)*a^(k*x)*a^(l
*x)
```

3.503.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx=\frac{4\,a^{kx}a^{lx}kl+(kl+l^2)a^{2\,kx}+(k^2+kl)a^{2\,lx}}{2\,(k^2l+kl^2)\log{(a)}}$$

input
```
integrate((a^(k*x)+a^(l*x))^2,x, algorithm="fricas")
```

output
```
1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*a^(2*l*x)
)/((k^2*l + k*l^2)*log(a))
```

3.503.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(41) = 82.

Time = 0.46 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.72

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx$$
$$=\begin{cases} 4x \\ \frac{a^{2lx}}{2l\log(a)}+\frac{2a^{lx}}{l\log(a)}+x \\ \frac{a^{2lx}}{2l\log(a)}+2x-\frac{a^{-2lx}}{2l\log(a)} \\ \frac{a^{2kx}}{2k\log(a)}+\frac{2a^{kx}}{k\log(a)}+x \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} \end{cases}$$

input
```
integrate((a**(k*x)+a**(l*x))**2,x)
```

output
```
Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
 (a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l
*x)/(2*l*log(a)) + 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -l)), (a**(2*k*x
)/(2*k*log(a)) + 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*
k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*
l**2*log(a)) + 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a))
 + a**(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2
*k**2*l*log(a) + 2*k*l**2*log(a)), True))
```

3.503.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx=\frac{2\,a^{kx+lx}}{(k+l)\log(a)}+\frac{a^{2\,kx}}{2\,k\log(a)}+\frac{a^{2\,lx}}{2\,l\log(a)}$$

input
```
integrate((a^(k*x)+a^(l*x))^2,x, algorithm="maxima")
```

output
```
2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x
)/(l*log(a))
```

3.503.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 691, normalized size of antiderivative = 13.04

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)+a^(l*x))^2,x, algorithm="giac")
```

output
```
(2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k
*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*
k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi
*l*x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi
*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(
a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I
*e^(I*pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a)))
 + I*e^(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(ab
s(a)))) - 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*
sgn(a) - I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I
*pi*l*sgn(a) + I*pi*l + 2*l*log(abs(a)))) + 4*(2*(k*log(abs(a)) + l*log(ab
s(a)))*cos(-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*
x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log
(abs(a)))^2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*s
gn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*
sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(
abs(a)) + l*log(abs(a)))*x) + 2*I*(I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x
*sgn(a) - 1/2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*
pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-1/2*I*pi*k*x*sg
n(a) - 1/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(...
```

3.503.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \frac{a^{2\,k\,x}}{2\,k\,\ln\left(a\right)}+\frac{\frac{a^{2\,l\,x}\,k^2}{2}+l\,\left(2\,a^{k\,x+l\,x}\,k+\frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln\left(a\right)\,\left(k+l\right)}$$

input
```
int((a^(k*x) + a^(l*x))^2,x)
```

output
```
a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 + l*(2*a^(k*x + l*x)*k + (a^(2
*l*x)*k)/2))/(k*l*log(a)*(k + l))
```

3.503. $\int \left(a^{kx} + a^{lx}\right)^2 \, dx$

3.504 $\int \left(a^{kx}+a^{lx}\right)^3 \, dx$

3.504.1 Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx=\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}$$

output
```
1/3*a^(3*k*x)/k/ln(a)+1/3*a^(3*l*x)/l/ln(a)+3*a^((2*k+l)*x)/(2*k+l)/ln(a)+
3*a^((k+2*l)*x)/(k+2*l)/ln(a)
```

3.504.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx=\frac{\frac{a^{3kx}}{k}+\frac{a^{3lx}}{l}+\frac{9a^{(2k+l)x}}{2k+l}+\frac{9a^{(k+2l)x}}{k+2l}}{3\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*
l)*x))/(k + 2*l))/(3*Log[a])
```

3.504.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int \left(a^{kx}+a^{lx}\right)^3 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \; 7293$$

$$\frac{\int \left(a^{3kx}+a^{3lx}+3a^{(2k+l)x}+3a^{(k+2l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3a^{x(2k+l)}}{2k+l}+\frac{3a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{3k}+\frac{a^{3lx}}{3l}}{\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/(3*k) + a^(3*l*x)/(3*l) + (3*a^((2*k + l)*x))/(2*k + l) + (3*a^
((k + 2*l)*x))/(k + 2*l))/Log[a]
```

3.504.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.504.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	si
risch	$\frac{a^{3kx}}{3k\ln(a)}+\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}+\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$	8
norman	$\frac{e^{3kx\ln(a)}}{3k\ln(a)}+\frac{e^{3lx\ln(a)}}{3l\ln(a)}+\frac{3\,e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)}+\frac{3\,e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$	9
meijerg	$-\frac{1-e^{3kx\ln(a)}}{3k\ln(a)}-\frac{3\left(1-e^{xk\ln(a)\left(2+\frac{l}{k}\right)}\right)}{k\ln(a)\left(2+\frac{l}{k}\right)}-\frac{3\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}-\frac{1-e^{3lx\ln(a)}}{3l\ln(a)}$	1
parallelrisch	$\frac{2a^{3kx}k^{2}l+5a^{3kx}k\,l^{2}+2a^{3kx}l^{3}+9a^{2kx}a^{lx}k^{2}l+18a^{2kx}a^{lx}k\,l^{2}+18a^{kx}a^{2lx}k^{2}l+9a^{kx}a^{2lx}k\,l^{2}+2a^{3lx}k^{3}+5a^{3lx}k^{2}l+2a^{3lx}k\,l^{2}}{3\ln(a)kl(k+2l)(2k+l)}$	1

input
```
int((a^(k*x)+a^(l*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/k/ln(a)*(a^(k*x))^3+1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a
^(l*x))^2+3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.65

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx$$
$$=\frac{9\,(2\,k^2l+kl^2)a^{kx}a^{2\,lx}+9\,(k^2l+2\,kl^2)a^{2\,kx}a^{lx}+(2\,k^2l+5\,kl^2+2\,l^3)a^{3\,kx}+(2\,k^3+5\,k^2l+2\,kl^2)a^{3\,lx}}{3\,(2\,k^3l+5\,k^2l^2+2\,kl^3)\log(a)}$$

input
```
integrate((a^(k*x)+a^(l*x))^3,x, algorithm="fricas")
```

output
```
1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*a^(2*k*x)
*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^3 + 5*k^2*l + 2*k*
l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

3.504.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(63) = 126$.

Time = 2.05 (sec) , antiderivative size = 665, normalized size of antiderivative = 8.42

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx$$
$$= \begin{cases} 8x \\ \frac{a^{3lx}}{3l\log(a)}+\frac{3a^{2lx}}{2l\log(a)}+\frac{3a^{lx}}{l\log(a)}+x \\ \frac{a^{3lx}}{3l\log(a)}+3x-\frac{a^{-3lx}}{l\log(a)}-\frac{a^{-6lx}}{6l\log(a)} \\ \frac{2a^{\frac{3lx}{2}}}{l\log(a)}+\frac{a^{3lx}}{3l\log(a)}+3x-\frac{2a^{-\frac{3lx}{2}}}{3l\log(a)} \\ \frac{a^{3kx}}{3k\log(a)}+\frac{3a^{2kx}}{2k\log(a)}+\frac{3a^{kx}}{k\log(a)}+x \\ \frac{2a^{3kx}k^2l}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{5a^{3kx}kl^2}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{2a^{3kx}l^3}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)}+\frac{}{6k^3l\log(} \end{cases}$$

input
```
integrate((a**(k*x)+a**(l*x))**3,x)
```

output
```
Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
 (a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) + 3*a**(l*x)/(l*log(
a)) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) + 3*x - 1/(a**(3*l*x)*l*log(a
)) - 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) +
 a**(3*l*x)/(3*l*log(a)) + 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -l/2))
, (a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log
(a)) + x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2
*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*
k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3
*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(2*k*x)*a**(l*x
)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a*
*(k*x)*a**(2*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) + 9*a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*lo
g(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**
2*log(a) + 6*k*l**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k*
*2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), True))
```

3.504.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log{(a)}}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log{(a)}}+\frac{a^{3\,kx}}{3\,k\log{(a)}}+\frac{a^{3\,lx}}{3\,l\log{(a)}}$$

input
```
integrate((a^(k*x)+a^(l*x))^3,x, algorithm="maxima")
```

output
```
3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a)
) + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

3.504.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1033, normalized size of antiderivative = 13.08

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((a^(k*x)+a^(l*x))^3,x, algorithm="giac")
```

output
```
2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a
))^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(
a) + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3
*k*x) + 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*l
og(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*
l*x*sgn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*a
bs(a)^(3*l*x) + I*abs(a)^(3*k*x)*(I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a
) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) + I*abs
(a)^(3*l*x)*(I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a) - 3
*I*pi*l + 6*l*log(abs(a))) - I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*x)/(-3
*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) + 6*(2*(2*k*log(abs(a)) + l*
log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)
/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l
*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi*k
*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi*
l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^((
2*k*log(abs(a)) + l*log(abs(a)))*x) + 3*I*(I*e^(I*pi*k*x*sgn(a) + 1/2*I*pi
*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a) -
2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-I*pi*k*x...
```

3.504.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx=\frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+2\,l\,\ln\left(a\right)}+\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{3\,k\,x}}{3\,k\,\ln\left(a\right)}+\frac{a^{3\,l\,x}}{3\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) + a^(l*x))^3,x)
```

output
```
(3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) + (3*a^(2*k*x)*a^(l*x))/(2*k
*log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) + a^(3*l*x)/(3*l*log(a))
```

3.505 $\int \left(a^{kx}+a^{lx}\right)^4\,dx$

3.505.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}+\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}+\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

output
```
1/4*a^(4*k*x)/k/ln(a)+1/4*a^(4*l*x)/l/ln(a)+3*a^(2*(k+l)*x)/(k+l)/ln(a)+4*
a^((3*k+l)*x)/(3*k+l)/ln(a)+4*a^((k+3*l)*x)/(k+3*l)/ln(a)
```

3.505.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\frac{\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}+\frac{12a^{2(k+l)x}}{k+l}+\frac{16a^{(3k+l)x}}{3k+l}+\frac{16a^{(k+3l)x}}{k+3l}}{4\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a^((3*k +
l)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

3.505.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^4 \, dx$$

$$\Big\downarrow \ 7281$$

$$\frac{\int \left(a^{kx}+a^{lx}\right)^4 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \ 7293$$

$$\frac{\int \left(a^{4kx}+a^{4lx}+6a^{2(k+l)x}+4a^{(3k+l)x}+4a^{(k+3l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{3a^{2x(k+l)}}{k+l}+\frac{4a^{x(3k+l)}}{3k+l}+\frac{4a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{4k}+\frac{a^{4lx}}{4l}}{\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/(4*k) + a^(4*l*x)/(4*l) + (3*a^(2*(k + l)*x))/(k + l) + (4*a^((
3*k + l)*x))/(3*k + l) + (4*a^((k + 3*l)*x))/(k + 3*l))/Log[a]
```

3.505.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.505.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
risch	$\frac{a^{4kx}}{4k\ln(a)}+\frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)}+\frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)}+\frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)}+\frac{a^{4lx}}{4l\ln(a)}$
meijerg	$-\frac{1-e^{4kx\ln(a)}}{4k\ln(a)}-\frac{4\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}-\frac{3\left(1-e^{2xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{4\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}-$
parallelrisch	$\frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}k\,l^3+3a^{4kx}l^4+16a^{3kx}a^{lx}k^3l+64a^{3kx}a^{lx}k^2l^2+48a^{3kx}a^{lx}k\,l^3+36a^{2kx}a^{2lx}k^3l+120a^{2kx}a^{2lx}k^2l}{4\ln(a)k(3k+l)(k+l)(}$

input
```
int((a^(k*x)+a^(l*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4/ln(a)/k*(a^(k*x))^4+4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/
ln(a)/(k+l)*(a^(l*x))^2+4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a
^(l*x))^4
```

3.505.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(94) = 188$.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.09

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx$$
$$=\frac{16\,(3\,k^3l+4\,k^2l^2+kl^3)a^{kx}a^{3\,lx}+12\,(3\,k^3l+10\,k^2l^2+3\,kl^3)a^{2\,kx}a^{2\,lx}+16\,(k^3l+4\,k^2l^2+3\,kl^3)a^{3\,kx}a^{lx}+}{4\,(3\,k^4l+13\,k^3l^2+13\,k^2l^3+3\,kl^4)\,\mathrm{lo}}$$

input
```
integrate((a^(k*x)+a^(l*x))^4,x, algorithm="fricas")
```

output
```
1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) + 12*(3*k^3*l + 10
*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)
*a^(3*k*x)*a^(l*x) + (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) +
 (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l
^2 + 13*k^2*l^3 + 3*k*l^4)*log(a))
```

3.505.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(82) = 164$.

Time = 16.75 (sec) , antiderivative size = 1350, normalized size of antiderivative = 13.78

$$\int \left(a^{kx} + a^{lx}\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a**(k*x)+a**(l*x))**4,x)
```

output
```
Piecewise((16*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0)))
, (a**(4*l*x)/(4*l*log(a)) + 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*l
og(a)) + 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) +
4*x - 3/(2*a**(4*l*x)*l*log(a)) - 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12
*l*x)*l*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) + 2*a**(2*l*x)/(l*
log(a)) + 6*x - 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k,
 -l)), (3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*
l*x)/(4*l*log(a)) + 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(
4*k*x)/(4*k*log(a)) + 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a))
+ 4*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*lo
g(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*
a**(4*k*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**
3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52
*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)
*l**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k
*l**4*log(a)) + 16*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*
l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 64*a**(3*k*x)*a**(
l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(
a) + 12*k*l**4*log(a)) + 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) +
 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**...
```

3.505.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a^{kx} + a^{lx}\right)^4 \, dx = \frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log(a)} + \frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log(a)} + \frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log(a)} + \frac{a^{4\,kx}}{4\,k\log(a)} + \frac{a^{4\,lx}}{4\,l\log(a)}$$

input
```
integrate((a^(k*x)+a^(l*x))^4,x, algorithm="maxima")
```

output
```
4*a^(3*k*x + l*x)/((3*k + l)*log(a)) + 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a)
) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*
a^(4*l*x)/(l*log(a))
```

3.505.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1359, normalized size of antiderivative = 13.87

$$\int \left(a^{kx}+a^{lx}\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)+a^(l*x))^4,x, algorithm="giac")
```

output
```
1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2
*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) +
1/2*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2
*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) -
1/2*I*abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(
a) - 2*I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(
-2*I*pi*k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I
*e^(2*I*pi*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(
abs(a))) + I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*p
i*l + 4*l*log(abs(a)))) + 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*
pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(
a) + pi*l*sgn(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^
2) - (3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a)
- 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(
a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*lo
g(abs(a)) + l*log(abs(a)))*x) + 4*I*(I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l
*x*sgn(a) - 3/2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a)
- 3*I*pi*k - I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-3/2*I*...
```

3.505.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int (a^{kx}+a^{lx})^4\,dx = \frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln(a)+l\,\ln(a)}+\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln(a)+3\,l\,\ln(a)} + \frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln(a)+l\,\ln(a)}+\frac{a^{4\,k\,x}}{4\,k\,\ln(a)}+\frac{a^{4\,l\,x}}{4\,l\,\ln(a)}$$

input
```
int((a^(k*x) + a^(l*x))^4,x)
```

output
```
(3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) + (4*a^(k*x)*a^(3*l*x))/(k*l
og(a) + 3*l*log(a)) + (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4
*k*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

3.506 $\int (a^{kx}+a^{lx})^n\,dx$

3.506.1 Optimal result

Integrand size = 13, antiderivative size = 72

$$\int (a^{kx}+a^{lx})^n\,dx = \frac{(1+a^{(k-l)x})(a^{kx}+a^{lx})^n\,\text{Hypergeometric2F1}\left(1,1+\frac{kn}{k-l},1+\frac{ln}{k-l},-a^{(k-l)x}\right)}{ln\log(a)}$$

output
```
(1+a^((k-l)*x))*(a^(k*x)+a^(l*x))^n*hypergeom([1, 1+k*n/(k-l)],[1+l*n/(k-l
)],-a^((k-l)*x))/l/n/ln(a)
```

3.506.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int (a^{kx}+a^{lx})^n\,dx = \frac{(a^{kx}+a^{lx})^n(1+a^{(-k+l)x})\,\text{Hypergeometric2F1}\left(1,1+n+\frac{kn}{-k+l},1+\frac{kn}{-k+l},-a^{(-k+l)x}\right)}{kn\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^n,x]
```

output
```
((a^(k*x) + a^(l*x))^n*(1 + a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (
k*n)/(-k + l), 1 + (k*n)/(-k + l), -a^((-k + l)*x)])/(k*n*Log[a])
```

3.506.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2723, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^n dx$$

$$\Big\downarrow \text{2723}$$

$$a^{-knx}\left(a^{-(x(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n \int a^{knx}\left(a^{-((k-l)x)}+1\right)^n dx$$

$$\Big\downarrow \text{2681}$$

$$\frac{\left(a^{-(x(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},-a^{-((k-l)x)}\right)}{kn\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^n,x]
```

output
```
((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(
k - l), -a^(-((k - l)*x))])/((1 + a^(-((k - l)*x)))^n*k*n*Log[a])
```

3.506.3.1 Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2723
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Simp[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n)   Int[u*F^(n*v)*(a
 + b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Inte
gerQ[n] && LinearQ[{v, w}, x]
```

3.506.4 Maple [F]

$$\int \left(a^{kx}+a^{lx}\right)^n dx$$

input
```
int((a^(k*x)+a^(l*x))^n,x)
```

output
```
int((a^(k*x)+a^(l*x))^n,x)
```

3.506.5 Fricas [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx = \int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a^(k*x)+a^(l*x))^n,x, algorithm="fricas")
```

output
```
integral((a^(k*x) + a^(l*x))^n, x)
```

3.506.6 Sympy [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx = \int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a**(k*x)+a**(l*x))**n,x)
```

output
```
Integral((a**(k*x) + a**(l*x))**n, x)
```

3.506.7 Maxima [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a^(k*x)+a^(l*x))^n,x, algorithm="maxima")
```

output
```
integrate((a^(k*x) + a^(l*x))^n, x)
```

3.506.8 Giac [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a^(k*x)+a^(l*x))^n,x, algorithm="giac")
```

output
```
integrate((a^(k*x) + a^(l*x))^n, x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{k\,x}+a^{l\,x}\right)^n\,dx$$

input
```
int((a^(k*x) + a^(l*x))^n,x)
```

output
```
int((a^(k*x) + a^(l*x))^n, x)
```

3.507 $\int (a^{kx} - a^{lx}) \, dx$

3.507.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int (a^{kx} - a^{lx}) \, dx = \frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

output
```
a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)
```

3.507.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a^{kx} - a^{lx}) \, dx = \frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

input
```
Integrate[a^(k*x) - a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

3.507.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

input
```
Int[a^(k*x) - a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

3.507.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.507.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a^{kx} l-a^{lx} k}{\ln(a) k l}$	28
default	$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$	29
risch	$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$	29
parts	$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$	29
norman	$\frac{{\mathrm e}^{k x \ln(a)}}{k\ln(a)}-\frac{{\mathrm e}^{l x \ln(a)}}{l\ln(a)}$	31
meijerg	$-\frac{1-{\mathrm e}^{k x \ln(a)}}{k\ln(a)}+\frac{1-{\mathrm e}^{l x \ln(a)}}{l\ln(a)}$	39

input
```
int(a^(k*x)-a^(l*x),x,method=_RETURNVERBOSE)
```

output
```
(a^(k*x)*l-a^(l*x)*k)/ln(a)/k/l
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx} - a^{lx}\right) \, dx = -\frac{a^{lx}k - a^{kx}l}{kl \log\left(a\right)}$$

input
```
integrate(a^(k*x)-a^(l*x),x, algorithm="fricas")
```

output
```
-(a^(l*x)*k - a^(k*x)*l)/(k*l*log(a))
```

3.507.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \left(a^{kx} - a^{lx}\right) \, dx = \begin{cases} \frac{a^{kx}}{k \log\left(a\right)} & \text{for}\: k \log\left(a\right) \neq 0 \\ x & \text{otherwise} \end{cases} - \begin{cases} \frac{a^{lx}}{l \log\left(a\right)} & \text{for}\: l \log\left(a\right) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(a**(k*x)-a**(l*x),x)
```

output
```
Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Piecewise((
a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

3.507.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx} - a^{lx}\right) \, dx = \frac{a^{kx}}{k \log\left(a\right)} - \frac{a^{lx}}{l \log\left(a\right)}$$

input
```
integrate(a^(k*x)-a^(l*x),x, algorithm="maxima")
```

output
```
a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

3.507.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx} - a^{lx}\right) dx = \frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

input
```
integrate(a^(k*x)-a^(l*x),x, algorithm="giac")
```

output
```
a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

3.507.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(a^{kx} - a^{lx}\right) dx = \frac{a^{k\,x}\,l - a^{l\,x}\,k}{k\,l\,\ln(a)}$$

input
```
int(a^(k*x) - a^(l*x),x)
```

output
```
(a^(k*x)*l - a^(l*x)*k)/(k*l*log(a))
```

3.508 $\int \left(a^{kx}-a^{lx}\right)^2 \, dx$

3.508.1 Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

output
```
1/2*a^(2*k*x)/k/ln(a)+1/2*a^(2*l*x)/l/ln(a)-2*a^((k+l)*x)/(k+l)/ln(a)
```

3.508.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^2,x]
```

output
```
a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k +
l)*Log[a])
```

3.508.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^2 dx$$
$$\Big\downarrow \ 7281$$
$$\frac{\int \left(a^{kx}-a^{lx}\right)^2 d(x\log(a))}{\log(a)}$$
$$\Big\downarrow \ 7293$$
$$\frac{\int \left(a^{2kx}+a^{2lx}-2a^{(k+l)x}\right) d(x\log(a))}{\log(a)}$$
$$\Big\downarrow \ 2009$$
$$\frac{-\frac{2a^{x(k+l)}}{k+l}+\frac{a^{2kx}}{2k}+\frac{a^{2lx}}{2l}}{\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^2,x]
```

output
```
(a^(2*k*x)/(2*k) + a^(2*l*x)/(2*l) - (2*a^((k + l)*x))/(k + l))/Log[a]
```

3.508.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.508.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k\ln(a)}+\frac{a^{2lx}}{2l\ln(a)}-\frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{e^{2kx\ln(a)}}{2k\ln(a)}+\frac{e^{2lx\ln(a)}}{2l\ln(a)}-\frac{2\,e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$	59
parallelrisch	$\frac{a^{2kx}lk+a^{2kx}l^2-4a^{kx}a^{lx}kl+a^{2lx}k^2+a^{2lx}kl}{2\ln(a)kl(k+l)}$	75
meijerg	$-\frac{1-e^{2kx\ln(a)}}{2k\ln(a)}+\frac{2-2\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{1-e^{2lx\ln(a)}}{2l\ln(a)}$	77

input
```
int((a^(k*x)-a^(l*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2-2/ln(a)/(k+l)*a^(k*x)*a^(l
*x)
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx=-\frac{4\,a^{kx}a^{lx}kl-(kl+l^2)a^{2\,kx}-(k^2+kl)a^{2\,lx}}{2\,(k^2l+kl^2)\log{(a)}}$$

input
```
integrate((a^(k*x)-a^(l*x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)*a^(2*l*x
))/((k^2*l + k*l^2)*log(a))
```

3.508.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(41) = 82$.

Time = 0.44 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.68

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx = \begin{cases} 0 \\ \frac{a^{2lx}}{2l\log(a)} - \frac{2a^{lx}}{l\log(a)} + x \\ \frac{a^{2lx}}{2l\log(a)} - 2x - \frac{a^{-2lx}}{2l\log(a)} \\ \frac{a^{2kx}}{2k\log(a)} - \frac{2a^{kx}}{k\log(a)} + x \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} - \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} \end{cases}$$

input
```
integrate((a**(k*x)-a**(l*x))**2,x)
```

output
```
Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (
a**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x
)/(2*l*log(a)) - 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -l)), (a**(2*k*x)/
(2*k*log(a)) - 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k*
*2*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l*
*2*log(a)) - 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) +
 a**(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k
**2*l*log(a) + 2*k*l**2*log(a)), True))
```

3.508.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}-a^{lx}\right)^2\,dx = -\frac{2\,a^{kx+lx}}{(k+l)\log(a)} + \frac{a^{2\,kx}}{2\,k\log(a)} + \frac{a^{2\,lx}}{2\,l\log(a)}$$

input
```
integrate((a^(k*x)-a^(l*x))^2,x, algorithm="maxima")
```

output
```
-2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*
x)/(l*log(a))
```

3.508.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 691, normalized size of antiderivative = 13.04

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)-a^(l*x))^2,x, algorithm="giac")
```

output
```
(2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k
*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*
k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi
*l*x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi
*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(
a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I
*e^(I*pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a)))
 + I*e^(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(ab
s(a)))) - 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*
sgn(a) - I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I
*pi*l*sgn(a) + I*pi*l + 2*l*log(abs(a)))) - 4*(2*(k*log(abs(a)) + l*log(ab
s(a)))*cos(-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*
x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log
(abs(a)))^2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*s
gn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*
sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(
abs(a)) + l*log(abs(a)))*x) + 2*I*(-I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*
x*sgn(a) - 1/2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I
*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-1/2*I*pi*k*x*s
gn(a) - 1/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn...
```

3.508.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx = \frac{a^{2\,k\,x}}{2\,k\,\ln\left(a\right)}+\frac{\frac{a^{2\,l\,x}\,k^2}{2}-l\,\left(2\,a^{k\,x+l\,x}\,k-\frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln\left(a\right)\,\left(k+l\right)}$$

input
```
int((a^(k*x) - a^(l*x))^2,x)
```

output
```
a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 - l*(2*a^(k*x + l*x)*k - (a^(2
*l*x)*k)/2))/(k*l*log(a)*(k + l))
```

3.508. $\int \left(a^{kx} - a^{lx}\right)^2 \, dx$

3.509 $\int (a^{kx} - a^{lx})^3 \, dx$

3.509.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int (a^{kx} - a^{lx})^3 \, dx = \frac{a^{3kx}}{3k\log(a)} - \frac{a^{3lx}}{3l\log(a)} - \frac{3a^{(2k+l)x}}{(2k+l)\log(a)} + \frac{3a^{(k+2l)x}}{(k+2l)\log(a)}$$

output
```
1/3*a^(3*k*x)/k/ln(a)-1/3*a^(3*l*x)/l/ln(a)-3*a^((2*k+l)*x)/(2*k+l)/ln(a)+
3*a^((k+2*l)*x)/(k+2*l)/ln(a)
```

3.509.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int (a^{kx} - a^{lx})^3 \, dx = \frac{\frac{a^{3kx}}{k} - \frac{a^{3lx}}{l} - \frac{9a^{(2k+l)x}}{2k+l} + \frac{9a^{(k+2l)x}}{k+2l}}{3\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*
l)*x))/(k + 2*l))/(3*Log[a])
```

3.509.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^3 dx$$

$$\Big\downarrow \ 7281$$

$$\frac{\int \left(a^{kx}-a^{lx}\right)^3 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \ 7293$$

$$\frac{\int \left(a^{3kx}-a^{3lx}-3a^{(2k+l)x}+3a^{(k+2l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{3a^{x(2k+l)}}{2k+l}+\frac{3a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{3k}-\frac{a^{3lx}}{3l}}{\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/(3*k) - a^(3*l*x)/(3*l) - (3*a^((2*k + l)*x))/(2*k + l) + (3*a^
((k + 2*l)*x))/(k + 2*l))/Log[a]
```

3.509.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.509.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	s
risch	$\frac{a^{3kx}}{3k\ln(a)}-\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}-\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$	8
norman	$\frac{e^{3kx\ln(a)}}{3k\ln(a)}-\frac{e^{3lx\ln(a)}}{3l\ln(a)}+\frac{3\,e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)}-\frac{3\,e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$	9
meijerg	$-\frac{1-e^{3kx\ln(a)}}{3k\ln(a)}+\frac{3-3\,e^{xk\ln(a)\left(2+\frac{l}{k}\right)}}{k\ln(a)\left(2+\frac{l}{k}\right)}-\frac{3\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}+\frac{1-e^{3lx\ln(a)}}{3l\ln(a)}$	1
parallelrisch	$\frac{2a^{3kx}k^{2}l+5a^{3kx}k\,l^{2}+2a^{3kx}l^{3}-9a^{2kx}a^{lx}k^{2}l-18a^{2kx}a^{lx}k\,l^{2}+18a^{kx}a^{2lx}k^{2}l+9a^{kx}a^{2lx}k\,l^{2}-2a^{3lx}k^{3}-5a^{3lx}k^{2}l-2a^{3lx}k\,l^{2}}{3\ln(a)kl(k+2l)(2k+l)}$	1

input
```
int((a^(k*x)-a^(l*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/k/ln(a)*(a^(k*x))^3-1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a
^(l*x))^2-3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \left(a^{kx}-a^{lx}\right)^{3}\,dx$$
$$=\frac{9\,(2\,k^{2}l+kl^{2})a^{kx}a^{2\,lx}-9\,(k^{2}l+2\,kl^{2})a^{2\,kx}a^{lx}+(2\,k^{2}l+5\,kl^{2}+2\,l^{3})a^{3\,kx}-(2\,k^{3}+5\,k^{2}l+2\,kl^{2})a^{3\,lx}}{3\,(2\,k^{3}l+5\,k^{2}l^{2}+2\,kl^{3})\log{(a)}}$$

input
```
integrate((a^(k*x)-a^(l*x))^3,x, algorithm="fricas")
```

output
```
1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) - 9*(k^2*l + 2*k*l^2)*a^(2*k*x)
*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) - (2*k^3 + 5*k^2*l + 2*k*
l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

3.509.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(63) = 126$.

Time = 2.05 (sec) , antiderivative size = 663, normalized size of antiderivative = 8.39

$$\int \left(a^{kx} - a^{lx}\right)^3 dx$$

$$= \begin{cases} 0 \\ -\frac{a^{3lx}}{3l\log(a)} + \frac{3a^{2lx}}{2l\log(a)} - \frac{3a^{lx}}{l\log(a)} + x \\ -\frac{a^{3lx}}{3l\log(a)} + 3x + \frac{a^{-3lx}}{l\log(a)} - \frac{a^{-6lx}}{6l\log(a)} \\ \frac{2a^{\frac{3lx}{2}}}{l\log(a)} - \frac{a^{3lx}}{3l\log(a)} - 3x - \frac{2a^{-\frac{3lx}{2}}}{3l\log(a)} \\ \frac{a^{3kx}}{3k\log(a)} - \frac{3a^{2kx}}{2k\log(a)} + \frac{3a^{kx}}{k\log(a)} - x \\ \frac{2a^{3kx}k^2l}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)} + \frac{5a^{3kx}kl^2}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)} + \frac{2a^{3kx}l^3}{6k^3l\log(a)+15k^2l^2\log(a)+6kl^3\log(a)} - \frac{}{6k^3l\log(} \end{cases}$$

input
```
integrate((a**(k*x)-a**(l*x))**3,x)
```

output
```
Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (
-a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3*a**(l*x)/(l*log(a
)) + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) + 3*x + 1/(a**(3*l*x)*l*log(a
)) - 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) -
 a**(3*l*x)/(3*l*log(a)) - 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -l/2))
, (a**(3*k*x)/(3*k*log(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log
(a)) - x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2
*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*
k**2*l**2*log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3
*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 18*a**(2*k*x)*a**(l*x
)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a*
*(k*x)*a**(2*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) + 9*a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*lo
g(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**
2*log(a) + 6*k*l**3*log(a)) - 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k*
*2*l**2*log(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), True))
```

3.509.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(a^{kx} - a^{lx}\right)^3 \, dx = -\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log(a)} + \frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log(a)} + \frac{a^{3\,kx}}{3\,k\log(a)} - \frac{a^{3\,lx}}{3\,l\log(a)}$$

input
```
integrate((a^(k*x)-a^(l*x))^3,x, algorithm="maxima")
```

output
```
-3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a
)) + 1/3*a^(3*k*x)/(k*log(a)) - 1/3*a^(3*l*x)/(l*log(a))
```

3.509.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1033, normalized size of antiderivative = 13.08

$$\int \left(a^{kx} - a^{lx}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)-a^(l*x))^3,x, algorithm="giac")
```

output
```
2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a
))^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(
a) + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3
*k*x) - 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*l
og(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*
l*x*sgn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*a
bs(a)^(3*l*x) + I*abs(a)^(3*k*x)*(I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a
) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) + I*abs
(a)^(3*l*x)*(-I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a) -
3*I*pi*l + 6*l*log(abs(a))) + I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*x)/(-
3*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) - 6*(2*(2*k*log(abs(a)) + l
*log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x
)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) +
l*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi*
k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi
*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^(
(2*k*log(abs(a)) + l*log(abs(a)))*x) + 3*I*(-I*e^(I*pi*k*x*sgn(a) + 1/2*I*
pi*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a)
- 2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-I*pi*k...
```

3.509.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx=\frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+2\,l\,\ln\left(a\right)}-\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{3\,k\,x}}{3\,k\,\ln\left(a\right)}-\frac{a^{3\,l\,x}}{3\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) - a^(l*x))^3,x)
```

output
```
(3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) - (3*a^(2*k*x)*a^(l*x))/(2*k
*log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) - a^(3*l*x)/(3*l*log(a))
```

3.510 $\int \left(a^{kx}-a^{lx}\right)^4 dx$

3.510.1 Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \left(a^{kx}-a^{lx}\right)^4 dx=\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}-\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}-\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

output
```
1/4*a^(4*k*x)/k/ln(a)+1/4*a^(4*l*x)/l/ln(a)+3*a^(2*(k+l)*x)/(k+l)/ln(a)-4*
a^((3*k+l)*x)/(3*k+l)/ln(a)-4*a^((k+3*l)*x)/(k+3*l)/ln(a)
```

3.510.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}-a^{lx}\right)^4 dx=\frac{\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}+\frac{12a^{2(k+l)x}}{k+l}-\frac{16a^{(3k+l)x}}{3k+l}-\frac{16a^{(k+3l)x}}{k+3l}}{4\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a^((3*k +
l)*x))/(3*k + l) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

3.510.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^4 dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int \left(a^{kx}-a^{lx}\right)^4 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(a^{4kx}+a^{4lx}+6a^{2(k+l)x}-4a^{(3k+l)x}-4a^{(k+3l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3a^{2x(k+l)}}{k+l}-\frac{4a^{x(3k+l)}}{3k+l}-\frac{4a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{4k}+\frac{a^{4lx}}{4l}}{\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/(4*k) + a^(4*l*x)/(4*l) + (3*a^(2*(k + l)*x))/(k + l) - (4*a^((
3*k + l)*x))/(3*k + l) - (4*a^((k + 3*l)*x))/(k + 3*l))/Log[a]
```

3.510.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.510.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
risch	$\frac{a^{4kx}}{4k\ln(a)} - \frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)} + \frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)} - \frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)} + \frac{a^{4lx}}{4l\ln(a)}$
meijerg	$-\frac{1-e^{4kx\ln(a)}}{4k\ln(a)} + \frac{4-4\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)} - \frac{3\left(1-e^{2xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)} + \frac{4-4\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)} - \frac{1-e^{4l}}{4l\ln}$
parallelrisch	$\frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}k\,l^3+3a^{4kx}l^4-16a^{3kx}a^{lx}k^3l-64a^{3kx}a^{lx}k^2l^2-48a^{3kx}a^{lx}k\,l^3+36a^{2kx}a^{2lx}k^3l+120a^{2kx}a^{2lx}k^2l}{4\ln(a)k(3k+l)(k+l)(}$

input
```
int((a^(k*x)-a^(l*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/
ln(a)/(k+l)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a
^(l*x))^4
```

3.510.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(94) = 188.

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.11

$$\int \left(a^{kx} - a^{lx}\right)^4 \, dx = -\frac{16\,(3\,k^3l + 4\,k^2l^2 + kl^3)a^{kx}a^{3\,lx} - 12\,(3\,k^3l + 10\,k^2l^2 + 3\,kl^3)a^{2\,kx}a^{2\,lx} + 16\,(k^3l + 4\,k^2l^2 + 3\,kl^3)a^{3\,kx}a^{lx}}{4\,(3\,k^4l + 13\,k^3l^2 + 13\,k^2l^3 + 3\,kl^4}$$

input
```
integrate((a^(k*x)-a^(l*x))^4,x, algorithm="fricas")
```

output
```
-1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) - 12*(3*k^3*l + 1
0*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3
)*a^(3*k*x)*a^(l*x) - (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x)
- (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*
l^2 + 13*k^2*l^3 + 3*k*l^4)*log(a))
```

3.510.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(82) = 164$.

Time = 16.58 (sec) , antiderivative size = 1348, normalized size of antiderivative = 13.76

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a**(k*x)-a**(l*x))**4,x)
```

output
```
Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (
a**(4*l*x)/(4*l*log(a)) - 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*log(
a)) - 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) - 4*x
 - 3/(2*a**(4*l*x)*l*log(a)) + 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12*l*
x)*l*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) - 2*a**(2*l*x)/(l*log
(a)) + 6*x + 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k, -l
)), (-3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*l*
x)/(4*l*log(a)) - 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(4*
k*x)/(4*k*log(a)) - 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a)) -
4*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*log(
a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a*
*(4*k*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*
log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52*k
**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)*l
**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l
**4*log(a)) - 16*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l*
*2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) - 64*a**(3*k*x)*a**(l*
x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a)
 + 12*k*l**4*log(a)) - 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) + 5
2*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2...
```

3.510.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx = -\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log{(a)}} - \frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log{(a)}} + \frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log{(a)}} + \frac{a^{4\,kx}}{4\,k\log{(a)}} + \frac{a^{4\,lx}}{4\,l\log{(a)}}$$

input
```
integrate((a^(k*x)-a^(l*x))^4,x, algorithm="maxima")
```

output
```
-4*a^(3*k*x + l*x)/((3*k + l)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a
)) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4
*a^(4*l*x)/(l*log(a))
```

3.510.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 1359, normalized size of antiderivative = 13.87

$$\int \left(a^{kx} - a^{lx}\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)-a^(l*x))^4,x, algorithm="giac")
```

output
```
1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2
*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) +
1/2*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2
*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) -
1/2*I*abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(
a) - 2*I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(
-2*I*pi*k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I
*e^(2*I*pi*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(
abs(a))) + I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*p
i*l + 4*l*log(abs(a)))) - 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*
pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(
a) + pi*l*sgn(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^
2) - (3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a)
- 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(
a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*lo
g(abs(a)) + l*log(abs(a)))*x) + 4*I*(-I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*
l*x*sgn(a) - 3/2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a)
 - 3*I*pi*k - I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-3/2*I...
```

3.510.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx=\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+l\,\ln\left(a\right)}-\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln\left(a\right)+3\,l\,\ln\left(a\right)}-\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{4\,k\,x}}{4\,k\,\ln\left(a\right)}+\frac{a^{4\,l\,x}}{4\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) - a^(l*x))^4,x)
```

output
```
(3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) - (4*a^(k*x)*a^(3*l*x))/(k*l
og(a) + 3*l*log(a)) - (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4
*k*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

3.511 $\int \left(a^{kx}-a^{lx}\right)^n\,dx$

3.511.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \left(a^{kx}-a^{lx}\right)^n\,dx = \frac{\left(1-a^{(k-l)x}\right)\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(1,1+\frac{kn}{k-l},1+\frac{ln}{k-l},a^{(k-l)x}\right)}{ln\log(a)}$$

output
```
(1-a^((k-l)*x))*(a^(k*x)-a^(l*x))^n*hypergeom([1, 1+k*n/(k-l)],[1+l*n/(k-l
)],a^((k-l)*x))/l/n/ln(a)
```

3.511.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}-a^{lx}\right)^n\,dx = \frac{\left(a^{kx}-a^{lx}\right)^n\left(1-a^{(-k+l)x}\right) \text{Hypergeometric2F1}\left(1,1+n+\frac{kn}{-k+l},1+\frac{kn}{-k+l},a^{(-k+l)x}\right)}{kn\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^n,x]
```

output
```
((a^(k*x) - a^(l*x))^n*(1 - a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (
k*n)/(-k + l), 1 + (k*n)/(-k + l), a^((-k + l)*x)])/(k*n*Log[a])
```

3.511.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2723, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^n dx$$

$$\Big\downarrow \text{2723}$$

$$a^{-knx}\left(1-a^{-(x(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n \int a^{knx}\left(1-a^{-((k-l)x)}\right)^n dx$$

$$\Big\downarrow \text{2681}$$

$$\frac{\left(1-a^{-(x(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},a^{-((k-l)x)}\right)}{kn\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^n,x]
```

output
```
((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(
k - l), a^(-((k - l)*x))])/((1 - a^(-((k - l)*x)))^n*k*n*Log[a])
```

3.511.3.1 Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2723
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Simp[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n)   Int[u*F^(n*v)*(a
 + b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Inte
gerQ[n] && LinearQ[{v, w}, x]
```

3.511.4 Maple [F]

$$\int \left(a^{kx}-a^{lx}\right)^n dx$$

input
```
int((a^(k*x)-a^(l*x))^n,x)
```

output
```
int((a^(k*x)-a^(l*x))^n,x)
```

3.511.5 Fricas [F]

$$\int \left(a^{kx}-a^{lx}\right)^n dx = \int \left(a^{kx}-a^{lx}\right)^n dx$$

input
```
integrate((a^(k*x)-a^(l*x))^n,x, algorithm="fricas")
```

output
```
integral((a^(k*x) - a^(l*x))^n, x)
```

3.511.6 Sympy [F]

$$\int \left(a^{kx}-a^{lx}\right)^n dx = \int \left(a^{kx}-a^{lx}\right)^n dx$$

input
```
integrate((a**(k*x)-a**(l*x))**n,x)
```

output
```
Integral((a**(k*x) - a**(l*x))**n, x)
```

3.511.7 Maxima [F]

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{kx} - a^{lx}\right)^n \, dx$$

input
```
integrate((a^(k*x)-a^(l*x))^n,x, algorithm="maxima")
```

output
```
integrate((a^(k*x) - a^(l*x))^n, x)
```

3.511.8 Giac [F]

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{kx} - a^{lx}\right)^n \, dx$$

input
```
integrate((a^(k*x)-a^(l*x))^n,x, algorithm="giac")
```

output
```
integrate((a^(k*x) - a^(l*x))^n, x)
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx = \int \left(a^{k\,x} - a^{l\,x}\right)^n \, dx$$

input
```
int((a^(k*x) - a^(l*x))^n,x)
```

output
```
int((a^(k*x) - a^(l*x))^n, x)
```

3.512 $\int (1+a^{mx})\,dx$

3.512.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int (1+a^{mx})\,dx = x + \frac{a^{mx}}{m\log(a)}$$

output
```
x+a^(m*x)/m/ln(a)
```

3.512.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1+a^{mx})\,dx = x + \frac{a^{mx}}{m\log(a)}$$

input
```
Integrate[1 + a^(m*x),x]
```

output
```
x + a^(m*x)/(m*Log[a])
```

3.512.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)\, dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^{mx}}{m\log(a)}+x$$

input
```
Int[1 + a^(m*x),x]
```

output
```
x + a^(m*x)/(m*Log[a])
```

3.512.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.512.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$x+\frac{a^{mx}}{m\ln(a)}$	16
risch	$x+\frac{a^{mx}}{m\ln(a)}$	16
parallelrisch	$x+\frac{a^{mx}}{m\ln(a)}$	16
parts	$x+\frac{a^{mx}}{m\ln(a)}$	16
norman	$x+\frac{e^{mx\ln(a)}}{m\ln(a)}$	17
derivativedivides	$\frac{a^{mx}+\ln(a^{mx})}{m\ln(a)}$	21

input
```
int(1+a^(m*x),x,method=_RETURNVERBOSE)
```

output
```
x+a^(m*x)/m/ln(a)
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int (1 + a^{mx})\,dx = \frac{mx\log(a) + a^{mx}}{m\log(a)}$$

input
```
integrate(1+a^(m*x),x, algorithm="fricas")
```

output
```
(m*x*log(a) + a^(m*x))/(m*log(a))
```

3.512.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\,dx = x + \begin{cases} \frac{a^{mx}}{m\log(a)} & \text{for } m\log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(1+a**(m*x),x)
```

output
```
x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))
```

3.512.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\,dx = x + \frac{a^{mx}}{m\log(a)}$$

input
```
integrate(1+a^(m*x),x, algorithm="maxima")
```

output
```
x + a^(m*x)/(m*log(a))
```

3.512.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1+a^{mx})\ dx = x + \frac{a^{mx}}{m\log(a)}$$

input
```
integrate(1+a^(m*x),x, algorithm="giac")
```

output
```
x + a^(m*x)/(m*log(a))
```

3.512.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1+a^{mx})\ dx = x + \frac{a^{m\,x}}{m\ \ln(a)}$$

input
```
int(a^(m*x) + 1,x)
```

output
```
x + a^(m*x)/(m*log(a))
```

3.513 $\int (1 + a^{mx})^2 \, dx$

3.513.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int (1 + a^{mx})^2 \, dx = x + \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)}$$

output
```
x+2*a^(m*x)/m/ln(a)+1/2*a^(2*m*x)/m/ln(a)
```

3.513.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (1 + a^{mx})^2 \, dx = \frac{\frac{a^{mx}(4+a^{mx})}{2m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^2,x]
```

output
```
((a^(m*x)*(4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]
```

3.513.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^2\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^2\,da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int (a^{-mx}+a^{mx}+2)\,da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{mx}+\frac{1}{2}a^{2mx}+\log(a^{mx})}{m\log(a)}$$

input
```
Int[(1 + a^(m*x))^2,x]
```

output
```
(2*a^(m*x) + a^(2*m*x)/2 + Log[a^(m*x)])/(m*Log[a])
```

3.513.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.513.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
parallelrisch	$\frac{2mx\ln(a)+a^{2mx}+4a^{mx}}{2\ln(a)m}$	31
derivativedivides	$\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
risch	$x+\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$	33
norman	$x+\frac{2\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}$	35

input
```
int((1+a^(m*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*m*x*ln(a)+(a^(m*x))^2+4*a^(m*x))/ln(a)/m
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1+a^{mx})^2\,dx=\frac{2\,mx\log{(a)}+a^{2\,mx}+4\,a^{mx}}{2\,m\log{(a)}}$$

input
```
integrate((1+a^(m*x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

3.513.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (1+a^{mx})^2\,dx = x + \begin{cases} \frac{a^{2mx}m\log(a)+4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } m^2\log(a)^2 \neq 0 \\ 3x & \text{otherwise} \end{cases}$$

input
```
integrate((1+a**(m*x))**2,x)
```

output
```
x + Piecewise(((a**(2*m*x)*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**2*log(a)*
*2), Ne(m**2*log(a)**2, 0)), (3*x, True))
```

3.513.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int (1+a^{mx})^2\,dx = x + \frac{a^{2\,mx}}{2\,m\log(a)} + \frac{2\,a^{mx}}{m\log(a)}$$

input
```
integrate((1+a^(m*x))^2,x, algorithm="maxima")
```

output
```
x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))
```

3.513.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (1+a^{mx})^2\,dx = \frac{2\,mx\log(|a|) + a^{2\,mx} + 4\,a^{mx}}{2\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^2,x, algorithm="giac")
```

output
```
1/2*(2*m*x*log(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

3.513.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (1+a^{mx})^2 \, dx = x + \frac{2\,a^{m\,x} + \frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

input
```
int((a^(m*x) + 1)^2,x)
```

output
```
x + (2*a^(m*x) + a^(2*m*x)/2)/(m*log(a))
```

3.514 $\int (1+a^{mx})^3\, dx$

3.514.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (1+a^{mx})^3\, dx = x+\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}+\frac{a^{3mx}}{3m\log(a)}$$

output
```
x+3*a^(m*x)/m/ln(a)+3/2*a^(2*m*x)/m/ln(a)+1/3*a^(3*m*x)/m/ln(a)
```

3.514.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (1+a^{mx})^3\, dx = \frac{\frac{a^{mx}(18+9a^{mx}+2a^{2mx})}{6m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^3,x]
```

output
```
((a^(m*x)*(18 + 9*a^(m*x) + 2*a^(2*m*x)))/(6*m) + Log[a^(m*x)]/m)/Log[a]
```

3.514.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^3 \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^3 \, da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^{-mx}+3a^{mx}+a^{2mx}+3\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3a^{mx}+\frac{3}{2}a^{2mx}+\frac{1}{3}a^{3mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 + a^(m*x))^3,x]
```

output
```
(3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3 + Log[a^(m*x)])/(m*Log[a])
```

3.514.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.514.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
default	$\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
parallelrisch	$\frac{2a^{3mx}+6mx\ln(a)+9a^{2mx}+18a^{mx}}{6\ln(a)m}$	42
risch	$x+\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}+\frac{a^{3mx}}{3m\ln(a)}$	49
norman	$x+\frac{3\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}+\frac{\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}$	52

input
```
int((1+a^(m*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(1/3*(a^(m*x))^3+3/2*(a^(m*x))^2+3*a^(m*x)+ln(a^(m*x)))
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1+a^{mx})^3\,dx=\frac{6\,mx\log(a)+2\,a^{3\,mx}+9\,a^{2\,mx}+18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^3,x, algorithm="fricas")
```

output
```
1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

3.514.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int (1+a^{mx})^3\,dx = x + \begin{cases} \frac{2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2+18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } m^3\log(a)^3 \neq 0 \\ 7x & \text{otherwise} \end{cases}$$

input
```
integrate((1+a**(m*x))**3,x)
```

output
```
x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
+ 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)),
(7*x, True))
```

3.514.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (1+a^{mx})^3\,dx = x + \frac{a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{2\,m\log(a)} + \frac{3\,a^{mx}}{m\log(a)}$$

input
```
integrate((1+a^(m*x))^3,x, algorithm="maxima")
```

output
```
x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*x)/(m*log
(a))
```

3.514.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (1+a^{mx})^3\,dx = \frac{6\,mx\log(|a|) + 2\,a^{3\,mx} + 9\,a^{2\,mx} + 18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^3,x, algorithm="giac")
```

output
```
1/6*(6*m*x*log(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a)
)
```

3.514.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int (1+a^{mx})^3\,dx = x + \frac{3\,a^{m\,x} + \frac{3\,a^{2\,m\,x}}{2} + \frac{a^{3\,m\,x}}{3}}{m\,\ln(a)}$$

input
```
int((a^(m*x) + 1)^3,x)
```

output
```
x + (3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

3.515 $\int (1 + a^{mx})^4 \, dx$

3.515.1 Optimal result

Integrand size = 9, antiderivative size = 65

$$\int (1 + a^{mx})^4 \, dx = x + \frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} + \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)}$$

output
```
x+4*a^(m*x)/m/ln(a)+3*a^(2*m*x)/m/ln(a)+4/3*a^(3*m*x)/m/ln(a)+1/4*a^(4*m*x
)/m/ln(a)
```

3.515.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (1 + a^{mx})^4 \, dx = \frac{\frac{a^{mx}(48 + 36a^{mx} + 16a^{2mx} + 3a^{3mx})}{12m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^4,x]
```

output
```
((a^(m*x)*(48 + 36*a^(m*x) + 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^(
m*x)]/m)/Log[a]
```

3.515.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^4\, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^4\, da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^{-mx}+6a^{mx}+4a^{2mx}+a^{3mx}+4\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a^{mx}+3a^{2mx}+\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 + a^(m*x))^4,x]
```

output
```
(4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4 + Log[a^(m*x)])/(
m*Log[a])
```

3.515.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.515.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
parallelrisch	$\frac{3a^{4mx}+16a^{3mx}+12mx\ln(a)+36a^{2mx}+48a^{mx}}{12\ln(a)m}$	51
risch	$x+\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}+\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$	65
norman	$x+\frac{4\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{m\ln(a)}+\frac{4\,\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}+\frac{\mathrm{e}^{4mx\ln(a)}}{4m\ln(a)}$	69

input
```
int((1+a^(m*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(1/4*(a^(m*x))^4+4/3*(a^(m*x))^3+3*(a^(m*x))^2+4*a^(m*x)+ln(a^(m
*x)))
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1+a^{mx})^4 \, dx = \frac{12\,mx\log(a)+3\,a^{4\,mx}+16\,a^{3\,mx}+36\,a^{2\,mx}+48\,a^{mx}}{12\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^4,x, algorithm="fricas")
```

output
```
1/12*(12*m*x*log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*
x))/(m*log(a))
```

3.515.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (1+a^{mx})^4 \, dx = x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3+16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3+48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } m^4\log(a)^4 \neq 0 \\ 15x & \text{otherwise} \end{cases}$$

input
```
integrate((1+a**(m*x))**4,x)
```

output
```
x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 + 16*a**(3*m*x)*m**3*log(a)**3
 + 36*a**(2*m*x)*m**3*log(a)**3 + 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log
(a)**4), Ne(m**4*log(a)**4, 0)), (15*x, True))
```

3.515.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (1+a^{mx})^4 \, dx = x + \frac{a^{4mx}}{4m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{mx}}{m\log(a)}$$

input
```
integrate((1+a^(m*x))^4,x, algorithm="maxima")
```

output
```
x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*l
og(a)) + 4*a^(m*x)/(m*log(a))
```

3.515.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int (1+a^{mx})^4 \, dx = \frac{12mx\log(|a|) + 3a^{4mx} + 16a^{3mx} + 36a^{2mx} + 48a^{mx}}{12m\log(a)}$$

input
```
integrate((1+a^(m*x))^4,x, algorithm="giac")
```

output
```
1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*
a^(m*x))/(m*log(a))
```

3.515.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int (1 + a^{mx})^4 \, dx = x + \frac{4\,a^{m\,x} + 3\,a^{2\,m\,x} + \frac{4\,a^{3\,m\,x}}{3} + \frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

input
```
int((a^(m*x) + 1)^4,x)
```

output
```
x + (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4)/(m*log(a))
```

3.516 $\int (1 + a^{mx})^n \, dx$

3.516.1 Optimal result

Integrand size = 9, antiderivative size = 40

$$\int (1 + a^{mx})^n \, dx = -\frac{(1 + a^{mx})^{1+n} \text{Hypergeometric2F1}(1, 1 + n, 2 + n, 1 + a^{mx})}{m(1 + n) \log(a)}$$

output
```
-(1+a^(m*x))^(1+n)*hypergeom([1, 1+n],[2+n],1+a^(m*x))/m/(1+n)/ln(a)
```

3.516.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})^n \, dx = -\frac{(1 + a^{mx})^{1+n} \text{Hypergeometric2F1}(1, 1 + n, 2 + n, 1 + a^{mx})}{m(1 + n) \log(a)}$$

input
```
Integrate[(1 + a^(m*x))^n,x]
```

output
```
-(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/
(m*(1 + n)*Log[a]))
```

3.516.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^n\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int a^{-mx}(a^{mx}+1)^n\,da^{mx}}{m\log(a)}$$

$$\Big\downarrow \ 75$$

$$-\frac{(a^{mx}+1)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,a^{mx}+1\right)}{m(n+1)\log(a)}$$

input
```
Int[(1 + a^(m*x))^n,x]
```

output
```
-(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/
(m*(1 + n)*Log[a]))
```

3.516.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.516.4 Maple [F]

$$\int (1+a^{mx})^n \, dx$$

input
```
int((1+a^(m*x))^n,x)
```

output
```
int((1+a^(m*x))^n,x)
```

3.516.5 Fricas [F]

$$\int (1+a^{mx})^n \, dx = \int (a^{mx}+1)^n \, dx$$

input
```
integrate((1+a^(m*x))^n,x, algorithm="fricas")
```

output
```
integral((a^(m*x) + 1)^n, x)
```

3.516.6 Sympy [F]

$$\int (1+a^{mx})^n \, dx = \int (a^{mx}+1)^n \, dx$$

input
```
integrate((1+a**(m*x))**n,x)
```

output
```
Integral((a**(m*x) + 1)**n, x)
```

3.516.7 Maxima [F]

$$\int (1+a^{mx})^n \, dx = \int (a^{mx}+1)^n \, dx$$

input
```
integrate((1+a^(m*x))^n,x, algorithm="maxima")
```

output
```
integrate((a^(m*x) + 1)^n, x)
```

3.516.8 Giac [F]

$$\int (1+a^{mx})^n \, dx = \int (a^{mx}+1)^n \, dx$$

input
```
integrate((1+a^(m*x))^n,x, algorithm="giac")
```

output
```
integrate((a^(m*x) + 1)^n, x)
```

3.516.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int (1+a^{mx})^n \, dx = \frac{(a^{m\,x}+1)^n \, {}_2\mathrm{F}_1\left(-n,-n;\ 1-n;\ -\frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\,\left(\frac{1}{a^{m\,x}}+1\right)^n}$$

input
```
int((a^(m*x) + 1)^n,x)
```

output
```
((a^(m*x) + 1)^n*hypergeom([-n, -n], 1 - n, -1/a^(m*x)))/(m*n*log(a)*(1/a^
(m*x) + 1)^n)
```

3.517 $\int (1 - a^{mx}) \, dx$

3.517.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (1 - a^{mx}) \, dx = x - \frac{a^{mx}}{m \log(a)}$$

output
```
x-a^(m*x)/m/ln(a)
```

3.517.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx}) \, dx = x - \frac{a^{mx}}{m \log(a)}$$

input
```
Integrate[1 - a^(m*x),x]
```

output
```
x - a^(m*x)/(m*Log[a])
```

3.517.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1 - a^{mx})\, dx$$
$$\Big\downarrow \text{ 2009}$$
$$x - \frac{a^{mx}}{m \log(a)}$$

input
```
Int[1 - a^(m*x),x]
```

output
```
x - a^(m*x)/(m*Log[a])
```

3.517.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.517.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$x - \frac{a^{mx}}{m \ln(a)}$	17
risch	$x - \frac{a^{mx}}{m \ln(a)}$	17
parallelrisch	$x - \frac{a^{mx}}{m \ln(a)}$	17
parts	$x - \frac{a^{mx}}{m \ln(a)}$	17
norman	$x - \frac{{\rm e}^{mx \ln(a)}}{m \ln(a)}$	18
derivativedivides	$\frac{-a^{mx}+\ln(a^{mx})}{m \ln(a)}$	23

input
```
int(1-a^(m*x),x,method=_RETURNVERBOSE)
```

output
```
x-a^(m*x)/m/ln(a)
```

3.517.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (1 - a^{mx})\, dx = \frac{mx\log(a) - a^{mx}}{m\log(a)}$$

input
```
integrate(1-a^(m*x),x, algorithm="fricas")
```

output
```
(m*x*log(a) - a^(m*x))/(m*log(a))
```

3.517.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (1 - a^{mx})\, dx = x + \begin{cases} -\frac{a^{mx}}{m\log(a)} & \text{for } m\log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate(1-a**(m*x),x)
```

output
```
x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True))
```

3.517.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

input
```
integrate(1-a^(m*x),x, algorithm="maxima")
```

output
```
x - a^(m*x)/(m*log(a))
```

3.517.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m \log(a)}$$

input
```
integrate(1-a^(m*x),x, algorithm="giac")
```

output
```
x - a^(m*x)/(m*log(a))
```

3.517.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{m\,x}}{m\,\ln(a)}$$

input
```
int(1 - a^(m*x),x)
```

output
```
x - a^(m*x)/(m*log(a))
```

3.518 $\int (1 - a^{mx})^2 \, dx$

3.518.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (1 - a^{mx})^2 \, dx = x - \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)}$$

output
```
x-2*a^(m*x)/m/ln(a)+1/2*a^(2*m*x)/m/ln(a)
```

3.518.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (1 - a^{mx})^2 \, dx = \frac{\frac{a^{mx}(-4+a^{mx})}{2m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^2,x]
```

output
```
((a^(m*x)*(-4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]
```

3.518.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^2 \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^2 da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int (a^{-mx}+a^{mx}-2) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-2a^{mx}+\frac{1}{2}a^{2mx}+\log(a^{mx})}{m\log(a)}$$

input
```
Int[(1 - a^(m*x))^2,x]
```

output
```
(-2*a^(m*x) + a^(2*m*x)/2 + Log[a^(m*x)])/(m*Log[a])
```

3.518.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.518.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
parallelrisch	$\frac{2mx\ln(a)+a^{2mx}-4a^{mx}}{2\ln(a)m}$	31
derivativedivides	$\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$	32
risch	$x-\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$	33
norman	$x-\frac{2\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}$	35

input
```
int((1-a^(m*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*m*x*ln(a)+(a^(m*x))^2-4*a^(m*x))/ln(a)/m
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1-a^{mx})^2\,dx=\frac{2\,mx\log(a)+a^{2\,mx}-4\,a^{mx}}{2\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

3.518.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (1-a^{mx})^2\,dx = x + \begin{cases} \frac{a^{2mx}m\log(a)-4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } m^2\log(a)^2 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate((1-a**(m*x))**2,x)
```

output
```
x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**2*log(a)*
*2), Ne(m**2*log(a)**2, 0)), (-x, True))
```

3.518.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int (1-a^{mx})^2\,dx = x + \frac{a^{2\,mx}}{2\,m\log(a)} - \frac{2\,a^{mx}}{m\log(a)}$$

input
```
integrate((1-a^(m*x))^2,x, algorithm="maxima")
```

output
```
x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))
```

3.518.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (1-a^{mx})^2\,dx = \frac{2\,mx\log(|a|) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^2,x, algorithm="giac")
```

output
```
1/2*(2*m*x*log(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

3.518.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (1-a^{mx})^2 \, dx = x - \frac{2\,a^{m\,x} - \frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

input
```
int((a^(m*x) - 1)^2,x)
```

output
```
x - (2*a^(m*x) - a^(2*m*x)/2)/(m*log(a))
```

3.519 $\int (1-a^{mx})^3 \, dx$

3.519.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int (1-a^{mx})^3 \, dx = x - \frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} - \frac{a^{3mx}}{3m\log(a)}$$

output
```
x-3*a^(m*x)/m/ln(a)+3/2*a^(2*m*x)/m/ln(a)-1/3*a^(3*m*x)/m/ln(a)
```

3.519.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (1-a^{mx})^3 \, dx = \frac{-\frac{a^{mx}(18-9a^{mx}+2a^{2mx})}{6m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^3,x]
```

output
```
(-1/6*(a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/m + Log[a^(m*x)]/m)/Log[a]
```

3.519.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^3 \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^3 \, da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^{-mx}+3a^{mx}-a^{2mx}-3\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-3a^{mx}+\frac{3}{2}a^{2mx}-\frac{1}{3}a^{3mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 - a^(m*x))^3,x]
```

output
```
(-3*a^(m*x) + (3*a^(2*m*x))/2 - a^(3*m*x)/3 + Log[a^(m*x)])/(m*Log[a])
```

3.519.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.519.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
default	$\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln(a^{mx})}{m\ln(a)}$	41
parallelrisch	$\frac{6mx\ln(a)-2a^{3mx}+9a^{2mx}-18a^{mx}}{6\ln(a)m}$	42
risch	$x-\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}-\frac{a^{3mx}}{3m\ln(a)}$	49
norman	$x-\frac{3\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}-\frac{\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}$	52

input
```
int((1-a^(m*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(-1/3*(a^(m*x))^3+3/2*(a^(m*x))^2-3*a^(m*x)+ln(a^(m*x)))
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1-a^{mx})^3\,dx=\frac{6\,mx\log(a)-2\,a^{3\,mx}+9\,a^{2\,mx}-18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^3,x, algorithm="fricas")
```

output
```
1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

3.519.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int (1-a^{mx})^3\, dx = x + \begin{cases} \frac{-2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2-18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } m^3\log(a)^3 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate((1-a**(m*x))**3,x)
```

output
```
x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
 - 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)),
 (-x, True))
```

3.519.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (1-a^{mx})^3\, dx = x - \frac{a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{2\,m\log(a)} - \frac{3\,a^{mx}}{m\log(a)}$$

input
```
integrate((1-a^(m*x))^3,x, algorithm="maxima")
```

output
```
x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*x)/(m*log
(a))
```

3.519.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (1-a^{mx})^3\, dx = \frac{6\,mx\log(|a|) - 2\,a^{3\,mx} + 9\,a^{2\,mx} - 18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^3,x, algorithm="giac")
```

output
```
1/6*(6*m*x*log(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a)
)
```

3.519.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int (1 - a^{mx})^3 \, dx = x - \frac{3\, a^{m\,x} - \frac{3\, a^{2\,m\,x}}{2} + \frac{a^{3\,m\,x}}{3}}{m\, \ln(a)}$$

input
```
int(-(a^(m*x) - 1)^3,x)
```

output
```
x - (3*a^(m*x) - (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

3.520 $\int (1-a^{mx})^4 \, dx$

3.520.1 Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (1-a^{mx})^4 \, dx = x - \frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} - \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)}$$

output
```
x-4*a^(m*x)/m/ln(a)+3*a^(2*m*x)/m/ln(a)-4/3*a^(3*m*x)/m/ln(a)+1/4*a^(4*m*x
)/m/ln(a)
```

3.520.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (1-a^{mx})^4 \, dx = \frac{\frac{a^{mx}(-48+36a^{mx}-16a^{2mx}+3a^{3mx})}{12m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^4,x]
```

output
```
((a^(m*x)*(-48 + 36*a^(m*x) - 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^
(m*x)]/m)/Log[a]
```

3.520.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^4 \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^4 da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^{-mx}+6a^{mx}-4a^{2mx}+a^{3mx}-4\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-4a^{mx}+3a^{2mx}-\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 - a^(m*x))^4,x]
```

output
```
(-4*a^(m*x) + 3*a^(2*m*x) - (4*a^(3*m*x))/3 + a^(4*m*x)/4 + Log[a^(m*x)])/
(m*Log[a])
```

3.520.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.520.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$	50
parallelrisch	$\frac{3a^{4mx}-16a^{3mx}+12mx\ln(a)+36a^{2mx}-48a^{mx}}{12\ln(a)m}$	51
risch	$x-\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}-\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$	65
norman	$x-\frac{4\,e^{mx\ln(a)}}{m\ln(a)}+\frac{3\,e^{2mx\ln(a)}}{m\ln(a)}-\frac{4\,e^{3mx\ln(a)}}{3m\ln(a)}+\frac{e^{4mx\ln(a)}}{4m\ln(a)}$	69

input
```
int((1-a^(m*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(1/4*(a^(m*x))^4-4/3*(a^(m*x))^3+3*(a^(m*x))^2-4*a^(m*x)+ln(a^(m
*x)))
```

3.520.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1-a^{mx})^4\,dx = \frac{12\,mx\log(a)+3\,a^{4\,mx}-16\,a^{3\,mx}+36\,a^{2\,mx}-48\,a^{mx}}{12\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^4,x, algorithm="fricas")
```

output
```
1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*
x))/(m*log(a))
```

3.520.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (1-a^{mx})^4\,dx$$
$$= x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3-16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3-48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } m^4\log(a)^4 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate((1-a**(m*x))**4,x)
```

output
```
x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 - 16*a**(3*m*x)*m**3*log(a)**3
 + 36*a**(2*m*x)*m**3*log(a)**3 - 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log
(a)**4), Ne(m**4*log(a)**4, 0)), (-x, True))
```

3.520.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (1-a^{mx})^4\,dx = x + \frac{a^{4\,mx}}{4\,m\log(a)} - \frac{4\,a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{m\log(a)} - \frac{4\,a^{mx}}{m\log(a)}$$

input
```
integrate((1-a^(m*x))^4,x, algorithm="maxima")
```

output
```
x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*l
og(a)) - 4*a^(m*x)/(m*log(a))
```

3.520.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int (1-a^{mx})^4\,dx = \frac{12\,mx\log(|a|) + 3\,a^{4\,mx} - 16\,a^{3\,mx} + 36\,a^{2\,mx} - 48\,a^{mx}}{12\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^4,x, algorithm="giac")
```

output
```
1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*
a^(m*x))/(m*log(a))
```

3.520.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int (1-a^{mx})^4 \, dx = x - \frac{4\,a^{m\,x} - 3\,a^{2\,m\,x} + \frac{4\,a^{3\,m\,x}}{3} - \frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

input
```
int((a^(m*x) - 1)^4,x)
```

output
```
x - (4*a^(m*x) - 3*a^(2*m*x) + (4*a^(3*m*x))/3 - a^(4*m*x)/4)/(m*log(a))
```

3.521 $\int (1-a^{mx})^n \, dx$

3.521.1 Optimal result

Integrand size = 11, antiderivative size = 44

$$\int (1-a^{mx})^n \, dx = -\frac{(1-a^{mx})^{1+n} \operatorname{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

output
```
-(1-a^(m*x))^(1+n)*hypergeom([1, 1+n],[2+n],1-a^(m*x))/m/(1+n)/ln(a)
```

3.521.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (1-a^{mx})^n \, dx = -\frac{(1-a^{mx})^{1+n} \operatorname{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^n,x]
```

output
```
-(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/
(m*(1 + n)*Log[a]))
```

3.521.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^n\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int a^{-mx}(1-a^{mx})^n\,da^{mx}}{m\log(a)}$$

$$\Big\downarrow \; 75$$

$$-\frac{(1-a^{mx})^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,1-a^{mx}\right)}{m(n+1)\log(a)}$$

input
```
Int[(1 - a^(m*x))^n,x]
```

output
```
-(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/
(m*(1 + n)*Log[a]))
```

3.521.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.521.4 Maple [F]

$$\int (1-a^{mx})^n \, dx$$

input
```
int((1-a^(m*x))^n,x)
```

output
```
int((1-a^(m*x))^n,x)
```

3.521.5 Fricas [F]

$$\int (1-a^{mx})^n \, dx = \int (-a^{mx}+1)^n \, dx$$

input
```
integrate((1-a^(m*x))^n,x, algorithm="fricas")
```

output
```
integral((-a^(m*x) + 1)^n, x)
```

3.521.6 Sympy [F]

$$\int (1-a^{mx})^n \, dx = \int (1-a^{mx})^n \, dx$$

input
```
integrate((1-a**(m*x))**n,x)
```

output
```
Integral((1 - a**(m*x))**n, x)
```

3.521.7 Maxima [F]

$$\int (1-a^{mx})^n \, dx = \int (-a^{mx}+1)^n \, dx$$

input
```
integrate((1-a^(m*x))^n,x, algorithm="maxima")
```

output
```
integrate((-a^(m*x) + 1)^n, x)
```

3.521.8 Giac [F]

$$\int (1-a^{mx})^n \, dx = \int (-a^{mx}+1)^n \, dx$$

input
```
integrate((1-a^(m*x))^n,x, algorithm="giac")
```

output
```
integrate((-a^(m*x) + 1)^n, x)
```

3.521.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (1-a^{mx})^n \, dx = \frac{(1-a^{m\,x})^n \, {}_2F_1\left(-n,-n;\ 1-n;\ \frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\,\left(1-\frac{1}{a^{m\,x}}\right)^n}$$

input
```
int((1 - a^(m*x))^n,x)
```

output
```
((1 - a^(m*x))^n*hypergeom([-n, -n], 1 - n, 1/a^(m*x)))/(m*n*log(a)*(1 - 1
/a^(m*x))^n)
```

3.522 $\int \frac{1}{b+ae^{nx}}\,dx$

3.522.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{x}{b}-\frac{\log\left(b+ae^{nx}\right)}{bn}$$

output
```
x/b-ln(b+a*exp(n*x))/b/n
```

3.522.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{\log\left(e^{nx}\right)}{bn}-\frac{\log\left(b^2n+abe^{nx}n\right)}{bn}$$

input
```
Integrate[(b + a*E^(n*x))^(-1),x]
```

output
```
Log[E^(n*x)]/(b*n) - Log[b^2*n + a*b*E^(n*x)*n]/(b*n)
```

3.522.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ae^{nx}+b}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{e^{-nx}}{e^{nx}a+b}de^{nx}}{n}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{\int e^{-nx}de^{nx}}{b}-\frac{a\int \frac{1}{e^{nx}a+b}de^{nx}}{b}}{n}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{\log(e^{nx})}{b}-\frac{a\int \frac{1}{e^{nx}a+b}de^{nx}}{b}}{n}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\log(e^{nx})}{b}-\frac{\log(ae^{nx}+b)}{b}}{n}$$

input
```
Int[(b + a*E^(n*x))^(-1),x]
```

output
```
(Log[E^(n*x)]/b - Log[b + a*E^(n*x)]/b)/n
```

3.522.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.522.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{-nx+\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b n}$	23
norman	$\frac{x}{b}-\frac{\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b n}$	24
risch	$\frac{x}{b}-\frac{\ln\left({\mathrm e}^{n x}+\frac{b}{a}\right)}{b n}$	26
derivativedivides	$\frac{-\frac{\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b}+\frac{\ln\left({\mathrm e}^{n x}\right)}{b}}{n}$	29
default	$\frac{-\frac{\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b}+\frac{\ln\left({\mathrm e}^{n x}\right)}{b}}{n}$	29

input
```
int(1/(b+a*exp(n*x)),x,method=_RETURNVERBOSE)
```

output
```
-(-n*x+ln(b+a*exp(n*x)))/b/n
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{nx-\log\left(ae^{(nx)}+b\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x, algorithm="fricas")
```

output
```
(n*x - log(a*e^(n*x) + b))/(b*n)
```

3.522.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{x}{b} - \frac{\log\left(e^{nx}+\frac{b}{a}\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x)
```

output
```
x/b - log(exp(n*x) + b/a)/(b*n)
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{x}{b} - \frac{\log\left(ae^{(nx)}+b\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x, algorithm="maxima")
```

output
```
x/b - log(a*e^(n*x) + b)/(b*n)
```

3.522.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{\frac{nx}{b} - \frac{\log\left(\left|ae^{(nx)}+b\right|\right)}{b}}{n}$$

input
```
integrate(1/(b+a*exp(n*x)),x, algorithm="giac")
```

output
```
(n*x/b - log(abs(a*e^(n*x) + b))/b)/n
```

3.522.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{b+ae^{nx}}\,dx = -\frac{\ln\left(b+a\,\mathrm{e}^{n\,x}\right)-n\,x}{b\,n}$$

input
```
int(1/(b + a*exp(n*x)),x)
```

output
```
-(log(b + a*exp(n*x)) - n*x)/(b*n)
```

3.523 $\int \frac{e^x}{b+ae^{3x}}\,dx$

3.523.1 Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{e^x}{b+ae^{3x}}\,dx=-\frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{2\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b+ae^{3x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
1/2*ln(b^(1/3)+a^(1/3)*exp(x))/a^(1/3)/b^(2/3)-1/6*ln(b+a*exp(3*x))/a^(1/3
)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*exp(x))/b^(1/3)*3^(1/2))/a^(1/
3)/b^(2/3)*3^(1/2)
```

3.523.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{e^x}{b+ae^{3x}}\,dx=-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)+\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[E^x/(b + a*E^(3*x)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1
/3) + a^(1/3)*E^x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)])
/(a^(1/3)*b^(2/3))
```

3.523.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2679, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{ae^{3x}+b}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\int \frac{1}{ae^{3x}+b}de^x$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{3b^{2/3}}+\frac{\int \frac{1}{e^x\sqrt[3]{a}+\sqrt[3]{b}}de^x}{3b^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}e^x\right)}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}e^x\right)}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x+\frac{3\int \frac{1}{-3-e^{2x}}d\left(1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[E^x/(b + a*E^(3*x)),x]
```

output
```
Log[b^(1/3) + a^(1/3)*E^x]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (
2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3) - a^(1/3)*b^(1/3)
*E^x + a^(2/3)*E^(2*x)]/(2*a^(1/3)))/(3*b^(2/3))
```

3.523.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.523. $\int \frac{e^x}{b+ae^{3x}}\,dx$

3.523.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}(27b^2a\_Z^3-1)} \_R\ln\left(3b\_R+\mathrm{e}^x\right)$	26
default	$\frac{\ln\left(\mathrm{e}^x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\mathrm{e}^{2x}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{e}^x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,\mathrm{e}^x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	95

input
```
int(exp(x)/(b+a*exp(3*x)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(3*b*_R+exp(x)),_R=RootOf(27*_Z^3*a*b^2-1))
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.11

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abe^{(3\,x)}-3\,(ab^2)^{\frac{1}{3}}be^x-b^2+3\sqrt{\frac{1}{3}}\left(2\,abe^{(2\,x)}+(ab^2)^{\frac{2}{3}}e^x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ae^{(3\,x)}+b}\right)-(ab^2)^{\frac{2}{3}}\log\left(abe^{\cdots}\right.}{6\,ab^2}\right.$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*e^(3*x) - 3*(a*b^2
)^(1/3)*b*e^x - b^2 + 3*sqrt(1/3)*(2*a*b*e^(2*x) + (a*b^2)^(2/3)*e^x - (a*
b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*e^(3*x) + b)) - (a*b^2)^(2/3)*log
(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(
a*b*e^x + (a*b^2)^(2/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)
/a)*arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^
(1/3)/a)/b^2) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2
)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2)]
```

3.523.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \operatorname{RootSum}\left(27z^3ab^2-1,\left(i\mapsto i\log\left(3ib+e^x\right)\right)\right)$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x)
```

output
```
RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\,e^x\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*e^x)/(b/a)^(1/3))/(a*(b/a
)^(2/3)) - 1/6*log(-(b/a)^(1/3)*e^x + (b/a)^(2/3) + e^(2*x))/(a*(b/a)^(2/3
)) + 1/3*log((b/a)^(1/3) + e^x)/(a*(b/a)^(2/3))
```

3.523.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{e^x}{b+ae^{3x}}\,dx = -\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right|\right)}{3\,b}+\frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\,e^x\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{(-a^2b)^{\frac{1}{3}}\log\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(-\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,ab}$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="giac")
```

output
```
-1/3*(-b/a)^(1/3)*log(abs(-(-b/a)^(1/3) + e^x))/b + 1/3*sqrt(3)*(-a^2*b)^(
1/3)*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*e^x)/(-b/a)^(1/3))/(a*b) + 1/6*(
-a^2*b)^(1/3)*log((-b/a)^(1/3)*e^x + (-b/a)^(2/3) + e^(2*x))/(a*b)
```

3.523.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{\ln\left(\frac{b^{1/3}}{a^{7/3}}+\frac{e^x}{a^2}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(\frac{e^x}{a^2}+\frac{b^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2\,a^{7/3}}\right)\left(-1+\sqrt{3}\,1i\right)}{6\,a^{1/3}\,b^{2/3}}-\frac{\ln\left(\frac{e^x}{a^2}-\frac{b^{1/3}\left(1+\sqrt{3}\,1i\right)}{2\,a^{7/3}}\right)\left(1+\sqrt{3}\,1i\right)}{6\,a^{1/3}\,b^{2/3}}$$

input
```
int(exp(x)/(b + a*exp(3*x)),x)
```

output
```
log(b^(1/3)/a^(7/3) + exp(x)/a^2)/(3*a^(1/3)*b^(2/3)) + (log(exp(x)/a^2 +
(b^(1/3)*(3^(1/2)*1i - 1))/(2*a^(7/3)))*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/
3)) - (log(exp(x)/a^2 - (b^(1/3)*(3^(1/2)*1i + 1))/(2*a^(7/3)))*(3^(1/2)*1
i + 1))/(6*a^(1/3)*b^(2/3))
```

3.524 $\int \frac{-1+e^x}{1+e^x} \, dx$

3.524.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2\log\left(1+e^x\right)$$

output
```
-x+2*ln(1+exp(x))
```

3.524.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{-1+e^x}{1+e^x} \, dx = -\log\left(e^x\right) + 2\log\left(1+e^x\right)$$

input
```
Integrate[(-1 + E^x)/(1 + E^x),x]
```

output
```
-Log[E^x] + 2*Log[1 + E^x]
```

3.524.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x-1}{e^x+1}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{e^{-x}(1-e^x)}{e^x+1}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{e^{-x}(1-e^x)}{1+e^x}de^x$$

$$\Big\downarrow \text{86}$$

$$-\int \left(e^{-x}-\frac{2}{1+e^x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$2\log(e^x+1)-\log(e^x)$$

input
```
Int[(-1 + E^x)/(1 + E^x),x]
```

output
```
-Log[E^x] + 2*Log[1 + E^x]
```

3.524.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.524.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
norman	$-x+2\ln\left(1+{\mathrm e}^{x}\right)$	12
risch	$-x+2\ln\left(1+{\mathrm e}^{x}\right)$	12
parallelrisch	$-x+2\ln\left(1+{\mathrm e}^{x}\right)$	12
derivativedivides	$2\ln\left(1+{\mathrm e}^{x}\right)-\ln\left({\mathrm e}^{x}\right)$	14
default	$2\ln\left(1+{\mathrm e}^{x}\right)-\ln\left({\mathrm e}^{x}\right)$	14

input
```
int((-1+exp(x))/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-x+2*ln(1+exp(x))
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx=-x+2\,\log\left(e^x+1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x, algorithm="fricas")
```

output
```
-x + 2*log(e^x + 1)
```

3.524.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x + 2\log\left(e^x + 1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x)
```

output
```
-x + 2*log(exp(x) + 1)
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x + 2\,\log\left(e^x + 1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x, algorithm="maxima")
```

output
```
-x + 2*log(e^x + 1)
```

3.524.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x + 2\,\log\left(e^x + 1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x, algorithm="giac")
```

output
```
-x + 2*log(e^x + 1)
```

3.524.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = 2\,\ln\left(\mathrm{e}^x+1\right)-x$$

input
```
int((exp(x) - 1)/(exp(x) + 1),x)
```

output
```
2*log(exp(x) + 1) - x
```

3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx$

3.525.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx=-\frac{\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)$$

output
```
1/12*ln(1-2*exp(2*x)+3*exp(4*x))-1/12*arctan(1/2*(1-3*exp(2*x))*2^(1/2))*2
^(1/2)
```

3.525.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx=\frac{1}{12}\left(-\sqrt{2}\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)+\log\left(1-2e^{2x}+3e^{4x}\right)\right)$$

input
```
Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

output
```
(-(Sqrt[2]*ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]) + Log[1 - 2*E^(2*x) + 3*E^(4*x
)])/12
```

3.525.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{-2e^{2x}+3e^{4x}+1}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\int \frac{e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{1-2e^{2x}+3e^{4x}}de^{2x}+\frac{1}{6}\int -\frac{2(1-3e^{2x})}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{1-2e^{2x}+3e^{4x}}de^{2x}-\frac{1}{3}\int \frac{1-3e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{1}{-8-e^{4x}}d(-2+6e^{2x})-\frac{1}{3}\int \frac{1-3e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6e^{2x}-2}{2\sqrt{2}}\right)}{3\sqrt{2}}-\frac{1}{3}\int \frac{1-3e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6e^{2x}-2}{2\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{6}\log\left(-2e^{2x}+3e^{4x}+1\right)\right)$$

input
```
Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

output
```
(ArcTan[(-2 + 6*E^(2*x))/(2*Sqrt[2])]/(3*Sqrt[2]) + Log[1 - 2*E^(2*x) + 3*
E^(4*x)]/6)/2
```

3.525. $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}dx$

3.525.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.525.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln\left(1-2\,\mathrm{e}^{2x}+3\,\mathrm{e}^{4x}\right)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(6\,\mathrm{e}^{2x}-2\right)\sqrt{2}}{4}\right)}{12}$	38
risch	$\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)}{12}+\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)}{12}-\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}$	70

input
```
int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x)^2-2)*2^
(1/2))
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3}{2}\,\sqrt{2}e^{(2\,x)}-\frac{1}{2}\,\sqrt{2}\right)+\frac{1}{12}\,\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*arctan(3/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)) + 1/12*log(3*e^(4*x
) - 2*e^(2*x) + 1)
```

3.525.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \operatorname{RootSum}\left(96z^2-16z+1,\left(i\mapsto i\log\left(8i+e^{2x}-1\right)\right)\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

output
```
RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) - 1)))
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(3\,e^{(2\,x)}-1\right)\right)+\frac{1}{12}\,\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*
e^(2*x) + 1)
```

3.525.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\big(3\,e^{(2\,x)}-1\big)\right)+\frac{1}{12}\,\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*
e^(2*x) + 1)
```

3.525.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{\ln\left(3\,\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)}{12}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\,\sqrt{2}\,\mathrm{e}^{2\,x}}{2}\right)}{12}$$

input
```
int(exp(4*x)/(3*exp(4*x) - 2*exp(2*x) + 1),x)
```

output
```
log(3*exp(4*x) - 2*exp(2*x) + 1)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)
*exp(2*x))/2))/12
```

3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}} \, dx$

3.526.1 Optimal result

Integrand size = 29, antiderivative size = 39

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}} \, dx = e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(1-e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

output
```
exp(x)+1/2*exp(2*x)-arctan(exp(x))+ln(1-exp(x))-1/2*ln(1+exp(2*x))
```

3.526.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}} \, dx = e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(-1+e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

input
```
Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

output
```
E^x + E^(2*x)/2 - ArcTan[E^x] + Log[-1 + E^x] - Log[1 + E^(2*x)]/2
```

3.526.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2720, 25, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x+e^{5x}}{e^x-e^{2x}+e^{3x}-1}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{e^{4x}+1}{-e^x+e^{2x}-e^{3x}+1}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1+e^{4x}}{1-e^x+e^{2x}-e^{3x}}de^x$$

$$\Big\downarrow \text{2462}$$

$$-\int \left(\frac{1+e^x}{1+e^{2x}}-e^x+\frac{1}{1-e^x}-1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$-\arctan\left(e^x\right)+e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\frac{1}{2}\log\left(e^{2x}+1\right)$$

input
```
Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

output
```
E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

3.526.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.526.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\ln(1+e^{2x})}{2}-\arctan\left(e^{x}\right)+\ln\left(-1+e^{x}\right)+\frac{e^{2x}}{2}+e^{x}$	29
risch	$\frac{e^{2x}}{2}+e^{x}-\frac{\ln(e^{x}-i)}{2}+\frac{i\ln(e^{x}-i)}{2}-\frac{\ln(e^{x}+i)}{2}-\frac{i\ln(e^{x}+i)}{2}+\ln\left(-1+e^{x}\right)$	49

input
```
int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x,method=_RETURNVERBOS
E)
```

output
```
-1/2*ln(1+exp(x)^2)-arctan(exp(x))+ln(-1+exp(x))+1/2*exp(x)^2+exp(x)
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^{x}+e^{5x}}{-1+e^{x}-e^{2x}+e^{3x}}\,dx=-\arctan\left(e^{x}\right)+\frac{1}{2}\,e^{(2\,x)}+e^{x}-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(e^{x}-1\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="fr
icas")
```

output
```
-arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

3.526.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = \frac{e^{2x}}{2}+e^x+\log\left(e^x-1\right) + \mathrm{RootSum}\left(2z^2+2z+1,\left(i\mapsto i\log\left(\frac{4i^2}{5}-\frac{6i}{5}+e^x-\frac{3}{5}\right)\right)\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

output
```
exp(2*x)/2 + exp(x) + log(exp(x) - 1) + RootSum(2*_z**2 + 2*_z + 1, Lambda
(_i, _i*log(4*_i**2/5 - 6*_i/5 + exp(x) - 3/5)))
```

3.526.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = -\arctan\left(e^x\right)+\frac{1}{2}\,e^{(2\,x)}+e^x-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(e^x-1\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="ma
xima")
```

output
```
-arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

3.526.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = -\arctan\left(e^x\right)+\frac{1}{2}\,e^{(2\,x)}+e^x-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(|e^x-1|\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="gi
ac")
```

output
```
-arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(abs(e^x - 1)
)
```

3.526.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = \frac{e^{2x}}{2}-\frac{\ln\left(e^{2x}+1\right)}{2}-\mathrm{atan}(e^x)+\ln\left(e^x-1\right)+e^x$$

input
```
int(-(exp(5*x) + exp(x))/(exp(2*x) - exp(3*x) - exp(x) + 1),x)
```

output
```
exp(2*x)/2 - log(exp(2*x) + 1)/2 - atan(exp(x)) + log(exp(x) - 1) + exp(x)
```

3.527 $\int e^{nx}(a+be^{nx})^{r/s}\,dx$

3.527.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{(a+be^{nx})^{\frac{r+s}{s}}s}{bn(r+s)}$$

output
```
(a+b*exp(n*x))^((r+s)/s)*s/b/n/(r+s)
```

3.527.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{(a+be^{nx})^{1+\frac{r}{s}}s}{bnr+bns}$$

input
```
Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

output
```
((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)
```

3.527.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx$$

$$\Big\downarrow \; 2676$$

$$\frac{\int (a+be^{nx})^{r/s}\,de^{nx}}{n}$$

$$\Big\downarrow \; 17$$

$$\frac{s(a+be^{nx})^{\frac{r+s}{s}}}{bn(r+s)}$$

input
```
Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

output
```
((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

3.527.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

3.527.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{(a+b\,\mathrm{e}^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
default	$\frac{(a+b\,\mathrm{e}^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
risch	$\frac{s(a+b\,\mathrm{e}^{nx})(a+b\,\mathrm{e}^{nx})^{\frac{r}{s}}}{bn(r+s)}$	36
parallelrisch	$\frac{\mathrm{e}^{nx}(a+b\,\mathrm{e}^{nx})^{\frac{r}{s}}bs+(a+b\,\mathrm{e}^{nx})^{\frac{r}{s}}as}{bn(r+s)}$	52
norman	$\frac{s\,\mathrm{e}^{nx}\mathrm{e}^{\frac{r\ln(a+b\,\mathrm{e}^{nx})}{s}}}{n(r+s)}+\frac{as\,\mathrm{e}^{\frac{r\ln(a+b\,\mathrm{e}^{nx})}{s}}}{bn(r+s)}$	60

input
```
int(exp(n*x)*(a+b*exp(n*x))^(r/s),x,method=_RETURNVERBOSE)
```

output
```
1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(bse^{(nx)}+as\right)\left(be^{(nx)}+a\right)^{\frac{r}{s}}}{bnr+bns}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="fricas")
```

output
```
(b*s*e^(n*x) + a*s)*(b*e^(n*x) + a)^(r/s)/(b*n*r + b*n*s)
```

3.527.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(20) = 40.

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.13

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\begin{cases}\frac{x}{a} & \text{for } b=0\wedge n=0\wedge r=-s\\ \frac{a^{\frac{r}{s}}e^{nx}}{n} & \text{for } b=0\\ x(a+b)^{\frac{r}{s}} & \text{for } n=0\\ \frac{\log\left(\frac{a}{b}+e^{nx}\right)}{bn} & \text{for } r=-s\\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns}+\frac{bs(a+be^{nx})^{\frac{r}{s}}e^{nx}}{bnr+bns} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n*x)/n, Eq
(b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*x))/(b*n), Eq(r, -
s)), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*s) + b*s*(a + b*exp(n*x))**
(r/s)*exp(n*x)/(b*n*r + b*n*s), True))
```

3.527.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="maxima")
```

output
```
(b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

3.527.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="giac")
```

output
```
(b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

3.527.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{s\,\left(a+b\,\mathrm{e}^{n\,x}\right)^{\frac{r}{s}+1}}{b\,n\,\left(r+s\right)}$$

input
```
int(exp(n*x)*(a + b*exp(n*x))^(r/s),x)
```

output
```
(s*(a + b*exp(n*x))^(r/s + 1))/(b*n*(r + s))
```

3.528 $\int \sqrt[4]{1-2e^{x/3}}\,dx$

3.528.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

output
```
12*(1-2*exp(1/3*x))^(1/4)-6*arctan((1-2*exp(1/3*x))^(1/4))-6*arctanh((1-2*
exp(1/3*x))^(1/4))
```

3.528.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

input
```
Integrate[(1 - 2*E^(x/3))^(1/4),x]
```

output
```
12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1
- 2*E^(x/3))^(1/4)]
```

3.528.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 60, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{1-2e^{x/3}}\,dx$$

$$\Big\downarrow \ 2720$$

$$3\int e^{-x/3}\sqrt[4]{1-2e^{x/3}}de^{x/3}$$

$$\Big\downarrow \ 60$$

$$3\left(\int \frac{e^{-x/3}}{\left(1-2e^{x/3}\right)^{3/4}}de^{x/3}+4\sqrt[4]{1-2e^{x/3}}\right)$$

$$\Big\downarrow \ 73$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\int \frac{1}{\frac{1}{2}-\frac{1}{2}e^{4x/3}}d\sqrt[4]{1-2e^{x/3}}\right)$$

$$\Big\downarrow \ 756$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\left(\int \frac{1}{1-e^{2x/3}}d\sqrt[4]{1-2e^{x/3}}+\int \frac{1}{1+e^{2x/3}}d\sqrt[4]{1-2e^{x/3}}\right)\right)$$

$$\Big\downarrow \ 216$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\left(\int \frac{1}{1-e^{2x/3}}d\sqrt[4]{1-2e^{x/3}}+\arctan\left(\sqrt[4]{1-2e^{x/3}}\right)\right)\right)$$

$$\Big\downarrow \ 219$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\left(\arctan\left(\sqrt[4]{1-2e^{x/3}}\right)+\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)\right)\right)$$

input
```
Int[(1 - 2*E^(x/3))^(1/4),x]
```

output
```
3*(4*(1 - 2*E^(x/3))^(1/4) - 2*(ArcTan[(1 - 2*E^(x/3))^(1/4)] + ArcTanh[(1
 - 2*E^(x/3))^(1/4)]))
```

3.528.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.528.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
derivativedivides	$12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\,\mathrm{e}\right.\right.$
default	$12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\,\mathrm{e}\right.\right.$

input
```
int((1-2*exp(1/3*x))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
12*(1-2*exp(1/3*x))^(1/4)+3*ln((1-2*exp(1/3*x))^(1/4)-1)-3*ln((1-2*exp(1/3
*x))^(1/4)+1)-6*arctan((1-2*exp(1/3*x))^(1/4))
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="fricas")
```

output
```
12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((
-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

3.528.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{\frac{x}{3}}} + 3\log\left(\sqrt[4]{1-2e^{\frac{x}{3}}}-1\right) - 3\log\left(\sqrt[4]{1-2e^{\frac{x}{3}}}+1\right) - 6\,\mathrm{atan}\left(\sqrt[4]{1-2e^{\frac{x}{3}}}\right)$$

input
```
integrate((1-2*exp(1/3*x))**(1/4),x)
```

output
```
12*(1 - 2*exp(x/3))**(1/4) + 3*log((1 - 2*exp(x/3))**(1/4) - 1) - 3*log((1
 - 2*exp(x/3))**(1/4) + 1) - 6*atan((1 - 2*exp(x/3))**(1/4))
```

3.528.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="maxima")
```

output
```
12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((
-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

3.528.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left|\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="giac")
```

output
```
12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((
-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log(abs((-2*e^(1/3*x) + 1)^(1/4) - 1))
```

3.528.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = \frac{12\left(2-4\,\mathrm{e}^{x/3}\right)^{1/4}{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{\mathrm{e}^{-\frac{x}{3}}}{2}\right)}{\left(2-\mathrm{e}^{-\frac{x}{3}}\right)^{1/4}}$$

input

```
int((1 - 2*exp(x/3))^(1/4),x)
```

output

```
(12*(2 - 4*exp(x/3))^(1/4)*hypergeom([-1/4, -1/4], 3/4, exp(-x/3)/2))/(2 -
 exp(-x/3))^(1/4)
```

3.529 $\int (a+be^{nx})^{r/s}\,dx$

3.529.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int (a+be^{nx})^{r/s}\,dx = -\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},2+\frac{r}{s},1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

output
```
-(a+b*exp(n*x))^((r+s)/s)*s*hypergeom([1, (r+s)/s],[2+r/s],1+b*exp(n*x)/a)
/a/n/(r+s)
```

3.529.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+be^{nx})^{r/s}\,dx = -\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},2+\frac{r}{s},1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

input
```
Integrate[(a + b*E^(n*x))^(r/s),x]
```

output
```
-(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s,
1 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

3.529.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2720, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^{nx})^{r/s}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-nx}(a+be^{nx})^{r/s}\,de^{nx}}{n}$$

$$\Big\downarrow \text{75}$$

$$-\frac{s(a+be^{nx})^{\frac{r+s}{s}}\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},\frac{r}{s}+2,\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

input
```
Int[(a + b*E^(n*x))^(r/s),x]
```

output
```
-(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s,
1 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

3.529.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.529.4 Maple [F]

$$\int (a+b\,\mathrm{e}^{nx})^{\frac{r}{s}}\,dx$$

input
```
int((a+b*exp(n*x))^(r/s),x)
```

output
```
int((a+b*exp(n*x))^(r/s),x)
```

3.529.5 Fricas [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))^(r/s),x, algorithm="fricas")
```

output
```
integral((b*e^(n*x) + a)^(r/s), x)
```

3.529.6 Sympy [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int (a+be^{nx})^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))**(r/s),x)
```

output
```
Integral((a + b*exp(n*x))**(r/s), x)
```

3.529.7 Maxima [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))^(r/s),x, algorithm="maxima")
```

output
```
integrate((b*e^(n*x) + a)^(r/s), x)
```

3.529.8 Giac [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))^(r/s),x, algorithm="giac")
```

output
```
integrate((b*e^(n*x) + a)^(r/s), x)
```

3.529.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int (a+be^{nx})^{r/s}\,dx=\frac{s\,(a+b\,e^{n\,x})^{r/s}\,{}_2F_1\left(-\frac{r}{s},-\frac{r}{s};\;1-\frac{r}{s};\;-\frac{a\,e^{-n\,x}}{b}\right)}{n\,r\,\left(\frac{a\,e^{-n\,x}}{b}+1\right)^{r/s}}$$

input
```
int((a + b*exp(n*x))^(r/s),x)
```

output
```
(s*(a + b*exp(n*x))^(r/s)*hypergeom([-r/s, -r/s], 1 - r/s, -(a*exp(-n*x))/
b))/(n*r*((a*exp(-n*x))/b + 1)^(r/s))
```

3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx$

3.530.1 Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

output
```
arctanh(exp(x)/(a^2+exp(2*x))^(1/2))
```

3.530.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = -\log\left(-e^x+\sqrt{a^2+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[a^2 + E^(2*x)],x]
```

output
```
-Log[-E^x + Sqrt[a^2 + E^(2*x)]]
```

3.530.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}dx$$
$$\Big\downarrow \; 2679$$
$$\int \frac{1}{\sqrt{a^2+e^{2x}}}de^x$$
$$\Big\downarrow \; 224$$
$$\int \frac{1}{1-e^{2x}}d\frac{e^x}{\sqrt{a^2+e^{2x}}}$$
$$\Big\downarrow \; 219$$
$$\text{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

input
```
Int[E^x/Sqrt[a^2 + E^(2*x)],x]
```

output
```
ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

3.530.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.530.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\ln\left(\mathrm{e}^{x}+\sqrt{a^{2}+\mathrm{e}^{2x}}\right)$	15

input
```
int(exp(x)/(a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+(a^2+exp(x)^2)^(1/2))
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{x}}{\sqrt{a^{2}+e^{2x}}}\,dx = -\log\left(\sqrt{a^{2}+e^{(2x)}}-e^{x}\right)$$

input
```
integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(a^2 + e^(2*x)) - e^x)
```

3.530.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \begin{cases} \log\left(2\sqrt{a^2+e^{2x}}+2e^x\right) & \text{for } a^2 \neq 0 \\ \frac{e^x \log(e^x)}{\sqrt{e^{2x}}} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)
```

output
```
Piecewise((log(2*sqrt(a**2 + exp(2*x)) + 2*exp(x)), Ne(a**2, 0)), (exp(x)*
log(exp(x))/sqrt(exp(2*x)), True))
```

3.530.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \operatorname{arsinh}\left(\frac{e^x}{a}\right)$$

input
```
integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(e^x/a)
```

3.530.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = -\log\left(\sqrt{a^2+e^{(2x)}}-e^x\right)$$

input
```
integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(a^2 + e^(2*x)) - e^x)
```

3.530.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx=\ln\left(\mathrm{e}^x+\sqrt{a^2+\mathrm{e}^{2\,x}}\right)$$

input
```
int(exp(x)/(exp(2*x) + a^2)^(1/2),x)
```

output
```
log(exp(x) + (exp(2*x) + a^2)^(1/2))
```

3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx$

3.531.1 Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)$$

output
```
arctanh(exp(x)/(-a^2+exp(2*x))^(1/2))
```

3.531.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(-e^x+\sqrt{-a^2+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

output
```
-Log[-E^x + Sqrt[-a^2 + E^(2*x)]]
```

3.531.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{e^{2x}-a^2}}\,dx$$
$$\Big\downarrow \text{ 2679}$$
$$\int \frac{1}{\sqrt{e^{2x}-a^2}}de^x$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{1-e^{2x}}d\frac{e^x}{\sqrt{e^{2x}-a^2}}$$
$$\Big\downarrow \text{ 219}$$
$$\operatorname{arctanh}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

input
```
Int[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

output
```
ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

3.531.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.531.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$\ln\left(e^{x}+\sqrt{-a^{2}+e^{2x}}\right)$	17

input
```
int(exp(x)/(-a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+(-a^2+exp(x)^2)^(1/2))
```

3.531.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{x}}{\sqrt{-a^{2}+e^{2x}}}\,dx = -\log\left(\sqrt{-a^{2}+e^{(2x)}}-e^{x}\right)$$

input
```
integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(-a^2 + e^(2*x)) - e^x)
```

3.531.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \begin{cases} \log\left(2\sqrt{-a^2+e^{2x}}+2e^x\right) & \text{for } a^2 \neq 0 \\ \frac{e^x \log(e^x)}{\sqrt{e^{2x}}} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)
```

output
```
Piecewise((log(2*sqrt(-a**2 + exp(2*x)) + 2*exp(x)), Ne(a**2, 0)), (exp(x)
*log(exp(x))/sqrt(exp(2*x)), True))
```

3.531.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \log\left(2\sqrt{-a^2+e^{(2x)}}+2e^x\right)$$

input
```
integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)
```

3.531.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(-\sqrt{-a^2+e^{(2x)}}+e^x\right)$$

input
```
integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(-sqrt(-a^2 + e^(2*x)) + e^x)
```

3.531.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2\,x}-a^2}\right)$$

input
```
int(exp(x)/(exp(2*x) - a^2)^(1/2),x)
```

output
```
log(exp(x) + (exp(2*x) - a^2)^(1/2))
```

3.532 $\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$

3.532.1 Optimal result

Integrand size = 39, antiderivative size = 40

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx=\frac{2}{3}\text{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)$$

output
```
2/3*arctanh(1/4*(2-5*exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2))
```

3.532.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx=-\frac{4}{3}\text{arctanh}\left(1-\frac{1}{2}e^{3x/4}+\frac{1}{2}\sqrt{-2+e^{3x/4}+e^{3x/2}}\right)$$

input
```
Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)
/2)]),x]
```

output
```
(-4*ArcTanh[1 - E^((3*x)/4)/2 + Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]/2])/3
```

3.532.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2720, 25, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3x/4}}{\left(e^{3x/4}-2\right)\sqrt{e^{3x/4}+e^{3x/2}-2}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{4}{3}\int -\frac{1}{\left(2-e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}de^{3x/4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4}{3}\int \frac{1}{\left(2-e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}de^{3x/4}$$

$$\Big\downarrow \text{1154}$$

$$\frac{8}{3}\int \frac{1}{16-e^{3x/2}}d\frac{2-5e^{3x/4}}{\sqrt{-2+e^{3x/4}+e^{3x/2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\text{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

input
```
Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),
x]
```

output
```
(2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])/
3
```

3.532.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.532.4 Maple [F]

$$\int \frac{e^{\frac{3x}{4}}}{\left(-2+e^{\frac{3x}{4}}\right)\sqrt{-2+e^{\frac{3x}{4}}+e^{\frac{3x}{2}}}}dx$$

input
```
int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

output
```
int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

3.532.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}\,x\right)}+e^{\left(\frac{3}{4}\,x\right)}-2}-e^{\left(\frac{3}{4}\,x\right)}+4\right)+\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}\,x\right)}+e^{\left(\frac{3}{4}\,x\right)}-2}-e^{\left(\frac{3}{4}\,x\right)}\right)$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, a
lgorithm="fricas")
```

output
```
-2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*log(sqrt(e
^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))
```

3.532.6 Sympy [F]

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\,dx$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
```

output
```
Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2) - 2)),
x)
```

3.532.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{2}{3}\log\left(\frac{4\sqrt{e^{\left(\frac{3}{2}\,x\right)}+e^{\left(\frac{3}{4}\,x\right)}-2}}{\left|e^{\left(\frac{3}{4}\,x\right)}-2\right|}+\frac{8}{\left|e^{\left(\frac{3}{4}\,x\right)}-2\right|}+5\right)$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, a
lgorithm="maxima")
```

output
```
-2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8/abs(e^(3
/4*x) - 2) + 5)
```

3.532.8 Giac [F]

$$\int \frac{e^{3x/4}}{(-2+e^{3x/4})\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{e^{\left(\frac{3}{4}x\right)}}{\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}\left(e^{\left(\frac{3}{4}x\right)}-2\right)}\,dx$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, a
lgorithm="giac")
```

output
```
integrate(e^(3/4*x)/(sqrt(e^(3/2*x) + e^(3/4*x) - 2)*(e^(3/4*x) - 2)), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3x/4}}{(-2+e^{3x/4})\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{\mathrm{e}^{\frac{3x}{4}}}{\left(\mathrm{e}^{\frac{3x}{4}}-2\right)\sqrt{\mathrm{e}^{\frac{3x}{2}}+\mathrm{e}^{\frac{3x}{4}}-2}}\,dx$$

input
```
int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/
2)),x)
```

output
```
int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/
2)), x)
```

3.533 $\int e^{-2x}\left(-3+e^{7x}\right)^{2/3} dx$

3.533.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3} dx = \frac{1}{6}e^{-2x}\left(-3+e^{7x}\right)^{5/3}\text{Hypergeometric2F1}\left(1,\frac{29}{21},\frac{5}{7},\frac{e^{7x}}{3}\right)$$

output
```
1/6*(-3+exp(7*x))^(5/3)*hypergeom([1, 29/21],[5/7],1/3*exp(7*x))/exp(2*x)
```

3.533.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3} dx = -\frac{e^{-2x}\left(-3+e^{7x}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$

input
```
Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

output
```
-1/2*((-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/
(E^(2*x)*(1 - E^(7*x)/3)^(2/3))
```

3.533.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2679, 858, 889, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x}\left(e^{7x}-3\right)^{2/3}dx$$

$$\Big\downarrow \text{2679}$$

$$-\int e^{-x}\left(-3+e^{7x}\right)^{2/3}de^{-x}$$

$$\Big\downarrow \text{858}$$

$$\int e^{3x}\left(e^{-7x}-3\right)^{2/3}de^{x}$$

$$\Big\downarrow \text{889}$$

$$\frac{3^{2/3}\left(e^{-7x}-3\right)^{2/3}\int \frac{e^{3x}\left(3-e^{-7x}\right)^{2/3}}{3^{2/3}}de^{x}}{\left(3-e^{-7x}\right)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(e^{-7x}-3\right)^{2/3}\int e^{3x}\left(3-e^{-7x}\right)^{2/3}de^{x}}{\left(3-e^{-7x}\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{3^{2/3}e^{2x}\left(e^{-7x}-3\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{-7x}}{3}\right)}{2\left(3-e^{-7x}\right)^{2/3}}$$

input
```
Int[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

output
```
-1/2*(3^(2/3)*E^(2*x)*(-3 + E^(-7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7,
5/7, 1/(3*E^(7*x))])/(3 - E^(-7*x))^(2/3)
```

3.533.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.533.4 Maple [F]

$$\int \left(-3+\mathrm{e}^{7x}\right)^{\frac{2}{3}} \mathrm{e}^{-2x}dx$$

input
```
int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

output
```
int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

3.533.5 Fricas [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

input
```
integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="fricas")
```

output
```
integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

3.533.6 Sympy [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{7x}-3\right)^{\frac{2}{3}}e^{-2x}\,dx$$

input
```
integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)
```

output
```
Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)
```

3.533.7 Maxima [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

input
```
integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="maxima")
```

output
```
integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

3.533.8 Giac [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

input
```
integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="giac")
```

output
```
integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{7\,x}-3\right)^{2/3}\,dx$$

input
```
int(exp(-2*x)*(exp(7*x) - 3)^(2/3),x)
```

output
```
int(exp(-2*x)*(exp(7*x) - 3)^(2/3), x)
```

3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$

3.534.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx=-216\sqrt[4]{3-e^{x/2}}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}$$

output
```
-216*(3-exp(1/2*x))^(1/4)+216/5*(3-exp(1/2*x))^(5/4)-8*(3-exp(1/2*x))^(9/4
)+8/13*(3-exp(1/2*x))^(13/4)
```

3.534.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx=-\frac{8}{65}\sqrt[4]{3-e^{x/2}}\left(1152+96e^{x/2}+20e^{x}+5e^{3x/2}\right)$$

input
```
Integrate[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

output
```
(-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/2)))/65
```

3.534.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$$

$$\Big\downarrow \text{2678}$$

$$2\int \frac{e^{3x/2}}{\left(3-e^{x/2}\right)^{3/4}}de^{x/2}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\left(3-e^{x/2}\right)^{9/4}+9\left(3-e^{x/2}\right)^{5/4}-27\sqrt[4]{3-e^{x/2}}+\frac{27}{\left(3-e^{x/2}\right)^{3/4}}\right)de^{x/2}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{4}{13}\left(3-e^{x/2}\right)^{13/4}-4\left(3-e^{x/2}\right)^{9/4}+\frac{108}{5}\left(3-e^{x/2}\right)^{5/4}-108\sqrt[4]{3-e^{x/2}}\right)$$

input
```
Int[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

output
```
2*(-108*(3 - E^(x/2))^(1/4) + (108*(3 - E^(x/2))^(5/4))/5 - 4*(3 - E^(x/2)
)^(9/4) + (4*(3 - E^(x/2))^(13/4))/13)
```

3.534.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

3.534.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{8\left(5\,\mathrm{e}^{\frac{3x}{2}}+20\,\mathrm{e}^{x}+96\,\mathrm{e}^{\frac{x}{2}}+1152\right)\left(-3+\mathrm{e}^{\frac{x}{2}}\right)}{65\left(3-\mathrm{e}^{\frac{x}{2}}\right)^{\frac{3}{4}}}$	37

input
```
int(exp(2*x)/(3-exp(1/2*x))^(3/4),x,method=_RETURNVERBOSE)
```

output
```
8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1152)*(-3+
exp(1/2*x))
```

3.534.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\frac{8}{65}\left(5\,e^{\left(\frac{3}{2}\,x\right)}+96\,e^{\left(\frac{1}{2}\,x\right)}+20\,e^{x}+1152\right)\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="fricas")
```

output
```
-8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) + 3)^(1/4)
```

3.534.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = \frac{8\left(3-e^{\frac{x}{2}}\right)^{\frac{13}{4}}}{13} - 8\left(3-e^{\frac{x}{2}}\right)^{\frac{9}{4}} + \frac{216\left(3-e^{\frac{x}{2}}\right)^{\frac{5}{4}}}{5} - 216\sqrt[4]{3-e^{\frac{x}{2}}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
```

output
```
8*(3 - exp(x/2))**(13/4)/13 - 8*(3 - exp(x/2))**(9/4) + 216*(3 - exp(x/2))
**(5/4)/5 - 216*(3 - exp(x/2))**(1/4)
```

3.534.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = \frac{8}{13}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{13}{4}} - 8\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{9}{4}} + \frac{216}{5}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{5}{4}} - 216\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="maxima")
```

output
```
8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(-e^(1/2*x
) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)
```

3.534.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = -\frac{8}{13}\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{3}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}} - 8\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{2}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}} + \frac{216}{5}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{5}{4}} - 216\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="giac")
```

output
```
-8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3)^2*(-e^(
1/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1
/4)
```

3.534.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\left(3-\mathrm{e}^{x/2}\right)^{1/4}\left(\frac{768\,\mathrm{e}^{x/2}}{65}+\frac{8\,\mathrm{e}^{\frac{3\,x}{2}}}{13}+\frac{32\,\mathrm{e}^{x}}{13}+\frac{9216}{65}\right)$$

input
```
int(exp(2*x)/(3 - exp(x/2))^(3/4),x)
```

output
```
-(3 - exp(x/2))^(1/4)*((768*exp(x/2))/65 + (8*exp((3*x)/2))/13 + (32*exp(x
))/13 + 9216/65)
```

3.535 $\int e^{-x/2}x^3\,dx$

3.535.1 Optimal result

Integrand size = 11, antiderivative size = 44

$$\int e^{-x/2}x^3\,dx = -96e^{-x/2} - 48e^{-x/2}x - 12e^{-x/2}x^2 - 2e^{-x/2}x^3$$

output
```
-96/exp(1/2*x)-48*x/exp(1/2*x)-12*x^2/exp(1/2*x)-2*x^3/exp(1/2*x)
```

3.535.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int e^{-x/2}x^3\,dx = e^{-x/2}\left(-96 - 48x - 12x^2 - 2x^3\right)$$

input
```
Integrate[x^3/E^(x/2),x]
```

output
```
(-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)
```

3.535.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x/2}x^3\,dx$$

$$\Big\downarrow \text{ 2607}$$

$$6\int e^{-x/2}x^2dx-2e^{-x/2}x^3$$

$$\Big\downarrow \text{ 2607}$$

$$6\left(4\int e^{-x/2}xdx-2e^{-x/2}x^2\right)-2e^{-x/2}x^3$$

$$\Big\downarrow \text{ 2607}$$

$$6\left(4\left(2\int e^{-x/2}dx-2e^{-x/2}x\right)-2e^{-x/2}x^2\right)-2e^{-x/2}x^3$$

$$\Big\downarrow \text{ 2624}$$

$$6\left(4\left(-2e^{-x/2}x-4e^{-x/2}\right)-2e^{-x/2}x^2\right)-2e^{-x/2}x^3$$

input
```
Int[x^3/E^(x/2),x]
```

output
```
(-2*x^3)/E^(x/2) + 6*((-2*x^2)/E^(x/2) + 4*(-4/E^(x/2) - (2*x)/E^(x/2)))
```

3.535.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.535.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

method	result	size
risch	$(-2x^3 - 12x^2 - 48x - 96)\, e^{-\frac{x}{2}}$	21
gosper	$-2(x^3 + 6x^2 + 24x + 48)\, e^{-\frac{x}{2}}$	22
norman	$(-2x^3 - 12x^2 - 48x - 96)\, e^{-\frac{x}{2}}$	23
meijerg	$96 - 4\left(\frac{1}{2}x^3 + 3x^2 + 12x + 24\right) e^{-\frac{x}{2}}$	24
parallelrisch	$-(2x^3 + 12x^2 + 48x + 96)\, e^{-\frac{x}{2}}$	24
derivativedivides	$-96\, e^{-\frac{x}{2}} - 48x\, e^{-\frac{x}{2}} - 12x^2 e^{-\frac{x}{2}} - 2x^3 e^{-\frac{x}{2}}$	41
default	$-96\, e^{-\frac{x}{2}} - 48x\, e^{-\frac{x}{2}} - 12x^2 e^{-\frac{x}{2}} - 2x^3 e^{-\frac{x}{2}}$	41

input
```
int(x^3/exp(1/2*x),x,method=_RETURNVERBOSE)
```

output
```
(-2*x^3-12*x^2-48*x-96)*exp(-1/2*x)
```

3.535.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3 + 6\,x^2 + 24\,x + 48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^3/exp(1/2*x),x, algorithm="fricas")
```

output
```
-2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

3.535.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int e^{-x/2}x^3\,dx = \left(-2x^3-12x^2-48x-96\right)e^{-\frac{x}{2}}$$

input
```
integrate(x**3/exp(1/2*x),x)
```

output
```
(-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)
```

3.535.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^3/exp(1/2*x),x, algorithm="maxima")
```

output
```
-2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

3.535.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^3/exp(1/2*x),x, algorithm="giac")
```

output
```
-2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

3.535.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-x/2}x^3\,dx = -16\,\mathrm{e}^{-\frac{x}{2}}\left(\frac{x^3}{8}+\frac{3\,x^2}{4}+3\,x+6\right)$$

input
```
int(x^3*exp(-x/2),x)
```

output
```
-16*exp(-x/2)*(3*x + (3*x^2)/4 + x^3/8 + 6)
```

3.536 $\int \frac{e^{-x/2}}{x^3} \, dx$

3.536.1 Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{e^{-x/2}}{x^3} \, dx = -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{8}$$

output
```
-1/2/exp(1/2*x)/x^2+1/4/exp(1/2*x)/x+1/8*Ei(-1/2*x)
```

3.536.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{e^{-x/2}}{x^3} \, dx = \frac{1}{8}\left(\frac{2e^{-x/2}(-2+x)}{x^2} + \text{ExpIntegralEi}\left(-\frac{x}{2}\right)\right)$$

input
```
Integrate[1/(E^(x/2)*x^3),x]
```

output
```
((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-1/2*x])/8
```

3.536.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x/2}}{x^3} \, dx$$

$$\Big\downarrow \ 2608$$

$$-\frac{1}{4}\int \frac{e^{-x/2}}{x^2}dx-\frac{e^{-x/2}}{2x^2}$$

$$\Big\downarrow \ 2608$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{e^{-x/2}}{x}dx+\frac{e^{-x/2}}{x}\right)-\frac{e^{-x/2}}{2x^2}$$

$$\Big\downarrow \ 2609$$

$$\frac{1}{4}\left(\frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{2}+\frac{e^{-x/2}}{x}\right)-\frac{e^{-x/2}}{2x^2}$$

input
```
Int[1/(E^(x/2)*x^3),x]
```

output
```
-1/2*1/(E^(x/2)*x^2) + (1/(E^(x/2)*x) + ExpIntegralEi[-1/2*x]/2)/4
```

3.536.3.1 Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.536.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\mathrm{Ei}_1\left(\frac{x}{2}\right)}{8}$	27
derivativedivides	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\mathrm{Ei}_1\left(\frac{x}{2}\right)}{8}$	31
default	$-\frac{e^{-\frac{x}{2}}}{2x^2}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\mathrm{Ei}_1\left(\frac{x}{2}\right)}{8}$	31
meijerg	$-\frac{1}{2x^2}+\frac{1}{2x}-\frac{3}{16}+\frac{\ln(x)}{8}-\frac{\ln(2)}{8}+\frac{\frac{9}{4}x^2-6x+6}{12x^2}-\frac{\left(-\frac{3x}{2}+3\right)e^{-\frac{x}{2}}}{6x^2}-\frac{\ln\left(\frac{x}{2}\right)}{8}-\frac{\mathrm{Ei}_1\left(\frac{x}{2}\right)}{8}$	63

input
```
int(1/exp(1/2*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-1/2*x)/x^2+1/4*exp(-1/2*x)/x-1/8*Ei(1,1/2*x)
```

3.536.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right)+2\,(x-2)e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

input
```
integrate(1/exp(1/2*x)/x^3,x, algorithm="fricas")
```

output
```
1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2
```

3.536.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{\mathrm{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{e^{-\frac{x}{2}}}{2x^2}$$

input
```
integrate(1/exp(1/2*x)/x**3,x)
```

output
```
Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

3.536.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.18

$$\int \frac{e^{-x/2}}{x^3}\,dx = -\frac{1}{4}\,\Gamma\left(-2, \frac{1}{2}\,x\right)$$

input
```
integrate(1/exp(1/2*x)/x^3,x, algorithm="maxima")
```

output
```
-1/4*gamma(-2, 1/2*x)
```

3.536.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right) + 2\,xe^{\left(-\frac{1}{2}\,x\right)} - 4\,e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

input
```
integrate(1/exp(1/2*x)/x^3,x, algorithm="giac")
```

output
```
1/8*(x^2*Ei(-1/2*x) + 2*x*e^(-1/2*x) - 4*e^(-1/2*x))/x^2
```

3.536.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{\mathrm{e}^{-\frac{x}{2}}\,\left(\frac{1}{x} - \frac{2}{x^2}\right)}{4} - \frac{\mathrm{expint}\left(\frac{x}{2}\right)}{8}$$

input
```
int(exp(-x/2)/x^3,x)
```

output
```
(exp(-x/2)*(1/x - 2/x^2))/4 - expint(x/2)/8
```

3.537 $\int a^{3x}x^2\,dx$

3.537.1 Optimal result

Integrand size = 9, antiderivative size = 44

$$\int a^{3x}x^2\,dx=\frac{2a^{3x}}{27\log^3(a)}-\frac{2a^{3x}x}{9\log^2(a)}+\frac{a^{3x}x^2}{3\log(a)}$$

output
```
2/27*a^(3*x)/ln(a)^3-2/9*a^(3*x)*x/ln(a)^2+1/3*a^(3*x)*x^2/ln(a)
```

3.537.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int a^{3x}x^2\,dx=\frac{a^{3x}(2-6x\log(a)+9x^2\log^2(a))}{27\log^3(a)}$$

input
```
Integrate[a^(3*x)*x^2,x]
```

output
```
(a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)
```

3.537.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 a^{3x} \, dx$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{x^2 a^{3x}}{3\log(a)} - \frac{2\int a^{3x} x dx}{3\log(a)}$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{x^2 a^{3x}}{3\log(a)} - \frac{2\left(\frac{x a^{3x}}{3\log(a)} - \frac{\int a^{3x} dx}{3\log(a)}\right)}{3\log(a)}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{x^2 a^{3x}}{3\log(a)} - \frac{2\left(\frac{x a^{3x}}{3\log(a)} - \frac{a^{3x}}{9\log^2(a)}\right)}{3\log(a)}$$

input
```
Int[a^(3*x)*x^2,x]
```

output
```
(a^(3*x)*x^2)/(3*Log[a]) - (2*(-1/9*a^(3*x)/Log[a]^2 + (a^(3*x)*x)/(3*Log[
a])))/(3*Log[a])
```

3.537.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.537.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(9x^{2}\ln\left(a\right)^{2}-6x\ln\left(a\right)+2\right)a^{3x}}{27\ln\left(a\right)^{3}}$	28
risch	$\frac{\left(9x^{2}\ln\left(a\right)^{2}-6x\ln\left(a\right)+2\right)a^{3x}}{27\ln\left(a\right)^{3}}$	28
meijerg	$-\frac{2-\frac{\left(27x^{2}\ln\left(a\right)^{2}-18x\ln\left(a\right)+6\right)e^{3x\ln\left(a\right)}}{3}}{27\ln\left(a\right)^{3}}$	33
parallelrisch	$\frac{9x^{2}a^{3x}\ln\left(a\right)^{2}-6x\,a^{3x}\ln\left(a\right)+2a^{3x}}{27\ln\left(a\right)^{3}}$	39
norman	$\frac{2\,e^{3x\ln\left(a\right)}}{27\ln\left(a\right)^{3}}-\frac{2x\,e^{3x\ln\left(a\right)}}{9\ln\left(a\right)^{2}}+\frac{x^{2}e^{3x\ln\left(a\right)}}{3\ln\left(a\right)}$	42

input
```
int(a^(3*x)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3
```

3.537.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^{2}\,dx=\frac{\left(9\,x^{2}\log\left(a\right)^{2}-6\,x\log\left(a\right)+2\right)a^{3\,x}}{27\,\log\left(a\right)^{3}}$$

input
```
integrate(a^(3*x)*x^2,x, algorithm="fricas")
```

output
```
1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

3.537.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int a^{3x}x^2\,dx = \begin{cases} \frac{a^{3x}\left(9x^2\log(a)^2-6x\log(a)+2\right)}{27\log(a)^3} & \text{for } \log(a)^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(a**(3*x)*x**2,x)
```

output
```
Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a)**3), Ne
(log(a)**3, 0)), (x**3/3, True))
```

3.537.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{(9\,x^2\log(a)^2 - 6\,x\log(a) + 2)a^{3\,x}}{27\,\log(a)^3}$$

input
```
integrate(a^(3*x)*x^2,x, algorithm="maxima")
```

output
```
1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

3.537.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 826, normalized size of antiderivative = 18.77

$$\int a^{3x}x^2\,dx = \text{Too large to display}$$

input
```
integrate(a^(3*x)*x^2,x, algorithm="giac")
```

output
```
-1/27*((6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(a)) - pi*x*sgn(a
) + pi*x)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a
))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^
2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^
2) - (9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(abs(a))^2 - 12*x*log(abs
(a)) + 4)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^
3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^
2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2))
*cos(-3/2*pi*x*sgn(a) + 3/2*pi*x) - ((9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*
x^2*log(abs(a))^2 - 12*x*log(abs(a)) + 4)*(pi^3*sgn(a) - 3*pi*log(abs(a))^
2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*s
gn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2
*log(abs(a)) + 2*log(abs(a))^3)^2) + 6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi
*x^2*log(abs(a)) - pi*x*sgn(a) + pi*x)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2
*log(abs(a)) + 2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a)
- pi^3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(a
bs(a)) + 2*log(abs(a))^3)^2))*sin(-3/2*pi*x*sgn(a) + 3/2*pi*x))*abs(a)^(3*
x) - 2*I*abs(a)^(3*x)*((-9*I*pi^2*x^2*sgn(a) + 18*pi*x^2*log(abs(a))*sgn(a
) + 9*I*pi^2*x^2 - 18*pi*x^2*log(abs(a)) - 18*I*x^2*log(abs(a))^2 - 6*pi*x
*sgn(a) + 6*pi*x + 12*I*x*log(abs(a)) - 4*I)*e^(3/2*I*pi*x*sgn(a) - 3/2...
```

3.537.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{a^{3\,x}\,\left(9\,x^2\,\ln\left(a\right)^2-6\,x\,\ln\left(a\right)+2\right)}{27\ln\left(a\right)^3}$$

input
```
int(a^(3*x)*x^2,x)
```

output
```
(a^(3*x)*(9*x^2*log(a)^2 - 6*x*log(a) + 2))/(27*log(a)^3)
```

3.538 $\int e^{x^2} x(1+x^2)\,dx$

3.538.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int e^{x^2} x(1+x^2)\,dx = \frac{1}{2}e^{x^2}x^2$$

output
```
1/2*exp(x^2)*x^2
```

3.538.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{x^2} x(1+x^2)\,dx = \frac{1}{2}e^{x^2}x^2$$

input
```
Integrate[E^x^2*x*(1 + x^2),x]
```

output
```
(E^x^2*x^2)/2
```

3.538.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x (x^2+1)\, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(e^{x^2} x + e^{x^2} x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} e^{x^2} x^2$$

input
```
Int[E^x^2*x*(1 + x^2),x]
```

output
```
(E^x^2*x^2)/2
```

3.538.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

3.538.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{e^{x^2} x^2}{2}$	10
derivativedivides	$\frac{e^{x^2} x^2}{2}$	10
default	$\frac{e^{x^2} x^2}{2}$	10
norman	$\frac{e^{x^2} x^2}{2}$	10
risch	$\frac{e^{x^2} x^2}{2}$	10
parallelrisch	$\frac{e^{x^2} x^2}{2}$	10
meijerg	$-\frac{(-2x^2+2)e^{x^2}}{4}+\frac{e^{x^2}}{2}$	21
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x^3}{2}+\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(\operatorname{erfi}(x)x^3+\operatorname{erfi}(x)x-\frac{e^{x^2}x^2}{\sqrt{\pi}}\right)}{2}$	48

input
```
int(exp(x^2)*x*(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x^2)*x^2
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{1}{2}\,x^2 e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x*(x^2+1),x, algorithm="fricas")
```

output
```
1/2*x^2*e^(x^2)
```

3.538.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int e^{x^2} x(1+x^2)\, dx = \frac{x^2 e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x*(x**2+1),x)
```

output
```
x**2*exp(x**2)/2
```

3.538.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int e^{x^2} x(1+x^2)\, dx = \frac{1}{2}\,(x^2-1)e^{(x^2)} + \frac{1}{2}\,e^{(x^2)}$$

input
```
integrate(exp(x^2)*x*(x^2+1),x, algorithm="maxima")
```

output
```
1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

3.538.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int e^{x^2} x(1+x^2)\, dx = \frac{1}{2}\,(x^2-1)e^{(x^2)} + \frac{1}{2}\,e^{(x^2)}$$

input
```
integrate(exp(x^2)*x*(x^2+1),x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

3.538.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{x^2\,\mathrm{e}^{x^2}}{2}$$

input
```
int(x*exp(x^2)*(x^2 + 1),x)
```

output
```
(x^2*exp(x^2))/2
```

3.539 $\int \frac{x}{(e^{-x}+e^x)^2} dx$

3.539.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{x}{(e^{-x}+e^x)^2} dx = \frac{x}{2} - \frac{x}{2(1+e^{2x})} - \frac{1}{4}\log\left(1+e^{2x}\right)$$

output
```
1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

3.539.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x}{(e^{-x}+e^x)^2} dx = \frac{e^{2x}x}{2+2e^{2x}} - \frac{1}{4}\log\left(1+e^{2x}\right)$$

input
```
Integrate[x/(E^(-x) + E^x)^2,x]
```

output
```
(E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

3.539.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2721, 2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(e^{-x}+e^x)^2} \, dx$$
$$\Big\downarrow \ 2721$$
$$\int \frac{e^{2x}x}{(e^{2x}+1)^2}dx$$
$$\Big\downarrow \ 2621$$
$$\frac{1}{2}\int \frac{1}{1+e^{2x}}dx-\frac{x}{2(e^{2x}+1)}$$
$$\Big\downarrow \ 2720$$
$$\frac{1}{4}\int \frac{e^{-2x}}{1+e^{2x}}de^{2x}-\frac{x}{2(e^{2x}+1)}$$
$$\Big\downarrow \ 47$$
$$\frac{1}{4}\left(\int e^{-2x}de^{2x}-\int \frac{1}{1+e^{2x}}de^{2x}\right)-\frac{x}{2(e^{2x}+1)}$$
$$\Big\downarrow \ 14$$
$$\frac{1}{4}\left(\log\left(e^{2x}\right)-\int \frac{1}{1+e^{2x}}de^{2x}\right)-\frac{x}{2(e^{2x}+1)}$$
$$\Big\downarrow \ 16$$
$$\frac{1}{4}\left(\log\left(e^{2x}\right)-\log\left(e^{2x}+1\right)\right)-\frac{x}{2(e^{2x}+1)}$$

input
```
Int[x/(E^(-x) + E^x)^2,x]
```

output
```
-1/2*x/(1 + E^(2*x)) + (Log[E^(2*x)] - Log[1 + E^(2*x)])/4
```

3.539.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

3.539. $\int \frac{x}{(e^{-x}+e^x)^2}\,dx$

3.539.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{x}{2}-\frac{x}{2(1+\mathrm{e}^{2x})}-\frac{\ln(1+\mathrm{e}^{2x})}{4}$	25
default	$-\frac{\ln(1+\mathrm{e}^{2x})}{4}+\frac{\mathrm{e}^{2x}x}{2+2\,\mathrm{e}^{2x}}$	26
norman	$-\frac{\ln(1+\mathrm{e}^{2x})}{4}+\frac{\mathrm{e}^{2x}x}{2+2\,\mathrm{e}^{2x}}$	26

input
```
int(x/(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{2\,xe^{(2\,x)}-\left(e^{(2\,x)}+1\right)\log\left(e^{(2\,x)}+1\right)}{4\left(e^{(2\,x)}+1\right)}$$

input
```
integrate(x/(exp(-x)+exp(x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*x*e^(2*x) - (e^(2*x) + 1)*log(e^(2*x) + 1))/(e^(2*x) + 1)
```

3.539.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = -\frac{x}{2}+\frac{x}{2+2e^{-2x}}-\frac{\log\left(1+e^{-2x}\right)}{4}$$

input
```
integrate(x/(exp(-x)+exp(x))**2,x)
```

output
```
-x/2 + x/(2 + 2*exp(-2*x)) - log(1 + exp(-2*x))/4
```

3.539.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{(e^{-x}+e^{x})^{2}}\,dx = \frac{xe^{(2\,x)}}{2\,(e^{(2\,x)}+1)} - \frac{1}{4}\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(x/(exp(-x)+exp(x))^2,x, algorithm="maxima")
```

output
```
1/2*x*e^(2*x)/(e^(2*x) + 1) - 1/4*log(e^(2*x) + 1)
```

3.539.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{x}{(e^{-x}+e^{x})^{2}}\,dx = \frac{2\,xe^{(2\,x)} - e^{(2\,x)}\log\left(e^{(2\,x)}+1\right) - \log\left(e^{(2\,x)}+1\right)}{4\,(e^{(2\,x)}+1)}$$

input
```
integrate(x/(exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
1/4*(2*x*e^(2*x) - e^(2*x)*log(e^(2*x) + 1) - log(e^(2*x) + 1))/(e^(2*x) +
 1)
```

3.539.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{(e^{-x}+e^{x})^{2}}\,dx = \frac{x\,\mathrm{e}^{2\,x}}{2\,\left(\mathrm{e}^{2\,x}+1\right)} - \frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{4}$$

input
```
int(x/(exp(-x) + exp(x))^2,x)
```

output
```
(x*exp(2*x))/(2*(exp(2*x) + 1)) - log(exp(2*x) + 1)/4
```

3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx$

3.540.1 Optimal result

Integrand size = 25, antiderivative size = 15

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

output
```
exp(x)*(-x^2+1)^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

input
```
Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

output
```
E^x*Sqrt[1 - x^2]
```

3.540.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(-x^2-x+1)}{\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \; 2726$$
$$e^x\sqrt{1-x^2}$$

input
```
Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

output
```
E^x*Sqrt[1 - x^2]
```

3.540.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.540.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

method	result	size
gosper	$-\frac{e^x(-1+x)(1+x)}{\sqrt{-x^2+1}}$	20

input
```
int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)*(-1+x)*(1+x)/(-x^2+1)^(1/2)
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{-x^2+1}e^x$$

input
```
integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-x^2 + 1)*e^x
```

3.540.6 Sympy [F]

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = -\int \left(-\frac{e^x}{\sqrt{1-x^2}}\right)dx - \int \frac{xe^x}{\sqrt{1-x^2}}\,dx - \int \frac{x^2e^x}{\sqrt{1-x^2}}\,dx$$

input
```
integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)
```

output
```
-Integral(-exp(x)/sqrt(1 - x**2), x) - Integral(x*exp(x)/sqrt(1 - x**2), x
) - Integral(x**2*exp(x)/sqrt(1 - x**2), x)
```

3.540.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = -\frac{(x^2-1)e^x}{\sqrt{x+1}\sqrt{-x+1}}$$

input
```
integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

3.540.8 Giac [F]

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \int -\frac{(x^2+x-1)e^x}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1), x)
```

3.540.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

input
```
int(-(exp(x)*(x + x^2 - 1))/(1 - x^2)^(1/2),x)
```

output
```
exp(x)*(1 - x^2)^(1/2)
```

3.541 $\int e^{-3x} \cos(2x) \, dx$

3.541.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-3x} \cos(2x) \, dx = -\frac{3}{13} e^{-3x} \cos(2x) + \frac{2}{13} e^{-3x} \sin(2x)$$

output
```
-3/13*cos(2*x)/exp(3*x)+2/13*sin(2*x)/exp(3*x)
```

3.541.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-3x} \cos(2x) \, dx = \frac{1}{13} e^{-3x} (-3\cos(2x) + 2\sin(2x))$$

input
```
Integrate[Cos[2*x]/E^(3*x),x]
```

output
```
(-3*Cos[2*x] + 2*Sin[2*x])/(13*E^(3*x))
```

3.541.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x}\cos(2x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

input
```
Int[Cos[2*x]/E^(3*x),x]
```

output
```
(-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

3.541.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.541.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{{\rm e}^{-3x}\left(-3\cos\left(2x\right)+2\sin\left(2x\right)\right)}{13}$	20
default	$-\frac{3\,{\rm e}^{-3x}\cos\left(2x\right)}{13}+\frac{2\,{\rm e}^{-3x}\sin\left(2x\right)}{13}$	22
norman	$\frac{\left(-\frac{3}{13}+\frac{3\left(\tan^{2}\left(x\right)\right)}{13}+\frac{4\tan\left(x\right)}{13}\right){\rm e}^{-3x}}{1+\tan^{2}\left(x\right)}$	28
risch	$-\frac{3\,{\rm e}^{\left(-3+2i\right)x}}{26}-\frac{i{\rm e}^{\left(-3+2i\right)x}}{13}-\frac{3\,{\rm e}^{\left(-3-2i\right)x}}{26}+\frac{i{\rm e}^{\left(-3-2i\right)x}}{13}$	36

input
```
int(cos(2*x)/exp(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/13*exp(-3*x)*(-3*cos(2*x)+2*sin(2*x))
```

3.541.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-3x}\cos(2x)\,dx = -\frac{3}{13}\cos(2x)\,e^{(-3x)} + \frac{2}{13}\,e^{(-3x)}\sin(2x)$$

input
```
integrate(cos(2*x)/exp(3*x),x, algorithm="fricas")
```

output
```
-3/13*cos(2*x)*e^(-3*x) + 2/13*e^(-3*x)*sin(2*x)
```

3.541.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{-3x}\cos(2x)\,dx = \frac{2e^{-3x}\sin(2x)}{13} - \frac{3e^{-3x}\cos(2x)}{13}$$

input
```
integrate(cos(2*x)/exp(3*x),x)
```

output
```
2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

3.541.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{1}{13}\,(3\cos(2x) - 2\sin(2x))e^{(-3x)}$$

input
```
integrate(cos(2*x)/exp(3*x),x, algorithm="maxima")
```

output
```
-1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

3.541.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x} \cos(2x) \, dx = -\frac{1}{13} (3 \cos(2x) - 2 \sin(2x)) e^{(-3x)}$$

input
```
integrate(cos(2*x)/exp(3*x),x, algorithm="giac")
```

output
```
-1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

3.541.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x} \cos(2x) \, dx = -\frac{e^{-3x} (3 \cos(2x) - 2 \sin(2x))}{13}$$

input
```
int(cos(2*x)*exp(-3*x),x)
```

output
```
-(exp(-3*x)*(3*cos(2*x) - 2*sin(2*x)))/13
```

3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$

3.542.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}+\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

output
```
-30/13*cos(1/2*x)/exp(x)^(1/3)+6/13*sin(1/2*x)/exp(x)^(1/3)
```

3.542.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=\frac{6\left(-5\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

input
```
Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

output
```
(6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))
```

3.542.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2717, 7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}dx$$

$$\Big\downarrow \text{2717}$$

$$\frac{e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)dx}{\sqrt[3]{e^x}}$$

$$\Big\downarrow \text{7281}$$

$$\frac{6e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)d\frac{x}{6}}{\sqrt[3]{e^x}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{6e^{x/3}\int \left(e^{-x/3}\cos\left(\frac{x}{2}\right)+e^{-x/3}\sin\left(\frac{x}{2}\right)\right)d\frac{x}{6}}{\sqrt[3]{e^x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6e^{x/3}\left(\frac{1}{13}e^{-x/3}\sin\left(\frac{x}{2}\right)-\frac{5}{13}e^{-x/3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt[3]{e^x}}$$

input
```
Int[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

output
```
(6*E^(x/3)*((-5*Cos[x/2])/(13*E^(x/3)) + Sin[x/2]/(13*E^(x/3))))/(E^x)^(1/
3)
```

3.542.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.542.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$\frac{-\frac{30 \cos\left(\frac{x}{2}\right)}{13}+\frac{6 \sin\left(\frac{x}{2}\right)}{13}}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$	18
default	$-\frac{30\,\mathrm{e}^{-\frac{x}{3}} \cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}} \sin\left(\frac{x}{2}\right)}{13}$	22
parts	$-\frac{30\,\mathrm{e}^{-\frac{x}{3}} \cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}} \sin\left(\frac{x}{2}\right)}{13}$	22
risch	$\frac{\left(-\frac{15}{169}-\frac{3i}{169}\right)\left((25-5i) \cos\left(\frac{x}{2}\right)+(-5+i) \sin\left(\frac{x}{2}\right)\right)}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$	26

input
```
int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
6/13/exp(x)^(1/3)*(-5*cos(1/2*x)+sin(1/2*x))
```

3.542.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=-\frac{30}{13}\,\cos\left(\frac{1}{2}\,x\right)e^{\left(-\frac{1}{3}\,x\right)}+\frac{6}{13}\,e^{\left(-\frac{1}{3}\,x\right)}\sin\left(\frac{1}{2}\,x\right)$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="fricas")
```

output
```
-30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)
```

3.542.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
```

output
```
6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))
```

3.542.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=-\frac{6}{13}\left(3\,\cos\left(\frac{1}{2}\,x\right)+2\,\sin\left(\frac{1}{2}\,x\right)\right)e^{\left(-\frac{1}{3}\,x\right)}-\frac{6}{13}\left(2\,\cos\left(\frac{1}{2}\,x\right)-3\,\sin\left(\frac{1}{2}\,x\right)\right)e^{\left(-\frac{1}{3}\,x\right)}$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="maxima")
```

output
```
-6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*si
n(1/2*x))*e^(-1/3*x)
```

3.542.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)} - \frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="giac")
```

output
```
-6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*si
n(1/2*x))*e^(-1/3*x)
```

3.542.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6\,e^{-\frac{x}{3}}\left(5\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{13}$$

input
```
int((cos(x/2) + sin(x/2))/exp(x)^(1/3),x)
```

output
```
-(6*exp(-x/3)*(5*cos(x/2) - sin(x/2)))/13
```

3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$

3.543.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)} + \frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

output
```
-4/3*cos(3/2*x)*ln(3)/(3^(3*x))^(1/4)/(4+ln(3)^2)+8/3*sin(3/2*x)/(3^(3*x))
^(1/4)/(4+ln(3)^2)
```

3.543.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\cos\left(\frac{3x}{2}\right)\log(3)-2\sin\left(\frac{3x}{2}\right)\right)}{3\sqrt[4]{27^x}\left(4+\log^2(3)\right)}$$

input
```
Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

output
```
(-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 + Log[3]^2)
)
```

3.543.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2717, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$$

$$\Big\downarrow \text{ 2717}$$

$$\frac{3^{3x/4}\int 3^{-3x/4}\cos\left(\frac{3x}{2}\right)dx}{\sqrt[4]{3^{3x}}}$$

$$\Big\downarrow \text{ 4933}$$

$$\frac{3^{3x/4}\left(\frac{8\ 3^{-\frac{3x}{4}-1}\sin\left(\frac{3x}{2}\right)}{4+\log^2(3)}-\frac{4\ 3^{-\frac{3x}{4}-1}\log(3)\cos\left(\frac{3x}{2}\right)}{4+\log^2(3)}\right)}{\sqrt[4]{3^{3x}}}$$

input
```
Int[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

output
```
(3^((3*x)/4)*((-4*3^(-1 - (3*x)/4)*Cos[(3*x)/2]*Log[3])/(4 + Log[3]^2) + (
8*3^(-1 - (3*x)/4)*Sin[(3*x)/2])/(4 + Log[3]^2)))/(3^(3*x))^(1/4)
```

3.543.3.1 Defintions of rubi rules used

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.543.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{4\left(\cos\left(\frac{3x}{2}\right)\ln(3)-2\sin\left(\frac{3x}{2}\right)\right)}{\left(27^{x}\right)^{\frac{1}{4}}\left(3\ln(3)^{2}+12\right)}$	32
risch	$-\frac{2\left(2\cos\left(\frac{3x}{2}\right)\ln(3)-4\sin\left(\frac{3x}{2}\right)\right)}{3(2i+\ln(3))(-2i+\ln(3))\left(27^{x}\right)^{\frac{1}{4}}}$	37

input
```
int(cos(3/2*x)/(3^(3*x))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4*(cos(3/2*x)*ln(3)-2*sin(3/2*x))/(27^x)^(1/4)/(3*ln(3)^2+12)
```

3.543.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.543.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^{2}+12\sqrt[4]{3^{3x}}} - \frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^{2}+12\sqrt[4]{3^{3x}}}$$

input
```
integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
```

output
```
8*sin(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4)) - 4*lo
g(3)*cos(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4))
```

3.543.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\cos\left(\frac{3}{2}\,x\right)\log\left(3\right) - 2\,\sin\left(\frac{3}{2}\,x\right)\right)}{3\left(\log\left(3\right)^{2} + 4\right)3^{\frac{3}{4}\,x}}$$

input
```
integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")
```

output
```
-4/3*(cos(3/2*x)*log(3) - 2*sin(3/2*x))/((log(3)^2 + 4)*3^(3/4*x))
```

3.543.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\frac{\cos\left(\frac{3}{2}\,x\right)\log\left(3\right)}{\log\left(3\right)^{2}+4} - \frac{2\,\sin\left(\frac{3}{2}\,x\right)}{\log\left(3\right)^{2}+4}\right)}{3\cdot 3^{\frac{3}{4}\,x}}$$

input
```
integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="giac")
```

output
```
-4/3*(cos(3/2*x)*log(3)/(log(3)^2 + 4) - 2*sin(3/2*x)/(log(3)^2 + 4))/3^(3
/4*x)
```

3.543.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{\frac{3\,\sin\left(\frac{3\,x}{2}\right)}{2} - \frac{3\,\cos\left(\frac{3\,x}{2}\right)\,\ln(3)}{4}}{3^{\frac{3\,x}{4}}\,\left(\frac{9\,\ln(3)^2}{16} + \frac{9}{4}\right)}$$

input
```
int(cos((3*x)/2)/(3^(3*x))^(1/4),x)
```

output
```
((3*sin((3*x)/2))/2 - (3*cos((3*x)/2)*log(3))/4)/(3^((3*x)/4)*((9*log(3)^2
)/16 + 9/4))
```

3.544 $\int e^{mx}\cos^2(x)\,dx$

3.544.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int e^{mx}\cos^2(x)\,dx=\frac{2e^{mx}}{m\,(4+m^2)}+\frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}$$

output
```
2*exp(m*x)/m/(m^2+4)+exp(m*x)*m*cos(x)^2/(m^2+4)+2*exp(m*x)*cos(x)*sin(x)/
(m^2+4)
```

3.544.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int e^{mx}\cos^2(x)\,dx=\frac{e^{mx}(4+m^2+m^2\cos(2x)+2m\sin(2x))}{2m\,(4+m^2)}$$

input
```
Integrate[E^(m*x)*Cos[x]^2,x]
```

output
```
(E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))
```

3.544.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\cos^2(x)\,dx$$

$$\Big\downarrow \text{4935}$$

$$\frac{2\int e^{mx}dx}{m^2+4}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2e^{mx}}{m\,(m^2+4)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

input
```
Int[E^(m*x)*Cos[x]^2,x]
```

output
```
(2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E^(m*x)*Co
s[x]*Sin[x])/(4 + m^2)
```

3.544.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

3.544.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{mx}(m^2\cos(2x)+2m\sin(2x)+m^2+4)}{2m(m^2+4)}$	37
risch	$\frac{e^{mx}}{2m}+\frac{e^{(2i+m)x}}{8i+4m}+\frac{e^{x(m-2i)}}{4m-8i}$	41
default	$\frac{e^{mx}}{2m}+\frac{m\,e^{mx}\cos(2x)}{2m^2+8}+\frac{e^{mx}\sin(2x)}{m^2+4}$	45
norman	$\frac{\frac{(m^2+2)e^{mx}}{m(m^2+4)}+\frac{(m^2+2)e^{mx}\left(\tan^4\left(\frac{x}{2}\right)\right)}{m(m^2+4)}+\frac{4\,e^{mx}\tan\left(\frac{x}{2}\right)}{m^2+4}-\frac{4\,e^{mx}\left(\tan^3\left(\frac{x}{2}\right)\right)}{m^2+4}-\frac{2(m^2-2)e^{mx}\left(\tan^2\left(\frac{x}{2}\right)\right)}{m(m^2+4)}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	122

input
```
int(exp(m*x)*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(m*x)*(m^2*cos(2*x)+2*m*sin(2*x)+m^2+4)/m/(m^2+4)
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx=\frac{2\,m\cos(x)\,e^{(mx)}\sin(x)+\left(m^2\cos(x)^2+2\right)e^{(mx)}}{m^3+4\,m}$$

input
```
integrate(exp(m*x)*cos(x)^2,x, algorithm="fricas")
```

output
```
(2*m*cos(x)*e^(m*x)*sin(x) + (m^2*cos(x)^2 + 2)*e^(m*x))/(m^3 + 4*m)
```

3.544.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.91

$$\int e^{mx}\cos^2(x)\,dx = \begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m=0 \\ -\frac{xe^{-2ix}\sin^2(x)}{4} + \frac{ixe^{-2ix}\sin(x)\cos(x)}{2} + \frac{xe^{-2ix}\cos^2(x)}{4} - \frac{e^{-2ix}\sin(x)\cos(x)}{4} + \frac{ie^{-2ix}\cos^2(x)}{2} & \text{for } m=-2i \\ -\frac{xe^{2ix}\sin^2(x)}{4} - \frac{ixe^{2ix}\sin(x)\cos(x)}{2} + \frac{xe^{2ix}\cos^2(x)}{4} - \frac{e^{2ix}\sin(x)\cos(x)}{4} - \frac{ie^{2ix}\cos^2(x)}{2} & \text{for } m=2i \\ \frac{m^2e^{mx}\cos^2(x)}{m^3+4m} + \frac{2me^{mx}\sin(x)\cos(x)}{m^3+4m} + \frac{2e^{mx}\sin^2(x)}{m^3+4m} + \frac{2e^{mx}\cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(m*x)*cos(x)**2,x)
```

output
```
Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, 0)), (-x
*exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)/2 + x*exp(-2*I*x)
*cos(x)**2/4 - exp(-2*I*x)*sin(x)*cos(x)/4 + I*exp(-2*I*x)*cos(x)**2/2, Eq
(m, -2*I)), (-x*exp(2*I*x)*sin(x)**2/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 +
x*exp(2*I*x)*cos(x)**2/4 - exp(2*I*x)*sin(x)*cos(x)/4 - I*exp(2*I*x)*cos(x
)**2/2, Eq(m, 2*I)), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*
sin(x)*cos(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x
)*cos(x)**2/(m**3 + 4*m), True))
```

3.544.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int e^{mx}\cos^2(x)\,dx = \frac{m^2\cos(2x)\,e^{(mx)} + 2\,me^{(mx)}\sin(2x) + (m^2+4)e^{(mx)}}{2\,(m^3+4\,m)}$$

input
```
integrate(exp(m*x)*cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*(m^2*cos(2*x)*e^(m*x) + 2*m*e^(m*x)*sin(2*x) + (m^2 + 4)*e^(m*x))/(m^3
 + 4*m)
```

3.544.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^{mx}\cos^2(x)\,dx = \frac{1}{2}\left(\frac{m\cos(2x)}{m^2+4} + \frac{2\sin(2x)}{m^2+4}\right)e^{(mx)} + \frac{e^{(mx)}}{2m}$$

input
```
integrate(exp(m*x)*cos(x)^2,x, algorithm="giac")
```

output
```
1/2*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4))*e^(m*x) + 1/2*e^(m*x)/m
```

3.544.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx = \frac{e^{m\,x}}{2\,m} + \frac{e^{m\,x}\,(2\,\sin(2\,x) + m\,\cos(2\,x))}{2\,(m^2+4)}$$

input
```
int(exp(m*x)*cos(x)^2,x)
```

output
```
exp(m*x)/(2*m) + (exp(m*x)*(2*sin(2*x) + m*cos(2*x)))/(2*(m^2 + 4))
```

3.545 $\int e^{mx}\sin^3(x)\,dx$

3.545.1 Optimal result

Integrand size = 10, antiderivative size = 82

$$\int e^{mx}\sin^3(x)\,dx=-\frac{6e^{mx}\cos(x)}{9+10m^2+m^4}+\frac{6e^{mx}m\sin(x)}{9+10m^2+m^4}-\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}$$

output
```
-6*exp(m*x)*cos(x)/(m^4+10*m^2+9)+6*exp(m*x)*m*sin(x)/(m^4+10*m^2+9)-3*exp
(m*x)*cos(x)*sin(x)^2/(m^2+9)+exp(m*x)*m*sin(x)^3/(m^2+9)
```

3.545.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int e^{mx}\sin^3(x)\,dx=\frac{e^{mx}(-3(9+m^2)\cos(x)+3(1+m^2)\cos(3x)-2m(-13-m^2+(1+m^2)\cos(2x))\sin(x))}{4(9+10m^2+m^4)}$$

input
```
Integrate[E^(m*x)*Sin[x]^3,x]
```

output
```
(E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 - m^2 + (1
 + m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

3.545.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4934, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx} \sin^3(x) \, dx$$

$$\Big\downarrow \text{4934}$$

$$\frac{6 \int e^{mx} \sin(x)dx}{m^2+9}+\frac{me^{mx} \sin^3(x)}{m^2+9}-\frac{3e^{mx} \sin^2(x) \cos(x)}{m^2+9}$$

$$\Big\downarrow \text{4932}$$

$$\frac{me^{mx} \sin^3(x)}{m^2+9}-\frac{3e^{mx} \sin^2(x) \cos(x)}{m^2+9}+\frac{6 \left(\frac{me^{mx} \sin(x)}{m^2+1}-\frac{e^{mx} \cos(x)}{m^2+1}\right)}{m^2+9}$$

input
```
Int[E^(m*x)*Sin[x]^3,x]
```

output
```
(-3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x)*m*Sin[x]^3)/(9 + m^2) +
(6*(-((E^(m*x)*Cos[x])/(1 + m^2)) + (E^(m*x)*m*Sin[x])/(1 + m^2)))/(9 + m^
2)
```

3.545.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

3.545.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{3\,\mathrm{e}^{m x}\left((m^2+1)\cos(3x)+\frac{(-m^3-m)\sin(3x)}{3}+(m^2+9)(m\sin(x)-\cos(x))\right)}{4(m^4+10m^2+9)}$
risch	$\frac{i\mathrm{e}^{(3i+m)x}}{24i+8m}-\frac{3i\mathrm{e}^{x(m+i)}}{8(m+i)}+\frac{3i\mathrm{e}^{x(m-i)}}{8(m-i)}-\frac{i\mathrm{e}^{x(m-3i)}}{8(m-3i)}$
default	$-\frac{3\,\mathrm{e}^{m x}\cos(x)}{4(m^2+1)}+\frac{3m\,\mathrm{e}^{m x}\sin(x)}{4(m^2+1)}+\frac{3\,\mathrm{e}^{m x}\cos(3x)}{4(m^2+9)}-\frac{m\,\mathrm{e}^{m x}\sin(3x)}{4(m^2+9)}$
norman	$\frac{-\frac{6\,\mathrm{e}^{m x}}{m^4+10m^2+9}+\frac{6\,\mathrm{e}^{m x}\left(\tan^6\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{12m\,\mathrm{e}^{m x}\tan\left(\frac{x}{2}\right)}{m^4+10m^2+9}+\frac{12m\,\mathrm{e}^{m x}\left(\tan^5\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}-\frac{6\left(2m^2+3\right)\mathrm{e}^{m x}\left(\tan^2\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}+\frac{6\left(2m^2+3\right)\mathrm{e}^{m x}\left(\tan^4\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$

input
```
int(exp(m*x)*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/4*exp(m*x)*((m^2+1)*cos(3*x)+1/3*(-m^3-m)*sin(3*x)+(m^2+9)*(m*sin(x)-cos
(x)))/(m^4+10*m^2+9)
```

3.545.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int e^{mx}\sin^3(x)\,dx = \frac{\left(m^3-(m^3+m)\cos{(x)}^2+7\,m\right)e^{(mx)}\sin{(x)}+3\left((m^2+1)\cos{(x)}^3-(m^2+3)\cos{(x)}\right)e^{(mx)}}{m^4+10\,m^2+9}$$

input
```
integrate(exp(m*x)*sin(x)^3,x, algorithm="fricas")
```

output
```
((m^3 - (m^3 + m)*cos(x)^2 + 7*m)*e^(m*x)*sin(x) + 3*((m^2 + 1)*cos(x)^3 -
 (m^2 + 3)*cos(x))*e^(m*x))/(m^4 + 10*m^2 + 9)
```

3.545.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 638, normalized size of antiderivative = 7.78

$$\int e^{mx}\sin^3(x)\,dx$$
$$= \begin{cases} \frac{xe^{-3ix}\sin^3(x)}{8} - \frac{3ixe^{-3ix}\sin^2(x)\cos(x)}{8} - \frac{3xe^{-3ix}\sin(x)\cos^2(x)}{8} + \frac{ixe^{-3ix}\cos^3(x)}{8} + \frac{7ie^{-3ix}\sin^3(x)}{24} + \frac{ie^{-3ix}\sin(x)\cos^2(x)}{4} + \\ \frac{3xe^{-ix}\sin^3(x)}{8} - \frac{3ixe^{-ix}\sin^2(x)\cos(x)}{8} + \frac{3xe^{-ix}\sin(x)\cos^2(x)}{8} - \frac{3ixe^{-ix}\cos^3(x)}{8} + \frac{5ie^{-ix}\sin^3(x)}{8} + \frac{3ie^{-ix}\sin(x)\cos^2(x)}{4} + \\ \frac{3xe^{ix}\sin^3(x)}{8} + \frac{3ixe^{ix}\sin^2(x)\cos(x)}{8} + \frac{3xe^{ix}\sin(x)\cos^2(x)}{8} + \frac{3ixe^{ix}\cos^3(x)}{8} - \frac{5ie^{ix}\sin^3(x)}{8} - \frac{3ie^{ix}\sin(x)\cos^2(x)}{4} + \frac{3e^{ix}\cos^3(}{8} \\ \frac{xe^{3ix}\sin^3(x)}{8} + \frac{3ixe^{3ix}\sin^2(x)\cos(x)}{8} - \frac{3xe^{3ix}\sin(x)\cos^2(x)}{8} - \frac{ixe^{3ix}\cos^3(x)}{8} - \frac{7ie^{3ix}\sin^3(x)}{24} - \frac{ie^{3ix}\sin(x)\cos^2(x)}{4} + \frac{e^{3ix}\cos}{8} \\ \frac{m^3e^{mx}\sin^3(x)}{m^4+10m^2+9} - \frac{3m^2e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} + \frac{7me^{mx}\sin^3(x)}{m^4+10m^2+9} + \frac{6me^{mx}\sin(x)\cos^2(x)}{m^4+10m^2+9} - \frac{9e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} - \frac{6e^{mx}\cos^3(x)}{m^4+10m^2+9} \end{cases}$$

input
```
integrate(exp(m*x)*sin(x)**3,x)
```

output
```
Piecewise((x*exp(-3*I*x)*sin(x)**3/8 - 3*I*x*exp(-3*I*x)*sin(x)**2*cos(x)/
8 - 3*x*exp(-3*I*x)*sin(x)*cos(x)**2/8 + I*x*exp(-3*I*x)*cos(x)**3/8 + 7*I
*exp(-3*I*x)*sin(x)**3/24 + I*exp(-3*I*x)*sin(x)*cos(x)**2/4 + exp(-3*I*x)
*cos(x)**3/8, Eq(m, -3*I)), (3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I*x)*s
in(x)**2*cos(x)/8 + 3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)*cos
(x)**3/8 + 5*I*exp(-I*x)*sin(x)**3/8 + 3*I*exp(-I*x)*sin(x)*cos(x)**2/4 +
3*exp(-I*x)*cos(x)**3/8, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8 + 3*I*x*exp
(I*x)*sin(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)**2/8 + 3*I*x*exp(I*x
)*cos(x)**3/8 - 5*I*exp(I*x)*sin(x)**3/8 - 3*I*exp(I*x)*sin(x)*cos(x)**2/4
 + 3*exp(I*x)*cos(x)**3/8, Eq(m, I)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*ex
p(3*I*x)*sin(x)**2*cos(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(
3*I*x)*cos(x)**3/8 - 7*I*exp(3*I*x)*sin(x)**3/24 - I*exp(3*I*x)*sin(x)*cos
(x)**2/4 + exp(3*I*x)*cos(x)**3/8, Eq(m, 3*I)), (m**3*exp(m*x)*sin(x)**3/(
m**4 + 10*m**2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9
) + 7*m*exp(m*x)*sin(x)**3/(m**4 + 10*m**2 + 9) + 6*m*exp(m*x)*sin(x)*cos(
x)**2/(m**4 + 10*m**2 + 9) - 9*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 +
 9) - 6*exp(m*x)*cos(x)**3/(m**4 + 10*m**2 + 9), True))
```

3.545.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int e^{mx}\sin^3(x)\,dx = \frac{3\,(m^2+1)\cos\left(3\,x\right)e^{(mx)} - 3\,(m^2+9)\cos\left(x\right)e^{(mx)} - (m^3+m)e^{(mx)}\sin\left(3\,x\right) + 3\,(m^3+9\,m)e^{(mx)}\sin\left(x\right)}{4\,(m^4+10\,m^2+9)}$$

input
```
integrate(exp(m*x)*sin(x)^3,x, algorithm="maxima")
```

output
```
1/4*(3*(m^2 + 1)*cos(3*x)*e^(m*x) - 3*(m^2 + 9)*cos(x)*e^(m*x) - (m^3 + m)
*e^(m*x)*sin(3*x) + 3*(m^3 + 9*m)*e^(m*x)*sin(x))/(m^4 + 10*m^2 + 9)
```

3.545.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{mx}\sin^3(x)\,dx = -\frac{1}{4}\left(\frac{m\sin\left(3\,x\right)}{m^2+9} - \frac{3\,\cos\left(3\,x\right)}{m^2+9}\right)e^{(mx)} + \frac{3}{4}\left(\frac{m\sin\left(x\right)}{m^2+1} - \frac{\cos\left(x\right)}{m^2+1}\right)e^{(mx)}$$

input
```
integrate(exp(m*x)*sin(x)^3,x, algorithm="giac")
```

output
```
-1/4*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9))*e^(m*x) + 3/4*(m*sin(x)
/(m^2 + 1) - cos(x)/(m^2 + 1))*e^(m*x)
```

3.545.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int e^{mx}\sin^3(x)\,dx = -\frac{\mathrm{e}^{m\,x}\,\left(\frac{3\,\left(\cos(x)-m\,\sin(x)\right)}{m^2+1}-\frac{3\,\cos(3\,x)-m\,\sin(3\,x)}{m^2+9}\right)}{4}$$

input
```
int(exp(m*x)*sin(x)^3,x)
```

output
```
-(exp(m*x)*((3*(cos(x) - m*sin(x)))/(m^2 + 1) - (3*cos(3*x) - m*sin(3*x))/
(m^2 + 9)))/4
```

3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$

3.546.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx=-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

output
```
-48/65*cos(1/3*x)/exp(x)^(1/2)-2/5*cos(1/3*x)^3/exp(x)^(1/2)+32/65*sin(1/3
*x)/exp(x)^(1/2)+4/5*cos(1/3*x)^2*sin(1/3*x)/exp(x)^(1/2)
```

3.546.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx=\frac{-135\cos\left(\frac{x}{3}\right)-13\cos(x)+90\sin\left(\frac{x}{3}\right)+26\sin(x)}{130\sqrt{e^x}}$$

input
```
Integrate[Cos[x/3]^3/Sqrt[E^x],x]
```

output
```
(-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E^x])
```

3.546.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2717, 4935, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$$

$$\Big\downarrow \text{2717}$$

$$\frac{e^{x/2}\int e^{-x/2}\cos^3\left(\frac{x}{3}\right)dx}{\sqrt{e^x}}$$

$$\Big\downarrow \text{4935}$$

$$\frac{e^{x/2}\left(\frac{8}{15}\int e^{-x/2}\cos\left(\frac{x}{3}\right)dx-\frac{2}{5}e^{-x/2}\cos^3\left(\frac{x}{3}\right)+\frac{4}{5}e^{-x/2}\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)\right)}{\sqrt{e^x}}$$

$$\Big\downarrow \text{4933}$$

$$\frac{e^{x/2}\left(-\frac{2}{5}e^{-x/2}\cos^3\left(\frac{x}{3}\right)+\frac{4}{5}e^{-x/2}\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)+\frac{8}{15}\left(\frac{12}{13}e^{-x/2}\sin\left(\frac{x}{3}\right)-\frac{18}{13}e^{-x/2}\cos\left(\frac{x}{3}\right)\right)\right)}{\sqrt{e^x}}$$

input
```
Int[Cos[x/3]^3/Sqrt[E^x],x]
```

output
```
(E^(x/2)*((-2*Cos[x/3]^3)/(5*E^(x/2)) + (4*Cos[x/3]^2*Sin[x/3])/(5*E^(x/2)
) + (8*((-18*Cos[x/3])/(13*E^(x/2)) + (12*Sin[x/3])/(13*E^(x/2))))/15))/Sq
rt[E^x]
```

3.546.3.1 Defintions of rubi rules used

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

3.546.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.35

method	result	size
parallelrisch	$\frac{-13\cos(x)-135\cos\left(\frac{x}{3}\right)+26\sin(x)+90\sin\left(\frac{x}{3}\right)}{130\sqrt{e^{x}}}$	28
default	$-\frac{e^{-\frac{x}{2}}\cos(x)}{10}+\frac{e^{-\frac{x}{2}}\sin(x)}{5}-\frac{27\,e^{-\frac{x}{2}}\cos\left(\frac{x}{3}\right)}{26}+\frac{9\,e^{-\frac{x}{2}}\sin\left(\frac{x}{3}\right)}{13}$	38
risch	$\frac{\left(-\frac{1}{1300}-\frac{i}{650}\right)\left(-52i e^{-ix}+65\,e^{ix}-39\,e^{-ix}+(270-540i)\cos\left(\frac{x}{3}\right)+(-180+360i)\sin\left(\frac{x}{3}\right)\right)}{\sqrt{e^{x}}}$	48

input
```
int(cos(1/3*x)^3/exp(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/130*(-13*cos(x)-135*cos(1/3*x)+26*sin(x)+90*sin(1/3*x))/exp(x)^(1/2)
```

3.546.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{4}{65}\left(13\cos\left(\frac{1}{3}x\right)^2+8\right)e^{\left(-\frac{1}{2}x\right)}\sin\left(\frac{1}{3}x\right) - \frac{2}{65}\left(13\cos\left(\frac{1}{3}x\right)^3+24\cos\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="fricas")
```

output
```
4/65*(13*cos(1/3*x)^2 + 8)*e^(-1/2*x)*sin(1/3*x) - 2/65*(13*cos(1/3*x)^3 +
 24*cos(1/3*x))*e^(-1/2*x)
```

3.546.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{32\sin^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{48\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{84\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{74\cos^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

input
```
integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
```

output
```
32*sin(x/3)**3/(65*sqrt(exp(x))) - 48*sin(x/3)**2*cos(x/3)/(65*sqrt(exp(x)
)) + 84*sin(x/3)*cos(x/3)**2/(65*sqrt(exp(x))) - 74*cos(x/3)**3/(65*sqrt(e
xp(x)))
```

3.546.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{1}{130}\left(135\cos\left(\frac{1}{3}x\right) + 13\cos(x) - 90\sin\left(\frac{1}{3}x\right) - 26\sin(x)\right)e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="maxima")
```

output
```
-1/130*(135*cos(1/3*x) + 13*cos(x) - 90*sin(1/3*x) - 26*sin(x))*e^(-1/2*x)
```

3.546.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{9}{26}\left(3\cos\left(\frac{1}{3}x\right) - 2\sin\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)} - \frac{1}{10}(\cos(x) - 2\sin(x))e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="giac")
```

output
```
-9/26*(3*cos(1/3*x) - 2*sin(1/3*x))*e^(-1/2*x) - 1/10*(cos(x) - 2*sin(x))*
e^(-1/2*x)
```

3.546.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{e^{-\frac{x}{2}}\left(\frac{8\cos\left(\frac{x}{3}\right)^3}{5} - \frac{16\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)^2}{5} + \frac{192\cos\left(\frac{x}{3}\right)}{65} - \frac{128\sin\left(\frac{x}{3}\right)}{65}\right)}{4}$$

input
```
int(cos(x/3)^3/exp(x)^(1/2),x)
```

output
```
-(exp(-x/2)*((192*cos(x/3))/65 - (128*sin(x/3))/65 - (16*cos(x/3)^2*sin(x/
3))/5 + (8*cos(x/3)^3)/5))/4
```

3.547 $\int e^{2x}\cos^2(x)\sin^2(x)\,dx$

3.547.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = \frac{e^{2x}}{16} - \frac{1}{80}e^{2x}\cos(4x) - \frac{1}{40}e^{2x}\sin(4x)$$

output
```
1/16*exp(2*x)-1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)
```

3.547.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}e^{2x}(-5+\cos(4x)+2\sin(4x))$$

input
```
Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

output
```
-1/80*(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))
```

3.547.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}\sin^2(x)\cos^2(x)\,dx$$

$$\big\downarrow \text{4972}$$

$$\int \left(\frac{e^{2x}}{8}-\frac{1}{8}e^{2x}\cos(4x)\right)dx$$

$$\big\downarrow \text{2009}$$

$$\frac{e^{2x}}{16}-\frac{1}{40}e^{2x}\sin(4x)-\frac{1}{80}e^{2x}\cos(4x)$$

input
```
Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

output
```
E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

3.547.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.547.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

method	result
parallelrisch	$-\frac{e^{2x}(-5+\cos(4x)+2\sin(4x))}{80}$
default	$\frac{e^{2x}}{16}-\frac{e^{2x}\cos(4x)}{80}-\frac{e^{2x}\sin(4x)}{40}$
risch	$\frac{e^{2x}}{16}-\frac{e^{(2+4i)x}}{160}+\frac{ie^{(2+4i)x}}{80}-\frac{e^{(2-4i)x}}{160}-\frac{ie^{(2-4i)x}}{80}$
norman	$\frac{-\frac{e^{2x}\tan\left(\frac{x}{2}\right)}{5}+\frac{3e^{2x}\left(\tan^2\left(\frac{x}{2}\right)\right)}{5}+\frac{7e^{2x}\left(\tan^3\left(\frac{x}{2}\right)\right)}{5}-\frac{e^{2x}\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\frac{7e^{2x}\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}+\frac{3e^{2x}\left(\tan^6\left(\frac{x}{2}\right)\right)}{5}+\frac{e^{2x}\left(\tan^7\left(\frac{x}{2}\right)\right)}{5}+\frac{e^{2x}(\tan}{20}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

input

```
int(exp(2*x)*cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/80*exp(2*x)*(-5+cos(4*x)+2*sin(4*x))
```

3.547.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{10}\left(2\cos(x)^3-\cos(x)\right)e^{(2x)}\sin(x) - \frac{1}{20}\left(2\cos(x)^4-2\cos(x)^2-1\right)e^{(2x)}$$

input

```
integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output

```
-1/10*(2*cos(x)^3 - cos(x))*e^(2*x)*sin(x) - 1/20*(2*cos(x)^4 - 2*cos(x)^2
 - 1)*e^(2*x)
```

3.547.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58.

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = \frac{e^{2x}\sin^4(x)}{20} + \frac{e^{2x}\sin^3(x)\cos(x)}{10} + \frac{e^{2x}\sin^2(x)\cos^2(x)}{5} - \frac{e^{2x}\sin(x)\cos^3(x)}{10} + \frac{e^{2x}\cos^4(x)}{20}$$

input
```
integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
```

output
```
exp(2*x)*sin(x)**4/20 + exp(2*x)*sin(x)**3*cos(x)/10 + exp(2*x)*sin(x)**2*
cos(x)**2/5 - exp(2*x)*sin(x)*cos(x)**3/10 + exp(2*x)*cos(x)**4/20
```

3.547.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}\cos(4x)e^{(2x)} - \frac{1}{40}e^{(2x)}\sin(4x) + \frac{1}{16}e^{(2x)}$$

input
```
integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/80*cos(4*x)*e^(2*x) - 1/40*e^(2*x)*sin(4*x) + 1/16*e^(2*x)
```

3.547.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}(\cos(4x) + 2\sin(4x))e^{(2x)} + \frac{1}{16}e^{(2x)}$$

input
```
integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
-1/80*(cos(4*x) + 2*sin(4*x))*e^(2*x) + 1/16*e^(2*x)
```

3.547.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{e^{2x}\,(\cos(4x)+2\sin(4x)-5)}{80}$$

input
```
int(exp(2*x)*cos(x)^2*sin(x)^2,x)
```

output
```
-(exp(2*x)*(cos(4*x) + 2*sin(4*x) - 5))/80
```

3.548 $\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx$

3.548.1 Optimal result

Integrand size = 22, antiderivative size = 36

$$\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx = \frac{e^{3x}}{24} - \frac{1}{120} e^{3x} \cos(6x) - \frac{1}{60} e^{3x} \sin(6x)$$

output
```
1/24*exp(3*x)-1/120*exp(3*x)*cos(6*x)-1/60*exp(3*x)*sin(6*x)
```

3.548.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx = -\frac{1}{120} e^{3x} (-5 + \cos(6x) + 2 \sin(6x))$$

input
```
Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

output
```
-1/120*(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))
```

3.548.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x} \sin^2\left(\frac{3x}{2}\right) \cos^2\left(\frac{3x}{2}\right) dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{e^{3x}}{8} - \frac{1}{8}e^{3x}\cos(6x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{3x}}{24} - \frac{1}{60}e^{3x}\sin(6x) - \frac{1}{120}e^{3x}\cos(6x)$$

input
```
Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

output
```
E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

3.548.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

3.548.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

method	result
parallelrisch	$-\frac{e^{3x}(-5+2\sin(6x)+\cos(6x))}{120}$
risch	$\frac{e^{3x}}{24}-\frac{e^{(3+6i)x}}{240}+\frac{ie^{(3+6i)x}}{120}-\frac{e^{(3-6i)x}}{240}-\frac{ie^{(3-6i)x}}{120}$
default	$-\frac{4(3\cos(x)+6\sin(x))e^{3x}(\cos^5(x))}{45}+\frac{2(3\cos(x)+4\sin(x))e^{3x}(\cos^3(x))}{15}-\frac{(3\cos(x)+2\sin(x))e^{3x}\cos(x)}{20}+\frac{e^{3x}}{20}$
norman	$\frac{-\frac{2e^{3x}\tan\left(\frac{3x}{4}\right)}{15}+\frac{2e^{3x}\left(\tan^2\left(\frac{3x}{4}\right)\right)}{5}+\frac{14e^{3x}\left(\tan^3\left(\frac{3x}{4}\right)\right)}{15}-\frac{e^{3x}\left(\tan^4\left(\frac{3x}{4}\right)\right)}{3}-\frac{14e^{3x}\left(\tan^5\left(\frac{3x}{4}\right)\right)}{15}+\frac{2e^{3x}\left(\tan^6\left(\frac{3x}{4}\right)\right)}{5}+\frac{2e^{3x}\left(\tan^7\left(\frac{3x}{4}\right.\right.}{15}}{\left(1+\tan^2\left(\frac{3x}{4}\right)\right)^4}$

input
```
int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*exp(3*x)*(-5+2*sin(6*x)+cos(6*x))
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{15}\left(2\cos\left(\frac{3}{2}x\right)^3-\cos\left(\frac{3}{2}x\right)\right)e^{(3x)}\sin\left(\frac{3}{2}x\right) - \frac{1}{30}\left(2\cos\left(\frac{3}{2}x\right)^4-2\cos\left(\frac{3}{2}x\right)^2-1\right)e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="fricas")
```

output
```
-1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*cos(3/2*x
)^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)
```

3.548.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(29) = 58.

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = \frac{e^{3x}\sin^4\left(\frac{3x}{2}\right)}{30} + \frac{e^{3x}\sin^3\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{15} + \frac{2e^{3x}\sin^2\left(\frac{3x}{2}\right)\cos^2\left(\frac{3x}{2}\right)}{15} - \frac{e^{3x}\sin\left(\frac{3x}{2}\right)\cos^3\left(\frac{3x}{2}\right)}{15} + \frac{e^{3x}\cos^4\left(\frac{3x}{2}\right)}{30}$$

input
```
integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

output
```
exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15 + 2*exp(3
*x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)*cos(3*x/2)**3/15
+ exp(3*x)*cos(3*x/2)**4/30
```

3.548.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}\cos(6x)\,e^{(3x)} - \frac{1}{60}\,e^{(3x)}\sin(6x) + \frac{1}{24}\,e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="maxima")
```

output
```
-1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)
```

3.548.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}\,(\cos(6x) + 2\sin(6x))e^{(3x)} + \frac{1}{24}\,e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="giac")
```

output
```
-1/120*(cos(6*x) + 2*sin(6*x))*e^(3*x) + 1/24*e^(3*x)
```

3.548.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx=-\frac{\mathrm{e}^{3\,x}\,\left(\cos\left(6\,x\right)+2\,\sin\left(6\,x\right)-5\right)}{120}$$

input
```
int(cos((3*x)/2)^2*sin((3*x)/2)^2*exp(3*x),x)
```

output
```
-(exp(3*x)*(cos(6*x) + 2*sin(6*x) - 5))/120
```

3.549 $\int e^{mx}\tan^2(x)\,dx$

3.549.1 Optimal result

Integrand size = 10, antiderivative size = 58

$$\int e^{mx}\tan^2(x)\,dx = -\frac{e^{mx}}{m}+\frac{4e^{(2i+m)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},-e^{2ix}\right)}{2i+m}$$

output
```
-exp(m*x)/m+4*exp((2*I+m)*x)*hypergeom([2, 1-1/2*I*m],[2-1/2*I*m],-exp(2*I
*x))/(2*I+m)
```

3.549.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int e^{mx}\tan^2(x)\,dx = \frac{e^{mx}\left(-1+\frac{ie^{2ix}m^2\,\text{Hypergeometric2F1}\left(1,1-\frac{im}{2},2-\frac{im}{2},-e^{2ix}\right)}{2i+m}-im\,\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)+m\tan(x)\right)}{m}$$

input
```
Integrate[E^(m*x)*Tan[x]^2,x]
```

output
```
(E^(m*x)*(-1 + (I*E^((2*I)*x)*m^2*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I
/2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[1, (-1/2*I)*m, 1 -
 (I/2)*m, -E^((2*I)*x)] + m*Tan[x]))/m
```

3.549.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\tan^2(x)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-\int \left(e^{mx}-\frac{4e^{mx}}{1+e^{2ix}}+\frac{4e^{mx}}{(1+e^{2ix})^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{mx}\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{4e^{mx}\text{Hypergeometric2F1}\left(2,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{e^{mx}}{m}$$

input
```
Int[E^(m*x)*Tan[x]^2,x]
```

output
```
-(E^(m*x)/m) + (4*E^(m*x)*Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I/2)*m, -E
^((2*I)*x)])/m - (4*E^(m*x)*Hypergeometric2F1[2, (-1/2*I)*m, 1 - (I/2)*m,
-E^((2*I)*x)])/m
```

3.549.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

3.549.4 Maple [F]

$$\int e^{mx} \left(\tan^{2}\left(x\right)\right) dx$$

input
```
int(exp(m*x)*tan(x)^2,x)
```

output
```
int(exp(m*x)*tan(x)^2,x)
```

3.549.5 Fricas [F]

$$\int e^{mx} \tan^2(x) \, dx = \int e^{(mx)} \tan\left(x\right)^{2} \, dx$$

input
```
integrate(exp(m*x)*tan(x)^2,x, algorithm="fricas")
```

output
```
integral(e^(m*x)*tan(x)^2, x)
```

3.549.6 Sympy [F]

$$\int e^{mx} \tan^2(x) \, dx = \int e^{mx} \tan^{2}\left(x\right) dx$$

input
```
integrate(exp(m*x)*tan(x)**2,x)
```

output
```
Integral(exp(m*x)*tan(x)**2, x)
```

3.549.7 Maxima [F]

$$\int e^{mx}\tan^2(x)\,dx=\int e^{(mx)}\tan(x)^2\,dx$$

input
```
integrate(exp(m*x)*tan(x)^2,x, algorithm="maxima")
```

output
```
-((m^4 + 20*m^2 + 64)*cos(4*x)^2*e^(m*x) - 4*(m^4 + 12*m^2 - 64)*cos(2*x)^
2*e^(m*x) + (m^4 + 20*m^2 + 64)*e^(m*x)*sin(4*x)^2 - 4*(m^4 + 12*m^2 - 64)
*e^(m*x)*sin(2*x)^2 - 16*(11*m^2 - 16)*cos(2*x)*e^(m*x) + 8*(5*m^3 - 16*m)
*e^(m*x)*sin(2*x) + 2*(8*(m^2 + 16)*cos(2*x)*e^(m*x) + 4*(m^3 + 16*m)*e^(m
*x)*sin(2*x) + (m^4 - 28*m^2 + 64)*e^(m*x))*cos(4*x) + (m^4 - 76*m^2 + 64)
*e^(m*x) - 16*(m^6 + 20*m^4 + (m^6 + 20*m^4 + 64*m^2)*cos(4*x)^2 + 4*(m^6
+ 20*m^4 + 64*m^2)*cos(2*x)^2 + (m^6 + 20*m^4 + 64*m^2)*sin(4*x)^2 + 4*(m^
6 + 20*m^4 + 64*m^2)*sin(4*x)*sin(2*x) + 4*(m^6 + 20*m^4 + 64*m^2)*sin(2*x
)^2 + 64*m^2 + 2*(m^6 + 20*m^4 + 64*m^2 + 2*(m^6 + 20*m^4 + 64*m^2)*cos(2*
x))*cos(4*x) + 4*(m^6 + 20*m^4 + 64*m^2)*cos(2*x))*integrate(-(6*m*cos(6*x
)*e^(m*x) + 18*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m
*x)*sin(6*x) - 3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x)
 + 6*m*e^(m*x))/(m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 +
64)*cos(4*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*si
n(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*sin(4*x)^2 + 18*(m^4 + 20*m^2 + 64)*sin(4
*x)*sin(2*x) + 9*(m^4 + 20*m^2 + 64)*sin(2*x)^2 + 20*m^2 + 2*(m^4 + 20*m^2
 + 3*(m^4 + 20*m^2 + 64)*cos(4*x) + 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*c
os(6*x) + 6*(m^4 + 20*m^2 + 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x)
+ 6*(m^4 + 20*m^2 + 64)*cos(2*x) + 6*((m^4 + 20*m^2 + 64)*sin(4*x) + (m^4
+ 20*m^2 + 64)*sin(2*x))*sin(6*x) + 64), x) - 8*((m^3 + 16*m)*cos(2*x)*...
```

3.549.8 Giac [F]

$$\int e^{mx}\tan^2(x)\,dx=\int e^{(mx)}\tan(x)^2\,dx$$

input
```
integrate(exp(m*x)*tan(x)^2,x, algorithm="giac")
```

output
```
integrate(e^(m*x)*tan(x)^2, x)
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int e^{mx} \tan^2(x)\,dx = \int e^{m\,x}\,\tan(x)^2\,dx$$

input
```
int(exp(m*x)*tan(x)^2,x)
```

output
```
int(exp(m*x)*tan(x)^2, x)
```

3.550 $\int e^{mx} \csc^2(x)\,dx$

3.550.1 Optimal result

Integrand size = 10, antiderivative size = 45

$$\int e^{mx}\csc^2(x)\,dx = -\frac{4e^{(2i+m)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{2i+m}$$

output
```
-4*exp((2*I+m)*x)*hypergeom([2, 1-1/2*I*m],[2-1/2*I*m],exp(2*I*x))/(2*I+m)
```

3.550.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.00

$$\int e^{mx}\csc^2(x)\,dx$$
$$= \frac{e^{mx}\left(e^{2ix}m\,\text{Hypergeometric2F1}\left(1,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)+(2i+m)\left(-i\cot(x)+\text{Hypergeometric2F1}\left(1,-\right.\right.\right.}{-2+im}$$

input
```
Integrate[E^(m*x)*Csc[x]^2,x]
```

output
```
(E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, E^(
(2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1, (-1/2*I)*m, 1 -
(I/2)*m, E^((2*I)*x)])))/(-2 + I*m)
```

3.550.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\csc^2(x)\,dx$$

$$\Big\downarrow \text{4953}$$

$$-\frac{4e^{(m+2i)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{m+2i}$$

input
```
Int[E^(m*x)*Csc[x]^2,x]
```

output
```
(-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I
)*x)])/(2*I + m)
```

3.550.3.1 Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

3.550.4 Maple [F]

$$\int \frac{e^{mx}}{\sin(x)^2}dx$$

input
```
int(exp(m*x)/sin(x)^2,x)
```

output
```
int(exp(m*x)/sin(x)^2,x)
```

3.550.5 Fricas [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

input
```
integrate(exp(m*x)/sin(x)^2,x, algorithm="fricas")
```

output
```
integral(-e^(m*x)/(cos(x)^2 - 1), x)
```

3.550.6 Sympy [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{mx}}{\sin^2(x)}\,dx$$

input
```
integrate(exp(m*x)/sin(x)**2,x)
```

output
```
Integral(exp(m*x)/sin(x)**2, x)
```

3.550.7 Maxima [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

input
```
integrate(exp(m*x)/sin(x)^2,x, algorithm="maxima")
```

output
```
4*(2*(m^3 + 16*m)*cos(2*x)^2*e^(m*x) + 2*(m^3 + 16*m)*e^(m*x)*sin(2*x)^2 -
 (m^3 + 64*m)*cos(2*x)*e^(m*x) + 2*(5*m^2 - 16)*e^(m*x)*sin(2*x) - ((m^3 +
 16*m)*cos(2*x)*e^(m*x) - 2*(m^2 + 16)*e^(m*x)*sin(2*x) - 24*m*e^(m*x))*co
s(4*x) + 24*m*e^(m*x) - 4*(m^5 + 20*m^3 + (m^5 + 20*m^3 + 64*m)*cos(4*x)^2
 + 4*(m^5 + 20*m^3 + 64*m)*cos(2*x)^2 + (m^5 + 20*m^3 + 64*m)*sin(4*x)^2 -
 4*(m^5 + 20*m^3 + 64*m)*sin(4*x)*sin(2*x) + 4*(m^5 + 20*m^3 + 64*m)*sin(2
*x)^2 + 2*(m^5 + 20*m^3 - 2*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*cos(4*x
) - 4*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*integrate(-(6*m*cos(6*x)*e^(m
*x) - 18*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m*x)*si
n(6*x) + 3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x) - 6*m
*e^(m*x))/(m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*co
s(4*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*sin(6*x)
^2 + 9*(m^4 + 20*m^2 + 64)*sin(4*x)^2 - 18*(m^4 + 20*m^2 + 64)*sin(4*x)*si
n(2*x) + 9*(m^4 + 20*m^2 + 64)*sin(2*x)^2 + 20*m^2 - 2*(m^4 + 20*m^2 + 3*(
m^4 + 20*m^2 + 64)*cos(4*x) - 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(6*x
) + 6*(m^4 + 20*m^2 - 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x) - 6*(m
^4 + 20*m^2 + 64)*cos(2*x) - 6*((m^4 + 20*m^2 + 64)*sin(4*x) - (m^4 + 20*m
^2 + 64)*sin(2*x))*sin(6*x) + 64), x) - (2*(m^2 + 16)*cos(2*x)*e^(m*x) + (
m^3 + 16*m)*e^(m*x)*sin(2*x) + 4*(m^2 - 8)*e^(m*x))*sin(4*x))/(m^4 + (m^4
+ 20*m^2 + 64)*cos(4*x)^2 + 4*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 2...
```

3.550.8 Giac [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

input
```
integrate(exp(m*x)/sin(x)^2,x, algorithm="giac")
```

output
```
integrate(e^(m*x)/sin(x)^2, x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{m\,x}}{\sin(x)^2}\,dx$$

input
```
int(exp(m*x)/sin(x)^2,x)
```

output
```
int(exp(m*x)/sin(x)^2, x)
```

3.551 $\int e^{mx}\sec^3(x)\,dx$

3.551.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int e^{mx}\sec^3(x)\,dx=\frac{8e^{(3i+m)x}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}(3-im),\frac{1}{2}(5-im),-e^{2ix}\right)}{3i+m}$$

output
```
8*exp((3*I+m)*x)*hypergeom([3, 3/2-1/2*I*m],[5/2-1/2*I*m],-exp(2*I*x))/(3*
I+m)
```

3.551.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int e^{mx}\sec^3(x)\,dx=\frac{1}{2}e^{mx}\left(2e^{ix}(-i+m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{im}{2},\frac{3}{2}-\frac{im}{2},-e^{2ix}\right)+\sec(x)(-m+\tan(x))\right)$$

input
```
Integrate[E^(m*x)*Sec[x]^3,x]
```

output
```
(E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m, 3/2 - (I/
2)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x])))/2
```

3.551.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\sec^3(x)\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{1}{2}(m^2+1)\int e^{mx}\sec(x)dx-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

$$\Big\downarrow \text{4951}$$

$$\frac{(m^2+1)\,e^{(m+i)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-im),\frac{1}{2}(3-im),-e^{2ix}\right)}{m+i}-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

input
```
Int[E^(m*x)*Sec[x]^3,x]
```

output
```
(E^((I + m)*x)*(1 + m^2)*Hypergeometric2F1[1, (1 - I*m)/2, (3 - I*m)/2, -E
^((2*I)*x)])/(I + m) - (E^(m*x)*m*Sec[x])/2 + (E^(m*x)*Sec[x]*Tan[x])/2
```

3.551.3.1 Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.551.4 Maple [F]

$$\int \frac{\mathrm{e}^{mx}}{\cos(x)^3} dx$$

input
```
int(exp(m*x)/cos(x)^3,x)
```

output
```
int(exp(m*x)/cos(x)^3,x)
```

3.551.5 Fricas [F]

$$\int e^{mx} \sec^3(x) \, dx = \int \frac{e^{(mx)}}{\cos(x)^3} \, dx$$

input
```
integrate(exp(m*x)/cos(x)^3,x, algorithm="fricas")
```

output
```
integral(e^(m*x)/cos(x)^3, x)
```

3.551.6 Sympy [F]

$$\int e^{mx} \sec^3(x) \, dx = \int \frac{e^{mx}}{\cos^3(x)} \, dx$$

input
```
integrate(exp(m*x)/cos(x)**3,x)
```

output
```
Integral(exp(m*x)/cos(x)**3, x)
```

3.551.7 Maxima [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

input
```
integrate(exp(m*x)/cos(x)^3,x, algorithm="maxima")
```

output
```
8*(48*m*cos(x)*e^(m*x) + 6*(m^2 - 15)*e^(m*x)*sin(x) + ((m^3 + 25*m)*cos(3
*x)*e^(m*x) + 48*m*cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2
 - 15)*e^(m*x)*sin(x))*cos(6*x) + 3*((m^3 + 25*m)*cos(3*x)*e^(m*x) + 48*m*
cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2 - 15)*e^(m*x)*sin(
x))*cos(4*x) + (3*(m^3 + 25*m)*cos(2*x)*e^(m*x) + 9*(m^2 + 25)*e^(m*x)*sin
(2*x) + (m^3 + 25*m)*e^(m*x))*cos(3*x) + 18*(8*m*cos(x)*e^(m*x) + (m^2 - 1
5)*e^(m*x)*sin(x))*cos(2*x) - 6*(m^4 + (m^4 + 34*m^2 + 225)*cos(6*x)^2 + 9
*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 9*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^
4 + 34*m^2 + 225)*sin(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 18*(m^4
 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 9*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 3
4*m^2 + 2*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(4*x) + 3*(m^4 + 34*m^
2 + 225)*cos(2*x) + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 22
5)*cos(2*x) + 225)*cos(4*x) + 6*(m^4 + 34*m^2 + 225)*cos(2*x) + 6*((m^4 +
34*m^2 + 225)*sin(4*x) + (m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225)*in
tegrate(((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x) + ((m^2 - 15)*cos(
x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*cos(8*x) + 4*((m^2 - 15)*cos(x)*e^(m*x) -
 8*m*e^(m*x)*sin(x))*cos(6*x) + 6*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)
*sin(x))*cos(4*x) + 4*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*cos
(2*x) + (8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(8*x) + 4*(8*m
*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(6*x) + 6*(8*m*cos(x)*e...
```

3.551.8 Giac [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

input
```
integrate(exp(m*x)/cos(x)^3,x, algorithm="giac")
```

output
```
integrate(e^(m*x)/cos(x)^3, x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{m\,x}}{\cos(x)^3}\,dx$$

input
```
int(exp(m*x)/cos(x)^3,x)
```

output
```
int(exp(m*x)/cos(x)^3, x)
```

3.552 $\int \frac{e^x}{1+\cos(x)}\,dx$

3.552.1 Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{e^x}{1+\cos(x)}\,dx = (1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

output
```
(1-I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-exp(I*x))
```

3.552.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+\cos(x)}\,dx = (1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

input
```
Integrate[E^x/(1 + Cos[x]),x]
```

output
```
(1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

3.552.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4957, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\cos(x)+1}\,dx$$

$$\Big\downarrow \text{ 4957}$$

$$\frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{ 4951}$$

$$(1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

input
```
Int[E^x/(1 + Cos[x]),x]
```

output
```
(1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

3.552.3.1 Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 4957
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]
^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && IL
tQ[n, 0]
```

3.552.4 Maple [F]

$$\int \frac{e^x}{\cos(x)+1}dx$$

input
```
int(exp(x)/(cos(x)+1),x)
```

output
```
int(exp(x)/(cos(x)+1),x)
```

3.552.5 Fricas [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x, algorithm="fricas")
```

output
```
integral(e^x/(cos(x) + 1), x)
```

3.552.6 Sympy [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x)
```

output
```
Integral(exp(x)/(cos(x) + 1), x)
```

3.552.7 Maxima [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x, algorithm="maxima")
```

output
```
-2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)
```

3.552.8 Giac [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x, algorithm="giac")
```

output
```
integrate(e^x/(cos(x) + 1), x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
int(exp(x)/(cos(x) + 1),x)
```

output
```
int(exp(x)/(cos(x) + 1), x)
```

3.553 $\int \frac{e^x}{1-\cos(x)}\,dx$

3.553.1 Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{e^x}{1-\cos(x)}\,dx = (-1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

output
```
(-1+I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],exp(I*x))
```

3.553.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(26) = 52.

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \frac{e^x}{1-\cos(x)}\,dx$$
$$= \frac{(1+i)e^x\sin\left(\frac{x}{2}\right)\left((1-i)\cos\left(\frac{x}{2}\right)+(1+i)\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\sin\left(\frac{x}{2}\right)+e^{ix}\,\text{Hypergeome}\right.}{-1+\cos(x)}$$

input
```
Integrate[E^x/(1 - Cos[x]),x]
```

output
```
((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2F1[-I, 1,
 1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(
I*x)]*Sin[x/2]))/(-1 + Cos[x])
```

3.553.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4958, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-\cos(x)} \, dx$$
$$\Big\downarrow \text{4958}$$
$$\frac{1}{2} \int e^x \csc^2\left(\frac{x}{2}\right) dx$$
$$\Big\downarrow \text{4953}$$
$$(-1+i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

input
```
Int[E^x/(1 - Cos[x]),x]
```

output
```
(-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]
```

3.553.3.1 Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

rule 4958
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Sin[d/2 + e*(x/2)]
^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f + g, 0] && IL
tQ[n, 0]
```

3.553.4 Maple [F]

$$\int \frac{e^x}{1-\cos(x)}dx$$

input
```
int(exp(x)/(1-cos(x)),x)
```

output
```
int(exp(x)/(1-cos(x)),x)
```

3.553.5 Fricas [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-cos(x)),x, algorithm="fricas")
```

output
```
integral(-e^x/(cos(x) - 1), x)
```

3.553.6 Sympy [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = -\int \frac{e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-cos(x)),x)
```

output
```
-Integral(exp(x)/(cos(x) - 1), x)
```

3.553.7 Maxima [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos{(x)}-1}\,dx$$

input
```
integrate(exp(x)/(1-cos(x)),x, algorithm="maxima")
```

output
```
2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

3.553.8 Giac [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos{(x)}-1}\,dx$$

input
```
integrate(exp(x)/(1-cos(x)),x, algorithm="giac")
```

output
```
integrate(-e^x/(cos(x) - 1), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1-\cos(x)}\,dx = -\int \frac{e^x}{\cos{(x)}-1}\,dx$$

input
```
int(-exp(x)/(cos(x) - 1),x)
```

output
```
-int(exp(x)/(cos(x) - 1), x)
```

3.554 $\int \frac{e^x}{1+\sin(x)} \, dx$

3.554.1 Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{e^x}{1+\sin(x)} \, dx = (-1+i)e^{(1-i)x} \text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)$$

output
```
(-1+I)*exp((1-I)*x)*hypergeom([2, 1+I],[2+I],-I/exp(I*x))
```

3.554.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{e^x}{1+\sin(x)} \, dx = \frac{2e^x \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)} - (1-i)(1 - (1-i) \text{Hypergeometric2F1}(-i,1,1-i,i\cos(x)-\sin(x)))(\cosh(x) + \sinh(x))$$

input
```
Integrate[E^x/(1 + Sin[x]),x]
```

output
```
(2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hypergeometr
ic2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[x])
```

3.554.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4956, 4952}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sin(x)+1} \, dx$$

$$\Big\downarrow \text{4956}$$

$$\frac{1}{2} \int e^x \csc^2\left(\frac{x}{2}+\frac{\pi}{4}\right) dx$$

$$\Big\downarrow \text{4952}$$

$$(-1-i) e^{(1+i) x} \text{Hypergeometric2F1}\left(1-i,2,2-i,i e^{i x}\right)$$

input
```
Int[E^x/(1 + Sin[x]),x]
```

output
```
(-1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, I*E^(I*x)]
```

3.554.3.1 Defintions of rubi rules used

rule 4952
```
Int[Csc[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_
))), x_Symbol] :> Simp[(-2*I)^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b
*x))/(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)),
 1 + n/2 - I*b*c*(Log[F]/(2*e)), E^(2*I*k*Pi)*E^(2*I*(d + e*x))], x] /; Fre
eQ[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

rule 4956
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4
*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[
f^2 - g^2, 0] && ILtQ[n, 0]
```

3.554.4 Maple [F]

$$\int \frac{e^x}{\sin(x) + 1} dx$$

input
```
int(exp(x)/(sin(x)+1),x)
```

output
```
int(exp(x)/(sin(x)+1),x)
```

3.554.5 Fricas [F]

$$\int \frac{e^x}{1 + \sin(x)} dx = \int \frac{e^x}{\sin(x) + 1} dx$$

input
```
integrate(exp(x)/(1+sin(x)),x, algorithm="fricas")
```

output
```
integral(e^x/(sin(x) + 1), x)
```

3.554.6 Sympy [F]

$$\int \frac{e^x}{1 + \sin(x)} dx = \int \frac{e^x}{\sin(x) + 1} dx$$

input
```
integrate(exp(x)/(1+sin(x)),x)
```

output
```
Integral(exp(x)/(sin(x) + 1), x)
```

3.554.7 Maxima [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^x
/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x)
 + 1)
```

3.554.8 Giac [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+sin(x)),x, algorithm="giac")
```

output
```
integrate(e^x/(sin(x) + 1), x)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}\,dx$$

input
```
int(exp(x)/(sin(x) + 1),x)
```

output
```
int(exp(x)/(sin(x) + 1), x)
```

3.555 $\int \frac{e^x}{1-\sin(x)}\,dx$

3.555.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{e^x}{1-\sin(x)}\,dx = (1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

output
```
(1+I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-I*exp(I*x))
```

3.555.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{e^x}{1-\sin(x)}\,dx = \frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)} + (1+i)(1 - (1+i)\,\text{Hypergeometric2F1}(-i,1,1-i,-i\cos(x)+\sin(x)))(\cosh(x) + \sinh(x))$$

input
```
Integrate[E^x/(1 - Sin[x]),x]
```

output
```
(2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hypergeometr
ic2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Sinh[x])
```

3.555.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4956, 4950}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-\sin(x)} \, dx$$

$$\Big\downarrow \text{4956}$$

$$\frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{4950}$$

$$(1+i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

input
```
Int[E^x/(1 - Sin[x]),x]
```

output
```
(1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]
```

3.555.3.1 Defintions of rubi rules used

rule 4950
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

rule 4956
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4
*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[
f^2 - g^2, 0] && ILtQ[n, 0]
```

3.555.4 Maple [F]

$$\int \frac{\mathrm{e}^{x}}{-\sin \left(x \right)+1}d x$$

input
```
int(exp(x)/(-sin(x)+1),x)
```

output
```
int(exp(x)/(-sin(x)+1),x)
```

3.555.5 Fricas [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^{x}}{\sin\left(x\right) - 1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x, algorithm="fricas")
```

output
```
integral(-e^x/(sin(x) - 1), x)
```

3.555.6 Sympy [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = -\int \frac{e^{x}}{\sin\left(x\right) - 1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x)
```

output
```
-Integral(exp(x)/(sin(x) - 1), x)
```

3.555.7 Maxima [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x, algorithm="maxima")
```

output
```
2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x/
(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1)
```

3.555.8 Giac [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x, algorithm="giac")
```

output
```
integrate(-e^x/(sin(x) - 1), x)
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx$$

input
```
int(-exp(x)/(sin(x) - 1),x)
```

output
```
-int(exp(x)/(sin(x) - 1), x)
```

3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)} \, dx$

3.556.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)} \, dx = -\frac{e^x\sin(x)}{1-\cos(x)}$$

output
```
-exp(x)*sin(x)/(1-cos(x))
```

3.556.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)} \, dx = \frac{e^x\sin(x)}{-1+\cos(x)}$$

input
```
Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

output
```
(E^x*Sin[x])/(-1 + Cos[x])
```

3.556.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$$
$$\Big\downarrow \; 2726$$
$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

input
```
Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

output
```
-((E^x*Sin[x])/(1 - Cos[x]))
```

3.556.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.556.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$-\frac{e^{x}}{\tan\left(\frac{x}{2}\right)}$	11
risch	$-i e^{x}-\frac{2 i e^{x}}{e^{i x}-1}$	21
norman	$\frac{-e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-e^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	33

input
```
int(exp(x)*(-sin(x)+1)/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)/tan(1/2*x)
```

3.556.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)+1)e^x}{\sin(x)}$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)*e^x/sin(x)
```

3.556.6 Sympy [F]

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = \int \frac{(\sin(x)-1)\,e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)
```

output
```
Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)
```

3.556.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{2\,e^x\sin(x)}{\cos(x)^2+\sin(x)^2-2\,\cos(x)+1}$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="maxima")
```

output
```
-2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="giac")
```

output
```
-e^x/tan(1/2*x)
```

3.556.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)\,\mathrm{e}^x$$

input
```
int((exp(x)*(sin(x) - 1))/(cos(x) - 1),x)
```

output
```
-cot(x/2)*exp(x)
```

3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$

3.557.1 Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = (-2+2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)+\frac{e^x\sin(x)}{1-\cos(x)}$$

output
```
(-2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],exp(I*x))+exp(x)*sin(x)/(1-
cos(x))
```

3.557.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. 2(41) = 82.

Time = 0.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$$
$$= \frac{2e^x\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+2i\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1+i)e^{ix}\,\text{Hypergeometric2F1}\left(1\right.\right.}{(-1+\cos(x))\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}$$

input
```
Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

output
```
(2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]
*Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Si
n[x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2] + Sin[x/2])^2)
```

3.557.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4966, 2726, 4964, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\sin(x)+1)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{4966}$$

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}dx+2\int \frac{e^x\sin(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{2726}$$

$$2\int \frac{e^x\sin(x)}{1-\cos(x)}dx-\frac{e^x\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \text{4964}$$

$$2\int e^x\cot\left(\frac{x}{2}\right)dx-\frac{e^x\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \text{4943}$$

$$-\frac{e^x\sin(x)}{1-\cos(x)}-2i\int \left(\frac{2e^x}{1-e^{ix}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^x\sin(x)}{1-\cos(x)}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\right)$$

input
```
Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]) - (E^x*Sin[
x])/(1 - Cos[x])
```

3.557.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4964
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Simp[f^n   Int[F^(c*(a
+ b*x))*Cot[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
&& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4966
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
 f*i, 0]
```

3.557.4 Maple [F]

$$\int \frac{\mathrm{e}^{x}(\sin(x)+1)}{1-\cos(x)}dx$$

input
```
int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

output
```
int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

3.557.5 Fricas [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx=\int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="fricas")
```

output
```
integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

3.557.6 Sympy [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx=-\int \frac{e^x}{\cos(x)-1}\,dx-\int \frac{e^x\sin(x)}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

output
```
-Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x) - 1), x
)
```

3.557.7 Maxima [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx=\int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="maxima")
```

output
```
2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x
) + 1)
```

3.557.8 Giac [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="giac")
```

output
```
integrate(-(sin(x) + 1)*e^x/(cos(x) - 1), x)
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{e^x(\sin(x)+1)}{\cos(x)-1}\,dx$$

input
```
int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1),x)
```

output
```
int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1), x)
```

3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx$

3.558.1 Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x\sin(x)}{1+\cos(x)}$$

output
```
exp(x)*sin(x)/(1+cos(x))
```

3.558.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x\sin(x)}{1+\cos(x)}$$

input
```
Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

output
```
(E^x*Sin[x])/(1 + Cos[x])
```

3.558.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\sin(x)+1)}{\cos(x)+1} \, dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{e^x \sin(x)}{\cos(x)+1}$$

input
```
Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

output
```
(E^x*Sin[x])/(1 + Cos[x])
```

3.558.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.558.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

method	result	size
norman	$e^x \tan\left(\frac{x}{2}\right)$	8
parallelrisch	$e^x \tan\left(\frac{x}{2}\right)$	8
risch	$-i e^x + \frac{2i e^x}{e^{ix}+1}$	21

input
```
int(exp(x)*(sin(x)+1)/(cos(x)+1),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*tan(1/2*x)
```

3.558.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x \sin{(x)}}{\cos{(x)}+1}$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
e^x*sin(x)/(cos(x) + 1)
```

3.558.6 Sympy [F]

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \int \frac{(\sin{(x)}+1)\,e^x}{\cos{(x)}+1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)
```

output
```
Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{2\,e^x \sin{(x)}}{\cos{(x)}^2+\sin{(x)}^2+2\,\cos{(x)}+1}$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

3.558.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = e^x\tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="giac")
```

output
```
e^x*tan(1/2*x)
```

3.558.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)\,\mathrm{e}^x$$

input
```
int((exp(x)*(sin(x) + 1))/(cos(x) + 1),x)
```

output
```
tan(x/2)*exp(x)
```

3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$

3.559.1 Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = (2-2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)-\frac{e^x\sin(x)}{1+\cos(x)}$$

output
```
(2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-exp(I*x))-exp(x)*sin(x)/(1+
cos(x))
```

3.559.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\frac{2e^x\cos\left(\frac{x}{2}\right)\left(2i\cos\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)-(1+i)e^{ix}\cos\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\right.}{1+\cos(x)}$$

input
```
Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

output
```
(-2*E^x*Cos[x/2]*((2*I)*Cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]
 - (1 + I)*E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I, 2 - I, -E^(I*x)] -
 Sin[x/2]))/(1 + Cos[x])
```

3.559.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4966, 2726, 4963, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\sin(x))}{\cos(x)+1}dx$$

$$\Big\downarrow \text{4966}$$

$$\int \frac{e^x(\sin(x)+1)}{\cos(x)+1}dx-2\int \frac{e^x\sin(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2\int \frac{e^x\sin(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{4963}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2\int e^x\tan\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{4942}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2i\int \left(\frac{2e^x}{1+e^{ix}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)\right)$$

input
```
Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]) + (E^x*Sin
[x])/(1 + Cos[x])
```

3.559.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 4963
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Simp[f^n   Int[F^(c*(a
+ b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
&& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4966
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
 f*i, 0]
```

3.559.4 Maple [F]

$$\int \frac{\mathrm{e}^{x}(-\sin(x)+1)}{\cos(x)+1}dx$$

input
```
int(exp(x)*(-sin(x)+1)/(cos(x)+1),x)
```

output
```
int(exp(x)*(-sin(x)+1)/(cos(x)+1),x)
```

3.559.5 Fricas [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

3.559.6 Sympy [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\int \left(-\frac{e^x}{\cos(x)+1}\right)dx - \int \frac{e^x\sin(x)}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

output
```
-Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(x) + 1),
x)
```

3.559.7 Maxima [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
-2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2
+ sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(
x) + 1)
```

3.559.8 Giac [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="giac")
```

output
```
integrate(-(sin(x) - 1)*e^x/(cos(x) + 1), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\int \frac{e^x\,(\sin(x)-1)}{\cos(x)+1}\,dx$$

input
```
int(-(exp(x)*(sin(x) - 1))/(cos(x) + 1),x)
```

output
```
-int((exp(x)*(sin(x) - 1))/(cos(x) + 1), x)
```

3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)} \, dx$

3.560.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)} \, dx = (2+2i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right) - \frac{e^x\cos(x)}{1-\sin(x)}$$

output
```
(2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-I*exp(I*x))-exp(x)*cos(x)/(
1-sin(x))
```

3.560.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)} \, dx = \frac{1}{2}(-1+\cos(x))\csc^2\left(\frac{x}{2}\right)\left(-\frac{e^x\left((1-2i)+(1+2i)\cot\left(\frac{x}{2}\right)\right)}{-1+\cot\left(\frac{x}{2}\right)} + 4i\,\text{Hypergeometric2F1}(-i,1,1-i,-i\cos(x)+\sin(x))(\cosh(x)+\sinh(x))\right)$$

input
```
Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

output
```
((-1 + Cos[x])*Csc[x/2]^2*(-((E^x*((1 - 2*I) + (1 + 2*I)*Cot[x/2]))/(-1 +
Cot[x/2])) + (4*I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]]*(
Cosh[x] + Sinh[x])))/2
```

3.560.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4965, 2726, 4962, 25, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)} \, dx$$

$$\Big\downarrow \text{4965}$$

$$\int \frac{e^x(\cos(x)+1)}{1-\sin(x)}dx-2\int \frac{e^x\cos(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2\int \frac{e^x\cos(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{4962}$$

$$2\int -e^x\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx+\frac{e^x\cos(x)}{1-\sin(x)}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2\int e^x\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{4942}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2i\int \left(\frac{2e^x}{1+e^{\frac{1}{2}i(2x+\pi)}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-ie^{ix}\right)\right)$$

input
```
Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x)]) + (E^x
*Cos[x])/(1 - Sin[x])
```

3.560.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 4962
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Simp[g^n   Int[F^(c*(a
+ b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d
, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4965
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
 f*i, 0]
```

3.560.4 Maple [F]

$$\int \frac{\mathrm{e}^{x}(1-\cos(x))}{-\sin(x)+1}dx$$

input
```
int(exp(x)*(1-cos(x))/(-sin(x)+1),x)
```

output
```
int(exp(x)*(1-cos(x))/(-sin(x)+1),x)
```

3.560.5 Fricas [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="fricas")
```

output
```
integral((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

3.560.6 Sympy [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)\,e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

output
```
Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)
```

3.560.7 Maxima [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="maxima")
```

output
```
2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^
x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x
) + 1)
```

3.560.8 Giac [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="giac")
```

output
```
integrate((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{e^x\,(\cos(x)-1)}{\sin(x)-1}\,dx$$

input
```
int((exp(x)*(cos(x) - 1))/(sin(x) - 1),x)
```

output
```
int((exp(x)*(cos(x) - 1))/(sin(x) - 1), x)
```

3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)} \, dx$

3.561.1 Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)} \, dx = \frac{e^x \cos(x)}{1-\sin(x)}$$

output
```
exp(x)*cos(x)/(1-sin(x))
```

3.561.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)} \, dx = -\frac{e^x \cos(x)}{-1+\sin(x)}$$

input
```
Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

output
```
-((E^x*Cos[x])/(-1 + Sin[x]))
```

3.561.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\cos(x)+1)}{1-\sin(x)} \, dx$$
$$\Big\downarrow \; 2726$$
$$\frac{e^x\cos(x)}{1-\sin(x)}$$

input
```
Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

output
```
(E^x*Cos[x])/(1 - Sin[x])
```

3.561.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.561.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

method	result	size
parallelrisch	$-\frac{e^{x}\left(1+\tan\left(\frac{x}{2}\right)\right)}{\tan\left(\frac{x}{2}\right)-1}$	19
risch	$-i e^{x}+\frac{2\,e^{x}}{e^{i x}-i}$	21
norman	$\frac{-e^{x}\tan\left(\frac{x}{2}\right)-e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)-e^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-e^{x}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(\tan\left(\frac{x}{2}\right)-1\right)}$	53

input
```
int(exp(x)*(cos(x)+1)/(-sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)*(1+tan(1/2*x))/(tan(1/2*x)-1)
```

3.561.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{(\cos(x)+1)e^x + e^x\sin(x)}{\cos(x)-\sin(x)+1}$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="fricas")
```

output
```
((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

3.561.6 Sympy [F]

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx - \int \frac{e^x\cos(x)}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)
```

output
```
-Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x) - 1), x
)
```

3.561.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{2\cos(x)e^x}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="maxima")
```

output
```
2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.561.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\tan\left(\frac{1}{2}x\right)+e^x}{\tan\left(\frac{1}{2}x\right)-1}$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="giac")
```

output
```
-(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)
```

3.561.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{\mathrm{e}^x\left(-1+\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)}{\mathrm{e}^{x\,1\mathrm{i}}-\mathrm{i}}$$

input
```
int(-(exp(x)*(cos(x) + 1))/(sin(x) - 1),x)
```

output
```
-(exp(x)*(exp(x*1i)*1i - 1))/(exp(x*1i) - 1i)
```

3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx$

3.562.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = (-2-2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,ie^{ix}\right)+\frac{e^x\cos(x)}{1+\sin(x)}$$

output
```
(-2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],I*exp(I*x))+exp(x)*cos(x)/(
1+sin(x))
```

3.562.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \frac{1}{2}(1+\cos(x))\sec^2\left(\frac{x}{2}\right)\left(-4i\,\text{Hypergeometric2F1}(-i,1,1-i,i\cos(x)-\sin(x))(\cosh(x)+\sinh(x))+\frac{e^x\left((-1+2i)+(1+2i)\tan\left(\frac{x}{2}\right)\right)}{1+\tan\left(\frac{x}{2}\right)}\right)$$

input
```
Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

output
```
((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x]
- Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 + 2*I)*Tan[x/2]))/(1
 + Tan[x/2])))/2
```

3.562.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4965, 2726, 4962, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\cos(x)+1)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{4965}$$

$$\int \frac{e^x(1-\cos(x))}{\sin(x)+1}dx+2\int \frac{e^x\cos(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{2726}$$

$$2\int \frac{e^x\cos(x)}{\sin(x)+1}dx-\frac{e^x\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{4962}$$

$$2\int e^x\cot\left(\frac{x}{2}+\frac{\pi}{4}\right)dx-\frac{e^x\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{4943}$$

$$-\frac{e^x\cos(x)}{\sin(x)+1}-2i\int \left(\frac{2e^x}{1-e^{\frac{1}{2}i(2x+\pi)}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^x\cos(x)}{\sin(x)+1}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,ie^{ix}\right)\right)$$

input
```
Int[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, I*E^(I*x)]) - (E^x*Co
s[x])/(1 + Sin[x])
```

3.562.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4962
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Simp[g^n   Int[F^(c*(a
+ b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d
, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4965
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
 f*i, 0]
```

3.562.4 Maple [F]

$$\int \frac{e^{x}(\cos (x)+1)}{\sin (x)+1}dx$$

input
```
int(exp(x)*(cos(x)+1)/(sin(x)+1),x)
```

output
```
int(exp(x)*(cos(x)+1)/(sin(x)+1),x)
```

3.562.5 Fricas [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="fricas")
```

output
```
integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

3.562.6 Sympy [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)\,e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

output
```
Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)
```

3.562.7 Maxima [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e
^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(
x) + 1)
```

3.562.8 Giac [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="giac")
```

output
```
integrate((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{e^x\,(\cos(x)+1)}{\sin(x)+1}\,dx$$

input
```
int((exp(x)*(cos(x) + 1))/(sin(x) + 1),x)
```

output
```
int((exp(x)*(cos(x) + 1))/(sin(x) + 1), x)
```

3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx$

3.563.1 Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

output
```
-exp(x)*cos(x)/(1+sin(x))
```

3.563.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

input
```
Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
```

output
```
-((E^x*Cos[x])/(1 + Sin[x]))
```

3.563.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\cos(x))}{\sin(x)+1}\,dx$$

$$\Big\downarrow \text{2726}$$

$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

input
```
Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
```

output
```
-((E^x*Cos[x])/(1 + Sin[x]))
```

3.563.3.1 Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

3.563.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
parallelrisch	$\frac{e^x(\tan(\frac{x}{2})-1)}{1+\tan(\frac{x}{2})}$	18
risch	$-ie^x-\frac{2\,e^x}{i+e^{ix}}$	21
norman	$\frac{e^x\tan(\frac{x}{2})+e^x(\tan^3(\frac{x}{2}))-e^x(\tan^2(\frac{x}{2}))-e^x}{(1+\tan^2(\frac{x}{2}))(1+\tan(\frac{x}{2}))}$	51

input
```
int(exp(x)*(1-cos(x))/(sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*(tan(1/2*x)-1)/(1+tan(1/2*x))
```

3.563.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{(\cos(x)+1)e^x - e^x\sin(x)}{\cos(x)+\sin(x)+1}$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="fricas")
```

output
```
-((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)
```

3.563.6 Sympy [F]

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\int \left(-\frac{e^x}{\sin(x)+1}\right)dx - \int \frac{e^x\cos(x)}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

output
```
-Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(x) + 1),
x)
```

3.563.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{2\cos(x)e^x}{\cos(x)^2+\sin(x)^2+2\sin(x)+1}$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

3.563.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = \frac{e^x \tan\left(\frac{1}{2}\,x\right) - e^x}{\tan\left(\frac{1}{2}\,x\right) + 1}$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="giac")
```

output
```
(e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)
```

3.563.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\mathrm{e}^x\,1\mathrm{i} - \frac{2\,\mathrm{e}^x}{\mathrm{e}^{x\,1\mathrm{i}} + 1\mathrm{i}}$$

input
```
int(-(exp(x)*(cos(x) - 1))/(sin(x) + 1),x)
```

output
```
- exp(x)*1i - (2*exp(x))/(exp(x*1i) + 1i)
```

3.564 $\int e^x x \cos(x) \, dx$

3.564.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)$$

output
```
1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

3.564.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x (x \cos(x) + (-1 + x) \sin(x))$$

input
```
Integrate[E^x*x*Cos[x],x]
```

output
```
(E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

3.564.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \cos(x) \, dx$$

$$\Big\downarrow \; 4969$$

$$-\int \left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right) dx + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

input
```
Int[E^x*x*Cos[x],x]
```

output
```
(E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

3.564.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.564.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{\mathrm{e}^{x}\left(\left(-1+x\right)\sin\left(x\right)+x\cos\left(x\right)\right)}{2}$	16
default	$\frac{\mathrm{e}^{x} x \cos\left(x\right)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right) \mathrm{e}^{x} \sin\left(x\right)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)\left(-1+i+2x\right)\mathrm{e}^{\left(1+i\right)x}+\left(\frac{1}{8}+\frac{i}{8}\right)\left(-1-i+2x\right)\mathrm{e}^{\left(1-i\right)x}$	36
norman	$\frac{\mathrm{e}^{x} x \tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^{x} x}{2}-\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x} x \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	45

input
```
int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*((-1+x)*sin(x)+x*cos(x))
```

3.564.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^{x} x \cos(x)\, dx = \frac{1}{2}\, x \cos\left(x\right) e^{x} + \frac{1}{2}\,(x-1)e^{x} \sin\left(x\right)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

3.564.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{x} x \cos(x)\, dx = \frac{xe^{x} \sin\left(x\right)}{2} + \frac{xe^{x} \cos\left(x\right)}{2} - \frac{e^{x} \sin\left(x\right)}{2}$$

input
```
integrate(exp(x)*x*cos(x),x)
```

output
```
x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

3.564.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x\cos(x)\,dx = \frac{1}{2}\,x\cos\left(x\right)e^x + \frac{1}{2}\,(x-1)e^x\sin\left(x\right)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

3.564.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^x x\cos(x)\,dx = \frac{1}{2}\,(x\cos\left(x\right) + (x-1)\sin\left(x\right))e^x$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

output
```
1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.564.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x\cos(x)\,dx = \frac{\mathrm{e}^x\,\left(x\,\cos\left(x\right)-\sin\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

input
```
int(x*exp(x)*cos(x),x)
```

output
```
(exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.565 $\int e^x x^2 \sin(x)\,dx$

3.565.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) - \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x x^2\sin(x)$$

output
```
-1/2*exp(x)*cos(x)+exp(x)*x*cos(x)-1/2*exp(x)*x^2*cos(x)-1/2*exp(x)*sin(x)
+1/2*exp(x)*x^2*sin(x)
```

3.565.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = \frac{1}{2}e^x\left(-(-1+x)^2\cos(x) + \left(-1+x^2\right)\sin(x)\right)$$

input
```
Integrate[E^x*x^2*Sin[x],x]
```

output
```
(E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

3.565.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^2 \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-2 \int -\frac{1}{2} x (e^x \cos(x) - e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\int x (e^x \cos(x) - e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{2010}$$

$$\int (e^x x \cos(x) - e^x x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x) - \frac{1}{2} e^x \sin(x) + e^x x \cos(x) - \frac{1}{2} e^x \cos(x)$$

input
```
Int[E^x*x^2*Sin[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (
E^x*x^2*Sin[x])/2
```

3.565.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.565.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.44

method	result	size
parallelrisch	$-\frac{e^{x}\left(\left(-1+x\right)\cos\left(x\right)-\sin\left(x\right)\left(1+x\right)\right)\left(-1+x\right)}{2}$	22
default	$\left(-\frac{1}{2}x^{2}+x-\frac{1}{2}\right)e^{x}\cos\left(x\right)+\left(\frac{x^{2}}{2}-\frac{1}{2}\right)e^{x}\sin\left(x\right)$	27
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^{2}+ix-x-i\right)e^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^{2}-ix-x+i\right)e^{(1-i)x}$	48
norman	$\frac{e^{x}x+e^{x}x^{2}\tan\left(\frac{x}{2}\right)-\frac{e^{x}x^{2}}{2}-e^{x}\tan\left(\frac{x}{2}\right)+\frac{e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-e^{x}x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{e^{x}x^{2}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	80

input
```
int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(x)*((-1+x)*cos(x)-sin(x)*(1+x))*(-1+x)
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^{x}x^{2}\sin(x)\,dx=-\frac{1}{2}\left(x^{2}-2\,x+1\right)\cos\left(x\right)e^{x}+\frac{1}{2}\left(x^{2}-1\right)e^{x}\sin\left(x\right)$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
```

output
```
-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

3.565.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int e^x x^2 \sin(x) \, dx = \frac{x^2 e^x \sin(x)}{2} - \frac{x^2 e^x \cos(x)}{2} + x e^x \cos(x) - \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

input
```
integrate(exp(x)*x**2*sin(x),x)
```

output
```
x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin
(x)/2 - exp(x)*cos(x)/2
```

3.565.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x) \, dx = -\frac{1}{2} \left(x^2 - 2x + 1\right) \cos(x) e^x + \frac{1}{2} \left(x^2 - 1\right) e^x \sin(x)$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x) \, dx = -\frac{1}{2} \left(\left(x^2 - 2x + 1\right) \cos(x) - \left(x^2 - 1\right) \sin(x) \right) e^x$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

output
```
-1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

3.565.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.42

$$\int e^x x^2 \sin(x)\, dx = \frac{e^x (x-1)\ (\cos(x) + \sin(x) - x\ \cos(x) + x\ \sin(x))}{2}$$

input
```
int(x^2*exp(x)*sin(x),x)
```

output
```
(exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

3.566 $\int e^{-3x}x^2\sin(x)\,dx$

3.566.1 Optimal result

Integrand size = 11, antiderivative size = 75

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{13}{250}e^{-3x}\cos(x) - \frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{9}{250}e^{-3x}\sin(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3x}x^2\sin(x)$$

output
```
-13/250*cos(x)/exp(3*x)-3/25*x*cos(x)/exp(3*x)-1/10*x^2*cos(x)/exp(3*x)-9/
250*sin(x)/exp(3*x)-4/25*x*sin(x)/exp(3*x)-3/10*x^2*sin(x)/exp(3*x)
```

3.566.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int e^{-3x}x^2\sin(x)\,dx = \frac{1}{250}e^{-3x}\left(-\left(\left(13+30x+25x^2\right)\cos(x)\right)-\left(9+40x+75x^2\right)\sin(x)\right)$$

input
```
Integrate[(x^2*Sin[x])/E^(3*x),x]
```

output
```
(-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(250*E^(3*x)
)
```

3.566.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x} x^2 \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-2 \int -\frac{1}{10} x \left(e^{-3x} \cos(x) + 3 e^{-3x} \sin(x)\right) dx - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5} \int x \left(e^{-3x} \cos(x) + 3 e^{-3x} \sin(x)\right) dx - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x)$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{5} \int \left(e^{-3x} x \cos(x) + 3 e^{-3x} x \sin(x)\right) dx - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x) + \frac{1}{5} \left(-\frac{9}{50} e^{-3x} \sin(x) - \frac{4}{5} e^{-3x} x \sin(x) - \frac{13}{50} e^{-3x} \cos(x) - \frac{3}{5} e^{-3x} x \cos(x)\right)$$

input
```
Int[(x^2*Sin[x])/E^(3*x),x]
```

output
```
-1/10*(x^2*Cos[x])/E^(3*x) - (3*x^2*Sin[x])/(10*E^(3*x)) + ((-13*Cos[x])/(
50*E^(3*x)) - (3*x*Cos[x])/(5*E^(3*x)) - (9*Sin[x])/(50*E^(3*x)) - (4*x*Si
n[x])/(5*E^(3*x)))/5
```

3.566.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.566.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

method	result
parallelrisch	$-\frac{\mathrm{e}^{-3x}\left(\left(x^{2}+\frac{6}{5}x+\frac{13}{25}\right)\cos\left(x\right)+3\sin\left(x\right)\left(x^{2}+\frac{8}{15}x+\frac{3}{25}\right)\right)}{10}$
default	$\left(-\frac{1}{10}x^{2}-\frac{3}{25}x-\frac{13}{250}\right)\mathrm{e}^{-3x}\cos\left(x\right)+\left(-\frac{3}{10}x^{2}-\frac{4}{25}x-\frac{9}{250}\right)\mathrm{e}^{-3x}\sin\left(x\right)$
risch	$\left(-\frac{1}{500}+\frac{3i}{500}\right)\left(25x^{2}+5ix+15x+3i+4\right)\mathrm{e}^{\left(-3+i\right)x}+\left(-\frac{1}{500}-\frac{3i}{500}\right)\left(25x^{2}-5ix+15x-3i+\right.$
norman	$\frac{\left(-\frac{13}{250}-\frac{3x}{25}-\frac{x^{2}}{10}+\frac{13\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{250}-\frac{8x\tan\left(\frac{x}{2}\right)}{25}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{25}-\frac{3x^{2}\tan\left(\frac{x}{2}\right)}{5}+\frac{x^{2}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{10}-\frac{9\tan\left(\frac{x}{2}\right)}{125}\right)\mathrm{e}^{-3x}}{1+\tan^{2}\left(\frac{x}{2}\right)}$

input
```
int(x^2*sin(x)/exp(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/10*exp(-3*x)*((x^2+6/5*x+13/25)*cos(x)+3*sin(x)*(x^2+8/15*x+3/25))
```

3.566.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(25\,x^2+30\,x+13\right)\cos\left(x\right)e^{(-3\,x)} - \frac{1}{250}\left(75\,x^2+40\,x+9\right)e^{(-3\,x)}\sin\left(x\right)$$

input
```
integrate(x^2*sin(x)/exp(3*x),x, algorithm="fricas")
```

output
```
-1/250*(25*x^2 + 30*x + 13)*cos(x)*e^(-3*x) - 1/250*(75*x^2 + 40*x + 9)*e^
(-3*x)*sin(x)
```

3.566.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{3x^2e^{-3x}\sin\left(x\right)}{10} - \frac{x^2e^{-3x}\cos\left(x\right)}{10} - \frac{4xe^{-3x}\sin\left(x\right)}{25} - \frac{3xe^{-3x}\cos\left(x\right)}{25} - \frac{9e^{-3x}\sin\left(x\right)}{250} - \frac{13e^{-3x}\cos\left(x\right)}{250}$$

input
```
integrate(x**2*sin(x)/exp(3*x),x)
```

output
```
-3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(-3*x)*sin
(x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/250 - 13*exp(-3*x)*c
os(x)/250
```

3.566.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{(-3\,x)}$$

input
```
integrate(x^2*sin(x)/exp(3*x),x, algorithm="maxima")
```

output
```
-1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

3.566.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{(-3\,x)}$$

input
```
integrate(x^2*sin(x)/exp(3*x),x, algorithm="giac")
```

output
```
-1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

3.566.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.52

$$\int e^{-3x}x^2\sin(x)\,dx$$
$$= -\frac{\mathrm{e}^{-3\,x}\,\left(13\,\cos\left(x\right)+9\,\sin\left(x\right)+25\,x^2\,\cos\left(x\right)+75\,x^2\,\sin\left(x\right)+30\,x\,\cos\left(x\right)+40\,x\,\sin\left(x\right)\right)}{250}$$

input
```
int(x^2*exp(-3*x)*sin(x),x)
```

output
```
-(exp(-3*x)*(13*cos(x) + 9*sin(x) + 25*x^2*cos(x) + 75*x^2*sin(x) + 30*x*c
os(x) + 40*x*sin(x)))/250
```

3.567 $\int e^{x/2}x^2\cos^3(x)\,dx$

3.567.1 Optimal result

Integrand size = 15, antiderivative size = 187

$$\int e^{x/2}x^2\cos^3(x)\,dx = -\frac{132}{125}e^{x/2}\cos(x)$$
$$+\frac{18}{25}e^{x/2}x\cos(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)-\frac{428e^{x/2}\cos(3x)}{50653}+\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{24}{125}e^{x/2}\sin(x)-$$

output
```
-132/125*exp(1/2*x)*cos(x)+18/25*exp(1/2*x)*x*cos(x)+48/185*exp(1/2*x)*x^2
*cos(x)+2/37*exp(1/2*x)*x^2*cos(x)^3-428/50653*exp(1/2*x)*cos(3*x)+70/1369
*exp(1/2*x)*x*cos(3*x)-24/125*exp(1/2*x)*sin(x)-24/25*exp(1/2*x)*x*sin(x)+
96/185*exp(1/2*x)*x^2*sin(x)+12/37*exp(1/2*x)*x^2*cos(x)^2*sin(x)-792/5065
3*exp(1/2*x)*sin(3*x)-24/1369*exp(1/2*x)*x*sin(3*x)
```

3.567.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{e^{x/2}(151959(-88+60x+25x^2)\cos(x)+125(-856+5180x+1369x^2)\cos(3x)+30}{12663250}$$

input
```
Integrate[E^(x/2)*x^2*Cos[x]^3,x]
```

output
```
(E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*x + 1369*
x^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750*(-264 - 296*x + 1
369*x^2)*Sin[3*x]))/12663250
```

3.567.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4969, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x/2}x^2\cos^3(x)\,dx$$

$$\Big\downarrow \text{4969}$$

$$-2\int \frac{2}{185}x\left(5e^{x/2}\cos^3(x)+30e^{x/2}\sin(x)\cos^2(x)+24e^{x/2}\cos(x)+48e^{x/2}\sin(x)\right)dx+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{185}\int x\left(5e^{x/2}\cos^3(x)+30e^{x/2}\sin(x)\cos^2(x)+24e^{x/2}\cos(x)+48e^{x/2}\sin(x)\right)dx+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{2010}$$

$$-\frac{4}{185}\int \left(5e^{x/2}x\cos^3(x)+30e^{x/2}x\sin(x)\cos^2(x)+24e^{x/2}x\cos(x)+48e^{x/2}x\sin(x)\right)dx+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{4}{185}\left(\frac{304668e^{x/2}\sin(x)}{34225}+\frac{8139}{185}e^{x/2}x\sin(x)+\frac{1020e^{x/2}\sin(3x)}{1369}+\frac{15}{37}e^{x/2}x\sin(3x)-\frac{20e^{x/2}\cos^3(x)}{1369}+\frac{10}{37}e^{x/2}x\,\text{co}\right.$$

input
```
Int[E^(x/2)*x^2*Cos[x]^3,x]
```

output
```
(48*E^(x/2)*x^2*Cos[x])/185 + (2*E^(x/2)*x^2*Cos[x]^3)/37 + (96*E^(x/2)*x^
2*Sin[x])/185 + (12*E^(x/2)*x^2*Cos[x]^2*Sin[x])/37 - (4*((1671924*E^(x/2)
*Cos[x])/34225 - (6198*E^(x/2)*x*Cos[x])/185 - (20*E^(x/2)*Cos[x]^3)/1369
+ (10*E^(x/2)*x*Cos[x]^3)/37 + (540*E^(x/2)*Cos[3*x])/1369 - (90*E^(x/2)*x
*Cos[3*x])/37 + (304668*E^(x/2)*Sin[x])/34225 + (8139*E^(x/2)*x*Sin[x])/18
5 - (120*E^(x/2)*Cos[x]^2*Sin[x])/1369 + (60*E^(x/2)*x*Cos[x]^2*Sin[x])/37
 + (1020*E^(x/2)*Sin[3*x])/1369 + (15*E^(x/2)*x*Sin[3*x])/37))/185
```

3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.567.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

method	result
parallelrisch	$\frac{3\left(\frac{5\left(x^{2}+\frac{140}{37}x-\frac{856}{1369}\right)\cos(3x)}{111}+\frac{10\left(x^{2}-\frac{8}{37}x-\frac{264}{1369}\right)\sin(3x)}{37}+\left(x^{2}+\frac{12}{5}x-\frac{88}{25}\right)\cos(x)+2\sin(x)\left(x^{2}-\frac{8}{5}x-\frac{8}{25}\right)\right)\mathrm{e}^{\frac{x}{2}}}{10}$
default	$\frac{\left(\frac{2}{37}x^{2}+\frac{280}{1369}x-\frac{1712}{50653}\right)\mathrm{e}^{\frac{x}{2}}\cos(3x)}{4}-\frac{\left(-\frac{12}{37}x^{2}+\frac{96}{1369}x+\frac{3168}{50653}\right)\mathrm{e}^{\frac{x}{2}}\sin(3x)}{4}+\frac{3\left(\frac{2}{5}x^{2}+\frac{24}{25}x-\frac{176}{125}\right)\mathrm{e}^{\frac{x}{2}}\cos(x)}{4}-\frac{3\left(-\frac{4}{5}x^{2}+\frac{32}{25}x\right.}{}$
risch	$\left(\frac{1}{202612}-\frac{3i}{101306}\right)\left(1369x^{2}+888ix-148x-96i-280\right)\mathrm{e}^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{3}{500}-\frac{3i}{250}\right)\left(25x^{2}+40ix\right.$

input
```
int(exp(1/2*x)*x^2*cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/10*(5/111*(x^2+140/37*x-856/1369)*cos(3*x)+10/37*(x^2-8/37*x-264/1369)*s
in(3*x)+(x^2+12/5*x-88/25)*cos(x)+2*sin(x)*(x^2-8/5*x-8/25))*exp(1/2*x)
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{12}{6331625}\left(125\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2+273800\,x^2-497280\,x-93056\right)e^{\left(\frac{1}{2}\,x\right.}$$
$$+\frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3+24\left(34225\,x^2+74740\,x-135952\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="fricas")
```

output
```
12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 - 497280*x
- 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x
)^3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))*e^(1/2*x)
```

3.567.6 Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{96x^2e^{\frac{x}{2}}\sin^3\left(x\right)}{185}+\frac{48x^2e^{\frac{x}{2}}\sin^2\left(x\right)\cos\left(x\right)}{185}$$
$$+\frac{156x^2e^{\frac{x}{2}}\sin\left(x\right)\cos^2\left(x\right)}{185}+\frac{58x^2e^{\frac{x}{2}}\cos^3\left(x\right)}{185}-\frac{32256xe^{\frac{x}{2}}\sin^3\left(x\right)}{34225}$$
$$+\frac{19392xe^{\frac{x}{2}}\sin^2\left(x\right)\cos\left(x\right)}{34225}-\frac{34656xe^{\frac{x}{2}}\sin\left(x\right)\cos^2\left(x\right)}{34225}$$
$$+\frac{26392xe^{\frac{x}{2}}\cos^3\left(x\right)}{34225}-\frac{1116672e^{\frac{x}{2}}\sin^3\left(x\right)}{6331625}-\frac{6525696e^{\frac{x}{2}}\sin^2\left(x\right)\cos\left(x\right)}{6331625}$$
$$-\frac{1512672e^{\frac{x}{2}}\sin\left(x\right)\cos^2\left(x\right)}{6331625}-\frac{6739696e^{\frac{x}{2}}\cos^3\left(x\right)}{6331625}$$

input
```
integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

output
```
96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x)/185 + 1
56*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)*cos(x)**3/185 - 3
2256*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x/2)*sin(x)**2*cos(x)/34225
- 34656*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 26392*x*exp(x/2)*cos(x)**3/342
25 - 1116672*exp(x/2)*sin(x)**3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x
)/6331625 - 1512672*exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*c
os(x)**3/6331625
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{250}\left(25\,x^2+60\,x-88\right)\cos\left(x\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{50653}\left(1369\,x^2-296\,x-264\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(3\,x\right) + \frac{3}{125}\left(25\,x^2-40\,x-8\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="maxima")
```

output
```
1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25*x^2 + 60
*x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin
(3*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="giac")
```

output
```
1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*
sin(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*
x - 8)*sin(x))*e^(1/2*x)
```

3.567.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.44

$$\int e^{x/2}x^2\cos^3(x)\,dx =$$

$$-\frac{e^{x/2}\,(107000\,\cos(3\,x)+198000\,\sin(3\,x)+13372392\,\cos(x)+2431344\,\sin(x)-647500\,x\,\cos(3\,x)-379$$

input
```
int(x^2*exp(x/2)*cos(x)^3,x)
```

output
```
-(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) + 13372392*cos(x) + 2431344*
sin(x) - 647500*x*cos(3*x) - 3798975*x^2*cos(x) + 222000*x*sin(3*x) - 7597
950*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) - 9117540*x*co
s(x) + 12156720*x*sin(x)))/12663250
```

3.568 $\int e^{2x} x^2 \sin(4x)\, dx$

3.568.1 Optimal result

Integrand size = 13, antiderivative size = 87

$$\int e^{2x} x^2 \sin(4x)\, dx = \frac{1}{250} e^{2x} \cos(4x) + \frac{2}{25} e^{2x} x \cos(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) - \frac{11}{500} e^{2x} \sin(4x) + \frac{3}{50} e^{2x} x \sin(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x)$$

output
```
1/250*exp(2*x)*cos(4*x)+2/25*exp(2*x)*x*cos(4*x)-1/5*exp(2*x)*x^2*cos(4*x)
-11/500*exp(2*x)*sin(4*x)+3/50*exp(2*x)*x*sin(4*x)+1/10*exp(2*x)*x^2*sin(4
*x)
```

3.568.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int e^{2x} x^2 \sin(4x)\, dx = \frac{1}{500} e^{2x} \left(\left(2 + 40x - 100x^2\right) \cos(4x) + \left(-11 + 30x + 50x^2\right) \sin(4x) \right)$$

input
```
Integrate[E^(2*x)*x^2*Sin[4*x],x]
```

output
```
(E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*Sin[4*x]))
/500
```

3.568.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x} x^2 \sin(4x) \, dx$$

$$\Big\downarrow \text{ 4968}$$

$$-2 \int -\frac{1}{10} x \left(2 e^{2x} \cos(4x) - e^{2x} \sin(4x)\right) dx + \frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5} \int x \left(2 e^{2x} \cos(4x) - e^{2x} \sin(4x)\right) dx + \frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x)$$

$$\Big\downarrow \text{ 2010}$$

$$\frac{1}{5} \int \left(2 e^{2x} x \cos(4x) - e^{2x} x \sin(4x)\right) dx + \frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{1}{5} \left(-\frac{11}{100} e^{2x} \sin(4x) + \frac{3}{10} e^{2x} x \sin(4x) + \frac{1}{50} e^{2x} \cos(4x) + \frac{2}{5} e^{2x} x \cos(4x)\right)$$

input
```
Int[E^(2*x)*x^2*Sin[4*x],x]
```

output
```
-1/5*(E^(2*x)*x^2*Cos[4*x]) + (E^(2*x)*x^2*Sin[4*x])/10 + ((E^(2*x)*Cos[4*
x])/50 + (2*E^(2*x)*x*Cos[4*x])/5 - (11*E^(2*x)*Sin[4*x])/100 + (3*E^(2*x)
*x*Sin[4*x])/10)/5
```

3.568.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

3.568.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

method	result
default	$\left(-\frac{1}{5}x^2+\frac{2}{25}x+\frac{1}{250}\right)\mathrm{e}^{2x}\cos\left(4x\right)+\left(\frac{1}{10}x^2+\frac{3}{50}x-\frac{11}{500}\right)\mathrm{e}^{2x}\sin\left(4x\right)$
parallelrisch	$\frac{(-100x^2+40x+2)\mathrm{e}^{2x}\cos(4x)}{500}+\frac{\mathrm{e}^{2x}\sin(4x)\left(x^2+\frac{3}{5}x-\frac{11}{50}\right)}{10}$
risch	$\left(-\frac{1}{500}-\frac{i}{1000}\right)\left(50x^2+20ix-10x-4i-3\right)\mathrm{e}^{(2+4i)x}+\left(-\frac{1}{500}+\frac{i}{1000}\right)\left(50x^2-20ix-10x+\right.$
norman	$\frac{\frac{2\,\mathrm{e}^{2x}x}{25}-\frac{\mathrm{e}^{2x}x^2}{5}-\frac{11\,\mathrm{e}^{2x}\tan(2x)}{250}-\frac{\mathrm{e}^{2x}\left(\tan^2(2x)\right)}{250}+\frac{3\,\mathrm{e}^{2x}x\tan(2x)}{25}-\frac{2\,\mathrm{e}^{2x}x\left(\tan^2(2x)\right)}{25}+\frac{\mathrm{e}^{2x}x^2\tan(2x)}{5}+\frac{\mathrm{e}^{2x}x^2\left(\tan^2(2x)\right)}{5}+\frac{\mathrm{e}^{2x}}{250}}{1+\tan^2(2x)}$

input
```
int(exp(2*x)*x^2*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500)*exp(2*x
)*sin(4*x)
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)} + \frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

input
```
integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="fricas")
```

output
```
-1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e
^(2*x)*sin(4*x)
```

3.568.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int e^{2x}x^2\sin(4x)\,dx = \frac{x^2e^{2x}\sin\left(4x\right)}{10} - \frac{x^2e^{2x}\cos\left(4x\right)}{5} + \frac{3xe^{2x}\sin\left(4x\right)}{50} + \frac{2xe^{2x}\cos\left(4x\right)}{25} - \frac{11e^{2x}\sin\left(4x\right)}{500} + \frac{e^{2x}\cos\left(4x\right)}{250}$$

input
```
integrate(exp(2*x)*x**2*sin(4*x),x)
```

output
```
x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*x)*sin(4*
x)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/500 + exp(2*x)*cos
(4*x)/250
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)} + \frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

input
```
integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="maxima")
```

output
```
-1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e
^(2*x)*sin(4*x)
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int e^{2x}x^2\sin(4x)\,dx$$
$$= -\frac{1}{500}\left(2\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)-\left(50\,x^2+30\,x-11\right)\sin\left(4\,x\right)\right)e^{(2\,x)}$$

input
```
integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="giac")
```

output
```
-1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin(4*x))*e^
(2*x)
```

3.568.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int e^{2x}x^2\sin(4x)\,dx$$
$$= \frac{e^{2\,x}\,\left(2\,\cos\left(4\,x\right)-11\,\sin\left(4\,x\right)+40\,x\,\cos\left(4\,x\right)+30\,x\,\sin\left(4\,x\right)-100\,x^2\,\cos\left(4\,x\right)+50\,x^2\,\sin\left(4\,x\right)\right)}{500}$$

input
```
int(x^2*sin(4*x)*exp(2*x),x)
```

output
```
(exp(2*x)*(2*cos(4*x) - 11*sin(4*x) + 40*x*cos(4*x) + 30*x*sin(4*x) - 100*
x^2*cos(4*x) + 50*x^2*sin(4*x)))/500
```

3.569 $\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$

3.569.1 Optimal result

Integrand size = 17, antiderivative size = 185

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx=-\frac{44}{125}e^{x/2}\cos(x)$$
$$+\frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)+\frac{428e^{x/2}\cos(3x)}{50653}-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{125}e^{x/2}\sin(x)-$$

output
```
-44/125*exp(1/2*x)*cos(x)+6/25*exp(1/2*x)*x*cos(x)+1/10*exp(1/2*x)*x^2*cos
(x)+428/50653*exp(1/2*x)*cos(3*x)-70/1369*exp(1/2*x)*x*cos(3*x)-1/74*exp(1
/2*x)*x^2*cos(3*x)-8/125*exp(1/2*x)*sin(x)-8/25*exp(1/2*x)*x*sin(x)+1/5*ex
p(1/2*x)*x^2*sin(x)+792/50653*exp(1/2*x)*sin(3*x)+24/1369*exp(1/2*x)*x*sin
(3*x)-3/37*exp(1/2*x)*x^2*sin(3*x)
```

3.569.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx=\frac{e^{x/2}(50653((-88+60x+25x^2)\cos(x)+2(-8-40x+25x^2)\sin(x))-125((-}{1266325}$$

input
```
Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25*x^2)*Sin
[x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-264 - 296*x + 1369*x
^2)*Sin[3*x])))/12663250
```

3.569.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4973, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x/2}x^2\sin^2(x)\cos(x)\,dx$$

$$\Big\downarrow \text{4973}$$

$$\int \left(\frac{1}{4}e^{x/2}x^2\cos(x)-\frac{1}{4}e^{x/2}x^2\cos(3x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin(x)+\frac{792e^{x/2}\sin(3x)}{50653}+\frac{6}{25}e^{x/2}x\cos(x)-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{44}{125}e^{x/2}\cos(x)+\frac{428e^{x/2}\cos(3x)}{50653}$$

input
```
Int[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2*Cos[x])/
10 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*x])/1369 - (E^(x/2
)*x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (8*E^(x/2)*x*Sin[x])/25 + (E
^(x/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Sin[3*x])/50653 + (24*E^(x/2)*x*Sin[3*
x])/1369 - (3*E^(x/2)*x^2*Sin[3*x])/37
```

3.569.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4973
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.569.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

method	result
default	$\frac{\left(\frac{2}{5}x^2+\frac{24}{25}x-\frac{176}{125}\right)e^{\frac{x}{2}}\cos(x)}{4}-\frac{\left(-\frac{4}{5}x^2+\frac{32}{25}x+\frac{32}{125}\right)e^{\frac{x}{2}}\sin(x)}{4}-\frac{\left(\frac{2}{37}x^2+\frac{280}{1369}x-\frac{1712}{50653}\right)e^{\frac{x}{2}}\cos(3x)}{4}+\frac{\left(-\frac{12}{37}x^2+\frac{96}{1369}x+\frac{3168}{50653}\right)e^{\frac{x}{2}}}{4}$
risch	$\left(-\frac{1}{202612}+\frac{3i}{101306}\right)\left(1369x^2+888ix-148x-96i-280\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{1}{500}-\frac{i}{250}\right)\left(25x^2+40ix-2\right.$

input
```
int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-1/4*(-4/5*x^2+32/25*x+32/1
25)*exp(1/2*x)*sin(x)-1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(
3*x)+1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx =$$
$$-\frac{4}{6331625}\left(375\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2-444925\,x^2+534280\,x+126056\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)$$
$$-\frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3-\left(444925\,x^2+1245420\,x-1194616\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

output
```
-4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 + 534280*x
+ 126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(
x)^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x))*e^(1/2*x)
```

3.569.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{52x^2e^{\frac{x}{2}}\sin^3(x)}{185} + \frac{26x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} - \frac{8x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{16x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{11552xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{13464xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{9152xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{6464xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{504224e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{2389232e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{108224e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{2175232e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

input
```
integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

output
```
52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x)/185 - 8
*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*cos(x)**3/185 - 115
52*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2)*sin(x)**2*cos(x)/34225 -
9152*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 6464*x*exp(x/2)*cos(x)**3/34225 -
 504224*exp(x/2)*sin(x)**3/6331625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/633
1625 - 108224*exp(x/2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)*
*3/6331625
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.42

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos(3\,x)\,e^{\left(\frac{1}{2}\,x\right)} + \frac{1}{250}\left(25\,x^2+60\,x-88\right)\cos(x)\,e^{\left(\frac{1}{2}\,x\right)} - \frac{3}{50653}\left(1369\,x^2-296\,x-264\right)e^{\left(\frac{1}{2}\,x\right)}\sin(3\,x) + \frac{1}{125}\left(25\,x^2-40\,x-8\right)e^{\left(\frac{1}{2}\,x\right)}\sin(x)$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(25*x^2 + 6
0*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*si
n(3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{1}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

output
```
-1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)
*sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40
*x - 8)*sin(x))*e^(1/2*x)
```

3.569.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{e^{x/2}\left(107000\,\cos\left(3\,x\right)+198000\,\sin\left(3\,x\right)-4457464\,\cos\left(x\right)-810448\,\sin\left(x\right)-\right.}{}$$

input
```
int(x^2*exp(x/2)*cos(x)*sin(x)^2,x)
```

output
```
(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) - 4457464*cos(x) - 810448*sin
(x) - 647500*x*cos(3*x) + 1266325*x^2*cos(x) + 222000*x*sin(3*x) + 2532650
*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) + 3039180*x*cos(x
) - 4052240*x*sin(x)))/12663250
```

3.570 $\int \cosh(x) \, dx$

3.570.1 Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cosh(x) \, dx = \sinh(x)$$

output
```
sinh(x)
```

3.570.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x) \, dx = \sinh(x)$$

input
```
Integrate[Cosh[x],x]
```

output
```
Sinh[x]
```

3.570.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sinh(x)$$

input
```
Int[Cosh[x],x]
```

output
```
Sinh[x]
```

3.570.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.570.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sinh(x)$	3
default	$\sinh(x)$	3
meijerg	$\sinh(x)$	3
parallelrisch	$\sinh(x)$	3
risch	$-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$	12

input
```
int(cosh(x),x,method=_RETURNVERBOSE)
```

output
```
sinh(x)
```

3.570.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
integrate(cosh(x),x, algorithm="fricas")
```

output
```
sinh(x)
```

3.570.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
integrate(cosh(x),x)
```

output
```
sinh(x)
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh{(x)}$$

input
```
integrate(cosh(x),x, algorithm="maxima")
```

output
```
sinh(x)
```

3.570.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \cosh(x)\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(cosh(x),x, algorithm="giac")
```

output
```
-1/2*e^(-x) + 1/2*e^x
```

3.570.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
int(cosh(x),x)
```

output
```
sinh(x)
```

3.571 $\int \sinh(x)\,dx$

3.571.1 Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \sinh(x)\,dx = \cosh(x)$$

output
```
cosh(x)
```

3.571.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

input
```
Integrate[Sinh[x],x]
```

output
```
Cosh[x]
```

3.571.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -i\sin(ix)dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \sin(ix)dx$$
$$\Big\downarrow \text{ 3118}$$
$$\cosh(x)$$

input
```
Int[Sinh[x],x]
```

output
```
Cosh[x]
```

3.571.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.571.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\cosh(x)$	3
default	$\cosh(x)$	3
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}$	12
parallelrisch	$-\frac{2}{\tanh^{2}\left(\frac{x}{2}\right)-1}$	13
meijerg	$-\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x)}{\sqrt{\pi}}\right)$	17

input
```
int(sinh(x),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)
```

3.571.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

input
```
integrate(sinh(x),x, algorithm="fricas")
```

output
```
cosh(x)
```

3.571.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

input
```
integrate(sinh(x),x)
```

output
```
cosh(x)
```

3.571.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

input
```
integrate(sinh(x),x, algorithm="maxima")
```

output
```
cosh(x)
```

3.571.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \sinh(x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(sinh(x),x, algorithm="giac")
```

output
```
1/2*e^(-x) + 1/2*e^x
```

3.571.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

input
```
int(sinh(x),x)
```

output
```
cosh(x)
```

3.572 $\int \tanh(x)\,dx$

3.572.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

output
```
ln(cosh(x))
```

3.572.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

input
```
Integrate[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

3.572.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix)dx$$

$$\Big\downarrow \text{3956}$$

$$\log(\cosh(x))$$

input
```
Int[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

3.572.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.572.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln\left(\cosh\left(x\right)\right)$	4
derivativedivides	$\ln\left(\cosh\left(x\right)\right)$	4
default	$\ln\left(\cosh\left(x\right)\right)$	4
risch	$-x+\ln\left(1+{\mathrm e}^{2 x}\right)$	12
parallelrisch	$-\ln\left(1-\tanh\left(x\right)\right)-x$	14

input
```
int(tanh(x),x,method=_RETURNVERBOSE)
```

output
```
ln(cosh(x))
```

3.572.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x)\, dx = -x + \log\left(\frac{2 \cosh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right)$$

input
```
integrate(tanh(x),x, algorithm="fricas")
```

output
```
-x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.572.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x)\, dx = x - \log\left(\tanh\left(x\right) + 1\right)$$

input
```
integrate(tanh(x),x)
```

output
```
x - log(tanh(x) + 1)
```

3.572.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log\left(\cosh\left(x\right)\right)$$

input
```
integrate(tanh(x),x, algorithm="maxima")
```

output
```
log(cosh(x))
```

3.572.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x)\,dx = -x + \log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(tanh(x),x, algorithm="giac")
```

output
```
-x + log(e^(2*x) + 1)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \ln\left(\cosh(x)\right)$$

input
```
int(tanh(x),x)
```

output
```
log(cosh(x))
```

3.573 $\int \coth(x)\,dx$

3.573.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \coth(x)\,dx = \log(\sinh(x))$$

output
```
ln(sinh(x))
```

3.573.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \coth(x)\,dx = \log(\cosh(x)) + \log(\tanh(x))$$

input
```
Integrate[Coth[x],x]
```

output
```
Log[Cosh[x]] + Log[Tanh[x]]
```

3.573.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i \tan\left(\frac{\pi}{2} + ix\right) dx$$

$$\Big\downarrow \text{26}$$

$$-i \int \tan\left(ix + \frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{3956}$$

$$\log(\sinh(x))$$

input
```
Int[Coth[x],x]
```

output
```
Log[Sinh[x]]
```

3.573.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.573.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
derivativedivides	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x+\ln(e^{2x}-1)$	12
parallelrisch	$\ln(\tanh(x))-\ln(1-\tanh(x))-x$	17

input
```
int(coth(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sinh(x))
```

3.573.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \coth(x)\,dx = -x+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(coth(x),x, algorithm="fricas")
```

output
```
-x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

3.573.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = x-\log(\tanh(x)+1)+\log(\tanh(x))$$

input
```
integrate(coth(x),x)
```

output
```
x - log(tanh(x) + 1) + log(tanh(x))
```

3.573.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \log\left(\sinh\left(x\right)\right)$$

input
```
integrate(coth(x),x, algorithm="maxima")
```

output
```
log(sinh(x))
```

3.573.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = -x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate(coth(x),x, algorithm="giac")
```

output
```
-x + log(abs(e^(2*x) - 1))
```

3.573.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \ln\left(\sinh(x)\right)$$

input
```
int(coth(x),x)
```

output
```
log(sinh(x))
```

3.574 $\int \operatorname{sech}(x)\,dx$

3.574.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

output
```
arctan(sinh(x))
```

3.574.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

input
```
Integrate[Sech[x],x]
```

output
```
ArcTan[Sinh[x]]
```

3.574.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(x) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(\frac{\pi}{2}+ix\right) dx$$

$$\Big\downarrow \; 4257$$

$$\arctan(\sinh(x))$$

input
```
Int[Sech[x],x]
```

output
```
ArcTan[Sinh[x]]
```

3.574.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.574.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\arctan\left(\sinh\left(x\right)\right)$	4
default	$\arctan\left(\sinh\left(x\right)\right)$	4
risch	$i\ln\left({\mathrm e}^{x}+i\right)-i\ln\left({\mathrm e}^{x}-i\right)$	20
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)$	23

input
```
int(sech(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))
```

3.574.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \operatorname{sech}(x)\,dx=2\,\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)$$

input
```
integrate(sech(x),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))
```

3.574.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \operatorname{sech}(x)\,dx=2\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(sech(x),x)
```

output
```
2*atan(tanh(x/2))
```

3.574.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

input
```
integrate(sech(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x))
```

3.574.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\arctan(e^x)$$

input
```
integrate(sech(x),x, algorithm="giac")
```

output
```
2*arctan(e^x)
```

3.574.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\operatorname{atan}(e^x)$$

input
```
int(1/cosh(x),x)
```

output
```
2*atan(exp(x))
```

3.575 $\int \operatorname{csch}(x)\,dx$

3.575.1 Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \operatorname{csch}(x)\,dx = -\operatorname{arctanh}(\cosh(x))$$

output
```
-arctanh(cosh(x))
```

3.575.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \operatorname{csch}(x)\,dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x],x]
```

output
```
-Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

3.575.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int i\csc(ix)dx$$
$$\Big\downarrow \; 26$$
$$i\int \csc(ix)dx$$
$$\Big\downarrow \; 4257$$
$$-\operatorname{arctanh}(\cosh(x))$$

input
```
Int[Csch[x],x]
```

output
```
-ArcTanh[Cosh[x]]
```

3.575.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.575.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
lookup	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
parallelrisch	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
risch	$\ln\left(-1+\mathrm{e}^{x}\right)-\ln\left(1+\mathrm{e}^{x}\right)$	14

input
```
int(csch(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(1/2*x))
```

3.575.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \operatorname{csch}(x)\,dx = -\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

input
```
integrate(csch(x),x, algorithm="fricas")
```

output
```
-log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

3.575.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(csch(x),x)
```

output
```
log(tanh(x/2))
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

input
```
integrate(csch(x),x, algorithm="maxima")
```

output
```
log(tanh(1/2*x))
```

3.575.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \operatorname{csch}(x)\,dx = -\log\left(e^x+1\right) + \log\left(|e^x-1|\right)$$

input
```
integrate(csch(x),x, algorithm="giac")
```

output
```
-log(e^x + 1) + log(abs(e^x - 1))
```

3.575.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
int(1/sinh(x),x)
```

output
```
log(tanh(x/2))
```

3.576 $\int \cosh^2(x)\,dx$

3.576.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cosh(x)\sinh(x)$$

output
```
1/2*x+1/2*cosh(x)*sinh(x)
```

3.576.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sinh(2x)$$

input
```
Integrate[Cosh[x]^2,x]
```

output
```
x/2 + Sinh[2*x]/4
```

3.576.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(\frac{\pi}{2}+ix\right)^2 dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

$$\Big\downarrow \; 24$$

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

input
```
Int[Cosh[x]^2,x]
```

output
```
x/2 + (Cosh[x]*Sinh[x])/2
```

3.576.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.576.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cosh(x)\sinh(x)}{2}$	11
parallelrisch	$\frac{x}{2}+\frac{\sinh(2x)}{4}$	11
risch	$\frac{x}{2}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}$	17

input
```
int(cosh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cosh(x)*sinh(x)
```

3.576.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cosh^2(x)\,dx=\frac{1}{2}\cosh(x)\sinh(x)+\frac{1}{2}x$$

input
```
integrate(cosh(x)^2,x, algorithm="fricas")
```

output
```
1/2*cosh(x)*sinh(x) + 1/2*x
```

3.576.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx=-\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}$$

input
```
integrate(cosh(x)**2,x)
```

output
```
-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
```

3.576.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cosh^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(cosh(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.576.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx = -\frac{1}{8}\,\left(2\,e^{(2\,x)} + 1\right)e^{(-2\,x)} + \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)^2,x, algorithm="giac")
```

output
```
-1/8*(2*e^(2*x) + 1)*e^(-2*x) + 1/2*x + 1/8*e^(2*x)
```

3.576.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{\sinh(2\,x)}{4}$$

input
```
int(cosh(x)^2,x)
```

output
```
x/2 + sinh(2*x)/4
```

3.577 $\int \sinh^5(x)\,dx$

3.577.1 Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \sinh^5(x)\,dx = \cosh(x) - \frac{2\cosh^3(x)}{3} + \frac{\cosh^5(x)}{5}$$

output
```
cosh(x)-2/3*cosh(x)^3+1/5*cosh(x)^5
```

3.577.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sinh^5(x)\,dx = \frac{5\cosh(x)}{8} - \frac{5}{48}\cosh(3x) + \frac{1}{80}\cosh(5x)$$

input
```
Integrate[Sinh[x]^5,x]
```

output
```
(5*Cosh[x])/8 - (5*Cosh[3*x])/48 + Cosh[5*x]/80
```

3.577.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -i\sin(ix)^5dx$$
$$\Big\downarrow \ 26$$
$$-i\int \sin(ix)^5dx$$
$$\Big\downarrow \ 3113$$
$$\int \left(\cosh^4(x)-2\cosh^2(x)+1\right)d\cosh(x)$$
$$\Big\downarrow \ 2009$$
$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

input
```
Int[Sinh[x]^5,x]
```

output
```
Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

3.577.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.577.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\left(\frac{8}{15}+\frac{\left(\sinh^{4}\left(x\right)\right)}{5}-\frac{4\left(\sinh^{2}\left(x\right)\right)}{15}\right)\cosh\left(x\right)$	18
risch	$\frac{{\mathrm e}^{5 x}}{160}-\frac{5\,{\mathrm e}^{3 x}}{96}+\frac{5\,{\mathrm e}^{x}}{16}+\frac{5\,{\mathrm e}^{-x}}{16}-\frac{5\,{\mathrm e}^{-3 x}}{96}+\frac{{\mathrm e}^{-5 x}}{160}$	36
parallelrisch	$\frac{-18\cosh\left(7 x\right)+90\cosh\left(5 x\right)-162\cosh\left(3 x\right)+90\cosh\left(x\right)+3\cosh\left(8 x\right)-180\cosh\left(4 x\right)+20\cosh\left(6 x\right)+492\cosh\left(2 x\right)-335}{480\cosh\left(3 x\right)+7200\cosh\left(x\right)-2880\cosh\left(2 x\right)-4800}$	70

input
```
int(sinh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
(8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \sinh^5(x)\,dx = \frac{1}{80}\cosh\left(x\right)^{5}+\frac{1}{16}\cosh\left(x\right)\sinh\left(x\right)^{4}-\frac{5}{48}\cosh\left(x\right)^{3}+\frac{1}{16}\left(2\cosh\left(x\right)^{3}-5\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+\frac{5}{8}\cosh\left(x\right)$$

input
```
integrate(sinh(x)^5,x, algorithm="fricas")
```

output
```
1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 5/48*cosh(x)^3 + 1/16*(2*cosh(x)
^3 - 5*cosh(x))*sinh(x)^2 + 5/8*cosh(x)
```

3.577.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \sinh^5(x)\,dx = \sinh^4(x)\cosh(x) - \frac{4\sinh^2(x)\cosh^3(x)}{3} + \frac{8\cosh^5(x)}{15}$$

input
```
integrate(sinh(x)**5,x)
```

output
```
sinh(x)**4*cosh(x) - 4*sinh(x)**2*cosh(x)**3/3 + 8*cosh(x)**5/15
```

3.577.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \sinh^5(x)\,dx = \frac{1}{160}\,e^{(5\,x)} - \frac{5}{96}\,e^{(3\,x)} + \frac{5}{16}\,e^{(-x)} - \frac{5}{96}\,e^{(-3\,x)} + \frac{1}{160}\,e^{(-5\,x)} + \frac{5}{16}\,e^{x}$$

input
```
integrate(sinh(x)^5,x, algorithm="maxima")
```

output
```
1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/160*e^(-5*x
) + 5/16*e^x
```

3.577.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sinh^5(x)\,dx = \frac{1}{480}\left(150\,e^{(4\,x)} - 25\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{160}\,e^{(5\,x)} - \frac{5}{96}\,e^{(3\,x)} + \frac{5}{16}\,e^{x}$$

input
```
integrate(sinh(x)^5,x, algorithm="giac")
```

output
```
1/480*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 1/160*e^(5*x) - 5/96*e^(3*
x) + 5/16*e^x
```

3.577.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sinh^5(x)\,dx = \frac{\cosh(x)^5}{5} - \frac{2\cosh(x)^3}{3} + \cosh(x)$$

input
```
int(sinh(x)^5,x)
```

output
```
cosh(x) - (2*cosh(x)^3)/3 + cosh(x)^5/5
```

3.578 $\int \tanh^4(x)\,dx$

3.578.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tanh^4(x)\,dx = x - \tanh(x) - \frac{\tanh^3(x)}{3}$$

output
```
x-tanh(x)-1/3*tanh(x)^3
```

3.578.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tanh^4(x)\,dx = \operatorname{arctanh}(\tanh(x)) - \tanh(x) - \frac{\tanh^3(x)}{3}$$

input
```
Integrate[Tanh[x]^4,x]
```

output
```
ArcTanh[Tanh[x]] - Tanh[x] - Tanh[x]^3/3
```

3.578.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^4dx$$

$$\Big\downarrow \text{3954}$$

$$-\int -\tanh^2(x)dx-\frac{1}{3}\tanh^3(x)$$

$$\Big\downarrow \text{25}$$

$$\int \tanh^2(x)dx-\frac{\tanh^3(x)}{3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(x)}{3}+\int -\tan(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\tanh^3(x)-\int \tan(ix)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

$$\Big\downarrow \text{24}$$

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

input
```
Int[Tanh[x]^4,x]
```

output
```
x - Tanh[x] - Tanh[x]^3/3
```

3.578.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.578.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$x-\tanh\left(x\right)-\frac{\left(\tanh^{3}\left(x\right)\right)}{3}$	13
derivativedivides	$-\frac{\left(\tanh^{3}\left(x\right)\right)}{3}-\tanh\left(x\right)-\frac{\ln\left(-1+\tanh\left(x\right)\right)}{2}+\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	26
default	$-\frac{\left(\tanh^{3}\left(x\right)\right)}{3}-\tanh\left(x\right)-\frac{\ln\left(-1+\tanh\left(x\right)\right)}{2}+\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	26
risch	$x+\frac{4\,{\mathrm e}^{4x}+4\,{\mathrm e}^{2x}+\frac{8}{3}}{\left(1+{\mathrm e}^{2x}\right)^{3}}$	27

input
```
int(tanh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tanh(x)-1/3*tanh(x)^3
```

3.578.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.86

$$\int \tanh^4(x)\,dx$$
$$= \frac{(3x+4)\cosh(x)^3 + 3(3x+4)\cosh(x)\sinh(x)^2 - 12\cosh(x)^2\sinh(x) - 4\sinh(x)^3 + 3(3x+4)\cosh}{3\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\cosh(x)\right)}$$

input
```
integrate(tanh(x)^4,x, algorithm="fricas")
```

output
```
1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cosh(x)^2*si
nh(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^
2 + 3*cosh(x))
```

3.578.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \tanh^4(x)\,dx = x - \frac{\tanh^3(x)}{3} - \tanh(x)$$

input
```
integrate(tanh(x)**4,x)
```

output
```
x - tanh(x)**3/3 - tanh(x)
```

3.578.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \tanh^4(x)\,dx = x - \frac{4\left(3e^{(-2x)} + 3e^{(-4x)} + 2\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)}$$

input
```
integrate(tanh(x)^4,x, algorithm="maxima")
```

output
```
x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x)
+ 1)
```

3.578.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \tanh^4(x)\,dx = x + \frac{4\left(3\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)}{3\left(e^{(2\,x)} + 1\right)^3}$$

input
```
integrate(tanh(x)^4,x, algorithm="giac")
```

output
```
x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3
```

3.578.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tanh^4(x)\,dx = -\frac{\tanh(x)^3}{3} - \tanh(x) + x$$

input
```
int(tanh(x)^4,x)
```

output
```
x - tanh(x) - tanh(x)^3/3
```

3.579 $\int \mathrm{csch}^3(x)\,dx$

3.579.1 Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \mathrm{csch}^3(x)\,dx = \frac{1}{2}\mathrm{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\mathrm{csch}(x)$$

output
```
1/2*arctanh(cosh(x))-1/2*coth(x)*csch(x)
```

3.579.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \mathrm{csch}^3(x)\,dx = -\frac{1}{8}\mathrm{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\mathrm{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Csch[x]^3,x]
```

output
```
-1/8*Csch[x/2]^2 + Log[Cosh[x/2]]/2 - Log[Sinh[x/2]]/2 - Sech[x/2]^2/8
```

3.579.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ix)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ix)^3dx$$

$$\Big\downarrow \text{4255}$$

$$-i\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

input
```
Int[Csch[x]^3,x]
```

output
```
(-I)*((I/2)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x])
```

3.579.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.579.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{\coth(x)\operatorname{csch}(x)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	11
parallelrisch	$\ln\left(\frac{1}{\sqrt{\coth(x)-\operatorname{csch}(x)}}\right)-\frac{\coth(x)\operatorname{csch}(x)}{2}$	18
risch	$-\frac{\mathrm{e}^{x}\left(1+\mathrm{e}^{2x}\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}}+\frac{\ln(1+\mathrm{e}^{x})}{2}-\frac{\ln(-1+\mathrm{e}^{x})}{2}$	34

input
```
int(csch(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(x)*csch(x)+arctanh(exp(x))
```

3.579.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 13.19

$$\int \operatorname{csch}^3(x)\,dx =$$
$$-\frac{2\cosh(x)^3 + 6\cosh(x)\sinh(x)^2 + 2\sinh(x)^3 - \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\,(3\,\mathrm{c}\right.}{}$$

input
```
integrate(csch(x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*
cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1
) + 2*(3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x
)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)
^3 - cosh(x))*sinh(x) + 1)
```

3.579.6 Sympy [F]

$$\int \operatorname{csch}^3(x)\,dx = \int \operatorname{csch}^3(x)\,dx$$

input
```
integrate(csch(x)**3,x)
```

output
```
Integral(csch(x)**3, x)
```

3.579.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \operatorname{csch}^3(x)\,dx = \frac{e^{(-x)} + e^{(-3x)}}{2\,e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{2}\log\left(e^{(-x)} + 1\right) - \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(csch(x)^3,x, algorithm="maxima")
```

output
```
(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*log(e^(-x) + 1) - 1/
2*log(e^(-x) - 1)
```

3.579.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \operatorname{csch}^3(x)\,dx = -\frac{e^{(-x)} + e^{x}}{\left(e^{(-x)} + e^{x}\right)^2 - 4} + \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate(csch(x)^3,x, algorithm="giac")
```

output
```
-(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + 1/4*log(e^(-x) + e^x + 2) - 1/4*l
og(e^(-x) + e^x - 2)
```

3.579.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^3(x)\,dx = -\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{\cosh(x)}{2\sinh(x)^2}$$

input
```
int(1/sinh(x)^3,x)
```

output
```
- log(tanh(x/2))/2 - cosh(x)/(2*sinh(x)^2)
```

3.580 $\int \operatorname{sech}^5(x)\,dx$

3.580.1 Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \operatorname{sech}^5(x)\,dx=\frac{3}{8}\arctan(\sinh(x))+\frac{3}{8}\operatorname{sech}(x)\tanh(x)+\frac{1}{4}\operatorname{sech}^3(x)\tanh(x)$$

output
```
3/8*arctan(sinh(x))+3/8*sech(x)*tanh(x)+1/4*sech(x)^3*tanh(x)
```

3.580.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^5(x)\,dx=\frac{3}{8}\arctan(\sinh(x))+\frac{3}{8}\operatorname{sech}(x)\tanh(x)+\frac{1}{4}\operatorname{sech}^3(x)\tanh(x)$$

input
```
Integrate[Sech[x]^5,x]
```

output
```
(3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

3.580.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+ix\right)^5 dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\int \operatorname{sech}^3(x)dx+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\left(\frac{\int \operatorname{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\operatorname{sech}(x)\right)+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{4}\left(\frac{1}{2}\tanh(x)\operatorname{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\operatorname{sech}(x)\right)+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)$$

input
```
Int[Sech[x]^5,x]
```

output
```
(Sech[x]^3*Tanh[x])/4 + (3*(ArcTan[Sinh[x]]/2 + (Sech[x]*Tanh[x])/2))/4
```

3.580.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.580.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\left(\frac{\operatorname{sech}(x)^{3}}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh\left(x\right)+\frac{3\arctan(\mathrm{e}^{x})}{4}$	21
parallelrisch	$\frac{3i\ln(i+\coth(x)-\operatorname{csch}(x))}{8}-\frac{3i\ln(-i+\coth(x)-\operatorname{csch}(x))}{8}+\frac{3\operatorname{sech}(x)\tanh(x)}{8}+\frac{\operatorname{sech}(x)^{3}\tanh(x)}{4}$	42
risch	$\frac{\mathrm{e}^{x}\left(3\,\mathrm{e}^{6x}+11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}-3\right)}{4(1+\mathrm{e}^{2x})^{4}}+\frac{3i\ln(\mathrm{e}^{x}+i)}{8}-\frac{3i\ln(\mathrm{e}^{x}-i)}{8}$	52

input
```
int(1/cosh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))
```

3.580.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 461, normalized size of antiderivative = 17.73

$$\int \operatorname{sech}^5(x)\,dx$$
$$=\frac{3\cosh\left(x\right)^{7}+21\cosh\left(x\right)\sinh\left(x\right)^{6}+3\sinh\left(x\right)^{7}+\left(63\cosh\left(x\right)^{2}+11\right)\sinh\left(x\right)^{5}+11\cosh\left(x\right)^{5}+5\left(21\,\mathrm{co}\right.}{}$$

input
```
integrate(1/cosh(x)^5,x, algorithm="fricas")
```

output
```
1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11
)*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (10
5*cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5
 + 110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x
)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(
x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^
4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4
*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 +
8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(co
sh(x) + sinh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x
) - 3*cosh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)
^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 +
2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)
^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 +
 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3
*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.580.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(27) = 54$.

Time = 1.17 (sec) , antiderivative size = 422, normalized size of antiderivative = 16.23

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\tanh^8\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} - \frac{5\tanh^7\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} + \frac{12\tanh^6\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} + \frac{3\tanh^5\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} + \frac{18\tanh^4\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} - \frac{3\tanh^3\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} + \frac{12\tanh^2\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} + \frac{5\tanh\left(\frac{x}{2}\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4} + \frac{3\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{4\tanh^8\left(\frac{x}{2}\right)+16\tanh^6\left(\frac{x}{2}\right)+24\tanh^4\left(\frac{x}{2}\right)+16\tanh^2\left(\frac{x}{2}\right)+4}$$

input
```
integrate(1/cosh(x)**5,x)
```

output
```
3*tanh(x/2)**8*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh
(x/2)**4 + 16*tanh(x/2)**2 + 4) - 5*tanh(x/2)**7/(4*tanh(x/2)**8 + 16*tanh
(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 12*tanh(x/2)**6*atan(t
anh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/
2)**2 + 4) + 3*tanh(x/2)**5/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/
2)**4 + 16*tanh(x/2)**2 + 4) + 18*tanh(x/2)**4*atan(tanh(x/2))/(4*tanh(x/2
)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 3*tanh(x
/2)**3/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)*
*2 + 4) + 12*tanh(x/2)**2*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**
6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 5*tanh(x/2)/(4*tanh(x/2)**8 +
 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*atan(tanh(x/
2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2
+ 4)
```

3.580. $\int \operatorname{sech}^5(x)\,dx$

3.580.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)} - \frac{3}{4}\arctan\left(e^{(-x)}\right)$$

input
```
integrate(1/cosh(x)^5,x, algorithm="maxima")
```

output
```
1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^
(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))
```

3.580.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \operatorname{sech}^5(x)\,dx = \frac{3}{16}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^3 + 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^2} + \frac{3}{8}\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate(1/cosh(x)^5,x, algorithm="giac")
```

output
```
3/16*pi - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
+ 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.580.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\,\mathrm{atan}(\mathrm{e}^{x})}{4} + \frac{3\sinh(x)}{8\cosh\left(x\right)^2} + \frac{\sinh(x)}{4\cosh\left(x\right)^4}$$

input
```
int(1/cosh(x)^5,x)
```

output
```
(3*atan(exp(x)))/4 + (3*sinh(x))/(8*cosh(x)^2) + sinh(x)/(4*cosh(x)^4)
```

3.581 $\int \sinh^4(x)\tanh(x)\,dx$

3.581.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \sinh^4(x)\tanh(x)\,dx = -\cosh^2(x) + \frac{\cosh^4(x)}{4} + \log(\cosh(x))$$

output
```
-cosh(x)^2+1/4*cosh(x)^4+ln(cosh(x))
```

3.581.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sinh^4(x)\tanh(x)\,dx = -\cosh^2(x) + \frac{\cosh^4(x)}{4} + \log(\cosh(x))$$

input
```
Integrate[Sinh[x]^4*Tanh[x],x]
```

output
```
-Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

3.581.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(x)\tanh(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ix)^4\tan(ix)dx$$

$$\Big\downarrow \ 26$$

$$-i\int \sin(ix)^4\tan(ix)dx$$

$$\Big\downarrow \ 3070$$

$$\int \left(1-\cosh^2(x)\right)^2\operatorname{sech}(x)d\cosh(x)$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \left(1-\cosh^2(x)\right)^2\operatorname{sech}(x)d\cosh^2(x)$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\cosh^2(x)+\operatorname{sech}(x)-2\right)d\cosh^2(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\cosh^4(x)}{2}-2\cosh^2(x)+\log\left(\cosh^2(x)\right)\right)$$

input
```
Int[Sinh[x]^4*Tanh[x],x]
```

output
```
(-2*Cosh[x]^2 + Cosh[x]^4/2 + Log[Cosh[x]^2])/2
```

3.581.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.581.4 Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(\sinh^4(x))}{4}-\frac{(\sinh^2(x))}{2}+\ln(\cosh(x))$	17
risch	$-x+\frac{e^{4x}}{64}-\frac{3\,e^{2x}}{16}-\frac{3\,e^{-2x}}{16}+\frac{e^{-4x}}{64}+\ln(1+e^{2x})$	36

input
```
int(tanh(x)^5/sech(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(x)^4-1/2*sinh(x)^2+ln(cosh(x))
```

3.581.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 14.28

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\cosh(x)^2 - 3\right)\sinh(x)^6 - 12\cosh(x)^6 + 8\left(7\cosh(x)^3\right.}{}$$

input
```
integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")
```

output
```
1/64*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*si
nh(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 - 64*x*cosh
(x)^4 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 - 32*x)*sinh(x)^4 + 8*(7*cosh(x)^5
- 30*cosh(x)^3 - 32*x*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 -
 96*x*cosh(x)^2 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 64*(cosh(x)^4 + 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)*log(2
*cosh(x)/(cosh(x) - sinh(x))) + 8*(cosh(x)^7 - 9*cosh(x)^5 - 32*x*cosh(x)^
3 - 3*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2
*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

3.581.6 Sympy [F]

$$\int \sinh^4(x)\tanh(x)\,dx = \int \frac{\tanh^5(x)}{\operatorname{sech}^4(x)}\,dx$$

input
```
integrate(tanh(x)**5/sech(x)**4,x)
```

output
```
Integral(tanh(x)**5/sech(x)**4, x)
```

3.581.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sinh^4(x)\tanh(x)\,dx = -\frac{1}{64}\left(12\,e^{(-2\,x)}-1\right)e^{(4\,x)}+x-\frac{3}{16}\,e^{(-2\,x)}+\frac{1}{64}\,e^{(-4\,x)}+\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")
```

output
```
-1/64*(12*e^(-2*x) - 1)*e^(4*x) + x - 3/16*e^(-2*x) + 1/64*e^(-4*x) + log(
e^(-2*x) + 1)
```

3.581.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{1}{64}\left(48\,e^{(4\,x)}-12\,e^{(2\,x)}+1\right)e^{(-4\,x)}-x+\frac{1}{64}\,e^{(4\,x)}-\frac{3}{16}\,e^{(2\,x)}+\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(tanh(x)^5/sech(x)^4,x, algorithm="giac")
```

output
```
1/64*(48*e^(4*x) - 12*e^(2*x) + 1)*e^(-4*x) - x + 1/64*e^(4*x) - 3/16*e^(2
*x) + log(e^(2*x) + 1)
```

3.581.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sinh^4(x)\tanh(x)\,dx = \ln\left(\mathrm{e}^{2\,x}+1\right)-x-\frac{3\,\mathrm{e}^{-2\,x}}{16}-\frac{3\,\mathrm{e}^{2\,x}}{16}+\frac{\mathrm{e}^{-4\,x}}{64}+\frac{\mathrm{e}^{4\,x}}{64}$$

input
```
int(cosh(x)^4*tanh(x)^5,x)
```

output
```
log(exp(2*x) + 1) - x - (3*exp(-2*x))/16 - (3*exp(2*x))/16 + exp(-4*x)/64
+ exp(4*x)/64
```

3.582 $\int \mathrm{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$

3.582.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \mathrm{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = -\frac{4}{3}\mathrm{sech}^{\frac{3}{4}}(x)+\frac{8}{11}\mathrm{sech}^{\frac{11}{4}}(x)-\frac{4}{19}\mathrm{sech}^{\frac{19}{4}}(x)$$

output
```
-4/3*sech(x)^(3/4)+8/11*sech(x)^(11/4)-4/19*sech(x)^(19/4)
```

3.582.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \mathrm{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \mathrm{sech}^{\frac{3}{4}}(x)\left(-\frac{4}{3}+\frac{8\mathrm{sech}^2(x)}{11}-\frac{4\mathrm{sech}^4(x)}{19}\right)$$

input
```
Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]
```

output
```
Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)
```

3.582.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 3102, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(x)\text{sech}^{\frac{23}{4}}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sec(ix)^{23/4}}{\csc(ix)^5}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sec(ix)^{23/4}}{\csc(ix)^5}dx$$

$$\Big\downarrow \text{3102}$$

$$-\int \frac{\left(1-\text{sech}^2(x)\right)^2}{\sqrt[4]{\text{sech}(x)}}d\text{sech}(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\text{sech}^{\frac{15}{4}}(x)-2\text{sech}^{\frac{7}{4}}(x)+\frac{1}{\sqrt[4]{\text{sech}(x)}}\right)d\text{sech}(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{19}\text{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\text{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\text{sech}^{\frac{3}{4}}(x)$$

input
```
Int[Sech[x]^(23/4)*Sinh[x]^5,x]
```

output
```
(-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))/19
```

3.582.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.582.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-\frac{4\,\mathrm{sech}(x)^{\frac{3}{4}}}{3}+\frac{8\,\mathrm{sech}(x)^{\frac{11}{4}}}{11}-\frac{4\,\mathrm{sech}(x)^{\frac{19}{4}}}{19}$	20
default	$-\frac{4\,\mathrm{sech}(x)^{\frac{3}{4}}}{3}+\frac{8\,\mathrm{sech}(x)^{\frac{11}{4}}}{11}-\frac{4\,\mathrm{sech}(x)^{\frac{19}{4}}}{19}$	20

input
```
int(sech(x)^(3/4)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-4/3*sech(x)^(3/4)+8/11*sech(x)^(11/4)-4/19*sech(x)^(19/4)
```

3.582.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(19) = 38.

Time = 0.24 (sec) , antiderivative size = 359, normalized size of antiderivative = 11.58

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx =$$

$$-\frac{4\cdot 2^{\frac{3}{4}}\left(209\,\cosh\left(x\right)^{8}+1672\,\cosh\left(x\right)\sinh\left(x\right)^{7}+209\,\sinh\left(x\right)^{8}+76\,\left(77\,\cosh\left(x\right)^{2}+5\right)\sinh\left(x\right)^{6}+380\,\cdots\right)}{627\,\left(\cosh\left(x\right)^{8}+8\,\cosh\left(x\right)\sinh\left(x\right)^{7}+\sinh\left(x\right)^{8}+4\cdots\right)}$$

input
```
integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")
```

output
```
-4/627*2^(3/4)*(209*cosh(x)^8 + 1672*cosh(x)*sinh(x)^7 + 209*sinh(x)^8 + 7
6*(77*cosh(x)^2 + 5)*sinh(x)^6 + 380*cosh(x)^6 + 152*(77*cosh(x)^3 + 15*co
sh(x))*sinh(x)^5 + 10*(1463*cosh(x)^4 + 570*cosh(x)^2 + 87)*sinh(x)^4 + 87
0*cosh(x)^4 + 8*(1463*cosh(x)^5 + 950*cosh(x)^3 + 435*cosh(x))*sinh(x)^3 +
 4*(1463*cosh(x)^6 + 1425*cosh(x)^4 + 1305*cosh(x)^2 + 95)*sinh(x)^2 + 380
*cosh(x)^2 + 8*(209*cosh(x)^7 + 285*cosh(x)^5 + 435*cosh(x)^3 + 95*cosh(x)
)*sinh(x) + 209)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 + 1))^(3/4)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cos
h(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^
5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cos
h(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)
^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.582.6 Sympy [A] (verification not implemented)

Time = 39.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = -\frac{4\tanh^4(x)\operatorname{sech}^{\frac{3}{4}}(x)}{19}-\frac{64\tanh^2(x)\operatorname{sech}^{\frac{3}{4}}(x)}{209}-\frac{512\operatorname{sech}^{\frac{3}{4}}(x)}{627}$$

input
```
integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

output
```
-4*tanh(x)**4*sech(x)**(3/4)/19 - 64*tanh(x)**2*sech(x)**(3/4)/209 - 512*s
ech(x)**(3/4)/627
```

3.582.7 Maxima [F]

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \int \operatorname{sech}(x)^{\frac{3}{4}}\tanh(x)^5\,dx$$

input
```
integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")
```

output
```
integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

3.582.8 Giac [F]

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \int \operatorname{sech}(x)^{\frac{3}{4}}\tanh(x)^5\,dx$$

input
```
integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="giac")
```

output
```
integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

3.582.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \frac{32\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{11\,(e^{2x}+1)} - \frac{1312\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{209\,(e^{2x}+1)^2} + \frac{128\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{19\,(e^{2x}+1)^3} - \frac{64\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{19\,(e^{2x}+1)^4} - \frac{4\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{3}$$

input
```
int(tanh(x)^5*(1/cosh(x))^(3/4),x)
```

output
```
(32*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(11*(exp(2*x) + 1)) - (1312*(1/(exp(
-x)/2 + exp(x)/2))^(3/4))/(209*(exp(2*x) + 1)^2) + (128*(1/(exp(-x)/2 + ex
p(x)/2))^(3/4))/(19*(exp(2*x) + 1)^3) - (64*(1/(exp(-x)/2 + exp(x)/2))^(3/
4))/(19*(exp(2*x) + 1)^4) - (4*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/3
```

3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$

3.583.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
2*arctanh((a-b)*tanh(1/2*x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

3.583.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cosh(x)}\,dx = -\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(a + b*Cosh[x])^(-1),x]
```

output
```
(-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

3.583.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 3138$$

$$2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cosh[x])^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

3.583.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.583.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}$	36
risch	$\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	109

input
```
int(1/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.583.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.27

$$\int \frac{1}{a+b\cosh(x)}\,dx = \left[\frac{\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{\sqrt{a^2-b^2}},\ -\frac{2\,\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

input
```
integrate(1/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2
*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(
b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))/
sqrt(a^2 - b^2), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a)/(a^2 - b^2))/(a^2 - b^2)]
```

3.583.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(31) = 62.

Time = 1.83 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.07

$$\int \frac{1}{a+b\cosh(x)}\,dx = \begin{cases} \tilde{\infty}\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a=0\wedge b=0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ -\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for } a=-b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2)/b, Eq(a,
b)), (-1/(b*tanh(x/2)), Eq(a, -b)), (-log(-sqrt(a/(a - b) + b/(a - b)) + t
anh(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a - b)))
+ log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a -
b)) - b*sqrt(a/(a - b) + b/(a - b))), True))
```

3.583.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.583.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
integrate(1/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.583.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{b^2-a^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

input
```
int(1/(a + b*cosh(x)),x)
```

output
```
(2*atan(a/(b^2 - a^2)^(1/2) + (b*exp(x))/(b^2 - a^2)^(1/2)))/(b^2 - a^2)^(
1/2)
```

3.584 $\int \frac{1}{(1+\cosh(x))^2} \, dx$

3.584.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))}$$

output
```
1/3*sinh(x)/(1+cosh(x))^2+1/3*sinh(x)/(1+cosh(x))
```

3.584.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = \frac{(2+\cosh(x))\sinh(x)}{3(1+\cosh(x))^2}$$

input
```
Integrate[(1 + Cosh[x])^(-2),x]
```

output
```
((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

3.584.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{3}\int \frac{1}{\cosh(x)+1}dx+\frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(x)}{3(\cosh(x)+1)}+\frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

input
```
Int[(1 + Cosh[x])^(-2),x]
```

output
```
Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

3.584.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.584.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

method	result	size
risch	$-\frac{2(1+3\,e^{x})}{3(1+e^{x})^{3}}$	15
default	$-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$	16
parallelrisch	$-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$	16

input
```
int(1/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+3*exp(x))/(1+exp(x))^3
```

3.584.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1}{(1+\cosh(x))^2}\,dx =$$
$$-\frac{2\,(3\,\cosh\,(x)+3\,\sinh\,(x)+1)}{3\,\left(\cosh\,(x)^{3}+3\,(\cosh\,(x)+1)\sinh\,(x)^{2}+\sinh\,(x)^{3}+3\,\cosh\,(x)^{2}+3\,\left(\cosh\,(x)^{2}+2\,\cosh\,(x)+1\right)\sinh$$

input
```
integrate(1/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(x) + 3*sinh(x) + 1)/(cosh(x)^3 + 3*(cosh(x) + 1)*sinh(x)^2 +
sinh(x)^3 + 3*cosh(x)^2 + 3*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x) + 3*cosh(x
) + 1)
```

3.584.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{\tanh^3\left(\frac{x}{2}\right)}{6} + \frac{\tanh\left(\frac{x}{2}\right)}{2}$$

input
```
integrate(1/(1+cosh(x))**2,x)
```

output
```
-tanh(x/2)**3/6 + tanh(x/2)/2
```

3.584.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = \frac{2\,e^{(-x)}}{3\,e^{(-x)} + 3\,e^{(-2\,x)} + e^{(-3\,x)} + 1} + \frac{2}{3\,(3\,e^{(-x)} + 3\,e^{(-2\,x)} + e^{(-3\,x)} + 1)}$$

input
```
integrate(1/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2*
x) + e^(-3*x) + 1)
```

3.584.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{2\,(3\,e^x + 1)}{3\,(e^x + 1)^3}$$

input
```
integrate(1/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^x + 1)/(e^x + 1)^3
```

3.584.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{2\,(3\,e^x+1)}{3\,(e^x+1)^3}$$

input
```
int(1/(cosh(x) + 1)^2,x)
```

output
```
-(2*(3*exp(x) + 1))/(3*(exp(x) + 1)^3)
```

3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$

3.585.1 Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

3.585.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\tanh(x)}\,dx =\frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(a+b\tanh(x))}{2(a-b)(a+b)}$$

input
```
Integrate[(a + b*Tanh[x])^(-1),x]
```

output
```
((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[a + b*Tanh
[x]])/(2*(a - b)*(a + b))
```

3.585.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tanh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\tan(ix)}dx$$

$$\Big\downarrow \text{3965}$$

$$\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\tanh(x))}{a+b\tanh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\tanh(x)}{a+b\tanh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan(ix)}{a-ib\tan(ix)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

input
```
Int[(a + b*Tanh[x])^(-1),x]
```

output
```
(a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

3.585.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.585.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$-\frac{-\ln(1-\tanh(x))b+b\ln(a+b\tanh(x))-ax-bx}{a^2-b^2}$	42
derivativedivides	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}-\frac{\ln(-1+\tanh(x))}{2a+2b}$	55
default	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}-\frac{\ln(-1+\tanh(x))}{2a+2b}$	55
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}-\frac{b\ln\left(\mathrm{e}^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

input
```
int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
-(-ln(1-tanh(x))*b+b*ln(a+b*tanh(x))-a*x-b*x)/(a^2-b^2)
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(a+b)x - b\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

output
```
((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

3.585.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(29) = 58.

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.74

$$\int \frac{1}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)+\log(\tanh(x))) & \text{for } a=0 \wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b}+\frac{x}{2b\tanh(x)-2b}+\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}-\frac{b\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2}+\frac{b\log(\tanh(x)+1)}{a^2-b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tanh(x)),x)
```

output
```
Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0))
, (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b)
 + 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(
2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2)
- b*x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) +
 1)/(a**2 - b**2), True))
```

3.585.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

input
```
integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

3.585.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

input
```
integrate(1/(a+b*tanh(x)),x, algorithm="giac")
```

output
```
-b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.585.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{a\,x-b\,(x-\ln(\tanh(x)+1)+\ln(a+b\tanh(x)))}{a^2-b^2}$$

input
```
int(1/(a + b*tanh(x)),x)
```

output
```
(a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

3.586 $\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$

3.586.1 Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

output
```
arctanh(a*tanh(x)/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(1/2)
```

3.586.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

input
```
Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

3.586.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a^2+b^2\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$
$$\Big\downarrow \text{ 3660}$$
$$\int \frac{1}{a^2-(a^2+b^2)\coth^2(x)}d\coth(x)$$
$$\Big\downarrow \text{ 221}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{a^2+b^2}\coth(x)}{a}\right)}{a\sqrt{a^2+b^2}}$$

input
```
Int[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^2 + b^2]*Coth[x])/a]/(a*Sqrt[a^2 + b^2])
```

3.586.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.586.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(27) = 54$.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

method	result	size
default	$\frac{\ln\left(\sqrt{a^2+b^2}\,(\tanh^2(\frac{x}{2}))+2a\tanh(\frac{x}{2})+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(\sqrt{a^2+b^2}\,(\tanh^2(\frac{x}{2}))-2a\tanh(\frac{x}{2})+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}$	98
risch	$\frac{\ln\left(e^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}-2a^3-2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(e^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}+2a^3+2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}$	146

input
```
int(1/(a^2+b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1/2*x)+(a^
2+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2-2*a*t
anh(1/2*x)+(a^2+b^2)^(1/2))
```

3.586.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 9.29

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}\log\left(\frac{b^4\cosh(x)^4+4b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8a^4+8a^2b^2+b^4+2(2a^2b^2+b^4)\cosh(x)^2+2\left(3b^4\cosh(x)^2+2a^2b^2+b^4\right)\sinh}{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2(2a^2+b^2)\cosh(x)^2+2\left(3b\right.}\right.}{2(a^3+ab^2)}$$

input
```
integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a^2 + b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sin
h(x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(3*b^
4*cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + (2*a^2*b^2 +
 b^4)*cosh(x))*sinh(x) - 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*
b^2*sinh(x)^2 + 2*a^3 + a*b^2)*sqrt(a^2 + b^2))/(b^2*cosh(x)^4 + 4*b^2*cos
h(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2)*cosh(x)^2 + 2*(3*b^2*cosh
(x)^2 + 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*co
sh(x))*sinh(x)))/(a^3 + a*b^2)
```

3.586.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.58 (sec) , antiderivative size = 1129, normalized size of antiderivative = 36.42

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a**2+b**2*cosh(x)**2),x)
```

output
```
Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(
x/2)/(2*b**2) - 1/(2*b**2*tanh(x/2)), Eq(a, I*b) | Eq(a, -I*b)), (2*tanh(x
/2)/(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-a*sqrt(a/(a - I*b) + I*b/(a -
I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(
a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqr
t(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt
(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh
(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a
 + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I
*b/(a + I*b))) - a*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(-sqrt(a/(a - I*b)
 + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*s
qrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*
b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt(a/(a + I*b) - I*b/(a + I*b
))*log(sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a -
I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/
(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + I*b*sqrt(a
/(a - I*b) + I*b/(a - I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(
x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a
+ I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*
b/(a + I*b))) - I*b*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I...
```

3.586.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(27) = 54.

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)} \, dx = -\frac{\log\left(\frac{b^2 e^{(-2\,x)} + 2\,a^2 + b^2 - 2\,\sqrt{a^2 + b^2}a}{b^2 e^{(-2\,x)} + 2\,a^2 + b^2 + 2\,\sqrt{a^2 + b^2}a}\right)}{2\,\sqrt{a^2 + b^2}a}$$

input
```
integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log((b^2*e^(-2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*a)/(b^2*e^(-2*x)
+ 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*a))/(sqrt(a^2 + b^2)*a)
```

3.586.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \frac{1}{a^2 + b^2\cosh^2(x)}\,dx = \frac{\log\left(\frac{b^2e^{(2\,x)}+2\,a^2+b^2-2\,\sqrt{a^2+b^2}|a|}{b^2e^{(2\,x)}+2\,a^2+b^2+2\,\sqrt{a^2+b^2}|a|}\right)}{2\,\sqrt{a^2+b^2}|a|}$$

input
```
integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="giac")
```

output
```
1/2*log((b^2*e^(2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*abs(a))/(b^2*e^(2*x
) + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 + b^2)*abs(a))
```

3.586.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{a^2 + b^2\cosh^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{2\,a^2\,\left(-a^4-a^2\,b^2\right)^{3/2}+b^2\,\left(-a^4-a^2\,b^2\right)^{3/2}+b^2\,\mathrm{e}^{2\,x}\,\left(-a^4-a^2\,b^2\right)^{3/2}}{2\,a^8+4\,a^6\,b^2+2\,a^4\,b^4}\right)}{\sqrt{-a^4-a^2\,b^2}}$$

input
```
int(1/(b^2*cosh(x)^2 + a^2),x)
```

output
```
atan((2*a^2*(- a^4 - a^2*b^2)^(3/2) + b^2*(- a^4 - a^2*b^2)^(3/2) + b^2*ex
p(2*x)*(- a^4 - a^2*b^2)^(3/2))/(2*a^8 + 2*a^4*b^4 + 4*a^6*b^2))/(- a^4 -
a^2*b^2)^(1/2)
```

3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$

3.587.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

output
```
arctanh(a*tanh(x)/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(1/2)
```

3.587.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

input
```
Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

3.587.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2-b^2\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a^2-b^2\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{a^2-(a^2-b^2)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a^2-b^2}\coth(x)}{a}\right)}{a\sqrt{a^2-b^2}}$$

input
```
Int[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^2 - b^2]*Coth[x])/a]/(a*Sqrt[a^2 - b^2])
```

3.587.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.587.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$	74
risch	$\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}-2a^3+2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}-\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}+2a^3-2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}$	166

input
```
int(1/(a^2-b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a
/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.587.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 11.09

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4-8\,a^2b^2+b^4-2\,(2\,a^2b^2-b^4)\cosh(x)^2+2\left(3\,b^4\cosh(x)^2-2\,a^2b^2+b^4\right)\mathrm{s}}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4-2\,(2\,a^2-b^2)\cosh(x)^2+2\,(}\right.}{2\,(a^3-ab}\right.$$

input
```
integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a^2 - b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*si
nh(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 + 2*(3*b
^4*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - (2*a^2*b^2
- b^4)*cosh(x))*sinh(x) + 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a
*b^2*sinh(x)^2 - 2*a^3 + a*b^2)*sqrt(a^2 - b^2))/(b^2*cosh(x)^4 + 4*b^2*co
sh(x)*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cos
h(x)^2 - 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*c
osh(x))*sinh(x)))/(a^3 - a*b^2), sqrt(-a^2 + b^2)*arctan(-1/2*(b^2*cosh(x)
^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)
/(a^3 - a*b^2))/(a^3 - a*b^2)]
```

3.587.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(27) = 54$.

Time = 14.55 (sec) , antiderivative size = 874, normalized size of antiderivative = 24.97

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a**2-b**2*cosh(x)**2),x)
```

output
```
Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (tanh(x
/2)/(2*b**2) + 1/(2*b**2*tanh(x/2)), Eq(a, b) | Eq(a, -b)), (-2*tanh(x/2)/
(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-a*sqrt(a/(a - b) + b/(a - b))*log(
-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a -
b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqr
t(a/(a + b) - b/(a + b))) + a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a +
b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a
 + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) -
 b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(a -
b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a
 + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)))
 + a*sqrt(a/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/
2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*
b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(
a - b) + b/(a - b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*
sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/
(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a - b) + b/(
a - b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b
) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(
a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a + b) - b/(a + b))*lo...
```

3.587.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.587.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = -\frac{\arctan\left(\frac{b^2 e^{(2\,x)} - 2\,a^2 + b^2}{2\,\sqrt{-a^2 + b^2}a}\right)}{\sqrt{-a^2 + b^2}a}$$

input
```
integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="giac")
```

output
```
-arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sqrt(-a^2 +
 b^2)*a)
```

3.587.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = -\frac{\operatorname{atan}\left(\frac{b^2\,(a^2\,b^2 - a^4)^{3/2} - 2\,a^2\,(a^2\,b^2 - a^4)^{3/2} + b^2\,e^{2\,x}\,(a^2\,b^2 - a^4)^{3/2}}{2\,a^8 - 4\,a^6\,b^2 + 2\,a^4\,b^4}\right)}{\sqrt{a^2\,b^2 - a^4}}$$

input
```
int(-1/(b^2*cosh(x)^2 - a^2),x)
```

output
```
-atan((b^2*(a^2*b^2 - a^4)^(3/2) - 2*a^2*(a^2*b^2 - a^4)^(3/2) + b^2*exp(2
*x)*(a^2*b^2 - a^4)^(3/2))/(2*a^8 + 2*a^4*b^4 - 4*a^6*b^2))/(a^2*b^2 - a^4
)^(1/2)
```

3.588 $\int \frac{1}{1-\sinh^4(x)}\,dx$

3.588.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

output
```
1/4*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)
```

3.588.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right) + 2\tanh(x)\right)$$

input
```
Integrate[(1 - Sinh[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

3.588.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3688, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sinh^4(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{1-\sin(ix)^4}dx$$
$$\Big\downarrow \; 3688$$
$$\int \frac{1-\tanh^2(x)}{1-2\tanh^2(x)}d\tanh(x)$$
$$\Big\downarrow \; 299$$
$$\frac{1}{2}\int \frac{1}{1-2\tanh^2(x)}d\tanh(x)+\frac{\tanh(x)}{2}$$
$$\Big\downarrow \; 219$$
$$\frac{\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

input
```
Int[(1 - Sinh[x]^4)^(-1),x]
```

output
```
ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

3.588.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.588.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

method	result	size
risch	$-\frac{1}{1+\mathrm{e}^{2x}}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3+2\sqrt{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{8}$	46
default	$\frac{\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)+1}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{4}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{4}$	55

input
```
int(1/(1-sinh(x)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+exp(2*x))+1/8*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/8*2^(1/2)*ln(exp(2*
x)-3-2*2^(1/2))
```

3.588.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)}{\cosh(x)^2+\sinh(x)^2-}\right.}{8\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}$$

input
```
integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

output
```
1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 +
sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sin
h(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2
 - 3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

3.588.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. 2(20) = 40.

Time = 2.48 (sec) , antiderivative size = 908, normalized size of antiderivative = 36.32

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-sinh(x)**4),x)
```

output
```
3064704*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x
/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sq
rt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2
)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 3064704*log(
tanh(x/2) - 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/
2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh(x/2) - 1 +
 sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1225881
6*sqrt(2) + 17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/
(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2)
+ 17336584) + 2167073*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1
2258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) +
17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/
2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqr
t(2)*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 1733658
4*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x
/2) - sqrt(2) - 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*
tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt
(2) - 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)*
*2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2
) - 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1225881...
```

3.588.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) + \frac{1}{e^{(-2x)}+1}$$

input
```
integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

3.588.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{1}{1-\sinh^4(x)}\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right)-\frac{1}{e^{(2\,x)}+1}$$

input
```
integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/(e^(2*x) + 1)
```

3.588.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\sqrt{2}\,\ln\left(2\,e^{2\,x}+\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{8}\right)}{8}-\frac{\sqrt{2}\,\ln\left(2\,e^{2\,x}-\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{8}\right)}{8}-\frac{1}{e^{2\,x}+1}$$

input
```
int(-1/(sinh(x)^4 - 1),x)
```

output
```
(2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log
(2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)
```

3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx$

3.589.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx = -\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{3(1+\tanh(x))}$$

output
```
-4/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/3/(1+tanh(x))
```

3.589.2 Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx = \frac{1}{18}\left(8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right) - 3\cosh(2x) + 3\sinh(2x)\right)$$

input
```
Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2*x])/1
8
```

3.589.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\sinh^3(x)+\cosh^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3-i\sin(ix)^3}{i\sin(ix)^3+\cos(ix)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tanh^2(x)+\tanh(x)+1}{\tanh^4(x)+\tanh^3(x)+\tanh(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2}{3\left(\tanh^2(x)-\tanh(x)+1\right)}+\frac{1}{3(\tanh(x)+1)^2}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(\tanh(x)+1)}$$

input
```
Int[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[x]))
```

3.589.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.589.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{{\mathrm e}^{-2 x}}{6}+\frac{2 i \sqrt{3}\, \ln\left({\mathrm e}^{2 x}+i \sqrt{3}\right)}{9}-\frac{2 i \sqrt{3}\, \ln\left({\mathrm e}^{2 x}-i \sqrt{3}\right)}{9}$
default	$\frac{2 i \sqrt{3}\, \ln\left(\tanh^{2}\left(\frac{x}{2}\right)+\left(-1-i \sqrt{3}\right) \tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2 i \sqrt{3}\, \ln\left(\tanh^{2}\left(\frac{x}{2}\right)+\left(-1+i \sqrt{3}\right) \tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2}{3 \left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{2}{3 (\tanh(\frac{x}{2}$

input
```
int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*exp(-2*x)+2/9*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2))-2/9*I*3^(1/2)*ln(exp(2
*x)-I*3^(1/2))
```

3.589.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = -\frac{8\left(\sqrt{3}\cosh(x)^2 + 2\sqrt{3}\cosh(x)\sinh(x) + \sqrt{3}\sinh(x)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right) + 3}{18\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

input
```
integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas
")
```

output
```
-1/18*(8*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^
2)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) +
3)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

3.589.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.09

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{2\sqrt{3}\sinh(x)}{3\cosh(x)} - \frac{\sqrt{3}}{3}\right)}{9\sinh(x) + 9\cosh(x)} + \frac{3\sinh(x)}{9\sinh(x) + 9\cosh(x)} + \frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{2\sqrt{3}\sinh(x)}{3\cosh(x)} - \frac{\sqrt{3}}{3}\right)}{9\sinh(x) + 9\cosh(x)}$$

input
```
integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

output
```
4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(
x) + 9*cosh(x)) + 3*sinh(x)/(9*sinh(x) + 9*cosh(x)) + 4*sqrt(3)*cosh(x)*at
an(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x) + 9*cosh(x))
```

3.589.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} + 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} - 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{1}{6}e^{(-2x)}$$

input
```
integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima
")
```

output
```
4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) - 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sq
rt(2))) - 1/6*e^(-2*x)
```

3.589.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right) - \frac{1}{6}e^{(-2x)}$$

input
```
integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

output
```
4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6*e^(-2*x)
```

3.589.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2x}}{3}\right)}{9} - \frac{\mathrm{e}^{-2x}}{6}$$

input
```
int((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x)
```

output
```
(4*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9 - exp(-2*x)/6
```

3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$

3.590.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sinh(2x) + \frac{1}{16}\sinh(4x) + \frac{1}{24}\sinh(6x)$$

output
```
1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

3.590.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sinh(2x) + \frac{1}{16}\sinh(4x) + \frac{1}{24}\sinh(6x)$$

input
```
Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

output
```
x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

3.590.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ix)\cos(2ix)\cos(3ix)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(4x)+\frac{1}{4}\cosh(6x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

input
```
Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

output
```
x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

3.590.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.590.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
default	$\frac{x}{4}+\frac{\sinh(2x)}{8}+\frac{\sinh(4x)}{16}+\frac{\sinh(6x)}{24}$
risch	$\frac{x}{4}+\frac{e^{6x}}{48}+\frac{e^{4x}}{32}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}-\frac{e^{-4x}}{32}-\frac{e^{-6x}}{48}$
parallelrisch	$\frac{(-24+24\cosh(x))\ln(1-\coth(x)+\operatorname{csch}(x))+(-24\cosh(x)+24)\ln(\coth(x)-\operatorname{csch}(x)+1)+48x\cosh(x)-48x-4\sinh(6x)+2\sinh(}{96\cosh(x)-96}$

input
```
int(cosh(x)*cosh(2*x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

3.590.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{1}{4}\cosh(x)\sinh(x)^5 + \frac{1}{12}\left(10\cosh(x)^3+3\cosh(x)\right)\sinh(x)^3 + \frac{1}{4}\left(\cosh(x)^5+\cosh(x)^3+\cosh(x)\right)\sinh(x)+\frac{1}{4}x$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="fricas")
```

output
```
1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 1/4*(c
osh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x
```

3.590.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44.

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.87

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x\sinh(x)\sinh(2x)\cosh(3x)}{4} - \frac{x\sinh(x)\sinh(3x)\cosh(2x)}{4} - \frac{x\sinh(2x)\sinh(3x)\cosh(x)}{4} + \frac{x\cosh(x)\cosh(2x)\cosh(3x)}{4} - \frac{3\sinh(x)\sinh(2x)\sinh(3x)}{8} + \frac{\sinh(x)\cosh(2x)\cosh(3x)}{3} + \frac{5\sinh(2x)\cosh(x)\cosh(3x)}{24}$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

output
```
x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4 - x*sinh
(2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x)/4 - 3*sinh(x)*si
nh(2*x)*sinh(3*x)/8 + sinh(x)*cosh(2*x)*cosh(3*x)/3 + 5*sinh(2*x)*cosh(x)*
cosh(3*x)/24
```

3.590.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{1}{96}\left(3e^{(-2x)} + 6e^{(-4x)} + 2\right)e^{(6x)} + \frac{1}{4}x - \frac{1}{16}e^{(-2x)} - \frac{1}{32}e^{(-4x)} - \frac{1}{48}e^{(-6x)}$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="maxima")
```

output
```
1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x) - 1/32*
e^(-4*x) - 1/48*e^(-6*x)
```

3.590.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = -\frac{1}{96}\left(22\,e^{(6\,x)} + 6\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{4}\,x + \frac{1}{48}\,e^{(6\,x)} + \frac{1}{32}\,e^{(4\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="giac")
```

output
```
-1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)*e^(-6*x) + 1/4*x + 1/48*e^(
6*x) + 1/32*e^(4*x) + 1/16*e^(2*x)
```

3.590.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} - \frac{e^{-2\,x}}{16} + \frac{e^{2\,x}}{16} - \frac{e^{-4\,x}}{32} + \frac{e^{4\,x}}{32} - \frac{e^{-6\,x}}{48} + \frac{e^{6\,x}}{48}$$

input
```
int(cosh(2*x)*cosh(3*x)*cosh(x),x)
```

output
```
x/4 - exp(-2*x)/16 + exp(2*x)/16 - exp(-4*x)/32 + exp(4*x)/32 - exp(-6*x)/
48 + exp(6*x)/48
```

3.591 $\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$

3.591.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx=-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

output
```
-1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

3.591.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx=-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

input
```
Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

output
```
-1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

3.591.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ix)\sin\left(\frac{5ix}{2}\right)\left(-\cos\left(\frac{3ix}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos\left(\frac{3ix}{2}\right)\sin(ix)\sin\left(\frac{5ix}{2}\right)dx$$

$$\Big\downarrow \text{4855}$$

$$-\int \left(-\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(3x)-\frac{1}{4}\cosh(5x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

input
```
Int[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

output
```
-1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

3.591.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.591.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
default	$-\frac{x}{4}+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$
risch	$-\frac{x}{4}+\frac{e^{5x}}{40}-\frac{e^{3x}}{24}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}+\frac{e^{-3x}}{24}-\frac{e^{-5x}}{40}$
parallelrisch	$\frac{(-40\cosh(\frac{x}{2})+40)\ln(1-\tanh(\frac{3x}{4}))+(40\cosh(\frac{x}{2})-40)\ln(\tanh(\frac{3x}{4})+1)-120x\cosh(\frac{x}{2})+120x+6\sinh(\frac{11x}{2})-30\sinh(2x)+20}{-240+240\cosh(\frac{x}{2})}$

input
```
int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

3.591.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(22) = 44.

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.70

$$\begin{aligned}
&\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx \\
&= 6\,\cosh\left(\frac{1}{2}\,x\right)^{3}\sinh\left(\frac{1}{2}\,x\right)^{7}+\frac{1}{2}\,\cosh\left(\frac{1}{2}\,x\right)\sinh\left(\frac{1}{2}\,x\right)^{9} \\
&\quad+\frac{1}{10}\left(126\,\cosh\left(\frac{1}{2}\,x\right)^{5}-5\,\cosh\left(\frac{1}{2}\,x\right)\right)\sinh\left(\frac{1}{2}\,x\right)^{5} \\
&\quad+\frac{1}{6}\left(36\,\cosh\left(\frac{1}{2}\,x\right)^{7}-10\,\cosh\left(\frac{1}{2}\,x\right)^{3}+3\,\cosh\left(\frac{1}{2}\,x\right)\right)\sinh\left(\frac{1}{2}\,x\right)^{3} \\
&\quad+\frac{1}{2}\left(\cosh\left(\frac{1}{2}\,x\right)^{9}-\cosh\left(\frac{1}{2}\,x\right)^{5}+\cosh\left(\frac{1}{2}\,x\right)^{3}\right)\sinh\left(\frac{1}{2}\,x\right)-\frac{1}{4}\,x
\end{aligned}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="fricas")
```

output
```
6*cosh(1/2*x)^3*sinh(1/2*x)^7 + 1/2*cosh(1/2*x)*sinh(1/2*x)^9 + 1/10*(126*
cosh(1/2*x)^5 - 5*cosh(1/2*x))*sinh(1/2*x)^5 + 1/6*(36*cosh(1/2*x)^7 - 10*
cosh(1/2*x)^3 + 3*cosh(1/2*x))*sinh(1/2*x)^3 + 1/2*(cosh(1/2*x)^9 - cosh(1
/2*x)^5 + cosh(1/2*x)^3)*sinh(1/2*x) - 1/4*x
```

3.591.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(22) = 44$.

Time = 0.94 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.63

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{x\sinh(x)\sinh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4} + \frac{x\sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)}{4} + \frac{x\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\cosh(x)}{4} - \frac{x\cosh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4} - \frac{3\sinh(x)\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)}{20} + \frac{5\sinh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{12} - \frac{\sinh\left(\frac{5x}{2}\right)\cosh(x)\cosh\left(\frac{3x}{2}\right)}{15}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

output
```
-x*sinh(x)*sinh(3*x/2)*cosh(5*x/2)/4 + x*sinh(x)*sinh(5*x/2)*cosh(3*x/2)/4
 + x*sinh(3*x/2)*sinh(5*x/2)*cosh(x)/4 - x*cosh(x)*cosh(3*x/2)*cosh(5*x/2)
/4 - 3*sinh(x)*sinh(3*x/2)*sinh(5*x/2)/20 + 5*sinh(x)*cosh(3*x/2)*cosh(5*x
/2)/12 - sinh(5*x/2)*cosh(x)*cosh(3*x/2)/15
```

3.591.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{1}{240}\left(10\,e^{(-2\,x)} - 15\,e^{(-3\,x)} - 6\right)e^{(5\,x)} - \frac{1}{4}\,x - \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)} - \frac{1}{40}\,e^{(-5\,x)}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="maxima")
```

output
```
-1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(-2*x) + 1
/24*e^(-3*x) - 1/40*e^(-5*x)
```

3.591.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = \frac{1}{240}\left(137\,e^{(5\,x)} - 15\,e^{(3\,x)} + 10\,e^{(2\,x)} - 6\right)e^{(-5\,x)} - \frac{1}{4}\,x + \frac{1}{40}\,e^{(5\,x)} - \frac{1}{24}\,e^{(3\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="giac")
```

output
```
1/240*(137*e^(5*x) - 15*e^(3*x) + 10*e^(2*x) - 6)*e^(-5*x) - 1/4*x + 1/40*
e^(5*x) - 1/24*e^(3*x) + 1/16*e^(2*x)
```

3.591.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = \frac{e^{2\,x}}{16} - \frac{e^{-2\,x}}{16} - \frac{x}{4} + \frac{e^{-3\,x}}{24} - \frac{e^{3\,x}}{24} - \frac{e^{-5\,x}}{40} + \frac{e^{5\,x}}{40}$$

input
```
int(cosh((3*x)/2)*sinh((5*x)/2)*sinh(x),x)
```

output
```
exp(2*x)/16 - exp(-2*x)/16 - x/4 + exp(-3*x)/24 - exp(3*x)/24 - exp(-5*x)/
40 + exp(5*x)/40
```

3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x)+\sinh(2x))}\,dx$

3.592.1 Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = \sqrt{2}\arctan\left(\operatorname{sech}(x)\sqrt{\cosh(x)\sinh(x)}\right) + \frac{1}{6}\arctan\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right) - \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\operatorname{sech}(x)\sqrt{\cosh(x)\sinh(x)}\right) + \frac{\cosh(x)}{\sqrt{\sinh(2x)}}$$

output
```
1/6*arctan(sinh(x)/sinh(2*x)^(1/2))+arctan(sech(x)*(cosh(x)*sinh(x))^(1/2)
)*2^(1/2)-1/3*arctanh(sech(x)*(cosh(x)*sinh(x))^(1/2))*2^(1/2)+cosh(x)/sin
h(2*x)^(1/2)
```

3.592.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(69) = 138.

Time = 18.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.32

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = \frac{\sqrt{\sinh(2x)}\left(6\sqrt{2}\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right)+\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}\right)-2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right)+\frac{3\sqrt{\cosh(x)\text{sech}}}{\sqrt{\tanh\left(\frac{x}{2}\right.}}\right.}{6(1+\cosh(x))\sqrt{\tanh\left(\frac{x}{2}\right)}\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}$$

input
```
Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + S
inh[2*x])),x]
```

output
```
(Sqrt[Sinh[2*x]]*(6*Sqrt[2]*ArcTan[Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[
x])]] + ArcTan[Sqrt[Tanh[x/2]]/Sqrt[1 + Tanh[x/2]^2]] - 2*Sqrt[2]*ArcTanh[
Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[x])]] + (3*Sqrt[Cosh[x]*Sech[x/2]^2
])/Sqrt[Tanh[x/2]]))/(6*(1 + Cosh[x])*Sqrt[Tanh[x/2]]*Sqrt[1 + Tanh[x/2]^2
])
```

3.592.3 Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4890, 26, 4889, 25, 2035, 2247, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)(\tanh(x)-\cosh(2x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)(-\cos(2ix)-i\tan(ix))}{(-\sin(ix)^2-i\sin(2ix))\sqrt{-i\sin(2ix)}}dx$$

$$\Big\downarrow \text{4890}$$

$$\frac{i\sinh(x)\int -\frac{i\cos(ix)\text{csch}(x)(\cos(2ix)+i\tan(ix))\sqrt{\tanh(x)}}{\sin(ix)^2+i\sin(2ix)}dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh(x)\int \frac{\cos(ix)\text{csch}(x)(\cos(2ix)+i\tan(ix))\sqrt{\tanh(x)}}{\sin(ix)^2+i\sin(2ix)}dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{\sinh(x)\int -\frac{\tanh^3(x)+\tanh^2(x)-\tanh(x)+1}{\tanh^{\frac{3}{2}}(x)(\tanh(x)+2)(1-\tanh^2(x))}d\tanh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sinh(x)\int \frac{\tanh^3(x)+\tanh^2(x)-\tanh(x)+1}{\tanh^{\frac{3}{2}}(x)(\tanh(x)+2)(1-\tanh^2(x))}d\tanh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 2035}$$

$$-\frac{2\sinh(x)\int \frac{\coth^2(x)(\tanh^3(x)+\tanh^2(x)-\tanh(x)+1)}{(\tanh(x)+2)(1-\tanh^2(x))}d\sqrt{\tanh(x)}}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 2247}$$

$$-\frac{2\sinh(x)\int \left(\frac{\coth^2(x)}{2}+\frac{1}{-\tanh(x)-1}-\frac{1}{3(\tanh(x)-1)}-\frac{1}{6(\tanh(x)+2)}\right)d\sqrt{\tanh(x)}}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sinh(x)\left(-\arctan\left(\sqrt{\tanh(x)}\right)-\frac{\arctan\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{3}\text{arctanh}\left(\sqrt{\tanh(x)}\right)-\frac{\coth(x)}{2}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

input
```
Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*
x])),x]
```

output
```
(-2*(-ArcTan[Sqrt[Tanh[x]]] - ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]/(6*Sqrt[2]) +
ArcTanh[Sqrt[Tanh[x]]]/3 - Coth[x]/2)*Sinh[x])/(Sqrt[Sinh[2*x]]*Sqrt[Tanh[
x]])
```

3.592. $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x)+\sinh(2x))}\,dx$

3.592.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2247
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d + e*x^2)^q*(a + c
*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && PolyQ[Px, x] && Integ
erQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

3.592. $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x)+\sinh(2x))}\,dx$

3.592.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(53) = 106.

Time = 1.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.42

method	result
default	$\frac{\sqrt{\frac{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)-1\right)^2}}\left(\tanh^2\left(\frac{x}{2}\right)-1\right)\left(2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}\,\sqrt{2}}{2\tanh\left(\frac{x}{2}\right)}\right)\tanh\left(\frac{x}{2}\right)+6\sqrt{2}\arctan\left(\frac{\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}}{2\tanh\left(\frac{x}{2}\right)}\right.}{6\sqrt{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}\,\tanh\left(\frac{x}{2}\right)}$

input
```
int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/6*((tanh(1/2*x)^2+1)*tanh(1/2*x)/(tanh(1/2*x)^2-1)^2)^(1/2)*(tanh(1/2*x)
^2-1)*(2*2^(1/2)*arctanh(1/2/tanh(1/2*x)*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)
*2^(1/2))*tanh(1/2*x)+6*2^(1/2)*arctan(1/2/tanh(1/2*x)*(tanh(1/2*x)^3+tanh
(1/2*x))^(1/2)*2^(1/2))*tanh(1/2*x)+arctan(1/tanh(1/2*x)*(tanh(1/2*x)^3+ta
nh(1/2*x))^(1/2))*tanh(1/2*x)-3*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2))/((tanh(
1/2*x)^2+1)*tanh(1/2*x))^(1/2)/tanh(1/2*x)
```

3.592.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.45

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx =$$

$$-\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\arctan\left(\frac{\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+3\sqrt{2}\right)\sqrt{\frac{}{c}}}{2\left(\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4c\right.}\right.}{}$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/
2),x, algorithm="fricas")
```

output
```
-1/12*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + 3*sqrt(2))*sq
rt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4
 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 - 1)) + 6*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*
sinh(x)^2 - sqrt(2))*arctan(2*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(
x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)) - (sqrt(2)*cosh(x)^2 + 2*sqrt
(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(2*cosh(x)^4 + 8*cos
h(x)^3*sinh(x) + 12*cosh(x)^2*sinh(x)^2 + 8*cosh(x)*sinh(x)^3 + 2*sinh(x)^
4 - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(cosh(x)*sinh(x)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1) - 12*sqrt(2)*sqrt(cosh(x)*s
inh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)
```

3.592.6 Sympy [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = -\int \frac{\cosh\left(x\right)\cosh\left(2x\right)}{\sinh^2\left(x\right)\sqrt{\sinh\left(2x\right)}+\sinh^{\frac{3}{2}}\left(2x\right)}\,dx - \int \left(-\frac{\cosh\left(x\right)\tanh\left(x\right)}{\sinh^2\left(x\right)\sqrt{\sinh\left(2x\right)}+\sinh^{\frac{3}{2}}\left(2x\right)}\right)dx$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(
1/2),x)
```

output
```
-Integral(cosh(x)*cosh(2*x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(3/2)
), x) - Integral(-cosh(x)*tanh(x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)*
*(3/2)), x)
```

3.592.7 Maxima [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = \int -\frac{(\cosh(2x)-\tanh(x))\cosh(x)}{(\sinh(x)^2+\sinh(2x))\sqrt{\sinh(2x)}}\,dx$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/
2),x, algorithm="maxima")
```

output
```
-integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sin
h(2*x))), x)
```

3.592.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = \sqrt{2}\arctan\left(\sqrt{e^{(4x)}-1}-e^{(2x)}\right) + \frac{1}{6}\sqrt{2}\log\left(-\sqrt{e^{(4x)}-1}+e^{(2x)}\right) + \frac{\sqrt{2}}{\sqrt{e^{(4x)}-1}-e^{(2x)}+1} + \frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\sqrt{e^{(4x)}-1}-3e^{(2x)}-1\right)\right)$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/
2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(sqrt(e^(4*x) - 1) - e^(2*x)) + 1/6*sqrt(2)*log(-sqrt(e^(4*x
) - 1) + e^(2*x)) + sqrt(2)/(sqrt(e^(4*x) - 1) - e^(2*x) + 1) + 1/6*arctan
(1/4*sqrt(2)*(3*sqrt(e^(4*x) - 1) - 3*e^(2*x) - 1))
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = -\int \frac{\cosh(x)\,\left(\cosh(2\,x)-\tanh(x)\right)}{\sqrt{\sinh\left(2\,x\right)}\,\left({\sinh\left(x\right)}^2+\sinh\left(2\,x\right)\right)}\,dx$$

input
```
int(-(cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)
^2)),x)
```

output
```
-int((cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)
^2)), x)
```

3.593 $\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}}\,dx$

3.593.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}}\,dx = -\frac{\cosh(x)}{27\,(-9+4\cosh^2(x))^{3/2}}+\frac{2\cosh(x)}{243\sqrt{-9+4\cosh^2(x)}}$$

output
```
-1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

3.593.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}}\,dx = \frac{\cosh(x)(-23+4\cosh(2x))}{243(-7+2\cosh(2x))^{3/2}}$$

input
```
Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

output
```
(Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))
```

3.593.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3669, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\left(4\cosh^2(x)-9\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cos\left(\frac{\pi}{2}+ix\right)}{\left(-9+4\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)}{\left(4\sin\left(ix+\frac{\pi}{2}\right)^2-9\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\int \frac{1}{\left(4\cosh^2(x)-9\right)^{5/2}}d\cosh(x)$$

$$\Big\downarrow \text{ 209}$$

$$-\frac{2}{27}\int \frac{1}{\left(4\cosh^2(x)-9\right)^{3/2}}d\cosh(x)-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

input
```
Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

output
```
-1/27*Cosh[x]/(-9 + 4*Cosh[x]^2)^(3/2) + (2*Cosh[x])/(243*Sqrt[-9 + 4*Cosh
[x]^2])
```

3.593.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.593.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\cosh(x)}{27(-9+4(\cosh^2(x)))^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4(\cosh^2(x))}}$	30
default	$-\frac{\cosh(x)}{27(-9+4(\cosh^2(x)))^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4(\cosh^2(x))}}$	30

input
```
int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

3.593. $\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}}\,dx$

3.593.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 12.81

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \frac{2\cosh(x)^8+16\cosh(x)\sinh(x)^7+2\sinh(x)^8+28\left(2\cosh(x)^2-1\right)\sinh(x)^6 \cdots}{\cdots}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/486*(2*cosh(x)^8 + 16*cosh(x)*sinh(x)^7 + 2*sinh(x)^8 + 28*(2*cosh(x)^2
- 1)*sinh(x)^6 - 28*cosh(x)^6 + 56*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2
*(70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 102*cosh(x)^4 + 8*(14*cos
h(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 4*(14*cosh(x)^6 - 105*cosh
(x)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 28*cosh(x)^2 + 8*(2*cosh(x)^7 - 21*
cosh(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + (2*cosh(x)^6 + 12*cosh(x)*
sinh(x)^5 + 2*sinh(x)^6 + 3*(10*cosh(x)^2 - 7)*sinh(x)^4 - 21*cosh(x)^4 +
4*(10*cosh(x)^3 - 21*cosh(x))*sinh(x)^3 + 3*(10*cosh(x)^4 - 42*cosh(x)^2 -
 7)*sinh(x)^2 - 21*cosh(x)^2 + 6*(2*cosh(x)^5 - 14*cosh(x)^3 - 7*cosh(x))*
sinh(x) + 2)*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 2)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 1
4*(2*cosh(x)^2 - 1)*sinh(x)^6 - 14*cosh(x)^6 + 28*(2*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^5 + (70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 51*cosh(x)^4
 + 4*(14*cosh(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^
6 - 105*cosh(x)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(2*cos
h(x)^7 - 21*cosh(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)
```

3.593.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.593.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(29) = 58.

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{1855\,e^{(-2\,x)}-8485\,e^{(-4\,x)}+5285\,e^{(-6\,x)}-980\,e^{(-8\,x)}+56\,e^{(-10\,x)}-106}{12150\,\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{980\,e^{(-2\,x)}-5285\,e^{(-4\,x)}+8485\,e^{(-6\,x)}-1855\,e^{(-8\,x)}+106\,e^{(-10\,x)}-56}{12150\,\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12150*(1855*e^(-2*x) - 8485*e^(-4*x) + 5285*e^(-6*x) - 980*e^(-8*x) + 5
6*e^(-10*x) - 106)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) + e^(-2*x)
+ 1)^(5/2)) + 1/12150*(980*e^(-2*x) - 5285*e^(-4*x) + 8485*e^(-6*x) - 1855
*e^(-8*x) + 106*e^(-10*x) - 56)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x
) + e^(-2*x) + 1)^(5/2))
```

3.593.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(2\,e^{(2\,x)}-21\right)e^{(2\,x)}-21\right)e^{(2\,x)}+2}{486\,\left(e^{(4\,x)}-7\,e^{(2\,x)}+1\right)^{\frac{3}{2}}}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/486*(((2*e^(2*x) - 21)*e^(2*x) - 21)*e^(2*x) + 2)/(e^(4*x) - 7*e^(2*x) +
 1)^(3/2)
```

3.593.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{e^x\sqrt{4\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)^2-9}\,\left(21\,e^{2\,x}+21\,e^{4\,x}-2\,e^{6\,x}-2\right)}{486\,\left(e^{4\,x}-7\,e^{2\,x}+1\right)^2}$$

input
```
int(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x)
```

output
```
-(exp(x)*(4*(exp(-x)/2 + exp(x)/2)^2 - 9)^(1/2)*(21*exp(2*x) + 21*exp(4*x)
 - 2*exp(6*x) - 2))/(486*(exp(4*x) - 7*exp(2*x) + 1)^2)
```

3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx$

3.594.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = \frac{2}{\sqrt{1-\sinh^2(x)}} + 2\sqrt{1-\sinh^2(x)}$$

output
```
2/(1-sinh(x)^2)^(1/2)+2*(1-sinh(x)^2)^(1/2)
```

3.594.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = \frac{5-\cosh(2x)}{\sqrt{1-\sinh^2(x)}}$$

input
```
Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

output
```
(5 - Cosh[2*x])/Sqrt[1 - Sinh[x]^2]
```

3.594.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4878, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^2\sin(2ix)}{(1+\sin(ix)^2)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ix)^2\sin(2ix)}{(\sin(ix)^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{4878}$$

$$i\int -\frac{2i\sinh^3(x)}{\left(1-\sinh^2(x)\right)^{3/2}}d\sinh(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sinh^3(x)}{\left(1-\sinh^2(x)\right)^{3/2}}d\sinh(x)$$

$$\Big\downarrow \text{243}$$

$$\int \frac{\sinh^2(x)}{\left(1-\sinh^2(x)\right)^{3/2}}d\sinh^2(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{\left(1-\sinh^2(x)\right)^{3/2}}-\frac{1}{\sqrt{1-\sinh^2(x)}}\right)d\sinh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

input
```
Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

output
```
2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

3.594.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.594. $\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx$

3.594.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{2(\sinh^2(x))}{\sqrt{1-(\sinh^2(x))}}+\frac{4}{\sqrt{1-(\sinh^2(x))}}$	30
default	$-\frac{2(\sinh^2(x))}{\sqrt{1-(\sinh^2(x))}}+\frac{4}{\sqrt{1-(\sinh^2(x))}}$	30

input
```
int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sinh(x)^2/(1-sinh(x)^2)^(1/2)+4/(1-sinh(x)^2)^(1/2)
```

3.594.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.55

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{2}\left(\cosh\left(x\right)^4+4\,\cosh\left(x\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4+2\left(3\,\cosh\left(x\right)^2-5\right)\sinh}{\cosh\left(x\right)^5+5\,\cosh\left(x\right)\sinh\left(x\right)^4+\sinh\left(x\right)^5+2\left(5\,\cosh\left(x\right)^2-3\right)\sinh\left(x\right)^3-6\,\mathrm{co}}$$

input
```
integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 5)
*sinh(x)^2 - 10*cosh(x)^2 + 4*(cosh(x)^3 - 5*cosh(x))*sinh(x) + 1)*sqrt(-(
cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(c
osh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + 2*(5*cosh(x)^2 - 3)*sinh(x)^3
 - 6*cosh(x)^3 + 2*(5*cosh(x)^3 - 9*cosh(x))*sinh(x)^2 + (5*cosh(x)^4 - 18
*cosh(x)^2 + 1)*sinh(x) + cosh(x))
```

3.594.6 Sympy [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{\sinh^2(x)\sinh(2x)}{\left(-(\sinh(x)-1)(\sinh(x)+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
```

output
```
Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/2), x)
```

3.594.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(25) = 50$.

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.10

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = -\frac{16\,e^{(-x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{62\,e^{(-3\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} - \frac{16\,e^{(-5\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{e^{(-7\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{e^{x}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}$$

input
```
integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-16*e^(-x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2
)) + 62*e^(-3*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1
)^(3/2)) - 16*e^(-5*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*
x) + 1)^(3/2)) + e^(-7*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(
-2*x) + 1)^(3/2)) + e^x/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2
*x) + 1)^(3/2))
```

3.594.8 Giac [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{\sinh\left(2\,x\right)\sinh\left(x\right)^2}{\left(-\sinh\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2), x)
```

3.594.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{2\sqrt{1-\left(\frac{e^{-x}}{2}-\frac{e^x}{2}\right)^2}\left(e^{4\,x}-10\,e^{2\,x}+1\right)}{e^{4\,x}-6\,e^{2\,x}+1}$$

input
```
int((sinh(2*x)*sinh(x)^2)/(1 - sinh(x)^2)^(3/2),x)
```

output
```
(2*(1 - (exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x) - 10*exp(2*x) + 1))/(exp
(4*x) - 6*exp(2*x) + 1)
```

3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$

3.595.1 Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

output
```
1/2*arcsinh(sinh(x)*2^(1/2))*2^(1/2)
```

3.595.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

input
```
Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

output
```
ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

3.595.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4856, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)}{\sqrt{\cos(2ix)}}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{1}{\sqrt{2\sinh^2(x)+1}}d\sinh(x)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\text{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

input
```
Int[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

output
```
ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

3.595.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.595.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(12) = 24$.

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.20

method	result	size
default	$\frac{\sqrt{\left(2\left(\cosh^2(x)\right)-1\right)\left(\sinh^2(x)\right)}\,\ln\left(\sqrt{2}\,\left(\sinh^2(x)\right)+\frac{\sqrt{2}}{4}+\sqrt{2\left(\sinh^4(x)\right)+\sinh^2(x)}\right)\sqrt{2}}{4\sinh(x)\sqrt{2\left(\cosh^2(x)\right)-1}}$	63

input
```
int(cosh(x)/cosh(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+1/4*2^(1/2)+(2*
sinh(x)^4+sinh(x)^2)^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2-1)^(1/2)
```

3.595.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 482, normalized size of antiderivative = 32.13

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

$$= \frac{1}{8}\sqrt{2}\log\left(-\frac{\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + \left(28\cosh(x)^2 - 3\right)\sinh(x)^6 - 3\cosh(x)^6 + 2}{}\right.$$

$$+\frac{1}{8}\sqrt{2}\log\left(\frac{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + \left(6\cosh(x)^2 + 1\right)\sinh(x)^2 + \sqrt{2}\left(\cosh(x)^2 + \right.}{\cosh(x}\right.$$

input
```
integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x
)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(
x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4
+ 30*cosh(x)^2 - 4)*sinh(x)^2 + sqrt(2)*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 +
 sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3
- 3*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 18*cosh(x)^2 + 4)*sinh(x)^2 + 4*c
osh(x)^2 + 2*(3*cosh(x)^5 - 6*cosh(x)^3 + 4*cosh(x))*sinh(x) - 4)*sqrt((co
sh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*cosh
(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) +
 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)
^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
+ 1/8*sqrt(2)*log((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x
)^2 + 1)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + 1)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2))
```

3.595.6 Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{\cosh{(2x)}}}\,dx$$

input
```
integrate(cosh(x)/cosh(2*x)**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

3.595.7 Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{\cosh{(2\,x)}}}\,dx$$

input
```
integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

3.595.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.87

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = -\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

input
```
integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
 - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{\cosh(2\,x)}}\,dx$$

input
```
int(cosh(x)/cosh(2*x)^(1/2),x)
```

output
```
int(cosh(x)/cosh(2*x)^(1/2), x)
```

3.596 $\int x \tanh^2(x)\,dx$

3.596.1 Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x \tanh^2(x)\,dx = \frac{x^2}{2} + \log(\cosh(x)) - x\tanh(x)$$

output
```
1/2*x^2+ln(cosh(x))-x*tanh(x)
```

3.596.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x \tanh^2(x)\,dx = \frac{x^2}{2} + \log(\cosh(x)) - x\tanh(x)$$

input
```
Integrate[x*Tanh[x]^2,x]
```

output
```
x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

3.596.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tanh^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\tan(ix)^2dx$$

$$\Big\downarrow \text{4203}$$

$$\int xdx-i\int i\tanh(x)dx-x\tanh(x)$$

$$\Big\downarrow \text{15}$$

$$-i\int i\tanh(x)dx+\frac{x^2}{2}-x\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \tanh(x)dx+\frac{x^2}{2}-x\tanh(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)dx+\frac{x^2}{2}-x\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix)dx+\frac{x^2}{2}-x\tanh(x)$$

$$\Big\downarrow \text{3956}$$

$$\frac{x^2}{2}-x\tanh(x)+\log(\cosh(x))$$

input
```
Int[x*Tanh[x]^2,x]
```

output
```
x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

3.596.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.596.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
parallelrisch	$\frac{x^2}{2} - x\tanh(x) - x - \ln(1-\tanh(x))$	24
risch	$\frac{x^2}{2} - 2x + \frac{2x}{1+e^{2x}} + \ln(1+e^{2x})$	28

input
```
int(x*tanh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-x*tanh(x)-x-ln(1-tanh(x))
```

3.596.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int x\tanh^2(x)\,dx$$
$$= \frac{(x^2-4x)\cosh(x)^2 + 2(x^2-4x)\cosh(x)\sinh(x) + (x^2-4x)\sinh(x)^2 + x^2 + 2\left(\cosh(x)^2 + 2\cosh(x\right.}{2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}$$

input
```
integrate(x*tanh(x)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*s
inh(x)^2 + x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*c
osh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)
```

3.596.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x\tanh^2(x)\,dx = \frac{x^2}{2} - x\tanh(x) + x - \log(\tanh(x) + 1)$$

input
```
integrate(x*tanh(x)**2,x)
```

output
```
x**2/2 - x*tanh(x) + x - log(tanh(x) + 1)
```

3.596.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int x\tanh^2(x)\,dx = -\frac{xe^{(2x)}}{e^{(2x)} + 1} + \frac{x^2 + (x^2 - 2x)e^{(2x)}}{2\left(e^{(2x)} + 1\right)} + \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(x*tanh(x)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) +
 log(e^(2*x) + 1)
```

3.596.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int x\tanh^2(x)\,dx = \frac{x^2e^{(2x)} + x^2 - 4\,xe^{(2x)} + 2\,e^{(2x)}\log\left(e^{(2x)} + 1\right) + 2\,\log\left(e^{(2x)} + 1\right)}{2\left(e^{(2x)} + 1\right)}$$

input
```
integrate(x*tanh(x)^2,x, algorithm="giac")
```

output
```
1/2*(x^2*e^(2*x) + x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) + 1) + 2*log(
e^(2*x) + 1))/(e^(2*x) + 1)
```

3.596.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x\tanh^2(x)\,dx = \ln\left(e^{2x}+1\right) - x - x\tanh(x) + \frac{x^2}{2}$$

input
```
int(x*tanh(x)^2,x)
```

output
```
log(exp(2*x) + 1) - x - x*tanh(x) + x^2/2
```

3.597 $\int x \coth^2(x) \, dx$

3.597.1 Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x \coth^2(x) \, dx = \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

output
```
1/2*x^2-x*coth(x)+ln(sinh(x))
```

3.597.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x \coth^2(x) \, dx = \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

input
```
Integrate[x*Coth[x]^2,x]
```

output
```
x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

3.597.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^2(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x \tan \left(\frac{\pi}{2}+i x\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x \tan \left(i x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4203}$$

$$\int xdx-i \int i \coth (x)dx-x \coth (x)$$

$$\Big\downarrow \text{15}$$

$$-i \int i \coth (x)dx+\frac{x^2}{2}-x \coth (x)$$

$$\Big\downarrow \text{26}$$

$$\int \coth (x)dx+\frac{x^2}{2}-x \coth (x)$$

$$\Big\downarrow \text{3042}$$

$$\int -i \tan \left(i x+\frac{\pi}{2}\right)dx+\frac{x^2}{2}-x \coth (x)$$

$$\Big\downarrow \text{26}$$

$$-i \int \tan \left(i x+\frac{\pi}{2}\right)dx+\frac{x^2}{2}-x \coth (x)$$

$$\Big\downarrow \text{3956}$$

$$\frac{x^2}{2}-x \coth (x)+\log (\sinh (x))$$

input
```
Int[x*Coth[x]^2,x]
```

output
```
x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

3.597.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

3.597.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

method	result	size
risch	$\frac{x^2}{2} - 2x - \frac{2x}{e^{2x}-1} + \ln\left(e^{2x}-1\right)$	28
parallelrisch	$\frac{-2\ln(1-\tanh(x))\tanh(x)+2\ln(\tanh(x))\tanh(x)+x(-2+(-2+x)\tanh(x))}{2\tanh(x)}$	36

input
```
int(x*coth(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-2*x-2*x/(exp(2*x)-1)+ln(exp(2*x)-1)
```

3.597.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.94

$$\int x\coth^2(x)\,dx$$
$$= \frac{(x^2-4x)\cosh(x)^2 + 2(x^2-4x)\cosh(x)\sinh(x) + (x^2-4x)\sinh(x)^2 - x^2 + 2\left(\cosh(x)^2 + 2\cosh(x\right.}{2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)}$$

input
```
integrate(x*coth(x)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*s
inh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*s
inh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)
```

3.597.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x\coth^2(x)\,dx=\frac{x^2}{2}+x-\frac{x}{\tanh(x)}-\log(\tanh(x)+1)+\log(\tanh(x))$$

input
```
integrate(x*coth(x)**2,x)
```

output
```
x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))
```

3.597.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\coth^2(x)\,dx=-\frac{xe^{(2x)}}{e^{(2x)}-1}-\frac{x^2-(x^2-2x)e^{(2x)}}{2(e^{(2x)}-1)}+\log(e^x+1)+\log(e^x-1)$$

input
```
integrate(x*coth(x)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*x) - 1) +
 log(e^x + 1) + log(e^x - 1)
```

3.597.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\coth^2(x)\,dx=\frac{x^2e^{(2x)}-x^2-4\,xe^{(2x)}+2\,e^{(2x)}\log\left(e^{(2x)}-1\right)-2\,\log\left(e^{(2x)}-1\right)}{2\,(e^{(2x)}-1)}$$

input
```
integrate(x*coth(x)^2,x, algorithm="giac")
```

output
```
1/2*(x^2*e^(2*x) - x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) - 1) - 2*log(
e^(2*x) - 1))/(e^(2*x) - 1)
```

3.597.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int x\coth^2(x)\,dx = \ln\left(\mathrm{e}^{2\,x}-1\right)-2\,x-\frac{2\,x}{\mathrm{e}^{2\,x}-1}+\frac{x^2}{2}$$

input
```
int(x*coth(x)^2,x)
```

output
```
log(exp(2*x) - 1) - 2*x - (2*x)/(exp(2*x) - 1) + x^2/2
```

3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

3.598.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=-e^x+\frac{e^{2x}}{2}+e^xx$$

output
```
-exp(x)+1/2*exp(2*x)+exp(x)*x
```

3.598.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{1}{2}\cosh(2x)+(-1+x)\sinh(x)+\cosh(x)(-1+x+\sinh(x))$$

input
```
Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

output
```
Cosh[2*x]/2 + (-1 + x)*Sinh[x] + Cosh[x]*(-1 + x + Sinh[x])
```

3.598.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6182, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sinh(x)+\cosh(x)}{\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \text{6182}$$

$$\int e^x(x+\sinh(x)+\cosh(x))dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(e^x x+e^x\sinh(x)+e^x\cosh(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^x x-e^x+\frac{e^{2x}}{2}$$

input
```
Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

output
```
-E^x + E^(2*x)/2 + E^x*x
```

3.598.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6182
```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.598.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
risch	$(-1+x)\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2 x}}{2}$	14
default	$-\cosh\left(x\right)+x\sinh\left(x\right)+x\cosh\left(x\right)-\sinh\left(x\right)+\cosh\left(x\right)\sinh\left(x\right)+\cosh^{2}\left(x\right)$	27

input
```
int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*exp(x)+1/2*exp(2*x)
```

3.598.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{2\,x+\cosh\left(x\right)+\sinh\left(x\right)-2}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(2*x + cosh(x) + sinh(x) - 2)/(cosh(x) - sinh(x))
```

3.598.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{x}{-\sinh\left(x\right)+\cosh\left(x\right)}+\frac{\sinh\left(x\right)}{-\sinh\left(x\right)+\cosh\left(x\right)}-\frac{1}{-\sinh\left(x\right)+\cosh\left(x\right)}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

output
```
x/(-sinh(x) + cosh(x)) + sinh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(x) + cosh
(x))
```

3.598.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=(x-1)e^{x}+\frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

output
```
(x - 1)*e^x + 1/2*e^(2*x)
```

3.598.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{1}{2}\,(2\,x+e^{x}-2)e^{x}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
```

output
```
1/2*(2*x + e^x - 2)*e^x
```

3.598.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\mathrm{e}^{x}\left(x+\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}-1\right)$$

input
```
int((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
```

output
```
exp(x)*(x + exp(-x)/2 + exp(x)/2 - 1)
```

3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx$

3.599.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx = x-(1-x)\tanh\left(\frac{x}{2}\right)$$

output
```
x-(1-x)*tanh(1/2*x)
```

3.599.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx = \frac{\left(-1+x+x\coth\left(\frac{x}{2}\right)\right)\sinh(x)}{1+\cosh(x)}$$

input
```
Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

output
```
((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

3.599.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sinh(x)+\cosh(x)}{\cosh(x)+1}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x+\cosh(x)}{\cosh(x)+1}+\tanh\left(\frac{x}{2}\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

input
```
Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

output
```
x - (1 - x)*Tanh[x/2]
```

3.599.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.599.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$2x-\frac{2(-1+x)}{1+e^{x}}$	16

input
```
int((x+cosh(x)+sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

output
```
2*x-2*(-1+x)/(1+exp(x))
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = \frac{2\left(x\cosh\left(x\right)+x\sinh\left(x\right)+1\right)}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)
```

3.599.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = x\tanh\left(\frac{x}{2}\right)+x-\tanh\left(\frac{x}{2}\right)$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

output
```
x*tanh(x/2) + x - tanh(x/2)
```

3.599.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(10) = 20$.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{x + \cosh(x) + \sinh(x)}{1 + \cosh(x)}\,dx = x + \frac{2\,xe^{x}}{e^{x} + 1} - \frac{2}{e^{(-x)} + 1} + \log\left(\cosh\left(x\right) + 1\right) - 2\,\log\left(e^{x} + 1\right)$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
x + 2*x*e^x/(e^x + 1) - 2/(e^(-x) + 1) + log(cosh(x) + 1) - 2*log(e^x + 1)
```

3.599.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x + \cosh(x) + \sinh(x)}{1 + \cosh(x)}\,dx = \frac{2\left(xe^{x} + 1\right)}{e^{x} + 1}$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
2*(x*e^x + 1)/(e^x + 1)
```

3.599.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x + \cosh(x) + \sinh(x)}{1 + \cosh(x)}\,dx = 2\,x - \frac{2\,x - 2}{\mathrm{e}^{x} + 1}$$

input
```
int((x + cosh(x) + sinh(x))/(cosh(x) + 1),x)
```

output
```
2*x - (2*x - 2)/(exp(x) + 1)
```

3.600 $\int e^{2x}\text{csch}^4(x)\,dx$

3.600.1 Optimal result

Integrand size = 10, antiderivative size = 20

$$\int e^{2x}\text{csch}^4(x)\,dx = \frac{8e^{6x}}{3(1-e^{2x})^3}$$

output
```
8/3*exp(6*x)/(1-exp(2*x))^3
```

3.600.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{2x}\text{csch}^4(x)\,dx = \frac{8e^{6x}}{3(1-e^{2x})^3}$$

input
```
Integrate[E^(2*x)*Csch[x]^4,x]
```

output
```
(8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

3.600.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}\text{csch}^4(x)\,dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \frac{16e^{5x}}{(1-e^{2x})^4}de^x$$
$$\Big\downarrow \text{ 27}$$
$$16\int \frac{e^{5x}}{(1-e^{2x})^4}de^x$$
$$\Big\downarrow \text{ 242}$$
$$\frac{8e^{6x}}{3(1-e^{2x})^3}$$

input
```
Int[E^(2*x)*Csch[x]^4,x]
```

output
```
(8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

3.600.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.600.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$-\frac{e^{2x} \operatorname{csch}(x)^2 (\coth(x)+1)}{3}$	15
default	$-\frac{1}{3 \tanh(x)^3} - \frac{1}{\tanh(x)^2} - \frac{1}{\tanh(x)}$	20
risch	$-\frac{8(3 e^{4x}-3 e^{2x}+1)}{3(e^{2x}-1)^3}$	25

input
```
int(exp(2*x)/sinh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*exp(2*x)*csch(x)^2*(coth(x)+1)
```

3.600.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.75

$$\int e^{2x} \operatorname{csch}^4(x)\, dx =$$

$$-\frac{8 \left(4 \cosh(x)^2 + 4 \cosh(x) \sinh(x) + 4 \sinh(x)^2 - 3\right)}{3 \left(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2 \left(3 \cosh(x)^2 - 2\right) \sinh(x)^2 - 4 \cosh(x)^2 + 4 (\cosh(x\right.}$$

input
```
integrate(exp(2*x)/sinh(x)^4,x, algorithm="fricas")
```

output
```
-8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x)^2 - 4*cosh(x)^2
+ 4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)
```

3.600.6 Sympy [F]

$$\int e^{2x}\operatorname{csch}^4(x)\,dx = \int \frac{e^{2x}}{\sinh^4(x)}\,dx$$

input
```
integrate(exp(2*x)/sinh(x)**4,x)
```

output
```
Integral(exp(2*x)/sinh(x)**4, x)
```

3.600.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int e^{2x}\operatorname{csch}^4(x)\,dx = \frac{8}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)}$$

input
```
integrate(exp(2*x)/sinh(x)^4,x, algorithm="maxima")
```

output
```
8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)
```

3.600.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int e^{2x}\operatorname{csch}^4(x)\,dx = -\frac{8\left(3\,e^{(4\,x)} - 3\,e^{(2\,x)} + 1\right)}{3\left(e^{(2\,x)} - 1\right)^3}$$

input
```
integrate(exp(2*x)/sinh(x)^4,x, algorithm="giac")
```

output
```
-8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

3.600.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int e^{2x}\operatorname{csch}^4(x)\,dx = -\frac{8\left(3\,\mathrm{e}^{4\,x}-3\,\mathrm{e}^{2\,x}+1\right)}{3\left(\mathrm{e}^{2\,x}-1\right)^3}$$

input
```
int(exp(2*x)/sinh(x)^4,x)
```

output
```
-(8*(3*exp(4*x) - 3*exp(2*x) + 1))/(3*(exp(2*x) - 1)^3)
```

3.601 $\int e^{-2x}\text{sech}^4(x)\,dx$

3.601.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int e^{-2x}\text{sech}^4(x)\,dx=-\frac{8}{3\left(1+e^{2x}\right)^3}$$

output
```
-8/3/(1+exp(2*x))^3
```

3.601.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2x}\text{sech}^4(x)\,dx=-\frac{8}{3\left(1+e^{2x}\right)^3}$$

input
```
Integrate[Sech[x]^4/E^(2*x),x]
```

output
```
-8/(3*(1 + E^(2*x))^3)
```

3.601.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2720, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x}\text{sech}^4(x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{16e^x}{(e^{2x}+1)^4}de^x$$

$$\Big\downarrow \text{27}$$

$$16\int \frac{e^x}{(1+e^{2x})^4}de^x$$

$$\Big\downarrow \text{241}$$

$$-\frac{8}{3\,(e^{2x}+1)^3}$$

input
```
Int[Sech[x]^4/E^(2*x),x]
```

output
```
-8/(3*(1 + E^(2*x))^3)
```

3.601.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.601.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{(-1+\tanh(x))^3}{3}$	9
risch	$-\frac{8}{3(1+e^{2x})^3}$	11
parallelrisch	$\frac{\operatorname{sech}(x)^2(-1+\tanh(x))e^{-2x}}{3}$	15

input
```
int(1/exp(2*x)/cosh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1+tanh(x))^3
```

3.601.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 7.85

$$\int e^{-2x}\operatorname{sech}^4(x)\,dx =$$

$$-\frac{}{3\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2+1\right)\sinh(x)^4+3\cosh(x)^4+4\left(5\cosh\right.\right.}$$

input
```
integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="fricas")
```

output
```
-8/3/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*si
nh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(
x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)
^3 + cosh(x))*sinh(x) + 1)
```

3.601.6 Sympy [F]

$$\int e^{-2x}\text{sech}^4(x)\,dx = \int \frac{e^{-2x}}{\cosh^4(x)}\,dx$$

input
```
integrate(1/exp(2*x)/cosh(x)**4,x)
```

output
```
Integral(exp(-2*x)/cosh(x)**4, x)
```

3.601.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(10) = 20.

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.77

$$\int e^{-2x}\text{sech}^4(x)\,dx = \frac{8\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} + \frac{8\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} + \frac{8}{3\,(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1)}$$

input
```
integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="maxima")
```

output
```
8*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*e^(-4*x)/(3*e^(-2*
x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x)
+ 1)
```

3.601.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{8}{3\,(e^{(2\,x)}+1)^3}$$

input
```
integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="giac")
```

output
```
-8/3/(e^(2*x) + 1)^3
```

3.601.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int e^{-2x}\operatorname{sech}^4(x)\,dx = -\frac{e^{-3x}}{3\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)^3}$$

input
```
int(exp(-2*x)/cosh(x)^4,x)
```

output
```
-exp(-3*x)/(3*(exp(-x)/2 + exp(x)/2)^3)
```

3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx$

3.602.1 Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx=\frac{e^{2x}}{2}$$

output
```
1/2*exp(2*x)
```

3.602.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx=\frac{e^{2x}}{2}$$

input
```
Integrate[E^x/(Cosh[x] - Sinh[x]),x]
```

output
```
E^(2*x)/2
```

3.602.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2720, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^x de^x$$

$$\Big\downarrow \text{15}$$

$$\frac{e^{2x}}{2}$$

input
```
Int[E^x/(Cosh[x] - Sinh[x]),x]
```

output
```
E^(2*x)/2
```

3.602.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.602.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{e^{2x}}{2}$	7
gosper	$\frac{e^{x}}{2\cosh(x)-2\sinh(x)}$	14
default	$\frac{2}{\tanh(\frac{x}{2})-1}+\frac{2}{(\tanh(\frac{x}{2})-1)^{2}}$	22

input
```
int(exp(x)/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*x)
```

3.602.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx=\frac{\cosh(x)+\sinh(x)}{2(\cosh(x)-\sinh(x))}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

3.602.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx=\frac{e^x}{-2\sinh(x)+2\cosh(x)}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x)
```

output
```
exp(x)/(-2*sinh(x) + 2*cosh(x))
```

3.602.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

output
```
1/2*e^(2*x)
```

3.602.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
```

output
```
1/2*e^(2*x)
```

3.602.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{\mathrm{e}^{2\,x}}{2}$$

input
```
int(exp(x)/(cosh(x) - sinh(x)),x)
```

output
```
exp(2*x)/2
```

3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx$

3.603.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{e^{(-1+m)x}}{-1+m}$$

output
```
exp((-1+m)*x)/(-1+m)
```

3.603.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{e^{mx}(\cosh(x)-\sinh(x))}{-1+m}$$

input
```
Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

output
```
(E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)
```

3.603.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6182, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{mx}}{\sinh(x)+\cosh(x)}\,dx$$

$$\Big\downarrow \text{6182}$$

$$\int e^{mx-x}dx$$

$$\Big\downarrow \text{2624}$$

$$-\frac{e^{-((1-m)x)}}{1-m}$$

input
```
Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

output
```
-(1/(E^((1 - m)*x)*(1 - m)))
```

3.603.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6182
```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

3.603.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{(-1+m)x}}{-1+m}$	13
gosper	$\frac{e^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$	18
default	$\frac{\sinh((-1+m)x)}{-1+m}+\frac{\cosh((-1+m)x)}{-1+m}$	26

input
```
int(exp(m*x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
exp((-1+m)*x)/(-1+m)
```

3.603.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{\cosh(mx)+\sinh(mx)}{(m-1)\cosh(x)+(m-1)\sinh(x)}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
(cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))
```

3.603.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(8) = 16.

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \begin{cases} \frac{e^{mx}}{m\sinh(x)+m\cosh(x)-\sinh(x)-\cosh(x)} & \text{for } m \neq 1 \\ \frac{xe^{x}}{\sinh(x)+\cosh(x)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
```

output
```
Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)), Ne(m, 1))
, (x*exp(x)/(sinh(x) + cosh(x)), True))
```

3.603.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-m>0)', see `assume?` for more d
etails)Is
```

3.603.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{e^{(mx)}}{me^x - e^x}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
e^(m*x)/(m*e^x - e^x)
```

3.603.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \frac{\mathrm{e}^{m\,x-x}}{m-1}$$

input
```
int(exp(m*x)/(cosh(x) + sinh(x)),x)
```

output
```
exp(m*x - x)/(m - 1)
```

3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx$

3.604.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

output
```
x
```

3.604.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

input
```
Integrate[E^x/(Cosh[x] + Sinh[x]),x]
```

output
```
x
```

3.604.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 4.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sinh(x)+\cosh(x)}dx$$
$$\Big\downarrow \text{2720}$$
$$\int e^{-x}de^x$$
$$\Big\downarrow \text{14}$$
$$\log\left(e^x\right)$$

input
```
Int[E^x/(Cosh[x] + Sinh[x]),x]
```

output
```
Log[E^x]
```

3.604.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.604.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2

input
```
int(exp(x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
x
```

3.604.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(0) = 0$.

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 10.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = \frac{xe^x}{\sinh{(x)}+\cosh{(x)}}$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x)
```

output
```
x*exp(x)/(sinh(x) + cosh(x))
```

3.604.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)} \, dx = x$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
x
```

3.604.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)} \, dx = x$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
x
```

3.604.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)} \, dx = x$$

input
```
int(exp(x)/(cosh(x) + sinh(x)),x)
```

output
```
x
```

3.605 $\int \frac{e^x}{1-\cosh(x)}\,dx$

3.605.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\frac{2}{1-e^x}-2\log\left(1-e^x\right)$$

output
```
-2/(1-exp(x))-2*ln(1-exp(x))
```

3.605.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{4\left(\frac{1}{1-e^x}+\log\left(1-e^x\right)\right)\sinh^2\left(\frac{x}{2}\right)}{1-\cosh(x)}$$

input
```
Integrate[E^x/(1 - Cosh[x]),x]
```

output
```
(4*((1 - E^x)^(-1) + Log[1 - E^x])*Sinh[x/2]^2)/(1 - Cosh[x])
```

3.605.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-\cosh (x)} \, dx$$
$$\Big\downarrow \text{2720}$$
$$\int -\frac{2 e^x}{\left(1-e^x\right)^2}de^x$$
$$\Big\downarrow \text{27}$$
$$-2 \int \frac{e^x}{\left(1-e^x\right)^2}de^x$$
$$\Big\downarrow \text{49}$$
$$-2 \int \left(\frac{1}{-1+e^x}+\frac{1}{\left(-1+e^x\right)^2}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$-2 \left(\frac{1}{1-e^x}+\log \left(1-e^x\right)\right)$$

input
```
Int[E^x/(1 - Cosh[x]),x]
```

output
```
-2*((1 - E^x)^(-1) + Log[1 - E^x])
```

3.605.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.605.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2}{-1+e^{x}}-2\ln\left(-1+e^{x}\right)$	17
default	$2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	24

input
```
int(exp(x)/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(-1+exp(x))-2*ln(-1+exp(x))
```

3.605.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\frac{2\left(\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)-1\right)}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

input
```
integrate(exp(x)/(1-cosh(x)),x, algorithm="fricas")
```

output
```
-2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sin
h(x) - 1)
```

3.605.6 Sympy [F]

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\int \frac{e^x}{\cosh{(x)}-1}\,dx$$

input
```
integrate(exp(x)/(1-cosh(x)),x)
```

output
```
-Integral(exp(x)/(cosh(x) - 1), x)
```

3.605.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\log{(e^x-1)}$$

input
```
integrate(exp(x)/(1-cosh(x)),x, algorithm="maxima")
```

output
```
2/(e^x - 1) - 2*log(e^x - 1)
```

3.605.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\log{(|e^x-1|)}$$

input
```
integrate(exp(x)/(1-cosh(x)),x, algorithm="giac")
```

output
```
2/(e^x - 1) - 2*log(abs(e^x - 1))
```

3.605.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\ln\left(e^x-1\right)$$

input
```
int(-exp(x)/(cosh(x) - 1),x)
```

output
```
2/(exp(x) - 1) - 2*log(exp(x) - 1)
```

3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx$

3.606.1 Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = e^x+\frac{2}{1+e^x}$$

output
```
exp(x)+2/(1+exp(x))
```

3.606.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2+e^x+e^{2x}}{1+e^x}$$

input
```
Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

output
```
(2 + E^x + E^(2*x))/(1 + E^x)
```

3.606.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 25, 1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\sinh(x)+1)}{\cosh(x)+1} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int -\frac{-2e^x-e^{2x}+1}{(e^x+1)^2}de^x$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1-2e^x-e^{2x}}{(1+e^x)^2}de^x$$

$$\Big\downarrow \text{ 1107}$$

$$-\int \left(\frac{2}{(1+e^x)^2}-1\right)de^x$$

$$\Big\downarrow \text{ 2009}$$

$$e^x+\frac{2}{e^x+1}$$

input
```
Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

output
```
E^x + 2/(1 + E^x)
```

3.606.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] &&  !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.606.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
risch	$\mathrm{e}^{x}+\frac{2}{1+\mathrm{e}^{x}}$	12
default	$-\tanh\left(\frac{x}{2}\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	18

input
```
int(exp(x)*(1+sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+2/(1+exp(x))
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx=\frac{3\,\cosh\left(x\right)-\sinh\left(x\right)+1}{\cosh\left(x\right)-\sinh\left(x\right)+1}$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
(3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)
```

3.606.6 Sympy [F]

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \int \frac{(\sinh(x)+1)\,e^x}{\cosh(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

output
```
Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)
```

3.606.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2}{e^x+1} + e^x$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
2/(e^x + 1) + e^x
```

3.606.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2}{e^x+1} + e^x$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
2/(e^x + 1) + e^x
```

3.606.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \mathrm{e}^x + \frac{2}{\mathrm{e}^x+1}$$

input
```
int((exp(x)*(sinh(x) + 1))/(cosh(x) + 1),x)
```

output
```
exp(x) + 2/(exp(x) + 1)
```

3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} \, dx$

3.607.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} \, dx = e^x - \frac{2}{1-e^x}$$

output
```
exp(x)-2/(1-exp(x))
```

3.607.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} \, dx = \frac{2-e^x+e^{2x}}{-1+e^x}$$

input
```
Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

output
```
(2 - E^x + E^(2*x))/(-1 + E^x)
```

3.607.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2720, 25, 1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^x-e^{2x}+1}{(1-e^x)^2}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1+2e^x-e^{2x}}{(1-e^x)^2}de^x$$

$$\Big\downarrow \text{1107}$$

$$-\int \left(\frac{2}{(-1+e^x)^2}-1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$e^x-\frac{2}{1-e^x}$$

input
```
Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

output
```
E^x - 2/(1 - E^x)
```

3.607.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] &&  !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.607.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\mathrm{e}^{x}+\frac{2}{-1+\mathrm{e}^{x}}$	12
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh\left(\frac{x}{2}\right)}$	18

input
```
int(exp(x)*(1-sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+2/(-1+exp(x))
```

3.607.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx=-\frac{3\,\cosh\left(x\right)-\sinh\left(x\right)-1}{\cosh\left(x\right)-\sinh\left(x\right)-1}$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

output
```
-(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)
```

3.607.6 Sympy [F]

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \int \frac{(\sinh(x)-1)\,e^x}{\cosh(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

output
```
Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)
```

3.607.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} + e^x$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

output
```
2/(e^x - 1) + e^x
```

3.607.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} + e^x$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

output
```
2/(e^x - 1) + e^x
```

3.607.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = e^x + \frac{2}{e^x-1}$$

input
```
int((exp(x)*(sinh(x) - 1))/(cosh(x) - 1),x)
```

output
```
exp(x) + 2/(exp(x) - 1)
```

3.608 $\int x^m \log(x)\,dx$

3.608.1 Optimal result

Integrand size = 6, antiderivative size = 26

$$\int x^m \log(x)\,dx = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log(x)}{1+m}$$

output
```
-x^(1+m)/(1+m)^2+x^(1+m)*ln(x)/(1+m)
```

3.608.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int x^m \log(x)\,dx = \frac{x^{1+m}(-1+(1+m)\log(x))}{(1+m)^2}$$

input
```
Integrate[x^m*Log[x],x]
```

output
```
(x^(1 + m)*(-1 + (1 + m)*Log[x]))/(1 + m)^2
```

3.608.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log(x)\,dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

input
```
Int[x^m*Log[x],x]
```

output
```
-(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

3.608.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.608.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{x(m\ln(x)+\ln(x)-1)x^m}{(1+m)^2}$	19
norman	$\frac{x\ln(x)\mathrm{e}^{m\ln(x)}}{1+m} - \frac{x\,\mathrm{e}^{m\ln(x)}}{m^2+2m+1}$	34
parallelrisch	$\frac{x\,x^m\ln(x)m+x^m\ln(x)x-x\,x^m}{m^2+2m+1}$	34

input
```
int(x^m*ln(x),x,method=_RETURNVERBOSE)
```

output
```
x*(m*ln(x)+ln(x)-1)/(1+m)^2*x^m
```

3.608.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^m \log(x)\,dx = \frac{((m+1)x\log(x) - x)x^m}{m^2 + 2m + 1}$$

input
```
integrate(x^m*log(x),x, algorithm="fricas")
```

output
```
((m + 1)*x*log(x) - x)*x^m/(m^2 + 2*m + 1)
```

3.608.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int x^m \log(x)\,dx = \begin{cases} \frac{mxx^m\log(x)}{m^2+2m+1} + \frac{xx^m\log(x)}{m^2+2m+1} - \frac{xx^m}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*ln(x),x)
```

output
```
Piecewise((m*x*x**m*log(x)/(m**2 + 2*m + 1) + x*x**m*log(x)/(m**2 + 2*m +
1) - x*x**m/(m**2 + 2*m + 1), Ne(m, -1)), (log(x)**2/2, True))
```

3.608.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m \log(x)\,dx = \frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

input
```
integrate(x^m*log(x),x, algorithm="maxima")
```

output
```
x^(m + 1)*log(x)/(m + 1) - x^(m + 1)/(m + 1)^2
```

3.608.8 Giac [F]

$$\int x^m \log(x)\,dx = \int x^m \log(x)\;dx$$

input
```
integrate(x^m*log(x),x, algorithm="giac")
```

output
```
integrate(x^m*log(x), x)
```

3.608.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int x^m \log(x)\,dx = \begin{cases} \frac{\ln(x)^2}{2} & \text{if } m = -1 \\ \frac{x^{m+1}\,(\ln(x)\,(m+1)-1)}{(m+1)^2} & \text{if } m \neq -1 \end{cases}$$

input
```
int(x^m*log(x),x)
```

output
```
piecewise(m == -1, log(x)^2/2, m ~= -1, (x^(m + 1)*(log(x)*(m + 1) - 1))/(
m + 1)^2)
```

3.609 $\int x^m \log^2(x)\,dx$

3.609.1 Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x^m \log^2(x)\,dx = \frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m}\log(x)}{(1+m)^2} + \frac{x^{1+m}\log^2(x)}{1+m}$$

output
```
2*x^(1+m)/(1+m)^3-2*x^(1+m)*ln(x)/(1+m)^2+x^(1+m)*ln(x)^2/(1+m)
```

3.609.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int x^m \log^2(x)\,dx = \frac{x^{1+m}(2-2(1+m)\log(x)+(1+m)^2\log^2(x))}{(1+m)^3}$$

input
```
Integrate[x^m*Log[x]^2,x]
```

output
```
(x^(1 + m)*(2 - 2*(1 + m)*Log[x] + (1 + m)^2*Log[x]^2))/(1 + m)^3
```

3.609.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log^2(x)\,dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{x^{m+1}\log^2(x)}{m+1}-\frac{2\int x^m\log(x)dx}{m+1}$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^{m+1}\log^2(x)}{m+1}-\frac{2\left(\frac{x^{m+1}\log(x)}{m+1}-\frac{x^{m+1}}{(m+1)^2}\right)}{m+1}$$

input
```
Int[x^m*Log[x]^2,x]
```

output
```
(x^(1 + m)*Log[x]^2)/(1 + m) - (2*(-(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log
[x])/(1 + m)))/(1 + m)
```

3.609.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.609.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{x\left(m^{2} \ln(x)^{2}+2 m \ln(x)^{2}-2 m \ln(x)+\ln(x)^{2}-2 \ln(x)+2\right) x^{m}}{(1+m)^{3}}$	41
norman	$\frac{x \ln(x)^{2} \mathrm{e}^{m \ln(x)}}{1+m}+\frac{2 x \,\mathrm{e}^{m \ln(x)}}{m^{3}+3 m^{2}+3 m+1}-\frac{2 x \ln(x) \mathrm{e}^{m \ln(x)}}{m^{2}+2 m+1}$	61
parallelrisch	$\frac{x \,x^{m} \ln(x)^{2} m^{2}+2 x \,x^{m} \ln(x)^{2} m+x^{m} \ln(x)^{2} x-2 x \,x^{m} \ln(x) m-2 x^{m} \ln(x) x+2 x \,x^{m}}{m^{3}+3 m^{2}+3 m+1}$	73

input
```
int(x^m*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(m^2*ln(x)^2+2*m*ln(x)^2-2*m*ln(x)+ln(x)^2-2*ln(x)+2)/(1+m)^3*x^m
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int x^m \log^2(x)\,dx = \frac{((m^2+2\,m+1)x\log(x)^2-2\,(m+1)x\log(x)+2\,x)x^m}{m^3+3\,m^2+3\,m+1}$$

input
```
integrate(x^m*log(x)^2,x, algorithm="fricas")
```

output
```
((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3 + 3*m^2 +
 3*m + 1)
```

3.609.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(39) = 78.

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int x^m \log^2(x)\,dx$$
$$= \begin{cases} \frac{m^2xx^m\log(x)^2}{m^3+3m^2+3m+1}+\frac{2mxx^m\log(x)^2}{m^3+3m^2+3m+1}-\frac{2mxx^m\log(x)}{m^3+3m^2+3m+1}+\frac{xx^m\log(x)^2}{m^3+3m^2+3m+1}-\frac{2xx^m\log(x)}{m^3+3m^2+3m+1}+\frac{2xx^m}{m^3+3m^2+3m+1} & \text{for } m \neq \\ \frac{\log(x)^3}{3} & \text{otherw} \end{cases}$$

input
```
integrate(x**m*ln(x)**2,x)
```

output
```
Piecewise((m**2*x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) + 2*m*x*x**m*lo
g(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*m*x*x**m*log(x)/(m**3 + 3*m**2 + 3*m
 + 1) + x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*x*x**m*log(x)/(m**3
 + 3*m**2 + 3*m + 1) + 2*x*x**m/(m**3 + 3*m**2 + 3*m + 1), Ne(m, -1)), (lo
g(x)**3/3, True))
```

3.609.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int x^m \log^2(x)\,dx = \frac{x^{m+1}\log(x)^2}{m+1} - \frac{2\,x^{m+1}\log(x)}{(m+1)^2} + \frac{2\,x^{m+1}}{(m+1)^3}$$

input
```
integrate(x^m*log(x)^2,x, algorithm="maxima")
```

output
```
x^(m + 1)*log(x)^2/(m + 1) - 2*x^(m + 1)*log(x)/(m + 1)^2 + 2*x^(m + 1)/(m
 + 1)^3
```

3.609.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

$$\int x^m \log^2(x)\,dx = -\frac{2\,mxx^m\log(x)}{(m^2+2\,m+1)(m+1)} + \frac{x^{m+1}\log(x)^2}{m+1} - \frac{2\,xx^m\log(x)}{(m^2+2\,m+1)(m+1)} + \frac{2\,xx^m}{(m^2+2\,m+1)(m+1)}$$

input
```
integrate(x^m*log(x)^2,x, algorithm="giac")
```

output
```
-2*m*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + x^(m + 1)*log(x)^2/(m + 1) -
 2*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + 2*x*x^m/((m^2 + 2*m + 1)*(m +
1))
```

3.609.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int x^m \log^2(x)\, dx = \begin{cases} \frac{\ln(x)^3}{3} & \text{if } m = -1 \\ \frac{x^{m+1} \left(\ln(x)^2 (m+1)^2 - 2 \ln(x) (m+1) + 2\right)}{(m+1)^3} & \text{if } m \neq -1 \end{cases}$$

input
```
int(x^m*log(x)^2,x)
```

output
```
piecewise(m == -1, log(x)^3/3, m ~= -1, (x^(m + 1)*(- 2*log(x)*(m + 1) + l
og(x)^2*(m + 1)^2 + 2))/(m + 1)^3)
```

3.610 $\int \frac{\log^2(x)}{x^{5/2}} \, dx$

3.610.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\log^2(x)}{x^{5/2}} \, dx = -\frac{16}{27x^{3/2}} - \frac{8\log(x)}{9x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}}$$

output
```
-16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

3.610.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)}{x^{5/2}} \, dx = -\frac{2\left(8 + 12\log(x) + 9\log^2(x)\right)}{27x^{3/2}}$$

input
```
Integrate[Log[x]^2/x^(5/2),x]
```

output
```
(-2*(8 + 12*Log[x] + 9*Log[x]^2))/(27*x^(3/2))
```

3.610.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)}{x^{5/2}} \, dx$$

$$\Big\downarrow \; 2742$$

$$\frac{4}{3} \int \frac{\log(x)}{x^{5/2}}dx - \frac{2\log^2(x)}{3x^{3/2}}$$

$$\Big\downarrow \; 2741$$

$$\frac{4}{3}\left(-\frac{4}{9x^{3/2}} - \frac{2\log(x)}{3x^{3/2}}\right) - \frac{2\log^2(x)}{3x^{3/2}}$$

input
```
Int[Log[x]^2/x^(5/2),x]
```

output
```
(-2*Log[x]^2)/(3*x^(3/2)) + (4*(-4/(9*x^(3/2)) - (2*Log[x])/(3*x^(3/2))))/
3
```

3.610.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))    Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.610.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^{2}}{3x^{\frac{3}{2}}}$	23
default	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^{2}}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^{2}}{3x^{\frac{3}{2}}}$	23

input
```
int(ln(x)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

3.610.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\left(9\,\log\left(x\right)^{2}+12\,\log\left(x\right)+8\right)}{27\,x^{\frac{3}{2}}}$$

input
```
integrate(log(x)^2/x^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)
```

3.610.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\log\left(x\right)^{2}}{3x^{\frac{3}{2}}}-\frac{8\log\left(x\right)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

input
```
integrate(ln(x)**2/x**(5/2),x)
```

output
```
-2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/2))
```

3.610.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\,\log\left(x\right)^2}{3\,x^{\frac{3}{2}}} - \frac{8\,\log\left(x\right)}{9\,x^{\frac{3}{2}}} - \frac{16}{27\,x^{\frac{3}{2}}}$$

input
```
integrate(log(x)^2/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

3.610.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\,\log\left(x\right)^2}{3\,x^{\frac{3}{2}}} - \frac{8\,\log\left(x\right)}{9\,x^{\frac{3}{2}}} - \frac{16}{27\,x^{\frac{3}{2}}}$$

input
```
integrate(log(x)^2/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

3.610.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{18\ln\left(x\right)^2 + 24\,\ln\left(x\right) + 16}{27\,x^{3/2}}$$

input
```
int(log(x)^2/x^(5/2),x)
```

output
```
-(24*log(x) + 18*log(x)^2 + 16)/(27*x^(3/2))
```

3.611 $\int (a+bx)\log(x)\,dx$

3.611.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int (a+bx)\log(x)\,dx=-ax-\frac{bx^2}{4}+ax\log(x)+\frac{1}{2}bx^2\log(x)$$

output
```
-a*x-1/4*b*x^2+a*x*ln(x)+1/2*b*x^2*ln(x)
```

3.611.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx)\log(x)\,dx=-ax-\frac{bx^2}{4}+ax\log(x)+\frac{1}{2}bx^2\log(x)$$

input
```
Integrate[(a + b*x)*Log[x],x]
```

output
```
-(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

3.611.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)(a+bx)\,dx$$

$$\Big\downarrow \ 2750$$

$$\frac{\log(x)(a+bx)^2}{2b}-\int \frac{(a+bx)^2}{2bx}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\log(x)(a+bx)^2}{2b}-\frac{\int \frac{(a+bx)^2}{x}dx}{2b}$$

$$\Big\downarrow \ 49$$

$$\frac{\log(x)(a+bx)^2}{2b}-\frac{\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{2b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\log(x)(a+bx)^2}{2b}-\frac{a^2\log(x)+2abx+\frac{b^2x^2}{2}}{2b}$$

input
```
Int[(a + b*x)*Log[x],x]
```

output
```
((a + b*x)^2*Log[x])/(2*b) - (2*a*b*x + (b^2*x^2)/2 + a^2*Log[x])/(2*b)
```

3.611.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2750
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n])
  u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b,
 c, d, e, n, r}, x] && IGtQ[q, 0]
```

3.611.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
norman	$-ax-\frac{x^{2}b}{4}+ax\ln\left(x\right)+\frac{b\,x^{2}\ln\left(x\right)}{2}$	25
risch	$\left(\frac{1}{2}x^{2}b+ax\right)\ln\left(x\right)-\frac{x^{2}b}{4}-ax$	25
parallelrisch	$-ax-\frac{x^{2}b}{4}+ax\ln\left(x\right)+\frac{b\,x^{2}\ln\left(x\right)}{2}$	25
parts	$-ax-\frac{x^{2}b}{4}+ax\ln\left(x\right)+\frac{b\,x^{2}\ln\left(x\right)}{2}$	25
default	$b\left(-\frac{x^{2}}{4}+\frac{x^{2}\ln\left(x\right)}{2}\right)+a\left(-x+x\ln\left(x\right)\right)$	27

input
```
int((b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-a*x-1/4*x^2*b+a*x*ln(x)+1/2*b*x^2*ln(x)
```

3.611.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int(a+bx)\log(x)\,dx=-\frac{1}{4}\,bx^{2}-ax+\frac{1}{2}\,\left(bx^{2}+2\,ax\right)\log\left(x\right)$$

input
```
integrate((b*x+a)*log(x),x, algorithm="fricas")
```

output
```
-1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

3.611.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (a+bx)\log(x)\,dx = -ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log(x)$$

input
```
integrate((b*x+a)*ln(x),x)
```

output
```
-a*x - b*x**2/4 + (a*x + b*x**2/2)*log(x)
```

3.611.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(x)\,dx = -\frac{1}{4}\,bx^2 - ax + \frac{1}{2}\,\left(bx^2 + 2\,ax\right)\log(x)$$

input
```
integrate((b*x+a)*log(x),x, algorithm="maxima")
```

output
```
-1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

3.611.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)\log(x)\,dx = \frac{1}{2}\,bx^2\log(x) - \frac{1}{4}\,bx^2 + ax\log(x) - ax$$

input
```
integrate((b*x+a)*log(x),x, algorithm="giac")
```

output
```
1/2*b*x^2*log(x) - 1/4*b*x^2 + a*x*log(x) - a*x
```

3.611.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (a+bx)\log(x)\,dx=-\frac{x\,\left(4\,a+b\,x-4\,a\,\ln\left(x\right)-2\,b\,x\,\ln\left(x\right)\right)}{4}$$

input
```
int(log(x)*(a + b*x),x)
```

output
```
-(x*(4*a + b*x - 4*a*log(x) - 2*b*x*log(x)))/4
```

3.612 $\int (a+bx)^3 \log(x)\,dx$

3.612.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int (a+bx)^3 \log(x)\,dx = -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} - \frac{a^4\log(x)}{4b} + \frac{(a+bx)^4\log(x)}{4b}$$

output
```
-a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a
)^4*ln(x)/b
```

3.612.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int (a+bx)^3 \log(x)\,dx = -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} + a^3x\log(x) + \frac{3}{2}a^2bx^2\log(x) + ab^2x^3\log(x) + \frac{1}{4}b^3x^4\log(x)$$

input
```
Integrate[(a + b*x)^3*Log[x],x]
```

output
```
-(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 + a^3*x*Log[x] +
 (3*a^2*b*x^2*Log[x])/2 + a*b^2*x^3*Log[x] + (b^3*x^4*Log[x])/4
```

3.612.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)(a+bx)^3 \, dx$$

$$\Big\downarrow \text{ 2750}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\int \frac{(a+bx)^4}{4bx}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\frac{\int \frac{(a+bx)^4}{x}dx}{4b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\frac{\int \left(\frac{a^4}{x}+4ba^3+6b^2xa^2+4b^3x^2a+b^4x^3\right)dx}{4b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\frac{a^4\log(x)+4a^3bx+3a^2b^2x^2+\frac{4}{3}ab^3x^3+\frac{b^4x^4}{4}}{4b}$$

input
```
Int[(a + b*x)^3*Log[x],x]
```

output
```
((a + b*x)^4*Log[x])/(4*b) - (4*a^3*b*x + 3*a^2*b^2*x^2 + (4*a*b^3*x^3)/3
+ (b^4*x^4)/4 + a^4*Log[x])/(4*b)
```

3.612.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2750
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n])
  u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b,
 c, d, e, n, r}, x] && IGtQ[q, 0]
```

3.612.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result
risch	$-a^{3}x-\frac{3a^{2}b\,x^{2}}{4}-\frac{a\,b^{2}x^{3}}{3}-\frac{b^{3}x^{4}}{16}-\frac{a^{4}\ln(x)}{4b}+\frac{(bx+a)^{4}\ln(x)}{4b}$
default	$b^{3}\left(-\frac{x^{4}}{16}+\frac{x^{4}\ln(x)}{4}\right)+3b^{2}a\left(-\frac{x^{3}}{9}+\frac{x^{3}\ln(x)}{3}\right)+3a^{2}b\left(-\frac{x^{2}}{4}+\frac{x^{2}\ln(x)}{2}\right)+a^{3}(-x+x\ln(x))$
norman	$a^{3}x\ln(x)+a\,b^{2}x^{3}\ln(x)-a^{3}x-\frac{b^{3}x^{4}}{16}-\frac{a\,b^{2}x^{3}}{3}-\frac{3a^{2}b\,x^{2}}{4}+\frac{b^{3}x^{4}\ln(x)}{4}+\frac{3a^{2}b\,x^{2}\ln(x)}{2}$
parallelrisch	$a^{3}x\ln(x)+a\,b^{2}x^{3}\ln(x)-a^{3}x-\frac{b^{3}x^{4}}{16}-\frac{a\,b^{2}x^{3}}{3}-\frac{3a^{2}b\,x^{2}}{4}+\frac{b^{3}x^{4}\ln(x)}{4}+\frac{3a^{2}b\,x^{2}\ln(x)}{2}$
parts	$\frac{b^{3}x^{4}\ln(x)}{4}+a\,b^{2}x^{3}\ln(x)+\frac{3a^{2}b\,x^{2}\ln(x)}{2}+a^{3}x\ln(x)+\frac{a^{4}\ln(x)}{4b}-\frac{\frac{b^{4}x^{4}}{4}+\frac{4a\,b^{3}x^{3}}{3}+3a^{2}b^{2}x^{2}+4a^{3}bx+a^{4}\ln(x)}{4b}$

input
```
int((b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a
)^4*ln(x)/b
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx)^3 \log(x)\,dx = -\frac{1}{16}\,b^3x^4 - \frac{1}{3}\,ab^2x^3 - \frac{3}{4}\,a^2bx^2 - a^3x + \frac{1}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x\right)\log\left(x\right)$$

input
```
integrate((b*x+a)^3*log(x),x, algorithm="fricas")
```

output
```
-1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a
*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

3.612.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\,dx = -a^3x - \frac{3a^2bx^2}{4} - \frac{ab^2x^3}{3} - \frac{b^3x^4}{16} + \left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right)\log\left(x\right)$$

input
```
integrate((b*x+a)**3*ln(x),x)
```

output
```
-a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*x + 3*a**
2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*log(x)
```

3.612.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx)^3 \log(x)\,dx = -\frac{1}{16}\,b^3x^4 - \frac{1}{3}\,ab^2x^3 - \frac{3}{4}\,a^2bx^2 - a^3x + \frac{1}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x\right)\log\left(x\right)$$

input
```
integrate((b*x+a)^3*log(x),x, algorithm="maxima")
```

output
```
-1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a
*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

3.612.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\,dx = \frac{1}{4}\,b^3x^4\log\left(x\right) - \frac{1}{16}\,b^3x^4 + ab^2x^3\log\left(x\right) - \frac{1}{3}\,ab^2x^3 + \frac{3}{2}\,a^2bx^2\log\left(x\right) - \frac{3}{4}\,a^2bx^2 + a^3x\log\left(x\right) - a^3x$$

input
```
integrate((b*x+a)^3*log(x),x, algorithm="giac")
```

output
```
1/4*b^3*x^4*log(x) - 1/16*b^3*x^4 + a*b^2*x^3*log(x) - 1/3*a*b^2*x^3 + 3/2
*a^2*b*x^2*log(x) - 3/4*a^2*b*x^2 + a^3*x*log(x) - a^3*x
```

3.612.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\,dx = a^3\,x\,\ln\left(x\right) - \frac{b^3\,x^4}{16} - \frac{3\,a^2\,b\,x^2}{4} - \frac{a\,b^2\,x^3}{3} - a^3\,x + \frac{b^3\,x^4\,\ln\left(x\right)}{4} + \frac{3\,a^2\,b\,x^2\,\ln\left(x\right)}{2} + a\,b^2\,x^3\,\ln\left(x\right)$$

input
```
int(log(x)*(a + b*x)^3,x)
```

output
```
a^3*x*log(x) - (b^3*x^4)/16 - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - a^3*x + (b
^3*x^4*log(x))/4 + (3*a^2*b*x^2*log(x))/2 + a*b^2*x^3*log(x)
```

3.613 $\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx$

3.613.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=-35x+34x\log(x)-17x\log^2(x)+3x\log^3(x)$$

output
```
-35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

3.613.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=-35x+34x\log(x)-17x\log^2(x)+3x\log^3(x)$$

input
```
Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

output
```
-35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

3.613.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3\log^3(x) - 8\log^2(x) - 1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-35x + 3x\log^3(x) - 17x\log^2(x) + 34x\log(x)$$

input
```
Int[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

output
```
-35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

3.613.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.613.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24
norman	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24
risch	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24
parallelrisch	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24
parts	$-35x + 34x\ln(x) - 17x\ln(x)^2 + 3x\ln(x)^3$	24

input
```
int(-1-8*ln(x)^2+3*ln(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

3.613.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3\,x\log\left(x\right)^3-17\,x\log\left(x\right)^2+34\,x\log\left(x\right)-35\,x$$

input
```
integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="fricas")
```

output
```
3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

3.613.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3x\log\left(x\right)^3-17x\log\left(x\right)^2+34x\log\left(x\right)-35x$$

input
```
integrate(-1-8*ln(x)**2+3*ln(x)**3,x)
```

output
```
3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

3.613.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3\,\left(\log\left(x\right)^3-3\,\log\left(x\right)^2+6\,\log\left(x\right)-6\right)x-8\,\left(\log\left(x\right)^2-2\,\log\left(x\right)+2\right)x-x$$

input
```
integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="maxima")
```

output
```
3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(x) + 2)*x
 - x
```

3.613.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=3\,x\log\left(x\right)^3-17\,x\log\left(x\right)^2+34\,x\log\left(x\right)-35\,x$$

input
```
integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="giac")
```

output
```
3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

3.613.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=x\,\left(3\ln\left(x\right)^3-17\ln\left(x\right)^2+34\,\ln\left(x\right)-35\right)$$

input
```
int(3*log(x)^3 - 8*log(x)^2 - 1,x)
```

output
```
x*(34*log(x) - 17*log(x)^2 + 3*log(x)^3 - 35)
```

3.614 $\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right) \, dx$

3.614.1 Optimal result

Integrand size = 16, antiderivative size = 60

$$\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right) \, dx = -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x)-3x\log^2(x) -\frac{3}{25}x^5\log^2(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

output
```
-3*x+169/625*x^5+4*x*ln(x)-44/125*x^5*ln(x)-3*x*ln(x)^2-3/25*x^5*ln(x)^2+x
*ln(x)^3+1/5*x^5*ln(x)^3
```

3.614.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right) \, dx = -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x)-3x\log^2(x) -\frac{3}{25}x^5\log^2(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

input
```
Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

output
```
-3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (
3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

3.614.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+1\right) \left(\log^3(x)-2 \log(x)+1\right) \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(x^4+\left(x^4+1\right) \log^3(x)-2 \left(x^4+1\right) \log(x)+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{169 x^5}{625}+\frac{1}{5} x^5 \log^3(x)-\frac{3}{25} x^5 \log^2(x)-\frac{44}{125} x^5 \log(x)-3 x+x \log^3(x)-3 x \log^2(x)+4 x \log(x)$$

input
```
Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

output
```
-3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (
3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

3.614.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.614.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
risch	$\left(\frac{1}{5}x^{5}+x\right)\ln\left(x\right)^{3}+\left(-\frac{3}{25}x^{5}-3x\right)\ln\left(x\right)^{2}+\left(-\frac{44}{125}x^{5}+4x\right)\ln\left(x\right)+\frac{169x^{5}}{625}-3x$	48
default	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
norman	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
parallelrisch	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53
parts	$-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+x\ln\left(x\right)^{3}+\frac{x^{5}\ln(x)^{3}}{5}$	53

input
```
int((x^4+1)*(1-2*ln(x)+ln(x)^3),x,method=_RETURNVERBOSE)
```

output
```
(1/5*x^5+x)*ln(x)^3+(-3/25*x^5-3*x)*ln(x)^2+(-44/125*x^5+4*x)*ln(x)+169/62
5*x^5-3*x
```

3.614.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \left(1+x^4\right)\left(1-2\log(x)+\log^3(x)\right)\,dx = \frac{169}{625}\,x^{5}+\frac{1}{5}\,\left(x^{5}+5\,x\right)\log\left(x\right)^{3} - \frac{3}{25}\,\left(x^{5}+25\,x\right)\log\left(x\right)^{2} - \frac{4}{125}\,\left(11\,x^{5}-125\,x\right)\log\left(x\right)-3\,x$$

input
```
integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="fricas")
```

output
```
169/625*x^5 + 1/5*(x^5 + 5*x)*log(x)^3 - 3/25*(x^5 + 25*x)*log(x)^2 - 4/12
5*(11*x^5 - 125*x)*log(x) - 3*x
```

3.614.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{169x^5}{625} - 3x + \left(-\frac{44x^5}{125} + 4x\right)\log(x) + \left(-\frac{3x^5}{25} - 3x\right)\log(x)^2 + \left(\frac{x^5}{5} + x\right)\log(x)^3$$

input
```
integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

output
```
169*x**5/625 - 3*x + (-44*x**5/125 + 4*x)*log(x) + (-3*x**5/25 - 3*x)*log(
x)**2 + (x**5/5 + x)*log(x)**3
```

3.614.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{1}{625} (125 \log(x)^3 - 75 \log(x)^2 + 30 \log(x) - 6)x^5 - \frac{2}{25} x^5 (5 \log(x) - 1) + \frac{1}{5} x^5 + (\log(x)^3 - 3 \log(x)^2 + 6 \log(x) - 6)x - 2x(\log(x) - 1) + x$$

input
```
integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="maxima")
```

output
```
1/625*(125*log(x)^3 - 75*log(x)^2 + 30*log(x) - 6)*x^5 - 2/25*x^5*(5*log(x
) - 1) + 1/5*x^5 + (log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 2*x*(log(x)
- 1) + x
```

3.614.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right) dx = \frac{1}{5} x^5 \log(x)^3 - \frac{3}{25} x^5 \log(x)^2 - \frac{44}{125} x^5 \log(x) + \frac{169}{625} x^5 + x \log(x)^3 - 3 x \log(x)^2 + 4 x \log(x) - 3 x$$

input
```
integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="giac")
```

output
```
1/5*x^5*log(x)^3 - 3/25*x^5*log(x)^2 - 44/125*x^5*log(x) + 169/625*x^5 + x
*log(x)^3 - 3*x*log(x)^2 + 4*x*log(x) - 3*x
```

3.614.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right) dx = \frac{x \left(125 x^4 \ln(x)^3 - 75 x^4 \ln(x)^2 - 220 x^4 \ln(x) + 169 x^4 + 625 \ln(x)^3 - 1875 \ln(x)^2 + 2500 \ln(x) - 1875\right)}{625}$$

input
```
int((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x)
```

output
```
(x*(2500*log(x) - 220*x^4*log(x) - 1875*log(x)^2 + 625*log(x)^3 - 75*x^4*l
og(x)^2 + 125*x^4*log(x)^3 + 169*x^4 - 1875))/625
```

3.615 $\int \frac{1}{x^3 \log^4(x)} dx$

3.615.1 Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{x^3 \log^4(x)} dx = -\frac{4}{3} \text{ExpIntegralEi}(-2\log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

output
```
-4/3*Ei(-2*ln(x))-1/3/x^2/ln(x)^3+1/3/x^2/ln(x)^2-2/3/x^2/ln(x)
```

3.615.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \log^4(x)} dx = -\frac{4}{3} \text{ExpIntegralEi}(-2\log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

input
```
Integrate[1/(x^3*Log[x]^4),x]
```

output
```
(-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2)
- 2/(3*x^2*Log[x])
```

3.615.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2743, 2743, 2743, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log^4(x)} \, dx$$

$$\Big\downarrow \text{ 2743}$$

$$-\frac{2}{3} \int \frac{1}{x^3 \log^3(x)}dx - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{ 2743}$$

$$-\frac{2}{3} \left(-\int \frac{1}{x^3 \log^2(x)}dx - \frac{1}{2x^2 \log^2(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{ 2743}$$

$$-\frac{2}{3} \left(2\int \frac{1}{x^3 \log(x)}dx - \frac{1}{2x^2 \log^2(x)} + \frac{1}{x^2 \log(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{ 2746}$$

$$-\frac{2}{3} \left(2\int \frac{1}{x^2 \log(x)}d\log(x) - \frac{1}{2x^2 \log^2(x)} + \frac{1}{x^2 \log(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{ 2609}$$

$$-\frac{2}{3} \left(2\,\text{ExpIntegralEi}(-2\log(x)) - \frac{1}{2x^2 \log^2(x)} + \frac{1}{x^2 \log(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

input
```
Int[1/(x^3*Log[x]^4),x]
```

output
```
(-2*(2*ExpIntegralEi[-2*Log[x]] - 1/(2*x^2*Log[x]^2) + 1/(x^2*Log[x])))/3
- 1/(3*x^2*Log[x]^3)
```

3.615.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

3.615.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{2\ln(x)^2-\ln(x)+1}{3x^2\ln(x)^3}+\frac{4\,\mathrm{Ei}_1(2\ln(x))}{3}$	31
default	$-\frac{1}{3x^2\ln(x)^3}+\frac{1}{3x^2\ln(x)^2}-\frac{2}{3x^2\ln(x)}+\frac{4\,\mathrm{Ei}_1(2\ln(x))}{3}$	37

input
```
int(1/x^3/ln(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*ln(x)^2-ln(x)+1)/x^2/ln(x)^3+4/3*Ei(1,2*ln(x))
```

3.615.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\log^4(x)}\,dx = -\frac{4\,x^2\log(x)^3\,\mathrm{log\_integral}\left(\frac{1}{x^2}\right)+2\,\log(x)^2-\log(x)+1}{3\,x^2\log(x)^3}$$

input
```
integrate(1/x^3/log(x)^4,x, algorithm="fricas")
```

output
```
-1/3*(4*x^2*log(x)^3*log_integral(x^(-2)) + 2*log(x)^2 - log(x) + 1)/(x^2*
log(x)^3)
```

3.615.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\log^4(x)}\,dx = -\frac{4\,\mathrm{Ei}\left(-2\log\left(x\right)\right)}{3} + \frac{-2\log\left(x\right)^2 + \log\left(x\right) - 1}{3x^2\log\left(x\right)^3}$$

input
```
integrate(1/x**3/ln(x)**4,x)
```

output
```
-4*Ei(-2*log(x))/3 + (-2*log(x)**2 + log(x) - 1)/(3*x**2*log(x)**3)
```

3.615.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\log^4(x)}\,dx = -8\,\Gamma\left(-3, 2\,\log\left(x\right)\right)$$

input
```
integrate(1/x^3/log(x)^4,x, algorithm="maxima")
```

output
```
-8*gamma(-3, 2*log(x))
```

3.615.8 Giac [F]

$$\int \frac{1}{x^3\log^4(x)}\,dx = \int \frac{1}{x^3\log\left(x\right)^4}\,dx$$

input
```
integrate(1/x^3/log(x)^4,x, algorithm="giac")
```

output
```
integrate(1/(x^3*log(x)^4), x)
```

3.615.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -\frac{4 \operatorname{ei}(-2 \ln(x))}{3} - \frac{\frac{2 \ln(x)^2}{3} - \frac{\ln(x)}{3} + \frac{1}{3}}{x^2 \ln(x)^3}$$

input
```
int(1/(x^3*log(x)^4),x)
```

output
```
- (4*ei(-2*log(x)))/3 - ((2*log(x)^2)/3 - log(x)/3 + 1/3)/(x^2*log(x)^3)
```

3.616 $\int \frac{\log(x)}{a+bx}\,dx$

3.616.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}$$

output
```
ln(x)*ln(1+b*x/a)/b+polylog(2,-b*x/a)/b
```

3.616.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{\log(x)\log\left(\frac{a+bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}$$

input
```
Integrate[Log[x]/(a + b*x),x]
```

output
```
(Log[x]*Log[(a + b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

3.616.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{a+bx} \, dx$$

$$\Big\downarrow \; 2754$$

$$\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}-\frac{\int \frac{\log\left(\frac{bx}{a}+1\right)}{x}dx}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

input
```
Int[Log[x]/(a + b*x),x]
```

output
```
(Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

3.616.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.616.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln\left(x\right)\ln\left(\frac{bx+a}{a}\right)}{b}$	32
risch	$\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln\left(x\right)\ln\left(\frac{bx+a}{a}\right)}{b}$	32
parts	$\frac{\ln\left(bx+a\right)\ln\left(x\right)}{b}-\frac{\operatorname{dilog}\left(-\frac{bx}{a}\right)+\ln\left(bx+a\right)\ln\left(-\frac{bx}{a}\right)}{b}$	43

input
```
int(ln(x)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

3.616.5 Fricas [F]

$$\int \frac{\log(x)}{a+bx}\,dx=\int \frac{\log\left(x\right)}{bx+a}\,dx$$

input
```
integrate(log(x)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(log(x)/(b*x + a), x)
```

3.616.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.10

$$\int \frac{\log(x)}{a+bx}\,dx = \begin{cases} -\frac{\operatorname{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b} + \frac{i\pi\log\left(\frac{a}{b}+x\right)}{b} - \frac{\operatorname{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{\operatorname{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} - \frac{i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| \frac{a}{b}+x\right)}{b} + \frac{G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| \frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} + \frac{i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| \frac{a}{b}+x\right)}{b} \end{cases}$$

input
```
integrate(ln(x)/(b*x+a),x)
```

output
```
Piecewise((-polylog(2, b*(a/b + x)/a)/b, (Abs(a/b + x) < 1) & (1/Abs(a/b +
 x) < 1)), (log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylog(2, b*(
a/b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b + x))/b - I*pi*log
(1/(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, 1/Abs(a/b + x) < 1), (-meij
erg(((), (1, 1)), ((0, 0), ()), a/b + x)*log(a/b)/b - I*pi*meijerg(((), (1
, 1)), ((0, 0), ()), a/b + x)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b
+ x)*log(a/b)/b + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - po
lylog(2, b*(a/b + x)/a)/b, True))
```

3.616.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{\log\left(\frac{bx}{a}+1\right)\log(x) + \operatorname{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

input
```
integrate(log(x)/(b*x+a),x, algorithm="maxima")
```

output
```
(log(b*x/a + 1)*log(x) + dilog(-b*x/a))/b
```

3.616.8 Giac [F]

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\log(x)}{bx+a}\,dx$$

input
```
integrate(log(x)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(x)/(b*x + a), x)
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\ln(x)}{a+b\,x}\,dx$$

input
```
int(log(x)/(a + b*x),x)
```

output
```
int(log(x)/(a + b*x), x)
```

3.617 $\int \frac{\log(x)}{(a+bx)^2}\,dx$

3.617.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

output
```
x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

3.617.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{\frac{x\log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

input
```
Integrate[Log[x]/(a + b*x)^2,x]
```

output
```
((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a
```

3.617.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{(a+bx)^2} dx$$

$$\Big\downarrow \text{2751}$$

$$\frac{x\log(x)}{a(a+bx)} - \frac{\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

input
```
Int[Log[x]/(a + b*x)^2,x]
```

output
```
(x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

3.617.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, 
x] && EqQ[r*(q + 1) + 1, 0]
```

3.617.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x\ln(x)}{a(bx+a)}-\frac{\ln(bx+a)}{ab}$	30
norman	$\frac{x\ln(x)}{a(bx+a)}-\frac{\ln(bx+a)}{ab}$	30
parts	$-\frac{\ln(x)}{b(bx+a)}+\frac{\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}}{b}$	38
parallelrisch	$\frac{-b\ln(bx+a)x+bx\ln(x)-\ln(bx+a)a}{ab(bx+a)}$	40
risch	$-\frac{\ln(x)}{b(bx+a)}-\frac{\ln(bx+a)}{ab}+\frac{\ln(-x)}{ab}$	41

input
```
int(ln(x)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

3.617.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\log(x)}{(a+bx)^2}\,dx=\frac{bx\log{(x)}-(bx+a)\log{(bx+a)}}{ab^2x+a^2b}$$

input
```
integrate(log(x)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)
```

3.617.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\log(x)}{(a+bx)^2}\,dx=-\frac{\log{(x)}}{ab+b^2x}+\frac{\log{(x)}-\log{\left(\frac{a}{b}+x\right)}}{ab}$$

input
```
integrate(ln(x)/(b*x+a)**2,x)
```

output
```
-log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)
```

3.617.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = -\frac{\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}}{b} - \frac{\log(x)}{(bx+a)b}$$

input
```
integrate(log(x)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)
```

3.617.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(29) = 58.

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.76

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = b^2\left(\frac{\log\left(\frac{(bx+a)^2|b|\left|\frac{a}{bx+a}-1\right|}{b^2|bx+a|}\right)}{ab^3} + \frac{\log\left(-\frac{a+\frac{(bx+a)b\left(\frac{a}{bx+a}-1\right)-ab}{b}}{b}\right)}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)b^3} - \frac{\log\left(\left|-(bx+a)\left(\frac{a}{bx+a}-1\right)+a\right|\right)}{ab^3}\right)$$

input
```
integrate(log(x)/(b*x+a)^2,x, algorithm="giac")
```

output
```
b^2*(log((b*x + a)^2*abs(b)*abs(a/(b*x + a) - 1)/(b^2*abs(b*x + a)))/(a*b^
3) + log(-(a + ((b*x + a)*b*(a/(b*x + a) - 1) - a*b)/b)/b)/(((b*x + a)*(a/
(b*x + a) - 1) - a)*b^3) - log(abs(-(b*x + a)*(a/(b*x + a) - 1) + a))/(a*b
^3))
```

3.617.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{x^2\,\ln(x)}{a\,\left(b\,x^2+a\,x\right)} - \frac{\ln(a+b\,x)}{a\,b}$$

input
```
int(log(x)/(a + b*x)^2,x)
```

output
```
(x^2*log(x))/(a*(a*x + b*x^2)) - log(a + b*x)/(a*b)
```

3.618 $\int \frac{\log^n(x)}{x} \, dx$

3.618.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{\log^n(x)}{x} \, dx = \frac{\log^{1+n}(x)}{1+n}$$

output
```
ln(x)^(1+n)/(1+n)
```

3.618.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x} \, dx = \frac{\log^{1+n}(x)}{1+n}$$

input
```
Integrate[Log[x]^n/x,x]
```

output
```
Log[x]^(1 + n)/(1 + n)
```

3.618.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(x)}{x} \, dx$$

$$\Big\downarrow \; 2739$$

$$\int \log^n(x) d\log(x)$$

$$\Big\downarrow \; 15$$

$$\frac{\log^{n+1}(x)}{n+1}$$

input
```
Int[Log[x]^n/x,x]
```

output
```
Log[x]^(1 + n)/(1 + n)
```

3.618.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.618.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+n}}{1+n}$	13
default	$\frac{\ln(x)^{1+n}}{1+n}$	13
risch	$\frac{\ln(x)\ln(x)^{n}}{1+n}$	13
norman	$\frac{\ln(x)\mathrm{e}^{n\ln(\ln(x))}}{1+n}$	15

input
```
int(ln(x)^n/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)^(1+n)/(1+n)
```

3.618.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^n \log(x)}{n+1}$$

input
```
integrate(log(x)^n/x,x, algorithm="fricas")
```

output
```
log(x)^n*log(x)/(n + 1)
```

3.618.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log^n(x)}{x}\,dx = \begin{cases} \frac{\log(x)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(x)**n/x,x)
```

output
```
Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), True))
```

3.618.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^{n+1}}{n+1}$$

input
```
integrate(log(x)^n/x,x, algorithm="maxima")
```

output
```
log(x)^(n + 1)/(n + 1)
```

3.618.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^{n+1}}{n+1}$$

input
```
integrate(log(x)^n/x,x, algorithm="giac")
```

output
```
log(x)^(n + 1)/(n + 1)
```

3.618.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\log^n(x)}{x}\,dx = \begin{cases} \ln(\ln(x)) & \text{if } n = -1 \\ \frac{\ln(x)^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

input
```
int(log(x)^n/x,x)
```

output
```
piecewise(n == -1, log(log(x)), n ~= -1, log(x)^(n + 1)/(n + 1))
```

3.619 $\int \frac{(a+b\log(x))^n}{x}\,dx$

3.619.1 Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{1+n}}{b(1+n)}$$

output
```
(a+b*ln(x))^(1+n)/b/(1+n)
```

3.619.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[(a + b*Log[x])^n/x,x]
```

output
```
(a + b*Log[x])^(1 + n)/(b*(1 + n))
```

3.619.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(x))^n}{x}\,dx$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int (a+b\log(x))^n d(a+b\log(x))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

input
```
Int[(a + b*Log[x])^n/x,x]
```

output
```
(a + b*Log[x])^(1 + n)/(b*(1 + n))
```

3.619.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.619.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$	20
default	$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$	20
risch	$\frac{(a+b\ln(x))(a+b\ln(x))^{n}}{b(1+n)}$	24
parallelrisch	$\frac{\ln(x)(a+b\ln(x))^{n}b+(a+b\ln(x))^{n}a}{b(1+n)}$	33
norman	$\frac{\ln(x)e^{n\ln(a+b\ln(x))}}{1+n}+\frac{a\,e^{n\ln(a+b\ln(x))}}{b(1+n)}$	40

input
```
int((a+b*ln(x))^n/x,x,method=_RETURNVERBOSE)
```

output
```
(a+b*ln(x))^(1+n)/b/(1+n)
```

3.619.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log(x)+a)(b\log(x)+a)^n}{bn+b}$$

input
```
integrate((a+b*log(x))^n/x,x, algorithm="fricas")
```

output
```
(b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)
```

3.619.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\log(x))^n}{x}\,dx = -\begin{cases} -a^n\log(x) & \text{for } b=0 \\ -\frac{\begin{cases} \frac{(a+b\log(x))^{n+1}}{n+1} & \text{for } n\neq -1 \\ \log(a+b\log(x)) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln(x))**n/x,x)
```

output
```
-Piecewise((-a**n*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(x))**(n + 1)/
(n + 1), Ne(n, -1)), (log(a + b*log(x)), True))/b, True))
```

3.619.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*log(x))^n/x,x, algorithm="maxima")
```

output
```
(b*log(x) + a)^(n + 1)/(b*(n + 1))
```

3.619.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*log(x))^n/x,x, algorithm="giac")
```

output
```
(b*log(x) + a)^(n + 1)/(b*(n + 1))
```

3.619.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\ln(x))^{n+1}}{b\,(n+1)}$$

input
```
int((a + b*log(x))^n/x,x)
```

output
```
(a + b*log(x))^(n + 1)/(b*(n + 1))
```

3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$

3.620.1 Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(a+b\log(x))}{b}$$

output
```
ln(a+b*ln(x))/b
```

3.620.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(a+b\log(x))}{b}$$

input
```
Integrate[1/(x*(a + b*Log[x])),x]
```

output
```
Log[a + b*Log[x]]/b
```

3.620.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\log(x))}dx$$
$$\Big\downarrow \text{2739}$$
$$\frac{\int \frac{1}{a+b\log(x)}d(a+b\log(x))}{b}$$
$$\Big\downarrow \text{14}$$
$$\frac{\log(a+b\log(x))}{b}$$

input
```
Int[1/(x*(a + b*Log[x])),x]
```

output
```
Log[a + b*Log[x]]/b
```

3.620.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.620.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\ln(x))}{b}$	12
default	$\frac{\ln(a+b\ln(x))}{b}$	12
norman	$\frac{\ln(a+b\ln(x))}{b}$	12
parallelrisch	$\frac{\ln(a+b\ln(x))}{b}$	12
risch	$\frac{\ln\left(\ln(x)+\frac{a}{b}\right)}{b}$	14

input
```
int(1/x/(a+b*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*ln(x))/b
```

3.620.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(b\log\left(x\right)+a\right)}{b}$$

input
```
integrate(1/x/(a+b*log(x)),x, algorithm="fricas")
```

output
```
log(b*log(x) + a)/b
```

3.620.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(\frac{a}{b}+\log\left(x\right)\right)}{b}$$

input
```
integrate(1/x/(a+b*ln(x)),x)
```

output
```
log(a/b + log(x))/b
```

3.620.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(b\log(x)+a)}{b}$$

input
```
integrate(1/x/(a+b*log(x)),x, algorithm="maxima")
```

output
```
log(b*log(x) + a)/b
```

3.620.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(\frac{1}{4}\pi^2b^2(\mathrm{sgn}(x)-1)^2+(b\log(|x|)+a)^2\right)}{2b}$$

input
```
integrate(1/x/(a+b*log(x)),x, algorithm="giac")
```

output
```
1/2*log(1/4*pi^2*b^2*(sgn(x) - 1)^2 + (b*log(abs(x)) + a)^2)/b
```

3.620.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\ln(a+b\ln(x))}{b}$$

input
```
int(1/(x*(a + b*log(x))),x)
```

output
```
log(a + b*log(x))/b
```

3.621 $\int \frac{(a+b\log(x))^{-n}}{x}\,dx$

3.621.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

output
```
(a+b*ln(x))^(1-n)/b/(1-n)
```

3.621.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

input
```
Integrate[1/(x*(a + b*Log[x])^n),x]
```

output
```
(a + b*Log[x])^(1 - n)/(b*(1 - n))
```

3.621.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx$$

$$\Big\downarrow \ 2739$$

$$\frac{\int (a+b\log(x))^{-n}d(a+b\log(x))}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

input
```
Int[1/(x*(a + b*Log[x])^n),x]
```

output
```
(a + b*Log[x])^(1 - n)/(b*(1 - n))
```

3.621.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.621.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
default	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
risch	$-\frac{(a+b\ln(x))(a+b\ln(x))^{-n}}{b(-1+n)}$	27
parallelrisch	$\frac{(-\ln(x)bn-an)(a+b\ln(x))^{-n}}{nb(-1+n)}$	34
norman	$\left(-\frac{\ln(x)}{-1+n}-\frac{a}{b(-1+n)}\right)e^{-n\ln(a+b\ln(x))}$	35

input
```
int(1/x/((a+b*ln(x))^n),x,method=_RETURNVERBOSE)
```

output
```
(a+b*ln(x))^(1-n)/b/(1-n)
```

3.621.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{b\log(x)+a}{(bn-b)(b\log(x)+a)^n}$$

input
```
integrate(1/x/((a+b*log(x))^n),x, algorithm="fricas")
```

output
```
-(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)
```

3.621.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(14) = 28.

Time = 5.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=1 \\ a^{-n}\log(x) & \text{for } b=0 \\ \frac{\log\left(\frac{a}{b}+\log(x)\right)}{b} & \text{for } n=1 \\ -\frac{a}{bn(a+b\log(x))^n - b(a+b\log(x))^n} - \frac{b\log(x)}{bn(a+b\log(x))^n - b(a+b\log(x))^n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/((a+b*ln(x))**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (log(x)/a**n, Eq(b, 0)), (log(a
/b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**
n) - b*log(x)/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n), True))
```

3.621.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/((a+b*log(x))^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-n>0)', see `assume?` for more d
etails)Is
```

3.621.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{(b\log(x)+a)^{-n+1}}{b(n-1)}$$

input
```
integrate(1/x/((a+b*log(x))^n),x, algorithm="giac")
```

output
```
-(b*log(x) + a)^(-n + 1)/(b*(n - 1))
```

3.621.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{(a+b\,\ln(x))^{1-n}}{b\,(n-1)}$$

input
```
int(1/(x*(a + b*log(x))^n),x)
```

output
```
-(a + b*log(x))^(1 - n)/(b*(n - 1))
```

3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$

3.622.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

output
```
arctanh(ln(x)/(a^2+ln(x)^2)^(1/2))
```

3.622.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
-1/2*Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[a^2 + Log[
x]^2]]/2
```

3.622.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3039, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{a^2+\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{\log^2(x)}{a^2+\log^2(x)}}d\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

input
```
Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

3.622.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.622.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$	15
default	$\ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$	15

input
```
int(1/x/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x)+(a^2+ln(x)^2)^(1/2))
```

3.622.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{a^2+\log(x)^2}-\log(x)\right)$$

input
```
integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(a^2 + log(x)^2) - log(x))
```

3.622.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx=\int \frac{1}{x\sqrt{a^2+\log{(x)}^2}}\,dx$$

input
```
integrate(1/x/(a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a**2 + log(x)**2)), x)
```

3.622.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx=\operatorname{arsinh}\left(\frac{\log{(x)}}{a}\right)$$

input
```
integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(log(x)/a)
```

3.622.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx=-\log\left(\sqrt{a^2+\log{(x)}^2}-\log{(x)}\right)$$

input
```
integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(a^2 + log(x)^2) - log(x))
```

3.622.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$$

input
```
int(1/(x*(log(x)^2 + a^2)^(1/2)),x)
```

output
```
log(log(x) + (log(x)^2 + a^2)^(1/2))
```

3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$

3.623.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)$$

output
```
arctanh(ln(x)/(-a^2+ln(x)^2)^(1/2))
```

3.623.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
-1/2*Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-a^2 + Lo
g[x]^2]]/2
```

3.623.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3039, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\log^2(x)-a^2}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{\log^2(x)-a^2}}d\log(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{\log^2(x)}{\log^2(x)-a^2}}d\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

input
```
Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

3.623.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.623.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\ln\left(\ln\left(x\right)+\sqrt{-a^{2}+\ln\left(x\right)^{2}}\right)$	17
default	$\ln\left(\ln\left(x\right)+\sqrt{-a^{2}+\ln\left(x\right)^{2}}\right)$	17

input
```
int(1/x/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x)+(-a^2+ln(x)^2)^(1/2))
```

3.623.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{-a^{2}+\log\left(x\right)^{2}}-\log\left(x\right)\right)$$

input
```
integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(-a^2 + log(x)^2) - log(x))
```

3.623.6 Sympy [F]

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx=\int \frac{1}{x\sqrt{-(a-\log(x))\,(a+\log(x))}}\,dx$$

input
```
integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))), x)
```

3.623.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx=\log\left(2\sqrt{-a^2+\log(x)^2}+2\log(x)\right)$$

input
```
integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))
```

3.623.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.623.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{\ln(x)^2-a^2}\right)$$

input
```
int(1/(x*(log(x)^2 - a^2)^(1/2)),x)
```

output
```
log(log(x) + (log(x)^2 - a^2)^(1/2))
```

3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$

3.624.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

output
```
arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

3.624.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

3.624.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{a^2-\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{\frac{\log^2(x)}{a^2-\log^2(x)}+1}d\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

input
```
Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

3.624.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.624.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$	17
default	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$	17

input
```
int(1/x/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

3.624.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = -2\arctan\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)$$

input
```
integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))
```

3.624.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx=\int \frac{1}{x\sqrt{(a-\log(x))\,(a+\log(x))}}\,dx$$

input
```
integrate(1/x/(a**2-ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a - log(x))*(a + log(x)))), x)
```

3.624.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx=\arcsin\left(\frac{\log(x)}{a}\right)$$

input
```
integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(log(x)/a)
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx=\arcsin\left(\frac{\log(x)}{a}\right)\mathrm{sgn}(a)$$

input
```
integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arcsin(log(x)/a)*sgn(a)
```

3.624.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \operatorname{atan}\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$$

input
```
int(1/(x*(a^2 - log(x)^2)^(1/2)),x)
```

output
```
atan(log(x)/(a^2 - log(x)^2)^(1/2))
```

3.625 $\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx$

3.625.1 Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

output
```
-arctanh((a^2+ln(x)^2)^(1/2)/a)/a
```

3.625.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

3.625.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3039, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}dx$$
$$\Big\downarrow \text{ 3039}$$
$$\int \frac{1}{\log(x)\sqrt{a^2+\log^2(x)}}d\log(x)$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{\log^2(x)\sqrt{a^2+\log^2(x)}}d\log^2(x)$$
$$\Big\downarrow \text{ 73}$$
$$\int \frac{1}{\log^4(x)-a^2}d\sqrt{a^2+\log^2(x)}$$
$$\Big\downarrow \text{ 220}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

input
```
Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

3.625.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.625.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37

input
```
int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+ln(x)^2)^(1/2))/ln(x))
```

3.625.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\log\left(a+\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)-\log\left(-a+\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(log(a + sqrt(a^2 + log(x)^2) - log(x)) - log(-a + sqrt(a^2 + log(x)^2) -
 log(x)))/a
```

3.625.6 Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{a^2+\log\left(x\right)^2}\log\left(x\right)}\,dx$$

input
```
integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)
```

3.625.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{|\log(x)|}\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/abs(log(x)))/a
```

3.625.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.625.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a^2+\ln(x)^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(x*log(x)*(log(x)^2 + a^2)^(1/2)),x)
```

output
```
atan((log(x)^2 + a^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

3.626 $\int \frac{1}{x \log(x) \sqrt{a^2-\log^2(x)}} \, dx$

3.626.1 Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{1}{x \log(x) \sqrt{a^2-\log^2(x)}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

output
```
-arctanh((a^2-ln(x)^2)^(1/2)/a)/a
```

3.626.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x) \sqrt{a^2-\log^2(x)}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

3.626.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3039, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log (x) \sqrt{a^2-\log ^2(x)}} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log (x) \sqrt{a^2-\log ^2(x)}}d\log (x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{\log ^2(x) \sqrt{a^2-\log ^2(x)}}d\log ^2(x)$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{a^2-\log ^4(x)}d\sqrt{a^2-\log ^2(x)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a^2-\log ^2(x)}}{a}\right)}{a}$$

input
```
Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

3.626.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.626.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39
default	$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39

input
```
int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-ln(x)^2)^(1/2))/ln(x))
```

3.626.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(-(a - sqrt(a^2 - log(x)^2))/log(x))/a
```

3.626.6 Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}\log(x)}\,dx$$

input
```
integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a - log(x))*(a + log(x)))*log(x)), x)
```

3.626.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\log\left(\frac{2a^2}{|\log(x)|}+\frac{2\sqrt{a^2-\log(x)^2}a}{|\log(x)|}\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*a^2/abs(log(x)) + 2*sqrt(a^2 - log(x)^2)*a/abs(log(x)))/a
```

3.626.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.626.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-\ln(x)^2}}{a}\right)}{a}$$

input
```
int(1/(x*log(x)*(a^2 - log(x)^2)^(1/2)),x)
```

output
```
-atanh((a^2 - log(x)^2)^(1/2)/a)/a
```

3.627 $\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}} dx$

3.627.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}} dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}$$

output
```
arctan((-a^2+ln(x)^2)^(1/2)/a)/a
```

3.627.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}} dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

3.627.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3039, 243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)\sqrt{\log^2(x)-a^2}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log(x)\sqrt{\log^2(x)-a^2}}d\log(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{\log^2(x)\sqrt{\log^2(x)-a^2}}d\log^2(x)$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{a^2+\log^4(x)}d\sqrt{\log^2(x)-a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

3.627.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.627.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$	43
default	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$	43

input
```
int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+ln(x)^2)^(1/2))/ln(x))
```

3.627.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}-\log(x)}{a}\right)}{a}$$

input
```
integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan((sqrt(-a^2 + log(x)^2) - log(x))/a)/a
```

3.627.6 Sympy [F]

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-(a-\log(x))(a+\log(x))}\log(x)}\,dx$$

input
```
integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))*log(x)), x)
```

3.627.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = -\frac{\arcsin\left(\frac{a}{|\log(x)|}\right)}{a}$$

input
```
integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(a/abs(log(x)))/a
```

3.627.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}}{a}\right)}{a}$$

input
```
integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(-a^2 + log(x)^2)/a)/a
```

3.627.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{\ln(x)^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*log(x)*(log(x)^2 - a^2)^(1/2)),x)
```

output
```
atan((log(x)^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

3.628 $\int \frac{\log(\log(x))}{x} \, dx$

3.628.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

output
```
-ln(x)+ln(x)*ln(ln(x))
```

3.628.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

input
```
Integrate[Log[Log[x]]/x,x]
```

output
```
-Log[x] + Log[x]*Log[Log[x]]
```

3.628.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x))}{x} \, dx$$
$$\Big\downarrow \text{3001}$$
$$\log(x)\log(\log(x)) - \log(x)$$

input
```
Int[Log[Log[x]]/x,x]
```

output
```
-Log[x] + Log[x]*Log[Log[x]]
```

3.628.3.1 Defintions of rubi rules used

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.628.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
default	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
norman	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
risch	$-\ln(x)+\ln(x)\ln(\ln(x))$	12

input
```
int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(x)*ln(ln(x))
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x)) - log(x)
```

3.628.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(ln(ln(x))/x,x)
```

output
```
log(x)*log(log(x)) - log(x)
```

3.628.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="maxima")
```

output
```
log(x)*log(log(x)) - log(x)
```

3.628.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x)) - log(x)
```

3.628.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(x))}{x} \, dx = \ln(x) \, (\ln(\ln(x)) - 1)$$

input
```
int(log(log(x))/x,x)
```

output
```
log(x)*(log(log(x)) - 1)
```

3.629 $\int \frac{\log^2(\log(x))}{x} \, dx$

3.629.1 Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{\log^2(\log(x))}{x} \, dx = 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))$$

output
```
2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

3.629.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x} \, dx = 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))$$

input
```
Integrate[Log[Log[x]]^2/x,x]
```

output
```
2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

3.629.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3039, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^2(\log(x))d\log(x)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^2(\log(x)) - 2\int \log(\log(x))d\log(x)$$

$$\Big\downarrow \text{2732}$$

$$\log(x)\log^2(\log(x)) - 2(\log(x)\log(\log(x)) - \log(x))$$

input
```
Int[Log[Log[x]]^2/x,x]
```

output
```
Log[x]*Log[Log[x]]^2 - 2*(-Log[x] + Log[x]*Log[Log[x]])
```

3.629.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.629.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^{2}$	21
default	$2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^{2}$	21
norman	$2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^{2}$	21
risch	$2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^{2}$	21

input
```
int(ln(ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

3.629.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\log(\log(x))^2 - 2\,\log(x)\log(\log(x)) + 2\,\log(x)$$

input
```
integrate(log(log(x))^2/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

3.629.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

input
```
integrate(ln(ln(x))**2/x,x)
```

output
```
log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)
```

3.629.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x}\,dx = \left(\log\left(\log\left(x\right)\right)^2 - 2\,\log\left(\log\left(x\right)\right) + 2\right)\log\left(x\right)$$

input
```
integrate(log(log(x))^2/x,x, algorithm="maxima")
```

output
```
(log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

3.629.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log\left(x\right)\log\left(\log\left(x\right)\right)^2 - 2\,\log\left(x\right)\log\left(\log\left(x\right)\right) + 2\,\log\left(x\right)$$

input
```
integrate(log(log(x))^2/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

3.629.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x}\,dx = \ln\left(x\right)\,\left(\ln\left(\ln\left(x\right)\right)^2 - 2\,\ln\left(\ln\left(x\right)\right) + 2\right)$$

input
```
int(log(log(x))^2/x,x)
```

output
```
log(x)*(log(log(x))^2 - 2*log(log(x)) + 2)
```

3.630 $\int \frac{\log^3(\log(x))}{x}\,dx$

3.630.1 Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \frac{\log^3(\log(x))}{x}\,dx = -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))$$

output
```
-6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

3.630.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))$$

input
```
Integrate[Log[Log[x]]^3/x,x]
```

output
```
-6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log
[x]]^3
```

3.630.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3039, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^3(\log(x))d\log(x)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^3(\log(x)) - 3\int \log^2(\log(x))d\log(x)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^3(\log(x)) - 3\left(\log(x)\log^2(\log(x)) - 2\int \log(\log(x))d\log(x)\right)$$

$$\Big\downarrow \text{2732}$$

$$\log(x)\log^3(\log(x)) - 3(\log(x)\log^2(\log(x)) - 2(\log(x)\log(\log(x)) - \log(x)))$$

input
```
Int[Log[Log[x]]^3/x,x]
```

output
```
Log[x]*Log[Log[x]]^3 - 3*(Log[x]*Log[Log[x]]^2 - 2*(-Log[x] + Log[x]*Log[L
og[x]]))
```

3.630.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.630.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
default	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
norman	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30
risch	$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$	30

input
```
int(ln(ln(x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
-6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

3.630.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

input
```
integrate(log(log(x))^3/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*l
og(x)
```

3.630.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

input
```
integrate(ln(ln(x))**3/x,x)
```

output
```
log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(log(x)) - 6
*log(x)
```

3.630.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\log^3(\log(x))}{x}\,dx = \left(\log(\log(x))^3 - 3\,\log(\log(x))^2 + 6\,\log(\log(x)) - 6\right)\log(x)$$

input
```
integrate(log(log(x))^3/x,x, algorithm="maxima")
```

output
```
(log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

3.630.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\,\log(x)\log(\log(x))^2 + 6\,\log(x)\log(\log(x)) - 6\,\log(x)$$

input
```
integrate(log(log(x))^3/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*l
og(x)
```

3.630.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \ln(x)\,\ln(\ln(x))^3 - 3\,\ln(x)\,\ln(\ln(x))^2 + 6\,\ln(x)\,\ln(\ln(x)) - 6\,\ln(x)$$

input
```
int(log(log(x))^3/x,x)
```

output
```
6*log(log(x))*log(x) - 6*log(x) - 3*log(log(x))^2*log(x) + log(log(x))^3*l
og(x)
```

3.631 $\int \frac{\log^4(\log(x))}{x} \, dx$

3.631.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \frac{\log^4(\log(x))}{x} \, dx = 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))$$

output
```
24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)
*ln(ln(x))^4
```

3.631.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x} \, dx = 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))$$

input
```
Integrate[Log[Log[x]]^4/x,x]
```

output
```
24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log
[Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

3.631.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3039, 2733, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^4(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^4(\log(x))d\log(x)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^4(\log(x))-4\int \log^3(\log(x))d\log(x)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^4(\log(x))-4\left(\log(x)\log^3(\log(x))-3\int \log^2(\log(x))d\log(x)\right)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^4(\log(x))-4\left(\log(x)\log^3(\log(x))-3\left(\log(x)\log^2(\log(x))-2\int \log(\log(x))d\log(x)\right)\right)$$

$$\Big\downarrow \text{2732}$$

$$\log(x)\log^4(\log(x))-4(\log(x)\log^3(\log(x))-3(\log(x)\log^2(\log(x))-2(\log(x)\log(\log(x))-\log(x))))$$

input
```
Int[Log[Log[x]]^4/x,x]
```

output
```
Log[x]*Log[Log[x]]^4 - 4*(Log[x]*Log[Log[x]]^3 - 3*(Log[x]*Log[Log[x]]^2 -
 2*(-Log[x] + Log[x]*Log[Log[x]])))
```

3.631.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.631.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
derivativedivides	$24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln(\text{l}$
default	$24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln(\text{l}$
norman	$24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln(\text{l}$
risch	$24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln(\text{l}$

input
```
int(ln(ln(x))^4/x,x,method=_RETURNVERBOSE)
```

output
```
24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)
*ln(ln(x))^4
```

3.631.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

input
```
integrate(log(log(x))^4/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 -
24*log(x)*log(log(x)) + 24*log(x)
```

3.631.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

input
```
integrate(ln(ln(x))**4/x,x)
```

output
```
log(x)*log(log(x))**4 - 4*log(x)*log(log(x))**3 + 12*log(x)*log(log(x))**2
 - 24*log(x)*log(log(x)) + 24*log(x)
```

3.631.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\log^4(\log(x))}{x}\,dx = \left(\log(\log(x))^4 - 4\log(\log(x))^3 + 12\log(\log(x))^2 - 24\log(\log(x)) + 24\right)\log(x)$$

input
```
integrate(log(log(x))^4/x,x, algorithm="maxima")
```

output
```
(log(log(x))^4 - 4*log(log(x))^3 + 12*log(log(x))^2 - 24*log(log(x)) + 24)
*log(x)
```

3.631.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x} \, dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

input
```
integrate(log(log(x))^4/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 -
24*log(x)*log(log(x)) + 24*log(x)
```

3.631.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x} \, dx = \ln(x)\ \ln(\ln(x))^4 - 4\ln(x)\ \ln(\ln(x))^3 + 12\ln(x)\ \ln(\ln(x))^2 - 24\ln(x)\ \ln(\ln(x)) + 24\ln(x)$$

input
```
int(log(log(x))^4/x,x)
```

output
```
24*log(x) - 24*log(log(x))*log(x) + 12*log(log(x))^2*log(x) - 4*log(log(x)
)^3*log(x) + log(log(x))^4*log(x)
```

3.632 $\int \frac{\log^n(\log(x))}{x} \, dx$

3.632.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{\log^n(\log(x))}{x} \, dx = \Gamma(1+n, -\log(\log(x)))(-\log(\log(x)))^{-n} \log^n(\log(x))$$

output
```
GAMMA(1+n,-ln(ln(x)))*ln(ln(x))^n/((-ln(ln(x)))^n)
```

3.632.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x} \, dx = \Gamma(1+n, -\log(\log(x)))(-\log(\log(x)))^{-n} \log^n(\log(x))$$

input
```
Integrate[Log[Log[x]]^n/x,x]
```

output
```
(Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

3.632.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3039, 2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^n(\log(x))d\log(x)$$

$$\Big\downarrow \text{2736}$$

$$\int x\log^n(\log(x))d\log(\log(x))$$

$$\Big\downarrow \text{2612}$$

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\Gamma(n+1,-\log(\log(x)))$$

input
```
Int[Log[Log[x]]^n/x,x]
```

output
```
(Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

3.632.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.632.4 Maple [F]

$$\int \frac{\ln(\ln(x))^n}{x}dx$$

input
```
int(ln(ln(x))^n/x,x)
```

output
```
int(ln(ln(x))^n/x,x)
```

3.632.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{\log^n(\log(x))}{x}\,dx = e^{(-i\,\pi n)}\Gamma(n+1,-\log(\log(x)))$$

input
```
integrate(log(log(x))^n/x,x, algorithm="fricas")
```

output
```
e^(-I*pi*n)*gamma(n + 1, -log(log(x)))
```

3.632.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x}\,dx = (-\log(\log(x)))^{-n}\log(\log(x))^n\Gamma(n+1,-\log(\log(x)))$$

input
```
integrate(ln(ln(x))**n/x,x)
```

output
```
log(log(x))**n*uppergamma(n + 1, -log(log(x)))/(-log(log(x)))**n
```

3.632.7 Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\log^n(\log(x))}{x} \, dx = -(-\log(\log(x)))^{-n-1} \log(\log(x))^{n+1} \Gamma(n+1, -\log(\log(x)))$$

input
```
integrate(log(log(x))^n/x,x, algorithm="maxima")
```

output
```
-(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(log(x)))
```

3.632.8 Giac [F]

$$\int \frac{\log^n(\log(x))}{x} \, dx = \int \frac{\log(\log(x))^n}{x} \, dx$$

input
```
integrate(log(log(x))^n/x,x, algorithm="giac")
```

output
```
integrate(log(log(x))^n/x, x)
```

3.632.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x} \, dx = \frac{\ln(\ln(x))^n \Gamma(n+1, -\ln(\ln(x)))}{(-\ln(\ln(x)))^n}$$

input
```
int(log(log(x))^n/x,x)
```

output
```
(log(log(x))^n*igamma(n + 1, -log(log(x))))/(-log(log(x)))^n
```

3.633 $\int \frac{\cot(x)}{\log(\sin(x))} \, dx$

3.633.1 Optimal result

Integrand size = 8, antiderivative size = 4

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(\log(\sin(x)))$$

output
```
ln(ln(sin(x)))
```

3.633.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(\log(\sin(x)))$$

input
```
Integrate[Cot[x]/Log[Sin[x]],x]
```

output
```
Log[Log[Sin[x]]]
```

3.633.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4838, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\log(\sin(x))} dx$$

$$\Big\downarrow \; 4838$$

$$\int \frac{\csc(x)}{\log(\sin(x))} d\sin(x)$$

$$\Big\downarrow \; 2739$$

$$\int \csc(x) d\log(\sin(x))$$

$$\Big\downarrow \; 14$$

$$\log(\log(\sin(x)))$$

input
```
Int[Cot[x]/Log[Sin[x]],x]
```

output
```
Log[Log[Sin[x]]]
```

3.633.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.633.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\ln(\ln(\sin(x)))$
default	$\ln(\ln(\sin(x)))$
risch	$\ln\left(-\frac{i\pi\,\mathrm{csgn}(i(\mathrm{e}^{2ix}-1))\,\mathrm{csgn}(i\mathrm{e}^{-ix})\,\mathrm{csgn}(\sin(x))}{2}-\frac{i\pi\,\mathrm{csgn}(i(\mathrm{e}^{2ix}-1))\,\mathrm{csgn}(\sin(x))^2}{2}-\frac{i\pi\,\mathrm{csgn}(i\mathrm{e}^{-ix})\,\mathrm{csgn}(\sin\right.$

input
```
int(cot(x)/ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(sin(x)))
```

3.633.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log(\log(\sin(x)))$$

input
```
integrate(cot(x)/log(sin(x)),x, algorithm="fricas")
```

output
```
log(log(sin(x)))
```

3.633.6 Sympy [F]

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \int \frac{\cot(x)}{\log(\sin(x))}\,dx$$

input
```
integrate(cot(x)/ln(sin(x)),x)
```

output
```
Integral(cot(x)/log(sin(x)), x)
```

3.633.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log\left(\log\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cot(x)/log(sin(x)),x, algorithm="maxima")
```

output
```
log(log(sin(x)))
```

3.633.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log\left(\left|\log\left(\sin\left(x\right)\right)\right|\right)$$

input
```
integrate(cot(x)/log(sin(x)),x, algorithm="giac")
```

output
```
log(abs(log(sin(x))))
```

3.633.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \ln\left(\ln\left(\sin\left(x\right)\right)\right)$$

input
```
int(cot(x)/log(sin(x)),x)
```

output
```
log(log(sin(x)))
```

3.634 $\int (\cos(x) + \sec(x)) \tan(x) \, dx$

3.634.1 Optimal result

Integrand size = 8, antiderivative size = 7

$$\int (\cos(x) + \sec(x)) \tan(x) \, dx = -\cos(x) + \sec(x)$$

output
```
-cos(x)+sec(x)
```

3.634.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\cos(x) + \sec(x)) \tan(x) \, dx = -\cos(x) + \sec(x)$$

input
```
Integrate[(Cos[x] + Sec[x])*Tan[x],x]
```

output
```
-Cos[x] + Sec[x]
```

3.634.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 4724, 3042, 4879, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)(\cos(x)+\sec(x))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(x)(\cos(x)+\sec(x))dx$$
$$\Big\downarrow \text{4724}$$
$$\int \left(\cos^2(x)+1\right)\tan(x)\sec(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^2+1}{\cos(x)\cot(x)}dx$$
$$\Big\downarrow \text{4879}$$
$$-\int \left(\sec^2(x)+1\right)d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$\sec(x)-\cos(x)$$

input
```
Int[(Cos[x] + Sec[x])*Tan[x],x]
```

output
```
-Cos[x] + Sec[x]
```

3.634.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4724
```
Int[(u_)*((A_.) + cos[(a_.) + (b_.)*(x_)]*(B_.) + (C_.)*sec[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Int[ActivateTrig[u]*((C + A*Cos[a + b*x] + B*Cos[a + b*
x]^2)/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B, C}, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.634.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{1}{\cos(x)} - \cos(x)$	10
parts	$\frac{1}{\cos(x)} - \cos(x)$	10
risch	$-\frac{e^{3ix} - \cos(x) - 3i\sin(x)}{2(e^{2ix}+1)}$	27

input
```
int((1/cos(x)+cos(x))*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/cos(x)-cos(x)
```

3.634.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = -\frac{\cos{(x)}^2-1}{\cos{(x)}}$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="fricas")
```

output
```
-(cos(x)^2 - 1)/cos(x)
```

3.634.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = -\cos{(x)}+\frac{1}{\cos{(x)}}$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x)
```

output
```
-cos(x) + 1/cos(x)
```

3.634.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = \frac{1}{\cos{(x)}}-\cos{(x)}$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="maxima")
```

output
```
1/cos(x) - cos(x)
```

3.634.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = \frac{1}{\cos(x)} - \cos(x)$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="giac")
```

output
```
1/cos(x) - cos(x)
```

3.634.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = \frac{1}{\cos(x)} - \cos(x)$$

input
```
int(tan(x)*(cos(x) + 1/cos(x)),x)
```

output
```
1/cos(x) - cos(x)
```

3.635 $\int \log(\cosh(x)) \sinh(x) \, dx$

3.635.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \log(\cosh(x)) \sinh(x) \, dx = -\cosh(x) + \cosh(x) \log(\cosh(x))$$

output
```
-cosh(x)+cosh(x)*ln(cosh(x))
```

3.635.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x)) \sinh(x) \, dx = -\cosh(x) + \cosh(x) \log(\cosh(x))$$

input
```
Integrate[Log[Cosh[x]]*Sinh[x],x]
```

output
```
-Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

3.635.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3034, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\log(\cosh(x))\,dx$$

$$\Big\downarrow \text{ 3034}$$

$$\cosh(x)\log(\cosh(x))-\int \sinh(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cosh(x)\log(\cosh(x))-\int -i\sin(ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$\cosh(x)\log(\cosh(x))+i\int \sin(ix)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\cosh(x)\log(\cosh(x))-\cosh(x)$$

input
```
Int[Log[Cosh[x]]*Sinh[x],x]
```

output
```
-Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

3.635.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.635.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\cosh(x)+\cosh(x)\ln(\cosh(x))$
default	$-\cosh(x)+\cosh(x)\ln(\cosh(x))$
risch	$-\frac{(1+e^{2x})e^{-x}\ln(e^{x})}{2}-\frac{\left(-i\pi\,\mathrm{csgn}\left(i\left(1+e^{2x}\right)\right)\mathrm{csgn}\left(ie^{-x}\left(1+e^{2x}\right)\right)^{2}-i\pi\,\mathrm{csgn}\left(i\left(1+e^{2x}\right)\right)\mathrm{csgn}\left(ie^{-x}\left(1+e^{2x}\right)\right)^{2}e^{2x}+i\pi\right.}{}$

input
```
int(ln(cosh(x))*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)+cosh(x)*ln(cosh(x))
```

3.635.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(11) = 22.

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.18

$$\int \log(\cosh(x))\sinh(x)\,dx = -\frac{\cosh(x)^{2}-\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}+1\right)\log(\cosh(x))+2\cosh(x)\sinh(x)+\sinh}{2\left(\cosh(x)+\sinh(x)\right)}$$

input
```
integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")
```

output
```
-1/2*(cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(cosh
(x)) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

3.635.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \log(\cosh(x))\sinh(x)\,dx = \log\left(\cosh\left(x\right)\right)\cosh\left(x\right) - \cosh\left(x\right)$$

input
```
integrate(ln(cosh(x))*sinh(x),x)
```

output
```
log(cosh(x))*cosh(x) - cosh(x)
```

3.635.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh\left(x\right)\log\left(\cosh\left(x\right)\right) - \cosh\left(x\right)$$

input
```
integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")
```

output
```
cosh(x)*log(cosh(x)) - cosh(x)
```

3.635.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(11) = 22.

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.45

$$\int \log(\cosh(x))\sinh(x)\,dx = \frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\log\left(\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\right) - \frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}$$

input
```
integrate(log(cosh(x))*sinh(x),x, algorithm="giac")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x)*log(1/2*(e^(2*x) + 1)*e^(-x)) - 1/2*(e^(2*x) + 1)
*e^(-x)
```

3.635.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \log(\cosh(x)) \sinh(x) \, dx = \cosh(x) \ (\ln(\cosh(x)) - 1)$$

input
```
int(log(cosh(x))*sinh(x),x)
```

output
```
cosh(x)*(log(cosh(x)) - 1)
```

3.636 $\int \log(\cosh(x)) \tanh(x) \, dx$

3.636.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \log(\cosh(x)) \tanh(x) \, dx = \frac{1}{2} \log^2(\cosh(x))$$

output
```
1/2*ln(cosh(x))^2
```

3.636.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x)) \tanh(x) \, dx = \frac{1}{2} \log^2(\cosh(x))$$

input
```
Integrate[Log[Cosh[x]]*Tanh[x],x]
```

output
```
Log[Cosh[x]]^2/2
```

3.636.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4841, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\log(\cosh(x))\,dx$$
$$\Big\downarrow \text{4841}$$
$$\int \operatorname{sech}(x)\log(\cosh(x))d\cosh(x)$$
$$\Big\downarrow \text{2738}$$
$$\frac{1}{2}\log^2(\cosh(x))$$

input
```
Int[Log[Cosh[x]]*Tanh[x],x]
```

output
```
Log[Cosh[x]]^2/2
```

3.636.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 4841
```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Cosh[c*(a
 + b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

3.636.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\cosh(x))^2}{2}$
default	$\frac{\ln(\cosh(x))^2}{2}$
risch	$(x-\ln(1+e^{2x}))\ln(e^x)+\frac{\ln(1+e^{2x})^2}{2}+\frac{i\ln(1+e^{2x})\pi\,\mathrm{csgn}(i(1+e^{2x}))\,\mathrm{csgn}(ie^{-x}(1+e^{2x}))^2}{2}+\frac{i\pi\,\mathrm{csgn}(i$

input
```
int(ln(cosh(x))*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(cosh(x))^2
```

3.636.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log(\cosh(x))^2$$

input
```
integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")
```

output
```
1/2*log(cosh(x))^2
```

3.636.6 Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\log(\cosh(x))^2}{2}$$

input
```
integrate(ln(cosh(x))*tanh(x),x)
```

output
```
log(cosh(x))**2/2
```

3.636.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log\left(\cosh\left(x\right)\right)^2$$

input
```
integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")
```

output
```
1/2*log(cosh(x))^2
```

3.636.8 Giac [F]

$$\int \log(\cosh(x))\tanh(x)\,dx = \int \log\left(\cosh\left(x\right)\right)\tanh\left(x\right)\,dx$$

input
```
integrate(log(cosh(x))*tanh(x),x, algorithm="giac")
```

output
```
integrate(log(cosh(x))*tanh(x), x)
```

3.636.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\ln\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)^2}{2}$$

input
```
int(log(cosh(x))*tanh(x),x)
```

output
```
log(exp(-x)/2 + exp(x)/2)^2/2
```

3.637 $\int \log \left(x-\sqrt{1+x^2}\right) \, dx$

3.637.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \log \left(x-\sqrt{1+x^2}\right) \, dx=\sqrt{1+x^2}+x \log \left(x-\sqrt{1+x^2}\right)$$

output
```
x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

3.637.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log \left(x-\sqrt{1+x^2}\right) \, dx=\sqrt{1+x^2}+x \log \left(x-\sqrt{1+x^2}\right)$$

input
```
Integrate[Log[x - Sqrt[1 + x^2]],x]
```

output
```
Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

3.637.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3014, 25, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x-\sqrt{x^2+1}\right) dx$$

$$\Big\downarrow \text{3014}$$

$$x \log \left(x-\sqrt{x^2+1}\right)-\int -\frac{x}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x}{\sqrt{x^2+1}}dx+x \log \left(x-\sqrt{x^2+1}\right)$$

$$\Big\downarrow \text{241}$$

$$\sqrt{x^2+1}+x \log \left(x-\sqrt{x^2+1}\right)$$

input
```
Int[Log[x - Sqrt[1 + x^2]],x]
```

output
```
Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

3.637.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3014
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Simp[a*c*f^2   Int[x/(d*e*(
a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
 f}, x] && EqQ[e^2 - c*f^2, 0]
```

3.637.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$x\ln\left(-\sqrt{x^2+1}+x\right)+\sqrt{x^2+1}$	23
parts	$x\ln\left(-\sqrt{x^2+1}+x\right)-\frac{x^2\sqrt{x^2+1}}{3}+\frac{2\sqrt{x^2+1}}{3}+\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	46

input
```
int(ln(-(x^2+1)^(1/2)+x),x,method=_RETURNVERBOSE)
```

output
```
x*ln(-(x^2+1)^(1/2)+x)+(x^2+1)^(1/2)
```

3.637.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx = x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
integrate(log(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

3.637.6 Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx = x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
integrate(ln(x-(x**2+1)**(1/2)),x)
```

output
```
x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

3.637.7 Maxima [F]

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx=\int \log\left(x-\sqrt{x^2+1}\right)\,dx$$

input
```
integrate(log(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x^2 + 1)^(
3/2) + x), x)
```

3.637.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx=x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
integrate(log(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

3.637.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log\left(x-\sqrt{1+x^2}\right)\,dx=x\,\ln\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
int(log(x - (x^2 + 1)^(1/2)),x)
```

output
```
x*log(x - (x^2 + 1)^(1/2)) + (x^2 + 1)^(1/2)
```

3.638 $\int \frac{\log(-1+x)}{x^3} dx$

3.638.1 Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{\log(-1+x)}{x^3} dx = \frac{1}{2x} + \frac{1}{2}\log(1-x) - \frac{\log(-1+x)}{2x^2} - \frac{\log(x)}{2}$$

output
```
1/2/x+1/2*ln(1-x)-1/2*ln(-1+x)/x^2-1/2*ln(x)
```

3.638.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\log(-1+x)}{x^3} dx = \frac{1}{2}\left(\frac{1}{x} + \log(1-x) - \frac{\log(-1+x)}{x^2} - \log(x)\right)$$

input
```
Integrate[Log[-1 + x]/x^3,x]
```

output
```
(x^(-1) + Log[1 - x] - Log[-1 + x]/x^2 - Log[x])/2
```

3.638.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2842, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x-1)}{x^3}\,dx$$

$$\Big\downarrow \ 2842$$

$$\frac{1}{2}\int -\frac{1}{(1-x)x^2}dx-\frac{\log(x-1)}{2x^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \frac{1}{(1-x)x^2}dx-\frac{\log(x-1)}{2x^2}$$

$$\Big\downarrow \ 54$$

$$-\frac{1}{2}\int \left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{1-x}\right)dx-\frac{\log(x-1)}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{1}{x}+\log(1-x)-\log(x)\right)-\frac{\log(x-1)}{2x^2}$$

input
```
Int[Log[-1 + x]/x^3,x]
```

output
```
-1/2*Log[-1 + x]/x^2 + (x^(-1) + Log[1 - x] - Log[x])/2
```

3.638.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

3.638.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{\ln(x)}{2}+\frac{1}{2x}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
default	$-\frac{\ln(x)}{2}+\frac{1}{2x}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
parts	$-\frac{\ln(-1+x)}{2x^2}+\frac{\ln(-1+x)}{2}+\frac{1}{2x}-\frac{\ln(x)}{2}$	26
norman	$\frac{\frac{x}{2}+\frac{x^2\ln(-1+x)}{2}-\frac{\ln(-1+x)}{2}}{x^2}-\frac{\ln(x)}{2}$	29
risch	$-\frac{\ln(-1+x)}{2x^2}+\frac{\ln(-1+x)x-x\ln(x)+1}{2x}$	29
parallelrisch	$-\frac{x^2\ln(x)-x^2\ln(-1+x)-x+\ln(-1+x)}{2x^2}$	29

input
```
int(ln(-1+x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x)+1/2/x+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2
```

3.638.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\log(-1+x)}{x^3}\,dx=-\frac{x^2\log(x)-(x^2-1)\log(x-1)-x}{2\,x^2}$$

input
```
integrate(log(-1+x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(x^2*log(x) - (x^2 - 1)*log(x - 1) - x)/x^2
```

3.638.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\log(-1+x)}{x^3}\,dx = -\frac{\log(x)}{2} + \frac{\log(x-1)}{2} + \frac{1}{2x} - \frac{\log(x-1)}{2x^2}$$

input
```
integrate(ln(-1+x)/x**3,x)
```

output
```
-log(x)/2 + log(x - 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

3.638.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{1}{2\,x} - \frac{\log(x-1)}{2\,x^2} + \frac{1}{2}\,\log(x-1) - \frac{1}{2}\,\log(x)$$

input
```
integrate(log(-1+x)/x^3,x, algorithm="maxima")
```

output
```
1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(x - 1) - 1/2*log(x)
```

3.638.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{1}{2\,x} - \frac{\log(x-1)}{2\,x^2} + \frac{1}{2}\,\log(|x-1|) - \frac{1}{2}\,\log(|x|)$$

input
```
integrate(log(-1+x)/x^3,x, algorithm="giac")
```

output
```
1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(abs(x - 1)) - 1/2*log(abs(x))
```

3.638.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{x-\ln\left(x-1\right)+x^2\,\ln\left(1-\frac{1}{x}\right)}{2\,x^2}$$

input
```
int(log(x - 1)/x^3,x)
```

output
```
(x - log(x - 1) + x^2*log(1 - 1/x))/(2*x^2)
```

3.639 $\int (-e^{-x} + e^x) \log(1 + e^{2x}) \, dx$

3.639.1 Optimal result

Integrand size = 20, antiderivative size = 32

$$\int (-e^{-x} + e^x) \log(1 + e^{2x}) \, dx = -2e^x + e^{-x} \log(1 + e^{2x}) + e^x \log(1 + e^{2x})$$

output
```
-2*exp(x)+ln(1+exp(2*x))/exp(x)+exp(x)*ln(1+exp(2*x))
```

3.639.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int (-e^{-x} + e^x) \log(1 + e^{2x}) \, dx = -2e^x + (e^{-x} + e^x) \log(1 + e^{2x})$$

input
```
Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

output
```
-2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

3.639.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 25, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right) \log \left(e^{2 x}+1\right) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int -e^{-2 x} \left(1-e^{2 x}\right) \log \left(e^{2 x}+1\right)de^x$$

$$\Big\downarrow \text{25}$$

$$-\int e^{-2 x} \left(1-e^{2 x}\right) \log \left(1+e^{2 x}\right)de^x$$

$$\Big\downarrow \text{2926}$$

$$-\int \left(e^{-2 x} \log \left(1+e^{2 x}\right)-\log \left(1+e^{2 x}\right)\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$-2 e^x+e^{-x} \log \left(e^{2 x}+1\right)+e^x \log \left(e^{2 x}+1\right)$$

input
```
Int[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

output
```
-2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

3.639.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

3.639.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
risch	$\left(1+{\mathrm e}^{2 x}\right) {\mathrm e}^{-x} \ln \left(1+{\mathrm e}^{2 x}\right)-2 \,{\mathrm e}^{x}$	24
norman	$\left({\mathrm e}^{2 x} \ln \left(1+{\mathrm e}^{2 x}\right)-2 \,{\mathrm e}^{2 x}+\ln \left(1+{\mathrm e}^{2 x}\right)\right) {\mathrm e}^{-x}$	32

input
```
int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
(1+exp(2*x))*exp(-x)*ln(1+exp(2*x))-2*exp(x)
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(-e^{-x}+e^x\right) \log \left(1+e^{2 x}\right) \, dx=\left({\left(e^{\left(2 \, x\right)} + 1\right)} \log \left(e^{\left(2 \, x\right)} + 1\right) - 2 \, e^{\left(2 \, x\right)}\right)e^{\left(-x\right)}$$

input
```
integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="fricas")
```

output
```
((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)
```

3.639.6 Sympy [A] (verification not implemented)

Time = 81.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(-e^{-x}+e^{x}\right)\log\left(1+e^{2x}\right)\,dx=e^{x}\log\left(e^{2x}+1\right)-2e^{x}+e^{-x}\log\left(e^{2x}+1\right)$$

input
```
integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

output
```
exp(x)*log(exp(2*x) + 1) - 2*exp(x) + exp(-x)*log(exp(2*x) + 1)
```

3.639.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \left(-e^{-x}+e^{x}\right)\log\left(1+e^{2x}\right)\,dx=\left(e^{(-x)}+e^{x}\right)\log\left(e^{(2x)}+1\right)-2\,e^{x}$$

input
```
integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="maxima")
```

output
```
(e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

3.639.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \left(-e^{-x}+e^{x}\right)\log\left(1+e^{2x}\right)\,dx=\left(e^{(-x)}+e^{x}\right)\log\left(e^{(2x)}+1\right)-2\,e^{x}$$

input
```
integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="giac")
```

output
```
(e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

3.639.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(-e^{-x}+e^x\right)\log\left(1+e^{2x}\right)\,dx = 2\,\ln\left(\mathrm{e}^{2\,x}+1\right)\,\cosh(x)-\frac{\mathrm{e}^{2\,x}+1}{\cosh\left(x\right)}$$

input
```
int(-log(exp(2*x) + 1)*(exp(-x) - exp(x)),x)
```

output
```
2*log(exp(2*x) + 1)*cosh(x) - (exp(2*x) + 1)/cosh(x)
```

3.640 $\int e^{3x/2} \log(-1+e^x)\, dx$

3.640.1 Optimal result

Integrand size = 14, antiderivative size = 52

$$\int e^{3x/2} \log(-1+e^x)\, dx = -\frac{4e^{x/2}}{3} - \frac{4}{9}e^{3x/2} + \frac{4}{3}\text{arctanh}\left(e^{x/2}\right) + \frac{2}{3}e^{3x/2}\log(-1+e^x)$$

output
```
-4/3*exp(1/2*x)-4/9*exp(3/2*x)+4/3*arctanh(exp(1/2*x))+2/3*exp(3/2*x)*ln(-
1+exp(x))
```

3.640.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2} \log(-1+e^x)\, dx = \frac{2}{9}\left(6\text{arctanh}\left(e^{x/2}\right) + e^{x/2}(-2(3+e^x) + 3e^x\log(-1+e^x))\right)$$

input
```
Integrate[E^((3*x)/2)*Log[-1 + E^x],x]
```

output
```
(2*(6*ArcTanh[E^(x/2)] + E^(x/2)*(-2*(3 + E^x) + 3*E^x*Log[-1 + E^x])))/9
```

3.640.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3034, 27, 25, 2678, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x/2}\log\left(e^x-1\right)\,dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)-\int \frac{2e^{5x/2}}{3\left(-1+e^x\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)-\frac{2}{3}\int -\frac{e^{5x/2}}{1-e^x}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\int \frac{e^{5x/2}}{1-e^x}dx+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

$$\Big\downarrow \text{2678}$$

$$\frac{4}{3}\int \frac{e^{2x}}{1-e^x}de^{x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{4}{3}\int \left(-e^x-1+\frac{1}{1-e^x}\right)de^{x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{3}\left(\text{arctanh}\left(e^{x/2}\right)-e^{x/2}-\frac{1}{3}e^{3x/2}\right)+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

input
```
Int[E^((3*x)/2)*Log[-1 + E^x],x]
```

output
```
(4*(-E^(x/2) - E^((3*x)/2)/3 + ArcTanh[E^(x/2)]))/3 + (2*E^((3*x)/2)*Log[-
1 + E^x])/3
```

3.640.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

3.640.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{2\,e^{\frac{3x}{2}}\ln(-1+e^{x})}{3}-\frac{4\,e^{\frac{3x}{2}}}{9}-\frac{4\,e^{\frac{x}{2}}}{3}+\frac{2\ln\left(e^{\frac{x}{2}}+1\right)}{3}-\frac{2\ln\left(-1+e^{\frac{x}{2}}\right)}{3}$	43

input
```
int(exp(3/2*x)*ln(-1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*exp(3/2*x)*ln(-1+exp(x))-4/9*exp(3/2*x)-4/3*exp(1/2*x)+2/3*ln(exp(1/2*
x)+1)-2/3*ln(-1+exp(1/2*x))
```

3.640.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^x-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}-1\right)$$

input
```
integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="fricas")
```

output
```
2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/
2*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

3.640.6 Sympy [F(-1)]

Timed out.

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\text{Timed out}$$

input
```
integrate(exp(3/2*x)*ln(-1+exp(x)),x)
```

output
```
Timed out
```

3.640.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^x-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}-1\right)$$

input
```
integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="maxima")
```

output
```
2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/
2*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

3.640.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{3x/2}\log\left(-1+e^{x}\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(\left|e^{\left(\frac{1}{2}\,x\right)}-1\right|\right)$$

input
```
integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="giac")
```

output
```
2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/
2*x) + 1) - 2/3*log(abs(e^(1/2*x) - 1))
```

3.640.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{3x/2}\log\left(-1+e^{x}\right)\,dx=\frac{4\,\mathrm{atanh}\left(\sqrt{\mathrm{e}^{x}}\right)}{3}-\frac{4\,\mathrm{e}^{\frac{3\,x}{2}}}{9}-\frac{4\,\mathrm{e}^{x/2}}{3}+\frac{2\,\mathrm{e}^{\frac{3\,x}{2}}\,\ln\left(\mathrm{e}^{x}-1\right)}{3}$$

input
```
int(exp((3*x)/2)*log(exp(x) - 1),x)
```

output
```
(4*atanh(exp(x)^(1/2)))/3 - (4*exp((3*x)/2))/9 - (4*exp(x/2))/3 + (2*exp((
3*x)/2)*log(exp(x) - 1))/3
```

3.641 $\int \cos^3(x)\log(\sin(x))\,dx$

3.641.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \cos^3(x)\log(\sin(x))\,dx = -\sin(x)+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\frac{1}{3}\log(\sin(x))\sin^3(x)$$

output
```
-sin(x)+ln(sin(x))*sin(x)+1/9*sin(x)^3-1/3*ln(sin(x))*sin(x)^3
```

3.641.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\log(\sin(x))\,dx = -\sin(x)+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\frac{1}{3}\log(\sin(x))\sin^3(x)$$

input
```
Integrate[Cos[x]^3*Log[Sin[x]],x]
```

output
```
-Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

3.641.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3034, 27, 3042, 4856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{6}\cos(x)(\cos(2x)+5)dx-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}\int \cos(x)(\cos(2x)+5)dx-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\int \cos(x)(\cos(2x)+5)dx-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{4856}$$

$$-\frac{1}{6}\int \left(6-2\sin^2(x)\right)d\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{2\sin^3(x)}{3}-6\sin(x)\right)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

input
```
Int[Cos[x]^3*Log[Sin[x]],x]
```

output
```
Log[Sin[x]]*Sin[x] - (Log[Sin[x]]*Sin[x]^3)/3 + (-6*Sin[x] + (2*Sin[x]^3)/
3)/6
```

3.641.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.641.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{(3\ln(\sin(x))-1)\sin(3x)}{36}+\frac{3\ln(\sin(x))\sin(x)}{4}-\frac{11\sin(x)}{12}$	26
derivativedivides	$-\sin(x)+\ln(\sin(x))\sin(x)+\frac{(\sin^3(x))}{9}-\frac{\ln(\sin(x))(\sin^3(x))}{3}$	27
default	$-\sin(x)+\ln(\sin(x))\sin(x)+\frac{(\sin^3(x))}{9}-\frac{\ln(\sin(x))(\sin^3(x))}{3}$	27
risch	Expression too large to display	577

input
```
int(cos(x)^3*ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*ln(sin(x))-1)*sin(3*x)+3/4*ln(sin(x))*sin(x)-11/12*sin(x)
```

3.641.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{1}{3}\left(\cos\left(x\right)^2 + 2\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right) - \frac{1}{9}\left(\cos\left(x\right)^2 + 8\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*log(sin(x))*sin(x) - 1/9*(cos(x)^2 + 8)*sin(x)
```

3.641.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{2\log\left(\sin\left(x\right)\right)\sin^3\left(x\right)}{3} + \log\left(\sin\left(x\right)\right)\sin\left(x\right)\cos^2\left(x\right) - \frac{8\sin^3\left(x\right)}{9} - \sin\left(x\right)\cos^2\left(x\right)$$

input
```
integrate(cos(x)**3*ln(sin(x)),x)
```

output
```
2*log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x)*cos(x)**2 - 8*sin(x)**3/9 -
 sin(x)*cos(x)**2
```

3.641.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{1}{9}\sin\left(x\right)^3 - \frac{1}{3}\left(\sin\left(x\right)^3 - 3\sin\left(x\right)\right)\log\left(\sin\left(x\right)\right) - \sin\left(x\right)$$

input
```
integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")
```

output
```
1/9*sin(x)^3 - 1/3*(sin(x)^3 - 3*sin(x))*log(sin(x)) - sin(x)
```

3.641.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \cos^3(x)\log(\sin(x))\,dx = -\frac{1}{3}\log(\sin(x))\sin(x)^3 + \frac{1}{9}\sin(x)^3 + \log(\sin(x))\sin(x) - \sin(x)$$

input
```
integrate(cos(x)^3*log(sin(x)),x, algorithm="giac")
```

output
```
-1/3*log(sin(x))*sin(x)^3 + 1/9*sin(x)^3 + log(sin(x))*sin(x) - sin(x)
```

3.641.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(x)\log(\sin(x))\,dx = \int \ln(\sin(x))\cos(x)^3\,dx$$

input
```
int(log(sin(x))*cos(x)^3,x)
```

output
```
int(log(sin(x))*cos(x)^3, x)
```

3.642 $\int \log(\tan(x)) \sec^4(x) \, dx$

3.642.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \log(\tan(x)) \sec^4(x) \, dx = -\tan(x) + \log(\tan(x)) \tan(x) - \frac{\tan^3(x)}{9} + \frac{1}{3} \log(\tan(x)) \tan^3(x)$$

output
```
-tan(x)+ln(tan(x))*tan(x)-1/9*tan(x)^3+1/3*ln(tan(x))*tan(x)^3
```

3.642.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \log(\tan(x)) \sec^4(x) \, dx = \frac{1}{9} \left(-8 + (-1 + 6 \log(\tan(x)) + 3 \cos(2x) \log(\tan(x))) \sec^2(x)\right) \tan(x)$$

input
```
Integrate[Log[Tan[x]]*Sec[x]^4,x]
```

output
```
((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Tan[x])/9
```

3.642.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3034, 27, 3042, 4889, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(x) \log(\tan(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{3}(\cos(2x)+2)\sec^4(x)dx+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int (\cos(2x)+2)\sec^4(x)dx+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{\cos(2x)+2}{\cos(x)^4}dx+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{4889}$$

$$-\frac{1}{3}\int \left(\tan^2(x)+3\right)d\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{1}{3}\tan^3(x)-3\tan(x)\right)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

input
```
Int[Log[Tan[x]]*Sec[x]^4,x]
```

output
```
Log[Tan[x]]*Tan[x] + (Log[Tan[x]]*Tan[x]^3)/3 + (-3*Tan[x] - Tan[x]^3/3)/3
```

3.642.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.642.4 Maple [A] (verified)

Time = 10.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\tan\left(x\right)+\ln\left(\tan\left(x\right)\right)\tan\left(x\right)-\frac{\left(\tan^{3}\left(x\right)\right)}{9}+\frac{\ln\left(\tan\left(x\right)\right)\left(\tan^{3}\left(x\right)\right)}{3}$	27
default	$-\tan\left(x\right)+\ln\left(\tan\left(x\right)\right)\tan\left(x\right)-\frac{\left(\tan^{3}\left(x\right)\right)}{9}+\frac{\ln\left(\tan\left(x\right)\right)\left(\tan^{3}\left(x\right)\right)}{3}$	27
risch	Expression too large to display	782

input
```
int(ln(tan(x))/cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-tan(x)+ln(tan(x))*tan(x)-1/9*tan(x)^3+1/3*ln(tan(x))*tan(x)^3
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{3\left(2\cos(x)^2+1\right)\log\left(\frac{\sin(x)}{\cos(x)}\right)\sin(x)-\left(8\cos(x)^2+1\right)\sin(x)}{9\cos(x)^3}$$

input
```
integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")
```

output
```
1/9*(3*(2*cos(x)^2 + 1)*log(sin(x)/cos(x))*sin(x) - (8*cos(x)^2 + 1)*sin(x
))/cos(x)^3
```

3.642.6 Sympy [A] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{\log(\tan(x))\tan^3(x)}{3}+\log(\tan(x))\tan(x)-\frac{\sin^3(x)}{9\cos^3(x)}+\frac{\sin(x)}{3\cos(x)}-\frac{4\tan(x)}{3}$$

input
```
integrate(ln(tan(x))/cos(x)**4,x)
```

output
```
log(tan(x))*tan(x)**3/3 + log(tan(x))*tan(x) - sin(x)**3/(9*cos(x)**3) + s
in(x)/(3*cos(x)) - 4*tan(x)/3
```

3.642.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \log(\tan(x))\sec^4(x)\,dx = -\frac{1}{9}\tan(x)^3+\frac{1}{3}\left(\tan(x)^3+3\tan(x)\right)\log(\tan(x))-\tan(x)$$

input
```
integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")
```

output
```
-1/9*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*log(tan(x)) - tan(x)
```

3.642.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{1}{3}\log(\tan(x))\tan(x)^3 - \frac{1}{9}\tan(x)^3 + \log(\tan(x))\tan(x) - \tan(x)$$

input
```
integrate(log(tan(x))/cos(x)^4,x, algorithm="giac")
```

output
```
1/3*log(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + log(tan(x))*tan(x) - tan(x)
```

3.642.9 Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.93

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{\ln\left(-\frac{8\,e^{x\,2i}}{3}-\frac{8}{3}\right)2i}{3} - \frac{\ln\left(\frac{8}{3}-\frac{8\,e^{x\,2i}}{3}\right)2i}{3} + \frac{8i}{9\left(3\,e^{x\,2i}+3\,e^{x\,4i}+e^{x\,6i}+1\right)} - \frac{4i}{3\left(2\,e^{x\,2i}+e^{x\,4i}+1\right)} - \frac{4i}{3\left(e^{x\,2i}+1\right)} + \frac{\ln\left(-\frac{e^{x\,2i}\,1i-i}{e^{x\,2i}+1}\right)\left(e^{x\,2i}\,4i+\frac{4}{3}i\right)}{3\,e^{x\,2i}+3\,e^{x\,4i}+e^{x\,6i}+1}$$

input
```
int(log(tan(x))/cos(x)^4,x)
```

output
```
(log(- (8*exp(x*2i))/3 - 8/3)*2i)/3 - (log(8/3 - (8*exp(x*2i))/3)*2i)/3 +
8i/(9*(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)) - 4i/(3*(2*exp(x*2i) +
exp(x*4i) + 1)) - 4i/(3*(exp(x*2i) + 1)) + (log(-(exp(x*2i)*1i - 1i)/(exp(
x*2i) + 1))*(exp(x*2i)*4i + 4i/3))/(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i)
+ 1)
```

3.643 $\int \frac{\log(\cos(\frac{x}{2}))}{1+\cos(x)}\,dx$

3.643.1 Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{x}{2}+\frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)}+\tan\left(\frac{x}{2}\right)$$

output
```
-1/2*x+ln(cos(1/2*x))*sin(x)/(1+cos(x))+tan(1/2*x)
```

3.643.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{\left(x\cot\left(\frac{x}{2}\right)-2\left(1+\log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right)\sin(x)}{2(1+\cos(x))}$$

input
```
Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

output
```
-1/2*((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(1 + Cos[x])
```

3.643.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3034, 27, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\cos \left(\frac{x}{2}\right)\right)}{\cos (x)+1} \, dx$$

$$\Big\downarrow \text{ 3034}$$

$$\frac{\sin (x) \log \left(\cos \left(\frac{x}{2}\right)\right)}{\cos (x)+1}-\int -\frac{1}{2} \tan ^2\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \int \tan ^2\left(\frac{x}{2}\right)dx+\frac{\sin (x) \log \left(\cos \left(\frac{x}{2}\right)\right)}{\cos (x)+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2} \int \tan \left(\frac{x}{2}\right)^2dx+\frac{\sin (x) \log \left(\cos \left(\frac{x}{2}\right)\right)}{\cos (x)+1}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{1}{2} \left(2 \tan \left(\frac{x}{2}\right)-\int 1dx\right)+\frac{\sin (x) \log \left(\cos \left(\frac{x}{2}\right)\right)}{\cos (x)+1}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{2} \left(2 \tan \left(\frac{x}{2}\right)-x\right)+\frac{\sin (x) \log \left(\cos \left(\frac{x}{2}\right)\right)}{\cos (x)+1}$$

input
```
Int[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

output
```
(Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + (-x + 2*Tan[x/2])/2
```

3.643.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.643.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.86

method	result
risch	$-\frac{2i\ln\left(\mathrm{e}^{\frac{ix}{2}}\right)}{\mathrm{e}^{ix}+1}+\frac{\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{ix}+1\right)\right)\mathrm{csgn}\left(i\mathrm{e}^{-\frac{ix}{2}}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)-\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{ix}+1\right)\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)^{2}-\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-\frac{ix}{2}}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right.\right.}{\mathrm{e}^{ix}+1}$

input
```
int(ln(cos(1/2*x))/(cos(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-2*I/(exp(I*x)+1)*ln(exp(1/2*I*x))+(Pi*csgn(I*(exp(I*x)+1))*csgn(I*exp(-1/
2*I*x))*csgn(I*cos(1/2*x))-Pi*csgn(I*(exp(I*x)+1))*csgn(I*cos(1/2*x))^2-Pi
*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))^2+Pi*csgn(I*cos(1/2*x))^3-I*ln(e
xp(I*x)+1)*exp(I*x)-x*exp(I*x)-2*I*ln(2)+I*ln(exp(I*x)+1)+2*I-x)/(exp(I*x)
+1)
```

3.643.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{x\cos\left(\frac{1}{2}\,x\right) - 2\,\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right) - 2\,\sin\left(\frac{1}{2}\,x\right)}{2\,\cos\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
-1/2*(x*cos(1/2*x) - 2*log(cos(1/2*x))*sin(1/2*x) - 2*sin(1/2*x))/cos(1/2*
x)
```

3.643.6 Sympy [F]

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}\,dx$$

input
```
integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

output
```
Integral(log(cos(x/2))/(cos(x) + 1), x)
```

3.643.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin(x)}{\cos(x)+1} - \frac{x\cos(x)^2 + x\sin(x)^2 + 2\,x\cos(x) + x - 4\,\sin(x)}{2\,\left(\cos(x)^2 + \sin(x)^2 + 2\,\cos(x) + 1\right)}$$

input
```
integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
log(cos(1/2*x))*sin(x)/(cos(x) + 1) - 1/2*(x*cos(x)^2 + x*sin(x)^2 + 2*x*c
os(x) + x - 4*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

3.643.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{1}{2}\,x - \frac{2\,\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)} + \tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="giac")
```

output
```
-1/2*x - 2*log(cos(1/2*x))*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1)
) + tan(1/2*x)
```

3.643.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right) - x + \tan\left(\frac{x}{2}\right)\,\ln\left(\cos\left(\frac{x}{2}\right)\right) + \ln\left(\cos\left(\frac{x}{2}\right)\right)\,1\mathrm{i} - \ln\left(\cos\left(x\right) + 1 + \sin\left(x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(log(cos(x/2))/(cos(x) + 1),x)
```

output
```
tan(x/2) - x + log(cos(x/2))*1i - log(cos(x) + sin(x)*1i + 1)*1i + tan(x/2
)*log(cos(x/2))
```

3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$

3.644.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} + \frac{8\sin(x)}{9(1+\cos(x))} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}$$

output
```
-2/3*x-1/9*sin(x)/(1+cos(x))^2+8/9*sin(x)/(1+cos(x))-1/3*ln(sin(x))*sin(x)
/(1+cos(x))^2+2/3*ln(sin(x))*sin(x)/(1+cos(x))
```

3.644.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{1}{18}\sec^3\left(\frac{x}{2}\right)\left(9x\cos\left(\frac{x}{2}\right) + 3x\cos\left(\frac{3x}{2}\right) - (7+3\log(\sin(x)) + \cos(x)(8+6\log(\sin(x))))\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

output
```
-1/18*(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]] +
Cos[x]*(8 + 6*Log[Sin[x]]))*Sin[x/2]))
```

3.644.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3034, 27, 3042, 3447, 3042, 3498, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)\log(\sin(x))}{(\cos(x)+1)^2}dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{\cos(x)(2\cos(x)+1)}{3(\cos(x)+1)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{\cos(x)(2\cos(x)+1)}{(\cos(x)+1)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{\sin\left(x+\frac{\pi}{2}\right)\left(2\sin\left(x+\frac{\pi}{2}\right)+1\right)}{\left(\sin\left(x+\frac{\pi}{2}\right)+1\right)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{1}{3}\int \frac{2\cos^2(x)+\cos(x)}{(\cos(x)+1)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{2\sin\left(x+\frac{\pi}{2}\right)^2+\sin\left(x+\frac{\pi}{2}\right)}{\left(\sin\left(x+\frac{\pi}{2}\right)+1\right)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3498}$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{2(1-3\cos(x))}{\cos(x)+1}dx-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{1-3\cos(x)}{\cos(x)+1}dx-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{1-3\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)+1}dx-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \textcolor{blue}{3214}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(4\int \frac{1}{\cos(x)+1}dx-3x\right)-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(4\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx-3x\right)-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \textcolor{blue}{3127}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(\frac{4\sin(x)}{\cos(x)+1}-3x\right)-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

input
```
Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

output
```
-1/3*(Log[Sin[x]]*Sin[x])/(1 + Cos[x])^2 + (2*Log[Sin[x]]*Sin[x])/(3*(1 +
Cos[x])) + (-1/3*Sin[x]/(1 + Cos[x])^2 + (2*(-3*x + (4*Sin[x])/(1 + Cos[x]
)))/3)/3
```

3.644.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.644.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{-6x\cos(2x)+(6\ln(\sin(x))+8)\sin(2x)-24x\cos(x)+6\ln(\sin(x))\sin(x)-18x+14\sin(x)}{9\cos(2x)+27+36\cos(x)}$
default	$\frac{6\ln\left(\frac{\sin(x)}{2}\right)\cos(x)\sin(x)+12\arctan(-\csc(x)+\cot(x))\left(\cos^2(x)\right)+6\ln(2)\cos(x)\sin(x)+3\ln\left(\frac{\sin(x)}{2}\right)\sin(x)+24\arctan(-\csc(x}{9(\cos(x)+1)^2}$
risch	Expression too large to display

input
```
int(cos(x)*ln(sin(x))/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-6*x*cos(2*x)+(6*ln(sin(x))+8)*sin(2*x)-24*x*cos(x)+6*ln(sin(x))*sin(x)-1
8*x+14*sin(x))/(9*cos(2*x)+27+36*cos(x))
```

3.644.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{6\,x\cos{(x)}^2 - 3\,(2\,\cos{(x)} + 1)\log{(\sin{(x)})}\sin{(x)} + 12\,x\cos{(x)} - (8\,\cos{(x)} + 7)\sin{(x)} + 6\,x}{9\left(\cos{(x)}^2 + 2\,\cos{(x)} + 1\right)}$$

input
```
integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
-1/9*(6*x*cos(x)^2 - 3*(2*cos(x) + 1)*log(sin(x))*sin(x) + 12*x*cos(x) - (
8*cos(x) + 7)*sin(x) + 6*x)/(cos(x)^2 + 2*cos(x) + 1)
```

3.644.6 Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3} - \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^3\left(\frac{x}{2}\right)}{6} + \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{2} - \frac{\log\left(2\right)\tan^3\left(\frac{x}{2}\right)}{6} - \frac{\tan^3\left(\frac{x}{2}\right)}{18} + \frac{\log\left(2\right)\tan\left(\frac{x}{2}\right)}{2} + \frac{5\tan\left(\frac{x}{2}\right)}{6}$$

input
```
integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
```

output
```
-2*x/3 - log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**3/6 + log(tan(x/2)/(tan
(x/2)**2 + 1))*tan(x/2)/2 - log(2)*tan(x/2)**3/6 - tan(x/2)**3/18 + log(2)
*tan(x/2)/2 + 5*tan(x/2)/6
```

3.644.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = \frac{1}{6}\left(\frac{3\,\sin{(x)}}{\cos{(x)}+1} - \frac{\sin{(x)}^3}{(\cos{(x)}+1)^3}\right)\log\left(\frac{2\,\sin{(x)}}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos{(x)}+1)}\right) + \frac{5\,\sin{(x)}}{6\,(\cos{(x)}+1)} - \frac{\sin{(x)}^3}{18\,(\cos{(x)}+1)^3} - \frac{4}{3}\,\arctan\left(\frac{\sin{(x)}}{\cos{(x)}+1}\right)$$

input
```
integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)*log(2*sin(x)/((sin(x
)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))) + 5/6*sin(x)/(cos(x) + 1) - 1/18*si
n(x)^3/(cos(x) + 1)^3 - 4/3*arctan(sin(x)/(cos(x) + 1))
```

3.644.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{1}{18}\tan\left(\frac{1}{2}x\right)^3 - \frac{1}{6}\left(\tan\left(\frac{1}{2}x\right)^3 - 3\tan\left(\frac{1}{2}x\right)\right)\log\left(\sin\left(x\right)\right) - \frac{2}{3}x + \frac{5}{6}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="giac")
```

output
```
-1/18*tan(1/2*x)^3 - 1/6*(tan(1/2*x)^3 - 3*tan(1/2*x))*log(sin(x)) - 2/3*x
 + 5/6*tan(1/2*x)
```

3.644.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = \frac{\frac{4\sin(2x)}{9} - \frac{\ln\left(-2\sin(x)^2+\sin(2x)\,1\mathrm{i}\right)7\mathrm{i}}{3} - \frac{14x}{3} + \frac{\ln(\sin(x))\,7\mathrm{i}}{3} + \frac{7\sin(x)}{9} + \frac{\sin(2x)\ln(\sin(x))}{3} - \frac{\sin(x)^2\,8\mathrm{i}}{9} + \sin\left(\frac{x}{2}\right)^2\left(\frac{16x}{3} + \right.}{}$$

input
```
int((log(sin(x))*cos(x))/(cos(x) + 1)^2,x)
```

output
```
((4*sin(2*x))/9 - (log(sin(2*x)*1i - 2*sin(x)^2)*7i)/3 - (14*x)/3 + (log(s
in(x))*7i)/3 + (7*sin(x))/9 + (sin(2*x)*log(sin(x)))/3 - (sin(x)^2*8i)/9 +
 sin(x/2)^2*((16*x)/3 + (log(sin(2*x)*1i - 2*sin(x)^2)*8i)/3 - (log(sin(x)
)*8i)/3 - 32i/9) + (log(sin(2*x)*1i - 2*sin(x)^2)*(2*sin(x)^2 - 1)*1i)/3 +
 (log(sin(x))*sin(x))/3 - (log(sin(x))*(2*sin(x)^2 - 1)*1i)/3 + (2*x*(2*si
n(x)^2 - 1))/3 + 32i/9)/(2*sin(x/2)^2 - 2)^2
```

3.644. $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$

3.645 $\int \frac{\arccos(x)^2}{x^5} \, dx$

3.645.1 Optimal result

Integrand size = 8, antiderivative size = 65

$$\int \frac{\arccos(x)^2}{x^5} \, dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} + \frac{\sqrt{1-x^2}\arccos(x)}{3x} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}$$

output
```
-1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1
/3*arccos(x)*(-x^2+1)^(1/2)/x
```

3.645.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\arccos(x)^2}{x^5} \, dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}(1+2x^2)\arccos(x)}{6x^3} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}$$

input
```
Integrate[ArcCos[x]^2/x^5,x]
```

output
```
-1/12*1/x^2 + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - ArcCos[x]^2/
(4*x^4) + Log[x]/3
```

3.645.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5139, 5205, 15, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(x)^2}{x^5}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{1}{2}\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}dx-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{5205}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx+\frac{\int \frac{1}{x^3}dx}{3}+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}\right)-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{1}{6x^2}\right)-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{5187}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(-\int \frac{1}{x}dx-\frac{\sqrt{1-x^2}\arccos(x)}{x}\right)+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{1}{6x^2}\right)-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(-\frac{\sqrt{1-x^2}\arccos(x)}{x}-\log(x)\right)+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{1}{6x^2}\right)-\frac{\arccos(x)^2}{4x^4}$$

input
```
Int[ArcCos[x]^2/x^5,x]
```

output
```
-1/4*ArcCos[x]^2/x^4 + (-1/6*1/x^2 + (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (
2*(-((Sqrt[1 - x^2]*ArcCos[x])/x) - Log[x]))/3)/2
```

3.645.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

3.645.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{12x^{2}}-\frac{\arccos\left(x\right)^{2}}{4x^{4}}+\frac{\ln\left(x\right)}{3}+\frac{\arccos\left(x\right)\sqrt{-x^{2}+1}}{6x^{3}}+\frac{\arccos\left(x\right)\sqrt{-x^{2}+1}}{3x}$	52

input
```
int(arccos(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1
/3*arccos(x)*(-x^2+1)^(1/2)/x
```

3.645.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \frac{4\,x^4\log\left(x\right) + 2\left(2\,x^3 + x\right)\sqrt{-x^2+1}\arccos\left(x\right) - x^2 - 3\,\arccos\left(x\right)^2}{12\,x^4}$$

input
```
integrate(arccos(x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*x^4*log(x) + 2*(2*x^3 + x)*sqrt(-x^2 + 1)*arccos(x) - x^2 - 3*arcc
os(x)^2)/x^4
```

3.645.6 Sympy [F]

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{\operatorname{acos}^2\left(x\right)}{x^5}\,dx$$

input
```
integrate(acos(x)**2/x**5,x)
```

output
```
Integral(acos(x)**2/x**5, x)
```

3.645.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \frac{1}{6}\left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3}\right)\arccos\left(x\right) - \frac{1}{12\,x^2} - \frac{\arccos\left(x\right)^2}{4\,x^4} + \frac{1}{3}\log\left(x\right)$$

input
```
integrate(arccos(x)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) - 1/12/x^2 - 1/4*a
rccos(x)^2/x^4 + 1/3*log(x)
```

3.645.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(51) = 102$.

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{\arccos(x)^2}{x^5}\,dx$$
$$= -\frac{1}{48}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) - \frac{2x^2+1}{12x^2} - \frac{\arccos(x)^2}{4x^4} + \frac{1}{6}\log\left(x^2\right)$$

input
```
integrate(arccos(x)^2/x^5,x, algorithm="giac")
```

output
```
-1/48*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(
sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) - 1/12*(2*x^
2 + 1)/x^2 - 1/4*arccos(x)^2/x^4 + 1/6*log(x^2)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{\mathrm{acos}(x)^2}{x^5}\,dx$$

input
```
int(acos(x)^2/x^5,x)
```

output
```
int(acos(x)^2/x^5, x)
```

3.646 $\int x^2 \arcsin(x)^2 \, dx$

3.646.1 Optimal result

Integrand size = 8, antiderivative size = 61

$$\int x^2 \arcsin(x)^2 \, dx = -\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9}\sqrt{1-x^2}\arcsin(x) + \frac{2}{9}x^2\sqrt{1-x^2}\arcsin(x) + \frac{1}{3}x^3\arcsin(x)^2$$

output
```
-4/9*x-2/27*x^3+1/3*x^3*arcsin(x)^2+4/9*arcsin(x)*(-x^2+1)^(1/2)+2/9*x^2*a
rcsin(x)*(-x^2+1)^(1/2)
```

3.646.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{27}\left(-2x(6+x^2) + 6\sqrt{1-x^2}(2+x^2)\arcsin(x) + 9x^3\arcsin(x)^2\right)$$

input
```
Integrate[x^2*ArcSin[x]^2,x]
```

output
```
(-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*ArcSin[x]^2)
/27
```

3.646.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5138, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arcsin(x)^2 \, dx$$

$$\Big\downarrow \text{ 5138}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\int \frac{x^3 \arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 5210}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(\frac{2}{3}\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}dx + \frac{\int x^2dx}{3} - \frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(\frac{2}{3}\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}dx - \frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x) + \frac{x^3}{9}\right)$$

$$\Big\downarrow \text{ 5182}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(\frac{2}{3}\left(\int 1dx - \sqrt{1-x^2} \arcsin(x)\right) - \frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x) + \frac{x^3}{9}\right)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(-\frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x) + \frac{2}{3}\left(x - \sqrt{1-x^2} \arcsin(x)\right) + \frac{x^3}{9}\right)$$

input
```
Int[x^2*ArcSin[x]^2,x]
```

output
```
(x^3*ArcSin[x]^2)/3 - (2*(x^3/9 - (x^2*Sqrt[1 - x^2]*ArcSin[x])/3 + (2*(x
- Sqrt[1 - x^2]*ArcSin[x]))/3))/3
```

3.646.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

3.646.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^3 \arcsin(x)^2}{3}+\frac{2 \arcsin(x)(x^2+2)\sqrt{-x^2+1}}{9}-\frac{2x^3}{27}-\frac{4x}{9}$	37

input
```
int(x^2*arcsin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsin(x)^2+2/9*arcsin(x)*(x^2+2)*(-x^2+1)^(1/2)-2/27*x^3-4/9*x
```

3.646.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{3} x^3 \arcsin(x)^2 - \frac{2}{27} x^3 + \frac{2}{9} (x^2+2) \sqrt{-x^2+1} \arcsin(x) - \frac{4}{9} x$$

input
```
integrate(x^2*arcsin(x)^2,x, algorithm="fricas")
```

output
```
1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*sqrt(-x^2 + 1)*arcsin(x) -
4/9*x
```

3.646.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^2 \arcsin(x)^2 \, dx = \frac{x^3 \operatorname{asin}^2(x)}{3} - \frac{2x^3}{27} + \frac{2x^2\sqrt{1-x^2} \operatorname{asin}(x)}{9} - \frac{4x}{9} + \frac{4\sqrt{1-x^2} \operatorname{asin}(x)}{9}$$

input
```
integrate(x**2*asin(x)**2,x)
```

output
```
x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(1 - x**2)*asin(x)/9 - 4*x/9 +
4*sqrt(1 - x**2)*asin(x)/9
```

3.646.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^2 \arcsin(x)^2 \, dx$$
$$= \frac{1}{3} x^3 \arcsin(x)^2 - \frac{2}{27} x^3 + \frac{2}{9} \left(\sqrt{-x^2+1} x^2 + 2\sqrt{-x^2+1}\right) \arcsin(x) - \frac{4}{9} x$$

input
```
integrate(x^2*arcsin(x)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(sqrt(-x^2 + 1)*x^2 + 2*sqrt(-x^2 + 1
))*arcsin(x) - 4/9*x
```

3.646.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^2 \arcsin(x)^2\,dx = \frac{1}{3}\left(x^2-1\right)x \arcsin\left(x\right)^2 + \frac{1}{3}\,x \arcsin\left(x\right)^2 - \frac{2}{9}\left(-x^2+1\right)^{\frac{3}{2}} \arcsin\left(x\right) - \frac{2}{27}\left(x^2-1\right)x + \frac{2}{3}\sqrt{-x^2+1} \arcsin\left(x\right) - \frac{14}{27}\,x$$

input
```
integrate(x^2*arcsin(x)^2,x, algorithm="giac")
```

output
```
1/3*(x^2 - 1)*x*arcsin(x)^2 + 1/3*x*arcsin(x)^2 - 2/9*(-x^2 + 1)^(3/2)*arc
sin(x) - 2/27*(x^2 - 1)*x + 2/3*sqrt(-x^2 + 1)*arcsin(x) - 14/27*x
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arcsin(x)^2\,dx = \int x^2 \operatorname{asin}(x)^2\,dx$$

input
```
int(x^2*asin(x)^2,x)
```

output
```
int(x^2*asin(x)^2, x)
```

3.647 $\int x^3 \arctan(x)^2 \, dx$

3.647.1 Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^2}{12} + \frac{1}{2} x \arctan(x) - \frac{1}{6} x^3 \arctan(x) - \frac{\arctan(x)^2}{4} + \frac{1}{4} x^4 \arctan(x)^2 - \frac{1}{3} \log\left(1 + x^2\right)$$

output
```
1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(
x)^2-1/3*ln(x^2+1)
```

3.647.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{12}\left(x^2 - 2x\left(-3 + x^2\right) \arctan(x) + 3\left(-1 + x^4\right) \arctan(x)^2 - 4 \log\left(1 + x^2\right)\right)$$

input
```
Integrate[x^3*ArcTan[x]^2,x]
```

output
```
(x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Log[1 + x^2
])/12
```

3.647.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5361, 5451, 5361, 243, 49, 2009, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(x)^2 \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4 \arctan(x)^2 - \frac{1}{2}\int \frac{x^4 \arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx - \int x^2 \arctan(x)dx\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx + \frac{1}{3}\int \frac{x^3}{x^2+1}dx - \frac{1}{3}x^3 \arctan(x)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx + \frac{1}{6}\int \frac{x^2}{x^2+1}dx^2 - \frac{1}{3}x^3 \arctan(x)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx + \frac{1}{6}\int \left(1 + \frac{1}{-x^2-1}\right)dx^2 - \frac{1}{3}x^3 \arctan(x)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx - \frac{1}{3}x^3 \arctan(x) + \frac{1}{6}\left(x^2 - \log\left(x^2+1\right)\right)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2+1}dx + \int \arctan(x)dx - \frac{1}{3}x^3 \arctan(x) + \frac{1}{6}\left(x^2 - \log\left(x^2+1\right)\right)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2+1}dx - \int \frac{x}{x^2+1}dx - \frac{1}{3}x^3 \arctan(x) + x\arctan(x) + \frac{1}{6}\left(x^2 - \log\left(x^2+1\right)\right)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2+1}dx-\frac{1}{3}x^3\arctan(x)+x\arctan(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{4}x^4\arctan(x)^2$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{4}x^4\arctan(x)^2+\frac{1}{2}\left(-\frac{1}{3}x^3\arctan(x)+x\arctan(x)-\frac{\arctan(x)^2}{2}+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)\right)$$

input
```
Int[x^3*ArcTan[x]^2,x]
```

output
```
(x^4*ArcTan[x]^2)/4 + (x*ArcTan[x] - (x^3*ArcTan[x])/3 - ArcTan[x]^2/2 + (
x^2 - Log[1 + x^2])/6 - Log[1 + x^2]/2)/2
```

3.647.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.647.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result
default	$\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{\arctan(x)^2}{4}+\frac{x^4\arctan(x)^2}{4}-\frac{\ln(x^2+1)}{3}$
parts	$\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{\arctan(x)^2}{4}+\frac{x^4\arctan(x)^2}{4}-\frac{\ln(x^2+1)}{3}$
parallelrisch	$\frac{x^4\arctan(x)^2}{4}-\frac{x^3\arctan(x)}{6}+\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{\arctan(x)^2}{4}-\frac{\ln(x^2+1)}{3}-\frac{1}{12}$
risch	$-\frac{\left(\frac{x^4}{4}-\frac{1}{4}\right)\ln(ix+1)^2}{4}-\frac{\left(-\frac{x^4\ln(-ix+1)}{2}-\frac{ix^3}{3}+ix+\frac{\ln(-ix+1)}{2}\right)\ln(ix+1)}{4}-\frac{x^4\ln(-ix+1)^2}{16}+\frac{\ln(-ix+1)^2}{16}-\frac{ix^3\ln(-i}{12}$

input
```
int(x^3*arctan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(
x)^2-1/3*ln(x^2+1)
```

3.647.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{4} \left(x^4 - 1\right) \arctan\left(x\right)^2 + \frac{1}{12} x^2 - \frac{1}{6} \left(x^3 - 3x\right) \arctan\left(x\right) - \frac{1}{3} \log\left(x^2 + 1\right)$$

input
```
integrate(x^3*arctan(x)^2,x, algorithm="fricas")
```

output
```
1/4*(x^4 - 1)*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x)*arctan(x) - 1/3*log
(x^2 + 1)
```

3.647.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^4 \operatorname{atan}^2(x)}{4} - \frac{x^3 \operatorname{atan}(x)}{6} + \frac{x^2}{12} + \frac{x \operatorname{atan}(x)}{2} - \frac{\log(x^2 + 1)}{3} - \frac{\operatorname{atan}^2(x)}{4}$$

input
```
integrate(x**3*atan(x)**2,x)
```

output
```
x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(x**2 + 1)
/3 - atan(x)**2/4
```

3.647.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\begin{aligned} \int x^3 \arctan(x)^2 \, dx = {} & \frac{1}{4} x^4 \arctan\left(x\right)^2 + \frac{1}{12} x^2 - \frac{1}{6} \left(x^3 - 3x + 3 \arctan\left(x\right)\right) \arctan\left(x\right) \\ & + \frac{1}{4} \arctan\left(x\right)^2 - \frac{1}{3} \log\left(x^2 + 1\right) \end{aligned}$$

input
```
integrate(x^3*arctan(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*arctan(x) +
 1/4*arctan(x)^2 - 1/3*log(x^2 + 1)
```

3.647.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2\,dx = \frac{1}{4}\,x^4 \arctan\left(x\right)^2 - \frac{1}{6}\,x^3 \arctan\left(x\right) + \frac{1}{12}\,x^2 + \frac{1}{2}\,x \arctan\left(x\right) - \frac{1}{4}\,\arctan\left(x\right)^2 - \frac{1}{3}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3*arctan(x)^2,x, algorithm="giac")
```

output
```
1/4*x^4*arctan(x)^2 - 1/6*x^3*arctan(x) + 1/12*x^2 + 1/2*x*arctan(x) - 1/4
*arctan(x)^2 - 1/3*log(x^2 + 1)
```

3.647.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2\,dx = \frac{x^4\,\mathrm{atan}(x)^2}{4} - \frac{x^3\,\mathrm{atan}(x)}{6} - \frac{\mathrm{atan}(x)^2}{4} - \frac{\ln\left(x^2+1\right)}{3} + \frac{x\,\mathrm{atan}(x)}{2} + \frac{x^2}{12}$$

input
```
int(x^3*atan(x)^2,x)
```

output
```
(x^4*atan(x)^2)/4 - (x^3*atan(x))/6 - atan(x)^2/4 - log(x^2 + 1)/3 + (x*at
an(x))/2 + x^2/12
```

3.648 $\int \frac{\arctan(x)^2}{x^5}\,dx$

3.648.1 Optimal result

Integrand size = 8, antiderivative size = 61

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)$$

output
```
-1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*arctan(x)^
2/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

3.648.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{1}{12x^2}+\frac{(-1+3x^2)\arctan(x)}{6x^3}+\frac{(-1+x^4)\arctan(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)$$

input
```
Integrate[ArcTan[x]^2/x^5,x]
```

output
```
-1/12*1/x^2 + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcTan[x]^2)/
(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

3.648.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.625, Rules used = {5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)^2}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\int \frac{\arctan(x)}{x^4(x^2+1)}dx-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^4}dx-\int \frac{\arctan(x)}{x^2(x^2+1)}dx\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{3}\int \frac{1}{x^3(x^2+1)}dx-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2}dx+\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx-\int \frac{1}{x\left(x^2+1\right)}dx-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log \left(x^2\right)+\log \left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{1}{x^2\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log \left(x^2\right)+\log \left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 47$$

$$\frac{1}{2}\Bigg(\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2-\int \frac{1}{x^2}dx^2\right)-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\Bigg(-\frac{1}{x^2}-\log \left(x^2\right)+\log \big(x^2+1 \qquad \frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 14$$

$$\frac{1}{2}\Bigg(\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2-\log \left(x^2\right)\right)-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\Bigg(-\frac{1}{x^2}-\log \left(x^2\right)+\log \left(x^2+1\right) \qquad \frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\Bigg(\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log \left(x^2+1\right)-\log \left(x^2\right)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log \left(x^2\right)+\log \left(x^2+1\right)\right) \qquad \frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 5419$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)^2}{2}+\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log \left(x^2+1\right)-\log \left(x^2\right)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log \left(x^2\right)+\log \left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

input
```
Int[ArcTan[x]^2/x^5,x]
```

output
```
-1/4*ArcTan[x]^2/x^4 + (-1/3*ArcTan[x]/x^3 + ArcTan[x]/x + ArcTan[x]^2/2 +
 (-Log[x^2] + Log[1 + x^2])/2 + (-x^(-2) - Log[x^2] + Log[1 + x^2])/6)/2
```

3.648. $\int \frac{\arctan(x)^2}{x^5}\,dx$

3.648.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.648. $\int \frac{\arctan(x)^2}{x^5} \, dx$

3.648.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\ln(x)}{3}+\frac{\ln(x^2+1)}{3}$
parts	$-\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\ln(x)}{3}+\frac{\ln(x^2+1)}{3}$
parallelrisch	$-\frac{-3x^4\arctan(x)^2+8x^4\ln(x)-4\ln(x^2+1)x^4-6x^3\arctan(x)+x^2+2x\arctan(x)+3\arctan(x)^2}{12x^4}$
risch	$-\frac{(x^4-1)\ln(ix+1)^2}{16x^4}+\frac{(3x^4\ln(-ix+1)-6ix^3+2ix-3\ln(-ix+1))\ln(ix+1)}{24x^4}-\frac{3x^4\ln(-ix+1)^2+32x^4\ln(x)-16\ln(x^2+1)x}{}$

input
```
int(arctan(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/x^2-1/6/x^3*arctan(x)+1/2/x*arctan(x)+1/4*arctan(x)^2-1/4*arctan(x)^
2/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

3.648.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{4\,x^4\log\left(x^2+1\right)-8\,x^4\log\left(x\right)+3\left(x^4-1\right)\arctan\left(x\right)^2-x^2+2\left(3\,x^3-x\right)\arctan\left(x\right)}{12\,x^4}$$

input
```
integrate(arctan(x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*x^4*log(x^2 + 1) - 8*x^4*log(x) + 3*(x^4 - 1)*arctan(x)^2 - x^2 +
2*(3*x^3 - x)*arctan(x))/x^4
```

3.648.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{2\log(x)}{3} + \frac{\log(x^2+1)}{3} + \frac{\operatorname{atan}^2(x)}{4} + \frac{\operatorname{atan}(x)}{2x} - \frac{1}{12x^2} - \frac{\operatorname{atan}(x)}{6x^3} - \frac{\operatorname{atan}^2(x)}{4x^4}$$

input
```
integrate(atan(x)**2/x**5,x)
```

output
```
-2*log(x)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1/(12*x**2)
 - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

3.648.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{1}{6}\left(\frac{3\,x^2-1}{x^3} + 3\arctan(x)\right)\arctan(x) - \frac{3\,x^2\arctan(x)^2 - 4\,x^2\log(x^2+1) + 8\,x^2\log(x) + 1}{12\,x^2} - \frac{\arctan(x)^2}{4\,x^4}$$

input
```
integrate(arctan(x)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*((3*x^2 - 1)/x^3 + 3*arctan(x))*arctan(x) - 1/12*(3*x^2*arctan(x)^2 -
4*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2 - 1/4*arctan(x)^2/x^4
```

3.648.8 Giac [F]

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \int \frac{\arctan(x)^2}{x^5}\,dx$$

input
```
integrate(arctan(x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(arctan(x)^2/x^5, x)
```

3.648.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{\ln\left(x^2+1\right)}{3} - \frac{2\,\ln\left(x\right)}{3} - \mathrm{atan}(x)^2\left(\frac{1}{4\,x^4} - \frac{1}{4}\right) - \frac{1}{12\,x^2} + \frac{\mathrm{atan}(x)\,\left(\frac{x^2}{2} - \frac{1}{6}\right)}{x^3}$$

input
```
int(atan(x)^2/x^5,x)
```

output
```
log(x^2 + 1)/3 - (2*log(x))/3 - atan(x)^2*(1/(4*x^4) - 1/4) - 1/(12*x^2) +
 (atan(x)*(x^2/2 - 1/6))/x^3
```

3.649 $\int x^3 \csc^{-1}(x)^2 \, dx$

3.649.1 Optimal result

Integrand size = 8, antiderivative size = 63

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}} x \csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x) + \frac{1}{4}x^4 \csc^{-1}(x)^2 + \frac{\log(x)}{3}$$

output
```
1/12*x^2+1/4*x^4*arccsc(x)^2+1/3*ln(x)+1/3*x*arccsc(x)*(1-1/x^2)^(1/2)+1/6
*x^3*arccsc(x)*(1-1/x^2)^(1/2)
```

3.649.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{12}\left(x^2 + 2\sqrt{1-\frac{1}{x^2}} x (2+x^2) \csc^{-1}(x) + 3x^4 \csc^{-1}(x)^2 + 4\log(x)\right)$$

input
```
Integrate[x^3*ArcCsc[x]^2,x]
```

output
```
(x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]^2 + 4*Lo
g[x])/12
```

3.649.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {5746, 4245, 3042, 4673, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \csc^{-1}(x)^2 \, dx$$

$$\Big\downarrow \text{5746}$$

$$-\int \sqrt{1-\frac{1}{x^2}} x^5 \csc^{-1}(x)^2 d\csc^{-1}(x)$$

$$\Big\downarrow \text{4245}$$

$$\frac{1}{4} x^4 \csc^{-1}(x)^2-\frac{1}{2} \int x^4 \csc^{-1}(x) d\csc^{-1}(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4} x^4 \csc^{-1}(x)^2-\frac{1}{2} \int \csc^{-1}(x) \csc\left(\csc^{-1}(x)\right)^4 d\csc^{-1}(x)$$

$$\Big\downarrow \text{4673}$$

$$\frac{1}{2} \left(-\frac{2}{3} \int x^2 \csc^{-1}(x) d\csc^{-1}(x)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+\frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \left(-\frac{2}{3} \int \csc^{-1}(x) \csc\left(\csc^{-1}(x)\right)^2 d\csc^{-1}(x)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+\frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{4672}$$

$$\frac{1}{2} \left(-\frac{2}{3} \left(\int \sqrt{1-\frac{1}{x^2}} x d\csc^{-1}(x)-\sqrt{1-\frac{1}{x^2}} x \csc^{-1}(x)\right)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+ \frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \left(-\frac{2}{3} \left(\int -\tan\left(\csc^{-1}(x)+\frac{\pi}{2}\right) d\csc^{-1}(x)-\sqrt{1-\frac{1}{x^2}} x \csc^{-1}(x)\right)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+ \frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(-\int \tan\left(\csc^{-1}(x)+\frac{\pi}{2}\right)d\csc^{-1}(x)-\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)\right)+\frac{x^2}{6}+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)\right)+\frac{1}{4}x^4\csc^{-1}(x)^2$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{4}x^4\csc^{-1}(x)^2+\frac{1}{2}\left(\frac{x^2}{6}-\frac{2}{3}\left(\log\left(\frac{1}{x}\right)-\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)\right)+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)\right)$$

input
```
Int[x^3*ArcCsc[x]^2,x]
```

output
```
(x^4*ArcCsc[x]^2)/4 + (x^2/6 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x])/3 - (2*(-(
Sqrt[1 - x^(-2)]*x*ArcCsc[x]) + Log[x^(-1)]))/3)/2
```

3.649.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4245
```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^p/(b*n*p)),
 x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /;
FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.649.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{x^{4} \operatorname{arccsc}\left(x\right)^{2}}{4}+\frac{x^{3} \operatorname{arccsc}\left(x\right) \sqrt{\frac{x^{2}-1}{x^{2}}}}{6}+\frac{x^{2}}{12}+\frac{\operatorname{arccsc}\left(x\right) \sqrt{\frac{x^{2}-1}{x^{2}}}\, x}{3}-\frac{\ln\left(\frac{1}{x}\right)}{3}$	56

input
```
int(x^3*arccsc(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arccsc(x)^2+1/6*x^3*arccsc(x)*((x^2-1)/x^2)^(1/2)+1/12*x^2+1/3*arc
csc(x)*((x^2-1)/x^2)^(1/2)*x-1/3*ln(1/x)
```

3.649.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int x^3 \csc^{-1}(x)^2\,dx = \frac{1}{4}\,x^4\operatorname{arccsc}\left(x\right)^2 + \frac{1}{6}\,\left(x^2+2\right)\sqrt{x^2-1}\operatorname{arccsc}\left(x\right) + \frac{1}{12}\,x^2 + \frac{1}{3}\,\log\left(x\right)$$

input
```
integrate(x^3*arccsc(x)^2,x, algorithm="fricas")
```

output
```
1/4*x^4*arccsc(x)^2 + 1/6*(x^2 + 2)*sqrt(x^2 - 1)*arccsc(x) + 1/12*x^2 + 1
/3*log(x)
```

3.649.6 Sympy [F]

$$\int x^3 \csc^{-1}(x)^2\,dx = \int x^3 \operatorname{acsc}^2(x)\,dx$$

input
```
integrate(x**3*acsc(x)**2,x)
```

output
```
Integral(x**3*acsc(x)**2, x)
```

3.649.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int x^3 \csc^{-1}(x)^2\,dx = \frac{1}{4}\,x^4 \operatorname{arccsc}(x)^2 + \frac{2\,x^4 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + 2\,x^2 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + \left(x^2 + 2\,\log\left(x^2\right)\right)\sqrt{x+1}\sqrt{x-1} - 4}{12\,\sqrt{x+1}\sqrt{x-1}}$$

input
```
integrate(x^3*arccsc(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arccsc(x)^2 + 1/12*(2*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 2*
x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + (x^2 + 2*log(x^2))*sqrt(x + 1)*s
qrt(x - 1) - 4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x -
1))
```

3.649.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int x^3 \csc^{-1}(x)^2\,dx = \frac{1}{4}\,x^4 \arcsin\left(\frac{1}{x}\right)^2 + \frac{1}{12}\,x^2\left(\frac{2}{x^2} + 1\right) + \frac{1}{48}\left(x^3\left(\sqrt{-\frac{1}{x^2} + 1} - 1\right)^3 + 9\,x\left(\sqrt{-\frac{1}{x^2} + 1} - 1\right) - \frac{9\,x^2\left(\sqrt{-\frac{1}{x^2} + 1} - 1\right)^2 + 1}{x^3\left(\sqrt{-\frac{1}{x^2} + 1} - 1\right)^3}\right)\arcsin\left(\frac{1}{x}\right) - \frac{1}{6}\,\log\left(\frac{1}{x^2}\right)$$

input
```
integrate(x^3*arccsc(x)^2,x, algorithm="giac")
```

output
```
1/4*x^4*arcsin(1/x)^2 + 1/12*x^2*(2/x^2 + 1) + 1/48*(x^3*(sqrt(-1/x^2 + 1)
 - 1)^3 + 9*x*(sqrt(-1/x^2 + 1) - 1) - (9*x^2*(sqrt(-1/x^2 + 1) - 1)^2 + 1
)/(x^3*(sqrt(-1/x^2 + 1) - 1)^3))*arcsin(1/x) - 1/6*log(x^(-2))
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}(x)^2 \, dx = \int x^3 \operatorname{asin}\left(\frac{1}{x}\right)^2 \, dx$$

input
```
int(x^3*asin(1/x)^2,x)
```

output
```
int(x^3*asin(1/x)^2, x)
```

3.650 $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$

3.650.1 Optimal result

Integrand size = 8, antiderivative size = 148

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = -\frac{3}{128x^4}-\frac{45}{128x^2}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3}-\frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} -\frac{45}{128}\sec^{-1}(x)^2+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{9\sec^{-1}(x)^2}{16x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} +\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{3}{32}\sec^{-1}(x)^4-\frac{\sec^{-1}(x)^4}{4x^4}$$

output
```
-3/128/x^4-45/128/x^2-45/128*arcsec(x)^2+3/16*arcsec(x)^2/x^4+9/16*arcsec(
x)^2/x^2+3/32*arcsec(x)^4-1/4*arcsec(x)^4/x^4-3/32*arcsec(x)*(1-1/x^2)^(1/
2)/x^3-45/64*arcsec(x)*(1-1/x^2)^(1/2)/x+1/4*arcsec(x)^3*(1-1/x^2)^(1/2)/x
^3+3/8*arcsec(x)^3*(1-1/x^2)^(1/2)/x
```

3.650.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$
$$= \frac{-3-45x^2-6\sqrt{1-\frac{1}{x^2}}x(2+15x^2)\sec^{-1}(x)+(24+72x^2-45x^4)\sec^{-1}(x)^2+16\sqrt{1-\frac{1}{x^2}}x(2+3x^2)\sec}{128x^4}$$

input
```
Integrate[ArcSec[x]^4/x^5,x]
```

output
```
(-3 - 45*x^2 - 6*Sqrt[1 - x^(-2)]*x*(2 + 15*x^2)*ArcSec[x] + (24 + 72*x^2
- 45*x^4)*ArcSec[x]^2 + 16*Sqrt[1 - x^(-2)]*x*(2 + 3*x^2)*ArcSec[x]^3 + 4*
(-8 + 3*x^4)*ArcSec[x]^4)/(128*x^4)
```

3.650.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$, Rules used = {5745, 3925, 3042, 3792, 3042, 3791, 3042, 3791, 15, 3792, 15, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(x)^4}{x^5} \, dx$$

$$\Big\downarrow \text{ 5745}$$

$$\int \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^4}{x^3}d\sec^{-1}(x)$$

$$\Big\downarrow \text{ 3925}$$

$$\int \frac{\sec^{-1}(x)^3}{x^4}d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^4d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{3}{8}\int \frac{\sec^{-1}(x)}{x^4}d\sec^{-1}(x)+\frac{3}{4}\int \frac{\sec^{-1}(x)^3}{x^2}d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)-\frac{3}{8}\int \sec^{-1}(x)\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^4d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3}{8}\left(\frac{3}{4}\int \frac{\sec^{-1}(x)}{x^2}d\sec^{-1}(x)+\frac{1}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)+$$
$$\frac{3}{4}\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{8}\left(\frac{3}{4}\int \sec^{-1}(x)\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)+\frac{1}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)+$$
$$\frac{3}{4}\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec^{-1}(x)d\sec^{-1}(x)+\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}\right)+\frac{1}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)+$$
$$\frac{3}{4}\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-$$
$$\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$$\Big\downarrow \text{3792}$$

$$\frac{3}{4}\left(-\frac{3}{2}\int \frac{\sec^{-1}(x)}{x^2}d\sec^{-1}(x)+\frac{1}{2}\int \sec^{-1}(x)^3d\sec^{-1}(x)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}\right)-$$
$$\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-$$
$$\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$$\Big\downarrow \text{15}$$

3.650. $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$

$$\frac{3}{4}\left(-\frac{3}{2}\int \frac{\sec^{-1}(x)}{x^2}d\sec^{-1}(x)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}+\frac{1}{8}\sec^{-1}(x)^4\right)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(-\frac{3}{2}\int \sec^{-1}(x)\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}+\frac{1}{8}\sec^{-1}(x)^4\right)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}\int \sec^{-1}(x)d\sec^{-1}(x)+\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}+\frac{1}{8}\sec^{-1}(x)\right.$$
$$-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}-\frac{3}{2}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{1}{8}\sec^{-1}(x)^4\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

3.650. $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$

input
```
Int[ArcSec[x]^4/x^5,x]
```

output
```
(3*ArcSec[x]^2)/(16*x^4) + (Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(4*x^3) - ArcSec
[x]^4/(4*x^4) + (3*((3*ArcSec[x]^2)/(4*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x]^
3)/(2*x) + ArcSec[x]^4/8 - (3*(1/(4*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x])/(2
*x) + ArcSec[x]^2/4))/2))/4 - (3*(1/(16*x^4) + (Sqrt[1 - x^(-2)]*ArcSec[x]
)/(4*x^3) + (3*(1/(4*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x])/(2*x) + ArcSec[x]
^2/4))/4))/8
```

3.650.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.650.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\operatorname{arcsec}(x)^4}{4x^4}+\frac{\operatorname{arcsec}(x)^3\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-1}{x^2}}\right)}{8x^3}+\frac{3\operatorname{arcsec}(x)^2}{16x^4}-\frac{3\operatorname{arcsec}(x)\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}\right.}{64x^3}$

input
```
int(arcsec(x)^4/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arcsec(x)^4/x^4+1/8*arcsec(x)^3*(3*arcsec(x)*x^3+3*x^2*((x^2-1)/x^2)^
(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+3/16*arcsec(x)^2/x^4-3/64*arcsec(x)*(3*ar
csec(x)*x^3+3*x^2*((x^2-1)/x^2)^(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+45/128*ar
csec(x)^2-3/512*(3*x^2+2)^2/x^4+9/16*arcsec(x)^2/x^2-9/16*arcsec(x)*(x*arc
sec(x)+((x^2-1)/x^2)^(1/2))/x+9/32-9/32/x^2-9/32*arcsec(x)^4
```

3.650.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \frac{4\,(3\,x^4-8)\operatorname{arcsec}(x)^4-3\,(15\,x^4-24\,x^2-8)\operatorname{arcsec}(x)^2-45\,x^2+2\left(8\,(3\,x^2+2)\operatorname{arcsec}(x)^3-3\,(15\,x^2\right.}{128\,x^4}$$

input
```
integrate(arcsec(x)^4/x^5,x, algorithm="fricas")
```

output
```
1/128*(4*(3*x^4 - 8)*arcsec(x)^4 - 3*(15*x^4 - 24*x^2 - 8)*arcsec(x)^2 - 4
5*x^2 + 2*(8*(3*x^2 + 2)*arcsec(x)^3 - 3*(15*x^2 + 2)*arcsec(x))*sqrt(x^2
- 1) - 3)/x^4
```

3.650.6 Sympy [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \int \frac{\operatorname{asec}^4(x)}{x^5}\,dx$$

input
```
integrate(asec(x)**4/x**5,x)
```

output
```
Integral(asec(x)**4/x**5, x)
```

3.650.7 Maxima [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \int \frac{\operatorname{arcsec}(x)^4}{x^5}\,dx$$

input
```
integrate(arcsec(x)^4/x^5,x, algorithm="maxima")
```

output
```
1/64*(64*x^4*integrate(1/8*(12*(x^2 - 1)*log(x^2)^2*log(x)^2 - 16*(x^2 - 1
)*log(x^2)*log(x)^3 + 8*(x^2 - 1)*log(x)^4 + (x^2 - 4*(x^2 - 1)*log(x) - 1
)*log(x^2)^3 - 12*(4*(x^2 - 1)*log(x)^2 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*l
og(x^2))*arctan(sqrt(x + 1)*sqrt(x - 1))^2 + 2*(4*arctan(sqrt(x + 1)*sqrt(
x - 1))^3 - 3*arctan(sqrt(x + 1)*sqrt(x - 1))*log(x^2)^2)*sqrt(x + 1)*sqrt
(x - 1))/(x^7 - x^5), x) - 16*arctan(sqrt(x + 1)*sqrt(x - 1))^4 + 24*arcta
n(sqrt(x + 1)*sqrt(x - 1))^2*log(x^2)^2 - log(x^2)^4)/x^4
```

3.650.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{\sec^{-1}(x)^4}{x^5}\,dx &= \frac{3}{32}\arccos\left(\frac{1}{x}\right)^4 + \frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{8\,x} - \frac{45}{128}\arccos\left(\frac{1}{x}\right)^2 \\ &\quad - \frac{45\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{64\,x} + \frac{\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{4\,x^3} \\ &\quad + \frac{9\arccos\left(\frac{1}{x}\right)^2}{16\,x^2} - \frac{\arccos\left(\frac{1}{x}\right)^4}{4\,x^4} - \frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{32\,x^3} \\ &\quad - \frac{45}{128\,x^2} + \frac{3\arccos\left(\frac{1}{x}\right)^2}{16\,x^4} - \frac{3}{128\,x^4} + \frac{189}{1024}\end{aligned}$$

input
```
integrate(arcsec(x)^4/x^5,x, algorithm="giac")
```

output
```
3/32*arccos(1/x)^4 + 3/8*sqrt(-1/x^2 + 1)*arccos(1/x)^3/x - 45/128*arccos(
1/x)^2 - 45/64*sqrt(-1/x^2 + 1)*arccos(1/x)/x + 1/4*sqrt(-1/x^2 + 1)*arcco
s(1/x)^3/x^3 + 9/16*arccos(1/x)^2/x^2 - 1/4*arccos(1/x)^4/x^4 - 3/32*sqrt(
-1/x^2 + 1)*arccos(1/x)/x^3 - 45/128/x^2 + 3/16*arccos(1/x)^2/x^4 - 3/128/
x^4 + 189/1024
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)^4}{x^5} dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^4}{x^5} dx$$

input
```
int(acos(1/x)^4/x^5,x)
```

output
```
int(acos(1/x)^4/x^5, x)
```

3.651 $\int \sqrt{1-x^2} \arcsin(x)\,dx$

3.651.1 Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arcsin(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)^2}{4}$$

output
```
-1/4*x^2+1/4*arcsin(x)^2+1/2*x*arcsin(x)*(-x^2+1)^(1/2)
```

3.651.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arcsin(x)\,dx = \frac{1}{4}\left(-x^2 + 2x\sqrt{1-x^2}\arcsin(x) + \arcsin(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcSin[x],x]
```

output
```
(-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

3.651.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\arcsin(x)\,dx$$

$$\Big\downarrow \text{5156}$$

$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)-\frac{x^2}{4}$$

$$\Big\downarrow \text{5152}$$

$$\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcSin[x],x]
```

output
```
-1/4*x^2 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

3.651.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

3.651.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\arcsin(x)\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}$	31

input
```
int(arcsin(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2-1/4*x^2
```

3.651.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arcsin(x)\,dx=\frac{1}{2}\sqrt{-x^2+1}x\arcsin\left(x\right)-\frac{1}{4}x^2+\frac{1}{4}\arcsin\left(x\right)^2$$

input
```
integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2
```

3.651.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = -\frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}\right)\operatorname{asin}(x) - \frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate(asin(x)*(-x**2+1)**(1/2),x)
```

output
```
-x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*asin(x) - asin(x)**2/4
```

3.651.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = -\frac{1}{4}\,x^2 + \frac{1}{2}\left(\sqrt{-x^2+1}x + \arcsin(x)\right)\arcsin(x) - \frac{1}{4}\arcsin(x)^2$$

input
```
integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

3.651.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arcsin(x) - \frac{1}{4}\,x^2 + \frac{1}{4}\arcsin(x)^2 + \frac{1}{8}$$

input
```
integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2}\arcsin(x)\,dx=\int \operatorname{asin}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(asin(x)*(1 - x^2)^(1/2),x)
```

output
```
int(asin(x)*(1 - x^2)^(1/2), x)
```

3.652 $\int \sqrt{1-x^2} \arccos(x) \, dx$

3.652.1 Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arccos(x) - \frac{\arccos(x)^2}{4}$$

output
```
1/4*x^2-1/4*arccos(x)^2+1/2*x*arccos(x)*(-x^2+1)^(1/2)
```

3.652.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{4} \left(x^2 + 2x\sqrt{1-x^2} \arccos(x) - \arccos(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

3.652.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\arccos(x)\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{1-x^2}\arccos(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\arccos(x)+\frac{x^2}{4}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2}\sqrt{1-x^2}x\arccos(x)-\frac{\arccos(x)^2}{4}+\frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

3.652.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

3.652.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-x\sqrt{-x^2+1}+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$	33

input
```
int(arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

3.652.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

3.652.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}\right)\operatorname{acos}(x) + \frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate(acos(x)*(-x**2+1)**(1/2),x)
```

output
```
x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*acos(x) + asin(x)**2/4
```

3.652.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{-x^2+1}x + \arcsin(x)\right)\arccos(x) + \frac{1}{4}\arcsin(x)^2$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

3.652.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2 - \frac{1}{8}$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \operatorname{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(acos(x)*(1 - x^2)^(1/2), x)
```

3.653 $\int x\sqrt{1-x^2}\arccos(x)\,dx$

3.653.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=-\frac{x}{3}+\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)$$

output
```
-1/3*x+1/9*x^3-1/3*(-x^2+1)^(3/2)*arccos(x)
```

3.653.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{9}\left(-3x+x^3-3\left(1-x^2\right)^{3/2}\arccos(x)\right)$$

input
```
Integrate[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(-3*x + x^3 - 3*(1 - x^2)^(3/2)*ArcCos[x])/9
```

3.653.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-x^2}\arccos(x)\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{1}{3}\int \left(1-x^2\right)dx-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^3}{3}-x\right)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)$$

input
```
Int[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(-x + x^3/3)/3 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

3.653.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.653.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arccos(x)}{3}+\frac{(x^2-3)x}{9}$	28

input
```
int(x*arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2-1)*(-x^2+1)^(1/2)*arccos(x)+1/9*(x^2-3)*x
```

3.653.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{9}\,x^3+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}\arccos\left(x\right)-\frac{1}{3}\,x$$

input
```
integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/9*x^3 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x) - 1/3*x
```

3.653.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=\frac{x^3}{9}+\frac{x^2\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{3}-\frac{x}{3}-\frac{\sqrt{1-x^2}\,\mathrm{acos}\,(x)}{3}$$

input
```
integrate(x*acos(x)*(-x**2+1)**(1/2),x)
```

output
```
x**3/9 + x**2*sqrt(1 - x**2)*acos(x)/3 - x/3 - sqrt(1 - x**2)*acos(x)/3
```

3.653.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3 - \frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

input
```
integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

3.653.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3 - \frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

input
```
integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \int x\,\mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(x*acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(x*acos(x)*(1 - x^2)^(1/2), x)
```

3.654 $\int (1-x^2)^{3/2} \arcsin(x) \, dx$

3.654.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (1-x^2)^{3/2} \arcsin(x) \, dx = -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8} x \sqrt{1-x^2} \arcsin(x) + \frac{1}{4} x (1-x^2)^{3/2} \arcsin(x) + \frac{3 \arcsin(x)^2}{16}$$

output
```
-5/16*x^2+1/16*x^4+1/4*x*(-x^2+1)^(3/2)*arcsin(x)+3/16*arcsin(x)^2+3/8*x*a
rcsin(x)*(-x^2+1)^(1/2)
```

3.654.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int (1-x^2)^{3/2} \arcsin(x) \, dx = \frac{1}{16} \left(-5x^2 + x^4 - 2x\sqrt{1-x^2} (-5+2x^2) \arcsin(x) + 3 \arcsin(x)^2 \right)$$

input
```
Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/
16
```

3.654.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right)^{3/2} \arcsin(x) \, dx$$

$$\Big\downarrow \text{5158}$$

$$\frac{3}{4} \int \sqrt{1-x^2} \arcsin(x)dx-\frac{1}{4} \int x \left(1-x^2\right) dx+\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4} \int \sqrt{1-x^2} \arcsin(x)dx-\frac{1}{4} \int \left(x-x^3\right) dx+\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4} \int \sqrt{1-x^2} \arcsin(x)dx+\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)+\frac{1}{4} \left(\frac{x^4}{4}-\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{5156}$$

$$\frac{3}{4} \left(\frac{1}{2} \int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{\int xdx}{2}+\frac{1}{2} x \sqrt{1-x^2} \arcsin(x)\right)+\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)+\frac{1}{4} \left(\frac{x^4}{4}-\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4} \left(\frac{1}{2} \int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{1}{2} \sqrt{1-x^2} x \arcsin(x)-\frac{x^2}{4}\right)+\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)+\frac{1}{4} \left(\frac{x^4}{4}-\frac{x^2}{2}\right)$$

$$\Big\downarrow \text{5152}$$

$$\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)+\frac{3}{4} \left(\frac{1}{2} \sqrt{1-x^2} x \arcsin(x)+\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}\right)+\frac{1}{4} \left(\frac{x^4}{4}-\frac{x^2}{2}\right)$$

input
```
Int[(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(-1/2*x^2 + x^4/4)/4 + (x*(1 - x^2)^(3/2)*ArcSin[x])/4 + (3*(-1/4*x^2 + (x
*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4))/4
```

3.654.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x],
x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

3.654. $\int (1-x^2)^{3/2} \arcsin(x)\, dx$

3.654.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3+5x\sqrt{-x^2+1}+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{(2x^2-5)^2}{64}$	54

input
```
int((-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*
arcsin(x)^2+1/64*(2*x^2-5)^2
```

3.654.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \left(1-x^2\right)^{3/2}\arcsin(x)\,dx = \frac{1}{16}\,x^4 - \frac{1}{8}\left(2\,x^3-5\,x\right)\sqrt{-x^2+1}\arcsin\left(x\right) - \frac{5}{16}\,x^2 + \frac{3}{16}\,\arcsin\left(x\right)^2$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

output
```
1/16*x^4 - 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 1)*arcsin(x) - 5/16*x^2 + 3/16*ar
csin(x)^2
```

3.654.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \left(1-x^2\right)^{3/2}\arcsin(x)\,dx = \frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\,\operatorname{asin}\left(x\right)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{1-x^2}\,\operatorname{asin}\left(x\right)}{8} + \frac{3\operatorname{asin}^2\left(x\right)}{16}$$

input
```
integrate((-x**2+1)**(3/2)*asin(x),x)
```

output
```
x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(1 - x**2)*a
sin(x)/8 + 3*asin(x)**2/16
```

3.654.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int (1-x^2)^{3/2} \arcsin(x)\,dx = \frac{1}{16}x^4 - \frac{5}{16}x^2 + \frac{1}{8}\left(2\left(-x^2+1\right)^{\frac{3}{2}}x + 3\sqrt{-x^2+1}x + 3\arcsin(x)\right)\arcsin(x) - \frac{3}{16}\arcsin(x)^2$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

output
```
1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2 + 1)^(3/2)*x + 3*sqrt(-x^2 + 1)*x + 3*a
rcsin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

3.654.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int (1-x^2)^{3/2} \arcsin(x)\,dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin(x) + \frac{3}{8}\sqrt{-x^2+1}x\arcsin(x) + \frac{1}{16}\left(x^2-1\right)^2 - \frac{3}{16}x^2 + \frac{3}{16}\arcsin(x)^2 + \frac{9}{128}$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

output
```
1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(
x^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int (1-x^2)^{3/2} \arcsin(x)\, dx = \int \operatorname{asin}(x)\, (1-x^2)^{3/2}\, dx$$

input
```
int(asin(x)*(1 - x^2)^(3/2),x)
```

output
```
int(asin(x)*(1 - x^2)^(3/2), x)
```

3.655 $\int x(1-x^2)^{3/2} \arcsin(x) \, dx$

3.655.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int x(1-x^2)^{3/2} \arcsin(x) \, dx = \frac{x}{5} - \frac{2x^3}{15} + \frac{x^5}{25} - \frac{1}{5}(1-x^2)^{5/2} \arcsin(x)$$

output
```
1/5*x-2/15*x^3+1/25*x^5-1/5*(-x^2+1)^(5/2)*arcsin(x)
```

3.655.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int x(1-x^2)^{3/2} \arcsin(x) \, dx = \frac{1}{5}\left(x - \frac{2x^3}{3} + \frac{x^5}{5} - (1-x^2)^{5/2} \arcsin(x)\right)$$

input
```
Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(x - (2*x^3)/3 + x^5/5 - (1 - x^2)^(5/2)*ArcSin[x])/5
```

3.655.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5182, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-x^2\right)^{3/2}\arcsin(x)\,dx$$

$$\Big\downarrow \text{5182}$$

$$\frac{1}{5}\int \left(1-x^2\right)^2 dx-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)$$

$$\Big\downarrow \text{210}$$

$$\frac{1}{5}\int \left(x^4-2x^2+1\right) dx-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{x^5}{5}-\frac{2x^3}{3}+x\right)-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)$$

input
```
Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(x - (2*x^3)/3 + x^5/5)/5 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

3.655.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.655.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{(x^2-1)^2\sqrt{-x^2+1}\,\arcsin(x)}{5}+\frac{(3x^4-10x^2+15)x}{75}$	37

input
```
int(x*(-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arcsin(x)+1/75*(3*x^4-10*x^2+15)*x
```

3.655.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx=\frac{1}{25}x^5-\frac{2}{15}x^3-\frac{1}{5}\left(x^4-2x^2+1\right)\sqrt{-x^2+1}\arcsin(x)+\frac{1}{5}x$$

input
```
integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

output
```
1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) + 1/5
*x
```

3.655.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx=\frac{x^5}{25}-\frac{x^4\sqrt{1-x^2}\operatorname{asin}(x)}{5}-\frac{2x^3}{15}+\frac{2x^2\sqrt{1-x^2}\operatorname{asin}(x)}{5}+\frac{x}{5}-\frac{\sqrt{1-x^2}\operatorname{asin}(x)}{5}$$

input
```
integrate(x*(-x**2+1)**(3/2)*asin(x),x)
```

output
```
x**5/25 - x**4*sqrt(1 - x**2)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqrt(1 - x**2
)*asin(x)/5 + x/5 - sqrt(1 - x**2)*asin(x)/5
```

3.655.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{1}{5}\,(-x^2+1)^{\frac{5}{2}}\arcsin(x) - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

input
```
integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

output
```
1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x
```

3.655.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{1}{5}\,(x^2-1)^2\sqrt{-x^2+1}\arcsin(x) - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

input
```
integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

output
```
1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 + 1/5*x
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \int x\,\mathrm{asin}(x)\,\left(1-x^2\right)^{3/2}\,dx$$

input
```
int(x*asin(x)*(1 - x^2)^(3/2),x)
```

output
```
int(x*asin(x)*(1 - x^2)^(3/2), x)
```

3.656 $\int x^3(1-x^2)^{3/2}\arccos(x)\,dx$

3.656.1 Optimal result

Integrand size = 17, antiderivative size = 61

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx=-\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49}-\frac{1}{5}(1-x^2)^{5/2}\arccos(x)+\frac{1}{7}(1-x^2)^{7/2}\arccos(x)$$

output
```
-2/35*x-1/105*x^3+8/175*x^5-1/49*x^7-1/5*(-x^2+1)^(5/2)*arccos(x)+1/7*(-x^
2+1)^(7/2)*arccos(x)
```

3.656.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx=-\frac{x(210+35x^2-168x^4+75x^6)}{3675}-\frac{1}{35}(1-x^2)^{5/2}(2+5x^2)\arccos(x)$$

input
```
Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

output
```
-1/3675*(x*(210 + 35*x^2 - 168*x^4 + 75*x^6)) - ((1 - x^2)^(5/2)*(2 + 5*x^
2)*ArcCos[x])/35
```

3.656.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5195, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(1-x^2\right)^{3/2} \arccos(x) \, dx$$

$$\Big\downarrow \text{5195}$$

$$\int -\frac{1}{35} \left(1-x^2\right)^2 \left(5x^2+2\right) dx + \frac{1}{7}\left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5}\left(1-x^2\right)^{5/2} \arccos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35} \int \left(1-x^2\right)^2 \left(5x^2+2\right) dx + \frac{1}{7}\left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5}\left(1-x^2\right)^{5/2} \arccos(x)$$

$$\Big\downarrow \text{290}$$

$$-\frac{1}{35} \int \left(5x^6-8x^4+x^2+2\right) dx + \frac{1}{7}\left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5}\left(1-x^2\right)^{5/2} \arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}\left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5}\left(1-x^2\right)^{5/2} \arccos(x) + \frac{1}{35}\left(-\frac{5x^7}{7}+\frac{8x^5}{5}-\frac{x^3}{3}-2x\right)$$

input
```
Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

output
```
(-2*x - x^3/3 + (8*x^5)/5 - (5*x^7)/7)/35 - ((1 - x^2)^(5/2)*ArcCos[x])/5
+ ((1 - x^2)^(7/2)*ArcCos[x])/7
```

3.656.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeBQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

3.656.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{(x^2-1)^2\sqrt{-x^2+1}\,\arccos(x)}{5}-\frac{(3x^4-10x^2+15)x}{75}-\frac{(x^2-1)^3\sqrt{-x^2+1}\,\arccos(x)}{7}-\frac{(5x^6-21x^4+35x^2-35)x}{245}$	77

input
```
int(x^3*(-x^2+1)^(3/2)*arccos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arccos(x)-1/75*(3*x^4-10*x^2+15)*x-1/7*(x^2-
1)^3*(-x^2+1)^(1/2)*arccos(x)-1/245*(5*x^6-21*x^4+35*x^2-35)*x
```

3.656.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}\,x^7+\frac{8}{175}\,x^5-\frac{1}{105}\,x^3 - \frac{1}{35}\left(5\,x^6-8\,x^4+x^2+2\right)\sqrt{-x^2+1}\arccos\left(x\right)-\frac{2}{35}\,x$$

input
```
integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="fricas")
```

output
```
-1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*x^6 - 8*x^4 + x^2 + 2)*sqrt(-x
^2 + 1)*arccos(x) - 2/35*x
```

3.656.6 Sympy [A] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{x^7}{49} - \frac{x^6\sqrt{1-x^2}\operatorname{acos}(x)}{7} + \frac{8x^5}{175} + \frac{8x^4\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{x^3}{105} - \frac{x^2\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{2x}{35} - \frac{2\sqrt{1-x^2}\operatorname{acos}(x)}{35}$$

input
```
integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

output
```
-x**7/49 - x**6*sqrt(1 - x**2)*acos(x)/7 + 8*x**5/175 + 8*x**4*sqrt(1 - x*
*2)*acos(x)/35 - x**3/105 - x**2*sqrt(1 - x**2)*acos(x)/35 - 2*x/35 - 2*sq
rt(1 - x**2)*acos(x)/35
```

3.656.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}x^7 + \frac{8}{175}x^5 - \frac{1}{105}x^3 - \frac{1}{35}\left(5\left(-x^2+1\right)^{\frac{5}{2}}x^2 + 2\left(-x^2+1\right)^{\frac{5}{2}}\right)\arccos(x) - \frac{2}{35}x$$

input
```
integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="maxima")
```

output
```
-1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(-x^2 + 1)^(5/2)*x^2 + 2*(-x^2
 + 1)^(5/2))*arccos(x) - 2/35*x
```

3.656.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}x^7 + \frac{8}{175}x^5 - \frac{1}{105}x^3 - \frac{1}{35}\left(5\left(x^2-1\right)^3\sqrt{-x^2+1} + 7\left(x^2-1\right)^2\sqrt{-x^2+1}\right)\arccos(x) - \frac{2}{35}x$$

input
```
integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="giac")
```

output
```
-1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 7
*(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x
```

3.656.9 Mupad [F(-1)]

Timed out.

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx=\int x^3\,\mathrm{acos}(x)\,\left(1-x^2\right)^{3/2}\,dx$$

input
```
int(x^3*acos(x)*(1 - x^2)^(3/2),x)
```

output
```
int(x^3*acos(x)*(1 - x^2)^(3/2), x)
```

3.657 $\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx$

3.657.1 Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x) + 2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)$$

output
```
4/3*x-1/9*x^3+1/3*(-x^2+1)^(3/2)*arccos(x)+2*I*arccos(x)*arctan(x+I*(-x^2+
1)^(1/2))-I*polylog(2,-I*(x+I*(-x^2+1)^(1/2)))+I*polylog(2,I*(x+I*(-x^2+1)
^(1/2)))+arccos(x)*(-x^2+1)^(1/2)
```

3.657.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = x+\sqrt{1-x^2}\arccos(x) + \frac{1}{36}\left(9x+12(1-x^2)^{3/2}\arccos(x)-\cos(3\arccos(x))\right)-\arccos(x)\log\left(1-ie^{i\arccos(x)}\right) + \arccos(x)\log\left(1+ie^{i\arccos(x)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)$$

input
```
Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

output
```
x + Sqrt[1 - x^2]*ArcCos[x] + (9*x + 12*(1 - x^2)^(3/2)*ArcCos[x] - Cos[3*
ArcCos[x]])/36 - ArcCos[x]*Log[1 - I*E^(I*ArcCos[x])] + ArcCos[x]*Log[1 +
I*E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E
^(I*ArcCos[x])]
```

3.657.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx$$

$$\Big\downarrow \text{5203}$$

$$\int \frac{\sqrt{1-x^2}\arccos(x)}{x}dx+\frac{1}{3}\int (1-x^2)\,dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sqrt{1-x^2}\arccos(x)}{x}dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{5199}$$

$$\int \frac{\arccos(x)}{x\sqrt{1-x^2}}dx+\int 1dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{\arccos(x)}{x\sqrt{1-x^2}}dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{5219}$$

$$-\int \frac{\arccos(x)}{x}d\arccos(x)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(x)\csc\left(\arccos(x)+\frac{\pi}{2}\right)d\arccos(x)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{4669}$$

$$\int \log\left(1-ie^{i\arccos(x)}\right)d\arccos(x)-\int \log\left(1+ie^{i\arccos(x)}\right)d\arccos(x)+ 2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{2715}$$

$$-i\int e^{-i\arccos(x)}\log\left(1-ie^{i\arccos(x)}\right)de^{i\arccos(x)}+i\int e^{-i\arccos(x)}\log\left(1+ie^{i\arccos(x)}\right)de^{i\arccos(x)}+ 2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{2838}$$

$$2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)-i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\mathrm{PolyLog}\left(2,ie^{i\arccos(x)}\right)+ \frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

input
```
Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

output
```
x + (x - x^3/3)/3 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/
3 + (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCo
s[x])] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

3.657.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.657. $\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.657. $\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx$

3.657.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
default	$-\frac{5\left(-\sqrt{-x^2+1}+ix\right)(\arccos(x)+i)}{8}+\frac{5\left(ix+\sqrt{-x^2+1}\right)(\arccos(x)-i)}{8}+\arccos\left(x\right)\ln\left(1+i\left(i\sqrt{-x^2+1}+x\right)\right)-$

input
```
int((-x^2+1)^(3/2)*arccos(x)/x,x,method=_RETURNVERBOSE)
```

output
```
-5/8*(-(-x^2+1)^(1/2)+I*x)*(arccos(x)+I)+5/8*(I*x+(-x^2+1)^(1/2))*(arccos(
x)-I)+arccos(x)*ln(1+I*(I*(-x^2+1)^(1/2)+x))-arccos(x)*ln(1-I*(I*(-x^2+1)^
(1/2)+x))-I*dilog(1+I*(I*(-x^2+1)^(1/2)+x))+I*dilog(1-I*(I*(-x^2+1)^(1/2)+
x))-1/36*cos(3*arccos(x))-1/12*arccos(x)*sin(3*arccos(x))
```

3.657.5 Fricas [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx=\int \frac{(-x^2+1)^{\frac{3}{2}}\arccos\left(x\right)}{x}\,dx$$

input
```
integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="fricas")
```

output
```
integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)
```

3.657.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx=\text{Timed out}$$

input
```
integrate((-x**2+1)**(3/2)*acos(x)/x,x)
```

output
```
Timed out
```

3.657.7 Maxima [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{(-x^2+1)^{\frac{3}{2}}\arccos{(x)}}{x}\,dx$$

input
```
integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="maxima")
```

output
```
integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

3.657.8 Giac [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{(-x^2+1)^{\frac{3}{2}}\arccos{(x)}}{x}\,dx$$

input
```
integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="giac")
```

output
```
integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{\mathrm{acos}(x)\,(1-x^2)^{3/2}}{x}\,dx$$

input
```
int((acos(x)*(1 - x^2)^(3/2))/x,x)
```

output
```
int((acos(x)*(1 - x^2)^(3/2))/x, x)
```

3.658 $\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx$

3.658.1 Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{\log(x)}{5}$$

output
```
-1/20/x^4+1/5/x^2-1/5*(-x^2+1)^(5/2)*arcsin(x)/x^5+1/5*ln(x)
```

3.658.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{x-4x^3+4(1-x^2)^{5/2}\arcsin(x)-4x^5\log(x)}{20x^5}$$

input
```
Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

output
```
-1/20*(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/x^5
```

3.658.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5186, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx$$

$$\Big\downarrow \text{ 5186}$$

$$\frac{1}{5}\int \frac{(1-x^2)^2}{x^5}dx-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{10}\int \frac{(1-x^2)^2}{x^6}dx^2-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{10}\int \left(\frac{1}{x^2}-\frac{2}{x^4}+\frac{1}{x^6}\right)dx^2-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{10}\left(-\frac{1}{2x^4}+\frac{2}{x^2}+\log\left(x^2\right)\right)-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

input
```
Int[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

output
```
-1/5*((1 - x^2)^(5/2)*ArcSin[x])/x^5 + (-1/2*1/x^4 + 2/x^2 + Log[x^2])/10
```

3.658.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5186
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

3.658.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.90

method	result
default	$-\frac{2i\arcsin(x)}{5}+\frac{\left(-x^4\sqrt{-x^2+1}+ix^5+2x^2\sqrt{-x^2+1}-\sqrt{-x^2+1}\right)\left(20\arcsin(x)x^8-4ix^8-4\sqrt{-x^2+1}\,x^7-40\arcsin(x)x^6+ix^6+9\sqrt{-\right.}{20(5x^8-10x^6+10x^4-5x^2+1)x^5}$

input
```
int((-x^2+1)^(3/2)*arcsin(x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*arcsin(x)+1/20*(-x^4*(-x^2+1)^(1/2)+I*x^5+2*x^2*(-x^2+1)^(1/2)-(-x^
2+1)^(1/2))*(20*arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^(1/2)*x^7-40*arcsin(x)*x^
6+I*x^6+9*(-x^2+1)^(1/2)*x^5+40*arcsin(x)*x^4-6*(-x^2+1)^(1/2)*x^3-20*x^2*
arcsin(x)+x*(-x^2+1)^(1/2)+4*arcsin(x))/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+
1/5*ln((I*x+(-x^2+1)^(1/2))^2-1)
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \frac{4\,x^5\log(x)+4\,x^3-4\,(x^4-2\,x^2+1)\sqrt{-x^2+1}\arcsin(x)-x}{20\,x^5}$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")
```

output
```
1/20*(4*x^5*log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x)
- x)/x^5
```

3.658.6 Sympy [F]

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \int \frac{(-(x-1)\,(x+1))^{\frac{3}{2}}\operatorname{asin}(x)}{x^6}\,dx$$

input
```
integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

output
```
Integral((-(x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)
```

3.658.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}\arcsin(x)}{5\,x^5}+\frac{4\,x^2-1}{20\,x^4}+\frac{1}{10}\log(x^2)$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(-x^2 + 1)^(5/2)*arcsin(x)/x^5 + 1/20*(4*x^2 - 1)/x^4 + 1/10*log(x^2)
```

3.658.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(31) = 62$.

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.29

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{1}{160}\left(\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-\frac{10\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{\left(\sqrt{-x^2+1}-1\right)^5}+\frac{10\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{5\left(\sqrt{-x^2+1}-1\right)^3}{x^3}+\frac{\left(\sqrt{-x^2+\right.}{x^5}\right.$$
$$-\frac{3\,x^4-4\,x^2+1}{20\,x^4}+\frac{1}{10}\log\left(x^2\right)$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="giac")
```

output
```
-1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) - 1)^4/x^4
- 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x - 5*(sqrt(-x^2 + 1
) - 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcsin(x) - 1/20*(3*x^4 - 4*x^2
 + 1)/x^4 + 1/10*log(x^2)
```

3.658.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \int \frac{\mathrm{asin}(x)\,\left(1-x^2\right)^{3/2}}{x^6}\,dx$$

input
```
int((asin(x)*(1 - x^2)^(3/2))/x^6,x)
```

output
```
int((asin(x)*(1 - x^2)^(3/2))/x^6, x)
```

3.659 $\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx$

3.659.1 Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)^2}{4}$$

output
```
1/4*x^2+1/4*arcsin(x)^2-1/2*x*arcsin(x)*(-x^2+1)^(1/2)
```

3.659.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{4}\left(x^2 - 2x\sqrt{1-x^2}\arcsin(x) + \arcsin(x)^2\right)$$

input
```
Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
(x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

3.659.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \text{5210}$$
$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)$$
$$\Big\downarrow \text{15}$$
$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{x^2}{4}$$
$$\Big\downarrow \text{5152}$$
$$-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}$$

input
```
Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

3.659.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

3.659.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\arcsin(x)\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}$	32

input
```
int(x^2*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)*(-x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2+1/4*x^2
```

3.659.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2$$

input
```
integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2
```

3.659.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{x^2}{4} - \frac{x\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{2} + \frac{\mathrm{asin}^2\,(x)}{4}$$

input
```
integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
```

output
```
x**2/4 - x*sqrt(1 - x**2)*asin(x)/2 + asin(x)**2/4
```

3.659.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{4}\,x^2 - \frac{1}{2}\left(\sqrt{-x^2+1}x - \arcsin\,(x)\right)\arcsin\,(x) - \frac{1}{4}\,\arcsin\,(x)^2$$

input
```
integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

3.659.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\,\sqrt{-x^2+1}x\arcsin\,(x) + \frac{1}{4}\,x^2 + \frac{1}{4}\,\arcsin\,(x)^2 - \frac{1}{8}$$

input
```
integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8
```

3.659.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \operatorname{asin}(x)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^2*asin(x))/(1 - x^2)^(1/2),x)
```

output
```
int((x^2*asin(x))/(1 - x^2)^(1/2), x)
```

3.660 $\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} dx$

3.660.1 Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} dx = \frac{3x^2}{16} + \frac{x^4}{16} - \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) - \frac{1}{4}x^3\sqrt{1-x^2}\arcsin(x) + \frac{3\arcsin(x)^2}{16}$$

output
```
3/16*x^2+1/16*x^4+3/16*arcsin(x)^2-3/8*x*arcsin(x)*(-x^2+1)^(1/2)-1/4*x^3*
arcsin(x)*(-x^2+1)^(1/2)
```

3.660.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} dx = \frac{1}{16}\left(x^2(3+x^2) - 2x\sqrt{1-x^2}(3+2x^2)\arcsin(x) + 3\arcsin(x)^2\right)$$

input
```
Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
(x^2*(3 + x^2) - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/
16
```

3.660.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5210}$$

$$\frac{3}{4}\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}dx+\frac{\int x^3dx}{4}-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}dx-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

$$\Big\downarrow \text{5210}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)\right)-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{x^2}{4}\right)-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

$$\Big\downarrow \text{5152}$$

$$\frac{3}{4}\left(-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}\right)-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

input
```
Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
x^4/16 - (x^3*Sqrt[1 - x^2]*ArcSin[x])/4 + (3*(x^2/4 - (x*Sqrt[1 - x^2]*Ar
cSin[x])/2 + ArcSin[x]^2/4))/4
```

3.660.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

3.660.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3-3x\sqrt{-x^2+1}+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{(2x^2+3)^2}{64}$	54

input
```
int(x^4*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3-3*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*
arcsin(x)^2+1/64*(2*x^2+3)^2
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \frac{1}{16}\, x^4 - \frac{1}{8}\left(2\, x^3 + 3\, x\right)\sqrt{-x^2+1} \arcsin\left(x\right) + \frac{3}{16}\, x^2 + \frac{3}{16}\, \arcsin\left(x\right)^2$$

input
```
integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*x^4 - 1/8*(2*x^3 + 3*x)*sqrt(-x^2 + 1)*arcsin(x) + 3/16*x^2 + 3/16*ar
csin(x)^2
```

3.660.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{8} + \frac{3\operatorname{asin}^2\left(x\right)}{16}$$

input
```
integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

output
```
x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 + 3*x**2/16 - 3*x*sqrt(1 - x**2)*a
sin(x)/8 + 3*asin(x)**2/16
```

3.660.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\, dx = {} & \frac{1}{16}\, x^4 + \frac{3}{16}\, x^2 \\ & - \frac{1}{8}\left(2\sqrt{-x^2+1}x^3 + 3\sqrt{-x^2+1}x - 3\, \arcsin\left(x\right)\right)\arcsin\left(x\right) \\ & - \frac{3}{16}\, \arcsin\left(x\right)^2\end{aligned}$$

input
```
integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/16*x^4 + 3/16*x^2 - 1/8*(2*sqrt(-x^2 + 1)*x^3 + 3*sqrt(-x^2 + 1)*x - 3*a
rcsin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

3.660.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}} x \arcsin(x) - \frac{5}{8}\sqrt{-x^2+1}x \arcsin(x) + \frac{1}{16}\left(x^2-1\right)^2 + \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2 - \frac{23}{128}$$

input
```
integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) - 5/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(
x^2 - 1)^2 + 5/16*x^2 + 3/16*arcsin(x)^2 - 23/128
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \int \frac{x^4 \operatorname{asin}(x)}{\sqrt{1-x^2}}\, dx$$

input
```
int((x^4*asin(x))/(1 - x^2)^(1/2),x)
```

output
```
int((x^4*asin(x))/(1 - x^2)^(1/2), x)
```

3.661 $\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} dx$

3.661.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

output
```
-arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)
```

3.661.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

input
```
Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

3.661.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5182, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{5182}$$

$$\frac{\arcsin(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2}dx$$

$$\Big\downarrow \text{219}$$

$$\frac{\arcsin(x)}{\sqrt{1-x^2}} - \text{arctanh}(x)$$

input
```
Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

3.661.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.661.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

method	result	size
default	$-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+\frac{x}{\sqrt{-x^2+1}}\right)$	46

input
```
int(x*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+1/(-x^2+1)^(1/2)*x)
```

3.661.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)+2\sqrt{-x^2+1}\arcsin(x)}{2(x^2-1)}$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 + 1)*arcsi
n(x))/(x^2 - 1)
```

3.661.6 Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\log(x-1)}{2}-\frac{\log(x+1)}{2}+\frac{\operatorname{asin}(x)}{\sqrt{1-x^2}}$$

input
```
integrate(x*asin(x)/(-x**2+1)**(3/2),x)
```

output
```
log(x - 1)/2 - log(x + 1)/2 + asin(x)/sqrt(1 - x**2)
```

3.661.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

3.661.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{asin}(x)}{(1-x^2)^{3/2}}\,dx$$

input
```
int((x*asin(x))/(1 - x^2)^(3/2),x)
```

output
```
int((x*asin(x))/(1 - x^2)^(3/2), x)
```

3.662 $\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx$

3.662.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arccos(x)}{\sqrt{1-x^2}} + \operatorname{arctanh}(x)$$

output
```
arctanh(x)+arccos(x)/(-x^2+1)^(1/2)
```

3.662.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx = \frac{1}{2} \left(\frac{2 \arccos(x)}{\sqrt{1-x^2}} - \log(1-x) + \log(1+x) \right)$$

input
```
Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

output
```
((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2
```

3.662.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5183, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{5183}$$

$$\int \frac{1}{1-x^2}dx+\frac{\arccos(x)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arccos(x)}{\sqrt{1-x^2}}+\text{arctanh}(x)$$

input
```
Int[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

output
```
ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

3.662.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.662.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

method	result	size
default	$-\frac{\sqrt{-x^2+1}\,\arccos(x)}{x^2-1}-\ln\left(-\frac{x}{\sqrt{-x^2+1}}+\frac{1}{\sqrt{-x^2+1}}\right)$	47

input
```
int(x*arccos(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(-1/(-x^2+1)^(1/2)*x+1/(-x^2+1)^(1/2))
```

3.662.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\sqrt{-x^2+1}\arccos(x)}{2(x^2-1)}$$

input
```
integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 + 1)*arccos
(x))/(x^2 - 1)
```

3.662.6 Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = -\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}+\frac{\operatorname{acos}(x)}{\sqrt{1-x^2}}$$

input
```
integrate(x*acos(x)/(-x**2+1)**(3/2),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 + acos(x)/sqrt(1 - x**2)
```

3.662.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arccos{(x)}}{\sqrt{-x^2+1}} + \frac{1}{2}\log{(x+1)} - \frac{1}{2}\log{(x-1)}$$

input
```
integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.662.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arccos{(x)}}{\sqrt{-x^2+1}} + \frac{1}{2}\log{(|x+1|)} - \frac{1}{2}\log{(|x-1|)}$$

input
```
integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
arccos(x)/sqrt(-x^2 + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{acos}(x)}{(1-x^2)^{3/2}}\,dx$$

input
```
int((x*acos(x))/(1 - x^2)^(3/2),x)
```

output
```
int((x*acos(x))/(1 - x^2)^(3/2), x)
```

3.663 $\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx$

3.663.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{1}{6(1-x^2)}+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}+\frac{2x\arcsin(x)}{3\sqrt{1-x^2}}+\frac{1}{3}\log\left(1-x^2\right)$$

output
```
-1/6/(-x^2+1)+1/3*x*arcsin(x)/(-x^2+1)^(3/2)+1/3*ln(-x^2+1)+2/3*x*arcsin(x
)/(-x^2+1)^(1/2)
```

3.663.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{1}{6}\left(\frac{1}{-1+x^2}-\frac{2x(-3+2x^2)\arcsin(x)}{(1-x^2)^{3/2}}+2\log\left(1-x^2\right)\right)$$

input
```
Integrate[ArcSin[x]/(1 - x^2)^(5/2),x]
```

output
```
((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) + 2*Log[1
- x^2])/6
```

3.663.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5162, 241, 5160, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5162}$$

$$\frac{2}{3}\int \frac{\arcsin(x)}{(1-x^2)^{3/2}}dx-\frac{1}{3}\int \frac{x}{(1-x^2)^2}dx+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2}{3}\int \frac{\arcsin(x)}{(1-x^2)^{3/2}}dx+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}-\frac{1}{6(1-x^2)}$$

$$\Big\downarrow \text{5160}$$

$$\frac{2}{3}\left(\frac{x\arcsin(x)}{\sqrt{1-x^2}}-\int \frac{x}{1-x^2}dx\right)+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}-\frac{1}{6(1-x^2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\arcsin(x)}{\sqrt{1-x^2}}+\frac{1}{2}\log\left(1-x^2\right)\right)-\frac{1}{6(1-x^2)}$$

input
```
Int[ArcSin[x]/(1 - x^2)^(5/2),x]
```

output
```
-1/6*1/(1 - x^2) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*((x*ArcSin[x])/S
qrt[1 - x^2] + Log[1 - x^2]/2))/3
```

3.663.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5160
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSin[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5162
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

3.663.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{1}{6x^2-6}+\frac{\sqrt{-x^2+1}\,\arcsin(x)x}{3(x^2-1)^2}+\frac{\ln(-x^2+1)}{3}-\frac{2\sqrt{-x^2+1}\,\arcsin(x)x}{3(x^2-1)}$	63

input
```
int(arcsin(x)/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(x^2-1)+1/3*(-x^2+1)^(1/2)/(x^2-1)^2*arcsin(x)*x+1/3*ln(-x^2+1)-2/3*(-
x^2+1)^(1/2)/(x^2-1)*arcsin(x)*x
```

3.663.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{2\,(2\,x^3-3\,x)\sqrt{-x^2+1}\arcsin(x)-x^2-2\,(x^4-2\,x^2+1)\log(x^2-1)+1}{6\,(x^4-2\,x^2+1)}$$

input
```
integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(2*(2*x^3 - 3*x)*sqrt(-x^2 + 1)*arcsin(x) - x^2 - 2*(x^4 - 2*x^2 + 1)
*log(x^2 - 1) + 1)/(x^4 - 2*x^2 + 1)
```

3.663.6 Sympy [A] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \left(\begin{cases} \frac{x^3}{3(1-x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1-x^2}} & \text{for}\ x>-1\wedge x<1 \end{cases}\right)\operatorname{asin}(x) - \begin{cases} \text{NaN} & \text{for}\ x<-1 \\ -\frac{2x^2\log(1-x^2)}{6x^2-6}-\frac{x^2}{6x^2-6}+\frac{2\log(1-x^2)}{6x^2-6} & \text{for}\ x<1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate(asin(x)/(-x**2+1)**(5/2),x)
```

output
```
Piecewise((x**3/(3*(1 - x**2)**(3/2)) + x/sqrt(1 - x**2), (x > -1) & (x <
1)))*asin(x) - Piecewise((nan, x < -1), (-2*x**2*log(1 - x**2)/(6*x**2 - 6
) - x**2/(6*x**2 - 6) + 2*log(1 - x**2)/(6*x**2 - 6), x < 1), (nan, True))
```

3.663.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{-x^2+1}}+\frac{x}{(-x^2+1)^{\frac{3}{2}}}\right)\arcsin(x)+\frac{1}{6\,(x^2-1)}+\frac{1}{3}\log(-3\,x^2+3)$$

input
```
integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/sqrt(-x^2 + 1) + x/(-x^2 + 1)^(3/2))*arcsin(x) + 1/6/(x^2 - 1) +
1/3*log(-3*x^2 + 3)
```

3.663.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{(2\,x^2-3)\sqrt{-x^2+1}x\arcsin{(x)}}{3\,(x^2-1)^2} - \frac{2\,x^2-3}{6\,(x^2-1)} + \frac{1}{3}\log{(|x^2-1|)}$$

input
```
integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*x^2 - 3)/
(x^2 - 1) + 1/3*log(abs(x^2 - 1))
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \int \frac{\mathrm{asin}(x)}{(1-x^2)^{5/2}}\,dx$$

input
```
int(asin(x)/(1 - x^2)^(5/2),x)
```

output
```
int(asin(x)/(1 - x^2)^(5/2), x)
```

3.664 $\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} dx$

3.664.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} dx = -x + \frac{\arcsin(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \arcsin(x) - \operatorname{arctanh}(x)$$

output
```
-x-arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)+arcsin(x)*(-x^2+1)^(1/2)
```

3.664.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} dx = \frac{1}{2}\left(-2x - \frac{2(-2+x^2)\arcsin(x)}{\sqrt{1-x^2}} + \log(1-x) - \log(1+x)\right)$$

input
```
Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
(-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/
2
```

3.664.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5194, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 5194$$

$$-\int \frac{2-x^2}{1-x^2}dx + \sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \; 299$$

$$-\int \frac{1}{1-x^2}dx + \sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)}{\sqrt{1-x^2}} - x$$

$$\Big\downarrow \; 219$$

$$\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)}{\sqrt{1-x^2}} - \text{arctanh}(x) - x$$

input
```
Int[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
-x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[x]
```

3.664.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5194
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

3.664.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

method	result
default	$\frac{(\arcsin(x)+i)\left(ix+\sqrt{-x^2+1}\right)}{2}-\frac{\left(-\sqrt{-x^2+1}+ix\right)(\arcsin(x)-i)}{2}-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\ln\left(ix+\sqrt{-x^2+1}+i\right)+\mathrm{l}$

input
```
int(x^3*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arcsin(x)+I)*(I*x+(-x^2+1)^(1/2))-1/2*(-(-x^2+1)^(1/2)+I*x)*(arcsin(x
)-I)-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(I*x+(-x^2+1)^(1/2)+I)+ln(I*x+(-x^
2+1)^(1/2)-I)
```

3.664.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{x^3\arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\frac{2\,x^3-2\,(x^2-2)\sqrt{-x^2+1}\arcsin\left(x\right)+(x^2-1)\log\left(x+1\right)-(x^2-1)\log\left(x-1\right)-2\,x}{2\,(x^2-1)}$$

input
```
integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*x^3 - 2*(x^2 - 2)*sqrt(-x^2 + 1)*arcsin(x) + (x^2 - 1)*log(x + 1)
- (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

3.664.6 Sympy [A] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -x - \left(-\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right)\operatorname{asin}(x) + \frac{\log(x-1)}{2} - \frac{\log(x+1)}{2}$$

input
```
integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
```

output
```
-x - (-sqrt(1 - x**2) - 1/sqrt(1 - x**2))*asin(x) + log(x - 1)/2 - log(x +
 1)/2
```

3.664.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\left(\frac{x^2}{\sqrt{-x^2+1}} - \frac{2}{\sqrt{-x^2+1}}\right)\arcsin(x) - x - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

input
```
integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-(x^2/sqrt(-x^2 + 1) - 2/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(x + 1) +
1/2*log(x - 1)
```

3.664.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \left(\sqrt{-x^2+1} + \frac{1}{\sqrt{-x^2+1}}\right)\arcsin(x) - x - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(-x^2 + 1) + 1/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(abs(x + 1)) +
1/2*log(abs(x - 1))
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{asin}(x)}{(1-x^2)^{3/2}}\,dx$$

input
```
int((x^3*asin(x))/(1 - x^2)^(3/2),x)
```

output
```
int((x^3*asin(x))/(1 - x^2)^(3/2), x)
```

3.665 $\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx$

3.665.1 Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - 2\arcsin(x)\operatorname{arctanh}\left(e^{i\arcsin(x)}\right) - \operatorname{arctanh}(x) + i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)$$

output
```
-2*arcsin(x)*arctanh(I*x+(-x^2+1)^(1/2))-arctanh(x)+I*polylog(2,-I*x-(-x^2
+1)^(1/2))-I*polylog(2,I*x+(-x^2+1)^(1/2))+arcsin(x)/(-x^2+1)^(1/2)
```

3.665.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} + \arcsin(x)\log\left(1-e^{i\arcsin(x)}\right) - \arcsin(x)\log\left(1+e^{i\arcsin(x)}\right) + \log\left(\cos\left(\frac{\arcsin(x)}{2}\right)-\sin\left(\frac{\arcsin(x)}{2}\right)\right) - \log\left(\cos\left(\frac{\arcsin(x)}{2}\right)+\sin\left(\frac{\arcsin(x)}{2}\right)\right) + i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)$$

input
```
Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - ArcSin[x]*L
og[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[ArcSin[x]/2]] - Log[C
os[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*P
olyLog[2, E^(I*ArcSin[x])]
```

3.665.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5208, 219, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5208}$$

$$\int \frac{\arcsin(x)}{x\sqrt{1-x^2}}dx-\int \frac{1}{1-x^2}dx+\frac{\arcsin(x)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{\arcsin(x)}{x\sqrt{1-x^2}}dx+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{ 5218}$$

$$\int \frac{\arcsin(x)}{x}d\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \arcsin(x)\csc(\arcsin(x))d\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{ 4671}$$

$$-\int \log\left(1-e^{i\arcsin(x)}\right)d\arcsin(x)+\int \log\left(1+e^{i\arcsin(x)}\right)d\arcsin(x)-2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{ 2715}$$

$$i\int e^{-i\arcsin(x)}\log\left(1-e^{i\arcsin(x)}\right)de^{i\arcsin(x)}-i\int e^{-i\arcsin(x)}\log\left(1+e^{i\arcsin(x)}\right)de^{i\arcsin(x)}-2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \ 2838$$

$$-2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right)+i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right)-i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

input
```
Int[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] - ArcTanh[x
] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

3.665.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.665. $\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx$

rule 5208
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218
```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]]   Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.665.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(76) = 152$.

Time = 0.69 (sec) , antiderivative size = 430, normalized size of antiderivative = 6.94

method	result
default	$-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\frac{i\left(\ln\left(ix+\sqrt{-x^2+1}+1\right)-\ln\left(ix+\sqrt{-x^2+1}-1\right)-2\arctan\left(ix+\sqrt{-x^2+1}\right)\right)}{2}+\frac{i\left(i\arcsin(x)\ln\left(ix+\sqrt{-x^2\right.\right.}{}$

input
```
int(arcsin(x)/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-1/2*I*(ln(I*x+(-x^2+1)^(1/2)+1)-ln(I*x+(
-x^2+1)^(1/2)-1)-2*arctan(I*x+(-x^2+1)^(1/2)))+1/2*I*(I*arcsin(x)*ln(I*x+(
-x^2+1)^(1/2)+1)+arcsin(x)*ln(1+I*(I*x+(-x^2+1)^(1/2)))-arcsin(x)*ln(1-I*(
I*x+(-x^2+1)^(1/2)))-I*dilog(1+I*(I*x+(-x^2+1)^(1/2)))+I*dilog(1-I*(I*x+(-
x^2+1)^(1/2)))+dilog(I*x+(-x^2+1)^(1/2)+1)+dilog(I*x+(-x^2+1)^(1/2)))+1/2*
I*(I*arcsin(x)*ln(I*x+(-x^2+1)^(1/2)+1)-arcsin(x)*ln(1+I*(I*x+(-x^2+1)^(1/
2)))+arcsin(x)*ln(1-I*(I*x+(-x^2+1)^(1/2)))+I*dilog(1+I*(I*x+(-x^2+1)^(1/2
)))-I*dilog(1-I*(I*x+(-x^2+1)^(1/2)))+dilog(I*x+(-x^2+1)^(1/2)+1)+dilog(I*
x+(-x^2+1)^(1/2)))+1/2*I*(ln(I*x+(-x^2+1)^(1/2)+1)-ln(I*x+(-x^2+1)^(1/2)-1
)+2*arctan(I*x+(-x^2+1)^(1/2)))
```

3.665.5 Fricas [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx = \int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^2 + 1)*arcsin(x)/(x^5 - 2*x^3 + x), x)
```

3.665.6 Sympy [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx = \int \frac{\operatorname{asin}\left(x\right)}{x\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

output
```
Integral(asin(x)/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

3.665.7 Maxima [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx = \int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

3.665.8 Giac [F]

$$\int \frac{\arcsin(x)}{x\,(1-x^2)^{3/2}}\,dx = \int \frac{\arcsin(x)}{(-x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

3.665.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{x\,(1-x^2)^{3/2}}\,dx = \int \frac{\mathrm{asin}(x)}{x\,(1-x^2)^{3/2}}\,dx$$

input
```
int(asin(x)/(x*(1 - x^2)^(3/2)),x)
```

output
```
int(asin(x)/(x*(1 - x^2)^(3/2)), x)
```

3.666 $\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx$

3.666.1 Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{6x^2}-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{2\sqrt{1-x^2}\arccos(x)}{3x}-\frac{2\log(x)}{3}$$

output
```
1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^
(1/2)/x
```

3.666.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{x-2\sqrt{1-x^2}(1+2x^2)\arccos(x)-4x^3\log(x)}{6x^3}$$

input
```
Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

output
```
(x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3)
```

3.666.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5205, 15, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 5205}$$

$$\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx-\frac{\int \frac{1}{x^3}dx}{3}-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}+\frac{1}{6x^2}$$

$$\Big\downarrow \text{ 5187}$$

$$\frac{2}{3}\left(-\int \frac{1}{x}dx-\frac{\sqrt{1-x^2}\arccos(x)}{x}\right)-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}+\frac{1}{6x^2}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{2}{3}\left(-\frac{\sqrt{1-x^2}\arccos(x)}{x}-\log(x)\right)-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}+\frac{1}{6x^2}$$

input
```
Int[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

output
```
1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) + (2*(-((Sqrt[1 - x^2]*ArcCo
s[x])/x) - Log[x]))/3
```

3.666.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

3.666.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{6x^2}-\frac{2\ln(x)}{3}-\frac{\arccos(x)\sqrt{-x^2+1}}{3x^3}-\frac{2\arccos(x)\sqrt{-x^2+1}}{3x}$	43

input
```
int(arccos(x)/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^
(1/2)/x
```

3.666.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{4\,x^3\log(x) + 2\,(2\,x^2+1)\sqrt{-x^2+1}\arccos(x) - x}{6\,x^3}$$

input
```
integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(4*x^3*log(x) + 2*(2*x^2 + 1)*sqrt(-x^2 + 1)*arccos(x) - x)/x^3
```

3.666.6 Sympy [A] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \left(\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for}\ x > -1 \wedge x < 1 \end{cases}\right)\operatorname{acos}(x) + \begin{cases} \text{NaN} & \text{for}\ x < -1 \\ -\frac{2\log(x)}{3} + \frac{1}{6x^2} & \text{for}\ x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
 1)))*acos(x) + Piecewise((nan, x < -1), (-2*log(x)/3 + 1/(6*x**2), x < 1)
, (nan, True))
```

3.666.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x) + \frac{1}{6\,x^2} - \frac{2}{3}\log(x)$$

input
```
integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) + 1/6/x^2 - 2/3*l
og(x)
```

3.666.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) + \frac{2\,x^2+1}{6\,x^2} - \frac{1}{3}\log\left(x^2\right)$$

input
```
integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(s
qrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) + 1/6*(2*x^2
+ 1)/x^2 - 1/3*log(x^2)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \int \frac{\operatorname{acos}(x)}{x^4\,\sqrt{1-x^2}}\,dx$$

input
```
int(acos(x)/(x^4*(1 - x^2)^(1/2)),x)
```

output
```
int(acos(x)/(x^4*(1 - x^2)^(1/2)), x)
```

3.667 $\int x\sqrt{1-x^2}\arccos(x)^2\,dx$

3.667.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx=\frac{4\sqrt{1-x^2}}{9}+\frac{2}{27}\left(1-x^2\right)^{3/2}-\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

output
```
2/27*(-x^2+1)^(3/2)-2/3*x*arccos(x)+2/9*x^3*arccos(x)-1/3*(-x^2+1)^(3/2)*a
rccos(x)^2+4/9*(-x^2+1)^(1/2)
```

3.667.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx=\frac{1}{27}\left(-2\sqrt{1-x^2}\left(-7+x^2\right)+6x\left(-3+x^2\right)\arccos(x)-9\left(1-x^2\right)^{3/2}\arccos(x)^2\right)$$

input
```
Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

output
```
(-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 - x^2)^(3/2
)*ArcCos[x]^2)/27
```

3.667.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5183, 5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{2}{3}\int (1-x^2)\arccos(x)dx-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{5155}$$

$$-\frac{2}{3}\left(\int \frac{x(3-x^2)}{3\sqrt{1-x^2}}dx-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}\left(\frac{1}{3}\int \frac{x(3-x^2)}{\sqrt{1-x^2}}dx-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{353}$$

$$-\frac{2}{3}\left(\frac{1}{6}\int \frac{3-x^2}{\sqrt{1-x^2}}dx^2-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{53}$$

$$-\frac{2}{3}\left(\frac{1}{6}\int \left(\sqrt{1-x^2}+\frac{2}{\sqrt{1-x^2}}\right)dx^2-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2-\frac{2}{3}\left(-\frac{1}{3}x^3\arccos(x)+x\arccos(x)+\frac{1}{6}\left(-\frac{2}{3}(1-x^2)^{3/2}-4\sqrt{1-x^2}\right)\right)$$

input
```
Int[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

output
```
-1/3*((1 - x^2)^(3/2)*ArcCos[x]^2) - (2*((-4*Sqrt[1 - x^2] - (2*(1 - x^2)^
(3/2))/3)/6 + x*ArcCos[x] - (x^3*ArcCos[x])/3))/3
```

3.667.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.667.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{(x^{2}-1)\sqrt{-x^{2}+1}\,\arccos(x)^{2}}{3}+\frac{2\arccos(x)(x^{2}-3)x}{9}-\frac{2(x^{2}-1)\sqrt{-x^{2}+1}}{27}+\frac{4\sqrt{-x^{2}+1}}{9}$	59

input
```
int(x*arccos(x)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2-1)*(-x^2+1)^(1/2)*arccos(x)^2+2/9*arccos(x)*(x^2-3)*x-2/27*(x^2-1
)*(-x^2+1)^(1/2)+4/9*(-x^2+1)^(1/2)
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2}{9}\left(x^3-3\,x\right)\arccos\left(x\right) + \frac{1}{27}\left(9\left(x^2-1\right)\arccos\left(x\right)^2-2\,x^2+14\right)\sqrt{-x^2+1}$$

input
```
integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(x^3 - 3*x)*arccos(x) + 1/27*(9*(x^2 - 1)*arccos(x)^2 - 2*x^2 + 14)*sq
rt(-x^2 + 1)
```

3.667.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2x^3\operatorname{acos}\left(x\right)}{9}+\frac{x^2\sqrt{1-x^2}\operatorname{acos}^2\left(x\right)}{3}-\frac{2x^2\sqrt{1-x^2}}{27} - \frac{2x\operatorname{acos}\left(x\right)}{3}-\frac{\sqrt{1-x^2}\operatorname{acos}^2\left(x\right)}{3}+\frac{14\sqrt{1-x^2}}{27}$$

input
```
integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
```

output
```
2*x**3*acos(x)/9 + x**2*sqrt(1 - x**2)*acos(x)**2/3 - 2*x**2*sqrt(1 - x**2
)/27 - 2*x*acos(x)/3 - sqrt(1 - x**2)*acos(x)**2/3 + 14*sqrt(1 - x**2)/27
```

3.667.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = -\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 + \frac{2}{9}\left(x^3-3\,x\right)\arccos\left(x\right) + \frac{14}{27}\sqrt{-x^2+1}$$

input
```
integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 + 2/9*(x^3 - 3
*x)*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

3.667.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2}{9}\,x^3\arccos\left(x\right) - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 - \frac{2}{3}\,x\arccos\left(x\right) + \frac{14}{27}\sqrt{-x^2+1}$$

input
```
integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2/9*x^3*arccos(x) - 1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)
*x^2 - 2/3*x*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \int x\,\mathrm{acos}(x)^2\,\sqrt{1-x^2}\,dx$$

input
```
int(x*acos(x)^2*(1 - x^2)^(1/2),x)
```

output
```
int(x*acos(x)^2*(1 - x^2)^(1/2), x)
```

3.668 $\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}} \, dx$

3.668.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}} \, dx = -\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2}\arcsin(x) - \frac{3\arcsin(x)^2}{8} + \frac{3}{4}x^2\arcsin(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{\arcsin(x)^4}{8}$$

output
```
-3/8*x^2-3/8*arcsin(x)^2+3/4*x^2*arcsin(x)^2+1/8*arcsin(x)^4+3/4*x*arcsin(
x)*(-x^2+1)^(1/2)-1/2*x*arcsin(x)^3*(-x^2+1)^(1/2)
```

3.668.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}} \, dx = \frac{1}{8}\left(-3x^2 + 6x\sqrt{1-x^2}\arcsin(x) + (-3+6x^2)\arcsin(x)^2 - 4x\sqrt{1-x^2}\arcsin(x)^3 + \arcsin(x)^4\right)$$

input
```
Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

output
```
(-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2 - 4*x*Sqr
t[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8
```

3.668.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5210, 5138, 5152, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5210}$$

$$\frac{1}{2}\int \frac{\arcsin(x)^3}{\sqrt{1-x^2}}dx+\frac{3}{2}\int x\arcsin(x)^2dx-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3$$

$$\Big\downarrow \text{5138}$$

$$\frac{3}{2}\left(\frac{1}{2}x^2\arcsin(x)^2-\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}dx\right)+\frac{1}{2}\int \frac{\arcsin(x)^3}{\sqrt{1-x^2}}dx-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3$$

$$\Big\downarrow \text{5152}$$

$$\frac{3}{2}\left(\frac{1}{2}x^2\arcsin(x)^2-\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}dx\right)-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{\arcsin(x)^4}{8}$$

$$\Big\downarrow \text{5210}$$

$$\frac{3}{2}\left(-\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{\int xdx}{2}+\frac{1}{2}x^2\arcsin(x)^2+\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)\right)-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{\arcsin(x)^4}{8}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{2}\left(-\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}x^2\arcsin(x)^2+\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)-\frac{x^2}{4}\right)-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{\arcsin(x)^4}{8}$$

$$\Big\downarrow \text{5152}$$

$$-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{3}{2}\left(\frac{1}{2}x^2\arcsin(x)^2+\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)-\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}\right)+\frac{\arcsin(x)^4}{8}$$

input
```
Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

3.668. $\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}dx$

output
```
-1/2*(x*Sqrt[1 - x^2]*ArcSin[x]^3) + ArcSin[x]^4/8 + (3*(-1/4*x^2 + (x*Sqr
t[1 - x^2]*ArcSin[x])/2 - ArcSin[x]^2/4 + (x^2*ArcSin[x]^2)/2))/2
```

3.668.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

3.668.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

method	result
default	$\frac{\arcsin(x)^3\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}+\frac{3\arcsin(x)^2(x^2-1)}{4}+\frac{3\arcsin(x)\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{4}-\frac{3\arcsin(x)^2}{8}-\frac{3x^2}{8}-3$

input
```
int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)^3*(-x*(-x^2+1)^(1/2)+arcsin(x))+3/4*arcsin(x)^2*(x^2-1)+3/4*
arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-3/8*arcsin(x)^2-3/8*x^2-3/8*arcsin(
x)^4
```

3.668.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{1}{8}\arcsin(x)^4 + \frac{3}{8}\left(2x^2-1\right)\arcsin(x)^2 - \frac{3}{8}x^2 - \frac{1}{4}\left(2x\arcsin(x)^3 - 3x\arcsin(x)\right)\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*x*arcsin(
x)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

3.668.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{3x^2\operatorname{asin}^2(x)}{4} - \frac{3x^2}{8} - \frac{x\sqrt{1-x^2}\operatorname{asin}^3(x)}{2} + \frac{3x\sqrt{1-x^2}\operatorname{asin}(x)}{4} + \frac{\operatorname{asin}^4(x)}{8} - \frac{3\operatorname{asin}^2(x)}{8}$$

input
```
integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

output
```
3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(1 - x**2)*asin(x)**3/2 + 3*x*sqrt(
1 - x**2)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

3.668.7 Maxima [F]

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \arcsin\left(x\right)^3}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1), x)
```

3.668.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x\arcsin\left(x\right)^3 + \frac{1}{8}\arcsin\left(x\right)^4 + \frac{3}{4}\left(x^2-1\right)\arcsin\left(x\right)^2 + \frac{3}{4}\sqrt{-x^2+1}x\arcsin\left(x\right) - \frac{3}{8}x^2 + \frac{3}{8}\arcsin\left(x\right)^2 + \frac{3}{16}$$

input
```
integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 - 1)*arcsin
(x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*arcsin(x)^2 + 3/16
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \operatorname{asin}(x)^3}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^2*asin(x)^3)/(1 - x^2)^(1/2),x)
```

output
```
int((x^2*asin(x)^3)/(1 - x^2)^(1/2), x)
```

3.669 $\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx$

3.669.1 Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x}{4(1+x^2)} + \frac{\arctan(x)}{4} - \frac{\arctan(x)}{2(1+x^2)}$$

output
```
1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)
```

3.669.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x + (-1+x^2)\arctan(x)}{4(1+x^2)}$$

input
```
Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))
```

3.669.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(x)}{(x^2+1)^2} \, dx$$

$$\Big\downarrow \; 5465$$

$$\frac{1}{2} \int \frac{1}{(x^2+1)^2} dx - \frac{\arctan(x)}{2(x^2+1)}$$

$$\Big\downarrow \; 215$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{x}{2(x^2+1)} \right) - \frac{\arctan(x)}{2(x^2+1)}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2} \left(\frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)} \right) - \frac{\arctan(x)}{2(x^2+1)}$$

input
```
Int[(x*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(x/(2*(1 + x^2)) + ArcTan[x]/2)/2 - ArcTan[x]/(2*(1 + x^2))
```

3.669.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.669.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{x^2 \arctan(x)+x-\arctan(x)}{4x^2+4}$	22
default	$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$	27
parts	$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$	27
risch	$\frac{i\ln(ix+1)}{4x^2+4}-\frac{i(2\ln(-ix+1)-\ln(x+i)x^2-\ln(x+i)+x^2\ln(x-i)+\ln(x-i)+2ix)}{8(x-i)(x+i)}$	79

input
```
int(x*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*arctan(x)+x-arctan(x))/(x^2+1)
```

3.669.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{(x^2-1)\arctan(x)+x}{4(x^2+1)}$$

input
```
integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)
```

3.669.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{x^2\,\mathrm{atan}\,(x)}{4x^2+4} + \frac{x}{4x^2+4} - \frac{\mathrm{atan}\,(x)}{4x^2+4}$$

input
```
integrate(x*atan(x)/(x**2+1)**2,x)
```

output
```
x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)
```

3.669.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{x}{4\,(x^2+1)} - \frac{\arctan\,(x)}{2\,(x^2+1)} + \frac{1}{4}\arctan\,(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

3.669.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{x}{4\,(x^2+1)} - \frac{\arctan\,(x)}{2\,(x^2+1)} + \frac{1}{4}\arctan\,(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

3.669.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x\arctan(x)}{(1+x^2)^2}\,dx = \frac{\operatorname{atan}(x)}{4} + \frac{\frac{x}{4} - \frac{\operatorname{atan}(x)}{2}}{x^2+1}$$

input
```
int((x*atan(x))/(x^2 + 1)^2,x)
```

output
```
atan(x)/4 + (x/4 - atan(x)/2)/(x^2 + 1)
```

3.670 $\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx$

3.670.1 Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} + \frac{3\arctan(x)}{32} - \frac{\arctan(x)}{4(1+x^2)^2}$$

output
```
1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

3.670.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{x(5+3x^2) + (-5+6x^2+3x^4)\arctan(x)}{32(1+x^2)^2}$$

input
```
Integrate[(x*ArcTan[x])/(1 + x^2)^3,x]
```

output
```
(x*(5 + 3*x^2) + (-5 + 6*x^2 + 3*x^4)*ArcTan[x])/(32*(1 + x^2)^2)
```

3.670.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5465, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(x)}{(x^2+1)^3} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{1}{4}\int \frac{1}{(x^2+1)^3}dx - \frac{\arctan(x)}{4(x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{1}{(x^2+1)^2}dx + \frac{x}{4(x^2+1)^2}\right) - \frac{\arctan(x)}{4(x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{x}{2(x^2+1)}\right) + \frac{x}{4(x^2+1)^2}\right) - \frac{\arctan(x)}{4(x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)}\right) + \frac{x}{4(x^2+1)^2}\right) - \frac{\arctan(x)}{4(x^2+1)^2}$$

input
```
Int[(x*ArcTan[x])/(1 + x^2)^3,x]
```

output
```
(x/(4*(1 + x^2)^2) + (3*(x/(2*(1 + x^2)) + ArcTan[x]/2))/4)/4 - ArcTan[x]/
(4*(1 + x^2)^2)
```

3.670.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.670.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	si
default	$\frac{x}{16(x^2+1)^2}+\frac{3x}{32(x^2+1)}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4(x^2+1)^2}$	37
parallelrisch	$\frac{3\arctan(x)x^4+3x^3+6x^2\arctan(x)+5x-5\arctan(x)}{32(x^2+1)^2}$	37
parts	$\frac{x}{16(x^2+1)^2}+\frac{3x}{32(x^2+1)}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4(x^2+1)^2}$	37
risch	$\frac{i\ln(ix+1)}{8(x^2+1)^2}-\frac{i(8\ln(-ix+1)+3\ln(x-i)x^4+6x^2\ln(x-i)+3\ln(x-i)-3\ln(x+i)x^4-6\ln(x+i)x^2-3\ln(x+i)+6ix^3+10ix)}{64(x+i)^2(x-i)^2}$	10

input
```
int(x*arctan(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

3.670.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3\,x^3+(3\,x^4+6\,x^2-5)\arctan(x)+5\,x}{32\,(x^4+2\,x^2+1)}$$

input
```
integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

3.670.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(37) = 74.

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3x^4\operatorname{atan}(x)}{32x^4+64x^2+32}+\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\operatorname{atan}(x)}{32x^4+64x^2+32} + \frac{5x}{32x^4+64x^2+32}-\frac{5\operatorname{atan}(x)}{32x^4+64x^2+32}$$

input
```
integrate(x*atan(x)/(x**2+1)**3,x)
```

output
```
3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x**2 + 32)
+ 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4 + 64*x**2 + 32) -
 5*atan(x)/(32*x**4 + 64*x**2 + 32)
```

3.670.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3\,x^3+5\,x}{32\,(x^4+2\,x^2+1)}-\frac{\arctan(x)}{4\,(x^2+1)^2}+\frac{3}{32}\arctan(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*ar
ctan(x)
```

3.670.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{3\,x^3 + 5\,x}{32\,(x^2+1)^2} - \frac{\arctan(x)}{4\,(x^2+1)^2} + \frac{3}{32}\arctan(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="giac")
```

output
```
1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x
)
```

3.670.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x \arctan(x)}{(1+x^2)^3} \, dx = \frac{3\,\mathrm{atan}(x)}{32} + \frac{\frac{5\,x}{32} - \frac{\mathrm{atan}(x)}{4} + \frac{3\,x^3}{32}}{(x^2+1)^2}$$

input
```
int((x*atan(x))/(x^2 + 1)^3,x)
```

output
```
(3*atan(x))/32 + ((5*x)/32 - atan(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

3.671 $\int \frac{x^2 \arctan(x)}{1+x^2} \, dx$

3.671.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

output
```
x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

3.671.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

3.671.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\int \arctan(x) dx - \int \frac{\arctan(x)}{x^2+1} dx$$

$$\Big\downarrow \text{5345}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx - \int \frac{x}{x^2+1} dx + x \arctan(x)$$

$$\Big\downarrow \text{240}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + x \arctan(x) - \frac{1}{2} \log \left(x^2+1\right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{1}{2} \arctan(x)^2 + x \arctan(x) - \frac{1}{2} \log \left(x^2+1\right)$$

input
```
Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

3.671.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.671.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$x \arctan \left(x \right)-\frac{\arctan \left(x \right)^{2}}{2}-\frac{\ln \left(x^{2}+1\right)}{2}$	20
parallelrisch	$x \arctan \left(x \right)-\frac{\arctan \left(x \right)^{2}}{2}-\frac{\ln \left(x^{2}+1\right)}{2}$	20
parts	$x \arctan \left(x \right)-\frac{\arctan \left(x \right)^{2}}{2}-\frac{\ln \left(x^{2}+1\right)}{2}$	20
risch	$\frac{\ln \left(i x +1\right)^{2}}{8}+\frac{i \left(-x +\frac{i \ln \left(-i x +1\right)}{2}\right) \ln \left(i x +1\right)}{2}+\frac{\ln \left(-i x +1\right)^{2}}{8}+\frac{i x \ln \left(-i x +1\right)}{2}-\frac{\ln \left(x^{2}+1\right)}{2}$	67

input
```
int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

3.671.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan \left(x\right) - \frac{1}{2} \, \arctan \left(x\right)^{2} - \frac{1}{2} \, \log \left(x^{2} + 1\right)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.671.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

input
```
integrate(x**2*atan(x)/(x**2+1),x)
```

output
```
x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

3.671.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = (x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
```

output
```
(x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.671.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.671.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2}\, dx = -\frac{\operatorname{atan}(x)^2}{2} + x \operatorname{atan}(x) - \frac{\ln(x^2+1)}{2}$$

input
```
int((x^2*atan(x))/(x^2 + 1),x)
```

output
```
x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

3.672 $\int \frac{x^3 \arctan(x)}{1+x^2} \, dx$

3.672.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = -\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2 +\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)$$

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)+1/2*I*arctan(x)^2+arctan(x)*ln(2/(1
+I*x))+1/2*I*polylog(2,1-2/(1+I*x))
```

3.672.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \frac{1}{2}\left(-x+i\arctan(x)^2+\arctan(x)\left(1+x^2+2\log\left(-\frac{2i}{-i+x}\right)\right) +i\,\text{PolyLog}\left(2,\frac{i+x}{-i+x}\right)\right)$$

input
```
Integrate[(x^3*ArcTan[x])/(1 + x^2),x]
```

output
```
(-x + I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[(-2*I)/(-I + x)]) + I*Pol
yLog[2, (I + x)/(-I + x)])/2
```

3.672.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\int x \arctan(x) dx - \int \frac{x \arctan(x)}{x^2+1} dx$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx - \frac{1}{2} \int \frac{x^2}{x^2+1} dx + \frac{1}{2} x^2 \arctan(x)$$

$$\Big\downarrow \text{262}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx + \frac{1}{2} \left(\int \frac{1}{x^2+1} dx - x \right) + \frac{1}{2} x^2 \arctan(x)$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx + \frac{1}{2} x^2 \arctan(x) + \frac{1}{2} (\arctan(x) - x)$$

$$\Big\downarrow \text{5455}$$

$$\int \frac{\arctan(x)}{i-x} dx + \frac{1}{2} x^2 \arctan(x) + \frac{1}{2} i \arctan(x)^2 + \frac{1}{2} (\arctan(x) - x)$$

$$\Big\downarrow \text{5379}$$

$$-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1} dx + \frac{1}{2} x^2 \arctan(x) + \frac{1}{2} i \arctan(x)^2 + \frac{1}{2} (\arctan(x) - x) + \arctan(x) \log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2849}$$

$$i \int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}} d\frac{1}{ix+1} + \frac{1}{2} x^2 \arctan(x) + \frac{1}{2} i \arctan(x)^2 + \frac{1}{2} (\arctan(x) - x) + \arctan(x) \log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

input
```
Int[(x^3*ArcTan[x])/(1 + x^2),x]
```

output
```
(x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + (-x + ArcTan[x])/2 + ArcTan[x]*Log
[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

3.672.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.672. $\int \frac{x^3\arctan(x)}{1+x^2}\,dx$

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.672.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(53) = 106$.

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

method	result
default	$\frac{x^2\arctan(x)}{2} - \frac{\arctan(x)\ln(x^2+1)}{2} - \frac{x}{2} + \frac{\arctan(x)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(x+i)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right.\right.}{4}$
parts	$\frac{x^2\arctan(x)}{2} - \frac{\arctan(x)\ln(x^2+1)}{2} - \frac{x}{2} + \frac{\arctan(x)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(x+i)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right.\right.}{4}$
risch	$-\frac{i\ln(ix+1)}{4} - \frac{i\ln(-ix+1)^2}{8} + \frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4} - \frac{x}{2} + \frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4} - \frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4} + \frac{i\ln(ix+1)^2}{8} + ix^2$

input
```
int(x^3*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x)-1/2*arctan(x)*ln(x^2+1)-1/2*x+1/2*arctan(x)-1/4*I*(ln(x-
I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2*I*(x+I)))+1
/4*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*
(x-I)))
```

3.672.5 Fricas [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan(x)}{x^2+1} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(x)/(x^2 + 1), x)
```

3.672.6 Sympy [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{atan}(x)}{x^2+1} \, dx$$

input
```
integrate(x**3*atan(x)/(x**2+1),x)
```

output
```
Integral(x**3*atan(x)/(x**2 + 1), x)
```

3.672.7 Maxima [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan(x)}{x^2+1} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1), x)
```

3.672.8 Giac [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan{(x)}}{x^2+1} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{atan}(x)}{x^2+1} \, dx$$

input
```
int((x^3*atan(x))/(x^2 + 1),x)
```

output
```
int((x^3*atan(x))/(x^2 + 1), x)
```

3.673 $\int \frac{x^2 \arctan(x)}{\left(1+x^2\right)^2} \, dx$

3.673.1 Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x^2 \arctan(x)}{\left(1+x^2\right)^2} \, dx = -\frac{1}{4\left(1+x^2\right)} - \frac{x \arctan(x)}{2\left(1+x^2\right)} + \frac{\arctan(x)^2}{4}$$

output
```
-1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

3.673.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arctan(x)}{\left(1+x^2\right)^2} \, dx = \frac{-1 - 2x \arctan(x) + \left(1+x^2\right) \arctan(x)^2}{4\left(1+x^2\right)}$$

input
```
Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))
```

3.673.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(x)}{(x^2+1)^2} dx$$

$$\Big\downarrow \ 5469$$

$$\frac{1}{2}\int \frac{\arctan(x)}{x^2+1}dx - \frac{x\arctan(x)}{2(x^2+1)} - \frac{1}{4(x^2+1)}$$

$$\Big\downarrow \ 5419$$

$$-\frac{x\arctan(x)}{2(x^2+1)} + \frac{\arctan(x)^2}{4} - \frac{1}{4(x^2+1)}$$

input
```
Int[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
-1/4*1/(1 + x^2) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

3.673.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

3.673.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{4(x^2+1)}-\frac{x\arctan(x)}{2(x^2+1)}+\frac{\arctan(x)^2}{4}$	29
parts	$-\frac{1}{4(x^2+1)}-\frac{x\arctan(x)}{2(x^2+1)}+\frac{\arctan(x)^2}{4}$	29
risch	$-\frac{\ln(ix+1)^2}{16}+\frac{(x^2\ln(-ix+1)+\ln(-ix+1)+2ix)\ln(ix+1)}{8x^2+8}-\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+4ix\ln(-ix+1)+4}{16(x+i)(x-i)}$	101

input
```
int(x^2*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

3.673.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2\arctan(x)}{(1+x^2)^2}\,dx=\frac{(x^2+1)\arctan{(x)}^2-2\,x\arctan{(x)}-1}{4\,(x^2+1)}$$

input
```
integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)
```

3.673.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2\arctan(x)}{(1+x^2)^2}\,dx=\text{Exception raised: RecursionError}$$

input
```
integrate(x**2*atan(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

3.673.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{1}{2} \left(\frac{x}{x^2+1} - \arctan(x) \right) \arctan(x) - \frac{(x^2+1) \arctan(x)^2 + 1}{4 \, (x^2+1)}$$

input
```
integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(x)^2 + 1)
/(x^2 + 1)
```

3.673.8 Giac [F]

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^2 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(x)/(x^2 + 1)^2, x)
```

3.673.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \frac{\operatorname{atan}(x)^2}{4} - \frac{\frac{x \operatorname{atan}(x)}{2} + \frac{1}{4}}{x^2+1}$$

input
```
int((x^2*atan(x))/(x^2 + 1)^2,x)
```

output
```
atan(x)^2/4 - ((x*atan(x))/2 + 1/4)/(x^2 + 1)
```

3.674 $\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx$

3.674.1 Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{x}{4(1+x^2)} - \frac{\arctan(x)}{4} + \frac{\arctan(x)}{2(1+x^2)} - \frac{1}{2} i \arctan(x)^2 - \arctan(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

output
```
-1/4*x/(x^2+1)-1/4*arctan(x)+1/2*arctan(x)/(x^2+1)-1/2*I*arctan(x)^2-arcta
n(x)*ln(2/(1+I*x))-1/2*I*polylog(2,1-2/(1+I*x))
```

3.674.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \frac{1}{2} i \arctan(x)^2 + \frac{1}{4} \arctan(x) \cos(2 \arctan(x)) - \arctan(x) \log\left(1 + e^{2i \arctan(x)}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2i \arctan(x)}\right) - \frac{1}{8} \sin(2 \arctan(x))$$

input
```
Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Log[1 + E^(
(2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x])] - Sin[2*ArcTan[
x]]/8
```

3.674.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5499, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(x)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\int \frac{x\arctan(x)}{x^2+1}dx-\int \frac{x\arctan(x)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{5455}$$

$$-\int \frac{x\arctan(x)}{(x^2+1)^2}dx-\int \frac{\arctan(x)}{i-x}dx-\frac{1}{2}i\arctan(x)^2$$

$$\Big\downarrow \text{5379}$$

$$-\int \frac{x\arctan(x)}{(x^2+1)^2}dx+\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2849}$$

$$-\int \frac{x\arctan(x)}{(x^2+1)^2}dx-i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\int \frac{x\arctan(x)}{(x^2+1)^2}dx-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5465}$$

$$-\frac{1}{2}\int \frac{1}{(x^2+1)^2}dx+\frac{\arctan(x)}{2(x^2+1)}-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x}{2\left(x^2+1\right)}\right)+\frac{\arctan(x)}{2\left(x^2+1\right)}-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(x)}{2\left(x^2+1\right)}+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{x}{2\left(x^2+1\right)}\right)-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

input
```
Int[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(-1/2*x/(1 + x^2) - ArcTan[x]/2)/2 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*ArcTa
n[x]^2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

3.674.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.674. $\int \frac{x^3\arctan(x)}{\left(1+x^2\right)^2}\,dx$

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.674.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

method	result
default	$\frac{\arctan(x)}{2x^2+2}+\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{4}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(}{}\right.\right.}{4}$
parts	$\frac{\arctan(x)}{2x^2+2}+\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{4}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(}{}\right.\right.}{4}$
risch	$\frac{i\ln(-ix+1)^2}{8}+\frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4}+\frac{i\ln(ix+1)}{16ix-16}+\frac{i}{-8ix+8}+\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}-\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}-\frac{i}{8(ix+1)}-\frac{\arctan(}{8}$

3.674. $\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx$

input
```
int(x^3*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)/(x^2+1)+1/2*arctan(x)*ln(x^2+1)-1/4*x/(x^2+1)-1/4*arctan(x)+
1/4*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2
*I*(x+I)))-1/4*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+
I)*ln(1/2*I*(x-I)))
```

3.674.5 Fricas [F]

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^3 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

3.674.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \text{Exception raised: RecursionError}$$

input
```
integrate(x**3*atan(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded while
 calling a Python object
```

3.674.7 Maxima [F]

$$\int \frac{x^3 \arctan(x)}{\left(1+x^2\right)^2} \, dx = \int \frac{x^3 \arctan\left(x\right)}{\left(x^2+1\right)^2} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

3.674.8 Giac [F]

$$\int \frac{x^3 \arctan(x)}{\left(1+x^2\right)^2} \, dx = \int \frac{x^3 \arctan\left(x\right)}{\left(x^2+1\right)^2} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

3.674.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(x)}{\left(1+x^2\right)^2} \, dx = \int \frac{x^3 \operatorname{atan}(x)}{\left(x^2+1\right)^2} \, dx$$

input
```
int((x^3*atan(x))/(x^2 + 1)^2,x)
```

output
```
int((x^3*atan(x))/(x^2 + 1)^2, x)
```

3.675 $\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx$

3.675.1 Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3\arctan(x)}{4} + \frac{1}{2}x^2\arctan(x) - \frac{\arctan(x)}{2(1+x^2)} + i\arctan(x)^2 + 2\arctan(x)\log\left(\frac{2}{1+ix}\right) + i\,\text{PolyLog}\left(2, 1-\frac{2}{1+ix}\right)$$

output
```
-1/2*x+1/4*x/(x^2+1)+3/4*arctan(x)+1/2*x^2*arctan(x)-1/2*arctan(x)/(x^2+1)
+I*arctan(x)^2+2*arctan(x)*ln(2/(1+I*x))+I*polylog(2,1-2/(1+I*x))
```

3.675.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \frac{1}{8}\left(-4x + 4(1+x^2)\arctan(x) - 8i\arctan(x)^2 - 2\arctan(x)\cos(2\arctan(x)) + 16\arctan(x)\log\left(1+e^{2i\arctan(x)}\right) - 8i\,\text{PolyLog}\left(2, -e^{2i\arctan(x)}\right) + \sin(2\arctan(x))\right)$$

input
```
Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*Cos[2*ArcT
an[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] - (8*I)*PolyLog[2, -E^(
(2*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8
```

3.675.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.308, Rules used = {5499, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752, 5499, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(x)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\int \frac{x^3 \arctan(x)}{x^2+1}dx-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{5451}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+\int x \arctan(x)dx$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx-\frac{1}{2}\int \frac{x^2}{x^2+1}dx+\frac{1}{2}x^2 \arctan(x)$$

$$\Big\downarrow \text{262}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+\frac{1}{2}\left(\int \frac{1}{x^2+1}dx-x\right)+\frac{1}{2}x^2 \arctan(x)$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}(\arctan(x)-x)$$

$$\Big\downarrow \text{5455}$$

$$-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+\int \frac{\arctan(x)}{i-x}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)$$

$$\Big\downarrow \text{5379}$$

$$-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2849}$$

$$-\int \frac{x^3\arctan(x)}{(x^2+1)^2}dx+i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}+\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\int \frac{x^3\arctan(x)}{(x^2+1)^2}dx+\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5499}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx-\int \frac{x\arctan(x)}{x^2+1}dx+\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5455}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx+\int \frac{\arctan(x)}{i-x}dx+\frac{1}{2}x^2\arctan(x)+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5379}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx+\frac{1}{2}x^2\arctan(x)+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{2849}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx+i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}+\frac{1}{2}x^2\arctan(x)+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{2752}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx+\frac{1}{2}x^2\arctan(x)+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

3.675. $\int \frac{x^5\arctan(x)}{(1+x^2)^2}\,dx$

$$\Big\downarrow \text{5465}$$

$$\frac{1}{2}\int \frac{1}{(x^2+1)^2}dx+\frac{1}{2}x^2\arctan(x)-\frac{\arctan(x)}{2(x^2+1)}+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+ 2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}\right)+\frac{1}{2}x^2\arctan(x)-\frac{\arctan(x)}{2(x^2+1)}+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+ 2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\left(\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}\right)-\frac{\arctan(x)}{2(x^2+1)}+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+ 2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

input
```
Int[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(x/(2*(1 + x^2)) + ArcTan[x]/2)/2 + (x^2*ArcTan[x])/2 - ArcTan[x]/(2*(1 +
x^2)) + I*ArcTan[x]^2 + (-x + ArcTan[x])/2 + 2*ArcTan[x]*Log[2/(1 + I*x)]
+ I*PolyLog[2, 1 - 2/(1 + I*x)]
```

3.675.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.675. $\int \frac{x^5\arctan(x)}{(1+x^2)^2}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.675. $\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.675.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.65

method	result
default	$\frac{x^2\arctan(x)}{2}-\arctan(x)\ln(x^2+1)-\frac{\arctan(x)}{2(x^2+1)}-\frac{x}{2}+\frac{x}{4x^2+4}+\frac{3\arctan(x)}{4}-i\Big(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\mathrm{d}$
parts	$\frac{x^2\arctan(x)}{2}-\arctan(x)\ln(x^2+1)-\frac{\arctan(x)}{2(x^2+1)}-\frac{x}{2}+\frac{x}{4x^2+4}+\frac{3\arctan(x)}{4}-i\Big(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\mathrm{d}$
risch	$-\frac{x}{2}-\frac{i\ln(ix+1)}{4}-\frac{i\ln(-ix+1)^2}{4}+\frac{i\ln(\frac{1}{2}-\frac{ix}{2})\ln(ix+1)}{2}+\frac{\arctan(x)}{8}+\frac{\ln(-ix+1)x}{-16ix-16}-\frac{i}{8(-ix+1)}+\frac{i\,\mathrm{dilog}(\frac{1}{2}-\frac{ix}{2})}{2}-$

input
```
int(x^5*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x)-arctan(x)*ln(x^2+1)-1/2*arctan(x)/(x^2+1)-1/2*x+1/4*x/(x
^2+1)+3/4*arctan(x)-1/2*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x
+I))-ln(x-I)*ln(-1/2*I*(x+I)))+1/2*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilo
g(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*(x-I)))
```

3.675.5 Fricas [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

3.675.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \text{Exception raised: RecursionError}$$

input
```
integrate(x**5*atan(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

3.675.7 Maxima [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

3.675.8 Giac [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

3.675.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \operatorname{atan}(x)}{(x^2+1)^2} \, dx$$

input
```
int((x^5*atan(x))/(x^2 + 1)^2,x)
```

output
```
int((x^5*atan(x))/(x^2 + 1)^2, x)
```

3.676 $\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx$

3.676.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

output
```
-arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)
```

3.676.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

input
```
Integrate[((1 + x^2)*ArcTan[x])/x^2,x]
```

output
```
-(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

3.676.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5345, 240, 5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)\arctan(x)}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$\int \frac{\arctan(x)}{x^2}dx + \int \arctan(x)dx$$

$$\Big\downarrow \text{5345}$$

$$\int \frac{\arctan(x)}{x^2}dx - \int \frac{x}{x^2+1}dx + x\arctan(x)$$

$$\Big\downarrow \text{240}$$

$$\int \frac{\arctan(x)}{x^2}dx + x\arctan(x) - \frac{1}{2}\log(x^2+1)$$

$$\Big\downarrow \text{5361}$$

$$\int \frac{1}{x(x^2+1)}dx + x\arctan(x) - \frac{\arctan(x)}{x} - \frac{1}{2}\log(x^2+1)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2 + x\arctan(x) - \frac{\arctan(x)}{x} - \frac{1}{2}\log(x^2+1)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2 - \int \frac{1}{x^2+1}dx^2\right) + x\arctan(x) - \frac{\arctan(x)}{x} - \frac{1}{2}\log(x^2+1)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log(x^2) - \int \frac{1}{x^2+1}dx^2\right) + x\arctan(x) - \frac{\arctan(x)}{x} - \frac{1}{2}\log(x^2+1)$$

$$\Big\downarrow \text{16}$$

$$x\arctan(x) - \frac{\arctan(x)}{x} + \frac{1}{2}\left(\log(x^2) - \log(x^2+1)\right) - \frac{1}{2}\log(x^2+1)$$

input
```
Int[((1 + x^2)*ArcTan[x])/x^2,x]
```

output
```
-(ArcTan[x]/x) + x*ArcTan[x] + (Log[x^2] - Log[1 + x^2])/2 - Log[1 + x^2]/
2
```

3.676.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.676. $\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.676.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{\arctan(x)}{x}+x\arctan\left(x\right)+\ln\left(x\right)-\ln\left(x^{2}+1\right)$	23
parts	$-\frac{\arctan(x)}{x}+x\arctan\left(x\right)+\ln\left(x\right)-\ln\left(x^{2}+1\right)$	23
parallelrisch	$\frac{x^{2}\arctan(x)+x\ln(x)-x\ln(x^{2}+1)-\arctan(x)}{x}$	29
meijerg	$\ln\left(x\right)-\frac{\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}-\ln\left(x^{2}+1\right)+\frac{x^{2}\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}$	40
risch	$-\frac{i(x^{2}-1)\ln(ix+1)}{2x}+\frac{i(-2i\ln(x)x+2i\ln(x^{2}+1)x+x^{2}\ln(-ix+1)-\ln(-ix+1))}{2x}$	63

input
```
int((x^2+1)*arctan(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*arctan(x)+x*arctan(x)+ln(x)-ln(x^2+1)
```

3.676.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx=\frac{(x^{2}-1)\arctan\left(x\right)-x\log\left(x^{2}+1\right)+x\log\left(x\right)}{x}$$

input
```
integrate((x^2+1)*arctan(x)/x^2,x, algorithm="fricas")
```

output
```
((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x
```

3.676.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = x\,\mathrm{atan}\,(x) + \log\,(x) - \log\,(x^2+1) - \frac{\mathrm{atan}\,(x)}{x}$$

input
```
integrate((x**2+1)*atan(x)/x**2,x)
```

output
```
x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x
```

3.676.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \left(x - \frac{1}{x}\right)\arctan\,(x) - \log\,(x^2+1) + \log\,(x)$$

input
```
integrate((x^2+1)*arctan(x)/x^2,x, algorithm="maxima")
```

output
```
(x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)
```

3.676.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \left(x - \frac{1}{x}\right)\arctan\,(x) - \log\,(x^2+1) + \frac{1}{2}\log\,(x^2)$$

input
```
integrate((x^2+1)*arctan(x)/x^2,x, algorithm="giac")
```

output
```
(x - 1/x)*arctan(x) - log(x^2 + 1) + 1/2*log(x^2)
```

3.676.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \ln(x) - \ln(x^2+1) - \frac{\operatorname{atan}(x)}{x} + x\operatorname{atan}(x)$$

input
```
int((atan(x)*(x^2 + 1))/x^2,x)
```

output
```
log(x) - log(x^2 + 1) - atan(x)/x + x*atan(x)
```

3.677 $\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx$

3.677.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{1}{12x^3}-\frac{1}{4x}-\frac{(1+x^2)^2\arctan(x)}{4x^4}$$

output
```
-1/12/x^3-1/4/x-1/4*(x^2+1)^2*arctan(x)/x^4
```

3.677.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{2x^2}-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x^2\right)}{12x^3} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x^2\right)}{2x}$$

input
```
Integrate[((1 + x^2)*ArcTan[x])/x^5,x]
```

output
```
-1/4*ArcTan[x]/x^4 - ArcTan[x]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2,
-x^2]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x)
```

3.677.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5479, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)\arctan(x)}{x^5}\,dx$$
$$\Big\downarrow \ 5479$$
$$\frac{1}{4}\int \frac{x^2+1}{x^4}dx-\frac{(x^2+1)^2\arctan(x)}{4x^4}$$
$$\Big\downarrow \ 244$$
$$\frac{1}{4}\int \left(\frac{1}{x^2}+\frac{1}{x^4}\right)dx-\frac{(x^2+1)^2\arctan(x)}{4x^4}$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{4}\left(-\frac{1}{3x^3}-\frac{1}{x}\right)-\frac{(x^2+1)^2\arctan(x)}{4x^4}$$

input
```
Int[((1 + x^2)*ArcTan[x])/x^5,x]
```

output
```
(-1/3*1/x^3 - x^(-1))/4 - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

3.677.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.677.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{4}-\frac{1}{12x^3}-\frac{1}{4x}$	30
parts	$-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{4}-\frac{1}{12x^3}-\frac{1}{4x}$	30
parallelrisch	$-\frac{3\arctan(x)x^4+3x^3+6x^2\arctan(x)+x+3\arctan(x)}{12x^4}$	31
meijerg	$-\frac{1}{12x^3}-\frac{1}{4x}-\frac{2\left(-\frac{3x^4}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{2x^2}$	47
risch	$\frac{i(2x^2+1)\ln(ix+1)}{8x^4}+\frac{i(3\ln(x-i)x^4-3\ln(x+i)x^4+6ix^3-6x^2\ln(-ix+1)+2ix-3\ln(-ix+1))}{24x^4}$	80

input
```
int((x^2+1)*arctan(x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)/x^2-1/4*arctan(x)/x^4-1/4*arctan(x)-1/12/x^3-1/4/x
```

3.677.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx=-\frac{3\,x^3+3\,(x^4+2\,x^2+1)\arctan(x)+x}{12\,x^4}$$

input
```
integrate((x^2+1)*arctan(x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + x)/x^4
```

3.677.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\operatorname{atan}(x)}{4} - \frac{1}{4x} - \frac{\operatorname{atan}(x)}{2x^2} - \frac{1}{12x^3} - \frac{\operatorname{atan}(x)}{4x^4}$$

input
```
integrate((x**2+1)*atan(x)/x**5,x)
```

output
```
-atan(x)/4 - 1/(4*x) - atan(x)/(2*x**2) - 1/(12*x**3) - atan(x)/(4*x**4)
```

3.677.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^2+1}{12\,x^3} - \frac{(2\,x^2+1)\arctan(x)}{4\,x^4} - \frac{1}{4}\arctan(x)$$

input
```
integrate((x^2+1)*arctan(x)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

3.677.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^2+1}{12\,x^3} - \frac{(2\,x^2+1)\arctan(x)}{4\,x^4} - \frac{1}{4}\arctan(x)$$

input
```
integrate((x^2+1)*arctan(x)/x^5,x, algorithm="giac")
```

output
```
-1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

3.677.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\mathrm{atan}(x)}{4} - \frac{\frac{x}{12} + \frac{\mathrm{atan}(x)}{4} + \frac{x^2\,\mathrm{atan}(x)}{2} + \frac{x^3}{4}}{x^4}$$

input
```
int((atan(x)*(x^2 + 1))/x^5,x)
```

output
```
- atan(x)/4 - (x/12 + atan(x)/4 + (x^2*atan(x))/2 + x^3/4)/x^4
```

3.678 $\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx$

3.678.1 Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = -\frac{1}{12x^3} - \frac{3}{4x} - \frac{3\arctan(x)}{4} - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)$$

output
```
-1/12/x^3-3/4/x-3/4*arctan(x)-1/4*arctan(x)/x^4-arctan(x)/x^2+1/2*I*polylo
g(2,-I*x)-1/2*I*polylog(2,I*x)
```

3.678.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.29

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = -\frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} - \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x^2\right)}{12x^3} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x^2\right)}{x} + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)$$

input
```
Integrate[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

output
```
-1/4*ArcTan[x]/x^4 - ArcTan[x]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x^2
]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/x + (I/2)*PolyLog[2, (-
I)*x] - (I/2)*PolyLog[2, I*x]
```

3.678.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2 \arctan(x)}{x^5} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(\frac{\arctan(x)}{x^5}+\frac{2\arctan(x)}{x^3}+\frac{\arctan(x)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{x^2}-\frac{3\arctan(x)}{4}+\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)-\frac{1}{12x^3}-\frac{3}{4x}$$

input
```
Int[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

output
```
-1/12*1/x^3 - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^
2 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

3.678.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.678.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

method	result
default	$\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}-\frac{\arctan(x)}{4x^4}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}$
parts	$\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}-\frac{\arctan(x)}{4x^4}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}$
meijerg	$-\frac{1}{12x^3}-\frac{3}{4x}-\frac{2\left(-\frac{3x^4}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{ix\,\mathrm{Li}_2\left(i\sqrt{x^2}\right)}{2\sqrt{x^2}}+\frac{ix\,\mathrm{Li}_2\left(-i\sqrt{x^2}\right)}{2\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{x^2}$
risch	$-\frac{3}{4x}-\frac{1}{12x^3}+\frac{3i\ln(-ix)}{8}-\frac{3\arctan(x)}{4}-\frac{i\ln(-ix+1)}{8x^4}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{i\ln(-ix+1)}{2x^2}-\frac{3i\ln(ix)}{8}+\frac{i\ln(ix+1)}{8x^4}+$

input
```
int((x^2+1)^2*arctan(x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
arctan(x)*ln(x)-arctan(x)/x^2-1/4*arctan(x)/x^4+1/2*I*ln(x)*ln(1+I*x)-1/2*
I*ln(x)*ln(1-I*x)+1/2*I*dilog(1+I*x)-1/2*I*dilog(1-I*x)-1/12/x^3-3/4/x-3/4
*arctan(x)
```

3.678.5 Fricas [F]

$$\int \frac{(1+x^2)^2\arctan(x)}{x^5}\,dx=\int \frac{(x^2+1)^2\arctan(x)}{x^5}\,dx$$

input
```
integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="fricas")
```

output
```
integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)
```

3.678.6 Sympy [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = \int \frac{(x^2+1)^2 \operatorname{atan}(x)}{x^5} \, dx$$

input
```
integrate((x**2+1)**2*atan(x)/x**5,x)
```

output
```
Integral((x**2 + 1)**2*atan(x)/x**5, x)
```

3.678.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = -\frac{3\,\pi x^4 \log(x^2+1) - 12\,x^4 \arctan(x)\log(x) + 6i\,x^4 \mathrm{Li}_2(i\,x+1) - 6i\,x^4 \mathrm{Li}_2(-i\,x+1) + 9\,x^3 + 3\,(3\,x^4 + 4}{12\,x^4}$$

input
```
integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*pi*x^4*log(x^2 + 1) - 12*x^4*arctan(x)*log(x) + 6*I*x^4*dilog(I*x
 + 1) - 6*I*x^4*dilog(-I*x + 1) + 9*x^3 + 3*(3*x^4 + 4*x^2 + 1)*arctan(x)
+ x)/x^4
```

3.678.8 Giac [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = \int \frac{(x^2+1)^2 \arctan(x)}{x^5} \, dx$$

input
```
integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="giac")
```

output
```
integrate((x^2 + 1)^2*arctan(x)/x^5, x)
```

3.678.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx = \frac{x^2-\frac{1}{3}}{4\,x^3} - \frac{\operatorname{atan}(x)}{x^2} - \frac{\operatorname{atan}(x)}{4\,x^4} - \frac{3\operatorname{atan}(x)}{4} - \frac{1}{x} - \frac{\operatorname{Li}_2(1-x\,1\mathrm{i})\ 1\mathrm{i}}{2} + \frac{\operatorname{polylog}(2,-x\,1\mathrm{i})\ 1\mathrm{i}}{2}$$

input
```
int((atan(x)*(x^2 + 1)^2)/x^5,x)
```

output
```
(polylog(2, -x*1i)*1i)/2 - (3*atan(x))/4 - atan(x)/x^2 - atan(x)/(4*x^4) -
 (dilog(1 - x*1i)*1i)/2 + (x^2 - 1/3)/(4*x^3) - 1/x
```

3.679 $\int \frac{\arctan(x)}{x^2(1+x^2)} \, dx$

3.679.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\arctan(x)}{x^2(1+x^2)} \, dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

3.679.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)}{x^2(1+x^2)} \, dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcTan[x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

3.679.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)}{x^2(x^2+1)}dx$$

$$\Big\downarrow \text{5453}$$

$$\int \frac{\arctan(x)}{x^2}dx-\int \frac{\arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\int \frac{1}{x(x^2+1)}dx-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{47}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{14}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\log\left(x^2\right)-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{1}{2}\arctan(x)^2-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)$$

input
```
Int[ArcTan[x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcTan[x]/x) - ArcTan[x]^2/2 + (Log[x^2] - Log[1 + x^2])/2
```

3.679.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.679. $\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx$

3.679.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}+\ln(x)-\frac{\ln(x^{2}+1)}{2}$	25
parts	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}+\ln(x)-\frac{\ln(x^{2}+1)}{2}$	25
parallelrisch	$\frac{-x\arctan(x)^{2}+2x\ln(x)-x\ln(x^{2}+1)-2\arctan(x)}{2x}$	32
risch	$\frac{\ln(ix+1)^{2}}{8}-\frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x}-\frac{-x\ln(-ix+1)^{2}-8x\ln(x)+4x\ln(x^{2}+1)+4i\ln(-ix+1)}{8x}$	79

input
```
int(arctan(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x*arctan(x)-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx=-\frac{x\arctan(x)^2+x\log(x^2+1)-2\,x\log(x)+2\,\arctan(x)}{2\,x}$$

input
```
integrate(arctan(x)/x^2/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/x
```

3.679.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx=\log(x)-\frac{\log(x^2+1)}{2}-\frac{\operatorname{atan}^2(x)}{2}-\frac{\operatorname{atan}(x)}{x}$$

input
```
integrate(atan(x)/x**2/(x**2+1),x)
```

output
```
log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

3.679.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\left(\frac{1}{x} + \arctan(x)\right)\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log(x^2+1) + \log(x)$$

input
```
integrate(arctan(x)/x^2/(x^2+1),x, algorithm="maxima")
```

output
```
-(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1) + log(x)
```

3.679.8 Giac [F]

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = \int \frac{\arctan(x)}{(x^2+1)x^2}\,dx$$

input
```
integrate(arctan(x)/x^2/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arctan(x)/((x^2 + 1)*x^2), x)
```

3.679.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}(x)^2}{2}$$

input
```
int(atan(x)/(x^2*(x^2 + 1)),x)
```

output
```
log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2/2
```

3.680 $\int \frac{\arctan(x)^2}{x^3} \, dx$

3.680.1 Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{\arctan(x)^2}{x^3} \, dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

3.680.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(x)^2}{x^3} \, dx = -\frac{\arctan(x)}{x} + \frac{(-1-x^2)\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcTan[x]^2/x^3,x]
```

output
```
-(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[1 + x^2]/
2
```

3.680.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)^2}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$\int \frac{\arctan(x)}{x^2(x^2+1)}dx-\frac{\arctan(x)^2}{2x^2}$$

$$\Big\downarrow \text{5453}$$

$$\int \frac{\arctan(x)}{x^2}dx-\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)^2}{2x^2}$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\int \frac{1}{x(x^2+1)}dx-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{47}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{14}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\log(x^2)-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log(x^2)-\log(x^2+1)\right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{2}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log(x^2)-\log(x^2+1)\right)$$

input
```
Int[ArcTan[x]^2/x^3,x]
```

output
```
-(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + (Log[x^2] - Log[1 +
 x^2])/2
```

3.680.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.680. $\int \frac{\arctan(x)^2}{x^3} \, dx$

3.680.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}-\frac{\arctan(x)^{2}}{2x^{2}}+\ln\left(x\right)-\frac{\ln(x^{2}+1)}{2}$
parts	$-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}-\frac{\arctan(x)^{2}}{2x^{2}}+\ln\left(x\right)-\frac{\ln(x^{2}+1)}{2}$
parallelrisch	$\frac{-x^{2}\arctan(x)^{2}+2x^{2}\ln(x)-\ln(x^{2}+1)x^{2}-2x\arctan(x)-\arctan(x)^{2}}{2x^{2}}$
risch	$\frac{(x^{2}+1)\ln(ix+1)^{2}}{8x^{2}}-\frac{(x^{2}\ln(-ix+1)-2ix+\ln(-ix+1))\ln(ix+1)}{4x^{2}}+\frac{x^{2}\ln(-ix+1)^{2}-4ix\ln(-ix+1)+8x^{2}\ln(x)-4\ln(x^{2}+1)}{8x^{2}}$

input
```
int(arctan(x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/x*arctan(x)-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int\frac{\arctan(x)^{2}}{x^{3}}\,dx=-\frac{\left(x^{2}+1\right)\arctan\left(x\right)^{2}+x^{2}\log\left(x^{2}+1\right)-2\,x^{2}\log\left(x\right)+2\,x\arctan\left(x\right)}{2\,x^{2}}$$

input
```
integrate(arctan(x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*arctan(x)^2 + x^2*log(x^2 + 1) - 2*x^2*log(x) + 2*x*arctan
(x))/x^2
```

3.680.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int\frac{\arctan(x)^{2}}{x^{3}}\,dx=\log\left(x\right)-\frac{\log\left(x^{2}+1\right)}{2}-\frac{\operatorname{atan}^{2}\left(x\right)}{2}-\frac{\operatorname{atan}\left(x\right)}{x}-\frac{\operatorname{atan}^{2}\left(x\right)}{2x^{2}}$$

input
```
integrate(atan(x)**2/x**3,x)
```

output
```
log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x)**2/(2*x**2)
```

3.680.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(x)^2}{x^3}\,dx = -\left(\frac{1}{x} + \arctan(x)\right)\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{\arctan(x)^2}{2\,x^2} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

input
```
integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

output
```
-(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/x^2 - 1/2
*log(x^2 + 1) + log(x)
```

3.680.8 Giac [F]

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \int \frac{\arctan(x)^2}{x^3}\,dx$$

input
```
integrate(arctan(x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arctan(x)^2/x^3, x)
```

3.680.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{\mathrm{atan}(x)}{x} - \mathrm{atan}(x)^2\left(\frac{1}{2\,x^2} + \frac{1}{2}\right)$$

input
```
int(atan(x)^2/x^3,x)
```

output
```
log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2*(1/(2*x^2) + 1/2)
```

3.681 $\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx$

3.681.1 Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{1}{12x^2} - \frac{\arctan(x)}{6x^3} - \frac{\arctan(x)}{2x} - \frac{(1+x^2)^2\arctan(x)^2}{4x^4} + \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^2\right)$$

output
```
-1/12/x^2-1/6*arctan(x)/x^3-1/2*arctan(x)/x-1/4*(x^2+1)^2*arctan(x)^2/x^4+
1/3*ln(x)-1/6*ln(x^2+1)
```

3.681.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{-2(x+3x^3)\arctan(x) - 3(1+x^2)^2\arctan(x)^2 + x^2(-1+4x^2\log(x) - 2x^2\log(1+x^2))}{12x^4}$$

input
```
Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

output
```
(-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 + 4*x^2*Lo
g[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)
```

3.681.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5479, 5485, 5361, 243, 47, 14, 16, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)\arctan(x)^2}{x^5}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{1}{2}\int \frac{(x^2+1)\arctan(x)}{x^4}dx-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5485}$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^4}dx+\int \frac{\arctan(x)}{x^2}dx\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\int \frac{1}{x(x^2+1)}dx+\frac{1}{3}\int \frac{1}{x^3(x^2+1)}dx-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\log\left(x^2\right)-\int \frac{1}{x^2+1}dx^2\right)+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{6}\int \frac{1}{x^4\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2}\left(\frac{1}{6}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

input
```
Int[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

output
```
-1/4*((1 + x^2)^2*ArcTan[x]^2)/x^4 + (-1/3*ArcTan[x]/x^3 - ArcTan[x]/x + (
Log[x^2] - Log[1 + x^2])/2 + (-x^(-2) - Log[x^2] + Log[1 + x^2])/6)/2
```

3.681.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.681.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)^2}{4}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{1}{12x^2}+\frac{\ln(x)}{3}-\frac{\ln(x^2+1)}{6}$
parts	$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)^2}{4}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{1}{12x^2}+\frac{\ln(x)}{3}-\frac{\ln(x^2+1)}{6}$
parallelrisch	$\frac{-3x^4\arctan(x)^2+4x^4\ln(x)-2\ln(x^2+1)x^4-6x^3\arctan(x)-6x^2\arctan(x)^2-x^2-2x\arctan(x)-3\arctan(x)^2}{12x^4}$
risch	$\frac{(x^4+2x^2+1)\ln(ix+1)^2}{16x^4}-\frac{(3x^4\ln(-ix+1)-6ix^3+6x^2\ln(-ix+1)-2ix+3\ln(-ix+1))\ln(ix+1)}{24x^4}+\frac{3x^4\ln(-ix+1)^2-12ix^3}{}$

3.681. $\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx$

input
```
int((x^2+1)*arctan(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)^2/x^2-1/4*arctan(x)^2/x^4-1/4*arctan(x)^2-1/6/x^3*arctan(x)
-1/2/x*arctan(x)-1/12/x^2+1/3*ln(x)-1/6*ln(x^2+1)
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{2\,x^4\log\left(x^2+1\right)-4\,x^4\log\left(x\right)+3\,\left(x^4+2\,x^2+1\right)\arctan\left(x\right)^2+x^2+2\,\left(3\,x^3+x\right)\arctan\left(x\right)}{12\,x^4}$$

input
```
integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(2*x^4*log(x^2 + 1) - 4*x^4*log(x) + 3*(x^4 + 2*x^2 + 1)*arctan(x)^2
 + x^2 + 2*(3*x^3 + x)*arctan(x))/x^4
```

3.681.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{\log\left(x\right)}{3}-\frac{\log\left(x^2+1\right)}{6}-\frac{\operatorname{atan}^2\left(x\right)}{4}-\frac{\operatorname{atan}\left(x\right)}{2x}-\frac{\operatorname{atan}^2\left(x\right)}{2x^2}-\frac{1}{12x^2}-\frac{\operatorname{atan}\left(x\right)}{6x^3}-\frac{\operatorname{atan}^2\left(x\right)}{4x^4}$$

input
```
integrate((x**2+1)*atan(x)**2/x**5,x)
```

output
```
log(x)/3 - log(x**2 + 1)/6 - atan(x)**2/4 - atan(x)/(2*x) - atan(x)**2/(2*
x**2) - 1/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

3.681.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{1}{6}\left(\frac{3\,x^2+1}{x^3}+3\,\arctan(x)\right)\arctan(x) + \frac{3\,x^2\arctan(x)^2-2\,x^2\log(x^2+1)+4\,x^2\log(x)-1}{12\,x^2} - \frac{(2\,x^2+1)\arctan(x)^2}{4\,x^4}$$

input
```
integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/6*((3*x^2 + 1)/x^3 + 3*arctan(x))*arctan(x) + 1/12*(3*x^2*arctan(x)^2 -
 2*x^2*log(x^2 + 1) + 4*x^2*log(x) - 1)/x^2 - 1/4*(2*x^2 + 1)*arctan(x)^2/
x^4
```

3.681.8 Giac [F]

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \int \frac{(x^2+1)\arctan(x)^2}{x^5}\,dx$$

input
```
integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="giac")
```

output
```
integrate((x^2 + 1)*arctan(x)^2/x^5, x)
```

3.681.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{\ln(x)}{3} - \frac{\ln(x^2+1)}{6} - \mathrm{atan}(x)^2\left(\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}+\frac{1}{4}\right) - \frac{1}{12\,x^2} - \frac{\mathrm{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{6}\right)}{x^3}$$

input
```
int((atan(x)^2*(x^2 + 1))/x^5,x)
```

output
```
log(x)/3 - log(x^2 + 1)/6 - atan(x)^2*((x^2/2 + 1/4)/x^4 + 1/4) - 1/(12*x^
2) - (atan(x)*(x^2/2 + 1/6))/x^3
```

3.681. $\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx$

3.682 $\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx$

3.682.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = -\frac{1}{32(1+x^2)^2} + \frac{5}{32(1+x^2)} + \frac{x^3 \arctan(x)}{8(1+x^2)^2} + \frac{3x \arctan(x)}{16(1+x^2)} - \frac{3\arctan(x)^2}{32} + \frac{x^4 \arctan(x)^2}{4(1+x^2)^2}$$

output
```
-1/32/(x^2+1)^2+5/32/(x^2+1)+1/8*x^3*arctan(x)/(x^2+1)^2+3/16*x*arctan(x)/
(x^2+1)-3/32*arctan(x)^2+1/4*x^4*arctan(x)^2/(x^2+1)^2
```

3.682.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = \frac{4 + 5x^2 + 2x(3 + 5x^2)\arctan(x) + (-3 - 6x^2 + 5x^4)\arctan(x)^2}{32(1+x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

output
```
(4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*ArcTan[x]^2)
/(32*(1 + x^2)^2)
```

3.682.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5479, 5473, 5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(x)^2}{(x^2+1)^3} \, dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(x)^2}{4 (x^2+1)^2} - \frac{1}{2} \int \frac{x^4 \arctan(x)}{(x^2+1)^3} dx$$

$$\Big\downarrow \text{5473}$$

$$\frac{1}{2} \left(-\frac{3}{4} \int \frac{x^2 \arctan(x)}{(x^2+1)^2} dx + \frac{x^3 \arctan(x)}{4 (x^2+1)^2} - \frac{x^4}{16 (x^2+1)^2} \right) + \frac{x^4 \arctan(x)^2}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{5469}$$

$$\frac{1}{2} \left(-\frac{3}{4} \left(\frac{1}{2} \int \frac{\arctan(x)}{x^2+1} dx - \frac{x \arctan(x)}{2 (x^2+1)} - \frac{1}{4 (x^2+1)} \right) + \frac{x^3 \arctan(x)}{4 (x^2+1)^2} - \frac{x^4}{16 (x^2+1)^2} \right) + \frac{x^4 \arctan(x)^2}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x^4 \arctan(x)^2}{4 (x^2+1)^2} + \frac{1}{2} \left(-\frac{3}{4} \left(-\frac{x \arctan(x)}{2 (x^2+1)} + \frac{\arctan(x)^2}{4} - \frac{1}{4 (x^2+1)} \right) + \frac{x^3 \arctan(x)}{4 (x^2+1)^2} - \frac{x^4}{16 (x^2+1)^2} \right)$$

input
```
Int[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

output
```
(x^4*ArcTan[x]^2)/(4*(1 + x^2)^2) + (-1/16*x^4/(1 + x^2)^2 + (x^3*ArcTan[x
])/(4*(1 + x^2)^2) - (3*(-1/4*1/(1 + x^2) - (x*ArcTan[x])/(2*(1 + x^2)) +
ArcTan[x]^2/4))/4)/2
```

3.682.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.682.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

method	result
default	$\frac{\arctan(x)^2}{4(x^2+1)^2}-\frac{\arctan(x)^2}{2(x^2+1)}+\frac{5x^3\arctan(x)}{16(x^2+1)^2}+\frac{3x\arctan(x)}{16(x^2+1)^2}+\frac{5\arctan(x)^2}{32}-\frac{1}{32(x^2+1)^2}+\frac{5}{32(x^2+1)}$
parts	$\frac{\arctan(x)^2}{4(x^2+1)^2}-\frac{\arctan(x)^2}{2(x^2+1)}+\frac{5x^3\arctan(x)}{16(x^2+1)^2}+\frac{3x\arctan(x)}{16(x^2+1)^2}+\frac{5\arctan(x)^2}{32}-\frac{1}{32(x^2+1)^2}+\frac{5}{32(x^2+1)}$
risch	$-\frac{(5x^4-6x^2-3)\ln(ix+1)^2}{128(x^2+1)^2}+\frac{(-6x^2\ln(-ix+1)-3\ln(-ix+1)+5x^4\ln(-ix+1)-10ix^3-6ix)\ln(ix+1)}{64(x+i)^2(x-i)^2}-\frac{5x^4\ln(-ix+1)^2-6x^2\ln}{}$

3.682. $\int \frac{x^3\arctan(x)^2}{(1+x^2)^3}\,dx$

input
```
int(x^3*arctan(x)^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(x)^2/(x^2+1)^2-1/2*arctan(x)^2/(x^2+1)+5/16*x^3*arctan(x)/(x^2+
1)^2+3/16*x*arctan(x)/(x^2+1)^2+5/32*arctan(x)^2-1/32/(x^2+1)^2+5/32/(x^2+
1)
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx = \frac{(5\,x^4-6\,x^2-3)\arctan{(x)}^2+5\,x^2+2\,(5\,x^3+3\,x)\arctan{(x)}+4}{32\,(x^4+2\,x^2+1)}$$

input
```
integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/32*((5*x^4 - 6*x^2 - 3)*arctan(x)^2 + 5*x^2 + 2*(5*x^3 + 3*x)*arctan(x)
+ 4)/(x^4 + 2*x^2 + 1)
```

3.682.6 Sympy [F]

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx = \int \frac{x^3 \operatorname{atan}^2{(x)}}{(x^2+1)^3}\,dx$$

input
```
integrate(x**3*atan(x)**2/(x**2+1)**3,x)
```

output
```
Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)
```

3.682.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \frac{x^3 \arctan(x)^2}{\left(1+x^2\right)^3} \, dx = \frac{1}{16}\left(\frac{5\,x^3+3\,x}{x^4+2\,x^2+1}+5\,\arctan\left(x\right)\right)\arctan\left(x\right) - \frac{\left(2\,x^2+1\right)\arctan\left(x\right)^2}{4\left(x^4+2\,x^2+1\right)} - \frac{5\left(x^4+2\,x^2+1\right)\arctan\left(x\right)^2-5\,x^2-4}{32\left(x^4+2\,x^2+1\right)}$$

input
```
integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*((5*x^3 + 3*x)/(x^4 + 2*x^2 + 1) + 5*arctan(x))*arctan(x) - 1/4*(2*x^
2 + 1)*arctan(x)^2/(x^4 + 2*x^2 + 1) - 1/32*(5*(x^4 + 2*x^2 + 1)*arctan(x)
^2 - 5*x^2 - 4)/(x^4 + 2*x^2 + 1)
```

3.682.8 Giac [F]

$$\int \frac{x^3 \arctan(x)^2}{\left(1+x^2\right)^3} \, dx = \int \frac{x^3 \arctan\left(x\right)^2}{\left(x^2+1\right)^3} \, dx$$

input
```
integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(x)^2/(x^2 + 1)^3, x)
```

3.682.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int \frac{x^3 \arctan(x)^2}{\left(1+x^2\right)^3} \, dx = -\frac{-5\,x^4\,\mathrm{atan}(x)^2+4\,x^4-10\,x^3\,\mathrm{atan}(x)+6\,x^2\,\mathrm{atan}(x)^2+3\,x^2-6\,x\,\mathrm{atan}(x)+3\,\mathrm{atan}(x)^2}{32\left(x^2+1\right)^2}$$

input
```
int((x^3*atan(x)^2)/(x^2 + 1)^3,x)
```

output
```
-(3*atan(x)^2 - 10*x^3*atan(x) + 6*x^2*atan(x)^2 - 5*x^4*atan(x)^2 - 6*x*a
tan(x) + 3*x^2 + 4*x^4)/(32*(x^2 + 1)^2)
```

3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

3.683.1 Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx=-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x}+\frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}$$

output
```
-(x^2)^(1/2)/x^2-2*I*arcsec(x)*arctan(1/x+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x
+I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-I*polylog(2,I*(1/x+
I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-arcsec(x)*(x^2-1)^(1/2)/x
```

3.683.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}\left(1+\sqrt{1-\frac{1}{x^2}}x\sec^{-1}(x)-x\sec^{-1}(x)\log\left(1-ie^{i\sec^{-1}(x)}\right)+x\sec^{-1}(x)\log\left(1+ie^{i\sec^{-1}(x)}\right)-\right.}{\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

output
```
-((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[x]*Log[1
- I*E^(I*ArcSec[x])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] - I*x*PolyLo
g[2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E^(I*ArcSec[x])]))/Sqrt[-1 +
 x^2])
```

3.683.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5765, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}\sec^{-1}(x)}{x^2}dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \sqrt{1-\frac{1}{x^2}}x\arccos\left(\frac{1}{x}\right)d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5199}$$

$$-\frac{\sqrt{x^2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\int 1d\frac{1}{x}+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sqrt{x^2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)+\frac{1}{x}\right)}{x}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{\sqrt{x^2}\left(-\int x\arccos\left(\frac{1}{x}\right)d\arccos\left(\frac{1}{x}\right)+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)+\frac{1}{x}\right)}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{x^2}\left(-\int \arccos\left(\frac{1}{x}\right)\csc\left(\arccos\left(\frac{1}{x}\right)+\frac{\pi}{2}\right)d\arccos\left(\frac{1}{x}\right)+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)+\frac{1}{x}\right)}{x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\sqrt{x^2}\left(\int \log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)-\int \log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)+2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\arccos(\right.\right.}{x}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\sqrt{x^2}\left(-i\int x\log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+i\int x\log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+2i\arccos\left(\frac{1}{x}\right)\arctan\left(e\right.\right.}{x}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\sqrt{x^2}\left(2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\arccos\left(\frac{1}{x}\right)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos\left(\frac{1}{x}\right)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos\left(\frac{1}{x}\right)}\right)+\sqrt{1-\frac{1}{x^2}}\,\text{a}\right.}{x}$$

input
```
Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

output
```
-((Sqrt[x^2]*(x^(-1) + Sqrt[1 - x^(-2)]*ArcCos[x^(-1)] + (2*I)*ArcCos[x^(-
1)]*ArcTan[E^(I*ArcCos[x^(-1)])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x^(-1)])]
 + I*PolyLog[2, I*E^(I*ArcCos[x^(-1)])]))/x)
```

3.683.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.683. $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

3.683.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\left(\sqrt{\frac{x^2-1}{x^2}}\,x-i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arcsec}(x)+i)}{2x}-\frac{\left(\sqrt{\frac{x^2-1}{x^2}}\,x+i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arcsec}(x)-i)}{2x}-\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\Big($

input
```
int(arcsec(x)*(x^2-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(((x^2-1)/x^2)^(1/2)*x-I)/x*csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)+I)-1/2
*(((x^2-1)/x^2)^(1/2)*x+I)/x*csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)-I)-csgn(x*
(1-1/x^2)^(1/2))*(arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2)))-arcsec(x)*ln(1
-I*(1/x+I*(1-1/x^2)^(1/2)))-I*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))+I*dilog(1
-I*(1/x+I*(1-1/x^2)^(1/2))))
```

3.683.5 Fricas [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x^2}\,dx$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

3.683.6 Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}\,\mathrm{asec}\,(x)}{x^2}\,dx$$

input
```
integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**2, x)
```

3.683.7 Maxima [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x^2}\,dx$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

3.683.8 Giac [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x^2}\,dx$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

3.683.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)\sqrt{x^2-1}}{x^2}\,dx$$

input
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^2,x)
```

output
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^2, x)
```

3.684 $\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx$

3.684.1 Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{3+2x^4}{12x\sqrt{x^2}} - \frac{5\sqrt{-1+x^2}\csc^{-1}(x)}{2x^2} - \frac{5(-1+x^2)^{3/2}\csc^{-1}(x)}{3x^2} + \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{3x^2} - \frac{5x\csc^{-1}(x)^2}{4\sqrt{x^2}} - \frac{7x\log(x)}{3\sqrt{x^2}}$$

output
```
-5/3*(x^2-1)^(3/2)*arccsc(x)/x^2+1/3*(x^2-1)^(5/2)*arccsc(x)/x^2+1/12*(2*x
^4+3)/x/(x^2)^(1/2)-5/4*x*arccsc(x)^2/(x^2)^(1/2)-7/3*x*ln(x)/(x^2)^(1/2)-
5/2*arccsc(x)*(x^2-1)^(1/2)/x^2
```

3.684.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{\sqrt{-1+x^2}\left(4x^2 - 30\csc^{-1}(x)^2 - 3\cos\left(2\csc^{-1}(x)\right) + 48\log\left(\frac{1}{x}\right) - 8\log(x) + \right.}{24\sqrt{1-\frac{1}{x^2}}\,x}$$

input
```
Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

output
```
(Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48*Log[x^(-
1)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x^2) - 6*Sin[2*ArcC
sc[x]])))/(24*Sqrt[1 - x^(-2)]*x)
```

3.684.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {5766, 5200, 243, 49, 2009, 5200, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2-1\right)^{5/2}\csc^{-1}(x)}{x^3}dx$$

$$\Big\downarrow \text{5766}$$

$$-\frac{\sqrt{x^2}\int \left(1-\frac{1}{x^2}\right)^{5/2}x^4\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5200}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{1}{3}\int \left(1-\frac{1}{x^2}\right)^2x^3d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{1}{6}\int \left(1-\frac{1}{x^2}\right)^2x^2d\frac{1}{x^2}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{1}{6}\int \left(x^2-2x+1\right)d\frac{1}{x^2}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)+\frac{1}{6}\left(\frac{1}{x^2}-2\log\left(\frac{1}{x^2}\right)-x\right)\right)}{x}$$

$$\Big\downarrow \text{5200}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\int \sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\int \left(1-\frac{1}{x^2}\right)xd\frac{1}{x}-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\cdots}{x}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\int \sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\int \left(x-\frac{1}{x}\right)d\frac{1}{x}-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\cdots}{x}$$

3.684. $\int \frac{\left(-1+x^2\right)^{5/2}\csc^{-1}(x)}{x^3}dx$

$\downarrow$ 2009

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\int\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right.}{x}$$

$\downarrow$ 5156

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\left(\frac{1}{2}\int\frac{\arcsin\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{1}{2}\int\frac{1}{x}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)}{2x}\right)-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(\right.\right.}{x}$$

$\downarrow$ 15

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\left(\frac{1}{2}\int\frac{\arcsin\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)}{2x}-\frac{1}{4x^2}\right)-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\right.\right.}{x}$$

$\downarrow$ 5152

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-3\left(\frac{\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)}{2x}+\frac{1}{4}\arcsin\left(\frac{1}{x}\right)^2-\frac{1}{4x^2}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right.\right.}{x}$$

input
```
Int[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

output
```
-((Sqrt[x^2]*(-1/3*((1 - x^(-2))^(5/2)*x^3*ArcSin[x^(-1)]) + (x^(-2) - x -
 2*Log[x^(-2)])/6 - (5*(-1/2*1/x^2 - (1 - x^(-2))^(3/2)*x*ArcSin[x^(-1)] -
 3*(-1/4*1/x^2 + (Sqrt[1 - x^(-2)]*ArcSin[x^(-1)])/(2*x) + ArcSin[x^(-1)]^
2/4) + Log[x^(-1)]))/3))/x)
```

3.684.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

3.684. $\int \frac{\left(-1+x^2\right)^{5/2}\csc^{-1}(x)}{x^3}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5200
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5766
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

3.684. $\int \frac{\left(-1+x^2\right)^{5/2}\csc^{-1}(x)}{x^3}\,dx$

3.684.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.58

method	result
default	$-\frac{5\,\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\mathrm{arccsc}(x)^2}{4}+\frac{\left(-2\sqrt{\frac{x^2-1}{x^2}}\,x+ix^2-2i\right)\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(2\,\mathrm{arccsc}(x)+i)}{16x^2}-\frac{\left(2\sqrt{\frac{x^2-1}{x^2}}\,x+ix^2-2i\right)\mathrm{csgn}\left(x\right.}{16x^2}$

input
```
int((x^2-1)^(5/2)*arccsc(x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-5/4*csgn(x*(1-1/x^2)^(1/2))*arccsc(x)^2+1/16*(-2*((x^2-1)/x^2)^(1/2)*x+I*
x^2-2*I)/x^2*csgn(x*(1-1/x^2)^(1/2))*(2*arccsc(x)+I)-1/16*(2*((x^2-1)/x^2)
^(1/2)*x+I*x^2-2*I)*csgn(x*(1-1/x^2)^(1/2))*(-I+2*arccsc(x))/x^2-14/3*I*cs
gn(x*(1-1/x^2)^(1/2))*arccsc(x)+1/6*(((x^2-1)/x^2)^(1/2)*x^3-7*((x^2-1)/x^
2)^(1/2)*x+7*I)*csgn(x*(1-1/x^2)^(1/2))*(2*arccsc(x)*x^4+((x^2-1)/x^2)^(1/
2)*x^3-30*arccsc(x)*x^2-7*((x^2-1)/x^2)^(1/2)*x+126*arccsc(x)-7*I)/(x^4-15
*x^2+63)+7/3*csgn(x*(1-1/x^2)^(1/2))*ln((I/x+(1-1/x^2)^(1/2))^2-1)
```

3.684.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{2\,x^4-15\,x^2\,\mathrm{arccsc}\,(x)^2-28\,x^2\log{(x)}+2\,(2\,x^4-14\,x^2-3)\sqrt{x^2-1}\,\mathrm{arccsc}\,(}{12\,x^2}$$

input
```
integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")
```

output
```
1/12*(2*x^4 - 15*x^2*arccsc(x)^2 - 28*x^2*log(x) + 2*(2*x^4 - 14*x^2 - 3)*
sqrt(x^2 - 1)*arccsc(x) + 3)/x^2
```

3.684.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
```

output
```
Timed out
```

3.684.7 Maxima [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

input
```
integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

3.684.8 Giac [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

input
```
integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)\,(x^2-1)^{5/2}}{x^3}\,dx$$

input
```
int((asin(1/x)*(x^2 - 1)^(5/2))/x^3,x)
```

output
```
int((asin(1/x)*(x^2 - 1)^(5/2))/x^3, x)
```

3.685 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx$

3.685.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = -\frac{1}{9\left(x^2\right)^{3/2}}+\frac{1}{3\sqrt{x^2}}+\frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

output
```
1/3*(x^2-1)^(3/2)*arcsec(x)/x^3+1/3/(x^2)^(1/2)-1/9/x^2/(x^2)^(1/2)
```

3.685.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}x(-1+3x^2)+3(-1+x^2)^2\sec^{-1}(x)}{9x^3\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x^3*Sqrt[-
1 + x^2])
```

3.685.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5761, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}\sec^{-1}(x)}{x^4}\,dx$$

$$\Big\downarrow \text{5761}$$

$$\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}-\frac{x\int -\frac{1-x^2}{3x^4}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\int \frac{1-x^2}{x^4}dx}{3\sqrt{x^2}}+\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{244}$$

$$\frac{x\int \left(\frac{1}{x^4}-\frac{1}{x^2}\right)dx}{3\sqrt{x^2}}+\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{x}-\frac{1}{3x^3}\right)x}{3\sqrt{x^2}}+\frac{\left(x^2-1\right)^{3/2}\sec^{-1}(x)}{3x^3}$$

input
```
Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

output
```
((-1/3*1/x^3 + x^(-1))*x)/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x])/(3*
x^3)
```

3.685.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.685.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.07

method	result	size
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(3\,\operatorname{arcsec}(x)x^4+3\sqrt{\frac{x^2-1}{x^2}}\,x^3-6\,\operatorname{arcsec}(x)x^2-\sqrt{\frac{x^2-1}{x^2}}\,x+3\,\operatorname{arcsec}(x)\right)}{9(x^2-1)x^2}$	85

input
```
int(arcsec(x)*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)/x^2*(3*arcsec(x)*x
^4+3*((x^2-1)/x^2)^(1/2)*x^3-6*arcsec(x)*x^2-((x^2-1)/x^2)^(1/2)*x+3*arcse
c(x))
```

3.685. $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx$

3.685.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{3\,(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)+3\,x^2-1}{9\,x^3}$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/9*(3*(x^2 - 1)^(3/2)*arcsec(x) + 3*x^2 - 1)/x^3
```

3.685.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.685.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)}{3\,x^3}+\frac{3\,x^2-1}{9\,x^3}$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(x^2 - 1)^(3/2)*arcsec(x)/x^3 + 1/9*(3*x^2 - 1)/x^3
```

3.685.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = -\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{3\,\mathrm{sgn}\,(x)} + \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^4+1\right)\arccos\left(\frac{1}{x}\right)}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3} + \frac{3\,x^2-1}{9\,x^3\mathrm{sgn}\,(x)}$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2/3*arctan(-x + sqrt(x^2 - 1))/sgn(x) + 2/3*(3*(x - sqrt(x^2 - 1))^4 + 1)
*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2 - 1)/(x^3*sgn(x))
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)\sqrt{x^2-1}}{x^4}\,dx$$

input
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^4,x)
```

output
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^4, x)
```

3.686 $\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

3.686.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{\sqrt{x^2}}{6(1-x^2)} + \frac{5}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}$$

output
```
5/6*arccoth((x^2)^(1/2))-1/3*x*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/(-x
^2+1)+2/3*x*arcsec(x)/(x^2-1)^(1/2)
```

3.686.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{4x(-3+2x^2)\sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(-2x-5(-1+x^2)\log(1-x)+5(-1+x^2)\log(1}{12(-1+x^2)^{3/2}}$$

input
```
Integrate[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

output
```
(4*x*(-3 + 2*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x - 5*(-1 + x^2)*Log[
1 - x] + 5*(-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

3.686.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(x)}{(x^2-1)^{5/2}}dx$$

$$\Big\downarrow \text{5751}$$

$$-\frac{x\int -\frac{3-2x^2}{3(1-x^2)^2}dx}{\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\int \frac{3-2x^2}{(1-x^2)^2}dx}{3\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{x\left(\frac{5}{2}\int \frac{1}{1-x^2}dx+\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(\frac{5\text{arctanh}(x)}{2}+\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

input
```
Int[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

output
```
-1/3*(x*ArcSec[x])/(-1 + x^2)^(3/2) + (2*x*ArcSec[x])/(3*Sqrt[-1 + x^2]) +
 (x*(x/(2*(1 - x^2)) + (5*ArcTanh[x])/2))/(3*Sqrt[x^2])
```

3.686.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.686.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

method	result
default	$\frac{\sqrt{\frac{x^2-1}{x^2}}\, x^2 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(4 \operatorname{arcsec}(x) x^2-\sqrt{\frac{x^2-1}{x^2}}\, x-6 \operatorname{arcsec}(x)\right)}{6(x^2-1)^2}+\frac{5 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right) \ln\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^2}}+1\right)}{6}-\frac{5 \operatorname{csgn}\left(x\sqrt{1\right.}}{}$

input
```
int(arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((x^2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(4*arcsec(x)
*x^2-((x^2-1)/x^2)^(1/2)*x-6*arcsec(x))+5/6*csgn(x*(1-1/x^2)^(1/2))*ln(1/x
+I*(1-1/x^2)^(1/2)+1)-5/6*csgn(x*(1-1/x^2)^(1/2))*ln(1/x+I*(1-1/x^2)^(1/2)
-1)
```

3.686.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx =$$
$$-\frac{2\,x^3-4\,(2\,x^3-3\,x)\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)-5\,(x^4-2\,x^2+1)\log{(x+1)}+5\,(x^4-2\,x^2+1)\log{(x-1)}-2\,x}{12\,(x^4-2\,x^2+1)}$$

input
```
integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(2*x^3 - 4*(2*x^3 - 3*x)*sqrt(x^2 - 1)*arcsec(x) - 5*(x^4 - 2*x^2 +
1)*log(x + 1) + 5*(x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

3.686.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

3.686.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{x^2-1}}-\frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\mathrm{arcsec}\,(x) - \frac{x}{6\,(x^2-1)}+\frac{5}{12}\,\log{(x+1)}-\frac{5}{12}\,\log{(x-1)}$$

input
```
integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/sqrt(x^2 - 1) - x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) +
5/12*log(x + 1) - 5/12*log(x - 1)
```

3.686.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{(2\,x^2-3)x\arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}} + \frac{5\,\log\left(|x+1|\right)}{12\,\mathrm{sgn}\left(x\right)} - \frac{5\,\log\left(|x-1|\right)}{12\,\mathrm{sgn}\left(x\right)} - \frac{x}{6\,(x^2-1)\mathrm{sgn}\left(x\right)}$$

input
```
integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*x^2 - 3)*x*arccos(1/x)/(x^2 - 1)^(3/2) + 5/12*log(abs(x + 1))/sgn(x
) - 5/12*log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

3.686.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

input
```
int(acos(1/x)/(x^2 - 1)^(5/2),x)
```

output
```
int(acos(1/x)/(x^2 - 1)^(5/2), x)
```

3.687 $\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

3.687.1 Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{1}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}}$$

output
```
-1/6*arccoth((x^2)^(1/2))-1/3*x^3*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/
(-x^2+1)
```

3.687.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{-4x^3 \sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}\,x(-2x + (-1+x^2)\log(1-x) - (-1+x^2)\log(1+x))}{12(-1+x^2)^{3/2}}$$

input
```
Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
(-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 - x] - (-1
 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

3.687.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5761, 27, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sec^{-1}(x)}{(x^2-1)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5761}$$

$$-\frac{x \int -\frac{x^2}{3(1-x^2)^2}dx}{\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x \int \frac{x^2}{(1-x^2)^2}dx}{3\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{x \left(\frac{x}{2(1-x^2)}-\frac{1}{2}\int \frac{1}{1-x^2}dx\right)}{3\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x \left(\frac{x}{2(1-x^2)}-\frac{\text{arctanh}(x)}{2}\right)}{3\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

input
```
Int[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-1/3*(x^3*ArcSec[x])/(-1 + x^2)^(3/2) + (x*(x/(2*(1 - x^2)) - ArcTanh[x]/2
))/(3*Sqrt[x^2])
```

3.687.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.687.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.63

method	result
default	$-\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(2\operatorname{arcsec}(x)x^4\sqrt{\frac{x^2-1}{x^2}}-\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right)x^4+2\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right)x^2+x^3-\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right.\right.}{6(x^2-1)^2}$

input
```
int(x^2*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(x*(1-1/x^2)^(1/2))*(2*arcsec(x)*x^4*((x^2-1)/x^2)^(1/2)-ln(1/(1-
1/x^2)^(1/2)-1/x/(1-1/x^2)^(1/2))*x^4+2*ln(1/(1-1/x^2)^(1/2)-1/x/(1-1/x^2)
^(1/2))*x^2+x^3-ln(1/(1-1/x^2)^(1/2)-1/x/(1-1/x^2)^(1/2))-x)/(x^2-1)^2
```

3.687.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = -\frac{4\sqrt{x^2-1}x^3 \operatorname{arcsec}(x) + 2x^3 + (x^4-2x^2+1)\log(x+1) - (x^4-2x^2+1)\log(x-1) - 2x}{12(x^4-2x^2+1)}$$

input
```
integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(x^2 - 1)*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*log(x + 1
) - (x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

3.687.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

3.687.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = -\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}} + \frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\operatorname{arcsec}(x) - \frac{x}{6(x^2-1)} - \frac{1}{12}\log(x+1) + \frac{1}{12}\log(x-1)$$

input
```
integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(x/sqrt(x^2 - 1) + x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) - 1
/12*log(x + 1) + 1/12*log(x - 1)
```

3.687.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = -\frac{x^3 \arccos\left(\frac{1}{x}\right)}{3(x^2-1)^{\frac{3}{2}}} - \frac{\log(|x+1|)}{12\operatorname{sgn}(x)} + \frac{\log(|x-1|)}{12\operatorname{sgn}(x)} - \frac{x}{6(x^2-1)\operatorname{sgn}(x)}$$

input
```
integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x^3*arccos(1/x)/(x^2 - 1)^(3/2) - 1/12*log(abs(x + 1))/sgn(x) + 1/12*
log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = \int \frac{x^2 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\, dx$$

input
```
int((x^2*acos(1/x))/(x^2 - 1)^(5/2),x)
```

output
```
int((x^2*acos(1/x))/(x^2 - 1)^(5/2), x)
```

3.688 $\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

3.688.1 Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{x}{6\sqrt{x^2}\,(1-x^2)} - \frac{\sec^{-1}(x)}{3\,(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log\left(-1+x^2\right)}{3\sqrt{x^2}}$$

output
```
-1/3*arcsec(x)/(x^2-1)^(3/2)+1/6*x/(-x^2+1)/(x^2)^(1/2)-2/3*x*ln(x)/(x^2)^
(1/2)+1/3*x*ln(x^2-1)/(x^2)^(1/2)-arcsec(x)/(x^2-1)^(1/2)
```

3.688.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{-2(-2+3x^2)\sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}\,x(-1+2(-1+x^2)\log(-1+x) - 4(-1+x^2)\log(}{6\,(-1+x^2)^{3/2}}$$

input
```
Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
(-2*(-2 + 3*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-1 + 2*(-1 + x^2)*Log[-1
+ x] - 4*(-1 + x^2)*Log[x] - 2*Log[1 + x] + 2*x^2*Log[1 + x]))/(6*(-1 + x^
2)^(3/2))
```

3.688.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5761, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sec^{-1}(x)}{(x^2-1)^{5/2}}dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{x\int \frac{2-3x^2}{3x(1-x^2)^2}dx}{\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\int \frac{2-3x^2}{x(1-x^2)^2}dx}{3\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{354}$$

$$-\frac{x\int \frac{2-3x^2}{x^2(1-x^2)^2}dx^2}{6\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{x\int \left(\frac{2}{x^2}-\frac{2}{x^2-1}-\frac{1}{(x^2-1)^2}\right)dx^2}{6\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x\left(\frac{1}{x^2-1}+2\log\left(x^2\right)-2\log\left(1-x^2\right)\right)}{6\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

input
```
Int[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-1/3*ArcSec[x]/(-1 + x^2)^(3/2) - ArcSec[x]/Sqrt[-1 + x^2] - (x*((-1 + x^2
)^(-1) + 2*Log[x^2] - 2*Log[1 - x^2]))/(6*Sqrt[x^2])
```

3.688.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.688.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^{2}}}\right)\left(6x^{3}\,\mathrm{arcsec}(x)\sqrt{\frac{x^{2}-1}{x^{2}}}-2\ln\left(1-\frac{1}{x^{2}}\right)x^{4}+x^{4}-4\,\mathrm{arcsec}(x)x\sqrt{\frac{x^{2}-1}{x^{2}}}+4\ln\left(1-\frac{1}{x^{2}}\right)x^{2}-x^{2}-2\ln\left(1-\frac{1}{x^{2}}\right)\right)}{6\left(x^{2}-1\right)^{2}}$	101

input
```
int(x^3*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

3.688. $\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

output
```
-1/6*csgn(x*(1-1/x^2)^(1/2))*(6*x^3*arcsec(x)*((x^2-1)/x^2)^(1/2)-2*ln(1-1
/x^2)*x^4+x^4-4*arcsec(x)*x*((x^2-1)/x^2)^(1/2)+4*ln(1-1/x^2)*x^2-x^2-2*ln
(1-1/x^2))/(x^2-1)^2
```

3.688.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{2\,(3\,x^2-2)\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)+x^2-2\,(x^4-2\,x^2+1)\log\left(x^2-1\right)+4\,(x^4-2\,x^2+1)\log\left(x\right)-1}{6\,(x^4-2\,x^2+1)}$$

input
```
integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^2 + 1)*lo
g(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^2 + 1)
```

3.688.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

3.688.7 Maxima [F]

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2), x)
```

3.688.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{(3\,x^2-2)\arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}} - \frac{\log(x^2)}{3\operatorname{sgn}(x)} + \frac{\log(|x^2-1|)}{3\operatorname{sgn}(x)} - \frac{2\,x^2-1}{6\,(x^2-1)\operatorname{sgn}(x)}$$

input
```
integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*log(x^2)/sgn(x) + 1/3*l
og(abs(x^2 - 1))/sgn(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sgn(x))
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

input
```
int((x^3*acos(1/x))/(x^2 - 1)^(5/2),x)
```

output
```
int((x^3*acos(1/x))/(x^2 - 1)^(5/2), x)
```

3.689 $\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$

3.689.1 Optimal result

Integrand size = 15, antiderivative size = 175

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx = \frac{\sqrt{x^2}(2-3x^2)}{6(-1+x^2)} - \frac{13}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{5x^3 \sec^{-1}(x)}{6(-1+x^2)^{3/2}} + \frac{x^5 \sec^{-1}(x)}{2(-1+x^2)^{3/2}} - \frac{5x \sec^{-1}(x)}{2\sqrt{-1+x^2}} - \frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{5i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x} - \frac{5i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}$$

output
```
-13/6*arccoth((x^2)^(1/2))-5/6*x^3*arcsec(x)/(x^2-1)^(3/2)+1/2*x^5*arcsec(
x)/(x^2-1)^(3/2)+1/6*(-3*x^2+2)*(x^2)^(1/2)/(x^2-1)-5*I*arcsec(x)*arctan(1
/x+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x+5/2*I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1
/2)))*(x^2)^(1/2)/x-5/2*I*polylog(2,I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)
/x-5/2*x*arcsec(x)/(x^2-1)^(1/2)
```

3.689.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 383 vs. 2(175) = 350.

Time = 1.25 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.19

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx =$$
$$-\frac{x^5\left(22\sec^{-1}(x)+40\sec^{-1}(x)\cos\left(2\sec^{-1}(x)\right)-30\sec^{-1}(x)\cos\left(4\sec^{-1}(x)\right)-30\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)\log\left(1\right.\right.}{}$$

input
```
Integrate[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-1/96*(x^5*(22*ArcSec[x] + 40*ArcSec[x]*Cos[2*ArcSec[x]] - 30*ArcSec[x]*Co
s[4*ArcSec[x]] - 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]
+ 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] + 26*Sqrt[1 - x
^(-2)]*Log[Cos[ArcSec[x]/2]] - 26*Sqrt[1 - x^(-2)]*Log[Sin[ArcSec[x]/2]] +
 16*Sin[2*ArcSec[x]] - (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec
[x])]*Sin[2*ArcSec[x]]^2 + (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSe
c[x])]*Sin[2*ArcSec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*
ArcSec[x]] + 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 13
*Log[Cos[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/2]]*Sin[3*A
rcSec[x]] - 4*Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*S
in[5*ArcSec[x]] - 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]]
 - 13*Log[Cos[ArcSec[x]/2]]*Sin[5*ArcSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Si
n[5*ArcSec[x]]))/(-1 + x^2)^(3/2)
```

3.689.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5765, 5205, 253, 264, 219, 5209, 215, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \sec^{-1}(x)}{(x^2-1)^{5/2}}\,dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \frac{x^3\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5205}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{\left(1-\frac{1}{x^2}\right)^2}d\frac{1}{x}-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right)}{x}$$

$$\Big\downarrow \text{253}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}+\frac{1}{2}\left(-\frac{3}{2}\int \frac{x^2}{1-\frac{1}{x^2}}d\frac{1}{x}-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right)}{x}$$

$$\Big\downarrow \text{264}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}+\frac{1}{2}\left(-\frac{3}{2}\left(\int \frac{1}{1-\frac{1}{x^2}}d\frac{1}{x}-x\right)-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right)}{x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-x\right)-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)\right)}{x}$$

$$\Big\downarrow \text{5209}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}+\frac{1}{3}\int \frac{1}{\left(1-\frac{1}{x^2}\right)^2}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-x\right)-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)\cdots\right.}{x}$$

$$\Big\downarrow \text{215}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}+\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{1}{x}+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-x\cdots\right.\right.\right.}{x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-\cdots\right.\right.\right.}{x}$$

3.689. $\int \frac{x^6\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

$$\Big\downarrow 5209$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\int \frac{1}{1-\frac{1}{x^2}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)\right)\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}-}{x}$$

$$\Big\downarrow 219$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)+\text{arctanh}\left(\frac{1}{x}\right)\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right.}{x}$$

$$\Big\downarrow 5219$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(-\int x\arccos\left(\frac{1}{x}\right)d\arccos\left(\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)+\text{arctanh}\left(\frac{1}{x}\right)\right)\right.\right.}{x}$$

$$\Big\downarrow 3042$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(-\int \arccos\left(\frac{1}{x}\right)\csc\left(\arccos\left(\frac{1}{x}\right)+\frac{\pi}{2}\right)d\arccos\left(\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right.}\right.\right.\right.\right.}{x}$$

$$\Big\downarrow 4669$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(\int \log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)-\int \log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)+2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\,\text{ar}}\right.\right.\right.\right.}{}$$

$$\Big\downarrow 2715$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(-i\int x\log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+i\int x\log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+2i\arccos\left(\frac{1}{x}\right)\text{arctan}\right.\right.\right.}{}$$

$$\Big\downarrow 2838$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\left(2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\arccos\left(\frac{1}{x}\right)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos\left(\frac{1}{x}\right)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos\left(\frac{1}{x}\right)}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}\right.\right.\right.}{}$$

3.689. $\int \frac{x^6\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

input
```
Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-((Sqrt[x^2]*(-1/2*(x^2*ArcCos[x^(-1)])/(1 - x^(-2))^(3/2) + (-1/2*x/(1 -
x^(-2)) - (3*(-x + ArcTanh[x^(-1)]))/2)/2 + (5*(ArcCos[x^(-1)]/(3*(1 - x^(
-2))^(3/2)) + ArcCos[x^(-1)]/Sqrt[1 - x^(-2)] + (2*I)*ArcCos[x^(-1)]*ArcTa
n[E^(I*ArcCos[x^(-1)])] + (1/(2*(1 - x^(-2))*x) + ArcTanh[x^(-1)]/2)/3 + A
rcTanh[x^(-1)] - I*PolyLog[2, (-I)*E^(I*ArcCos[x^(-1)])] + I*PolyLog[2, I*
E^(I*ArcCos[x^(-1)])]))/2))/x)
```

3.689.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.689. $\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

3.689.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.41

method	result
default	$\frac{\sqrt{\frac{x^2-1}{x^2}}\, x^2 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(3 \operatorname{arcsec}(x) x^4-3\sqrt{\frac{x^2-1}{x^2}}\, x^3-20 \operatorname{arcsec}(x) x^2+2\sqrt{\frac{x^2-1}{x^2}}\, x+15 \operatorname{arcsec}(x)\right)}{6(x^2-1)^2}+\frac{i \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(15 i \operatorname{a}\right.}{}$

input
```
int(x^6*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((x^2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(3*arcsec(x)
*x^4-3*((x^2-1)/x^2)^(1/2)*x^3-20*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x+15
*arcsec(x))+1/6*I*csgn(x*(1-1/x^2)^(1/2))*(15*I*arcsec(x)*ln(1+I*(1/x+I*(1
-1/x^2)^(1/2)))-15*I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))+13*I*ln(1/x
+I*(1-1/x^2)^(1/2)+1)-13*I*ln(1/x+I*(1-1/x^2)^(1/2)-1)+15*dilog(1+I*(1/x+I
*(1-1/x^2)^(1/2)))-15*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2))))
```

3.689.5 Fricas [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\, dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*x^6*arcsec(x)/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

3.689.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

3.689.7 Maxima [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

3.689.8 Giac [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

input
```
int((x^6*acos(1/x))/(x^2 - 1)^(5/2),x)
```

output
```
int((x^6*acos(1/x))/(x^2 - 1)^(5/2), x)
```

3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$

3.690.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{1}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}$$

output
```
1/(x^2)^(1/2)+arcsec(x)*(x^2-1)^(1/2)/x
```

3.690.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}x + (-1+x^2)\sec^{-1}(x)}{x\sqrt{-1+x^2}}$$

input
```
Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])
```

3.690.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5761, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{x^2-1}}dx$$

$$\Big\downarrow \text{5761}$$

$$\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}-\frac{x\int \frac{1}{x^2}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

input
```
Int[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

3.690.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.690.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

method	result	size
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(\operatorname{arcsec}(x)x^2-\operatorname{arcsec}(x)+\sqrt{\frac{x^2-1}{x^2}}\,x\right)}{x^2-1}$	56

input
```
int(arcsec(x)/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)*(arcsec(x)*x^2-arcsec(
x)+((x^2-1)/x^2)^(1/2)*x)
```

3.690.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)+1}{x}$$

input
```
integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(x^2 - 1)*arcsec(x) + 1)/x
```

3.690.6 Sympy [F]

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{asec}(x)}{x^2\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(asec(x)/x**2/(x**2-1)**(1/2),x)
```

output
```
Integral(asec(x)/(x**2*sqrt((x - 1)*(x + 1))), x)
```

3.690.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x} + \frac{1}{x}$$

input
```
integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*arcsec(x)/x + 1/x
```

3.690.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(19) = 38.

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{2\arccos\left(\frac{1}{x}\right)}{\left(x-\sqrt{x^2-1}\right)^2+1} - \frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\mathrm{sgn}\,(x)} + \frac{1}{x\mathrm{sgn}\,(x)}$$

input
```
integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
2*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1) - 2*arctan(-x + sqrt(x^2 - 1))/s
gn(x) + 1/(x*sgn(x))
```

3.690.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{x^2\,\sqrt{x^2-1}}\,dx$$

input
```
int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)),x)
```

output
```
int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)), x)
```

3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}} \, dx$

3.691.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}} \, dx = -\frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2}}{6(-1+x^2)} - \frac{11}{6}\coth^{-1}\left(\sqrt{x^2}\right) + \frac{(3-12x^2+8x^4)\csc^{-1}(x)}{3x(-1+x^2)^{3/2}}$$

output
```
-11/6*arccoth((x^2)^(1/2))+1/3*(8*x^4-12*x^2+3)*arccsc(x)/x/(x^2-1)^(3/2)-
1/(x^2)^(1/2)+1/6*(x^2)^(1/2)/(x^2-1)
```

3.691.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}} \, dx = \frac{4(3-12x^2+8x^4)\csc^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(12-10x^2+11x(-1+x^2)\log(1-x) - 1}{12x(-1+x^2)^{3/2}}$$

input
```
Integrate[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

output
```
(4*(3 - 12*x^2 + 8*x^4)*ArcCsc[x] + Sqrt[1 - x^(-2)]*x*(12 - 10*x^2 + 11*x
*(-1 + x^2)*Log[1 - x] - 11*x*(-1 + x^2)*Log[1 + x]))/(12*x*(-1 + x^2)^(3/
2))
```

3.691.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5762, 27, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{-1}(x)}{x^2 (x^2-1)^{5/2}} dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{x\int \frac{8x^4-12x^2+3}{3x^2(1-x^2)^2}dx}{\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\int \frac{8x^4-12x^2+3}{x^2(1-x^2)^2}dx}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{1582}$$

$$\frac{x\left(-\frac{1}{2}\int -\frac{6-17x^2}{x^2(1-x^2)}dx-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(\frac{1}{2}\int \frac{6-17x^2}{x^2(1-x^2)}dx-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{x\left(\frac{1}{2}\left(-11\int \frac{1}{1-x^2}dx-\frac{6}{x}\right)-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(\frac{1}{2}\left(-11\text{arctanh}(x)-\frac{6}{x}\right)-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}}$$

input
```
Int[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

output
```
ArcCsc[x]/(x*(-1 + x^2)^(3/2)) - (4*x*ArcCsc[x])/(3*(-1 + x^2)^(3/2)) + (8
*x*ArcCsc[x])/(3*Sqrt[-1 + x^2]) + (x*(-1/2*x/(1 - x^2) + (-6/x - 11*ArcTa
nh[x])/2))/(3*Sqrt[x^2])
```

3.691.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.691. $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx$

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.691.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arccsc}(x)+i)\left(\sqrt{\frac{x^2-1}{x^2}}\,x+i\right)}{2x}+\frac{\left(\sqrt{\frac{x^2-1}{x^2}}\,x-i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arccsc}(x)-i)}{2x}+\frac{\sqrt{\frac{x^2-1}{x^2}}\,x^2\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(}{}$

input
```
int(arccsc(x)/x^2/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(x*(1-1/x^2)^(1/2))*(arccsc(x)+I)*(((x^2-1)/x^2)^(1/2)*x+I)/x+1/2*
(((x^2-1)/x^2)^(1/2)*x-I)/x*csgn(x*(1-1/x^2)^(1/2))*(arccsc(x)-I)+1/6*((x^
2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(10*arccsc(x)*x^2+((
x^2-1)/x^2)^(1/2)*x-12*arccsc(x))-11/6*csgn(x*(1-1/x^2)^(1/2))*ln(I/x+(1-1
/x^2)^(1/2)+I)+11/6*csgn(x*(1-1/x^2)^(1/2))*ln(I/x+(1-1/x^2)^(1/2)-I)
```

3.691.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\,dx = -\frac{10\,x^4-4\,(8\,x^4-12\,x^2+3)\sqrt{x^2-1}\operatorname{arccsc}(x)-22\,x^2+11\,(x^5-2\,x^3+x)\log(x+1)-11\,(x^5-2\,x^3+x}{12\,(x^5-2\,x^3+x)}$$

input
```
integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*sqrt(x^2 - 1)*arccsc(x) - 22*x^2 +
11*(x^5 - 2*x^3 + x)*log(x + 1) - 11*(x^5 - 2*x^3 + x)*log(x - 1) + 12)/(x
^5 - 2*x^3 + x)
```

3.691.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

3.691.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(56) = 112.

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\, dx = \frac{32\, x^4 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) - (x^3-x)\sqrt{x+1}\sqrt{x-1}\left(\frac{2\,(5\,x^2-6)}{x^3-x} + 11\,\log\left(x+\right.\right.}{12\,(x^3-x)}$$

input
```
integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(32*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - (x^3 - x)*sqrt(x + 1)*s
qrt(x - 1)*(2*(5*x^2 - 6)/(x^3 - x) + 11*log(x + 1) - 11*log(x - 1)) - 48*
x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 12*arctan2(1, sqrt(x + 1)*sqrt(x
 - 1)))/((x^3 - x)*sqrt(x + 1)*sqrt(x - 1))
```

3.691.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{(5x^2-6)x}{(x^2-1)^{\frac{3}{2}}} + \frac{6}{\left(x-\sqrt{x^2-1}\right)^2+1}\right)\arcsin\left(\frac{1}{x}\right) + \frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\operatorname{sgn}(x)} - \frac{11\log(|x+1|)}{12\operatorname{sgn}(x)} + \frac{11\log(|x-1|)}{12\operatorname{sgn}(x)} - \frac{5x^2-6}{6(x^3-x)\operatorname{sgn}(x)}$$

input
```
integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
1/3*((5*x^2 - 6)*x/(x^2 - 1)^(3/2) + 6/((x - sqrt(x^2 - 1))^2 + 1))*arcsin
(1/x) + 2*arctan(-x + sqrt(x^2 - 1))/sgn(x) - 11/12*log(abs(x + 1))/sgn(x)
 + 11/12*log(abs(x - 1))/sgn(x) - 1/6*(5*x^2 - 6)/((x^3 - x)*sgn(x))
```

3.691.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)}{x^2(x^2-1)^{5/2}}\,dx$$

input
```
int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)),x)
```

output
```
int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)), x)
```

3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$

3.692.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24\sqrt{-1+x^2}}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}-\frac{12\sqrt{-1+x^2}\csc^{-1}(x)^2}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}+\frac{\sqrt{-1+x^2}\csc^{-1}(x)^4}{x}$$

output
```
24*arccsc(x)/(x^2)^(1/2)-4*arccsc(x)^3/(x^2)^(1/2)+24*(x^2-1)^(1/2)/x-12*a
rccsc(x)^2*(x^2-1)^(1/2)/x+arccsc(x)^4*(x^2-1)^(1/2)/x
```

3.692.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24(-1+x^2)+24\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)-12(-1+x^2)\csc^{-1}(x)^2-4\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)^3+(-1+x^2)\csc^{-1}}{x\sqrt{-1+x^2}}$$

input
```
Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
(24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*ArcCsc[x]
^2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCsc[x]^4)/(x*Sqrt[-1
 + x^2])
```

3.692.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5766, 5182, 5130, 5182, 5130, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{x^2-1}}dx$$

$$\Big\downarrow \text{5766}$$

$$-\frac{\sqrt{x^2}\int \frac{\arcsin\left(\frac{1}{x}\right)^4}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5182}$$

$$-\frac{\sqrt{x^2}\left(4\int \arcsin\left(\frac{1}{x}\right)^3d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{5130}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\int \frac{\arcsin\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{5182}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\left(2\int \arcsin\left(\frac{1}{x}\right)d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^2\right)\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{5130}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\left(2\left(\frac{\arcsin\left(\frac{1}{x}\right)}{x}-\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^2\right)\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\left(2\left(\frac{\arcsin\left(\frac{1}{x}\right)}{x}+\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^2\right)\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

input
```
Int[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
-((Sqrt[x^2]*(-(Sqrt[1 - x^(-2)]*ArcSin[x^(-1)]^4) + 4*(ArcSin[x^(-1)]^3/x
 - 3*(-(Sqrt[1 - x^(-2)]*ArcSin[x^(-1)]^2) + 2*(Sqrt[1 - x^(-2)] + ArcSin[
x^(-1)]/x)))))/x)
```

3.692.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5766
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

3.692.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(\operatorname{arccsc}(x)^4x^2-\operatorname{arccsc}(x)^4-12\operatorname{arccsc}(x)^2x^2+12\operatorname{arccsc}(x)^2-4\sqrt{\frac{x^2-1}{x^2}}\operatorname{arccsc}(x)^3x+24x^2-24+24\operatorname{arccsc}(x\right.}{x^2-1}$

input
```
int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)*(arccsc(x)^4*x^2-arccs
c(x)^4-12*arccsc(x)^2*x^2+12*arccsc(x)^2-4*((x^2-1)/x^2)^(1/2)*arccsc(x)^3
*x+24*x^2-24+24*arccsc(x)*((x^2-1)/x^2)^(1/2)*x)
```

3.692.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = -\frac{4\,\mathrm{arccsc}\,(x)^3-\left(\mathrm{arccsc}\,(x)^4-12\,\mathrm{arccsc}\,(x)^2+24\right)\sqrt{x^2-1}-24\,\mathrm{arccsc}\,(x)}{x}$$

input
```
integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-(4*arccsc(x)^3 - (arccsc(x)^4 - 12*arccsc(x)^2 + 24)*sqrt(x^2 - 1) - 24*a
rccsc(x))/x
```

3.692.6 Sympy [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\mathrm{acsc}^4\,(x)}{x^2\sqrt{(x-1)\,(x+1)}}\,dx$$

input
```
integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

output
```
Integral(acsc(x)**4/(x**2*sqrt((x - 1)*(x + 1))), x)
```

3.692.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\operatorname{arccsc}(x)^4}{x} - 12\sqrt{-\frac{1}{x^2}+1}\operatorname{arccsc}(x)^2 - \frac{4\operatorname{arccsc}(x)^3}{x} + 24\sqrt{-\frac{1}{x^2}+1} + \frac{24\operatorname{arccsc}(x)}{x}$$

input
```
integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4*arccsc(x
)^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

3.692.8 Giac [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{arccsc}(x)^4}{\sqrt{x^2-1}x^2}\,dx$$

input
```
integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2), x)
```

3.692.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)^4}{x^2\sqrt{x^2-1}}\,dx$$

input
```
int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)),x)
```

output
```
int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)), x)
```

3.693 $\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx$

3.693.1 Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \frac{\sqrt{-1+x^2}(-2+17x^2)}{64x^4} - \frac{3\sec^{-1}(x)}{8x\sqrt{x^2}} + \frac{9x\sec^{-1}(x)}{64\sqrt{x^2}} + \frac{(-1+x^2)^2 \sec^{-1}(x)}{8x^3\sqrt{x^2}} - \frac{3\sqrt{-1+x^2}\sec^{-1}(x)^2}{8x^2} - \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{4x^4} + \frac{x\sec^{-1}(x)^3}{8\sqrt{x^2}}$$

output
```
-1/4*(x^2-1)^(3/2)*arcsec(x)^2/x^4-3/8*arcsec(x)/x/(x^2)^(1/2)+9/64*x*arcs
ec(x)/(x^2)^(1/2)+1/8*(x^2-1)^2*arcsec(x)/x^3/(x^2)^(1/2)+1/8*x*arcsec(x)^
3/(x^2)^(1/2)+1/64*(17*x^2-2)*(x^2-1)^(1/2)/x^4-3/8*arcsec(x)^2*(x^2-1)^(1
/2)/x^2
```

3.693.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \frac{\sqrt{-1+x^2}(32\sec^{-1}(x)^3 + 4\sec^{-1}(x)\left(-16\cos\left(2\sec^{-1}(x)\right) + \cos\left(4\sec^{-1}(x)\right)\right)}{}$$

input
```
Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

output
```
(Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x]] + Cos[
4*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] + 8*ArcSec[x]^2*(-8
*Sin[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sqrt[1 - x^(-2)]*x)
```

3.693.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {5765, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-1)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^2 d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5159}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\int \sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2 d\frac{1}{x}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^2}{4x}\right)}{x}$$

$$\Big\downarrow \text{5157}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{1}{2}\int \frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\int \frac{\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}\right)+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^2}{4x}\right)}{x}$$

$$\Big\downarrow \text{5139}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{1}{2}\int \frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}\right)+\frac{\left(1-\frac{1}{x^2}\right)^{3}}{}\right)}{x}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{1}{2}\int \frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}\right)\right)}{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{x^2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}+\frac{1}{2}\left(\frac{1}{2}\arcsin\left(\frac{1}{x}\right)-\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)\right)+\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}\right.}{x}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}-\frac{1}{6}\arccos\left(\frac{1}{x}\right)^3+\frac{1}{2}\left(\frac{1}{2}\arcsin\left(\frac{1}{x}\right)-\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)\right)+\right.}{x}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(-\frac{1}{4}\int\left(1-\frac{1}{x^2}\right)^{3/2}d\frac{1}{x}-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}-\frac{1}{6}\arccos\left(\frac{1}{x}\right)^3+\frac{1}{2}\left(\right.\right.\right.}{x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{3}{4}\int\sqrt{1-\frac{1}{x^2}}d\frac{1}{x}-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4x}\right)-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}-\frac{1}{6}\text{ar}\right.\right.}{x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4x}\right)-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\text{ar}\right.\right.}{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\arcsin\left(\frac{1}{x}\right)+\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4x}\right)-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\text{arcc}}{2}\right.\right.}{x}$$

input
```
Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

3.693. $\int\frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$

output
```
-((Sqrt[x^2]*(((1 - x^(-2))^(3/2)*ArcCos[x^(-1)]^2)/(4*x) + (-1/4*((1 - x^
(-2))^2*ArcCos[x^(-1)]) + (-1/4*(1 - x^(-2))^(3/2)/x - (3*(Sqrt[1 - x^(-2)
]/(2*x) + ArcSin[x^(-1)]/2))/4)/4)/2 + (3*(ArcCos[x^(-1)]/(2*x^2) + (Sqrt[
1 - x^(-2)]*ArcCos[x^(-1)]^2)/(2*x) - ArcCos[x^(-1)]^3/6 + (-1/2*Sqrt[1 -
x^(-2)]/x + ArcSin[x^(-1)]/2)/2))/4))/x)
```

3.693.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

3.693. $\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

3.693.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(8\operatorname{arcsec}(x)^3x^4-40\operatorname{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x^3+17\operatorname{arcsec}(x)x^4+16\operatorname{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x+17\sqrt{\frac{x^2-1}{x^2}}\,x^3-40\operatorname{arcsec}(x)x^2}{64x^4}$

input
```
int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x,method=_RETURNVERBOSE)
```

3.693. $\int \frac{\left(-1+x^2\right)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$

output
```
1/64*csgn(x*(1-1/x^2)^(1/2))*(8*arcsec(x)^3*x^4-40*arcsec(x)^2*((x^2-1)/x^
2)^(1/2)*x^3+17*arcsec(x)*x^4+16*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x+17*((x^
2-1)/x^2)^(1/2)*x^3-40*arcsec(x)*x^2-2*((x^2-1)/x^2)^(1/2)*x+8*arcsec(x))/
x^4
```

3.693.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{8\,x^4\operatorname{arcsec}(x)^3 + (17\,x^4 - 40\,x^2 + 8)\operatorname{arcsec}(x) - (8\,(5\,x^2 - 2)\operatorname{arcsec}(x)^2 -}{64\,x^4}$$

input
```
integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")
```

output
```
1/64*(8*x^4*arcsec(x)^3 + (17*x^4 - 40*x^2 + 8)*arcsec(x) - (8*(5*x^2 - 2)
*arcsec(x)^2 - 17*x^2 + 2)*sqrt(x^2 - 1))/x^4
```

3.693.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

output
```
Timed out
```

3.693.7 Maxima [F]

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \int \frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)^2}{x^5}\,dx$$

input
```
integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

3.693.8 Giac [F]

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx=\int \frac{(x^2-1)^{\frac{3}{2}}\,\mathrm{arcsec}\,(x)^2}{x^5}\,dx$$

input
```
integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

3.693.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx=\int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^2(x^2-1)^{3/2}}{x^5}\,dx$$

input
```
int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5,x)
```

output
```
int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5, x)
```

3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$

3.694.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{2(1-21x^2)}{27\left(x^2\right)^{3/2}} - \frac{4\sqrt{-1+x^2}\sec^{-1}(x)}{3x} - \frac{2(-1+x^2)^{3/2}\sec^{-1}(x)}{9x^3} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{(-1+x^2)\sec^{-1}(x)^2}{3\left(x^2\right)^{3/2}} + \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^3}{3x^3}$$

output
```
-2/9*(x^2-1)^(3/2)*arcsec(x)/x^3+1/3*(x^2-1)^(3/2)*arcsec(x)^3/x^3+2/27*(-
21*x^2+1)/x^2/(x^2)^(1/2)+2/3*arcsec(x)^2/(x^2)^(1/2)+1/3*(x^2-1)*arcsec(x
)^2/x^2/(x^2)^(1/2)-4/3*arcsec(x)*(x^2-1)^(1/2)/x
```

3.694.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{2\sqrt{1-\frac{1}{x^2}}x(1-21x^2) - 6(1-8x^2+7x^4)\sec^{-1}(x) + 9\sqrt{1-\frac{1}{x^2}}x(-1+3x^2)\sec^{-1}(x)^2 + 9(-1+x^2)^2\sec}{27x^3\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec[x] + 9*S
qrt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2)^2*ArcSec[x]^3)/(
27*x^3*Sqrt[-1 + x^2])
```

3.694.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5765, 5183, 5159, 5131, 5183, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}\sec^{-1}(x)^3}{x^4}dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^3}{x}d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{\sqrt{x^2}\left(-\int \left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3\right)}{x}$$

$$\Big\downarrow \text{5159}$$

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\int \frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}-\frac{2}{3}\int \arccos\left(\frac{1}{x}\right)^2d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}\right)}{x}$$

$$\Big\downarrow \text{5131}$$

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\left(2\int \frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)-\frac{2}{3}\int \frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}\right.}{x}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\left(2\left(-\int 1d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)\right)+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)-\frac{2}{3}\left(-\frac{1}{3}\int \left(1-\frac{1}{x^2}\right)d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)\right)\right.}{x}$$

$$\Big\downarrow \; 24$$

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\left(-\frac{1}{3}\int\left(1-\frac{1}{x^2}\right)d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}-\frac{2}{3}\left(2\left(\right.\right.\right.}{x}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{x^2}\left(-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}-\frac{2}{3}\left(2\left(-\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)-\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3.}\right.\right.\right.}{x}$$

input
```
Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

output
```
-((Sqrt[x^2]*(-1/3*((1 - x^(-2))*ArcCos[x^(-1)]^2)/x - ((1 - x^(-2))^(3/2)
*ArcCos[x^(-1)]^3)/3 - (2*((1/(3*x^3) - x^(-1))/3 - ((1 - x^(-2))^(3/2)*Ar
cCos[x^(-1)])/3))/3 - (2*(ArcCos[x^(-1)]^2/x + 2*(-x^(-1) - Sqrt[1 - x^(-2
)]*ArcCos[x^(-1)])))/3))/x)
```

3.694.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

3.694. $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

3.694.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

method	result
default	$\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(9\,\mathrm{arcsec}(x)^3x^4+27\,\mathrm{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x^3-18x^2\,\mathrm{arcsec}(x)^3-42\,\mathrm{arcsec}(x)x^4-9\,\mathrm{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x-42\sqrt{}\right.}{27(x^2-1)x^2}$

input
```
int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/27*csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)/x^2*(9*arcsec(x)^
3*x^4+27*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x^3-18*x^2*arcsec(x)^3-42*arcsec(
x)*x^4-9*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x-42*((x^2-1)/x^2)^(1/2)*x^3+9*ar
csec(x)^3+48*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x-6*arcsec(x))
```

3.694.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$$
$$=\frac{9\,(3\,x^2-1)\,\mathrm{arcsec}\,(x)^2-42\,x^2+3\,\left(3\,(x^2-1)\,\mathrm{arcsec}\,(x)^3-2\,(7\,x^2-1)\,\mathrm{arcsec}\,(x)\right)\sqrt{x^2-1}+2}{27\,x^3}$$

input
```
integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(x)^3 - 2*
(7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

3.694.6 Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\sqrt{(x-1)(x+1)}\operatorname{asec}^3(x)}{x^4}\,dx$$

input
```
integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))*asec(x)**3/x**4, x)
```

3.694.7 Maxima [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)^3}{3\,x^3} + \frac{(3\,x^2-1)\operatorname{arcsec}(x)^2}{3\,x^3} - \frac{2\left((21\,x^2-1)\sqrt{x+1}\sqrt{x-1} + 3\,(7\,x^4-8\,x^2+1)\arctan\left(\sqrt{x+1}\sqrt{x-1}\right)\right)}{27\,\sqrt{x+1}\sqrt{x-1}x^3}$$

input
```
integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2/x^3 - 2/
27*((21*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 3*(7*x^4 - 8*x^2 + 1)*arctan(sq
rt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1)*x^3)
```

3.694.8 Giac [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\left(x\right)^3}{x^4}\,dx$$

input
```
integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4, x)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^3\sqrt{x^2-1}}{x^4}\,dx$$

input
```
int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4,x)
```

output
```
int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4, x)
```

3.695 $\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$

3.695.1 Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=-\frac{\sqrt{2}a\sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}}+(a+x)\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)$$

output
```
(a+x)*arcsin(((-a+x)/(a+x))^(1/2))-a*2^(1/2)*((-a+x)/(a+x))^(1/2)/(a/(a+x)
)^(1/2)
```

3.695.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.80

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=x\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{\frac{a}{a+x}}\left(2a-2x+\sqrt{2}\sqrt{a}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{2}\sqrt{a}}\right)\right)}{\sqrt{2}\sqrt{\frac{-a+x}{a+x}}}$$

input
```
Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x + Sqrt[2]*S
qrt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(Sqrt[2]*Sqrt
[(-a + x)/(a + x)])
```

3.695.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. 2(55) = 110.

Time = 0.78 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5339, 27, 2045, 7268, 2044, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin\left(\sqrt{\frac{x-a}{a+x}}\right)dx$$

$$\Big\downarrow \text{ 5339}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{2}a\sqrt{\frac{x-a}{a+x}}}dx$$

$$\Big\downarrow \text{ 27}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{-\frac{a-x}{a+x}}}dx}{\sqrt{2}a}$$

$$\Big\downarrow \text{ 2045}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\int \frac{x}{\sqrt{-\frac{a-x}{a+x}}\left(\frac{x}{a}+1\right)^{3/2}}dx}{\sqrt{2}a}$$

$$\Big\downarrow \text{ 7268}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\int \left(1-\frac{a-x}{a+x}\right)\left(\frac{1}{\frac{a-x}{a+x}+1}\right)^{3/2}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{ 2044}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\sqrt{\frac{1}{\frac{a-x}{a+x}+1}}\sqrt{\frac{a-x}{a+x}+1}\int \frac{1-\frac{a-x}{a+x}}{\left(\frac{a-x}{a+x}+1\right)^{3/2}}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{ 298}$$

$$x \arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-$$
$$a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\sqrt{\frac{1}{\frac{a-x}{a+x}+1}}\sqrt{\frac{a-x}{a+x}+1}\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\sqrt{\frac{a-x}{a+x}+1}}-\int\frac{1}{\sqrt{\frac{a-x}{a+x}+1}}d\sqrt{-\frac{a-x}{a+x}}\right)$$

$$\Big\downarrow \ 223$$

$$x \arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-$$
$$a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\sqrt{\frac{1}{\frac{a-x}{a+x}+1}}\sqrt{\frac{a-x}{a+x}+1}\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\sqrt{\frac{a-x}{a+x}+1}}-\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)\right)$$

input
```
Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
-(a*Sqrt[a/(a + x)]*Sqrt[1 + x/a]*Sqrt[(1 + (a - x)/(a + x))^(-1)]*Sqrt[1
+ (a - x)/(a + x)]*((2*Sqrt[-((a - x)/(a + x))])/Sqrt[1 + (a - x)/(a + x)]
 - ArcSin[Sqrt[-((a - x)/(a + x))]])) + x*ArcSin[Sqrt[-((a - x)/(a + x))]]
```

3.695.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

rule 5339
```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

3.695.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

method	result	size
default	$x \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{2\sqrt{-\frac{a-x}{a+x}}}$	86
parts	$x \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{2\sqrt{-\frac{a-x}{a+x}}}$	86

input
```
int(arcsin(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arcsin(((-a+x)/(a+x))^(1/2))+1/2/(-(a-x)/(a+x))^(1/2)*(-a+x)^(1/2)*2^(1/
2)*(a/(a+x))^(1/2)*(-2*(-a+x)^(1/2)+a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1/2
)*2^(1/2)/a^(1/2)))
```

3.695.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = -\sqrt{2}(a+x)\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}} + (a+x)\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

output
```
-sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)*arcsin(s
qrt(-(a - x)/(a + x)))
```

3.695.6 Sympy [F]

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \operatorname{asin}\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$

input
```
integrate(asin(((-a+x)/(a+x))**(1/2)),x)
```

output
```
Integral(asin(sqrt((-a + x)/(a + x))), x)
```

3.695.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = a\left(\frac{2\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1} + \frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}+1} + \frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}-1}\right)$$

input
```
integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

output
```
a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt((a - x)/(
a + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt
(-(a - x)/(a + x)) - 1))
```

3.695.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.71

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$
$$= x\arcsin\left(\frac{\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)}{a+x}\right)$$
$$+ \frac{\sqrt{2}\left(\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a^2+ax}}{2a}\right)-\sqrt{2}\left(a\arctan\left(\frac{i|a|}{a}\right)-2i|a|\right)-2\sqrt{-a^2+ax}\right)a}{2|a|}$$

input
```
integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

output
```
x*arcsin(sqrt(-a^2 + x^2)*sgn(a + x)/(a + x)) + 1/2*sqrt(2)*(sqrt(2)*a*arc
tan(1/2*sqrt(2)*sqrt(-a^2 + a*x)/a) - sqrt(2)*(a*arctan(I*abs(a)/a) - 2*I*
abs(a)) - 2*sqrt(-a^2 + a*x))*a/abs(a)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \operatorname{asin}\left(\sqrt{-\frac{a-x}{a+x}}\right)\,dx$$

input
```
int(asin((-(a - x)/(a + x))^(1/2)),x)
```

output
```
int(asin((-(a - x)/(a + x))^(1/2)), x)
```

3.696 $\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

3.696.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

output
```
x*arctan(((-a+x)/(a+x))^(1/2))-a*arctanh(((-a+x)/(a+x))^(1/2))
```

3.696.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x\arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) - \frac{a\sqrt{-a+x}\text{arctanh}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)}{\sqrt{\frac{-a+x}{a+x}}\sqrt{a+x}}$$

input
```
Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*ArcTanh[Sqrt[a + x]/Sqr
t[-a + x]])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

3.696.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5726, 27, 2055, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\sqrt{\frac{x-a}{a+x}}\right)dx$$

$$\Big\downarrow \text{ 5726}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\int \frac{a}{2\sqrt{\frac{x-a}{a+x}}(a+x)}dx$$

$$\Big\downarrow \text{ 27}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{1}{2}a\int \frac{1}{\sqrt{-\frac{a-x}{a+x}}(a+x)}dx$$

$$\Big\downarrow \text{ 2055}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-2a^2\int \frac{1}{\frac{2(a-x)a}{a+x}+2a}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{ 221}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
Int[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

3.696.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 5726
```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

3.696.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

method	result	size
default	$x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x) a \ln\left(x+\sqrt{-a^{2}+x^{2}}\right)}{2 \sqrt{-\frac{a-x}{a+x}}\, \sqrt{-(a-x)(a+x)}}$	66
parts	$x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x) a \ln\left(x+\sqrt{-a^{2}+x^{2}}\right)}{2 \sqrt{-\frac{a-x}{a+x}}\, \sqrt{-(a-x)(a+x)}}$	66

input
```
int(arctan(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(((-a+x)/(a+x))^(1/2))+1/2*(a-x)*a*ln(x+(-a^2+x^2)^(1/2))/(-(a-x)/
(a+x))^(1/2)/(-(a-x)*(a+x))^(1/2)
```

3.696.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{1}{2} a \log\left(\sqrt{-\frac{a-x}{a+x}} + 1\right) + \frac{1}{2} a \log\left(\sqrt{-\frac{a-x}{a+x}} - 1\right)$$

input
```
integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

output
```
x*arctan(sqrt(-(a - x)/(a + x))) - 1/2*a*log(sqrt(-(a - x)/(a + x)) + 1) +
 1/2*a*log(sqrt(-(a - x)/(a + x)) - 1)
```

3.696.6 Sympy [F]

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = \int \operatorname{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$$

input
```
integrate(atan(((-a+x)/(a+x))**(1/2)),x)
```

output
```
Integral(atan(sqrt((-a + x)/(a + x))), x)
```

3.696.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = \frac{1}{2} a \left(\frac{4 \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x} + 1} - 2 \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \log\left(\sqrt{-\frac{a-x}{a+x}} + 1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}} - 1\right)\right)$$

input
```
integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

output
```
1/2*a*(4*arctan(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) - 2*arctan(s
qrt(-(a - x)/(a + x))) - log(sqrt(-(a - x)/(a + x)) + 1) + log(sqrt(-(a -
x)/(a + x)) - 1))
```

3.696.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \frac{1}{2}\,a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\mathrm{sgn}(a+x) + x\arctan\left(\frac{\sqrt{-a^2+x^2}\mathrm{sgn}(a+x)}{a+x}\right)$$

input
```
integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

output
```
1/2*a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + x*arctan(sqrt(-a^2 + x^
2)*sgn(a + x)/(a + x))
```

3.696.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\,\mathrm{atan}\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
int(atan((-(a - x)/(a + x))^(1/2)),x)
```

output
```
x*atan((-(a - x)/(a + x))^(1/2)) - a*atanh((-(a - x)/(a + x))^(1/2))
```

3.697 $\int \frac{\arctan(x)}{(1+x)^3}\,dx$

3.697.1 Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{1}{4(1+x)} - \frac{\arctan(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{8}\log\left(1+x^2\right)$$

output
```
-1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

3.697.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = \frac{1}{8}\left(-\frac{2}{1+x} - \frac{4\arctan(x)}{(1+x)^2} + 2\log(1+x) - \log\left(1+x^2\right)\right)$$

input
```
Integrate[ArcTan[x]/(1 + x)^3,x]
```

output
```
(-2/(1 + x) - (4*ArcTan[x])/(1 + x)^2 + 2*Log[1 + x] - Log[1 + x^2])/8
```

3.697.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5387, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)}{(x+1)^3} dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{1}{2}\int \frac{1}{(x+1)^2 (x^2+1)}dx - \frac{\arctan(x)}{2(x+1)^2}$$

$$\Big\downarrow \text{480}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{(x+1)(x^2+1)}dx - \frac{1}{2(x+1)}\right) - \frac{\arctan(x)}{2(x+1)^2}$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \left(\frac{1}{x+1} - \frac{x}{x^2+1}\right)dx - \frac{1}{2(x+1)}\right) - \frac{\arctan(x)}{2(x+1)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\log(x+1) - \frac{1}{2}\log\left(x^2+1\right)\right) - \frac{1}{2(x+1)}\right) - \frac{\arctan(x)}{2(x+1)^2}$$

input
```
Int[ArcTan[x]/(1 + x)^3,x]
```

output
```
-1/2*ArcTan[x]/(1 + x)^2 + (-1/2*1/(1 + x) + (Log[1 + x] - Log[1 + x^2]/2)
/2)/2
```

3.697.3.1 Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.697.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{8}$
parts	$-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{8}$
parallelrisch	$\frac{2\ln(1+x)x^2-\ln(x^2+1)x^2-2+4\ln(1+x)x-2x\ln(x^2+1)+2\ln(1+x)-\ln(x^2+1)-2x-4\arctan(x)}{8(1+x)^2}$
risch	$\frac{i\ln(ix+1)}{4(1+x)^2}-\frac{i(2i\ln(1+x)x^2-i\ln(x^2+1)x^2+4i\ln(1+x)x-2i\ln(x^2+1)x+2i\ln(1+x)-i\ln(x^2+1)-2ix-2i+2\ln(-ix+1))}{8(1+x)^2}$

input
```
int(arctan(x)/(1+x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

3.697.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{(x^2+2x+1)\log(x^2+1)-2(x^2+2x+1)\log(x+1)+2x+4\arctan(x)+2}{8(x^2+2x+1)}$$

input
```
integrate(arctan(x)/(1+x)^3,x, algorithm="fricas")
```

output
```
-1/8*((x^2 + 2*x + 1)*log(x^2 + 1) - 2*(x^2 + 2*x + 1)*log(x + 1) + 2*x +
4*arctan(x) + 2)/(x^2 + 2*x + 1)
```

3.697.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.92

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = \frac{2x^2\log(x+1)}{8x^2+16x+8} - \frac{x^2\log(x^2+1)}{8x^2+16x+8} + \frac{4x\log(x+1)}{8x^2+16x+8} - \frac{2x\log(x^2+1)}{8x^2+16x+8} - \frac{2x}{8x^2+16x+8} + \frac{2\log(x+1)}{8x^2+16x+8} - \frac{\log(x^2+1)}{8x^2+16x+8} - \frac{4\operatorname{atan}(x)}{8x^2+16x+8} - \frac{2}{8x^2+16x+8}$$

input
```
integrate(atan(x)/(1+x)**3,x)
```

output
```
2*x**2*log(x + 1)/(8*x**2 + 16*x + 8) - x**2*log(x**2 + 1)/(8*x**2 + 16*x
+ 8) + 4*x*log(x + 1)/(8*x**2 + 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 + 16
*x + 8) - 2*x/(8*x**2 + 16*x + 8) + 2*log(x + 1)/(8*x**2 + 16*x + 8) - log
(x**2 + 1)/(8*x**2 + 16*x + 8) - 4*atan(x)/(8*x**2 + 16*x + 8) - 2/(8*x**2
 + 16*x + 8)
```

3.697.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{1}{4(x+1)} - \frac{\arctan(x)}{2(x+1)^2} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{4}\log(x+1)$$

input
```
integrate(arctan(x)/(1+x)^3,x, algorithm="maxima")
```

output
```
-1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(x + 1)
```

3.697.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = -\frac{1}{4\,(x+1)} - \frac{\arctan(x)}{2\,(x+1)^2} - \frac{1}{8}\,\log\left(x^2+1\right) + \frac{1}{4}\,\log\left(|x+1|\right)$$

input
```
integrate(arctan(x)/(1+x)^3,x, algorithm="giac")
```

output
```
-1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(abs(x
+ 1))
```

3.697.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx = \frac{\ln(x+1)}{4} - \frac{\ln(x^2+1)}{8} - \frac{\frac{x}{4} + \frac{\operatorname{atan}(x)}{2} + \frac{1}{4}}{(x+1)^2}$$

input
```
int(atan(x)/(x + 1)^3,x)
```

output
```
log(x + 1)/4 - log(x^2 + 1)/8 - (x/4 + atan(x)/2 + 1/4)/(x + 1)^2
```

3.698 $\int -\frac{\arctan(a-x)}{a+x}\,dx$

3.698.1 Optimal result

Integrand size = 13, antiderivative size = 122

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \arctan(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1+\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)$$

output
```
arctan(a-x)*ln(2/(1-I*(a-x)))-arctan(a-x)*ln(-2*(a+x)/(I-2*a)/(1-I*(a-x)))
-1/2*I*polylog(2,1-2/(1-I*(a-x)))+1/2*I*polylog(2,1+2*(a+x)/(I-2*a)/(1-I*(
a-x)))
```

3.698.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\frac{1}{2}i\left(-\log(1+i(a-x))\log\left(\frac{a+x}{-i+2a}\right) + \log(1-ia+ix)\log\left(\frac{a+x}{i+2a}\right) + \text{PolyLog}\left(2,\frac{i+a-x}{i+2a}\right) - \text{PolyLog}\left(2,\frac{i-a+x}{i-2a}\right)\right)$$

input
```
Integrate[-(ArcTan[a - x]/(a + x)),x]
```

output
```
(-1/2*I)*(-(Log[1 + I*(a - x)]*Log[(a + x)/(-I + 2*a)]) + Log[1 - I*a + I*
x]*Log[(a + x)/(I + 2*a)] + PolyLog[2, (I + a - x)/(I + 2*a)] - PolyLog[2,
 (I - a + x)/(I - 2*a)])
```

3.698.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {25, 5570, 5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\arctan(a-x)}{a+x}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\arctan(a-x)}{a+x}dx$$

$$\Big\downarrow \text{5570}$$

$$\int \frac{\arctan(a-x)}{a+x}d(a-x)$$

$$\Big\downarrow \text{5381}$$

$$-\int \frac{\log\left(\frac{2}{1-i(a-x)}\right)}{(a-x)^2+1}d(a-x)+\int \frac{\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)}{(a-x)^2+1}d(a-x)+\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)$$

$$\Big\downarrow \text{2849}$$

$$-i\int \frac{\log\left(\frac{2}{1-i(a-x)}\right)}{1-\frac{2}{1-i(a-x)}}d\frac{1}{1-i(a-x)}+\int \frac{\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)}{(a-x)^2+1}d(a-x)+\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)$$

$$\Big\downarrow \text{2752}$$

$$\int \frac{\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)}{(a-x)^2+1}d(a-x)+\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)$$

$$\Big\downarrow \text{2897}$$

$$\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\operatorname{PolyLog}\left(2,\frac{2(a+x)}{(i-2a)(1-i(a-x))}+1\right)$$

input
```
Int[-(ArcTan[a - x]/(a + x)),x]
```

output
```
ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I
- 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)
*PolyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]
```

3.698.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.698.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-$ i dilog
default	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-$ i dilog
parts	$-\ln(a+x)\arctan(a-x)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-$ i dilog
risch	$-\frac{i\,\mathrm{dilog}\left(\frac{ia+ix}{2ia-1}\right)}{2}-\frac{i\ln(-ia+ix+1)\ln\left(\frac{ia+ix}{2ia-1}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{-ia-ix}{-2ia-1}\right)}{2}+\frac{i\ln(ia-ix+1)\ln\left(\frac{-ia-ix}{-2ia-1}\right)}{2}$

input
```
int(-arctan(a-x)/(a+x),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+x)*arctan(a-x)+1/2*I*ln(a+x)*ln((I+a-x)/(2*a+I))-1/2*I*ln(a+x)*ln((I
-a+x)/(I-2*a))+1/2*I*dilog((I+a-x)/(2*a+I))-1/2*I*dilog((I-a+x)/(I-2*a))
```

3.698.5 Fricas [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \int -\frac{\arctan(a-x)}{a+x}\,dx$$

input
```
integrate(-arctan(a-x)/(a+x),x, algorithm="fricas")
```

output
```
integral(arctan(-a + x)/(a + x), x)
```

3.698.6 Sympy [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\int \frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

input
```
integrate(-atan(a-x)/(a+x),x)
```

output
```
-Integral(atan(a - x)/(a + x), x)
```

3.698.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\begin{aligned}\int -\frac{\arctan(a-x)}{a+x}\,dx = & -\frac{1}{2}\arctan\left(\frac{a+x}{4a^2+1}, \frac{2(a^2+ax)}{4a^2+1}\right)\log\left(a^2-2ax+x^2+1\right) \\ & + \frac{1}{2}\arctan(-a+x)\log\left(\frac{a^2+2ax+x^2}{4a^2+1}\right) \\ & - \frac{1}{2}i\operatorname{Li}_2\left(-\frac{-ia+ix+1}{2ia-1}\right) + \frac{1}{2}i\operatorname{Li}_2\left(-\frac{-ia+ix-1}{2ia+1}\right)\end{aligned}$$

input
```
integrate(-arctan(a-x)/(a+x),x, algorithm="maxima")
```

output
```
-1/2*arctan2((a + x)/(4*a^2 + 1), 2*(a^2 + a*x)/(4*a^2 + 1))*log(a^2 - 2*a
*x + x^2 + 1) + 1/2*arctan(-a + x)*log((a^2 + 2*a*x + x^2)/(4*a^2 + 1)) -
1/2*I*dilog(-(-I*a + I*x + 1)/(2*I*a - 1)) + 1/2*I*dilog(-(-I*a + I*x - 1)
/(2*I*a + 1))
```

3.698.8 Giac [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \int -\frac{\arctan\left(a-x\right)}{a+x}\,dx$$

input
```
integrate(-arctan(a-x)/(a+x),x, algorithm="giac")
```

output
```
integrate(-arctan(a - x)/(a + x), x)
```

3.698.9 Mupad [F(-1)]

Timed out.

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\int \frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

input
```
int(-atan(a - x)/(a + x),x)
```

output
```
-int(atan(a - x)/(a + x), x)
```

3.699 $\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

3.699.1 Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

output
```
-1/2*arcsin((-x^2+1)^(1/2))^2*(x^2)^(1/2)/x
```

3.699.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

input
```
Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

output
```
-1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

3.699.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5333, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5333}$$

$$-\frac{\sqrt{x^2}\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{x^2}}d\sqrt{1-x^2}}{x}$$

$$\Big\downarrow \text{5152}$$

$$-\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

input
```
Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

output
```
-1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

3.699.3.1 Defintions of rubi rules used

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5333
```
Int[ArcSin[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Simp[Sqrt[(-b)*x^2]/(b*x)   Subst[Int[ArcSin[x]^n/Sqrt[1 - x^2], x], x,
Sqrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

3.699.4 Maple [F]

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}dx$$

input
```
int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

output
```
int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

3.699.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\arcsin\left(\sqrt{-x^2+1}\right)^2$$

input
```
integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arcsin(sqrt(-x^2 + 1))^2
```

3.699.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \frac{x\,\mathrm{asin}^2(x)}{2\sqrt{x^2}} + \mathrm{asin}(x)\,\mathrm{asin}\left(\sqrt{1-x^2}\right)$$

input
```
integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

output
```
x*asin(x)**2/(2*sqrt(x**2)) + asin(x)*asin(sqrt(1 - x**2))
```

3.699.7 Maxima [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

3.699.8 Giac [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

3.699.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\mathrm{asin}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

input
```
int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2),x)
```

output
```
int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2), x)
```

3.700 $\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

3.700.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} \arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2} \log\left(2+x^2\right)$$

output
```
-1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

3.700.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} \arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2} \log\left(2+x^2\right)$$

input
```
Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

output
```
Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

3.700.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5730, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan\left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{5730}$$

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\int \frac{x}{x^2+2}dx$$

$$\Big\downarrow \text{240}$$

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

input
```
Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

output
```
Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

3.700.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

3.700.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$	26
default	$-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$	26

input
```
int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x\arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx=\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

3.700.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x\arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx=\sqrt{x^2+1}\,\mathrm{atan}\left(\sqrt{x^2+1}\right)-\frac{\log\left(x^2+2\right)}{2}$$

input
```
integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

output
```
sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

3.700.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan \left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\, dx = \sqrt{x^2+1} \arctan \left(\sqrt{x^2+1}\right) - \frac{1}{2} \log \left(x^2+2\right)$$

input
```
integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

3.700.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan \left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\, dx = \sqrt{x^2+1} \arctan \left(\sqrt{x^2+1}\right) - \frac{1}{2} \log \left(x^2+2\right)$$

input
```
integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

3.700.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan \left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\, dx = \mathrm{atan}\left(\sqrt{x^2+1}\right)\, \sqrt{x^2+1} - \frac{\ln \left(x^2+2\right)}{2}$$

input
```
int((x*atan((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
```

output
```
atan((x^2 + 1)^(1/2))*(x^2 + 1)^(1/2) - log(x^2 + 2)/2
```

3.701 $\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx$

3.701.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = -\frac{\sqrt{1+x}}{3(1-x)} + \frac{2\arcsin(x)}{3(1-x)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

output
```
2/3*arcsin(x)/(1-x)^(3/2)-1/6*arctanh(1/2*2^(1/2)*(1+x)^(1/2))*2^(1/2)-1/3
*(1+x)^(1/2)/(1-x)
```

3.701.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{1}{6}\left(-\frac{2\left(\sqrt{1-x^2}-2\arcsin(x)\right)}{(1-x)^{3/2}} - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}}\right)\right)$$

input
```
Integrate[ArcSin[x]/(1 - x)^(5/2),x]
```

output
```
((-2*(Sqrt[1 - x^2] - 2*ArcSin[x]))/(1 - x)^(3/2) - Sqrt[2]*ArcTanh[Sqrt[1
 - x^2]/Sqrt[2 - 2*x]])/6
```

3.701.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5242, 456, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}dx$$

$$\Big\downarrow \text{5242}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\left(\frac{1}{4}\int \frac{1}{(1-x)\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{2(1-x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x+1}+\frac{\sqrt{x+1}}{2(1-x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\sqrt{x+1}}{2(1-x)}\right)$$

input
```
Int[ArcSin[x]/(1 - x)^(5/2),x]
```

output
```
(2*ArcSin[x])/(3*(1 - x)^(3/2)) - (2*(Sqrt[1 + x]/(2*(1 - x)) + ArcTanh[Sq
rt[1 + x]/Sqrt[2]]/(2*Sqrt[2])))/3
```

3.701.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 5242
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

3.701.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^{2}+2-2x}}$	70
default	$\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^{2}+2-2x}}$	70

input
```
int(arcsin(x)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arcsin(x)/(1-x)^(3/2)-1/6/(1-x)^(1/2)*(1+x)^(1/2)*(2^(1/2)*arctanh(2^(
1/2)/(1+x)^(1/2))*(1-x)+2*(1+x)^(1/2))/(-(1-x)^2+2-2*x)^(1/2)
```

3.701.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(40) = 80$.

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{\sqrt{2}(x^2-2\,x+1)\log\left(-\frac{x^2+2\,\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2\,x-3}{x^2-2\,x+1}\right)-4\,\sqrt{-x+1}\left(\sqrt{-x^2+1}-2\,\arcsin\left(\right.\right.}{12\,(x^2-2\,x+1)}$$

input
```
integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x
 + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x + 1)*(sqrt(-x^2 + 1) - 2*arc
sin(x)))/(x^2 - 2*x + 1)
```

3.701.6 Sympy [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\operatorname{asin}(x)}{(1-x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(asin(x)/(1-x)**(5/2),x)
```

output
```
Integral(asin(x)/(1 - x)**(5/2), x)
```

3.701.7 Maxima [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\arcsin(x)}{(-x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*(x - 1)*sqrt(-x + 1)*integrate(1/3*sqrt(x + 1)*x^2/(x^5 - x^4 - x^
3 + x^2 + (x^3 - x^2 - x + 1)*e^(log(x + 1) + log(-x + 1))), x) + arctan2(
x, sqrt(x + 1)*sqrt(-x + 1)))/((x - 1)*sqrt(-x + 1))
```

3.701.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right) + \frac{\sqrt{x+1}}{3\,(x-1)} - \frac{2\arcsin(x)}{3\,(x-1)\sqrt{-x+1}}$$

input
```
integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 1/3*sq
rt(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))
```

3.701.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\mathrm{asin}(x)}{(1-x)^{5/2}}\,dx$$

input
```
int(asin(x)/(1 - x)^(5/2),x)
```

output
```
int(asin(x)/(1 - x)^(5/2), x)
```

3.702 $\int (-1+x)^{5/2} \csc^{-1}(x) \, dx$

3.702.1 Optimal result

Integrand size = 10, antiderivative size = 82

$$\int (-1+x)^{5/2} \csc^{-1}(x) \, dx = \frac{4x\sqrt{-1+x^2}(83-19x+3x^2)}{105\sqrt{-1+x}\sqrt{x^2}} + \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x) + \frac{4x\operatorname{arctanh}\left(\frac{\sqrt{-1+x^2}}{\sqrt{-1+x}}\right)}{7\sqrt{x^2}}$$

output
```
2/7*(-1+x)^(7/2)*arccsc(x)+4/7*x*arctanh((x^2-1)^(1/2)/(-1+x)^(1/2))/(x^2)
^(1/2)+4/105*x*(3*x^2-19*x+83)*(x^2-1)^(1/2)/(-1+x)^(1/2)/(x^2)^(1/2)
```

3.702.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int (-1+x)^{5/2} \csc^{-1}(x) \, dx = \frac{4\sqrt{1-\frac{1}{x^2}}x(83-19x+3x^2)}{105\sqrt{-1+x}} + \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x) + \frac{4}{7}\operatorname{arctanh}\left(\frac{\sqrt{1-\frac{1}{x^2}}x}{\sqrt{-1+x}}\right)$$

input
```
Integrate[(-1 + x)^(5/2)*ArcCsc[x],x]
```

output
```
(4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (2*(-1 + x
)^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/Sqrt[-1 + x]])/7
```

3.702.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5750, 1898, 586, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-1)^{5/2}\csc^{-1}(x)\,dx$$

$$\Big\downarrow \ 5750$$

$$\frac{2}{7}\int \frac{(x-1)^{7/2}}{\sqrt{1-\frac{1}{x^2}}x^2}dx+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

$$\Big\downarrow \ 1898$$

$$\frac{2\sqrt{x^2-1}\int \frac{(x-1)^{7/2}}{x\sqrt{x^2-1}}dx}{7\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

$$\Big\downarrow \ 586$$

$$\frac{2\sqrt{x+1}\sqrt{x-1}\int -\frac{(1-x)^3}{x\sqrt{x+1}}dx}{7\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

$$\Big\downarrow \ 25$$

$$\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)-\frac{2\sqrt{x-1}\sqrt{x+1}\int \frac{(1-x)^3}{x\sqrt{x+1}}dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$\Big\downarrow \ 99$$

$$\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)-\frac{2\sqrt{x-1}\sqrt{x+1}\int \left(-(x+1)^{3/2}+5\sqrt{x+1}+\frac{1}{x\sqrt{x+1}}-\frac{7}{\sqrt{x+1}}\right)dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)-\frac{2\sqrt{x-1}\sqrt{x+1}\left(-2\text{arctanh}\left(\sqrt{x+1}\right)-\frac{2}{5}(x+1)^{5/2}+\frac{10}{3}(x+1)^{3/2}-14\sqrt{x+1}\right)}{7\sqrt{1-\frac{1}{x^2}}x}$$

input
```
Int[(-1 + x)^(5/2)*ArcCsc[x],x]
```

output
```
(2*(-1 + x)^(7/2)*ArcCsc[x])/7 - (2*Sqrt[-1 + x]*Sqrt[1 + x]*(-14*Sqrt[1 +
 x] + (10*(1 + x)^(3/2))/3 - (2*(1 + x)^(5/2))/5 - 2*ArcTanh[Sqrt[1 + x]])
)/(7*Sqrt[1 - x^(-2)]*x)
```

3.702.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 586
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]/((c + d*x)^FracPart[p]*(a/c + (
b*x)/d)^FracPart[p])   Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.702.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result	siz
derivativedivides	$\frac{2(-1+x)^{\frac{7}{2}}\operatorname{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^{2}\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15\operatorname{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^{2}}}\,x}$	76
default	$\frac{2(-1+x)^{\frac{7}{2}}\operatorname{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^{2}\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15\operatorname{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^{2}}}\,x}$	76

input
```
int((-1+x)^(5/2)*arccsc(x),x,method=_RETURNVERBOSE)
```

output
```
2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+x)^2*(1+x
)^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67*(1+x)^(1/2))/((-1
+x)*(1+x)/x^2)^(1/2)/x
```

3.702.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int(-1+x)^{5/2}\csc^{-1}(x)\,dx=\frac{2\left(15\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)\sqrt{x-1}\operatorname{arccsc}\left(x\right)+2\left(3\,x^{2}-19\,x+83\right)\sqrt{x^{2}-1}\sqrt{x-}\right.}{105\,(x-1}$$

input
```
integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="fricas")
```

output
```
2/105*(15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x - 1)*arccsc(x) + 2*(3*x^2
 - 19*x + 83)*sqrt(x^2 - 1)*sqrt(x - 1) + 15*(x - 1)*log((x^2 + sqrt(x^2 -
 1)*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*(x - 1)*log(-(x^2 - sqrt(x^2 - 1)*sqr
t(x - 1) - 1)/(x^2 - 1)))/(x - 1)
```

3.702.6 Sympy [F(-1)]

Timed out.

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \text{Timed out}$$

input
```
integrate((-1+x)**(5/2)*acsc(x),x)
```

output
```
Timed out
```

3.702.7 Maxima [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{4}{35}(x+1)^{\frac{5}{2}} - \frac{20}{21}(x+1)^{\frac{3}{2}} + \frac{2}{7}\left(x^3\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right) - 3x^2\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right) + 3x\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right) - \text{ar}\right. + 4\sqrt{x+1} + \frac{2}{7}\log\left(\sqrt{x+1}+1\right) - \frac{2}{7}\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="maxima")
```

output
```
4/35*(x + 1)^(5/2) - 20/21*(x + 1)^(3/2) + 2/7*(x^3*arctan2(1, sqrt(x + 1)
*sqrt(x - 1)) - 3*x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 3*x*arctan2(1,
 sqrt(x + 1)*sqrt(x - 1)) - arctan2(1, sqrt(x + 1)*sqrt(x - 1)))*sqrt(x -
1) + 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x + 1) - 1)
```

3.702.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(62) = 124$.

Time = 0.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.78

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{2}{35}\left(5(x-1)^{\frac{7}{2}}+21(x-1)^{\frac{5}{2}}+35(x-1)^{\frac{3}{2}}+35\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) - \frac{2}{5}\left(3(x-1)^{\frac{5}{2}}+10(x-1)^{\frac{3}{2}}+15\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) + 2\left((x-1)^{\frac{3}{2}}+3\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) - 2\sqrt{x-1}\arcsin\left(\frac{1}{x}\right) + \frac{4\left(3(x+1)^{\frac{5}{2}}-4(x+1)^{\frac{3}{2}}+21\sqrt{x+1}\right)}{105\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}}+\sqrt{x-1}\right)} - \frac{4\left((x+1)^{\frac{3}{2}}+\sqrt{x+1}\right)}{5\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}}+\sqrt{x-1}\right)} + \frac{2\log\left(\sqrt{x+1}+1\right)}{7\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}}+\sqrt{x-1}\right)} - \frac{2\log\left(\sqrt{x+1}-1\right)}{7\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}}+\sqrt{x-1}\right)} + \frac{4\sqrt{x+1}}{\mathrm{sgn}\left((x-1)^{\frac{3}{2}}+\sqrt{x-1}\right)}$$

input
```
integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="giac")
```

output
```
2/35*(5*(x - 1)^(7/2) + 21*(x - 1)^(5/2) + 35*(x - 1)^(3/2) + 35*sqrt(x -
1))*arcsin(1/x) - 2/5*(3*(x - 1)^(5/2) + 10*(x - 1)^(3/2) + 15*sqrt(x - 1)
)*arcsin(1/x) + 2*((x - 1)^(3/2) + 3*sqrt(x - 1))*arcsin(1/x) - 2*sqrt(x -
 1)*arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 4*(x + 1)^(3/2) + 21*sqrt(x + 1
))/sgn((x - 1)^(3/2) + sqrt(x - 1)) - 4/5*((x + 1)^(3/2) + sqrt(x + 1))/sg
n((x - 1)^(3/2) + sqrt(x - 1)) + 2/7*log(sqrt(x + 1) + 1)/sgn((x - 1)^(3/2
) + sqrt(x - 1)) - 2/7*log(sqrt(x + 1) - 1)/sgn((x - 1)^(3/2) + sqrt(x - 1
)) + 4*sqrt(x + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1))
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \int \mathrm{asin}\left(\frac{1}{x}\right)(x-1)^{5/2}\,dx$$

input
```
int(asin(1/x)*(x - 1)^(5/2),x)
```

output
```
int(asin(1/x)*(x - 1)^(5/2), x)
```

3.703 $\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx$

3.703.1 Optimal result

Integrand size = 8, antiderivative size = 49

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = -\frac{2}{3}\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right)+\frac{1}{6}\text{sech}(x)\sqrt{1-\sinh^2(x)} + \arcsin(\sinh(x))\tanh(x)-\frac{1}{3}\arcsin(\sinh(x))\tanh^3(x)$$

output
```
-2/3*arcsin(1/2*cosh(x)*2^(1/2))+1/6*sech(x)*(1-sinh(x)^2)^(1/2)+arcsin(si
nh(x))*tanh(x)-1/3*arcsin(sinh(x))*tanh(x)^3
```

3.703.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \frac{1}{12}\Big(8i\log\left(i\sqrt{2}\cosh(x)+\sqrt{3-\cosh(2x)}\right) + \sqrt{6-2\cosh(2x)}\text{sech}(x) + 4\arcsin(\sinh(x))(2+\cosh(2x))\text{sech}^2(x)\tanh(x)\Big)$$

input
```
Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

output
```
((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]] + Sqrt[6 - 2*Cosh[2*x]
]*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*Tanh[x])/12
```

3.703.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5343, 27, 3042, 26, 4857, 358, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(x)\arcsin(\sinh(x))\,dx$$

$$\Big\downarrow \; 5343$$

$$-\int \frac{(\cosh(2x)+2)\operatorname{sech}(x)\tanh(x)}{3\sqrt{1-\sinh^2(x)}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{3}\int \frac{(\cosh(2x)+2)\operatorname{sech}(x)\tanh(x)}{\sqrt{1-\sinh^2(x)}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{3}\int -\frac{i(\cos(2ix)+2)\sin(ix)}{\cos(ix)^2\sqrt{\sin(ix)^2+1}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \; 26$$

$$\frac{1}{3}i\int \frac{(\cos(2ix)+2)\sin(ix)}{\cos(ix)^2\sqrt{\sin(ix)^2+1}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \; 4857$$

$$-\frac{1}{3}\int \frac{(2\cosh^2(x)+1)\operatorname{sech}^2(x)}{\sqrt{2-\cosh^2(x)}}d\cosh(x)-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \; 358$$

$$\frac{1}{3}\left(\frac{1}{2}\sqrt{2-\cosh^2(x)}\operatorname{sech}(x)-2\int \frac{1}{\sqrt{2-\cosh^2(x)}}d\cosh(x)\right)-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \; 223$$

$$-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))+\frac{1}{3}\left(\frac{1}{2}\sqrt{2-\cosh^2(x)}\operatorname{sech}(x)-2\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right)\right)$$

input
```
Int[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

output
```
(-2*ArcSin[Cosh[x]/Sqrt[2]] + (Sqrt[2 - Cosh[x]^2]*Sech[x])/2)/3 + ArcSin[
Sinh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

3.703.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 5343
```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcSin[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/Sqrt[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b},
 x] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /;
 FreeQ[{c, d, m}, x]]
```

3.703.4 Maple [F]

$$\int \arcsin(\sinh(x))\,\mathrm{sech}(x)^4\,dx$$

input
```
int(arcsin(sinh(x))*sech(x)^4,x)
```

output
```
int(arcsin(sinh(x))*sech(x)^4,x)
```

3.703.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 519, normalized size of antiderivative = 10.59

$$\int \arcsin(\sinh(x))\mathrm{sech}^4(x)\,dx = \text{Too large to display}$$

input
```
integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(
-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
- 4*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sin
h(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x
)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^
3 + cosh(x))*sinh(x) + 1)*arctan(sqrt(2)*(3*cosh(x)^2 + 6*cosh(x)*sinh(x)
+ 3*sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(
cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x)
 + 1)) + 8*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 1)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sin
h(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 +
4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5
+ sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3
 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*
cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.703.6 Sympy [F(-1)]

Timed out.

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \text{Timed out}$$

input
```
integrate(asin(sinh(x))*sech(x)**4,x)
```

output
```
Timed out
```

3.703.7 Maxima [F]

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \int \arcsin\left(\sinh\left(x\right)\right)\text{sech}\left(x\right)^4\,dx$$

input
```
integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

output
```
-1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x - 1)*sqr
t(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)*integra
te(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x) + 2*e^x - 1) + 1/2*log(-e
^(2*x) + 2*e^x + 1))/((e^(8*x) - 4*e^(6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e
^(log(e^(2*x) + 2*e^x - 1) + log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(
10*x) - e^(8*x) + 12*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*
e^(4*x) + 3*e^(2*x) + 1)
```

3.703.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.45

$$\begin{aligned}
&\int \arcsin(\sinh(x))\operatorname{sech}^4(x)\,dx \\
&= -\frac{16\left(-8i\sqrt{2}\arctan\left(-i\right)-3\sqrt{2}+32\arctan\left(-i\right)-3i\right)}{96i\sqrt{2}-384} \\
&\quad+\frac{\sqrt{2}+\frac{2\sqrt{2}-\sqrt{-e^{(4x)}+6e^{(2x)}-1}}{e^{(2x)}-3}}{6\left(\frac{\sqrt{2}\left(2\sqrt{2}-\sqrt{-e^{(4x)}+6e^{(2x)}-1}\right)}{e^{(2x)}-3}+\frac{\left(2\sqrt{2}-\sqrt{-e^{(4x)}+6e^{(2x)}-1}\right)^2}{\left(e^{(2x)}-3\right)^2}+1\right)} \\
&\quad-\frac{4\left(3e^{(2x)}+1\right)\arcsin\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)}{3\left(e^{(2x)}+1\right)^3} \\
&\quad-\frac{4}{3}\arctan\left(-2\sqrt{2}-\frac{3\left(2\sqrt{2}-\sqrt{-e^{(4x)}+6e^{(2x)}-1}\right)}{e^{(2x)}-3}\right)
\end{aligned}$$

input
```
integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="giac")
```

output
```
-16*(-8*I*sqrt(2)*arctan(-I) - 3*sqrt(2) + 32*arctan(-I) - 3*I)/(96*I*sqrt
(2) - 384) + 1/6*(sqrt(2) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(
e^(2*x) - 3))/(sqrt(2)*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^(2*
x) - 3) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))^2/(e^(2*x) - 3)^2 +
 1) - 4/3*(3*e^(2*x) + 1)*arcsin(1/2*(e^(2*x) - 1)*e^(-x))/(e^(2*x) + 1)^3
 - 4/3*arctan(-2*sqrt(2) - 3*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/
(e^(2*x) - 3))
```

3.703.9 Mupad [F(-1)]

Timed out.

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \int \frac{\text{asin}(\sinh(x))}{\cosh(x)^4}\,dx$$

input
```
int(asin(sinh(x))/cosh(x)^4,x)
```

output
```
int(asin(sinh(x))/cosh(x)^4, x)
```

3.704 $\int \cot^{-1}(\cosh(x))\coth(x)\mathbf{csch}^3(x)\,dx$

3.704.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)$$

output
```
1/6*coth(x)-1/3*arccot(cosh(x))*csch(x)^3+1/12*arctanh(1/2*2^(1/2)*tanh(x)
)*2^(1/2)
```

3.704.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.00

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = \frac{1}{48}\operatorname{csch}^3(x)\left(-16\cot^{-1}(\cosh(x))-2\cosh(x)+2\cosh(3x)-3i\sqrt{2}\arctan\left(1-i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(x)+3i\sqrt{2}\arctan\left(1+i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(x)+i\sqrt{2}\arctan\left(1-i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(3x)-i\sqrt{2}\arctan\left(1+i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(3x)\right)$$

input
```
Integrate[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

output
```
(Csch[x]^3*(-16*ArcCot[Cosh[x]] - 2*Cosh[x] + 2*Cosh[3*x] - (3*I)*Sqrt[2]*
ArcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[x] + (3*I)*Sqrt[2]*ArcTan[1 + I*Sqrt[
2]*Tanh[x/2]]*Sinh[x] + I*Sqrt[2]*ArcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[3*x
] - I*Sqrt[2]*ArcTan[1 + I*Sqrt[2]*Tanh[x/2]]*Sinh[3*x]))/48
```

3.704.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5731, 27, 3042, 25, 4889, 27, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\text{csch}^3(x)\cot^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{5731}$$

$$\int -\frac{2\text{csch}^2(x)}{3(\cosh(2x)+3)}dx-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}\int \frac{\text{csch}^2(x)}{\cosh(2x)+3}dx-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))-\frac{2}{3}\int -\frac{1}{(\cos(2ix)+3)\sin(ix)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))+\frac{2}{3}\int \frac{1}{(\cos(2ix)+3)\sin(ix)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{2}{3}\int -\frac{\coth^2(x)\left(1-\tanh^2(x)\right)}{2\left(2-\tanh^2(x)\right)}d\tanh(x)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{\coth^2(x)\left(1-\tanh^2(x)\right)}{2-\tanh^2(x)}d\tanh(x)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 359$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{2-\tanh^2(x)}d\tanh(x)+\frac{\coth(x)}{2}\right)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\coth(x)}{2}\right)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

input
```
Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

output
```
(ArcTanh[Tanh[x]/Sqrt[2]]/(2*Sqrt[2]) + Coth[x]/2)/3 - (ArcCot[Cosh[x]]*Cs
ch[x]^3)/3
```

3.704.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.704. $\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx$

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

3.704.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 850, normalized size of antiderivative = 23.61

Expression too large to display

input
```
int(arccot(cosh(x))*cosh(x)/sinh(x)^4,x)
```

output
```
4/3*I*exp(3*x)/(exp(2*x)-1)^3*ln(exp(2*x)+1+2*I*exp(x))-1/24*(-8-16*Pi*csg
n(-I*exp(2*x)+2*exp(x)-I)*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*
x)-16*Pi*csgn(I*exp(2*x)+I+2*exp(x))*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1
))^2*exp(3*x)-16*Pi*csgn(I*exp(-x))*csgn(I*exp(2*x)+I+2*exp(x))*csgn(I*exp
(-x)*(2*I*exp(x)-exp(2*x)-1))*exp(3*x)+16*exp(2*x)+16*Pi*csgn(I*exp(-x)*(2
*I*exp(x)-exp(2*x)-1))*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))*exp(3*x)-16*P
i*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp
(x)))*exp(3*x)-8*exp(4*x)+16*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)+1
+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))*csg
n(exp(-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(exp(2*
x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*c
sgn(I*exp(-x))*csgn(-I*exp(2*x)+2*exp(x)-I)*csgn(I*exp(-x)*(exp(2*x)+1+2*I
*exp(x)))*exp(3*x)-2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)-16*Pi*csgn(I*exp(-x)
)*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x)+2^(1/2)*ln(exp(2*x)+(
2^(1/2)-1)^2)+16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)-16*Pi
*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3*x)+16*Pi*csgn(exp(-x)*(2*I*
exp(x)-exp(2*x)-1))^3*exp(3*x)+16*Pi*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))
^2*exp(3*x)-ln(exp(2*x)+(2^(1/2)-1)^2)*2^(1/2)*exp(6*x)+32*I*exp(3*x)*ln(e
xp(2*x)+1-2*I*exp(x))-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(
3*x)+ln(exp(2*x)+(1+2^(1/2))^2)*2^(1/2)*exp(6*x)+3*ln(exp(2*x)+(2^(1/2)...
```

3.704.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 11.75

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx$$

$$= \frac{8\,\cosh(x)^4 + 32\,\cosh(x)\sinh(x)^3 + 8\,\sinh(x)^4 + 16\left(3\,\cosh(x)^2 - 1\right)\sinh(x)^2 - 64\left(\cosh(x)^3 + 3\,\cosh\right.}{}$$

input
```
integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="fricas")
```

output
```
1/24*(8*cosh(x)^4 + 32*cosh(x)*sinh(x)^3 + 8*sinh(x)^4 + 16*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 64*(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2
+ sinh(x)^3)*arctan(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) - 16*cosh(x)^2 + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*s
inh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(
x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 +
 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(2
)*cosh(x))*sinh(x) - sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqr
t(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(
cosh(x)^2 + sinh(x)^2 + 3)) + 32*(cosh(x)^3 - cosh(x))*sinh(x) + 8)/(cosh(
x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3
*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*co
sh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x
))*sinh(x) - 1)
```

3.704.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(34) = 68$.

Time = 78.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 5.94

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = -\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{24} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{24} - \frac{\tanh^3\left(\frac{x}{2}\right)\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{24} + \frac{\tanh\left(\frac{x}{2}\right)\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{8} + \frac{\tanh\left(\frac{x}{2}\right)}{12} - \frac{\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{8\tanh\left(\frac{x}{2}\right)} + \frac{1}{12\tanh\left(\frac{x}{2}\right)} + \frac{\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{24\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

output
```
-sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/24 + sqrt(2)*log(4*
tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/24 - tanh(x/2)**3*acot(tanh(x/2)**
2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/24 + tanh(x/2)*acot(tanh(x/2)
**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/8 + tanh(x/2)/12 - acot(tan
h(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/(8*tanh(x/2)) + 1/(12
*tanh(x/2)) + acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))
/(24*tanh(x/2)**3)
```

3.704.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = -\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right) - \frac{1}{3\left(e^{(-2x)}-1\right)} - \frac{\operatorname{arccot}\left(\cosh\left(x\right)\right)}{3\sinh\left(x\right)^3}$$

input
```
integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="maxima")
```

output
```
-1/24*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
- 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)^3
```

3.704.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = \frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) + \frac{1}{3\left(e^{(2x)}-1\right)} + \frac{8\arctan\left(\frac{2}{e^{(-x)}+e^{x}}\right)}{3\left(e^{(-x)}-e^{x}\right)^3}$$

input
```
integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="giac")
```

output
```
1/24*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1
/3/(e^(2*x) - 1) + 8/3*arctan(2/(e^(-x) + e^x))/(e^(-x) - e^x)^3
```

3.704.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \cot^{-1}(\cosh(x)) \coth(x)\operatorname{csch}^3(x)\, dx = \frac{\sqrt{2}\,\ln\left(-\frac{2\,e^{2\,x}}{3}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{24}\right)}{24} - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{24}-\frac{2\,e^{2\,x}}{3}\right)}{24} + \frac{1}{3\,\left(e^{2\,x}-1\right)} - \frac{8\,e^{3\,x}\,\mathrm{acot}\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}{3\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}$$

input
```
int((acot(cosh(x))*cosh(x))/sinh(x)^4,x)
```

output
```
(2^(1/2)*log(- (2*exp(2*x))/3 - (2^(1/2)*(12*exp(2*x) + 4))/24))/24 - (2^(
1/2)*log((2^(1/2)*(12*exp(2*x) + 4))/24 - (2*exp(2*x))/3))/24 + 1/(3*(exp(
2*x) - 1)) - (8*exp(3*x)*acot(exp(-x)/2 + exp(x)/2))/(3*(3*exp(2*x) - 3*ex
p(4*x) + exp(6*x) - 1))
```

3.705 $\int e^x \arcsin(\tanh(x))\,dx$

3.705.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int e^x \arcsin(\tanh(x))\,dx = e^x \arcsin(\tanh(x)) - \cosh(x)\log\left(1+e^{2x}\right)\sqrt{\operatorname{sech}^2(x)}$$

output
```
exp(x)*arcsin(tanh(x))-cosh(x)*ln(1+exp(2*x))*(sech(x)^2)^(1/2)
```

3.705.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.89

$$\int e^x \arcsin(\tanh(x))\,dx = -e^{-x}\sqrt{\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)\log\left(1+e^{2x}\right) - ie^x\log\left(\frac{-ie^{-x}+ie^x+2\left(e^{-x}+e^x\right)\sqrt{\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}}{e^{-x}+e^x}\right)$$

input
```
Integrate[E^x*ArcSin[Tanh[x]],x]
```

output
```
-((Sqrt[E^(2*x)/(1 + E^(2*x))^2]*(1 + E^(2*x))*Log[1 + E^(2*x)])/E^x) - I*
E^x*Log[((-I)/E^x + I*E^x + 2*(E^(-x) + E^x)*Sqrt[E^(2*x)/(1 + E^(2*x))^2]
)/(E^(-x) + E^x)]
```

3.705.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5343, 7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \arcsin(\tanh(x))\,dx$$

$$\Big\downarrow \text{5343}$$

$$e^x \arcsin(\tanh(x)) - \int e^x \sqrt{\text{sech}^2(x)}dx$$

$$\Big\downarrow \text{7271}$$

$$e^x \arcsin(\tanh(x)) - \cosh(x)\sqrt{\text{sech}^2(x)}\int e^x \text{sech}(x)dx$$

$$\Big\downarrow \text{2720}$$

$$e^x \arcsin(\tanh(x)) - \cosh(x)\sqrt{\text{sech}^2(x)}\int \frac{2e^x}{1+e^{2x}}de^x$$

$$\Big\downarrow \text{27}$$

$$e^x \arcsin(\tanh(x)) - 2\cosh(x)\sqrt{\text{sech}^2(x)}\int \frac{e^x}{1+e^{2x}}de^x$$

$$\Big\downarrow \text{240}$$

$$e^x \arcsin(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\text{sech}^2(x)}$$

input
```
Int[E^x*ArcSin[Tanh[x]],x]
```

output
```
E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

3.705.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5343
```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcSin[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/Sqrt[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b},
 x] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /;
 FreeQ[{c, d, m}, x]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.705.4 Maple [F]

$$\int \mathrm{e}^{x} \arcsin \left(\tanh \left(x\right)\right)dx$$

input
```
int(exp(x)*arcsin(tanh(x)),x)
```

output
```
int(exp(x)*arcsin(tanh(x)),x)
```

3.705.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int e^x \arcsin(\tanh(x))\,dx = (\cosh(x) + \sinh(x))\arctan(\sinh(x)) - \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

input
```
integrate(exp(x)*arcsin(tanh(x)),x, algorithm="fricas")
```

output
```
(cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.705.6 Sympy [F]

$$\int e^x \arcsin(\tanh(x))\,dx = \int e^x \operatorname{asin}(\tanh(x))\,dx$$

input
```
integrate(exp(x)*asin(tanh(x)),x)
```

output
```
Integral(exp(x)*asin(tanh(x)), x)
```

3.705.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^x \arcsin(\tanh(x))\,dx = \arcsin(\tanh(x))\,e^x - \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(exp(x)*arcsin(tanh(x)),x, algorithm="maxima")
```

output
```
arcsin(tanh(x))*e^x - log(e^(2*x) + 1)
```

3.705.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int e^x \arcsin(\tanh(x))\,dx = \arcsin\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)e^x - \log\left(e^{(2x)}+1\right)$$

input
```
integrate(exp(x)*arcsin(tanh(x)),x, algorithm="giac")
```

output
```
arcsin((e^(2*x) - 1)/(e^(2*x) + 1))*e^x - log(e^(2*x) + 1)
```

3.705.9 Mupad [F(-1)]

Timed out.

$$\int e^x \arcsin(\tanh(x))\,dx = \int \operatorname{asin}(\tanh(x))\,\mathrm{e}^x\,dx$$

input
```
int(asin(tanh(x))*exp(x),x)
```

output
```
int(asin(tanh(x))*exp(x), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```