Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

0-Independent-test-suites/11-Welz-Problems

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [116]. This is test number [11].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	94.83 (110)	5.17 (6)
Mathematica	87.93 (102)	12.07 (14)
Fricas	77.59 (90)	22.41 (26)
Maple	75.86 (88)	24.14 (28)
Mupad	31.90 (37)	68.10 (79)
Giac	31.03 (36)	68.97 (80)
Sympy	25.00 (29)	75.00 (87)
Maxima	17.24 (20)	82.76 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

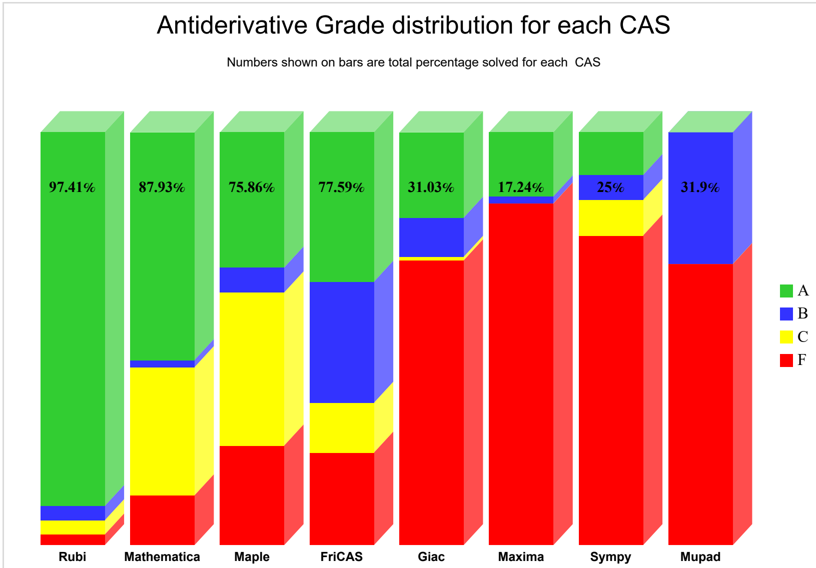
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

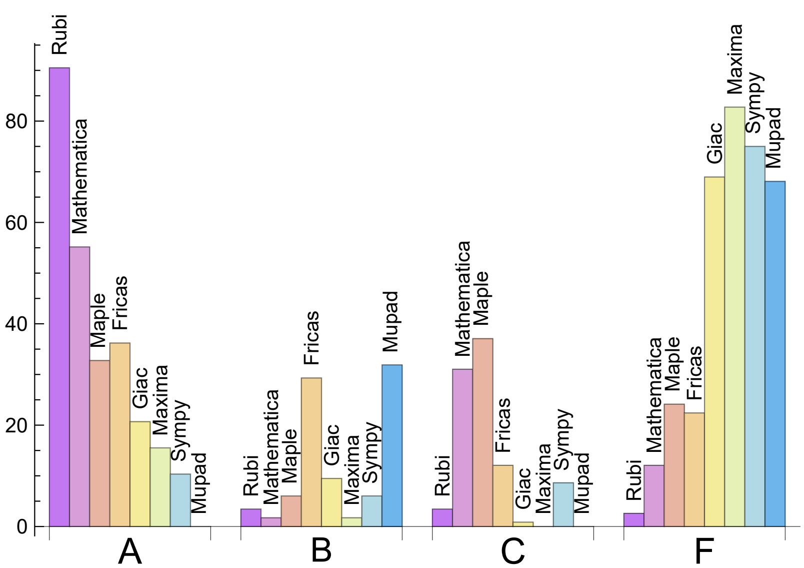
System	% A grade	% B grade	% C grade	% F grade
Rubi	87.931	3.448	3.448	5.172
Mathematica	55.172	1.724	31.034	12.069
Fricas	36.207	29.310	12.069	22.414
Maple	32.759	6.034	37.069	24.138
Giac	20.690	9.483	0.862	68.966
Maxima	15.517	1.724	0.000	82.759
Sympy	10.345	6.034	8.621	75.000
Mupad	0.000	31.897	0.000	68.103

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00	0.00	0.00
Mathematica	14	100.00	0.00	0.00
Fricas	26	42.31	26.92	30.77
Maple	28	100.00	0.00	0.00
Mupad	79	0.00	100.00	0.00
Giac	80	98.75	1.25	0.00
Sympy	87	90.80	8.05	1.15
Maxima	96	98.96	0.00	1.04

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.25
Rubi	0.36
Sympy	0.72
Mupad	0.92
Giac	1.46
Fricas	2.66
Mathematica	3.80
Maple	4.56

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	81.80	1.16	62.00	1.05
Mathematica	132.28	1.16	83.50	1.07
Rubi	151.02	1.29	90.00	1.00
Mupad	151.70	1.71	76.00	1.09
Sympy	208.41	5.09	37.00	0.82
Giac	247.89	1.54	69.50	1.11
Maple	336.27	3.08	111.00	1.18
Fricas	459.56	3.39	194.50	1.75

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

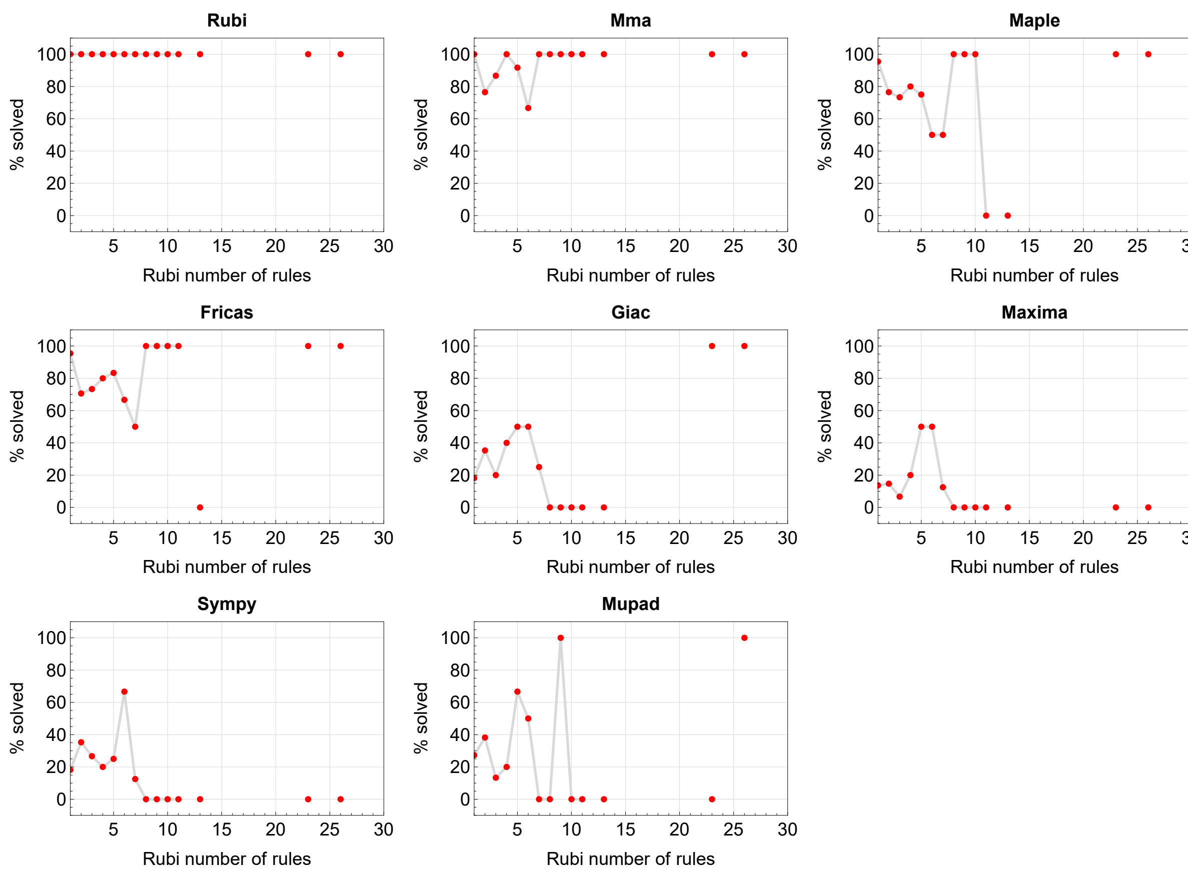


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

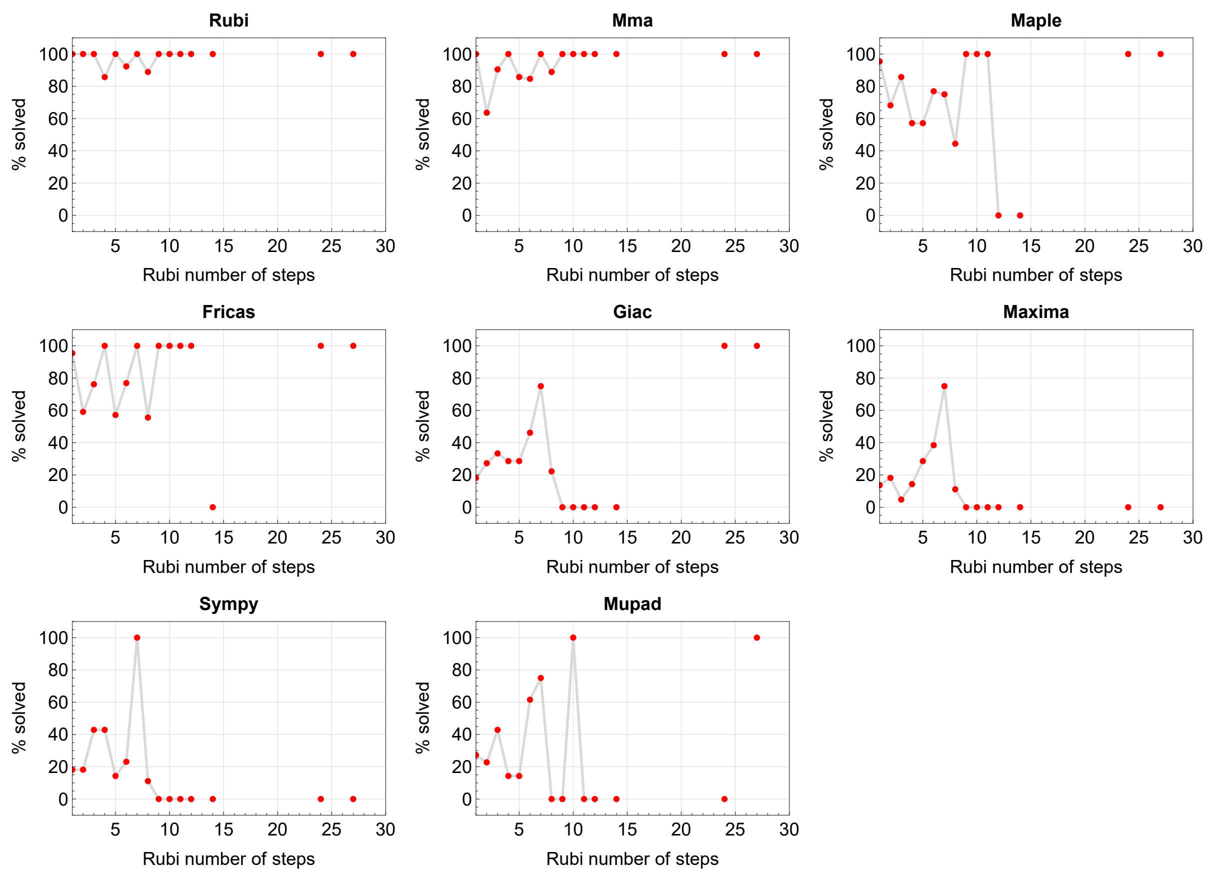


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

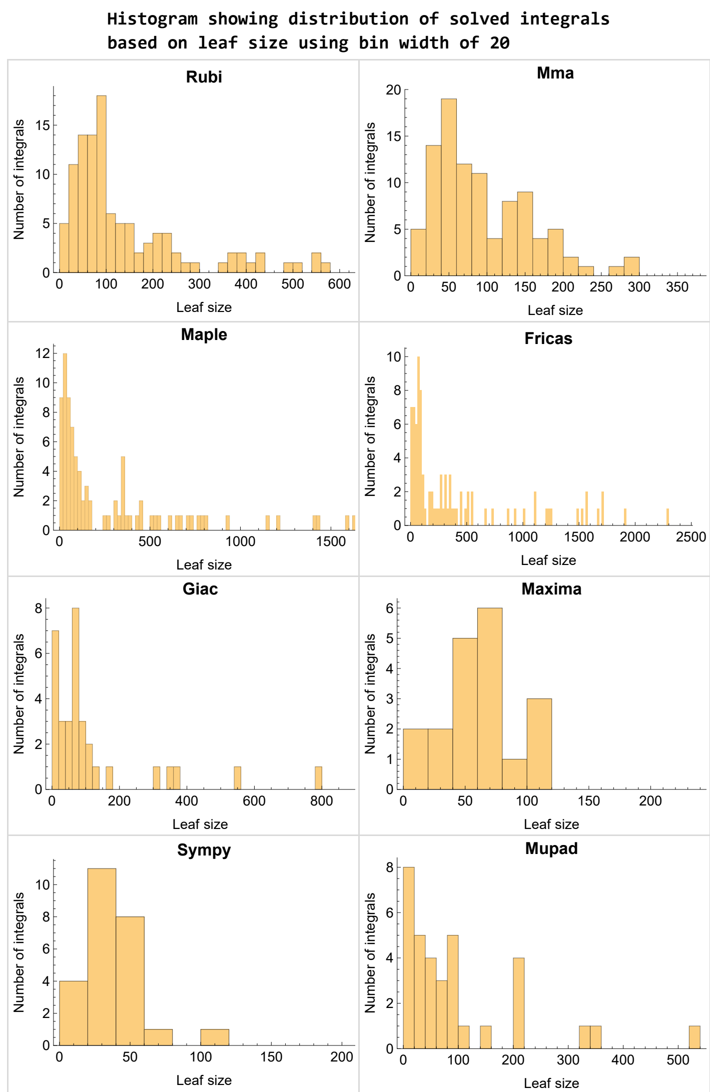


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

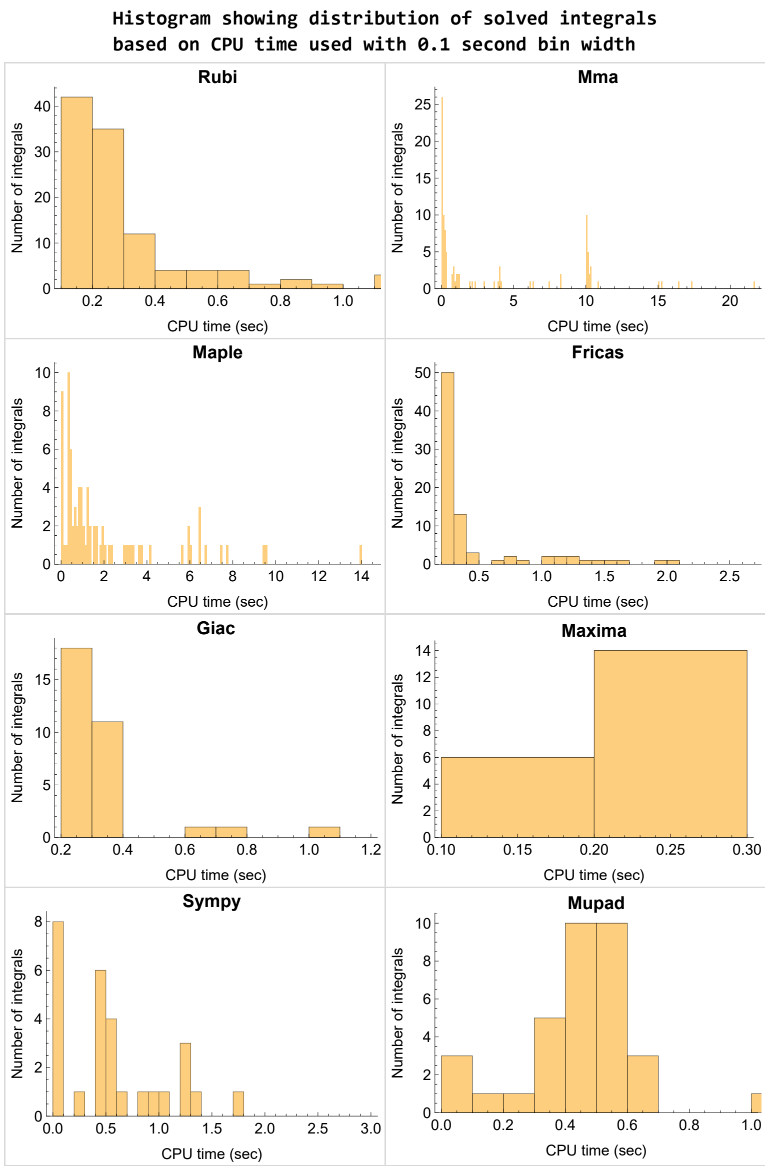


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

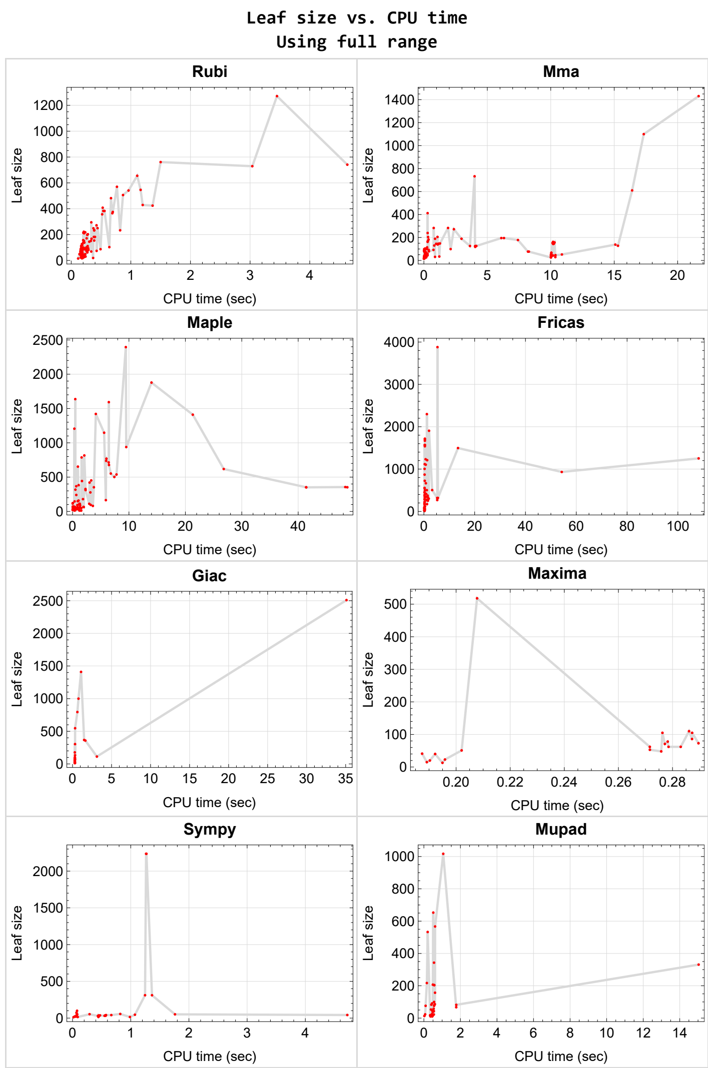


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{40, 41, 42, 82\}$

Mathematica $\{53, 54, 69, 70, 71, 72, 77, 78, 79, 80, 115\}$

Maple $\{37, 38, 39, 52, 59, 74, 76, 77, 100, 101, 102, 116\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

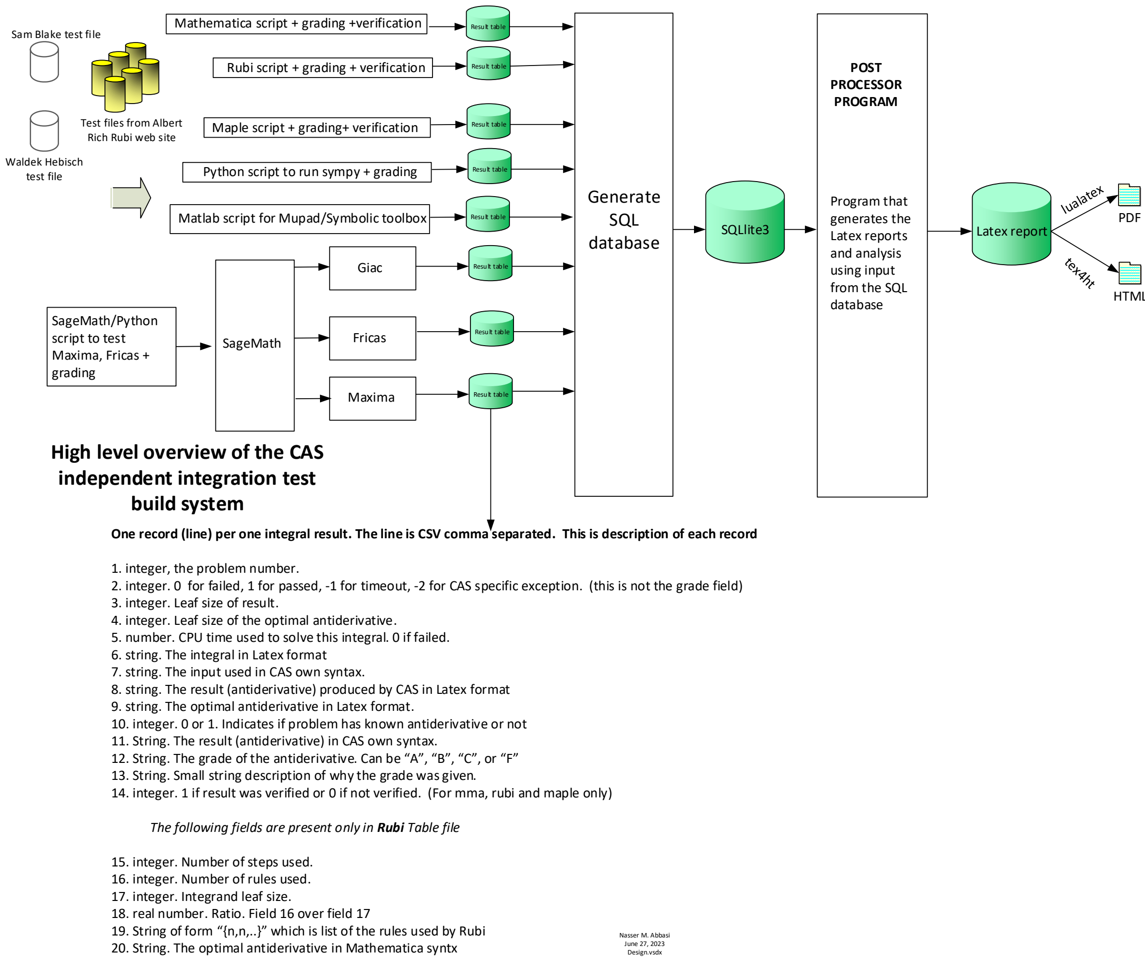
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

B grade { 10, 100, 101, 102 }

C grade { 2, 52, 82, 83 }

F normal fail { 43, 44, 45, 48, 49, 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 43, 44, 52, 55, 56, 57, 62, 66, 67, 81, 82, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 113, 116 }

B grade { 12, 13 }

C grade { 2, 15, 24, 40, 47, 48, 49, 50, 51, 53, 54, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 94, 96, 106, 114, 115 }

F normal fail { 38, 45, 46, 58, 59, 60, 61, 93, 95, 108, 109, 110, 111, 112 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 4, 6, 7, 8, 11, 16, 19, 20, 21, 22, 23, 25, 26, 30, 32, 33, 36, 41, 44, 47, 48, 49, 50, 51, 62, 63, 66, 67, 81, 94, 96, 97, 99, 103, 104, 105, 113 }

B grade { 5, 9, 10, 17, 24, 31, 85 }

C grade { 2, 3, 15, 28, 34, 35, 37, 38, 39, 40, 52, 55, 56, 57, 59, 64, 65, 68, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 106, 107, 110, 116 }

F normal fail { 12, 13, 14, 18, 27, 29, 42, 43, 45, 46, 53, 54, 58, 60, 61, 69, 70, 71, 72, 80, 95, 98, 108, 109, 111, 112, 114, 115 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade { 4, 7, 9, 10, 12, 13, 14, 35, 37, 39, 41, 42, 43, 45, 55, 74, 76, 78, 80, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 100, 101, 102 }

C grade { 24, 47, 48, 49, 50, 51, 59, 68, 73, 75, 77, 79, 82, 110 }

F normal fail { 2, 29, 61, 103, 104, 105, 109, 111, 112, 114, 115 }

F(-1) timedout fail { 44, 69, 70, 71, 72, 95, 108 }

F(-2) exception fail { 38, 46, 53, 54, 58, 60, 84, 92 }

2.1.5 Maxima

A grade { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 96, 99, 107 }

B grade { 2, 106 }

C grade { }

F normal fail { 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-1) timedout fail { }

F(-2) exception fail { 11 }

2.1.6 Giac

A grade { 1, 6, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 36, 41, 49, 57, 62, 63, 64, 96, 99, 107 }

B grade { 2, 3, 4, 5, 7, 9, 10, 11, 24, 47, 48 }

C grade { 50 }

F normal fail { 12, 13, 14, 15, 17, 18, 27, 28, 29, 31, 32, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-1) timedout fail { 51 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 5, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 35, 36, 41, 47, 48, 49, 57, 62, 63, 64, 73, 74, 75, 81, 82, 84, 85, 96, 99, 106, 107 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 27, 28, 29, 31, 32, 37, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 14, 15, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade { 8, 16, 17, 19, 20, 30, 31 }

C grade { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F normal fail { 3, 4, 5, 6, 7, 9, 12, 13, 18, 24, 29, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F(-1) timedout fail { 10, 32, 47, 48, 49, 51, 52 }

F(-2) exception fail { 11 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.118	0.002	0.315	0.195	0.242	0.017	0.300	0.029

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	49	37	34	41	0	42	41	43
N.S.	1	3.27	2.47	2.27	2.73	0.00	2.80	2.73	2.87
time (sec)	N/A	0.238	0.019	0.342	0.187	0.000	4.721	0.283	0.547

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	60	73	0	100	0	177	204
N.S.	1	1.00	0.73	0.89	0.00	1.22	0.00	2.16	2.49
time (sec)	N/A	0.251	0.188	0.201	0.000	0.247	0.000	0.292	0.568

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	52	0	80	0	94	0
N.S.	1	1.00	1.65	1.21	0.00	1.86	0.00	2.19	0.00
time (sec)	N/A	0.155	0.074	0.460	0.000	0.243	0.000	0.299	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	115	0	105	0	133	82
N.S.	1	1.00	0.99	1.55	0.00	1.42	0.00	1.80	1.11
time (sec)	N/A	0.239	0.065	0.024	0.000	0.240	0.000	0.323	1.773

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	70	65	53	89	0	84	0
N.S.	1	1.00	1.09	1.02	0.83	1.39	0.00	1.31	0.00
time (sec)	N/A	0.184	0.186	0.424	0.272	0.243	0.000	0.286	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	37	0	83	0	101	0
N.S.	1	1.00	1.15	0.77	0.00	1.73	0.00	2.10	0.00
time (sec)	N/A	0.150	0.104	0.416	0.000	0.238	0.000	0.298	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	21	0	18	53	20	21
N.S.	1	1.00	0.87	0.70	0.00	0.60	1.77	0.67	0.70
time (sec)	N/A	0.242	0.181	0.329	0.000	0.235	0.292	0.281	0.438

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	365	195	1206	0	450	0	367	0
N.S.	1	1.66	0.89	5.48	0.00	2.05	0.00	1.67	0.00
time (sec)	N/A	0.694	6.105	0.324	0.000	0.245	0.000	1.489	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	541	195	1637	0	450	0	358	0
N.S.	1	2.46	0.89	7.44	0.00	2.05	0.00	1.63	0.00
time (sec)	N/A	0.954	6.314	0.485	0.000	0.250	0.000	1.649	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	126	146	0	161	0	547	0
N.S.	1	1.00	0.91	1.06	0.00	1.17	0.00	3.96	0.00
time (sec)	N/A	0.221	3.633	0.366	0.000	0.252	0.000	0.331	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	151	272	0	0	502	0	0	0
N.S.	1	1.21	2.18	0.00	0.00	4.02	0.00	0.00	0.00
time (sec)	N/A	0.334	2.366	0.000	0.000	1.171	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	205	0	0	504	0	0	0
N.S.	1	1.00	2.53	0.00	0.00	6.22	0.00	0.00	0.00
time (sec)	N/A	0.287	1.074	0.000	0.000	3.271	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	44	0	0	60	15	0	0
N.S.	1	1.00	1.42	0.00	0.00	1.94	0.48	0.00	0.00
time (sec)	N/A	0.190	0.134	0.000	0.000	0.368	0.986	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	57	22	0	29	15	0	0
N.S.	1	1.00	1.73	0.67	0.00	0.88	0.45	0.00	0.00
time (sec)	N/A	0.196	0.158	0.358	0.000	0.385	0.444	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	28	56	15	15
N.S.	1	1.00	1.00	0.84	0.79	1.47	2.95	0.79	0.79
time (sec)	N/A	0.370	0.009	0.411	0.189	0.240	0.820	0.308	0.476

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	43	120	0	32	2236	0	0
N.S.	1	0.98	0.83	2.31	0.00	0.62	43.00	0.00	0.00
time (sec)	N/A	0.186	0.144	0.040	0.000	0.248	1.264	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	55	50	0	0	33	0	0	0
N.S.	1	0.98	0.89	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.184	0.140	0.000	0.000	0.255	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	15	311	15	15
N.S.	1	1.00	1.00	0.94	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.183	0.022	0.332	0.000	0.240	1.365	0.303	0.343

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	18	41	18	18
N.S.	1	1.00	1.00	0.95	0.00	0.90	2.05	0.90	0.90
time (sec)	N/A	0.186	0.023	0.356	0.000	0.245	0.666	0.286	0.342

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	39	43	40	52	36	47	58
N.S.	1	1.07	0.93	1.02	0.95	1.24	0.86	1.12	1.38
time (sec)	N/A	0.187	0.040	0.056	0.192	0.239	0.068	0.280	0.534

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	20	19	20	19	24
N.S.	1	1.00	1.00	0.91	0.91	0.86	0.91	0.86	1.09
time (sec)	N/A	0.169	0.022	0.046	0.190	0.254	0.055	0.292	0.436

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	50	51	58	51	74	47
N.S.	1	1.00	0.79	0.81	0.82	0.94	0.82	1.19	0.76
time (sec)	N/A	0.281	0.071	0.104	0.202	0.267	0.081	0.294	0.454

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	94	239	158	0	285	0	304	0
N.S.	1	1.09	2.78	1.84	0.00	3.31	0.00	3.53	0.00
time (sec)	N/A	0.291	0.299	1.065	0.000	0.256	0.000	0.322	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	0	15	15	15	15
N.S.	1	1.00	1.00	0.84	0.00	0.79	0.79	0.79	0.79
time (sec)	N/A	0.199	0.048	0.435	0.000	0.243	0.090	0.292	0.484

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	27	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	1.04	0.85	0.85
time (sec)	N/A	0.221	0.068	0.343	0.000	0.242	0.454	0.294	0.546

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	69	56	0	0	198	46	0	0
N.S.	1	1.10	0.89	0.00	0.00	3.14	0.73	0.00	0.00
time (sec)	N/A	0.336	0.105	0.000	0.000	0.258	1.072	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	92	82	25	0	216	51	0	0
N.S.	1	1.12	1.00	0.30	0.00	2.63	0.62	0.00	0.00
time (sec)	N/A	0.224	0.013	0.023	0.000	0.261	1.760	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	606	741	412	0	518	0	0	0	0
N.S.	1	1.22	0.68	0.00	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	4.520	0.295	0.000	0.208	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	15	311	15	15
N.S.	1	1.00	1.00	0.94	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.184	0.043	0.367	0.000	0.245	1.246	0.305	0.355

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	46	120	0	32	2236	0	0
N.S.	1	0.98	0.88	2.31	0.00	0.62	43.00	0.00	0.00
time (sec)	N/A	0.184	0.209	0.036	0.000	0.259	1.270	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	48	48	0	0	0
N.S.	1	1.00	0.97	1.29	1.41	1.41	0.00	0.00	0.00
time (sec)	N/A	0.212	0.196	0.723	0.276	0.246	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	61	77	63	62	64	36	64	86
N.S.	1	1.05	1.33	1.09	1.07	1.10	0.62	1.10	1.48
time (sec)	N/A	0.171	0.069	1.300	0.283	0.242	0.447	0.287	0.603

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	62	77	48	62	64	37	64	76
N.S.	1	1.07	1.33	0.83	1.07	1.10	0.64	1.10	1.31
time (sec)	N/A	0.173	0.054	1.234	0.279	0.244	0.472	0.298	0.557

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	78	82	29	0	10
N.S.	1	1.00	1.76	0.24	1.59	1.67	0.59	0.00	0.20
time (sec)	N/A	0.141	0.043	1.540	0.278	0.241	0.436	0.000	0.367

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	61	79	63	62	64	32	63	80
N.S.	1	1.11	1.44	1.15	1.13	1.16	0.58	1.15	1.45
time (sec)	N/A	0.173	0.056	1.997	0.272	0.245	0.453	0.289	0.575

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	97	148	769	0	301	0	0	0
N.S.	1	1.00	1.53	7.93	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.197	1.271	6.006	0.000	1.929	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	145	145	0	1878	0	0	0	0	0
N.S.	1	1.00	0.00	12.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.000	13.997	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	120	162	1593	0	277	0	0	0
N.S.	1	1.09	1.47	14.48	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.187	0.294	6.433	0.000	1.226	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	85	653	0	120	0	0	0
N.S.	1	1.06	1.05	8.06	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.215	0.015	0.950	0.000	0.404	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	86	130	92	0	415	0	67	37
N.S.	1	1.30	1.97	1.39	0.00	6.29	0.00	1.02	0.56
time (sec)	N/A	0.210	0.818	0.849	0.000	0.898	0.000	0.319	0.483

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	94	145	0	0	665	0	0	0
N.S.	1	1.19	1.84	0.00	0.00	8.42	0.00	0.00	0.00
time (sec)	N/A	0.213	0.968	0.000	0.000	0.783	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	0	189	0	0	1496	0	0	0
N.S.	1	0.00	1.60	0.00	0.00	12.68	0.00	0.00	0.00
time (sec)	N/A	0.000	2.966	0.000	0.000	13.441	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	0	128	111	0	0	0	0	0
N.S.	1	0.00	1.15	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	15.299	1.215	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	B	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	932	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	5.30	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	54.282	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	493	570	0	0	0	0	0	0	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	506	178	277	0	555	0	795	343
N.S.	1	1.24	0.44	0.68	0.00	1.36	0.00	1.95	0.84
time (sec)	N/A	0.874	7.416	3.160	0.000	0.272	0.000	0.623	0.554

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	C	F(-1)	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	0	610	352	0	1005	0	1000	567
N.S.	1	0.00	0.94	0.54	0.00	1.55	0.00	1.54	0.88
time (sec)	N/A	0.000	16.413	3.783	0.000	0.284	0.000	0.765	0.614

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	C	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1058	0	1100	502	0	1905	0	1410	1017
N.S.	1	0.00	1.04	0.47	0.00	1.80	0.00	1.33	0.96
time (sec)	N/A	0.000	17.330	7.409	0.000	2.036	0.000	1.091	1.060

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	424	733	452	0	869	0	2509	0
N.S.	1	1.12	1.94	1.20	0.00	2.30	0.00	6.64	0.00
time (sec)	N/A	1.341	3.996	3.310	0.000	0.285	0.000	35.108	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	0	1431	552	0	1569	0	0	0
N.S.	1	0.00	2.24	0.87	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.000	21.655	6.780	0.000	0.322	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	429	99	787	0	546	0	0	0
N.S.	1	6.50	1.50	11.92	0.00	8.27	0.00	0.00	0.00
time (sec)	N/A	1.220	2.110	1.605	0.000	0.297	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	198	203	145	0	0	0	0	0	0
N.S.	1	1.03	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	10.240	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	153	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	10.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	168	2395	0	171	0	0	0
N.S.	1	1.00	1.73	24.69	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.172	0.312	9.443	0.000	1.244	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	76	99	15	105	96	32	0	0
N.S.	1	1.04	1.36	0.21	1.44	1.32	0.44	0.00	0.00
time (sec)	N/A	0.168	0.240	1.189	0.276	0.257	0.568	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	75	88	49	71	73	37	72	83
N.S.	1	1.12	1.31	0.73	1.06	1.09	0.55	1.07	1.24
time (sec)	N/A	0.196	0.074	1.553	0.277	0.245	0.561	0.295	0.407

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	482	482	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.665	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	280	408	0	1410	0	3880	0	0	0
N.S.	1	1.46	0.00	5.04	0.00	13.86	0.00	0.00	0.00
time (sec)	N/A	0.546	0.000	21.318	0.000	5.355	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	23	23	22	23	23
N.S.	1	1.00	1.00	0.88	0.92	0.92	0.88	0.92	0.92
time (sec)	N/A	0.172	0.009	0.022	0.196	0.264	0.038	0.304	0.071

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	43	73	40	53
N.S.	1	1.00	1.46	0.88	0.00	0.73	1.24	0.68	0.90
time (sec)	N/A	0.184	0.025	0.046	0.000	0.249	0.067	0.311	0.389

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	85	99	70	0	66	100	64	76
N.S.	1	1.09	1.27	0.90	0.00	0.85	1.28	0.82	0.97
time (sec)	N/A	0.257	0.027	0.036	0.000	0.247	0.078	0.302	0.096

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	57	55	0	56	0	0	0
N.S.	1	1.00	1.16	1.12	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.149	0.289	0.904	0.000	0.280	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	62	0	61	0	0	0
N.S.	1	1.00	0.83	1.17	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.159	0.291	0.862	0.000	0.285	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	101	79	89	0	359	0	0	0
N.S.	1	1.35	1.05	1.19	0.00	4.79	0.00	0.00	0.00
time (sec)	N/A	0.265	0.399	1.347	0.000	0.313	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	92	149	0	229	0	0	0
N.S.	1	1.00	0.54	0.87	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.263	0.380	0.925	0.000	0.328	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	80	88	157	0	0	0	0	0	0
N.S.	1	1.10	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	10.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	96	162	0	0	0	0	0	0
N.S.	1	1.09	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	10.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	149	151	144	0	0	0	0	0	0
N.S.	1	1.01	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	10.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	137	152	0	0	0	0	0	0
N.S.	1	1.01	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1116	0	0	653
N.S.	1	1.00	0.22	1.29	0.00	8.79	0.00	0.00	5.14
time (sec)	N/A	0.187	10.018	5.903	0.000	0.338	0.000	0.000	0.515

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	1669	0	0	331
N.S.	1	1.00	0.34	1.53	0.00	10.63	0.00	0.00	2.11
time (sec)	N/A	0.206	10.025	0.665	0.000	0.391	0.000	0.000	15.033

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	421	0	723	0	0	533
N.S.	1	1.00	0.65	5.69	0.00	9.77	0.00	0.00	7.20
time (sec)	N/A	0.429	10.017	3.015	0.000	0.348	0.000	0.000	0.208

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	497	0	0	0
N.S.	1	1.00	0.31	3.72	0.00	4.83	0.00	0.00	0.00
time (sec)	N/A	0.650	10.022	1.086	0.000	0.423	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	81	81	126	538	0	1210	0	0	0
N.S.	1	1.00	1.56	6.64	0.00	14.94	0.00	0.00	0.00
time (sec)	N/A	0.161	4.134	7.796	0.000	1.301	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	126	443	0	345	0	0	0
N.S.	1	1.00	1.56	5.47	0.00	4.26	0.00	0.00	0.00
time (sec)	N/A	0.165	4.070	1.669	0.000	1.009	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	938	0	1232	0	0	0
N.S.	1	1.00	1.04	8.30	0.00	10.90	0.00	0.00	0.00
time (sec)	N/A	0.172	4.047	9.519	0.000	0.681	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1103	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	10.12	0.00	0.00	0.00
time (sec)	N/A	0.169	4.032	0.000	0.000	0.739	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	29	33	0	85	0	0	217
N.S.	1	1.00	0.33	0.38	0.00	0.98	0.00	0.00	2.49
time (sec)	N/A	0.499	10.362	0.759	0.000	0.253	0.000	0.000	0.163

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	F	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	546	46	317	0	70	0	0	207
N.S.	1	7.69	0.65	4.46	0.00	0.99	0.00	0.00	2.92
time (sec)	N/A	1.173	10.372	0.506	0.000	0.264	0.000	0.000	0.490

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	233	52	367	0	63	0	0	0
N.S.	1	5.07	1.13	7.98	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.818	10.879	0.655	0.000	0.276	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	111	0	0	0	0	67
N.S.	1	1.00	1.06	3.47	0.00	0.00	0.00	0.00	2.09
time (sec)	N/A	0.228	1.217	2.957	0.000	0.000	0.000	0.000	1.770

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	42	0	44	0	0	204
N.S.	1	1.00	1.35	1.83	0.00	1.91	0.00	0.00	8.87
time (sec)	N/A	0.186	0.879	0.817	0.000	0.268	0.000	0.000	0.567

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	47	355	0	1569	0	0	0
N.S.	1	1.00	0.22	1.63	0.00	7.20	0.00	0.00	0.00
time (sec)	N/A	0.232	10.046	48.349	0.000	0.426	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	50	352	0	1702	0	0	0
N.S.	1	1.00	0.24	1.68	0.00	8.10	0.00	0.00	0.00
time (sec)	N/A	0.216	10.051	48.707	0.000	0.393	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	65	351	0	1718	0	0	0
N.S.	1	1.00	0.29	1.58	0.00	7.74	0.00	0.00	0.00
time (sec)	N/A	0.218	10.053	41.432	0.000	0.396	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	68	352	0	1538	0	0	0
N.S.	1	1.00	0.32	1.64	0.00	7.19	0.00	0.00	0.00
time (sec)	N/A	0.222	10.045	41.404	0.000	0.391	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	73	77	327	0	323	0	0	0
N.S.	1	1.12	1.18	5.03	0.00	4.97	0.00	0.00	0.00
time (sec)	N/A	0.265	8.275	2.295	0.000	0.360	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	70	77	311	0	112	0	0	0
N.S.	1	1.11	1.22	4.94	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.269	8.204	2.308	0.000	0.380	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	92	816	0	0	0	0	0
N.S.	1	1.00	1.74	15.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.757	2.089	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	0	1421	0	267	0	0	0
N.S.	1	1.03	0.00	13.16	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.275	0.000	4.127	0.000	1.044	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	189	111	0	1252	0	0	0
N.S.	1	1.00	1.93	1.13	0.00	12.78	0.00	0.00	0.00
time (sec)	N/A	0.178	0.882	1.293	0.000	108.172	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	100	182	92	110	387	0	113	157
N.S.	1	1.04	1.90	0.96	1.15	4.03	0.00	1.18	1.64
time (sec)	N/A	0.207	0.370	0.911	0.286	0.263	0.000	3.121	0.611

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	95	0	253	0	0	0
N.S.	1	1.00	1.30	1.08	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.160	0.258	3.221	0.000	1.607	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	249	283	0	0	373	0	0	0
N.S.	1	1.07	1.21	0.00	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.369	0.775	0.000	0.000	1.485	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	104	80	86	90	0	87	100
N.S.	1	1.05	1.27	0.98	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.185	0.097	3.605	0.287	0.245	0.000	0.315	0.558

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	383	145	677	0	318	0	0	0
N.S.	1	2.84	1.07	5.01	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.540	1.163	6.457	0.000	5.537	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	383	145	737	0	318	0	0	0
N.S.	1	2.84	1.07	5.46	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.564	1.122	5.957	0.000	5.248	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	357	139	714	0	268	0	0	0
N.S.	1	3.00	1.17	6.00	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.525	1.088	6.401	0.000	5.265	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	0	0	0	0	0
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	10.181	0.862	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	0	0	0	0	0
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	10.141	0.606	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0	0
N.S.	1	1.00	1.10	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	10.017	0.534	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	70	101	12	105	94	31	0	10
N.S.	1	1.04	1.51	0.18	1.57	1.40	0.46	0.00	0.15
time (sec)	N/A	0.154	0.082	1.276	0.287	0.250	0.550	0.000	0.457

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	90	66	73	75	41	74	91
N.S.	1	1.09	1.29	0.94	1.04	1.07	0.59	1.06	1.30
time (sec)	N/A	0.177	0.050	1.934	0.290	0.249	0.576	0.295	0.449

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	384	375	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	199	250	0	618	0	2298	0	0	0
N.S.	1	1.26	0.00	3.11	0.00	11.55	0.00	0.00	0.00
time (sec)	N/A	0.456	0.000	26.803	0.000	1.184	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	182	0	0	0	0	0	0	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	182	0	0	0	0	0	0	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	138	204	179	0	191	0	0	0
N.S.	1	1.05	1.55	1.36	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.203	0.331	1.854	0.000	0.225	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	272	26	0	0	0	0	0	0
N.S.	1	1.09	0.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	10.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	383	655	138	0	0	0	0	0	0
N.S.	1	1.71	0.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.115	15.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	272	295	283	1147	0	341	0	0	0
N.S.	1	1.08	1.04	4.22	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.335	1.905	5.600	0.000	1.540	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [47] had the largest ratio of [1.30000000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	10	0.100
2	C	5	4	3.27	37	0.108
3	A	2	2	1.00	15	0.133
4	A	5	4	1.00	19	0.211
5	A	2	2	1.00	17	0.118
6	A	8	7	1.00	17	0.412
7	A	4	3	1.00	17	0.176
8	A	2	2	1.00	23	0.087
9	A	2	2	1.66	27	0.074
10	B	2	2	2.46	39	0.051
11	A	1	1	1.00	45	0.022
12	A	5	4	1.21	32	0.125
13	A	4	3	1.00	32	0.094
14	A	3	2	1.00	27	0.074
15	A	3	2	1.00	29	0.069
16	A	2	2	1.00	30	0.067
17	A	4	3	0.98	13	0.231
18	A	4	3	0.98	15	0.200
19	A	3	2	1.00	23	0.087
20	A	3	2	1.00	25	0.080
21	A	4	3	1.07	11	0.273
22	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	1.00	20	0.300
24	A	8	7	1.09	31	0.226
25	A	3	2	1.00	29	0.069
26	A	3	2	1.00	35	0.057
27	A	7	6	1.10	32	0.188
28	A	8	7	1.12	21	0.333
29	A	5	5	1.22	14	0.357
30	A	3	2	1.00	23	0.087
31	A	4	3	0.98	13	0.231
32	A	2	2	1.00	33	0.061
33	A	6	5	1.05	15	0.333
34	A	6	5	1.07	15	0.333
35	A	1	1	1.00	11	0.091
36	A	6	5	1.11	15	0.333
37	A	1	1	1.00	17	0.059
38	A	3	3	1.00	18	0.167
39	A	3	3	1.09	16	0.188
40	A	6	5	1.06	17	0.294
41	A	6	5	1.30	13	0.385
42	A	6	5	1.19	16	0.312
43	F	0	0	N/A	0.000	N/A
44	F	0	0	N/A	0.000	N/A
45	F	0	0	N/A	0.000	N/A
46	A	2	2	1.16	32	0.062
47	A	27	26	1.24	20	1.300
48	F	0	0	N/A	0.000	N/A
49	F	0	0	N/A	0.000	N/A
50	A	24	23	1.12	23	1.000
51	F	0	0	N/A	0.000	N/A
52	C	11	10	6.50	48	0.208
53	A	8	7	1.03	24	0.292
54	A	8	7	1.00	24	0.292
55	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	2	2	1.04	13	0.154
57	A	7	6	1.12	15	0.400
58	A	2	2	1.00	17	0.118
59	A	2	2	1.46	22	0.091
60	A	2	2	1.00	17	0.118
61	A	2	2	1.00	21	0.095
62	A	1	1	1.00	38	0.026
63	A	1	1	1.00	33	0.030
64	A	3	3	1.09	38	0.079
65	A	1	1	1.00	19	0.053
66	A	5	4	1.00	19	0.211
67	A	5	4	1.35	24	0.167
68	A	1	1	1.00	24	0.042
69	A	8	7	1.10	24	0.292
70	A	8	7	1.09	24	0.292
71	A	3	3	1.01	26	0.115
72	A	3	3	1.01	22	0.136
73	A	1	1	1.00	22	0.045
74	A	1	1	1.00	23	0.043
75	A	10	9	1.00	18	0.500
76	A	9	8	1.00	23	0.348
77	A	1	1	1.00	21	0.048
78	A	1	1	1.00	19	0.053
79	A	1	1	1.00	19	0.053
80	A	1	1	1.00	19	0.053
81	A	6	5	1.00	34	0.147
82	C	6	5	7.69	40	0.125
83	C	8	7	5.07	51	0.137
84	A	3	2	1.00	29	0.069
85	A	3	2	1.00	18	0.111
86	A	1	1	1.00	25	0.040
87	A	1	1	1.00	25	0.040
88	A	1	1	1.00	25	0.040
89	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	3	2	1.12	40	0.050
91	A	3	2	1.11	40	0.050
92	A	1	1	1.00	18	0.056
93	A	3	3	1.03	15	0.200
94	A	1	1	1.00	21	0.048
95	A	2	2	1.00	23	0.087
96	A	6	5	1.04	24	0.208
97	A	1	1	1.00	19	0.053
98	A	12	11	1.07	20	0.550
99	A	6	5	1.05	22	0.227
100	B	2	2	2.84	25	0.080
101	B	3	3	2.84	24	0.125
102	B	2	2	3.00	23	0.087
103	A	2	2	1.00	20	0.100
104	A	2	2	1.00	25	0.080
105	A	3	3	1.00	19	0.158
106	A	2	2	1.04	11	0.182
107	A	7	6	1.09	15	0.400
108	A	6	6	0.98	19	0.316
109	A	2	2	1.00	22	0.091
110	A	2	2	1.26	27	0.074
111	A	5	5	1.03	17	0.294
112	A	6	6	1.03	27	0.222
113	A	3	3	1.05	19	0.158
114	A	14	13	1.09	20	0.650
115	A	2	2	1.71	24	0.083
116	A	11	10	1.08	19	0.526

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

output
```
-2*(-a*x+1)^(1/2)/a
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

input
```
Integrate[1/Sqrt[1 - a*x],x]
```

output
```
(-2*Sqrt[1 - a*x])/a
```

3.1.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-ax}}\,dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2\sqrt{1-ax}}{a}$$

input
```
Int[1/Sqrt[1 - a*x],x]
```

output
```
(-2*Sqrt[1 - a*x])/a
```

3.1.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2\sqrt{-ax+1}}{a}$	14
derivativedivides	$-\frac{2\sqrt{-ax+1}}{a}$	14
default	$-\frac{2\sqrt{-ax+1}}{a}$	14
trager	$-\frac{2\sqrt{-ax+1}}{a}$	14
pseudoelliptic	$-\frac{2\sqrt{-ax+1}}{a}$	14
risch	$\frac{2ax-2}{a\sqrt{-ax+1}}$	19
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-ax+1}}{\sqrt{\pi}\,a}$	28

input
```
int(1/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-a*x+1)^(1/2)/a
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\,\sqrt{-ax+1}}{a}$$

input
```
integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*x + 1)/a
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax+1}}{a}$$

input
```
integrate(1/(-a*x+1)**(1/2),x)
```

output
```
-2*sqrt(-a*x + 1)/a
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\,\sqrt{-ax+1}}{a}$$

input
```
integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-a*x + 1)/a
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax+1}}{a}$$

input
```
integrate(1/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-a*x + 1)/a
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-a\,x}}{a}$$

input
```
int(1/(1 - a*x)^(1/2),x)
```

output
```
-(2*(1 - a*x)^(1/2))/a
```

3.2 $\int \frac{-2\log(-\sqrt{-1+ax})+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

3.2.1 Optimal result

Integrand size = 37, antiderivative size = 15

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=-\frac{2\sqrt{1-ax}}{a}$$

output
```
-2*(-a*x+1)^(1/2)/a
```

3.2.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=\frac{\sqrt{-1+ax}\left(-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)\right)}{a\pi}$$

input
```
Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),
x]
```

output
```
(Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

3.2.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {27, 25, 7267, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ax-1)-2\log\left(-\sqrt{ax-1}\right)}{2\pi\sqrt{ax-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{2\log\left(-\sqrt{ax-1}\right)-\log(ax-1)}{\sqrt{ax-1}}dx}{2\pi}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2\log\left(-\sqrt{ax-1}\right)-\log(ax-1)}{\sqrt{ax-1}}dx}{2\pi}$$

$$\Big\downarrow \text{7267}$$

$$-\frac{\int \left(2\log\left(-\sqrt{ax-1}\right)-\log(ax-1)\right)d\sqrt{ax-1}}{\pi a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)-\sqrt{ax-1}\log(ax-1)}{\pi a}$$

input
```
Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

output
```
-((2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]] - Sqrt[-1 + a*x]*Log[-1 + a*x])/(
a*Pi))
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.2.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

method	result	size
gosper	$\frac{\sqrt{ax-1}\,(\ln(ax-1)-2\ln(-\sqrt{ax-1}))}{a\pi}$	34
derivativedivides	$\frac{-2\ln(-\sqrt{ax-1})\sqrt{ax-1}+\sqrt{ax-1}\,\ln(ax-1)}{\pi a}$	42
default	$\frac{-2\ln(-\sqrt{ax-1})\sqrt{ax-1}+\sqrt{ax-1}\,\ln(ax-1)}{\pi a}$	42
meijerg	$\frac{i\sqrt{-\operatorname{signum}(ax-1)}\,(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-ax+1})}{\sqrt{\pi}\,\sqrt{\operatorname{signum}(ax-1)}\,a}$	47
parts	$\frac{-2\ln(-\sqrt{ax-1})\sqrt{ax-1}+2\sqrt{ax-1}}{a\pi}+\frac{\sqrt{ax-1}\,\ln(ax-1)-2\sqrt{ax-1}}{\pi a}$	68

input
```
int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
(a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi
```

3.2.5 Fricas [F]

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=\int \frac{\log\left(ax-1\right)-2\,\log\left(-\sqrt{ax-1}\right)}{2\,\pi\sqrt{ax-1}}\,dx$$

input
```
integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algor
ithm="fricas")
```

output
```
0
```

3.2.6 Sympy [A] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.80

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=\frac{\begin{cases}\frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log\left(ax-1\right)}{a} & \text{for } a\neq 0\\ \pi x & \text{otherwise}\end{cases}}{\pi}$$

input
```
integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

output
```
Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
 1))/a, Ne(a, 0)), (pi*x, True))/pi
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx=\frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

input
```
integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algor
ithm="maxima")
```

output
```
(sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

input
```
integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algor
ithm="giac")
```

output
```
(sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = -\frac{2\,\ln\left(-\sqrt{a\,x-1}\right)\,\sqrt{a\,x-1}-\ln\left(a\,x-1\right)\,\sqrt{a\,x-1}}{\Pi\,a}$$

input
```
int((log(a*x - 1)/2 - log(-(a*x - 1)^(1/2)))/(Pi*(a*x - 1)^(1/2)),x)
```

output
```
-(2*log(-(a*x - 1)^(1/2))*(a*x - 1)^(1/2) - log(a*x - 1)*(a*x - 1)^(1/2))/
(Pi*a)
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$

3.3.1 Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

output
```
4/3*x/(-3*x^2+1)-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(1/2*3^(1/2)*(x
^2+1)^(1/2))*3^(1/2)-2/3*(x^2+1)^(1/2)/(-3*x^2+1)
```

3.3.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\frac{6\left(-2x+\sqrt{1+x^2}\right)-2\sqrt{3}(-1+3x^2)\operatorname{arctanh}\left(\frac{x-\sqrt{1+x^2}}{\sqrt{3}}\right)}{-9+27x^2}$$

input
```
Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

output
```
(6*(-2*x + Sqrt[1 + x^2]) - 2*Sqrt[3]*(-1 + 3*x^2)*ArcTanh[(x - Sqrt[1 + x
^2])/Sqrt[3]])/(-9 + 27*x^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x^2+1}+2x\right)^2}dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(-\frac{4\sqrt{x^2+1}x}{(3x^2-1)^2}+\frac{5}{3(3x^2-1)}+\frac{8}{3(3x^2-1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}+\frac{4x}{3(1-3x^2)}-\frac{2\sqrt{x^2+1}}{3(1-3x^2)}$$

input
```
Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

output
```
(4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3
]*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.3.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

method	result
trager	$-\frac{4x}{3(3x^2-1)}+\frac{2\sqrt{x^2+1}}{3(3x^2-1)}+\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{3\sqrt{x^2+1}+2\,\mathrm{RootOf}\left(\_Z^2-3\right)}{\mathrm{RootOf}\left(\_Z^2-3\right)x+1}\right)}{9}$
default	$-\frac{x}{2(3x^2-1)}-\frac{\operatorname{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\sqrt{3}\left(-\frac{\left(\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}\right)^{\frac{3}{2}}}{12\left(x-\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\left(\frac{\sqrt{9\left(x-\frac{\sqrt{3}}{3}\right)^2+6\sqrt{3}\left(\right.}}{3}\right.}{}\right.$

input
```
int(1/(2*x+(x^2+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*x/(3*x^2-1)+2/3/(3*x^2-1)*(x^2+1)^(1/2)+1/9*RootOf(_Z^2-3)*ln(-(3*(x^
2+1)^(1/2)+2*RootOf(_Z^2-3))/(RootOf(_Z^2-3)*x+1))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$$
$$=\frac{\sqrt{3}(3\,x^2-1)\log\left(\frac{3\,x^2-2\,\sqrt{3}x+1}{3\,x^2-1}\right)+\sqrt{3}(3\,x^2-1)\log\left(\frac{3\,x^2+4\,\sqrt{3}\sqrt{x^2+1}+7}{3\,x^2-1}\right)-24\,x+12\,\sqrt{x^2+1}}{18\,(3\,x^2-1)}$$

input
```
integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqr
t(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) -
24*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

3.3.6 Sympy [F]

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

input
```
integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

output
```
Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

3.3.7 Maxima [F]

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{x^2+1}\right)^2}\,dx$$

input
```
integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \frac{1}{18}\,\sqrt{3}\log\left(\frac{\left|6\,x-2\,\sqrt{3}\right|}{\left|6\,x+2\,\sqrt{3}\right|}\right) - \frac{1}{18}\,\sqrt{3}\log\left(-\frac{\left|-6\,x-8\,\sqrt{3}+6\,\sqrt{x^2+1}-\frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3\,x-4\,\sqrt{3}-3\,\sqrt{x^2+1}+\frac{3}{x-\sqrt{x^2+1}}\right)}\right) - \frac{4\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)}{3\left(3\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)^2-16\right)} - \frac{4\,x}{3\,(3\,x^2-1)}$$

input
```
integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
```

output
```
1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)
*log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/
(3*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sq
rt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x
^2 + 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.49

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \frac{\sqrt{3}\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{18} - \frac{4\,x}{9\,\left(x^2-\frac{1}{3}\right)} + \frac{\sqrt{3}\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{18} - \frac{\sqrt{3}\left(6\,\ln\left(x-\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{54} - \frac{\sqrt{3}\left(6\,\ln\left(x+\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{54} + \frac{\sqrt{3}\,\sqrt{x^2+1}}{9\,\left(x-\frac{\sqrt{3}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{x^2+1}}{9\,\left(x+\frac{\sqrt{3}}{3}\right)} + \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{9}$$

input
```
int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

output
```
(3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
 (3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log
(x + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*
log(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2
)*(6*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3
^(1/2)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9
*(x + 3^(1/2)/3))
```

3.3. $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$

3.4 $\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx$

3.4.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx=\frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}+\frac{5}{16}\text{arctanh}\left(\frac{x}{2\sqrt{-1+x^2}}\right)$$

output
```
5/16*arctanh(1/2*x/(x^2-1)^(1/2))+3/8*x*(x^2-1)^(1/2)/(-3*x^2+4)
```

3.4.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx=-\frac{3x\sqrt{-1+x^2}}{8\left(-4+3x^2\right)}+\frac{5}{32}\log\left(2-x^2+x\sqrt{-1+x^2}\right)-\frac{5}{32}\log\left(2-3x^2+3x\sqrt{-1+x^2}\right)$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

output
```
(-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*Log[2 - x^2 + x*Sqrt[-1 + x^2]
])/32 - (5*Log[2 - 3*x^2 + 3*x*Sqrt[-1 + x^2]])/32
```

3.4.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {296, 25, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\left(3x^2-4\right)^2}dx$$

$$\Big\downarrow \; 296$$

$$\frac{3x\sqrt{x^2-1}}{8\left(4-3x^2\right)}-\frac{5}{8}\int -\frac{1}{\left(4-3x^2\right)\sqrt{x^2-1}}dx$$

$$\Big\downarrow \; 25$$

$$\frac{5}{8}\int \frac{1}{\left(4-3x^2\right)\sqrt{x^2-1}}dx+\frac{3\sqrt{x^2-1}x}{8\left(4-3x^2\right)}$$

$$\Big\downarrow \; 291$$

$$\frac{5}{8}\int \frac{1}{4-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}+\frac{3\sqrt{x^2-1}x}{8\left(4-3x^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{5}{16}\text{arctanh}\left(\frac{x}{2\sqrt{x^2-1}}\right)+\frac{3\sqrt{x^2-1}x}{8\left(4-3x^2\right)}$$

input
```
Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

output
```
(3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/1
6
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

method	result
trager	$-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)}+\frac{5\ln\left(-\frac{4\sqrt{x^2-1}\,x+5x^2-4}{3x^2-4}\right)}{32}$
pseudoelliptic	$\frac{(-15x^2+20)\ln\left(\frac{2\sqrt{x^2-1}-x}{x}\right)+15\ln\left(\frac{x+2\sqrt{x^2-1}}{x}\right)x^2-12\sqrt{x^2-1}\,x-20\ln\left(\frac{x+2\sqrt{x^2-1}}{x}\right)}{96x^2-128}$
risch	$-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)}-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}+\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}$
default	$-\frac{\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}+\frac{1}{3}}}{16\left(x+\frac{2\sqrt{3}}{3}\right)}-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x-\frac{2\sqrt{3}}{3}\right)^2+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}}}{16\left(x-\frac{2\sqrt{3}}{3}\right)}$

input
```
int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x/(3*x^2-4)*(x^2-1)^(1/2)+5/32*ln(-(4*(x^2-1)^(1/2)*x+5*x^2-4)/(3*x^2
-4))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(33) = 66$.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = -\frac{12\,x^2+5\,(3\,x^2-4)\log\left(3\,x^2-3\,\sqrt{x^2-1}x-2\right)-5\,(3\,x^2-4)\log\left(x^2-\sqrt{x^2-1}x-2\right)+12\,\sqrt{x^2-1}x}{32\,(3\,x^2-4)}$$

input
```
integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^
2 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 -
4)
```

3.4.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\,(3x^2-4)^2}\,dx$$

input
```
integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

3.4.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \int \frac{1}{(3\,x^2-4)^2\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)} - \frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right) + \frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

input
```
integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(
x^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(a
bs((x - sqrt(x^2 - 1))^2 - 3))
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2-1}\,(3\,x^2-4)^2}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

3.5.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx=\frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}-\frac{8\text{arcsinh}\left(\sqrt{x}\right)}{9}+\frac{10}{9}\text{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\log(1-3x)$$

output
```
8/9/(1-3*x)-8/9*arcsinh(x^(1/2))+10/9*arctanh(2*x^(1/2)/(1+x)^(1/2))+5/9*l
n(1-3*x)-4/3*x^(1/2)*(1+x)^(1/2)/(1-3*x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx=\frac{2\left(-4+6\sqrt{x}\sqrt{1+x}+(1-3x)\log\left(-\sqrt{x}+\sqrt{1+x}\right)+5(-1+3x)\log\left(1-x+\sqrt{x}\sqrt{1+x}\right)\right)}{-9+27x}$$

input
```
Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

output
```
(2*(-4 + 6*Sqrt[x]*Sqrt[1 + x] + (1 - 3*x)*Log[-Sqrt[x] + Sqrt[1 + x]] + 5
*(-1 + 3*x)*Log[1 - x + Sqrt[x]*Sqrt[1 + x]]))/(-9 + 27*x)
```

3.5.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{4\sqrt{x}\sqrt{x+1}}{(3x-1)^2}+\frac{5}{3(3x-1)}+\frac{8}{3(3x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8}{9}\text{arcsinh}\left(\sqrt{x}\right)+\frac{10}{9}\text{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)$$

input
```
Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

output
```
8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[
x]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(54) = 108.

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

method	result
default	$-\frac{8}{9(-1+3x)}+\frac{5\ln(-1+3x)}{9}-\frac{\sqrt{x}\,\sqrt{1+x}\,\left(12\ln\left(\frac{1}{2}+x+\sqrt{x(1+x)}\right)x-15\,\text{arctanh}\left(\frac{5x+1}{4\sqrt{x(1+x)}}\right)x-4\ln\left(\frac{1}{2}+x+\sqrt{x(1+x)}\right)+5\,\text{arc}}{9\sqrt{x(1+x)}\,(-1+3x)}$

input
```
int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-8/9/(-1+3*x)+5/9*ln(-1+3*x)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(x*(1+x)
)^(1/2))*x-15*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))*x-4*ln(1/2+x+(x*(1+x))^
(1/2))+5*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2))/(x*(1+x)
)^(1/2)/(-1+3*x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx =$$
$$-\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x}\right.}{9\,(3\,x-1)}$$

input
```
integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2
*sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x
+ 1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x
 - 1)
```

3.5.6 Sympy [F]

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}\,dx$$

input
```
integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

3.5.7 Maxima [F]

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \int \frac{1}{\left(\sqrt{x+1}+2\sqrt{x}\right)^2}\,dx$$

input
```
integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(54) = 108.

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = &-\frac{8\left(5\left(\sqrt{x+1}-\sqrt{x}\right)^2-3\right)}{9\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^4-10\left(\sqrt{x+1}-\sqrt{x}\right)^2+3\right)} \\ &-\frac{5\,x+1}{3\,(3\,x-1)}+\frac{4}{9}\log\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2\right) \\ &-\frac{5}{9}\log\left(\left|3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right|\right) \\ &+\frac{5}{9}\log\left(\left|\left(\sqrt{x+1}-\sqrt{x}\right)^2-3\right|\right)+\frac{5}{9}\log\left(|3\,x-1|\right)\end{aligned}$$

input
```
integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
-8/9*(5*(sqrt(x + 1) - sqrt(x))^2 - 3)/(3*(sqrt(x + 1) - sqrt(x))^4 - 10*(
sqrt(x + 1) - sqrt(x))^2 + 3) - 1/3*(5*x + 1)/(3*x - 1) + 4/9*log((sqrt(x
+ 1) - sqrt(x))^2) - 5/9*log(abs(3*(sqrt(x + 1) - sqrt(x))^2 - 1)) + 5/9*l
og(abs((sqrt(x + 1) - sqrt(x))^2 - 3)) + 5/9*log(abs(3*x - 1))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx = \frac{10\,\mathrm{atanh}\left(\frac{2662400\,\sqrt{x}}{81\,\left(\frac{665600\,x}{81\,{\left(\sqrt{x+1}-1\right)}^2}+\frac{665600}{81}\right)\,\left(\sqrt{x+1}-1\right)}\right)}{9}+\frac{5\,\ln\left(x-\frac{1}{3}\right)}{9}-\frac{16\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9}-\frac{8}{27\,\left(x-\frac{1}{3}\right)}+\frac{4\,\sqrt{x}\,\sqrt{x+1}}{3\,\left(3\,x-1\right)}$$

input
```
int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

output
```
(10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 6
65600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2
)/((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(
3*(3*x - 1))
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx$

3.6.1 Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx=\frac{\sqrt{-1+x^2}}{i-x}-\frac{i\arctan\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
arctanh(x/(x^2-1)^(1/2))-1/2*I*arctan(1/2*(1-I*x)*2^(1/2)/(x^2-1)^(1/2))*2
^(1/2)+(x^2-1)^(1/2)/(I-x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx=\frac{\sqrt{-1+x^2}}{i-x}-\sqrt{2}\operatorname{arctanh}\left(\frac{1+ix-i\sqrt{-1+x^2}}{\sqrt{2}}\right)-\log\left(-x+\sqrt{-1+x^2}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

output
```
Sqrt[-1 + x^2]/(I - x) - Sqrt[2]*ArcTanh[(1 + I*x - I*Sqrt[-1 + x^2])/Sqrt
[2]] - Log[-x + Sqrt[-1 + x^2]]
```

3.6.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {492, 25, 605, 224, 219, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(x-i)^2}dx$$

$$\Big\downarrow \text{492}$$

$$\int -\frac{x}{(i-x)\sqrt{x^2-1}}dx+\frac{\sqrt{x^2-1}}{-x+i}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{x^2-1}}{-x+i}-\int \frac{x}{(i-x)\sqrt{x^2-1}}dx$$

$$\Big\downarrow \text{605}$$

$$\int \frac{1}{\sqrt{x^2-1}}dx-i\int \frac{1}{(i-x)\sqrt{x^2-1}}dx+\frac{\sqrt{x^2-1}}{-x+i}$$

$$\Big\downarrow \text{224}$$

$$-i\int \frac{1}{(i-x)\sqrt{x^2-1}}dx+\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}+\frac{\sqrt{x^2-1}}{-x+i}$$

$$\Big\downarrow \text{219}$$

$$-i\int \frac{1}{(i-x)\sqrt{x^2-1}}dx+\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{\sqrt{x^2-1}}{-x+i}$$

$$\Big\downarrow \text{488}$$

$$i\int \frac{1}{-\frac{(1-ix)^2}{x^2-1}-2}d\frac{1-ix}{\sqrt{x^2-1}}+\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{\sqrt{x^2-1}}{-x+i}$$

$$\Big\downarrow \text{217}$$

$$-\frac{i\arctan\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}+\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{\sqrt{x^2-1}}{-x+i}$$

input
```
Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

output
```
Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sq
rt[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\sqrt{x^2-1}}{x-i}+\ln\left(x+\sqrt{x^2-1}\right)+\frac{i\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2+2i(x-i)-2}}\right)}{2}$
default	$\frac{\left((x-i)^2+2i(x-i)-2\right)^{\frac{3}{2}}}{2x-2i}-\frac{i\left(\sqrt{(x-i)^2+2i(x-i)-2}+i\ln\left(x+\sqrt{(x-i)^2+2i(x-i)-2}\right)-\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2+2i(x-i)-2}}\right)\right)}{2}-\frac{x}{}$

input
```
int((x^2-1)^(1/2)/(x-I)^2,x,method=_RETURNVERBOSE)
```

output
```
-(x^2-1)^(1/2)/(x-I)+ln(x+(x^2-1)^(1/2))+1/2*I*2^(1/2)*arctan(1/4*(-4+2*I*
(x-I))*2^(1/2)/((x-I)^2+2*I*(x-I)-2)^(1/2))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = -\frac{\sqrt{2}(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+2\,(x-i)\log\left(\right.}{2\,(x-i)}$$

input
```
integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x
 - I)*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + 2*(x - I)*log(-x + sqrt(x^
2 - 1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(x - I)
```

3.6.6 Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}}{(x-i)^2}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \frac{1}{2}i\,\sqrt{2}\arcsin\left(\frac{i\,x}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

output
```
1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I
) + log(2*x + 2*sqrt(x^2 - 1))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = i\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(x-\sqrt{x^2-1}-i\right)\right) + \frac{2\,\left(i\,x-i\,\sqrt{x^2-1}-1\right)}{\left(x-\sqrt{x^2-1}\right)^2-2i\,x+2i\,\sqrt{x^2-1}+1}-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
```

output
```
I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) + 2*(I*x - I*sqrt(x
^2 - 1) - 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log
(abs(-x + sqrt(x^2 - 1)))
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx = \int \frac{\sqrt{x^2-1}}{(x-\mathrm{i})^2}\,dx$$

input
```
int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
```

output
```
int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx$

3.7.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx = -\frac{x\sqrt{-1+x^2}}{4(1+x^2)} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}$$

output
```
3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

3.7.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx = \frac{1}{8}\left(-\frac{2x\sqrt{-1+x^2}}{1+x^2} + 3\sqrt{2}\text{arctanh}\left(\frac{1+x^2-x\sqrt{-1+x^2}}{\sqrt{2}}\right)\right)$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

output
```
((-2*x*Sqrt[-1 + x^2])/(1 + x^2) + 3*Sqrt[2]*ArcTanh[(1 + x^2 - x*Sqrt[-1
+ x^2])/Sqrt[2]])/8
```

3.7.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \; 296$$

$$\frac{3}{4}\int \frac{1}{\sqrt{x^2-1}\left(x^2+1\right)}dx-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

$$\Big\downarrow \; 291$$

$$\frac{3}{4}\int \frac{1}{1-\frac{2x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{3\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

input
```
Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

output
```
-1/4*(x*Sqrt[-1 + x^2])/(1 + x^2) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]]
)/(4*Sqrt[2])
```

3.7.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.7.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{3 \operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2-1}}{4(x^2+1)}$	37
default	$-\frac{x}{8\sqrt{x^2-1}\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)}+\frac{3 \operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}$	45
pseudoelliptic	$\frac{(3x^2+3)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x^2-1}}{2x}\right)-2\sqrt{x^2-1}\,x}{8x^2+8}$	49
trager	$-\frac{x\sqrt{x^2-1}}{4(x^2+1)}+\frac{3 \operatorname{RootOf}\left(\_Z^2-2\right) \ln\left(\frac{3 \operatorname{RootOf}\left(\_Z^2-2\right) x^2+4\sqrt{x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2-2\right)}{x^2+1}\right)}{16}$	66

input
```
int(1/(x^2+1)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \frac{3\sqrt{2}(x^2+1)\log\left(\frac{9\,x^2+2\sqrt{2}(3\,x^2-1)+2\sqrt{x^2-1}\left(3\sqrt{2}x+4\,x\right)-3}{x^2+1}\right)-4\,x^2-4\sqrt{x^2-1}x-4}{16\,(x^2+1)}$$

input
```
integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2
- 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/
(x^2 + 1)
```

3.7.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\,(x^2+1)^2}\,dx$$

input
```
integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

3.7.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \int \frac{1}{(x^2+1)^2\sqrt{x^2-1}}\,dx$$

input
```
integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = -\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right) - \frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

input
```
integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^
2 - 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2-1}\,(x^2+1)^2}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2), x)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx$

3.8.1 Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}$$

output
```
4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = -\frac{4x^{3/2}}{3}+\frac{2}{3}\sqrt{-1+x}(1+2x)$$

input
```
Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

output
```
(-4*x^(3/2))/3 + (2*Sqrt[-1 + x]*(1 + 2*x))/3
```

3.8.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x-1}+\sqrt{x}\right)^2 \sqrt{x-1}}dx$$

$$\Big\downarrow \text{7240}$$

$$\int \left(\frac{2x}{\sqrt{x-1}}-2\sqrt{x}-\frac{1}{\sqrt{x-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

input
```
Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

output
```
2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{4(-1+x)^{\frac{3}{2}}}{3}-\frac{4x^{\frac{3}{2}}}{3}+2\sqrt{-1+x}$	21

input
```
int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx=\frac{2}{3}\,(2\,x+1)\sqrt{x-1}-\frac{4}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
```

output
```
2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx=-\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1}+6x-3}$$

input
```
integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
```

output
```
-4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sq
rt(x - 1) + 6*x - 3)
```

3.8.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x}\right)^2}\,dx$$

input
```
integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \frac{4}{3}\,(x-1)^{\frac{3}{2}} - \frac{4}{3}\,x^{\frac{3}{2}} + 2\,\sqrt{x-1}$$

input
```
integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")
```

output
```
4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \frac{4\,x\,\sqrt{x-1}}{3} + \frac{2\,\sqrt{x-1}}{3} - \frac{4\,x^{3/2}}{3}$$

input
```
int(1/(((x - 1)^(1/2) + x^(1/2))^2*(x - 1)^(1/2)),x)
```

output
```
(4*x*(x - 1)^(1/2))/3 + (2*(x - 1)^(1/2))/3 - (4*x^(3/2))/3
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

3.9.1 Optimal result

Integrand size = 27, antiderivative size = 220

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx &= \frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right)\\ &\quad-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+2\sqrt{5}}\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right)\\ &\quad-\frac{1}{25}\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}\sqrt{x}\right)\\ &\quad-\frac{1}{50}\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right)\end{aligned}$$

output
```
1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2
))^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/
2)*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(
-2+2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+
2*5^(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 6.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2}+\sqrt{-110+50\sqrt{5}}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right)-\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)-\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right)+\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

input
```
Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

output
```
((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
 50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5
]]*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqr
t[5]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*Arc
Tanh[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

3.9.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2x}{\sqrt{x^2-1}\,(x^2-x-1)^2}-\frac{2\sqrt{x}}{(x^2-x-1)^2}+\frac{1}{\sqrt{x^2-1}\,(x^2-x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\arctan\left(\frac{2-(1-\sqrt{5})\,x}{\sqrt{2\,(\sqrt{5}-1)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\,(\sqrt{5}-1)}}\arctan\left(\frac{2-(1-\sqrt{5})\,x}{\sqrt{2\,(\sqrt{5}-1)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\text{arctanh}\left(\frac{2-(1+\sqrt{5})\,x}{\sqrt{2\,(1+\sqrt{5})}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\,(1+\sqrt{5})}}\text{arctanh}\left(\frac{2-(1+\sqrt{5})\,x}{\sqrt{2\,(1+\sqrt{5})}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)-\frac{2\sqrt{x^2-1}(1-2x)}{5\,(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5\,(-x^2+x+1)}$$

input
```
Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

output
```
(2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*
(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5
])]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/
(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*Arc
Tan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (S
qrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sq
rt[2/(5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5]
)]*Sqrt[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5]
)*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. 2(158) = 316.

Time = 0.32 (sec) , antiderivative size = 1206, normalized size of antiderivative = 5.48

method	result	size
default	Expression too large to display	1206

input
```
int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-6/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2
*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1
)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))-6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)
*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)
/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(
1/2))+(2/5+2/5*5^(1/2))*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/
2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/
4*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(
5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)
^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+(2/5-2/5*5^(1/2)
)*(-1/4/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(1/2)+1)/(1/2-1
/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5
^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*
(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)))+1/5*(5^(1/2)+1)^2*(-1/4/(1/2+1/2*
5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(
1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/4*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^
(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^
(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+
2*5^(1/2))^(1/2)))+1/5*(5^(1/2)-1)^2*(-1/4/(1/2-1/2*5^(1/2))/(x+1/2*5^(...
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(153) = 306.

Time = 0.24 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx =$$

$$-\frac{\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}(\sqrt{5}-3)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}(x^2-}{}$$

input
```
integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/50*(sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 2
2)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sqrt(5)*(x^2 -
 x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) + 4*
sqrt(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5
) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) + sqrt(5)*(
x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3
) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5)
 + 3)*sqrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sq
rt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5) + 3)*sqrt(-10*sqrt
(5) + 22) + 4*sqrt(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(
-(sqrt(5) + 3)*sqrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1)
+ 2) + sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(-(sqrt(5) + 3)*sqr
t(-10*sqrt(5) + 22) + 4*sqrt(x)) + 40*x^2 + 20*sqrt(x^2 - 1)*(2*x - 1) - 2
0*(2*x - 1)*sqrt(x) - 40*x - 40)/(x^2 - x - 1)
```

3.9.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

input
```
integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
```

output
```
Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

3.9.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

input
```
integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(153) = 306$.

Time = 1.49 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$
$$= \frac{2}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)$$
$$+\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x+22956\sqrt{5}\sqrt{50\sqrt{5}+110}+76520\sqrt{5}+153040\sqrt{x^2-1}-3826\right.\right.$$
$$-\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x-22956\sqrt{5}\sqrt{50\sqrt{5}+110}+76520\sqrt{5}+153040\sqrt{x^2-1}+3826\right.\right.$$
$$+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\sqrt{x}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$+\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\left|\sqrt{x}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$
$$+\frac{4\left(\left(x-\sqrt{x^2-1}\right)^3+2\left(x-\sqrt{x^2-1}\right)^2+3x-3\sqrt{x^2-1}-2\right)}{5\left(\left(x-\sqrt{x^2-1}\right)^4-2\left(x-\sqrt{x^2-1}\right)^3-2\left(x-\sqrt{x^2-1}\right)^2-2x+2\sqrt{x^2-1}+1\right)}$$
$$+\frac{2\left(2x^{\frac{3}{2}}-\sqrt{x}\right)}{5\left(x^2-x-1\right)}$$

input
```
integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

output
```
2/5*sqrt(1/10)*sqrt(5*sqrt(5) - 11)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1
) - 1)/sqrt(2*sqrt(5) - 2)) + 1/5*sqrt(1/10)*sqrt(5*sqrt(5) + 11)*log(abs(
-153040*x + 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 76520*sqrt(5) + 153040*
sqrt(x^2 - 1) - 38260*sqrt(50*sqrt(5) + 110) + 76520)) - 1/5*sqrt(1/10)*sq
rt(5*sqrt(5) + 11)*log(abs(-153040*x - 22956*sqrt(5)*sqrt(50*sqrt(5) + 110
) + 76520*sqrt(5) + 153040*sqrt(x^2 - 1) + 38260*sqrt(50*sqrt(5) + 110) +
76520)) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*sqrt(5) - 1/
2)) - 1/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5) + 1/2)) +
 1/50*sqrt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1/2*sqrt(5) + 1/2))) +
 4/5*((x - sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2 + 3*x - 3*sqrt(x^2 -
 1) - 2)/((x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))^3 - 2*(x - sqrt(x^
2 - 1))^2 - 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2) - sqrt(x))/(x^2 -
x - 1)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx=\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

input
```
int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
```

output
```
int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$

3.10.1 Optimal result

Integrand size = 39, antiderivative size = 220

$$\begin{aligned}\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx &= \frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)} \\ &+ \frac{1}{25}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right) \\ &- \frac{1}{50}\sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+2\sqrt{5}}\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right) \\ &- \frac{1}{25}\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}\sqrt{x}\right) \\ &- \frac{1}{50}\sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{-1+x^2}}{2-x-\sqrt{5}x}\right)\end{aligned}$$

output
```
1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2
))^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/
2)*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(
-2+2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+
2*5^(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx = \frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2} + \sqrt{-110+50\sqrt{5}}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right) - \sqrt{-110+50\sqrt{5}}\arctan\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right) - \sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right) + \sqrt{110+50\sqrt{5}}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

input
```
Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

output
```
((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
 50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5
]]*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqr
t[5]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*Arc
Tanh[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

3.10.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 541 vs. 2(220) = 440.

Time = 0.95 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x}-\sqrt{x^2-1}\right)^2}{(-x^2+x+1)^2\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^2}{\sqrt{x^2-1}\,(x^2-x-1)^2}+\frac{x}{\sqrt{x^2-1}\,(x^2-x-1)^2}-\frac{2\sqrt{x}}{(x^2-x-1)^2}-\frac{1}{\sqrt{x^2-1}\,(x^2-x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\arctan\left(\frac{2-(1-\sqrt{5})\,x}{\sqrt{2\,(\sqrt{5}-1)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\arctan\left(\frac{2-(1-\sqrt{5})\,x}{\sqrt{2\,(\sqrt{5}-1)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\arctan\left(\frac{2-(1-\sqrt{5})\,x}{\sqrt{2\,(\sqrt{5}-1)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2-(1+\sqrt{5})\,x}{\sqrt{2\,(1+\sqrt{5})}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2-(1+\sqrt{5})\,x}{\sqrt{2\,(1+\sqrt{5})}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\operatorname{arctanh}\left(\frac{2-(1+\sqrt{5})\,x}{\sqrt{2\,(1+\sqrt{5})}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)-\frac{\sqrt{x^2-1}(1-2x)}{5\,(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5\,(-x^2+x+1)}-\frac{(3-x)\sqrt{x^2-1}}{5\,(-x^2+x+1)}+\frac{(x+2)\sqrt{x^2-1}}{5\,(-x^2+x+1)}$$

input
```
Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

output
```
(2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
 + x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[
-1 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[
2/(-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1
 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5
*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1
+ x^2])])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[
2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcT
anh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[
(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqr
t[-1 + x^2])])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x
)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

3.10. $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx$

3.10.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(158) = 316$.

Time = 0.48 (sec) , antiderivative size = 1637, normalized size of antiderivative = 7.44

method	result	size
default	Expression too large to display	1637

input
```
int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*
5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)
*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+2/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*
arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/
(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1
/2))+(2/5+2/5*5^(1/2))*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2
*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/4
*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5
^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^
2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+(2/5-2/5*5^(1/2))
*(-1/4/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1
/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(1/2)+1)/(1/2-1/
2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^
(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(
x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)-8/
25*(-5/2+5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/
2))+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-8/25*(5/2+5^(1/2))/(2+2*5^(1/2))^(1/2)
*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/
2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(
1/2))^(3/2)+1/10*(5^(1/2)+1)/(1/2+1/2*5^(1/2))*(1/2*(4*(x-1/2*5^(1/2)-1...
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(153) = 306$.

Time = 0.25 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.05

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = -\frac{\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}(\sqrt{5}-3)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}(x^2-}{}$$

input
```
integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorith
m="fricas")
```

output
```
-1/50*(sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 2
2)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sqrt(5)*(x^2 -
 x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) + 4*
sqrt(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5
) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) + sqrt(5)*(
x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3
) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5)
 + 3)*sqrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1) + 2) - sq
rt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log((sqrt(5) + 3)*sqrt(-10*sqrt
(5) + 22) + 4*sqrt(x)) - sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(
-(sqrt(5) + 3)*sqrt(-10*sqrt(5) + 22) - 4*x - 2*sqrt(5) + 4*sqrt(x^2 - 1)
+ 2) + sqrt(5)*(x^2 - x - 1)*sqrt(-10*sqrt(5) + 22)*log(-(sqrt(5) + 3)*sqr
t(-10*sqrt(5) + 22) + 4*sqrt(x)) + 40*x^2 + 20*sqrt(x^2 - 1)*(2*x - 1) - 2
0*(2*x - 1)*sqrt(x) - 40*x - 40)/(x^2 - x - 1)
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = \text{Timed out}$$

input
```
integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
```

output
```
Timed out
```

3.10.7 Maxima [F]

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx=\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{(x^2-x-1)^2\sqrt{x^2-1}}\,dx$$

input
```
integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorith
m="maxima")
```

output
```
-2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x
))/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x +
 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(153) = 306$.

Time = 1.65 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.63

$$\begin{aligned}
\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx &= \frac{2}{5}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\arctan\left(\frac{2\,x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)\\
&+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\sqrt{x}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)\\
&-\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left|-520\,x-78\sqrt{5}\sqrt{50\sqrt{5}+110}+260\sqrt{5}+520\sqrt{x^2-1}+130\sqrt{50\sqrt{5}+110}+2\right.\right.\\
&+\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left|-1040\,x+156\sqrt{5}\sqrt{50\sqrt{5}+110}+520\sqrt{5}+1040\sqrt{x^2-1}-260\sqrt{50\sqrt{5}+110}\right.\right.\\
&+\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\left|\sqrt{x}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)\\
&+\frac{4\left(\left(x-\sqrt{x^2-1}\right)^3+2\left(x-\sqrt{x^2-1}\right)^2+3\,x-3\sqrt{x^2-1}-2\right)}{5\left(\left(x-\sqrt{x^2-1}\right)^4-2\left(x-\sqrt{x^2-1}\right)^3-2\left(x-\sqrt{x^2-1}\right)^2-2\,x+2\sqrt{x^2-1}+1\right)}\\
&+\frac{2\left(2\,x^{\frac{3}{2}}-\sqrt{x}\right)}{5\,(x^2-x-1)}
\end{aligned}$$

input
```
integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorith
m="giac")
```

output
```
2/5*sqrt(1/2*sqrt(5) - 11/10)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1) - 1)
/sqrt(2*sqrt(5) - 2)) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/
2*sqrt(5) - 1/2)) - 1/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqr
t(5) + 1/2)) - 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-520*x - 78*sqrt(5)*s
qrt(50*sqrt(5) + 110) + 260*sqrt(5) + 520*sqrt(x^2 - 1) + 130*sqrt(50*sqrt
(5) + 110) + 260)) + 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-1040*x + 156*s
qrt(5)*sqrt(50*sqrt(5) + 110) + 520*sqrt(5) + 1040*sqrt(x^2 - 1) - 260*sqr
t(50*sqrt(5) + 110) + 520)) + 1/50*sqrt(50*sqrt(5) + 110)*log(abs(sqrt(x)
- sqrt(1/2*sqrt(5) + 1/2))) + 4/5*((x - sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2
 - 1))^2 + 3*x - 3*sqrt(x^2 - 1) - 2)/((x - sqrt(x^2 - 1))^4 - 2*(x - sqrt
(x^2 - 1))^3 - 2*(x - sqrt(x^2 - 1))^2 - 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*
(2*x^(3/2) - sqrt(x))/(x^2 - x - 1)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx=\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\sqrt{x^2-1}\,(-x^2+x+1)^2}\,dx$$

input
```
int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2),x)
```

output
```
int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2), x)
```

3.11 $\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx$

3.11.1 Optimal result

Integrand size = 45, antiderivative size = 138

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx = -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+\frac{\operatorname{arctanh}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{i-x}{\sqrt{1+i}\sqrt{i+x^2}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

output
```
1/2*arctanh((I+x)/(1-I)^(1/2)/(-I+x^2)^(1/2))/(1-I)^(3/2)*2^(1/2)-1/2*arct
anh((I-x)/(1+I)^(1/2)/(I+x^2)^(1/2))/(1+I)^(3/2)*2^(1/2)-(1/4+1/4*I)*(-I+x
^2)^(1/2)/(1+x)*2^(1/2)+(-1/4+1/4*I)*(I+x^2)^(1/2)/(1+x)*2^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx = -\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{-i+x^2}+\sqrt{i+x^2}+\frac{2(1+x)\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\left(1+x-\sqrt{-i+x^2}\right)\right)}{\sqrt{1-i}}+(1+i)^{3/2}(1+x)\arctan\left(\sqrt{-\frac{1}{2}\right.\right.}{\sqrt{2}(1+x)}$$

input
```
Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt
[I + x^2]),x]
```

output
```
((-1/2 + I/2)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2] + (2*(1 + x)*ArcTan[Sqrt[-
1/2 - I/2]*(1 + x - Sqrt[-I + x^2])])/Sqrt[1 - I] + (1 + I)^(3/2)*(1 + x)*
ArcTan[Sqrt[-1/2 + I/2]*(1 + x - Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

3.11.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\sqrt{2}(x+1)^2\sqrt{x^2+i}}+\frac{1}{\sqrt{2}(x+1)^2\sqrt{x^2-i}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}$$

input
```
Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x
^2]),x]
```

output
```
((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^
2])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1
- I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I
)^(3/2)*Sqrt[2])
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{2}\left(\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\sqrt{(1+x)^2-2x-1-i}}{1+x}+\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\ln\left(\frac{-2i-2x+2\sqrt{1-i}\,\sqrt{(1+x)^2-2x-1-i}}{1+x}\right)}{\sqrt{1-i}}\right)}{2}+\frac{\sqrt{2}\left(\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)\sqrt{(1+x)^2-2x-1+i}}{1+x}+\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)}{}\right.}{2}$

input
```
int(1/2/(1+x)^2*2^(1/2)/(x^2-I)^(1/2)+1/2/(1+x)^2*2^(1/2)/(x^2+I)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*((-1/2-1/2*I)/(1+x)*((1+x)^2-2*x-1-I)^(1/2)-(1/2+1/2*I)/(1-I)^
(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((1+x)^2-2*x-1-I)^(1/2))/(1+x)))+1/2*2^(1
/2)*((-1/2+1/2*I)/(1+x)*((1+x)^2-2*x-1+I)^(1/2)+(-1/2+1/2*I)/(1+I)^(1/2)*l
n((2*I-2*x+2*(1+I)^(1/2)*((1+x)^2-2*x-1+I)^(1/2))/(1+x)))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

$$\int\left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)dx$$
$$=\frac{\sqrt{-\frac{1}{2}i+\frac{1}{2}}(-(i-1)\ x-i+1)\log\left(\sqrt{2}\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}((i-1)\ x+i-1)\,\mathrm{lo}}{}$$

input
```
integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(
1/2),x, algorithm="fricas")
```

output
```
(sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) -
x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(
2)*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I +
 1)*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) +
 sqrt(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2)
- x + sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2
 + I) - I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

3.11.6 Sympy [F(-2)]

Exception generated.

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+
x**2)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.11.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(
1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  1which is not of the expected type LIST
```

3.11.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(82) = 164.

Time = 0.33 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.96

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx$$

$$=\sqrt{2}\left(\frac{-(i-1)\,\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+(2i-2)\,x+2i+2}{\left(\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2\,x\right)^2-4\,\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+8\,x-4i}-\cdots\right.$$

$$+\sqrt{2}\left(\frac{(i+1)\,\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-(2i+2)\,x-2i+2}{\left(\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2\,x\right)^2-4\,\sqrt{2\,x^2+2\,\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+8\,x+\cdots}\right.$$

input
```
integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(
1/2),x, algorithm="giac")
```

output
```
sqrt(2)*((-(I - 1)*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt(x^4 + 1))
+ 1) + (2*I - 2)*x + 2*I + 2)/((sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sq
rt(x^4 + 1)) + 1) - 2*x)^2 - 4*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqr
t(x^4 + 1)) + 1) + 8*x - 4*I) - (I - 1)*arctan((sqrt(2)*(sqrt(2*x^2 + 2*sq
rt(x^4 + 1))*(I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x) - sqrt(2*x^2 + 2*sqrt(x^
4 + 1))*(I/(x^2 + sqrt(x^4 + 1)) + 1) + 2*x - 2*sqrt(2) + 2)/(sqrt(2)*sqrt
(2*sqrt(2) - 2) - (I + 1)*sqrt(2*sqrt(2) - 2)))/(sqrt(2*sqrt(2) - 2)*(-I/(
sqrt(2) - 1) + 1))) + sqrt(2)*(((I + 1)*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(-I/
(x^2 + sqrt(x^4 + 1)) + 1) - (2*I + 2)*x - 2*I + 2)/((sqrt(2*x^2 + 2*sqrt(
x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x)^2 - 4*sqrt(2*x^2 + 2*sqrt(
x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) + 8*x + 4*I) + (I + 1)*arctan((sq
rt(2)*(sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x)
 - sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) + 2*x - 2*
sqrt(2) + 2)/(sqrt(2)*sqrt(2*sqrt(2) - 2) + (I - 1)*sqrt(2*sqrt(2) - 2)))/
(sqrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1)))
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx$$
$$=\int \frac{\sqrt{2}}{2\,\sqrt{x^2-\mathrm{i}}\,{\left(x+1\right)}^2}+\frac{\sqrt{2}}{2\,\sqrt{x^2+1\mathrm{i}}\,{\left(x+1\right)}^2}\,dx$$

input
```
int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(
x + 1)^2),x)
```

output
```
int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(
x + 1)^2), x)
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

3.12.1 Optimal result

Integrand size = 32, antiderivative size = 125

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) -\frac{1}{4}(1+i)^{3/2}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

output
```
-1/4*(1-I)^(3/2)*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))-1/4*(1+I)^(3
/2)*arctanh((1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))-1/2*(1-I*x^2)^(1/2)/(1+x)
-1/2*(1+I*x^2)^(1/2)/(1+x)
```

3.12.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(125) = 250.

Time = 2.37 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \frac{1}{2}\left(\frac{-1-2x^4-\sqrt{1+x^4}-x^2\left(1+2\sqrt{1+x^4}\right)}{(1+x)\left(x^2+\sqrt{1+x^4}\right)^{3/2}} + \frac{\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}} - \sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right) - \frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}} + \sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

input

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

output

```
((-1 - 2*x^4 - Sqrt[1 + x^4] - x^2*(1 + 2*Sqrt[1 + x^4]))/((1 + x)*(x^2 +
Sqrt[1 + x^4])^(3/2)) + ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]
]/Sqrt[-1 + Sqrt[2]] - Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*
Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])] - ArcTanh[Sqrt[-1 +
Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[1 + Sqrt[2]] + Sqrt[-1 + Sqrt[2]]
*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sq
rt[1 + x^4])])/2
```

3.12.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2558, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(x+1)^2\sqrt{x^4+1}}\,dx$$
$$\Big\downarrow \text{2558}$$
$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(x+1)^2\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(x+1)^2\sqrt{ix^2+1}}dx$$
$$\Big\downarrow \text{491}$$
$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{1-ix^2}}dx-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-ix^2}}{x+1}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{ix^2+1}}dx-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+ix^2}}{x+1}\right)$$
$$\Big\downarrow \text{488}$$
$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1-i)-\frac{(ix+1)^2}{1-ix^2}}d\frac{ix+1}{\sqrt{1-ix^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-ix^2}}{x+1}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+i)-\frac{(1-ix)^2}{ix^2+1}}d\frac{1-ix}{\sqrt{ix^2+1}}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+ix^2}}{x+1}\right)$$
$$\Big\downarrow \text{219}$$
$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{1}{2}\sqrt{1-i}\,\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-ix^2}}{x+1}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{1}{2}\sqrt{1+i}\,\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+ix^2}}{x+1}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

output
```
(1/2 - I/2)*(((-1/2 - I/2)*Sqrt[1 - I*x^2])/(1 + x) - (Sqrt[1 - I]*ArcTanh
[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2) + (1/2 + I/2)*(((-1/2 + I/2)
*Sqrt[1 + I*x^2])/(1 + x) - (Sqrt[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sq
rt[1 + I*x^2])])/2)
```

3.12. $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

3.12.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

3.12.4 Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)^2\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(81) = 162$.

Time = 1.17 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.02

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = -\frac{(x+1)\sqrt{-\sqrt{2}-1}\log\left(-\frac{\sqrt{2}(x^2+1)\sqrt{-\sqrt{2}-1}+\left(2x^3+\sqrt{2}(x^3-x^2-x-1)-\sqrt{x^4+1}\left(\sqrt{2}(x-1)+2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-2\sqrt{x^4+1}\cdots}{x^2+2x+1}\right.}{}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/8*((x + 1)*sqrt(-sqrt(2) - 1)*log(-(sqrt(2)*(x^2 + 1)*sqrt(-sqrt(2) - 1
) + (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
+ 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - 2*sqrt(x^4 + 1)*sqrt(-sqrt(2) - 1)
)/(x^2 + 2*x + 1)) - (x + 1)*sqrt(-sqrt(2) - 1)*log((sqrt(2)*(x^2 + 1)*sqr
t(-sqrt(2) - 1) - (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sq
rt(2)*(x - 1) + 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - 2*sqrt(x^4 + 1)*sqrt
(-sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqrt(2) - 1)*log(-((2*x^3
- sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)
*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1))*sqrt(sq
rt(2) - 1))/(x^2 + 2*x + 1)) + (x + 1)*sqrt(sqrt(2) - 1)*log(-((2*x^3 - sq
rt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2
) - 1))/(x^2 + 2*x + 1)) - 4*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1
) - 1))/(x + 1)
```

3.12.6 Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

3.12.7 Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

3.12.8 Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

3.13.1 Optimal result

Integrand size = 32, antiderivative size = 81

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = -\frac{1}{2}\sqrt{1-i}\operatorname{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\operatorname{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

output
```
-1/2*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))*(1-I)^(1/2)-1/2*arctanh(
(1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))*(1+I)^(1/2)
```

3.13.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(81) = 162.

Time = 1.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \frac{\sqrt{-1+\sqrt{2}}\left(\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)-\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{1+\sqrt{2}}\operatorname{arcta}}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[-1 + Sqrt[2]]*(ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] -
 ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sq
rt[1 + x^4])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + S
qrt[1 + x^4]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x
^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/Sqrt[2]
```

3.13.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = $\{2558, 488, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(x+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{488}$$

$$\left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1-i)-\frac{(ix+1)^2}{1-ix^2}}d\frac{ix+1}{\sqrt{1-ix^2}}-\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+i)-\frac{(1-ix)^2}{ix^2+1}}d\frac{1-ix}{\sqrt{ix^2+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\sqrt{1-i}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

output
```
-1/2*(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) - (Sqr
t[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

3.13.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

3.13.4 Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(49) = 98$.

Time = 3.27 (sec) , antiderivative size = 504, normalized size of antiderivative = 6.22

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{8}\sqrt{-2\sqrt{2}+2}\log\left(-\frac{\sqrt{x^4+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+2}+\left(2x^3+\sqrt{2}(x^3-x^2-x-1)-\sqrt{x^4+1}\left(\sqrt{2}(x-\right.\right.}{x^2+2x+1}\right.$$
$$-\frac{1}{8}\sqrt{-2\sqrt{2}+2}\log\left(\frac{\sqrt{x^4+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+2}-\left(2x^3+\sqrt{2}(x^3-x^2-x-1)-\sqrt{x^4+1}\left(\sqrt{2}(x\right.\right.}{x^2+2x+}\right.$$
$$-\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+}{x^2+2x+1}\right.$$
$$+\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-}{x^2+2x+1}\right.$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/8*sqrt(-2*sqrt(2) + 2)*log(-(sqrt(x^4 + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2)
 + 2) + (2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)*(x -
 1) + 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 + sqrt(2)*(x^2 + 1) + 1)*
sqrt(-2*sqrt(2) + 2))/(x^2 + 2*x + 1)) - 1/8*sqrt(-2*sqrt(2) + 2)*log((sqr
t(x^4 + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 2) - (2*x^3 + sqrt(2)*(x^3 - x^
2 - x - 1) - sqrt(x^4 + 1)*(sqrt(2)*(x - 1) + 2*x) - 2)*sqrt(x^2 + sqrt(x^
4 + 1)) - (x^2 + sqrt(2)*(x^2 + 1) + 1)*sqrt(-2*sqrt(2) + 2))/(x^2 + 2*x +
 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1)
+ sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (
x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2)
+ 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - sqrt(2)*(x
^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 +
sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2) - 2) +
1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

3.13.6 Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

3.13.7 Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

3.13.8 Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

3.14.1 Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(x*2^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\log\left(x^2+\sqrt{1+x^4}+\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
Log[x^2 + Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

3.14.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2557, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 2557}$$

$$\int \frac{1}{1-\frac{2x^2}{\sqrt{x^4+1}+x^2}}d\frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

3.14.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

3.14.4 Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqr
t(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix}\middle|\, x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.14.7 Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.14.8 Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

3.15.1 Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(x*2^(1/2)/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

3.15.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=-\frac{i\log\left(ix^2-i\sqrt{1+x^4}+\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
((-I)*Log[I*x^2 - I*Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]])
/Sqrt[2]
```

3.15.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2557, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2557}$$

$$\int \frac{1}{\frac{2x^2}{\sqrt{x^4+1}-x^2}+1}d\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

3.15.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
meijerg	$-\frac{\sqrt{2}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{5}{4};\frac{3}{2},\frac{3}{2};-\frac{1}{x^4}\right)}{4x^2}$	22

input
```
int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|\,x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.15.7 Maxima [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.15.8 Giac [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

3.16.1 Optimal result

Integrand size = 30, antiderivative size = 19

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx=-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

output
```
-2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx=-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

input
```
Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),
x]
```

output
```
-2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

3.16.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x-1)^{3/2}+(x+1)^{3/2}}{(x-1)^{3/2}(x+1)^{3/2}}dx$$
$$\Big\downarrow \text{7239}$$
$$\int \left(\frac{1}{(x+1)^{3/2}}+\frac{1}{(x-1)^{3/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

input
```
Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

output
```
-2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.16.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$	16
meijerg	$\frac{2\sqrt{\pi}-\frac{2\sqrt{\pi}}{\sqrt{1+x}}}{\sqrt{\pi}}-\frac{2(-\operatorname{signum}(-1+x))^{\frac{3}{2}}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1-x}}\right)}{\sqrt{\pi}\operatorname{signum}(-1+x)^{\frac{3}{2}}}$	56

input
```
int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx=-\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}(x-1)\right)}{x^2-1}$$

input
```
integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm
="fricas")
```

output
```
-2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.82 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx=-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

input
```
integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

output
```
-2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(
x**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

input
```
integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm
="maxima")
```

output
```
-2/sqrt(x + 1) - 2/sqrt(x - 1)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

input
```
integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm
="giac")
```

output
```
-2/sqrt(x + 1) - 2/sqrt(x - 1)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{2}{\sqrt{x-1}} - \frac{2}{\sqrt{x+1}}$$

input
```
int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
```

output
```
- 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b \, dx$

3.17.1 Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \left(x+\sqrt{a+x^2}\right)^b \, dx = -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

output
```
-1/2*a*(x+(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x+(x^2+a)^(1/2))^(1+b)/(1+b)
```

3.17.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(x+\sqrt{a+x^2}\right)^b \, dx = \frac{\left(x+\sqrt{a+x^2}\right)^{-1+b}\left(ab+(-1+b)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+b^2}$$

input
```
Integrate[(x + Sqrt[a + x^2])^b,x]
```

output
```
((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1
+ b^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{a+x^2}+x\right)^b dx$$

$$\Big\downarrow \ 2542$$

$$\frac{1}{2}\int \left(x+\sqrt{x^2+a}\right)^{b-2}\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \ 244$$

$$\frac{1}{2}\int \left(a\left(x+\sqrt{x^2+a}\right)^{b-2}+\left(x+\sqrt{x^2+a}\right)^b\right)d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{b+1}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{1-b}\right)$$

input
```
Int[(x + Sqrt[a + x^2])^b,x]
```

output
```
(-((a*(x + Sqrt[a + x^2])^(-1 + b))/(1 - b)) + (x + Sqrt[a + x^2])^(1 + b)
/(1 + b))/2
```

3.17.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result	size
meijerg	$\frac{a^{\frac{b}{2}+\frac{1}{2}} b \left(\frac{8\sqrt{\pi}\, x^{1+b} a^{-\frac{b}{2}-\frac{1}{2}} \left(\frac{ab}{x^{2}}+b-1\right) \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+b}}{(1+b) b (2b-2)}+\frac{4\sqrt{\pi}\, x^{1+b} a^{-\frac{b}{2}-\frac{1}{2}} \sqrt{1+\frac{a}{x^{2}}} \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+b}}{(1+b) b}\right)}{4\sqrt{\pi}}$	120

input
```
int((x+(x^2+a)^(1/2))^b,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(a
*b/x^2+b-1)/(2*b-2)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+b
)*a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \left(x+\sqrt{a+x^2}\right)^b \, dx = \frac{\left(\sqrt{x^2+a}b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

input
```
integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

output
```
(sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. 2(37) = 74.

Time = 1.26 (sec) , antiderivative size = 2236, normalized size of antiderivative = 43.00

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx = \text{Too large to display}$$

input
```
integrate((x+(x**2+a)**(1/2))**b,x)
```

output
```
Piecewise((2*a**(9/2)*a**(b/2 + 1/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/s
qrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma
(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1
- b/2)) - 2*a**(9/2)*a**(b/2 + 1/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamm
a(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/
2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**(7/2)*a**(b/2 + 1/2)*b*x**2*sq
rt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/
(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b
**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) + 4*a**(7/2)*a**
(b/2 + 1/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b
/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/
2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**(7/2)
*a**(b/2 + 1/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*
a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)
*x**2*gamma(1 - b/2)) - 2*a**(7/2)*a**(b/2 + 1/2)*x**2*sqrt(a/x**2 + 1)*si
nh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*
gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**(7/2)*a**(b/2 + 1/2)*x**2*
cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**
2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gam...
```

3.17.7 Maxima [F]

$$\int \left(x+\sqrt{a+x^2}\right)^b\,dx = \int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^b, x)
```

3.17.8 Giac [F]

$$\int \left(x+\sqrt{a+x^2}\right)^b dx = \int \left(x+\sqrt{x^2+a}\right)^b dx$$

input
```
integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + a))^b, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \left(x+\sqrt{a+x^2}\right)^b dx = \int \left(x+\sqrt{x^2+a}\right)^b dx$$

input
```
int((x + (a + x^2)^(1/2))^b,x)
```

output
```
int((x + (a + x^2)^(1/2))^b, x)
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b \, dx$

3.18.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \left(x-\sqrt{a+x^2}\right)^b \, dx = -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

output
```
-1/2*a*(x-(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x-(x^2+a)^(1/2))^(1+b)/(1+b)
```

3.18.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(x-\sqrt{a+x^2}\right)^b \, dx = \frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+b}\left(\frac{a}{-1+b}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+b}\right)$$

input
```
Integrate[(x - Sqrt[a + x^2])^b,x]
```

output
```
((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b))
)/2
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x-\sqrt{a+x^2}\right)^b dx$$

$$\Big\downarrow \text{ 2542}$$

$$\frac{1}{2}\int \left(x-\sqrt{x^2+a}\right)^{b-2}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{ 244}$$

$$\frac{1}{2}\int \left(a\left(x-\sqrt{x^2+a}\right)^{b-2}+\left(x-\sqrt{x^2+a}\right)^b\right)d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{b+1}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{1-b}\right)$$

input
```
Int[(x - Sqrt[a + x^2])^b,x]
```

output
```
(-((a*(x - Sqrt[a + x^2])^(-1 + b))/(1 - b)) + (x - Sqrt[a + x^2])^(1 + b)
/(1 + b))/2
```

3.18.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.18.4 Maple [F]

$$\int \left(x - \sqrt{x^2 + a}\right)^b dx$$

input
```
int((x-(x^2+a)^(1/2))^b,x)
```

output
```
int((x-(x^2+a)^(1/2))^b,x)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \left(x - \sqrt{a + x^2}\right)^b dx = -\frac{\left(\sqrt{x^2 + a}b + x\right)\left(x - \sqrt{x^2 + a}\right)^b}{b^2 - 1}$$

input
```
integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

output
```
-(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

3.18.6 Sympy [F]

$$\int \left(x - \sqrt{a + x^2}\right)^b dx = \int \left(x - \sqrt{a + x^2}\right)^b dx$$

input
```
integrate((x-(x**2+a)**(1/2))**b,x)
```

output
```
Integral((x - sqrt(a + x**2))**b, x)
```

3.18.7 Maxima [F]

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx=\int \left(x-\sqrt{x^2+a}\right)^b\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^b, x)
```

3.18.8 Giac [F]

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx=\int \left(x-\sqrt{x^2+a}\right)^b\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
```

output
```
integrate((x - sqrt(x^2 + a))^b, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx=\int \left(x-\sqrt{x^2+a}\right)^b\,dx$$

input
```
int((x - (a + x^2)^(1/2))^b,x)
```

output
```
int((x - (a + x^2)^(1/2))^b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx$

3.19.1 Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx = \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

output
```
(x+(x^2+a)^(1/2))^b/b
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx = \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

input
```
Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

output
```
(x + Sqrt[a + x^2])^b/b
```

3.19.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{a+x^2}+x\right)^b}{\sqrt{a+x^2}}\,dx$$

$$\Big\downarrow \ 2547$$

$$\int \left(\sqrt{a+x^2}+x\right)^{b-1}d\left(\sqrt{a+x^2}+x\right)$$

$$\Big\downarrow \ 15$$

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

input
```
Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

output
```
(x + Sqrt[a + x^2])^b/b
```

3.19.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.19.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$	16
default	$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$	16

input
```
int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x+(x^2+a)^(1/2))^b/b
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

input
```
integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
(x + sqrt(x^2 + a))^b/b
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(12) = 24.

Time = 1.37 (sec) , antiderivative size = 311, normalized size of antiderivative = 18.29

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$$
$$=\begin{cases}\frac{\sqrt{a}a^{\frac{b}{2}}\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}}-\frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)}+\frac{a^{\frac{b}{2}}x\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}b}+\frac{a^{\frac{b}{2}}x\sinh\left(b\right.}{} \\ \frac{a^{\frac{b}{2}}\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}}-\frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)}+\frac{a^{\frac{b}{2}}x^2\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}}+\frac{a^{\frac{b}{2}}x\cosh\left(b\operatorname{as}\right.}{}\end{cases}$$

input
```
integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a)*a**(b/2)*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(b
*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/
(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a))
)/(sqrt(a)*b) + a**(b/2)*x*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sq
rt(a)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(b/2)*sinh(b*asinh(x/sqrt
(a)) - asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/
sqrt(a)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) + a**(b/2)*x**2*sinh(b*asinh(x
/sqrt(a)) - asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(b*a
sinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

3.19.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

input
```
integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(x + sqrt(x^2 + a))^b/b
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

input
```
int((x + (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
```

output
```
(x + (a + x^2)^(1/2))^b/b
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} dx$

3.20.1 Optimal result

Integrand size = 25, antiderivative size = 20

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} dx = -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

output
```
-(x-(x^2+a)^(1/2))^b/b
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} dx = -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

input
```
Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

output
```
-((x - Sqrt[a + x^2])^b/b)
```

3.20.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx$$

$$\Big\downarrow \text{2547}$$

$$-\int \left(x-\sqrt{x^2+a}\right)^{b-1}d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{15}$$

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

input
```
Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

output
```
-((x - Sqrt[a + x^2])^b/b)
```

3.20.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.20.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{\left(x-\sqrt{x^{2}+a}\right)^{b}}{b}$	19
default	$-\frac{\left(x-\sqrt{x^{2}+a}\right)^{b}}{b}$	19

input
```
int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x-(x^2+a)^(1/2))^b/b
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

input
```
integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(x - sqrt(x^2 + a))^b/b
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx = \begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \log\left(2x+2\sqrt{a+x^2}\right) & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{x^2}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

output
```
Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((log(2*x + 2*
sqrt(a + x**2)), Ne(a, 0)), (x*log(x)/sqrt(x**2), True)), True))
```

3.20.7 Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx=-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

input
```
integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(x - sqrt(x^2 + a))^b/b
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx=-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

input
```
int((x - (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
```

output
```
-(x - (a + x^2)^(1/2))^b/b
```

3.21 $\int \frac{1}{(a+be^{px})^2}\,dx$

3.21.1 Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{1}{a(a+be^{px})p} + \frac{x}{a^2} - \frac{\log(a+be^{px})}{a^2p}$$

output
```
1/a/(a+b*exp(p*x))/p+x/a^2-ln(a+b*exp(p*x))/a^2/p
```

3.21.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{\frac{a}{a+be^{px}} + \log(e^{px}) - \log(a+be^{px})}{a^2p}$$

input
```
Integrate[(a + b*E^(p*x))^(-2),x]
```

output
```
(a/(a + b*E^(p*x)) + Log[E^(p*x)] - Log[a + b*E^(p*x)])/(a^2*p)
```

3.21.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+be^{px})^2} \, dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{e^{-px}}{(a+be^{px})^2}de^{px}}{p}$$

$$\Big\downarrow \; 54$$

$$\frac{\int \left(-\frac{b}{a^2(a+be^{px})}-\frac{b}{a(a+be^{px})^2}+\frac{e^{-px}}{a^2}\right)de^{px}}{p}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\log(a+be^{px})}{a^2}+\frac{\log(e^{px})}{a^2}+\frac{1}{a(a+be^{px})}}{p}$$

input
```
Int[(a + b*E^(p*x))^(-2),x]
```

output
```
(1/(a*(a + b*E^(p*x))) + Log[E^(p*x)]/a^2 - Log[a + b*E^(p*x)]/a^2)/p
```

3.21.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.21.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\,e^{px})}{a^2}+\frac{1}{a(a+b\,e^{px})}+\frac{\ln(e^{px})}{a^2}}{p}$	43
default	$\frac{-\frac{\ln(a+b\,e^{px})}{a^2}+\frac{1}{a(a+b\,e^{px})}+\frac{\ln(e^{px})}{a^2}}{p}$	43
risch	$\frac{x}{a^2}+\frac{1}{a(a+b\,e^{px})p}-\frac{\ln\left(e^{px}+\frac{a}{b}\right)}{a^2 p}$	43
norman	$\frac{\frac{x}{a}+\frac{bx\,e^{px}}{a^2}-\frac{b\,e^{px}}{a^2 p}}{a+b\,e^{px}}-\frac{\ln(a+b\,e^{px})}{a^2 p}$	59
parallelrisch	$-\frac{-b^2 e^{px} xp+\ln(a+b\,e^{px})e^{px} b^2-xabp+\ln(a+b\,e^{px})ab-ab}{a^2 bp(a+b\,e^{px})}$	73

input
```
int(1/(a+b*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/p*(-1/a^2*ln(a+b*exp(p*x))+1/a/(a+b*exp(p*x))+1/a^2*ln(exp(p*x)))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{bpxe^{(px)}+apx-\left(be^{(px)}+a\right)\log\left(be^{(px)}+a\right)+a}{a^2bpe^{(px)}+a^3p}$$

input
```
integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
```

output
```
(b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*
e^(p*x) + a^3*p)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{1}{a^2p+abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b}+e^{px}\right)}{a^2p}$$

input
```
integrate(1/(a+b*exp(p*x))**2,x)
```

output
```
1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{x}{a^2} + \frac{1}{(abe^{(px)}+a^2)p} - \frac{\log\left(be^{(px)}+a\right)}{a^2p}$$

input
```
integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
```

output
```
x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(px)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(px)}+a)ab}\right)}{p}$$

input
```
integrate(1/(a+b*exp(p*x))^2,x, algorithm="giac")
```

output
```
b*(log(abs(-a/(b*e^(p*x) + a) + 1))/(a^2*b) + 1/((b*e^(p*x) + a)*a*b))/p
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a+be^{px})^2}\,dx = \frac{\frac{x}{a}+\frac{b\,x\,\mathrm{e}^{p\,x}}{a^2}-\frac{b\,\mathrm{e}^{p\,x}}{a^2\,p}}{a+b\,\mathrm{e}^{p\,x}}-\frac{\ln\left(a+b\,\mathrm{e}^{p\,x}\right)}{a^2\,p}$$

input
```
int(1/(a + b*exp(p*x))^2,x)
```

output
```
(x/a + (b*x*exp(p*x))/a^2 - (b*exp(p*x))/(a^2*p))/(a + b*exp(p*x)) - log(a
 + b*exp(p*x))/(a^2*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

3.22.1 Optimal result

Integrand size = 18, antiderivative size = 22

$$\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx = -\frac{1}{2a\left(b+ae^{2px}\right)p}$$

output
```
-1/2/a/(b+a*exp(2*p*x))/p
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx = -\frac{1}{2a\left(b+ae^{2px}\right)p}$$

input
```
Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

output
```
-1/2*1/(a*(b + a*E^(2*p*x))*p)
```

3.22.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2720, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ae^{px} + be^{-px})^2} dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{e^{px}}{(e^{2px}a+b)^2} de^{px}}{p}$$
$$\Big\downarrow \text{241}$$
$$-\frac{1}{2ap\,(ae^{2px} + b)}$$

input
```
Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

output
```
-1/2*1/(a*(b + a*E^(2*p*x))*p)
```

3.22.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.22.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{1}{2a(b+a\,e^{2px})p}$	20
derivativedivides	$-\frac{1}{2a(b+a\,e^{2px})p}$	21
default	$-\frac{1}{2a(b+a\,e^{2px})p}$	21
norman	$-\frac{1}{2a(b+a\,e^{2px})p}$	21
parallelrisch	$-\frac{1}{2a(b+a\,e^{2px})p}$	21

input
```
int(1/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/(b+a*exp(2*p*x))/p
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx = -\frac{1}{2\,(a^2pe^{(2\,px)}+abp)}$$

input
```
integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

output
```
-1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx = \frac{1}{2abp+2b^2pe^{-2px}}$$

input
```
integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
```

output
```
1/(2*a*b*p + 2*b**2*p*exp(-2*p*x))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(be^{-px} + ae^{px})^2} \, dx = \frac{1}{2 \left(b^2 e^{(-2 \, px)} + ab\right) p}$$

input
```
integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

output
```
1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{(be^{-px} + ae^{px})^2} \, dx = -\frac{1}{2 \left(ae^{(2 \, px)} + b\right) ap}$$

input
```
integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

output
```
-1/2/((a*e^(2*p*x) + b)*a*p)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(be^{-px} + ae^{px})^2} \, dx = \frac{\mathrm{e}^{2\,p\,x}}{2\,b\,p\,\left(b + a\,\mathrm{e}^{2\,p\,x}\right)}$$

input
```
int(1/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

output
```
exp(2*p*x)/(2*b*p*(b + a*exp(2*p*x)))
```

3.23 $\int \frac{x}{\left(be^{-px}+ae^{px}\right)^2} \, dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x}{\left(be^{-px}+ae^{px}\right)^2} \, dx=\frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}$$

output
```
1/2*x/a/b/p-1/2*x/a/(b+a*exp(2*p*x))/p-1/4*ln(b+a*exp(2*p*x))/a/b/p^2
```

3.23.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{x}{\left(be^{-px}+ae^{px}\right)^2} \, dx=\frac{\frac{2e^{2px}px}{b+ae^{2px}}-\frac{\log\left(b+ae^{2px}\right)}{a}}{4bp^2}$$

input
```
Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

output
```
((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2721, 2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ae^{px}+be^{-px})^2}dx$$

$$\Big\downarrow \text{2721}$$

$$\int \frac{xe^{2px}}{(ae^{2px}+b)^2}dx$$

$$\Big\downarrow \text{2621}$$

$$\frac{\int \frac{1}{e^{2px}a+b}dx}{2ap}-\frac{x}{2ap\,(ae^{2px}+b)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{-2px}}{e^{2px}a+b}de^{2px}}{4ap^2}-\frac{x}{2ap\,(ae^{2px}+b)}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int e^{-2px}de^{2px}}{b}-\frac{a\int \frac{1}{e^{2px}a+b}de^{2px}}{b}}{4ap^2}-\frac{x}{2ap\,(ae^{2px}+b)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(e^{2px})}{b}-\frac{a\int \frac{1}{e^{2px}a+b}de^{2px}}{b}}{4ap^2}-\frac{x}{2ap\,(ae^{2px}+b)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(e^{2px})}{b}-\frac{\log(ae^{2px}+b)}{b}}{4ap^2}-\frac{x}{2ap\,(ae^{2px}+b)}$$

input
```
Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

output
```
-1/2*x/(a*(b + a*E^(2*p*x))*p) + (Log[E^(2*p*x)]/b - Log[b + a*E^(2*p*x)]/
b)/(4*a*p^2)
```

3.23.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

3.23.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{2px}\right)}{4ba}+\frac{px\,\mathrm{e}^{2px}}{2b\left(b+a\,\mathrm{e}^{2px}\right)}}{p^2}$	50
default	$\frac{-\frac{\ln\left(b+a\,\mathrm{e}^{2px}\right)}{4ba}+\frac{px\,\mathrm{e}^{2px}}{2b\left(b+a\,\mathrm{e}^{2px}\right)}}{p^2}$	50
norman	$\frac{x\,\mathrm{e}^{2px}}{2pb(b+a\,\mathrm{e}^{2px})}-\frac{\ln(b+a\,\mathrm{e}^{2px})}{4ab\,p^2}$	51
risch	$\frac{x}{2abp}-\frac{x}{2a(b+a\,\mathrm{e}^{2px})p}-\frac{\ln\left(\mathrm{e}^{2px}+\frac{b}{a}\right)}{4ab\,p^2}$	57
parallelrisch	$-\frac{-2\,\mathrm{e}^{2px}apx+\ln(b+a\,\mathrm{e}^{2px})\mathrm{e}^{2px}a+\ln(b+a\,\mathrm{e}^{2px})b}{4ab\,p^2(b+a\,\mathrm{e}^{2px})}$	68

input
```
int(x/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/p^2*(-1/4/b/a*ln(a*exp(p*x)^2+b)+1/2*p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{2\,apxe^{(2\,px)}-\left(ae^{(2\,px)}+b\right)\log\left(ae^{(2\,px)}+b\right)}{4\left(a^2bp^2e^{(2\,px)}+ab^2p^2\right)}$$

input
```
integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^
2*e^(2*p*x) + a*b^2*p^2)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{x}{2abp+2b^2pe^{-2px}} - \frac{x}{2abp} - \frac{\log\left(\frac{a}{b}+e^{-2px}\right)}{4abp^2}$$

input
```
integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

output
```
x/(2*a*b*p + 2*b**2*p*exp(-2*p*x)) - x/(2*a*b*p) - log(a/b + exp(-2*p*x))/
(4*a*b*p**2)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{xe^{(2\,px)}}{2\,(abpe^{(2\,px)}+b^2p)} - \frac{\log\left(\frac{ae^{(2\,px)}+b}{a}\right)}{4\,abp^2}$$

input
```
integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

output
```
1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(
a*b*p^2)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx = \frac{2\,apxe^{(2\,px)} - ae^{(2\,px)}\log\left(-ae^{(2\,px)}-b\right) - b\log\left(-ae^{(2\,px)}-b\right)}{4\,(a^2bp^2e^{(2\,px)}+ab^2p^2)}$$

input
```
integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

output
```
1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2
*p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{x}{(be^{-px} + ae^{px})^2} \, dx = \frac{x \, e^{2\,p\,x}}{2\,b\,p\,\left(b + a\,e^{2\,p\,x}\right)} - \frac{\ln\left(b + a\,e^{2\,p\,x}\right)}{4\,a\,b\,p^2}$$

input
```
int(x/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

output
```
(x*exp(2*p*x))/(2*b*p*(b + a*exp(2*p*x))) - log(b + a*exp(2*p*x))/(4*a*b*p
^2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx$

3.24.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx=\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\arctan\left(\frac{\sqrt{2}(1+x)}{\sqrt{1-x+x^2}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

output
```
arctan((1+x)*2^(1/2)/(x^2-x+1)^(1/2))*2^(1/2)-1/6*arctanh(1/3*(1-x)*6^(1/2
)/(x^2-x+1)^(1/2))*6^(1/2)+(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)
```

3.24.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.78

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2} dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} - \text{RootSum}\Big[3+6\#1+\#1^2-2\#1^3 + \#1^4\&, \frac{19\log\left(-x+\sqrt{1-x+x^2}-\#1\right)+6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1}{3+\#1-3\#1^2+2\#1^3}\&\Big] - \frac{1}{2}\text{RootSum}\Big[3+6\#1+\#1^2-2\#1^3 + \#1^4\&, \frac{-36\log\left(-x+\sqrt{1-x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1-x\right.}}{3+\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

output
```
((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) - RootSum[3 + 6*#1 + #1^2 - 2*#1
^3 + #1^4 & , (19*Log[-x + Sqrt[1 - x + x^2] - #1] + 6*Log[-x + Sqrt[1 - x
 + x^2] - #1]*#1)/(3 + #1 - 3*#1^2 + 2*#1^3) & ] - RootSum[3 + 6*#1 + #1^2
 - 2*#1^3 + #1^4 & , (-36*Log[-x + Sqrt[1 - x + x^2] - #1] - 6*Log[-x + Sq
rt[1 - x + x^2] - #1]*#1 + Log[-x + Sqrt[1 - x + x^2] - #1]*#1^2)/(3 + #1
- 3*#1^2 + 2*#1^3) & ]/2
```

3.24.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-x+1}{\sqrt{x^2-x+1}\left(x^2+x+1\right)^2}dx$$

$$\Big\downarrow \; 2135$$

$$\frac{1}{12}\int \frac{6(3-x)}{\sqrt{x^2-x+1}\left(x^2+x+1\right)}dx + \frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{3-x}{\sqrt{x^2-x+1}\,(x^2+x+1)}dx+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{4(x+1)}{\sqrt{x^2-x+1}\,(x^2+x+1)}dx-\frac{1}{4}\int -\frac{8(1-x)}{\sqrt{x^2-x+1}\,(x^2+x+1)}dx\right)+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(2\int \frac{1-x}{\sqrt{x^2-x+1}\,(x^2+x+1)}dx+\int \frac{x+1}{\sqrt{x^2-x+1}\,(x^2+x+1)}dx\right)+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

$$\Big\downarrow \text{1362}$$

$$\frac{1}{2}\left(2\int \frac{1}{3-\frac{2(1-x)^2}{x^2-x+1}}d\left(-\frac{1-x}{\sqrt{x^2-x+1}}\right)-12\int \frac{1}{-\frac{18(x+1)^2}{x^2-x+1}-9}d\frac{3(x+1)}{\sqrt{x^2-x+1}}\right)+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(2\int \frac{1}{3-\frac{2(1-x)^2}{x^2-x+1}}d\left(-\frac{1-x}{\sqrt{x^2-x+1}}\right)+2\sqrt{2}\arctan\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right)\right)+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\sqrt{2}\arctan\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right)-\sqrt{\frac{2}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)\right)+\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}$$

input
```
Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

output
```
((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + (2*Sqrt[2]*ArcTan[(Sqrt[2]*(1
+ x))/Sqrt[1 - x + x^2]] - Sqrt[2/3]*ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 -
x + x^2]])/2
```

3.24. $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}(1+x+x^2)^2}\,dx$

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(71) = 142$.

Time = 1.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.84

method	result
risch	$\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}+\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(6\sqrt{2}\arctan\left(\frac{2\sqrt{2}(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)-\sqrt{6}\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}}{4}\right)\right)}{6\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}$
default	$\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(3\sqrt{2}\arctan\left(\frac{2\sqrt{2}(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)-\sqrt{6}\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}}{4}\right)\right)}{2\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}-\frac{\frac{9\sqrt{2}\arctan\left(\frac{2\sqrt{2}(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}}{(1-x)^2}}{}$
trager	$\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}-\mathrm{RootOf}\left(576\_Z^4+528\_Z^2+169\right)\ln\left(-\frac{-3456x\,\mathrm{RootOf}\left(576\_Z^4+528\_Z^2+169\right)^5-6312x}{}\right.$

input
```
int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)+1/6*((1+x)^2/(1-x)^2+3)^(1/2)*(6*2^(1/2)*a
rctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)/(1-x))-6^(1/2)*arctanh(1/4
*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2)))/((((1+x)^2/(1-x)^2+3)/((1+x)/(1-x)+1)^
2)^(1/2)/((1+x)/(1-x)+1)
```

3.24.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.31

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx$$
$$=\frac{\sqrt{6}(x^2+x+1)\sqrt{4i\sqrt{3}-11}\log\left(\sqrt{6}(5i\sqrt{3}-9)\sqrt{4i\sqrt{3}-11}-78x-39i\sqrt{3}+78\sqrt{x^2-x+1}-39\right)}{}$$

input
```
integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(6)*(x^2 + x + 1)*sqrt(4*I*sqrt(3) - 11)*log(sqrt(6)*(5*I*sqrt(3
) - 9)*sqrt(4*I*sqrt(3) - 11) - 78*x - 39*I*sqrt(3) + 78*sqrt(x^2 - x + 1)
 - 39) - sqrt(6)*(x^2 + x + 1)*sqrt(-4*I*sqrt(3) - 11)*log(sqrt(6)*(5*I*sq
rt(3) + 9)*sqrt(-4*I*sqrt(3) - 11) - 78*x + 39*I*sqrt(3) + 78*sqrt(x^2 - x
 + 1) - 39) - sqrt(6)*(x^2 + x + 1)*sqrt(4*I*sqrt(3) - 11)*log(sqrt(6)*sqr
t(4*I*sqrt(3) - 11)*(-5*I*sqrt(3) + 9) - 78*x - 39*I*sqrt(3) + 78*sqrt(x^2
 - x + 1) - 39) + sqrt(6)*(x^2 + x + 1)*sqrt(-4*I*sqrt(3) - 11)*log(sqrt(6
)*sqrt(-4*I*sqrt(3) - 11)*(-5*I*sqrt(3) - 9) - 78*x + 39*I*sqrt(3) + 78*sq
rt(x^2 - x + 1) - 39) + 12*x^2 + 12*sqrt(x^2 - x + 1)*(x + 1) + 12*x + 12)
/(x^2 + x + 1)
```

3.24.6 Sympy [F]

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \int \frac{3x^2-x+1}{\sqrt{x^2-x+1}\,(x^2+x+1)^2}\,dx$$

input
```
integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

output
```
Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

3.24.7 Maxima [F]

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \int \frac{3\,x^2-x+1}{(x^2+x+1)^2\sqrt{x^2-x+1}}\,dx$$

input
```
integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(69) = 138$.

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.53

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx = -\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x+\sqrt{6}-2\sqrt{x^2-x+1}+1}{\sqrt{3}+\sqrt{2}}\right) + \frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x-\sqrt{6}-2\sqrt{x^2-x+1}+1}{\sqrt{3}-\sqrt{2}}\right) + \frac{1}{12}\sqrt{6}\log\left(4\left(\sqrt{6}\sqrt{3}+3\sqrt{3}\right)^2+36\left(2x+\sqrt{6}-2\sqrt{x^2-x+1}+1\right)^2\right) - \frac{1}{12}\sqrt{6}\log\left(4\left(\sqrt{6}\sqrt{3}-3\sqrt{3}\right)^2+36\left(2x-\sqrt{6}-2\sqrt{x^2-x+1}+1\right)^2\right) + \frac{\left(x-\sqrt{x^2-x+1}\right)^3+4\left(x-\sqrt{x^2-x+1}\right)^2-10x+10\sqrt{x^2-x+1}+5}{\left(x-\sqrt{x^2-x+1}\right)^4+2\left(x-\sqrt{x^2-x+1}\right)^3+\left(x-\sqrt{x^2-x+1}\right)^2-6x+6\sqrt{x^2-x+1}+3}$$

input
```
integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(6)*sqrt(3)*arctan(-(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)/(sq
rt(3) + sqrt(2))) + 1/3*sqrt(6)*sqrt(3)*arctan(-(2*x - sqrt(6) - 2*sqrt(x^
2 - x + 1) + 1)/(sqrt(3) - sqrt(2))) + 1/12*sqrt(6)*log(4*(sqrt(6)*sqrt(3)
 + 3*sqrt(3))^2 + 36*(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)^2) - 1/12*s
qrt(6)*log(4*(sqrt(6)*sqrt(3) - 3*sqrt(3))^2 + 36*(2*x - sqrt(6) - 2*sqrt(
x^2 - x + 1) + 1)^2) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x +
1))^2 - 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*(x
 - sqrt(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 - x
 + 1) + 3)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx = \int \frac{3x^2-x+1}{\sqrt{x^2-x+1}\left(x^2+x+1\right)^2}\,dx$$

input
```
int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2),x)
```

output
```
int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2), x)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx$

3.25.1 Optimal result

Integrand size = 29, antiderivative size = 19

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

output
```
2*(x+(a^2+x^2)^(1/2))^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

input
```
Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

output
```
2*Sqrt[x + Sqrt[a^2 + x^2]]
```

3.25.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a^2+x^2}}dx$$

$$\Big\downarrow \text{ 2547}$$

$$\int \frac{1}{\sqrt{\sqrt{a^2+x^2}+x}}d\left(\sqrt{a^2+x^2}+x\right)$$

$$\Big\downarrow \text{ 15}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

input
```
Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

output
```
2*Sqrt[x + Sqrt[a^2 + x^2]]
```

3.25.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.25.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{a^2+x^2}}$	16
default	$2\sqrt{x+\sqrt{a^2+x^2}}$	16

input
```
int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+(a^2+x^2)^(1/2))^(1/2)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x + sqrt(a^2 + x^2))
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

input
```
integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
```

output
```
2*sqrt(x + sqrt(a**2 + x**2))
```

3.25.7 Maxima [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x + sqrt(a^2 + x^2))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx = 2\sqrt{x+\sqrt{a^2+x^2}}$$

input
```
int((x + (a^2 + x^2)^(1/2))^(1/2)/(a^2 + x^2)^(1/2),x)
```

output
```
2*(x + (a^2 + x^2)^(1/2))^(1/2)
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

3.26.1 Optimal result

Integrand size = 35, antiderivative size = 26

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

output
```
2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

3.26.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

input
```
Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

output
```
(2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

3.26.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{a+b^2x^2}+bx}}{\sqrt{a+b^2x^2}}dx$$

$$\Big\downarrow \text{2547}$$

$$\frac{\int \frac{1}{\sqrt{bx+\sqrt{b^2x^2+a}}}d\left(bx+\sqrt{b^2x^2+a}\right)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

input
```
Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

output
```
(2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

3.26.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.26.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2\sqrt{bx+\sqrt{x^2b^2+a}}}{b}$	23
default	$\frac{2\sqrt{bx+\sqrt{x^2b^2+a}}}{b}$	23

input
```
int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

input
```
integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx = \begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

output
```
Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tr
ue))
```

3.26.7 Maxima [F]

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

input
```
integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

input
```
integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx=\frac{2\sqrt{\sqrt{b^2\,x^2+a}+b\,x}}{b}$$

input
```
int(((a + b^2*x^2)^(1/2) + b*x)^(1/2)/(a + b^2*x^2)^(1/2),x)
```

output
```
(2*((a + b^2*x^2)^(1/2) + b*x)^(1/2))/b
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

3.27.1 Optimal result

Integrand size = 32, antiderivative size = 63

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-2*arctanh((x+(a^2+x^2
)^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = -\frac{2\left(\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

output
```
(-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
 + x^2]]/Sqrt[a]]))/a^(3/2)
```

3.27.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2545, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{\sqrt{a^2+x^2}+x}}dx$$

$$\Big\downarrow \text{ 2545}$$

$$2\int -\frac{1}{\sqrt{x+\sqrt{a^2+x^2}}\left(a^2-\left(x+\sqrt{a^2+x^2}\right)^2\right)}d\left(x+\sqrt{a^2+x^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{1}{\sqrt{x+\sqrt{a^2+x^2}}\left(a^2-\left(x+\sqrt{a^2+x^2}\right)^2\right)}d\left(x+\sqrt{a^2+x^2}\right)$$

$$\Big\downarrow \text{ 266}$$

$$-4\int \frac{1}{a^2-\left(x+\sqrt{a^2+x^2}\right)^2}d\sqrt{x+\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{ 756}$$

$$-4\left(\frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}+\frac{\int \frac{1}{a+x+\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-4\left(\frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}+\frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-4\left(\frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}\right)$$

input
```
Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

output
```
-4*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]/(2*a^(3/2)) + ArcTanh[Sqrt[x
 + Sqrt[a^2 + x^2]]/Sqrt[a]]/(2*a^(3/2)))
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2545
```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m   Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^
2)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g,
i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m]
&& (IntegerQ[m] || GtQ[i/c, 0])
```

3.27.4 Maple [F]

$$\int \frac{1}{x\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}}dx$$

input
```
int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

output
```
int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

3.27.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.14

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\right.}{}\right.}{}\right.$$

input
```
integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
[-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2
+ sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x +
 sqrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2
+ x^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) -
sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = -\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

output
```
-gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_pola
r(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

3.27.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}x}\,dx$$

input
```
integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

3.27.8 Giac [F]

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}x}\,dx$$

input
```
integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = \int \frac{1}{x\,\sqrt{x+\sqrt{a^2+x^2}}\,\sqrt{a^2+x^2}}\,dx$$

input
```
int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)),x)
```

output
```
int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)), x)
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

output
```
-2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2
)^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

output
```
2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/S
qrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

3.28.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2544, 25, 363, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{a^2+x^2}+x}}{x}dx$$

$$\Big\downarrow \text{2544}$$

$$\int -\frac{\left(\sqrt{a^2+x^2}+x\right)^2+a^2}{\sqrt{\sqrt{a^2+x^2}+x}\left(a^2-\left(\sqrt{a^2+x^2}+x\right)^2\right)}d\left(\sqrt{a^2+x^2}+x\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{a^2+\left(x+\sqrt{a^2+x^2}\right)^2}{\sqrt{x+\sqrt{a^2+x^2}}\left(a^2-\left(x+\sqrt{a^2+x^2}\right)^2\right)}d\left(x+\sqrt{a^2+x^2}\right)$$

$$\Big\downarrow \text{363}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-2a^2\int \frac{1}{\sqrt{x+\sqrt{a^2+x^2}}\left(a^2-\left(x+\sqrt{a^2+x^2}\right)^2\right)}d\left(x+\sqrt{a^2+x^2}\right)$$

$$\Big\downarrow \text{266}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\int \frac{1}{a^2-\left(x+\sqrt{a^2+x^2}\right)^2}d\sqrt{x+\sqrt{a^2+x^2}}$$

$$\Big\downarrow \text{756}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\left(\frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}+\frac{\int \frac{1}{a+x+\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}\right)$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\left(\frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}}d\sqrt{x+\sqrt{a^2+x^2}}}{2a}+\frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$2\sqrt{\sqrt{a^2+x^2}+x}-4a^2\left(\frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}}\right)$$

input
```
Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

output
```
2*Sqrt[x + Sqrt[a^2 + x^2]] - 4*a^2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt
[a]]/(2*a^(3/2)) + ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]/(2*a^(3/2)))
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

3.28.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.30

method	result	size
meijerg	$2\sqrt{2}\,\sqrt{x}\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{4};\frac{1}{2},\frac{3}{4};-\frac{a^2}{x^2}\right)$	25

input
```
int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) + \sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right) + \sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}a+\left(\sqrt{-a}(a+x)-\sqrt{a^2+x^2}\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x +
sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt
(-a)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a
 + (sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2))
)/x) + 2*sqrt(x + sqrt(a^2 + x^2))]
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \frac{\sqrt{x}\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

output
```
sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**
2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

3.28.7 Maxima [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.28.8 Giac [F]

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

input
```
integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

input
```
int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

output
```
int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

3.29 $\int x^3 \log^3(2+x) \log(3+x)\,dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 606

$$
\begin{aligned}
\int x^3 \log^3(2+x)\log(3+x)\,dx = & -\frac{302177x}{1152}+\frac{8029x^2}{2304}-\frac{763x^3}{3456}+\frac{3x^4}{256}+\frac{377}{64}(2+x)^2 \\
& -\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\log(2+x) \\
& -\frac{187}{64}x^2\log(2+x)+\frac{83}{288}x^3\log(2+x) \\
& -\frac{3}{128}x^4\log(2+x)+\frac{6733}{32}(2+x)\log(2+x) \\
& -\frac{377}{32}(2+x)^2\log(2+x)+\frac{71}{72}(2+x)^3\log(2+x) \\
& -\frac{3}{64}(2+x)^4\log(2+x)-\frac{43}{12}\log^2(2+x)-\frac{17}{48}x^3\log^2(2+x) \\
& +\frac{3}{64}x^4\log^2(2+x)-\frac{1251}{16}(2+x)\log^2(2+x) \\
& +\frac{273}{32}(2+x)^2\log^2(2+x)-\frac{3}{4}(2+x)^3\log^2(2+x) \\
& +\frac{3}{64}(2+x)^4\log^2(2+x)+\frac{65}{4}(2+x)\log^3(2+x) \\
& -\frac{33}{8}(2+x)^2\log^3(2+x)+\frac{3}{4}(2+x)^3\log^3(2+x) \\
& -\frac{1}{16}(2+x)^4\log^3(2+x)+\frac{3891}{128}\log(3+x) \\
& -\frac{115}{48}x^2\log(3+x)+\frac{37}{144}x^3\log(3+x)-\frac{3}{128}x^4\log(3+x) \\
& +\frac{415}{12}(3+x)\log(3+x)-\frac{4083}{32}\log(2+x)\log(3+x) \\
& -25x\log(2+x)\log(3+x)+\frac{13}{4}x^2\log(2+x)\log(3+x) \\
& -\frac{7}{12}x^3\log(2+x)\log(3+x)+\frac{3}{32}x^4\log(2+x)\log(3+x) \\
& +\frac{963}{16}\log^2(2+x)\log(3+x)+6x\log^2(2+x)\log(3+x) \\
& -\frac{3}{2}x^2\log^2(2+x)\log(3+x)+\frac{1}{2}x^3\log^2(2+x)\log(3+x) \\
& -\frac{3}{16}x^4\log^2(2+x)\log(3+x)-\frac{81}{4}\log^3(2+x)\log(3+x) \\
& +\frac{1}{4}x^4\log^3(2+x)\log(3+x)-\frac{5609\,\mathrm{PolyLog}(2,-2-x)}{96} \\
& +\frac{563}{8}\log(2+x)\,\mathrm{PolyLog}(2,-2-x) \\
& -\frac{195}{4}\log^2(2+x)\,\mathrm{PolyLog}(2,-2-x) \\
& -\frac{563\,\mathrm{PolyLog}(3,-2-x)}{8} \\
& +\frac{195}{2}\log(2+x)\,\mathrm{PolyLog}(3,-2-x) \\
& -\frac{195\,\mathrm{PolyLog}(4,-2-x)}{2}
\end{aligned}
$$

output
```
-302177/1152*x-43/12*ln(2+x)^2-5609/96*polylog(2,-2-x)-563/8*polylog(3,-2-
x)-195/2*polylog(4,-2-x)+377/64*(2+x)^2-71/216*(2+x)^3+3/256*(2+x)^4+2069/
144*ln(2+x)+3891/128*ln(3+x)+3/256*x^4+8029/2304*x^2-763/3456*x^3+963/16*l
n(2+x)^2*ln(3+x)-81/4*ln(2+x)^3*ln(3+x)+563/8*ln(2+x)*polylog(2,-2-x)-195/
4*ln(2+x)^2*polylog(2,-2-x)+195/2*ln(2+x)*polylog(3,-2-x)-187/64*x^2*ln(2+
x)+83/288*x^3*ln(2+x)-3/128*x^4*ln(2+x)+6733/32*(2+x)*ln(2+x)-377/32*(2+x)
^2*ln(2+x)+71/72*(2+x)^3*ln(2+x)-3/64*(2+x)^4*ln(2+x)-17/48*x^3*ln(2+x)^2+
3/64*x^4*ln(2+x)^2-1251/16*(2+x)*ln(2+x)^2+273/32*(2+x)^2*ln(2+x)^2-3/4*(2
+x)^3*ln(2+x)^2+3/64*(2+x)^4*ln(2+x)^2+65/4*(2+x)*ln(2+x)^3-33/8*(2+x)^2*l
n(2+x)^3+3/4*(2+x)^3*ln(2+x)^3-1/16*(2+x)^4*ln(2+x)^3-115/48*x^2*ln(3+x)+3
7/144*x^3*ln(3+x)-3/128*x^4*ln(3+x)+415/12*(3+x)*ln(3+x)-4083/32*ln(2+x)*l
n(3+x)+1/2*x^3*ln(2+x)^2*ln(3+x)+13/4*x^2*ln(2+x)*ln(3+x)-25*x*ln(2+x)*ln(
3+x)-3/2*x^2*ln(2+x)^2*ln(3+x)+6*x*ln(2+x)^2*ln(3+x)-7/12*x^3*ln(2+x)*ln(3
+x)+3/32*x^4*ln(2+x)*ln(3+x)-3/16*x^4*ln(2+x)^2*ln(3+x)+1/4*x^4*ln(2+x)^3*
ln(3+x)
```

3.29.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.68

$$\int x^3 \log^3(2+x)\log(3+x)\,dx$$
$$= \frac{-195984 - 558290x + 17705x^2 - 1050x^3 + 54x^4 + 910528\log(2+x) + 400008x\log(2+x) - 22836x^2\operatorname{lo}}{}$$

input
```
Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

output
```
(-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] +
400008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4
*Log[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[
2 + x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]
^3 + 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 -
 144*x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*
Log[3 + x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Lo
g[3 + x] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x]
- 1344*x^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*
Log[2 + x]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2
 + x]^2*Log[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]
^2*Log[3 + x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3
 + x] - 24*(5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x]
 + 288*(-563 + 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 -
 x])/2304
```

3.29.3 Rubi [A] (verified)

Time = 4.52 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2889, 2863, 2009, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log^3(x+2)\log(x+3)\,dx$$

$$\Big\downarrow \ 2889$$

$$-\frac{1}{4}\int \frac{x^4\log^3(x+2)}{x+3}dx-\frac{3}{4}\int \frac{x^4\log^2(x+2)\log(x+3)}{x+2}dx+\frac{1}{4}x^4\log^3(x+2)\log(x+3)$$

$$\Big\downarrow \ 2863$$

$$-\frac{3}{4}\int \frac{x^4\log^2(x+2)\log(x+3)}{x+2}dx-\frac{1}{4}\int \left(x^3\log^3(x+2)-3x^2\log^3(x+2)+9x\log^3(x+2)+\frac{81\log^3(x+2)}{x+3}-27\log^3(x+2)\right)dx+\frac{1}{4}x^4\log^3(x+2)\log(x+3)$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{4}\int \frac{x^4\log^2(x+2)\log(x+3)}{x+2}dx+$$
$$\frac{1}{4}\left(-486\,\text{PolyLog}(4,-x-2)-243\,\text{PolyLog}(2,-x-2)\log^2(x+2)+486\,\text{PolyLog}(3,-x-2)\log(x+2)+\frac{3}{128}(x\right.$$
$$\frac{1}{4}x^4\log^3(x+2)\log(x+3)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{3}{4}\int \left(\log^2(x+2)\log(x+3)x^3-2\log^2(x+2)\log(x+3)x^2+4\log^2(x+2)\log(x+3)x+\frac{16\log^2(x+2)\log(x+}{x+2}\right.$$
$$\frac{1}{4}\left(-486\,\text{PolyLog}(4,-x-2)-243\,\text{PolyLog}(2,-x-2)\log^2(x+2)+486\,\text{PolyLog}(3,-x-2)\log(x+2)+\frac{3}{128}(x\right.$$
$$\frac{1}{4}x^4\log^3(x+2)\log(x+3)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{4}\left(\frac{5609\,\text{PolyLog}(2,-x-2)}{72}+\frac{563\,\text{PolyLog}(3,-x-2)}{6}-32\,\text{PolyLog}(4,-x-2)-16\,\text{PolyLog}(2,-x-2)\log^2(\right.$$
$$\frac{1}{4}\left(-486\,\text{PolyLog}(4,-x-2)-243\,\text{PolyLog}(2,-x-2)\log^2(x+2)+486\,\text{PolyLog}(3,-x-2)\log(x+2)+\frac{3}{128}(x\right.$$
$$\frac{1}{4}x^4\log^3(x+2)\log(x+3)$$

input
```
Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

3.29. $\int x^3\log^3(2+x)\log(3+x)\,dx$

output
```
(x^4*Log[2 + x]^3*Log[3 + x])/4 + (-390*x + (99*(2 + x)^2)/8 - (2*(2 + x)^
3)/3 + (3*(2 + x)^4)/128 + 390*(2 + x)*Log[2 + x] - (99*(2 + x)^2*Log[2 +
x])/4 + 2*(2 + x)^3*Log[2 + x] - (3*(2 + x)^4*Log[2 + x])/32 - 195*(2 + x)
*Log[2 + x]^2 + (99*(2 + x)^2*Log[2 + x]^2)/4 - 3*(2 + x)^3*Log[2 + x]^2 +
 (3*(2 + x)^4*Log[2 + x]^2)/16 + 65*(2 + x)*Log[2 + x]^3 - (33*(2 + x)^2*L
og[2 + x]^3)/2 + 3*(2 + x)^3*Log[2 + x]^3 - ((2 + x)^4*Log[2 + x]^3)/4 - 8
1*Log[2 + x]^3*Log[3 + x] - 243*Log[2 + x]^2*PolyLog[2, -2 - x] + 486*Log[
2 + x]*PolyLog[3, -2 - x] - 486*PolyLog[4, -2 - x])/4 - (3*((189857*x)/864
 - (8029*x^2)/1728 + (763*x^3)/2592 - x^4/64 - (179*(2 + x)^2)/48 + (35*(2
 + x)^3)/162 - (2 + x)^4/128 - (2069*Log[2 + x])/108 + (187*x^2*Log[2 + x]
)/48 - (83*x^3*Log[2 + x])/216 + (x^4*Log[2 + x])/32 - (3613*(2 + x)*Log[2
 + x])/24 + (179*(2 + x)^2*Log[2 + x])/24 - (35*(2 + x)^3*Log[2 + x])/54 +
 ((2 + x)^4*Log[2 + x])/32 + (43*Log[2 + x]^2)/9 + (17*x^3*Log[2 + x]^2)/3
6 - (x^4*Log[2 + x]^2)/16 + (157*(2 + x)*Log[2 + x]^2)/4 - (25*(2 + x)^2*L
og[2 + x]^2)/8 - (1297*Log[3 + x])/32 + (115*x^2*Log[3 + x])/36 - (37*x^3*
Log[3 + x])/108 + (x^4*Log[3 + x])/32 - (415*(3 + x)*Log[3 + x])/9 + (1361
*Log[2 + x]*Log[3 + x])/8 + (100*x*Log[2 + x]*Log[3 + x])/3 - (13*x^2*Log[
2 + x]*Log[3 + x])/3 + (7*x^3*Log[2 + x]*Log[3 + x])/9 - (x^4*Log[2 + x]*L
og[3 + x])/8 - (321*Log[2 + x]^2*Log[3 + x])/4 - 8*x*Log[2 + x]^2*Log[3 +
x] + 2*x^2*Log[2 + x]^2*Log[3 + x] - (2*x^3*Log[2 + x]^2*Log[3 + x])/3 ...
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2889
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Simp[g*j*(m/(r + 1))   Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
 + j*x)), x], x] - Simp[b*e*n*(p/(r + 1))   Int[x^(r + 1)*(a + b*Log[c*(d +
 e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[
{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (E
qQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.29.4 Maple [F]

$$\int x^3 \ln(2+x)^3 \ln(3+x)\,dx$$

input
```
int(x^3*ln(2+x)^3*ln(3+x),x)
```

output
```
int(x^3*ln(2+x)^3*ln(3+x),x)
```

3.29.5 Fricas [F]

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = \int x^3 \log(x+3)\log(x+2)^3\,dx$$

input
```
integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

output
```
integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

3.29.6 Sympy [F]

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = \left(\frac{x^4\log(x+2)^3}{4} - \frac{3x^4\log(x+2)^2}{16} + \frac{3x^4\log(x+2)}{32} - \frac{3x^4}{128} + \frac{x^3\log(x+2)^2}{2} - \frac{7x^3\log(x+2)}{12} + \frac{37x^3}{144} - \frac{3x^2\log(x+2)^2}{2} + \frac{13x^2\log(x+2)}{4} - \frac{115x^2}{48} + 6x\log(x+2)^2 - 25x\log(x+2) + \frac{415x}{12} - 4\log(x+2)^3 + 25\log(x+2)^2 - \frac{415\log(x+2)}{6} + \frac{10955281}{240000}\right)\log(x+3)$$
$$- \frac{\int \frac{24900000x}{x+3}\,dx + \int\left(-\frac{1725000x^2}{x+3}\right)dx + \int \frac{185000x^3}{x+3}\,dx + \int\left(-\frac{16875x^4}{x+3}\right)dx + \int\left(-\frac{49800000\log(x+2)}{x+3}\right)dx + \int \ldots}{\ldots}$$

input
```
integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

output
```
(x**4*log(x + 2)**3/4 - 3*x**4*log(x + 2)**2/16 + 3*x**4*log(x + 2)/32 - 3
*x**4/128 + x**3*log(x + 2)**2/2 - 7*x**3*log(x + 2)/12 + 37*x**3/144 - 3*
x**2*log(x + 2)**2/2 + 13*x**2*log(x + 2)/4 - 115*x**2/48 + 6*x*log(x + 2)
**2 - 25*x*log(x + 2) + 415*x/12 - 4*log(x + 2)**3 + 25*log(x + 2)**2 - 41
5*log(x + 2)/6 + 10955281/240000)*log(x + 3) - (Integral(24900000*x/(x + 3
), x) + Integral(-1725000*x**2/(x + 3), x) + Integral(185000*x**3/(x + 3),
 x) + Integral(-16875*x**4/(x + 3), x) + Integral(-49800000*log(x + 2)/(x
+ 3), x) + Integral(18000000*log(x + 2)**2/(x + 3), x) + Integral(-2880000
*log(x + 2)**3/(x + 3), x) + Integral(-18000000*x*log(x + 2)/(x + 3), x) +
 Integral(4320000*x*log(x + 2)**2/(x + 3), x) + Integral(2340000*x**2*log(
x + 2)/(x + 3), x) + Integral(-1080000*x**2*log(x + 2)**2/(x + 3), x) + In
tegral(-420000*x**3*log(x + 2)/(x + 3), x) + Integral(360000*x**3*log(x +
2)**2/(x + 3), x) + Integral(67500*x**4*log(x + 2)/(x + 3), x) + Integral(
-135000*x**4*log(x + 2)**2/(x + 3), x) + Integral(180000*x**4*log(x + 2)**
3/(x + 3), x) + Integral(32865843/(x + 3), x))/720000
```

3.29. $\int x^3 \log^3(2+x) \log(3+x) \, dx$

3.29.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.85

$$\int x^3 \log^3(2+x) \log(3+x) \, dx$$
$$= \frac{3}{128} x^4 + \frac{1}{16} \left(4 x^4 \log(x+3) - x^4 + 4 x^3 - 18 x^2 + 108 x - 324 \log(x+3)\right) \log(x+2)^3$$
$$- \frac{65}{4} \log(x+3) \log(x+2)^3 + \frac{195}{4} \log(x+3) \log(x+2)^2 \log(-x-2) - \frac{175}{384} x^3$$
$$+ \frac{1}{96} \left(9 x^4 - 70 x^3 + 495 x^2 - 6 \left(3 x^4 - 8 x^3 + 24 x^2 - 96 x\right) \log(x+3) + 4680 \log(x+3) \log(-x-2) -$$
$$+2)^2 + \frac{195}{4} \mathrm{Li}_2(x+3) \log(x+2)^2 - \frac{195}{4} \mathrm{Li}_2(-x-2) \log(x+2)^2 + \frac{563}{16} \log(x+3) \log(x+2)^2$$
$$+ 21 \log(x+2)^3 + \frac{17705}{2304} x^2 + \frac{1}{8} \left(780 \log(x+2)^2 - 563 \log(x+2)\right) \mathrm{Li}_2(-x-2)$$
$$- \frac{1}{1152} \left(27 x^4 - 296 x^3 - 18720 \log(x+2)^3 + 2760 x^2 + 40536 \log(x+2)^2 - 39840 x - 67308 \log(x+$$
$$+3)$$
$$- \frac{1}{1152} \left(81 x^4 - 1036 x^3 + 56160 \log(x+3) \log(x+2)^2 + 112320 \log(x+3) \log(x+2) \log(-x-2) +$$
$$+2) + \frac{563}{8} \mathrm{Li}_2(-x-2) \log(x+2)$$
$$- \frac{5609}{96} \log(x+3) \log(x+2) + \frac{1573}{12} \log(x+2)^2 - \frac{279145}{1152} x - \frac{5609}{96} \mathrm{Li}_2(-x-2)$$
$$+ \frac{17171}{128} \log(x+3) + \frac{14227}{36} \log(x+2) - \frac{195}{2} \mathrm{Li}_4(-x-2) - \frac{563}{8} \mathrm{Li}_3(-x-2)$$

input
```
integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

output
```
3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*lo
g(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*l
og(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6
*(3*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2)
- 4950*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x +
2)^2 + 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2
+ 563/16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*
(780*log(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x
^3 - 18720*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*
log(x + 2))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(
x + 2)^2 + 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^
4 - 56*x^3 + 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x
 + 3) + 112320*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) -
 81072*log(x + 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog
(-x - 2) + 146988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x -
2))*log(x + 2) + 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x
 + 2) + 1573/12*log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 171
71/128*log(x + 3) + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8
*polylog(3, -x - 2)
```

3.29.8 Giac [F]

$$\int x^3 \log^3(2+x) \log(3+x) \, dx = \int x^3 \log(x+3) \log(x+2)^3 \, dx$$

input
```
integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
```

output
```
integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \log^3(2+x) \log(3+x) \, dx = \int x^3 \ln(x+2)^3 \ln(x+3) \, dx$$

input
```
int(x^3*log(x + 2)^3*log(x + 3),x)
```

output
```
int(x^3*log(x + 2)^3*log(x + 3), x)
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}} \, dx$

3.30.1 Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}} \, dx=\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

output
```
(x+(x^2+b)^(1/2))^a/a
```

3.30.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}} \, dx=\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

input
```
Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

output
```
(x + Sqrt[b + x^2])^a/a
```

3.30.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{b+x^2}+x\right)^a}{\sqrt{b+x^2}}dx$$

$$\Big\downarrow \text{2547}$$

$$\int \left(\sqrt{b+x^2}+x\right)^{a-1}d\left(\sqrt{b+x^2}+x\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

input
```
Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

output
```
(x + Sqrt[b + x^2])^a/a
```

3.30.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.30.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$	16
default	$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$	16

input
```
int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x+(x^2+b)^(1/2))^a/a
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

input
```
integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
```

output
```
(x + sqrt(x^2 + b))^a/a
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(12) = 24.

Time = 1.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 18.29

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$$
$$=\begin{cases} \frac{\sqrt{b}b^{\frac{a}{2}}\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}}+\frac{b^{\frac{a}{2}}x\cosh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}}+\frac{b^{\frac{a}{2}}x\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}}-\frac{2b^{\frac{a}{2}}\cosh \\ \frac{b^{\frac{a}{2}}\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}}+\frac{b^{\frac{a}{2}}x^2\sinh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}}+\frac{b^{\frac{a}{2}}x\cosh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}}-\frac{2b^{\frac{a}{2}}\cosh\left(\end{cases}$$

input
```
integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
```

output
```
Piecewise((sqrt(b)*b**(a/2)*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a
*x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b
)))/(a*sqrt(b)) + b**(a/2)*x*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(
a*sqrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1
- a/2)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (b**(a/2)*sinh(a*asinh(x/sqrt
(b)) - asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) + b**(a/2)*x**2*sinh(a*asinh
(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(a
*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asin
h(x/sqrt(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

3.30.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx=\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

input
```
integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

input
```
integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
```

output
```
(x + sqrt(x^2 + b))^a/a
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

input
```
int((x + (b + x^2)^(1/2))^a/(b + x^2)^(1/2),x)
```

output
```
(x + (b + x^2)^(1/2))^a/a
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a dx$

3.31.1 Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \left(x+\sqrt{b+x^2}\right)^a dx=-\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

output
```
-1/2*b*(x+(x^2+b)^(1/2))^(-1+a)/(1-a)+1/2*(x+(x^2+b)^(1/2))^(1+a)/(1+a)
```

3.31.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(x+\sqrt{b+x^2}\right)^a dx=\frac{1}{2}\left(x+\sqrt{b+x^2}\right)^{-1+a}\left(\frac{b}{-1+a}+\frac{\left(x+\sqrt{b+x^2}\right)^2}{1+a}\right)$$

input
```
Integrate[(x + Sqrt[b + x^2])^a,x]
```

output
```
((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a))
)/2
```

3.31.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b+x^2}+x\right)^a dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{1}{2}\int \left(x+\sqrt{x^2+b}\right)^{a-2}\left(\left(x+\sqrt{x^2+b}\right)^2+b\right)d\left(x+\sqrt{x^2+b}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}\int \left(b\left(x+\sqrt{x^2+b}\right)^{a-2}+\left(x+\sqrt{x^2+b}\right)^a\right)d\left(x+\sqrt{x^2+b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{a+1}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{1-a}\right)$$

input
```
Int[(x + Sqrt[b + x^2])^a,x]
```

output
```
(-((b*(x + Sqrt[b + x^2])^(-1 + a))/(1 - a)) + (x + Sqrt[b + x^2])^(1 + a)
/(1 + a))/2
```

3.31.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result	size
meijerg	$\frac{b^{\frac{a}{2}+\frac{1}{2}} a \left(\frac{8\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}} \left(\frac{ab}{x^2}+a-1\right) \left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}}{(1+a)a(2a-2)}+\frac{4\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}} \sqrt{1+\frac{b}{x^2}} \left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}}{(1+a)a}\right)}{4\sqrt{\pi}}$	120

input
```
int((x+(x^2+b)^(1/2))^a,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(a
*b/x^2+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(1+a)/a*x^(1+a)
*b^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \left(x+\sqrt{b+x^2}\right)^a \, dx = \frac{\left(\sqrt{x^2+b}a-x\right)\left(x+\sqrt{x^2+b}\right)^a}{a^2-1}$$

input
```
integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
```

output
```
(sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. 2(37) = 74.

Time = 1.27 (sec) , antiderivative size = 2236, normalized size of antiderivative = 43.00

$$\int \left(x+\sqrt{b+x^2}\right)^a dx = \text{Too large to display}$$

input
```
integrate((x+(x**2+b)**(1/2))**a,x)
```

output
```
Piecewise((-a**2*b**4*b**(a/2 + 1/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqr
t(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*
gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2
)) - a**2*b**3*b**(a/2 + 1/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)
))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamm
a(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) +
 2*a*b**(9/2)*b**(a/2 + 1/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*g
amma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma
(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) -
2*a*b**(9/2)*b**(a/2 + 1/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2)
 + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**(7/2)*b**(a/2 + 1/2)*x**2*sqrt(b/x**2
+ 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b*
*(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*g
amma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 4*a*b**(7/2)*b**(a/2 + 1
/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**
2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/
2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**(7/2)*b**(a/2
 + 1/2)*x**2*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7
/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*g...
```

3.31.7 Maxima [F]

$$\int \left(x+\sqrt{b+x^2}\right)^a dx = \int \left(x+\sqrt{x^2+b}\right)^a dx$$

input
```
integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + b))^a, x)
```

3.31.8 Giac [F]

$$\int \left(x+\sqrt{b+x^2}\right)^a\,dx=\int \left(x+\sqrt{x^2+b}\right)^a\,dx$$

input
```
integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + b))^a, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \left(x+\sqrt{b+x^2}\right)^a\,dx=\int \left(x+\sqrt{x^2+b}\right)^a\,dx$$

input
```
int((x + (b + x^2)^(1/2))^a,x)
```

output
```
int((x + (b + x^2)^(1/2))^a, x)
```

3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx$

3.32.1 Optimal result

Integrand size = 33, antiderivative size = 34

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx = \frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}$$

output
```
1/6*x^(1+a)*(6+3*x^a+2*x^(2*a))^(1+1/a)/(1+a)
```

3.32.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx = \frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6+6a}$$

input
```
Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

output
```
(x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

3.32.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2028, 2285}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}}\left(x^{2a}+x^{3a}+x^a\right)dx$$

$$\Big\downarrow \text{2028}$$

$$\int x^a\left(x^{2a}+x^a+1\right)\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}}dx$$

$$\Big\downarrow \text{2285}$$

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

input
```
Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

output
```
(x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

3.32.3.1 Defintions of rubi rules used

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2285
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[d*(g*x)^(m
 + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*g*(m + 1))), x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

3.32.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{x\,x^{a}\left(6+3x^{a}+2x^{2a}\right)\left(6+3x^{a}+2x^{2a}\right)^{\frac{1}{a}}}{6+6a}$	44

input
```
int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x,method=_RETURNVERBOS
E)
```

output
```
1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx=\frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a})(2\,x^{2\,a}+3\,x^{a}+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

input
```
integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fr
icas")
```

output
```
1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
 + 1)
```

3.32.6 Sympy [F(-1)]

Timed out.

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}}\left(x^a+x^{2a}+x^{3a}\right)\,dx=\text{Timed out}$$

input
```
integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

output
```
Timed out
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right)\, dx = \frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a})(2\,x^{2\,a}+3\,x^{a}+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

input
```
integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="ma
xima")
```

output
```
1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
 + 1)
```

3.32.8 Giac [F]

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right)\, dx = \int \left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)} \left(x^{3\,a}+x^{2\,a}+x^{a}\right) dx$$

input
```
integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="gi
ac")
```

output
```
integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right)\, dx = \int \left(x^{a}+x^{2\,a}+x^{3\,a}\right)\,\left(3\,x^{a}+2\,x^{2\,a}+6\right)^{1/a} dx$$

input
```
int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a),x)
```

output
```
int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a), x)
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

3.33.1 Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

output
```
-1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^
(1/2))*3^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{4}\left(2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^2}\right) - \log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

input
```
Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^
(1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

3.33.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{1-x^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt[3]{1-x^2}}dx^2$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}+\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-3\int \frac{1}{-x^4-3}d\left(2\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log\left(x^2\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)$$

input
```
Int[1/(x*(1 - x^2)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x^2]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/2)/2
```

3.33.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.33.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\arctan\left(\frac{\left(1+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}+\frac{\ln\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)}{2}$
meijerg	$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln\left(3\right)}{2}+2\ln\left(x\right)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{2}{}_{3}F_{2}\left(1,1,\frac{4}{3};2,2;x^{2}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$
trager	$\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{2}+15\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(-x^{2}+1\right)^{\frac{2}{3}}-\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)\ldots}{\ldots}\right.$

input
```
int(1/x/(-x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln((-x^2+1)^(2/3)+(-x^2+1)^(1/3)+1)+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3
))*3^(1/2))*3^(1/2)+1/2*ln((-x^2+1)^(1/3)-1)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right) - \frac{1}{4}\,\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left((-x^2+1)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

3.33.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = -\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(-x**2+1)**(1/3),x)
```

output
```
-exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*ga
mma(4/3))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{\ln\left(\frac{9\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{9}{4}\right)}{2} + \ln\left(\frac{9\,{\left(1-x^2\right)}^{1/3}}{4}-9\,{\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}^2\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right) - \ln\left(\frac{9\,{\left(1-x^2\right)}^{1/3}}{4}-9\,{\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}^2\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

input

```
int(1/(x*(1 - x^2)^(1/3)),x)
```

output

```
log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/4 - 9*((3^(1/
2)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(1 - x^2)^(1/3))/4 - 9*
((3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)
```

3.33. $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

3.34 $\int \frac{1}{x(1-x^2)^{2/3}}\,dx$

3.34.1 Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{1}{x(1-x^2)^{2/3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

output
```
-1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))-1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^
(1/2))*3^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(1-x^2)^{2/3}}\,dx = \frac{1}{4}\left(-2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+2\log\left(-1+\sqrt[3]{1-x^2}\right)-\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

input
```
Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)
^(1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

3.34.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 69, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-x^2)^{2/3}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(1-x^2)^{2/3}}dx^2$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{2}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}-\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(-\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(3\int \frac{1}{-x^4-3}d\left(2\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log\left(x^2\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)$$

input
```
Int[1/(x*(1 - x^2)^(2/3)),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]) - Log[x^2]/2 + (3*Log[
1 - (1 - x^2)^(1/3)])/2)/2
```

3.34.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.34.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result
meijerg	$\frac{\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{2\Gamma\left(\frac{2}{3}\right)x^2{}_3F_2\left(1,1,\frac{5}{3};2,2;x^2\right)}{3}}{2\Gamma\left(\frac{2}{3}\right)}$
pseudoelliptic	$-\frac{\ln\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)}{4}-\frac{\arctan\left(\frac{\left(1+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}+\frac{\ln\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)}{2}$
trager	$\frac{\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2+15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootO}}{x^2}\right.}{2}$

input
```
int(1/x/(-x^2+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3)+2/3*GAM
MA(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log(
(-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

3.34.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

$$\int \frac{1}{x(1-x^2)^{2/3}}\,dx = -\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(-x**2+1)**(2/3),x)
```

output
```
-exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*
gamma(5/3))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-x^2)^{2/3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")
```

output
```
-1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2
+ 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(1-x^2)^{2/3}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")
```

output
```
-1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2
+ 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\,(1-x^2)^{2/3}}\,dx = \frac{\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9}{4}\right)}{2} + \ln\left(\frac{9\,(1-x^2)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right) - \ln\left(\frac{9\,(1-x^2)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

input
```
int(1/(x*(1 - x^2)^(2/3)),x)
```

output
```
log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/2 - (3^(1/2)*
9i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(1 - x^2)^(1
/3))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

3.35.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
1/2*ln(x+(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=\frac{\arctan\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)$$

input
```
Integrate[(1 - x^3)^(-1/3),x]
```

output
```
ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^
3)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

3.35.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$$

$$\Big\downarrow \text{ 769}$$

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 - x^3)^(-1/3),x]
```

output
```
-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)
^(1/3)]/2
```

3.35.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

3.35.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.24

method	result
meijerg	$x{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x^3\right)$
pseudoelliptic	$\frac{\ln\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)}{3}-\frac{\ln\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}-x\left(-x^3+1\right)^{\frac{1}{3}}+x^2}{x^2}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(-2\left(-x^3+1\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)}{3}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(\_Z^2+\_Z\right.\right.}{3}$

input
```
int(1/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/3],[4/3],x^3)
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3
*log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

3.35.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(-x**3+1)**(1/3),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right) + \frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right) - \frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

input
```
integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^
3 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 +
 1)
```

3.35.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(-1/3), x)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

input
```
int(1/(1 - x^3)^(1/3),x)
```

output
```
x*hypergeom([1/3, 1/3], 4/3, x^3)
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

3.36.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx=\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

output
```
-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^
(1/2))*3^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx=\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right)-\frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

input
```
Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

output
```
ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1 + (1 - x^
3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

3.36.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-\frac{\log\left(x^3\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

input
```
Int[1/(x*(1 - x^3)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - Log[x^3]/2 + (3*Log[1 -
 (1 - x^3)^(1/3)])/2)/3
```

3.36.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.36.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)}{3}$
meijerg	$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln\left(3\right)}{2}+3\ln\left(x\right)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{3}\,{}_{3}F_{2}\left(1,1,\frac{4}{3};2,2;x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$
trager	$\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)\ln\left(\frac{-1438\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{3}-9855\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)x^{3}+5502\operatorname{RootOf}\left(\_Z^{2}+\_Z+\right.}{}\right.$

input
```
int(1/x/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3
))*3^(1/2))*3^(1/2)+1/3*ln((-x^3+1)^(1/3)-1)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((
-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

3.36.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = -\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(-x**3+1)**(1/3),x)
```

output
```
-exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3
))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3} + \ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x*(1 - x^3)^(1/3)),x)
```

output
```
log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6
)^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/
6)^2)*((3^(1/2)*1i)/6 + 1/6)
```

3.36. $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

3.37.1 Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
-1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
-1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

3.37.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$
$$=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-\right.}{4\sqrt[3]{2}}$$

input
```
Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] + 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] - Log[2^(2/3)
 - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x^3)^
(2/3)])/(4*2^(1/3))
```

3.37.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{2574}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

input
```
Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

output
```
-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(
1/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)])/(4*2^(1/3))
```

3.37.3.1 Defintions of rubi rules used

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.37.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.01 (sec) , antiderivative size = 769, normalized size of antiderivative = 7.93

method	result	size
trager	Expression too large to display	769

input
```
int(1/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(4*(-x^3+1)^(2
/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-1
0*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x
-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-
9*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootO
f(_Z^3-4)*x+2*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x+9*(-x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)-2*(-x^3+1)^(1/3)
*RootOf(_Z^3-4)^2-35*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x
^2-7*RootOf(_Z^3-4)*x^2+26*(-x^3+1)^(2/3)-30*RootOf(RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+4*_Z^2)*x-6*RootOf(_Z^3-4)*x-35*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)-7*RootOf(_Z^3-4))/(1+x)^2)+1/4*RootOf(_Z^3-4)*ln(
(8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Root
Of(_Z^3-4)^2-8*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootO
f(_Z^3-4)^2*x-10*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootO
f(_Z^3-4)^3*x+26*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)*RootOf(_Z^3-4)*x+4*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-26*(-x^3+1)
^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)-
4*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*x^2+35*RootOf(_Z^3-4)*x^2-36*(-x^3+1)^(2/3)+8*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+10*RootOf(_Z^3-4)*x+28*RootOf(...
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(71) = 142.

Time = 1.93 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.10

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(13\,x^6+2\,x^5+19\,x^4-4\,x^3+19\,x^2+2\,x+13)-4\sqrt{2}(5\,x^5-5\,x^4+6\,x^3-6\right.}{6\,(3\,x^6-18\,x^5-3\,x^4-28\,x^3}\right.$$
$$-\frac{1}{24}$$
$$\cdot 2^{\frac{2}{3}}\log\left(\frac{4\cdot 2^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}(x^2+1)+2^{\frac{1}{3}}(5\,x^4+6\,x^2+5)-2\,(3\,x^3-x^2+x-3)(-x^3+1)^{\frac{1}{3}}}{x^4+4\,x^3+6\,x^2+4\,x+1}\right)$$
$$+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+2\,x+1)-2\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4\,(-x^3+1)^{\frac{2}{3}}}{x^2+2\,x+1}\right)$$

input
```
integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
 19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 -
6*x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x
+ 1)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3
)) - 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x
^4 + 6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 +
 6*x^2 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)
*(-x^3 + 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

3.37.6 Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.37.7 Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.37.8 Giac [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x+1)}\,dx$$

input
```
int(1/((1 - x^3)^(1/3)*(x + 1)),x)
```

output
```
int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

3.38.1 Optimal result

Integrand size = 18, antiderivative size = 145

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/8*ln((1-x)*(1+x)^2)*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))-3/8*ln(-1+x+2^(2/3)
*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)+1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(
2/3)
```

3.38.2 Mathematica [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

input
```
Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

output
```
Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

3.38.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2577, 769, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)\sqrt[3]{1-x^3}}\,dx$$

$$\Big\downarrow \text{2577}$$

$$\int \frac{1}{\sqrt[3]{1-x^3}}dx - \int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{769}$$

$$-\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

$$\Big\downarrow \text{2574}$$

$$\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right) - \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} + \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

input
```
Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/
3)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1
 + x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3
)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

3.38.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

rule 2577
```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Simp[f/d   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[(d*e - c*f
)/d   Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x]
```

3.38.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.00 (sec) , antiderivative size = 1878, normalized size of antiderivative = 12.95

method	result	size
trager	Expression too large to display	1878

input
```
int(x/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln(-(8*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+1
2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x
-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+
8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootO
f(_Z^3+4)*x-9*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+9*(-x^3+1)^(1/3)
*RootOf(_Z^3+4)^2+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x
^2-14*RootOf(_Z^3+4)*x^2+36*(-x^3+1)^(2/3)+12*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*x-4*RootOf(_Z^3+4)*x+42*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)-14*RootOf(_Z^3+4))/(1+x)^2)-1/4*ln((8*RootOf(Roo
tOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-
12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*
x-10*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*
x+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*Roo
tOf(_Z^3+4)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-13*(-x^3+1)^(
1/3)*RootOf(_Z^3+4)^2+42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)*x^2+35*RootOf(_Z^3+4)*x^2-52*(-x^3+1)^(2/3)+36*RootOf(RootOf(_Z^3+4)^2+
2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+30*RootOf(_Z^3+4)*x+42*RootOf(RootOf(_Z^3...
```

3.38.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.38.6 Sympy [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

input
```
integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.38.7 Maxima [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.38.8 Giac [F]

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{x}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

input
```
int(x/((1 - x^3)^(1/3)*(x + 1)),x)
```

output
```
int(x/((1 - x^3)^(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

3.39.1 Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{2-3x+x^2}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}+\frac{3\log\left(2-x-2^{2/3}\sqrt[3]{2-3x+x^2}\right)}{4\sqrt[3]{2}}$$

output
```
-1/8*ln(2-x)*2^(2/3)-1/4*ln(x)*2^(2/3)+3/8*ln(2-x-2^(2/3)*(x^2-3*x+2)^(1/3
))*2^(2/3)+1/4*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-x)/(x^2-3*x+2)^(1/3)*3^(
1/2))*3^(1/2)*2^(2/3)
```

3.39.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2-3x+x^2}}{2\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{2-3x+x^2}}\right)+2\log\left(-2\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{2-3x+x^2}\right)-\log\left(4\,2^{2/3}-4\,2^{2/}\right.}{4\sqrt[3]{2}}$$

input
```
Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*(2 - 3*x + x^2)^(1/3))/(2*2^(1/3) - 2^(1/3)*x +
 (2 - 3*x + x^2)^(1/3))] + 2*Log[-2*2^(1/3) + 2^(1/3)*x + 2*(2 - 3*x + x^2
)^(1/3)] - Log[4*2^(2/3) - 4*2^(2/3)*x + 2^(2/3)*x^2 - 2*2^(1/3)*(-2 + x)*
(2 - 3*x + x^2)^(1/3) + 4*(2 - 3*x + x^2)^(2/3)])/(4*2^(1/3))
```

3.39.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1177, 27, 133}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{x^2-3x+2}}dx$$

$$\Big\downarrow \text{1177}$$

$$\frac{2^{2/3}\sqrt[3]{x-2}\sqrt[3]{x-1}\int \frac{1}{2^{2/3}\sqrt[3]{x-2}\sqrt[3]{x-1}x}dx}{\sqrt[3]{x^2-3x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\int \frac{1}{\sqrt[3]{x-2}\sqrt[3]{x-1}x}dx}{\sqrt[3]{x^2-3x+2}}$$

$$\Big\downarrow \text{133}$$

$$\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(-\frac{(x-2)^{2/3}}{2^{2/3}}-\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}\right)}{\sqrt[3]{x^2-3x+2}}$$

input
```
Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

output
```
((-2 + x)^(1/3)*(-1 + x)^(1/3)*(-1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(1/3)*
(-2 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/2^(1/3) + (3*Log[-((-2 + x)^(2/
3)/2^(2/3)) - (-1 + x)^(1/3)])/(4*2^(1/3)) - Log[x]/(2*2^(1/3))))/(2 - 3*x
 + x^2)^(1/3)
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 133
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp[-Log[
a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2*q*((c
 + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Simp[3*(
Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

rule 1177
```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(b + q + 2*c*x)^(1/3)*((b - q
+ 2*c*x)^(1/3)/(a + b*x + c*x^2)^(1/3))   Int[1/((d + e*x)*(b + q + 2*c*x)^
(1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c
^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

3.39.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.43 (sec) , antiderivative size = 1593, normalized size of antiderivative = 14.48

method	result	size
trager	Expression too large to display	1593

input
```
int(1/x/(x^2-3*x+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*RootOf(_Z^3-4)*ln(-(-12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*
_Z^2)*RootOf(_Z^3-4)^3*x^2+112*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+54*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-504*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-237*(x^2-3*x+2)^(1/3)*
RootOf(_Z^3-4)^2*x-258*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-54*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3
-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+504*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2+474*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2+
516*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*
RootOf(_Z^3-4)-3*RootOf(_Z^3-4)*x^2+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*x^2-516*(x^2-3*x+2)^(2/3)-72*RootOf(_Z^3-4)*x+672*RootOf(
RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+72*RootOf(_Z^3-4)-672*RootO
f(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)-1/4*ln((12*RootOf(Roo
tOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+136*RootOf(
RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+216*(x
^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf
(_Z^3-4)^2-54*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_
Z^3-4)^3*x-612*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*Ro...
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(81) = 162$.

Time = 1.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx =$$
$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+36x^5-612x^4+2880x^3-5760x^2+5184x-1728)+12\sqrt{2}(x^5-3\right.}{6\left(x^6-108x^5+972x^4\right.}\right.$$
$$+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2+6\cdot 2^{\frac{1}{3}}(x^2-3x+2)^{\frac{1}{3}}(x-2)+12(x^2-3x+2)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{24}$$
$$\cdot 2^{\frac{2}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^2-3x+2)^{\frac{2}{3}}(x^2-6x+6)+2^{\frac{1}{3}}(x^4-36x^3+180x^2-288x+144)-6(x^3-14x^2+\right.}{x^4}\right.$$

input
```
integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 -
612*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4
+ 252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4
 - 6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x
^3 + 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3
)*(x^2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^
(2/3)*log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4
 - 36*x^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3
*x + 2)^(1/3))/x^4)
```

3.39.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{x\sqrt[3]{(x-2)\,(x-1)}}\,dx$$

input
```
integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

output
```
Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

3.39.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{(x^2-3\,x+2)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

3.39.8 Giac [F]

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{(x^2-3\,x+2)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \int \frac{1}{x\,(x^2-3\,x+2)^{1/3}}\,dx$$

input
```
int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
```

output
```
int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

3.40.1 Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\sqrt[3]{-5+7x-3x^2+x^3}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\left(1-x+\sqrt[3]{-5+7x-3x^2+x^3}\right)$$

output
```
1/4*ln(1-x)-3/4*ln(1-x+(x^3-3*x^2+7*x-5)^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3
*(-1+x)/(x^3-3*x^2+7*x-5)^(1/3)*3^(1/2))*3^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx=\frac{3\sqrt[3]{(2-i)+ix}\sqrt[3]{i(-1+x)}((-1+2i)+x)\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{1}{4}i((-1+2i)+x),-\frac{1}{2}i((-1+2i)+x\right.}{4\sqrt[3]{-5+7x-3x^2+x^3}}$$

input
```
Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

output
```
(3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
 1/3, 1/3, 5/3, (-1/4*I)*((-1 + 2*I) + x), (-1/2*I)*((-1 + 2*I) + x)])/(4*
(-5 + 7*x - 3*x^2 + x^3)^(1/3))
```

3.40.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2481, 1917, 266, 807, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}dx$$

$$\Big\downarrow \text{2481}$$

$$\int \frac{1}{\sqrt[3]{(x-1)^3+4(x-1)}}d(x-1)$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\int \frac{1}{\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}}d(x-1)}{\sqrt[3]{(x-1)^3+4(x-1)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\int \frac{\sqrt[3]{x-1}}{\sqrt[3]{(x-1)^2+4}}d\sqrt[3]{x-1}}{\sqrt[3]{(x-1)^3+4(x-1)}}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\int \frac{1}{\sqrt[3]{x+3}}d(x-1)^{2/3}}{2\sqrt[3]{(x-1)^3+4(x-1)}}$$

$$\Big\downarrow \text{769}$$

$$\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\left(\frac{\arctan\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{x+3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{x+3}+1\right)\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}}$$

input
```
Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

output
```
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*(ArcTan[(1 + (2*(-1 + x)^(2/3))/(
3 + x)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x + (3 + x)^(1/3)]/2))/(2*(4*(-1
+ x) + (-1 + x)^3)^(1/3))
```

3.40.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2481
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, c/(3
*d) + x]] /; FreeQ[p, x] && PolyQ[Px, x, 3]
```

3.40.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.95 (sec) , antiderivative size = 653, normalized size of antiderivative = 8.06

method	result
trager	$\text{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-304\,\text{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+624\,\text{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-3x^2+7x-5\right)^{\frac{2}{3}}+624\,\text{RootOf}\left(\_\right.\right.$

input
```
int(1/(x^3-3*x^2+7*x-5)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(_Z^2-_Z+1)*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2+624*RootOf(_Z^2-_Z+1
)*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x+
608*RootOf(_Z^2-_Z+1)^2*x+928*RootOf(_Z^2-_Z+1)*x^2+51*(x^3-3*x^2+7*x-5)^(
2/3)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+51*(x^3-3*x^2+7*x-5)^(1
/3)*x-1856*RootOf(_Z^2-_Z+1)*x-253*x^2-51*(x^3-3*x^2+7*x-5)^(1/3)+2356*Roo
tOf(_Z^2-_Z+1)+506*x-713)-1/2*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2-624*RootOf(_
Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^
(1/3)*x+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^2+675*(x^3-3*x^2
+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+675*(x^3-3*x^2
+7*x-5)^(1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x^2+7*x-5)^(1/3
)-2356*RootOf(_Z^2-_Z+1)-742*x+1643)*RootOf(_Z^2-_Z+1)+1/2*ln(-304*RootOf(
_Z^2-_Z+1)^2*x^2-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(
_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(
_Z^2-_Z+1)*x^2+675*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^
2+7*x-5)^(1/3)+675*(x^3-3*x^2+7*x-5)^(1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x
^2-675*(x^3-3*x^2+7*x-5)^(1/3)-2356*RootOf(_Z^2-_Z+1)-742*x+1643)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx =$$
$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{22791076\sqrt{3}(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}(x-1)+\sqrt{3}(20389537\,x^2-40779074\,x+53222437}{7204617\,x^2-14409234\,x-20666867}\right.$$
$$-\frac{1}{4}\log\left(3\left(x^3-3\,x^2+7\,x-5\right)^{\frac{1}{3}}(x-1)-3\left(x^3-3\,x^2+7\,x-5\right)^{\frac{2}{3}}+4\right)$$

input
```
integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1
) + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3
 - 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*lo
g(3*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3
) + 4)
```

3.40.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

input
```
integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

output
```
Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

3.40.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \int \frac{1}{(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

3.40.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{1/3}}\,dx$$

input
```
int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3),x)
```

output
```
int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$

3.41.1 Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\sqrt[3]{x\left(-q+x^2\right)}}\right) + \frac{\log(x)}{4} - \frac{3}{4}\log\left(-x+\sqrt[3]{x\left(-q+x^2\right)}\right)$$

output
```
1/4*ln(x)-3/4*ln(-x+(x*(x^2-q))^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*x/(x*(x^
2-q))^(1/3)*3^(1/2))*3^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx = \frac{\sqrt[3]{x}\sqrt[3]{-q+x^2}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{-q+x^2}}\right)-2\log\left(-x^{2/3}+\sqrt[3]{-q+x^2}\right)+\log\left(x^{4/3}+x^{2/3}\sqrt[3]{-q+\right.\right.}{4\sqrt[3]{-qx+x^3}}$$

input
```
Integrate[(x*(-q + x^2))^(-1/3),x]
```

output
```
(x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x^(2/3))/(x^(2/3) + 2
*(-q + x^2)^(1/3))] - 2*Log[-x^(2/3) + (-q + x^2)^(1/3)] + Log[x^(4/3) + x
^(2/3)*(-q + x^2)^(1/3) + (-q + x^2)^(2/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

3.41.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2078, 1917, 266, 807, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x(x^2-q)}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt[3]{x^3-qx}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-q}\int \frac{1}{\sqrt[3]{x}\sqrt[3]{x^2-q}}dx}{\sqrt[3]{x^3-qx}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\int \frac{\sqrt[3]{x}}{\sqrt[3]{x^2-q}}d\sqrt[3]{x}}{\sqrt[3]{x^3-qx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\int \frac{1}{\sqrt[3]{x-q}}dx^{2/3}}{2\sqrt[3]{x^3-qx}}$$

$$\Big\downarrow \text{769}$$

$$\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\left(\frac{\arctan\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x-q}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x-q}-x^{2/3}\right)\right)}{2\sqrt[3]{x^3-qx}}$$

input
```
Int[(x*(-q + x^2))^(-1/3),x]
```

output
```
(3*x^(1/3)*(-q + x^2)^(1/3)*(ArcTan[(1 + (2*x^(2/3))/(-q + x)^(1/3))/Sqrt[
3]]/Sqrt[3] - Log[-x^(2/3) + (-q + x)^(1/3)]/2))/(2*(-(q*x) + x^3)^(1/3))
```

3.41.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.41.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\ln\left(\frac{\left(-x\left(-x^{2}+q\right)\right)^{\frac{2}{3}}+\left(-x\left(-x^{2}+q\right)\right)^{\frac{1}{3}}x+x^{2}}{x^{2}}\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(x+2\left(-x\left(-x^{2}+q\right)\right)^{\frac{1}{3}}\right)\sqrt{3}}{3x}\right)}{2}-\frac{\ln\left(\frac{\left(-x\left(-x^{2}+q\right)\right)^{\frac{1}{3}}-x}{x}\right)}{2}$

input
```
int(1/(x*(x^2-q))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(((-x*(-x^2+q))^(2/3)+(-x*(-x^2+q))^(1/3)*x+x^2)/x^2)-1/2*3^(1/2)*ar
ctan(1/3*(x+2*(-x*(-x^2+q))^(1/3))*3^(1/2)/x)-1/2*ln(((-x*(-x^2+q))^(1/3)-
x)/x)
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(52) = 104$.

Time = 0.90 (sec) , antiderivative size = 415, normalized size of antiderivative = 6.29

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^{2}\right)}}\,dx$$
$$=\frac{1}{2}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(q^{12}-15\,q^{10}+90\,q^{8}-351\,q^{6}+810\,q^{4}-1215\,q^{2}+729)(x^{3}-qx)^{\frac{1}{3}}x-2\sqrt{3}(q^{12}+6\,q^{1}}{}\right.$$
$$-\frac{1}{4}\log\left(-3\left(x^{3}-qx\right)^{\frac{1}{3}}x+q+3\left(x^{3}-qx\right)^{\frac{2}{3}}\right)$$

input
```
integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
 - 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^1
0 - 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1
215*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q
^11 - 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q
^4 - 1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351
*q^6 + 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*
q^2 + 729*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 +
6561*q^7 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 -
 15*q^10 - 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^
3 - 1215*q^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q
*x)^(1/3)*x + q + 3*(x^3 - q*x)^(2/3))
```

3.41.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx = \int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx$$

input
```
integrate(1/(x*(x**2-q))**(1/3),x)
```

output
```
Integral((x*(-q + x**2))**(-1/3), x)
```

3.41.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx = \int \frac{1}{((x^2-q)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x^2 - q)*x)^(-1/3), x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{4}\log\left(\left(-\frac{q}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{2}\log\left(\left|\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

output
```
-1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-q/x^2 + 1)^(1/3) + 1)) + 1/4*log((-q/
x^2 + 1)^(2/3) + (-q/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((-q/x^2 + 1)^(1/3)
- 1))
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx = \frac{3\,x\left(1-\frac{x^2}{q}\right)^{1/3}{}_2F_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{x^2}{q}\right)}{2\left(x^3-q\,x\right)^{1/3}}$$

input
```
int(1/(-x*(q - x^2))^(1/3),x)
```

output
```
(3*x*(1 - x^2/q)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2/q))/(2*(x^3 - q*x)^(
1/3))
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\left(1-x+\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}\right)$$

output
```
1/4*ln(1-x)-3/4*ln(1-x+((-1+x)*(x^2+q-2*x))^(1/3))+1/2*arctan(1/3*3^(1/2)+
2/3*(-1+x)/((-1+x)*(x^2+q-2*x))^(1/3)*3^(1/2))*3^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx$$
$$=\frac{\sqrt[3]{-1+x}\sqrt[3]{q+(-2+x)x}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}(-1+x)^{2/3}}{(-1+x)^{2/3}+2\sqrt[3]{q+(-2+x)x}}\right)-2\log\left(-(-1+x)^{2/3}+\sqrt[3]{q+(\right.\right.}{4\sqrt[3]{(-1+x)(q+(-2+}}$$

input
```
Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

output
```
((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x)
^(2/3))/((-1 + x)^(2/3) + 2*(q + (-2 + x)*x)^(1/3))] - 2*Log[-(-1 + x)^(2/
3) + (q + (-2 + x)*x)^(1/3)] + Log[(-1 + x)^(4/3) + (-1 + x)^(2/3)*(q + (-
2 + x)*x)^(1/3) + (q + (-2 + x)*x)^(2/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))
^(1/3))
```

3.42.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2481, 1917, 266, 807, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)(q+x^2-2x)}}dx$$

$$\Big\downarrow \text{ 2481}$$

$$\int \frac{1}{\sqrt[3]{(x-1)^3-(1-q)(x-1)}}d(x-1)$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\int \frac{1}{\sqrt[3]{(x-1)^2+q-1}\sqrt[3]{x-1}}d(x-1)}{\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\int \frac{\sqrt[3]{x-1}}{\sqrt[3]{(x-1)^2+q-1}}d\sqrt[3]{x-1}}{\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\int \frac{1}{\sqrt[3]{q+x-2}}d(x-1)^{2/3}}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\left(\frac{\arctan\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+x-2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{q+x-2}-x+1\right)\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

input
```
Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

output
```
(3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*(ArcTan[(1 + (2*(-1 + x)^(2/
3))/(-2 + q + x)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x + (-2 + q + x)^(1/3)]
/2))/(2*(-((1 - q)*(-1 + x)) + (-1 + x)^3)^(1/3))
```

3.42.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2481
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, c/(3
*d) + x]] /; FreeQ[p, x] && PolyQ[Px, x, 3]
```

3.42.4 Maple [F]

$$\int \frac{1}{((-1+x)(x^2+q-2x))^{\frac{1}{3}}}dx$$

input
```
int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

output
```
int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(65) = 130$.

Time = 0.78 (sec) , antiderivative size = 665, normalized size of antiderivative = 8.42

$$\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx$$
$$= \frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt{3}(q^{12}-18q^{11}+117q^{10}-346q^9+414q^8-18q^7+69q^6-774q^5-234q^4+1058q^3+}{}\right.$$
$$-\frac{1}{4}\log\left(3\left(x^3+(q+2)x-3x^2-q\right)^{\frac{1}{3}}(x-1)+q-3\left(x^3+(q+2)x-3x^2-q\right)^{\frac{2}{3}}-1\right)$$

input
```
integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q
^8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 53
9)*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^
10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291
*q^2 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54
*q^5 - 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q
 + 2)*x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10
 - 434*q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343
*q^3 + (q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 +
882*q^5 - 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5
 - 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22
*q^12 + 249*q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6
 - 3546*q^5 - 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^
9 + 78*q^8 + 186*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 60
2*q - 147)*x^2 + 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^
8 + 186*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147
)*x - 995*q + 8)) - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1)
+ q - 3*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) - 1)
```

3.42.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx=\int \frac{1}{\sqrt[3]{(x-1)\left(q+x^2-2x\right)}}\,dx$$

input
```
integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

output
```
Integral(((x - 1)*(q + x**2 - 2*x))**(-1/3), x)
```

3.42.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx=\int \frac{1}{\left((x^2+q-2\,x)(x-1)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

3.42.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx=\int \frac{1}{\left((x^2+q-2\,x)(x-1)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="giac")
```

output
```
integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx=\int \frac{1}{\left((x-1)\,\left(x^2-2\,x+q\right)\right)^{1/3}}\,dx$$

input
```
int(1/((x - 1)*(q - 2*x + x^2))^(1/3),x)
```

output
```
int(1/((x - 1)*(q - 2*x + x^2))^(1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$

3.43.1 Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{q}(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)(q-2qx+x^2)}}\right)}{2\sqrt[3]{q}} + \frac{\log(1-x)}{4\sqrt[3]{q}} + \frac{\log(x)}{2\sqrt[3]{q}} - \frac{3\log\left(-\sqrt[3]{q}(-1+x)+\sqrt[3]{(-1+x)(q-2qx+x^2)}\right)}{4\sqrt[3]{q}}$$

output
```
1/4*ln(1-x)/q^(1/3)+1/2*ln(x)/q^(1/3)-3/4*ln(-q^(1/3)*(-1+x)+((-1+x)*(-2*q
*x+x^2+q))^(1/3))/q^(1/3)+1/2*arctan(1/3*3^(1/2)+2/3*q^(1/3)*(-1+x)/((-1+x
)*(-2*q*x+x^2+q))^(1/3)*3^(1/2))*3^(1/2)/q^(1/3)
```

3.43.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.60

$$\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx = \frac{\sqrt[3]{-1+x}\sqrt[3]{q-2qx+x^2}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{q}(-1+x)^{2/3}}{\sqrt[3]{q}(-1+x)^{2/3}+2\sqrt[3]{q-2qx+x^2}}\right)-2\log\left(-\sqrt[3]{q}(-1+x)^{2/3}+\sqrt[3]{q}\right.\right.}{4\sqrt[3]{q}\sqrt[3]{(-1+x)(q-}}$$

input
```
Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

output
```
((-1 + x)^(1/3)*(q - 2*q*x + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*q^(1/3)
*(-1 + x)^(2/3))/(q^(1/3)*(-1 + x)^(2/3) + 2*(q - 2*q*x + x^2)^(1/3))] - 2
*Log[-(q^(1/3)*(-1 + x)^(2/3)) + (q - 2*q*x + x^2)^(1/3)] + Log[q^(2/3)*(-
1 + x)^(4/3) + q^(1/3)*(-1 + x)^(2/3)*(q - 2*q*x + x^2)^(1/3) + (q - 2*q*x
 + x^2)^(2/3)]))/(4*q^(1/3)*((-1 + x)*(q - 2*q*x + x^2))^(1/3))
```

3.43.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}dx$$

$$\Big\downarrow \text{2490}$$

$$\int \frac{1}{\left(\frac{1}{3}(-2q-1)+\frac{1}{3}(2q+1)+x\right)\sqrt[3]{\left(\frac{1}{3}(-2q-1)+x\right)^3-\frac{1}{3}(1-4q)(1-q)\left(\frac{1}{3}(-2q-1)+x\right)-\frac{2}{27}(1-q)^2(8q}}$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{\left(\frac{1}{3}(-2q-1)+\frac{1}{3}(2q+1)+x\right)\sqrt[3]{\left(\frac{1}{3}(-2q-1)+x\right)^3-\frac{1}{3}(1-4q)(1-q)\left(\frac{1}{3}(-2q-1)+x\right)-\frac{2}{27}(1-q)^2(8q}}$$

input
```
Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.43.4 Maple [F]

$$\int \frac{1}{x((-1+x)\left(-2qx+x^2+q\right))^{\frac{1}{3}}}dx$$

input
```
int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

output
```
int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(100) = 200$.

Time = 13.44 (sec) , antiderivative size = 1496, normalized size of antiderivative = 12.68

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

output
```
[1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 +
54*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x +
540*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1
)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x -
 q)^(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 +
9*(7*q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^
2 + x^3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4
 - 6*(7*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q
+ 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5
+ 5*(q^3 + 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*
q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2
+ 3*q - 1)*x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x
 - 108*(4*q^3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqr
t((-q)^(1/3)/q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x
^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6
*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2
+ 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(...
```

3.43.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx=\int \frac{1}{x\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}\,dx$$

input
```
integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
```

output
```
Integral(1/(x*((x - 1)*(-2*q*x + q + x**2))**(1/3)), x)
```

3.43.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)\,(q-2qx+x^2)}}\,dx = \int \frac{1}{\left(-(2\,qx-x^2-q)(x-1)\right)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

3.43.8 Giac [F]

$$\int \frac{1}{x\sqrt[3]{(-1+x)\,(q-2qx+x^2)}}\,dx = \int \frac{1}{\left(-(2\,qx-x^2-q)(x-1)\right)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{(-1+x)\,(q-2qx+x^2)}}\,dx = \int \frac{1}{x\left((x-1)\,\left(x^2-2\,q\,x+q\right)\right)^{1/3}}\,dx$$

input
```
int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
```

output
```
int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$

3.44.1 Optimal result

Integrand size = 36, antiderivative size = 111

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{k}x}{\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{\sqrt[3]{k}} + \frac{\log(x)}{2\sqrt[3]{k}} + \frac{\log(1-(1+k)x)}{2\sqrt[3]{k}} - \frac{3\log\left(-\sqrt[3]{k}x+\sqrt[3]{(1-x)x(1-kx)}\right)}{2\sqrt[3]{k}}$$

output
```
1/2*ln(x)/k^(1/3)+1/2*ln(1-(1+k)*x)/k^(1/3)-3/2*ln(-k^(1/3)*x+((1-x)*x*(-k
*x+1))^(1/3))/k^(1/3)+arctan(1/3*(1+2*k^(1/3)*x/((1-x)*x*(-k*x+1))^(1/3))*
3^(1/2))*3^(1/2)/k^(1/3)
```

3.44.2 Mathematica [A] (verified)

Time = 15.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{k}x}{\sqrt[3]{k}x+2\sqrt[3]{(-1+x)x(-1+kx)}}\right)-2\log\left(-\sqrt[3]{k}x+\sqrt[3]{(-1+x)x(-1+kx)}\right)+\log\left(k^{2/3}x^2\right.}{2\sqrt[3]{k}}$$

input
```
Integrate[(2 - (1 + k)*x)/((((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*k^(1/3)*x)/(k^(1/3)*x + 2*((-1 + x)*x*(-1 + k*x
))^(1/3))] - 2*Log[-(k^(1/3)*x) + ((-1 + x)*x*(-1 + k*x))^(1/3)] + Log[k^(
2/3)*x^2 + k^(1/3)*x*((-1 + x)*x*(-1 + k*x))^(1/3) + ((-1 + x)*x*(-1 + k*x
))^(2/3)])/(2*k^(1/3))
```

3.44.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-(k+1)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(k+1)x)}\,dx$$
$$\Big\downarrow \text{2467}$$
$$\frac{\sqrt[3]{x}\sqrt[3]{kx^2-(k+1)x+1}\int \frac{2-(k+1)x}{\sqrt[3]{x}(1-(k+1)x)\sqrt[3]{kx^2-(k+1)x+1}}dx}{\sqrt[3]{(1-x)x(1-kx)}}$$
$$\Big\downarrow \text{2035}$$
$$\frac{3\sqrt[3]{x}\sqrt[3]{kx^2-(k+1)x+1}\int \frac{\sqrt[3]{x}(2-(k+1)x)}{(1-(k+1)x)\sqrt[3]{kx^2-(k+1)x+1}}d\sqrt[3]{x}}{\sqrt[3]{(1-x)x(1-kx)}}$$
$$\Big\downarrow \text{7276}$$
$$\frac{3\sqrt[3]{x}\sqrt[3]{kx^2-(k+1)x+1}\int \left(\frac{\sqrt[3]{x}}{(1-(k+1)x)\sqrt[3]{kx^2-(k+1)x+1}}+\frac{\sqrt[3]{x}}{\sqrt[3]{kx^2-(k+1)x+1}}\right)d\sqrt[3]{x}}{\sqrt[3]{(1-x)x(1-kx)}}$$
$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt[3]{x}\,\sqrt[3]{kx^2-(k+1)x+1}\left(\int \frac{\sqrt[3]{x}}{((-k-1)x+1)\sqrt[3]{kx^2+(-k-1)x+1}}d\sqrt[3]{x}+\frac{\sqrt[3]{1-x}x^{2/3}\sqrt[3]{1-kx}\,\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{}{k+\sqrt{}}\right.}{2\sqrt[3]{kx^2-(k+1)x}}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

input
```
Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

output
```
$Aborted
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.44.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(k^{\frac{1}{3}}x+2((-1+x)x(kx-1))^{\frac{1}{3}}\right)}{3k^{\frac{1}{3}}x}\right)-\frac{\ln\left(\frac{k^{\frac{2}{3}}x^2+k^{\frac{1}{3}}((-1+x)x(kx-1))^{\frac{1}{3}}x+((-1+x)x(kx-1))^{\frac{2}{3}}}{x^2}\right)}{2}+\ln\left(\frac{-k^{\frac{1}{3}}x+((-1}{}\right.}{k^{\frac{1}{3}}}$

input
```
int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x,method=_RETURNVERBO
SE)
```

output
```
-(3^(1/2)*arctan(1/3*3^(1/2)*(k^(1/3)*x+2*((-1+x)*x*(k*x-1))^(1/3))/k^(1/3
)/x)-1/2*ln((k^(2/3)*x^2+k^(1/3)*((-1+x)*x*(k*x-1))^(1/3)*x+((-1+x)*x*(k*x
-1))^(2/3))/x^2)+ln((-k^(1/3)*x+((-1+x)*x*(k*x-1))^(1/3))/x))/k^(1/3)
```

3.44.5 Fricas [F(-1)]

Timed out.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \text{Timed out}$$

input
```
integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="f
ricas")
```

output
```
Timed out
```

3.44.6 Sympy [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{kx+x-2}{\sqrt[3]{x(x-1)(kx-1)}\,(kx+x-1)}\,dx$$

input
```
integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

output
```
Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

3.44.7 Maxima [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

input
```
integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="m
axima")
```

output
```
integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x
)
```

3.44.8 Giac [F]

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

input
```
integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="g
iac")
```

output
```
integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x
)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx = \int \frac{x\,(k+1)-2}{(x\,(k+1)-1)\,(x\,(k\,x-1)\,(x-1))^{1/3}}\,dx$$

input
```
int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

output
```
int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

3.45.1 Optimal result

Integrand size = 33, antiderivative size = 176

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(-1+kx+2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}\right)}{2\,2^{2/3}\sqrt[3]{1-k}}$$

output
```
1/2*ln(1-(2-k)*x)*2^(1/3)/(1-k)^(1/3)+1/4*ln(-k*x+1)*2^(1/3)/(1-k)^(1/3)-3
/4*ln(-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(-k*x+1))^(1/3))*2^(1/3)/(1-k)^(
1/3)-1/2*arctan(1/3*(1+2^(1/3)*(-k*x+1)/(1-k)^(1/3)/((1-x)*x*(-k*x+1))^(1/
3))*3^(1/2))*3^(1/2)*2^(1/3)/(1-k)^(1/3)
```

3.45.2 Mathematica [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx=\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

input
```
Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

output
```
Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

3.45.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-kx}{((k-2)x+1)((1-x)x(1-kx))^{2/3}}\,dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{x^{2/3}\left(kx^2-(k+1)x+1\right)^{2/3}\int \frac{1-kx}{x^{2/3}(1-(2-k)x)\left(kx^2-(k+1)x+1\right)^{2/3}}dx}{((1-x)x(1-kx))^{2/3}}$$

$$\Big\downarrow \text{ 2035}$$

$$\frac{3x^{2/3}\left(kx^2-(k+1)x+1\right)^{2/3}\int \frac{1-kx}{(1-(2-k)x)\left(kx^2-(k+1)x+1\right)^{2/3}}d\sqrt[3]{x}}{((1-x)x(1-kx))^{2/3}}$$

$$\Big\downarrow \text{ 1395}$$

$$\frac{3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}(1-(2-k)x)}d\sqrt[3]{x}}{((1-x)x(1-kx))^{2/3}}$$

$$\Big\downarrow \text{ 1028}$$

$$\frac{3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(\frac{k\int \frac{1}{(1-x)^{2/3}(1-kx)^{2/3}}d\sqrt[3]{x}}{2-k}+\frac{2(1-k)\int \frac{1}{(1-x)^{2/3}(1-(2-k)x)(1-kx)^{2/3}}d\sqrt[3]{x}}{2-k}\right)}{((1-x)x(1-kx))^{2/3}}$$

$$\Big\downarrow \text{ 905}$$

$$\frac{3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(\frac{2(1-k)\int \frac{1}{(1-x)^{2/3}(1-(2-k)x)(1-kx)^{2/3}}d\sqrt[3]{x}}{2-k}+\frac{k\sqrt[3]{x}\left(\frac{1-x}{1-kx}\right)^{2/3}\sqrt[3]{1-kx}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\right.}{(2-k)(1-x)^{2/3}}\right.}{((1-x)x(1-kx))^{2/3}}$$

$\big\downarrow$ 1030

$$\frac{3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(\frac{2(1-k)\int \left(\frac{1}{3\left(1-\sqrt[3]{2-k}\sqrt[3]{x}\right)(1-x)^{2/3}(1-kx)^{2/3}}+\frac{1}{3\left(\sqrt[3]{-1}\sqrt[3]{2-k}\sqrt[3]{x}+1\right)(1-x)^{2/3}(1-kx)^{2/3}}+\frac{}{3\left(1-\right.}\right.}{2-k}\right.}{((1-x)x(1-kx))^{2}}$$

$\big\downarrow$ 2009

$$\frac{3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(\frac{2(1-k)\left(\frac{1}{3}\int \frac{1}{\left(1-\sqrt[3]{2-k}\sqrt[3]{x}\right)(1-x)^{2/3}(1-kx)^{2/3}}d\sqrt[3]{x}+\frac{1}{3}\int \frac{1}{\left(\sqrt[3]{-1}\sqrt[3]{2-k}\sqrt[3]{x}+1\right)(1-x)^{2/3}(1-kx)^{2/}}\right.}{2-k}\right.}{((1-x)x(1}$$

input
```
Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

output
```
$Aborted
```

3.45.3.1 Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

rule 1028
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_
.)*(x_)^(n_))^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^n)^(p + 1)*(c + d*
x^n)^(q - 1)*(e + f*x^n)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^n)^p
*(c + d*x^n)^(q - 1)*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n, r
}, x] && ILtQ[p, 0] && GtQ[q, 0]
```

rule 1030
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_
.)*(x_)^(n_))^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^n)^p*(c
 + d*x^n)^q*(e + f*x^n)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IGtQ[n, 0]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.45. $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

3.45.4 Maple [F]

$$\int \frac{-kx+1}{(1+(-2+k)\,x)\,((1-x)\,x\,(-kx+1))^{\frac{2}{3}}}dx$$

input
```
int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

output
```
int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(132) = 264$.

Time = 54.28 (sec) , antiderivative size = 932, normalized size of antiderivative = 5.30

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fri
cas")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/3*(24*sqrt(3)*2^(1/3)*((k^5 - 3*k^4 - 4*k^3 +
 22*k^2 - 24*k + 8)*x^4 - 2*(k^4 - 10*k^3 + 27*k^2 - 26*k + 8)*x^3 - 6*(k^
3 - 4*k^2 + 4*k - 1)*x^2 - 2*(k^2 - 1)*x + k - 1)*(k*x^3 - (k + 1)*x^2 + x
)^(2/3)/(k - 1)^(1/3) - 6*sqrt(3)*2^(2/3)*((k^6 + 27*k^5 - 40*k^4 - 20*k^3
 + 48*k^2 - 16*k)*x^5 - (33*k^5 + 55*k^4 - 220*k^3 + 132*k^2 + 16*k - 16)*
x^4 + 2*(55*k^4 - 55*k^3 - 66*k^2 + 82*k - 16)*x^3 - 2*(55*k^3 - 99*k^2 +
38*k + 6)*x^2 + (33*k^2 - 61*k + 28)*x - k + 1)*(k*x^3 - (k + 1)*x^2 + x)^
(1/3)/(k - 1)^(2/3) + sqrt(3)*((k^6 - 48*k^5 - 192*k^4 + 416*k^3 - 48*k^2
- 192*k + 64)*x^6 + 6*(7*k^5 + 104*k^4 - 80*k^3 - 176*k^2 + 176*k - 32)*x^
5 - 3*(139*k^4 + 256*k^3 - 768*k^2 + 352*k + 16)*x^4 + 4*(203*k^3 - 192*k^
2 - 120*k + 104)*x^3 - 3*(139*k^2 - 208*k + 64)*x^2 + 6*(7*k - 8)*x + 1))/
((k^6 + 96*k^5 - 48*k^4 - 160*k^3 + 240*k^2 - 192*k + 64)*x^6 - 6*(17*k^5
+ 64*k^4 - 112*k^3 + 80*k^2 - 80*k + 32)*x^5 + 3*(149*k^4 + 32*k^3 - 96*k^
2 - 160*k + 80)*x^4 - 4*(157*k^3 - 24*k^2 - 168*k + 40)*x^3 + 3*(149*k^2 -
 128*k - 16)*x^2 - 6*(17*k - 16)*x + 1))/(k - 1)^(1/3) - 1/12*2^(1/3)*log(
(12*2^(2/3)*(k*x^3 - (k + 1)*x^2 + x)^(2/3)*((k^3 + k^2 - 4*k + 2)*x^2 - 2
*(2*k^2 - 3*k + 1)*x + k - 1)/(k - 1)^(2/3) + 6*((k^3 + 8*k^2 - 8*k)*x^3 -
 (11*k^2 - 8)*x^2 + (11*k - 8)*x - 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3) + 2^
(1/3)*((k^4 + 28*k^3 - 12*k^2 - 32*k + 16)*x^4 - 4*(8*k^3 + 15*k^2 - 30*k
+ 8)*x^3 + 6*(13*k^2 - 10*k - 2)*x^2 - 4*(8*k - 7)*x + 1)/(k - 1)^(1/3)...
```

3.45.6 Sympy [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = -\int \frac{kx}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\,dx - \int \left(-\frac{1}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\right)dx$$

input
```
integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

output
```
-Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x
**2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integr
al(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x*
*2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

3.45.7 Maxima [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \int -\frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}\,((k-2)x+1)}\,dx$$

input
```
integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="max
ima")
```

output
```
-integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

3.45.8 Giac [F]

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \int -\frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}\,((k-2)x+1)}\,dx$$

input
```
integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="gia
c")
```

output
```
integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = -\int \frac{k\,x-1}{(x\,(k-2)+1)\,\left(x\,(k\,x-1)\,(x-1)\right)^{2/3}}\,dx$$

input
```
int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
```

output
```
-int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

3.46 $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

3.46.1 Optimal result

Integrand size = 32, antiderivative size = 493

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{(a+b)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{(a-c)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(b+c)\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{(a+b)\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{(a-c)\log(1+x^3)}{6\sqrt[3]{2}} - \frac{(b+c)\log(1+x^3)}{6\sqrt[3]{2}} + \frac{(a+b)\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{(a+b)\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{(a-c)\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) - \frac{(a+b)\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/24*(a+b)*ln((1-x)*(1+x)^2)*2^(2/3)-1/12*(a-c)*ln(x^3+1)*2^(2/3)-1/12*(b+
c)*ln(x^3+1)*2^(2/3)+1/12*(a+b)*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3
)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*(a+b)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3
))*2^(2/3)+1/4*(b+c)*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/4*(a-c)*ln(-2^(1
/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/2*c*ln(x+(-x^3+1)^(1/3))-1/8*(a+b)*ln(-1+x
+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*(a+b)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*(a+b)*arctan(1/3*(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/3*c*arctan(1/3*(1-2*x/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)-1/6*(a-c)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^
(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/6*(b+c)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(
1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.46.2 Mathematica [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

input
```
Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

output
```
Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

3.46.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{(x^2-x+1)\sqrt[3]{1-x^3}}\,dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(\frac{x(a+b)}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{a}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{x^2(b+c)}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{cx^3}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{(a+b)\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{(a+b)\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{(a+b)\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}-\frac{a\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{a\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)(b+c)}{\sqrt[3]{2}\sqrt{3}}-\frac{c\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{(b+c)\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{c\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{c\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{1}{2}c\log\left(\sqrt[3]{1-x^3}+x\right)$$

input
```
Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

output
```
((a + b)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]) + ((a + b)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt
[3]])/(2*2^(1/3)*Sqrt[3]) - (c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]
)/Sqrt[3] - (a*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]) + (c*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(
1/3)*Sqrt[3]) + ((b + c)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2
^(1/3)*Sqrt[3]) + ((a + b)*Log[(1 - x)*(1 + x)^2])/(12*2^(1/3)) - (a*Log[1
 + x^3])/(6*2^(1/3)) + (c*Log[1 + x^3])/(6*2^(1/3)) - ((b + c)*Log[1 + x^3
])/(6*2^(1/3)) + ((a + b)*Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2
^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/(6*2^(1/3)) - ((a + b)*Log[1 + (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3)])/(3*2^(1/3)) + ((b + c)*Log[2^(1/3) - (1 - x^3)^
(1/3)])/(2*2^(1/3)) + (a*Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3))
- (c*Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) + (c*Log[x + (1 - x^
3)^(1/3)])/2 - ((a + b)*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

3.46. $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.46.4 Maple [F]

$$\int \frac{c\,x^{2}+b x +a}{\left(x^{2}-x +1\right) \left(-x^{3}+1\right)^{\frac{1}{3}}}d x$$

input
```
int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

output
```
int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

3.46.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.46.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx=\int \frac{a+bx+cx^2}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x^2-x+1)}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

output
```
Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x +
1)), x)
```

3.46.7 Maxima [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx=\int \frac{cx^2+bx+a}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

input
```
integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.46.8 Giac [F]

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx=\int \frac{cx^2+bx+a}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

input
```
integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{c\,x^2+b\,x+a}{{\left(1-x^3\right)}^{1/3}\,\left(x^2-x+1\right)}\,dx$$

input
```
int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
```

output
```
int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

3.47.1 Optimal result

Integrand size = 20, antiderivative size = 407

$$\begin{aligned}\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx &= -\frac{19255}{395136(3-2x)^{9/2}}-\frac{462025}{30118144(3-2x)^{7/2}}\\
&-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38225}{240945152\sqrt{3-2x}}\\
&+\frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4}+\frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3}\\
&+\frac{1387+3049x}{32928(3-2x)^{9/2}\left(1+x+2x^2\right)^2}+\frac{5(3049+4377x)}{153664(3-2x)^{9/2}\left(1+x+2x^2\right)}\\
&+\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{3373232128}\\
&-\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\arctan\left(\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{3373232128}\\
&+\frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256}\\
&-\frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{6746464256}\end{aligned}$$

output
```
-19255/395136/(3-2*x)^(9/2)-462025/30118144/(3-2*x)^(7/2)-38491/8605184/(3
-2*x)^(5/2)-141045/120472576/(3-2*x)^(3/2)+1/28*x/(3-2*x)^(9/2)/(2*x^2+x+1
)^4+1/1176*(23+73*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^3+1/32928*(1387+3049*x)/(3-
2*x)^(9/2)/(2*x^2+x+1)^2+5/153664*(3049+4377*x)/(3-2*x)^(9/2)/(2*x^2+x+1)-
38225/240945152/(3-2*x)^(1/2)+5/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2
)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)-5/13492
928512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-29809300795
4+81630132224*14^(1/2))^(1/2)+5/6746464256*arctan((-2*(3-2*x)^(1/2)+(7+2*1
4^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))
^(1/2)-5/6746464256*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14
^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)
```

3.47.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.42 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.44

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{-\frac{14(40289347-429812744x+135202154x^2-1073855156x^3+1627773523x^4-1470758860x^5+2888}{(3-2x}}{}$$

input
```
Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

output
```
((-14*(40289347 - 429812744*x + 135202154*x^2 - 1073855156*x^3 + 162777352
3*x^4 - 1470758860*x^5 + 2888625656*x^6 - 3106712560*x^7 + 2343370048*x^8
- 2443779648*x^9 + 1873554048*x^10 - 677249280*x^11 + 88070400*x^12))/((3
- 2*x)^(9/2)*(1 + x + 2*x^2)^4) - 45*Sqrt[149046503977 + (12577271771*I)*S
qrt[7]]*ArcTan[(Sqrt[-1 - I/Sqrt[7]]*Sqrt[3 - 2*x])/2] - 45*Sqrt[149046503
977 - (12577271771*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 + I/Sqrt[7]]*Sqrt[3 - 2*x])
/2])/30359089152
```

3.47.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.24, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {1165, 27, 1235, 27, 1235, 27, 1235, 27, 1198, 27, 1198, 27, 1198, 27, 1198, 27, 1198, 27, 1197, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-2x)^{11/2} (2x^2+x+1)^5} \, dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{784} \int \frac{28(25-23x)}{(3-2x)^{11/2} (2x^2+x+1)^4} dx + \frac{x}{28(3-2x)^{9/2} (2x^2+x+1)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28} \int \frac{25-23x}{(3-2x)^{11/2} (2x^2+x+1)^4} dx + \frac{x}{28(3-2x)^{9/2} (2x^2+x+1)^4}$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{28} \left(\frac{1}{588} \int \frac{14(831-1387x)}{(3-2x)^{11/2} (2x^2+x+1)^3} dx + \frac{73x+23}{42(3-2x)^{9/2} (2x^2+x+1)^3} \right) + \frac{x}{28(3-2x)^{9/2} (2x^2+x+1)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28} \left(\frac{1}{42} \int \frac{831-1387x}{(3-2x)^{11/2} (2x^2+x+1)^3} dx + \frac{73x+23}{42(3-2x)^{9/2} (2x^2+x+1)^3} \right) + \frac{x}{28(3-2x)^{9/2} (2x^2+x+1)^4}$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{28} \left(\frac{1}{42} \left(\frac{1}{392} \int \frac{210(664-3049x)}{(3-2x)^{11/2} (2x^2+x+1)^2} dx + \frac{3049x+1387}{28(3-2x)^{9/2} (2x^2+x+1)^2} \right) + \frac{73x+23}{42(3-2x)^{9/2} (2x^2+x+1)^3} \right) + \frac{x}{28(3-2x)^{9/2} (2x^2+x+1)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\int\frac{664-3049x}{(3-2x)^{11/2}\left(2x^2+x+1\right)^2}dx+\frac{3049x+1387}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^2}\Bigg)+\frac{73x+23}{42(3-2x)^{9/2}\left(2x^2+x+1\right)^3}\Bigg)$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 1235

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\Bigg(\frac{1}{196}\int-\frac{14(48147x+22129)}{(3-2x)^{11/2}\left(2x^2+x+1\right)}dx+\frac{4377x+3049}{14(3-2x)^{9/2}\left(2x^2+x+1\right)}\Bigg)+\frac{3049x+1387}{28(3-2x)^{9/2}\left(2x^2+x+\right.}$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 27

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\Bigg(\frac{4377x+3049}{14(3-2x)^{9/2}\left(2x^2+x+1\right)}-\frac{1}{14}\int\frac{48147x+22129}{(3-2x)^{11/2}\left(2x^2+x+1\right)}dx\Bigg)+\frac{3049x+1387}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^2}$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 1198

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\Bigg(\frac{1}{14}\Bigg(-\frac{1}{28}\int\frac{14(26957x+5767)}{(3-2x)^{9/2}\left(2x^2+x+1\right)}dx-\frac{26957}{18(3-2x)^{9/2}}\Bigg)+\frac{4377x+3049}{14(3-2x)^{9/2}\left(2x^2+x+1\right)}\Bigg)+\frac{\quad}{28(3}$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 27

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\Bigg(\frac{1}{14}\Bigg(-\frac{1}{2}\int\frac{26957x+5767}{(3-2x)^{9/2}\left(2x^2+x+1\right)}dx-\frac{26957}{18(3-2x)^{9/2}}\Bigg)+\frac{4377x+3049}{14(3-2x)^{9/2}\left(2x^2+x+1\right)}\Bigg)+\frac{\quad}{28(3-}$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 1198

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\Bigg(\frac{1}{14}\Bigg(\frac{1}{2}\Bigg(-\frac{1}{28}\int-\frac{2(3889-92405x)}{(3-2x)^{7/2}\left(2x^2+x+1\right)}dx-\frac{92405}{98(3-2x)^{7/2}}\Bigg)-\frac{26957}{18(3-2x)^{9/2}}\Bigg)+\frac{4377x}{14(3-2x)^{9/2}}$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 27

$$\frac{1}{28}\Bigg(\frac{1}{42}\Bigg(\frac{15}{28}\Bigg(\frac{1}{14}\Bigg(\frac{1}{2}\Bigg(\frac{1}{14}\int\frac{3889-92405x}{(3-2x)^{7/2}\left(2x^2+x+1\right)}dx-\frac{92405}{98(3-2x)^{7/2}}\Bigg)-\frac{26957}{18(3-2x)^{9/2}}\Bigg)+\frac{4377x+3}{14(3-2x)^{9/2}(2x}$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

3.47. $\int\frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

$$\Big\downarrow \text{1198}$$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{28}\int \frac{14(15423-38491x)}{(3-2x)^{5/2}\left(2x^2+x+1\right)}dx-\frac{38491}{10(3-2x)^{5/2}}\right)-\frac{92405}{98(3-2x)^{7/2}}\right)-\frac{26957}{18(3-2x)^{9/}}\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\int \frac{15423-38491x}{(3-2x)^{5/2}\left(2x^2+x+1\right)}dx-\frac{38491}{10(3-2x)^{5/2}}\right)-\frac{92405}{98(3-2x)^{7/2}}\right)-\frac{26957}{18(3-2x)^{9/2}}\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{1198}$$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{28}\int \frac{2(100183-84627x)}{(3-2x)^{3/2}\left(2x^2+x+1\right)}dx-\frac{28209}{14(3-2x)^{3/2}}\right)-\frac{38491}{10(3-2x)^{5/2}}\right)-\frac{9240}{98(3-2}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\int \frac{100183-84627x}{(3-2x)^{3/2}\left(2x^2+x+1\right)}dx-\frac{28209}{14(3-2x)^{3/2}}\right)-\frac{38491}{10(3-2x)^{5/2}}\right)-\frac{9240}{98(3-2}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{1198}$$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{28}\int \frac{14(69337-7645x)}{\sqrt{3-2x}\left(2x^2+x+1\right)}dx-\frac{7645}{2\sqrt{3-2x}}\right)-\frac{28209}{14(3-2x)^{3/2}}\right)-\frac{38491}{10(3-2x)}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\int \frac{69337-7645x}{\sqrt{3-2x}\left(2x^2+x+1\right)}dx-\frac{7645}{2\sqrt{3-2x}}\right)-\frac{28209}{14(3-2x)^{3/2}}\right)-\frac{38491}{10(3-2x)^{5/}}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{1197}$$

3.47. $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\int -\frac{7645(3-2x)+115739}{2\left((3-2x)^2-7(3-2x)+14\right)}d\sqrt{3-2x}-\frac{7645}{2\sqrt{3-2x}}\right)-\frac{28209}{14(3-2x)^{3/2}}\right)\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 27

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(-\frac{1}{2}\int \frac{7645(3-2x)+115739}{(3-2x)^2-7(3-2x)+14}d\sqrt{3-2x}-\frac{7645}{2\sqrt{3-2x}}\right)-\frac{28209}{14(3-2x)^{3/2}}\right)-\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 1483

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(-\frac{\int \frac{115739\sqrt{7+2\sqrt{14}}-\left(115739-7645\sqrt{14}\right)\sqrt{3-2x}}{-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3}d\sqrt{3-2x}}{2\sqrt{14\left(7+2\sqrt{14}\right)}}-\frac{\int \frac{\left(115739-7645\sqrt{14}\right)}{-2x+\sqrt{7+2}}}{2\sqrt{}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 1142

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{7+2\sqrt{14}}\left(115739+7645\sqrt{14}\right)\int \frac{1}{-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3}d\sqrt{3-}}{2\sqrt{14\left(7+\right.}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 25

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{7+2\sqrt{14}}\left(115739+7645\sqrt{14}\right)\int \frac{1}{-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3}d\sqrt{3-}}{2\sqrt{14\left(7+\right.}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 1083

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\left(115739-7645\sqrt{14}\right)\int \frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3}d\sqrt{3-2x}-\sqrt{7+}}{2\sqrt{14\left(7+\right.}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

3.47. $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

$\big\downarrow$ 217

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}(115739-7645\sqrt{14})\int\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3}d\sqrt{3-2x}+\sqrt{\frac{7+2\sqrt{}}{2\sqrt{14}}}}{2\sqrt{14\left(7+2\sqrt{14}\right)}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

$\big\downarrow$ 1103

$$\frac{1}{28}\left(\frac{1}{42}\left(\frac{15}{28}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(-\frac{\sqrt{\frac{7+2\sqrt{14}}{2\sqrt{14}-7}}(115739+7645\sqrt{14})\arctan\left(\frac{2\sqrt{3-2x}-\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)-\frac{1}{2}(115739}{2\sqrt{14\left(7+2\sqrt{1}\right.}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}$$

input
```
Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

output
```
x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + ((23 + 73*x)/(42*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)^3) + ((1387 + 3049*x)/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2
)^2) + (15*((3049 + 4377*x)/(14*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)) + (-26957
/(18*(3 - 2*x)^(9/2)) + (-92405/(98*(3 - 2*x)^(7/2)) + (-38491/(10*(3 - 2*
x)^(5/2)) + (-28209/(14*(3 - 2*x)^(3/2)) + (-7645/(2*Sqrt[3 - 2*x]) + (-1/
2*(Sqrt[(7 + 2*Sqrt[14])/(-7 + 2*Sqrt[14])]*(115739 + 7645*Sqrt[14])*ArcTa
n[(-Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]] - ((115
739 - 7645*Sqrt[14])*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x]
 - 2*x])/2)/Sqrt[14*(7 + 2*Sqrt[14])] - (Sqrt[(7 + 2*Sqrt[14])/(-7 + 2*Sqr
t[14])]*(115739 + 7645*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 -
 2*x])/Sqrt[-7 + 2*Sqrt[14]]] + ((115739 - 7645*Sqrt[14])*Log[3 + Sqrt[14]
 + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/2)/(2*Sqrt[14*(7 + 2*Sqrt[14
])]))/2)/14)/2)/14)/2)/14))/28)/42)/28
```

3.47. $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.47.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{5\left(\left(x-\frac{3}{2}\right)^{4}\left(x^{2}+\frac{1}{2}x+\frac{1}{2}\right)^{4}\left(\frac{\sqrt{7+2\sqrt{14}}\,\left(146319\sqrt{14}-569986\right)\left(\ln\left(3-2x+\sqrt{14}-\sqrt{3-2x}\,\sqrt{7+2\sqrt{14}}\right)-\ln\left(3-2x+\sqrt{14}+\sqrt{3-2x}\,\sqrt{7+\right.\right.}{2}\right.\right.}{}$
derivativedivides	$\frac{\frac{567651623\sqrt{3-2x}}{32}-\frac{6194606411(3-2x)^{\frac{3}{2}}}{192}+\frac{9801432515(3-2x)^{\frac{5}{2}}}{384}-\frac{8763772549(3-2x)^{\frac{7}{2}}}{768}+\frac{149630663(3-2x)^{\frac{9}{2}}}{48}-\frac{200063633(3-2x)^{\frac{11}{2}}}{384}}{6588344\left((3-2x)^{2}-7+14x\right)^{4}}$
default	$\frac{\frac{567651623\sqrt{3-2x}}{32}-\frac{6194606411(3-2x)^{\frac{3}{2}}}{192}+\frac{9801432515(3-2x)^{\frac{5}{2}}}{384}-\frac{8763772549(3-2x)^{\frac{7}{2}}}{768}+\frac{149630663(3-2x)^{\frac{9}{2}}}{48}-\frac{200063633(3-2x)^{\frac{11}{2}}}{384}}{6588344\left((3-2x)^{2}-7+14x\right)^{4}}$
trager	Expression too large to display
risch	$-\frac{88070400x^{12}-677249280x^{11}+1873554048x^{10}-2443779648x^{9}+2343370048x^{8}-3106712560x^{7}+2888625656x^{6}-1470}{2168506368(2x-3)^{4}\sqrt{3-2x}\,(2x^{2}+x+}$

input
```
int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/26353376/(3-2*x)^(9/2)*((x-3/2)^4*(x^2+1/2*x+1/2)^4*(1/2*(7+2*14^(1/2))
^(1/2)*(146319*14^(1/2)-569986)*(ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(
1/2))^(1/2))-ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)))*(-7+2*
14^(1/2))^(1/2)+(115739*14^(1/2)+107030)*(arctan((2*(3-2*x)^(1/2)-(7+2*14^
(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))+arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2)
)^(1/2))/(-7+2*14^(1/2))^(1/2))))*(3-2*x)^(1/2)+214060*(-7+2*14^(1/2))^(1/
2)*(1626349/76450*x^10+x^12-24405799/2001600*x^3+1627773523/88070400*x^4+3
61078207/11008800*x^6+67601077/44035200*x^2-53726593/11008800*x+36615157/1
376100*x^8-73537943/4403520*x^5+40289347/88070400-38833907/1100880*x^7-424
2673/152900*x^9-58789/7645*x^11))/(-7+2*14^(1/2))^(1/2)/(2*x^2+x+1)^4
```

3.47.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.36

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{9\left(512\,x^{13}-2816\,x^{12}+5632\,x^{11}-5888\,x^{10}+6848\,x^{9}-8992\,x^{8}+6112\right.}{}$$

input
```
integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

output
```
1/60718178304*(9*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9
- 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^
2 - 162*x - 243)*sqrt(314431794275*I*sqrt(7) - 3726162599425)*log(sqrt(314
431794275*I*sqrt(7) - 3726162599425)*(146319*I*sqrt(7) - 115739) + 4081506
61120*sqrt(-2*x + 3)) - 9*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 +
6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
- 1242*x^2 - 162*x - 243)*sqrt(314431794275*I*sqrt(7) - 3726162599425)*log
(sqrt(314431794275*I*sqrt(7) - 3726162599425)*(-146319*I*sqrt(7) + 115739)
 + 408150661120*sqrt(-2*x + 3)) - 9*(512*x^13 - 2816*x^12 + 5632*x^11 - 58
88*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4
+ 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(-314431794275*I*sqrt(7) - 3726162
599425)*log((146319*I*sqrt(7) + 115739)*sqrt(-314431794275*I*sqrt(7) - 372
6162599425) + 408150661120*sqrt(-2*x + 3)) + 9*(512*x^13 - 2816*x^12 + 563
2*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5
- 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(-314431794275*I*sqrt(7
) - 3726162599425)*log((-146319*I*sqrt(7) - 115739)*sqrt(-314431794275*I*s
qrt(7) - 3726162599425) + 408150661120*sqrt(-2*x + 3)) + 28*(88070400*x^12
 - 677249280*x^11 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^8 - 31
06712560*x^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 10738551
56*x^3 + 135202154*x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*x...
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
```

output
```
Timed out
```

3.47.7 Maxima [F]

$$\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx = \int \frac{1}{(2x^2+x+1)^5(-2x+3)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(298) = 596$.

Time = 0.62 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.95

$$\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

output
```
-5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sq
rt(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) +
 53515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt
(2*sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14)
 + 8) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1
/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) +
1/2)) - 5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqr
t(-2*sqrt(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14)
 + 8) + 53515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/
4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*s
qrt(14) + 8) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4
)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqr
t(14) + 1/2)) - 5/3022415986688*sqrt(7)*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt
(14) + 8)*(sqrt(14) + 4) + 22935*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sq
rt(14) - 4) - 160545*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) - 53515
*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 925912*14^(1/4)*sqrt(7)*s
qrt(2*sqrt(14) + 8) - 6481384*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)
*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 5/302
2415986688*sqrt(7)*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) +
 4) + 22935*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) - 1605...
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx =$$

$$-\frac{\frac{272\,x}{441}-\frac{164\,(2\,x-3)^2}{441}+\frac{1966\,(2\,x-3)^3}{3087}-\frac{9091\,(2\,x-3)^4}{3087}-\frac{32070727\,(2\,x-3)^5}{5531904}-\frac{41014777\,(2\,x-3)^6}{11063808}-\frac{141921511\,(2\,x-3)^7}{154893312}+\frac{2326265}{3097}}{38416\,(3-2\,x)^{9/2}-76832\,(3-2\,x)^{11/2}+68600\,(3-2\,x)^{13/2}-35672\,(3-2\,x)^{15/2}+11809\,(3-2\,x)^{17/}}$$

$$+\frac{\mathrm{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977+\sqrt{7}\,12577271771\mathrm{i}}\,1572158971375\mathrm{i}}{39166305625367605393385O624\left(-\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{19583152812683802696692531 2}\right)}-\frac{1572158971375\,\sqrt{7}\,\sqrt{3-}}{391663056253676053933850624\left(-\frac{}{279}\right.}\right.}{3373232128}$$

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977-\sqrt{7}\,12577271771\mathrm{i}}\,1572158971375\mathrm{i}}{391663056253676053933850624\left(\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{195831528126838026966925312}\right)}+\frac{1572158971375\,\sqrt{7}\,\sqrt{3-2}}{391663056253676053933850624\left(\frac{2}{27975}\right.}\right.}{3373232128}$$

input
```
int(1/((3 - 2*x)^(11/2)*(x + 2*x^2 + 1)^5),x)
```

output
```
(atan(((3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*1572158
971375i)/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/19583
1528126838026966925312 - 230036728532618625/27975932589548289566703616)) -
 (1572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 1490465039
77)^(1/2))/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/195
831528126838026966925312 - 230036728532618625/27975932589548289566703616))
)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128 - ((272*x)/44
1 - (164*(2*x - 3)^2)/441 + (1966*(2*x - 3)^3)/3087 - (9091*(2*x - 3)^4)/3
087 - (32070727*(2*x - 3)^5)/5531904 - (41014777*(2*x - 3)^6)/11063808 - (
141921511*(2*x - 3)^7)/154893312 + (23262655*(2*x - 3)^8)/309786624 + (157
1659*(2*x - 3)^9)/15059072 + (468427*(2*x - 3)^10)/17210368 + (394105*(2*x
 - 3)^11)/120472576 + (38225*(2*x - 3)^12)/240945152 - 520/441)/(38416*(3
- 2*x)^(9/2) - 76832*(3 - 2*x)^(11/2) + 68600*(3 - 2*x)^(13/2) - 35672*(3
- 2*x)^(15/2) + 11809*(3 - 2*x)^(17/2) - 2548*(3 - 2*x)^(19/2) + 350*(3 -
2*x)^(21/2) - 28*(3 - 2*x)^(23/2) + (3 - 2*x)^(25/2)) - (atan(((3 - 2*x)^(
1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2)*1572158971375i)/(391663
056253676053933850624*((7^(1/2)*181960107187971125i)/195831528126838026966
925312 + 230036728532618625/27975932589548289566703616)) + (1572158971375*
7^(1/2)*(3 - 2*x)^(1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2))/(39
1663056253676053933850624*((7^(1/2)*181960107187971125i)/19583152812683...
```

3.48 $\int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 648

$$\int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx = \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{3029508823715}{1555033025150976(3-2x)^{15/2}} - \frac{13515743021825}{13476952884641792(3-2x)^{13/2}} - \frac{5846828446875}{14513641568075776(3-2x)^{11/2}} - \frac{37283626871975}{261245548225363968(3-2x)^{9/2}} - \frac{132355162272575}{2844673747342852096(3-2x)^{7/2}} - \frac{11557581705725}{812763927812243456(3-2x)^{5/2}} - \frac{46601678385075}{11378694989371408384(3-2x)^{3/2}} - \frac{24229218097975}{22757389978742816768\sqrt{3-2x}} + \frac{x}{63(3-2x)^{19/2}(1+x+2x^2)^9} + \frac{53+173x}{7056(3-2x)^{19/2}(1+x+2x^2)^8} + \frac{8477+21409x}{691488(3-2x)^{19/2}(1+x+2x^2)^7} + \frac{5(21409+47471x)}{6453888(3-2x)^{19/2}(1+x+2x^2)^6} + \frac{41(47471+92875x)}{90354432(3-2x)^{19/2}(1+x+2x^2)^5} + \frac{41(3436375+5677637x)}{5059848192(3-2x)^{19/2}(1+x+2x^2)^4} + \frac{451(811091+998691x)}{10119696384(3-2x)^{19/2}(1+x+2x^2)^3} + \frac{451(28962039+14627273x)}{283351498752(3-2x)^{19/2}(1+x+2x^2)^2} + \frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}(1+x+2x^2)} + \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{318603459702399434752} - \frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\arctan\left(\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right)}{318603459702399434752} + \frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\log\left(3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{637206919404798869504} - \frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\log\left(3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right)}{637206919404798869504}$$

output
```
4718120139975/351733660450816/(3-2*x)^(19/2)-815900548375/629418129227776/
(3-2*x)^(17/2)-3029508823715/1555033025150976/(3-2*x)^(15/2)-1351574302182
5/13476952884641792/(3-2*x)^(13/2)-5846828446875/14513641568075776/(3-2*x)
^(11/2)-37283626871975/261245548225363968/(3-2*x)^(9/2)-132355162272575/28
44673747342852096/(3-2*x)^(7/2)-11557581705725/812763927812243456/(3-2*x)^
(5/2)-46601678385075/11378694989371408384/(3-2*x)^(3/2)+1/63*x/(3-2*x)^(19
/2)/(2*x^2+x+1)^9+1/7056*(53+173*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^8+1/691488*
(8477+21409*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^7+5/6453888*(21409+47471*x)/(3-2
*x)^(19/2)/(2*x^2+x+1)^6+41/90354432*(47471+92875*x)/(3-2*x)^(19/2)/(2*x^2
+x+1)^5+41/5059848192*(3436375+5677637*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^4+451
/10119696384*(811091+998691*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^3+451/2833514987
52*(28962039+14627273*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^2+11275/3966920982528*
(14627273-35058731*x)/(3-2*x)^(19/2)/(2*x^2+x+1)-24229218097975/2275738997
8742816768/(3-2*x)^(1/2)+11275/127441383880959773 9008*ln(3-2*x+14^(1/2)-(3
-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*
14^(1/2))^(1/2)-11275/127441383880959773 9008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/
2)*(7+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))
^(1/2)+11275/63720691940479886 9504*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(
1/2))^(1/2)-11275/63720691940479886 9504*arctan((2*(3-2*x)^(1/2)+(7+2*14...
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.41 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.94

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\frac{20776+67816x}{1568(3-2x)^{19/2}(1+x+2x^2)^8} + \frac{\frac{46521776+117492592x}{1372(3-2x)^{19/2}\left(1+x+2x^2\right)^7} + \frac{\frac{74020332960+164128134240x}{1176(3-2x)^{19/2}\left(1+x+2x^2\right)^6} + \frac{\frac{94209549053760+184316990760000x}{980(3-2x)^{19/2}\left(1+x+2x^2\right)^5} + \frac{1}{980}\left(\frac{9547620121368\cdots}{784(3-\cdots}\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

output
```
x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + ((20776 + 67816*x)/(1568*(3 -
2*x)^(19/2)*(1 + x + 2*x^2)^8) + ((46521776 + 117492592*x)/(1372*(3 - 2*x)
^(19/2)*(1 + x + 2*x^2)^7) + ((74020332960 + 164128134240*x)/(1176*(3 - 2*
x)^(19/2)*(1 + x + 2*x^2)^6) + ((94209549053760 + 184316990760000*x)/(980*
(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^5) + ((9547620121368000 + 15774739736793
4080*x)/(784*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^4) + ((728792975839855449 60
+ 8973579855213300960*x)/(588*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^3) + ((364
327342121659983897 60 + 1840034637954157784832 0*x)/(392*(3 - 2*x)^(19/2)*(1
 + x + 2*x^2)^2) + ((6440121232839552246912000 - 1543571914665913655846400
0*x)/(196*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (3947992688254522195411 2000/
(19*(3 - 2*x)^(19/2)) + (-908021664138480966930240000/(17*(3 - 2*x)^(17/2)
) + (-1910552049302324858274620 1600/(3 - 2*x)^(15/2) + (-26849557435537239
465884310720000/(13*(3 - 2*x)^(13/2)) + (-1509944238585987965392741200000 0
0/(3 - 2*x)^(11/2) + (-8237718113587514139784976619840000/(3 - 2*x)^(9/2)
+ (-33838931203656046646004407284704 0000/(3 - 2*x)^(7/2) + (-1013530552857
651055083639451564896 0000/(3 - 2*x)^(5/2) + (-2043343757384956488128059567
91073600000/(3 - 2*x)^(3/2) + (-223099486651988979682856103640622880 0000/S
qrt[3 - 2*x] + ((Sqrt[(7 - I*Sqrt[7])/2]*(-312339281312784571555998545096 8
7203200000 - (71750597240923349846054347713013891200000*I)*Sqrt[7])*ArcTan
h[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +...
```

3.48.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-2x)^{21/2}\left(2x^2+x+1\right)^{10}}dx$$

$$\Big\downarrow \; 1165$$

$$\frac{\int \frac{28(60-53x)}{(3-2x)^{21/2}(2x^2+x+1)^9}dx}{1764}+\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{63}\int \frac{60-53x}{(3-2x)^{21/2}\left(2x^2+x+1\right)^9}dx+\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{63}\left(\frac{\int \frac{14(6466-8477x)}{(3-2x)^{21/2}(2x^2+x+1)^8}dx}{1568}+\frac{173x+53}{112(3-2x)^{19/2}\left(2x^2+x+1\right)^8}\right)+\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{63}\left(\frac{1}{112}\int \frac{6466-8477x}{(3-2x)^{21/2}\left(2x^2+x+1\right)^8}dx+\frac{173x+53}{112(3-2x)^{19/2}\left(2x^2+x+1\right)^8}\right)+\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 1235$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{\int \frac{630(13031-21409x)}{(3-2x)^{21/2}(2x^2+x+1)^7}dx}{1372}+\frac{21409x+8477}{98(3-2x)^{19/2}\left(2x^2+x+1\right)^7}\right)+\frac{173x+53}{112(3-2x)^{19/2}\left(2x^2+x+1\right)^8}\right)+\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\int \frac{13031-21409x}{(3-2x)^{21/2}\left(2x^2+x+1\right)^7}dx+\frac{21409x+8477}{98(3-2x)^{19/2}\left(2x^2+x+1\right)^7}\right)+\frac{173x+53}{112(3-2x)^{19/2}\left(2x^2+x+1\right.}\right.\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 1235$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{\int \frac{574(22702-47471x)}{(3-2x)^{21/2}(2x^2+x+1)^6}dx}{1176}+\frac{47471x+21409}{84(3-2x)^{19/2}\left(2x^2+x+1\right)^6}\right)+\frac{21409x+8477}{98(3-2x)^{19/2}\left(2x^2+x+1\right)^7}\right)+\right.\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\int \frac{22702-47471x}{(3-2x)^{21/2}\left(2x^2+x+1\right)^6}dx+\frac{47471x+21409}{84(3-2x)^{19/2}\left(2x^2+x+1\right)^6}\right)+\frac{21409x+8477}{98(3-2x)^{19/2}\left(2x^2+x\right.}\right.\right.\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 1235$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{980}\int \frac{14(1120631-3436375x)}{(3-2x)^{21/2}\left(2x^2+x+1\right)^5}dx+\frac{92875x+47471}{70(3-2x)^{19/2}\left(2x^2+x+1\right)^5}\right)+\frac{47471x+21}{84(3-2x)^{19/2}\left(2x^2\right.}\right.\right.\right.\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \ 27$$

3.48. $\int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\int\frac{1120631-3436375x}{(3-2x)^{21/2}\left(2x^2+x+1\right)^5}dx+\frac{92875x+47471}{70(3-2x)^{19/2}\left(2x^2+x+1\right)^5}\right)+\frac{47471x+214}{84(3-2x)^{19/2}\left(2x^2\right.}\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 1235

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{1}{784}\int\frac{3234(93800-811091x)}{(3-2x)^{21/2}\left(2x^2+x+1\right)^4}dx+\frac{5677637x+3436375}{56(3-2x)^{19/2}\left(2x^2+x+1\right)^4}\right)+\frac{92875x}{70(3-2x)^{19/2}}\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 27

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\int\frac{93800-811091x}{(3-2x)^{21/2}\left(2x^2+x+1\right)^4}dx+\frac{5677637x+3436375}{56(3-2x)^{19/2}\left(2x^2+x+1\right)^4}\right)+\frac{92875x}{70(3-2x)^{19/2}}\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 1235

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{588}\int-\frac{14(28962039x+7167383)}{(3-2x)^{21/2}\left(2x^2+x+1\right)^3}dx+\frac{998691x+811091}{42(3-2x)^{19/2}\left(2x^2+x+1\right)^3}\right)+\frac{567}{56(3-}\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 27

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{998691x+811091}{42(3-2x)^{19/2}\left(2x^2+x+1\right)^3}-\frac{1}{42}\int\frac{28962039x+7167383}{(3-2x)^{21/2}\left(2x^2+x+1\right)^3}dx\right)+\frac{567763}{56(3-2x)}\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 1235

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{1}{392}\int\frac{350(14627273x+24843002)}{(3-2x)^{21/2}\left(2x^2+x+1\right)^2}dx\right)+\frac{}{42(}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 27

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\int\frac{14627273x+24843002}{(3-2x)^{21/2}\left(2x^2+x+1\right)^2}dx\right)+\frac{9}{42(3-}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

3.48. $\int\frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{1}{196}\int \frac{42(158887401-245411117x)}{(3-2x)^{21/2}\left(2x^2+x+1\right)}dx-\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\int \frac{158887401-245411117x}{(3-2x)^{21/2}\left(2x^2+x+1\right)}dx-\frac{1}{14(3}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{1198}$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{28}\int \frac{2(880960721-418458549x)}{(3-2x)^{19/2}\left(2x^2+x+1\right)}dx\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\int \frac{880960721-418458549x}{(3-2x)^{19/2}\left(2x^2+x+1\right)}dx-\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{1198}$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{28}\int \frac{14(72363685x+563185919}{(3-2x)^{17/2}\left(2x^2+x+1\right)}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\int \frac{72363685x+563185919}{(3-2x)^{17/2}\left(2x^2+x+1\right)}dx\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{1198}$$

3.48. $\int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{28}\int\frac{2(1343462893x+21803}{(3-2x)^{15/2}\left(2x^2+x\right.}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\downarrow 27$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\int\frac{1343462893x+2180379}{(3-2x)^{15/2}\left(2x^2+x+\right.}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\downarrow 1198$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{28}\int\frac{14(1198735523x+}{(3-2x)^{13/2}\left(2x^2\right.}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\downarrow 27$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\int\frac{1198735523x+1054}{(3-2x)^{13/2}\left(2x^2+\right.}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\downarrow 1198$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{28}\int\frac{2(5704222875x}{(3-2x)^{11/2}(}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\downarrow 27$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\int\frac{5704222875x+}{(3-2x)^{11/2}(2x}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\downarrow 1198$$

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{28}\int\frac{14(3306751}{(3-2x)}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

3.48. $\int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx$

$\downarrow$ 27

$$\frac{1}{63}\left(\frac{1}{112}\left(\frac{45}{98}\left(\frac{41}{84}\left(\frac{1}{70}\left(\frac{33}{8}\left(\frac{1}{42}\left(\frac{14627273x+28962039}{28(3-2x)^{19/2}\left(2x^2+x+1\right)^2}-\frac{25}{28}\left(\frac{3}{14}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{2}\int\frac{3306751829x}{(3-2x)^{9/2}}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}$$

input
```
Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

output
```
$Aborted
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

3.48.4 Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$\frac{11275\left(x-\frac{3}{2}\right)^{9}\left(\sqrt{7+2\sqrt{14}}\left(18352320711\sqrt{14}-69111417106\right)\left(\ln\left(3-2x+\sqrt{14}-\sqrt{3-2x}\,\sqrt{7+2\sqrt{14}}\right)-\ln\left(3-2x+\sqrt{14}+\sqrt{3-2x}\,\sqrt{7+2\sqrt{14}\right.\right.\right.\right.}{48615}$
derivativedivides	$\frac{93649997064789087411 37(3-2x)^{\frac{5}{2}}}{2048}-\frac{238519057729032790543 47(3-2x)^{\frac{7}{2}}}{4096}+\frac{1929836137953835410413 17(3-2x)^{\frac{9}{2}}}{36864}-\frac{5775842147534844}{163}$
default	$\frac{93649997064789087411 37(3-2x)^{\frac{5}{2}}}{2048}-\frac{238519057729032790543 47(3-2x)^{\frac{7}{2}}}{4096}+\frac{1929836137953835410413 17(3-2x)^{\frac{9}{2}}}{36864}-\frac{5775842147534844}{163}$
trager	Expression too large to display
risch	$-\frac{2400312049377144274944 00x^{27}-2621948941596237063782400x^{26}+12365045055896811105484800x^{25}-3396989006}{}$

input
```
int(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

output
```
11275/4861502986181632*((x-3/2)^9*((7+2*14^(1/2))^(1/2)*(18352320711*14^(1
/2)-69111417106)*(ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))-ln
(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)))*(-7+2*14^(1/2))^(1/2)
+2*(9756589235*14^(1/2)+30085060166)*(arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2
))^(1/2))/(-7+2*14^(1/2))^(1/2))+arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1
/2))/(-7+2*14^(1/2))^(1/2))))*(x^2+1/2*x+1/2)^9*(3-2*x)^(1/2)+120340240664
*(-7+2*14^(1/2))^(1/2)*(-9651082208977600419673/266701338819682697216*x^6+
22855828001615591421921/26670133881968269721600*x+18366515291389111269346
3/120015602468857213747200*x^5-134998393682507368342493/400052008229524045
82400*x^4+996043194154916251217/175461407118212300800*x^3+8479300658909318
16713/1818418219225109299200*x^2-465471892878599/515743888560*x^20-3801437
0445393391293/31762259291597120*x^18-299208867441559564523/254098074332776
960*x^16-20254438577741909746663/81311383786488627200*x^10-112900187480730
3405453/2032784594662215680*x^12-10928359993529274103333/11434413344974963
200*x^14+193096157908388472533/152458844599666176*x^17+271924352600651293/
257184285761920*x^19+3324327068969447/4916758404272*x^21-63047885074067/14
1829569354*x^22+21065437682057/77361583284*x^23-1216492052933/8595731476*x
^24+112774755927521146576673/650491070291909017600*x^9+1219889689554254358
5363/28698135454054809600*x^11+21932125545555763373243/3049176891993323520
0*x^13+297725881275469254209863/6667533470492067430400*x^7-221517107732...
```

3.48.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.55

$$\int \frac{1}{(3-2x)^{21/2}\,(1+x+2x^2)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

output
```
1/240806866912267540774256664*(37791*(524288*x^28 - 5505024*x^27 + 24772608
*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 -
 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 49658
1120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x
^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 191
05065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 73483
2*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(388244949319992410911898l875*I
*sqrt(7) - 291499433615861543069724120625)*log(sqrt(3882449493199924109118
981875*I*sqrt(7) - 291499433615861543069724120625)*(18352320711*I*sqrt(7)
- 9756589235) + 138279123449649741430784400*sqrt(-2*x + 3)) - 37791*(524288
*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 19
4052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 51559424
0*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15
 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 836
21482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 22
35114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt
(3882449493199924109118981875*I*sqrt(7) - 291499433615861543069724120625)*
log(sqrt(3882449493199924109118981875*I*sqrt(7) - 291499433615861543069724
120625)*(-18352320711*I*sqrt(7) + 9756589235) + 138279123449649741430784400
*sqrt(-2*x + 3)) - 37791*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - ...
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Timed out}$$

input
```
integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

output
```
Timed out
```

3.48.7 Maxima [F]

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \int \frac{1}{\left(2\,x^2+x+1\right)^{10}(-2\,x+3)^{\frac{21}{2}}}\,dx$$

input
```
integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

3.48.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(491) = 982.

Time = 0.76 (sec) , antiderivative size = 1000, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

output
```
-11275/1427343499466749467688 96*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt
(14) + 4)*sqrt(-2*sqrt(14) + 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) -
4)*sqrt(-2*sqrt(14) + 8) - 15042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt
(14) + 4) - 45127590249*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 780
52713880*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sq
rt(2*sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14)
 + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 11275/1427343499466
74946768896*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sq
rt(14) + 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14)
+ 8) - 15042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 45127590
249*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 78052713880*14^(1/4)*sq
rt(7)*sqrt(-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*sqrt(14) + 8))*
arctan(-1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x
 + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 11275/285468699893349893537792*sqrt(7)
*(2148932869*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 644679
8607*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 45127590249*14
^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqrt(1
4) - 4)*sqrt(-2*sqrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt(1
4) + 8) + 546368997160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1
/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 11275/285...
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.88

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \text{Too large to display}$$

input
```
int(1/((3 - 2*x)^(21/2)*(x + 2*x^2 + 1)^10),x)
```

output
```
((184192*(2*x - 3)^2)/47481 - (18944*x)/2261 - (15552*(2*x - 3)^3)/4199 +
(5666272*(2*x - 3)^4)/1440257 - (63490768*(2*x - 3)^5)/12962313 + (5334956
72*(2*x - 3)^6)/70572593 - (1111521492*(2*x - 3)^7)/70572593 + (7800732315
8*(2*x - 3)^8)/1482024453 - (250239440467*(2*x - 3)^9)/494008151 + (111869
3654785651073*(2*x - 3)^10)/453254454575104 + (1624300450152249301*(2*x -
3)^11)/97125954551808 + (3504865352067494889 7*(2*x - 3)^12)/90650890915020
8 + (9552751196743757791 5*(2*x - 3)^13)/1813017818300416 + (56406629997314
15610547*(2*x - 3)^14)/114220122552926208 + (1737142288764447500149*(2*x -
 3)^15)/50764498912411648 + (1297121066722909760105 5*(2*x - 3)^16)/7107029
84773763072 + (327234412069467956652 35*(2*x - 3)^17)/4264217908642578432 +
 (1026457970347777106813 25*(2*x - 3)^18)/3979936714733073203 2 + (146093178
743020066531 5*(2*x - 3)^19)/2094703534070038528 + (687618468821894139745*(
2*x - 3)^20)/4528256169239642112 + (399689956766038477 25*(2*x - 3)^21)/150
9418723079880704 + (594013294361384987 5*(2*x - 3)^22)/1625527855624486912
+ (571797850362001037 5*(2*x - 3)^23)/14629750700620382208 + (1780569958183
25525*(2*x - 3)^24)/5689347494685704192 + (179665281323275*(2*x - 3)^25)/1
01595490976530432 + (1433237383402275*(2*x - 3)^26)/22757389978742816768 +
 (24229218097975*(2*x - 3)^27)/22757389978742816768 + 37120/2261)/(2066104
6784*(3 - 2*x)^(19/2) - 92974710528*(3 - 2*x)^(21/2) + 199231522560*(3 - 2
*x)^(23/2) - 270069397248*(3 - 2*x)^(25/2) + 259475340096*(3 - 2*x)^(27...
```

3.49 $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 1058

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \text{Too large to display}$$

output
```
1115516822297077423237689l145/16851663325326165602473542240l7408/(3-2*x)^(
23/2)+14011818498091020272474956375/10110997995195699361484125344104448/(3
-2*x)^(21/2)-13056959628363355534285785425/106924014357253562723941220352/
(3-2*x)^(39/2)-304688229262620222736480811/537361713180043545997243056128/
(3-2*x)^(35/2)+2124315846756567455653862925/168885109856585114456276389068
8/(3-2*x)^(33/2)+47657515074514118796095929535/666328524343253997036581389
59872/(3-2*x)^(31/2)+34911619993974714062172751985/12466791745777010267136
0389021696/(3-2*x)^(29/2)+149066309808794760843017404825/16249818206564516
83095663001731072/(3-2*x)^(27/2)+15848613964169066543734380171/60184511876
1648771516912222863360/(3-2*x)^(25/2)-3948194343291401740321996415/2028814
63139404195937734623232/(3-2*x)^(37/2)-22724090823469905152713519545/16042
78348571050965355481221264572416/(3-2*x)^(17/2)-10119027441277961867857327
5245/3963511214116714149701777134888943616/(3-2*x)^(15/2)-4605031904169582
83087439337135/34350430522344855964082068502370844672/(3-2*x)^(13/2)-22116
19588790911794826342607495/406920484649315986036049119181931544576/(3-2*x)
^(11/2)+173441368149804378661935869705/89650848890735201005159244717726105
6/(3-2*x)^(19/2)-927027754781476746208047620505/58004665448193406009502274
443388060172288/(3-2*x)^(1/2)-405965372440630510720926890227/2071595194578
335928910795515835287863296/(3-2*x)^(5/2)-4986681479187781853417316522775/
87006998172290109014253411665082090258432/(3-2*x)^(3/2)-143401467550777...
```

3.49.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.33 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

output
```
x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 -
 2*x)^(39/2)*(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136
*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875
840*x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((9408813737133390720
+ 20924013532366815360*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2
7243065619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)
*(1 + x + 2*x^2)^13) + ((72110377354780278913835520 + 1452953429486831061б
4016640*x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((172901458108932B
96335179801600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^(39/2)*
(1 + x + 2*x^2)^11) + ((370557652515461812186329087129600 + 64580296723188
63068265404244480000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((6961
755986759734387590105775549440000 + 1088028437838790621809440473088716800*x
)/(1764*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((111196506347124401548924B1
63496668569600 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*
x)^(39/2)*(1 + x + 2*x^2)^8) + ((142763602303895852541818962327603916021 76
00 + 141022945428029359210858021724843234795 5200*x)/(1372*(3 - 2*x)^(39/2)
*(1 + x + 2*x^2)^7) + ((128330880339506716881880799769607343663923 2000 + 4
21439161286999121770135584246204836237312000*x)/(1176*(3 - 2*x)^(39/2)*(1
+ x + 2*x^2)^6) + ((35990904373909724999169578894625893014666 4448000 - ...
```

3.49.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{20}}dx$$

$$\Big\downarrow \ 1165$$

$$\frac{\int \frac{28(130-113x)}{(3-2x)^{41/2}(2x^2+x+1)^{19}}dx}{3724}+\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{133}\int \frac{130-113x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{19}}dx+\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{133}\left(\frac{\int \frac{14(33336-40657x)}{(3-2x)^{41/2}(2x^2+x+1)^{18}}dx}{3528}+\frac{373x+113}{252(3-2x)^{39/2}\left(2x^2+x+1\right)^{18}}\right)+\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{133}\left(\frac{1}{252}\int \frac{33336-40657x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{18}}dx+\frac{373x+113}{252(3-2x)^{39/2}\left(2x^2+x+1\right)^{18}}\right)+\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{\int \frac{210(539991-751303x)}{(3-2x)^{41/2}(2x^2+x+1)^{17}}dx}{3332}+\frac{107329x+40657}{238(3-2x)^{39/2}\left(2x^2+x+1\right)^{17}}\right)+\frac{373x+113}{252(3-2x)^{39/2}\left(2x^2+x+1\right)^{18}}\right)+\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\int \frac{539991-751303x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{17}}dx+\frac{107329x+40657}{238(3-2x)^{39/2}\left(2x^2+x+1\right)^{17}}\right)+\frac{373x+113}{252(3-2x)^{39/2}\left(2x^2+\right.}\right.\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{\int \frac{14(122225856-184959785x)}{(3-2x)^{41/2}(2x^2+x+1)^{16}}dx}{3136}+\frac{1831285x+751303}{224(3-2x)^{39/2}\left(2x^2+x+1\right)^{16}}\right)+\frac{107329x+40657}{238(3-2x)^{39/2}\left(2x^2+x+\right.}\right.\right.\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

3.49. $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\int\frac{122225856-184959785x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{16}}dx+\frac{1831285x+751303}{224(3-2x)^{39/2}\left(2x^2+x+1\right)^{16}}\right)+\frac{107329x+}{238(3-2x)^{39/2}\,(2}\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{\int\frac{14(25488953979-41652915209x)}{(3-2x)^{41/2}(2x^2+x+1)^{15}}dx}{2940}+\frac{429411497x+184959785}{210(3-2x)^{39/2}\left(2x^2+x+1\right)^{15}}\right)+\frac{1831285x+}{224(3-2x)^{39/2}\,(2}\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 27

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\int\frac{25488953979-41652915209x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{15}}dx+\frac{429411497x+184959785}{210(3-2x)^{39/2}\left(2x^2+x+1\right)^{15}}\right)+\frac{1831}{224(3-2x}\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{\int\frac{42(1614300418670-2871555518177x)}{(3-2x)^{41/2}(2x^2+x+1)^{14}}dx}{2744}+\frac{92630823167x+41652915209}{196(3-2x)^{39/2}\left(2x^2+x+1\right)^{14}}\right)+\frac{4294}{210(3-}\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 27

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\int\frac{1614300418670-2871555518177x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{14}}dx+\frac{92630823167x+41652915209}{196(3-2x)^{39/2}\left(2x^2+x+1\right)^{14}}\right)+\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{\int\frac{98(39357458161627-77559130805859x)}{(3-2x)^{41/2}(2x^2+x+1)^{13}}dx}{2548}+\frac{6100156355517x+2871555518177}{182(3-2x)^{39/2}\left(2x^2+x+1\right)^{13}}\right)\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 27

3.49. $\int\frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\int \frac{39357458161627-77559130805859x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{13}}dx+\frac{6100156355517x+287155551}{182(3-2x)^{39/2}\left(2x^2+x+1\right.}\right.\right.\right.\right.\right.\right. \frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{\int \frac{70(1182110687469684-2656658801194921x)}{(3-2x)^{41/2}(2x^2+x+1)^{12}}dx}{2352}+\frac{156274047129113x+775591}{168(3-2x)^{39/2}\left(2x^2+x\right.}\right.\right.\right.\right.\right.\right.\right. \frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\int \frac{1182110687469684-2656658801194921x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{12}}dx+\frac{156274047129113x}{168(3-2x)^{39/2}}\right.\right.\right.\right.\right.\right.\right.\right. \frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{\int \frac{126(16782494726084327-45187921585208601x)}{(3-2x)^{41/2}(2x^2+x+1)^{11}}dx}{2156}+\frac{5020880176134289x}{154(3-2x)^{39/}}\right.\right.\right.\right.\right.\right.\right.\right.\right. \frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\int \frac{16782494726084327-45187921585208601x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{11}}dx+\frac{502088017}{154(3}\right.\right.\right.\right.\right.\right.\right.\right.\right. \frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{\int \frac{14(170659923427279660 6-6063974149878048635x)}{(3-2x)^{41/2}(2x^2+x+1)^{10}}dx}{1960}+\frac{78752911}{140(}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right. \frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$$\Big\downarrow \; 27$$

3.49. $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\int \frac{1706599234272796606-6063974149878048635x}{(3-2x)^{41/2}\left(2x^2+x+1\right)^{10}}dx+\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{\int \frac{14(113832295864564863195-691833601144925854831x)}{(3-2x)^{41/2}(2x^2+x+1)^9}dx}{1764}+\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\downarrow$ 27

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\int \frac{113832295864564863195-691833601144925854}{(3-2x)^{41/2}\left(2x^2+x+1\right)^9}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{\int -\frac{966(919498192874055581221x+5605528390794056352)}{(3-2x)^{41/2}(2x^2+x+1)^8}}{1568}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\downarrow$ 27

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+691833601144925854}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+6918336011449258}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\downarrow$ 27

3.49. $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+6918336011449258548}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+691833601144925}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 27

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+6918336011449258548}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 1235

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+6918336011449258548}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

$\big\downarrow$ 27

$$\frac{1}{133}\left(\frac{1}{252}\left(\frac{15}{238}\left(\frac{1}{224}\left(\frac{1}{210}\left(\frac{3}{196}\left(\frac{1}{26}\left(\frac{5}{168}\left(\frac{9}{154}\left(\frac{1}{140}\left(\frac{1}{126}\left(\frac{919498192874055581221x+6918336011449258548}{112(3-2x)^{39/2}\left(2x^2+x+1\right)^8}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{x}{133(3-2x)^{39/2}\left(2x^2+x+1\right)^{19}}$$

input
```
Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

output
```
$Aborted
```

3.49. $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

3.49.4 Maple [A] (verified)

Time = 7.41 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	Expression too large to display	502
trager	Expression too large to display	733
risch	Expression too large to display	761
derivativedivides	Expression too large to display	820
default	Expression too large to display	820

input
```
int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x,method=_RETURNVERBOSE)
```

output
```
115/5908552821163231304184823545856/(3-2*x)^(39/2)/(-7+2*14^(1/2))^(1/2)*(
(x-3/2)^19*(1/2*(7+2*14^(1/2))^(1/2)*(6254156255679246494096078420 9*14^(1/
2)-23404402840488330765587709126 2)*(ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*1
4^(1/2))^(1/2))-ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2)))*(-7
+2*14^(1/2))^(1/2)+(302971189122193607250286930 61*14^(1/2)+112855552756005
86475576231901 8)*(arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(
1/2))^(1/2))+arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))
^(1/2))))*(x^2+1/2*x+1/2)^19*(3-2*x)^(1/2)+2257111055120117295115246380 36*
(-7+2*14^(1/2))^(1/2)*(440996520277951008903098744562486494852026613994907
/4419113301684085775522691718105806971018139402240 0*x^6+351285717820454846
300261178016875702440838740531 71/451731581949928768164541820073038045926 29
8694451200*x+21057456255216559133478662993664670665427161768062 9/188221492
479136986735225758363765852469291122688 00*x^5+328358483183097302410675984 8
201028935360810239060 41/112932895487482192041135455018259511481574673612 80
0*x^4+2642283729075540788996525685897250893153780413276 5/180692632779971 50
72658167280292152183705194777804 8*x^3+1025886174418270548567999652588213 91
6487932789974 3/4106650744999352437859471091573073144784533585920 0*x^2-3396
92530351150840696302021012775910383197972648611749049/2202191462005902744 8
02141372856060473890706135449600*x^20-5228125068107468722151644334616418 35
72170004539439383783/92723851031827483991669110436044651532240258334720...
```

3.49.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 1905, normalized size of antiderivative = 1.80

$$\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
```

output
```
1/33686407791258635613529170249611497401907447855054848O*(207411935445*(54
9755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 63063863800
6272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376576
000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 188583312363
618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 77811789
6260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 19
19929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^
41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 363409846710252
3392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 34617982
12247617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 2
477632938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x
^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 79051102413508
9152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 2828450695
99813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 781163
67732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200
795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 229684
9711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182213447
974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302*
x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 375574
0313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 +...
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \text{Timed out}$$

input
```
integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

output
```
Timed out
```

3.49.7 Maxima [F]

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \int \frac{1}{\left(2\,x^2+x+1\right)^{20}(-2\,x+3)^{\frac{41}{2}}}\,dx$$

input
```
integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

3.49.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 1410, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="giac")
```

output
```
-115/363805261691069042491598265308929913400590336*sqrt(7)*(24183332733429
828161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 80
61110911143276053983022787*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14
) + 8) - 5642777637800293237788115950 9*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt
(14) + 4) - 16928332913400879713364347852 7*14^(3/4)*sqrt(2*sqrt(14) + 8)*(
sqrt(14) - 4) + 24237695129775488580022954448 8*14^(1/4)*sqrt(7)*sqrt(-2*sq
rt(14) + 8) - 16966386590842842006016068114 16*14^(1/4)*sqrt(2*sqrt(14) + 8
))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2
*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 115/363805261691069042491598265308092
991340059 0336*sqrt(7)*(24183332733429828161949068361*14^(3/4)*sqrt(7)*(sqr
t(14) + 4)*sqrt(-2*sqrt(14) + 8) + 8061110911143276053983022787*14^(3/4)*s
qrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) - 564277763780029323778811595 0
9*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 169283329134008797133643 4
78527*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 242376951297754885800
229544488*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 169663865908428420060 16
06811416*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)*sq
rt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2))
- 115/727610523382138084983196530617859826801180672*sqrt(7)*(8061110911143
276053983022787*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 241
83332733429828161949068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(...
```

3.49.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 1017, normalized size of antiderivative = 0.96

$$\int \frac{1}{(3-2x)^{41/2} (1+x+2x^2)^{20}} \, dx = \text{Too large to display}$$

input
```
int(1/((3 - 2*x)^(41/2)*(x + 2*x^2 + 1)^20),x)
```

output
```
((64356352*(2*x - 3)^2)/38073 - (5767168*x)/1443 - (7517962240*(2*x - 3)^3
)/5444439 + (1357449428992*(2*x - 3)^4)/1181443263 - (34130408095744*(2*x
- 3)^5)/34261854627 + (1965832636456960*(2*x - 3)^6)/2158496841501 - (9552
588571922432*(2*x - 3)^7)/10792484207505 + (69571472879183872*(2*x - 3)^8)
/75547389452535 - (5204838729946112*(2*x - 3)^9)/5036492630169 + (32508205
2781755904*(2*x - 3)^10)/257635969158645 - (461538785202937088*(2*x - 3)^1
1)/272428464995505 + (17726678744562203264*(2*x - 3)^12)/6992330601551295
- (1432471149647610304*(2*x - 3)^13)/332968123883395 + (204346360124338870
4*(2*x - 3)^14)/241114848329355 - (969727684773439768l6*(2*x - 3)^15)/4840
844262612435 + (1083387067012254592765 6*(2*x - 3)^16)/181389282075536535 -
 (443401570498323057293 24*(2*x - 3)^17)/181389282075536535 + (691509778132
186261807282*(2*x - 3)^18)/423241658176251915 - (135773583315370822397034 0
7*(2*x - 3)^19)/423241658176251915 + (509495943858959939640753039465067261
4981*(2*x - 3)^20)/2035946169792430536236256 46080 + (474753402737241482257
498862608846325264 03*(2*x - 3)^21)/2035946169792430536236256 46080 + (54736
24067276673458681762307546007523414 99*(2*x - 3)^22)/518240843219891409 2237
74371840 + (1363217399168846741803250531443496167647559*(2*x - 3)^23)/4385
114827245235001124244 68480 + (40035704814224807138997531075224002020138815
9*(2*x - 3)^24)/598568173918974577653459399 47520 + (167803532186710618 7517
785121743165245085532 91*(2*x - 3)^25)/149642043479743644413364849 86880 ...
```

3.50 $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

3.50.1 Optimal result

Integrand size = 23, antiderivative size = 378

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{30316369-15043110x}{6860000000(3-2x+x^2)^{5/2}} - \frac{63043297-29625922x}{41160000000(3-2x+x^2)^{3/2}} - \frac{31(7434109-3088870x)}{411600000000\sqrt{3-2x+x^2}} - \frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1+x+2x^2)^3} + \frac{5485+8878x}{117600(3-2x+x^2)^{9/2}(1+x+2x^2)^2} + \frac{3(8822+8233x)}{343000(3-2x+x^2)^{9/2}(1+x+2x^2)} + \frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{5}{7\left(151363871237318045+110320475741093888\sqrt{2}\right)}}\left(308108\right.}{\sqrt{3-2x}}\right.}{137200000000} - \frac{\sqrt{\frac{1}{70}\left(-151363871237318045+110320475741093888\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{5}{7\left(-151363871237318045+110320475741093888\sqrt{2}\right)}}\left(3\right.}{\sqrt{3}}\right.}{137200000000}$$

output
```
1/123480000*(-3450497+2004270*x)/(x^2-2*x+3)^(9/2)+1/411600000*(-4878869+2
578034*x)/(x^2-2*x+3)^(7/2)+1/6860000000*(-30316369+15043110*x)/(x^2-2*x+3
)^(5/2)+1/41160000000*(-63043297+29625922*x)/(x^2-2*x+3)^(3/2)+1/280*(-1+1
0*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/1050*(28+67*x)/(x^2-2*x+3)^(9/2)/(2
*x^2+x+1)^3+1/117600*(5485+8878*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^2+3/34300
0*(8822+8233*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)-31/411600000000*(7434109-308
8870*x)/(x^2-2*x+3)^(1/2)-1/9604000000000*arctanh(1/7*(308108167+x*(932587
773-620347970*2^(1/2))-312239803*2^(1/2))*35^(1/2)/(-151363871237318045+11
0320475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-105954709866122263150+
77224333018765721 60*2^(1/2))^(1/2)+1/9604000000000*arctan(1/7*(308108167+3
12239803*2^(1/2)+x*(932587773+620347970*2^(1/2)))*35^(1/2)/(15136387123731
8045+110320475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(105954709866122
63150+77224333018765721 60*2^(1/2))^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.00 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.94

$$\int \frac{1}{(3-2x+x^2)^{11/2}\,(1+x+2x^2)^5}\,dx = \frac{-53205422447+261702502714x-266966654968x^2+1002897791524x^3-1409335257371x^4+\cdots}{\cdots}$$

input
```
Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

output
```
((-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3 - 1
409335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952*x
^7 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 39996561
32532*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 +
 188603773872*x^15 - 38639385552*x^16 + 4596238560*x^17)/((3 - 2*x + x^2)^
(9/2)*(1 + x + 2*x^2)^4) - 49392*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4
& , (-6014*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 10727*Log[-x + Sqrt[3 - 2*
x + x^2] - #1]*#1 + 3229*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*
#1 - 3*#1^2 + 4*#1^3) & ] - 56448*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4
 & , (73781*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 60407*Log[-x + Sqrt[3 - 2
*x + x^2] - #1]*#1 + 13104*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 1
0*#1 - 3*#1^2 + 4*#1^3) & ] - 504*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4
 & , (275935046*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 208696097*Log[-x + Sq
rt[3 - 2*x + x^2] - #1]*#1 + 50007219*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#
1^2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ] + 1440*RootSum[14 + 7*#1 - 5*#1^2 -
 #1^3 + #1^4 & , (3276009822*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 24478316
21*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1 + 590084719*Log[-x + Sqrt[3 - 2*x
 + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ] - 18*RootSum[14 + 7*
#1 - 5*#1^2 - #1^3 + #1^4 & , (254137663854*Log[-x + Sqrt[3 - 2*x + x^2] -
 #1] - 189631531133*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1 + 45801521671...
```

3.50.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.12, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {1305, 27, 2135, 27, 2135, 27, 2135, 27, 2135, 27, 2135, 27, 2135, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x+3)^{11/2}(2x^2+x+1)^5}dx$$

$$\Big\downarrow \text{1305}$$

$$-\frac{\int -\frac{5(160x^2-267x+247)}{(x^2-2x+3)^{11/2}(2x^2+x+1)^4}dx}{1400}-\frac{1-10x}{280(x^2-2x+3)^{9/2}(2x^2+x+1)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\int \frac{160x^2-267x+247}{(x^2-2x+3)^{11/2}(2x^2+x+1)^4}dx-\frac{1-10x}{280(x^2-2x+3)^{9/2}(2x^2+x+1)^4}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{280}\left(\frac{\int \frac{70(3752x^2-3814x+2901)}{(x^2-2x+3)^{11/2}(2x^2+x+1)^3}dx}{1050}+\frac{4(67x+28)}{15\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^3}\right)-\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{1}{15}\int \frac{3752x^2-3814x+2901}{\left(x^2-2x+3\right)^{11/2}\left(2x^2+x+1\right)^3}dx+\frac{4(67x+28)}{15\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^3}\right)-\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{1}{700}\int \frac{75\left(35512x^2-23581x+12713\right)}{\left(x^2-2x+3\right)^{11/2}\left(2x^2+x+1\right)^2}dx+\frac{8878x+5485}{28\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^2}\right)+\frac{4(67x}{15\left(x^2-2x+3\right)^{9/}}\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\int \frac{35512x^2-23581x+12713}{\left(x^2-2x+3\right)^{11/2}\left(2x^2+x+1\right)^2}dx+\frac{8878x+5485}{28\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^2}\right)+\frac{4(67x+}{15\left(x^2-2x+3\right)^{9/2}}\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{350}\int -\frac{10\left(-987960x^2-28350x+486617\right)}{\left(x^2-2x+3\right)^{11/2}\left(2x^2+x+1\right)}dx+\frac{12(8233x+8822)}{35\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}\right)+\frac{}{28\left(x^2-\right.}\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{12(8233x+8822)}{35\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}-\frac{1}{35}\int \frac{-987960x^2-28350x+486617}{\left(x^2-2x+3\right)^{11/2}\left(2x^2+x+1\right)}dx\right)+\frac{8878x}{28\left(x^2-2x+3\right)}\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{2135}$$

3.50. $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(-\frac{\int \frac{60\left(-10689440x^2+3332642x+2083763\right)}{(x^2-2x+3)^{9/2}(2x^2+x+1)}dx}{1800}-\frac{3450497-2004270x}{90\left(x^2-2x+3\right)^{9/2}}\right)+\frac{12(8233x+8822)}{35\left(x^2-2x+3\right)^{9/2}\left(2x^2\right.}\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 27

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(-\frac{1}{30}\int\frac{-10689440x^2+3332642x+2083763}{\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}dx-\frac{3450497-2004270x}{90\left(x^2-2x+3\right)^{9/2}}\right)+\frac{12(8233x}{35\left(x^2-2x+3\right)}\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 2135

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(-\frac{\int -\frac{420\left(10312136x^2-5581162x+620039\right)}{(x^2-2x+3)^{7/2}(2x^2+x+1)}dx}{1400}-\frac{4878869-2578034x}{10\left(x^2-2x+3\right)^{7/2}}\right)-\frac{3450497-2004270x}{90\left(x^2-2x+3\right)^{9/2}}\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 27

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\int\frac{10312136x^2-5581162x+620039}{\left(x^2-2x+3\right)^{7/2}\left(2x^2+x+1\right)}dx-\frac{4878869-2578034x}{10\left(x^2-2x+3\right)^{7/2}}\right)-\frac{3450497-20042}{90\left(x^2-2x+3\right.}\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 2135

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{\int \frac{100\left(24068976x^2-18512030x+9163631\right)}{(x^2-2x+3)^{5/2}(2x^2+x+1)}dx}{1000}-\frac{30316369-15043110x}{50\left(x^2-2x+3\right)^{5/2}}\right)-\frac{4878869-257}{10\left(x^2-2x+\right.}\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 27

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\int\frac{24068976x^2-18512030x+9163631}{\left(x^2-2x+3\right)^{5/2}\left(2x^2+x+1\right)}dx-\frac{30316369-15043110x}{50\left(x^2-2x+3\right)^{5/2}}\right)-\frac{487886}{10\left(x^2\right.}\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

3.50. $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{600}\int \frac{20\left(118503688x^2-141252406x+125053685\right)}{\left(x^2-2x+3\right)^{3/2}\left(2x^2+x+1\right)}dx-\frac{63043297-2962592}{30\left(x^2-2x+3\right)^{3/}}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\int \frac{118503688x^2-141252406x+125053685}{\left(x^2-2x+3\right)^{3/2}\left(2x^2+x+1\right)}dx-\frac{63043297-29625922x}{30\left(x^2-2x+3\right)^{3/2}}\right)\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{1}{200}\int \frac{60(132636591-89801606x)}{\sqrt{x^2-2x+3}\left(2x^2+x+1\right)}dx-\frac{31(7434109-3088870x)}{10\sqrt{x^2-2x+3}}\right)-\frac{630}{3}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{3}{10}\int \frac{132636591-89801606x}{\sqrt{x^2-2x+3}\left(2x^2+x+1\right)}dx-\frac{31(7434109-3088870x)}{10\sqrt{x^2-2x+3}}\right)-\frac{6304}{30}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{\int -\frac{5\left(-\left(\left(42834985-89801606\sqrt{2}\right)x\right)-132636591\sqrt{2}+222438197\right)}{\sqrt{x^2-2x+3}\left(2x^2+x+1\right)}dx}{10\sqrt{2}}-\frac{\int -\frac{5\left(-\right.}{}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$$\Big\downarrow \text{27}$$

3.50. $\displaystyle\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{\int \frac{-\left(\left(42834985+89801606\sqrt{2}\right)x\right)+132636591\sqrt{2}+222438197}{\sqrt{x^2-2x+3}(2x^2+x+1)}dx}{2\sqrt{2}} - \frac{\int \frac{-\left(\left(4283498\right.\right.}{}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 1362

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{\left(151363871237318045-110320475741093888\sqrt{2}\right)\int \frac{}{-\frac{5\left(\left(932587773-620\right.\right.}{}}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 217

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{\left(151363871237318045-110320475741093888\sqrt{2}\right)\int \frac{}{-\frac{5\left(\left(932587773-620\right.\right.}{}}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

$\downarrow$ 219

$$\frac{1}{280}\left(\frac{1}{15}\left(\frac{3}{28}\left(\frac{1}{35}\left(\frac{1}{30}\left(\frac{3}{10}\left(\frac{1}{10}\left(\frac{1}{30}\left(\frac{3}{10}\left(\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\arctan\left(\frac{\sqrt{}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}$$

input
```
Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

3.50. $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

output
```
-1/280*(1 - 10*x)/((3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + ((4*(28 + 67
*x))/(15*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + ((5485 + 8878*x)/(28*(
3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*((12*(8822 + 8233*x))/(35*(3
- 2*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (-1/90*(3450497 - 2004270*x)/(3 - 2*
x + x^2)^(9/2) + (-1/10*(4878869 - 2578034*x)/(3 - 2*x + x^2)^(7/2) + (3*(
-1/50*(30316369 - 15043110*x)/(3 - 2*x + x^2)^(5/2) + (-1/30*(63043297 - 2
9625922*x)/(3 - 2*x + x^2)^(3/2) + ((-31*(7434109 - 3088870*x))/(10*Sqrt[3
 - 2*x + x^2]) + (3*(Sqrt[(151363871237318045 + 110320475741093888*Sqrt[2]
)/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093888*Sqrt[2]))
]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt[2])*x))/Sqr
t[3 - 2*x + x^2]] + ((151363871237318045 - 110320475741093888*Sqrt[2])*Arc
Tanh[(Sqrt[5/(7*(-151363871237318045 + 110320475741093888*Sqrt[2]))]*(3081
08167 - 312239803*Sqrt[2] + (932587773 - 620347970*Sqrt[2])*x))/Sqrt[3 - 2
*x + x^2]])/Sqrt[70*(-151363871237318045 + 110320475741093888*Sqrt[2])]))/
10)/30)/10))/10)/30)/35))/28)/15)/280
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.50. $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.50.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.20

method	result
risch	$\frac{4596238560x^{17}-38639385552x^{16}+188603773872x^{15}-606785954952x^{14}+1459208021718x^{13}-2679143870481x^{12}+3999656132532x}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/1234800000000*(4596238560*x^17-38639385552*x^16+188603773872*x^15-606785
954952*x^14+1459208021718*x^13-2679143870481*x^12+3999656132532*x^11-49157
97913008*x^10+5380603084494*x^9-5134334619701*x^8+4591320676952*x^7-335981
3871472*x^6+2503427226914*x^5-1409335257371*x^4+1002897791524*x^3-26696665
4968*x^2+261702502714*x-53205422447)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/268
912000000000*4^(1/2)*((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)*2^(1/2)*(96
25722625*(-6050+4280*2^(1/2))^(1/2)*arctan(1/49*(-6050+4280*2^(1/2))^(1/2)
/((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)*(40*2^(1/2)+57)*(2^(1/2)-1+x)/(
2^(1/2)+1-x))*(-350+280*2^(1/2))^(1/2)*2^(1/2)+13664181884*(-6050+4280*2^(
1/2))^(1/2)*arctan(1/49*(-6050+4280*2^(1/2))^(1/2)/((2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+1)^(1/2)*(40*2^(1/2)+57)*(2^(1/2)-1+x)/(2^(1/2)+1-x))*(-350+280*
2^(1/2))^(1/2)+456968008770*arctanh(7*((2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1)^
(1/2)/(-350+280*2^(1/2))^(1/2))*2^(1/2)-607941010600*arctanh(7*((2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2+1)^(1/2)/(-350+280*2^(1/2))^(1/2)))/(((2^(1/2)-1+x)^
2/(2^(1/2)+1-x)^2+1)/((2^(1/2)-1+x)/(2^(1/2)+1-x)+1)^2)^(1/2)/((2^(1/2)-1+
x)/(2^(1/2)+1-x)+1)/(-350+280*2^(1/2))^(1/2)
```

3.50.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.30

$$\int \frac{1}{(3-2x+x^2)^{11/2}\,(1+x+2x^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

output
```
1/86436000000000*(321736699200*x^18 - 2573893593600*x^17 + 12386862919200*
x^16 - 39091008952800*x^15 + 93484619661300*x^14 - 169997628439800*x^13 +
255076876834500*x^12 - 312647637447600*x^11 + 349667466399300*x^10 - 33175
0753962600*x^9 + 310817759970900*x^8 - 219424428854400*x^7 + 1848980593215
00*x^6 - 88356941017800*x^5 + 88618352085900*x^4 - 7962983305200*x^3 - 9*s
qrt(35)*(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13
 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^
7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqr
t(14293820940408247*I*sqrt(7) - 151363871237318045)*log(sqrt(35)*sqrt(1429
3820940408247*I*sqrt(7) - 151363871237318045)*(932587773*I*sqrt(7) + 29984
4895) - 3088973320750628864*x + 772243330187657216*I*sqrt(7) + 30889733207
50628864*sqrt(x^2 - 2*x + 3) - 772243330187657216) + 9*sqrt(35)*(16*x^18 -
 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15
548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 439
4*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(142938209404082
47*I*sqrt(7) - 151363871237318045)*log(sqrt(35)*sqrt(14293820940408247*I*sq
rt(7) - 151363871237318045)*(-932587773*I*sqrt(7) - 299844895) - 308897332
0750628864*x + 772243330187657216*I*sqrt(7) + 3088973320750628864*sqrt(x^2
 - 2*x + 3) - 772243330187657216) + 9*sqrt(35)*(16*x^18 - 128*x^17 + 616*x
^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 173...
```

3.50.6 Sympy [F]

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = \int \frac{1}{(x^2-2x+3)^{\frac{11}{2}}(2x^2+x+1)^5}\,dx$$

input
```
integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

output
```
Integral(1/((x**2 - 2*x + 3)**(11/2)*(2*x**2 + x + 1)**5), x)
```

3.50.7 Maxima [F]

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = \int \frac{1}{(2\,x^2+x+1)^5(x^2-2\,x+3)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

3.50.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 35.11 (sec) , antiderivative size = 2509, normalized size of antiderivative = 6.64

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

output
```
1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
log(3136*(247430153598830145135914226638091465128017779071251327216101236l
81293485559300330785024470114864584026604284622700*sqrt(7)*sqrt(2)*sqrt(77
22433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt
(2) - 151363871237318045)^2 + 144334256265984251329283298872220021324677O3
7791563274209392387772421199909591859624597607567004340682185832696575O*sq
rt(7)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 28866851253196
850265856659774440042649354075583126548418784775544842399819183719249l952
1513400868136437166539315OO*sqrt(2)*(110320475741093888*sqrt(2) - 15136387
1237318045)^3 + 206191794665691787613261855531742887606681482559376106O134
17696817744571299416942320853725095720486688836903852250*sqrt(772243330187
6572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513
63871237318045)^3 - 104913854112296962573522080729623041436733404265622592
321084093289259251415027575686933144355006438004151420024881229481000*sqrt
(7)*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 10491385
411229696257352208072962304143673340426562259232108409328925925141502757 56
869331443550064380041514200248812294810O*sqrt(7)*sqrt(7722433301876572160*
sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 - 209827708224593925147044161459246082873466808531245184642168l8
657851850283005515137386628871001287600830284004976245896200*sqrt(2)*sq...
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{11/2}\,(1+x+2x^2)^5}\,dx=\int \frac{1}{(2\,x^2+x+1)^5\,(x^2-2\,x+3)^{11/2}}\,dx$$

input
```
int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)),x)
```

output
```
int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

3.51 $\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

3.51.1 Optimal result

Integrand size = 23, antiderivative size = 638

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}}$$

$$+\frac{476849951294984711-125181871472148210x}{10427372048800000000\left(3-2x+x^2\right)^{17/2}}$$

$$+\frac{7851758375483333511+1942164996204584234x}{156410580732000000000\left(3-2x+x^2\right)^{15/2}}$$

$$-\frac{11(75023251063082010 89-78139863797265168 86x)}{406667509903200000000\left(3-2x+x^2\right)^{13/2}}$$

$$-\frac{3(69053268515296359011-44840736195018286006x)}{1147010925368000000000\left(3-2x+x^2\right)^{11/2}}$$

$$-\frac{83851943938029533565 7-466189390555853643870x}{9384634843920000000000\left(3-2x+x^2\right)^{9/2}}$$

$$-\frac{1117646664729238460189-568839749685437871554x}{3128211614640000000000\left(3-2x+x^2\right)^{7/2}}$$

$$-\frac{6551405511565449301689-3127298559983309301910x}{52136860244000000000000\left(3-2x+x^2\right)^{5/2}}$$

$$-\frac{4179039782398459850819-1886993445589652402694x}{104273720488000000000000\left(3-2x+x^2\right)^{3/2}}$$

$$-\frac{12105495874518671061833-5117656435043679338190x}{104273720488000000000000\sqrt{3-2x+x^2}}$$

$$-\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{9}}+\frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{8}}$$

$$+\frac{14453+29371x}{1080450\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{7}}+\frac{8837931+17459234x}{605052000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{6}}$$

$$+\frac{447940041+813432205x}{26471025000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{5}}$$

$$+\frac{592729157441+911061463974x}{29647548000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{4}}$$

$$+\frac{277010166219+310705340015x}{12353145000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{3}}$$

$$+\frac{5488221294349+1384103301166x}{276710448000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^{2}}$$

$$-\frac{37857197792117+146548895467025x}{2421216420000000\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)}$$

$$+\frac{\sqrt{\frac{1}{70}\left(810422259212746896054789447978008548464 05+5730592252300170712602636387866650030899 2\right.}}{}$$

$$-\frac{\sqrt{\frac{1}{70}\left(810422259212746896054789447978008548464 05+5730592252300170712602636387866650030899\right.}}{}$$

output
```
1/1840124479200000000*(37358055634422583-14024622879097678*x)/(x^2-2*x+3)^
(19/2)+1/10427372048800000000*(476849951294984711-12518187147214821 0*x)/(
x^2-2*x+3)^(17/2)+1/15641058073200000000000*(7851758375483333511+194216499
6204584234*x)/(x^2-2*x+3)^(15/2)-11/40666750990320000000000*(750232510630
8201089-7813986379726516886*x)/(x^2-2*x+3)^(13/2)-3/114701092536800000000
000*(69053268515296359011-44840736195018286006*x)/(x^2-2*x+3)^(11/2)+1/938
463484392000000000000*(-838519439380295335657+466189390555853643870*x)/(x
^2-2*x+3)^(9/2)+1/3128211614640000000000000*(-1117646664729238460189+5688
39749685437871554*x)/(x^2-2*x+3)^(7/2)+1/5213686024400000000000000*(-655
1405511565449301689+3127298559983309301910*x)/(x^2-2*x+3)^(5/2)+1/10427372
04880000000000000000*(-41790397823984598508 19+18869934455896524026 94*x)/(x
^2-2*x+3)^(3/2)+1/630*(-1+10*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/88200*(
887+2218*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^8+1/1080450*(14453+29371*x)/(x^
2-2*x+3)^(19/2)/(2*x^2+x+1)^7+1/605052000*(8837931+17459234*x)/(x^2-2*x+3)
^(19/2)/(2*x^2+x+1)^6+1/26471025000*(447940041+813432205*x)/(x^2-2*x+3)^(1
9/2)/(2*x^2+x+1)^5+1/29647548000000*(592729157441+911061463974*x)/(x^2-2*x
+3)^(19/2)/(2*x^2+x+1)^4+1/12353145000000*(277010166219+310705340015*x)/(x
^2-2*x+3)^(19/2)/(2*x^2+x+1)^3+1/276710448000000*(5488221294349+1384103301
166*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^2+1/2421216420000000*(-3785719779211
7-146548895467025*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)+1/1042737204880000...
```

3.51.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.66 (sec) , antiderivative size = 1431, normalized size of antiderivative = 2.24

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

output
```
Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113
*x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(60562500000000*(3
 - 2*x + x^2)^8) + (1062937 + 1642511*x)/(1574625000000000*(3 - 2*x + x^2)
^7) + (7*(-678331 + 833371*x))/(2220625000000000*(3 - 2*x + x^2)^6) + (7*(
-73161291 + 43964675*x))/(90843750000000000*(3 - 2*x + x^2)^5) + (-1340879
383 + 430593031*x)/(181687500000000000*(3 - 2*x + x^2)^4) - (11*(162612572
3 + 112950205*x))/(302812500000000000*(3 - 2*x + x^2)^3) - (11*(331157064
7 + 15286717673*x))/(3633750000000000000*(3 - 2*x + x^2)^2) - (11*(-41152
1923277 + 484788625685*x))/(36337500000000000000*(3 - 2*x + x^2)) + (2519
43 + 221770*x)/(6300000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*
x))/(882000000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(10
804500000000000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(121
010400000000000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)
/(2647102500000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 12919659
7088670934*x)/(2964754800000000000000*(1 + x + 2*x^2)^4) + (628195598643
14747 + 169630389653846945*x)/(3705943500000000000000*(1 + x + 2*x^2)^3)
 + (1082422109196374795 + 4797048907791526114*x)/(830131344000000000000000
0*(1 + x + 2*x^2)^2) + (65571203144429922747 + 367152793968978953465*x)/(3
631824630000000000000000*(1 + x + 2*x^2))) + ((23244280795494674579 5*I +
 2163417783119192484 1*Sqrt[7])*ArcTan[(-135063738860435016899586558 9487...
```

3.51.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x+3)^{21/2}(2x^2+x+1)^{10}}\,dx$$

$$\Big\downarrow \text{1305}$$

$$-\frac{\int -\frac{20(90x^2-153x+148)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^9}dx}{3150}-\frac{1-10x}{630(x^2-2x+3)^{19/2}(2x^2+x+1)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{315}\int \frac{90x^2-153x+148}{(x^2-2x+3)^{21/2}(2x^2+x+1)^9}dx-\frac{1-10x}{630(x^2-2x+3)^{19/2}(2x^2+x+1)^9}$$

$$\Big\downarrow \text{2135}$$

$$\frac{2}{315}\left(\frac{\int \frac{5(75412x^2-86509x+80661)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^8}dx}{2800}+\frac{2218x+887}{560\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^8}\right)-\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{315}\left(\frac{1}{560}\int \frac{75412x^2-86509x+80661}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)^8}dx+\frac{2218x+887}{560\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^8}\right)-\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 2135$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{\int \frac{50(3759488x^2-3790178x+3715561)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^7}dx}{2450}+\frac{4(29371x+14453)}{49\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^7}\right)+\frac{2218x+887}{560\left(x^2-2x+3\right)^{19/2}\left(2\right.}\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\int \frac{3759488x^2-3790178x+3715561}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)^7}dx+\frac{4(29371x+14453)}{49\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^7}\right)+\frac{221}{560\left(x^2-2x+\right.}\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 2135$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{\int \frac{15(523777020x^2-494230435x+458962907)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^6}dx}{2100}+\frac{17459234x+8837931}{140\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^6}\right)+\frac{4(29}{49\left(x^2-2x\right.}\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\int \frac{523777020x^2-494230435x+458962907}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)^6}dx+\frac{17459234x+8837931}{140\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^6}\right)+\frac{}{4}\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 2135$$

3.51. $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{\int \frac{10(91104406960x^2-76561243634x+63390281609)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^5}dx}{1750}+\frac{4(813432205x+447940041)}{175\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^5}\right)+\right.\right.\right.\right.$$
$$\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9}$$

$\big\downarrow$ 27

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\int\frac{91104406960x^2-76561243634x+63390281609}{(x^2-2x+3)^{21/2}\,(2x^2+x+1)^5}dx+\frac{4(813432205x+4479400}{175\,(x^2-2x+3)^{19/2}\,(2x^2+}\right.\right.\right.\right.$$
$$\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9}$$

$\big\downarrow$ 2135

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{\int \frac{15(7895866021108x^2-5294487996061x+3622330118837)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^4}dx}{1400}+\frac{911061463974x+592729157}{280\,(x^2-2x+3)^{19/2}\,(2x^2+x}\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9}$$

$\big\downarrow$ 27

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\int\frac{7895866021108x^2-5294487996061x+3622330118837}{(x^2-2x+3)^{21/2}\,(2x^2+x+1)^4}dx+\frac{911061463974}{280\,(x^2-2x+3}\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9}$$

$\big\downarrow$ 2135

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{\int \frac{1050(5965542528288x^2-2041006971986x+660555973049)}{(x^2-2x+3)^{21/2}(2x^2+x+1)^3}dx}{1050}+\frac{4(310705340015x+277}{5\,(x^2-2x+3)^{19/2}\,(2x}\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9}$$

$\big\downarrow$ 27

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\int\frac{5965542528288x^2-2041006971986x+660555973049}{(x^2-2x+3)^{21/2}\,(2x^2+x+1)^3}dx+\frac{4(310705340015}{5\,(x^2-2x+3)}\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\,(x^2-2x+3)^{19/2}\,(2x^2+x+1)^9}$$

3.51. $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\Big\downarrow \text{2135}$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{700}\int -\frac{25\left(-30450272625652x^2-90242403939711x+57003619484663\right)}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)^2}dx+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(-\frac{1}{28}\int \frac{-30450272625652x^2-90242403939711x+57003619484663}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)^2}dx+\frac{4(31}{5\,(x}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{2135}$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(-\frac{1}{350}\int -\frac{10\left(-11723911637362000x^2+9423200395626322x+218632072\right.}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\int \frac{-11723911637362000x^2+9423200395626322x+21863207222336658}{\left(x^2-2x+3\right)^{21/2}\left(2x^2+x+1\right)}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{2135}$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{\int -\frac{60\left(168295474549172136x^2-211409077626196062x+28036472352531697\right)}{\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)}dx}{3800}+\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{37358055634422583-1402462287909767 8x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\int 1682954745491\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

3.51. $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\Big\downarrow \; 2135$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\left(\frac{\int \frac{20(40058198871}{}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\left(\frac{1}{170}\int \frac{40058198}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 2135$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\left(\frac{1}{170}\left(\frac{\int \frac{20(-54}{}}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\left(\frac{1}{170}\left(\frac{1}{150}\int \frac{-5}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 2135$$

$$\frac{2}{315}\left(\frac{1}{560}\left(\frac{1}{49}\left(\frac{1}{140}\left(\frac{1}{175}\left(\frac{3}{280}\left(\frac{1}{28}\left(\frac{1}{35}\left(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\left(\frac{1}{170}\left(\frac{1}{150}\left(\frac{\int}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$$\Big\downarrow \; 27$$

3.51. $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

$$\frac{2}{315}\Bigg(\frac{1}{560}\Bigg(\frac{1}{49}\Bigg(\frac{1}{140}\Bigg(\frac{1}{175}\Bigg(\frac{3}{280}\Bigg(\frac{1}{28}\Bigg(\frac{1}{35}\Bigg(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\Bigg(\frac{1}{170}\Bigg(\frac{1}{150}\Bigg(\frac{3}{2}\int$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 2135

$$\frac{2}{315}\Bigg(\frac{1}{560}\Bigg(\frac{1}{49}\Bigg(\frac{1}{140}\Bigg(\frac{1}{175}\Bigg(\frac{3}{280}\Bigg(\frac{1}{28}\Bigg(\frac{1}{35}\Bigg(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\Bigg(\frac{1}{170}\Bigg(\frac{1}{150}\Bigg(\frac{3}{2}$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 27

$$\frac{2}{315}\Bigg(\frac{1}{560}\Bigg(\frac{1}{49}\Bigg(\frac{1}{140}\Bigg(\frac{1}{175}\Bigg(\frac{3}{280}\Bigg(\frac{1}{28}\Bigg(\frac{1}{35}\Bigg(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\Bigg(\frac{1}{170}\Bigg(\frac{1}{150}\Bigg(\frac{3}{2}\Bigg($$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 2135

$$\frac{2}{315}\Bigg(\frac{1}{560}\Bigg(\frac{1}{49}\Bigg(\frac{1}{140}\Bigg(\frac{1}{175}\Bigg(\frac{3}{280}\Bigg(\frac{1}{28}\Bigg(\frac{1}{35}\Bigg(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\Bigg(\frac{1}{170}\Bigg(\frac{1}{150}\Bigg(\frac{3}{2}$$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

$\downarrow$ 27

$$\frac{2}{315}\Bigg(\frac{1}{560}\Bigg(\frac{1}{49}\Bigg(\frac{1}{140}\Bigg(\frac{1}{175}\Bigg(\frac{3}{280}\Bigg(\frac{1}{28}\Bigg(\frac{1}{35}\Bigg(\frac{37358055634422583-14024622879097678x}{190\left(x^2-2x+3\right)^{19/2}}-\frac{3}{190}\Bigg(\frac{1}{170}\Bigg(\frac{1}{150}\Bigg(\frac{3}{2}\Bigg($$
$$\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}$$

input
```
Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

output
```
$Aborted
```

3.51. $\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.51.4 Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 552, normalized size of antiderivative = 0.87

method	result
risch	$\underline{3372249001933422237824271360x^{37}-5350220539964003139479614 7712x^{36}+46914939408298970172949457 5872x^{35}-284749922091}$
trager	Expression too large to display
default	Expression too large to display

input
```
int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

output
```
1/134200278268056000000000000000*(33722490019334222378242713 60*x^37-5350
22053996400313947961477 12*x^36+4691493940829897017294945758 72*x^35-2847499
2209126677533830352990 72*x^34+13254252261100740556512388253568*x^33-497700
800585250776280642298325 76*x^32+15601073493700873938822088945 7760*x^31-417
5163988507543971301119197943 36*x^30+97153817191336525187370687335 3652*x^29
-19936532135755218378886012043802 28*x^28+3655553471852957606257345414 14003
1*x^27-605476999658173850375368615510 4785*x^26+915549415851386923027152974
6307221*x^25-12740106677685048178693605103009787*x^24+16442770202470076313
197215936814318*x^23-19772569734288744720189854470201506*x^22+222864376176
21909921609206629636086*x^21-23584986647560742443188031208946882*x^20+2357
9397211179175240196614296051673*x^19-22218747553941794885903840542461607*x
^18+19912295454080246583636391613811979*x^17-1680176080605339024299514 5349
148613*x^16+13613407965006475288139078599341572*x^15-102793056507331786692
2363402096 2076*x^14+76062883783034495243279389770408 24*x^13-50698382349927
5192947119042611524 8*x^12+35074259705961976800160782130309 77*x^11-19748144
8306134440527585109453473 5*x^10+135700238843005588183329355785228 3*x^9-566
969010759169461615951049236597*x^8+458426000073846882432457044306894*x^7-9
4704557665253489332536549937026*x^6+13518392042691323141520887230323 0*x^5-
102309531890177463840318627287 4*x^4+293980411535249733439176017421 51*x^3+1
9339571955700627087816291348 23*x^2+33974623503989478480635838434 61*x-80...
```

3.51.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1569, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

output
```
1/939401947876392000000000000000000*(2360574301353395566476989952O0*x^38 -
 36588901670977631280393344256O0*x^37 + 315136669230678308124678158592O0*x
^36 - 188019743102797956869892249676800*x^35 + 86122602667001945498454882 1
612800*x^34 - 318372131382470805922980682944960O*x^33 + 983137886463415569
01525296768384O0*x^32 - 25933999577342884900069590431438400*x^31 + 5953780
1053669957636238873995743000*x^30 - 120659917686431634864896285011569300*x
^29 + 218815205755728685314344512920641100*x^28 - 358981964316724903316209
908944483400*x^27 + 538611677703407694971607759062726400*x^26 - 7447470585
99416616000621120456199500*x^25 + 956690308445988798962145796617987300*x^2
4 - 1146215696378789191186353021849349200*x^23 + 128937354094227863787592 6
769729056200*x^22 - 1362598128377218516278181645663204500*x^21 + 136327109
2148660247173548652450625900*x^20 - 1285053072164246491655277964217182200*
x^19 + 1156090273753138114015372080442309200*x^18 - 976662031233628820573 8
07397218635500*x^17 + 798237355988012151640630610578068900*x^16 - 60237857
5789760029562840059112791200*x^15 + 453947813134211818985370625408991400*x
^14 - 299561768273477509253114104689745500*x^13 + 216090200276716466450059
917698391300*x^12 - 116372548125131610054621102465641400*x^11 + 8869828798
9963515100607660442952800*x^10 - 31524301955764963385813894907485700*x^9 +
 33341076472331463305896468245703500*x^8 - 304003426262053063050252423716 0
400*x^7 + 1159943887325514784157222044507020O*x^6 + 1565914164733200701...
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx=\text{Timed out}$$

input
```
integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
```

output
```
Timed out
```

3.51.7 Maxima [F]

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \int \frac{1}{(2\,x^2+x+1)^{10}(x^2-2\,x+3)^{\frac{21}{2}}}\,dx$$

input
```
integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

3.51.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \text{Timed out}$$

input
```
integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

output
```
Timed out
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = \int \frac{1}{(2\,x^2+x+1)^{10}\,(x^2-2\,x+3)^{21/2}}\,dx$$

input
```
int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)),x)
```

output
```
int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

3.52.1 Optimal result

Integrand size = 48, antiderivative size = 66

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx = -\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}(-a+x)}{\sqrt{(-a+x)(1+x^2)}}\right)$$

output
```
-arctan((-a+x)*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)/((-a+x)*(x^2+1))^(1/2))*2^
(1/2)*(a+(a^2+1)^(1/2))^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx = -\frac{\sqrt{2}\sqrt{-a+x}\sqrt{1+x^2}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\sqrt{-a+x}}{\sqrt{1+x^2}}\right)}{\sqrt{-a+\sqrt{1+a^2}}\sqrt{(-a+x)(1+x^2)}}$$

input
```
Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)
*(1 + x^2)]),x]
```

output
```
-((Sqrt[2]*Sqrt[-a + x]*Sqrt[1 + x^2]*ArcTan[(Sqrt[2]*Sqrt[-a + Sqrt[1 + a
^2]]*Sqrt[-a + x])/Sqrt[1 + x^2]])/(Sqrt[-a + Sqrt[1 + a^2]]*Sqrt[(-a + x)
*(1 + x^2)]))
```

3.52.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.50, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {7270, 2349, 510, 729, 25, 1416, 1534, 1416, 2212, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{a^2+1}-a+x}{\left(\sqrt{a^2+1}-a+x\right)\sqrt{(x^2+1)(x-a)}}dx$$

$$\Big\downarrow \; 7270$$

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\int \frac{a-x+\sqrt{a^2+1}}{\left(a-x-\sqrt{a^2+1}\right)\sqrt{x-a}\sqrt{x^2+1}}dx}{\sqrt{-\left((x^2+1)(a-x)\right)}}$$

$$\Big\downarrow \; 2349$$

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(2\sqrt{a^2+1}\int \frac{1}{\left(a-x-\sqrt{a^2+1}\right)\sqrt{x-a}\sqrt{x^2+1}}dx+\int \frac{1}{\sqrt{x-a}\sqrt{x^2+1}}dx\right)}{\sqrt{-\left((x^2+1)(a-x)\right)}}$$

$$\Big\downarrow \; 510$$

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(2\sqrt{a^2+1}\int \frac{1}{\left(a-x-\sqrt{a^2+1}\right)\sqrt{x-a}\sqrt{x^2+1}}dx+2\int \frac{1}{\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}\right)}{\sqrt{-\left((x^2+1)(a-x)\right)}}$$

$$\Big\downarrow \; 729$$

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(2\int \frac{1}{\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}+4\sqrt{a^2+1}\int -\frac{1}{\left(-a+x+\sqrt{a^2+1}\right)\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}\right)}{\sqrt{-\left((x^2+1)(a-x)\right)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(2\int \frac{1}{\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}-4\sqrt{a^2+1}\int \frac{1}{\left(-a+x+\sqrt{a^2+1}\right)\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}\right)}{\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}$$

$\Big\downarrow$ 1416

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(\frac{\sqrt[4]{a^2+1}\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)\sqrt{\frac{a^2+2a(x-a)+(x-a)^2+1}{\left(a^2+1\right)\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{x-a}}{\sqrt[4]{a^2+1}}\right),\frac{1}{2}\left(1-\frac{a}{\sqrt{a^2+1}}\right)\right)}{\sqrt{a^2+2a(x-a)+(x-a)^2+1}}-4\sqrt{a^2+1}\int\right.}{\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}$$

$\Big\downarrow$ 1534

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(4\sqrt{a^2+1}\left(-\frac{\int \frac{1}{\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}}{2\sqrt{a^2+1}}-\frac{\int \frac{a-x+\sqrt{a^2+1}}{\left(-a+x+\sqrt{a^2+1}\right)\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}}{2\sqrt{a^2+1}}\right)+\frac{\sqrt[4]{a^2+}}{}\right.}{\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}$$

$\Big\downarrow$ 1416

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(4\sqrt{a^2+1}\left(-\frac{\int \frac{a-x+\sqrt{a^2+1}}{\left(-a+x+\sqrt{a^2+1}\right)\sqrt{a^2+2(x-a)a+(x-a)^2+1}}d\sqrt{x-a}}{2\sqrt{a^2+1}}-\frac{\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)\sqrt{\frac{a^2+2a(x-a)+(x-a)^2+1}{\left(a^2+1\right)\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)^2}}\text{EllipticF}\left(\right.}{4\sqrt[4]{a^2+1}\sqrt{a^2+2a(x-}}\right.\right.}{\sqrt{-((}}$$

$\Big\downarrow$ 2212

$$\sqrt{x^2+1}\sqrt{x-a}\left(4\sqrt{a^2+1}\left(-\frac{1}{2}\int \frac{1}{\sqrt{a^2+1}-2\sqrt{a^2+1}\left(a-\sqrt{a^2+1}\right)(x-a)}d\frac{\sqrt{x-a}}{\sqrt{a^2+2(x-a)a+(x-a)^2+1}}-\frac{\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)\sqrt{\frac{a^2+2a(x-a)+}{\left(a^2+1\right)\left(\frac{x-}{\sqrt{a}}\right.}}}{4\sqrt[4]{}}\right.\right.$$

$\Big\downarrow$ 221

3.52. $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

$$\frac{\sqrt{x^2+1}\sqrt{x-a}\left(4\sqrt{a^2+1}\left(-\frac{\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)\sqrt{\frac{a^2+2a(x-a)+(x-a)^2+1}{(a^2+1)\left(\frac{x-a}{\sqrt{a^2+1}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{x-a}}{\sqrt[4]{a^2+1}}\right),\frac{1}{2}\left(1-\frac{a}{\sqrt{a^2+1}}\right)\right)}{4\sqrt[4]{a^2+1}\sqrt{a^2+2a(x-a)+(x-a)^2+1}}-\frac{\text{arctan}\cdots}{2\cdots}\right.\right.}{\sqrt{-((x^2+\cdots}}$$

input
```
Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 +
x^2)]),x]
```

output
```
(Sqrt[-a + x]*Sqrt[1 + x^2]*(((1 + a^2)^(1/4)*(1 + (-a + x)/Sqrt[1 + a^2])
*Sqrt[(1 + a^2 + 2*a*(-a + x) + (-a + x)^2)/((1 + a^2)*(1 + (-a + x)/Sqrt[
1 + a^2])^2)]*EllipticF[2*ArcTan[Sqrt[-a + x]/(1 + a^2)^(1/4)], (1 - a/Sqr
t[1 + a^2])/2])/Sqrt[1 + a^2 + 2*a*(-a + x) + (-a + x)^2] + 4*Sqrt[1 + a^2
]*(-1/2*ArcTanh[(Sqrt[2]*Sqrt[a - Sqrt[1 + a^2]]*Sqrt[-a + x])/Sqrt[1 + a^
2 + 2*a*(-a + x) + (-a + x)^2]]/(Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a - Sqrt[1 + a
^2]]) - ((1 + (-a + x)/Sqrt[1 + a^2])*Sqrt[(1 + a^2 + 2*a*(-a + x) + (-a +
 x)^2)/((1 + a^2)*(1 + (-a + x)/Sqrt[1 + a^2])^2)]*EllipticF[2*ArcTan[Sqrt
[-a + x]/(1 + a^2)^(1/4)], (1 - a/Sqrt[1 + a^2])/2])/(4*(1 + a^2)^(1/4)*Sq
rt[1 + a^2 + 2*a*(-a + x) + (-a + x)^2]))))/Sqrt[-((a - x)*(1 + x^2))]
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1534
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Simp[1/(2*d)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[1/(2*
d)   Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0
] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.52. $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

3.52.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.92

method	result
default	$\frac{2i\sqrt{-i(x+i)}\,\sqrt{\frac{-a+x}{-i-a}}\,\sqrt{i(x-i)}\,F\left(\frac{\sqrt{2}\,\sqrt{-i(x+i)}}{2},\sqrt{2}\,\sqrt{-\frac{i}{-i-a}}\right)}{\sqrt{-a\,x^2+x^3-a+x}} - \frac{2\sqrt{a^2+1}\,(2ax-x^2+1)\sqrt{-(a-x)(x^2+1)(a^2+1)}\left(-\frac{i\sqrt{a^2+1}\,\sqrt{-i}}{}\right.}{}$
elliptic	Expression too large to display

input
```
int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x,met
hod=_RETURNVERBOSE)
```

output
```
2*I*(-I*(x+I))^(1/2)*((-a+x)/(-I-a))^(1/2)*(I*(x-I))^(1/2)/(-a*x^2+x^3-a+x
)^(1/2)*EllipticF(1/2*2^(1/2)*(-I*(x+I))^(1/2),2^(1/2)*(-I/(-I-a))^(1/2))-
2*(a^2+1)^(1/2)*(2*a*x-x^2+1)*(-(a-x)*(x^2+1)*(a^2+1))^(1/2)/(-a+x+(a^2+1)
^(1/2))/((-(a-x)*(x^2+1))^(1/2)*a^2+(-(a-x)*(x^2+1)*(a^2+1))^(1/2)*a-(-(a-
x)*(x^2+1)*(a^2+1))^(1/2)*x+(-(a-x)*(x^2+1))^(1/2))*(-I*(a^2+1)^(1/2)*(1-I
*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a
^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*
(-I*(x+I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(a
^2+1)^(1/2)*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a
^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*Ellipti
cPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-I
-a))^(1/2))+I*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(
-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I
))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(1-I*x)^(1
/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(-
I-a+(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2
+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 546, normalized size of antiderivative = 8.27

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$$
$$=\left[\frac{1}{4}\sqrt{-2\,a-2\,\sqrt{a^2+1}}\log\left(-\frac{8\,ax^7+x^8+4\,(2\,a^2+15)x^6-8\,(4\,a^3+15\,a)x^5+2\,(8\,a^4+80\,a^2+67)x^4}{}\right.\right.$$
$$-\frac{1}{2}\sqrt{2\,a+2\,\sqrt{a^2+1}}\arctan\left(-\frac{\sqrt{-ax^2+x^3-a+x}\,(2\,a^2-2\,ax-x^2-2\,\sqrt{a^2+1}(a-x)-1)\,\sqrt{2\,a+}}{4\,(ax^2-x^3+a-x)}\right.$$

input
```
integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2)
,x, algorithm="fricas")
```

output
```
[1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
 - 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3
 + 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2
 + 3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*
a^2 + 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x
^2 + 8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*s
qrt(-2*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a
*x^4 - 7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1
)*x + a)*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3
- 3*a)*x^5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 -
 1)*x^2 - 8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-
a*x^2 + x^3 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*s
qrt(2*a + 2*sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx=\text{Timed out}$$

input
```
integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))*
*(1/2),x)
```

output
```
Timed out
```

3.52.7 Maxima [F]

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\,(1+x^2)}}\,dx = \int \frac{a-x+\sqrt{a^2+1}}{\sqrt{-(x^2+1)(a-x)}\left(a-x-\sqrt{a^2+1}\right)}\,dx$$

input
```
integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2)
,x, algorithm="maxima")
```

output
```
integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(
a^2 + 1))), x)
```

3.52.8 Giac [F]

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\,(1+x^2)}}\,dx = \int \frac{a-x+\sqrt{a^2+1}}{\sqrt{-(x^2+1)(a-x)}\left(a-x-\sqrt{a^2+1}\right)}\,dx$$

input
```
integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2)
,x, algorithm="giac")
```

output
```
integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(
a^2 + 1))), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\,(1+x^2)}}\,dx = \int -\frac{a-x+\sqrt{a^2+1}}{\sqrt{-\left(x^2+1\right)\,\left(a-x\right)}\,\left(x-a+\sqrt{a^2+1}\right)}\,dx$$

input
```
int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
 1)^(1/2))),x)
```

output
```
int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
 1)^(1/2))), x)
```

3.52. $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.53.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{a\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\sqrt{3}b\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}+\frac{a\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\operatorname{arctanh}(x)}{6\ 2^{2/3}}+\frac{a\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}-\frac{b\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

output
```
-1/12*a*arctanh(x)*2^(1/3)+1/4*a*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(
1/3)-1/8*b*ln(x^2+3)*2^(1/3)+3/8*b*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/12
*a*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*a*arctan((1-2^(1/3)*(-x^2+1)^(1/
3))*3^(1/2)/x)*2^(1/3)*3^(1/2)+1/4*b*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/
2))*3^(1/2)*2^(1/3)
```

3.53.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{6}bx^2\,\text{AppellF1}\left(1,\frac{1}{3},1,2,x^2,-\frac{x^2}{3}\right) - \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

input
```
Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (9*a*x*AppellF1[1/2, 1/3
, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3
, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2
] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

3.53.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1343, 305, 353, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{1343}$$

$$a\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx+b\int \frac{x}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{305}$$

$$b\int \frac{x}{\sqrt[3]{1-x^2}\,(x^2+3)}dx+$$
$$a\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}b\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2+$$
$$a\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{2}b\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}+\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}\right)+$$
$$a\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2}b\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)+$$
$$a\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \ 1082$$

3.53. $\int \frac{a+bx}{\sqrt[3]{1-x^2}(3+x^2)}dx$

$$\frac{1}{2}b\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)+$$
$$a\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow 217$$

$$a\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)+$$
$$\frac{1}{2}b\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
a*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1
 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[
x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))) + (b*((Sqrt[3]*ArcTan[(1 + 2
^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*
Log[2^(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3))))/2
```

3.53.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.53.4 Maple [F]

$$\int \frac{bx+a}{\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}dx$$

input
```
int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

output
```
int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

3.53.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (trace 0)
```

3.53.6 Sympy [F]

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{a+bx}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.53.7 Maxima [F]

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.53.8 Giac [F]

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{a+b\,x}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.54 $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

3.54.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{a\arctan(x)}{6\ 2^{2/3}}+\frac{a\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}b\arctan\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{b\log(3-x^2)}{4\ 2^{2/3}} - \frac{3b\log\left(2^{2/3}-\sqrt[3]{1+x^2}\right)}{4\ 2^{2/3}}$$

output
```
-1/12*a*arctan(x)*2^(1/3)+1/4*a*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3
)+1/8*b*ln(-x^2+3)*2^(1/3)-3/8*b*ln(2^(2/3)-(x^2+1)^(1/3))*2^(1/3)-1/12*a*
arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*a*arctanh((1-2^(1/3)*(x^2+1)^(1/3)
)*3^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*b*arctan(1/3*(1+2^(1/3)*(x^2+1)^(1/3))*3^
(1/2))*3^(1/2)*2^(1/3)
```

3.54.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.77

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \frac{1}{6}bx^2 \operatorname{AppellF1}\left(1,\frac{1}{3},1,2,-x^2,\frac{x^2}{3}\right) - \frac{9ax \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

output
```
(b*x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3,
1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1,
 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - Appe
llF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

3.54.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1343, 304, 353, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{x^2+1}}dx$$

$$\Big\downarrow \text{1343}$$

$$a\int \frac{1}{(3-x^2)\sqrt[3]{x^2+1}}dx+b\int \frac{x}{(3-x^2)\sqrt[3]{x^2+1}}dx$$

$$\Big\downarrow \text{304}$$

$$b\int \frac{x}{(3-x^2)\sqrt[3]{x^2+1}}dx+$$
$$a\left(\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}b\int \frac{1}{(3-x^2)\sqrt[3]{x^2+1}}dx^2+$$
$$a\left(\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}b\left(\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{x^2+1}}d\sqrt[3]{x^2+1}}{2\ 2^{2/3}}-\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{x^2+1}+2\sqrt[3]{2}}d\sqrt[3]{x^2+1}+\frac{\log(3-x^2)}{2\ 2^{2/3}}\right)+$$
$$a\left(\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}b\left(-\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{x^2+1}+2\sqrt[3]{2}}d\sqrt[3]{x^2+1}+\frac{\log(3-x^2)}{2\ 2^{2/3}}-\frac{3\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{2\ 2^{2/3}}\right)+$$
$$a\left(\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right)$$

$$\Big\downarrow \text{1082}$$

3.54. $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

$$\frac{1}{2}b\left(\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{x^2+1}+1\right)}{2^{2/3}}+\frac{\log\left(3-x^2\right)}{2\ 2^{2/3}}-\frac{3\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{2\ 2^{2/3}}\right)+$$
$$a\left(\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right)$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right)+$$
$$\frac{1}{2}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\log\left(3-x^2\right)}{2\ 2^{2/3}}-\frac{3\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

output
```
a*(-1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(
2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(
1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])) + (b*(-((Sqrt[3]*ArcTan[(1
+ 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/2^(2/3)) + Log[3 - x^2]/(2*2^(2/3)) -
 (3*Log[2^(2/3) - (1 + x^2)^(1/3)])/(2*2^(2/3))))/2
```

3.54.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 304
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.54.4 Maple [F]

$$\int \frac{bx+a}{(-x^2+3)\,(x^2+1)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

output
```
int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

3.54.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (trace 0)
```

3.54.6 Sympy [F]

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{a}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx - \int \frac{bx}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

input
```
integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

output
```
-Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(
b*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.54.7 Maxima [F]

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{bx+a}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

input
```
integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.54.8 Giac [F]

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{bx+a}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

input
```
integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{a+b\,x}{(x^2+1)^{1/3}\,(x^2-3)}\,dx$$

input
```
int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
```

output
```
int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

3.55.1 Optimal result

Integrand size = 18, antiderivative size = 97

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2\ 2^{2/3}} + \frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\ 2^{2/3}}$$

output
```
-1/4*ln(x)*2^(1/3)+1/4*ln(6-3*x-3*2^(1/3)*(3*x^2-6*x+4)^(1/3))*2^(1/3)+1/6
*arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(2-x)/(3*x^2-6*x+4)^(1/3)*3^(1/2))*2^(1/3
)*3^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{2\ 2^{2/3}-2^{2/3}x+\sqrt[3]{4-6x+3x^2}}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right) - 2\log\left(-22^{2/3}+2^{2/3}x+2\sqrt[3]{4-6x+3x^2}\right) + \log\left(-4\sqrt[3]{2}\right.}{6\ 2^{2/3}}$$

input
```
Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(2*2^(2/3) - 2^(2/3)*x + (4 - 6*x + 3*x^2)^(1/3))/(
Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))] - 2*Log[-2*2^(2/3) + 2^(2/3)*x + 2*(4 -
6*x + 3*x^2)^(1/3)] + Log[-4*2^(1/3) + 4*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)
*(-2 + x)*(4 - 6*x + 3*x^2)^(1/3) - 2*(4 - 6*x + 3*x^2)^(2/3)])/2^(2/3)
```

3.55.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1175}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

$$\Big\downarrow \; 1175$$

$$-\frac{\arctan\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\ 2^{2/3}}-\frac{\log(x)}{2\ 2^{2/3}}$$

input
```
Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

output
```
-(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/
(2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x
 + 3*x^2)^(1/3)]/(2*2^(2/3))
```

3.55.3.1 Defintions of rubi rules used

rule 1175
```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, Simp[(-Sqrt[3])*c*e*(ArcT
an[1/Sqrt[3] + 2*((c*d - b*e - c*e*x)/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3)))]
/q^2), x] + (-Simp[3*c*e*(Log[d + e*x]/(2*q^2)), x] + Simp[3*c*e*(Log[c*d -
 b*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)]/(2*q^2)), x])] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*e^2, 0] && PosQ[c*e^2
*(2*c*d - b*e)]
```

3.55.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.44 (sec) , antiderivative size = 2395, normalized size of antiderivative = 24.69

method	result	size
trager	Expression too large to display	2395

input
```
int(1/x/(3*x^2-6*x+4)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(600*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+30*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-300*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)*x^2-400*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-16
0*RootOf(_Z^3-2)+12*RootOf(_Z^3-2)*x^3-120*RootOf(_Z^3-2)*x^2+240*RootOf(_
Z^3-2)*x+20*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_
Z^3-2)^2*x^3+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(
_Z^3-2)^3*x^3-60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*Roo
tOf(_Z^3-2)^2*x^2-24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*R
ootOf(_Z^3-2)^3*x^2+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)^2*RootOf(_Z^3-2)^2*x+48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)^3*x-80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)^2*RootOf(_Z^3-2)^2-32*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)^3+30*(3*x^2-6*x+4)^(2/3)*x-60*(3*x^2-6*x+4)^(1/3)*RootO
f(_Z^3-2)^2+48*(3*x^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x-48*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+192*(3*x^2-6*x+4)^(1/3
)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-60*
(3*x^2-6*x+4)^(2/3)-96*(3*x^2-6*x+4)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*Ro
otOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-15*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^...
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(74) = 148$.

Time = 1.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = -\frac{1}{6} \cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}(3\,x^2-6\,x+4)^{\frac{2}{3}}(x-2)+4\,(3\,x^2-6\,x+4)^{\frac{1}{3}}(x^2-4\,x+4)\right)}{6\,(x^3-12\,x^2+24\,x-16)}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}(x-2)+2\,(3\,x^2-6\,x+4)^{\frac{1}{3}}}{x}\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(3\,x^2-6\,x+4)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^2-4\,x+4)-2\,(3\,x^2-6\,x+4)^{\frac{1}{3}}(x-2)}{x^2}\right)$$

input
```
integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(
3*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4)
)/(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x
^2 - 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3
) + 4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

3.55.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

input
```
integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

output
```
Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

3.55.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-6\,x+4)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

3.55.8 Giac [F]

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{(3\,x^2-6\,x+4)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx = \int \frac{1}{x\,(3\,x^2-6\,x+4)^{1/3}}\,dx$$

input
```
int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
```

output
```
int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

3.56.1 Optimal result

Integrand size = 13, antiderivative size = 73

$$\int x\sqrt[3]{1-x^3}\,dx=\frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)$$

output
```
1/3*x^2*(-x^3+1)^(1/3)-1/6*ln(-x-(-x^3+1)^(1/3))-1/9*arctan(1/3*(1-2*x/(-x
^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x\sqrt[3]{1-x^3}\,dx=\frac{1}{18}\left(6x^2\sqrt[3]{1-x^3}-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)-2\log\left(x+\sqrt[3]{1-x^3}\right)+\log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

input
```
Integrate[x*(1 - x^3)^(1/3),x]
```

output
```
(6*x^2*(1 - x^3)^(1/3) - 2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/
3))] - 2*Log[x + (1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3
)^(2/3)])/18
```

3.56.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {811, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[3]{1-x^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{3}\int \frac{x}{(1-x^3)^{2/3}}dx+\frac{1}{3}\sqrt[3]{1-x^3}x^2$$

$$\Big\downarrow \text{853}$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)\right)+\frac{1}{3}\sqrt[3]{1-x^3}x^2$$

input
```
Int[x*(1 - x^3)^(1/3),x]
```

output
```
(x^2*(1 - x^3)^(1/3))/3 + (-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/S
qrt[3]) - Log[-x - (1 - x^3)^(1/3)]/2)/3
```

3.56.3.1 Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{x^{2}{}_{2}F_{1}\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^{3}\right)}{2}$
risch	$-\frac{x^{2}\left(x^{3}-1\right)}{3\left(-x^{3}+1\right)^{\frac{2}{3}}}+\frac{\left(x^{3}-1\right)^{\frac{2}{3}}\left(-\operatorname{signum}\left(x^{3}-1\right)\right)^{\frac{2}{3}}x^{2}{}_{2}F_{1}\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};x^{3}\right)}{6\operatorname{signum}\left(x^{3}-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{2}{3}}}$
pseudoelliptic	$\frac{6x^{2}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\left(-2\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)-2\ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-x\left(-x^{3}+1\right)^{\frac{1}{3}}+x^{2}}{x^{2}}\right)}{18\left(x+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\left(\left(-x^{3}+1\right)^{\frac{2}{3}}-x\left(-x^{3}+1\right)^{\frac{1}{3}}+x^{2}\right)}$
trager	$\frac{x^{2}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3}-\frac{\ln\Big(-2\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)^{2}x^{3}+3\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}x-\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)}{9}$

input
```
int(x*(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*hypergeom([-1/3,2/3],[5/3],x^3)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int x\sqrt[3]{1-x^3}\,dx=\frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

input
```
integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*
(-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 -
 (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

3.56.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int x\sqrt[3]{1-x^3}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x*(-x**3+1)**(1/3),x)
```

output
```
x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(5/3))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\begin{aligned}\int x\sqrt[3]{1-x^3}\,dx = &-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{(-x^3+1)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)}\\ &-\frac{1}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)\end{aligned}$$

input
```
integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 +
1)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*
log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

3.56.8 Giac [F]

$$\int x\sqrt[3]{1-x^3}\,dx=\int \left(-x^3+1\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(1/3)*x, x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt[3]{1-x^3}\,dx=\int x\,\left(1-x^3\right)^{1/3}\,dx$$

input
```
int(x*(1 - x^3)^(1/3),x)
```

output
```
int(x*(1 - x^3)^(1/3), x)
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

3.57.1 Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

output
```
(-x^3+1)^(1/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1+2*(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = \sqrt[3]{1-x^3} - \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

input
```
Integrate[(1 - x^3)^(1/3)/x,x]
```

output
```
(1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1
 + (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

3.57.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 60, 69, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{1-x^3}}{x} \, dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}\int \frac{\sqrt[3]{1-x^3}}{x^3}dx^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{3}\left(\int \frac{1}{x^3\left(1-x^3\right)^{2/3}}dx^3+3\sqrt[3]{1-x^3}\right)$$

$$\Big\downarrow \; 69$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+3\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{3}\left(3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+3\sqrt[3]{1-x^3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\sqrt[3]{1-x^3}-\frac{\log\left(x^3\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

input
```
Int[(1 - x^3)^(1/3)/x,x]
```

output
```
(3*(1 - x^3)^(1/3) - Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - Log
[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2)/3
```

3.57.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.57. $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

3.57.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result
meijerg	$-\frac{-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\Gamma\left(\frac{2}{3}\right)x^{3}{}_{3}F_{2}\left(\frac{2}{3},1,1;2,2;x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}$
pseudoelliptic	$(-x^3+1)^{\frac{1}{3}}-\frac{\ln\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)}{6}-\frac{\arctan\left(\frac{\left(1+2(-x^3+1)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left((-x^3+1)^{\frac{1}{3}}-1\right)}{3}$
trager	$(-x^3+1)^{\frac{1}{3}}+\frac{\ln\left(-\frac{1438\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-6979\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)(-x\right.\right.}{}$

input
```
int((-x^3+1)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+G
AMMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+(-x^3+1)^{\frac{1}{3}} - \frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

input
```
integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 +
1)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^
3 + 1)^(1/3) - 1)
```

3.57.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((-x**3+1)**(1/3)/x,x)
```

output
```
-x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2
/3))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/
3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)
^(1/3) - 1)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/
3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
+ 1)^(1/3) - 1))
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt[3]{1-x^3}}{x}\,dx = \frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\left(1-x^3\right)^{1/3}$$

input
```
int((1 - x^3)^(1/3)/x,x)
```

output
```
log((1 - x^3)^(1/3) - 1)/3 + log(3*(1 - x^3)^(1/3) - (3^(1/2)*3i)/2 + 3/2)
*((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(1 - x^3)^(1/3) + 3/2)*((
3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(1/3)
```

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$

3.58.1 Optimal result

Integrand size = 17, antiderivative size = 482

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \sqrt[3]{1-x^3}+\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$
$$-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}$$
$$-\frac{1}{3}\sqrt[3]{2}\log\left(1+x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$
$$+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)-\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}}-\frac{1}{2}\log\left(-x-\right.$$

output
```
(-x^3+1)^(1/3)-1/3*2^(1/3)*ln(x^3+1)+1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))
*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1
/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(2^
(1/3)-(-x^3+1)^(1/3))*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/2*ln(-2^(1/3)*x-
(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3
^(1/2))*2^(1/3)*3^(1/2)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2
)-1/3*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.58.2 Mathematica [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

input
```
Integrate[(1 - x^3)^(1/3)/(1 + x),x]
```

output
```
Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

3.58.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{1-x^3}}{x+1}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{\sqrt[3]{1-x^3}x}{x^3+1}+\frac{\sqrt[3]{1-x^3}}{x^3+1}+\frac{\sqrt[3]{1-x^3}x^2}{x^3+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[3]{1-x^3}-\frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2^{2/3}}$$

input
```
Int[(1 - x^3)^(1/3)/(1 + x),x]
```

output
```
(1 - x^3)^(1/3) + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3
] + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
- (2^(1/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2^(1/
3)*Log[1 + x^3])/3 + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) -
Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^
(1/3)]/(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/
3 - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3
)^(1/3)]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(2/3) - Log[-x - (
1 - x^3)^(1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2/3)
```

3.58. $\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.58.4 Maple [F]

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{1+x}dx$$

input
```
int((-x^3+1)^(1/3)/(1+x),x)
```

output
```
int((-x^3+1)^(1/3)/(1+x),x)
```

3.58.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.58.6 Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x+1}\,dx$$

input
```
integrate((-x**3+1)**(1/3)/(1+x),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

3.58.7 Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

input
```
integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

3.58.8 Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

input
```
integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx = \int \frac{(1-x^3)^{1/3}}{x+1} \, dx$$

input
```
int((1 - x^3)^(1/3)/(x + 1),x)
```

output
```
int((1 - x^3)^(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$

3.59.1 Optimal result

Integrand size = 22, antiderivative size = 280

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx &= \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}(-1+x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\\
&\quad-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}\\
&\quad-\frac{\log\left(-3(-1+x)\left(1-x+x^2\right)\right)}{2\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\\
&\quad+\frac{3\log\left(-\sqrt[3]{2}(-1+x)+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\\
&\quad+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\end{aligned}$$

output
```
-1/4*ln(-3*(-1+x)*(x^2-x+1))*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3
)+3/4*ln(-2^(1/3)*(-1+x)+(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(x+(-x^3+1)^(1/3))-
1/4*ln(2^(1/3)*x+(-x^3+1)^(1/3))*2^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1
/3))*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*3^(1/2)+1/2*arctan(1/3*(1+2*2^(1/3)*(-1+x)/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)*2^(1/3)
```

3.59.2 Mathematica [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx=\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

input
```
Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

output
```
Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

3.59.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{1-x^3}}{x^2-x+1}\,dx$$

$$\Big\downarrow \; 2583$$

$$\int \left(\frac{\sqrt[3]{1-x^3}x}{x^3+1}+\frac{\sqrt[3]{1-x^3}}{x^3+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left(x^3+1\right)}{3\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2^{2/3}}$$

input
```
Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

output
```
(2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3
] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[
3]) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(1/3)*ArcTa
n[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[1 + x^3]/(3*
2^(2/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2
^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*
2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*
2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(
6*2^(2/3)) + Log[-x - (1 - x^3)^(1/3)]/2 - Log[-(2^(1/3)*x) - (1 - x^3)^(1
/3)]/2^(2/3)
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.59. $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$

3.59.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 21.32 (sec) , antiderivative size = 1410, normalized size of antiderivative = 5.04

method	result	size
trager	Expression too large to display	1410

input
```
int((-x^3+1)^(1/3)/(x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(RootOf(_Z^2+_Z+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x+3*Root
Of(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2-2*RootOf(_Z^2+_Z+1)*x^3+x^3+RootOf(_Z^2+_
Z+1)-1)*RootOf(_Z^2+_Z+1)-1/3*RootOf(_Z^2+_Z+1)*ln(RootOf(_Z^2+_Z+1)^2*x^3
+3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x-3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x
^2+4*RootOf(_Z^2+_Z+1)*x^3+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)+4*x^3-R
ootOf(_Z^2+_Z+1)-2)-1/3*ln(RootOf(_Z^2+_Z+1)^2*x^3+3*RootOf(_Z^2+_Z+1)*(-x
^3+1)^(2/3)*x-3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2+4*RootOf(_Z^2+_Z+1)*x
^3+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)+4*x^3-RootOf(_Z^2+_Z+1)-2)-1/18
*ln(-(12*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2
+_Z+1)*x^3+RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x^4-36*(-x^3+1)^(1/3)*
RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x^2+6*(-x^3+1)^(1
/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*x^3+2*x^3*RootOf(_Z^3-324*RootO
f(_Z^2+_Z+1)-162)^2+108*(-x^3+1)^(2/3)*x^2+12*(-x^3+1)^(1/3)*RootOf(_Z^3-3
24*RootOf(_Z^2+_Z+1)-162)*RootOf(_Z^2+_Z+1)*x-18*(-x^3+1)^(1/3)*RootOf(_Z^
3-324*RootOf(_Z^2+_Z+1)-162)*x^2-RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*
x^2-108*x*(-x^3+1)^(2/3)+6*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1
)-162)*x-2*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)^2*x+RootOf(_Z^3-324*Root
Of(_Z^2+_Z+1)-162)^2)/(x^2-x+1)^2)*RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)*
RootOf(_Z^2+_Z+1)-1/18*ln(-(12*(-x^3+1)^(1/3)*RootOf(_Z^3-324*RootOf(_Z^2+
_Z+1)-162)*RootOf(_Z^2+_Z+1)*x^3+RootOf(_Z^3-324*RootOf(_Z^2+_Z+1)-162)...
```

3.59.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.35 (sec) , antiderivative size = 3880, normalized size of antiderivative = 13.86

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

output
```
1/72*sqrt(3)*(sqrt(-3)*(-4)^(1/6) + (-4)^(1/6))*log(3*(6*sqrt(-3)*(-4)^(1/
3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 361508*x^9 - 363117*x^8 - 876
12*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39436*x^3 + 18254*x^2 + 36
52*x - 1278) + 48*(11866*x^10 - 16425*x^9 - 125794*x^8 + 251931*x^7 - 7118
7*x^6 - 79049*x^5 - 2745*x^4 + 52032*x^3 - 20629*x^2 - sqrt(3)*(3104*I*x^1
0 - 43815*I*x^9 + 84520*I*x^8 + 11329*I*x^7 - 92013*I*x^6 + 5291*I*x^5 + 5
3855*I*x^4 - 20262*I*x^3 - 2009*I*x^2 + 1278*I*x) + 2008*x)*(-x^3 + 1)^(2/
3) + sqrt(3)*(sqrt(-3)*(-4)^(5/6)*(31397*x^12 + 113940*x^11 - 831396*x^10
+ 973364*x^9 - 140709*x^8 + 407484*x^7 - 1009896*x^6 + 313212*x^5 + 248121
*x^4 - 75940*x^3 - 48198*x^2 + 19716*x - 2008) - (-4)^(5/6)*(31397*x^12 +
113940*x^11 - 831396*x^10 + 973364*x^9 - 140709*x^8 + 407484*x^7 - 1009896
*x^6 + 313212*x^5 + 248121*x^4 - 75940*x^3 - 48198*x^2 + 19716*x - 2008))
- 6*(-4)^(1/3)*(26655*x^12 - 185476*x^11 + 155872*x^10 + 361508*x^9 - 3631
17*x^8 - 87612*x^7 - 197936*x^6 + 492924*x^5 - 182367*x^4 - 39436*x^3 + 18
254*x^2 + 3652*x - 1278) - 6*(-x^3 + 1)^(1/3)*(sqrt(-3)*(-4)^(2/3)*(1459*x
^11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6 - 2639
73*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x) + (-4)^(2/3)*(1459*x^
11 + 94937*x^10 - 314364*x^9 + 204807*x^8 + 73586*x^7 + 103515*x^6 - 26397
3*x^5 + 67714*x^4 + 54774*x^3 - 25376*x^2 + 2008*x) + 2*sqrt(3)*(sqrt(-3)*
(-4)^(1/6)*(12049*x^11 - 48557*x^10 - 31048*x^9 + 203745*x^8 - 117748*x...
```

3.59.6 Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx = \int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x^2-x+1}\,dx$$

input
```
integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

3.59.7 Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^2-x+1} \, dx$$

input
```
integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

3.59.8 Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^2-x+1} \, dx$$

input
```
integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx = \int \frac{(1-x^3)^{1/3}}{x^2-x+1} \, dx$$

input
```
int((1 - x^3)^(1/3)/(x^2 - x + 1),x)
```

output
```
int((1 - x^3)^(1/3)/(x^2 - x + 1), x)
```

3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$

3.60.1 Optimal result

Integrand size = 17, antiderivative size = 232

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

$$= \sqrt[3]{1-x^3}+\frac{1}{2}x\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right)-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$
$$+\sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x^3}}{3\sqrt[6]{3}}\right)-\frac{\log(8+x^3)}{\sqrt[3]{3}}$$
$$+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-x-\sqrt[3]{1-x^3}\right)+\frac{1}{2}3^{2/3}\log\left(-\frac{1}{2}3^{2/3}x-\sqrt[3]{1-x^3}\right)$$

output
```
(-x^3+1)^(1/3)+1/2*x*AppellF1(1/3,-1/3,1,4/3,x^3,-1/8*x^3)-3^(1/6)*arctan(
2/9*(-x^3+1)^(1/3)*3^(5/6)+1/3*3^(1/2))+3^(1/6)*arctan(1/3*(1-3^(2/3)*x/(-
x^3+1)^(1/3))*3^(1/2))-1/3*ln(x^3+8)*3^(2/3)+1/2*3^(2/3)*ln(3^(2/3)-(-x^3+
1)^(1/3))-ln(-x-(-x^3+1)^(1/3))+1/2*3^(2/3)*ln(-1/2*3^(2/3)*x-(-x^3+1)^(1/
3))-2/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.60.2 Mathematica [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

input
```
Integrate[(1 - x^3)^(1/3)/(2 + x),x]
```

output
```
Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

3.60.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{1-x^3}}{x+2}\,dx$$

$$\Big\downarrow \ 2581$$

$$\int \left(-\frac{2\sqrt[3]{1-x^3}x}{x^3+8}+\frac{4\sqrt[3]{1-x^3}}{x^3+8}+\frac{\sqrt[3]{1-x^3}x^2}{x^3+8}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right)-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[6]{3}\arctan\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}}{3\sqrt[6]{3}}+\frac{1}{\sqrt{3}}\right)+\sqrt[3]{1-x^3}-\frac{\log\left(x^3+8\right)}{\sqrt[3]{3}}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{2}3^{2/3}\log\left(-\sqrt[3]{1-x^3}-\frac{1}{2}3^{2/3}x\right)$$

input
```
Int[(1 - x^3)^(1/3)/(2 + x),x]
```

output
```
(1 - x^3)^(1/3) + (x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -1/8*x^3])/2 - (2*Ar
cTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + 3^(1/6)*ArcTan[(1 - (
3^(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 3^(1/6)*ArcTan[1/Sqrt[3] + (2*(1 -
x^3)^(1/3))/(3*3^(1/6))] - Log[8 + x^3]/3^(1/3) + (3^(2/3)*Log[3^(2/3) - (
1 - x^3)^(1/3)])/2 - Log[-x - (1 - x^3)^(1/3)] + (3^(2/3)*Log[-1/2*(3^(2/3
)*x) - (1 - x^3)^(1/3)])/2
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.60.4 Maple [F]

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{2+x}dx$$

input
```
int((-x^3+1)^(1/3)/(2+x),x)
```

output
```
int((-x^3+1)^(1/3)/(2+x),x)
```

3.60.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.60.6 Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+2}\,dx$$

input
```
integrate((-x**3+1)**(1/3)/(2+x),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

3.60.7 Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x+2}\,dx$$

input
```
integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

3.60.8 Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x+2}\,dx$$

input
```
integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx = \int \frac{(1-x^3)^{1/3}}{x+2}\,dx$$

input
```
int((1 - x^3)^(1/3)/(x + 2),x)
```

output
```
int((1 - x^3)^(1/3)/(x + 2), x)
```

3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = -\frac{x^2 \operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}} + \frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\arctan\left(\frac{\sqrt[3]{3}+2\sqrt[3]{2+x^3}}{3^{5/6}}\right)}{3^{5/6}} + \frac{\log(1-x^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}} - \frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{\sqrt[3]{3}}$$

output
```
-1/4*x^2*AppellF1(2/3,1,1/3,5/3,x^3,-1/2*x^3)*2^(2/3)+1/3*arctan(1/3*(3^(1
/3)+2*(x^3+2)^(1/3))*3^(1/6))*3^(1/6)+2/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2
)^(1/3))*3^(1/2))*3^(1/6)+1/18*ln(-x^3+1)*3^(2/3)+1/6*ln(3^(1/3)-(x^3+2)^(
1/3))*3^(2/3)-1/3*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

3.61.2 Mathematica [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$$

input
```
Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

output
```
Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

3.61.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(x^2+x+1)\sqrt[3]{x^3+2}}\,dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(-\frac{x}{(1-x^3)\sqrt[3]{x^3+2}}+\frac{2}{(1-x^3)\sqrt[3]{x^3+2}}-\frac{x^2}{(1-x^3)\sqrt[3]{x^3+2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^2\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\arctan\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\arctan\left(\frac{2\sqrt[3]{x^3+2}+\sqrt[3]{3}}{3^{5/6}}\right)}{3^{5/6}}+\frac{\log(1-x^3)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}$$

input
```
Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

output
```
-1/2*(x^2*AppellF1[2/3, 1, 1/3, 5/3, x^3, -1/2*x^3])/2^(1/3) + (2*ArcTan[(
1 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]])/3^(5/6) + ArcTan[(3^(1/3) + 2
*(2 + x^3)^(1/3))/3^(5/6)]/3^(5/6) + Log[1 - x^3]/(6*3^(1/3)) + Log[3^(1/3
) - (2 + x^3)^(1/3)]/(2*3^(1/3)) - Log[3^(1/3)*x - (2 + x^3)^(1/3)]/3^(1/3
)
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.61.4 Maple [F]

$$\int \frac{2+x}{(x^2+x+1)(x^3+2)^{\frac{1}{3}}}dx$$

input
```
int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

output
```
int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

3.61.5 Fricas [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)
```

3.61.6 Sympy [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

input
```
integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)
```

output
```
Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

3.61.7 Maxima [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

3.61.8 Giac [F]

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \frac{x+2}{(x^3+2)^{1/3}\,(x^2+x+1)}\,dx$$

input
```
int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)),x)
```

output
```
int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)), x)
```

3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

3.62.1 Optimal result

Integrand size = 38, antiderivative size = 25

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

output
```
1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

input
```
Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4),x]
```

output
```
Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

3.62.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{160x^3 + 30x^2 - 3x + 3}{320x^4 + 80x^3 - 12x^2 + 24x + 9} \, dx$$

$$\Big\downarrow \ 2020$$

$$\frac{1}{8} \log \left(320x^4 + 80x^3 - 12x^2 + 24x + 9\right)$$

input
```
Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

output
```
Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

3.62.3.1 Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

3.62.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{\ln\left(x^4+\frac{1}{4}x^3-\frac{3}{80}x^2+\frac{3}{40}x+\frac{9}{320}\right)}{8}$	22
default	$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$	24
norman	$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$	24
risch	$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$	24

input
```
int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURN
VERBOSE)
```

output
```
1/8*ln(x^4+1/4*x^3-3/80*x^2+3/40*x+9/320)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorit
hm="fricas")
```

output
```
1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{\log\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$$

input
```
integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

output
```
log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorit
hm="maxima")
```

output
```
1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorit
hm="giac")
```

output
```
1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)}{8}$$

input
```
int((30*x^2 - 3*x + 160*x^3 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

output
```
log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9)/8
```

3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx$

3.63.1 Optimal result

Integrand size = 33, antiderivative size = 59

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx = -\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}} + \frac{\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

output
```
-1/22*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2)+1/22*arctan(1/66*(800*x^3-40
*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx = \frac{1}{8}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3 + 320\#1^4\&, \frac{3\log(x-\#1)+12\log(x-\#1)\#1+20\log(x-\#1)\#1^2}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

input
```
Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

output
```
RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*L
og[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

3.63.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2502}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{20x^2+12x+3}{320x^4+80x^3-12x^2+24x+9}\,dx$$

$$\Big\downarrow \; 2502$$

$$\frac{\arctan\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

input
```
Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

output
```
-1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 40*x^2
 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

3.63.3.1 Defintions of rubi rules used

rule 2502
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

3.63.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\sqrt{11}\,\arctan\left(-\frac{20\sqrt{11}\,x^{2}}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}\,x^{3}}{33}\right)}{22}+\frac{\sqrt{11}\,\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)}{22}$	52
default	$\frac{i\sqrt{11}\,\ln\left(80x^{2}+\left(10i\sqrt{11}+10\right)x+3i\sqrt{11}-9\right)}{44}-\frac{i\sqrt{11}\,\ln\left(80x^{2}+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)}{44}$	62

input
```
int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNVERBOSE
)
```

output
```
1/22*11^(1/2)*arctan(-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+40
0/33*11^(1/2)*x^3)+1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right) + \frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)$$

input
```
integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fri
cas")
```

output
```
1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*
sqrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\sqrt{11}\cdot\left(2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)+2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

input
```
integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

output
```
sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

3.63.7 Maxima [F]

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \int \frac{20\,x^2+12\,x+3}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

input
```
integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="max
ima")
```

output
```
integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{22}\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}(40\,x-7)\right)\right)$$

input
```
integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="gia
c")
```

output
```
1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arct
an(-1/55*sqrt(11)*(40*x - 7)))
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{\sqrt{11}\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)}{22} + \frac{\sqrt{11}\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)}{22}$$

input
```
int((12*x + 20*x^2 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

output
```
(11^(1/2)*atan((8*11^(1/2)*x)/11 - (7*11^(1/2))/55))/22 + (11^(1/2)*atan((
5*11^(1/2)*x)/11 + (19*11^(1/2))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)
*x^3)/33))/22
```

3.64 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} \, dx$

3.64.1 Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} \, dx = 2\sqrt{11}\arctan\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right) + 2\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

output
```
2*ln(320*x^4+80*x^3-12*x^2+24*x+9)+2*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/
2)-2*arctan(1/66*(800*x^3-40*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} \, dx = \frac{1}{2}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3+320\#1^4\&, \frac{-21\log(x-\#1)-144\log(x-\#1)\#1-100\log(x-\#1)\#1^2+640\log(x-\#1)\#1^3}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

input
```
Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
 320*x^4),x]
```

output
```
RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 14
4*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1
+ 30*#1^2 + 160*#1^3) & ]/2
```

3.64.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2525, 27, 2502}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2560x^3-400x^2-576x-84}{320x^4+80x^3-12x^2+24x+9}dx$$

$$\Big\downarrow \text{2525}$$

$$\frac{\int -\frac{56320(20x^2+12x+3)}{320x^4+80x^3-12x^2+24x+9}dx}{1280}+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

$$\Big\downarrow \text{27}$$

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-44\int \frac{20x^2+12x+3}{320x^4+80x^3-12x^2+24x+9}dx$$

$$\Big\downarrow \text{2502}$$

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-44\left(\frac{\arctan\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}\right)$$

input
```
Int[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4),x]
```

output
```
-44*(-1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 4
0*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])) + 2*Log[9 + 24*x - 12*x^2 + 8
0*x^3 + 320*x^4]
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2502
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

3.64.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result
default	$4\left(\frac{i\sqrt{11}}{4}+\frac{1}{2}\right)\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)+4\left(\frac{1}{2}-\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+\left(10i\sqrt{11}+\right.\right.$
risch	$2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\sqrt{11}\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\right.$

input
```
int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_R
ETURNVERBOSE)
```

output
```
4*(1/4*I*11^(1/2)+1/2)*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)+4*(
1/2-1/4*I*11^(1/2))*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$$
$$= -2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right) - 2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right) + 2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="fricas")
```

output
```
-2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(
11)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9)
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$$
$$= \sqrt{11}\left(-2\,\mathrm{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right) - 2\,\mathrm{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right) + 2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)$$

input
```
integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),
x)
```

output
```
sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x
**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(
x**4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

3.64.7 Maxima [F]

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \int \frac{4\,(640\,x^3-100\,x^2-144\,x-21)}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="maxima")
```

output
```
4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 +
24*x + 9), x)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -2\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}(40\,x-7)\right)\right) + 2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="giac")
```

output
```
-2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan
(-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\,\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right) - 2\,\sqrt{11}\,\mathrm{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right) - 2\,\sqrt{11}\,\mathrm{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}\,x^2}{33}+\frac{5\,\sqrt{11}\,x}{11}+\frac{19\,\sqrt{11}}{22}\right)$$

input
```
int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
 9),x)
```

output
```
2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*
x)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2
))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4} dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sqrt{1-x^4}}{1+x^4} dx = \frac{1}{2}\arctan\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right) + \frac{1}{2}\text{arctanh}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

output
```
1/2*arctan(x*(x^2+1)/(-x^4+1)^(1/2))+1/2*arctanh(x*(-x^2+1)/(-x^4+1)^(1/2)
)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-x^4}}{1+x^4} dx = \left(\frac{1}{4}-\frac{i}{4}\right)\arctan\left(\frac{(1+i)x}{\sqrt{1-x^4}}\right) - \left(\frac{1}{4}+\frac{i}{4}\right)\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-x^4}}{x}\right)$$

input
```
Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

output
```
(1/4 - I/4)*ArcTan[((1 + I)*x)/Sqrt[1 - x^4]] - (1/4 + I/4)*ArcTan[((1/2 +
 I/2)*Sqrt[1 - x^4])/x]
```

3.65.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {921}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

$$\Big\downarrow \text{921}$$

$$\frac{1}{2}\arctan\left(\frac{x(x^2+1)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

input
```
Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

output
```
ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4
]]/2
```

3.65.3.1 Defintions of rubi rules used

rule 921
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

3.65.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\ln\left(\frac{(1+i)\sqrt{-x^4+1}+2ix}{x^2+i}\right)-\arctan\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{-x^4+1}}{x}\right)+\ln\left(2\right)\right)$
default	$-\frac{\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}$
elliptic	$-\frac{\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)}{4}$
trager	$\operatorname{RootOf}\left(8\_Z^2+4\_Z+1\right)\ln\left(-\frac{-4\operatorname{RootOf}\left(8\_Z^2+4\_Z+1\right)x+\sqrt{-x^4+1}-2x}{4\operatorname{RootOf}\left(8\_Z^2+4\_Z+1\right)x^2+x^2-1}\right)-\frac{\ln\left(-\frac{4\operatorname{RootOf}\left(8\_Z\right.}{4\operatorname{RootOf}\left(8\_Z\right.}\right.}{}$

input
```
int((-x^4+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/4-1/4*I)*(ln(((1+I)*(-x^4+1)^(1/2)+2*I*x)/(x^2+I))-arctan((1/2-1/2*I)*(
-x^4+1)^(1/2)/x)+ln(2))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx=-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}x-1}{x^4+1}\right)$$

input
```
integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

output
```
-1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-
x^4 + 1)*x - 1)/(x^4 + 1))
```

3.65.6 Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}{x^4+1}\,dx$$

input
```
integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

3.65.7 Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

3.65.8 Giac [F]

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

input
```
int((1 - x^4)^(1/2)/(x^4 + 1),x)
```

output
```
int((1 - x^4)^(1/2)/(x^4 + 1), x)
```

3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} dx$

3.66.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sqrt{1+x^4}}{1-x^4} dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

output
```
1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*arctanh(x*2^(1/2)/(x^4+1)^
(1/2))*2^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1+x^4}}{1-x^4} dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

input
```
Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
```

output
```
(ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]] + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]])/(
2*Sqrt[2])
```

3.66.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {920, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+1}}{1-x^4}dx$$

$$\Big\downarrow \text{920}$$

$$\int \frac{1}{1-\frac{4x^4}{(x^4+1)^2}}d\frac{x}{\sqrt{x^4+1}}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{2x^2}{x^4+1}}d\frac{x}{\sqrt{x^4+1}}+\frac{1}{2}\int \frac{1}{\frac{2x^2}{x^4+1}+1}d\frac{x}{\sqrt{x^4+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{2x^2}{x^4+1}}d\frac{x}{\sqrt{x^4+1}}+\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

input
```
Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
 + x^4]]/(2*Sqrt[2])
```

3.66.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 920
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[a/c
  Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b
, c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

3.66.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	siz
default	$\frac{\sqrt{2}\left(2\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)+\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)}{8}$	62
pseudoelliptic	$\frac{\sqrt{2}\left(2\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)+\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)}{8}$	62
elliptic	$\frac{\left(-\frac{\ln\left(-1+\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)}{4}+\frac{\ln\left(1+\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)}{4}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{x^4+1}}{2x}\right)}{2}\right)\sqrt{2}}{2}$	65
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x+\sqrt{x^4+1}}{x^2+1}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{x^4+1}}{(-1+x)(1+x)}\right)}{4}$	72

input
```
int((x^4+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*(2*arctan(x*2^(1/2)/(x^4+1)^(1/2))-arctanh((x^2+x+1)*2^(1/2)/(
x^4+1)^(1/2))+arctanh((x^2-x+1)*2^(1/2)/(x^4+1)^(1/2)))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

input
```
integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqr
t(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

3.66.6 Sympy [F]

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = -\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

input
```
integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

output
```
-Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

3.66.7 Maxima [F]

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

input
```
integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

3.66.8 Giac [F]

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx=\int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

input
```
integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

output
```
integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx=-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

input
```
int(-(x^4 + 1)^(1/2)/(x^4 - 1),x)
```

output
```
-int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx$

3.67.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx = \frac{1}{4}\sqrt{2-p}\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2+p}\operatorname{arctanh}\left(\frac{\sqrt{2+p}x}{\sqrt{1+px^2+x^4}}\right)$$

output
```
1/4*arctan(x*(2-p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2-p)^(1/2)+1/4*arctanh(x*(2
+p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2+p)^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx = -\frac{1}{4}\sqrt{-2-p}\arctan\left(\frac{\sqrt{-2-p}x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2-p}\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{1+px^2+x^4}}\right)$$

input
```
Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

output
```
-1/4*(Sqrt[-2 - p]*ArcTan[(Sqrt[-2 - p]*x)/Sqrt[1 + p*x^2 + x^4]]) + (Sqrt
[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

3.67.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2517, 1406, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{px^2+x^4+1}}{1-x^4}dx$$

$$\Big\downarrow \text{2517}$$

$$\int \frac{1}{-\frac{(4-p^2)x^4}{(px^2+x^4+1)^2}-\frac{2px^2}{px^2+x^4+1}+1}d\frac{x}{\sqrt{px^2+x^4+1}}$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{4}(4-p^2)\int \frac{1}{-\frac{(4-p^2)x^2}{x^4+px^2+1}-p+2}d\frac{x}{\sqrt{x^4+px^2+1}}-\frac{1}{4}(4-p^2)\int \frac{1}{-\frac{(4-p^2)x^2}{x^4+px^2+1}-p-2}d\frac{x}{\sqrt{x^4+px^2+1}}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}(4-p^2)\int \frac{1}{-\frac{(4-p^2)x^2}{x^4+px^2+1}-p+2}d\frac{x}{\sqrt{x^4+px^2+1}}+\frac{(4-p^2)\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right)}{4\sqrt{2-p}(p+2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(4-p^2)\arctan\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right)}{4\sqrt{2-p}(p+2)}+\frac{(4-p^2)\text{arctanh}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)}{4(2-p)\sqrt{p+2}}$$

input
```
Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

output
```
((4 - p^2)*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/(4*Sqrt[2 - p]*(
2 + p)) + ((4 - p^2)*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/(4*(2
 - p)*Sqrt[2 + p])
```

3.67.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 2517
```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Simp[a/d   Subst[Int[1/(1 - 2*b*x^
2 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[
a*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

3.67.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\left(\frac{4\left(\frac{1}{4}+\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{4+2p}}\right)}{\sqrt{4+2p}}+\frac{4\left(\frac{1}{4}-\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{2p-4}}\right)}{\sqrt{2p-4}}\right)\sqrt{2}}{2}$
pseudoelliptic	$\frac{\left(\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}-2-2x^2+x(p-2)}{(1+x)^2}\right)+\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}+2+2x^2+x(p-2)}{(-1+x)^2}\right)+2\ln(2)\right)\sqrt{2+p}}{8}-\frac{\left(\ln\left(\frac{\sqrt{p-2}\,\sqrt{x^4+p}}{x^2}\right.\right.}{}$
default	$-\frac{\left(-\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}-2-2x^2+x(p-2)}{(1+x)^2}\right)-\ln\left(\frac{\sqrt{2+p}\,\sqrt{x^4+p\,x^2+1}+2+2x^2+x(p-2)}{(-1+x)^2}\right)-2\ln(2)\right)\sqrt{2+p}}{8}-\frac{\left(\ln\left(\frac{\sqrt{p-2}\,\sqrt{x}}{}\right.\right.}{}$

input
```
int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*(1/4+1/8*p)/(4+2*p)^(1/2)*arctanh((x^4+p*x^2+1)^(1/2)*2^(1/2)/x/(4+
2*p)^(1/2))+4*(1/4-1/8*p)/(2*p-4)^(1/2)*arctanh((x^4+p*x^2+1)^(1/2)*2^(1/2
)/x/(2*p-4)^(1/2)))*2^(1/2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.79

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2\,(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2\,x^2+1}\right)\right.$$
$$+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2\,(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2\,x^2+1}\right), \frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right.$$
$$+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2\,(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2\,x^2+1}\right),$$
$$-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)$$
$$+\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2\,(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2\,x^2+1}\right), \frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right.$$
$$\left.-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)\right]$$

input
```
integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

output
```
[1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
 - 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2
 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt
(-p + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*lo
g((x^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 -
 2*x^2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/
((p + 2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x
^2 + 1)*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqr
t(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*
x^2 + 1)*sqrt(-p - 2)/((p + 2)*x))]
```

3.67.6 Sympy [F]

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = -\int \frac{\sqrt{px^2+x^4+1}}{x^4-1}\,dx$$

input
```
integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

output
```
-Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

3.67.7 Maxima [F]

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+px^2+1}}{x^4-1}\,dx$$

input
```
integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

3.67.8 Giac [F]

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \int -\frac{\sqrt{x^4+px^2+1}}{x^4-1}\,dx$$

input
```
integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

output
```
integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = -\int \frac{\sqrt{x^4+p\,x^2+1}}{x^4-1}\,dx$$

input
```
int(-(p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1),x)
```

output
```
-int((p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1), x)
```

3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} dx$

3.68.1 Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} dx = -\frac{\sqrt{p+\sqrt{4+p^2}}\arctan\left(\frac{\sqrt{p+\sqrt{4+p^2}}x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4+p^2}}\text{arctanh}\left(\frac{\sqrt{-p+\sqrt{4+p^2}}x\left(p+\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}$$

output
```
1/4*arctanh(1/4*x*(p-2*x^2+(p^2+4)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
/(-x^4+p*x^2+1)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)-1/4*arctan(1/4*x*(
p-2*x^2-(p^2+4)^(1/2))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/(-x^4+p*x^2+1)^(1/2
))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} dx = \frac{1}{4}i\left(\sqrt{-2i-p}\arctan\left(\frac{\sqrt{-2i-p}x}{\sqrt{1+px^2-x^4}}\right) - \sqrt{2i-p}\arctan\left(\frac{\sqrt{2i-p}x}{\sqrt{1+px^2-x^4}}\right)\right)$$

input
```
Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

output
```
(I/4)*(Sqrt[-2*I - p]*ArcTan[(Sqrt[-2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]] - S
qrt[2*I - p]*ArcTan[(Sqrt[2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]])
```

3.68.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2518}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{px^2-x^4+1}}{x^4+1}dx$$

$$\Big\downarrow \text{2518}$$

$$\frac{\sqrt{\sqrt{p^2+4}-p}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{p^2+4}-p}x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\arctan\left(\frac{\sqrt{\sqrt{p^2+4}+p}x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

input
```
Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

output
```
-1/2*(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[
4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/Sqrt[2] + (Sqrt[-p
+ Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] -
2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

3.68.3.1 Defintions of rubi rules used

rule 2518
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(-a)*(Sqrt[b + q]/(2*Sqrt[2]*Rt[(-
a)*c, 2]*d))*ArcTan[Sqrt[b + q]*x*((b - q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x] + Simp[a*(Sqrt[-b + q]/(2*Sqrt[2]*Rt[(-a)
*c, 2]*d))*ArcTanh[Sqrt[-b + q]*x*((b + q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d
+ a*e, 0] && NegQ[a*c]
```

3.68.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{i\sqrt{2i+p}\,\sqrt{2i-p}\,\ln(2)+i\sqrt{2i+p}\,\sqrt{2i-p}\,\ln\left(\frac{\sqrt{2i+p}\,\sqrt{-x^4+p\,x^2+1}+2x\left(i+\frac{p}{2}\right)}{x^2+i}\right)+ip\arctan\left(\frac{\sqrt{-x^4+p\,x^2+1}}{x\sqrt{2i-p}}\right)+2\arctan\left(\frac{\sqrt{-x}}{x}\right.}{4\sqrt{2i-p}}$
default	$\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\,\sqrt{p^2+4}\,\ln\left(\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}-\frac{-x^4+p\,x^2+1}{x^2}-\sqrt{p^2+4}\right)}{16}+\frac{\sqrt{p^2+4}\left(p+\sqrt{p^2+4}\right)\arctan\left(\frac{2\sqrt{p+\sqrt{p^2+4}-}}{2\sqrt{-}}\right.}{8\sqrt{-p+\sqrt{p^2+4}}}\right.$
elliptic	$\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\,\sqrt{p^2+4}\,\ln\left(\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}-\frac{-x^4+p\,x^2+1}{x^2}-\sqrt{p^2+4}\right)}{16}+\frac{\sqrt{p^2+4}\left(p+\sqrt{p^2+4}\right)\arctan\left(\frac{2\sqrt{p+\sqrt{p^2+4}-}}{2\sqrt{-}}\right.}{8\sqrt{-p+\sqrt{p^2+4}}}\right.$

input
```
int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(I*(2*I+p)^(1/2)*(2*I-p)^(1/2)*ln(2)+I*(2*I+p)^(1/2)*(2*I-p)^(1/2)*ln
(((2*I+p)^(1/2)*(-x^4+p*x^2+1)^(1/2)+2*x*(I+1/2*p))/(x^2+I))+I*p*arctan((-
x^4+p*x^2+1)^(1/2)/x/(2*I-p)^(1/2))+2*arctan((-x^4+p*x^2+1)^(1/2)/x/(2*I-p
)^(1/2)))/(2*I-p)^(1/2)
```

3.68.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx$$
$$= \frac{1}{8}\sqrt{-p+2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2+i)-(i\,x^3-x)\sqrt{-p+2i}}{x^4+1}\right)$$
$$-\frac{1}{8}\sqrt{-p+2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2+i)-(-i\,x^3+x)\sqrt{-p+2i}}{x^4+1}\right)$$
$$-\frac{1}{8}\sqrt{-p-2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2-i)-(i\,x^3+x)\sqrt{-p-2i}}{x^4+1}\right)$$
$$+\frac{1}{8}\sqrt{-p-2i}\log\left(-\frac{\sqrt{-x^4+px^2+1}(x^2-i)-(-i\,x^3-x)\sqrt{-p-2i}}{x^4+1}\right)$$

input
```
integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(-p + 2*I)*log(-(sqrt(-x^4 + p*x^2 + 1)*(x^2 + I) - (I*x^3 - x)*sq
rt(-p + 2*I))/(x^4 + 1)) - 1/8*sqrt(-p + 2*I)*log(-(sqrt(-x^4 + p*x^2 + 1)
*(x^2 + I) - (-I*x^3 + x)*sqrt(-p + 2*I))/(x^4 + 1)) - 1/8*sqrt(-p - 2*I)*
log(-(sqrt(-x^4 + p*x^2 + 1)*(x^2 - I) - (I*x^3 + x)*sqrt(-p - 2*I))/(x^4
+ 1)) + 1/8*sqrt(-p - 2*I)*log(-(sqrt(-x^4 + p*x^2 + 1)*(x^2 - I) - (-I*x^
3 - x)*sqrt(-p - 2*I))/(x^4 + 1))
```

3.68.6 Sympy [F]

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{px^2-x^4+1}}{x^4+1}\,dx$$

input
```
integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
```

output
```
Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

3.68.7 Maxima [F]

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

input
```
integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

3.68.8 Giac [F]

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

input
```
integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = \int \frac{\sqrt{-x^4+p\,x^2+1}}{x^4+1}\,dx$$

input
```
int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1),x)
```

output
```
int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1), x)
```

3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

3.69.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{a\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\arctan\left(\sqrt[4]{-1+x^2}\right) + \frac{a\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\text{arctanh}\left(\sqrt[4]{-1+x^2}\right)$$

output
```
-b*arctan((x^2-1)^(1/4))+b*arctanh((x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(x^2-
1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/
2)
```

3.69.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.96

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{x\left(bx\sqrt[4]{1-x^2}(-2+x^2)\,\text{AppellF1}\left(1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right) - \frac{24a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)}{6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right)+\text{Ap}\right.}\right.}{4(-2+x^2)\sqrt[4]{-1+x^2}}$$

input
```
Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

output
```
(x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (2
4*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2,
x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2
, 5/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

3.69.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1343, 309, 353, 73, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{x^2-1}}dx$$

$$\Big\downarrow \text{1343}$$

$$a\int \frac{1}{(2-x^2)\sqrt[4]{x^2-1}}dx+b\int \frac{x}{(2-x^2)\sqrt[4]{x^2-1}}dx$$

$$\Big\downarrow \text{309}$$

$$b\int \frac{x}{(2-x^2)\sqrt[4]{x^2-1}}dx+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}b\int \frac{1}{(2-x^2)\sqrt[4]{x^2-1}}dx^2+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{73}$$

$$2b\int \frac{x^4}{1-x^8}d\sqrt[4]{x^2-1}+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{827}$$

$$2b\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{x^2-1}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{x^2-1}\right)+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{216}$$

$$2b\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{x^2-1}-\frac{1}{2}\arctan\left(\sqrt[4]{x^2-1}\right)\right)+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}\right)+2b\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{x^2-1}\right)\right)$$

input
```
Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

output
```
a*(ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(
-1 + x^2)^(1/4))]/(2*Sqrt[2])) + 2*b*(-1/2*ArcTan[(-1 + x^2)^(1/4)] + ArcT
anh[(-1 + x^2)^(1/4)]/2)
```

3.69.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.69.4 Maple [F]

$$\int \frac{bx+a}{\left(-x^{2}+2\right)\left(x^{2}-1\right)^{\frac{1}{4}}}dx$$

input
```
int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

output
```
int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.69.6 Sympy [F]

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\int \frac{a}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx - \int \frac{bx}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

input
```
integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
```

output
```
-Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(
b*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

3.69.7 Maxima [F]

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{bx+a}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

input
```
integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
-integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.69.8 Giac [F]

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{bx+a}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

input
```
integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{a+b\,x}{(x^2-1)^{1/4}\,(x^2-2)}\,dx$$

input
```
int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
```

output
```
int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)} \, dx$

3.70.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)} \, dx = \frac{a \arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b \arctan\left(\sqrt[4]{-1-x^2}\right) + \frac{a \text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b \text{arctanh}\left(\sqrt[4]{-1-x^2}\right)$$

output
```
b*arctan((-x^2-1)^(1/4))-b*arctanh((-x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(-x^
2-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(-x^2-1)^(1/4)*2^(1/2))*2^
(1/2)
```

3.70.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)} \, dx = \frac{x\left(bx\sqrt[4]{1+x^2}\,\text{AppellF1}\left(1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right) - \frac{24a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)}{(2+x^2)\left(-6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\right.\right.\right.}\right.}{4\sqrt[4]{-1-x^2}}$$

input
```
Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

output
```
(x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2] - (24*a*App
ellF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4,
 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^
2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(4*(-1 - x^2)^(1/4))
```

3.70.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1343, 309, 353, 73, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt[4]{-x^2-1}\,(x^2+2)}dx$$

$$\Big\downarrow \text{1343}$$

$$a\int \frac{1}{\sqrt[4]{-x^2-1}\,(x^2+2)}dx+b\int \frac{x}{\sqrt[4]{-x^2-1}\,(x^2+2)}dx$$

$$\Big\downarrow \text{309}$$

$$b\int \frac{x}{\sqrt[4]{-x^2-1}\,(x^2+2)}dx+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}b\int \frac{1}{\sqrt[4]{-x^2-1}\,(x^2+2)}dx^2+a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{73}$$

$$a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}\right)-2b\int \frac{x^4}{1-x^8}d\sqrt[4]{-x^2-1}$$

$$\Big\downarrow \text{827}$$

$$a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}\right)-2b\left(\frac{1}{2}\int\frac{1}{1-x^4}d\sqrt[4]{-x^2-1}-\frac{1}{2}\int\frac{1}{x^4+1}d\sqrt[4]{-x^2-1}\right)$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}\right)-2b\left(\frac{1}{2}\int\frac{1}{1-x^4}d\sqrt[4]{-x^2-1}-\frac{1}{2}\arctan\left(\sqrt[4]{-x^2-1}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}\right)-2b\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{-x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{-x^2-1}\right)\right)$$

input
```
Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

output
```
a*(ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(
-1 - x^2)^(1/4))]/(2*Sqrt[2])) - 2*b*(-1/2*ArcTan[(-1 - x^2)^(1/4)] + ArcT
anh[(-1 - x^2)^(1/4)]/2)
```

3.70.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.70.4 Maple [F]

$$\int \frac{bx+a}{\left(-x^{2}-1\right)^{\frac{1}{4}} \left(x^{2}+2\right)}dx$$

input
```
int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

output
```
int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

3.70.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

output
```
Timed out
```

3.70.6 Sympy [F]

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \int \frac{a+bx}{\sqrt[4]{-x^2-1}\,(x^2+2)}\,dx$$

input
```
integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
```

output
```
Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

3.70.7 Maxima [F]

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(-x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

3.70.8 Giac [F]

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx=\int \frac{bx+a}{(x^2+2)(-x^2-1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
```

output
```
integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx=\int \frac{a+b\,x}{(-x^2-1)^{1/4}\,(x^2+2)}\,dx$$

input
```
int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)),x)
```

output
```
int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)), x)
```

3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)}\,dx$

3.71.1 Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \frac{b\arctan\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\arctan\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\,\text{arctanh}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\,\text{arctanh}\left(\frac{1+\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)$$

output
```
1/2*a*arctan((1-(-x^2+1)^(1/2))/x/(-x^2+1)^(1/4))+1/2*a*arctanh((1+(-x^2+1
)^(1/2))/x/(-x^2+1)^(1/4))+1/2*b*arctan(1/2*(1-(-x^2+1)^(1/2))/(-x^2+1)^(1
/4)*2^(1/2))*2^(1/2)+1/2*b*arctanh(1/2*(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/4)*2
^(1/2))*2^(1/2)
```

3.71.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \frac{1}{4}bx^2\,\text{AppellF1}\left(1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right) - \frac{6ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\,(-2+x^2)\left(6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right)+\text{AppellF1}\left(\frac{3}{2},\frac{5}{4}\right.\right.\right.}$$

input
```
Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

output
```
(b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1
, 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3
/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1
[3/2, 5/4, 1, 5/2, x^2, x^2/2])))
```

3.71.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1343, 308, 348}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}dx$$

$$\Big\downarrow \text{1343}$$

$$a\int \frac{1}{\sqrt[4]{1-x^2}\,(2-x^2)}dx+b\int \frac{x}{\sqrt[4]{1-x^2}\,(2-x^2)}dx$$

$$\Big\downarrow \text{308}$$

$$b\int \frac{x}{\sqrt[4]{1-x^2}\,(2-x^2)}dx+a\left(\frac{1}{2}\arctan\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)\right)$$

$$\Big\downarrow \text{348}$$

$$a\left(\frac{1}{2}\arctan\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)\right)+b\left(\frac{\arctan\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{\text{arctanh}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}\right)$$

input
```
Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

output
```
b*(ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))]/Sqrt[2] + ArcTanh
[(1 + Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))]/Sqrt[2]) + a*(ArcTan[(1 -
Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))]/2 + ArcTanh[(1 + Sqrt[1 - x^2])/(x*(1
- x^2)^(1/4))]/2)
```

3.71.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

rule 348
```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.71.4 Maple [F]

$$\int \frac{bx + a}{(-x^2 + 1)^{\frac{1}{4}} (-x^2 + 2)} dx$$

input
```
int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

output
```
int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

3.71.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

output
```
Timed out
```

3.71.6 Sympy [F]

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = -\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx - \int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

input
```
integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

output
```
-Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(
b*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

3.71.7 Maxima [F]

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \int -\frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
```

output
```
-integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

3.71.8 Giac [F]

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \int -\frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
```

output
```
integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = \int -\frac{a+b\,x}{(1-x^2)^{1/4}\,(x^2-2)}\,dx$$

input
```
int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)),x)
```

output
```
int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)), x)
```

3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}(2+x^2)}\,dx$

3.72.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = -\frac{b\arctan\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}-\frac{1}{2}a\arctan\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)-\frac{1}{2}a\text{arctanh}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)-\frac{b\text{arctanh}\left(\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*a*arctan((1+(x^2+1)^(1/2))/x/(x^2+1)^(1/4))-1/2*a*arctanh((1-(x^2+1)^
(1/2))/x/(x^2+1)^(1/4))-1/2*b*arctan(1/2*(1-(x^2+1)^(1/2))/(x^2+1)^(1/4)*2
^(1/2))*2^(1/2)-1/2*b*arctanh(1/2*(1+(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(1/2))
*2^(1/2)
```

3.72.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \frac{1}{4}bx^2\,\text{AppellF1}\left(1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right)-\frac{6ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{1+x^2}\,(2+x^2)\left(-6\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\frac{x^2}{2}\right)+\text{AppellF}\right.\right.}$$

input
```
Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

output
```
(b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2])/4 - (6*a*x*AppellF1[1/2, 1/
4, 1, 3/2, -x^2, -1/2*x^2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1
/4, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2
*x^2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))
```

3.72.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1343, 308, 348}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt[4]{x^2+1}\,(x^2+2)}dx$$

$$\Big\downarrow \text{1343}$$

$$a\int \frac{1}{\sqrt[4]{x^2+1}\,(x^2+2)}dx+b\int \frac{x}{\sqrt[4]{x^2+1}\,(x^2+2)}dx$$

$$\Big\downarrow \text{308}$$

$$b\int \frac{x}{\sqrt[4]{x^2+1}\,(x^2+2)}dx+a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)\right)$$

$$\Big\downarrow \text{348}$$

$$a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)\right)+b\left(-\frac{\arctan\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}\right)$$

input
```
Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

output
```
a*(-1/2*ArcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))] - ArcTanh[(1 - Sqr
t[1 + x^2])/(x*(1 + x^2)^(1/4))]/2) + b*(-(ArcTan[(1 - Sqrt[1 + x^2])/(Sqr
t[2]*(1 + x^2)^(1/4))]/Sqrt[2]) - ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(1
+ x^2)^(1/4))]/Sqrt[2])
```

3.72.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

rule 348
```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.72.4 Maple [F]

$$\int \frac{bx+a}{(x^2+1)^{\frac{1}{4}}(x^2+2)}dx$$

input
```
int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

output
```
int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

3.72.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

output
```
Timed out
```

3.72.6 Sympy [F]

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{a+bx}{\sqrt[4]{x^2+1}\,(x^2+2)}\,dx$$

input
```
integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
```

output
```
Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

3.72.7 Maxima [F]

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

3.72.8 Giac [F]

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{bx+a}{(x^2+2)(x^2+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
```

output
```
integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = \int \frac{a+b\,x}{(x^2+1)^{1/4}\,(x^2+2)}\,dx$$

input
```
int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)),x)
```

output
```
int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)), x)
```

3.73 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)} dx$

3.73.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\operatorname{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

output
```
-1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(
1/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^
(1/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \frac{1}{8}x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right)$$

input
```
Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

output
```
(x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

3.73.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$$

$$\Big\downarrow \; 986$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

input
```
Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

output
```
-1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + A
rcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)
/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

3.73.3.1 Defintions of rubi rules used

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

3.73.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.90 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right.}{2\sqrt{-x^3+1}}\right.}{36}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right.}{2\sqrt{-x^3+1}}\right.}{36}$
trager	Expression too large to display

input
```
int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^
(1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha
^2+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*
3^(1/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootO
f(_Z^3-4))
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 1116, normalized size of antiderivative = 8.79

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752*x^6 - 518
4*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*x^
5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2*
x) - 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*
(x^7 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 + x^5
) + 144*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/(x^9 - 12*x^6 + 48*x^
3 - 64)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752
*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(
x^8 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7
+ x^4 - 2*x) - 2*x) - sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 -
 sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)
*x^5 + x^5) - 144*sqrt(3)*(-5*I*x^6 - 20*I*x^3 + 16*I)) + 2304)/(x^9 - 12*
x^6 + 48*x^3 - 64)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-(72
*x^9 + 4752*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 -
 sqrt(-3)*(x^8 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 + sqr
t(-3)*(x^7 + x^4 - 2*x) - 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7
 + 16*x^4 + sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) - 648*432^(1/6)*(-1)^(5/6
)*(sqrt(-3)*x^5 - x^5) + 144*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/
(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1
)*log(-(72*x^9 + 4752*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*...
```

3.73.6 Sympy [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

input
```
integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

3.73.7 Maxima [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.73.8 Giac [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.14

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

input
```
int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

output
```
- (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3
^(1/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2
)*(2^(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^
(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3
/2)/(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/
2)*1i)/2 + 1/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x
^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/
2)*1i)/2 - 1/2) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(...
```

3.73. $\int \frac{x}{\sqrt{1-x^3}(4-x^3)}\,dx$

3.74 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx$

3.74.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\frac{\arctan\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}} - \frac{\arctan\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

output
```
-1/6*arctan((1+2^(1/3)*d^(1/3)*x)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*ar
ctan((d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arctanh((1-2^(1/3)*d^(1/3)*x)*3
^(1/2)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/18*arctanh(1/3*(d*x^3-1)
^(1/2)*3^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \frac{x^2\sqrt{1-dx^3}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},dx^3,\frac{dx^3}{4}\right)}{8\sqrt{-1+dx^3}}$$

input
```
Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

output
```
(x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt
[-1 + d*x^3])
```

3.74.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {987}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(4-dx^3)\sqrt{dx^3-1}}dx$$

$$\Big\downarrow \text{987}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\arctan\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

input
```
Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

output
```
-1/3*ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(2^(2/3)*d^(2/3)) -
ArcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/
3)*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqr
t[-1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

3.74.3.1 Defintions of rubi rules used

rule 987
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(-q)*(ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]]/(9*2^(2/3
)*b*Rt[-c, 2])), x] + (-Simp[q*(ArcTan[Rt[-c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c
+ d*x^3])]/(3*2^(2/3)*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[Sqrt[c + d*x^3]/(
Sqrt[3]*Rt[-c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[
Sqrt[3]*Rt[-c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b
*Rt[-c, 2])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*
b*c - a*d, 0] && NegQ[c]
```

3.74.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.53

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\left(-2\_\alpha^{2}d+i\sqrt{3}\_\alpha d^{\frac{2}{3}}-i\sqrt{3}d^{\frac{1}{3}}+\_\alpha d^{\frac{2}{3}}\right.}{2\_\alpha}\right.}{9}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\left(-2\_\alpha^{2}d+i\sqrt{3}\_\alpha d^{\frac{2}{3}}-i\sqrt{3}d^{\frac{1}{3}}+\_\alpha d^{\frac{2}{3}}\right.}{2\_\alpha}\right.}{9}$

input
```
int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/
3))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1
/2*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*
_alpha^2*d+I*3^(1/2)*_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+_alpha*d^(2/3)+d^(1/
3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/
2)*d^(1/3))^(1/2),1/3*I*3^(1/2)*_alpha^2*d^(2/3)-1/6*I*3^(1/2)*_alpha*d^(1
/3)-1/6*I*3^(1/2)+1/2*_alpha*d^(1/3)-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)
-1/2*I*3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(110) = 220$.

Time = 0.39 (sec) , antiderivative size = 1669, normalized size of antiderivative = 10.63

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^
6 - 72*d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x + sqrt(-3)*(d^5
*x^7 + d^4*x^4 - 2*d^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d^3
*x^5 - 8*d^2*x^2 - sqrt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(-4))^(1
/3) + 6*sqrt(d*x^3 - 1)*(648*(1/432)^(5/6)*(sqrt(-3)*d^5*x^5 - d^5*x^5)*(d
^(-4))^(5/6) + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) -
(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x + sqrt(-3)*(d^3*x^7 + 16*d^2*x
^4 - 8*d*x))*(d^(-4))^(1/6)) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))
 + 1/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*
x^6 - 72*d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x + sqrt(-3)*(d
^5*x^7 + d^4*x^4 - 2*d^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d
^3*x^5 - 8*d^2*x^2 - sqrt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(-4))^
(1/3) - 6*sqrt(d*x^3 - 1)*(648*(1/432)^(5/6)*(sqrt(-3)*d^5*x^5 - d^5*x^5)*
(d^(-4))^(5/6) + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
- (1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x + sqrt(-3)*(d^3*x^7 + 16*d^2
*x^4 - 8*d*x))*(d^(-4))^(1/6)) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64
)) - 1/36*(1/432)^(1/6)*(sqrt(-3) - 1)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^
2*x^6 - 72*d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x - sqrt(-3)*
(d^5*x^7 + d^4*x^4 - 2*d^3*x))*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7
*d^3*x^5 - 8*d^2*x^2 + sqrt(-3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2))*(d^(...
```

3.74.6 Sympy [F]

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

input
```
integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
```

output
```
-Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

3.74.7 Maxima [F]

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3-1}(dx^3-4)}\,dx$$

input
```
integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

3.74.8 Giac [F]

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3-1}(dx^3-4)}\,dx$$

input
```
integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 15.03 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.11

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = \frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(54\sqrt{d\,x^3-1}+54\sqrt{3}-54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)\left(\sqrt{d\,x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)^3}{\left(2^{2/3}-d^{1/3}\,x\right)^6}\right)}{2916\,d^{2/3}} + \frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{d\,x^3-1}+2\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\sqrt{3}-108\sqrt{d\,x^3-1}+54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,162\mathrm{i}\right)}{\left(2^{2/3}+2\,d^{1/3}\,x-2^{2/3}\sqrt{3}\,1\mathrm{i}\right)^6}\right)}{2916\,d^{2/3}} + \frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{d\,x^3-1}-2\sqrt{3}-2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\sqrt{d\,x^3-1}+108\sqrt{3}+54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x-2^{1/3}\,d^{1/3}\,x\,162\mathrm{i}\right)}{\left(2^{2/3}+2\,d^{1/3}\,x+2^{2/3}\sqrt{3}\,1\mathrm{i}\right)^6}\right)}{2916\,d^{2/3}}$$

input

```
int(-x/((d*x^3 - 1)^(1/2)*(d*x^3 - 4)),x)
```

output

```
(3^(1/2)*314928^(1/3)*log(((54*(d*x^3 - 1)^(1/2) + 54*3^(1/2) - 54*2^(1/3)
*3^(1/2)*d^(1/3)*x)*((d*x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*d^(1/3)
*x)^3)/(2^(2/3) - d^(1/3)*x)^6))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*lo
g(((2*(d*x^3 - 1)^(1/2) + 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i + 2^(1/3)*3^(1/
2)*d^(1/3)*x)^3*(108*3^(1/2) - 108*(d*x^3 - 1)^(1/2) + 2^(1/3)*d^(1/3)*x*1
62i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3) - 2^(2/3)*3^(1/2)*1i + 2*d^(
1/3)*x)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*d^(2/3)) + (3^(1/2)*314928^
(1/3)*log(((2*(d*x^3 - 1)^(1/2) - 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i - 2^(1/
3)*3^(1/2)*d^(1/3)*x)^3*(108*(d*x^3 - 1)^(1/2) + 108*3^(1/2) - 2^(1/3)*d^(
1/3)*x*162i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3)*3^(1/2)*1i + 2^(2/3)
 + 2*d^(1/3)*x)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*d^(2/3))
```

3.75 $\int \frac{x}{\sqrt{-1+x^3}(8+x^3)} dx$

3.75.1 Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) + \frac{1}{18}\arctan\left(\frac{1}{3}\sqrt{-1+x^3}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

output
```
1/18*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))+1/18*arctan(1/3*(x^3-1)^(1/2))-1/18
*arctanh((1-x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \frac{x^2\sqrt{1-x^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right)}{16\sqrt{-1+x^3}}$$

input
```
Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

output
```
(x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -1/8*x^3])/(16*Sqrt[-1
+ x^3])
```

3.75.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {988, 25, 946, 73, 216, 2563, 216, 2570, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3-1}\,(x^3+8)}dx$$

$$\Big\downarrow \text{988}$$

$$-\frac{1}{12}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx-\frac{1}{12}\int -\frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx-\frac{1}{4}\int -\frac{x^2}{\sqrt{x^3-1}\,(x^3+8)}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{12}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx+\frac{1}{12}\int \frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx+\frac{1}{4}\int \frac{x^2}{\sqrt{x^3-1}\,(x^3+8)}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{12}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx+\frac{1}{12}\int \frac{1}{\sqrt{x^3-1}\,(x^3+8)}dx^3+\frac{1}{12}\int \frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{12}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx+\frac{1}{6}\int \frac{1}{x^6+9}d\sqrt{x^3-1}+\frac{1}{12}\int \frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{12}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx+\frac{1}{12}\int \frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx+\frac{1}{18}\arctan\left(\frac{\sqrt{x^3-1}}{3}\right)$$

$$\Big\downarrow \text{2563}$$

$$\frac{1}{6}\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}+\frac{1}{12}\int \frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx+\frac{1}{18}\arctan\left(\frac{\sqrt{x^3-1}}{3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{12}\int \frac{-x^2+2x+2}{(x^2-2x+4)\sqrt{x^3-1}}dx+\frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\arctan\left(\frac{\sqrt{x^3-1}}{3}\right)$$

$$\Big\downarrow \text{2570}$$

$$\frac{1}{3}\int \frac{1}{\frac{6(1-x)^2}{x^3-1}-2}d\frac{1-x}{\sqrt{x^3-1}}+\frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\arctan\left(\frac{\sqrt{x^3-1}}{3}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{18}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\arctan\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

input
```
Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

output
```
ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Ar
cTanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.75.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.02 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.69

method	result
default	$-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{x^3-1}}+\frac{i\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$
trager	$324\,\mathrm{RootOf}\left(104976\_Z^4-324\_Z^2+1\right)^3\ln\left(\frac{-629856\,\mathrm{RootOf}\left(104976\_Z^4-324\_Z^2+1\right)^5x^2+2519424x\,\mathrm{RootO}$
elliptic	$\frac{i\sqrt{-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{i\sqrt{3}+3}+\frac{i\sqrt{3}}{i\sqrt{3}+3}}\sqrt{3}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\left(1+i\sqrt{3}\right)\sqrt{3}}{6}+\frac{i\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+}{\frac{3}{2}-}}\right.}{6\sqrt{x^3-1}}$

input
```
int(x/(x^3+8)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)
,1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/9*I*
(1/2-1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/
2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2)
)/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),1/6*I*(1+I*3^(1/2))*3^(1/2)+1/3*I*3^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/9*I*(1/2+1/2*I*3^(1/2))*(-3/2-1
/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2
)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6
*I*(1-I*3^(1/2))*3^(1/2)-2/3*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

3.75.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 723, normalized size of antiderivative = 9.77

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx =$$

$$-\frac{1}{216}\sqrt{2}\sqrt{i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(7ix^3-12ix^2-12i}{x^6-}\right.$$

$$+\frac{1}{216}\sqrt{2}\sqrt{i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(-7ix^3+12ix^2+12}{x^6}\right.$$

$$+\frac{1}{216}\sqrt{2}\sqrt{-i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(7ix^3-12ix^2-12}{x^6}\right.$$

$$-\frac{1}{216}\sqrt{2}\sqrt{-i\sqrt{3}+1}\log\left(\frac{2x^6+6x^5-150x^4+176x^3+12x^2-3\sqrt{x^3-1}\big(\sqrt{3}\sqrt{2}(-7ix^3+12ix^2+}{x}\right.$$

$$+\frac{1}{54}\arctan\left(\frac{(x^3-12x^2-6x-10)\sqrt{x^3-1}}{6(x^4-x^3-x+1)}\right)$$

input
```
integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/216*sqrt(2)*sqrt(I*sqrt(3) + 1)*log((2*x^6 + 6*x^5 - 150*x^4 + 176*x^3
+ 12*x^2 - 3*sqrt(x^3 - 1)*(sqrt(3)*sqrt(2)*(7*I*x^3 - 12*I*x^2 - 12*I*x +
 8*I) + sqrt(2)*(2*x^4 - 5*x^3 - 24*x^2 + 28*x + 8))*sqrt(I*sqrt(3) + 1) -
 18*sqrt(3)*(-I*x^5 + I*x^4 + 6*I*x^3 - 2*I*x^2 - 4*I*x) + 168*x - 160)/(x
^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) + 1/216*sqrt(2)*sqrt(I
*sqrt(3) + 1)*log((2*x^6 + 6*x^5 - 150*x^4 + 176*x^3 + 12*x^2 - 3*sqrt(x^3
 - 1)*(sqrt(3)*sqrt(2)*(-7*I*x^3 + 12*I*x^2 + 12*I*x - 8*I) - sqrt(2)*(2*x
^4 - 5*x^3 - 24*x^2 + 28*x + 8))*sqrt(I*sqrt(3) + 1) - 18*sqrt(3)*(-I*x^5
+ I*x^4 + 6*I*x^3 - 2*I*x^2 - 4*I*x) + 168*x - 160)/(x^6 - 6*x^5 + 24*x^4
- 56*x^3 + 96*x^2 - 96*x + 64)) + 1/216*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log((
2*x^6 + 6*x^5 - 150*x^4 + 176*x^3 + 12*x^2 - 3*sqrt(x^3 - 1)*(sqrt(3)*sqrt
(2)*(7*I*x^3 - 12*I*x^2 - 12*I*x + 8*I) - sqrt(2)*(2*x^4 - 5*x^3 - 24*x^2
+ 28*x + 8))*sqrt(-I*sqrt(3) + 1) - 18*sqrt(3)*(I*x^5 - I*x^4 - 6*I*x^3 +
2*I*x^2 + 4*I*x) + 168*x - 160)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 -
96*x + 64)) - 1/216*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log((2*x^6 + 6*x^5 - 150*
x^4 + 176*x^3 + 12*x^2 - 3*sqrt(x^3 - 1)*(sqrt(3)*sqrt(2)*(-7*I*x^3 + 12*I
*x^2 + 12*I*x - 8*I) + sqrt(2)*(2*x^4 - 5*x^3 - 24*x^2 + 28*x + 8))*sqrt(-
I*sqrt(3) + 1) - 18*sqrt(3)*(I*x^5 - I*x^4 - 6*I*x^3 + 2*I*x^2 + 4*I*x) +
168*x - 160)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) + 1/54*
arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1))
```

3.75.6 Sympy [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+2)\,(x^2-2x+4)}\,dx$$

input
```
integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

3.75.7 Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \int \frac{x}{(x^3+8)\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

3.75.8 Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = \int \frac{x}{(x^3+8)\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 533, normalized size of antiderivative = 7.20

$$\begin{aligned}
&\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx \\
&= \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad - \frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\left(-1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad - \frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\left(1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int(x/((x^3 - 1)^(1/2)*(x^3 + 8)),x)
```

output
```
(((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(9*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (3^(1/2)*((3
^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1
i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i - 1)*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2)) - (3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(
(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2
)*1i + 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.75. $\int \frac{x}{\sqrt{-1+x^3}(8+x^3)}\,dx$

3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

3.76.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx=-\frac{\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

output
```
1/18*arctanh(1/3*(1+d^(1/3)*x)^2/(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctanh(1/3
*(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctan((1+d^(1/3)*x)*3^(1/2)/(d*x^3+1)^(1/2
))/d^(2/3)*3^(1/2)
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.31

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx=\frac{1}{16}x^2\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-dx^3,\frac{dx^3}{8}\right)$$

input
```
Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

output
```
(x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

3.76.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {988, 946, 73, 219, 2563, 219, 2570, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(8-dx^3)\sqrt{dx^3+1}}dx$$

$$\Big\downarrow \ 988$$

$$-\frac{\int \frac{-d^{4/3}x^2-2dx+2d^{2/3}}{\left(d^{2/3}x^2+2\sqrt[3]{d}x+4\right)\sqrt{dx^3+1}}dx}{12d}+\frac{\int \frac{\sqrt[3]{d}x+1}{\left(2-\sqrt[3]{d}x\right)\sqrt{dx^3+1}}dx}{12\sqrt[3]{d}}-\frac{1}{4}\sqrt[3]{d}\int \frac{x^2}{(8-dx^3)\sqrt{dx^3+1}}dx$$

$$\Big\downarrow \ 946$$

$$-\frac{\int \frac{-d^{4/3}x^2-2dx+2d^{2/3}}{\left(d^{2/3}x^2+2\sqrt[3]{d}x+4\right)\sqrt{dx^3+1}}dx}{12d}+\frac{\int \frac{\sqrt[3]{d}x+1}{\left(2-\sqrt[3]{d}x\right)\sqrt{dx^3+1}}dx}{12\sqrt[3]{d}}-\frac{1}{12}\sqrt[3]{d}\int \frac{1}{(8-dx^3)\sqrt{dx^3+1}}dx^3$$

$$\Big\downarrow \ 73$$

$$-\frac{\int \frac{1}{9-x^6}d\sqrt{dx^3+1}}{6d^{2/3}}-\frac{\int \frac{-d^{4/3}x^2-2dx+2d^{2/3}}{\left(d^{2/3}x^2+2\sqrt[3]{d}x+4\right)\sqrt{dx^3+1}}dx}{12d}+\frac{\int \frac{\sqrt[3]{d}x+1}{\left(2-\sqrt[3]{d}x\right)\sqrt{dx^3+1}}dx}{12\sqrt[3]{d}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\int \frac{-d^{4/3}x^2-2dx+2d^{2/3}}{\left(d^{2/3}x^2+2\sqrt[3]{d}x+4\right)\sqrt{dx^3+1}}dx}{12d}+\frac{\int \frac{\sqrt[3]{d}x+1}{\left(2-\sqrt[3]{d}x\right)\sqrt{dx^3+1}}dx}{12\sqrt[3]{d}}-\frac{\text{arctanh}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

$$\Big\downarrow \ 2563$$

$$\frac{\int \frac{1}{9-\frac{\left(\sqrt[3]{d}x+1\right)^4}{dx^3+1}}d\frac{\left(\sqrt[3]{d}x+1\right)^2}{\sqrt{dx^3+1}}}{6d^{2/3}}-\frac{\int \frac{-d^{4/3}x^2-2dx+2d^{2/3}}{\left(d^{2/3}x^2+2\sqrt[3]{d}x+4\right)\sqrt{dx^3+1}}dx}{12d}-\frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\int \frac{-d^{4/3}x^2-2dx+2d^{2/3}}{\left(d^{2/3}x^2+2\sqrt[3]{d}x+4\right)\sqrt{dx^3+1}}dx}{12d}+\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

$$\Big\downarrow \ 2570$$

$$\frac{1}{3}d^{4/3}\int \frac{1}{-\frac{6\left(\sqrt[3]{d}x+1\right)^2d^2}{dx^3+1}-2d^2}d\frac{\sqrt[3]{d}x+1}{\sqrt{dx^3+1}}+\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

$$\Big\downarrow \ 218$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{d}x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

input
```
Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

output
```
-1/6*ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(Sqrt[3]*d^(2/3)) +
 ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqr
t[1 + d*x^3]/3]/(18*d^(2/3))
```

3.76.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 988
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[d*(q/(4*b))   Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x
^3]), x], x] + (-Simp[q^2/(12*b)   Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]
), x], x] + Simp[1/(12*b*c)   Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x +
 q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[8*b*c + a*d, 0]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.76. $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

3.76.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.72

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8\right)}\frac{\left(-d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-\cdots\right.}{d}\right.}{2\left(-d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8\right)}\frac{\left(-d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-\cdots\right.}{d}\right.}{2\left(-d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/
(-3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1
/3)*_alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2)^(2/3)*3^(1/2)*_alpha-3*(-d^2)^(2/
3)*_alpha+I*3^(1/2)*d-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(73) = 146$.

Time = 0.42 (sec) , antiderivative size = 497, normalized size of antiderivative = 4.83

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$$
$$= \frac{2\sqrt{3}(d^2)^{\frac{1}{6}}d\arctan\left(-\frac{\left(9\sqrt{3}d^3x^5-\sqrt{3}(d^2x^6-40\,dx^3-32)\,(d^2)^{\frac{2}{3}}+3\sqrt{3}(5\,d^2x^4+8\,dx)\,(d^2)^{\frac{1}{3}}\right)\sqrt{dx^3+1}\,(d^2)^{\frac{1}{6}}}{9\,(d^4x^7-7\,d^3x^4-8\,d^2x)}\right)+2\,(d^2)^{\frac{2}{3}}\log\left(\frac{d^4\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d
^2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^
(1/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d
^2)^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d
*x^4 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^
4 + 160*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt
(d*x^3 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d
^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d
^3*x^6 - 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*
d^3*x^6 + 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2
*x^5 + d*x^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*
x^5 - 8*d*x^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 5
12)))/d^2
```

3.76.6 Sympy [F]

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

input
```
integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
```

output
```
-Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

3.76.7 Maxima [F]

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+1}(dx^3-8)}\,dx$$

input
```
integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

3.76.8 Giac [F]

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+1}(dx^3-8)}\,dx$$

input
```
integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\int \frac{x}{\sqrt{d\,x^3+1}\,(d\,x^3-8)}\,dx$$

input
```
int(-x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)),x)
```

output
```
-int(x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}(3-x^2)}\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx=\frac{1}{4}\arctan\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}$$

output
```
1/4*arctan((1-(-3*x^2+1)^(1/3))/x)+1/12*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/1
2*arctanh(1/9*(1-(-3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

3.77.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx=$$

$$-\frac{9x\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(-3+x^2)\left(9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3x^2,\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},3x^2,\frac{x^2}{3}\right)+3\operatorname{AppellF1}\right.\right.}$$

input
```
Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x
^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3
, 2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

3.77.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {307}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}dx$$

$$\Big\downarrow \textbf{307}$$

$$\frac{1}{4}\arctan\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\operatorname{arctanh}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

input
```
Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

output
```
ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - Arc
Tanh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

3.77.3.1 Defintions of rubi rules used

rule 307
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

3.77.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.80 (sec) , antiderivative size = 538, normalized size of antiderivative = 6.64

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{8\left(-3x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2-3\right)^2\operatorname{RootOf}\left(4\_Z\operatorname{RootOf}\left(\_Z^2-3\right)+48\_Z^2+1\right)x+192\left(-3x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_\right.}{}\right.}{}$

input
```
int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/12*RootOf(_Z^2-3)*ln((8*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)^2*RootOf(4*_Z*R
ootOf(_Z^2-3)+48*_Z^2+1)*x+192*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*RootOf(4*_Z
*RootOf(_Z^2-3)+48*_Z^2+1)^2*x-16*RootOf(_Z^2-3)^2*RootOf(4*_Z*RootOf(_Z^2
-3)+48*_Z^2+1)*x-384*RootOf(_Z^2-3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)^
2*x+12*RootOf(_Z^2-3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x^2+24*(-3*x^2
+1)^(1/3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)+6*(-3*x^2+1
)^(2/3)+12*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)-4*RootOf(_
Z^2-3)*x-96*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x+3*x^2+3)/(x^2-3))-1/12
*ln((2*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*R
ootOf(_Z^2-3)+48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)+4*RootOf(_Z^2-3)*x+96*RootOf
(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*Roo
tOf(_Z^2-3)-ln((2*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*Ro
otOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)+4*RootOf(_Z^2-3)*
x+96*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*x-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(
x^2-3))*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)
```

3.77.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 1210, normalized size of antiderivative = 14.94

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

output
```
1/144*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(-(42*x^5 + 276*x^3 + sqrt(6)*(x^6 +
 105*x^4 + 63*x^2 - 9)*sqrt(-I*sqrt(3) - 1) - 3*(40*x^3 + sqrt(6)*(x^4 + 1
2*x^2 - sqrt(3)*(-I*x^4 - 12*I*x^2 - 3*I) + 3)*sqrt(-I*sqrt(3) - 1) + 72*x
)*(-3*x^2 + 1)^(2/3) + 6*sqrt(3)*(-7*I*x^5 - 46*I*x^3 + 9*I*x) - 3*(2*x^5
+ 52*x^3 - sqrt(6)*(5*x^4 + 18*x^2 + sqrt(3)*(-5*I*x^4 - 18*I*x^2 + 3*I) -
 3)*sqrt(-I*sqrt(3) - 1) - 2*sqrt(3)*(-I*x^5 - 26*I*x^3 - 9*I*x) + 18*x)*(
-3*x^2 + 1)^(1/3) - 54*x)/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/144*sqrt(6)*sqr
t(-I*sqrt(3) - 1)*log(-(42*x^5 + 276*x^3 - sqrt(6)*(x^6 + 105*x^4 + 63*x^2
 - 9)*sqrt(-I*sqrt(3) - 1) - 3*(40*x^3 - sqrt(6)*(x^4 + 12*x^2 + sqrt(3)*(
I*x^4 + 12*I*x^2 + 3*I) + 3)*sqrt(-I*sqrt(3) - 1) + 72*x)*(-3*x^2 + 1)^(2/
3) + 6*sqrt(3)*(-7*I*x^5 - 46*I*x^3 + 9*I*x) - 3*(2*x^5 + 52*x^3 + sqrt(6)
*(5*x^4 + 18*x^2 - sqrt(3)*(5*I*x^4 + 18*I*x^2 - 3*I) - 3)*sqrt(-I*sqrt(3)
 - 1) - 2*sqrt(3)*(-I*x^5 - 26*I*x^3 - 9*I*x) + 18*x)*(-3*x^2 + 1)^(1/3) -
 54*x)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/144*sqrt(6)*sqrt(I*sqrt(3) - 1)*lo
g(-(42*x^5 + 276*x^3 - 24*(5*x^3 + 9*x)*(-3*x^2 + 1)^(2/3) + 6*sqrt(3)*(7*
I*x^5 + 46*I*x^3 - 9*I*x) - (3*sqrt(6)*(x^4 + 12*x^2 - sqrt(3)*(I*x^4 + 12
*I*x^2 + 3*I) + 3)*(-3*x^2 + 1)^(2/3) - 3*sqrt(6)*(5*x^4 + 18*x^2 + sqrt(3
)*(5*I*x^4 + 18*I*x^2 - 3*I) - 3)*(-3*x^2 + 1)^(1/3) - sqrt(6)*(x^6 + 105*
x^4 + 63*x^2 - 9))*sqrt(I*sqrt(3) - 1) - 6*(x^5 + 26*x^3 - sqrt(3)*(I*x^5
+ 26*I*x^3 + 9*I*x) + 9*x)*(-3*x^2 + 1)^(1/3) - 54*x)/(x^6 - 9*x^4 + 27...
```

3.77.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = -\int \frac{1}{x^2\sqrt[3]{1-3x^2}-3\sqrt[3]{1-3x^2}}\,dx$$

input
```
integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
```

output
```
-Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

3.77.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \int -\frac{1}{(x^2-3)(-3\,x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

3.77.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = \int -\frac{1}{(x^2-3)(-3\,x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
```

output
```
integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx = -\int \frac{1}{(x^2-3)\,\left(1-3\,x^2\right)^{1/3}}\,dx$$

input
```
int(-1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
```

output
```
-int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

3.78 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

3.78.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4}\text{arctanh}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

output
```
-1/4*arctanh((1-(3*x^2+1)^(1/3))/x)+1/12*arctan(1/3*x*3^(1/2))*3^(1/2)+1/1
2*arctan(1/9*(1-(3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

3.78.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3x^2,-\frac{x^2}{3}\right)}{(3+x^2)\sqrt[3]{1+3x^2}\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-3x^2,-\frac{x^2}{3}\right)+3\,\text{Ap}\right.\right.}$$

input
```
Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)
^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[
3/2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2,
 -1/3*x^2])))
```

3.78.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{306\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3)\sqrt[3]{3x^2+1}}dx$$

$$\Big\downarrow 306$$

$$\frac{\arctan\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{arctanh}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)$$

input
```
Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

output
```
ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3
]*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

3.78.3.1 Defintions of rubi rules used

rule 306
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 443, normalized size of antiderivative = 5.47

method	result
trager	$-\frac{\ln\left(-\frac{12\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)\left(3x^2+1\right)^{\frac{1}{3}}x-6\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)x^2+\left(3x^2+1\right)^{\frac{2}{3}}+12\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)\left(3x^2\right.}{x^2+3}\right.}{4}$

input
```
int(1/(x^2+3)/(3*x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x-6*RootOf(48*_Z^2+12
*_Z+1)*x^2+(3*x^2+1)^(2/3)+12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)+(3*x
^2+1)^(1/3)*x-24*RootOf(48*_Z^2+12*_Z+1)*x-x^2+(3*x^2+1)^(1/3)+6*RootOf(48
*_Z^2+12*_Z+1)-4*x+1)/(x^2+3))-ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(
1/3)*x-6*RootOf(48*_Z^2+12*_Z+1)*x^2+(3*x^2+1)^(2/3)+12*RootOf(48*_Z^2+12*
_Z+1)*(3*x^2+1)^(1/3)+(3*x^2+1)^(1/3)*x-24*RootOf(48*_Z^2+12*_Z+1)*x-x^2+(
3*x^2+1)^(1/3)+6*RootOf(48*_Z^2+12*_Z+1)-4*x+1)/(x^2+3))*RootOf(48*_Z^2+12
*_Z+1)+RootOf(48*_Z^2+12*_Z+1)*ln(-(-24*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^
(1/3)*x+12*RootOf(48*_Z^2+12*_Z+1)*x^2+2*(3*x^2+1)^(2/3)-24*RootOf(48*_Z^2
+12*_Z+1)*(3*x^2+1)^(1/3)-4*(3*x^2+1)^(1/3)*x+48*RootOf(48*_Z^2+12*_Z+1)*x
+x^2-4*(3*x^2+1)^(1/3)-12*RootOf(48*_Z^2+12*_Z+1)+4*x-1)/(x^2+3))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(59) = 118.

Time = 1.01 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.26

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$$
$$= \frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(3x^4-10x^3-36x^2+18x+9)(3x^2+1)^{\frac{2}{3}}-4\sqrt{3}(x^5+15x^4-26x^3-54x^2+9x\cdots}{x^6+126x^5-225x^4-828x^3-81\cdots}\right.$$
$$-\frac{1}{36}\sqrt{3}\arctan\left(\frac{2\left(2\sqrt{3}(23x^3+9x)(3x^2+1)^{\frac{2}{3}}+\sqrt{3}(x^5-80x^3-9x)(3x^2+1)^{\frac{1}{3}}+\sqrt{3}(11x^5+10x^3\cdots\right.}{x^6-657x^4-189x^2-27}\right.$$
$$+\frac{1}{24}\log\left(\frac{x^6+108x^5+549x^4+360x^3+99x^2+6(3x^4+32x^3+42x^2+3)(3x^2+1)^{\frac{2}{3}}+6(x^5+27x\cdots}{x^6+9x^4+27x^2+27}\right.$$

input
```
integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
 + 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2
+ 1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9)
)/(x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3
)*arctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80
*x^3 - 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 65
7*x^4 - 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*
x^2 + 6*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4
+ 70*x^3 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 +
 27*x^2 + 27))
```

3.78.6 Sympy [F]

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(x^2+3)\sqrt[3]{3x^2+1}}\,dx$$

input
```
integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

output
```
Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

3.78.7 Maxima [F]

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(3\,x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

input
```
integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

3.78.8 Giac [F]

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(3\,x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

input
```
integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx = \int \frac{1}{(x^2+3)\,(3\,x^2+1)^{1/3}}\,dx$$

input
```
int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
```

output
```
int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

3.79 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

output
```
-1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)
+1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1
/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

3.79.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=$$

$$-\frac{9x\,\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2
)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3
, 2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

3.79.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{305}$$

$$\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}$$

input
```
Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 -
x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(
1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

3.79.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.52 (sec) , antiderivative size = 938, normalized size of antiderivative = 8.30

method	result	size
trager	Expression too large to display	938

input
```
int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/432*ln((RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5-36*RootOf(_Z^6+
108)^2*(-x^2+1)^(1/3)*x^5+225*RootOf(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^
2*(-x^2+1)^(1/3)*x^4+72*RootOf(_Z^6+108)^4*x^3+648*(-x^2+1)^(2/3)*x^4-864*
RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-189*x^2*RootOf(_Z^6+108)^4-4536*(-x^
2+1)^(2/3)*x^3-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/
3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+27*RootOf(_Z^6+108)^4+1944*
(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((RootOf(_Z^6+108)^
4*x^6-72*RootOf(_Z^6+108)^4*x^5-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+2
25*RootOf(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4+72*Roo
tOf(_Z^6+108)^4*x^3+648*(-x^2+1)^(2/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)
^(1/3)*x^3-189*x^2*RootOf(_Z^6+108)^4-4536*(-x^2+1)^(2/3)*x^3-648*RootOf(_
Z^6+108)^2*(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/3)*x^2+324*RootOf(_Z^6+108)
^2*(-x^2+1)^(1/3)*x+27*RootOf(_Z^6+108)^4+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3
)*RootOf(_Z^6+108)-1/36*RootOf(_Z^6+108)*ln((486*RootOf(_Z^6+108)-27*RootO
f(_Z^6+108)^4-72*RootOf(_Z^6+108)^4*x^5+189*x^2*RootOf(_Z^6+108)^4-225*Roo
tOf(_Z^6+108)^4*x^4+1296*RootOf(_Z^6+108)*x^5-3402*RootOf(_Z^6+108)*x^2-12
96*RootOf(_Z^6+108)*x^3+72*RootOf(_Z^6+108)^4*x^3+4050*RootOf(_Z^6+108)*x^
4-RootOf(_Z^6+108)^4*x^6+18*RootOf(_Z^6+108)*x^6+1296*(-x^2+1)^(2/3)*x^4+9
072*(-x^2+1)^(2/3)*x^3+3888*(-x^2+1)^(2/3)*x^2-3888*(-x^2+1)^(2/3)*x-36*Ro
otOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/...
```

3.79.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 1232, normalized size of antiderivative = 10.90

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(
1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) +
 1728*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^
(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*
432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1
/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3)
 + 1)*log(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 -
69*x^4 + 63*x^2 - 27) - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqr
t(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*
x^2) - 9*x)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqr
t(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 -
sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
 - 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(
1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) +
 1728*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^
(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*
432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1
/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(...
```

3.79.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.79.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.79.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{\arctan(x)}{6\ 2^{2/3}} + \frac{\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

output
```
-1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/
12*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

3.80.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right)-\text{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)
*(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2,
 5/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

3.80.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-x^2)\sqrt[3]{x^2+1}}dx$$

$$\Big\downarrow \text{ 304}$$

$$\frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\arctan(x)}{6\ 2^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

input
```
Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

output
```
-1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3
)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3
)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

3.80.3.1 Defintions of rubi rules used

rule 304
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

3.80.4 Maple [F]

$$\int \frac{1}{(-x^2+3)(x^2+1)^{\frac{1}{3}}}dx$$

input
```
int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

output
```
int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(77) = 154$.

Time = 0.74 (sec) , antiderivative size = 1103, normalized size of antiderivative = 10.12

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

output
```
-1/10368*432^(5/6)*(sqrt(-3) + 1)*log((432^(5/6)*(x^6 + 69*x^4 + 63*x^2 +
sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 1728*(9*x^3 + sqrt(3)*(x^4 +
 9*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5
*x^5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 -
 sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3
)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(
-3) + 1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 + 69*x^4 +
 63*x^2 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)
^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9
*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 - sqrt(-3)*(x^5 + 18*x^3
+ 9*x) + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3)*(x^4 + 3*x^2))))/(x^6
- 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(-3) - 1)*log((432^(5/6)*
(x^6 + 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 172
8*(9*x^3 + sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x
^5 + 30*x^3 - sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)
*(2^(2/3)*(x^5 + 18*x^3 + sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/
6)*(x^4 + 3*x^2 + sqrt(-3)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) -
 1/10368*432^(5/6)*(sqrt(-3) - 1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 -
 sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4
+ 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^3 - sqrt(-3...
```

3.80.6 Sympy [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

input
```
integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

output
```
-Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.80.7 Maxima [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

input
```
integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.80.8 Giac [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

input
```
integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{(x^2+1)^{1/3}\,(x^2-3)}\,dx$$

input
```
int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
```

output
```
-int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

3.81.1 Optimal result

Integrand size = 34, antiderivative size = 87

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = -\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)\,x+x^2}\arctan\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

output
```
-2*arctan((1-a)*x^(1/2)/(a^2-(a^2+1)*x+x^2)^(1/2))*x^(1/2)*(a^2-(a^2+1)*x+
x^2)^(1/2)/(1-a)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 10.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = -\frac{2\arctan\left(\frac{(-1+a)x}{\sqrt{(-1+x)x(-a^2+x)}}\right)}{-1+a}$$

input
```
Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

output
```
(-2*ArcTan[((-1 + a)*x)/Sqrt[(-1 + x)*x*(-a^2 + x)]])/(-1 + a)
```

3.81.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2467, 25, 2035, 2212, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+x}{(x-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}dx$$

$$\Big\downarrow \ 2467$$

$$\frac{\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\int -\frac{a+x}{(a-x)\sqrt{x}\sqrt{a^2+x^2-(a^2+1)x}}dx}{\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\int \frac{a+x}{(a-x)\sqrt{x}\sqrt{a^2+x^2-(a^2+1)x}}dx}{\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

$$\Big\downarrow \ 2035$$

$$-\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\int \frac{a+x}{(a-x)\sqrt{a^2+x^2-(a^2+1)x}}d\sqrt{x}}{\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

$$\Big\downarrow \ 2212$$

$$-\frac{2a\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\int \frac{1}{ax(1-a)^2+a}d\frac{\sqrt{x}}{\sqrt{a^2+x^2-(a^2+1)x}}}{\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

$$\Big\downarrow \ 218$$

$$-\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\arctan\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

input
```
Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^
2 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.81.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.38

method	result
default	$\frac{2\arctan\left(\frac{\sqrt{-\left(a^{2}-x\right)x(-1+x)}}{x(a-1)}\right)}{a-1}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{-\left(a^{2}-x\right)x(-1+x)}}{x(a-1)}\right)}{a-1}$
elliptic	$-\frac{2a^{2}\sqrt{-\frac{-a^{2}+x}{a^{2}}}\,\sqrt{\frac{-1+x}{a^{2}-1}}\,\sqrt{\frac{x}{a^{2}}}\,F\left(\sqrt{-\frac{-a^{2}+x}{a^{2}}},\sqrt{\frac{a^{2}}{a^{2}-1}}\right)}{\sqrt{-a^{2}x^{2}+a^{2}x+x^{3}-x^{2}}}-\frac{4a^{3}\sqrt{-\frac{-a^{2}+x}{a^{2}}}\,\sqrt{\frac{-1+x}{a^{2}-1}}\,\sqrt{\frac{x}{a^{2}}}\,\Pi\left(\sqrt{-\frac{-a^{2}+x}{a^{2}}},\frac{a^{2}}{a^{2}-a},\sqrt{\frac{a^{2}}{a^{2}-1}}\right)}{\sqrt{-a^{2}x^{2}+a^{2}x+x^{3}-x^{2}}\,(a^{2}-a)}$

input
```
int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((-(a^2-x)*x*(-1+x))^(1/2)/x/(a-1))/(a-1)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{a^2x-(a^2+1)x^2+x^3}\,(a^2-2\,(a^2-a+1)x+x^2)}{2\,((a-1)x^3-(a^3-a^2+a-1)x^2+(a^3-a^2)x)}\right)}{a-1}$$

input
```
integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas"
)
```

output
```
arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^
2)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

3.81.6 Sympy [F]

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = \int \frac{a+x}{\sqrt{x\,(-a^2+x)\,(x-1)}\,(-a+x)}\,dx$$

input
```
integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

output
```
Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

3.81.7 Maxima [F]

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = \int -\frac{a+x}{\sqrt{a^2x-(a^2+1)x^2+x^3}(a-x)}\,dx$$

input
```
integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima"
)
```

output
```
-integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

3.81.8 Giac [F]

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = \int -\frac{a+x}{\sqrt{a^2x-(a^2+1)x^2+x^3}(a-x)}\,dx$$

input
```
integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.49

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = \frac{4\,a\,(a^2-1)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}\,\Pi\left(-\frac{a^2-1}{a-a^2};\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)}{(a-a^2)\,\sqrt{a^2\,x-x^2\,(a^2+1)+x^3}} - \frac{2\,(a^2-1)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}}{\sqrt{a^2\,x-x^2\,(a^2+1)+x^3}}$$

input
```
int(-(a + x)/((a - x)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)),x)
```

output
```
(4*a*(a^2 - 1)*(x/a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 -
1))^(1/2)*ellipticPi(-(a^2 - 1)/(a - a^2), asin((-(x - a^2)/(a^2 - 1))^(1/
2)), (a^2 - 1)/a^2))/((a - a^2)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)) - (2*
(a^2 - 1)*ellipticF(asin((-(x - a^2)/(a^2 - 1))^(1/2)), (a^2 - 1)/a^2)*(x/
a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1))^(1/2))/(a^2*x
- x^2*(a^2 + 1) + x^3)^(1/2)
```

3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}}\,dx$

3.82.1 Optimal result

Integrand size = 40, antiderivative size = 71

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \frac{\log\left(\frac{-a^2+2(-a+a^2)x+x^2-2a\sqrt{-((-2a+a^2)x)+(-1-2a+a^2)x^2+x^3}}{a^2-2ax+x^2}\right)}{a}$$

output
```
0
```

3.82.2 Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{(2a-a^2)x+(-1-2a+a^2)x^2+x^3}}{a(-1+x)}\right)}{a}$$

input
```
Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
 x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[(2*a - a^2)*x + (-1 - 2*a + a^2)*x^2 + x^3]/(a*(-1 + x))]
)/a
```

3.82.3 Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.17 (sec) , antiderivative size = 546, normalized size of antiderivative = 7.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2467, 2035, 2226, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+x-2}{(x-a)\sqrt{(a^2-2a-1)\,x^2+(2-a)ax+x^3}}\,dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{x}\sqrt{-(-a^2+2a+1)\,x+(2-a)a+x^2}\int \frac{-a-x+2}{(a-x)\sqrt{x}\sqrt{x^2-(-a^2+2a+1)x+(2-a)a}}dx}{\sqrt{-(-a^2+2a+1)\,x^2+(2-a)ax+x^3}}$$

$$\Big\downarrow \text{ 2035}$$

$$\frac{2\sqrt{x}\sqrt{-(-a^2+2a+1)\,x+(2-a)a+x^2}\int \frac{-a-x+2}{(a-x)\sqrt{x^2-(-a^2+2a+1)x+(2-a)a}}d\sqrt{x}}{\sqrt{-(-a^2+2a+1)\,x^2+(2-a)ax+x^3}}$$

$$\Big\downarrow \text{ 2226}$$

$$\frac{2\sqrt{x}\sqrt{-(-a^2+2a+1)\,x+(2-a)a+x^2}\left(\frac{\sqrt{(2-a)a}\int \frac{1}{\sqrt{x^2-\left(-a^2+2a+1\right)x+(2-a)a}}d\sqrt{x}}{a}+\left(\sqrt{2-a}-\sqrt{a}\right)\sqrt{2-a}\int \frac{\quad}{(a-x)}\right.}{\sqrt{-(-a^2+2a+1)\,x^2+(2-a)ax+x^3}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{2\sqrt{x}\sqrt{-(-a^2+2a+1)\,x+(2-a)a+x^2}\left(\left(\sqrt{2-a}-\sqrt{a}\right)\sqrt{2-a}\int \frac{\frac{x}{\sqrt{(2-a)a}}+1}{(a-x)\sqrt{x^2-(-a^2+2a+1)x+(2-a)a}}d\sqrt{x}+\frac{((2-a)a}{\quad}\right.}{\sqrt{-(-a^2+2a+1)\,x^2+(2-a)a}}$$

$$\Big\downarrow \text{ 2222}$$

$$\frac{\cdots}{2\sqrt{x}\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}\left(\left(\sqrt{2-a}-\sqrt{a}\right)\sqrt{2-a}\left(\frac{\sqrt[4]{(2-a)a}\left(1-\frac{a}{\sqrt{(2-a)a}}\right)\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-\left(-a^2+\cdots\right.}{(2-a)\cdots}}\cdots}{\cdots}\right.\right.}$$

input
```
Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3])
,x]
```

output
```
(2*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*((((2 - a)*a)^(3/4)*(
1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2 - a)
*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4
)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(2*a*Sqrt[(2 - a)*a - (1 + 2
*a - a^2)*x + x^2]) + (Sqrt[2 - a] - Sqrt[a])*Sqrt[2 - a]*(((1 + a/Sqrt[(2
 - a)*a])*ArcTanh[((1 - a)*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x
^2]])/(2*(1 - a)*a) + (((2 - a)*a)^(1/4)*(1 - a/Sqrt[(2 - a)*a])*(1 + x/Sq
rt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2 - a)*a*(1 +
x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(2*a - a^2 + Sqrt[(2 - a)*a])/(2*(2 - a)
*a), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a
)*a])/4])/(4*a*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]))))/Sqrt[(2 - a)*
a*x - (1 + 2*a - a^2)*x^2 + x^3]
```

3.82.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.82.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 1.

Time = 0.51 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.46

method	result
default	$\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{-1+x}{-a^2+2a-1}}\,\sqrt{\frac{x}{-a^2+2a}}\,F\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}-\frac{2(-2a+2)(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{-1+x}{-a^2+2a-1}}\,\sqrt{\frac{}{-}}}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3}}$
elliptic	$\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{-1+x}{-a^2+2a-1}}\,\sqrt{\frac{x}{-a^2+2a}}\,F\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}+\frac{2(2a-2)(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{-1+x}{-a^2+2a-1}}\,\sqrt{\frac{x}{-a^2}}}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3}}$

input
```
int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(
-a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a
^2-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))-2*(-2*a+2)*(a^
2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+
2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticP
i(((a^2-2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-
1))^(1/2))
```

3.82.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \frac{\log\left(-\frac{a^2-2\,(a^2-a)x-x^2+2\,\sqrt{(a^2-2\,a-1)x^2+x^3-(a^2-2\,a)x}a}{a^2-2\,ax+x^2}\right)}{a}$$

input
```
integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorit
hm="fricas")
```

output
```
log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2
- 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

3.82.6 Sympy [F]

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \int \frac{a+x-2}{\sqrt{x\,(x-1)\,(a^2-2a+x)}\,(-a+x)}\,dx$$

input
```
integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

output
```
Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

3.82.7 Maxima [F]

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \int -\frac{a+x-2}{\sqrt{-(a-2)ax+(a^2-2\,a-1)x^2+x^3}(a-x)}\,dx$$

input
```
integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorit
hm="maxima")
```

output
```
-integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a
- x)), x)
```

3.82.8 Giac [F]

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \int -\frac{a+x-2}{\sqrt{-(a-2)ax+(a^2-2\,a-1)x^2+x^3}(a-x)}\,dx$$

input
```
integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a
- x)), x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.92

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \frac{2\,\sqrt{\frac{x}{2\,a-a^2}}\,\sqrt{-\frac{x-1}{a^2-2\,a+1}}\,(a-1)^2\,\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\,\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)-2\,\Pi\left(-\frac{a^2-2\,a+1}{a-a^2};\mathrm{asin}\left(\sqrt{\frac{a^2}{a^2}}\right.\right.\right.}{a\,\sqrt{x^3+(a^2-2\,a-1)\,x^2+(2\,a-a^2)\,x}}$$

input
```
int(-(a + x - 2)/((a - x)*(x^3 - x^2*(2*a - a^2 + 1) - a*x*(a - 2))^(1/2))
,x)
```

output
```
(2*(x/(2*a - a^2))^(1/2)*(-(x - 1)/(a^2 - 2*a + 1))^(1/2)*(a - 1)^2*((x -
2*a + a^2)/(a^2 - 2*a + 1))^(1/2)*(a*ellipticF(asin(((x - 2*a + a^2)/(a^2
- 2*a + 1))^(1/2)), -(a^2 - 2*a + 1)/(2*a - a^2)) - 2*ellipticPi(-(a^2 - 2
*a + 1)/(a - a^2), asin(((x - 2*a + a^2)/(a^2 - 2*a + 1))^(1/2)), -(a^2 -
2*a + 1)/(2*a - a^2))))/(a*(x*(2*a - a^2) - x^2*(2*a - a^2 + 1) + x^3)^(1/
2))
```

3.82. $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}}\,dx$

3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$

3.83.1 Optimal result

Integrand size = 51, antiderivative size = 46

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \log\left(\frac{-a^2+2ax+x^2-2\left(x+\sqrt{(1-x)x\,(a^2+x-2ax)}\right)}{(a-x)^2}\right)$$

output
```
ln((-a^2+2*a*x+x^2-2*x-2*((1-x)*x*(a^2-2*a*x+x))^(1/2))/(a-x)^2)
```

3.83.2 Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = -2\text{arctanh}\left(\frac{\sqrt{a^2x+(1-2a-a^2)\,x^2+(-1+2a)x^3}}{-a^2+(-1+2a)x}\right)$$

input
```
Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2
+ (-1 + 2*a)*x^3]),x]
```

output
```
-2*ArcTanh[Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 + (-1 + 2*a)*x^3]/(-a^2 + (-1
+ 2*a)*x)]
```

3.83.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$, Rules used = {2467, 2035, 2228, 1417, 321, 1544, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2a-1)x-a}{(x-a)\sqrt{-\left(a^2+2a-1\right)x^2+a^2x+(2a-1)x^3}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\int \frac{a+(1-2a)x}{(a-x)\sqrt{x}\sqrt{a^2-(1-2a)x^2+\left(-a^2-2a+1\right)x}}dx}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-((1-2a)x^3)}}$$

$$\Big\downarrow \text{2035}$$

$$\frac{2\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\int \frac{a+(1-2a)x}{(a-x)\sqrt{a^2-(1-2a)x^2+\left(-a^2-2a+1\right)x}}d\sqrt{x}}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-((1-2a)x^3)}}$$

$$\Big\downarrow \text{2228}$$

$$\frac{2\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\left(2(1-a)a\int \frac{1}{(a-x)\sqrt{a^2-(1-2a)x^2+\left(-a^2-2a+1\right)x}}d\sqrt{x}-(1-2a)\int \frac{}{\sqrt{a^2-(1-}}\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-((1-2a)x^3)}}$$

$$\Big\downarrow \text{1417}$$

$$\frac{2\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\left(2(1-a)a\int \frac{1}{(a-x)\sqrt{a^2-(1-2a)x^2+\left(-a^2-2a+1\right)x}}d\sqrt{x}-\frac{(1-2a)\sqrt{1-x}\sqrt{\frac{(1-2a)x}{a^2}}}{\sqrt{(-a^2-2a+}}\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-((1-2a)x^3)}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\left(2(1-a)a\int \frac{1}{(a-x)\sqrt{a^2-(1-2a)x^2+(-a^2-2a+1)x}}d\sqrt{x}-\frac{(1-2a)\sqrt{1-x}\sqrt{\frac{(1-2a)x}{a^2}}}{\sqrt{(-a^2-2a}}\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-\left((1-2a)x^3\right)}}$$

$$\Big\downarrow \text{1544}$$

$$\frac{2\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\left(\frac{2(1-a)a\sqrt{1-x}\sqrt{\frac{(1-2a)x}{a^2}+1}\int \frac{1}{\sqrt{1-x}(a-x)\sqrt{\frac{(1-2a)x}{a^2}+1}}d\sqrt{x}}{\sqrt{(-a^2-2a+1)x+a^2-(1-2a)x^2}}-\frac{(1-2a)\sqrt{1-x}\sqrt{\frac{(1-2a)x}{a^2}}}{\sqrt{(-a^2-2a}}\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-\left((1-2a)x^3\right)}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2\sqrt{x}\sqrt{\left(-a^2-2a+1\right)x+a^2-(1-2a)x^2}\left(\frac{2(1-a)\sqrt{1-x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\text{EllipticPi}\left(\frac{1}{a},\arcsin\left(\sqrt{x}\right),-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x+a^2-(1-2a)x^2}}-\frac{(1-2a)\sqrt{1-x}\sqrt{\frac{(1-2a}{a^2}}}{\sqrt{(-a^2-}}\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-\left((1-2a)x^3\right)}}$$

input
```
Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1
+ 2*a)*x^3]),x]
```

output
```
(2*Sqrt[x]*Sqrt[a^2 + (1 - 2*a - a^2)*x - (1 - 2*a)*x^2]*(-(((1 - 2*a)*Sqr
t[1 - x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[ArcSin[Sqrt[x]], -((1 - 2*a
)/a^2)])/Sqrt[a^2 + (1 - 2*a - a^2)*x - (1 - 2*a)*x^2]) + (2*(1 - a)*Sqrt[
1 - x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticPi[a^(-1), ArcSin[Sqrt[x]], -((
1 - 2*a)/a^2)])/Sqrt[a^2 + (1 - 2*a - a^2)*x - (1 - 2*a)*x^2]))/Sqrt[a^2*x
 + (1 - 2*a - a^2)*x^2 - (1 - 2*a)*x^3]
```

3.83.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 1544
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/((d + e*x^2)*S
qrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& NegQ[c/a]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2228
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Simp[(e*A - d*B)/e   Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
 x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a
]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.83. $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$

3.83.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 367, normalized size of antiderivative = 7.98

method	result
elliptic	$-\frac{2a^2\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,F\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}-1\right)}}\right)}{\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}-\frac{4a^3(a-1)\sqrt{-\cdots}}{\cdots}$
default	$\frac{2a^2\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,F\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}-1\right)}}\right)}{(-1+2a)\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}-\frac{4a^3\sqrt{-\frac{\left(x-\frac{a^2}{-1+2\cdots}\right)}{\cdots}}}{\cdots}$

input
```
int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2*a^2*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1
/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)*
EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(-1+2*
a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*
((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a
^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*EllipticPi((-(x-a^2/(-1+2*a))
/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^2/(-1+2*a)/(a^2/(-1+
2*a)-1))^(1/2))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx$$
$$=\log\left(-\frac{a^2-2\,(a-1)x-x^2+2\,\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}}{a^2-2\,ax+x^2}\right)$$

input
```
integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2
),x, algorithm="fricas")
```

output
```
log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a
- 1)*x^2))/(a^2 - 2*a*x + x^2))
```

3.83.6 Sympy [F]

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx$$
$$=\int \frac{2ax-a-x}{\sqrt{x\,(x-1)\,(-a^2+2ax-x)}\,(-a+x)}\,dx$$

input
```
integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)*
*(1/2),x)
```

output
```
Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x
)
```

3.83.7 Maxima [F]

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx$$
$$=\int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}(a-x)}\,dx$$

input
```
integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2
),x, algorithm="maxima")
```

output
```
-integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)
*x^2)*(a - x)), x)
```

3.83.8 Giac [F]

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}(a-x)}\,dx$$

input
```
integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2
),x, algorithm="giac")
```

output
```
integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)
*x^2)*(a - x)), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \text{Hanged}$$

input
```
int((a - x*(2*a - 1))/((a - x)*(x^3*(2*a - 1) - x^2*(2*a + a^2 - 1) + a^2*
x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.84 $\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.84.1 Optimal result

Integrand size = 29, antiderivative size = 32

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

3.84.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\sqrt[3]{2}x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2562}$$

$$2\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

input
```
Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

3.84.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.84.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.96 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.47

method	result
trager	$-\frac{2^{\frac{1}{3}} \mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right) \ln\left(\frac{12\sqrt{x^3+1}\,x+3\,2^{\frac{2}{3}}x^2 \mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)-\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{x^3+1}\,2^{\frac{2}{3}}+6\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}} \cdots\right.}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{6}$
default	$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+1}\cdots}$
elliptic	$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+1}\cdots}$

input
```
int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/3)*RootOf(_Z^2+6*2^(1/3))*ln((12*(x^3+1)^(1/2)*x+3*2^(2/3)*x^2*R
ootOf(_Z^2+6*2^(1/3))-RootOf(_Z^2+6*2^(1/3))*x^3+6*(x^3+1)^(1/2)*2^(2/3)+6
*RootOf(_Z^2+6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x+2)^
3)
```

3.84.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.84.6 Sympy [F]

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\int \frac{\sqrt[3]{2}x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx - \int \left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx$$

input
```
integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - In
tegral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

3.84.7 Maxima [F]

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.84.8 Giac [F]

$$\int \frac{1-\sqrt[3]{2}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.09

$$\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

input
```
int(-(2^(1/3)*x - 1)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
(3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)*(3^(1/
2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3
```

3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

3.85.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

output
```
-2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

3.85.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

input
```
Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

3.85.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x-2)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2563}$$

$$-2\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2}{3}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

input
```
Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

3.85.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

method	result
trager	$-\frac{\ln\left(\frac{x^3+6\sqrt{x^3+1}\,x+12x^2+6\sqrt{x^3+1}-6x+10}{(-2+x)^3}\right)}{3}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$

input
```
int((1+x)/(-2+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln((x^3+6*(x^3+1)^(1/2)*x+12*x^2+6*(x^3+1)^(1/2)-6*x+10)/(-2+x)^3)
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

input
```
integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
 12*x - 8))
```

3.85.6 Sympy [F]

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\,(x^2-x+1)}\,(x-2)}\,dx$$

input
```
integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

3.85.7 Maxima [F]

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.85.8 Giac [F]

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 8.87

$$\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/
2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((
(x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))
^(1/2)
```

3.86 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

3.86.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

output
```
-1/12*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/
2))*3^(1/4)*2^(1/2)-1/18*arctan(1/6*(1-3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1
/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2)
)*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)-1/18*arctanh(1/2*3^(1
/4)*(1-2*x+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)
```

3.86.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx=\frac{x^2\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

input
```
Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

output
```
(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*S
qrt[3])
```

3.86.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3+1}\left(x^3+6\sqrt{3}+10\right)}\,dx$$

$$\Big\downarrow \text{989}$$

$$-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

input
```
Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

output
```
-1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1
 + x^3])])/(Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1
 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[
(3^(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)
) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1
 + x^3])])/(6*Sqrt[2]*3^(1/4))
```

3.86.3.1 Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

3.86.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 48.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.63

method	result
default	$\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(12+6\sqrt{3}\right)\sqrt{x^3+1}}-\frac{\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}(\_\ldots}\right.}{\ldots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{\left(-1-\sqrt{3}\right)^2}{3}+\frac{2\left(-1-\sqrt{3}\right)^2\sqrt{3}}{9}-\frac{2}{3}-\frac{\sqrt{3}}{9}-\frac{2\left(-1-\sqrt{3}\right)\sqrt{3}}{9}\right)\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\left(-1-\sqrt{3}\right)^{\ldots}}{3}\ldots\right.}{3\left(-1-\sqrt{3}\right)\sqrt{x^{\ldots}}}$
trager	Expression too large to display

input
```
int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-1-3^(1/2))/(12+6*3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2
*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi((
(1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/
2)+_alpha-2)/(-1+2*_alpha-3^(1/2))*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/
2)*((-I*3^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)
-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((1+x)/(3
/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+1/2*_alpha*3^(
1/2)-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(1/2)+4))
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(148) = 296$.

Time = 0.43 (sec) , antiderivative size = 1569, normalized size of antiderivative = 7.20

$$\int \frac{x}{\sqrt{1+x^3}\,(10+6\sqrt{3}+x^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2 + sqrt(3)*(x^2 - 10*x - 8) -
 18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1)) + 1/72*sqrt(7*sqrt(3) + 3
*sqrt(56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 + 3
2*x^3 - 44*x^2 + 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(2
6*x^7 + 12*x^6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2 + sqrt(3)*(15*x^7 + 7*x
^6 - 28*x^5 - 56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 97
) + ((336*x^5 + 33*x^4 - 132*x^3 - 474*x^2 + sqrt(3)*(194*x^5 + 19*x^4 - 7
6*x^3 - 274*x^2 - 152*x - 76) - 264*x - 132)*sqrt(x^3 + 1)*sqrt(56*sqrt(3)
 - 97) + (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 - 4*x^
5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x - 4) - 24*x - 4)*sqrt(x^3 + 1))*sqrt(7*
sqrt(3) + 3*sqrt(56*sqrt(3) - 97) - 12) + 8*x + 16)/(x^8 - 4*x^7 + 16*x^6
- 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) - 1/72*sqrt(7*sqrt(3) +
3*sqrt(56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*x^5 - 20*x^4 +
32*x^3 - 44*x^2 + 2*sqrt(3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(
26*x^7 + 12*x^6 - 48*x^5 - 98*x^4 - 96*x^3 - 48*x^2 + sqrt(3)*(15*x^7 + 7*
x^6 - 28*x^5 - 56*x^4 - 56*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(56*sqrt(3) - 9
7) - ((336*x^5 + 33*x^4 - 132*x^3 - 474*x^2 + sqrt(3)*(194*x^5 + 19*x^4 -
76*x^3 - 274*x^2 - 152*x - 76) - 264*x - 132)*sqrt(x^3 + 1)*sqrt(56*sqrt(3
) - 97) + (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2 + sqrt(3)*(3*x^6 - 4*x
^5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x - 4) - 24*x - 4)*sqrt(x^3 + 1))*sqr...
```

3.86.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^3+10+6\sqrt{3}\right)}\,dx$$

input
```
integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)
```

3.86.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.86.8 Giac [F]

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\sqrt{x^3+1}\,\left(x^3+6\sqrt{3}+10\right)}\,dx$$

input
```
int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)),x)
```

output
```
int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)), x)
```

3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$

3.87.1 Optimal result

Integrand size = 25, antiderivative size = 210

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

output

```
-1/18*arctan(1/2*3^(1/4)*(1-2*x-3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2)
)*3^(3/4)*2^(1/2)-1/36*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1
)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(
1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/6*
(1+3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)
```

3.87.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.24

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = -\frac{x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,\frac{1}{4}(5+3\sqrt{3})\,x^3\right)}{4\,(-5+3\sqrt{3})}$$

input
```
Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

output
```
-1/4*(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(-5 +
 3*Sqrt[3])
```

3.87.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3+1}\,(x^3-6\sqrt{3}+10)}\,dx$$

$$\Big\downarrow \; 989$$

$$-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(-2x-\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{(2+\sqrt{3})\arctan\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{(2+\sqrt{3})\,\text{arctanh}\left(\frac{(1+\sqrt{3})\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

input
```
Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

output
```
-1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 +
 x^3])])/(Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*
(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*Ar
cTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]
*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*
3^(3/4))])/(3*Sqrt[2]*3^(3/4))
```

3.87.3.1 Defintions of rubi rules used

rule 989
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

3.87.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 48.71 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.68

method	result
default	$\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(6\sqrt{3}-12\right)\sqrt{x^{3}+1}}-\frac{\sqrt{2}\left(\underset{\_\alpha=\text{RootOf}(\_}{}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{2\left(\sqrt{3}-1\right)^{2}\sqrt{3}}{9}-\frac{\left(\sqrt{3}-1\right)^{2}}{3}+\frac{2\left(\sqrt{3}-1\right)\sqrt{3}}{9}+\frac{\sqrt{3}}{9}-\frac{2}{3}\right)\Pi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{i\left(\sqrt{3}-1\right)^{2}\sqrt{3}}{6}\right.}{3\left(\sqrt{3}-1\right)\sqrt{x^{3}+1}}$
trager	Expression too large to display

input
```
int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3^(1/2)-1)/(6*3^(1/2)-12)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*
I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/
2)-_alpha+2)/(1-2*_alpha-3^(1/2))*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2
)*((-I*3^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-
3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha-1/2*_alpha*3^(1/
2)-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)),_alpha=RootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/2)+4))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. $2(146) = 292$.

Time = 0.39 (sec) , antiderivative size = 1702, normalized size of antiderivative = 8.10

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/72*sqrt(-7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*
x^6 - 16*x^5 - 20*x^4 + 32*x^3 - 44*x^2 + (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3
 - 42*x^2 - sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x - 4) +
 (336*x^5 + 33*x^4 - 132*x^3 - 474*x^2 - sqrt(3)*(194*x^5 + 19*x^4 - 76*x^
3 - 274*x^2 - 152*x - 76) - 264*x - 132)*sqrt(-56*sqrt(3) - 97) - 24*x - 4
)*sqrt(x^3 + 1)*sqrt(-7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12) - 2*sqrt(
3)*(x^7 - 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(26*x^7 + 12*x^6 - 48*x^5
- 98*x^4 - 96*x^3 - 48*x^2 - sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 - 56*x^4 - 5
6*x^3 - 28*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3) - 97) + 8*x + 16)/(x^8 - 4*
x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) - 1/72*sqrt
(-7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12)*log((x^8 - x^7 - 11*x^6 - 16*
x^5 - 20*x^4 + 32*x^3 - 44*x^2 - (5*x^6 - 6*x^5 - 33*x^4 - 44*x^3 - 42*x^2
 - sqrt(3)*(3*x^6 - 4*x^5 - 17*x^4 - 28*x^3 - 22*x^2 - 8*x - 4) + (336*x^5
 + 33*x^4 - 132*x^3 - 474*x^2 - sqrt(3)*(194*x^5 + 19*x^4 - 76*x^3 - 274*x
^2 - 152*x - 76) - 264*x - 132)*sqrt(-56*sqrt(3) - 97) - 24*x - 4)*sqrt(x^
3 + 1)*sqrt(-7*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) - 12) - 2*sqrt(3)*(x^7 -
 8*x^6 - 7*x^4 - 16*x^3 - 8*x - 8) + 3*(26*x^7 + 12*x^6 - 48*x^5 - 98*x^4
- 96*x^3 - 48*x^2 - sqrt(3)*(15*x^7 + 7*x^6 - 28*x^5 - 56*x^4 - 56*x^3 - 2
8*x^2 - 8*x) - 16*x)*sqrt(-56*sqrt(3) - 97) + 8*x + 16)/(x^8 - 4*x^7 + 16*
x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1/72*sqrt(-7*sq...
```

3.87.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^3-6\sqrt{3}+10\right)}\,dx$$

input
```
integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

3.87.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3-6\sqrt{3}+10)\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.87.8 Giac [F]

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = \int \frac{x}{(x^3-6\sqrt{3}+10)\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^3}\,(10-6\sqrt{3}+x^3)}\,dx = \int \frac{x}{\sqrt{x^3+1}\,(x^3-6\sqrt{3}+10)}\,dx$$

input
```
int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)),x)
```

output
```
int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)), x)
```

3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

3.88.1 Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

output
```
1/36*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2
))*3^(3/4)*2^(1/2)+1/18*arctan(1/2*3^(1/4)*(1+2*x+3^(1/2))*2^(1/2)/(x^3-1)
^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1-x)*(1+3^(1
/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/18*arctanh(1/6*(
1-3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)
```

3.88.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.29

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = -\frac{x^2\sqrt{1-x^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

output
```
-((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])]
)/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

3.88.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3-1}\left(x^3-6\sqrt{3}-10\right)}dx$$

$$\Big\downarrow \text{ 990}$$

$$\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

input
```
Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

output
```
((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x
^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] +
 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*Arc
Tanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]
*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]
*3^(3/4))])/(3*Sqrt[2]*3^(3/4))
```

3.88.3.1 Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

3.88.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 41.43 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.58

method	result
default	$\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(12+6\sqrt{3}\right)\sqrt{x^{3}-1}}-\frac{\sqrt{2}\left(\underline{}\alpha=\mathrm{Root}\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{\left(1+\sqrt{3}\right)^{2}}{3}-\frac{2\left(1+\sqrt{3}\right)^{2}\sqrt{3}}{9}+\frac{2}{3}+\frac{\sqrt{3}}{9}-\frac{2\left(1+\sqrt{3}\right)\sqrt{3}}{9}\right)\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{i\left(1+\sqrt{3}\right)^{2}}{3}\right.}{3\left(1+\sqrt{3}\right)\sqrt{x^{3}-1}}$
trager	Expression too large to display

input
```
int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-1-3^(1/2))/(12+6*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1
/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(
((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1
/2)+_alpha+2)/(-1-2*_alpha-3^(1/2))*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))
^(1/2)*((-I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1
/2)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((-1+x)
/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)-1/2*_alpha
*3^(1/2)+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)),_alpha=RootOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(146) = 292$.

Time = 0.40 (sec) , antiderivative size = 1718, normalized size of antiderivative = 7.74

$$\int \frac{x}{\sqrt{-1+x^3}\,\left(-10-6\sqrt{3}+x^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/72*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*
x^6 + 16*x^5 - 20*x^4 - 32*x^3 - 44*x^2 - 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 -
 16*x^3 - 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^
2 + sqrt(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 1
6*x)*sqrt(56*sqrt(3) - 97) + ((336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 + sqrt
(3)*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*sqrt
(x^3 - 1)*sqrt(56*sqrt(3) - 97) + (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^
2 + sqrt(3)*(3*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + 24*x -
4)*sqrt(x^3 - 1))*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12) - 8*x +
16)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)
) + 1/72*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12)*log((x^8 + x^7 -
11*x^6 + 16*x^5 - 20*x^4 - 32*x^3 - 44*x^2 - 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^
4 - 16*x^3 - 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48
*x^2 + sqrt(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x)
- 16*x)*sqrt(56*sqrt(3) - 97) - ((336*x^5 - 33*x^4 - 132*x^3 + 474*x^2 + s
qrt(3)*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x + 132)*s
qrt(x^3 - 1)*sqrt(56*sqrt(3) - 97) + (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42
*x^2 + sqrt(3)*(3*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + 24*x
 - 4)*sqrt(x^3 - 1))*sqrt(-7*sqrt(3) + 3*sqrt(56*sqrt(3) - 97) + 12) - 8*x
 + 16)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x...
```

3.88.6 Sympy [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-6\sqrt{3}-10\right)}\,dx$$

input
```
integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

3.88.7 Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.88.8 Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(-x^3+6\sqrt{3}+10\right)}\,dx$$

input
```
int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)),x)
```

output
```
int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)), x)
```

3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

3.89.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

output
```
-1/12*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/
2))*3^(1/4)*2^(1/2)+1/18*arctan(1/6*(1+3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1
/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/2*3^(1/4)*(1+2*x-3^(1/2))*
2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/36*arctanh(1/2*3^(1/4
)*(1-x)*(1+3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx=\frac{x^2\sqrt{1-x^3}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\left(-5+3\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

output
```
(x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3])
)])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

3.89.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {990}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3-1}\left(x^3+6\sqrt{3}-10\right)}dx$$

$$\Big\downarrow \; 990$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

input
```
Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

output
```
-1/2*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-
1 + x^3])])/(Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[
-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^
(1/4)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqr
t[-1 + x^3])])/(3*Sqrt[2]*3^(1/4))
```

3.89.3.1 Defintions of rubi rules used

rule 990
```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

3.89.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 41.40 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.64

method	result
default	$\frac{2\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\left(6\sqrt{3}-12\right)\sqrt{x^{3}-1}}-\frac{\sqrt{2}\left(\underset{\_\alpha=\operatorname{RootOf}\left(\_\right.}{}\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{2\left(1-\sqrt{3}\right)^{2}\sqrt{3}}{9}+\frac{\left(1-\sqrt{3}\right)^{2}}{3}+\frac{2\left(1-\sqrt{3}\right)\sqrt{3}}{9}+\frac{2}{3}-\frac{\sqrt{3}}{9}\right)\Pi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\left(1-\sqrt{3}\right)^{2}\sqrt{3}}{6}\right.}{3\left(1-\sqrt{3}\right)\sqrt{x^{3}-1}}$
trager	Expression too large to display

input
```
int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3^(1/2)-1)/(6*3^(1/2)-12)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/
2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi((
(-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2
)-_alpha-2)/(-3^(1/2)+2*_alpha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(
1/2)*((-I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2
)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha+1/2*_alpha*3^
(1/2)+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(
1/2)),_alpha=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)+4))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(148) = 296$.

Time = 0.39 (sec) , antiderivative size = 1538, normalized size of antiderivative = 7.19

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/36*sqrt(14*sqrt(3) + 24)*arctan(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8)
 + 18*x - 12)*sqrt(14*sqrt(3) + 24)/sqrt(x^3 - 1)) - 1/72*sqrt(7*sqrt(3) +
 3*sqrt(-56*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5 - 20*x^4
- 32*x^3 - 44*x^2 + (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 - sqrt(3)*(3
*x^6 + 4*x^5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + (336*x^5 - 33*x^4 - 1
32*x^3 + 474*x^2 - sqrt(3)*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x +
76) - 264*x + 132)*sqrt(-56*sqrt(3) - 97) + 24*x - 4)*sqrt(x^3 - 1)*sqrt(7
*sqrt(3) + 3*sqrt(-56*sqrt(3) - 97) + 12) + 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4
 - 16*x^3 - 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*
x^2 - sqrt(3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) -
 16*x)*sqrt(-56*sqrt(3) - 97) - 8*x + 16)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 +
 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) + 1/72*sqrt(7*sqrt(3) + 3*sqrt(-56
*sqrt(3) - 97) + 12)*log((x^8 + x^7 - 11*x^6 + 16*x^5 - 20*x^4 - 32*x^3 -
44*x^2 - (5*x^6 + 6*x^5 - 33*x^4 + 44*x^3 - 42*x^2 - sqrt(3)*(3*x^6 + 4*x^
5 - 17*x^4 + 28*x^3 - 22*x^2 + 8*x - 4) + (336*x^5 - 33*x^4 - 132*x^3 + 47
4*x^2 - sqrt(3)*(194*x^5 - 19*x^4 - 76*x^3 + 274*x^2 - 152*x + 76) - 264*x
 + 132)*sqrt(-56*sqrt(3) - 97) + 24*x - 4)*sqrt(x^3 - 1)*sqrt(7*sqrt(3) +
3*sqrt(-56*sqrt(3) - 97) + 12) + 2*sqrt(3)*(x^7 + 8*x^6 + 7*x^4 - 16*x^3 -
 8*x + 8) + 3*(26*x^7 - 12*x^6 - 48*x^5 + 98*x^4 - 96*x^3 + 48*x^2 - sqrt(
3)*(15*x^7 - 7*x^6 - 28*x^5 + 56*x^4 - 56*x^3 + 28*x^2 - 8*x) - 16*x)*s...
```

3.89.6 Sympy [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-10+6\sqrt{3}\right)}\,dx$$

input
```
integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

3.89.7 Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.89.8 Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx=\int \frac{x}{\sqrt{x^3-1}\left(x^3+6\sqrt{3}-10\right)}\,dx$$

input
```
int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)),x)
```

output
```
int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)), x)
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

3.90.1 Optimal result

Integrand size = 40, antiderivative size = 65

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

output
```
1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1
/2))*(-3+2*3^(1/2))^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 8.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-4+4\sqrt{3}x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x
^4]),x]
```

output
```
(Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
 + x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

3.90.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2278, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}x^2-4}}dx$$

$$\Big\downarrow \text{2278}$$

$$-4\left(2-\sqrt{3}\right)\int \frac{1}{\frac{4\left(x-\sqrt{3}+1\right)^4}{x^4+4\sqrt{3}x^2-4}+12\left(3-2\sqrt{3}\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{x^4+4\sqrt{3}x^2-4}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x
]
```

output
```
((2 - Sqrt[3])*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[
-4 + 4*Sqrt[3]*x^2 + x^4])])/Sqrt[3*(-3 + 2*Sqrt[3])]
```

3.90.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

3.90.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.03

method	result
elliptic	$\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(\frac{\sqrt{3}}{2}+1\right)x^2}\,F\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right), i\sqrt{1+4\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4\sqrt{3}\,x^2}} - 2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\left(-1-\sqrt{3}\right)^2\sqrt{3}-8+4\sqrt{3}\,x^2+}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\left(-1-\sqrt{3}\right)^2\sqrt{3}-}}\right)}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\left(-1-\sqrt{3}\right)}}\right.$

input
```
int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1/2*3^(1/2)+1)*
x^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),
I*(1+4*3^(1/2)*(1/2*3^(1/2)+1))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*(
-1-3^(1/2))^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(4*(-1-3^(1/2))^2*3^(1/2)-8+4*3
^(1/2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*(-1-3^(1/2))^2*3^(1/2)-
4)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2
))*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4+4*
3^(1/2)*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/
(-1-3^(1/2))^2,(1/2*3^(1/2)+1)^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(47) = 94$.

Time = 0.36 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.97

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$$
$$= \frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37\,x^{12}-204\,x^{11}+804\,x^{10}-2408\,x^9+3708\,x^8-5472\,x^7+6432\,x^6+10944\,x^5+1}{}\right.$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algo
rithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 +
3708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864
*x^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 10
08*x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320
*x^7 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256)
 - 1152*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqr
t(3)*(7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5
 - 3696*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 +
 12*x^11 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*
x^4 - 320*x^3 + 768*x^2 - 384*x + 64))
```

3.90.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

input
```
integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x
)
```

output
```
Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
 4)), x)
```

3.90.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) +
1)), x)
```

3.90.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) +
1)), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

input
```
int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

output
```
int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
 x)
```

3.91 $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

3.91.1 Optimal result

Integrand size = 40, antiderivative size = 63

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

output
```
-1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/
2))*(3+2*3^(1/2))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 8.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x
^4]),x]
```

output
```
-1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]
*x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

3.91.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2278, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}dx$$

$$\Big\downarrow \text{2278}$$

$$-4\left(2+\sqrt{3}\right)\int \frac{1}{\frac{4\left(x+\sqrt{3}+1\right)^4}{x^4-4\sqrt{3}x^2-4}+12\left(3+2\sqrt{3}\right)}d\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{x^4-4\sqrt{3}x^2-4}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x
]
```

output
```
-(((2 + Sqrt[3])*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*Sqrt[
-4 - 4*Sqrt[3]*x^2 + x^4])])/Sqrt[3*(3 + 2*Sqrt[3])])
```

3.91.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

3.91.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.31 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

method	result
elliptic	$\frac{\sqrt{1-\left(-\frac{\sqrt{3}}{2}-1\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\,F\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right), i\sqrt{1-4\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4\sqrt{3}\,x^2}}+2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\left(\sqrt{3}-1\right)^2\sqrt{3}-8-4\sqrt{3}\,x^2+2}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\left(\sqrt{3}-1\right)^2\sqrt{3}-4}}\right)}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\left(\sqrt{3}-1\right)^2}}\right.$

input
```
int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/(1/2*I+1/2*I*3^(1/2))*(1-(-1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*
x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),
I*(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*(3
^(1/2)-1)^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(-4*(3^(1/2)-1)^2*3^(1/2)-8-4*3^(
1/2)*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*(3^(1/2)-1)^2*3^(1/2)-4)^(1
/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1/2*3^(1/2)-1)^(1/2)/(3^(1/2)-1)*(1-
(-1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2
)*x^2)^(1/2)*EllipticPi((-1/2*3^(1/2)-1)^(1/2)*x,1/(-1/2*3^(1/2)-1)/(3^(1/
2)-1)^2,(1-1/2*3^(1/2))^(1/2)/(-1/2*3^(1/2)-1)^(1/2)))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(45) = 90$.

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$$
$$= \frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}(2x^4-10x^3+3x^2-10x+2)+24\right)\sqrt{x^4-4\sqrt{3}x}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right.$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algo
rithm="fricas")
```

output
```
1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^
4 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*
sqrt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

3.91.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

input
```
integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x
)
```

output
```
Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
 4)), x)
```

3.91.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) +
1)), x)
```

3.91.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) +
1)), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

output
```
int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
 x)
```

3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

3.92.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \sqrt{3}\arctan\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

output
```
ln(1+x)-3/2*ln(2+x-(x^3+2)^(1/3))+arctan(1/3*(1+2*(2+x)/(x^3+2)^(1/3))*3^(
1/2))*3^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = -\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2+x^3}}{4+2x+\sqrt[3]{2+x^3}}\right)-\log\left(-2-x+\sqrt[3]{2+x^3}\right)+\frac{1}{2}\log\left(4+4x+x^2+(2+x)\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)$$

input
```
Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

output
```
-(Sqrt[3]*ArcTan[(Sqrt[3]*(2 + x^3)^(1/3))/(4 + 2*x + (2 + x^3)^(1/3))]) -
 Log[-2 - x + (2 + x^3)^(1/3)] + Log[4 + 4*x + x^2 + (2 + x)*(2 + x^3)^(1/
3) + (2 + x^3)^(2/3)]/2
```

3.92.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2576}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

$$\Big\downarrow \; 2576$$

$$\sqrt{3}\arctan\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\log(x+1)$$

input
```
Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

output
```
Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (
3*Log[2 + x - (2 + x^3)^(1/3)])/2
```

3.92.3.1 Defintions of rubi rules used

rule 2576
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

3.92.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.09 (sec) , antiderivative size = 816, normalized size of antiderivative = 15.40

method	result	size
trager	Expression too large to display	816

input
```
int((-1+x)/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2-_Z+1)*ln(-(1239*RootOf(_Z^2-_Z+1)^2*x^3+4504*RootOf(_Z^2-_Z+1)
*(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2-2478*RootOf(_Z^2
-_Z+1)^2*x^2+3265*RootOf(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/
3)+335*x*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x+335*(x^3+2)
^(1/3)*x^2-4956*RootOf(_Z^2-_Z+1)^2*x+10816*RootOf(_Z^2-_Z+1)*x^2+1574*x^3
+670*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)+1340*x*(x^3+2)^(1
/3)+21632*RootOf(_Z^2-_Z+1)*x+7870*x^2+1340*(x^3+2)^(1/3)+17346*RootOf(_Z^
2-_Z+1)+15740*x+11018)/(1+x)^2)-ln((-1239*RootOf(_Z^2-_Z+1)^2*x^3+4504*Roo
tOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2+24
78*RootOf(_Z^2-_Z+1)^2*x^2+5743*RootOf(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+
1)*(x^3+2)^(2/3)-4839*x*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3
)*x-4839*(x^3+2)^(1/3)*x^2+4956*RootOf(_Z^2-_Z+1)^2*x+5860*RootOf(_Z^2-_Z+
1)*x^2-6078*x^3-9678*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)-1
9356*x*(x^3+2)^(1/3)+11720*RootOf(_Z^2-_Z+1)*x-16208*x^2-19356*(x^3+2)^(1/
3)+17346*RootOf(_Z^2-_Z+1)-32416*x-28364)/(1+x)^2)*RootOf(_Z^2-_Z+1)+ln((-
1239*RootOf(_Z^2-_Z+1)^2*x^3+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x+4504*R
ootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2+2478*RootOf(_Z^2-_Z+1)^2*x^2+5743*RootO
f(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)-4839*x*(x^3+2)^(2/3)
+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x-4839*(x^3+2)^(1/3)*x^2+4956*RootO
f(_Z^2-_Z+1)^2*x+5860*RootOf(_Z^2-_Z+1)*x^2-6078*x^3-9678*(x^3+2)^(2/3)...
```

3.92.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.92.6 Sympy [F]

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

input
```
integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
```

output
```
Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.92.7 Maxima [F]

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.92.8 Giac [F]

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{x-1}{(x^3+2)^{1/3}\,(x+1)}\,dx$$

input
```
int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
```

output
```
int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

3.93.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right) - \frac{1}{4}\log\left(-x+\sqrt[3]{2+x^3}\right)$$

output
```
-1/2*ln(1+x)+3/4*ln(2+x-(x^3+2)^(1/3))-1/4*ln(-x+(x^3+2)^(1/3))+1/6*arctan
(1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2*(2+x)/(x^3
+2)^(1/3))*3^(1/2))*3^(1/2)
```

3.93.2 Mathematica [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

input
```
Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

output
```
Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

3.93.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2575, 769, 2576}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}dx$$

$$\Big\downarrow \text{2575}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[3]{x^3+2}}dx+\frac{1}{2}\int \frac{1-x}{(x+1)\sqrt[3]{x^3+2}}dx$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{2}\int \frac{1-x}{(x+1)\sqrt[3]{x^3+2}}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+2}-x\right)\right)$$

$$\Big\downarrow \text{2576}$$

$$\frac{1}{2}\left(-\sqrt{3}\arctan\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\log(x+1)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+2}-x\right)\right)$$

input
```
Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]]) - Log[1 + x]
 + (3*Log[2 + x - (2 + x^3)^(1/3)])/2)/2 + (ArcTan[(1 + (2*x)/(2 + x^3)^(1
/3))/Sqrt[3]]/Sqrt[3] - Log[-x + (2 + x^3)^(1/3)]/2)/2
```

3.93.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 2575
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[1/(2*c)   Int[(c - d*x)/((
c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^
3 - a*d^3, 0]
```

rule 2576
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

3.93.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.13 (sec) , antiderivative size = 1421, normalized size of antiderivative = 13.16

method	result	size
trager	Expression too large to display	1421

input
```
int(1/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln((-4550781346817636-68457312523761*x^6-6728375859478224*x-6258954287
88672*x^5+4993190285176576*RootOf(_Z^2+_Z+1)^2*x^3+8816461926585488*RootOf
(_Z^2+_Z+1)*x^3+1055101552116528*RootOf(_Z^2+_Z+1)*x^2-21283128527537520*R
ootOf(_Z^2+_Z+1)*x+9094739448000192*RootOf(_Z^2+_Z+1)^2*x^2+58848314075295
36*RootOf(_Z^2+_Z+1)^2*x-234710785795752*x^4+2151515536461060*x^3-46942157
1591504*x^2+1326316169500028*RootOf(_Z^2+_Z+1)^2*x^6+4346750471470680*Root
Of(_Z^2+_Z+1)^2*x^5+4547369724000096*RootOf(_Z^2+_Z+1)^2*x^4+1538689763503
27*RootOf(_Z^2+_Z+1)*x^6-868588603920114*RootOf(_Z^2+_Z+1)*x^5-15559137585
059152*RootOf(_Z^2+_Z+1)+16295099853018372*x*(x^3+2)^(2/3)-912549035791293
6*(x^3+2)^(1/3)+527550776058264*RootOf(_Z^2+_Z+1)*x^4-14648813469281292*x*
(x^3+2)^(1/3)+10107087250606332*(x^3+2)^(2/3)-4682817420507954*(x^3+2)^(1/
3)*x^2-928201890361806*(x^3+2)^(2/3)*x^4-540325086981687*(x^3+2)^(1/3)*x^5
-1959537324097146*(x^3+2)^(2/3)*x^3-480288966205944*(x^3+2)^(1/3)*x^4+5569
211342170836*(x^3+2)^(2/3)*x^2+1200722415514860*(x^3+2)^(1/3)*x^3+71155808
83942020*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)+3372637147591320*RootOf(_Z^2+_Z+1
)*(x^3+2)^(1/3)+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+28844
9229728205*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^5-1306943427662820*RootOf(_
Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^3-601904942144643*RootOf(_Z^2+_Z+1)*(x^3+2)^(2
/3)*x^4+1286927332633530*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^4-58077394950
3594*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_...
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(84) = 168$.

Time = 1.04 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.47

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}(7114781247\,x^4 + 13663058416\,x^3 - 46178206896\,x^2 - 1268425}{}\right.$$
$$+ \frac{1}{12}\log\left(\frac{22\,x^6 + 6\,x^5 - 48\,x^4 + 44\,x^3 + 24\,x^2 + 3\,(7\,x^4 - 2\,x^3 - 32\,x^2 - 20\,x + 4)(x^3+2)^{\frac{2}{3}} + 3\,(7\,x^5 -}{x^6 + 6\,x^5 + 15\,x^4 + 20\,x^3 + 15\,x^2 + 6\,x + 1}\right.$$

input
```
integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 136
63058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(
2/3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26
102613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(
1/3) + sqrt(3)*(932925708335594356631323018885*x^6 + 3821515357110852788592
35047618*x^5 + 673924074224408772959625384792*x^4 + 889426563183087468015 5
80290048*x^3 + 888876515195959220955879945824*x^2 + 351260598258508240019 9
71964880*x - 476740009955972110578168843 04))/(789054348145647217457084648 8
3*x^6 + 337746705836458222863347934450*x^5 + 155989527760585878943360709 76
*x^4 - 895430525315100108684787964824*x^3 + 361667862240477028869533375352
*x^2 + 254180230101163251064597209033 6*x + 155481528682333409231448596888 0
)) + 1/12*log((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^
3 - 32*x^2 - 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x
 + 32)*(x^3 + 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*
x^2 + 6*x + 1))
```

3.93.6 Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

input
```
integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

output
```
Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.93.7 Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.93.8 Giac [F]

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{1/3}\,(x+1)}\,dx$$

input
```
int(1/((x^3 + 2)^(1/3)*(x + 1)),x)
```

output
```
int(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.94 $\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

output
```
1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3+a)^(1/3))/(a+b)^(1/
3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/(a+b)^(1/3)
*3^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{a+b}x}{\sqrt{3}\sqrt[3]{a+b}x-(3i+\sqrt{3})\sqrt[3]{a+bx^3}}\right)+(1+i\sqrt{3})\left(2\log\left(2\sqrt[3]{a+b}x+(1+i\sqrt{3})\sqrt[3]{a\cdots}\right.\right.}{12\sqrt[3]{a+b}}$$

input
```
Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

output
```
(-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(a + b)^(1/3)*x)/(Sqrt[3]*(a + b)^(
1/3)*x - (3*I + Sqrt[3])*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3])*(2*Log[2*(a
 + b)^(1/3)*x + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)] - Log[(-((a + b)^(1/3)*
x) + (a + b*x^3)^(1/3))*((2*I)*(a + b)^(1/3)*x + (I + Sqrt[3])*(a + b*x^3)
^(1/3))]))/(12*(a + b)^(1/3))
```

3.94.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{901}$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

input
```
Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

output
```
ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a +
b)^(1/3)) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3)*x - (a + b*
x^3)^(1/3)]/(2*(a + b)^(1/3))
```

3.94.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.94.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left((a+b)^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3(a+b)^{\frac{1}{3}}x}\right)+\ln\left(\frac{-(a+b)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{(a+b)^{\frac{2}{3}}x^{2}+(a+b)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}}{3(a+b)^{\frac{1}{3}}}$

input
```
int(1/(-x^3+1)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a+b)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*((a+b)^(1/3)*x+2*(b*x^3+a)^(1
/3))/(a+b)^(1/3)/x)+ln((-(a+b)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln(((a+b)^(
2/3)*x^2+(a+b)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(78) = 156.

Time = 108.17 (sec) , antiderivative size = 1252, normalized size of antiderivative = 12.78

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log(-((a^3 - 27*a^
2*b - 108*a*b^2 - 81*b^3)*x^9 - 3*(10*a^3 + 54*a^2*b + 45*a*b^2)*x^6 - 3*(
17*a^3 + 18*a^2*b)*x^3 - a^3 + 9*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a*b)*x^4
- a^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(1/3) + 9*((a^2 + 9*a*b + 9*b^2)*x^8 +
 (7*a^2 + 9*a*b)*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-a - b)^(2/3) + 3*sqrt(
1/3)*(3*((4*a^2 + 21*a*b + 18*b^2)*x^7 + (13*a^2 + 15*a*b)*x^4 + a^2*x)*(b
*x^3 + a)^(2/3)*(-a - b)^(2/3) + 3*((a^3 - 2*a^2*b - 12*a*b^2 - 9*b^3)*x^8
 - 5*(a^3 + 4*a^2*b + 3*a*b^2)*x^5 - 5*(a^3 + a^2*b)*x^2)*(b*x^3 + a)^(1/3
) + ((a^3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*(8*a^3 + 18*a^2*b + 9*a*
b^2)*x^6 + 3*a^3*x^3 - a^3)*(-a - b)^(1/3))*sqrt((-a - b)^(1/3)/(a + b)))/
(x^9 - 3*x^6 + 3*x^3 - 1)) - 2*(-a - b)^(2/3)*log(-(3*(b*x^3 + a)^(1/3)*(a
 + b)*(-a - b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a + b)*x + (a*x^3 - a)*(-a
 - b)^(2/3))/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2*a + 3*b)*x^4 + a*x)*(b
*x^3 + a)^(2/3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3*b^2)*x^5 + 2*(a^2 + a
*b)*x^2)*(b*x^3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x^6 + (7*a^2 + 9*a*b)*
x^3 + a^2)*(-a - b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b), 1/18*(6*sqrt(1/3)*
(a + b)*sqrt(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(6*((2*a^2 + 3*a*b)
*x^7 - (a^2 + 3*a*b)*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) - 6*((a
^3 + 10*a^2*b + 18*a*b^2 + 9*b^3)*x^8 + (7*a^3 + 16*a^2*b + 9*a*b^2)*x^5 +
 (a^3 + a^2*b)*x^2)*(b*x^3 + a)^(1/3) - ((a^3 - 9*a^2*b - 36*a*b^2 - 27...
```

3.94.6 Sympy [F]

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\int \frac{1}{x^3\sqrt[3]{a+bx^3} - \sqrt[3]{a+bx^3}}\,dx$$

input
```
integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

output
```
-Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

3.94.7 Maxima [F]

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{1}{3}}(x^3-1)}\,dx$$

input
```
integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

3.94.8 Giac [F]

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{1}{3}}(x^3-1)}\,dx$$

input
```
integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\int \frac{1}{(x^3-1)\,(b\,x^3+a)^{1/3}}\,dx$$

input
```
int(-1/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

output
```
-int(1/((x^3 - 1)*(a + b*x^3)^(1/3)), x)
```

3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

3.95.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

output
```
1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^
3+a)^(1/3))/(a+b)^(1/3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/(a+b)^(1/3)*3^(1/2)+1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1
/3))*3^(1/2))/(a+b)^(1/3)*3^(1/2)
```

3.95.2 Mathematica [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx=\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

input
```
Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

output
```
Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

3.95.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x^2+x+1)\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(\frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}-\frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

input
```
Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

output
```
ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a +
b)^(1/3)) + ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqr
t[3]*(a + b)^(1/3)) + Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)]/(2*(a + b)^(1
/3)) - Log[(a + b)^(1/3)*x - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3))
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.95.4 Maple [F]

$$\int \frac{1+x}{(x^2+x+1)\,(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

output
```
int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

3.95.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1+x}{(1+x+x^2)\,\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.95.6 Sympy [F]

$$\int \frac{1+x}{(1+x+x^2)\,\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{\sqrt[3]{a+bx^3}\,(x^2+x+1)}\,dx$$

input
```
integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
```

output
```
Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

3.95.7 Maxima [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

3.95.8 Giac [F]

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{(bx^3+a)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \frac{x+1}{(b\,x^3+a)^{1/3}\,(x^2+x+1)}\,dx$$

input
```
int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)),x)
```

output
```
int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)), x)
```

3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

3.96.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

output
```
1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)
-1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3))*3^(1/2))/(a+b)^(1/3)*3^(
1/2)
```

3.96.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1+\frac{\left(-1-i\sqrt{3}\right)\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right) - i(-i+\sqrt{3})\left(\log\left(\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)\left(2\sqrt[3]{a+b}+\sqrt[3]{\ldots}\right.\right.\right.}{12\sqrt[3]{a+b}}$$

input
```
Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

output
```
(2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(1 + ((-1 - I*Sqrt[3])*(a + b*x^3)^(1/3
))/(a + b)^(1/3))/Sqrt[3]] - I*(-I + Sqrt[3])*(Log[((a + b)^(1/3) - (a + b
*x^3)^(1/3))*(2*(a + b)^(1/3) + (a + b*x^3)^(1/3) - I*Sqrt[3]*(a + b*x^3)^
(1/3))] - 2*Log[2*(a + b)^(1/3) + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)]))/(12
*(a + b)^(1/3))
```

3.96.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {946, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{(1-x^3)\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt[3]{a+b}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a+b}} - \frac{3}{2}\int \frac{1}{x^6+(a+b)^{2/3}+\sqrt[3]{a+b}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a} + \frac{\log\left(1-x^3\right)}{2\sqrt[3]{a+b}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6+(a+b)^{2/3}+\sqrt[3]{a+b}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{\log(1-x^3)}{2\sqrt[3]{a+b}}-\frac{3\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a+b}}+1\right)}{\sqrt[3]{a+b}}+\frac{\log(1-x^3)}{2\sqrt[3]{a+b}}-\frac{3\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a+b}}+\frac{\log(1-x^3)}{2\sqrt[3]{a+b}}-\frac{3\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}\right)$$

input
```
Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]])/(a
+ b)^(1/3)) + Log[1 - x^3]/(2*(a + b)^(1/3)) - (3*Log[(a + b)^(1/3) - (a +
 b*x^3)^(1/3)])/(2*(a + b)^(1/3)))/3
```

3.96.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.96.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

method	result	siz
pseudoelliptic	$-\frac{\arctan\left(\frac{\left((a+b)^{\frac{1}{3}}+2\left(b x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3(a+b)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b x^{3}+a\right)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right)-\frac{\ln\left(\left(b x^{3}+a\right)^{\frac{2}{3}}+(a+b)^{\frac{1}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{2}}{3(a+b)^{\frac{1}{3}}}$	92

input
```
int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a+b)^(1/3)*(arctan(1/3*((a+b)^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/(a+b)
^(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-(a+b)^(1/3))-1/2*ln((b*x^3+a)^(2/3)+(a+
b)^(1/3)*(b*x^3+a)^(1/3)+(a+b)^(2/3)))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.

Time = 0.26 (sec) , antiderivative size = 387, normalized size of antiderivative = 4.03

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(a+b)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left((bx^3+a)^{\frac{1}{3}}(a+b)-(a+b)(-a-b)^{\frac{1}{3}}-2\,(bx^3+a)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}+3\,a-3\,(bx^3+a)^{\frac{1}{3}}}{x^3-1}\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(a+b)\sqrt{-\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{1}{3}}-(-a-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\right)-(-a-b)^{\frac{2}{3}}\log\left((bx^3\right.}{6\,(a+b)}$$

input
```
integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sq
rt(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a
)^(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)
^(1/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)
*log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a +
b)*sqrt(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (
-a - b)^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3
+ a)^(2/3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a -
b)^(2/3)*log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

3.96.6 Sympy [F]

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\int \frac{x^2}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

input
```
integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

output
```
-Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3(a+b)^{\frac{1}{3}}}\right)}{(a+b)^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{(a+b)^{\frac{1}{3}}}+\frac{2b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right)}{(a+b)^{\frac{1}{3}}}}{6b}$$

input
```
integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))
/(a + b)^(1/3))/(a + b)^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) + 2*b*log((b*x^3 + a)^(1/3)
 - (a + b)^(1/3))/(a + b)^(1/3))/b
```

3.96.8 Giac [A] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = -\frac{(a+b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{\sqrt{3}a+\sqrt{3}b} + \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{6\,(a+b)^{\frac{1}{3}}} - \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right|\right)}{3\,(a+b)^{\frac{1}{3}}}$$

input
```
integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
-(a + b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/(a
 + b)^(1/3))/(sqrt(3)*a + sqrt(3)*b) + 1/6*log((b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) - 1/3*log(abs((b*x
^3 + a)^(1/3) - (a + b)^(1/3)))/(a + b)^(1/3)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{9\,a+9\,b}{9\,(-a-b)^{2/3}}\right)}{3\,(-a-b)^{1/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}} - \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}$$

input
```
int(-x^2/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

output
```
log((a + b*x^3)^(1/3) - (9*a + 9*b)/(9*(- a - b)^(2/3)))/(3*(- a - b)^(1/3
)) + (log((a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)^2*(9*a + 9*b))/(36*(- a -
b)^(2/3)))*(3^(1/2)*1i - 1))/(6*(- a - b)^(1/3)) - (log((a + b*x^3)^(1/3)
- ((3^(1/2)*1i + 1)^2*(9*a + 9*b))/(36*(- a - b)^(2/3)))*(3^(1/2)*1i + 1))
/(6*(- a - b)^(1/3))
```

3.96. $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

3.97 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.97.1 Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arct
an(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3\right.}{6\sqrt[3]{2}}$$

input
```
Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[
2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) -
2^(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

3.97.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{901}$$

$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}$$

input
```
Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
 Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)
)
```

3.97.3.1 Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

3.97.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{2^{\frac{2}{3}}\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right)}{6}$	95
trager	Expression too large to display	780

input
```
int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(2/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+ln(
(2^(1/3)*x+(-x^3+1)^(1/3))/x)-1/2*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x
+(-x^3+1)^(2/3))/x^2))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(67) = 134.

Time = 1.61 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{18}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{}\right.}{6\,(109x^9-105x^6+3x^3+1)}\right.$$

$$+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) +
 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 +
 3*x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*
(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)
*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5
- x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

3.97.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.97.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.97.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.98 $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.98.1 Optimal result

Integrand size = 20, antiderivative size = 233

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))
*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2
^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2
^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

3.98.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{}\right.}{}$$

input
```
Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x
^3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2
^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3
)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2
^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + L
og[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*
(1 - x^3)^(2/3)])/(12*2^(1/3))
```

3.98.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \ 991$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \ 750$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$\downarrow$ 16

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$\downarrow$ 27

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$\downarrow$ 1142

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$\downarrow$ 25

3.98. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-$$
$$\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\ 2^{2/3}}\right)-$$
$$\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-$$
$$\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)-$$
$$\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-$$
$$\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)-$$
$$\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

3.98. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-$$

$$\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-$$

$$\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{2574}$$

3.98. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

$$-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+$$
$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right)-$$
$$\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

input
```
Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
-1/3*(2^(1/3)*(-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/
3)*(1 - x))/(1 - x^3)^(1/3)]/(2*2^(2/3)))) - Log[1 + (2^(1/3)*(1 - x))/(1
- x^3)^(1/3)]/(3*2^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) - (3
*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3
```

3.98.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.98. $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.98.4 Maple [F]

$$\int \frac{x}{\left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}dx$$

input
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

output
```
int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(171) = 342$.

Time = 1.48 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.60

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{36}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2}\right)(-x^{3}+1)^{\frac{2}{3}}+12\sqrt{6}(-1)^{\frac{1}{3}}\,(\right.}{6\,(x^{18}-}\right.$$

$$-\frac{1}{72}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{8}-4\,x^{5}+x^{2}\right)(-x^{3}+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{12}-32\,x^{9}+78\,x^{6}-32\,x^{3}+1\right)-}{x^{12}+4\,x^{9}+6\,x^{6}+4\,x^{3}+1}\right.$$

$$+\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\,(-x^{3}+1)^{\frac{2}{3}}x^{2}-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{4}-x\right)(-x^{3}+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{6}+2\,x^{3}+1\right)}{x^{6}+2\,x^{3}+1}\right)$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-
1)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(
6)*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 +
 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 +
1)) - 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x
^2)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3
 + 1) - 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6
*x^6 + 4*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2
 - 6*2^(1/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x
^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1))
```

3.98.6 Sympy [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.98.7 Maxima [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.98.8 Giac [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)} \, dx$

3.99.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(
1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)} \, dx = \frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - \log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

input
```
Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2
/3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)
^(2/3)])/(6*2^(1/3))
```

3.99.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {946, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(x^3+1)}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}dx^3$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}}{2\sqrt[3]{2}}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log(x^3+1)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{2}\sqrt[3]{1-x^3}+2^{2/3}}d\sqrt[3]{1-x^3}-\frac{\log(x^3+1)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(2^{2/3}\sqrt[3]{1-x^3}+1\right)}{\sqrt[3]{2}}-\frac{\log(x^3+1)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log(x^3+1)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\right)$$

input
```
Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/2^(1/3) - Log[1 +
 x^3]/(2*2^(1/3)) + (3*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*2^(1/3)))/3
```

3.99.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.99.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$\frac{2^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\right)}{12}$	80
trager	Expression too large to display	655

input
```
int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(2/3)*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+2*l
n((-x^3+1)^(1/3)-2^(1/3))-ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3)
))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3
+ 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3
 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.99.6 Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

input
```
integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

input
```
int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
(2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3)
- (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 -
 x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.100 $\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

3.100.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

output
```
1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2
/3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) - 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(2^{2/3}-2\ 2^{2/3}x+2^{2/3}x\right.}{2\sqrt[3]{2}}$$

input
```
Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (
1 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(
2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^
(2/3)])/(2*2^(1/3))
```

3.100.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 383 vs. $2(135) = 270$.

Time = 0.54 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x^2-x+1)\sqrt[3]{1-x^3}}\,dx$$

$$\Big\downarrow \ 2583$$

$$\int \left(\frac{2x}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{x^2}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

input
```
Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

output
```
(2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3
] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[
3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[
(1 - x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/
3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1
/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*
2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.100.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.46 (sec) , antiderivative size = 677, normalized size of antiderivative = 5.01

method	result	size
trager	Expression too large to display	677

input
```
int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(_Z^3+4)*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+
4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+2*(-x^3+1
)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)
*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^
2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+RootOf(_Z^3+4)*x^2+2*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-RootOf(_Z^3+4)*x-2*RootOf(
RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+RootOf(_Z^3+4)+2*RootOf(Roo
tOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))+RootOf(RootOf(_Z^3+4
)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_
Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-RootOf(RootOf(_Z^3+4)^2+2*_
Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)
^2*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2-RootOf(_Z^3+4)*x^2-2*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-2*(-x^3+1)^(2/3)+3*RootOf(_Z^3+4)*
x+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-RootOf(_Z^3+4)-2
*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.

Time = 5.54 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)(-x^3+1)^{\frac{2}{3}}-4\sqrt{2}(-1)^{\frac{1}{3}}\left(x^5-x^4\right.\right.}{6\left(3x^6-9x^5+6x^4\right.}\right.$$
$$-\frac{1}{12}$$
$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{2}{3}}(x^2-3x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4-3x^2+1\right)+4\left(-x^3+1\right)^{\frac{1}{3}}(x^2-x)}{x^4-2x^3+3x^2-2x+1}\right)$$
$$+\frac{1}{6}$$
$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2-x+1\right)-2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2-x+1}\right)$$

input
```
integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^
(2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1
/3)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 -
7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 +
 6*x^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^
3 + 1)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-
x^3 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(
-1)^(1/3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-
1)^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

3.100.6 Sympy [F]

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx=\int \frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x^2-x+1)}\,dx$$

input
```
integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

output
```
Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

3.100.7 Maxima [F]

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx=\int \frac{x+1}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

input
```
integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.100.8 Giac [F]

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{x+1}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

input
```
integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \frac{x+1}{(1-x^3)^{1/3}\,(x^2-x+1)}\,dx$$

input
```
int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
```

output
```
int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

3.101.1 Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

output
```
1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2
/3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) - 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(2^{2/3} - 2\ 2^{2/3}x + 2^{2/3}x^{2}\right.}{2\sqrt[3]{2}}$$

input
```
Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (
1 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(
2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^
(2/3)])/(2*2^(1/3))
```

3.101.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 383 vs. $2(135) = 270$.

Time = 0.56 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2019, 2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2}{\sqrt[3]{1-x^3}\,(x^3+1)}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{x+1}{(x^2-x+1)\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(\frac{2x}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{1}{\sqrt[3]{1-x^3}\,(x^3+1)}+\frac{x^2}{\sqrt[3]{1-x^3}\,(x^3+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

input
```
Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

output
```
(2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3
] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[
3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[
(1 - x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/
3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1
/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*
2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.101.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.96 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.46

method	result	size
trager	Expression too large to display	737

input
```
int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*RootOf(_Z^3+4)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z
^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^
2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-2*(-x^3+1)^(1/3)*RootOf(Roo
tOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-2*(-x^3+1)^(2/3)-6*RootOf(RootOf(
_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*Roo
tOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))-1/2*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^
2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(1/3)*RootOf(_
Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2)*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))
*RootOf(_Z^3+4)-ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*Ro
otOf(_Z^3+4)^2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^
(1/3)*RootOf(_Z^3+4)^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-2*RootOf(Ro
otOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.

Time = 5.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\,(x^4-4\,x^3+5\,x^2-4\,x+1)(-x^3+1)^{\frac{2}{3}}-4\,\sqrt{2}(-1)^{\frac{1}{3}}\,(x^5-x^4\right.}{6\,(3\,x^6-9\,x^5+6\,x^4}\right.$$

$$-\frac{1}{12}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(-x^3+1)^{\frac{2}{3}}(x^2-3\,x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^4-3\,x^2+1)+4\,(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2\,x^3+3\,x^2-2\,x+1}\right)$$

$$+\frac{1}{6}$$

$$\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^3+1)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2-x+1)-2\,(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

input
```
integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^
(2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1
/3)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 -
7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 +
 6*x^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^
3 + 1)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-
x^3 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(
-1)^(1/3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-
1)^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

3.101.6 Sympy [F]

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x+1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x^2-x+1)}\,dx$$

input
```
integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

3.101.7 Maxima [F]

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.101.8 Giac [F]

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\int \frac{(x+1)^2}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

input
```
int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

output
```
int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.102 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1+x)^2}{(1+x^3)^{2/3}}-\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{2\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{\sqrt[3]{2}}$$

output
```
-1/4*ln(1+2^(2/3)*(1+x)^2/(x^3+1)^(2/3)-2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/
3)+1/2*ln(1+2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3)-1/2*arctan(1/3*(1-2*2^(1/
3)*(1+x)/(x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

3.102.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x^3}}{-2\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{1+x^3}}\right)+2\log\left(\sqrt[3]{2}+\sqrt[3]{2}x+\sqrt[3]{1+x^3}\right)-\log\left(2^{2/3}+2\ 2^{2/3}x+2^{2/3}x^2-\right.}{2\sqrt[3]{2}}$$

input
```
Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x^3)^(1/3))/(-2*2^(1/3) - 2*2^(1/3)*x + (1
 + x^3)^(1/3))] + 2*Log[2^(1/3) + 2^(1/3)*x + (1 + x^3)^(1/3)] - Log[2^(2/
3) + 2*2^(2/3)*x + 2^(2/3)*x^2 - 2^(1/3)*(1 + x)*(1 + x^3)^(1/3) + (1 + x^
3)^(2/3)])/(2*2^(1/3))
```

3.102.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(119) = 238.

Time = 0.52 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{(x^2+x+1)\sqrt[3]{x^3+1}}dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(-\frac{2x}{(1-x^3)\sqrt[3]{x^3+1}}+\frac{1}{(1-x^3)\sqrt[3]{x^3+1}}+\frac{x^2}{(1-x^3)\sqrt[3]{x^3+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(1-x^3)}{3\sqrt[3]{2}}-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{(x^3+1)^{2/3}}-\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-2^{2/3}\sqrt[3]{x^3+1}+x+1\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)^2(x+1)\right)}{6\sqrt[3]{2}}$$

input
```
Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

output
```
ArcTan[(1 + (2*2^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - (2
^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
- ArcTan[(1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 + 2^(2/3)*(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[
(1 - x)^2*(1 + x)]/(6*2^(1/3)) + Log[1 - x^3]/(3*2^(1/3)) - Log[1 + (2^(2/
3)*(1 + x)^2)/(1 + x^3)^(2/3) - (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)]/(3*2^(1
/3)) + (2^(2/3)*Log[1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)])/3 - Log[2^(1/3
) - (1 + x^3)^(1/3)]/(2*2^(1/3)) - Log[2^(1/3)*x - (1 + x^3)^(1/3)]/(2*2^(
1/3)) + Log[1 + x - 2^(2/3)*(1 + x^3)^(1/3)]/(2*2^(1/3))
```

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.102.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.40 (sec) , antiderivative size = 714, normalized size of antiderivative = 6.00

method	result	size
trager	Expression too large to display	714

input
```
int((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*RootOf(_Z^3-4)^2+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2
*RootOf(_Z^3-4)^2*x-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-(x^3+1)^(1/3)*RootOf(
_Z^3-4)^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-2*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))*RootOf(_Z^3-4)-ln(-((x^3+1)^(2/3
)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+2*R
ootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-(x
^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2-2*RootOf(Roo
tOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-2*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2))/(x^2+x+1))*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/
2*RootOf(_Z^3-4)*ln(((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+2*(x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootO
f(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+2*(x^3+1)^(1/3)*RootOf(_Z^3-4)*R
ootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+2*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+2*(x^3+1)^(2/3)+6*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2))/(x^2+x+1))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(93) = 186$.

Time = 5.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.25

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$$

$$= \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+7x^5+10x^4+7x^3+10x^2+7x+1)-4\sqrt{2}(x^5+x^4-3x^3-3x^2+x}{6\left(3x^6+9x^5+6x^4+x^3+6x^2+}\right.$$

$$-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^3+1)^{\frac{2}{3}}(x^2+3x+1)-2^{\frac{1}{3}}(x^4-3x^2+1)-4(x^3+1)^{\frac{1}{3}}(x^2+x)}{x^4+2x^3+3x^2+2x+1}\right)$$

$$+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+x+1)+2\cdot 2^{\frac{1}{3}}(x^3+1)^{\frac{1}{3}}(x+1)+2(x^3+1)^{\frac{2}{3}}}{x^2+x+1}\right)$$

input
```
integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*
x^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x
 + 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1
)^(2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*l
og((2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) -
4*(x^3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3
)*log((2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3
+ 1)^(2/3))/(x^2 + x + 1))
```

3.102.6 Sympy [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = -\int \frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\right)dx$$

input
```
integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
```

output
```
-Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1
/3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x*
*3 + 1)**(1/3)), x)
```

3.102.7 Maxima [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \int -\frac{x-1}{(x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

3.102.8 Giac [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \int -\frac{x-1}{(x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = -\int \frac{x-1}{(x^3+1)^{1/3}\,(x^2+x+1)}\,dx$$

input
```
int(-(x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)),x)
```

output
```
-int((x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)), x)
```

3.103 $\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx$

3.103.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)$$

output
```
1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

3.103.2 Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \frac{(1+2x)\,(1-x^3)^{2/3}}{1+x+x^2} + x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

output
```
((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3
, 5/3, x^3]
```

3.103.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^3)^{2/3}}{(x^2+x+1)^2} \, dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(-\frac{2x}{(1-x^3)^{4/3}}+\frac{1}{(1-x^3)^{4/3}}+\frac{x^2}{(1-x^3)^{4/3}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x^2\left(-\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

input
```
Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

output
```
(1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3
, x^3]
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.103.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$	34

input
```
int((-x^3+1)^(2/3)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

3.103.5 Fricas [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx=\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

3.103.6 Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx=\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

input
```
integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

3.103.7 Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

3.103.8 Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2+x+1)^2}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \frac{(1-x^3)^{2/3}}{(x^2+x+1)^2}\,dx$$

input
```
int((1 - x^3)^(2/3)/(x + x^2 + 1)^2,x)
```

output
```
int((1 - x^3)^(2/3)/(x + x^2 + 1)^2, x)
```

3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

3.104.1 Optimal result

Integrand size = 25, antiderivative size = 43

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2 \text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)$$

output
```
1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

3.104.2 Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \frac{(1+2x)\left(1-x^3\right)^{2/3}}{1+x+x^2} + x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

output
```
((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3
, 5/3, x^3]
```

3.104.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{(x^2+x+1)\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(-\frac{2x}{(1-x^3)^{4/3}}+\frac{1}{(1-x^3)^{4/3}}+\frac{x^2}{(1-x^3)^{4/3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^2\left(-\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

input
```
Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

output
```
(1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3
, x^3]
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.104.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$	34

input
```
int((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

3.104.5 Fricas [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx=\int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

3.104.6 Sympy [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx=-\int \frac{x}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\,dx - \int \left(-\frac{1}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\right)dx$$

input
```
integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
```

output
```
-Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1
/3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1
- x**3)**(1/3)), x)
```

3.104.7 Maxima [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

3.104.8 Giac [F]

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \int -\frac{x-1}{(-x^3+1)^{\frac{1}{3}}(x^2+x+1)}\,dx$$

input
```
integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = -\int \frac{x-1}{(1-x^3)^{1/3}\,(x^2+x+1)}\,dx$$

input
```
int(-(x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)),x)
```

output
```
-int((x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)), x)
```

3.105 $\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

output
```
(1+(1-2*x)*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3, 2/3],[5/3],x^3)
```

3.105.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
```

output
```
(1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3
, x^3]
```

3.105.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2393, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}-\int -\frac{2x}{\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{x}{\sqrt[3]{1-x^3}}dx+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \text{ 888}$$

$$x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

input
```
Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

output
```
(1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x
^3]
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.105.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$	34
meijerg	$\frac{x}{(-x^3+1)^{\frac{1}{3}}}-x^2{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)+\frac{x^3{}_2F_1\left(1,\frac{4}{3};2;x^3\right)}{3}$	41

input
```
int((1-x)^2/(-x^3+1)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

3.105.5 Fricas [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx=\int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

3.105.6 Sympy [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((1-x)**2/(-x**3+1)**(4/3),x)
```

output
```
Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)
```

3.105.7 Maxima [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")
```

output
```
x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*
(-x + 1)^(1/3)), x)
```

3.105.8 Giac [F]

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(-x^3+1)^{\frac{4}{3}}}\,dx$$

input
```
integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="giac")
```

output
```
integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \int \frac{(x-1)^2}{(1-x^3)^{4/3}}\,dx$$

input
```
int((x - 1)^2/(1 - x^3)^(4/3),x)
```

output
```
int((x - 1)^2/(1 - x^3)^(4/3), x)
```

3.106 $\int (1-x^3)^{2/3}\,dx$

3.106.1 Optimal result

Integrand size = 11, antiderivative size = 67

$$\int (1-x^3)^{2/3}\,dx=\frac{1}{3}x(1-x^3)^{2/3}-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
1/3*x*(-x^3+1)^(2/3)+1/3*ln(x+(-x^3+1)^(1/3))-2/9*arctan(1/3*(1-2*x/(-x^3+
1)^(1/3))*3^(1/2))*3^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51

$$\int (1-x^3)^{2/3}\,dx=\frac{3(-1+x)(1-x^3)^{2/3}\,\text{AppellF1}\left(\frac{5}{3},-\frac{2}{3},-\frac{2}{3},\frac{8}{3},-\frac{-1+x}{1-(-1)^{2/3}},-\frac{-1+x}{1+\sqrt[3]{-1}}\right)}{5\left(1+\frac{-1+x}{1+\sqrt[3]{-1}}\right)^{2/3}\left(1+\frac{-1+x}{1-(-1)^{2/3}}\right)^{2/3}}$$

input
```
Integrate[(1 - x^3)^(2/3),x]
```

output
```
(3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
 (-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(
1/3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

3.106.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^3\right)^{2/3} dx$$

$$\Big\downarrow \text{ 748}$$

$$\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}dx+\frac{1}{3}\left(1-x^3\right)^{2/3} x$$

$$\Big\downarrow \text{ 769}$$

$$\frac{2}{3}\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{3}\left(1-x^3\right)^{2/3} x$$

input
```
Int[(1 - x^3)^(2/3),x]
```

output
```
(x*(1 - x^3)^(2/3))/3 + (2*(-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2))/3
```

3.106.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

3.106.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

method	result
meijerg	$x_2F_1\left(-\frac{2}{3}, \frac{1}{3}; \frac{4}{3}; x^3\right)$
risch	$-\frac{x(x^3-1)}{3(-x^3+1)^{\frac{1}{3}}} + \frac{2x_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x^3\right)}{3}$
pseudoelliptic	$\frac{3x(-x^3+1)^{\frac{2}{3}}+2\sqrt{3}\arctan\left(\frac{\left(-2(-x^3+1)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)+2\ln\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{(-x^3+1)^{\frac{2}{3}}-x(-x^3+1)^{\frac{1}{3}}+x^2}{x^2}\right)}{9\left(x+(-x^3+1)^{\frac{1}{3}}\right)\left((-x^3+1)^{\frac{2}{3}}-x(-x^3+1)^{\frac{1}{3}}+x^2\right)}$
trager	$\frac{x(-x^3+1)^{\frac{2}{3}}}{3} + \frac{2\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)(-x^3+1)^{\frac{2}{3}}x-5\,\mathrm{RootOf}\left(\_Z^2+\_Z+\right.\right.}{9}$

input
```
int((-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([-2/3,1/3],[4/3],x^3)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \left(1-x^3\right)^{2/3}\,dx = \frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x - \frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right) + \frac{2}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{9}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

output
```
1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-
x^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-
x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

3.106.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \left(1-x^3\right)^{2/3} dx = \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-x**3+1)**(2/3),x)
```

output
```
x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3))
```

3.106.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(52) = 104.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \left(1-x^3\right)^{2/3} dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right) - \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3\,x^2\left(\frac{x^3-1}{x^3}-1\right)} + \frac{2}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right) - \frac{1}{9}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

input
```
integrate((-x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 +
1)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9
*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

3.106.8 Giac [F]

$$\int \left(1-x^3\right)^{2/3} dx = \int \left(-x^3+1\right)^{\frac{2}{3}} dx$$

input
```
integrate((-x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3), x)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.15

$$\int \left(1-x^3\right)^{2/3} dx = x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

input
```
int((1 - x^3)^(2/3),x)
```

output
```
x*hypergeom([-2/3, 1/3], 4/3, x^3)
```

3.107 $\int \frac{(1-x^3)^{2/3}}{x}\,dx$

3.107.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{2}(1-x^3)^{2/3} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

output
```
1/2*(-x^3+1)^(2/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*
(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{6}\left(3(1-x^3)^{2/3} + 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^3}\right) - \log\left(1+\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right)\right)$$

input
```
Integrate[(1 - x^3)^(2/3)/x,x]
```

output
```
(3*(1 - x^3)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2
*Log[-1 + (1 - x^3)^(1/3)] - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/6
```

3.107.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 60, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-x^3\right)^{2/3}}{x}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{\left(1-x^3\right)^{2/3}}{x^3}dx^3$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\int \frac{1}{x^3\sqrt[3]{1-x^3}}dx^3+\frac{3}{2}\left(1-x^3\right)^{2/3}\right)$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^3}}d\sqrt[3]{1-x^3}+\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{3}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+\sqrt[3]{1-x^3}+1}d\sqrt[3]{1-x^3}+\frac{3}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3}\left(-3\int \frac{1}{-x^6-3}d\left(2\sqrt[3]{1-x^3}+1\right)+\frac{3}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(x^3\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+\frac{3}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(x^3\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\right)$$

input
```
Int[(1 - x^3)^(2/3)/x,x]
```

output
```
((3*(1 - x^3)^(2/3))/2 + Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] -
 Log[x^3]/2 + (3*Log[1 - (1 - x^3)^(1/3)])/2)/3
```

3.107.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.107.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

method	result
meijerg	$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{3}\,{}_{3}F_{2}\left(\frac{1}{3},1,1;2,2;x^{3}\right)}{3\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$
pseudoelliptic	$\frac{(-x^3+1)^{\frac{2}{3}}}{2}-\frac{\ln\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{\left(1+2(-x^3+1)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left((-x^3+1)^{\frac{1}{3}}-1\right)}{3}$
trager	$\frac{(-x^3+1)^{\frac{2}{3}}}{2}+\frac{\ln\left(\frac{-211\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-3126\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)(-x^3+1)\right.}{}}{}$

input
```
int((-x^3+1)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/9/Pi*3^(1/2)*GAMMA(2/3)*(-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*P
i*3^(1/2)/GAMMA(2/3)+2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,1],[2,
2],x^3))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}(-x^3+1)^{\frac{2}{3}} - \frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left((-x^3+1)^{\frac{1}{3}}-1\right)$$

input
```
integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
 + 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((
-x^3 + 1)^(1/3) - 1)
```

3.107.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = -\frac{x^2 e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((-x**3+1)**(2/3)/x,x)
```

output
```
-x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*ga
mma(1/3))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}} - 1\right)$$

input
```
integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^
(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 +
 1)^(1/3) - 1)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^
(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x
^3 + 1)^(1/3) - 1))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{(1-x^3)^{2/3}}{x}\,dx = \frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3} + \ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right.$$

input
```
int((1 - x^3)^(2/3)/x,x)
```

output
```
log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6
)^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/
6)^2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(2/3)/2
```

3.108 $\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$

3.108.1 Optimal result

Integrand size = 19, antiderivative size = 384

$$\begin{aligned}\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx &= \frac{(1-x^3)^{2/3}}{2b} - \frac{(a^3+b^3)\,x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2} \\ &+ \frac{a^2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} - \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a^3+b^3}x}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} \\ &+ \frac{(a^3+b^3)^{2/3}\arctan\left(\frac{1+\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^3} + \frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2} \\ &- \frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3} + \frac{(a^3+b^3)^{2/3}\log\left(-\frac{\sqrt[3]{a^3+b^3}x}{a}-\sqrt[3]{1-x^3}\right)}{2b^3} \\ &- \frac{a^2\log\left(x+\sqrt[3]{1-x^3}\right)}{2b^3} + \frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3}\end{aligned}$$

output
```
1/2*(-x^3+1)^(2/3)/b-1/2*(a^3+b^3)*x^2*AppellF1(2/3,1/3,1,5/3,x^3,-b^3*x^3
/a^3)/a^2/b^2+1/2*a*x^2*hypergeom([1/3, 2/3],[5/3],x^3)/b^2-1/3*(a^3+b^3)^
(2/3)*ln(b^3*x^3+a^3)/b^3+1/2*(a^3+b^3)^(2/3)*ln(-(a^3+b^3)^(1/3)*x/a-(-x^
3+1)^(1/3))/b^3-1/2*a^2*ln(x+(-x^3+1)^(1/3))/b^3+1/2*(a^3+b^3)^(2/3)*ln((a
^3+b^3)^(1/3)-b*(-x^3+1)^(1/3))/b^3+1/3*a^2*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))/b^3*3^(1/2)-1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1-2*(a^3+b^3)^(1/
3)*x/a/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)+1/3*(a^3+b^3)^(2/3)*arctan(1/3
*(1+2*b*(-x^3+1)^(1/3)/(a^3+b^3)^(1/3))*3^(1/2))/b^3*3^(1/2)
```

3.108.2 Mathematica [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

input
```
Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
```

output
```
Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

3.108.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2578, 888, 2577, 769, 2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

$$\Big\downarrow \text{2578}$$

$$\frac{\int \frac{b^2-a^2x}{(a+bx)\sqrt[3]{1-x^3}}dx}{b^2}+\frac{a\int \frac{x}{\sqrt[3]{1-x^3}}dx}{b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

$$\Big\downarrow \text{888}$$

$$\frac{\int \frac{b^2-a^2x}{(a+bx)\sqrt[3]{1-x^3}}dx}{b^2}+\frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

$$\Big\downarrow \text{2577}$$

$$\frac{\frac{(a^3+b^3)\int\frac{1}{(a+bx)\sqrt[3]{1-x^3}}dx}{b}-\frac{a^2\int\frac{1}{\sqrt[3]{1-x^3}}dx}{b}}{b^2}+\frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{(a^3+b^3)\int\frac{1}{(a+bx)\sqrt[3]{1-x^3}}dx}{b}-\frac{a^2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)}{b}}{b^2}+\frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

$$\Big\downarrow \text{2581}$$

$$\frac{\frac{(a^3+b^3)\int\left(\frac{a^2}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}-\frac{bxa}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}+\frac{b^2x^2}{\sqrt[3]{1-x^3}(a^3+b^3x^3)}\right)dx}{b}-\frac{a^2\left(\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}\right)}{b}}{b^2}+\frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^3+b^3)\left(-\frac{\arctan\left(\frac{1-\frac{2x\sqrt[3]{a^3+b^3}}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a^3+b^3}}+\frac{\arctan\left(\frac{\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a^3+b^3}}-\frac{\log(a^3+b^3x^3)}{3\sqrt[3]{a^3+b^3}}+\frac{\log\left(-\frac{x\sqrt[3]{a^3+b^3}}{a}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{a^3+b^3}}+\frac{\log\left(\sqrt[3]{a^3+b^3}-b\right.}{2\sqrt[3]{a^3+}}\right.}{b}}{b^2}$$

$$\frac{ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

input
```
Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

3.108. $\int\frac{(1-x^3)^{2/3}}{a+bx}\,dx$

output
```
(1 - x^3)^(2/3)/(2*b) + (a*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/(2*b
^2) + (-((a^2*(-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Lo
g[x + (1 - x^3)^(1/3)]/2))/b) + ((a^3 + b^3)*(-1/2*(b*x^2*AppellF1[2/3, 1/
3, 1, 5/3, x^3, -((b^3*x^3)/a^3)])/a^2 - ArcTan[(1 - (2*(a^3 + b^3)^(1/3)*
x)/(a*(1 - x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*(a^3 + b^3)^(1/3)) + ArcTan[(1 +
 (2*b*(1 - x^3)^(1/3))/(a^3 + b^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a^3 + b^3)^(1
/3)) - Log[a^3 + b^3*x^3]/(3*(a^3 + b^3)^(1/3)) + Log[-(((a^3 + b^3)^(1/3)
*x)/a) - (1 - x^3)^(1/3)]/(2*(a^3 + b^3)^(1/3)) + Log[(a^3 + b^3)^(1/3) -
b*(1 - x^3)^(1/3)]/(2*(a^3 + b^3)^(1/3))))/b)/b^2
```

3.108.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2577
```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Simp[f/d   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[(d*e - c*f
)/d   Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x]
```

rule 2578
```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Simp[1/d^2   Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
 + b*x^3)^(1/3)), x], x] - Simp[b*(c/d^2)   Int[x/(a + b*x^3)^(1/3), x], x]
) /; FreeQ[{a, b, c, d}, x]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.108.4 Maple [F]

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}dx$$

input
```
int((-x^3+1)^(2/3)/(b*x+a),x)
```

output
```
int((-x^3+1)^(2/3)/(b*x+a),x)
```

3.108.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \text{Timed out}$$

input
```
integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.108.6 Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{a+bx}\,dx$$

input
```
integrate((-x**3+1)**(2/3)/(b*x+a),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

3.108.7 Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

3.108.8 Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(1-x^3)^{2/3}}{a+b\,x}\,dx$$

input
```
int((1 - x^3)^(2/3)/(a + b*x),x)
```

output
```
int((1 - x^3)^(2/3)/(a + b*x), x)
```

3.109 $\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$

3.109.1 Optimal result

Integrand size = 22, antiderivative size = 234

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = -\frac{(1-x^3)^{2/3}}{3(1+x^3)} + \frac{x(1-x^3)^{2/3}}{3(1+x^3)} + \frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}$$

output
```
-1/3*(-x^3+1)^(2/3)/(x^3+1)+1/3*x*(-x^3+1)^(2/3)/(x^3+1)+2/3*x^2*(-x^3+1)^
(2/3)/(x^3+1)+1/3*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/6*ln(2^(1/3)-(-x^3
+1)^(1/3))*2^(2/3)+1/6*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/9*arctan(1/
3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/9*arctan(1/3*(
1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.109.2 Mathematica [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$$

input
```
Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

output
```
Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

3.109.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^3)^{2/3}}{(x^2-x+1)^2}\,dx$$

$$\Big\downarrow \ 2583$$

$$\int \left(\frac{2(1-x^3)^{2/3}x}{(x^3+1)^2}+\frac{(1-x^3)^{2/3}}{(x^3+1)^2}+\frac{(1-x^3)^{2/3}x^2}{(x^3+1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2^{2/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}x^2\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{(1-x^3)^{2/3}x}{3(x^3+1)}-\frac{(1-x^3)^{2/3}}{3(x^3+1)}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}+\frac{2(1-x^3)^{2/3}x^2}{3(x^3+1)}$$

input
```
Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

output
```
-1/3*(1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(3*(1 + x^3)) + (2*x^
2*(1 - x^3)^(2/3))/(3*(1 + x^3)) - (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 -
 x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3
)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/3 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-(2^(1/3)*x) - (1
- x^3)^(1/3)]/(3*2^(1/3))
```

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.109.4 Maple [F]

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}dx$$

input
```
int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

output
```
int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

3.109.5 Fricas [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

3.109.6 Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

input
```
integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)
```

3.109.7 Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

3.109.8 Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \frac{(1-x^3)^{2/3}}{(x^2-x+1)^2}\,dx$$

input
```
int((1 - x^3)^(2/3)/(x^2 - x + 1)^2,x)
```

output
```
int((1 - x^3)^(2/3)/(x^2 - x + 1)^2, x)
```

3.110 $\int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$

3.110.1 Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx = \frac{(1-x^3)^{2/3}}{1-x+x^2} - \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2^{2/3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} + \log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
(-x^3+1)^(2/3)/(x^2-x+1)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(-2^
(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+ln(x+(-x^3+1)^(1/3))-2/3*arctan(1/3*(1-2*x
/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(
1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3
^(1/2))*2^(2/3)*3^(1/2)
```

3.110.2 Mathematica [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx=\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

input
```
Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

output
```
Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

3.110.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2583, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

$$\Big\downarrow \text{2583}$$

$$\int \left(-\frac{2\left(1-x^3\right)^{2/3}x^3}{\left(x^3+1\right)^2}+\frac{\left(1-x^3\right)^{2/3}}{\left(x^3+1\right)^2}-\frac{3\left(1-x^3\right)^{2/3}x^2}{\left(x^3+1\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}x}{x^3+1}+\frac{\left(1-x^3\right)^{2/3}}{x^3+1}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)$$

input
```
Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

output
```
(1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(1 + x^3) - (2*ArcTan[(1 -
 (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 - (2*2^(1/3
)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1
- x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(1/3) +
 Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(3*2^(1/3)) - (2*2^(2/3)*Log[-(2^(1/3
)*x) - (1 - x^3)^(1/3)])/3 + Log[x + (1 - x^3)^(1/3)]
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.110.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 26.80 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.11

method	result
trager	$\frac{(-x^3+1)^{\frac{2}{3}}}{x^2-x+1} + \frac{2\ln\left(-\operatorname{RootOf}\left(\_Z^6+432\right)^6 x^3-36\operatorname{RootOf}\left(\_Z^6+432\right)^3(-x^3+1)^{\frac{2}{3}}x+36\operatorname{RootOf}\left(\_Z^6+432\right)^3 x^3-24\operatorname{RootOf}\left(\right.\right.}{3}$
risch	Expression too large to display

input
```
int((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-x^3+1)^(2/3)/(x^2-x+1)+2/3*ln(-RootOf(_Z^6+432)^6*x^3-36*RootOf(_Z^6+432
)^3*(-x^3+1)^(2/3)*x+36*RootOf(_Z^6+432)^3*x^3-24*RootOf(_Z^6+432)^3+432*x
*(-x^3+1)^(2/3)+864*x^2*(-x^3+1)^(1/3))+1/3*RootOf(_Z^6+432)*ln((RootOf(_Z
^6+432)^5*(-x^3+1)^(1/3)+RootOf(_Z^6+432)^4*x^2-RootOf(_Z^6+432)^4*x-RootO
f(_Z^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)+36*RootOf(_Z^6+432)*x^2
-36*RootOf(_Z^6+432)*x+144*(-x^3+1)^(2/3)-36*RootOf(_Z^6+432))/(x^2-x+1))+
1/72*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(_Z^6+432)^4*x-RootOf(_Z^6+432)^4+1
2*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*(-x^3+1)^(2/3))/(x^2-x+1))*RootOf
(_Z^6+432)^4-1/6*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(_Z^6+432)^4*x-RootOf(_
Z^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*(-x^3+1)^(2/3))/(x^2-
x+1))*RootOf(_Z^6+432)-1/36*ln(RootOf(_Z^6+432)^6*x^3+72*RootOf(_Z^6+432)^
3*(-x^3+1)^(2/3)*x+72*RootOf(_Z^6+432)^3*(-x^3+1)^(1/3)*x^2-24*RootOf(_Z^6
+432)^3+864*x*(-x^3+1)^(2/3)-864*x^2*(-x^3+1)^(1/3)-1296*x^3+864)*RootOf(_
Z^6+432)^3-1/3*ln(RootOf(_Z^6+432)^6*x^3+72*RootOf(_Z^6+432)^3*(-x^3+1)^(2
/3)*x+72*RootOf(_Z^6+432)^3*(-x^3+1)^(1/3)*x^2-24*RootOf(_Z^6+432)^3+864*x
*(-x^3+1)^(2/3)-864*x^2*(-x^3+1)^(1/3)-1296*x^3+864)
```

3.110.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 2298, normalized size of antiderivative = 11.55

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/36*(2*sqrt(3)*(-16)^(1/6)*(x^2 - x + 1)*log(9*(4*sqrt(3)*(-16)^(1/6)*(13
1*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112) + 12*2^(2/3)*(
15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) + 24*(31*x^4 +
107*x^3 - 243*x^2 - sqrt(3)*(-23*I*x^4 + 85*I*x^3 + 57*I*x^2 - 104*I*x + 4
*I) - 26*x + 50)*(-x^3 + 1)^(2/3) + 3*(-x^3 + 1)^(1/3)*(sqrt(3)*(-16)^(5/6
)*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23) - 8*(-2)^(1/3)*(50*x^5 -
93*x^4 - 88*x^3 - 7*x^2 + 150*x - 31)))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x
^2 - 3*x + 1)) - 2*sqrt(3)*(-16)^(1/6)*(x^2 - x + 1)*log(-9*(4*sqrt(3)*(-1
6)^(1/6)*(131*x^6 - 48*x^5 - 381*x^4 - 152*x^3 + 267*x^2 + 276*x - 112) -
12*2^(2/3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 + 157*x^2 - 124*x - 42) - 2
4*(31*x^4 + 107*x^3 - 243*x^2 - sqrt(3)*(23*I*x^4 - 85*I*x^3 - 57*I*x^2 +
104*I*x - 4*I) - 26*x + 50)*(-x^3 + 1)^(2/3) + 3*(-x^3 + 1)^(1/3)*(sqrt(3)
*(-16)^(5/6)*(4*x^5 + 69*x^4 - 58*x^3 - 77*x^2 + 12*x + 23) + 8*(-2)^(1/3)
*(50*x^5 - 93*x^4 - 88*x^3 - 7*x^2 + 150*x - 31)))/(x^6 - 3*x^5 + 6*x^4 -
7*x^3 + 6*x^2 - 3*x + 1)) - 24*sqrt(3)*(x^2 - x + 1)*arctan((4*sqrt(3)*(-x
^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x
^3 - 1)) + sqrt(3)*(sqrt(-3)*(-16)^(1/6)*(x^2 - x + 1) + (-16)^(1/6)*(x^2
- x + 1))*log(-9*(12*2^(2/3)*sqrt(-3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3
+ 157*x^2 - 124*x - 42) + 12*2^(2/3)*(15*x^6 - 200*x^5 + 5*x^4 + 216*x^3 +
 157*x^2 - 124*x - 42) - 48*(31*x^4 + 107*x^3 - 243*x^2 - sqrt(3)*(23*I...
```

3.110.6 Sympy [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = -\int\left(-\frac{\left(1-x^3\right)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\right)dx - \int\frac{2x\left(1-x^3\right)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\,dx$$

input
```
integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

output
```
-Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Inte
gral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

3.110.7 Maxima [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(2\,x-1)}{\left(x^2-x+1\right)^2}\,dx$$

input
```
integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
-integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

3.110.8 Giac [F]

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(2\,x-1)}{\left(x^2-x+1\right)^2}\,dx$$

input
```
integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = -\int \frac{(2\,x-1)\,\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

input
```
int(-((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2,x)
```

output
```
-int(((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2, x)
```

3.111 $\int \frac{(1-x^3)^{2/3}}{1+x} dx$

3.111.1 Optimal result

Integrand size = 17, antiderivative size = 177

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx = \frac{1}{2}(1-x^3)^{2/3} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log((1-x)(1+x)^2)}{2\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1
+x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3)
)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

3.111.2 Mathematica [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx=\int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

input
```
Integrate[(1 - x^3)^(2/3)/(1 + x),x]
```

output
```
Integrate[(1 - x^3)^(2/3)/(1 + x), x]
```

3.111.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2578, 888, 2577, 769, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^3)^{2/3}}{x+1}\,dx$$

$$\Big\downarrow \ 2578$$

$$\int \frac{x}{\sqrt[3]{1-x^3}}dx+\int \frac{1-x}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}(1-x^3)^{2/3}$$

$$\Big\downarrow \ 888$$

$$\int \frac{1-x}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{2}(1-x^3)^{2/3}$$

$$\Big\downarrow \ 2577$$

$$-\int \frac{1}{\sqrt[3]{1-x^3}}dx+2\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{2}(1-x^3)^{2/3}$$

$$\Big\downarrow \ 769$$

$$2\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{2}(1-x^3)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

$$\Big\downarrow \text{2574}$$

$$\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+$$

$$2\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right)+$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

input
```
Int[(1 - x^3)^(2/3)/(1 + x),x]
```

output
```
(1 - x^3)^(2/3)/2 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[x + (1 - x^3)^(1/3)]/2
 + 2*(-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
])/2^(1/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*
(1 - x^3)^(1/3)])/(4*2^(1/3)))
```

3.111.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

rule 2577
```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Simp[f/d   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[(d*e - c*f
)/d   Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x]
```

rule 2578
```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Simp[1/d^2   Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
 + b*x^3)^(1/3)), x], x] - Simp[b*(c/d^2)   Int[x/(a + b*x^3)^(1/3), x], x]
) /; FreeQ[{a, b, c, d}, x]
```

3.111.4 Maple [F]

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{1+x}dx$$

input
```
int((-x^3+1)^(2/3)/(1+x),x)
```

output
```
int((-x^3+1)^(2/3)/(1+x),x)
```

3.111.5 Fricas [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x+1}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

3.111.6 Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-(x-1)\,(x^2+x+1))^{\frac{2}{3}}}{x+1}\,dx$$

input
```
integrate((-x**3+1)**(2/3)/(1+x),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

3.111.7 Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x+1}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

3.111.8 Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x+1}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(1-x^3)^{2/3}}{x+1}\,dx$$

input
```
int((1 - x^3)^(2/3)/(x + 1),x)
```

output
```
int((1 - x^3)^(2/3)/(x + 1), x)
```

3.112 $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$

3.112.1 Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{1}{2}(1-x^3)^{2/3} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{2\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1
+x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3)
)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

3.112.2 Mathematica [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$$

input
```
Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

output
```
Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

3.112.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2019, 2578, 888, 2577, 769, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-x+1)(1-x^3)^{2/3}}{x^3+1}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{(1-x^3)^{2/3}}{x+1}dx$$

$$\Big\downarrow \text{2578}$$

$$\int \frac{x}{\sqrt[3]{1-x^3}}dx+\int \frac{1-x}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}(1-x^3)^{2/3}$$

$$\Big\downarrow \text{888}$$

$$\int \frac{1-x}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{2}(1-x^3)^{2/3}$$

$$\Big\downarrow \text{2577}$$

$$-\int \frac{1}{\sqrt[3]{1-x^3}}dx+2\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{2}(1-x^3)^{2/3}$$

$$\Big\downarrow \text{769}$$

$$2\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

$$\Big\downarrow \text{2574}$$

$$\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + 2\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}} + \frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}\right) + \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

input
```
Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

output
```
(1 - x^3)^(2/3)/2 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[x + (1 - x^3)^(1/3)]/2
 + 2*(-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
])/2^(1/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*
(1 - x^3)^(1/3)])/(4*2^(1/3)))
```

3.112. $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

rule 2577
```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Simp[f/d   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[(d*e - c*f
)/d   Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x]
```

rule 2578
```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Simp[1/d^2   Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
 + b*x^3)^(1/3)), x], x] - Simp[b*(c/d^2)   Int[x/(a + b*x^3)^(1/3), x], x]
) /; FreeQ[{a, b, c, d}, x]
```

3.112. $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$

3.112.4 Maple [F]

$$\int \frac{(x^2-x+1)(-x^3+1)^{\frac{2}{3}}}{x^3+1}dx$$

input
```
int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

3.112.5 Fricas [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}{x^3+1}\,dx$$

input
```
integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

3.112.6 Sympy [F]

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{x+1}\,dx$$

input
```
integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

3.112.7 Maxima [F]

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}{x^3+1}\,dx$$

input
```
integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

3.112.8 Giac [F]

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}{x^3+1}\,dx$$

input
```
integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{2/3}\,(x^2-x+1)}{x^3+1}\,dx$$

input
```
int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1),x)
```

output
```
int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1), x)
```

3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3} \, dx$

3.113.1 Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \frac{(1-x^3)^{2/3}}{1+x^3} \, dx = \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(1+x^3)}{3\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

output
```
-1/6*ln(x^3+1)*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+
(-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3
*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.55

$$\int \frac{(1-x^3)^{2/3}}{1+x^3} \, dx = \frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right) - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(x+\sqrt[3]{1-x^3}\right) + 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(x^2-x\sqrt[3]{1-x^3}\right.\right.$$

input
```
Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] - 2*2^(2/3)*Sqrt[3]
*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[x + (1 - x^3)^(
1/3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^
3)^(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/
3) - 2^(1/3)*(1 - x^3)^(2/3)])/6
```

3.113.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {916, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-x^3\right)^{2/3}}{x^3+1} \, dx$$

$$\Big\downarrow \text{916}$$

$$2\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx-\int \frac{1}{\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{769}$$

$$2\int \frac{1}{\sqrt[3]{1-x^3}\left(x^3+1\right)}dx+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

$$\Big\downarrow \text{901}$$

$$\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+2\left(-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}\right)-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

input
```
Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

output
```
ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + 2*(-(ArcTan[(1 - (2*
2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) - Log[1 + x^3]/(6*
2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))) - Log[x + (1 -
 x^3)^(1/3)]/2
```

3.113.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

3.113.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{3}+\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{3}-\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{6}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\right.}{}$

input
```
int((-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln((x+(-x^3+1)^(1/3))/x)+1/3*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)
-1/6*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)
+1/3*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+1/6
*ln(((-x^3+1)^(2/3)-x*(-x^3+1)^(1/3)+x^2)/x^2)-1/3*3^(1/2)*arctan(1/3*(-2*
(-x^3+1)^(1/3)+x)*3^(1/2)/x)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{\left(1-x^3\right)^{2/3}}{1+x^3}\,dx &= -\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)\\ &\quad+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\cdot 4^{\frac{1}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)\\ &\quad-\frac{1}{6}\cdot 4^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x+2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)\\ &\quad-\frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)\end{aligned}$$

input
```
integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(
1/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3)
)/x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*l
og((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2)
- 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x +
(-x^3 + 1)^(2/3))/x^2)
```

3.113.6 Sympy [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{(x+1)\left(x^2-x+1\right)}\,dx$$

input
```
integrate((-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.113.7 Maxima [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

3.113.8 Giac [F]

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

input
```
integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{2/3}}{x^3+1}\,dx$$

input
```
int((1 - x^3)^(2/3)/(x^3 + 1),x)
```

output
```
int((1 - x^3)^(2/3)/(x^3 + 1), x)
```

3.114 $\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx$

3.114.1 Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}2^{2/3}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

output
```
-1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/12*ln((1-x)*(1+x)^2)*2^(2/3)+1/
6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3
)-1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/4*ln(-1+x+2^(2/3)*(-x^3
+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2
))*3^(1/2)*2^(2/3)
```

3.114.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{1}{2}x^2\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)$$

input
```
Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

output
```
(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

3.114.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {984, 888, 991, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-x^3)^{2/3}}{x^3+1}dx$$

$$\Big\downarrow \; 984$$

$$2\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\int \frac{x}{\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \; 888$$

$$2\int \frac{x}{\sqrt[3]{1-x^3}\,(x^3+1)}dx-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 991$$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\int \frac{1}{\frac{2(1-x)^3}{1-x^3}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 750$$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{3}\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 16$$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\int \frac{\sqrt[3]{2}\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 27$$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 1142$$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-}}}{2\,2^{2/3}}\right.\right.$$
$$-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 25$$

3.114. $\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x}}}{2\ 2^{2/3}}\right.\right.$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$\big\downarrow$ 27

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x}}}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$\big\downarrow$ 1082

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}+\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}\right)\right.$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$\big\downarrow$ 217

3.114. $\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

$$2\left(-\frac{1}{3}\sqrt[3]{2}\left(\frac{\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}\right)-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{\log}{}\right.$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 1103$$

$$2\left(-\frac{1}{3}\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)-\frac{\log}{}\right.$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

$$\Big\downarrow \; 2574$$

$$2\left(-\frac{1}{3}\sqrt[3]{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}\right.\right.$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

3.114. $\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx$

output
```
-1/2*(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]) + 2*(-1/3*(2^(1/3)*(-((Sq
rt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3))
- Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3
)^(1/3)]/(2*2^(2/3)))) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(
1/3)) + ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)) + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) - (3*Log[-1 + x + 2^(2/3
)*(1 - x^3)^(1/3)])/(4*2^(1/3)))/3)
```

3.114.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 984
```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Simp[b/d   Int[x*(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   In
t[x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1,
 1, n, p, -1, x]
```

rule 991
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Simp[-q^2/(3*d)   Int[1/((1 - q*x)*(a + b*x^3)^(1/3)
), x], x] + Simp[q/d   Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x
^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c +
 a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.114. $\int \frac{x(1-x^3)^{2/3}}{1+x^3} \, dx$

3.114.4 Maple [F]

$$\int \frac{x(-x^3+1)^{\frac{2}{3}}}{x^3+1}dx$$

input
```
int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

3.114.5 Fricas [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\,dx$$

input
```
integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

3.114.6 Sympy [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{x(-(x-1)\,(x^2+x+1))^{\frac{2}{3}}}{(x+1)\,(x^2-x+1)}\,dx$$

input
```
integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.114.7 Maxima [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\,dx$$

input
```
integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

3.114.8 Giac [F]

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\,dx$$

input
```
integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{x\,(1-x^3)^{2/3}}{x^3+1}\,dx$$

input
```
int((x*(1 - x^3)^(2/3))/(x^3 + 1),x)
```

output
```
int((x*(1 - x^3)^(2/3))/(x^3 + 1), x)
```

3.115 $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} \, dx$

3.115.1 Optimal result

Integrand size = 24, antiderivative size = 383

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} \, dx = -\frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+x^3\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1}{\sqrt[3]{1-}}\right.}{3\sqrt[3]{2}}$$

output
```
1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/12*ln((1-x)*(1+x)^2)*2^(2/3)-1/6
*ln(x^3+1)*2^(2/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)+1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*
ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/4*ln(-1+x
+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/
3)*3^(1/2)
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 15.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.36

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx = -\frac{1}{2}x^2\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)$$
$$-\frac{4x\left(1-x^3\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)}{\left(1+x^3\right)\left(-4\,\text{AppellF1}\left(\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right)+2\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\right.\right.\right.}$$

input
```
Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

output
```
-1/2*(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]) - (4*x*(1 - x^3)^(2/3)*A
ppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3,
1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*App
ellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

3.115.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{x^3+1}\,dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(-\frac{2\left(1-x^3\right)^{2/3}}{3(-x-1)}+\frac{\left(-1-(-1)^{2/3}\right)\left(1-x^3\right)^{2/3}}{3\left(\sqrt[3]{-1}x-1\right)}+\frac{\left(\sqrt[3]{-1}-1\right)\left(1-x^3\right)^{2/3}}{3\left(-(-1)^{2/3}x-1\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2^{2/3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(1+(-1)^{2/3}\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\left(1-\sqrt[3]{-1}\right)\arctan\left(\frac{1-\frac{\sqrt[3]{2}\left(x+\sqrt[3]{-1}\right)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1+(-1)^{2/3}\right)\arctan\left(\frac{\frac{(-1)^{2/3}\sqrt[3]{2}\left(\sqrt[3]{-1}x+1\right)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{6}(-1)^{2/3}\left(1-\sqrt[3]{-1}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{6}\left(1-\sqrt[3]{-1}\right)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)+\frac{1}{3}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{1}{6}\left(1+(-1)^{2/3}\right)\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{1}{6}\left(1-\sqrt[3]{-1}\right)\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\left(1-\sqrt[3]{-1}\right)\log\left(-(-2)^{2/3}\sqrt[3]{1-x^3}-(-1)^{2/3}x+1\right)}{2\sqrt[3]{2}}+\frac{\log\left(-2^{2/3}\sqrt[3]{1-x^3}-x+1\right)}{\sqrt[3]{2}}+\frac{\left(1+(-1)^{2/3}\right)\log\left(\sqrt[3]{-1}2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{-1}x+1\right)}{2\sqrt[3]{2}}-\frac{\log\left(-\left((1-x)(x+1)^2\right)\right)}{3\sqrt[3]{2}}-\frac{\left(1+(-1)^{2/3}\right)\log\left(-(-1)^{2/3}\left(x+(-1)^{2/3}\right)^2\left(\sqrt[3]{-1}x+1\right)\right)}{6\sqrt[3]{2}}-\frac{\left(1-\sqrt[3]{-1}\right)\log\left((-1)^{2/3}\left(x+\sqrt[3]{-1}\right)\left((-1)^{2/3}x+1\right)^2\right)}{6\sqrt[3]{2}}$$

input
```
Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

3.115. $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

output
```
-((2^(2/3)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3
]) + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 - (
-1)^(1/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1
+ (-1)^(2/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (
(1 - (-1)^(1/3))*ArcTan[(1 - (2^(1/3)*((-1)^(1/3) + x))/(1 - x^3)^(1/3))/S
qrt[3]])/(2^(1/3)*Sqrt[3]) - ((1 + (-1)^(2/3))*ArcTan[(1 + ((-1)^(2/3)*2^(
1/3)*(1 + (-1)^(1/3)*x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]) + (x
^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/3 + ((1 - (-1)^(1/3))*x^2*Hyperg
eometric2F1[1/3, 2/3, 5/3, x^3])/6 + ((-1)^(2/3)*(1 - (-1)^(1/3))*x^2*Hype
rgeometric2F1[1/3, 2/3, 5/3, x^3])/6 - Log[-((1 - x)*(1 + x)^2)]/(3*2^(1/3
)) - ((1 + (-1)^(2/3))*Log[-((-1)^(2/3)*((-1)^(2/3) + x)^2*(1 + (-1)^(1/3)
*x))]])/(6*2^(1/3)) - ((1 - (-1)^(1/3))*Log[(-1)^(2/3)*((-1)^(1/3) + x)*(1
+ (-1)^(2/3)*x)^2]])/(6*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3 - ((1 - (-1)^
(1/3))*Log[x + (1 - x^3)^(1/3)])/6 - ((1 + (-1)^(2/3))*Log[x + (1 - x^3)^(
1/3)])/6 + ((1 - (-1)^(1/3))*Log[1 - (-1)^(2/3)*x - (-2)^(2/3)*(1 - x^3)^(
1/3)])/(2*2^(1/3)) + Log[1 - x - 2^(2/3)*(1 - x^3)^(1/3)]/2^(1/3) + ((1 +
(-1)^(2/3))*Log[1 + (-1)^(1/3)*x + (-1)^(1/3)*2^(2/3)*(1 - x^3)^(1/3)])/(2
*2^(1/3))
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.115.4 Maple [F]

$$\int \frac{(1-x)\,(-x^3+1)^{\frac{2}{3}}}{x^3+1}dx$$

input
```
int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

output
```
int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

3.115.5 Fricas [F]

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = \int -\frac{(-x^3+1)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

input
```
integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

output
```
integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

3.115.6 Sympy [F]

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = -\int \left(-\frac{(1-x^3)^{\frac{2}{3}}}{x^3+1}\right)dx - \int \frac{x(1-x^3)^{\frac{2}{3}}}{x^3+1}\,dx$$

input
```
integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
```

output
```
-Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)
/(x**3 + 1), x)
```

3.115.7 Maxima [F]

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = \int -\frac{(-x^3+1)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

input
```
integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

output
```
-integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

3.115.8 Giac [F]

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx=\int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

input
```
integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx=-\int \frac{\left(1-x^3\right)^{2/3}(x-1)}{x^3+1}\,dx$$

input
```
int(-((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1),x)
```

output
```
-int(((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1), x)
```

3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

3.116.1 Optimal result

Integrand size = 19, antiderivative size = 272

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx=\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)-\frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}$$

output
```
1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x
^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)
*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1
-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3
))*3^(1/2))*2^(1/3)*3^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{\ }\right.}{}$$

input
```
Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1
 - x^3)^(1/3))] + 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) +
 2*2^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^
(1/3)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) -
 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)]
 + Log[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3)
+ 4*(1 - x^3)^(2/3)])/2^(2/3)
```

3.116.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{1-x^3}}{x^3+1}\,dx$$

$$\Big\downarrow \; 927$$

$$-9\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}$$

$$\Big\downarrow \; 982$$

$$-9\left(\frac{1}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(4-\frac{(1-x)^3}{1-x^3}\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{2}{9}\int \frac{1-x}{\sqrt[3]{1-x^3}\left(\frac{2(1-x)^3}{1-x^3}+1\right)}d\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$\downarrow 821$$

$$-9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}\right)+\frac{1}{9}\left(\frac{\int \frac{1}{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\ 2^{2/3}}-\right.\right.$$

$$\downarrow 16$$

$$-9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}}{\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}}d\frac{1-}{\sqrt[3]{1-}}}{3\ 2^{2/3}}\right.\right.$$

$$\downarrow 1142$$

$$-9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}+\frac{\int -\frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\right.$$

$$\downarrow 25$$

3.116. $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

$$-9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\int \frac{\sqrt[3]{2}\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{2\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+$$

$$\Big\downarrow \text{27}$$

$$-9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)\right.$$

$$\Big\downarrow \text{1082}$$

$$-9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(\right.\right.$$

$$\Big\downarrow \text{217}$$

3.116. $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

$$-9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}d\frac{1-x}{\sqrt[3]{1-x^3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\,a\ldots}{}\right.\right.$$

$\big\downarrow$ 1103

$$-9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}\right)+\frac{1}{9}\left(-\frac{\sqrt{3}\arctan\left(\ldots\right.}{}\right.\right.$$

input
```
Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

output
```
-9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[
3]])/2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1
- x))/(1 - x^3)^(1/3)]/(2*2^(1/3)))/(3*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x)
)/(1 - x^3)^(1/3)]/(3*2^(2/3))))/9 + (-1/3*Log[2^(2/3) - (1 - x)/(1 - x^3)
^(1/3)]/2^(2/3) - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]] - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(
1 - x^3)^(1/3)]/2)/(3*2^(2/3)))/9)
```

3.116. $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

3.116.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.116. $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.116.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.60 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.22

method	result	size
trager	Expression too large to display	1147

input
```
int((-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^3-2)*ln((6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*RootOf(_Z^3-2)^4*x^3-18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)^2*RootOf(_Z^3-2)^3*x^3+RootOf(_Z^3-2)^2*x^6-3*RootOf(RootOf(_Z^3-2)^2
+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^6+18*(-x^3+1)^(2/3)*RootOf(_
Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^2-6*(-x^3+1
)^(1/3)*RootOf(_Z^3-2)*x^4-2*RootOf(_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^
2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3+12*(-x^3+1)^(2/3)*x^2+6*(
-x^3+1)^(1/3)*RootOf(_Z^3-2)*x+RootOf(_Z^3-2)^2-3*RootOf(RootOf(_Z^3-2)^2+
3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)-1/6*ln(-(
-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^4*x^
3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^
3*x^3+RootOf(_Z^3-2)^2*x^6-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9
*_Z^2)*RootOf(_Z^3-2)*x^6+18*(-x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf
(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^2-6*(-x^3+1)^(1/3)*RootOf(_Z^3-2)
*x^4-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
*x^4-6*RootOf(_Z^3-2)^2*x^3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)
+9*_Z^2)*RootOf(_Z^3-2)*x^3+6*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x+18*(-x^3+1)^
(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x+RootOf(_Z^3-2)
^2-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(
1+x)^2/(x^2-x+1)^2)*RootOf(_Z^3-2)-1/2*ln(-(-6*RootOf(RootOf(_Z^3-2)^2+...
```

3.116.5 Fricas [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$$
$$= \frac{1}{18}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{6\sqrt{3}2^{\frac{2}{3}}(x^{16}-33x^{13}+110x^{10}-110x^7+33x^4-x)(-x^3+1)^{\frac{1}{3}}-24\sqrt{3}2^{\frac{1}{3}}(x^{14}-2x}{3(x^{18}-102x^{15}+447x^{1}}\right.$$
$$+\frac{1}{18}\cdot 2^{\frac{1}{3}}\log\left(-\frac{12(-x^3+1)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}(x^6+2x^3+1)-6\cdot 2^{\frac{1}{3}}(x^4-x)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-\frac{1}{36}$$
$$\cdot 2^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^8-4x^5+x^2)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(x^{12}-32x^9+78x^6-32x^3+1)+6(x^{10}-11x^7+11x}{x^{12}+4x^9+6x^6+4x^3+1}\right.$$

input
```
integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

output
```
1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*
x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 -
 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15
- 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12
- 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/
3)*x^2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))
/(x^6 + 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x
^3 + 1)^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 -
11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

3.116.6 Sympy [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{(x+1)(x^2-x+1)}\,dx$$

input
```
integrate((-x**3+1)**(1/3)/(x**3+1),x)
```

output
```
Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.116.7 Maxima [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^3+1}\,dx$$

input
```
integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

3.116.8 Giac [F]

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{(-x^3+1)^{\frac{1}{3}}}{x^3+1}\,dx$$

input
```
integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

output
```
integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{1/3}}{x^3+1}\,dx$$

input
```
int((1 - x^3)^(1/3)/(x^3 + 1),x)
```

output
```
int((1 - x^3)^(1/3)/(x^3 + 1), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):    #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
      return expnType(expn.args[0])    #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
  return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
  return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

  #debug=False
  if debug:
     print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
    return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
  try:
    if expn.parent() is SR:
      return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
      return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
      return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

  except AttributeError as error:
    print("Exception,AttributeError in  is_atom")
    print ("cought exception" , type(error).__name__ )
    return False


def expnType(expn):

  if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
    return 1
  elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```