Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

0-Independent-test-suites/1-Apostol-Problems

Nasser M. Abbasi

December 4, 2023 Compiled on December 4, 2023 at 6:26pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [175]. This is test number [1].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (175)	0.00 (0)
Mathematica	100.00 (175)	0.00 (0)
Fricas	99.43 (174)	0.57 (1)
Maple	98.86 (173)	1.14 (2)
Giac	97.14 (170)	2.86 (5)
Mupad	96.57 (169)	3.43 (6)
Maxima	94.86 (166)	5.14 (9)
Sympy	94.29 (165)	5.71 (10)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

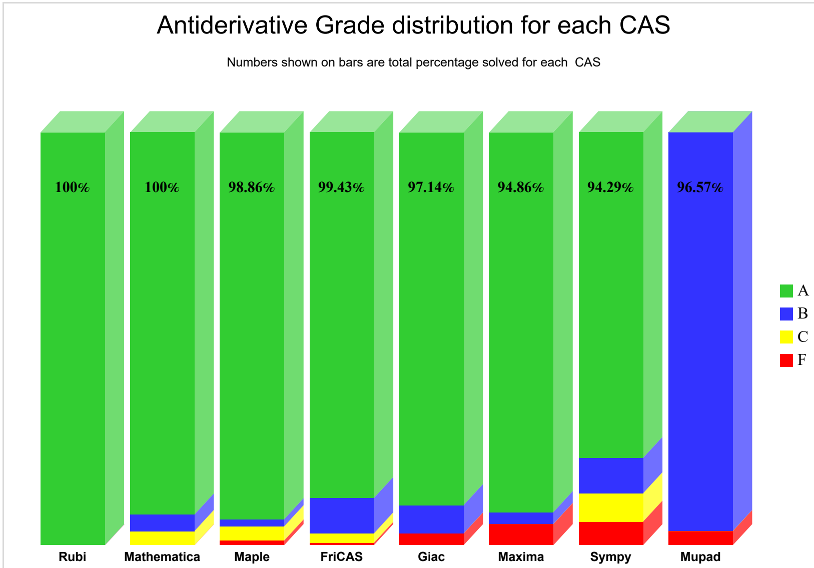
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

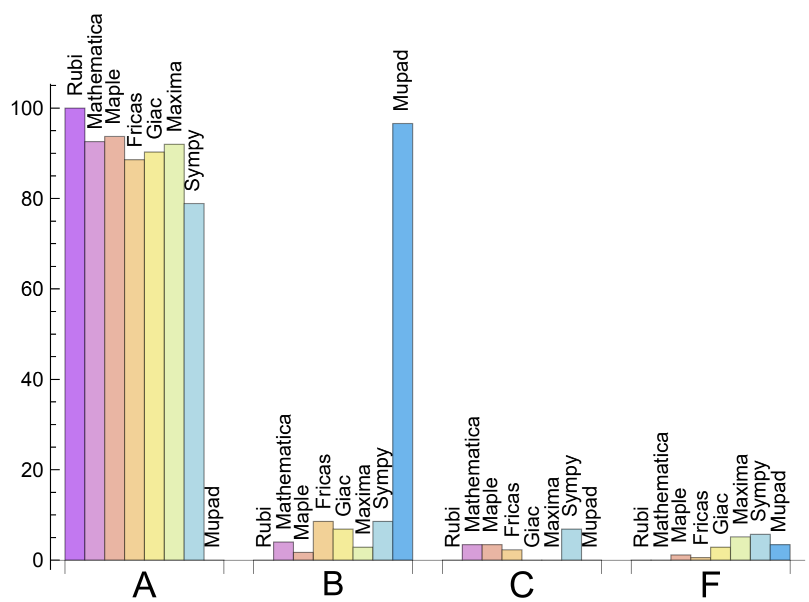
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	93.714	1.714	3.429	1.143
Mathematica	92.571	4.000	3.429	0.000
Maxima	92.000	2.857	0.000	5.143
Giac	90.286	6.857	0.000	2.857
Fricas	88.571	8.571	2.286	0.571
Sympy	78.857	8.571	6.857	5.714
Mupad	0.000	96.571	0.000	3.429

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	1	100.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Giac	5	100.00	0.00	0.00
Mupad	6	0.00	100.00	0.00
Maxima	9	66.67	0.00	33.33
Sympy	10	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.09
Mathematica	0.14
Rubi	0.16
Mupad	0.16
Maxima	0.21
Fricas	0.24
Giac	0.27
Sympy	1.44

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	17.84	0.90	15.00	0.82
Maxima	18.24	0.86	14.50	0.81
Giac	20.84	0.97	15.50	0.83
Fricas	21.74	1.04	17.00	0.85
Mathematica	21.81	1.04	18.00	1.00
Rubi	23.53	1.03	19.00	1.00
Mupad	31.83	1.11	14.00	0.79
Sympy	467.14	30.41	17.00	0.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

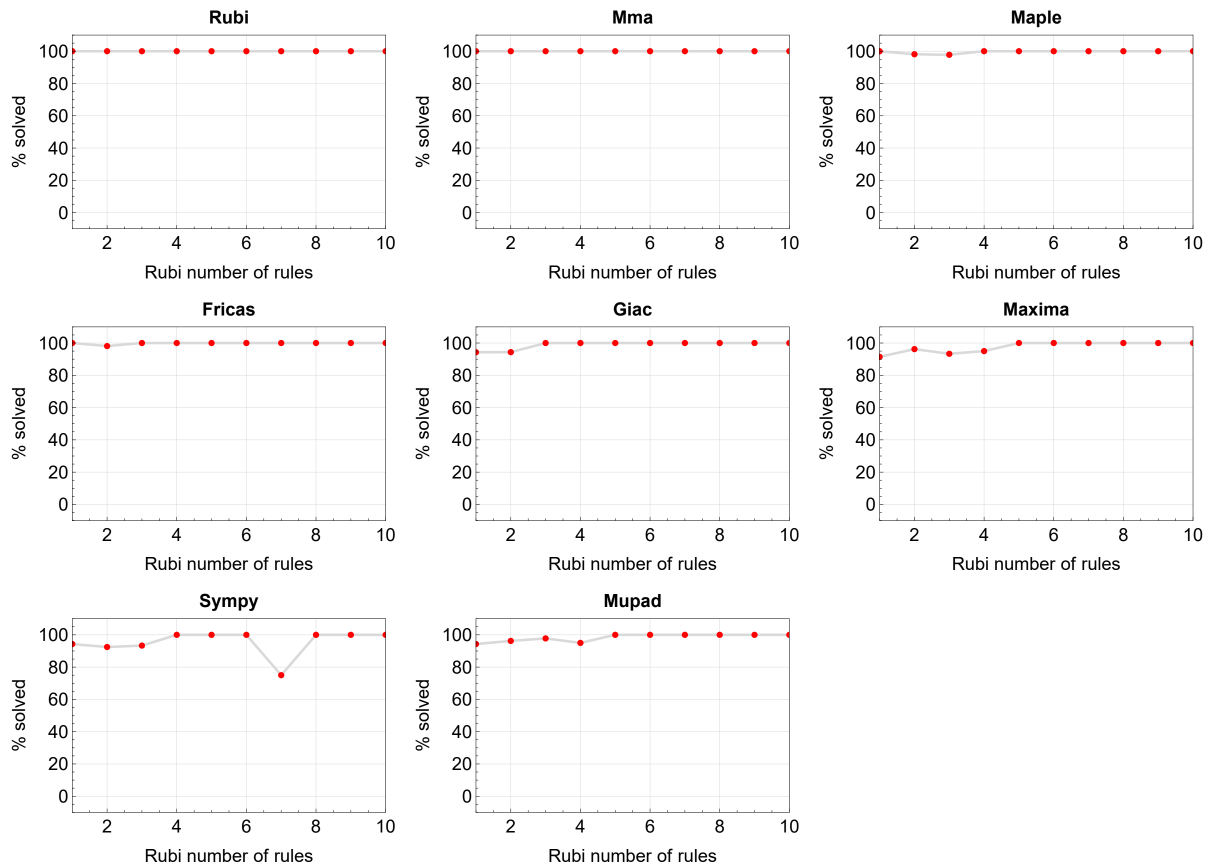


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

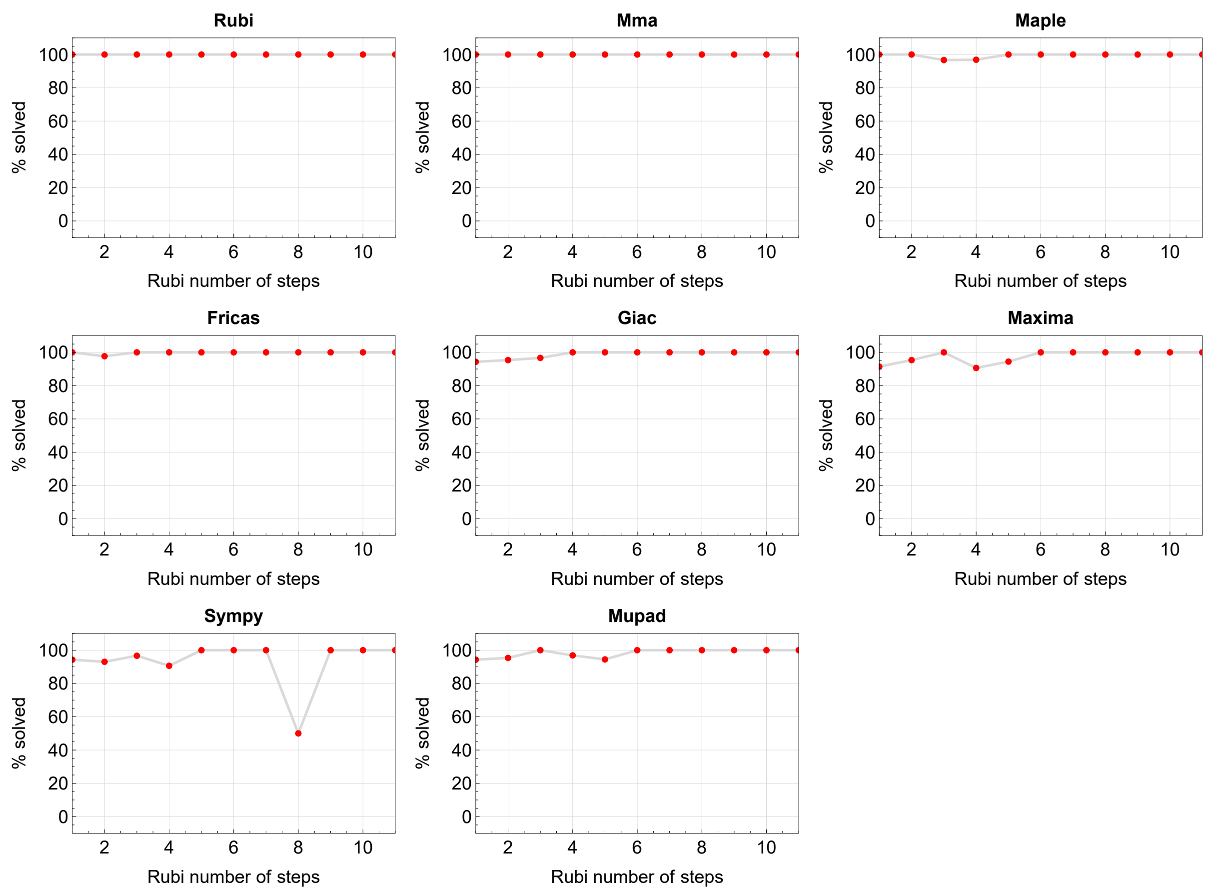


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

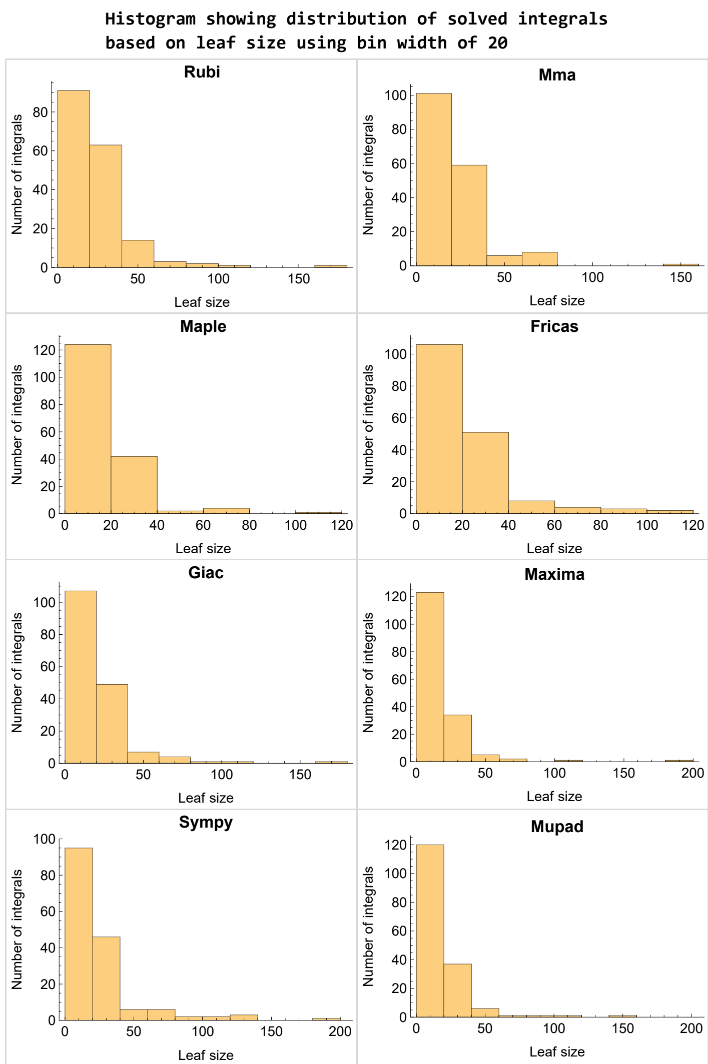


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

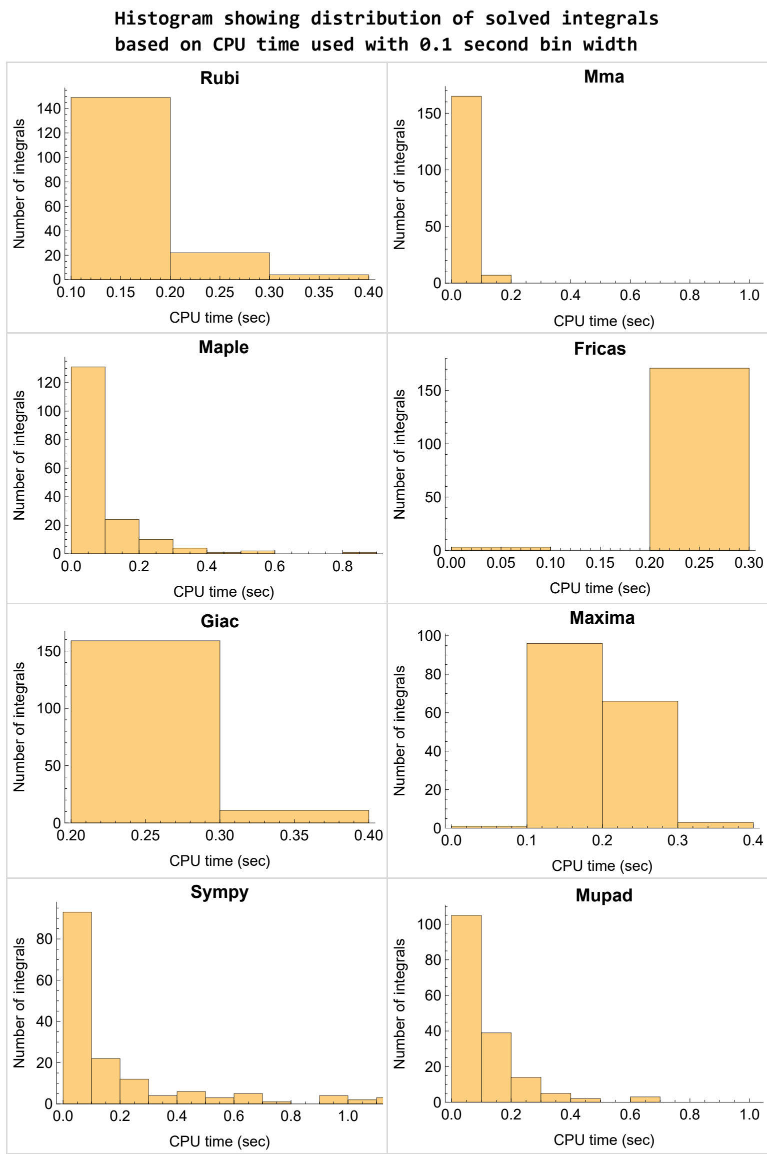


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

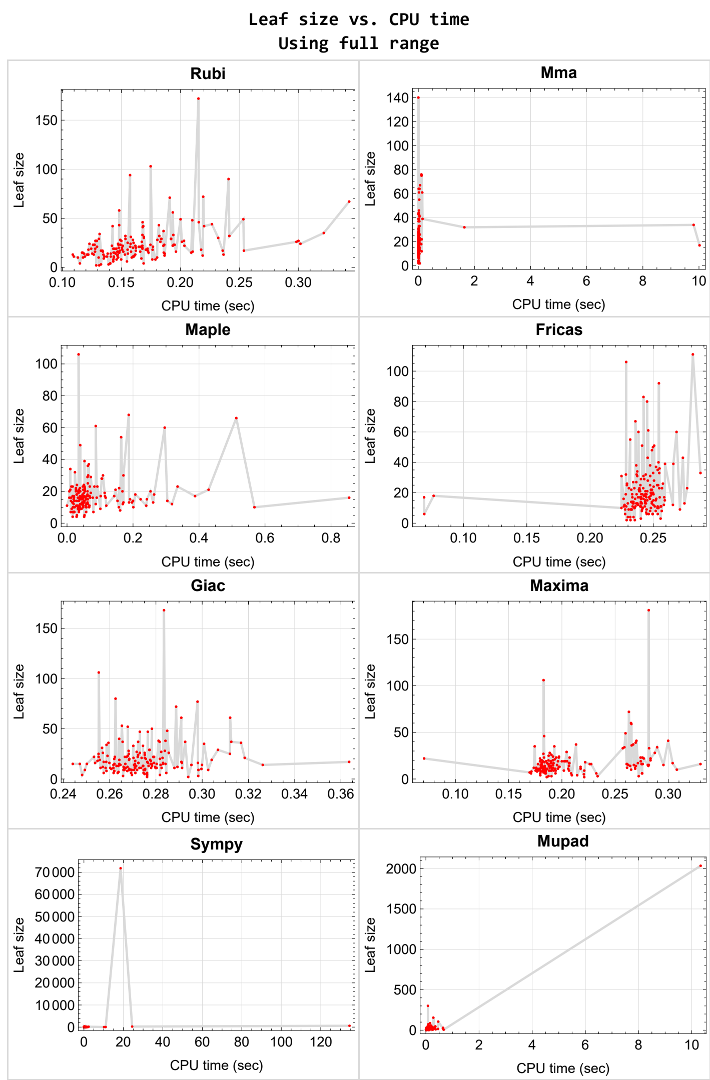


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the **Timofeev** test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

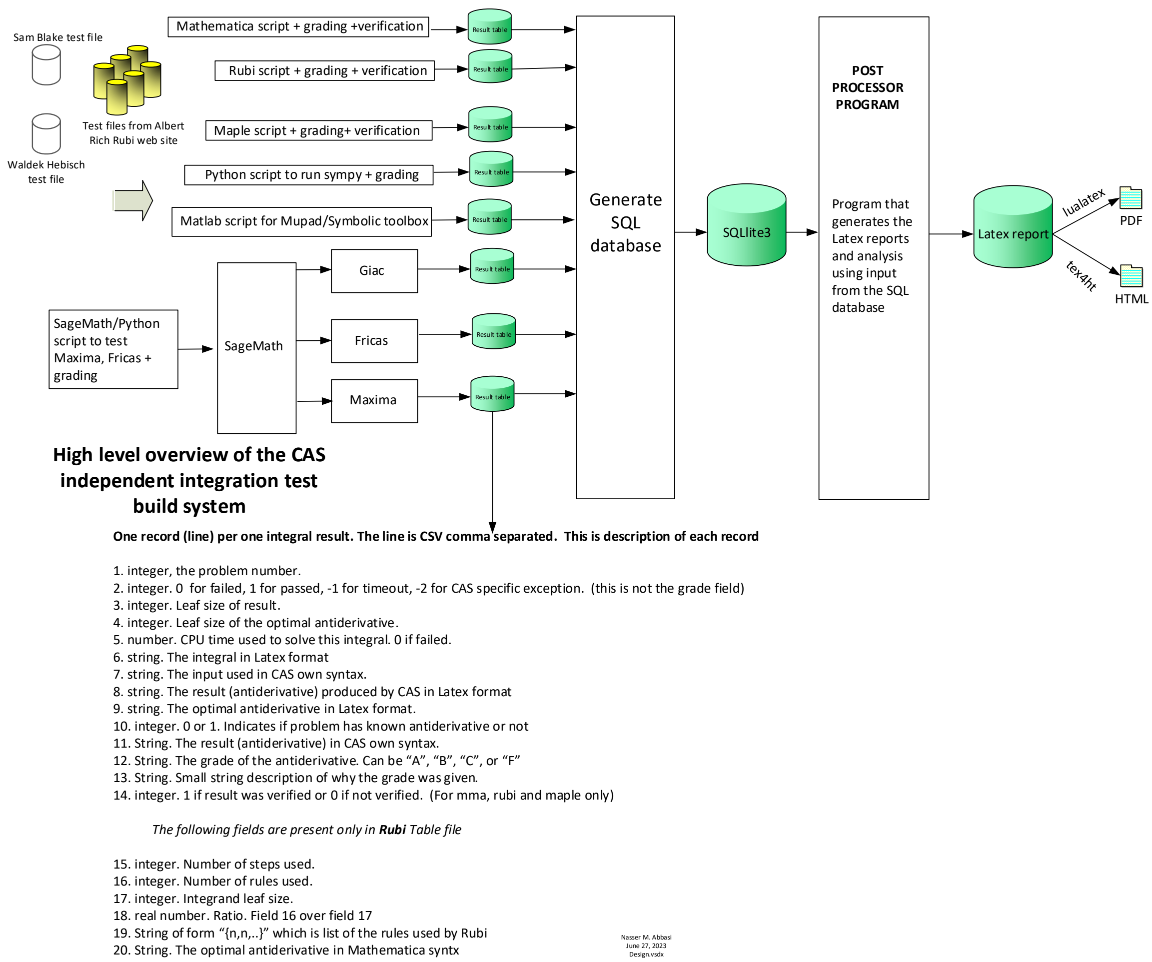
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { 50, 51, 61, 83, 84, 88, 154 }

C grade { 41, 44, 45, 98, 113, 175 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 173, 174 }

B grade { 51, 158, 170 }

C grade { 35, 41, 131, 137, 147, 175 }

F normal fail { 19, 172 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade { 16, 44, 45, 48, 50, 51, 61, 84, 88, 113, 114, 124, 131, 145, 146 }

C grade { 41, 137, 172, 175 }

F normal fail { 156 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173 }

B grade { 51, 83, 84, 113, 169 }

C grade { }

F normal fail { 19, 41, 98, 99, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { 104, 105, 141 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade { 44, 45, 51, 83, 84, 88, 105, 113, 136, 154, 155, 164 }

C grade { }

F normal fail { 41, 62, 156, 172, 175 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175 }

C grade { }

F normal fail { }

F(-1) timedout fail { 98, 99, 104, 105, 165, 174 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 84, 85, 86, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 147, 148, 149, 152, 153, 154, 157, 158, 159, 160, 161, 166, 167, 168, 170, 171, 172, 175 }

B grade { 9, 17, 42, 47, 48, 50, 51, 62, 90, 101, 114, 141, 144, 145, 146 }

C grade { 4, 7, 39, 80, 81, 83, 87, 89, 105, 150, 156, 169 }

F normal fail { 19, 103, 151, 155, 162, 163, 164, 165, 173, 174 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.268	0.041	0.568	0.194	0.231	0.030	0.270	0.198

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	19	39	19	14
N.S.	1	1.00	0.67	0.56	0.70	0.70	1.44	0.70	0.52
time (sec)	N/A	0.129	0.014	0.191	0.182	0.233	0.600	0.256	0.052

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	18	22	22	48	22	19
N.S.	1	1.00	0.62	0.53	0.65	0.65	1.41	0.65	0.56
time (sec)	N/A	0.133	0.055	0.207	0.179	0.233	0.783	0.255	0.042

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	14	19	14	60	19	14
N.S.	1	1.00	0.67	0.52	0.70	0.52	2.22	0.70	0.52
time (sec)	N/A	0.130	0.007	0.200	0.205	0.240	0.625	0.264	0.038

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	22	22	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.57	1.57	0.86	0.86
time (sec)	N/A	0.118	0.022	0.261	0.192	0.239	0.042	0.271	0.046

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.147	0.053	0.240	0.179	0.236	0.020	0.267	0.103

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	17	92	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.74	4.00	0.65	0.52
time (sec)	N/A	0.121	0.011	0.197	0.181	0.233	0.695	0.247	0.019

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.25	1.00	0.75	0.75
time (sec)	N/A	0.161	0.014	0.036	0.185	0.225	0.041	0.271	0.260

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	18	29	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.12	1.81	0.75	0.75
time (sec)	N/A	0.161	0.013	0.171	0.176	0.248	0.176	0.257	0.115

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.152	0.015	0.058	0.171	0.254	0.225	0.284	0.027

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	6	9
N.S.	1	1.00	1.00	0.92	0.83	1.00	1.00	0.50	0.75
time (sec)	N/A	0.237	0.118	0.119	0.185	0.250	0.294	0.275	0.117

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.176	0.021	0.101	0.177	0.252	0.124	0.259	0.104

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.167	0.012	0.157	0.190	0.271	1.484	0.275	0.130

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.115	0.015	0.318	0.206	0.240	0.098	0.258	0.216

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	15	34	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.141	0.009	0.046	0.192	0.244	0.619	0.267	0.019

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	22	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	2.00	0.73	0.82	0.82
time (sec)	N/A	0.110	0.024	0.033	0.180	0.249	0.421	0.275	0.048

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	0.73
time (sec)	N/A	0.119	0.009	0.087	0.180	0.229	0.096	0.274	0.115

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	1.00
time (sec)	N/A	0.169	0.044	0.169	0.182	0.251	0.126	0.278	0.149

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	24	32	0	0	23	0	15	47
N.S.	1	0.75	1.00	0.00	0.00	0.72	0.00	0.47	1.47
time (sec)	N/A	0.293	1.641	0.000	0.000	0.277	0.000	0.278	0.357

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.237	0.025	0.023	0.197	0.275	0.136	0.271	0.124

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	13	10	7	12	15	12	9
N.S.	1	1.00	0.81	0.62	0.44	0.75	0.94	0.75	0.56
time (sec)	N/A	0.130	0.005	0.058	0.170	0.238	0.589	0.266	0.107

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.167	0.010	0.040	0.185	0.247	0.070	0.262	0.049

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	15	17	15	15	17	15	15
N.S.	1	1.00	0.88	1.00	0.88	0.88	1.00	0.88	0.88
time (sec)	N/A	0.225	0.012	0.039	0.180	0.259	0.092	0.244	0.018

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	26	19	20	20	20	26	20	24
N.S.	1	1.13	0.83	0.87	0.87	0.87	1.13	0.87	1.04
time (sec)	N/A	0.289	0.012	0.040	0.187	0.249	0.140	0.276	0.018

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	27	20	23	21	21	26	21	23
N.S.	1	1.12	0.83	0.96	0.88	0.88	1.08	0.88	0.96
time (sec)	N/A	0.295	0.011	0.048	0.193	0.239	0.204	0.270	0.020

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.134	0.001	0.019	0.209	0.250	0.047	0.262	0.014

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	28	18	15	14	17	24	14	18
N.S.	1	1.22	0.78	0.65	0.61	0.74	1.04	0.61	0.78
time (sec)	N/A	0.170	0.002	0.043	0.180	0.247	0.107	0.282	0.068

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.134	0.006	0.031	0.184	0.254	0.022	0.300	0.036

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.146	0.001	0.000	0.176	0.256	0.028	0.280	0.002

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	22	17	16	19	24	16	16
N.S.	1	1.21	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.178	0.006	0.063	0.187	0.245	0.020	0.272	0.039

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	17	17	17	17	17
N.S.	1	1.00	1.10	0.81	0.81	0.81	0.81	0.81	0.81
time (sec)	N/A	0.151	0.006	0.068	0.181	0.245	0.027	0.259	0.045

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	44	30	23	24	25	36	22	22
N.S.	1	1.29	0.88	0.68	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.220	0.006	0.083	0.197	0.247	0.020	0.267	0.043

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	36	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.149	0.010	0.033	0.190	0.250	0.099	0.268	0.106

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	31	23	23	23	39	23	25
N.S.	1	0.97	0.94	0.70	0.70	0.70	1.18	0.70	0.76
time (sec)	N/A	0.233	0.007	0.069	0.183	0.242	0.139	0.259	0.116

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	46	29	19	26	29	56	26	28
N.S.	1	1.12	0.71	0.46	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.207	0.025	0.072	0.190	0.240	0.179	0.257	0.067

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.134	0.005	0.030	0.177	0.248	0.018	0.266	0.027

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	10	8	9	9
N.S.	1	1.00	1.00	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.170	0.003	0.048	0.184	0.242	0.020	0.268	0.037

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	29	22	17	16	19	24	16	16
N.S.	1	1.21	0.92	0.71	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.181	0.005	0.059	0.187	0.249	0.017	0.279	0.031

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	94	61	54	60	60	180	50	37
N.S.	1	1.12	0.73	0.64	0.71	0.71	2.14	0.60	0.44
time (sec)	N/A	0.155	0.140	0.164	0.265	0.238	2.398	0.278	0.222

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	25	21	34	21	39	28	20
N.S.	1	1.11	0.66	0.55	0.89	0.55	1.03	0.74	0.53
time (sec)	N/A	0.137	0.013	0.036	0.259	0.240	0.211	0.284	0.031

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	34	17	0	17	31	0	301
N.S.	1	1.00	0.20	0.10	0.00	0.10	0.18	0.00	1.75
time (sec)	N/A	0.211	9.809	0.102	0.000	0.069	0.383	0.000	0.078

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
N.S.	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.137	0.004	0.011	0.275	0.249	0.029	0.257	0.069

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	12	19	12	12
N.S.	1	1.00	1.14	0.93	0.86	0.86	1.36	0.86	0.86
time (sec)	N/A	0.177	0.004	0.016	0.269	0.258	0.030	0.280	0.033

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	13	10	20	8	18	8
N.S.	1	1.00	2.25	1.62	1.25	2.50	1.00	2.25	1.00
time (sec)	N/A	0.140	0.007	0.017	0.308	0.248	0.026	0.278	0.074

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	22	11	16	48	19	34	10
N.S.	1	1.00	1.83	0.92	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.180	0.005	0.028	0.261	0.249	0.028	0.259	0.024

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	26	18	20	18	26	18	20
N.S.	1	1.00	1.18	0.82	0.91	0.82	1.18	0.82	0.91
time (sec)	N/A	0.184	0.055	0.062	0.181	0.244	0.071	0.271	0.112

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.108	0.001	0.023	0.192	0.245	0.075	0.270	0.027

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	51	58	9	9
N.S.	1	1.00	0.85	0.77	0.69	3.92	4.46	0.69	0.69
time (sec)	N/A	0.112	0.001	0.049	0.189	0.241	0.018	0.266	0.108

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	14	19	19	15	14	14
N.S.	1	1.00	0.89	0.78	1.06	1.06	0.83	0.78	0.78
time (sec)	N/A	0.113	0.003	0.030	0.180	0.228	0.043	0.262	0.042

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	43	10	9	31	31	9	31
N.S.	1	1.00	3.91	0.91	0.82	2.82	2.82	0.82	2.82
time (sec)	N/A	0.112	0.001	0.039	0.178	0.225	0.017	0.275	0.027

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	140	106	106	106	131	106	106
N.S.	1	1.00	2.50	1.89	1.89	1.89	2.34	1.89	1.89
time (sec)	N/A	0.170	0.001	0.035	0.183	0.229	0.040	0.255	0.465

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.157	0.008	0.053	0.214	0.230	0.179	0.270	0.101

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	67	46	49	37	37	60	37	41
N.S.	1	1.08	0.74	0.79	0.60	0.60	0.97	0.60	0.66
time (sec)	N/A	0.328	0.023	0.040	0.213	0.252	0.381	0.266	0.244

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.131	0.002	0.044	0.171	0.257	0.104	0.268	0.055

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.178	0.010	0.188	0.189	0.266	0.940	0.289	0.208

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.108	0.000	0.021	0.180	0.241	0.022	0.266	0.075

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	15	16	12	15	15	15	12
N.S.	1	1.13	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.143	0.002	0.012	0.180	0.250	0.048	0.264	0.083

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.123	0.002	0.012	0.184	0.241	0.033	0.263	0.038

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.149	0.002	0.013	0.190	0.239	0.055	0.272	0.034

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00
time (sec)	N/A	0.103	0.000	0.030	0.180	0.232	0.020	0.275	0.016

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	11	3	4	3
N.S.	1	1.00	2.33	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.130	0.004	0.052	0.187	0.256	0.046	0.282	0.024

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	36	28	32	61	0	38
N.S.	1	1.00	0.75	1.29	1.00	1.14	2.18	0.00	1.36
time (sec)	N/A	0.138	0.005	0.063	0.189	0.244	0.348	0.000	0.219

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	33	28	23	17	22	26	22	17
N.S.	1	1.18	1.00	0.82	0.61	0.79	0.93	0.79	0.61
time (sec)	N/A	0.158	0.002	0.017	0.207	0.248	0.051	0.267	0.035

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.124	0.004	0.017	0.190	0.256	0.043	0.248	0.069

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.153	0.003	0.056	0.191	0.253	0.049	0.261	0.368

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
N.S.	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.166	0.019	0.047	0.190	0.227	1.148	0.257	0.180

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	49	39	32	23	31	37	31	23
N.S.	1	1.26	1.00	0.82	0.59	0.79	0.95	0.79	0.59
time (sec)	N/A	0.190	0.002	0.024	0.197	0.242	0.062	0.257	0.035

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.127	0.010	0.022	0.180	0.246	0.038	0.269	0.086

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	12	11	10	11	11
N.S.	1	1.00	1.00	0.86	0.86	0.79	0.71	0.79	0.79
time (sec)	N/A	0.135	0.003	0.048	0.174	0.256	0.068	0.256	0.117

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.149	0.006	0.161	0.183	0.244	0.127	0.264	0.112

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.139	0.012	0.063	0.176	0.240	0.138	0.260	0.020

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	10	9	13	15	9	9
N.S.	1	1.00	0.63	0.53	0.47	0.68	0.79	0.47	0.47
time (sec)	N/A	0.134	0.007	0.059	0.175	0.237	0.129	0.272	0.020

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	13	10	10	9	9	7	9	9
N.S.	1	1.30	1.00	1.00	0.90	0.90	0.70	0.90	0.90
time (sec)	N/A	0.128	0.009	0.021	0.202	0.241	0.040	0.269	0.049

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	6	6	5	6	6
N.S.	1	1.00	0.64	0.64	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.137	0.008	0.016	0.194	0.228	0.030	0.263	0.019

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
N.S.	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.139	0.009	0.019	0.221	0.232	0.029	0.249	0.023

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	21	12	12	11	11	10	11	11
N.S.	1	1.11	0.63	0.63	0.58	0.58	0.53	0.58	0.58
time (sec)	N/A	0.165	0.010	0.020	0.184	0.234	0.045	0.262	0.024

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	16	16	16	17	16	16
N.S.	1	1.00	0.59	0.50	0.50	0.50	0.53	0.50	0.50
time (sec)	N/A	0.171	0.012	0.021	0.178	0.242	0.038	0.298	0.077

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	25	16	16	11	11	20	11	11
N.S.	1	1.04	0.67	0.67	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.154	0.005	0.007	0.194	0.240	0.079	0.271	0.024

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	13	13	12	13	13
N.S.	1	1.00	0.62	0.54	0.50	0.50	0.46	0.50	0.50
time (sec)	N/A	0.157	0.016	0.023	0.182	0.244	0.039	0.264	0.109

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	28	27	31	139	36	27
N.S.	1	1.00	0.68	0.68	0.66	0.76	3.39	0.88	0.66
time (sec)	N/A	0.155	0.023	0.105	0.183	0.248	0.284	0.259	0.031

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	29	29	33	136	38	29
N.S.	1	1.00	0.69	0.69	0.69	0.79	3.24	0.90	0.69
time (sec)	N/A	0.154	0.022	0.072	0.204	0.250	0.275	0.284	0.020

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	21	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.40	0.87
time (sec)	N/A	0.130	0.003	0.036	0.198	0.237	0.071	0.283	0.109

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	64	22	35	33	17	37	21
N.S.	1	1.00	3.37	1.16	1.84	1.74	0.89	1.95	1.11
time (sec)	N/A	0.153	0.042	0.027	0.193	0.255	1.150	0.272	0.642

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	64	20	35	35	17	37	20
N.S.	1	1.00	3.76	1.18	2.06	2.06	1.00	2.18	1.18
time (sec)	N/A	0.150	0.024	0.007	0.174	0.253	1.180	0.282	0.205

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	25	24	23	23	22	23	22
N.S.	1	1.04	1.00	0.96	0.92	0.92	0.88	0.92	0.88
time (sec)	N/A	0.190	0.004	0.051	0.276	0.257	0.083	0.268	0.033

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	38	20
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91
time (sec)	N/A	0.152	0.003	0.010	0.257	0.266	1.097	0.281	0.025

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	28	14
N.S.	1	1.00	1.00	0.94	0.38	1.44	1.19	1.75	0.88
time (sec)	N/A	0.115	0.003	0.058	0.275	0.239	0.525	0.268	0.168

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	17	23	10	11	21	10	27	11
N.S.	1	1.70	2.30	1.00	1.10	2.10	1.00	2.70	1.10
time (sec)	N/A	0.127	0.050	0.201	0.269	0.244	0.262	0.282	0.091

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.112	0.002	0.068	0.265	0.235	0.050	0.275	0.040

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.117	0.009	0.056	0.282	0.236	0.057	0.250	0.105

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.123	0.004	0.388	0.261	0.252	0.051	0.254	0.078

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	20	16	16	15	13	15	15	14
N.S.	1	0.95	0.76	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.139	0.003	0.042	0.268	0.249	0.123	0.271	0.026

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	43	30	34	33	24	32	33	24
N.S.	1	1.08	0.75	0.85	0.82	0.60	0.80	0.82	0.60
time (sec)	N/A	0.167	0.011	0.010	0.282	0.250	0.121	0.274	0.030

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	34	25	29	29	29
N.S.	1	1.00	0.74	0.86	0.97	0.71	0.83	0.83	0.83
time (sec)	N/A	0.290	0.007	0.064	0.289	0.254	0.137	0.277	0.126

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	18	17	16	14	19	16	16
N.S.	1	1.32	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.142	0.012	0.017	0.276	0.244	0.447	0.276	0.061

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.164	0.006	0.022	0.184	0.245	0.448	0.276	0.670

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.116	0.034	0.264	0.304	0.237	0.106	0.276	0.077

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	37	16	0	15	31	24	0
N.S.	1	1.00	1.68	0.73	0.00	0.68	1.41	1.09	0.00
time (sec)	N/A	0.172	0.008	0.078	0.000	0.245	11.008	0.272	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	15	31	22	0
N.S.	1	1.00	1.00	0.80	0.00	0.75	1.55	1.10	0.00
time (sec)	N/A	0.154	0.005	0.041	0.000	0.252	10.172	0.279	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.115	0.006	0.049	0.281	0.245	0.041	0.283	0.029

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	3.75	0.75	0.75
time (sec)	N/A	0.134	0.013	0.029	0.272	0.240	0.054	0.266	0.102

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	33	27	25	19	28	19	21	22
N.S.	1	1.22	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.181	0.013	0.063	0.202	0.251	1.997	0.274	0.137

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	58	67	61	49	38	0	36	49
N.S.	1	1.38	1.60	1.45	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.140	0.062	0.087	0.260	0.237	0.000	0.317	0.070

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	75	68	0	80	114	61	0
N.S.	1	1.00	1.06	0.96	0.00	1.13	1.61	0.86	0.00
time (sec)	N/A	0.179	0.114	0.187	0.000	0.245	1.077	0.291	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	55	28	0	43	71	61	0
N.S.	1	1.00	1.72	0.88	0.00	1.34	2.22	1.91	0.00
time (sec)	N/A	0.144	0.033	0.106	0.000	0.246	0.901	0.312	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.134	0.003	0.053	0.182	0.234	0.041	0.262	0.054

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	14	15	13
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.74	0.79	0.68
time (sec)	N/A	0.135	0.003	0.052	0.181	0.230	0.047	0.278	0.040

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	21	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.91	0.83
time (sec)	N/A	0.181	0.003	0.062	0.200	0.241	0.056	0.275	0.046

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	19
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.83
time (sec)	N/A	0.183	0.005	0.030	0.192	0.259	0.069	0.269	0.205

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	18	26	19	24	18
N.S.	1	1.00	0.92	0.79	0.75	1.08	0.79	1.00	0.75
time (sec)	N/A	0.146	0.007	0.051	0.184	0.229	0.070	0.275	0.051

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	28	29	28	28	3	29	57
N.S.	1	1.04	1.00	1.04	1.00	1.00	0.11	1.04	2.04
time (sec)	N/A	0.176	0.021	0.062	0.287	0.246	0.064	0.307	0.192

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	61	37	36	51	14	37	53
N.S.	1	1.00	1.24	0.76	0.73	1.04	0.29	0.76	1.08
time (sec)	N/A	0.230	0.023	0.066	0.266	0.251	0.079	0.293	0.123

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	39	38	39	37	21
N.S.	1	1.00	1.14	0.90	1.86	1.81	1.86	1.76	1.00
time (sec)	N/A	0.152	0.018	0.114	0.269	0.247	0.239	0.313	0.339

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	14	55	48	14	87
N.S.	1	1.00	1.00	1.06	0.88	3.44	3.00	0.88	5.44
time (sec)	N/A	0.202	0.012	0.115	0.184	0.232	1.343	0.326	0.160

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.124	0.003	0.041	0.182	0.233	0.036	0.282	0.050

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.133	0.004	0.054	0.185	0.244	0.052	0.262	0.124

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	21	20	27	22	22	18
N.S.	1	1.00	0.93	0.70	0.67	0.90	0.73	0.73	0.60
time (sec)	N/A	0.158	0.006	0.027	0.191	0.234	0.035	0.253	0.043

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	20	26	30
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.74	0.96	1.11
time (sec)	N/A	0.188	0.004	0.030	0.207	0.235	0.062	0.275	0.095

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	16	20	32	17	16	15
N.S.	1	1.00	1.04	0.70	0.87	1.39	0.74	0.70	0.65
time (sec)	N/A	0.170	0.007	0.040	0.181	0.228	0.048	0.260	0.093

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	15	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88
time (sec)	N/A	0.145	0.003	0.047	0.270	0.233	0.037	0.280	0.044

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.136	0.005	0.049	0.261	0.238	0.063	0.268	0.039

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	1.00
time (sec)	N/A	0.129	0.002	0.040	0.196	0.227	0.039	0.277	0.104

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.134	0.006	0.054	0.192	0.237	0.040	0.277	0.037

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.150	0.012	0.053	0.183	0.242	0.087	0.268	0.097

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	1.00
time (sec)	N/A	0.121	0.003	0.033	0.185	0.227	0.027	0.291	0.030

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	19	17	14	17	13	10	16	13
N.S.	1	1.12	1.00	0.82	1.00	0.76	0.59	0.94	0.76
time (sec)	N/A	0.130	0.003	0.052	0.186	0.244	0.037	0.283	0.122

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	14	17	16	14
N.S.	1	1.00	1.00	0.75	0.70	0.70	0.85	0.80	0.70
time (sec)	N/A	0.135	0.003	0.063	0.188	0.234	0.045	0.291	0.054

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	16	8	13	12
N.S.	1	1.00	0.75	0.81	0.75	1.00	0.50	0.81	0.75
time (sec)	N/A	0.135	0.003	0.053	0.191	0.236	0.038	0.275	0.037

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	20	14	15	14	17	10	14	14
N.S.	1	1.43	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.148	0.006	0.242	0.281	0.240	0.078	0.300	0.093

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.150	0.005	0.029	0.207	0.240	0.080	0.262	0.083

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	23	23	34	20	25	17
N.S.	1	1.00	1.29	1.10	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.116	0.005	0.053	0.195	0.237	0.054	0.255	0.102

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.142	0.003	0.052	0.275	0.240	0.037	0.291	0.172

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	18	10	11	14	18	8	14	10
N.S.	1	1.80	1.00	1.10	1.40	1.80	0.80	1.40	1.00
time (sec)	N/A	0.146	0.004	0.037	0.190	0.243	0.068	0.271	0.030

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	21	19	25	19	21	16
N.S.	1	1.00	1.00	0.68	0.61	0.81	0.61	0.68	0.52
time (sec)	N/A	0.144	0.002	0.049	0.196	0.229	0.051	0.318	0.045

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	24
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	1.33
time (sec)	N/A	0.162	0.004	0.046	0.330	0.247	0.066	0.364	0.041

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.116	0.003	0.067	0.271	0.255	0.066	0.277	0.029

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	64	22	72	61	73	72	33
N.S.	1	1.06	0.75	0.26	0.85	0.72	0.86	0.85	0.39
time (sec)	N/A	0.225	0.013	0.054	0.263	0.246	0.073	0.289	0.118

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	15	16	15	26	14	15	15
N.S.	1	1.00	0.65	0.70	0.65	1.13	0.61	0.65	0.65
time (sec)	N/A	0.137	0.006	0.117	0.295	0.258	0.041	0.284	0.089

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.131	0.005	0.042	0.185	0.243	0.045	0.280	0.062

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	26	23	20	23	36	39	47	21
N.S.	1	0.58	0.51	0.44	0.51	0.80	0.87	1.04	0.47
time (sec)	N/A	0.171	0.027	0.168	0.274	0.255	0.228	0.276	0.100

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	0	111	110	53	28
N.S.	1	1.00	0.84	0.81	0.00	3.00	2.97	1.43	0.76
time (sec)	N/A	0.171	0.024	0.109	0.000	0.281	1.449	0.265	0.339

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	20	20	16	37	50	36	35	15
N.S.	1	0.36	0.36	0.29	0.66	0.89	0.64	0.62	0.27
time (sec)	N/A	0.149	0.015	0.070	0.267	0.250	0.165	0.301	0.271

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	16	19	23	32	40	32
N.S.	1	1.00	0.65	0.52	0.61	0.74	1.03	1.29	1.03
time (sec)	N/A	0.158	0.013	0.090	0.274	0.255	0.133	0.264	0.224

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	18	18	17	14	33	248	48	26
N.S.	1	0.50	0.50	0.47	0.39	0.92	6.89	1.33	0.72
time (sec)	N/A	0.206	0.105	0.143	0.264	0.287	24.438	0.285	0.212

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	71839	26	15
N.S.	1	1.00	1.00	1.07	1.00	2.87	4789.27	1.73	1.00
time (sec)	N/A	0.172	0.042	0.855	0.262	0.273	18.580	0.286	0.460

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	602	13	29
N.S.	1	1.00	1.00	0.82	0.82	2.29	35.41	0.76	1.71
time (sec)	N/A	0.152	0.027	0.304	0.181	0.259	134.571	0.278	0.649

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	20	41	11	22	25	34
N.S.	1	1.00	0.73	0.67	1.37	0.37	0.73	0.83	1.13
time (sec)	N/A	0.210	0.053	0.253	0.300	0.253	0.147	0.312	0.319

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	23	22	29	24	22	22
N.S.	1	1.00	1.41	0.79	0.76	1.00	0.83	0.76	0.76
time (sec)	N/A	0.123	0.045	0.335	0.277	0.243	0.088	0.279	0.037

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.112	0.001	0.059	0.186	0.240	0.071	0.273	0.194

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	43	33	30	41	40	87	47	35
N.S.	1	1.16	0.89	0.81	1.11	1.08	2.35	1.27	0.95
time (sec)	N/A	0.135	0.033	0.171	0.270	0.254	0.910	0.273	0.224

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	42	22	25	25	0	26	21
N.S.	1	1.00	1.91	1.00	1.14	1.14	0.00	1.18	0.95
time (sec)	N/A	0.135	0.034	0.158	0.188	0.253	0.000	0.273	0.090

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	21	20	25	24	25	20
N.S.	1	1.00	1.22	0.78	0.74	0.93	0.89	0.93	0.74
time (sec)	N/A	0.119	0.019	0.147	0.283	0.240	0.099	0.268	0.087

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	21	22	27	26	27	23
N.S.	1	1.00	1.22	0.78	0.81	1.00	0.96	1.00	0.85
time (sec)	N/A	0.139	0.044	0.429	0.283	0.237	0.250	0.272	0.054

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	39	7	15	17	15	33	11
N.S.	1	1.00	2.79	0.50	1.07	1.21	1.07	2.36	0.79
time (sec)	N/A	0.119	0.022	0.080	0.195	0.238	0.214	0.270	0.203

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	72	76	60	59	92	0	168	73
N.S.	1	1.06	1.12	0.88	0.87	1.35	0.00	2.47	1.07
time (sec)	N/A	0.200	0.110	0.296	0.265	0.254	0.000	0.283	0.094

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	0	73	0	13
N.S.	1	1.00	1.00	1.00	0.92	0.00	5.62	0.00	1.00
time (sec)	N/A	0.149	0.003	0.062	0.190	0.000	0.990	0.000	0.033

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	2	2	15	2	2
N.S.	1	1.00	1.00	1.00	0.13	0.13	1.00	0.13	0.13
time (sec)	N/A	0.199	0.010	0.223	0.186	0.235	0.073	0.276	0.125

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
N.S.	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.118	0.009	0.036	0.221	0.229	0.387	0.294	0.008

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4
N.S.	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.124	0.010	0.033	0.209	0.234	0.405	0.276	0.013

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	16	5	13	7	13	14
N.S.	1	1.00	1.00	1.45	0.45	1.18	0.64	1.18	1.27
time (sec)	N/A	0.150	0.011	0.033	0.221	0.240	0.599	0.274	0.025

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	9	13	10	18	9
N.S.	1	1.00	1.00	1.07	0.64	0.93	0.71	1.29	0.64
time (sec)	N/A	0.158	0.004	0.038	0.207	0.234	0.622	0.255	0.018

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	16	14	0	14	14
N.S.	1	1.00	1.00	1.13	1.07	0.93	0.00	0.93	0.93
time (sec)	N/A	0.144	0.029	0.068	0.228	0.234	0.000	0.265	0.029

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	10	0	10	10
N.S.	1	1.00	1.00	1.08	1.00	0.77	0.00	0.77	0.77
time (sec)	N/A	0.225	0.051	0.052	0.218	0.235	0.000	0.269	0.128

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	16	23	0	80	17
N.S.	1	1.00	1.00	1.16	0.84	1.21	0.00	4.21	0.89
time (sec)	N/A	0.162	0.040	0.039	0.226	0.231	0.000	0.263	0.125

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	19	0	16	0
N.S.	1	1.00	1.00	1.06	1.00	1.06	0.00	0.89	0.00
time (sec)	N/A	0.158	0.008	0.035	0.222	0.240	0.000	0.272	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
N.S.	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.141	0.009	0.018	0.188	0.228	0.031	0.303	0.019

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	29	16	15	14	14	12	14	14
N.S.	1	1.12	0.62	0.58	0.54	0.54	0.46	0.54	0.54
time (sec)	N/A	0.178	0.011	0.020	0.207	0.233	0.033	0.295	0.029

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	42	21	20	19	19	17	19	19
N.S.	1	1.17	0.58	0.56	0.53	0.53	0.47	0.53	0.53
time (sec)	N/A	0.211	0.012	0.027	0.203	0.237	0.039	0.304	0.022

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	66	181	60	360	77	2034
N.S.	1	1.00	0.81	1.38	3.77	1.25	7.50	1.60	42.38
time (sec)	N/A	0.195	0.150	0.513	0.282	0.268	0.409	0.298	10.335

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.115	0.053	0.014	0.234	0.232	0.184	0.298	0.010

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	6	14	7	11	10
N.S.	1	1.00	1.00	1.70	0.60	1.40	0.70	1.10	1.00
time (sec)	N/A	0.134	0.004	0.021	0.232	0.249	0.172	0.288	0.035

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	22	18	24	0	22
N.S.	1	1.00	1.00	0.00	1.00	0.82	1.09	0.00	1.00
time (sec)	N/A	0.156	0.019	0.000	0.071	0.076	0.274	0.000	0.063

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	11	9	0	9	9
N.S.	1	1.00	0.83	1.00	0.92	0.75	0.00	0.75	0.75
time (sec)	N/A	0.133	0.038	0.043	0.218	0.233	0.000	0.298	0.015

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	20	0	20	0
N.S.	1	1.00	1.00	1.05	0.00	0.91	0.00	0.91	0.00
time (sec)	N/A	0.186	0.120	0.092	0.000	0.250	0.000	0.282	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	17	14	0	6	27	0	155
N.S.	1	1.00	0.17	0.14	0.00	0.06	0.26	0.00	1.50
time (sec)	N/A	0.159	10.020	0.153	0.000	0.069	0.411	0.000	0.281

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [83] had the largest ratio of [2]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	9	0.111
2	A	2	2	1.00	11	0.182
3	A	2	2	1.00	11	0.182
4	A	2	2	1.00	11	0.182
5	A	1	1	1.00	14	0.071
6	A	4	3	1.00	4	0.750
7	A	2	2	1.00	9	0.222
8	A	5	4	1.00	7	0.571
9	A	4	3	1.00	17	0.176
10	A	4	3	1.00	9	0.333
11	A	8	7	1.00	11	0.636
12	A	5	4	1.00	16	0.250
13	A	4	3	1.00	10	0.300
14	A	1	1	1.00	15	0.067
15	A	2	2	1.00	9	0.222
16	A	1	1	1.00	9	0.111
17	A	1	1	1.00	15	0.067
18	A	2	2	1.00	17	0.118
19	A	4	3	0.75	20	0.150
20	A	1	1	1.00	26	0.038
21	A	1	1	1.00	20	0.050
22	A	4	4	1.00	4	1.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	7	1.00	6	1.167
24	A	10	10	1.13	6	1.667
25	A	9	9	1.12	6	1.500
26	A	4	3	1.00	5	0.600
27	A	4	4	1.22	6	0.667
28	A	3	3	1.00	4	0.750
29	A	4	3	1.00	4	0.750
30	A	5	5	1.21	4	1.250
31	A	4	3	1.00	4	0.750
32	A	7	7	1.29	4	1.750
33	A	3	3	1.00	6	0.500
34	A	6	6	0.97	6	1.000
35	A	6	6	1.12	8	0.750
36	A	3	3	1.00	4	0.750
37	A	4	3	1.00	4	0.750
38	A	5	5	1.21	4	1.250
39	A	6	5	1.12	13	0.385
40	A	4	3	1.11	13	0.231
41	A	2	2	1.00	13	0.154
42	A	3	3	1.00	4	0.750
43	A	5	5	1.00	4	1.250
44	A	3	3	1.00	4	0.750
45	A	5	5	1.00	4	1.250
46	A	4	4	1.00	10	0.400
47	A	1	1	1.00	11	0.091
48	A	1	1	1.00	9	0.111
49	A	1	1	1.00	13	0.077
50	A	1	1	1.00	11	0.091
51	A	2	2	1.00	11	0.182
52	A	4	3	1.00	8	0.375
53	A	11	10	1.08	8	1.250
54	A	1	1	1.00	10	0.100
55	A	5	4	1.00	17	0.235
56	A	1	1	1.00	7	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.13	4	0.500
58	A	1	1	1.00	4	0.250
59	A	2	2	1.00	6	0.333
60	A	1	1	1.00	5	0.200
61	A	3	3	1.00	2	1.500
62	A	1	1	1.00	8	0.125
63	A	2	2	1.18	8	0.250
64	A	3	2	1.00	8	0.250
65	A	3	2	1.00	14	0.143
66	A	4	3	1.00	14	0.214
67	A	3	3	1.26	8	0.375
68	A	1	1	1.00	9	0.111
69	A	1	1	1.00	13	0.077
70	A	3	2	1.00	9	0.222
71	A	1	1	1.00	6	0.167
72	A	1	1	1.00	6	0.167
73	A	5	4	1.30	7	0.571
74	A	2	2	1.00	5	0.400
75	A	2	2	1.00	7	0.286
76	A	3	3	1.11	7	0.429
77	A	3	3	1.00	9	0.333
78	A	4	3	1.04	7	0.429
79	A	2	2	1.00	11	0.182
80	A	1	1	1.00	10	0.100
81	A	1	1	1.00	10	0.100
82	A	2	2	1.00	2	1.000
83	A	5	4	1.00	2	2.000
84	A	5	4	1.00	2	2.000
85	A	3	3	1.04	4	0.750
86	A	5	4	1.00	6	0.667
87	A	3	2	1.00	13	0.154
88	A	3	2	1.70	14	0.143
89	A	1	1	1.00	9	0.111
90	A	1	1	1.00	9	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	3	2	1.00	10	0.200
92	A	3	3	0.95	4	0.750
93	A	5	4	1.08	6	0.667
94	A	5	5	1.00	6	0.833
95	A	5	4	1.32	6	0.667
96	A	1	1	1.00	17	0.059
97	A	2	2	1.00	11	0.182
98	A	1	1	1.00	15	0.067
99	A	1	1	1.00	14	0.071
100	A	2	2	1.00	11	0.182
101	A	3	2	1.00	13	0.154
102	A	7	6	1.22	10	0.600
103	A	4	3	1.38	15	0.200
104	A	5	4	1.00	15	0.267
105	A	4	3	1.00	15	0.200
106	A	2	2	1.00	16	0.125
107	A	2	2	1.00	16	0.125
108	A	3	3	1.00	18	0.167
109	A	3	3	1.00	23	0.130
110	A	2	2	1.00	19	0.105
111	A	7	6	1.04	18	0.333
112	A	4	4	1.00	31	0.129
113	A	4	3	1.00	7	0.429
114	A	4	3	1.00	22	0.136
115	A	2	2	1.00	16	0.125
116	A	2	2	1.00	17	0.118
117	A	2	2	1.00	12	0.167
118	A	3	3	1.00	21	0.143
119	A	2	2	1.00	20	0.100
120	A	4	4	1.00	16	0.250
121	A	3	3	1.00	16	0.188
122	A	2	2	1.00	11	0.182
123	A	4	3	1.00	11	0.273
124	A	2	2	1.00	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.00	7	0.286
126	A	7	6	1.12	9	0.667
127	A	2	2	1.00	12	0.167
128	A	3	3	1.00	14	0.214
129	A	5	4	1.43	21	0.190
130	A	3	3	1.00	18	0.167
131	A	2	2	1.00	7	0.286
132	A	4	4	1.00	11	0.364
133	A	4	3	1.80	16	0.188
134	A	3	3	1.00	11	0.273
135	A	2	2	1.00	18	0.111
136	A	3	3	1.00	7	0.429
137	A	9	8	1.06	7	1.143
138	A	4	3	1.00	14	0.214
139	A	1	1	1.00	16	0.062
140	A	5	4	0.58	12	0.333
141	A	4	3	1.00	8	0.375
142	A	4	3	0.36	8	0.375
143	A	2	2	1.00	10	0.200
144	A	6	5	0.50	13	0.385
145	A	4	3	1.00	19	0.158
146	A	2	2	1.00	11	0.182
147	A	6	5	1.00	11	0.455
148	A	2	2	1.00	11	0.182
149	A	1	1	1.00	13	0.077
150	A	5	4	1.16	15	0.267
151	A	4	3	1.00	13	0.231
152	A	2	2	1.00	9	0.222
153	A	4	3	1.00	12	0.250
154	A	3	2	1.00	9	0.222
155	A	8	7	1.06	18	0.389
156	A	2	2	1.00	8	0.250
157	A	6	6	1.00	5	1.200
158	A	1	1	1.00	7	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	1	1	1.00	9	0.111
160	A	2	2	1.00	7	0.286
161	A	2	2	1.00	5	0.400
162	A	1	1	1.00	14	0.071
163	A	3	2	1.00	14	0.143
164	A	2	2	1.00	9	0.222
165	A	2	2	1.00	8	0.250
166	A	2	2	1.00	7	0.286
167	A	3	3	1.12	9	0.333
168	A	4	4	1.17	9	0.444
169	A	2	2	1.00	21	0.095
170	A	1	1	1.00	4	0.250
171	A	2	2	1.00	4	0.500
172	A	3	2	1.00	8	0.250
173	A	1	1	1.00	11	0.091
174	A	2	2	1.00	16	0.125
175	A	1	1	1.00	9	0.111

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sqrt{1+2x}\,dx$

3.1.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

output
```
1/3*(1+2*x)^(3/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

input
```
Integrate[Sqrt[1 + 2*x],x]
```

output
```
(1 + 2*x)^(3/2)/3
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x+1}\,dx$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3}(2x+1)^{3/2}$$

input
```
Int[Sqrt[1 + 2*x],x]
```

output
```
(1 + 2*x)^(3/2)/3
```

3.1.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
default	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{1}{3}+\frac{2x}{3}\right)\sqrt{1+2x}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2+4x)\sqrt{1+2x}}{3}}{4\sqrt{\pi}}$	29

input
```
int((1+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1+2*x)^(3/2)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int\sqrt{1+2x}\,dx=\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

input
```
integrate((1+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*x + 1)^(3/2)
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int\sqrt{1+2x}\,dx=\frac{(2x+1)^{\frac{3}{2}}}{3}$$

input
```
integrate((1+2*x)**(1/2),x)
```

output
```
(2*x + 1)**(3/2)/3
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

input
```
integrate((1+2*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(2*x + 1)^(3/2)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

input
```
integrate((1+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*(2*x + 1)^(3/2)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1+2x}\,dx = \frac{(2\,x+1)^{3/2}}{3}$$

input
```
int((2*x + 1)^(1/2),x)
```

output
```
(2*x + 1)^(3/2)/3
```

3.2 $\int x\sqrt{1+3x}\,dx$

3.2.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int x\sqrt{1+3x}\,dx = -\frac{2}{27}(1+3x)^{3/2} + \frac{2}{45}(1+3x)^{5/2}$$

output
```
-2/27*(1+3*x)^(3/2)+2/45*(1+3*x)^(5/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x\sqrt{1+3x}\,dx = \frac{2}{135}(1+3x)^{3/2}(-2+9x)$$

input
```
Integrate[x*Sqrt[1 + 3*x],x]
```

output
```
(2*(1 + 3*x)^(3/2)*(-2 + 9*x))/135
```

3.2.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{3x+1}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{1}{3}(3x+1)^{3/2}-\frac{1}{3}\sqrt{3x+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

input
```
Int[x*Sqrt[1 + 3*x],x]
```

output
```
(-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

3.2.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2(1+3x)^{\frac{3}{2}}(9x-2)}{135}$	15
trager	$\left(\frac{2}{5}x^2+\frac{2}{45}x-\frac{4}{135}\right)\sqrt{1+3x}$	19
derivativedivides	$-\frac{2(1+3x)^{\frac{3}{2}}}{27}+\frac{2(1+3x)^{\frac{5}{2}}}{45}$	20
default	$-\frac{2(1+3x)^{\frac{3}{2}}}{27}+\frac{2(1+3x)^{\frac{5}{2}}}{45}$	20
risch	$\frac{2(27x^2+3x-2)\sqrt{1+3x}}{135}$	20
pseudoelliptic	$\frac{2(27x^2+3x-2)\sqrt{1+3x}}{135}$	20
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+3x)^{\frac{3}{2}}(-9x+2)}{15}}{18\sqrt{\pi}}$	29

input
```
int(x*(1+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/135*(1+3*x)^(3/2)*(9*x-2)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+3x}\,dx=\frac{2}{135}\left(27\,x^2+3\,x-2\right)\sqrt{3\,x+1}$$

input
```
integrate(x*(1+3*x)^(1/2),x, algorithm="fricas")
```

output
```
2/135*(27*x^2 + 3*x - 2)*sqrt(3*x + 1)
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x\sqrt{1+3x}\,dx = \frac{2x^2\sqrt{3x+1}}{5} + \frac{2x\sqrt{3x+1}}{45} - \frac{4\sqrt{3x+1}}{135}$$

input
```
integrate(x*(1+3*x)**(1/2),x)
```

output
```
2*x**2*sqrt(3*x + 1)/5 + 2*x*sqrt(3*x + 1)/45 - 4*sqrt(3*x + 1)/135
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+3x}\,dx = \frac{2}{45}\,(3\,x+1)^{\frac{5}{2}} - \frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+3*x)^(1/2),x, algorithm="maxima")
```

output
```
2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x\sqrt{1+3x}\,dx = \frac{2}{45}\,(3\,x+1)^{\frac{5}{2}} - \frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+3*x)^(1/2),x, algorithm="giac")
```

output
```
2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+3x}\,dx = \frac{2\,(3\,x+1)^{3/2}\,(9\,x-2)}{135}$$

input
```
int(x*(3*x + 1)^(1/2),x)
```

output
```
(2*(3*x + 1)^(3/2)*(9*x - 2))/135
```

3.3 $\int x^2\sqrt{1+x}\,dx$

3.3.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{7}(1+x)^{7/2}$$

output
```
2/3*(1+x)^(3/2)-4/5*(1+x)^(5/2)+2/7*(1+x)^(7/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{105}(1+x)^{3/2}\left(8-12x+15x^2\right)$$

input
```
Integrate[x^2*Sqrt[1 + x],x]
```

output
```
(2*(1 + x)^(3/2)*(8 - 12*x + 15*x^2))/105
```

3.3.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{x+1}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left((x+1)^{5/2}-2(x+1)^{3/2}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

input
```
Int[x^2*Sqrt[1 + x],x]
```

output
```
(2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

3.3.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(15x^2-12x+8)}{105}$	18
trager	$\left(\frac{2}{7}x^3+\frac{2}{35}x^2-\frac{8}{105}x+\frac{16}{105}\right)\sqrt{1+x}$	22
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{7}{2}}}{7}$	23
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{7}{2}}}{7}$	23
risch	$\frac{2(15x^3+3x^2-4x+8)\sqrt{1+x}}{105}$	23
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}\left(15x^2-12x+8\right)}{105}}{2\sqrt{\pi}}$	32

input
```
int(x^2*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(1+x)^(3/2)*(15*x^2-12*x+8)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{105}\left(15\,x^3+3\,x^2-4\,x+8\right)\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*x^3 + 3*x^2 - 4*x + 8)*sqrt(x + 1)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x^2\sqrt{1+x}\,dx = \frac{2x^3\sqrt{x+1}}{7} + \frac{2x^2\sqrt{x+1}}{35} - \frac{8x\sqrt{x+1}}{105} + \frac{16\sqrt{x+1}}{105}$$

input
```
integrate(x**2*(1+x)**(1/2),x)
```

output
```
2*x**3*sqrt(x + 1)/7 + 2*x**2*sqrt(x + 1)/35 - 8*x*sqrt(x + 1)/105 + 16*sq
rt(x + 1)/105
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{7}\,(x+1)^{\frac{7}{2}} - \frac{4}{5}\,(x+1)^{\frac{5}{2}} + \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate(x^2*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{1+x}\,dx = \frac{2}{7}\,(x+1)^{\frac{7}{2}} - \frac{4}{5}\,(x+1)^{\frac{5}{2}} + \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate(x^2*(1+x)^(1/2),x, algorithm="giac")
```

output
```
2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int x^2\sqrt{1+x}\,dx = -\frac{2\,(x+1)^{3/2}\,\left(42\,x-15\,(x+1)^2+7\right)}{105}$$

input
```
int(x^2*(x + 1)^(1/2),x)
```

output
```
-(2*(x + 1)^(3/2)*(42*x - 15*(x + 1)^2 + 7))/105
```

3.4 $\int \frac{x}{\sqrt{2-3x}} \, dx$

3.4.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{x}{\sqrt{2-3x}} \, dx = -\frac{4}{9}\sqrt{2-3x} + \frac{2}{27}(2-3x)^{3/2}$$

output
```
2/27*(2-3*x)^(3/2)-4/9*(2-3*x)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{2-3x}} \, dx = -\frac{2}{27}\sqrt{2-3x}(4+3x)$$

input
```
Integrate[x/Sqrt[2 - 3*x],x]
```

output
```
(-2*Sqrt[2 - 3*x]*(4 + 3*x))/27
```

3.4.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{2-3x}}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{2}{3\sqrt{2-3x}}-\frac{1}{3}\sqrt{2-3x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

input
```
Int[x/Sqrt[2 - 3*x],x]
```

output
```
(-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

3.4.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

method	result	size
trager	$\left(-\frac{2x}{9}-\frac{8}{27}\right)\sqrt{2-3x}$	14
gosper	$-\frac{2(3x+4)\sqrt{2-3x}}{27}$	15
pseudoelliptic	$-\frac{2(3x+4)\sqrt{2-3x}}{27}$	15
derivativedivides	$\frac{2(2-3x)^{\frac{3}{2}}}{27}-\frac{4\sqrt{2-3x}}{9}$	20
default	$\frac{2(2-3x)^{\frac{3}{2}}}{27}-\frac{4\sqrt{2-3x}}{9}$	20
risch	$\frac{2(-2+3x)(3x+4)}{27\sqrt{2-3x}}$	20
meijerg	$\frac{2\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(6x+8)\sqrt{1-\frac{3x}{2}}}{6}\right)}{9\sqrt{\pi}}$	32

input
```
int(x/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/9*x-8/27)*(2-3*x)^(1/2)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{2-3x}}\,dx = -\frac{2}{27}\,(3\,x+4)\sqrt{-3\,x+2}$$

input
```
integrate(x/(2-3*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/27*(3*x + 4)*sqrt(-3*x + 2)
```

3.4.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \begin{cases} -\frac{2ix\sqrt{3x-2}}{9} - \frac{8i\sqrt{3x-2}}{27} & \text{for } |x| > \frac{2}{3} \\ -\frac{2x\sqrt{2-3x}}{9} - \frac{8\sqrt{2-3x}}{27} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(2-3*x)**(1/2),x)
```

output
```
Piecewise((-2*I*x*sqrt(3*x - 2)/9 - 8*I*sqrt(3*x - 2)/27, Abs(x) > 2/3), (
-2*x*sqrt(2 - 3*x)/9 - 8*sqrt(2 - 3*x)/27, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}} - \frac{4}{9}\,\sqrt{-3\,x+2}$$

input
```
integrate(x/(2-3*x)^(1/2),x, algorithm="maxima")
```

output
```
2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{2-3x}}\,dx = \frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}} - \frac{4}{9}\,\sqrt{-3\,x+2}$$

input
```
integrate(x/(2-3*x)^(1/2),x, algorithm="giac")
```

output
```
2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{2-3x}}\,dx = -\frac{2\sqrt{2-3\,x}\,(3\,x+4)}{27}$$

input
```
int(x/(2 - 3*x)^(1/2),x)
```

output
```
-(2*(2 - 3*x)^(1/2)*(3*x + 4))/27
```

3.5 $\int \frac{1+x}{(2+2x+x^2)^3} \, dx$

3.5.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1+x}{(2+2x+x^2)^3} \, dx = -\frac{1}{4(2+2x+x^2)^2}$$

output
```
-1/4/(x^2+2*x+2)^2
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(2+2x+x^2)^3} \, dx = -\frac{1}{4(2+2x+x^2)^2}$$

input
```
Integrate[(1 + x)/(2 + 2*x + x^2)^3,x]
```

output
```
-1/4*1/(2 + 2*x + x^2)^2
```

3.5.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x^2+2x+2)^3}\,dx$$

$$\Big\downarrow \text{1104}$$

$$-\frac{1}{4(x^2+2x+2)^2}$$

input
```
Int[(1 + x)/(2 + 2*x + x^2)^3,x]
```

output
```
-1/4*1/(2 + 2*x + x^2)^2
```

3.5.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{4(x^2+2x+2)^2}$	13
default	$-\frac{1}{4(x^2+2x+2)^2}$	13
norman	$-\frac{1}{4(x^2+2x+2)^2}$	13
risch	$-\frac{1}{4(x^2+2x+2)^2}$	13
parallelrisch	$-\frac{1}{4(x^2+2x+2)^2}$	13

input
```
int((1+x)/(x^2+2*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^2+2*x+2)^2
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^4+4\,x^3+8\,x^2+8\,x+4)}$$

input
```
integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="fricas")
```

output
```
-1/4/(x^4 + 4*x^3 + 8*x^2 + 8*x + 4)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4x^4+16x^3+32x^2+32x+16}$$

input
```
integrate((1+x)/(x**2+2*x+2)**3,x)
```

output
```
-1/(4*x**4 + 16*x**3 + 32*x**2 + 32*x + 16)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^2+2\,x+2)^2}$$

input
```
integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="maxima")
```

output
```
-1/4/(x^2 + 2*x + 2)^2
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^2+2\,x+2)^2}$$

input
```
integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="giac")
```

output
```
-1/4/(x^2 + 2*x + 2)^2
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x}{(2+2x+x^2)^3}\,dx = -\frac{1}{4\,(x^2+2\,x+2)^2}$$

input
```
int((x + 1)/(2*x + x^2 + 2)^3,x)
```

output
```
-1/(4*(2*x + x^2 + 2)^2)
```

3.6 $\int \sin^3(x)\,dx$

3.6.1 Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x)\,dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

output
```
-cos(x)+1/3*cos(x)^3
```

3.6.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x)\,dx = -\frac{3\cos(x)}{4} + \frac{1}{12}\cos(3x)$$

input
```
Integrate[Sin[x]^3,x]
```

output
```
(-3*Cos[x])/4 + Cos[3*x]/12
```

3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(x)^3dx$$
$$\Big\downarrow \text{3113}$$
$$-\int \left(1-\cos^2(x)\right)d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\cos^3(x)}{3}-\cos(x)$$

input
```
Int[Sin[x]^3,x]
```

output
```
-Cos[x] + Cos[x]^3/3
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
parallelrisch	$-\frac{2}{3}-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	13
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

input
```
int(sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2+sin(x)^2)*cos(x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="fricas")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

input
```
integrate(sin(x)**3,x)
```

output
```
cos(x)**3/3 - cos(x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="maxima")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="giac")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x)\,dx = \frac{\cos(x)\,\left(\cos(x)^2 - 3\right)}{3}$$

input
```
int(sin(x)^3,x)
```

output
```
(cos(x)*(cos(x)^2 - 3))/3
```

3.7 $\int \sqrt[3]{-1+z}z\,dz$

3.7.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{4}(-1+z)^{4/3} + \frac{3}{7}(-1+z)^{7/3}$$

output
```
3/4*(-1+z)^(4/3)+3/7*(-1+z)^(7/3)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{28}(7+4(-1+z))(-1+z)^{4/3}$$

input
```
Integrate[(-1 + z)^(1/3)*z,z]
```

output
```
(3*(7 + 4*(-1 + z))*(-1 + z)^(4/3))/28
```

3.7.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{z-1}z\,dz$$

$$\Big\downarrow \text{53}$$

$$\int \left((z-1)^{4/3}+\sqrt[3]{z-1}\right)dz$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

input
```
Int[(-1 + z)^(1/3)*z,z]
```

output
```
(3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

3.7.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{3(-1+z)^{\frac{4}{3}}(4z+3)}{28}$	13
derivativedivides	$\frac{3(-1+z)^{\frac{4}{3}}}{4}+\frac{3(-1+z)^{\frac{7}{3}}}{7}$	16
default	$\frac{3(-1+z)^{\frac{4}{3}}}{4}+\frac{3(-1+z)^{\frac{7}{3}}}{7}$	16
trager	$\left(\frac{3}{7}z^{2}-\frac{3}{28}z-\frac{9}{28}\right)(-1+z)^{\frac{1}{3}}$	17
risch	$\frac{3(-1+z)^{\frac{1}{3}}(4z^{2}-z-3)}{28}$	18
meijerg	$\frac{\operatorname{signum}(-1+z)^{\frac{1}{3}}z^{2}{}_{2}F_{1}\left(-\frac{1}{3},2;3;z\right)}{2(-\operatorname{signum}(-1+z))^{\frac{1}{3}}}$	27

input
```
int((-1+z)^(1/3)*z,z,method=_RETURNVERBOSE)
```

output
```
3/28*(-1+z)^(4/3)*(4*z+3)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt[3]{-1+z}z\,dz=\frac{3}{28}\left(4z^{2}-z-3\right)(z-1)^{\frac{1}{3}}$$

input
```
integrate((-1+z)^(1/3)*z,z, algorithm="fricas")
```

output
```
3/28*(4*z^2 - z - 3)*(z - 1)^(1/3)
```

3.7.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \sqrt[3]{-1+z}z\,dz = \begin{cases} \frac{3z^2\sqrt[3]{z-1}}{7} - \frac{3z\sqrt[3]{z-1}}{28} - \frac{9\sqrt[3]{z-1}}{28} & \text{for } |z|>1 \\ \frac{3z^2\sqrt[3]{1-z}e^{\frac{i\pi}{3}}}{7} - \frac{3z\sqrt[3]{1-z}e^{\frac{i\pi}{3}}}{28} - \frac{9\sqrt[3]{1-z}e^{\frac{i\pi}{3}}}{28} & \text{otherwise} \end{cases}$$

input
```
integrate((-1+z)**(1/3)*z,z)
```

output
```
Piecewise((3*z**2*(z - 1)**(1/3)/7 - 3*z*(z - 1)**(1/3)/28 - 9*(z - 1)**(1
/3)/28, Abs(z) > 1), (3*z**2*(1 - z)**(1/3)*exp(I*pi/3)/7 - 3*z*(1 - z)**(
1/3)*exp(I*pi/3)/28 - 9*(1 - z)**(1/3)*exp(I*pi/3)/28, True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{7}(z-1)^{\frac{7}{3}} + \frac{3}{4}(z-1)^{\frac{4}{3}}$$

input
```
integrate((-1+z)^(1/3)*z,z, algorithm="maxima")
```

output
```
3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3}{7}(z-1)^{\frac{7}{3}} + \frac{3}{4}(z-1)^{\frac{4}{3}}$$

input
```
integrate((-1+z)^(1/3)*z,z, algorithm="giac")
```

output
```
3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{-1+z}z\,dz = \frac{3\,(4\,z+3)\,(z-1)^{4/3}}{28}$$

input
```
int(z*(z - 1)^(1/3),z)
```

output
```
(3*(4*z + 3)*(z - 1)^(4/3))/28
```

3.8 $\int \cot(x)\csc^2(x)\,dx$

3.8.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2}\csc^2(x)$$

output
```
-1/2*csc(x)^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2}\csc^2(x)$$

input
```
Integrate[Cot[x]*Csc[x]^2,x]
```

output
```
-1/2*Csc[x]^2
```

3.8.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\csc^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan\left(x-\frac{\pi}{2}\right)\left(-\sec\left(x-\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^2\tan\left(x-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3086$$

$$-\int \csc(x)d\csc(x)$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{2}\csc^2(x)$$

input
```
Int[Cot[x]*Csc[x]^2,x]
```

output
```
-1/2*Csc[x]^2
```

3.8.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.8.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{1}{2\sin(x)^2}$	7
default	$-\frac{1}{2\sin(x)^2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}$	17
norman	$\frac{-\frac{1}{8}-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}}{\tan\left(\frac{x}{2}\right)^2}$	18
parallelrisch	$\frac{-1-\left(\tan^4\left(\frac{x}{2}\right)\right)}{8\tan\left(\frac{x}{2}\right)^2}$	19

input
```
int(cos(x)/sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/sin(x)^2
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc^2(x)\,dx = \frac{1}{2\left(\cos\left(x\right)^2-1\right)}$$

input
```
integrate(cos(x)/sin(x)^3,x, algorithm="fricas")
```

output
```
1/2/(cos(x)^2 - 1)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2\sin^2{(x)}}$$

input
```
integrate(cos(x)/sin(x)**3,x)
```

output
```
-1/(2*sin(x)**2)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2\,\sin{(x)}^2}$$

input
```
integrate(cos(x)/sin(x)^3,x, algorithm="maxima")
```

output
```
-1/2/sin(x)^2
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^2(x)\,dx = -\frac{1}{2\,\sin{(x)}^2}$$

input
```
integrate(cos(x)/sin(x)^3,x, algorithm="giac")
```

output
```
-1/2/sin(x)^2
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^2(x)\,dx = -\frac{\cot(x)^2}{2}$$

input
```
int(cos(x)/sin(x)^3,x)
```

output
```
-cot(x)^2/2
```

3.9 $\int \cos(2x)\sqrt{4-\sin(2x)}\,dx$

3.9.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}(4-\sin(2x))^{3/2}$$

output
```
-1/3*(4-sin(2*x))^(3/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}(4-\sin(2x))^{3/2}$$

input
```
Integrate[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

output
```
-1/3*(4 - Sin[2*x])^(3/2)
```

3.9.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3147, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4-\sin(2x)}\cos(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{4-\sin(2x)}\cos(2x)dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{1}{2}\int \sqrt{4-\sin(2x)}d(-\sin(2x))$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

input
```
Int[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

output
```
-1/3*(4 - Sin[2*x])^(3/2)
```

3.9.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{(4-\sin(2x))^{\frac{3}{2}}}{3}$	13
default	$-\frac{(4-\sin(2x))^{\frac{3}{2}}}{3}$	13

input
```
int(cos(2*x)*(4-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4-sin(2*x))^(3/2)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = \frac{1}{3}\left(\sin\left(2\,x\right)-4\right)\sqrt{-\sin\left(2\,x\right)+4}$$

input
```
integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sin(2*x) - 4)*sqrt(-sin(2*x) + 4)
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = \frac{\sqrt{4-\sin\left(2x\right)}\sin\left(2x\right)}{3}-\frac{4\sqrt{4-\sin\left(2x\right)}}{3}$$

input
```
integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)
```

output
```
sqrt(4 - sin(2*x))*sin(2*x)/3 - 4*sqrt(4 - sin(2*x))/3
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

input
```
integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-sin(2*x) + 4)^(3/2)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

input
```
integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="giac")
```

output
```
-1/3*(-sin(2*x) + 4)^(3/2)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos(2x)\sqrt{4-\sin(2x)}\,dx = -\frac{\left(4-\sin\left(2\,x\right)\right)^{3/2}}{3}$$

input
```
int(cos(2*x)*(4 - sin(2*x))^(1/2),x)
```

output
```
-(4 - sin(2*x))^(3/2)/3
```

3.10 $\int \frac{\sin(x)}{(3+\cos(x))^2} \, dx$

3.10.1 Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{\sin(x)}{(3+\cos(x))^2} \, dx = \frac{1}{3+\cos(x)}$$

output
```
1/(3+cos(x))
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2} \, dx = \frac{1}{3+\cos(x)}$$

input
```
Integrate[Sin[x]/(3 + Cos[x])^2,x]
```

output
```
(3 + Cos[x])^(-1)
```

3.10.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3147, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(\cos(x)+3)^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(3-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3147$$

$$-\int \frac{1}{(\cos(x)+3)^2}d\cos(x)$$

$$\Big\downarrow \ 17$$

$$\frac{1}{\cos(x)+3}$$

input
```
Int[Sin[x]/(3 + Cos[x])^2,x]
```

output
```
(3 + Cos[x])^(-1)
```

3.10.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.10.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{1}{3+\cos(x)}$	7
default	$\frac{1}{3+\cos(x)}$	7
parallelrisch	$-\frac{1}{2(\tan^2(\frac{x}{2}))+4}$	15
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+6\,\mathrm{e}^{ix}+1}$	24
norman	$\frac{-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{x}{2}\right)+2\right)}$	32

input
```
int(sin(x)/(3+cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(3+cos(x))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+3}$$

input
```
integrate(sin(x)/(3+cos(x))^2,x, algorithm="fricas")
```

output
```
1/(cos(x) + 3)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+3}$$

input
```
integrate(sin(x)/(3+cos(x))**2,x)
```

output
```
1/(cos(x) + 3)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+3}$$

input
```
integrate(sin(x)/(3+cos(x))^2,x, algorithm="maxima")
```

output
```
1/(cos(x) + 3)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+3}$$

input
```
integrate(sin(x)/(3+cos(x))^2,x, algorithm="giac")
```

output
```
1/(cos(x) + 3)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+3}$$

input
```
int(sin(x)/(cos(x) + 3)^2,x)
```

output
```
1/(cos(x) + 3)
```

3.11 $\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx$

3.11.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

output
```
2*cos(x)/(cos(x)^3)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

input
```
Integrate[Sin[x]/Sqrt[Cos[x]^3],x]
```

output
```
(2*Cos[x])/Sqrt[Cos[x]^3]
```

3.11.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 25, 3686, 25, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^3}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^3}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$-\frac{\cos^{\frac{3}{2}}(x)\int -\frac{\sin(x)}{\cos^{\frac{3}{2}}(x)}dx}{\sqrt{\cos^3(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{\sin(x)}{\cos^{\frac{3}{2}}(x)}dx}{\sqrt{\cos^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{\sin(x)}{\cos(x)^{3/2}}dx}{\sqrt{\cos^3(x)}}$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{3}{2}}(x)}d\cos(x)}{\sqrt{\cos^3(x)}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

input
```
Int[Sin[x]/Sqrt[Cos[x]^3],x]
```

output
```
(2*Cos[x])/Sqrt[Cos[x]^3]
```

3.11.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.11.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{4\cos(x)}{\sqrt{\cos(3x)+3\cos(x)}}$	11
default	$\frac{4\cos(x)}{\sqrt{\cos(3x)+3\cos(x)}}$	11

input
```
int(sin(x)/(cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*cos(x)/(cos(x)^3)^(1/2)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\sqrt{\cos(x)^3}}{\cos(x)^2}$$

input
```
integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(cos(x)^3)/cos(x)^2
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

input
```
integrate(sin(x)/(cos(x)**3)**(1/2),x)
```

output
```
2*cos(x)/sqrt(cos(x)**3)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\cos(x)}{\sqrt{\cos(x)^3}}$$

input
```
integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
2*cos(x)/sqrt(cos(x)^3)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2}{\sqrt{\cos(x)}}$$

input
```
integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="giac")
```

output
```
2/sqrt(cos(x))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{2\,|\cos(x)|}{\cos(x)^{3/2}}$$

input
```
int(sin(x)/(cos(x)^3)^(1/2),x)
```

output
```
(2*abs(cos(x)))/cos(x)^(3/2)
```

3.12 $\int \frac{\sin(\sqrt{1+x})}{\sqrt{1+x}}\,dx$

3.12.1 Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{1+x}\right)$$

output
```
-2*cos((1+x)^(1/2))
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{1+x}\right)$$

input
```
Integrate[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

output
```
-2*Cos[Sqrt[1 + x]]
```

3.12.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3912, 30, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\sqrt{x+1}\right)}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{3912}$$

$$2\int \sin\left(\sqrt{x+1}\right)d\sqrt{x+1}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sin\left(\sqrt{x+1}\right)d\sqrt{x+1}$$

$$\Big\downarrow \text{3118}$$

$$-2\cos\left(\sqrt{x+1}\right)$$

input
```
Int[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

output
```
-2*Cos[Sqrt[1 + x]]
```

3.12.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

3.12.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-2\cos\left(\sqrt{1+x}\right)$	9
default	$-2\cos\left(\sqrt{1+x}\right)$	9

input
```
int(sin((1+x)^(1/2))/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*cos((1+x)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\,\cos\left(\sqrt{x+1}\right)$$

input
```
integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*cos(sqrt(x + 1))
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{x+1}\right)$$

input
```
integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)
```

output
```
-2*cos(sqrt(x + 1))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{x+1}\right)$$

input
```
integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-2*cos(sqrt(x + 1))
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\cos\left(\sqrt{x+1}\right)$$

input
```
integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*cos(sqrt(x + 1))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = -2\,\cos\left(\sqrt{x+1}\right)$$

input
```
int(sin((x + 1)^(1/2))/(x + 1)^(1/2),x)
```

output
```
-2*cos((x + 1)^(1/2))
```

3.13 $\int x^{-1+n} \sin(x^n)\, dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int x^{-1+n} \sin(x^n)\, dx = -\frac{\cos(x^n)}{n}$$

output
```
-cos(x^n)/n
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n} \sin(x^n)\, dx = -\frac{\cos(x^n)}{n}$$

input
```
Integrate[x^(-1 + n)*Sin[x^n],x]
```

output
```
-(Cos[x^n]/n)
```

3.13.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\sin\left(x^n\right)\,dx$$
$$\Big\downarrow \text{ 3860}$$
$$\frac{\int \sin\left(x^n\right)dx^n}{n}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sin\left(x^n\right)dx^n}{n}$$
$$\Big\downarrow \text{ 3118}$$
$$-\frac{\cos\left(x^n\right)}{n}$$

input
```
Int[x^(-1 + n)*Sin[x^n],x]
```

output
```
-(Cos[x^n]/n)
```

3.13.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.13.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{\cos(x^n)}{n}$	10
risch	$-\frac{\cos(x^n)}{n}$	10
norman	$\frac{2\left(\tan^2\left(\frac{e^{n\ln(x)}}{2}\right)\right)}{n\left(1+\tan^2\left(\frac{e^{n\ln(x)}}{2}\right)\right)}$	30
meijerg	$\frac{\sqrt{\pi}\left(\frac{2^{1-\frac{-1+n}{n}-\frac{1}{n}}(-1)^{\frac{1}{2}-\frac{-1+n}{2n}-\frac{1}{2n}}}{\sqrt{\pi}\,\Gamma\left(3-\frac{-1+n}{n}-\frac{1}{n}\right)}-\frac{(-1)^{\frac{1}{2}-\frac{-1+n}{2n}-\frac{1}{2n}}2^{1-\frac{-1+n}{n}-\frac{1}{n}}\cos(x^n)}{\sqrt{\pi}\,\Gamma\left(3-\frac{-1+n}{n}-\frac{1}{n}\right)}\right)}{n}$	126

input
```
int(x^(-1+n)*sin(x^n),x,method=_RETURNVERBOSE)
```

output
```
-cos(x^n)/n
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sin\left(x^n\right)\,dx = -\frac{\cos\left(x^n\right)}{n}$$

input
```
integrate(x^(-1+n)*sin(x^n),x, algorithm="fricas")
```

output
```
-cos(x^n)/n
```

3.13.6 Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x^{-1+n}\sin(x^n)\ dx = -\frac{\cos(x^n)}{n}$$

input
```
integrate(x**(-1+n)*sin(x**n),x)
```

output
```
-cos(x**n)/n
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sin(x^n)\ dx = -\frac{\cos(x^n)}{n}$$

input
```
integrate(x^(-1+n)*sin(x^n),x, algorithm="maxima")
```

output
```
-cos(x^n)/n
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sin(x^n)\ dx = -\frac{\cos(x^n)}{n}$$

input
```
integrate(x^(-1+n)*sin(x^n),x, algorithm="giac")
```

output
```
-cos(x^n)/n
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-1+n} \sin \left(x^n\right) \, dx = -\frac{\cos \left(x^n\right)}{n}$$

input
```
int(x^(n - 1)*sin(x^n),x)
```

output
```
-cos(x^n)/n
```

3.14 $\int \frac{x^5}{\sqrt{1-x^6}} \, dx$

3.14.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^5}{\sqrt{1-x^6}} \, dx = -\frac{1}{3}\sqrt{1-x^6}$$

output
```
-1/3*(-x^6+1)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{1-x^6}} \, dx = -\frac{1}{3}\sqrt{1-x^6}$$

input
```
Integrate[x^5/Sqrt[1 - x^6],x]
```

output
```
-1/3*Sqrt[1 - x^6]
```

3.14.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-x^6}} \, dx$$

$$\Big\downarrow \; 793$$

$$-\frac{1}{3}\sqrt{1-x^6}$$

input
```
Int[x^5/Sqrt[1 - x^6],x]
```

output
```
-1/3*Sqrt[1 - x^6]
```

3.14.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.14.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{-x^{6}+1}}{3}$	12
default	$-\frac{\sqrt{-x^{6}+1}}{3}$	12
trager	$-\frac{\sqrt{-x^{6}+1}}{3}$	12
pseudoelliptic	$-\frac{\sqrt{-x^{6}+1}}{3}$	12
risch	$\frac{x^{6}-1}{3\sqrt{-x^{6}+1}}$	17
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{6}+1}}{6\sqrt{\pi}}$	26
gosper	$\frac{(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{3\sqrt{-x^{6}+1}}$	32

input
```
int(x^5/(-x^6+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-x^6+1)^(1/2)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\sqrt{-x^6+1}$$

input
```
integrate(x^5/(-x^6+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-x^6 + 1)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{\sqrt{1-x^6}}{3}$$

input
```
integrate(x**5/(-x**6+1)**(1/2),x)
```

output
```
-sqrt(1 - x**6)/3
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\,\sqrt{-x^6+1}$$

input
```
integrate(x^5/(-x^6+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-x^6 + 1)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\,\sqrt{-x^6+1}$$

input
```
integrate(x^5/(-x^6+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(-x^6 + 1)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{\sqrt{1-x^6}}{3}$$

input
```
int(x^5/(1 - x^6)^(1/2),x)
```

output
```
-(1 - x^6)^(1/2)/3
```

3.15 $\int t\sqrt[4]{1+t}\,dt$

3.15.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int t\sqrt[4]{1+t}\,dt = -\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}$$

output
```
-4/5*(1+t)^(5/4)+4/9*(1+t)^(9/4)
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{45}(1+t)^{5/4}(-9+5(1+t))$$

input
```
Integrate[t*(1 + t)^(1/4),t]
```

output
```
(4*(1 + t)^(5/4)*(-9 + 5*(1 + t)))/45
```

3.15.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int t\sqrt[4]{t+1}\,dt$$

$$\Big\downarrow \text{53}$$

$$\int \left((t+1)^{5/4}-\sqrt[4]{t+1}\right)dt$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

input
```
Int[t*(1 + t)^(1/4),t]
```

output
```
(-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

3.15.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{4(1+t)^{\frac{5}{4}}(5t-4)}{45}$	13
meijerg	$\frac{t^{2}{}_{2}F_{1}\left(-\frac{1}{4},2;3;-t\right)}{2}$	15
derivativedivides	$-\frac{4(1+t)^{\frac{5}{4}}}{5}+\frac{4(1+t)^{\frac{9}{4}}}{9}$	16
default	$-\frac{4(1+t)^{\frac{5}{4}}}{5}+\frac{4(1+t)^{\frac{9}{4}}}{9}$	16
risch	$\frac{4(1+t)^{\frac{1}{4}}(5t^{2}+t-4)}{45}$	16
trager	$\left(\frac{4}{9}t^{2}+\frac{4}{45}t-\frac{16}{45}\right)(1+t)^{\frac{1}{4}}$	17

input
```
int(t*(1+t)^(1/4),t,method=_RETURNVERBOSE)
```

output
```
4/45*(1+t)^(5/4)*(5*t-4)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int t\sqrt[4]{1+t}\,dt=\frac{4}{45}\left(5\,t^{2}+t-4\right)(t+1)^{\frac{1}{4}}$$

input
```
integrate(t*(1+t)^(1/4),t, algorithm="fricas")
```

output
```
4/45*(5*t^2 + t - 4)*(t + 1)^(1/4)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int t\sqrt[4]{1+t}\,dt = \frac{4t^2\sqrt[4]{t+1}}{9} + \frac{4t\sqrt[4]{t+1}}{45} - \frac{16\sqrt[4]{t+1}}{45}$$

input
```
integrate(t*(1+t)**(1/4),t)
```

output
```
4*t**2*(t + 1)**(1/4)/9 + 4*t*(t + 1)**(1/4)/45 - 16*(t + 1)**(1/4)/45
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{9}\,(t+1)^{\frac{9}{4}} - \frac{4}{5}\,(t+1)^{\frac{5}{4}}$$

input
```
integrate(t*(1+t)^(1/4),t, algorithm="maxima")
```

output
```
4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int t\sqrt[4]{1+t}\,dt = \frac{4}{9}\,(t+1)^{\frac{9}{4}} - \frac{4}{5}\,(t+1)^{\frac{5}{4}}$$

input
```
integrate(t*(1+t)^(1/4),t, algorithm="giac")
```

output
```
4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int t\sqrt[4]{1+t}\,dt = \frac{4\,(5\,t-4)\,\left(t+1\right)^{5/4}}{45}$$

input
```
int(t*(t + 1)^(1/4),t)
```

output
```
(4*(5*t - 4)*(t + 1)^(5/4))/45
```

3.16 $\int \frac{1}{(1+x^2)^{3/2}} dx$

3.16.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{1+x^2}}$$

output
```
x/(x^2+1)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{1+x^2}}$$

input
```
Integrate[(1 + x^2)^(-3/2),x]
```

output
```
x/Sqrt[1 + x^2]
```

3.16.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{\sqrt{x^2+1}}$$

input
```
Int[(1 + x^2)^(-3/2),x]
```

output
```
x/Sqrt[1 + x^2]
```

3.16.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

3.16.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{x}{\sqrt{x^2+1}}$	10
default	$\frac{x}{\sqrt{x^2+1}}$	10
trager	$\frac{x}{\sqrt{x^2+1}}$	10
meijerg	$\frac{x}{\sqrt{x^2+1}}$	10
risch	$\frac{x}{\sqrt{x^2+1}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x^2+1}}$	10

input
```
int(1/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+1)^(1/2)
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x^2+\sqrt{x^2+1}x+1}{x^2+1}$$

input
```
integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(1/(x**2+1)**(3/2),x)
```

output
```
x/sqrt(x**2 + 1)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
x/sqrt(x^2 + 1)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
x/sqrt(x^2 + 1)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
int(1/(x^2 + 1)^(3/2),x)
```

output
```
x/(x^2 + 1)^(1/2)
```

3.17 $\int x^2(27+8x^3)^{2/3}\,dx$

3.17.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int x^2(27+8x^3)^{2/3}\,dx=\frac{1}{40}(27+8x^3)^{5/3}$$

output
```
1/40*(8*x^3+27)^(5/3)
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x^2(27+8x^3)^{2/3}\,dx=\frac{1}{40}(27+8x^3)^{5/3}$$

input
```
Integrate[x^2*(27 + 8*x^3)^(2/3),x]
```

output
```
(27 + 8*x^3)^(5/3)/40
```

3.17.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(8x^3+27\right)^{2/3} dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

input
```
Int[x^2*(27 + 8*x^3)^(2/3),x]
```

output
```
(27 + 8*x^3)^(5/3)/40
```

3.17.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.17.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
default	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
risch	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
pseudoelliptic	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
meijerg	$3x^3{}_2F_1\left(-\frac{2}{3},1;2;-\frac{8x^3}{27}\right)$	17
trager	$\left(\frac{x^3}{5}+\frac{27}{40}\right)(8x^3+27)^{\frac{2}{3}}$	18
gosper	$\frac{(3+2x)(4x^2-6x+9)(8x^3+27)^{\frac{2}{3}}}{40}$	27

input
```
int(x^2*(8*x^3+27)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*x^3+27)^(5/3)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2(27+8x^3)^{2/3}\,dx=\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

input
```
integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="fricas")
```

output
```
1/40*(8*x^3 + 27)^(5/3)
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{x^3(8x^3+27)^{\frac{2}{3}}}{5} + \frac{27(8x^3+27)^{\frac{2}{3}}}{40}$$

input
```
integrate(x**2*(8*x**3+27)**(2/3),x)
```

output
```
x**3*(8*x**3 + 27)**(2/3)/5 + 27*(8*x**3 + 27)**(2/3)/40
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{1}{40} \left(8\,x^3+27\right)^{\frac{5}{3}}$$

input
```
integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="maxima")
```

output
```
1/40*(8*x^3 + 27)^(5/3)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{1}{40} \left(8\,x^3+27\right)^{\frac{5}{3}}$$

input
```
integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="giac")
```

output
```
1/40*(8*x^3 + 27)^(5/3)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{\left(8\,x^3+27\right)^{5/3}}{40}$$

input
```
int(x^2*(8*x^3 + 27)^(2/3),x)
```

output
```
(8*x^3 + 27)^(5/3)/40
```

3.18 $\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx$

3.18.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

output
```
3/2*(-cos(x)+sin(x))^(2/3)
```

3.18.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

input
```
Integrate[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

output
```
(3*(-Cos[x] + Sin[x])^(2/3))/2
```

3.18.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 3624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)+\cos(x)}{\sqrt[3]{\sin(x)-\cos(x)}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(x)+\cos(x)}{\sqrt[3]{\sin(x)-\cos(x)}}dx$$
$$\Big\downarrow \; 3624$$
$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

input
```
Int[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

output
```
(3*(-Cos[x] + Sin[x])^(2/3))/2
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3624
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[(c*B - b*C)*((b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(b
^2 + c^2))), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{3(-\cos(x)+\sin(x))^{\frac{2}{3}}}{2}$	12
default	$\frac{3(-\cos(x)+\sin(x))^{\frac{2}{3}}}{2}$	12
risch	$\frac{\left(-\frac{3}{2}-\frac{3i}{2}\right)\left((1+i)\left(-\mathrm{e}^{4ix}+i\mathrm{e}^{2ix}\right)\right)^{\frac{1}{3}}\left(\mathrm{e}^{ix}-i\mathrm{e}^{-ix}\right)}{(-8\cos(x)+8\sin(x))^{\frac{1}{3}}\left((-1-i)\left(\mathrm{e}^{4ix}-i\mathrm{e}^{2ix}\right)\right)^{\frac{1}{3}}}$	72

input
```
int((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(-cos(x)+sin(x))^(2/3)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

input
```
integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="fricas")
```

output
```
3/2*(-cos(x) + sin(x))^(2/3)
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3(\sin\left(x\right)-\cos\left(x\right))^{\frac{2}{3}}}{2}$$

input
```
integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
```

output
```
3*(sin(x) - cos(x))**(2/3)/2
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

input
```
integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="maxima")
```

output
```
3/2*(-cos(x) + sin(x))^(2/3)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

input
```
integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="giac")
```

output
```
3/2*(-cos(x) + sin(x))^(2/3)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3\,2^{1/3}\,\left(-\cos\left(x+\frac{\pi}{4}\right)\right)^{2/3}}{2}$$

input
```
int((cos(x) + sin(x))/(sin(x) - cos(x))^(1/3),x)
```

output
```
(3*2^(1/3)*(-cos(x + pi/4))^(2/3))/2
```

3.19 $\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}} \, dx$

3.19.1 Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}} \, dx = \frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

output
```
2*((x^2+1)*(1+(x^2+1)^(1/2)))^(1/2)/(x^2+1)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}} \, dx = \frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

input
```
Integrate[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

output
```
(2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

3.19.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7266, 7267, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2+(x^2+1)^{3/2}+1}}dx$$
$$\Big\downarrow \text{ 7266}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+(x^2+1)^{3/2}+1}}dx^2$$
$$\Big\downarrow \text{ 7267}$$
$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4\left(\sqrt{x^2+1}+1\right)}}d\sqrt{x^2+1}$$
$$\Big\downarrow \text{ 2021}$$
$$\frac{2\sqrt{x^4\left(\sqrt{x^2+1}+1\right)}}{x^2}$$

input
```
Int[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

output
```
(2*Sqrt[x^4*(1 + Sqrt[1 + x^2])])/x^2
```

3.19.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.19.4 Maple [F]

$$\int \frac{x}{\sqrt{1+x^2+(x^2+1)^{\frac{3}{2}}}}dx$$

input
```
int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

output
```
int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \frac{2\sqrt{x^2+(x^2+1)^{\frac{3}{2}}+1}}{\sqrt{x^2+1}}$$

input
```
integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x^2 + (x^2 + 1)^(3/2) + 1)/sqrt(x^2 + 1)
```

3.19.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \int \frac{x}{\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}\,dx$$

input
```
integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)
```

output
```
Integral(x/sqrt((x**2 + 1)*(sqrt(x**2 + 1) + 1)), x)
```

3.19.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \int \frac{x}{\sqrt{x^2+(x^2+1)^{\frac{3}{2}}+1}}\,dx$$

input
```
integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1), x)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}-2$$

input
```
integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(sqrt(x^2 + 1) + 1) - 2
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx = \frac{2\,(x^2+1)\,\sqrt{\sqrt{x^2+1}+1}}{\left(\sqrt{\sqrt{x^2+1}+1}+1\right)\,\sqrt{(x^2+1)^{3/2}+x^2+1}}$$

input
```
int(x/((x^2 + 1)^(3/2) + x^2 + 1)^(1/2),x)
```

output
```
(2*(x^2 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/((((x^2 + 1)^(1/2) + 1)^(1/2) +
1)*((x^2 + 1)^(3/2) + x^2 + 1)^(1/2))
```

3.20 $\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx$

3.20.1 Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{1+\sqrt{1+x^2}}$$

output
```
2*(1+(x^2+1)^(1/2))^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{1+\sqrt{1+x^2}}$$

input
```
Integrate[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

output
```
2*Sqrt[1 + Sqrt[1 + x^2]]
```

3.20.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}}dx$$

$$\Big\downarrow \text{7237}$$

$$2\sqrt{\sqrt{x^2+1}+1}$$

input
```
Int[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

output
```
2*Sqrt[1 + Sqrt[1 + x^2]]
```

3.20.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\sqrt{1+\sqrt{x^2+1}}$	14
default	$2\sqrt{1+\sqrt{x^2+1}}$	14

input
```
int(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1+(x^2+1)^(1/2))^(1/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

input
```
integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(sqrt(x^2 + 1) + 1)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

input
```
integrate(x/(x**2+1)**(1/2)/(1+(x**2+1)**(1/2))**(1/2),x)
```

output
```
2*sqrt(sqrt(x**2 + 1) + 1)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

input
```
integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(sqrt(x^2 + 1) + 1)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

input
```
integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(sqrt(x^2 + 1) + 1)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{\sqrt{x^2+1}+1}$$

input
```
int(x/((x^2 + 1)^(1/2)*((x^2 + 1)^(1/2) + 1)^(1/2)),x)
```

output
```
2*((x^2 + 1)^(1/2) + 1)^(1/2)
```

3.21 $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 16

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\sqrt[5]{1-2x+x^2}$$

output
```
-5/2*(x^2-2*x+1)^(1/5)
```

3.21.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\sqrt[5]{(-1+x)^2}$$

input
```
Integrate[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

output
```
(-5*((-1 + x)^2)^(1/5))/2
```

3.21.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[5]{x^2-2x+1}}{1-x}\,dx$$
$$\Big\downarrow \ 1099$$
$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

input
```
Int[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

output
```
(-5*(1 - 2*x + x^2)^(1/5))/2
```

3.21.3.1 Defintions of rubi rules used

rule 1099
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[e^m*((a + b*x + c*x^2)^(p + (m + 1)/2)/(c^((m + 1)/2)*(m + 2*
p + 1))), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[
2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

3.21.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{5\left(\left(-1+x\right)^{2}\right)^{\frac{1}{5}}}{2}$	10
pseudoelliptic	$-\frac{5\left(\left(-1+x\right)^{2}\right)^{\frac{1}{5}}}{2}$	10
gosper	$-\frac{5\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{2}$	13
trager	$-\frac{5\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{2}$	13
meijerg	$\frac{\operatorname{signum}\left(-1+x\right)^{\frac{2}{5}} x\, {}_{2}F_{1}\left(\frac{3}{5},1;2;x\right)}{\left(-\operatorname{signum}\left(-1+x\right)\right)^{\frac{2}{5}}}$	24

input
```
int((x^2-2*x+1)^(1/5)/(1-x),x,method=_RETURNVERBOSE)
```

output
```
-5/2*((-1+x)^2)^(1/5)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\left(x^{2}-2\,x+1\right)^{\frac{1}{5}}$$

input
```
integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="fricas")
```

output
```
-5/2*(x^2 - 2*x + 1)^(1/5)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5\sqrt[5]{x^2-2x+1}}{2}$$

input
```
integrate((x**2-2*x+1)**(1/5)/(1-x),x)
```

output
```
-5*(x**2 - 2*x + 1)**(1/5)/2
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\,(x-1)^{\frac{2}{5}}$$

input
```
integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="maxima")
```

output
```
-5/2*(x - 1)^(2/5)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5}{2}\,\left(x^2-2\,x+1\right)^{\frac{1}{5}}$$

input
```
integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="giac")
```

output
```
-5/2*(x^2 - 2*x + 1)^(1/5)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\frac{5\left((x-1)^2\right)^{1/5}}{2}$$

input
```
int(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x)
```

output
```
-(5*((x - 1)^2)^(1/5))/2
```

3.22 $\int x \sin(x) \, dx$

3.22.1 Optimal result

Integrand size = 4, antiderivative size = 8

$$\int x \sin(x) \, dx = -x \cos(x) + \sin(x)$$

output
```
-x*cos(x)+sin(x)
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x) \, dx = -x \cos(x) + \sin(x)$$

input
```
Integrate[x*Sin[x],x]
```

output
```
-(x*Cos[x]) + Sin[x]
```

3.22.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin(x)dx$$

$$\Big\downarrow \text{3777}$$

$$\int \cos(x)dx - x\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)dx - x\cos(x)$$

$$\Big\downarrow \text{3117}$$

$$\sin(x) - x\cos(x)$$

input
```
Int[x*Sin[x],x]
```

output
```
-(x*Cos[x]) + Sin[x]
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.22.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$-x\cos(x)+\sin(x)$	9
risch	$-x\cos(x)+\sin(x)$	9
parallelrisch	$-x\cos(x)+\sin(x)$	9
parts	$-x\cos(x)+\sin(x)$	9
meijerg	$2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	22
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	30

input
```
int(x*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-x*cos(x)+sin(x)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x)+\sin(x)$$

input
```
integrate(x*sin(x),x, algorithm="fricas")
```

output
```
-x*cos(x) + sin(x)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

input
```
integrate(x*sin(x),x)
```

output
```
-x*cos(x) + sin(x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

input
```
integrate(x*sin(x),x, algorithm="maxima")
```

output
```
-x*cos(x) + sin(x)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

input
```
integrate(x*sin(x),x, algorithm="giac")
```

output
```
-x*cos(x) + sin(x)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sin(x)\,dx = \sin(x) - x\,\cos(x)$$

input
```
int(x*sin(x),x)
```

output
```
sin(x) - x*cos(x)
```

3.23 $\int x^2 \sin(x) \, dx$

3.23.1 Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^2 \sin(x) \, dx = 2\cos(x) - x^2 \cos(x) + 2x \sin(x)$$

output
```
2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

3.23.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) \, dx = -\left(\left(-2 + x^2\right) \cos(x)\right) + 2x \sin(x)$$

input
```
Integrate[x^2*Sin[x],x]
```

output
```
-((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

3.23.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin(x)\, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^2 \sin(x) dx$$

$$\Big\downarrow \text{ 3777}$$

$$2 \int x \cos(x) dx - x^2 \cos(x)$$

$$\Big\downarrow \text{ 3042}$$

$$2 \int x \sin\left(x + \frac{\pi}{2}\right) dx - x^2 \cos(x)$$

$$\Big\downarrow \text{ 3777}$$

$$2(\int -\sin(x) dx + x \sin(x)) - x^2 \cos(x)$$

$$\Big\downarrow \text{ 25}$$

$$2(x \sin(x) - \int \sin(x) dx) - x^2 \cos(x)$$

$$\Big\downarrow \text{ 3042}$$

$$2(x \sin(x) - \int \sin(x) dx) - x^2 \cos(x)$$

$$\Big\downarrow \text{ 3118}$$

$$2(x \sin(x) + \cos(x)) - x^2 \cos(x)$$

input
```
Int[x^2*Sin[x],x]
```

output
```
-(x^2*Cos[x]) + 2*(Cos[x] + x*Sin[x])
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.23.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
risch	$(-x^2+2)\cos(x)+2x\sin(x)$	17
default	$2\cos(x)-x^2\cos(x)+2x\sin(x)$	18
parts	$2\cos(x)-x^2\cos(x)+2x\sin(x)$	18
parallelrisch	$-x^2\cos(x)+2x\sin(x)+2\cos(x)+2$	19
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2}{2}+1\right)\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)$	34
norman	$\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)-x^2+4x\tan\left(\frac{x}{2}\right)+4}{1+\tan^2\left(\frac{x}{2}\right)}$	36

input
```
int(x^2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
(-x^2+2)*cos(x)+2*x*sin(x)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) \, dx = -\left(x^2 - 2\right) \cos\left(x\right) + 2 \, x \sin\left(x\right)$$

input
```
integrate(x^2*sin(x),x, algorithm="fricas")
```

output
```
-(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \sin(x) \, dx = -x^2 \cos\left(x\right) + 2x \sin\left(x\right) + 2 \cos\left(x\right)$$

input
```
integrate(x**2*sin(x),x)
```

output
```
-x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) \, dx = -\left(x^2 - 2\right) \cos\left(x\right) + 2 \, x \sin\left(x\right)$$

input
```
integrate(x^2*sin(x),x, algorithm="maxima")
```

output
```
-(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\,dx = -\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

input
```
integrate(x^2*sin(x),x, algorithm="giac")
```

output
```
-(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x)\,dx = 2\,x\,\sin\left(x\right)-\cos\left(x\right)\,\left(x^2-2\right)$$

input
```
int(x^2*sin(x),x)
```

output
```
2*x*sin(x) - cos(x)*(x^2 - 2)
```

3.24 $\int x^3 \cos(x)\,dx$

3.24.1 Optimal result

Integrand size = 6, antiderivative size = 23

$$\int x^3 \cos(x)\,dx = -6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)$$

output
```
-6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

3.24.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^3 \cos(x)\,dx = 3(-2+x^2)\cos(x) + x(-6+x^2)\sin(x)$$

input
```
Integrate[x^3*Cos[x],x]
```

output
```
3*(-2 + x^2)*Cos[x] + x*(-6 + x^2)*Sin[x]
```

3.24.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.667$, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sin\left(x+\frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{3777}$$

$$3\int -x^2 \sin(x)dx + x^3 \sin(x)$$

$$\Big\downarrow \text{25}$$

$$x^3 \sin(x) - 3\int x^2 \sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x^3 \sin(x) - 3\int x^2 \sin(x)dx$$

$$\Big\downarrow \text{3777}$$

$$x^3 \sin(x) - 3\left(2\int x\cos(x)dx - x^2 \cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x^3 \sin(x) - 3\left(2\int x\sin\left(x+\frac{\pi}{2}\right) dx - x^2 \cos(x)\right)$$

$$\Big\downarrow \text{3777}$$

$$x^3 \sin(x) - 3\left(2(\int -\sin(x)dx + x\sin(x)) - x^2 \cos(x)\right)$$

$$\Big\downarrow \text{25}$$

$$x^3 \sin(x) - 3\left(2(x\sin(x) - \int \sin(x)dx) - x^2 \cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x^3\sin(x) - 3\left(2(x\sin(x) - \int \sin(x)dx) - x^2\cos(x)\right)$$

$$\Big\downarrow \text{3118}$$

$$x^3\sin(x) - 3(2(x\sin(x) + \cos(x)) - x^2\cos(x))$$

input
```
Int[x^3*Cos[x],x]
```

output
```
x^3*Sin[x] - 3*(-(x^2*Cos[x]) + 2*(Cos[x] + x*Sin[x]))
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.24.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
risch	$3(x^2-2)\cos(x)+x(x^2-6)\sin(x)$	20
parallelrisch	$(3x^2-6)\cos(x)-6+(x^3-6x)\sin(x)$	23
default	$-6\cos(x)+3x^2\cos(x)-6x\sin(x)+x^3\sin(x)$	24
parts	$-6\cos(x)+3x^2\cos(x)-6x\sin(x)+x^3\sin(x)$	24
meijerg	$8\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2}{2}+3\right)\cos(x)}{4\sqrt{\pi}}-\frac{x\left(-\frac{x^2}{2}+3\right)\sin(x)}{4\sqrt{\pi}}\right)$	41
norman	$\frac{3x^2-12x\tan\left(\frac{x}{2}\right)-3x^2\left(\tan^2\left(\frac{x}{2}\right)\right)+2x^3\tan\left(\frac{x}{2}\right)-12}{1+\tan^2\left(\frac{x}{2}\right)}$	46

input
```
int(x^3*cos(x),x,method=_RETURNVERBOSE)
```

output
```
3*(x^2-2)*cos(x)+x*(x^2-6)*sin(x)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^3\cos(x)\,dx = 3\left(x^2-2\right)\cos(x)+\left(x^3-6\,x\right)\sin(x)$$

input
```
integrate(x^3*cos(x),x, algorithm="fricas")
```

output
```
3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^3\cos(x)\,dx = x^3\sin(x)+3x^2\cos(x)-6x\sin(x)-6\cos(x)$$

input
```
integrate(x**3*cos(x),x)
```

output
```
x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^3 \cos(x)\,dx = 3\left(x^2-2\right)\cos\left(x\right) + \left(x^3-6\,x\right)\sin\left(x\right)$$

input
```
integrate(x^3*cos(x),x, algorithm="maxima")
```

output
```
3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^3 \cos(x)\,dx = 3\left(x^2-2\right)\cos\left(x\right) + \left(x^3-6\,x\right)\sin\left(x\right)$$

input
```
integrate(x^3*cos(x),x, algorithm="giac")
```

output
```
3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int x^3 \cos(x)\,dx = \cos\left(x\right)\,\left(3\,x^2-6\right) - \sin\left(x\right)\,\left(6\,x-x^3\right)$$

input
```
int(x^3*cos(x),x)
```

output
```
cos(x)*(3*x^2 - 6) - sin(x)*(6*x - x^3)
```

3.25 $\int x^3 \sin(x) \, dx$

3.25.1 Optimal result

Integrand size = 6, antiderivative size = 24

$$\int x^3 \sin(x) \, dx = 6x \cos(x) - x^3 \cos(x) - 6 \sin(x) + 3x^2 \sin(x)$$

output
```
6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

3.25.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x^3 \sin(x) \, dx = -x(-6 + x^2) \cos(x) + 3(-2 + x^2) \sin(x)$$

input
```
Integrate[x^3*Sin[x],x]
```

output
```
-(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

3.25.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sin(x)dx$$

$$\Big\downarrow \text{3777}$$

$$3\int x^2 \cos(x)dx - x^3 \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$3\int x^2 \sin\left(x+\frac{\pi}{2}\right)dx - x^3 \cos(x)$$

$$\Big\downarrow \text{3777}$$

$$3\left(2\int -x\sin(x)dx + x^2\sin(x)\right) - x^3\cos(x)$$

$$\Big\downarrow \text{25}$$

$$3\left(x^2\sin(x) - 2\int x\sin(x)dx\right) - x^3\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$3\left(x^2\sin(x) - 2\int x\sin(x)dx\right) - x^3\cos(x)$$

$$\Big\downarrow \text{3777}$$

$$3\left(x^2\sin(x) - 2(\int \cos(x)dx - x\cos(x))\right) - x^3\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$3\left(x^2\sin(x) - 2\left(\int \sin\left(x+\frac{\pi}{2}\right)dx - x\cos(x)\right)\right) - x^3\cos(x)$$

$$\Big\downarrow \text{3117}$$

$$3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x)$$

input
```
Int[x^3*Sin[x],x]
```

output
```
-(x^3*Cos[x]) + 3*(x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x]))
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
risch	$(-x^3+6x)\cos(x)+3(x^2-2)\sin(x)$	23
default	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
parallelrisch	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
parts	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
meijerg	$8\sqrt{\pi}\left(\frac{x\left(-\frac{5x^2}{2}+15\right)\cos(x)}{20\sqrt{\pi}}-\frac{\left(-\frac{15x^2}{2}+15\right)\sin(x)}{20\sqrt{\pi}}\right)$	36
norman	$\frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)+6x-x^3-6x\left(\tan^2\left(\frac{x}{2}\right)\right)+6x^2\tan\left(\frac{x}{2}\right)-12\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	55

input
```
int(x^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
(-x^3+6*x)*cos(x)+3*(x^2-2)*sin(x)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\, dx = -\left(x^3 - 6\, x\right) \cos\left(x\right) + 3\,\left(x^2 - 2\right) \sin\left(x\right)$$

input
```
integrate(x^3*sin(x),x, algorithm="fricas")
```

output
```
-(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^3 \sin(x)\, dx = -x^3 \cos\left(x\right) + 3x^2 \sin\left(x\right) + 6x \cos\left(x\right) - 6 \sin\left(x\right)$$

input
```
integrate(x**3*sin(x),x)
```

output
```
-x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\, dx = -\left(x^3 - 6\, x\right) \cos\left(x\right) + 3\,\left(x^2 - 2\right) \sin\left(x\right)$$

input
```
integrate(x^3*sin(x),x, algorithm="maxima")
```

output
```
-(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int x^3 \sin(x)\, dx = -\left(x^3 - 6\, x\right) \cos\left(x\right) + 3 \left(x^2 - 2\right) \sin\left(x\right)$$

input
```
integrate(x^3*sin(x),x, algorithm="giac")
```

output
```
-(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int x^3 \sin(x)\, dx = \cos\left(x\right)\, \left(6\, x - x^3\right) + \sin\left(x\right)\, \left(3\, x^2 - 6\right)$$

input
```
int(x^3*sin(x),x)
```

output
```
cos(x)*(6*x - x^3) + sin(x)*(3*x^2 - 6)
```

3.26 $\int \cos(x)\sin(x)\,dx$

3.26.1 Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

output
```
1/2*sin(x)^2
```

3.26.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos^2(x)$$

input
```
Integrate[Cos[x]*Sin[x],x]
```

output
```
-1/2*Cos[x]^2
```

3.26.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)\cos(x)dx$$

$$\Big\downarrow \; 3044$$

$$\int \sin(x)d\sin(x)$$

$$\Big\downarrow \; 15$$

$$\frac{\sin^2(x)}{2}$$

input
```
Int[Cos[x]*Sin[x],x]
```

output
```
Sin[x]^2/2
```

3.26.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.26.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$\frac{1}{4}-\frac{\cos(2x)}{4}$	9
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

input
```
int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)^2
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\,\cos{(x)}^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)^2
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

input
```
integrate(cos(x)*sin(x),x)
```

output
```
sin(x)**2/2
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos(x)^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*cos(x)^2
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos(x)^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*cos(x)^2
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = \frac{\sin(x)^2}{2}$$

input
```
int(cos(x)*sin(x),x)
```

output
```
sin(x)^2/2
```

3.27 $\int x\cos(x)\sin(x)\,dx$

3.27.1 Optimal result

Integrand size = 6, antiderivative size = 23

$$\int x\cos(x)\sin(x)\,dx=-\frac{x}{4}+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

output
```
-1/4*x+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

3.27.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\cos(x)\sin(x)\,dx=-\frac{1}{4}x\cos(2x)+\frac{1}{8}\sin(2x)$$

input
```
Integrate[x*Cos[x]*Sin[x],x]
```

output
```
-1/4*(x*Cos[2*x]) + Sin[2*x]/8
```

3.27.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3924, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(x)\cos(x)\,dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{1}{2}x\sin^2(x)-\frac{1}{2}\int \sin^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x\sin^2(x)-\frac{1}{2}\int \sin(x)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

input
```
Int[x*Cos[x]*Sin[x],x]
```

output
```
(x*Sin[x]^2)/2 + (-1/2*x + (Cos[x]*Sin[x])/2)/2
```

3.27.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.27.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{x\cos(2x)}{4}+\frac{\sin(2x)}{8}$	15
parallelrisch	$-\frac{x\cos(2x)}{4}+\frac{\sin(2x)}{8}$	15
default	$-\frac{(\cos^2(x))x}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}$	18
meijerg	$\frac{\sqrt{\pi}\left(-\frac{x\cos(2x)}{\sqrt{\pi}}+\frac{\sin(2x)}{2\sqrt{\pi}}\right)}{4}$	26
norman	$\frac{-\frac{x}{4}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	48

input
```
int(x*cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x*cos(2*x)+1/8*sin(2*x)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\cos(x)\sin(x)\,dx=-\frac{1}{2}\,x\cos{(x)}^2+\frac{1}{4}\,\cos{(x)}\sin{(x)}+\frac{1}{4}\,x$$

input
```
integrate(x*cos(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*x*cos(x)^2 + 1/4*cos(x)*sin(x) + 1/4*x
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int x\cos(x)\sin(x)\,dx = \frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

input
```
integrate(x*cos(x)*sin(x),x)
```

output
```
x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int x\cos(x)\sin(x)\,dx = -\frac{1}{4}\,x\cos(2\,x) + \frac{1}{8}\,\sin(2\,x)$$

input
```
integrate(x*cos(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/4*x*cos(2*x) + 1/8*sin(2*x)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int x\cos(x)\sin(x)\,dx = -\frac{1}{4}\,x\cos(2\,x) + \frac{1}{8}\,\sin(2\,x)$$

input
```
integrate(x*cos(x)*sin(x),x, algorithm="giac")
```

output
```
-1/4*x*cos(2*x) + 1/8*sin(2*x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\cos(x)\sin(x)\,dx = \frac{\sin(2\,x)}{8} + \frac{x\,\left(2\sin(x)^2 - 1\right)}{4}$$

input
```
int(x*cos(x)*sin(x),x)
```

output
```
sin(2*x)/8 + (x*(2*sin(x)^2 - 1))/4
```

3.28 $\int \sin^2(x)\,dx$

3.28.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.28.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

input
```
Integrate[Sin[x]^2,x]
```

output
```
x/2 - Sin[2*x]/4
```

3.28.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)^2 dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\int 1dx}{2} - \frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Sin[x]^2,x]
```

output
```
x/2 - (Cos[x]*Sin[x])/2
```

3.28.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.28.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\,\cos(x)\sin(x) + \frac{1}{2}\,x$$

input
```
integrate(sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*cos(x)*sin(x) + 1/2*x
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(sin(x)**2,x)
```

output
```
x/2 - sin(x)*cos(x)/2
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\, dx = \frac{1}{2}\, x - \frac{1}{4}\, \sin\left(2\, x\right)$$

input
```
integrate(sin(x)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\, dx = \frac{1}{2}\, x - \frac{1}{4}\, \sin\left(2\, x\right)$$

input
```
integrate(sin(x)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\, dx = \frac{x}{2} - \frac{\sin\left(2\, x\right)}{4}$$

input
```
int(sin(x)^2,x)
```

output
```
x/2 - sin(2*x)/4
```

3.29 $\int \sin^3(x)\,dx$

3.29.1 Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x)\,dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

output
```
-cos(x)+1/3*cos(x)^3
```

3.29.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x)\,dx = -\frac{3\cos(x)}{4} + \frac{1}{12}\cos(3x)$$

input
```
Integrate[Sin[x]^3,x]
```

output
```
(-3*Cos[x])/4 + Cos[3*x]/12
```

3.29.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(x)^3dx$$
$$\Big\downarrow \text{3113}$$
$$-\int \left(1-\cos^2(x)\right)d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\cos^3(x)}{3}-\cos(x)$$

input
```
Int[Sin[x]^3,x]
```

output
```
-Cos[x] + Cos[x]^3/3
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.29.4 Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
parallelrisch	$-\frac{2}{3}-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	13
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

input
```
int(sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2+sin(x)^2)*cos(x)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="fricas")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x)$$

input
```
integrate(sin(x)**3,x)
```

output
```
cos(x)**3/3 - cos(x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="maxima")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x)\,dx = \frac{1}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^3,x, algorithm="giac")
```

output
```
1/3*cos(x)^3 - cos(x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x)\,dx = \frac{\cos(x)\,\left(\cos(x)^2 - 3\right)}{3}$$

input
```
int(sin(x)^3,x)
```

output
```
(cos(x)*(cos(x)^2 - 3))/3
```

3.30 $\int \sin^4(x)\,dx$

3.30.1 Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \sin^4(x)\,dx=\frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)$$

output
```
3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

3.30.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \sin^4(x)\,dx=\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

input
```
Integrate[Sin[x]^4,x]
```

output
```
(3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

3.30.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(x)^4dx$$
$$\Big\downarrow \text{3115}$$
$$\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)$$
$$\Big\downarrow \text{3115}$$
$$\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)$$
$$\Big\downarrow \text{24}$$
$$\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)$$

input
```
Int[Sin[x]^4,x]
```

output
```
-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4
```

3.30.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.30.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	siz
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

input
```
int(sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \sin^4(x)\,dx = \frac{1}{8}\left(2\,\cos\left(x\right)^3 - 5\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{8}\,x$$

input
```
integrate(sin(x)^4,x, algorithm="fricas")
```

output
```
1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{\sin^3\left(x\right)\cos\left(x\right)}{4} - \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

input
```
integrate(sin(x)**4,x)
```

output
```
3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin(4\,x) - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3\,x}{8} - \frac{\sin(2\,x)}{4} + \frac{\sin(4\,x)}{32}$$

input
```
int(sin(x)^4,x)
```

output
```
(3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

3.31 $\int \sin^5(x)\,dx$

3.31.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \sin^5(x)\,dx = -\cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5}$$

output
```
-cos(x)+2/3*cos(x)^3-1/5*cos(x)^5
```

3.31.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^5(x)\,dx = -\frac{5\cos(x)}{8} + \frac{5}{48}\cos(3x) - \frac{1}{80}\cos(5x)$$

input
```
Integrate[Sin[x]^5,x]
```

output
```
(-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

3.31.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^5dx$$

$$\Big\downarrow \text{3113}$$

$$-\int \left(\cos^4(x)-2\cos^2(x)+1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

input
```
Int[Sin[x]^5,x]
```

output
```
-Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\left(\frac{8}{3}+\sin^4(x)+\frac{4\left(\sin^2(x)\right)}{3}\right)\cos(x)}{5}$	17
risch	$-\frac{5\cos(x)}{8}-\frac{\cos(5x)}{80}+\frac{5\cos(3x)}{48}$	18
parallelrisch	$\frac{8}{15}-\frac{5\cos(x)}{8}+\frac{5\cos(3x)}{48}-\frac{\cos(5x)}{80}$	19
norman	$\frac{-\frac{32\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{16\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{16}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5}$	30

input
```
int(sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^5,x, algorithm="fricas")
```

output
```
-1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{\cos^5(x)}{5} + \frac{2\cos^3(x)}{3} - \cos(x)$$

input
```
integrate(sin(x)**5,x)
```

output
```
-cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^5,x, algorithm="maxima")
```

output
```
-1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

input
```
integrate(sin(x)^5,x, algorithm="giac")
```

output
```
-1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin^5(x)\,dx = -\frac{\cos(x)^5}{5} + \frac{2\cos(x)^3}{3} - \cos(x)$$

input
```
int(sin(x)^5,x)
```

output
```
(2*cos(x)^3)/3 - cos(x) - cos(x)^5/5
```

3.32 $\int \sin^6(x)\,dx$

3.32.1 Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x)$$

output
```
5/16*x-5/16*cos(x)*sin(x)-5/24*cos(x)*sin(x)^3-1/6*cos(x)*sin(x)^5
```

3.32.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

input
```
Integrate[Sin[x]^6,x]
```

output
```
(5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

3.32.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)$$

input
```
Int[Sin[x]^6,x]
```

output
```
-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Si
n[x])/2))/4))/6
```

3.32.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.32.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
default	$-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{85\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{33\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{33\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{85\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{5\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

input
```
int(sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/16*x-1/192*sin(6*x)+3/64*sin(4*x)-15/64*sin(2*x)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \sin^6(x)\,dx = -\frac{1}{48}\left(8\cos(x)^5 - 26\cos(x)^3 + 33\cos(x)\right)\sin(x) + \frac{5}{16}\,x$$

input
```
integrate(sin(x)^6,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \sin^6(x)\,dx = \frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

input
```
integrate(sin(x)**6,x)
```

output
```
5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \sin^6(x)\,dx = \frac{1}{48}\sin(2x)^3 + \frac{5}{16}\,x + \frac{3}{64}\sin(4x) - \frac{1}{4}\sin(2x)$$

input
```
integrate(sin(x)^6,x, algorithm="maxima")
```

output
```
1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \sin^6(x)\,dx = \frac{5}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) - \frac{15}{64}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^6,x, algorithm="giac")
```

output
```
5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \sin^6(x)\,dx = \frac{5\,x}{16} - \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} - \frac{\sin\left(6\,x\right)}{192}$$

input
```
int(sin(x)^6,x)
```

output
```
(5*x)/16 - (15*sin(2*x))/64 + (3*sin(4*x))/64 - sin(6*x)/192
```

3.33 $\int x \sin^2(x) \, dx$

3.33.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int x \sin^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2} x \cos(x) \sin(x) + \frac{\sin^2(x)}{4}$$

output
```
1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

3.33.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x \sin^2(x) \, dx = \frac{x^2}{4} - \frac{1}{8} \cos(2x) - \frac{1}{4} x \sin(2x)$$

input
```
Integrate[x*Sin[x]^2,x]
```

output
```
x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

3.33.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^2(x) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int x \sin(x)^2 dx$$

$$\Big\downarrow \; 3791$$

$$\frac{\int x dx}{2} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x)$$

$$\Big\downarrow \; 15$$

$$\frac{x^2}{4} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x)$$

input
```
Int[x*Sin[x]^2,x]
```

output
```
x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

3.33.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.33.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^2}{4}-\frac{\cos(2x)}{8}-\frac{x\sin(2x)}{4}$	20
default	$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin^2(x))}{4}$	25
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x^2+1}{2\sqrt{\pi}}-\frac{\cos(2x)}{2\sqrt{\pi}}-\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{4}$	38
norman	$\frac{\tan^2\left(\frac{x}{2}\right)+\left(\tan^3\left(\frac{x}{2}\right)\right)x+\frac{x^2}{4}-x\tan\left(\frac{x}{2}\right)+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{x^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

input
```
int(x*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/8*cos(2*x)-1/4*x*sin(2*x)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx=-\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x^2-\frac{1}{4}\,\cos\left(x\right)^2$$

input
```
integrate(x*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x\sin^2(x)\,dx = \frac{x^2\sin^2(x)}{4} + \frac{x^2\cos^2(x)}{4} - \frac{x\sin(x)\cos(x)}{2} - \frac{\cos^2(x)}{4}$$

input
```
integrate(x*sin(x)**2,x)
```

output
```
x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 - cos(x)**2/4
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{1}{4}\,x^2 - \frac{1}{4}\,x\sin(2\,x) - \frac{1}{8}\,\cos(2\,x)$$

input
```
integrate(x*sin(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{1}{4}\,x^2 - \frac{1}{4}\,x\sin(2\,x) - \frac{1}{8}\,\cos(2\,x)$$

input
```
integrate(x*sin(x)^2,x, algorithm="giac")
```

output
```
1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\sin^2(x)\,dx = \frac{\sin(x)^2}{4} - \frac{x\,\sin(2\,x)}{4} + \frac{x^2}{4}$$

input
```
int(x*sin(x)^2,x)
```

output
```
sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

3.34 $\int x \sin^3(x) \, dx$

3.34.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int x \sin^3(x) \, dx = -\frac{2}{3} x \cos(x) + \frac{2 \sin(x)}{3} - \frac{1}{3} x \cos(x) \sin^2(x) + \frac{\sin^3(x)}{9}$$

output
```
-2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

3.34.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int x \sin^3(x) \, dx = -\frac{3}{4} x \cos(x) + \frac{1}{12} x \cos(3x) + \frac{3 \sin(x)}{4} - \frac{1}{36} \sin(3x)$$

input
```
Integrate[x*Sin[x]^3,x]
```

output
```
(-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

3.34.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^3(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x \sin(x)^3 dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{2}{3} \int x \sin(x) dx + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3} \int x \sin(x) dx + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3} (\int \cos(x) dx - x \cos(x)) + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3} \left(\int \sin\left(x + \frac{\pi}{2}\right) dx - x \cos(x) \right) + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x)$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) + \frac{2}{3} (\sin(x) - x \cos(x))$$

input
```
Int[x*Sin[x]^3,x]
```

output
```
-1/3*(x*Cos[x]*Sin[x]^2) + Sin[x]^3/9 + (2*(-(x*Cos[x]) + Sin[x]))/3
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.34.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{x(2+\sin^2(x))\cos(x)}{3}+\frac{(\sin^3(x))}{9}+\frac{2\sin(x)}{3}$	23
risch	$-\frac{3x\cos(x)}{4}+\frac{3\sin(x)}{4}+\frac{x\cos(3x)}{12}-\frac{\sin(3x)}{36}$	24
parallelrisch	$-\frac{3x\cos(x)}{4}+\frac{3\sin(x)}{4}+\frac{x\cos(3x)}{12}-\frac{\sin(3x)}{36}$	24
norman	$\frac{-\frac{2x}{3}+\frac{32\left(\tan^3\left(\frac{x}{2}\right)\right)}{9}+\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)+2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	65

input
```
int(x*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{3}\,x\cos{(x)}^3 - x\cos{(x)} - \frac{1}{9}\,\left(\cos{(x)}^2 - 7\right)\sin{(x)}$$

input
```
integrate(x*sin(x)^3,x, algorithm="fricas")
```

output
```
1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x\sin^3(x)\,dx = -x\sin^2{(x)}\cos{(x)} - \frac{2x\cos^3{(x)}}{3} + \frac{7\sin^3{(x)}}{9} + \frac{2\sin{(x)}\cos^2{(x)}}{3}$$

input
```
integrate(x*sin(x)**3,x)
```

output
```
-x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2
/3
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{12}\,x\cos{(3\,x)} - \frac{3}{4}\,x\cos{(x)} - \frac{1}{36}\,\sin{(3\,x)} + \frac{3}{4}\,\sin{(x)}$$

input
```
integrate(x*sin(x)^3,x, algorithm="maxima")
```

output
```
1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x\sin^3(x)\,dx = \frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos\left(x\right) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin\left(x\right)$$

input
```
integrate(x*sin(x)^3,x, algorithm="giac")
```

output
```
1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\sin^3(x)\,dx = \frac{x\cos\left(x\right)^3}{3} - \frac{\sin\left(x\right)\,\cos\left(x\right)^2}{9} - x\,\cos\left(x\right) + \frac{7\,\sin\left(x\right)}{9}$$

input
```
int(x*sin(x)^3,x)
```

output
```
(7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

3.35 $\int x^2 \sin^2(x)\,dx$

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^2 \sin^2(x)\,dx = -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x)$$

output
```
-1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

3.35.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(x)\,dx = \frac{1}{24}\left(4x^3 - 6x\cos(2x) + \left(3 - 6x^2\right)\sin(2x)\right)$$

input
```
Integrate[x^2*Sin[x]^2,x]
```

output
```
(4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

3.35.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin(x)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{\int x^2 dx}{2} - \frac{1}{2}\int \sin^2(x)dx - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}\int \sin^2(x)dx + \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sin(x)^2 dx + \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x) - \frac{\int 1dx}{2}\right) + \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) + \frac{1}{2}x\sin^2(x) + \frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x) - \frac{x}{2}\right)$$

input
```
Int[x^2*Sin[x]^2,x]
```

output
```
x^3/6 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2 + (-1/2*x + (Cos[x]*Sin[x])
/2)/2
```

3.35.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.35.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

method	result	size
meijerg	$\frac{x^{5} {}_{2}F_{3}\left(1,\frac{5}{2};\frac{3}{2},2,\frac{7}{2};-x^{2}\right)}{5}$	19
risch	$\frac{x^{3}}{6}-\frac{x\cos\left(2x\right)}{4}-\frac{\left(2x^{2}-1\right)\sin\left(2x\right)}{8}$	27
default	$x^{2}\left(\frac{x}{2}-\frac{\cos\left(x\right)\sin\left(x\right)}{2}\right)-\frac{\left(\cos^{2}\left(x\right)\right)x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{4}+\frac{x}{4}-\frac{x^{3}}{3}$	37
norman	$\frac{x^{2}\left(\tan^{3}\left(\frac{x}{2}\right)\right)-\frac{x}{4}+\frac{x^{3}}{6}-\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}-x^{2}\tan\left(\frac{x}{2}\right)+\frac{x^{3}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{x^{3}\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{6}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	94

input
```
int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1,5/2],[3/2,2,7/2],-x^2)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(x)\,dx = \frac{1}{6}\,x^3 - \frac{1}{2}\,x\cos\left(x\right)^2 - \frac{1}{4}\,\left(2\,x^2 - 1\right)\cos\left(x\right)\sin\left(x\right) + \frac{1}{4}\,x$$

input
```
integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

output
```
1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int x^2 \sin^2(x)\,dx = \frac{x^3\sin^2\left(x\right)}{6} + \frac{x^3\cos^2\left(x\right)}{6} - \frac{x^2\sin\left(x\right)\cos\left(x\right)}{2} + \frac{x\sin^2\left(x\right)}{4} - \frac{x\cos^2\left(x\right)}{4} + \frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

input
```
integrate(x**2*sin(x)**2,x)
```

output
```
x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2\sin^2(x)\,dx = \frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\,\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

input
```
integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

output
```
1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2\sin^2(x)\,dx = \frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\,\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

input
```
integrate(x^2*sin(x)^2,x, algorithm="giac")
```

output
```
1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2\sin^2(x)\,dx = \frac{\sin\left(2\,x\right)}{8} - \frac{x\,\cos\left(2\,x\right)}{4} - \frac{x^2\,\sin\left(2\,x\right)}{4} + \frac{x^3}{6}$$

input
```
int(x^2*sin(x)^2,x)
```

output
```
sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

3.36 $\int \cos^2(x)\,dx$

3.36.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.36.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

3.36.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.36.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.36.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

input
```
integrate(cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(cos(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin\left(2\,x\right)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

3.37 $\int \cos^3(x)\,dx$

3.37.1 Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

output
```
sin(x)-1/3*sin(x)^3
```

3.37.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

input
```
Integrate[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

3.37.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{3113}$$

$$-\int \left(1-\sin^2(x)\right) d(-\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\sin(x)-\frac{\sin^3(x)}{3}$$

input
```
Int[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12
parallelrisch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12

input
```
int(cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2+cos(x)^2)*sin(x)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x)\,dx = \frac{1}{3}\left(\cos\left(x\right)^2+2\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*sin(x)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x)\,dx = -\frac{\sin^3\left(x\right)}{3}+\sin\left(x\right)$$

input
```
integrate(cos(x)**3,x)
```

output
```
-sin(x)**3/3 + sin(x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="maxima")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="giac")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin(x)^3}{3}$$

input
```
int(cos(x)^3,x)
```

output
```
sin(x) - sin(x)^3/3
```

3.38 $\int \cos^4(x)\,dx$

3.38.1 Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{3}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x)$$

output
```
3/8*x+3/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)
```

3.38.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

input
```
Integrate[Cos[x]^4,x]
```

output
```
(3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

3.38.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[Cos[x]^4,x]
```

output
```
(Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4
```

3.38.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.38.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x}{8}+\frac{\sin\left(4x\right)}{32}+\frac{\sin\left(2x\right)}{4}$	17
parallelrisch	$\frac{3x}{8}+\frac{\sin\left(4x\right)}{32}+\frac{\sin\left(2x\right)}{4}$	17
default	$\frac{\left(\cos^{3}\left(x\right)+\frac{3\cos\left(x\right)}{2}\right)\sin\left(x\right)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{8}+\frac{5\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{4}}$	82

input
```
int(cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*x+1/32*sin(4*x)+1/4*sin(2*x)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^4(x)\,dx = \frac{1}{8}\left(2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{3}{8}x$$

input
```
integrate(cos(x)^4,x, algorithm="fricas")
```

output
```
1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos^4(x)\,dx = \frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

input
```
integrate(cos(x)**4,x)
```

output
```
3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin(4\,x) + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\,dx = \frac{3\,x}{8} + \frac{\sin(2\,x)}{4} + \frac{\sin(4\,x)}{32}$$

input
```
int(cos(x)^4,x)
```

output
```
(3*x)/8 + sin(2*x)/4 + sin(4*x)/32
```

3.39 $\int (a^2 - x^2)^{5/2} \, dx$

3.39.1 Optimal result

Integrand size = 13, antiderivative size = 84

$$\int (a^2 - x^2)^{5/2} \, dx = \frac{5}{16} a^4 x \sqrt{a^2 - x^2} + \frac{5}{24} a^2 x (a^2 - x^2)^{3/2} + \frac{1}{6} x (a^2 - x^2)^{5/2} + \frac{5}{16} a^6 \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

output
```
5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(x/(a^2-x^
2)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int (a^2 - x^2)^{5/2} \, dx = \frac{1}{48} \sqrt{a^2 - x^2} (33 a^4 x - 26 a^2 x^3 + 8 x^5) + \frac{5}{16} a^6 \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

input
```
Integrate[(a^2 - x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 - x^2]*(33*a^4*x - 26*a^2*x^3 + 8*x^5))/48 + (5*a^6*ArcTan[x/Sqr
t[a^2 - x^2]])/16
```

3.39.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-x^2\right)^{5/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a^2\int \left(a^2-x^2\right)^{3/2} dx+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a^2\left(\frac{3}{4}a^2\int \sqrt{a^2-x^2}dx+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{a^2-x^2}}dx+\frac{1}{2}x\sqrt{a^2-x^2}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{\frac{x^2}{a^2-x^2}+1}d\frac{x}{\sqrt{a^2-x^2}}+\frac{1}{2}x\sqrt{a^2-x^2}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{6}a^2\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{1}{2}x\sqrt{a^2-x^2}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}$$

input
```
Int[(a^2 - x^2)^(5/2),x]
```

output
```
(x*(a^2 - x^2)^(5/2))/6 + (5*a^2*((x*(a^2 - x^2)^(3/2))/4 + (3*a^2*((x*Sqr
t[a^2 - x^2])/2 + (a^2*ArcTan[x/Sqrt[a^2 - x^2]])/2))/4))/6
```

3.39.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.39.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{x\left(33a^4-26a^2x^2+8x^4\right)\sqrt{a^2-x^2}}{48}+\frac{5a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{16}$	54
pseudoelliptic	$-\frac{5\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)a^6}{16}+\frac{11\sqrt{a^2-x^2}\left(a^4-\frac{26}{33}a^2x^2+\frac{8}{33}x^4\right)x}{16}$	54
default	$\frac{x\left(a^2-x^2\right)^{\frac{5}{2}}}{6}+\frac{5a^2\left(\frac{\left(a^2-x^2\right)^{\frac{3}{2}}x}{4}+\frac{3a^2\left(\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{2}\right)}{4}\right)}{6}$	75

input
```
int((a^2-x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(33*a^4-26*a^2*x^2+8*x^4)*(a^2-x^2)^(1/2)+5/16*a^6*arctan(x/(a^2-x^
2)^(1/2))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \left(a^2-x^2\right)^{5/2} dx = -\frac{5}{8}\,a^6\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\frac{1}{48}\left(33\,a^4x-26\,a^2x^3+8\,x^5\right)\sqrt{a^2-x^2}$$

input
```
integrate((a^2-x^2)^(5/2),x, algorithm="fricas")
```

output
```
-5/8*a^6*arctan(-(a - sqrt(a^2 - x^2))/x) + 1/48*(33*a^4*x - 26*a^2*x^3 +
8*x^5)*sqrt(a^2 - x^2)
```

3.39.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.14

$$\int \left(a^2-x^2\right)^{5/2} dx = \begin{cases} -\frac{5ia^6\operatorname{acosh}\left(\frac{x}{a}\right)}{16}-\frac{11ia^5x}{16\sqrt{-1+\frac{x^2}{a^2}}}+\frac{59ia^3x^3}{48\sqrt{-1+\frac{x^2}{a^2}}}-\frac{17iax^5}{24\sqrt{-1+\frac{x^2}{a^2}}}+\frac{ix^7}{6a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right|>1 \\ \frac{5a^6\operatorname{asin}\left(\frac{x}{a}\right)}{16}+\frac{11a^5x\sqrt{1-\frac{x^2}{a^2}}}{16}-\frac{13a^3x^3\sqrt{1-\frac{x^2}{a^2}}}{24}+\frac{ax^5\sqrt{1-\frac{x^2}{a^2}}}{6} & \text{otherwise} \end{cases}$$

input
```
integrate((a**2-x**2)**(5/2),x)
```

output
```
Piecewise((-5*I*a**6*acosh(x/a)/16 - 11*I*a**5*x/(16*sqrt(-1 + x**2/a**2))
 + 59*I*a**3*x**3/(48*sqrt(-1 + x**2/a**2)) - 17*I*a*x**5/(24*sqrt(-1 + x*
*2/a**2)) + I*x**7/(6*a*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1), (5*a**
6*asin(x/a)/16 + 11*a**5*x*sqrt(1 - x**2/a**2)/16 - 13*a**3*x**3*sqrt(1 -
x**2/a**2)/24 + a*x**5*sqrt(1 - x**2/a**2)/6, True))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int (a^2-x^2)^{5/2}\, dx = \frac{5}{16}\, a^6 \arcsin\left(\frac{x}{a}\right) + \frac{5}{16}\, \sqrt{a^2-x^2} a^4 x + \frac{5}{24}\, (a^2-x^2)^{\frac{3}{2}} a^2 x + \frac{1}{6}\, (a^2-x^2)^{\frac{5}{2}} x$$

input
```
integrate((a^2-x^2)^(5/2),x, algorithm="maxima")
```

output
```
5/16*a^6*arcsin(x/a) + 5/16*sqrt(a^2 - x^2)*a^4*x + 5/24*(a^2 - x^2)^(3/2)
*a^2*x + 1/6*(a^2 - x^2)^(5/2)*x
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int (a^2-x^2)^{5/2}\, dx = \frac{5}{16}\, a^6 \arcsin\left(\frac{x}{a}\right) \operatorname{sgn}(a) + \frac{1}{48}\, \left(33\, a^4 - 2\, \left(13\, a^2 - 4\, x^2\right) x^2\right) \sqrt{a^2-x^2} x$$

input
```
integrate((a^2-x^2)^(5/2),x, algorithm="giac")
```

output
```
5/16*a^6*arcsin(x/a)*sgn(a) + 1/48*(33*a^4 - 2*(13*a^2 - 4*x^2)*x^2)*sqrt(
a^2 - x^2)*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int (a^2-x^2)^{5/2}\, dx = \frac{x\, (a^2-x^2)^{5/2}\, {}_2\mathrm{F}_1\left(-\frac{5}{2}, \frac{1}{2};\ \frac{3}{2};\ \frac{x^2}{a^2}\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}$$

input
```
int((a^2 - x^2)^(5/2),x)
```

output
```
(x*(a^2 - x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, x^2/a^2))/(1 - x^2/a^2)^(
5/2)
```

3.40 $\int \frac{x^5}{\sqrt{5+x^2}}\,dx$

3.40.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = 25\sqrt{5+x^2}-\frac{10}{3}\left(5+x^2\right)^{3/2}+\frac{1}{5}\left(5+x^2\right)^{5/2}$$

output
```
-10/3*(x^2+5)^(3/2)+1/5*(x^2+5)^(5/2)+25*(x^2+5)^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{15}\sqrt{5+x^2}\left(200-20x^2+3x^4\right)$$

input
```
Integrate[x^5/Sqrt[5 + x^2],x]
```

output
```
(Sqrt[5 + x^2]*(200 - 20*x^2 + 3*x^4))/15
```

3.40.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{x^2+5}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{x^2+5}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\left(x^2+5\right)^{3/2}-10\sqrt{x^2+5}+\frac{25}{\sqrt{x^2+5}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(x^2+5\right)^{5/2}-\frac{20}{3}\left(x^2+5\right)^{3/2}+50\sqrt{x^2+5}\right)$$

input
```
Int[x^5/Sqrt[5 + x^2],x]
```

output
```
(50*Sqrt[5 + x^2] - (20*(5 + x^2)^(3/2))/3 + (2*(5 + x^2)^(5/2))/5)/2
```

3.40.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.40.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

method	result	size
trager	$\sqrt{x^{2}+5}\,\left(\frac{1}{5}x^{4}-\frac{4}{3}x^{2}+\frac{40}{3}\right)$	21
gosper	$\frac{\sqrt{x^{2}+5}\,\left(3x^{4}-20x^{2}+200\right)}{15}$	22
risch	$\frac{\sqrt{x^{2}+5}\,\left(3x^{4}-20x^{2}+200\right)}{15}$	22
pseudoelliptic	$\frac{\sqrt{x^{2}+5}\,\left(3x^{4}-20x^{2}+200\right)}{15}$	22
default	$\frac{x^{4}\sqrt{x^{2}+5}}{5}-\frac{4x^{2}\sqrt{x^{2}+5}}{3}+\frac{40\sqrt{x^{2}+5}}{3}$	35
meijerg	$\frac{25\sqrt{5}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{6}{25}x^{4}-\frac{8}{5}x^{2}+16\right)\sqrt{1+\frac{x^{2}}{5}}}{15}\right)}{2\sqrt{\pi}}$	41

input
```
int(x^5/(x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+5)^(1/2)*(1/5*x^4-4/3*x^2+40/3)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{15}\left(3\,x^4-20\,x^2+200\right)\sqrt{x^2+5}$$

input
```
integrate(x^5/(x^2+5)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^4 - 20*x^2 + 200)*sqrt(x^2 + 5)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{x^4\sqrt{x^2+5}}{5} - \frac{4x^2\sqrt{x^2+5}}{3} + \frac{40\sqrt{x^2+5}}{3}$$

input
```
integrate(x**5/(x**2+5)**(1/2),x)
```

output
```
x**4*sqrt(x**2 + 5)/5 - 4*x**2*sqrt(x**2 + 5)/3 + 40*sqrt(x**2 + 5)/3
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{5}\,\sqrt{x^2+5}x^4 - \frac{4}{3}\,\sqrt{x^2+5}x^2 + \frac{40}{3}\,\sqrt{x^2+5}$$

input
```
integrate(x^5/(x^2+5)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(x^2 + 5)*x^4 - 4/3*sqrt(x^2 + 5)*x^2 + 40/3*sqrt(x^2 + 5)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \frac{1}{5}\,\left(x^2+5\right)^{\frac{5}{2}} - \frac{10}{3}\,\left(x^2+5\right)^{\frac{3}{2}} + 25\,\sqrt{x^2+5}$$

input
```
integrate(x^5/(x^2+5)^(1/2),x, algorithm="giac")
```

output
```
1/5*(x^2 + 5)^(5/2) - 10/3*(x^2 + 5)^(3/2) + 25*sqrt(x^2 + 5)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{x^5}{\sqrt{5+x^2}}\,dx = \sqrt{x^2+5}\left(\frac{x^4}{5}-\frac{4\,x^2}{3}+\frac{40}{3}\right)$$

input
```
int(x^5/(x^2 + 5)^(1/2),x)
```

output
```
(x^2 + 5)^(1/2)*(x^4/5 - (4*x^2)/3 + 40/3)
```

3.41 $\int \frac{t^3}{\sqrt{4+t^3}} \, dt$

3.41.1 Optimal result

Integrand size = 13, antiderivative size = 172

$$\int \frac{t^3}{\sqrt{4+t^3}} \, dt = \frac{2}{5}t\sqrt{4+t^3} - \frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}(2^{2/3}+t)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{2^{2/3}\left(1-\sqrt{3}\right)+t}{2^{2/3}\left(1+\sqrt{3}\right)+t}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\sqrt{4+t^3}}$$

output
```
2/5*t*(t^3+4)^(1/2)-8/15*2^(2/3)*(2^(2/3)+t)*EllipticF((t+2^(2/3)*(1-3^(1/
2)))/(t+2^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((2*
2^(1/3)-2^(2/3)*t+t^2)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(t^3+4)^(1
/2)/((2^(2/3)+t)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.20

$$\int \frac{t^3}{\sqrt{4+t^3}} \, dt = \frac{2}{5}t\left(\sqrt{4+t^3} - 2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{t^3}{4}\right)\right)$$

input
```
Integrate[t^3/Sqrt[4 + t^3],t]
```

output
```
(2*t*(Sqrt[4 + t^3] - 2*Hypergeometric2F1[1/3, 1/2, 4/3, -1/4*t^3]))/5
```

3.41.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{t^3}{\sqrt{t^3+4}}dt$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8}{5}\int \frac{1}{\sqrt{t^3+4}}dt$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{t^3+4}}$$

input
```
Int[t^3/Sqrt[4 + t^3],t]
```

output
```
(2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t)*Sqrt[(2
*2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*EllipticF[ArcSi
n[(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)], -7 - 4*Sqrt[3]
])/(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t
^3])
```

3.41.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.41.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.10

method	result
meijerg	$\frac{t^{4}{}_{2}F_{1}\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};-\frac{t^{3}}{4}\right)}{8}$
default	$\frac{2t\sqrt{t^{3}+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\,F\left(\frac{\sqrt{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2}}{6}\right.}{15\sqrt{t^{3}+4}}$
risch	$\frac{2t\sqrt{t^{3}+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\,F\left(\frac{\sqrt{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2}}{6}\right.}{15\sqrt{t^{3}+4}}$
elliptic	$\frac{2t\sqrt{t^{3}+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\,F\left(\frac{\sqrt{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2}}{6}\right.}{15\sqrt{t^{3}+4}}$

input
```
int(t^3/(t^3+4)^(1/2),t,method=_RETURNVERBOSE)
```

output
```
1/8*t^4*hypergeom([1/2,4/3],[7/3],-1/4*t^3)
```

3.41.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.10

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}\sqrt{t^3+4}t - \frac{16}{5}\,\text{weierstrassPInverse}(0,-16,t)$$

input
```
integrate(t^3/(t^3+4)^(1/2),t, algorithm="fricas")
```

output
```
2/5*sqrt(t^3 + 4)*t - 16/5*weierstrassPInverse(0, -16, t)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.18

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{t^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{t^3e^{i\pi}}{4}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(t**3/(t**3+4)**(1/2),t)
```

output
```
t**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), t**3*exp_polar(I*pi)/4)/(6*gamma
(7/3))
```

3.41.7 Maxima [F]

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

input
```
integrate(t^3/(t^3+4)^(1/2),t, algorithm="maxima")
```

output
```
integrate(t^3/sqrt(t^3 + 4), t)
```

3.41.8 Giac [F]

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

input
```
integrate(t^3/(t^3+4)^(1/2),t, algorithm="giac")
```

output
```
integrate(t^3/sqrt(t^3 + 4), t)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.75

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2\,t\,\sqrt{t^3+4}}{5} - \frac{16\,\sqrt{-\frac{t-2^{2/3}\,(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})}{2^{2/3}+2^{2/3}\,(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})}}\,\sqrt{-\frac{t+2^{2/3}\,(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})}{2^{2/3}-2^{2/3}\,(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})}}\,\sqrt{\frac{t+2^{2/3}}{2^{2/3}+2^{2/3}\,(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})}}\,(2^{2/3}+2^{2/3}\,(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{5\,\sqrt{t^3+(2^{2/3}+2^{2/3}\,(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})-2^{2/3}\,(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}))\,t^2+(2\,2^{1/3}\,(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2})-2\,2^{1/3}\,(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}$$

input
```
int(t^3/(t^3 + 4)^(1/2),t)
```

output
```
(2*t*(t^3 + 4)^(1/2))/5 - (16*(-(t - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2
/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(-(t + 2^(2/3)*((3^(1/2)*1i)/
2 - 1/2))/(2^(2/3) - 2^(2/3)*((3^(1/2)*1i)/2 - 1/2)))^(1/2)*((t + 2^(2/3))
/(2^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(2^(2/3) + 2^(2/3)*((3^
(1/2)*1i)/2 + 1/2))*ellipticF(asin(((t + 2^(2/3))/(2^(2/3) + 2^(2/3)*((3^(
1/2)*1i)/2 + 1/2)))^(1/2)), (2^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^
(2/3) - 2^(2/3)*((3^(1/2)*1i)/2 - 1/2))))/(5*(t^2*(2^(2/3) + 2^(2/3)*((3^(
1/2)*1i)/2 - 1/2) - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)) - 4*((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) + t^3 - t*(2*2^(1/3)*((3^(1/2)*1i)/2 + 1/2) -
2*2^(1/3)*((3^(1/2)*1i)/2 - 1/2) + 2*2^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2)))^(1/2))
```

3.42 $\int \tan^2(x)\,dx$

3.42.1 Optimal result

Integrand size = 4, antiderivative size = 6

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

output
```
-x+tan(x)
```

3.42.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \tan^2(x)\,dx = -\arctan(\tan(x)) + \tan(x)$$

input
```
Integrate[Tan[x]^2,x]
```

output
```
-ArcTan[Tan[x]] + Tan[x]
```

3.42.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\tan(x)-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\tan(x)-x$$

input
```
Int[Tan[x]^2,x]
```

output
```
-x + Tan[x]
```

3.42.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.42.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
norman	$-x+\tan\left(x\right)$	7
parallelrisch	$-x+\tan\left(x\right)$	7
derivativedivides	$\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	9
default	$\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	9
risch	$-x+\frac{2i}{{\mathrm e}^{2ix}+1}$	17

input
```
int(tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(x)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx=-x+\tan\left(x\right)$$

input
```
integrate(tan(x)^2,x, algorithm="fricas")
```

output
```
-x + tan(x)
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \tan^2(x)\,dx=-x+\frac{\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
integrate(tan(x)**2,x)
```

output
```
-x + sin(x)/cos(x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

input
```
integrate(tan(x)^2,x, algorithm="maxima")
```

output
```
-x + tan(x)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

input
```
integrate(tan(x)^2,x, algorithm="giac")
```

output
```
-x + tan(x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = \tan(x) - x$$

input
```
int(tan(x)^2,x)
```

output
```
tan(x) - x
```

3.43 $\int \tan^4(x)\,dx$

3.43.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tan^4(x)\,dx = x - \tan(x) + \frac{\tan^3(x)}{3}$$

output
```
x-tan(x)+1/3*tan(x)^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tan^4(x)\,dx = \arctan(\tan(x)) - \tan(x) + \frac{\tan^3(x)}{3}$$

input
```
Integrate[Tan[x]^4,x]
```

output
```
ArcTan[Tan[x]] - Tan[x] + Tan[x]^3/3
```

3.43.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^3(x)}{3}-\int \tan^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(x)}{3}-\int \tan(x)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx+\frac{\tan^3(x)}{3}-\tan(x)$$

$$\Big\downarrow \text{24}$$

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

input
```
Int[Tan[x]^4,x]
```

output
```
x - Tan[x] + Tan[x]^3/3
```

3.43.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.43.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
norman	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
parallelrisch	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
derivativedivides	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
default	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
risch	$x-\frac{4i\left(3\,{\rm e}^{4ix}+3\,{\rm e}^{2ix}+2\right)}{3\left({\rm e}^{2ix}+1\right)^{3}}$	31

input
```
int(tan(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tan(x)+1/3*tan(x)^3
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\, dx = \frac{1}{3} \tan{(x)}^3 + x - \tan{(x)}$$

input
```
integrate(tan(x)^4,x, algorithm="fricas")
```

output
```
1/3*tan(x)^3 + x - tan(x)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \tan^4(x)\, dx = x + \frac{\sin^3{(x)}}{3\cos^3{(x)}} - \frac{\sin{(x)}}{\cos{(x)}}$$

input
```
integrate(tan(x)**4,x)
```

output
```
x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\, dx = \frac{1}{3} \tan{(x)}^3 + x - \tan{(x)}$$

input
```
integrate(tan(x)^4,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3 + x - tan(x)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan{(x)}^3 + x - \tan{(x)}$$

input
```
integrate(tan(x)^4,x, algorithm="giac")
```

output
```
1/3*tan(x)^3 + x - tan(x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{\tan(x)^3}{3} - \tan(x) + x$$

input
```
int(tan(x)^4,x)
```

output
```
x - tan(x) + tan(x)^3/3
```

3.44 $\int \cot^2(x)\,dx$

3.44.1 Optimal result

Integrand size = 4, antiderivative size = 8

$$\int \cot^2(x)\,dx = -x - \cot(x)$$

output
```
-x-cot(x)
```

3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cot^2(x)\,dx = -\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)$$

input
```
Integrate[Cot[x]^2,x]
```

output
```
-(Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2])
```

3.44.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx-\cot(x)$$

$$\Big\downarrow \text{24}$$

$$-x-\cot(x)$$

input
```
Int[Cot[x]^2,x]
```

output
```
-x - Cot[x]
```

3.44.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.44.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

method	result	size
norman	$\frac{-1-x\tan(x)}{\tan(x)}$	13
parallelrisch	$\frac{-1-x\tan(x)}{\tan(x)}$	13
derivativedivides	$-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))$	14
default	$-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))$	14
risch	$-x-\frac{2i}{e^{2ix}-1}$	17

input
```
int(cot(x)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1-x*tan(x))/tan(x)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \cot^2(x)\,dx = -\frac{x\sin(2x)+\cos(2x)+1}{\sin(2x)}$$

input
```
integrate(cot(x)^2,x, algorithm="fricas")
```

output
```
-(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot^2(x)\,dx = -x-\frac{\cos(x)}{\sin(x)}$$

input
```
integrate(cot(x)**2,x)
```

output
```
-x - cos(x)/sin(x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cot^2(x)\,dx = -x - \frac{1}{\tan(x)}$$

input
```
integrate(cot(x)^2,x, algorithm="maxima")
```

output
```
-x - 1/tan(x)
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cot^2(x)\,dx = -x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(cot(x)^2,x, algorithm="giac")
```

output
```
-x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot^2(x)\,dx = -x - \cot(x)$$

input
```
int(cot(x)^2,x)
```

output
```
- x - cot(x)
```

3.45 $\int \cot^4(x) \, dx$

3.45.1 Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \cot^4(x) \, dx = x + \cot(x) - \frac{\cot^3(x)}{3}$$

output
```
x+cot(x)-1/3*cot(x)^3
```

3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \cot^4(x) \, dx = -\frac{1}{3} \cot^3(x) \, \text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -\tan^2(x)\right)$$

input
```
Integrate[Cot[x]^4,x]
```

output
```
-1/3*(Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x]^2])
```

3.45.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3954}$$

$$-\int \cot^2(x)dx-\frac{1}{3}\cot^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^2 dx-\frac{1}{3}\cot^3(x)$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{1}{3}\cot^3(x)+\cot(x)$$

$$\Big\downarrow \text{24}$$

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

input
```
Int[Cot[x]^4,x]
```

output
```
x + Cot[x] - Cot[x]^3/3
```

3.45.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.45.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x+\cot \left(x \right)-\frac{\left(\cot^{3}\left(x \right)\right)}{3}$	11
derivativedivides	$-\frac{\left(\cot^{3}\left(x \right)\right)}{3}+\cot \left(x \right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot \left(x \right)\right)$	16
default	$-\frac{\left(\cot^{3}\left(x \right)\right)}{3}+\cot \left(x \right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot \left(x \right)\right)$	16
norman	$\frac{-\frac{1}{3}+\tan^{2}\left(x \right)+x \left(\tan^{3}\left(x \right)\right)}{\tan \left(x \right)^{3}}$	18
risch	$x+\frac{4 i \left(3 \,{\mathrm e}^{4 i x}-3 \,{\mathrm e}^{2 i x}+2\right)}{3 \left({\mathrm e}^{2 i x}-1\right)^{3}}$	31

input
```
int(cot(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x+cot(x)-1/3*cot(x)^3
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \cot^4(x)\,dx = \frac{4\,\cos\left(2\,x\right)^2 + 3\left(x\cos\left(2\,x\right) - x\right)\sin\left(2\,x\right) + 2\,\cos\left(2\,x\right) - 2}{3\left(\cos\left(2\,x\right) - 1\right)\sin\left(2\,x\right)}$$

input
```
integrate(cot(x)^4,x, algorithm="fricas")
```

output
```
1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*
x) - 1)*sin(2*x))
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cot^4(x)\,dx = x + \frac{\cos\left(x\right)}{\sin\left(x\right)} - \frac{\cos^3\left(x\right)}{3\sin^3\left(x\right)}$$

input
```
integrate(cot(x)**4,x)
```

output
```
x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot^4(x)\,dx = x + \frac{3\,\tan\left(x\right)^2 - 1}{3\,\tan\left(x\right)^3}$$

input
```
integrate(cot(x)^4,x, algorithm="maxima")
```

output
```
x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \cot^4(x)\,dx = \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(cot(x)^4,x, algorithm="giac")
```

output
```
1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(
1/2*x)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x)\,dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

input
```
int(cot(x)^4,x)
```

output
```
x + cot(x) - cot(x)^3/3
```

3.46 $\int (2+3x)\sin(5x)\,dx$

3.46.1 Optimal result

Integrand size = 10, antiderivative size = 22

$$\int (2+3x)\sin(5x)\,dx = -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{25}\sin(5x)$$

output
```
-1/5*(2+3*x)*cos(5*x)+3/25*sin(5*x)
```

3.46.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (2+3x)\sin(5x)\,dx = -\frac{2}{5}\cos(5x)-\frac{3}{5}x\cos(5x)+\frac{3}{25}\sin(5x)$$

input
```
Integrate[(2 + 3*x)*Sin[5*x],x]
```

output
```
(-2*Cos[5*x])/5 - (3*x*Cos[5*x])/5 + (3*Sin[5*x])/25
```

3.46.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)\sin(5x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (3x+2)\sin(5x)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{3}{5}\int \cos(5x)dx-\frac{1}{5}(3x+2)\cos(5x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{5}\int \sin\left(5x+\frac{\pi}{2}\right)dx-\frac{1}{5}(3x+2)\cos(5x)$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

input
```
Int[(2 + 3*x)*Sin[5*x],x]
```

output
```
-1/5*((2 + 3*x)*Cos[5*x]) + (3*Sin[5*x])/25
```

3.46.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
risch	$\left(-\frac{2}{5}-\frac{3x}{5}\right)\cos\left(5x\right)+\frac{3\sin(5x)}{25}$	18
derivativedivides	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3x\cos(5x)}{5}$	21
default	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3x\cos(5x)}{5}$	21
parts	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3x\cos(5x)}{5}$	21
norman	$\frac{-\frac{3x}{5}+\frac{3x\left(\tan^2\left(\frac{5x}{2}\right)\right)}{5}+\frac{6\tan\left(\frac{5x}{2}\right)}{25}-\frac{4}{5}}{1+\tan^2\left(\frac{5x}{2}\right)}$	32
parallelrisch	$\frac{15x\left(\tan^2\left(\frac{5x}{2}\right)\right)-20-15x+6\tan\left(\frac{5x}{2}\right)}{25\left(\tan^2\left(\frac{5x}{2}\right)\right)+25}$	34
meijerg	$\frac{2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(5x)}{\sqrt{\pi}}\right)}{5}+\frac{6\sqrt{\pi}\left(-\frac{5x\cos(5x)}{2\sqrt{\pi}}+\frac{\sin(5x)}{2\sqrt{\pi}}\right)}{25}$	45

input
```
int((2+3*x)*sin(5*x),x,method=_RETURNVERBOSE)
```

output
```
(-2/5-3/5*x)*cos(5*x)+3/25*sin(5*x)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (2+3x)\sin(5x)\,dx = -\frac{1}{5}\left(3\,x+2\right)\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

input
```
integrate((2+3*x)*sin(5*x),x, algorithm="fricas")
```

output
```
-1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (2+3x)\sin(5x)\,dx = -\frac{3x\cos(5x)}{5} + \frac{3\sin(5x)}{25} - \frac{2\cos(5x)}{5}$$

input
```
integrate((2+3*x)*sin(5*x),x)
```

output
```
-3*x*cos(5*x)/5 + 3*sin(5*x)/25 - 2*cos(5*x)/5
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (2+3x)\sin(5x)\,dx = -\frac{3}{5}\,x\cos(5\,x) - \frac{2}{5}\,\cos(5\,x) + \frac{3}{25}\,\sin(5\,x)$$

input
```
integrate((2+3*x)*sin(5*x),x, algorithm="maxima")
```

output
```
-3/5*x*cos(5*x) - 2/5*cos(5*x) + 3/25*sin(5*x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (2+3x)\sin(5x)\,dx = -\frac{1}{5}\,(3\,x+2)\cos(5\,x) + \frac{3}{25}\,\sin(5\,x)$$

input
```
integrate((2+3*x)*sin(5*x),x, algorithm="giac")
```

output
```
-1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (2+3x)\sin(5x)\,dx = \frac{3\,\sin\left(5\,x\right)}{25} - \frac{2\,\cos\left(5\,x\right)}{5} - \frac{3\,x\,\cos\left(5\,x\right)}{5}$$

input
```
int(sin(5*x)*(3*x + 2),x)
```

output
```
(3*sin(5*x))/25 - (2*cos(5*x))/5 - (3*x*cos(5*x))/5
```

3.47 $\int x\sqrt{1+x^2}\,dx$

3.47.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

output
```
1/3*(x^2+1)^(3/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[1 + x^2],x]
```

output
```
(1 + x^2)^(3/2)/3
```

3.47.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

input
```
Int[x*Sqrt[1 + x^2],x]
```

output
```
(1 + x^2)^(3/2)/3
```

3.47.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.47.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
default	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^2+2\right)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

input
```
int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+1)^(3/2)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int x\sqrt{1+x^2}\,dx = \frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

input
```
integrate(x*(x**2+1)**(1/2),x)
```

output
```
x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{(x^2+1)^{3/2}}{3}$$

input
```
int(x*(x^2 + 1)^(1/2),x)
```

output
```
(x^2 + 1)^(3/2)/3
```

3.48 $\int x(-1+x^2)^9\,dx$

3.48.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int x(-1+x^2)^9\,dx=\frac{1}{20}(1-x^2)^{10}$$

output
```
1/20*(-x^2+1)^10
```

3.48.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int x(-1+x^2)^9\,dx=\frac{1}{20}(-1+x^2)^{10}$$

input
```
Integrate[x*(-1 + x^2)^9,x]
```

output
```
(-1 + x^2)^10/20
```

3.48.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x^2-1)^9\,dx$$
$$\Big\downarrow \text{241}$$
$$\frac{1}{20}(1-x^2)^{10}$$

input
```
Int[x*(-1 + x^2)^9,x]
```

output
```
(1 - x^2)^10/20
```

3.48.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.48.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{(x^2-1)^{10}}{20}$	10
gosper	$\frac{x^2(x^{18}-10x^{16}+45x^{14}-120x^{12}+210x^{10}-252x^8+210x^6-120x^4+45x^2-10)}{20}$	51
norman	$-\frac{1}{2}x^2+\frac{9}{4}x^4-6x^6+\frac{21}{2}x^8-\frac{63}{5}x^{10}+\frac{21}{2}x^{12}-6x^{14}+\frac{9}{4}x^{16}-\frac{1}{2}x^{18}+\frac{1}{20}x^{20}$	52
parallelrisch	$-\frac{1}{2}x^2+\frac{9}{4}x^4-6x^6+\frac{21}{2}x^8-\frac{63}{5}x^{10}+\frac{21}{2}x^{12}-6x^{14}+\frac{9}{4}x^{16}-\frac{1}{2}x^{18}+\frac{1}{20}x^{20}$	52
risch	$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^8-6x^6+\frac{9}{4}x^4-\frac{1}{2}x^2+\frac{1}{20}$	53

input
```
int(x*(x^2-1)^9,x,method=_RETURNVERBOSE)
```

output
```
1/20*(x^2-1)^10
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(9) = 18$.

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.92

$$\int x(-1+x^2)^9\,dx = \frac{1}{20}\,x^{20} - \frac{1}{2}\,x^{18} + \frac{9}{4}\,x^{16} - 6\,x^{14} + \frac{21}{2}\,x^{12} - \frac{63}{5}\,x^{10} + \frac{21}{2}\,x^{8} - 6\,x^{6} + \frac{9}{4}\,x^{4} - \frac{1}{2}\,x^{2}$$

input
```
integrate(x*(x^2-1)^9,x, algorithm="fricas")
```

output
```
1/20*x^20 - 1/2*x^18 + 9/4*x^16 - 6*x^14 + 21/2*x^12 - 63/5*x^10 + 21/2*x^
8 - 6*x^6 + 9/4*x^4 - 1/2*x^2
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(7) = 14$.

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

$$\int x(-1+x^2)^9\,dx = \frac{x^{20}}{20} - \frac{x^{18}}{2} + \frac{9x^{16}}{4} - 6x^{14} + \frac{21x^{12}}{2} - \frac{63x^{10}}{5} + \frac{21x^{8}}{2} - 6x^{6} + \frac{9x^{4}}{4} - \frac{x^{2}}{2}$$

input
```
integrate(x*(x**2-1)**9,x)
```

output
```
x**20/20 - x**18/2 + 9*x**16/4 - 6*x**14 + 21*x**12/2 - 63*x**10/5 + 21*x*
*8/2 - 6*x**6 + 9*x**4/4 - x**2/2
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x(-1+x^2)^9\,dx = \frac{1}{20}\left(x^2-1\right)^{10}$$

input
```
integrate(x*(x^2-1)^9,x, algorithm="maxima")
```

output
```
1/20*(x^2 - 1)^10
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x(-1+x^2)^9\,dx = \frac{1}{20}\left(x^2-1\right)^{10}$$

input
```
integrate(x*(x^2-1)^9,x, algorithm="giac")
```

output
```
1/20*(x^2 - 1)^10
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x(-1+x^2)^9\,dx = \frac{\left(x^2-1\right)^{10}}{20}$$

input
```
int(x*(x^2 - 1)^9,x)
```

output
```
(x^2 - 1)^10/20
```

3.49 $\int \frac{3+2x}{(7+6x)^3} \, dx$

3.49.1 Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{3+2x}{(7+6x)^3} \, dx = -\frac{(3+2x)^2}{8(7+6x)^2}$$

output
```
-1/8*(3+2*x)^2/(7+6*x)^2
```

3.49.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{3+2x}{(7+6x)^3} \, dx = -\frac{4+3x}{9(7+6x)^2}$$

input
```
Integrate[(3 + 2*x)/(7 + 6*x)^3,x]
```

output
```
-1/9*(4 + 3*x)/(7 + 6*x)^2
```

3.49.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(6x+7)^3}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

input
```
Int[(3 + 2*x)/(7 + 6*x)^3,x]
```

output
```
-1/8*(3 + 2*x)^2/(7 + 6*x)^2
```

3.49.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

3.49.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
norman	$\frac{-\frac{x}{3}-\frac{4}{9}}{(7+6x)^2}$	14
gosper	$-\frac{3x+4}{9(7+6x)^2}$	15
risch	$\frac{-\frac{x}{3}-\frac{4}{9}}{(7+6x)^2}$	15
parallelrisch	$\frac{-12x-16}{36(7+6x)^2}$	15
default	$-\frac{1}{18(7+6x)}-\frac{1}{18(7+6x)^2}$	20
meijerg	$\frac{3x\left(\frac{6x}{7}+2\right)}{686\left(1+\frac{6x}{7}\right)^2}+\frac{x^2}{343\left(1+\frac{6x}{7}\right)^2}$	29

input
```
int((3+2*x)/(7+6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(7+6*x)^2*(-1/3*x-4/9)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(36\,x^2+84\,x+49)}$$

input
```
integrate((3+2*x)/(7+6*x)^3,x, algorithm="fricas")
```

output
```
-1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(7+6x)^3}\,dx = \frac{-3x-4}{324x^2+756x+441}$$

input
```
integrate((3+2*x)/(7+6*x)**3,x)
```

output
```
(-3*x - 4)/(324*x**2 + 756*x + 441)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(36\,x^2+84\,x+49)}$$

input
```
integrate((3+2*x)/(7+6*x)^3,x, algorithm="maxima")
```

output
```
-1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{3\,x+4}{9\,(6\,x+7)^2}$$

input
```
integrate((3+2*x)/(7+6*x)^3,x, algorithm="giac")
```

output
```
-1/9*(3*x + 4)/(6*x + 7)^2
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{(7+6x)^3} \, dx = -\frac{3\,x+4}{9\,\left(6\,x+7\right)^2}$$

input
```
int((2*x + 3)/(6*x + 7)^3,x)
```

output
```
-(3*x + 4)/(9*(6*x + 7)^2)
```

3.50 $\int x^4(1+x^5)^5 \, dx$

3.50.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int x^4(1+x^5)^5 \, dx = \frac{1}{30}(1+x^5)^6$$

output
```
1/30*(x^5+1)^6
```

3.50.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.91

$$\int x^4(1+x^5)^5 \, dx = \frac{x^5}{5} + \frac{x^{10}}{2} + \frac{2x^{15}}{3} + \frac{x^{20}}{2} + \frac{x^{25}}{5} + \frac{x^{30}}{30}$$

input
```
Integrate[x^4*(1 + x^5)^5,x]
```

output
```
x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

3.50.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(x^5+1\right)^5 \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{1}{30} \left(x^5+1\right)^6$$

input
```
Int[x^4*(1 + x^5)^5,x]
```

output
```
(1 + x^5)^6/30
```

3.50.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\left(x^{5}+1\right)^{6}}{30}$	10
gosper	$\frac{x^{5} \left(x^{25}+6 x^{20}+15 x^{15}+20 x^{10}+15 x^{5}+6\right)}{30}$	31
norman	$\frac{1}{5} x^{25}+\frac{1}{30} x^{30}+\frac{1}{2} x^{10}+\frac{2}{3} x^{15}+\frac{1}{2} x^{20}+\frac{1}{5} x^{5}$	32
parallelrisch	$\frac{1}{5} x^{25}+\frac{1}{30} x^{30}+\frac{1}{2} x^{10}+\frac{2}{3} x^{15}+\frac{1}{2} x^{20}+\frac{1}{5} x^{5}$	32
risch	$\frac{1}{30} x^{30}+\frac{1}{5} x^{25}+\frac{1}{2} x^{20}+\frac{2}{3} x^{15}+\frac{1}{2} x^{10}+\frac{1}{5} x^{5}+\frac{1}{30}$	33

input
```
int(x^4*(x^5+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/30*(x^5+1)^6
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int x^4 (1 + x^5)^5 \, dx = \frac{1}{30} x^{30} + \frac{1}{5} x^{25} + \frac{1}{2} x^{20} + \frac{2}{3} x^{15} + \frac{1}{2} x^{10} + \frac{1}{5} x^5$$

input
```
integrate(x^4*(x^5+1)^5,x, algorithm="fricas")
```

output
```
1/30*x^30 + 1/5*x^25 + 1/2*x^20 + 2/3*x^15 + 1/2*x^10 + 1/5*x^5
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(7) = 14$.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int x^4 (1 + x^5)^5 \, dx = \frac{x^{30}}{30} + \frac{x^{25}}{5} + \frac{x^{20}}{2} + \frac{2x^{15}}{3} + \frac{x^{10}}{2} + \frac{x^5}{5}$$

input
```
integrate(x**4*(x**5+1)**5,x)
```

output
```
x**30/30 + x**25/5 + x**20/2 + 2*x**15/3 + x**10/2 + x**5/5
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x^4 (1 + x^5)^5 \, dx = \frac{1}{30} (x^5 + 1)^6$$

input
```
integrate(x^4*(x^5+1)^5,x, algorithm="maxima")
```

output
```
1/30*(x^5 + 1)^6
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x^4(1+x^5)^5\,dx = \frac{1}{30}\,(x^5+1)^6$$

input
```
integrate(x^4*(x^5+1)^5,x, algorithm="giac")
```

output
```
1/30*(x^5 + 1)^6
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int x^4(1+x^5)^5\,dx = \frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2\,x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

input
```
int(x^4*(x^5 + 1)^5,x)
```

output
```
x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

3.51 $\int (1-x)^{20} x^4 \, dx$

3.51.1 Optimal result

Integrand size = 11, antiderivative size = 56

$$\int (1-x)^{20} x^4 \, dx = -\frac{1}{21}(1-x)^{21} + \frac{2}{11}(1-x)^{22} - \frac{6}{23}(1-x)^{23} + \frac{1}{6}(1-x)^{24} - \frac{1}{25}(1-x)^{25}$$

output
```
-1/21*(1-x)^21+2/11*(1-x)^22-6/23*(1-x)^23+1/6*(1-x)^24-1/25*(1-x)^25
```

3.51.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.

Time = 0.00 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.50

$$\int (1-x)^{20} x^4 \, dx = \frac{x^5}{5} - \frac{10x^6}{3} + \frac{190x^7}{7} - \frac{285x^8}{2} + \frac{1615x^9}{3} - \frac{7752x^{10}}{5} + \frac{38760x^{11}}{11} - 6460x^{12} + 9690x^{13} - \frac{83980x^{14}}{7} + \frac{184756x^{15}}{15} - \frac{20995x^{16}}{2} + 7410x^{17} - \frac{12920x^{18}}{3} + 2040x^{19} - \frac{3876x^{20}}{5} + \frac{1615x^{21}}{7} - \frac{570x^{22}}{11} + \frac{190x^{23}}{23} - \frac{5x^{24}}{6} + \frac{x^{25}}{25}$$

input
```
Integrate[(1 - x)^20*x^4,x]
```

output
```
x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10
)/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x
^15)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*
x^20)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^2
5/25
```

3.51.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{20}x^4\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left((1-x)^{24}-4(1-x)^{23}+6(1-x)^{22}-4(1-x)^{21}+(1-x)^{20}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

input
```
Int[(1 - x)^20*x^4,x]
```

output
```
-1/21*(1 - x)^21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^24/6 -
(1 - x)^25/25
```

3.51.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.51.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(46) = 92.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

method	result
gosper	$\frac{x^5(10626x^{20}-221375x^{19}+2194500x^{18}-13765500x^{17}+61289250x^{16}-205931880x^{15}+541926000x^{14}-1144066000x^{13}+19684}{}$
default	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}-$
norman	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}-$
risch	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}-$
parallelrisch	$\frac{1}{5}x^5-\frac{10}{3}x^6+\frac{190}{7}x^7-\frac{285}{2}x^8+\frac{1615}{3}x^9-\frac{7752}{5}x^{10}+\frac{38760}{11}x^{11}-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}-$

input
```
int((1-x)^20*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/265650*x^5*(10626*x^20-221375*x^19+2194500*x^18-13765500*x^17+61289250*x
^16-205931880*x^15+541926000*x^14-1144066000*x^13+1968466500*x^12-27886608
75*x^11+3272028760*x^10-3187041000*x^9+2574148500*x^8-1716099000*x^7+93605
4000*x^6-411863760*x^5+143008250*x^4-37855125*x^3+7210500*x^2-885500*x+531
30)
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(36) = 72.

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\begin{aligned}\int (1-x)^{20}x^4\,dx = {} & \frac{1}{25}\,x^{25}-\frac{5}{6}\,x^{24}+\frac{190}{23}\,x^{23}-\frac{570}{11}\,x^{22}+\frac{1615}{7}\,x^{21}-\frac{3876}{5}\,x^{20} \\ & +2040\,x^{19}-\frac{12920}{3}\,x^{18}+7410\,x^{17}-\frac{20995}{2}\,x^{16} \\ & +\frac{184756}{15}\,x^{15}-\frac{83980}{7}\,x^{14}+9690\,x^{13}-6460\,x^{12}+\frac{38760}{11}\,x^{11} \\ & -\frac{7752}{5}\,x^{10}+\frac{1615}{3}\,x^{9}-\frac{285}{2}\,x^{8}+\frac{190}{7}\,x^{7}-\frac{10}{3}\,x^{6}+\frac{1}{5}\,x^{5}\end{aligned}$$

input
```
integrate((1-x)^20*x^4,x, algorithm="fricas")
```

output
```
1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^
20 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15
- 83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 161
5/3*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.34

$$\int (1-x)^{20}x^4\,dx = \frac{x^{25}}{25} - \frac{5x^{24}}{6} + \frac{190x^{23}}{23} - \frac{570x^{22}}{11} + \frac{1615x^{21}}{7} - \frac{3876x^{20}}{5} + 2040x^{19} - \frac{12920x^{18}}{3} + 7410x^{17} - \frac{20995x^{16}}{2} + \frac{184756x^{15}}{15} - \frac{83980x^{14}}{7} + 9690x^{13} - 6460x^{12} + \frac{38760x^{11}}{11} - \frac{7752x^{10}}{5} + \frac{1615x^{9}}{3} - \frac{285x^{8}}{2} + \frac{190x^{7}}{7} - \frac{10x^{6}}{3} + \frac{x^{5}}{5}$$

input
```
integrate((1-x)**20*x**4,x)
```

output
```
x**25/25 - 5*x**24/6 + 190*x**23/23 - 570*x**22/11 + 1615*x**21/7 - 3876*x
**20/5 + 2040*x**19 - 12920*x**18/3 + 7410*x**17 - 20995*x**16/2 + 184756*
x**15/15 - 83980*x**14/7 + 9690*x**13 - 6460*x**12 + 38760*x**11/11 - 7752
*x**10/5 + 1615*x**9/3 - 285*x**8/2 + 190*x**7/7 - 10*x**6/3 + x**5/5
```

3.51.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\int (1-x)^{20}x^4\,dx = \frac{1}{25}\,x^{25} - \frac{5}{6}\,x^{24} + \frac{190}{23}\,x^{23} - \frac{570}{11}\,x^{22} + \frac{1615}{7}\,x^{21} - \frac{3876}{5}\,x^{20} + 2040\,x^{19} - \frac{12920}{3}\,x^{18} + 7410\,x^{17} - \frac{20995}{2}\,x^{16} + \frac{184756}{15}\,x^{15} - \frac{83980}{7}\,x^{14} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760}{11}\,x^{11} - \frac{7752}{5}\,x^{10} + \frac{1615}{3}\,x^{9} - \frac{285}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{10}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

input
```
integrate((1-x)^20*x^4,x, algorithm="maxima")
```

output
```
1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^
20 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15
- 83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 161
5/3*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\begin{aligned}\int (1-x)^{20}x^4\,dx = {} & \frac{1}{25}\,x^{25} - \frac{5}{6}\,x^{24} + \frac{190}{23}\,x^{23} - \frac{570}{11}\,x^{22} + \frac{1615}{7}\,x^{21} - \frac{3876}{5}\,x^{20} \\ & + 2040\,x^{19} - \frac{12920}{3}\,x^{18} + 7410\,x^{17} - \frac{20995}{2}\,x^{16} \\ & + \frac{184756}{15}\,x^{15} - \frac{83980}{7}\,x^{14} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760}{11}\,x^{11} \\ & - \frac{7752}{5}\,x^{10} + \frac{1615}{3}\,x^{9} - \frac{285}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{10}{3}\,x^{6} + \frac{1}{5}\,x^{5}\end{aligned}$$

input
```
integrate((1-x)^20*x^4,x, algorithm="giac")
```

output
```
1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^
20 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15
- 83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 161
5/3*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.89

$$\begin{aligned}\int (1-x)^{20}x^4\,dx = {} & \frac{x^{25}}{25} - \frac{5\,x^{24}}{6} + \frac{190\,x^{23}}{23} - \frac{570\,x^{22}}{11} + \frac{1615\,x^{21}}{7} - \frac{3876\,x^{20}}{5} + 2040\,x^{19} \\ & - \frac{12920\,x^{18}}{3} + 7410\,x^{17} - \frac{20995\,x^{16}}{2} + \frac{184756\,x^{15}}{15} - \frac{83980\,x^{14}}{7} + 9690\,x^{13} \\ & - 6460\,x^{12} + \frac{38760\,x^{11}}{11} - \frac{7752\,x^{10}}{5} + \frac{1615\,x^{9}}{3} - \frac{285\,x^{8}}{2} + \frac{190\,x^{7}}{7} - \frac{10\,x^{6}}{3} \\ & + \frac{x^{5}}{5}\end{aligned}$$

input
```
int(x^4*(x - 1)^20,x)
```

output
```
x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10
)/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x
^15)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*
x^20)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^2
5/25
```

3.51. $\int (1-x)^{20}x^4\,dx$

3.52 $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

3.52.1 Optimal result

Integrand size = 8, antiderivative size = 4

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

output
```
cos(1/x)
```

3.52.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

input
```
Integrate[Sin[x^(-1)]/x^2,x]
```

output
```
Cos[x^(-1)]
```

3.52.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx$$
$$\Big\downarrow \text{3860}$$
$$-\int \sin\left(\frac{1}{x}\right)d\frac{1}{x}$$
$$\Big\downarrow \text{3042}$$
$$-\int \sin\left(\frac{1}{x}\right)d\frac{1}{x}$$
$$\Big\downarrow \text{3118}$$
$$\cos\left(\frac{1}{x}\right)$$

input
```
Int[Sin[x^(-1)]/x^2,x]
```

output
```
Cos[x^(-1)]
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.52.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\cos\left(\frac{1}{x}\right)$	5
default	$\cos\left(\frac{1}{x}\right)$	5
risch	$\cos\left(\frac{1}{x}\right)$	5
parallelrisch	$1+\cos\left(\frac{1}{x}\right)$	7
norman	$\frac{2}{1+\tan^2\left(\frac{1}{2x}\right)}$	15
meijerg	$-\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{1}{x}\right)}{\sqrt{\pi}}\right)$	19

input
```
int(sin(1/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
cos(1/x)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx=\cos\left(\frac{1}{x}\right)$$

input
```
integrate(sin(1/x)/x^2,x, algorithm="fricas")
```

output
```
cos(1/x)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

input
```
integrate(sin(1/x)/x**2,x)
```

output
```
cos(1/x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

input
```
integrate(sin(1/x)/x^2,x, algorithm="maxima")
```

output
```
cos(1/x)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

input
```
integrate(sin(1/x)/x^2,x, algorithm="giac")
```

output
```
cos(1/x)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \cos\left(\frac{1}{x}\right)$$

input
```
int(sin(1/x)/x^2,x)
```

output
```
cos(1/x)
```

3.53 $\int \sin\left(\sqrt[4]{-1+x}\right)\,dx$

3.53.1 Optimal result

Integrand size = 8, antiderivative size = 62

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right) - 4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right) - 24\sin\left(\sqrt[4]{-1+x}\right) + 12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)$$

output
```
24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin(
(-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = -4\left(-6+\sqrt{-1+x}\right)\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right) + 12\left(-2+\sqrt{-1+x}\right)\sin\left(\sqrt[4]{-1+x}\right)$$

input
```
Integrate[Sin[(-1 + x)^(1/4)],x]
```

output
```
-4*(-6 + Sqrt[-1 + x])*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] + 12*(-2 + Sqrt[
-1 + x])*Sin[(-1 + x)^(1/4)]
```

3.53.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3842, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\sqrt[4]{x-1}\right)dx$$

$$\Big\downarrow \ 3842$$

$$4\int (x-1)^{3/4}\sin\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}$$

$$\Big\downarrow \ 3042$$

$$4\int (x-1)^{3/4}\sin\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}$$

$$\Big\downarrow \ 3777$$

$$4\left(3\int \sqrt{x-1}\cos\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

$$\Big\downarrow \ 3042$$

$$4\left(3\int \sqrt{x-1}\sin\left(\sqrt[4]{x-1}+\frac{\pi}{2}\right)d\sqrt[4]{x-1}-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

$$\Big\downarrow \ 3777$$

$$4\left(3\left(2\int -\sqrt[4]{x-1}\sin\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}+\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)\right)-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

$$\Big\downarrow \ 25$$

$$4\left(3\left(\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-2\int \sqrt[4]{x-1}\sin\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}\right)-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

$$\Big\downarrow \ 3042$$

$$4\left(3\left(\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-2\int \sqrt[4]{x-1}\sin\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}\right)-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

$$\Big\downarrow \ 3777$$

$$4\left(3\left(\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-2\left(\int \cos\left(\sqrt[4]{x-1}\right)d\sqrt[4]{x-1}-\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)\right)\right)-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(3\left(\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-2\left(\int\sin\left(\sqrt[4]{x-1}+\frac{\pi}{2}\right)d\sqrt[4]{x-1}-\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)\right)\right)-(x-1)^{3/4}\cos\left(\sqrt[4]{x-\right.$$

$$\Big\downarrow \text{3117}$$

$$4\left(3\left(\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-2\left(\sin\left(\sqrt[4]{x-1}\right)-\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)\right)\right)-(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)\right)$$

input
```
Int[Sin[(-1 + x)^(1/4)],x]
```

output
```
4*(-((-1 + x)^(3/4)*Cos[(-1 + x)^(1/4)]) + 3*(Sqrt[-1 + x]*Sin[(-1 + x)^(1
/4)] - 2*(-((-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)]) + Sin[(-1 + x)^(1/4)])))
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

3.53.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result
derivativedivides	$24(-1+x)^{\frac{1}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right)-4(-1+x)^{\frac{3}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right)-24\sin\left((-1+x)^{\frac{1}{4}}\right)+$
default	$24(-1+x)^{\frac{1}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right)-4(-1+x)^{\frac{3}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right)-24\sin\left((-1+x)^{\frac{1}{4}}\right)+$

input
```
int(sin((-1+x)^(1/4)),x,method=_RETURNVERBOSE)
```

output
```
24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin(
(-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx=-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

input
```
integrate(sin((-1+x)^(1/4)),x, algorithm="fricas")
```

output
```
-4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1)
- 2)*sin((x - 1)^(1/4))
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx=-4(x-1)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)+12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)$$

input
```
integrate(sin((-1+x)**(1/4)),x)
```

output
```
-4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x - 1)**(1/
4)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1)**(1/4))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[4]{-1+x}\right) dx = -4\left((x-1)^{\frac{3}{4}} - 6(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right) + 12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

input
```
integrate(sin((-1+x)^(1/4)),x, algorithm="maxima")
```

output
```
-4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1)
- 2)*sin((x - 1)^(1/4))
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[4]{-1+x}\right) dx = -4\left((x-1)^{\frac{3}{4}} - 6(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right) + 12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

input
```
integrate(sin((-1+x)^(1/4)),x, algorithm="giac")
```

output
```
-4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1)
- 2)*sin((x - 1)^(1/4))
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = 4\cos\left((x-1)^{1/4}\right)\left(6\,(x-1)^{1/4} - (x-1)^{3/4}\right) + 4\sin\left((x-1)^{1/4}\right)\left(3\sqrt{x-1}-6\right)$$

input
```
int(sin((x - 1)^(1/4)),x)
```

output
```
4*cos((x - 1)^(1/4))*(6*(x - 1)^(1/4) - (x - 1)^(3/4)) + 4*sin((x - 1)^(1/
4))*(3*(x - 1)^(1/2) - 6)
```

3.54 $\int x\cos(x^2)\sin(x^2)\,dx$

3.54.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x\cos(x^2)\sin(x^2)\,dx = \frac{1}{4}\sin^2(x^2)$$

output
```
1/4*sin(x^2)^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\cos(x^2)\sin(x^2)\,dx = -\frac{1}{4}\cos^2(x^2)$$

input
```
Integrate[x*Cos[x^2]*Sin[x^2],x]
```

output
```
-1/4*Cos[x^2]^2
```

3.54.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3922}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin\left(x^2\right) \cos\left(x^2\right) \, dx$$

$$\Big\downarrow \text{ 3922}$$

$$\frac{1}{4} \sin^2\left(x^2\right)$$

input
```
Int[x*Cos[x^2]*Sin[x^2],x]
```

output
```
Sin[x^2]^2/4
```

3.54.3.1 Defintions of rubi rules used

rule 3922
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{(\cos^2(x^2))}{4}$	9
default	$-\frac{(\cos^2(x^2))}{4}$	9
risch	$-\frac{\cos(2x^2)}{8}$	9
parallelrisch	$-\frac{\cos(2x^2)}{8}+\frac{1}{8}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x^2)}{\sqrt{\pi}}\right)}{8}$	21
norman	$\frac{\tan^2\left(\frac{x^2}{2}\right)}{\left(1+\tan^2\left(\frac{x^2}{2}\right)\right)^2}$	22

input
```
int(x*cos(x^2)*sin(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x^2)^2
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=-\frac{1}{4}\cos\left(x^2\right)^2$$

input
```
integrate(x*cos(x^2)*sin(x^2),x, algorithm="fricas")
```

output
```
-1/4*cos(x^2)^2
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=-\frac{\cos^2\left(x^2\right)}{4}$$

input
```
integrate(x*cos(x**2)*sin(x**2),x)
```

output
```
-cos(x**2)**2/4
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=-\frac{1}{4}\cos\left(x^2\right)^2$$

input
```
integrate(x*cos(x^2)*sin(x^2),x, algorithm="maxima")
```

output
```
-1/4*cos(x^2)^2
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=-\frac{1}{4}\cos\left(x^2\right)^2$$

input
```
integrate(x*cos(x^2)*sin(x^2),x, algorithm="giac")
```

output
```
-1/4*cos(x^2)^2
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(x^2\right)\sin\left(x^2\right)\,dx=\frac{\sin\left(x^2\right)^2}{4}$$

input
```
int(x*cos(x^2)*sin(x^2),x)
```

output
```
sin(x^2)^2/4
```

3.55 $\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx$

3.55.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

output
```
-2/9*(4-3*sin(x)^2)^(3/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

input
```
Integrate[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

output
```
(-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

3.55.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4878, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\sqrt{3\cos^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\sqrt{3\cos(x)^2+1}dx$$

$$\Big\downarrow \text{4878}$$

$$\int 2\sin(x)\sqrt{4-3\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \sin(x)\sqrt{4-3\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{241}$$

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

input
```
Int[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

output
```
(-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.55.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2(1+3(\cos^2(x)))^{\frac{3}{2}}}{9}$	13
default	$-\frac{2(1+3(\cos^2(x)))^{\frac{3}{2}}}{9}$	13

input
```
int(sin(2*x)*(1+3*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(1+3*cos(x)^2)^(3/2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(3\,\cos\left(x\right)^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(3*cos(x)^2 + 1)^(3/2)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2(3\cos^2{(x)}+1)^{\frac{3}{2}}}{9}$$

input
```
integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)
```

output
```
-2*(3*cos(x)**2 + 1)**(3/2)/9
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(3\,\cos{(x)}^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(3*cos(x)^2 + 1)^(3/2)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2}{9}\left(3\,\cos{(x)}^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(3*cos(x)^2 + 1)^(3/2)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx = -\frac{2\left(3\cos\left(x\right)^2+1\right)^{3/2}}{9}$$

input
```
int(sin(2*x)*(3*cos(x)^2 + 1)^(1/2),x)
```

output
```
-(2*(3*cos(x)^2 + 1)^(3/2))/9
```

3.56 $\int \frac{1}{2+3x} \, dx$

3.56.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3} \log(2+3x)$$

output
```
1/3*ln(2+3*x)
```

3.56.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3} \log(2+3x)$$

input
```
Integrate[(2 + 3*x)^(-1),x]
```

output
```
Log[2 + 3*x]/3
```

3.56.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x+2} \, dx$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3} \log(3x+2)$$

input
```
Int[(2 + 3*x)^(-1),x]
```

output
```
Log[2 + 3*x]/3
```

3.56.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.56.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\ln\left(\frac{2}{3}+x\right)}{3}$	7
default	$\frac{\ln(2+3x)}{3}$	9
norman	$\frac{\ln(2+3x)}{3}$	9
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
risch	$\frac{\ln(2+3x)}{3}$	9

input
```
int(1/(2+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(2/3+x)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(2+3*x),x, algorithm="fricas")
```

output
```
1/3*log(3*x + 2)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3x}\,dx = \frac{\log\left(3x+2\right)}{3}$$

input
```
integrate(1/(2+3*x),x)
```

output
```
log(3*x + 2)/3
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(2+3*x),x, algorithm="maxima")
```

output
```
1/3*log(3*x + 2)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(2+3*x),x, algorithm="giac")
```

output
```
1/3*log(abs(3*x + 2))
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{2+3x}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

input
```
int(1/(3*x + 2),x)
```

output
```
log(x + 2/3)/3
```

3.57 $\int \log^2(x)\,dx$

3.57.1 Optimal result

Integrand size = 4, antiderivative size = 15

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

output
```
2*x-2*x*ln(x)+x*ln(x)^2
```

3.57.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = 2x - 2x\log(x) + x\log^2(x)$$

input
```
Integrate[Log[x]^2,x]
```

output
```
2*x - 2*x*Log[x] + x*Log[x]^2
```

3.57.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2(x)\,dx$$

$$\Big\downarrow \text{2733}$$

$$x\log^2(x)-2\int \log(x)dx$$

$$\Big\downarrow \text{2732}$$

$$x\log^2(x)-2(x\log(x)-x)$$

input
```
Int[Log[x]^2,x]
```

output
```
x*Log[x]^2 - 2*(-x + x*Log[x])
```

3.57.3.1 Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.57.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$2x - 2x\ln(x) + x\ln(x)^2$	16
norman	$2x - 2x\ln(x) + x\ln(x)^2$	16
risch	$2x - 2x\ln(x) + x\ln(x)^2$	16
parallelrisch	$2x - 2x\ln(x) + x\ln(x)^2$	16

input
```
int(ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x-2*x*ln(x)+x*ln(x)^2
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2\,x\log(x) + 2\,x$$

input
```
integrate(log(x)^2,x, algorithm="fricas")
```

output
```
x*log(x)^2 - 2*x*log(x) + 2*x
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2x\log(x) + 2x$$

input
```
integrate(ln(x)**2,x)
```

output
```
x*log(x)**2 - 2*x*log(x) + 2*x
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = \left(\log(x)^2 - 2\log(x) + 2\right)x$$

input
```
integrate(log(x)^2,x, algorithm="maxima")
```

output
```
(log(x)^2 - 2*log(x) + 2)*x
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x)\,dx = x\log(x)^2 - 2\,x\log(x) + 2\,x$$

input
```
integrate(log(x)^2,x, algorithm="giac")
```

output
```
x*log(x)^2 - 2*x*log(x) + 2*x
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x)\,dx = x\left(\ln(x)^2 - 2\ln(x) + 2\right)$$

input
```
int(log(x)^2,x)
```

output
```
x*(log(x)^2 - 2*log(x) + 2)
```

3.58 $\int x \log(x)\,dx$

3.58.1 Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.58.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

input
```
Integrate[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.58.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log(x)\, dx$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

input
```
Int[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.58.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.58.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2 \ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2 \ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2 \ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2 \ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2 \ln(x)}{2}$	14

input
```
int(x*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="fricas")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log\left(x\right)}{2} - \frac{x^2}{4}$$

input
```
integrate(x*ln(x),x)
```

output
```
x**2*log(x)/2 - x**2/4
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="maxima")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="giac")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\left(\ln\left(x\right)-\frac{1}{2}\right)}{2}$$

input
```
int(x*log(x),x)
```

output
```
(x^2*(log(x) - 1/2))/2
```

3.59 $\int x \log^2(x) \, dx$

3.59.1 Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

3.59.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x \log^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

input
```
Integrate[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

3.59.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2(x) \, dx$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{1}{2}x^2 \log^2(x) - \int x \log(x)dx$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

input
```
Int[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

3.59.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

input
```
int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx=\frac{1}{2}\,x^2\log{(x)}^2-\frac{1}{2}\,x^2\log{(x)}+\frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="fricas")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx=\frac{x^2\log{(x)}^2}{2}-\frac{x^2\log{(x)}}{2}+\frac{x^2}{4}$$

input
```
integrate(x*ln(x)**2,x)
```

output
```
x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{1}{4}\left(2\,\log\left(x\right)^2 - 2\,\log\left(x\right) + 1\right)x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="maxima")
```

output
```
1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{x^2\left(2\ln\left(x\right)^2 - 2\,\ln\left(x\right) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.60 $\int \frac{1}{1+t} \, dt$

3.60.1 Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \frac{1}{1+t} \, dt = \log(1+t)$$

output
```
ln(1+t)
```

3.60.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t} \, dt = \log(1+t)$$

input
```
Integrate[(1 + t)^(-1),t]
```

output
```
Log[1 + t]
```

3.60.3 Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{t+1} \, dt$$
$$\Big\downarrow \; 16$$
$$\log(t+1)$$

input
```
Int[(1 + t)^(-1),t]
```

output
```
Log[1 + t]
```

3.60.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.60.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\ln(1+t)$	5
norman	$\ln(1+t)$	5
meijerg	$\ln(1+t)$	5
risch	$\ln(1+t)$	5
parallelrisch	$\ln(1+t)$	5

input
```
int(1/(1+t),t,method=_RETURNVERBOSE)
```

output
```
ln(1+t)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \log(t+1)$$

input
```
integrate(1/(1+t),t, algorithm="fricas")
```

output
```
log(t + 1)
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+t}\,dt = \log(t+1)$$

input
```
integrate(1/(1+t),t)
```

output
```
log(t + 1)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \log(t+1)$$

input
```
integrate(1/(1+t),t, algorithm="maxima")
```

output
```
log(t + 1)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{1+t}\,dt = \log\left(|t+1|\right)$$

input
```
integrate(1/(1+t),t, algorithm="giac")
```

output
```
log(abs(t + 1))
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+t}\,dt = \ln\left(t+1\right)$$

input
```
int(1/(t + 1),t)
```

output
```
log(t + 1)
```

3.61 $\int \cot(x)\,dx$

3.61.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x)\,dx = \log(\sin(x))$$

output
```
ln(sin(x))
```

3.61.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x)\,dx = \log(\cos(x)) + \log(\tan(x))$$

input
```
Integrate[Cot[x],x]
```

output
```
Log[Cos[x]] + Log[Tan[x]]
```

3.61.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\log(\sin(x))$$

input
```
Int[Cot[x],x]
```

output
```
Log[Sin[x]]
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.61.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right)+\ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(e^{2ix}-1)$	14

input
```
int(cot(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \cot(x)\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

input
```
integrate(cot(x),x, algorithm="fricas")
```

output
```
1/2*log(-1/2*cos(2*x) + 1/2)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log(\sin(x))$$

input
```
integrate(cot(x),x)
```

output
```
log(sin(x))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log(\sin(x))$$

input
```
integrate(cot(x),x, algorithm="maxima")
```

output
```
log(sin(x))
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \cot(x)\,dx = \log(|\sin(x)|)$$

input
```
integrate(cot(x),x, algorithm="giac")
```

output
```
log(abs(sin(x)))
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \ln\left(\sin\left(x\right)\right)$$

input
```
int(cot(x),x)
```

output
```
log(sin(x))
```

3.62 $\int x^n \log(ax)\,dx$

3.62.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int x^n \log(ax)\,dx = -\frac{x^{1+n}}{(1+n)^2} + \frac{x^{1+n}\log(ax)}{1+n}$$

output
```
-x^(1+n)/(1+n)^2+x^(1+n)*ln(a*x)/(1+n)
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int x^n \log(ax)\,dx = \frac{x^{1+n}(-1+(1+n)\log(ax))}{(1+n)^2}$$

input
```
Integrate[x^n*Log[a*x],x]
```

output
```
(x^(1 + n)*(-1 + (1 + n)*Log[a*x]))/(1 + n)^2
```

3.62.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n \log(ax)\,dx$$

$$\Big\downarrow \; 2741$$

$$\frac{x^{n+1}\log(ax)}{n+1} - \frac{x^{n+1}}{(n+1)^2}$$

input
```
Int[x^n*Log[a*x],x]
```

output
```
-(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

3.62.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.62.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

method	result
norman	$\frac{x \ln(ax) e^{n \ln(x)}}{1+n} - \frac{x\, e^{n \ln(x)}}{n^2+2n+1}$
parallelrisch	$\frac{x\, x^n \ln(ax) n + x^n \ln(ax) x - x\, x^n}{n^2+2n+1}$
risch	$\frac{x\left(-i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iax) n + i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}(iax)^2 n + i\pi\ \mathrm{csgn}(ix)\,\mathrm{csgn}(iax)^2 n - i\pi\,\mathrm{csgn}(iax)^3 n - i\pi\ \mathrm{csgn}(ia)\,\mathrm{csgn}(ix\right.}{2(1+n)}$

input
```
int(x^n*ln(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/(1+n)*x*ln(a*x)*exp(n*ln(x))-1/(n^2+2*n+1)*x*exp(n*ln(x))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int x^n \log(ax)\,dx = \frac{((n+1)x\log(a) + (n+1)x\log(x) - x)x^n}{n^2 + 2\,n + 1}$$

input
```
integrate(x^n*log(a*x),x, algorithm="fricas")
```

output
```
((n + 1)*x*log(a) + (n + 1)*x*log(x) - x)*x^n/(n^2 + 2*n + 1)
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int x^n \log(ax)\,dx = \begin{cases} \frac{nxx^n \log(ax)}{n^2+2n+1} + \frac{xx^n \log(ax)}{n^2+2n+1} - \frac{xx^n}{n^2+2n+1} & \text{for } n \neq -1 \\ \frac{\log(ax)^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**n*ln(a*x),x)
```

output
```
Piecewise((n*x*x**n*log(a*x)/(n**2 + 2*n + 1) + x*x**n*log(a*x)/(n**2 + 2*
n + 1) - x*x**n/(n**2 + 2*n + 1), Ne(n, -1)), (log(a*x)**2/2, True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^n \log(ax)\,dx = \frac{x^{n+1}\log(ax)}{n+1} - \frac{x^{n+1}}{(n+1)^2}$$

input
```
integrate(x^n*log(a*x),x, algorithm="maxima")
```

output
```
x^(n + 1)*log(a*x)/(n + 1) - x^(n + 1)/(n + 1)^2
```

3.62.8 Giac [F]

$$\int x^n \log(ax)\,dx = \int x^n \log(ax)\,dx$$

input
```
integrate(x^n*log(a*x),x, algorithm="giac")
```

output
```
integrate(x^n*log(a*x), x)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int x^n \log(ax)\,dx = \begin{cases} \frac{\ln(a\,x)^2}{2} & \text{if } n=-1 \\ \frac{x^{n+1}\left(\ln(a\,x)-\frac{1}{n+1}\right)}{n+1} & \text{if } n\neq -1 \end{cases}$$

input
```
int(x^n*log(a*x),x)
```

output
```
piecewise(n == -1, log(a*x)^2/2, n ~= -1, (x^(n + 1)*(log(a*x) - 1/(n + 1)
))/(n + 1))
```

3.63 $\int x^2 \log^2(x)\,dx$

3.63.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int x^2 \log^2(x)\,dx = \frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)$$

output
```
2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

3.63.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x^2 \log^2(x)\,dx = \frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)$$

input
```
Integrate[x^2*Log[x]^2,x]
```

output
```
(2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

3.63.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^2(x)\,dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{3}x^3\log^2(x)-\frac{2}{3}\int x^2\log(x)dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{3}x^3\log^2(x)-\frac{2}{3}\left(\frac{1}{3}x^3\log(x)-\frac{x^3}{9}\right)$$

input
```
Int[x^2*Log[x]^2,x]
```

output
```
(x^3*Log[x]^2)/3 - (2*(-1/9*x^3 + (x^3*Log[x])/3))/3
```

3.63.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23
norman	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23
risch	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23
parallelrisch	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23
parts	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23

input
```
int(x^2*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x^2\log^2(x)\,dx=\frac{1}{3}\,x^3\log(x)^2-\frac{2}{9}\,x^3\log(x)+\frac{2}{27}\,x^3$$

input
```
integrate(x^2*log(x)^2,x, algorithm="fricas")
```

output
```
1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^2\log^2(x)\,dx=\frac{x^3\log(x)^2}{3}-\frac{2x^3\log(x)}{9}+\frac{2x^3}{27}$$

input
```
integrate(x**2*ln(x)**2,x)
```

output
```
x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x^2 \log^2(x)\,dx = \frac{1}{27}\left(9\,\log\left(x\right)^2 - 6\,\log\left(x\right) + 2\right)x^3$$

input
```
integrate(x^2*log(x)^2,x, algorithm="maxima")
```

output
```
1/27*(9*log(x)^2 - 6*log(x) + 2)*x^3
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x^2 \log^2(x)\,dx = \frac{1}{3}\,x^3\log\left(x\right)^2 - \frac{2}{9}\,x^3\log\left(x\right) + \frac{2}{27}\,x^3$$

input
```
integrate(x^2*log(x)^2,x, algorithm="giac")
```

output
```
1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x^2 \log^2(x)\,dx = \frac{2\,x^3\left(\frac{9\ln(x)^2}{2} - 3\,\ln\left(x\right) + 1\right)}{27}$$

input
```
int(x^2*log(x)^2,x)
```

output
```
(2*x^3*((9*log(x)^2)/2 - 3*log(x) + 1))/27
```

3.64 $\int \frac{1}{x \log(x)} dx$

3.64.1 Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

output
```
ln(ln(x))
```

3.64.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

input
```
Integrate[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

3.64.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)}dx$$
$$\Big\downarrow \text{ 2739}$$
$$\int \frac{1}{\log(x)}d\log(x)$$
$$\Big\downarrow \text{ 14}$$
$$\log(\log(x))$$

input
```
Int[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

3.64.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

3.64.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4
parallelrisch	$\ln(\ln(x))$	4

input
```
int(1/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="fricas")
```

output
```
log(log(x))
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/ln(x),x)
```

output
```
log(log(x))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="maxima")
```

output
```
log(log(x))
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\log(x)}\,dx = \log(|\log(x)|)$$

input
```
integrate(1/x/log(x),x, algorithm="giac")
```

output
```
log(abs(log(x)))
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \ln(\ln(x))$$

input
```
int(1/(x*log(x)),x)
```

output
```
log(log(x))
```

3.65 $\int \frac{\log(1-t)}{1-t} \, dt$

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{\log(1-t)}{1-t} \, dt = -\frac{1}{2} \log^2(1-t)$$

output
```
-1/2*ln(1-t)^2
```

3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log(1-t)}{1-t} \, dt = -\frac{1}{2} \log^2(1-t)$$

input
```
Integrate[Log[1 - t]/(1 - t),t]
```

output
```
-1/2*Log[1 - t]^2
```

3.65.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2837, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-t)}{1-t} \, dt$$

$$\Big\downarrow \; 2837$$

$$-\int \frac{\log(1-t)}{1-t} d(1-t)$$

$$\Big\downarrow \; 2738$$

$$-\frac{1}{2} \log^2(1-t)$$

input
```
Int[Log[1 - t]/(1 - t),t]
```

output
```
-1/2*Log[1 - t]^2
```

3.65.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\ln(1-t)^2}{2}$	11
default	$-\frac{\ln(1-t)^2}{2}$	11
norman	$-\frac{\ln(1-t)^2}{2}$	11
risch	$-\frac{\ln(1-t)^2}{2}$	11
parts	$-\ln(-1+t)\ln(1-t)+\frac{\ln(-1+t)^2}{2}$	22

input
```
int(ln(1-t)/(1-t),t,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(1-t)^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\,\log\left(-t+1\right)^2$$

input
```
integrate(log(1-t)/(1-t),t, algorithm="fricas")
```

output
```
-1/2*log(-t + 1)^2
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{\log\left(1-t\right)^2}{2}$$

input
```
integrate(ln(1-t)/(1-t),t)
```

output
```
-log(1 - t)**2/2
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\log\left(-t+1\right)^2$$

input
```
integrate(log(1-t)/(1-t),t, algorithm="maxima")
```

output
```
-1/2*log(-t + 1)^2
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{1}{2}\log\left(-t+1\right)^2$$

input
```
integrate(log(1-t)/(1-t),t, algorithm="giac")
```

output
```
-1/2*log(-t + 1)^2
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-t)}{1-t}\,dt = -\frac{\ln\left(1-t\right)^2}{2}$$

input
```
int(-log(1 - t)/(t - 1),t)
```

output
```
-log(1 - t)^2/2
```

3.66 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = -2\sqrt{1+\log(x)} + \frac{2}{3}(1+\log(x))^{3/2}$$

output
```
2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}(-2+\log(x))\sqrt{1+\log(x)}$$

input
```
Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

output
```
(2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

3.66.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2812, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x\sqrt{\log(x)+1}}dx$$

$$\Big\downarrow \text{2812}$$

$$\int \frac{\log(x)}{\sqrt{\log(x)+1}}d\log(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(\sqrt{\log(x)+1}-\frac{1}{\sqrt{\log(x)+1}}\right)d\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

input
```
Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

output
```
-2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

3.66.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2812
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(d +
 e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

3.66.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3} - 2\sqrt{1+\ln(x)}$	18
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3} - 2\sqrt{1+\ln(x)}$	18

input
```
int(ln(x)/x/(1+ln(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,\sqrt{\log{(x)}+1}(\log{(x)}-2)$$

input
```
integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2(\log{(x)}+1)^{\frac{3}{2}}}{3} - 2\sqrt{\log{(x)}+1}$$

input
```
integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

output
```
2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

input
```
integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

input
```
integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
```

output
```
2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \sqrt{\ln(x)+1}\left(\frac{2\,\ln(x)}{3}-\frac{4}{3}\right)$$

input
```
int(log(x)/(x*(log(x) + 1)^(1/2)),x)
```

output
```
(log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

3.67 $\int x^3 \log^3(x)\,dx$

3.67.1 Optimal result

Integrand size = 8, antiderivative size = 39

$$\int x^3 \log^3(x)\,dx = -\frac{3x^4}{128} + \frac{3}{32}x^4\log(x) - \frac{3}{16}x^4\log^2(x) + \frac{1}{4}x^4\log^3(x)$$

output
```
-3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

3.67.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x^3 \log^3(x)\,dx = -\frac{3x^4}{128} + \frac{3}{32}x^4\log(x) - \frac{3}{16}x^4\log^2(x) + \frac{1}{4}x^4\log^3(x)$$

input
```
Integrate[x^3*Log[x]^3,x]
```

output
```
(-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

3.67.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log^3(x) \, dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{4}x^4 \log^3(x) - \frac{3}{4}\int x^3 \log^2(x)dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{4}x^4 \log^3(x) - \frac{3}{4}\left(\frac{1}{4}x^4 \log^2(x) - \frac{1}{2}\int x^3 \log(x)dx\right)$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{4}x^4 \log^3(x) - \frac{3}{4}\left(\frac{1}{4}x^4 \log^2(x) + \frac{1}{2}\left(\frac{x^4}{16} - \frac{1}{4}x^4 \log(x)\right)\right)$$

input
```
Int[x^3*Log[x]^3,x]
```

output
```
(x^4*Log[x]^3)/4 - (3*((x^4*Log[x]^2)/4 + (x^4/16 - (x^4*Log[x])/4)/2))/4
```

3.67.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))    Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.67.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
norman	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
risch	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
parallelrisch	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
parts	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32

input
```
int(x^3*ln(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^3\log^3(x)\,dx=\frac{1}{4}\,x^4\log\left(x\right)^3-\frac{3}{16}\,x^4\log\left(x\right)^2+\frac{3}{32}\,x^4\log\left(x\right)-\frac{3}{128}\,x^4$$

input
```
integrate(x^3*log(x)^3,x, algorithm="fricas")
```

output
```
1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^3\log^3(x)\,dx=\frac{x^4\log\left(x\right)^3}{4}-\frac{3x^4\log\left(x\right)^2}{16}+\frac{3x^4\log\left(x\right)}{32}-\frac{3x^4}{128}$$

input
```
integrate(x**3*ln(x)**3,x)
```

output
```
x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**4/128
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int x^3 \log^3(x)\, dx = \frac{1}{128} \left(32 \log{(x)}^3 - 24 \log{(x)}^2 + 12 \log{(x)} - 3\right)x^4$$

input
```
integrate(x^3*log(x)^3,x, algorithm="maxima")
```

output
```
1/128*(32*log(x)^3 - 24*log(x)^2 + 12*log(x) - 3)*x^4
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^3 \log^3(x)\, dx = \frac{1}{4}\, x^4 \log{(x)}^3 - \frac{3}{16}\, x^4 \log{(x)}^2 + \frac{3}{32}\, x^4 \log{(x)} - \frac{3}{128}\, x^4$$

input
```
integrate(x^3*log(x)^3,x, algorithm="giac")
```

output
```
1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int x^3 \log^3(x)\, dx = \frac{3\, x^4 \left(\frac{32 \ln{(x)}^3}{3} - 8 \ln{(x)}^2 + 4\, \ln{(x)} - 1\right)}{128}$$

input
```
int(x^3*log(x)^3,x)
```

output
```
(3*x^4*(4*log(x) - 8*log(x)^2 + (32*log(x)^3)/3 - 1))/128
```

3.68 $\int e^{x^3} x^2 \, dx$

3.68.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

output
```
1/3*exp(x^3)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

input
```
Integrate[E^x^3*x^2,x]
```

output
```
E^x^3/3
```

3.68.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^3} x^2 \, dx$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{e^{x^3}}{3}$$

input
```
Int[E^x^3*x^2,x]
```

output
```
E^x^3/3
```

3.68.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

3.68.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{x^3}}{3}$	7
derivativedivides	$\frac{e^{x^3}}{3}$	7
default	$\frac{e^{x^3}}{3}$	7
norman	$\frac{e^{x^3}}{3}$	7
risch	$\frac{e^{x^3}}{3}$	7
parallelrisch	$\frac{e^{x^3}}{3}$	7
meijerg	$-\frac{1}{3}+\frac{e^{x^3}}{3}$	9

input
```
int(exp(x^3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(x^3)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3}x^2\,dx = \frac{1}{3}\,e^{\left(x^3\right)}$$

input
```
integrate(exp(x^3)*x^2,x, algorithm="fricas")
```

output
```
1/3*e^(x^3)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

input
```
integrate(exp(x**3)*x**2,x)
```

output
```
exp(x**3)/3
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3} x^2 \, dx = \frac{1}{3} \, e^{\left(x^3\right)}$$

input
```
integrate(exp(x^3)*x^2,x, algorithm="maxima")
```

output
```
1/3*e^(x^3)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3} x^2 \, dx = \frac{1}{3} \, e^{\left(x^3\right)}$$

input
```
integrate(exp(x^3)*x^2,x, algorithm="giac")
```

output
```
1/3*e^(x^3)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

input
```
int(x^2*exp(x^3),x)
```

output
```
exp(x^3)/3
```

3.69 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$

3.69.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

output
```
2^(1+x^(1/2))/ln(2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

input
```
Integrate[2^Sqrt[x]/Sqrt[x],x]
```

output
```
2^(1 + Sqrt[x])/Log[2]
```

3.69.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

input
```
Int[2^Sqrt[x]/Sqrt[x],x]
```

output
```
2^(1 + Sqrt[x])/Log[2]
```

3.69.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
default	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
meijerg	$-\frac{2\left(1-{\mathrm e}^{\sqrt{x}\,\ln(2)}\right)}{\ln(2)}$	18

input
```
int(2^(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(x^(1/2))/ln(2)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx = \frac{2 \cdot 2^{(\sqrt{x})}}{\log(2)}$$

input
```
integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*2^sqrt(x)/log(2)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx = \frac{2 \cdot 2^{\sqrt{x}}}{\log(2)}$$

input
```
integrate(2**(x**(1/2))/x**(1/2),x)
```

output
```
2*2**(sqrt(x))/log(2)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx = \frac{2^{\sqrt{x}+1}}{\log(2)}$$

input
```
integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2^(sqrt(x) + 1)/log(2)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx = \frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\log\left(2\right)}$$

input
```
integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*2^sqrt(x)/log(2)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx = \frac{2\, 2^{\sqrt{x}}}{\ln\left(2\right)}$$

input
```
int(2^(x^(1/2))/x^(1/2),x)
```

output
```
(2*2^(x^(1/2)))/log(2)
```

3.70 $\int e^{2\sin(x)}\cos(x)\,dx$

3.70.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}e^{2\sin(x)}$$

output
```
1/2*exp(2*sin(x))
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}e^{2\sin(x)}$$

input
```
Integrate[E^(2*Sin[x])*Cos[x],x]
```

output
```
E^(2*Sin[x])/2
```

3.70.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\sin(x)}\cos(x)\,dx$$

$$\Big\downarrow \; 4834$$

$$\int e^{2\sin(x)}d\sin(x)$$

$$\Big\downarrow \; 2624$$

$$\frac{1}{2}e^{2\sin(x)}$$

input
```
Int[E^(2*Sin[x])*Cos[x],x]
```

output
```
E^(2*Sin[x])/2
```

3.70.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.70.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{e^{2\sin(x)}}{2}$	8
default	$\frac{e^{2\sin(x)}}{2}$	8
risch	$\frac{e^{2\sin(x)}}{2}$	8
parallelrisch	$\frac{e^{2\sin(x)}}{2}$	8
norman	$\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)e^{\frac{4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{2}+\frac{e^{\frac{4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	57

input
```
int(exp(2*sin(x))*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*sin(x))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx=\frac{1}{2}\,e^{(2\,\sin(x))}$$

input
```
integrate(exp(2*sin(x))*cos(x),x, algorithm="fricas")
```

output
```
1/2*e^(2*sin(x))
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx=\frac{e^{2\sin{(x)}}}{2}$$

input
```
integrate(exp(2*sin(x))*cos(x),x)
```

output
```
exp(2*sin(x))/2
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}\,e^{(2\,\sin(x))}$$

input
```
integrate(exp(2*sin(x))*cos(x),x, algorithm="maxima")
```

output
```
1/2*e^(2*sin(x))
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{1}{2}\,e^{(2\,\sin(x))}$$

input
```
integrate(exp(2*sin(x))*cos(x),x, algorithm="giac")
```

output
```
1/2*e^(2*sin(x))
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2\sin(x)}\cos(x)\,dx = \frac{\mathrm{e}^{2\,\sin(x)}}{2}$$

input
```
int(exp(2*sin(x))*cos(x),x)
```

output
```
exp(2*sin(x))/2
```

3.71 $\int e^x \sin(x)\,dx$

3.71.1 Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

output
```
-1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x(-\cos(x) + \sin(x))$$

input
```
Integrate[E^x*Sin[x],x]
```

output
```
(E^x*(-Cos[x] + Sin[x]))/2
```

3.71.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin(x)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

input
```
Int[E^x*Sin[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

3.71.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^{x}(-\cos(x)+\sin(x))}{2}$	12
default	$-\frac{e^{x}\cos(x)}{2}+\frac{e^{x}\sin(x)}{2}$	14
norman	$\frac{e^{x}\tan\left(\frac{x}{2}\right)+\frac{e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{i e^{(1-i)x}}{4}$	36

input
```
int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(-cos(x)+sin(x))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\cos(x)\,e^x + \frac{1}{2}\,e^x\sin(x)$$

input
```
integrate(exp(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\,dx = \frac{e^x\sin(x)}{2} - \frac{e^x\cos(x)}{2}$$

input
```
integrate(exp(x)*sin(x),x)
```

output
```
exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\,(\cos(x) - \sin(x))e^x$$

input
```
integrate(exp(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) - sin(x))*e^x
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^x$$

input
```
integrate(exp(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*(cos(x) - sin(x))*e^x
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{e^x\left(\cos\left(x\right) - \sin\left(x\right)\right)}{2}$$

input
```
int(exp(x)*sin(x),x)
```

output
```
-(exp(x)*(cos(x) - sin(x)))/2
```

3.72 $\int e^x \cos(x)\,dx$

3.72.1 Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \cos(x)\,dx = \frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

output
```
1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

3.72.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \cos(x)\,dx = \frac{1}{2}e^x(\cos(x) + \sin(x))$$

input
```
Integrate[E^x*Cos[x],x]
```

output
```
(E^x*(Cos[x] + Sin[x]))/2
```

3.72.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos(x)\, dx$$

$$\Big\downarrow \ 4933$$

$$\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x \cos(x)$$

input
```
Int[E^x*Cos[x],x]
```

output
```
(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

3.72.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{e^x(\cos(x)+\sin(x))}{2}$	10
default	$\frac{e^x\cos(x)}{2}+\frac{e^x\sin(x)}{2}$	14
norman	$\frac{e^x\tan\left(\frac{x}{2}\right)-\frac{e^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{e^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	34
risch	$\frac{e^{(1+i)x}}{4}-\frac{ie^{(1+i)x}}{4}+\frac{e^{(1-i)x}}{4}+\frac{ie^{(1-i)x}}{4}$	36

input
```
int(exp(x)*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(cos(x)+sin(x))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cos(x)\, dx = \frac{1}{2} \cos(x) e^x + \frac{1}{2} e^x \sin(x)$$

input
```
integrate(exp(x)*cos(x),x, algorithm="fricas")
```

output
```
1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \cos(x)\, dx = \frac{e^x \sin(x)}{2} + \frac{e^x \cos(x)}{2}$$

input
```
integrate(exp(x)*cos(x),x)
```

output
```
exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.47

$$\int e^x \cos(x)\, dx = \frac{1}{2} (\cos(x) + \sin(x)) e^x$$

input
```
integrate(exp(x)*cos(x),x, algorithm="maxima")
```

output
```
1/2*(cos(x) + sin(x))*e^x
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.47

$$\int e^x \cos(x)\,dx = \frac{1}{2}\left(\cos\left(x\right) + \sin\left(x\right)\right)e^x$$

input
```
integrate(exp(x)*cos(x),x, algorithm="giac")
```

output
```
1/2*(cos(x) + sin(x))*e^x
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.47

$$\int e^x \cos(x)\,dx = \frac{\mathrm{e}^x\left(\cos\left(x\right) + \sin\left(x\right)\right)}{2}$$

input
```
int(exp(x)*cos(x),x)
```

output
```
(exp(x)*(cos(x) + sin(x)))/2
```

3.73 $\int \frac{1}{1+e^x} dx$

3.73.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{1+e^x} dx = x - \log(1+e^x)$$

output
```
x-ln(1+exp(x))
```

3.73.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+e^x} dx = -2\text{arctanh}(1+2e^x)$$

input
```
Integrate[(1 + E^x)^(-1),x]
```

output
```
-2*ArcTanh[1 + 2*E^x]
```

3.73.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^x+1}dx$$

$$\Big\downarrow \; 2720$$

$$\int \frac{e^{-x}}{e^x+1}de^x$$

$$\Big\downarrow \; 47$$

$$\int e^{-x}de^x-\int \frac{1}{1+e^x}de^x$$

$$\Big\downarrow \; 14$$

$$\log \left(e^x\right)-\int \frac{1}{1+e^x}de^x$$

$$\Big\downarrow \; 16$$

$$\log \left(e^x\right)-\log \left(e^x+1\right)$$

input
```
Int[(1 + E^x)^(-1),x]
```

output
```
Log[E^x] - Log[1 + E^x]
```

3.73.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.73.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
norman	$x-\ln\left(1+{\mathrm e}^{x}\right)$	10
risch	$x-\ln\left(1+{\mathrm e}^{x}\right)$	10
parallelrisch	$x-\ln\left(1+{\mathrm e}^{x}\right)$	10
derivativedivides	$-\ln\left(1+{\mathrm e}^{x}\right)+\ln\left({\mathrm e}^{x}\right)$	12
default	$-\ln\left(1+{\mathrm e}^{x}\right)+\ln\left({\mathrm e}^{x}\right)$	12

input
```
int(1/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x-ln(1+exp(x))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x-\log\left(e^x+1\right)$$

input
```
integrate(1/(1+exp(x)),x, algorithm="fricas")
```

output
```
x - log(e^x + 1)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x)
```

output
```
x - log(exp(x) + 1)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x, algorithm="maxima")
```

output
```
x - log(e^x + 1)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x, algorithm="giac")
```

output
```
x - log(e^x + 1)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \ln\left(\mathrm{e}^x + 1\right)$$

input
```
int(1/(exp(x) + 1),x)
```

output
```
x - log(exp(x) + 1)
```

3.74 $\int e^x x\,dx$

3.74.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^x x\,dx = -e^x + e^x x$$

output
```
-exp(x)+exp(x)*x
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^x x\,dx = e^x(-1+x)$$

input
```
Integrate[E^x*x,x]
```

output
```
E^x*(-1 + x)
```

3.74.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \, dx$$

$$\Big\downarrow \text{2607}$$

$$e^x x - \int e^x dx$$

$$\Big\downarrow \text{2624}$$

$$e^x x - e^x$$

input
```
Int[E^x*x,x]
```

output
```
-E^x + E^x*x
```

3.74.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.74.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$(-1+x)\,\mathrm{e}^{x}$	7
risch	$(-1+x)\,\mathrm{e}^{x}$	7
default	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
norman	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
parallelrisch	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
parts	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
meijerg	$1-\frac{(-2x+2)\mathrm{e}^{x}}{2}$	12

input
```
int(exp(x)*x,x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*exp(x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^{x}x\,dx = (x-1)e^{x}$$

input
```
integrate(exp(x)*x,x, algorithm="fricas")
```

output
```
(x - 1)*e^x
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int e^{x}x\,dx = (x-1)\,e^{x}$$

input
```
integrate(exp(x)*x,x)
```

output
```
(x - 1)*exp(x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

input
```
integrate(exp(x)*x,x, algorithm="maxima")
```

output
```
(x - 1)*e^x
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

input
```
integrate(exp(x)*x,x, algorithm="giac")
```

output
```
(x - 1)*e^x
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = \mathrm{e}^x\,(x-1)$$

input
```
int(x*exp(x),x)
```

output
```
exp(x)*(x - 1)
```

3.75 $\int e^{-x}x\,dx$

3.75.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int e^{-x}x\,dx=-e^{-x}-e^{-x}x$$

output
```
-1/exp(x)-x/exp(x)
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int e^{-x}x\,dx=e^{-x}(-1-x)$$

input
```
Integrate[x/E^x,x]
```

output
```
(-1 - x)/E^x
```

3.75.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x} x \, dx$$
$$\Big\downarrow \text{2607}$$
$$\int e^{-x} dx - e^{-x} x$$
$$\Big\downarrow \text{2624}$$
$$-e^{-x} x - e^{-x}$$

input
```
Int[x/E^x,x]
```

output
```
-E^(-x) - x/E^x
```

3.75.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.75.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
gosper	$-(1+x)\,e^{-x}$	10
norman	$(-1-x)\,e^{-x}$	11
risch	$(-1-x)\,e^{-x}$	11
parallelrisch	$(-1-x)\,e^{-x}$	11
meijerg	$1-\frac{(2x+2)e^{-x}}{2}$	14
default	$-e^{-x}-x\,e^{-x}$	15

input
```
int(x/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-(1+x)/exp(x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -(x+1)e^{(-x)}$$

input
```
integrate(x/exp(x),x, algorithm="fricas")
```

output
```
-(x + 1)*e^(-x)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int e^{-x}x\,dx = (-x-1)\,e^{-x}$$

input
```
integrate(x/exp(x),x)
```

output
```
(-x - 1)*exp(-x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -(x+1)e^{(-x)}$$

input
```
integrate(x/exp(x),x, algorithm="maxima")
```

output
```
-(x + 1)*e^(-x)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -(x+1)e^{(-x)}$$

input
```
integrate(x/exp(x),x, algorithm="giac")
```

output
```
-(x + 1)*e^(-x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-x}x\,dx = -\mathrm{e}^{-x}\,(x+1)$$

input
```
int(x*exp(-x),x)
```

output
```
-exp(-x)*(x + 1)
```

3.76 $\int e^x x^2 \, dx$

3.76.1 Optimal result

Integrand size = 7, antiderivative size = 19

$$\int e^x x^2 \, dx = 2e^x - 2e^x x + e^x x^2$$

output
```
2*exp(x)-2*exp(x)*x+exp(x)*x^2
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x x^2 \, dx = e^x (2 - 2x + x^2)$$

input
```
Integrate[E^x*x^2,x]
```

output
```
E^x*(2 - 2*x + x^2)
```

3.76.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^2 \, dx$$

$$\Big\downarrow \; 2607$$

$$e^x x^2 - 2 \int e^x x dx$$

$$\Big\downarrow \; 2607$$

$$e^x x^2 - 2 \left(e^x x - \int e^x dx \right)$$

$$\Big\downarrow \; 2624$$

$$e^x x^2 - 2(e^x x - e^x)$$

input
```
Int[E^x*x^2,x]
```

output
```
E^x*x^2 - 2*(-E^x + E^x*x)
```

3.76.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.76.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$(x^2-2x+2)\,e^x$	12
risch	$(x^2-2x+2)\,e^x$	12
default	$2\,e^x-2\,e^x x+e^x x^2$	17
norman	$2\,e^x-2\,e^x x+e^x x^2$	17
meijerg	$-2+\frac{(3x^2-6x+6)\,e^x}{3}$	17
parallelrisch	$2\,e^x-2\,e^x x+e^x x^2$	17
parts	$2\,e^x-2\,e^x x+e^x x^2$	17

input
```
int(exp(x)*x^2,x,method=_RETURNVERBOSE)
```

output
```
(x^2-2*x+2)*exp(x)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2\,dx = \left(x^2-2\,x+2\right)e^x$$

input
```
integrate(exp(x)*x^2,x, algorithm="fricas")
```

output
```
(x^2 - 2*x + 2)*e^x
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int e^x x^2\,dx = \left(x^2-2x+2\right)e^x$$

input
```
integrate(exp(x)*x**2,x)
```

output
```
(x**2 - 2*x + 2)*exp(x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = (x^2 - 2x + 2)e^x$$

input
```
integrate(exp(x)*x^2,x, algorithm="maxima")
```

output
```
(x^2 - 2*x + 2)*e^x
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = (x^2 - 2x + 2)e^x$$

input
```
integrate(exp(x)*x^2,x, algorithm="giac")
```

output
```
(x^2 - 2*x + 2)*e^x
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x x^2 \, dx = \mathrm{e}^x \left(x^2 - 2x + 2\right)$$

input
```
int(x^2*exp(x),x)
```

output
```
exp(x)*(x^2 - 2*x + 2)
```

3.77 $\int e^{-2x}x^2\,dx$

3.77.1 Optimal result

Integrand size = 9, antiderivative size = 32

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}e^{-2x} - \frac{1}{2}e^{-2x}x - \frac{1}{2}e^{-2x}x^2$$

output
```
-1/4/exp(2*x)-1/2*x/exp(2*x)-1/2*x^2/exp(2*x)
```

3.77.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}e^{-2x}\left(1+2x+2x^2\right)$$

input
```
Integrate[x^2/E^(2*x),x]
```

output
```
-1/4*(1 + 2*x + 2*x^2)/E^(2*x)
```

3.77.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x}x^2\,dx$$

$$\Big\downarrow \; 2607$$

$$\int e^{-2x}xdx - \frac{1}{2}e^{-2x}x^2$$

$$\Big\downarrow \; 2607$$

$$\frac{1}{2}\int e^{-2x}dx - \frac{1}{2}e^{-2x}x^2 - \frac{1}{2}e^{-2x}x$$

$$\Big\downarrow \; 2624$$

$$-\frac{1}{2}e^{-2x}x^2 - \frac{1}{2}e^{-2x}x - \frac{e^{-2x}}{4}$$

input
```
Int[x^2/E^(2*x),x]
```

output
```
-1/4*1/E^(2*x) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

3.77.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.77.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

method	result	size
risch	$\left(-\frac{1}{2}x^{2}-\frac{1}{2}x-\frac{1}{4}\right) \mathrm{e}^{-2x}$	16
norman	$\left(-\frac{1}{2}x^{2}-\frac{1}{2}x-\frac{1}{4}\right) \mathrm{e}^{-2x}$	18
gosper	$-\frac{(2x^{2}+2x+1)\mathrm{e}^{-2x}}{4}$	19
meijerg	$\frac{1}{4}-\frac{(12x^{2}+12x+6)\mathrm{e}^{-2x}}{24}$	19
parallelrisch	$\frac{(-2x^{2}-2x-1)\mathrm{e}^{-2x}}{4}$	19
derivativedivides	$-\frac{\mathrm{e}^{-2x}}{4}-\frac{x\,\mathrm{e}^{-2x}}{2}-\frac{x^{2}\mathrm{e}^{-2x}}{2}$	30
default	$-\frac{\mathrm{e}^{-2x}}{4}-\frac{x\,\mathrm{e}^{-2x}}{2}-\frac{x^{2}\mathrm{e}^{-2x}}{2}$	30

input
```
int(x^2/exp(2*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^2-1/2*x-1/4)*exp(-2*x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx=-\frac{1}{4}\left(2\,x^{2}+2\,x+1\right)e^{(-2\,x)}$$

input
```
integrate(x^2/exp(2*x),x, algorithm="fricas")
```

output
```
-1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int e^{-2x}x^2\,dx = \frac{(-2x^2-2x-1)\,e^{-2x}}{4}$$

input
```
integrate(x**2/exp(2*x),x)
```

output
```
(-2*x**2 - 2*x - 1)*exp(-2*x)/4
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}\,(2\,x^2+2\,x+1)e^{(-2\,x)}$$

input
```
integrate(x^2/exp(2*x),x, algorithm="maxima")
```

output
```
-1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx = -\frac{1}{4}\,(2\,x^2+2\,x+1)e^{(-2\,x)}$$

input
```
integrate(x^2/exp(2*x),x, algorithm="giac")
```

output
```
-1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-2x}x^2\,dx = -\frac{e^{-2\,x}\,(4\,x^2+4\,x+2)}{8}$$

input
```
int(x^2*exp(-2*x),x)
```

output
```
-(exp(-2*x)*(4*x + 4*x^2 + 2))/8
```

3.78 $\int e^{\sqrt{x}}\,dx$

3.78.1 Optimal result

Integrand size = 7, antiderivative size = 24

$$\int e^{\sqrt{x}}\,dx = -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}$$

output
```
-2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}(-1+\sqrt{x})$$

input
```
Integrate[E^Sqrt[x],x]
```

output
```
2*E^Sqrt[x]*(-1 + Sqrt[x])
```

3.78.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2636, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 2636}$$

$$2\int e^{\sqrt{x}}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{ 2607}$$

$$2\left(e^{\sqrt{x}}\sqrt{x}-\int e^{\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2624}$$

$$2\left(e^{\sqrt{x}}\sqrt{x}-e^{\sqrt{x}}\right)$$

input
```
Int[E^Sqrt[x],x]
```

output
```
2*(-E^Sqrt[x] + E^Sqrt[x]*Sqrt[x])
```

3.78.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2636
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Simp[k/d   Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (
c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Int
egerQ[n]
```

3.78.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
meijerg	$2-\left(-2\sqrt{x}+2\right) \mathrm{e}^{\sqrt{x}}$	16
derivativedivides	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17
default	$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$	17

input
```
int(exp(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2-(-2*x^(1/2)+2)*exp(x^(1/2))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2)),x, algorithm="fricas")
```

output
```
2*(sqrt(x) - 1)*e^sqrt(x)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int e^{\sqrt{x}}\,dx = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

input
```
integrate(exp(x**(1/2)),x)
```

output
```
2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x} - 1\right)e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2)),x, algorithm="maxima")
```

output
```
2*(sqrt(x) - 1)*e^sqrt(x)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\left(\sqrt{x} - 1\right)e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2)),x, algorithm="giac")
```

output
```
2*(sqrt(x) - 1)*e^sqrt(x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int e^{\sqrt{x}}\,dx = 2\,\mathrm{e}^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

input
```
int(exp(x^(1/2)),x)
```

output
```
2*exp(x^(1/2))*(x^(1/2) - 1)
```

3.79 $\int e^{-x^2} x^3 \, dx$

3.79.1 Optimal result

Integrand size = 11, antiderivative size = 26

$$\int e^{-x^2} x^3 \, dx = -\frac{e^{-x^2}}{2} - \frac{1}{2} e^{-x^2} x^2$$

output
```
-1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-x^2} x^3 \, dx = -\frac{1}{2} e^{-x^2} \left(1 + x^2\right)$$

input
```
Integrate[x^3/E^x^2,x]
```

output
```
-1/2*(1 + x^2)/E^x^2
```

3.79.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^2}x^3\,dx$$

$$\Big\downarrow \text{2641}$$

$$\int e^{-x^2}xdx-\frac{1}{2}e^{-x^2}x^2$$

$$\Big\downarrow \text{2638}$$

$$-\frac{1}{2}e^{-x^2}x^2-\frac{e^{-x^2}}{2}$$

input
```
Int[x^3/E^x^2,x]
```

output
```
-1/2*1/E^x^2 - x^2/(2*E^x^2)
```

3.79.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

3.79.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{(x^2+1)\,\mathrm{e}^{-x^2}}{2}$	14
norman	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^2}$	15
risch	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^2}$	15
parallelrisch	$\frac{(-x^2-1)\,\mathrm{e}^{-x^2}}{2}$	16
meijerg	$\frac{1}{2}-\frac{(2x^2+2)\,\mathrm{e}^{-x^2}}{4}$	18
derivativedivides	$-\frac{\mathrm{e}^{-x^2}}{2}-\frac{x^2\mathrm{e}^{-x^2}}{2}$	21
default	$-\frac{\mathrm{e}^{-x^2}}{2}-\frac{x^2\mathrm{e}^{-x^2}}{2}$	21

input
```
int(x^3/exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2+1)/exp(x^2)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}\left(x^2+1\right)e^{(-x^2)}$$

input
```
integrate(x^3/exp(x^2),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + 1)*e^(-x^2)
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-x^2}x^3\,dx = \frac{(-x^2-1)\,e^{-x^2}}{2}$$

input
```
integrate(x**3/exp(x**2),x)
```

output
```
(-x**2 - 1)*exp(-x**2)/2
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}\left(x^2+1\right)e^{(-x^2)}$$

input
```
integrate(x^3/exp(x^2),x, algorithm="maxima")
```

output
```
-1/2*(x^2 + 1)*e^(-x^2)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}\left(x^2+1\right)e^{(-x^2)}$$

input
```
integrate(x^3/exp(x^2),x, algorithm="giac")
```

output
```
-1/2*(x^2 + 1)*e^(-x^2)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2} x^3 \, dx = -\frac{e^{-x^2} (x^2 + 1)}{2}$$

input
```
int(x^3*exp(-x^2),x)
```

output
```
-(exp(-x^2)*(x^2 + 1))/2
```

3.80 $\int e^{ax} \cos(bx)\,dx$

3.80.1 Optimal result

Integrand size = 10, antiderivative size = 41

$$\int e^{ax} \cos(bx)\,dx = \frac{ae^{ax}\cos(bx)}{a^2+b^2} + \frac{be^{ax}\sin(bx)}{a^2+b^2}$$

output
```
a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^{ax} \cos(bx)\,dx = \frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2}$$

input
```
Integrate[E^(a*x)*Cos[b*x],x]
```

output
```
(E^(a*x)*(a*Cos[b*x] + b*Sin[b*x]))/(a^2 + b^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{ax}\cos(bx)\,dx$$

$$\Big\downarrow \text{ 4933}$$

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

input
```
Int[E^(a*x)*Cos[b*x],x]
```

output
```
(a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

3.80.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.80.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{e^{ax}(\cos(bx)a+b\sin(bx))}{a^2+b^2}$	28
default	$\frac{a\,e^{ax}\cos(bx)}{a^2+b^2}+\frac{b\,e^{ax}\sin(bx)}{a^2+b^2}$	40
risch	$\frac{e^{x(ib+a)}}{2ib+2a}+\frac{e^{x(-ib+a)}}{-2ib+2a}$	40
norman	$\frac{\frac{a\,e^{ax}}{a^2+b^2}-\frac{a\,e^{ax}\left(\tan^2\left(\frac{bx}{2}\right)\right)}{a^2+b^2}+\frac{2b\,e^{ax}\tan\left(\frac{bx}{2}\right)}{a^2+b^2}}{1+\tan^2\left(\frac{bx}{2}\right)}$	73

input
```
int(exp(a*x)*cos(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2+b^2)*exp(a*x)*(cos(b*x)*a+b*sin(b*x))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int e^{ax}\cos(bx)\,dx = \frac{a\cos\left(bx\right)e^{(ax)} + be^{(ax)}\sin\left(bx\right)}{a^2+b^2}$$

input
```
integrate(exp(a*x)*cos(b*x),x, algorithm="fricas")
```

output
```
(a*cos(b*x)*e^(a*x) + b*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

3.80.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.39

$$\int e^{ax}\cos(bx)\,dx = \begin{cases} x & \text{for } a=0 \wedge b=0 \\ \frac{ixe^{-ibx}\sin\left(bx\right)}{2} + \frac{xe^{-ibx}\cos\left(bx\right)}{2} + \frac{ie^{-ibx}\cos\left(bx\right)}{2b} & \text{for } a=-ib \\ -\frac{ixe^{ibx}\sin\left(bx\right)}{2} + \frac{xe^{ibx}\cos\left(bx\right)}{2} - \frac{ie^{ibx}\cos\left(bx\right)}{2b} & \text{for } a=ib \\ \frac{ae^{ax}\cos\left(bx\right)}{a^2+b^2} + \frac{be^{ax}\sin\left(bx\right)}{a^2+b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a*x)*cos(b*x),x)
```

output
```
Piecewise((x, Eq(a, 0) & Eq(b, 0)), (I*x*exp(-I*b*x)*sin(b*x)/2 + x*exp(-I
*b*x)*cos(b*x)/2 + I*exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*b)), (-I*x*exp(I
*b*x)*sin(b*x)/2 + x*exp(I*b*x)*cos(b*x)/2 - I*exp(I*b*x)*cos(b*x)/(2*b),
Eq(a, I*b)), (a*exp(a*x)*cos(b*x)/(a**2 + b**2) + b*exp(a*x)*sin(b*x)/(a**
2 + b**2), True))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int e^{ax}\cos(bx)\,dx = \frac{(a\cos(bx) + b\sin(bx))e^{(ax)}}{a^2 + b^2}$$

input
```
integrate(exp(a*x)*cos(b*x),x, algorithm="maxima")
```

output
```
(a*cos(b*x) + b*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int e^{ax}\cos(bx)\,dx = \left(\frac{a\cos(bx)}{a^2 + b^2} + \frac{b\sin(bx)}{a^2 + b^2}\right)e^{(ax)}$$

input
```
integrate(exp(a*x)*cos(b*x),x, algorithm="giac")
```

output
```
(a*cos(b*x)/(a^2 + b^2) + b*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int e^{ax}\cos(bx)\,dx = \frac{\mathrm{e}^{a\,x}\,\left(a\,\cos(b\,x) + b\,\sin(b\,x)\right)}{a^2 + b^2}$$

input
```
int(exp(a*x)*cos(b*x),x)
```

output
```
(exp(a*x)*(a*cos(b*x) + b*sin(b*x)))/(a^2 + b^2)
```

3.81 $\int e^{ax}\sin(bx)\,dx$

3.81.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int e^{ax}\sin(bx)\,dx = -\frac{be^{ax}\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

output
```
-b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}(-b\cos(bx)+a\sin(bx))}{a^2+b^2}$$

input
```
Integrate[E^(a*x)*Sin[b*x],x]
```

output
```
(E^(a*x)*(-(b*Cos[b*x]) + a*Sin[b*x]))/(a^2 + b^2)
```

3.81.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{ax}\sin(bx)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

input
```
Int[E^(a*x)*Sin[b*x],x]
```

output
```
-((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

3.81.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.81.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{ax}(\sin(bx)a-b\cos(bx))}{a^2+b^2}$	29
default	$-\frac{b\,e^{ax}\cos(bx)}{a^2+b^2}+\frac{a\,e^{ax}\sin(bx)}{a^2+b^2}$	41
risch	$-\frac{ie^{x(ib+a)}}{2(ib+a)}+\frac{ie^{x(-ib+a)}}{-2ib+2a}$	42
norman	$\frac{\frac{b\,e^{ax}\left(\tan^2\left(\frac{bx}{2}\right)\right)}{a^2+b^2}-\frac{b\,e^{ax}}{a^2+b^2}+\frac{2a\,e^{ax}\tan\left(\frac{bx}{2}\right)}{a^2+b^2}}{1+\tan^2\left(\frac{bx}{2}\right)}$	73

input
```
int(exp(a*x)*sin(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2+b^2)*exp(a*x)*(sin(b*x)*a-b*cos(b*x))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int e^{ax}\sin(bx)\,dx = -\frac{b\cos(bx)\,e^{(ax)} - ae^{(ax)}\sin(bx)}{a^2+b^2}$$

input
```
integrate(exp(a*x)*sin(b*x),x, algorithm="fricas")
```

output
```
-(b*cos(b*x)*e^(a*x) - a*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

3.81.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.24

$$\int e^{ax}\sin(bx)\,dx = \begin{cases} 0 & \text{for } a = 0 \wedge b = 0 \\ \frac{xe^{-ibx}\sin(bx)}{2} - \frac{ixe^{-ibx}\cos(bx)}{2} - \frac{e^{-ibx}\cos(bx)}{2b} & \text{for } a = -ib \\ \frac{xe^{ibx}\sin(bx)}{2} + \frac{ixe^{ibx}\cos(bx)}{2} - \frac{e^{ibx}\cos(bx)}{2b} & \text{for } a = ib \\ \frac{ae^{ax}\sin(bx)}{a^2+b^2} - \frac{be^{ax}\cos(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(a*x)*sin(b*x),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*exp(-I*b*x)*sin(b*x)/2 - I*x*exp(-I
*b*x)*cos(b*x)/2 - exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*b)), (x*exp(I*b*x)
*sin(b*x)/2 + I*x*exp(I*b*x)*cos(b*x)/2 - exp(I*b*x)*cos(b*x)/(2*b), Eq(a,
 I*b)), (a*exp(a*x)*sin(b*x)/(a**2 + b**2) - b*exp(a*x)*cos(b*x)/(a**2 + b
**2), True))
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int e^{ax}\sin(bx)\,dx = -\frac{(b\cos(bx) - a\sin(bx))e^{(ax)}}{a^2+b^2}$$

input
```
integrate(exp(a*x)*sin(b*x),x, algorithm="maxima")
```

output
```
-(b*cos(b*x) - a*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{ax}\sin(bx)\,dx = -\left(\frac{b\cos(bx)}{a^2+b^2} - \frac{a\sin(bx)}{a^2+b^2}\right)e^{(ax)}$$

input
```
integrate(exp(a*x)*sin(b*x),x, algorithm="giac")
```

output
```
-(b*cos(b*x)/(a^2 + b^2) - a*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int e^{ax}\sin(bx)\,dx = -\frac{e^{a\,x}\,(b\,\cos(b\,x) - a\,\sin(b\,x))}{a^2+b^2}$$

input
```
int(exp(a*x)*sin(b*x),x)
```

output
```
-(exp(a*x)*(b*cos(b*x) - a*sin(b*x)))/(a^2 + b^2)
```

3.82 $\int \cot^{-1}(x)\,dx$

3.82.1 Optimal result

Integrand size = 2, antiderivative size = 15

$$\int \cot^{-1}(x)\,dx = x\cot^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)$$

output
```
x*arccot(x)+1/2*ln(x^2+1)
```

3.82.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(x)\,dx = x\cot^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcCot[x],x]
```

output
```
x*ArcCot[x] + Log[1 + x^2]/2
```

3.82.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(x)\,dx$$

$$\Big\downarrow \text{5346}$$

$$\int \frac{x}{x^2+1}dx + x\cot^{-1}(x)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\log\left(x^2+1\right) + x\cot^{-1}(x)$$

input
```
Int[ArcCot[x],x]
```

output
```
x*ArcCot[x] + Log[1 + x^2]/2
```

3.82.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.82.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
lookup	$x \operatorname{arccot}(x)+\frac{\ln(x^2+1)}{2}$	14
default	$x \operatorname{arccot}(x)+\frac{\ln(x^2+1)}{2}$	14
parallelrisch	$x \operatorname{arccot}(x)+\frac{\ln(x^2+1)}{2}$	14
parts	$x \operatorname{arccot}(x)+\frac{\ln(x^2+1)}{2}$	14
risch	$\frac{ix\ln(ix+1)}{2}-\frac{ix\ln(-ix+1)}{2}+\frac{\pi x}{2}+\frac{\ln(x^2+1)}{2}$	36

input
```
int(arccot(x),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(x)+1/2*ln(x^2+1)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(x)\,dx = x\operatorname{arccot}(x)+\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(arccot(x),x, algorithm="fricas")
```

output
```
x*arccot(x) + 1/2*log(x^2 + 1)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cot^{-1}(x)\,dx = x\operatorname{acot}(x)+\frac{\log(x^2+1)}{2}$$

input
```
integrate(acot(x),x)
```

output
```
x*acot(x) + log(x**2 + 1)/2
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(x)\,dx = x \operatorname{arccot}(x) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(arccot(x),x, algorithm="maxima")
```

output
```
x*arccot(x) + 1/2*log(x^2 + 1)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cot^{-1}(x)\,dx = x \arctan\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(\frac{1}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{1}{x^2}\right)$$

input
```
integrate(arccot(x),x, algorithm="giac")
```

output
```
x*arctan(1/x) + 1/2*log(1/x^2 + 1) - 1/2*log(x^(-2))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(x)\,dx = \frac{\ln\left(x^2+1\right)}{2} + x\,\mathrm{acot}(x)$$

input
```
int(acot(x),x)
```

output
```
log(x^2 + 1)/2 + x*acot(x)
```

3.83 $\int \sec^{-1}(x)\,dx$

3.83.1 Optimal result

Integrand size = 2, antiderivative size = 19

$$\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
x*arcsec(x)-arctanh((1-1/x^2)^(1/2))
```

3.83.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(19) = 38.

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \frac{\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

input
```
Integrate[ArcSec[x],x]
```

output
```
x*ArcSec[x] - (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt
[-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

3.83.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5737, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}(x)\,dx$$

$$\Big\downarrow \text{ 5737}$$

$$x\sec^{-1}(x)-\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}+x\sec^{-1}(x)$$

$$\Big\downarrow \text{ 73}$$

$$x\sec^{-1}(x)-\int \frac{1}{1-\frac{1}{x^4}}d\sqrt{1-\frac{1}{x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$x\sec^{-1}(x)-\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

input
```
Int[ArcSec[x],x]
```

output
```
x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

3.83.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5737
```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Simp[1/c   In
t[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

3.83.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
lookup	$x\,\operatorname{arcsec}\left(x\right)-\ln\left(x+x\sqrt{1-\frac{1}{x^{2}}}\right)$	22
default	$x\,\operatorname{arcsec}\left(x\right)-\ln\left(x+x\sqrt{1-\frac{1}{x^{2}}}\right)$	22
parts	$x\,\operatorname{arcsec}\left(x\right)-\frac{\sqrt{x^{2}-1}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{\sqrt{\frac{x^{2}-1}{x^{2}}}\,x}$	39

input
```
int(arcsec(x),x,method=_RETURNVERBOSE)
```

output
```
x*arcsec(x)-ln(x+x*(1-1/x^2)^(1/2))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \sec^{-1}(x)\,dx = (x-2)\,\mathrm{arcsec}\,(x) + 4\,\arctan\left(-x+\sqrt{x^2-1}\right) + \log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate(arcsec(x),x, algorithm="fricas")
```

output
```
(x - 2)*arcsec(x) + 4*arctan(-x + sqrt(x^2 - 1)) + log(-x + sqrt(x^2 - 1))
```

3.83.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sec^{-1}(x)\,dx = x\,\mathrm{asec}\,(x) - \begin{cases} \mathrm{acosh}\,(x) & \text{for } |x^2| > 1 \\ -i\,\mathrm{asin}\,(x) & \text{otherwise} \end{cases}$$

input
```
integrate(asec(x),x)
```

output
```
x*asec(x) - Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \sec^{-1}(x)\,dx = x\,\mathrm{arcsec}\,(x) - \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

input
```
integrate(arcsec(x),x, algorithm="maxima")
```

output
```
x*arcsec(x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) +
1)
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sec^{-1}(x)\,dx = x\arccos\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

input
```
integrate(arcsec(x),x, algorithm="giac")
```

output
```
x*arccos(1/x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1)
+ 1)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sec^{-1}(x)\,dx = x\,\mathrm{acos}\left(\frac{1}{x}\right) - \ln\left(x+\sqrt{x^2-1}\right)\,\mathrm{sign}(x)$$

input
```
int(acos(1/x),x)
```

output
```
x*acos(1/x) - log(x + (x^2 - 1)^(1/2))*sign(x)
```

3.84 $\int \csc^{-1}(x) \, dx$

3.84.1 Optimal result

Integrand size = 2, antiderivative size = 17

$$\int \csc^{-1}(x) \, dx = x \csc^{-1}(x) + \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{x^2}}\right)$$

output
```
x*arccsc(x)+arctanh((1-1/x^2)^(1/2))
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \csc^{-1}(x) \, dx = x \csc^{-1}(x) + \frac{\sqrt{-1+x^2}\left(-\log\left(1 - \frac{x}{\sqrt{-1+x^2}}\right) + \log\left(1 + \frac{x}{\sqrt{-1+x^2}}\right)\right)}{2\sqrt{1 - \frac{1}{x^2}}x}$$

input
```
Integrate[ArcCsc[x],x]
```

output
```
x*ArcCsc[x] + (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt
[-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

3.84.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5738, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^{-1}(x) \, dx$$

$$\Big\downarrow \text{ 5738}$$

$$\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}dx+x \csc^{-1}(x)$$

$$\Big\downarrow \text{ 798}$$

$$x \csc^{-1}(x)-\frac{1}{2} \int \frac{x^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 73}$$

$$\int \frac{1}{1-\frac{1}{x^4}}d\sqrt{1-\frac{1}{x^2}}+x \csc^{-1}(x)$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)+x \csc^{-1}(x)$$

input
```
Int[ArcCsc[x],x]
```

output
```
x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

3.84.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5738
```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Simp[1/c   In
t[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

3.84.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result	size
lookup	$x \operatorname{arccsc}(x)+\ln\left(x+x\sqrt{1-\frac{1}{x^2}}\right)$	20
default	$x \operatorname{arccsc}(x)+\ln\left(x+x\sqrt{1-\frac{1}{x^2}}\right)$	20
parts	$x \operatorname{arccsc}(x)+\frac{\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{\frac{x^2-1}{x^2}}\,x}$	38

input
```
int(arccsc(x),x,method=_RETURNVERBOSE)
```

output
```
x*arccsc(x)+ln(x+x*(1-1/x^2)^(1/2))
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \csc^{-1}(x)\,dx = (x-2)\operatorname{arccsc}(x) - 4\arctan\left(-x+\sqrt{x^2-1}\right) - \log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate(arccsc(x),x, algorithm="fricas")
```

output
```
(x - 2)*arccsc(x) - 4*arctan(-x + sqrt(x^2 - 1)) - log(-x + sqrt(x^2 - 1))
```

3.84.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \csc^{-1}(x)\,dx = x\operatorname{acsc}(x) + \begin{cases} \operatorname{acosh}(x) & \text{for } |x^2| > 1 \\ -i\operatorname{asin}(x) & \text{otherwise} \end{cases}$$

input
```
integrate(acsc(x),x)
```

output
```
x*acsc(x) + Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \csc^{-1}(x)\,dx = x\operatorname{arccsc}(x) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

input
```
integrate(arccsc(x),x, algorithm="maxima")
```

output
```
x*arccsc(x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) +
1)
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \csc^{-1}(x)\,dx = x\arcsin\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

input
```
integrate(arccsc(x),x, algorithm="giac")
```

output
```
x*arcsin(1/x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1)
+ 1)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \csc^{-1}(x)\,dx = x\,\mathrm{asin}\left(\frac{1}{x}\right) + \ln\left(x+\sqrt{x^2-1}\right)\,\mathrm{sign}(x)$$

input
```
int(asin(1/x),x)
```

output
```
x*asin(1/x) + log(x + (x^2 - 1)^(1/2))*sign(x)
```

3.85 $\int \arcsin(x)^2\,dx$

3.85.1 Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \arcsin(x)^2\,dx = -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

output
```
-2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \arcsin(x)^2\,dx = -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

input
```
Integrate[ArcSin[x]^2,x]
```

output
```
-2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

3.85.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5130, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(x)^2\,dx$$

$$\Big\downarrow \text{ 5130}$$

$$x\arcsin(x)^2-2\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 5182}$$

$$x\arcsin(x)^2-2\left(\int 1dx-\sqrt{1-x^2}\arcsin(x)\right)$$

$$\Big\downarrow \text{ 24}$$

$$x\arcsin(x)^2-2\left(x-\sqrt{1-x^2}\arcsin(x)\right)$$

input
```
Int[ArcSin[x]^2,x]
```

output
```
x*ArcSin[x]^2 - 2*(x - Sqrt[1 - x^2]*ArcSin[x])
```

3.85.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

3.85.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-2x + x \arcsin(x)^2 + 2 \arcsin(x) \sqrt{-x^2+1}$	24

input
```
int(arcsin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

input
```
integrate(arcsin(x)^2,x, algorithm="fricas")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = x \operatorname{asin}^2(x) - 2x + 2\sqrt{1-x^2}\operatorname{asin}(x)$$

input
```
integrate(asin(x)**2,x)
```

output
```
x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2 \, dx = x \arcsin{(x)}^2 + 2 \sqrt{-x^2 + 1} \arcsin{(x)} - 2 \, x$$

input
```
integrate(arcsin(x)^2,x, algorithm="maxima")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2 \, dx = x \arcsin{(x)}^2 + 2 \sqrt{-x^2 + 1} \arcsin{(x)} - 2 \, x$$

input
```
integrate(arcsin(x)^2,x, algorithm="giac")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2 \, dx = 2 \operatorname{asin}(x) \sqrt{1 - x^2} + x \left(\operatorname{asin}(x)^2 - 2\right)$$

input
```
int(asin(x)^2,x)
```

output
```
2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

3.86 $\int \frac{\arcsin(x)}{x^2}\,dx$

3.86.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-arcsin(x)/x-arctanh((-x^2+1)^(1/2))
```

3.86.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

input
```
Integrate[ArcSin[x]/x^2,x]
```

output
```
-(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

3.86.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5138, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{x^2} \, dx$$
$$\Big\downarrow \text{ 5138}$$
$$\int \frac{1}{x\sqrt{1-x^2}}dx - \frac{\arcsin(x)}{x}$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2 - \frac{\arcsin(x)}{x}$$
$$\Big\downarrow \text{ 73}$$
$$-\int \frac{1}{1-x^4}d\sqrt{1-x^2} - \frac{\arcsin(x)}{x}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{\arcsin(x)}{x} - \text{arctanh}\left(\sqrt{1-x^2}\right)$$

input
```
Int[ArcSin[x]/x^2,x]
```

output
```
-(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

3.86.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.86.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21
parts	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$	21

input
```
int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\arcsin(x)}{x^2}\,dx=-\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin\left(x\right)}{2\,x}$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))
/x
```

3.86.6 Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

input
```
integrate(asin(x)/x**2,x)
```

output
```
Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

output
```
-arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="giac")
```

output
```
-arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arcsin(x)}{x^2} \, dx = -\text{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{\text{asin}(x)}{x}$$

input
```
int(asin(x)/x^2,x)
```

output
```
- atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

3.87 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

3.87.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

output
```
arctan(x/(a^2-x^2)^(1/2))
```

3.87.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

input
```
Integrate[1/Sqrt[a^2 - x^2],x]
```

output
```
ArcTan[x/Sqrt[a^2 - x^2]]
```

3.87.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{\frac{x^2}{a^2-x^2}+1}d\frac{x}{\sqrt{a^2-x^2}}$$
$$\Big\downarrow \text{ 216}$$
$$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

input
```
Int[1/Sqrt[a^2 - x^2],x]
```

output
```
ArcTan[x/Sqrt[a^2 - x^2]]
```

3.87.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.87.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$	15
pseudoelliptic	$-\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)$	19

input
```
int(1/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/(a^2-x^2)^(1/2))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = -2 \arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

input
```
integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

3.87.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \begin{cases} -i \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a**2-x**2)**(1/2),x)
```

output
```
Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

input
```
integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(x/a)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{2}\,a^2\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a) + \frac{1}{2}\,\sqrt{a^2-x^2}x$$

input
```
integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*a^2*arcsin(x/a)*sgn(a) + 1/2*sqrt(a^2 - x^2)*x
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

input
```
int(1/(a^2 - x^2)^(1/2),x)
```

output
```
atan(x/(a^2 - x^2)^(1/2))
```

3.88 $\int \frac{1}{\sqrt{1-2x-x^2}}\,dx$

3.88.1 Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \arcsin\left(\frac{1+x}{\sqrt{2}}\right)$$

output
```
arcsin(1/2*(1+x)*2^(1/2))
```

3.88.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = 2\arctan\left(\frac{x}{-1+\sqrt{1-2x-x^2}}\right)$$

input
```
Integrate[1/Sqrt[1 - 2*x - x^2],x]
```

output
```
2*ArcTan[x/(-1 + Sqrt[1 - 2*x - x^2])]
```

3.88.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-2x+1}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{8}(-2x-2)^2}}d(-2x-2)}{2\sqrt{2}}$$
$$\Big\downarrow \text{223}$$
$$-\arcsin\left(\frac{-2x-2}{2\sqrt{2}}\right)$$

input
```
Int[1/Sqrt[1 - 2*x - x^2],x]
```

output
```
-ArcSin[(-2 - 2*x)/(2*Sqrt[2])]
```

3.88.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.88.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\arcsin\left(\frac{(1+x)\sqrt{2}}{2}\right)$	10
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x+1}-\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	39

input
```
int(1/(-x^2-2*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(1/2*(1+x)*2^(1/2))
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2-2\,x+1}-1}{x}\right)$$

input
```
integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(-x^2 - 2*x + 1) - 1)/x)
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{\sqrt{2}(x+1)}{2}\right)$$

input
```
integrate(1/(-x**2-2*x+1)**(1/2),x)
```

output
```
asin(sqrt(2)*(x + 1)/2)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = -\arcsin\left(-\frac{1}{2}\sqrt{2}(x+1)\right)$$

input
```
integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/2*sqrt(2)*(x + 1))
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2-2x+1}(x+1) + \arcsin\left(\frac{1}{2}\sqrt{2}(x+1)\right)$$

input
```
integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 2*x + 1)*(x + 1) + arcsin(1/2*sqrt(2)*(x + 1))
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{\sqrt{8}\,(2x+2)}{8}\right)$$

input
```
int(1/(1 - x^2 - 2*x)^(1/2),x)
```

output
```
asin((8^(1/2)*(2*x + 2))/8)
```

3.89 $\int \frac{1}{a^2+x^2} \, dx$

3.89.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

output
```
arctan(x/a)/a
```

3.89.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
Integrate[(a^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/a]/a
```

3.89.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+x^2} \, dx$$
$$\Big\downarrow \; 216$$
$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
Int[(a^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/a]/a
```

3.89.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.89.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
parallelrisch	$-\frac{i\ln\left(-ia+x\right)-i\ln\left(ia+x\right)}{2a}$	27

input
```
int(1/(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/a)/a
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="fricas")
```

output
```
arctan(x/a)/a
```

3.89.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

input
```
integrate(1/(a**2+x**2),x)
```

output
```
(-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="maxima")
```

output
```
arctan(x/a)/a
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="giac")
```

output
```
arctan(x/a)/a
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

input
```
int(1/(a^2 + x^2),x)
```

output
```
atan(x/a)/a
```

3.90 $\int \frac{1}{a+bx^2} dx$

3.90.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.90.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2} \, dx$$
$$\Big\downarrow \ 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.90.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.90.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.91 $\int \frac{1}{2-x+x^2} dx$

3.91.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{2-x+x^2} dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

output
```
-2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-x+x^2} dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[(2 - x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

3.91.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-x+2}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(2x-1)^2-7}d(2x-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Int[(2 - x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

3.91.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.91.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\sqrt{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

input
```
int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
```

output
```
2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="fricas")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

input
```
integrate(1/(x**2-x+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="maxima")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="giac")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2x-1)}{7}\right)}{7}$$

input
```
int(1/(x^2 - x + 2),x)
```

output
```
(2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

3.92 $\int x \arctan(x) \, dx$

3.92.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \arctan(x) \, dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2} x^2 \arctan(x)$$

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

3.92.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x \arctan(x) \, dx = \frac{1}{2} \left(-x + \left(1 + x^2\right) \arctan(x)\right)$$

input
```
Integrate[x*ArcTan[x],x]
```

output
```
(-x + (1 + x^2)*ArcTan[x])/2
```

3.92.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(x)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}\int\frac{x^2}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\int\frac{1}{x^2+1}dx-x\right)+\frac{1}{2}x^2\arctan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}(\arctan(x)-x)$$

input
```
Int[x*ArcTan[x],x]
```

output
```
(x^2*ArcTan[x])/2 + (-x + ArcTan[x])/2
```

3.92.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.92.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
parallelrisch	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
parts	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

input
```
int(x*arctan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x\arctan(x)\,dx=\frac{1}{2}\left(x^2+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

input
```
integrate(x*arctan(x),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{x^2\operatorname{atan}(x)}{2} - \frac{x}{2} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x*atan(x),x)
```

output
```
x**2*atan(x)/2 - x/2 + atan(x)/2
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{1}{2}\,x^2\arctan(x) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan(x)$$

input
```
integrate(x*arctan(x),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{1}{2}\,x^2\arctan(x) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan(x)$$

input
```
integrate(x*arctan(x),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x\arctan(x)\,dx = \operatorname{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

input
```
int(x*atan(x),x)
```

output
```
atan(x)*(x^2/2 + 1/2) - x/2
```

3.93 $\int x^2 \arccos(x)\,dx$

3.93.1 Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^2 \arccos(x)\,dx = -\frac{1}{3}\sqrt{1-x^2}+\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\arccos(x)$$

output
```
1/9*(-x^2+1)^(3/2)+1/3*x^3*arccos(x)-1/3*(-x^2+1)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int x^2 \arccos(x)\,dx = -\frac{1}{9}\sqrt{1-x^2}\left(2+x^2\right)+\frac{1}{3}x^3\arccos(x)$$

input
```
Integrate[x^2*ArcCos[x],x]
```

output
```
-1/9*(Sqrt[1 - x^2]*(2 + x^2)) + (x^3*ArcCos[x])/3
```

3.93.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5139, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(x) \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}dx + \frac{1}{3}x^3 \arccos(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}\int \frac{x^2}{\sqrt{1-x^2}}dx^2 + \frac{1}{3}x^3 \arccos(x)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}\int \left(\frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2}\right)dx^2 + \frac{1}{3}x^3 \arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \arccos(x) + \frac{1}{6}\left(\frac{2}{3}\left(1-x^2\right)^{3/2} - 2\sqrt{1-x^2}\right)$$

input
```
Int[x^2*ArcCos[x],x]
```

output
```
(-2*Sqrt[1 - x^2] + (2*(1 - x^2)^(3/2))/3)/6 + (x^3*ArcCos[x])/3
```

3.93.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.93.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^3 \arccos(x)}{3} - \frac{x^2\sqrt{-x^2+1}}{9} - \frac{2\sqrt{-x^2+1}}{9}$	34
parts	$\frac{x^3 \arccos(x)}{3} - \frac{x^2\sqrt{-x^2+1}}{9} - \frac{2\sqrt{-x^2+1}}{9}$	34

input
```
int(x^2*arccos(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccos(x)-1/9*x^2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(1/2)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arccos(x)\, dx = \frac{1}{3}\, x^3 \arccos\left(x\right) - \frac{1}{9}\, \left(x^2 + 2\right) \sqrt{-x^2 + 1}$$

input
```
integrate(x^2*arccos(x),x, algorithm="fricas")
```

output
```
1/3*x^3*arccos(x) - 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^2 \arccos(x)\,dx = \frac{x^3 \operatorname{acos}(x)}{3} - \frac{x^2\sqrt{1-x^2}}{9} - \frac{2\sqrt{1-x^2}}{9}$$

input
```
integrate(x**2*acos(x),x)
```

output
```
x**3*acos(x)/3 - x**2*sqrt(1 - x**2)/9 - 2*sqrt(1 - x**2)/9
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arccos(x)\,dx = \frac{1}{3}x^3 \arccos(x) - \frac{1}{9}\sqrt{-x^2+1}x^2 - \frac{2}{9}\sqrt{-x^2+1}$$

input
```
integrate(x^2*arccos(x),x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arccos(x)\,dx = \frac{1}{3}x^3 \arccos(x) - \frac{1}{9}\sqrt{-x^2+1}x^2 - \frac{2}{9}\sqrt{-x^2+1}$$

input
```
integrate(x^2*arccos(x),x, algorithm="giac")
```

output
```
1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arccos(x)\, dx = \frac{x^3 \operatorname{acos}(x)}{3} - \frac{\sqrt{1-x^2}\,(x^2+2)}{9}$$

input
```
int(x^2*acos(x),x)
```

output
```
(x^3*acos(x))/3 - ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.94 $\int x \arctan(x)^2 \, dx$

3.94.1 Optimal result

Integrand size = 6, antiderivative size = 35

$$\int x \arctan(x)^2 \, dx = -x \arctan(x) + \frac{\arctan(x)^2}{2} + \frac{1}{2}x^2 \arctan(x)^2 + \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

3.94.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int x \arctan(x)^2 \, dx = \frac{1}{2}\left(-2x \arctan(x) + \left(1+x^2\right)\arctan(x)^2 + \log\left(1+x^2\right)\right)$$

input
```
Integrate[x*ArcTan[x]^2,x]
```

output
```
(-2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2 + Log[1 + x^2])/2
```

3.94.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5361, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(x)^2 \, dx$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{2}x^2 \arctan(x)^2 - \int \frac{x^2 \arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \; 5451$$

$$\int \frac{\arctan(x)}{x^2+1}dx - \int \arctan(x)dx + \frac{1}{2}x^2 \arctan(x)^2$$

$$\Big\downarrow \; 5345$$

$$\int \frac{\arctan(x)}{x^2+1}dx + \int \frac{x}{x^2+1}dx + \frac{1}{2}x^2 \arctan(x)^2 - x \arctan(x)$$

$$\Big\downarrow \; 240$$

$$\int \frac{\arctan(x)}{x^2+1}dx + \frac{1}{2}x^2 \arctan(x)^2 - x \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \; 5419$$

$$\frac{1}{2}x^2 \arctan(x)^2 + \frac{\arctan(x)^2}{2} - x \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[x*ArcTan[x]^2,x]
```

output
```
-(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^2]/2
```

3.94.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.94.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result
default	$-x \arctan \left(x \right)+\frac{\arctan \left(x \right)^{2}}{2}+\frac{x^{2} \arctan \left(x \right)^{2}}{2}+\frac{\ln \left(x^{2}+1\right)}{2}$
parallelrisch	$-x \arctan \left(x \right)+\frac{\arctan \left(x \right)^{2}}{2}+\frac{x^{2} \arctan \left(x \right)^{2}}{2}+\frac{\ln \left(x^{2}+1\right)}{2}$
parts	$-x \arctan \left(x \right)+\frac{\arctan \left(x \right)^{2}}{2}+\frac{x^{2} \arctan \left(x \right)^{2}}{2}+\frac{\ln \left(x^{2}+1\right)}{2}$
risch	$-\frac{\left(\frac{x^{2}}{2}+\frac{1}{2}\right) \ln \left(i x +1\right)^{2}}{4}-\frac{\left(-x^{2} \ln \left(-i x +1\right)-2 i x -\ln \left(-i x +1\right)\right) \ln \left(i x +1\right)}{4}-\frac{x^{2} \ln \left(-i x +1\right)^{2}}{8}-\frac{\ln \left(-i x +1\right)^{2}}{8}-\frac{i x \ln \left(-i x +1\right)}{2}$

input
```
int(x*arctan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int x \arctan(x)^2\,dx = \frac{1}{2}\left(x^2+1\right)\arctan\left(x\right)^2 - x\arctan\left(x\right) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x*arctan(x)^2,x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*arctan(x)^2 - x*arctan(x) + 1/2*log(x^2 + 1)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x \arctan(x)^2\,dx = \frac{x^2\operatorname{atan}^2\left(x\right)}{2} - x\operatorname{atan}\left(x\right) + \frac{\log\left(x^2+1\right)}{2} + \frac{\operatorname{atan}^2\left(x\right)}{2}$$

input
```
integrate(x*atan(x)**2,x)
```

output
```
x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x \arctan(x)^2\,dx =& \frac{1}{2}x^2\arctan\left(x\right)^2 - \left(x-\arctan\left(x\right)\right)\arctan\left(x\right)\\ &- \frac{1}{2}\arctan\left(x\right)^2 + \frac{1}{2}\log\left(x^2+1\right)\end{aligned}$$

input
```
integrate(x*arctan(x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(x)^2 - (x - arctan(x))*arctan(x) - 1/2*arctan(x)^2 + 1/2*lo
g(x^2 + 1)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\arctan(x)^2\,dx = \frac{1}{2}\,x^2\arctan\left(x\right)^2 - x\arctan\left(x\right) + \frac{1}{2}\,\arctan\left(x\right)^2 + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x*arctan(x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*arctan(x)^2 - x*arctan(x) + 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\arctan(x)^2\,dx = \frac{\ln\left(x^2+1\right)}{2} + \frac{\mathrm{atan}(x)^2}{2} + \frac{x^2\,\mathrm{atan}(x)^2}{2} - x\,\mathrm{atan}(x)$$

input
```
int(x*atan(x)^2,x)
```

output
```
log(x^2 + 1)/2 + atan(x)^2/2 + (x^2*atan(x)^2)/2 - x*atan(x)
```

3.95 $\int \arctan(\sqrt{x})\,dx$

3.95.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \arctan\left(\sqrt{x}\right)\,dx = -\sqrt{x} + \arctan\left(\sqrt{x}\right) + x\arctan\left(\sqrt{x}\right)$$

output
```
arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \arctan\left(\sqrt{x}\right)\,dx = -\sqrt{x} + (1+x)\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[ArcTan[Sqrt[x]],x]
```

output
```
-Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

3.95.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5345, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan \left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \; 5345$$

$$x \arctan \left(\sqrt{x}\right)-\frac{1}{2} \int \frac{\sqrt{x}}{x+1}dx$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2} \left(\int \frac{1}{\sqrt{x} (x+1)}dx-2 \sqrt{x}\right)+x \arctan \left(\sqrt{x}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2} \left(2 \int \frac{1}{x+1}d\sqrt{x}-2 \sqrt{x}\right)+x \arctan \left(\sqrt{x}\right)$$

$$\Big\downarrow \; 216$$

$$x \arctan \left(\sqrt{x}\right)+\frac{1}{2} \left(2 \arctan \left(\sqrt{x}\right)-2 \sqrt{x}\right)$$

input
```
Int[ArcTan[Sqrt[x]],x]
```

output
```
x*ArcTan[Sqrt[x]] + (-2*Sqrt[x] + 2*ArcTan[Sqrt[x]])/2
```

3.95.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.95.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
default	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
parts	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

input
```
int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \arctan\left(\sqrt{x}\right)\,dx = (x+1)\arctan\left(\sqrt{x}\right) - \sqrt{x}$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arctan(sqrt(x)) - sqrt(x)
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \arctan\left(\sqrt{x}\right)\,dx = -\sqrt{x} + x\,\mathrm{atan}\left(\sqrt{x}\right) + \mathrm{atan}\left(\sqrt{x}\right)$$

input
```
integrate(atan(x**(1/2)),x)
```

output
```
-sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="giac")
```

output
```
x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = \operatorname{atan}\left(\sqrt{x}\right) + x\operatorname{atan}\left(\sqrt{x}\right) - \sqrt{x}$$

input
```
int(atan(x^(1/2)),x)
```

output
```
atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

3.96 $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}(1+x)}\,dx$

3.96.1 Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

output
```
arctan(x^(1/2))^2
```

3.96.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

input
```
Integrate[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

output
```
ArcTan[Sqrt[x]]^2
```

3.96.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(x+1)}\,dx$$

$$\Big\downarrow \text{ 7237}$$

$$\arctan\left(\sqrt{x}\right)^2$$

input
```
Int[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

output
```
ArcTan[Sqrt[x]]^2
```

3.96.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.96.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)^2$	7
default	$\arctan\left(\sqrt{x}\right)^2$	7

input
```
int(arctan(x^(1/2))/(1+x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))^2
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

input
```
integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(x))^2
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \operatorname{atan}^2\left(\sqrt{x}\right)$$

input
```
integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
```

output
```
atan(sqrt(x))**2
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

input
```
integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(x))^2
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)^2$$

input
```
integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(x))^2
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \operatorname{atan}\left(\sqrt{x}\right)^2$$

input
```
int(atan(x^(1/2))/(x^(1/2)*(x + 1)),x)
```

output
```
atan(x^(1/2))^2
```

3.97 $\int \sqrt{1-x^2}\,dx$

3.97.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.97.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

3.97.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.97.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

input
```
int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x - \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{1-x^2}\,dx = \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((-x**2+1)**(1/2),x)
```

output
```
x*sqrt(1 - x**2)/2 + asin(x)/2
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \frac{x\sqrt{1-x^2}}{2}$$

input
```
int((1 - x^2)^(1/2),x)
```

output
```
asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

3.98 $\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx$

3.98.1 Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = -\frac{e^{\arctan(x)}(1-x)}{2\sqrt{1+x^2}}$$

output
```
-1/2*exp(arctan(x))*(1-x)/(x^2+1)^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{1}{2}(1-ix)^{-\frac{1}{2}+\frac{i}{2}}(1+ix)^{-\frac{1}{2}-\frac{i}{2}}(-1+x)$$

input
```
Integrate[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

output
```
(-1 + x)/(2*(1 - I*x)^(1/2 - I/2)*(1 + I*x)^(1/2 + I/2))
```

3.98.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5600}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\arctan(x)}}{(x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 5600$$

$$-\frac{(1-x)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

input
```
Int[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

output
```
-1/2*(E^ArcTan[x]*(1 - x))/Sqrt[1 + x^2]
```

3.98.3.1 Defintions of rubi rules used

rule 5600
```
Int[(E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcTan[a*x])/(d*(n^2 + 1)*Sqrt[c + d*x^2])
), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

3.98.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{(-1+x)e^{\arctan(x)}}{2\sqrt{x^2+1}}$	16

input
```
int(exp(arctan(x))*x/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1+x)*exp(arctan(x))/(x^2+1)^(1/2)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{(x-1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

input
```
integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(x - 1)*e^arctan(x)/sqrt(x^2 + 1)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} - \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

input
```
integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)
```

output
```
x*exp(atan(x))/(2*sqrt(x**2 + 1)) - exp(atan(x))/(2*sqrt(x**2 + 1))
```

3.98.7 Maxima [F]

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \int \frac{xe^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

input
```
integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*(x/sqrt(x^2 + 1) - 1/sqrt(x^2 + 1))*e^arctan(x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(x)}x}{(1+x^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{e}^{\mathrm{atan}(x)}}{(x^2+1)^{3/2}}\,dx$$

input
```
int((x*exp(atan(x)))/(x^2 + 1)^(3/2),x)
```

output
```
int((x*exp(atan(x)))/(x^2 + 1)^(3/2), x)
```

3.99 $\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}} dx$

3.99.1 Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}} dx = \frac{e^{\arctan(x)}(1+x)}{2\sqrt{1+x^2}}$$

output
```
1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}} dx = \frac{e^{\arctan(x)}(1+x)}{2\sqrt{1+x^2}}$$

input
```
Integrate[E^ArcTan[x]/(1 + x^2)^(3/2),x]
```

output
```
(E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

3.99.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(x)}}{(x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 5592$$

$$\frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

input
```
Int[E^ArcTan[x]/(1 + x^2)^(3/2),x]
```

output
```
(E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

3.99.3.1 Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

3.99.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{e^{\arctan(x)}(1+x)}{2\sqrt{x^2+1}}$	16

input
```
int(exp(arctan(x))/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

input
```
integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(x + 1)*e^arctan(x)/sqrt(x^2 + 1)
```

3.99.6 Sympy [A] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} + \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

input
```
integrate(exp(atan(x))/(x**2+1)**(3/2),x)
```

output
```
x*exp(atan(x))/(2*sqrt(x**2 + 1)) + exp(atan(x))/(2*sqrt(x**2 + 1))
```

3.99.7 Maxima [F]

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \int \frac{e^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

input
```
integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*(x/sqrt(x^2 + 1) + 1/sqrt(x^2 + 1))*e^arctan(x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(x)}}{(1+x^2)^{3/2}}\,dx = \int \frac{e^{\mathrm{atan}(x)}}{(x^2+1)^{3/2}}\,dx$$

input
```
int(exp(atan(x))/(x^2 + 1)^(3/2),x)
```

output
```
int(exp(atan(x))/(x^2 + 1)^(3/2), x)
```

3.100 $\int \frac{x^2}{(1+x^2)^2} dx$

3.100.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x^2}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

3.100.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

input
```
Integrate[x^2/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

3.100.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)^2}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\int \frac{1}{x^2+1}dx - \frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)}$$

input
```
Int[x^2/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

3.100.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.100.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2x}{4(x^2+1)}$	52

input
```
int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)-x}{2\,(x^2+1)}$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x**2/(x**2+1)**2,x)
```

output
```
-x/(2*x**2 + 2) + atan(x)/2
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x)$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x)$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+x^2)^2} \, dx = \frac{\operatorname{atan}(x)}{2} - \frac{x}{2(x^2+1)}$$

input
```
int(x^2/(x^2 + 1)^2,x)
```

output
```
atan(x)/2 - x/(2*(x^2 + 1))
```

3.101 $\int \frac{e^x}{1+e^{2x}}\,dx$

3.101.1 Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

output
```
arctan(exp(x))
```

3.101.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
Integrate[E^x/(1 + E^(2*x)),x]
```

output
```
ArcTan[E^x]
```

3.101.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}+1} dx$$

$$\Big\downarrow \text{2679}$$

$$\int \frac{1}{e^{2x}+1} de^x$$

$$\Big\downarrow \text{216}$$

$$\arctan(e^x)$$

input
```
Int[E^x/(1 + E^(2*x)),x]
```

output
```
ArcTan[E^x]
```

3.101.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.101.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan\left(\mathrm{e}^{x}\right)$	4
risch	$\frac{i \ln\left(\mathrm{e}^{x}+i\right)}{2}-\frac{i \ln\left(\mathrm{e}^{x}-i\right)}{2}$	20

input
```
int(exp(x)/(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
arctan(exp(x))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx=\arctan\left(e^{x}\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
```

output
```
arctan(e^x)
```

3.101.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1+e^{2x}}\,dx=\operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
```

output
```
arctan(e^x)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
```

output
```
arctan(e^x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \mathrm{atan}(\mathrm{e}^x)$$

input
```
int(exp(x)/(exp(2*x) + 1),x)
```

output
```
atan(exp(x))
```

3.102 $\int e^{-x} \cot^{-1}(e^x) \, dx$

3.102.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2} \log\left(1 + e^{2x}\right)$$

output
```
-x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

3.102.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2} \log\left(1 + e^{2x}\right)$$

input
```
Integrate[ArcCot[E^x]/E^x,x]
```

output
```
-x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

3.102.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5731, 25, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\cot^{-1}\left(e^x\right)dx$$

$$\Big\downarrow \text{5731}$$

$$\int -\frac{1}{1+e^{2x}}dx-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{1+e^{2x}}dx-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}\int \frac{e^{-2x}}{1+e^{2x}}de^{2x}-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{1+e^{2x}}de^{2x}-\int e^{-2x}de^{2x}\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\int \frac{1}{1+e^{2x}}de^{2x}-\log\left(e^{2x}\right)\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(e^{2x}+1\right)-\log\left(e^{2x}\right)\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

input
```
Int[ArcCot[E^x]/E^x,x]
```

output
```
-(ArcCot[E^x]/E^x) + (-Log[E^(2*x)] + Log[1 + E^(2*x)])/2
```

3.102.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

3.102.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\operatorname{arccot}\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-\ln\left(\mathrm{e}^{x}\right)$	25
default	$-\operatorname{arccot}\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-\ln\left(\mathrm{e}^{x}\right)$	25
parallelrisch	$\frac{\left(\ln\left(1+\mathrm{e}^{2 x}\right) \mathrm{e}^{x}-2 \,\mathrm{e}^{x} x-2 \operatorname{arccot}\left(\mathrm{e}^{x}\right)\right) \mathrm{e}^{-x}}{2}$	28
risch	$-\frac{i \mathrm{e}^{-x} \ln\left(1+i \mathrm{e}^{x}\right)}{2}+\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-x+\frac{i \mathrm{e}^{-x} \ln\left(1-i \mathrm{e}^{x}\right)}{2}-\frac{\mathrm{e}^{-x} \pi}{2}$	51

input
```
int(arccot(exp(x))/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-arccot(exp(x))/exp(x)+1/2*ln(1+exp(x)^2)-ln(exp(x))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -\frac{1}{2}\left(2\,xe^{x} - e^{x}\log\left(e^{(2\,x)}+1\right) + 2\,\mathrm{arccot}\left(e^{x}\right)\right)e^{(-x)}$$

input
```
integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
```

output
```
-1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

3.102.6 Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -x + \frac{\log\left(e^{2x}+1\right)}{2} - e^{-x}\,\mathrm{acot}\left(e^{x}\right)$$

input
```
integrate(acot(exp(x))/exp(x),x)
```

output
```
-x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -\,\mathrm{arccot}\left(e^{x}\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
```

output
```
-arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -\arctan\left(e^{(-x)}\right) e^{(-x)} + \frac{1}{2} \log\left(e^{(-2x)} + 1\right)$$

input
```
integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
```

output
```
-arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-x} \cot^{-1}(e^x) \, dx = \frac{\ln(e^{2x} + 1)}{2} - x - \operatorname{acot}(e^x) \, e^{-x}$$

input
```
int(acot(exp(x))*exp(-x),x)
```

output
```
log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

3.103 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

3.103.1 Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + 2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)$$

output
```
2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x}+2a\sqrt{a-x}\arctan\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}$$

input
```
Integrate[Sqrt[(a + x)/(a - x)],x]
```

output
```
(Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt
[a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

3.103.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

$$\Big\downarrow \ 2051$$

$$4a\int \frac{a+x}{(a-x)\left(\frac{a+x}{a-x}+1\right)^2}d\sqrt{\frac{a+x}{a-x}}$$

$$\Big\downarrow \ 252$$

$$4a\left(\frac{1}{2}\int \frac{1}{\frac{a+x}{a-x}+1}d\sqrt{\frac{a+x}{a-x}}-\frac{\sqrt{\frac{a+x}{a-x}}}{2\left(\frac{a+x}{a-x}+1\right)}\right)$$

$$\Big\downarrow \ 216$$

$$4a\left(\frac{1}{2}\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{\sqrt{\frac{a+x}{a-x}}}{2\left(\frac{a+x}{a-x}+1\right)}\right)$$

input
```
Int[Sqrt[(a + x)/(a - x)],x]
```

output
```
4*a*(-1/2*Sqrt[(a + x)/(a - x)]/(1 + (a + x)/(a - x)) + ArcTan[Sqrt[(a + x
)/(a - x)]]/2)
```

3.103.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.103.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(\sqrt{a^2-x^2}-a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\right)}{\sqrt{(a-x)(a+x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{a+x}$	90

input
```
int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a+x)/(a-x))^(1/2)*(a-x)*((a^2-x^2)^(1/2)-a*arctan(x/(a^2-x^2)^(1/2)))/(
(a-x)*(a+x))^(1/2)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

output
```
2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

3.103.6 Sympy [F]

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \int \sqrt{\frac{a+x}{a-x}}\,dx$$

input
```
integrate(((a+x)/(a-x))**(1/2),x)
```

output
```
Integral(sqrt((a + x)/(a - x)), x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1} - \arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
 - x))))
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

output
```
a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right) - \frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

input
```
int(((a + x)/(a - x))^(1/2),x)
```

output
```
2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)
/(a - x) + 1)
```

3.104 $\int \sqrt{(b-x)(-a+x)}\,dx$

3.104.1 Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

output
```
-1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*
(-a*b+(a+b)*x-x^2)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x+\frac{(a-b)^2\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{b-x}\sqrt{-a+x}}\right)$$

input
```
Integrate[Sqrt[(b - x)*(-a + x)],x]
```

output
```
(Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x + ((a - b)^2*ArcTan[Sqrt[-a + x]/Sqr
t[b - x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

3.104.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2048, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(x-a)(b-x)}\,dx$$

$$\Big\downarrow \text{2048}$$

$$\int \sqrt{x(a+b)-ab-x^2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-x^2+(a+b)x-ab}}dx-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{4}(a-b)^2\int \frac{1}{-\frac{(a+b-2x)^2}{-x^2+(a+b)x-ab}-4}d\frac{a+b-2x}{\sqrt{-x^2+(a+b)x-ab}}-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

input
```
Int[Sqrt[(b - x)*(-a + x)],x]
```

output
```
-1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
 + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

3.104.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.104.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{(a+b-2x)\sqrt{-ab+(a+b)x-x^2}}{4}-\frac{\left(4ab-(a+b)^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{8}$	68
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\left(\frac{1}{4}ab-\frac{1}{8}b^2-\frac{1}{8}a^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	78

input
```
int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/8*(4*a*b-(a+b)^2)*arctan((x-1/2*
b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right) - \frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
 2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
 2*x)
```

3.104.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \sqrt{(b-x)(-a+x)}\,dx = \left(-\frac{ab}{2} + \frac{\left(\frac{a}{4}+\frac{b}{4}\right)(a+b)}{2}\right)\left(\begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for}\ ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}\right) + \left(-\frac{a}{4}-\frac{b}{4}+\frac{x}{2}\right)\sqrt{-ab-x^2+x\,(a+b)}$$

input
```
integrate(((b-x)*(-a+x))**(1/2),x)
```

output
```
(-a*b/2 + (a/4 + b/4)*(a + b)/2)*Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(
-a*b - x**2 + x*(a + b))), Ne(a*b - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*l
og(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/2 + x)**2), True)) + (-a/4 - b/4 + x/2)
*sqrt(-a*b - x**2 + x*(a + b))
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sq
rt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

input
```
int((-(a - x)*(b - x))^(1/2),x)
```

output
```
int((-(a - x)*(b - x))^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

3.105.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

output
```
-arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

3.105.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{2\sqrt{b-x}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{(a-x)(-b+x)}}$$

input
```
Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

output
```
(2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]])/Sqrt[(a - x)
*(-b + x)]
```

3.105.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2048, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(x-a)(b-x)}} \, dx$$

$$\Big\downarrow \; 2048$$

$$\int \frac{1}{\sqrt{x(a+b)-ab-x^2}}dx$$

$$\Big\downarrow \; 1092$$

$$2\int \frac{1}{-\frac{(a+b-2x)^2}{-x^2+(a+b)x-ab}-4}d\frac{a+b-2x}{\sqrt{-x^2+(a+b)x-ab}}$$

$$\Big\downarrow \; 217$$

$$-\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

input
```
Int[1/Sqrt[(b - x)*(-a + x)],x]
```

output
```
-ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

3.105.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.105.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
default	$\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	28

input
```
int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2x)}{2(ab-(a+b)x+x^2)}\right)$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

output
```
-arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
 x^2))
```

3.105.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b-x)*(-a+x))**(1/2),x)
```

output
```
Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-a*b - x**2 + x*(a + b))), Ne(a*b
 - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log(-a/2 - b/2 + x)/sqrt(-(-a/2 -
b/2 + x)**2), True))
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sq
rt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

input
```
int(1/(-(a - x)*(b - x))^(1/2),x)
```

output
```
int(1/(-(a - x)*(b - x))^(1/2), x)
```

3.106 $\int \frac{3+5x}{-3+2x+x^2} \, dx$

3.106.1 Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \frac{3+5x}{-3+2x+x^2} \, dx = 2\log(1-x) + 3\log(3+x)$$

output
```
2*ln(1-x)+3*ln(3+x)
```

3.106.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{3+5x}{-3+2x+x^2} \, dx = 2\log(1-x) + 3\log(3+x)$$

input
```
Integrate[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

output
```
2*Log[1 - x] + 3*Log[3 + x]
```

3.106.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x+3}{x^2+2x-3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(\frac{3}{x+3}-\frac{2}{1-x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(1-x)+3\log(x+3)$$

input
```
Int[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

output
```
2*Log[1 - x] + 3*Log[3 + x]
```

3.106.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.106.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$2\ln\left(-1+x\right)+3\ln\left(3+x\right)$	14
norman	$2\ln\left(-1+x\right)+3\ln\left(3+x\right)$	14
risch	$2\ln\left(-1+x\right)+3\ln\left(3+x\right)$	14
parallelrisch	$2\ln\left(-1+x\right)+3\ln\left(3+x\right)$	14

input
```
int((3+5*x)/(x^2+2*x-3),x,method=_RETURNVERBOSE)
```

output
```
2*ln(-1+x)+3*ln(3+x)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)$$

input
```
integrate((3+5*x)/(x^2+2*x-3),x, algorithm="fricas")
```

output
```
3*log(x + 3) + 2*log(x - 1)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 2\log\left(x-1\right)+3\log\left(x+3\right)$$

input
```
integrate((3+5*x)/(x**2+2*x-3),x)
```

output
```
2*log(x - 1) + 3*log(x + 3)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)$$

input
```
integrate((3+5*x)/(x^2+2*x-3),x, algorithm="maxima")
```

output
```
3*log(x + 3) + 2*log(x - 1)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 3\,\log\left(|x+3|\right)+2\,\log\left(|x-1|\right)$$

input
```
integrate((3+5*x)/(x^2+2*x-3),x, algorithm="giac")
```

output
```
3*log(abs(x + 3)) + 2*log(abs(x - 1))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+5x}{-3+2x+x^2}\,dx = 2\,\ln\left(x-1\right)+3\,\ln\left(x+3\right)$$

input
```
int((5*x + 3)/(2*x + x^2 - 3),x)
```

output
```
2*log(x - 1) + 3*log(x + 3)
```

3.107 $\int \frac{5+2x}{-3+2x+x^2} dx$

3.107.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{5+2x}{-3+2x+x^2} dx = \frac{7}{4}\log(1-x) + \frac{1}{4}\log(3+x)$$

output
```
7/4*ln(1-x)+1/4*ln(3+x)
```

3.107.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{5+2x}{-3+2x+x^2} dx = \frac{7}{4}\log(1-x) + \frac{1}{4}\log(3+x)$$

input
```
Integrate[(5 + 2*x)/(-3 + 2*x + x^2),x]
```

output
```
(7*Log[1 - x])/4 + Log[3 + x]/4
```

3.107.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+5}{x^2+2x-3}\,dx$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{1}{4(x+3)}-\frac{7}{4(1-x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

input
```
Int[(5 + 2*x)/(-3 + 2*x + x^2),x]
```

output
```
(7*Log[1 - x])/4 + Log[3 + x]/4
```

3.107.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.107.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
norman	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
risch	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
parallelrisch	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14

input
```
int((5+2*x)/(x^2+2*x-3),x,method=_RETURNVERBOSE)
```

output
```
7/4*ln(-1+x)+1/4*ln(3+x)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{5+2x}{-3+2x+x^2}\,dx=\frac{1}{4}\log\left(x+3\right)+\frac{7}{4}\log\left(x-1\right)$$

input
```
integrate((5+2*x)/(x^2+2*x-3),x, algorithm="fricas")
```

output
```
1/4*log(x + 3) + 7/4*log(x - 1)
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{5+2x}{-3+2x+x^2}\,dx=\frac{7\log\left(x-1\right)}{4}+\frac{\log\left(x+3\right)}{4}$$

input
```
integrate((5+2*x)/(x**2+2*x-3),x)
```

output
```
7*log(x - 1)/4 + log(x + 3)/4
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{1}{4}\log(x+3) + \frac{7}{4}\log(x-1)$$

input
```
integrate((5+2*x)/(x^2+2*x-3),x, algorithm="maxima")
```

output
```
1/4*log(x + 3) + 7/4*log(x - 1)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{1}{4}\log(|x+3|) + \frac{7}{4}\log(|x-1|)$$

input
```
integrate((5+2*x)/(x^2+2*x-3),x, algorithm="giac")
```

output
```
1/4*log(abs(x + 3)) + 7/4*log(abs(x - 1))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{5+2x}{-3+2x+x^2}\,dx = \frac{7\ln(x-1)}{4} + \frac{\ln(x+3)}{4}$$

input
```
int((2*x + 5)/(2*x + x^2 - 3),x)
```

output
```
(7*log(x - 1))/4 + log(x + 3)/4
```

3.108 $\int \frac{3x+x^3}{-3-2x+x^2}\,dx$

3.108.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx=2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)$$

output
```
2*x+1/2*x^2+9*ln(3-x)+ln(1+x)
```

3.108.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx=2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)$$

input
```
Integrate[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

output
```
2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

3.108.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2027, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+3x}{x^2-2x-3}\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(x^2+3)}{x^2-2x-3}dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{2(5x+3)}{x^2-2x-3}+x+2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

input
```
Int[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

output
```
2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.108.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20
norman	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20
risch	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20
parallelrisch	$2x+\frac{x^2}{2}+\ln(1+x)+9\ln(-3+x)$	20

input
```
int((x^3+3*x)/(x^2-2*x-3),x,method=_RETURNVERBOSE)
```

output
```
2*x+1/2*x^2+ln(1+x)+9*ln(-3+x)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx=\frac{1}{2}\,x^2+2\,x+\log(x+1)+9\,\log(x-3)$$

input
```
integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="fricas")
```

output
```
1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x+x^3}{-3-2x+x^2}\,dx=\frac{x^2}{2}+2x+9\log(x-3)+\log(x+1)$$

input
```
integrate((x**3+3*x)/(x**2-2*x-3),x)
```

output
```
x**2/2 + 2*x + 9*log(x - 3) + log(x + 1)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x + x^3}{-3 - 2x + x^2} \, dx = \frac{1}{2} \, x^2 + 2 \, x + \log{(x + 1)} + 9 \, \log{(x - 3)}$$

input
```
integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="maxima")
```

output
```
1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{3x + x^3}{-3 - 2x + x^2} \, dx = \frac{1}{2} \, x^2 + 2 \, x + \log{(|x + 1|)} + 9 \, \log{(|x - 3|)}$$

input
```
integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="giac")
```

output
```
1/2*x^2 + 2*x + log(abs(x + 1)) + 9*log(abs(x - 3))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3x + x^3}{-3 - 2x + x^2} \, dx = 2 \, x + \ln{(x + 1)} + 9 \, \ln{(x - 3)} + \frac{x^2}{2}$$

input
```
int(-(3*x + x^3)/(2*x - x^2 + 3),x)
```

output
```
2*x + log(x + 1) + 9*log(x - 3) + x^2/2
```

3.109 $\int \frac{-1+5x+2x^2}{-2x+x^2+x^3} \, dx$

3.109.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3} \, dx = 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)$$

output
```
2*ln(1-x)+1/2*ln(x)-1/2*ln(2+x)
```

3.109.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3} \, dx = 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)$$

input
```
Integrate[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

output
```
2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

3.109.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+5x-1}{x^3+x^2-2x} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{2x^2+5x-1}{x\left(x^2+x-2\right)}dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{1}{2x}-\frac{1}{2(x+2)}+\frac{2}{x-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

input
```
Int[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

output
```
2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.109.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18
norman	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18
risch	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18
parallelrisch	$2\ln\left(-1+x\right)-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}$	18

input
```
int((2*x^2+5*x-1)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
```

output
```
2*ln(-1+x)-1/2*ln(2+x)+1/2*ln(x)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = -\frac{1}{2}\,\log\left(x+2\right)+2\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
```

output
```
-1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = \frac{\log\left(x\right)}{2}+2\log\left(x-1\right)-\frac{\log\left(x+2\right)}{2}$$

input
```
integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)
```

output
```
log(x)/2 + 2*log(x - 1) - log(x + 2)/2
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = -\frac{1}{2}\,\log(x+2) + 2\,\log(x-1) + \frac{1}{2}\,\log(x)$$

input
```
integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
```

output
```
-1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = -\frac{1}{2}\,\log(|x+2|) + 2\,\log(|x-1|) + \frac{1}{2}\,\log(|x|)$$

input
```
integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 2)) + 2*log(abs(x - 1)) + 1/2*log(abs(x))
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx = 2\,\ln(x-1) + \operatorname{atanh}\left(\frac{135}{11\,(11\,x-5)} + \frac{16}{11}\right)$$

input
```
int((5*x + 2*x^2 - 1)/(x^2 - 2*x + x^3),x)
```

output
```
2*log(x - 1) + atanh(135/(11*(11*x - 5)) + 16/11)
```

3.110 $\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} \, dx$

3.110.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} \, dx = \frac{1}{1+x} + \frac{3}{2}\log(1-x) - \frac{1}{2}\log(1+x)$$

output
```
1/(1+x)+3/2*ln(1-x)-1/2*ln(1+x)
```

3.110.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} \, dx = \frac{1}{1+x} + \frac{3}{2}\log(-1+x) - \frac{1}{2}\log(1+x)$$

input
```
Integrate[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

output
```
(1 + x)^(-1) + (3*Log[-1 + x])/2 - Log[1 + x]/2
```

3.110.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x+3}{(x-1)(x+1)^2} \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(-\frac{1}{2(x+1)}-\frac{1}{(x+1)^2}+\frac{3}{2(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

input
```
Int[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

output
```
(1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

3.110.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.110.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
norman	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
risch	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
parallelrisch	$\frac{3\ln(-1+x)x-\ln(1+x)x+2+3\ln(-1+x)-\ln(1+x)}{2x+2}$	36

input
```
int((x^2+2*x+3)/(-1+x)/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(-1+x)+1/(1+x)-1/2*ln(1+x)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = -\frac{(x+1)\log(x+1)-3(x+1)\log(x-1)-2}{2(x+1)}$$

input
```
integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="fricas")
```

output
```
-1/2*((x + 1)*log(x + 1) - 3*(x + 1)*log(x - 1) - 2)/(x + 1)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{3\log(x-1)}{2}-\frac{\log(x+1)}{2}+\frac{1}{x+1}$$

input
```
integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)
```

output
```
3*log(x - 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{1}{x+1} - \frac{1}{2}\log(x+1) + \frac{3}{2}\log(x-1)$$

input
```
integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="maxima")
```

output
```
1/(x + 1) - 1/2*log(x + 1) + 3/2*log(x - 1)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{1}{x+1} + \log(|x+1|) + \frac{3}{2}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

input
```
integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="giac")
```

output
```
1/(x + 1) + log(abs(x + 1)) + 3/2*log(abs(-2/(x + 1) + 1))
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx = \frac{3\ln(x-1)}{2} - \frac{\ln(x+1)}{2} + \frac{1}{x+1}$$

input
```
int((2*x + x^2 + 3)/((x - 1)*(x + 1)^2),x)
```

output
```
(3*log(x - 1))/2 - log(x + 1)/2 + 1/(x + 1)
```

3.111 $\int \frac{-2+2x+3x^2}{-1+x^3}\,dx$

3.111.1 Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)$$

output
```
ln(-x^3+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)$$

input
```
Integrate[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

output
```
(4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

3.111.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2410, 25, 792, 2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2x-2}{x^3-1} \, dx$$
$$\Big\downarrow \text{ 2410}$$
$$\int \frac{2x-2}{x^3-1}dx+3\int -\frac{x^2}{1-x^3}dx$$
$$\Big\downarrow \text{ 25}$$
$$\int \frac{2x-2}{x^3-1}dx-3\int \frac{x^2}{1-x^3}dx$$
$$\Big\downarrow \text{ 792}$$
$$\int \frac{2x-2}{x^3-1}dx+\log\left(1-x^3\right)$$
$$\Big\downarrow \text{ 2019}$$
$$\int \frac{1}{\frac{x^2}{2}+\frac{x}{2}+\frac{1}{2}}dx+\log\left(1-x^3\right)$$
$$\Big\downarrow \text{ 1083}$$
$$\log\left(1-x^3\right)-2\int \frac{1}{-\left(x+\frac{1}{2}\right)^2-\frac{3}{4}}d\left(x+\frac{1}{2}\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{4\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(1-x^3\right)$$

input
```
Int[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

output
```
(4*ArcTan[(2*(1/2 + x))/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.111.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
default	$\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\ln\left(4x^{2}+4x+4\right)+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln\left(-1+x\right)$
meijerg	$-\frac{2x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\ln\left(-x^{3}+1\right)+$ $2x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\ln\left(1+(x\right.\right.$ …

input
```
int((3*x^2+2*x-2)/(x^3-1),x,method=_RETURNVERBOSE)
```

output
```
ln(-1+x)+ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x-1\right)$$

input
```
integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="fricas")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.11

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \log\left(x-1\right)$$

input
```
integrate((3*x**2+2*x-2)/(x**3-1),x)
```

output
```
log(x - 1)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x-1\right)$$

input
```
integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="maxima")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log\left(|x-1|\right)$$

input
```
integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="giac")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x -
 1))
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

$$\int \frac{-2+2x+3x^2}{-1+x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x-1\right) - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,2\mathrm{i}}{3} + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,2\mathrm{i}}{3}$$

input
```
int((2*x + 3*x^2 - 2)/(x^3 - 1),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x - 1)
 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*2i)/3 + (3^(1/2)*log(x + (3^(1/2
)*1i)/2 + 1/2)*2i)/3
```

3.112 $\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)(2+x^2)^2} dx$

3.112.1 Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)(2+x^2)^2} dx = \frac{1}{2(2+x^2)} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{1}{3}\log(1-x) + \frac{1}{3}\log\left(2+x^2\right)$$

output
```
1/2/(x^2+2)+1/3*ln(1-x)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)(2+x^2)^2} dx = \frac{1}{2(3+2(-1+x)+(-1+x)^2)} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{1}{3}\log\left(3+2(-1+x)+(-1+x)^2\right) + \frac{1}{3}\log(-1+x)$$

input
```
Integrate[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

output
```
1/(2*(3 + 2*(-1 + x) + (-1 + x)^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[
3 + 2*(-1 + x) + (-1 + x)^2]/3 + Log[-1 + x]/3
```

3.112.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2178, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2}dx$$

$$\Big\downarrow \text{2178}$$

$$\frac{1}{2(x^2+2)}-\frac{1}{4}\int \frac{4(x^2-x+1)}{(1-x)(x^2+2)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2(x^2+2)}-\int \frac{x^2-x+1}{(1-x)(x^2+2)}dx$$

$$\Big\downarrow \text{2160}$$

$$\frac{1}{2(x^2+2)}-\int \left(\frac{1-2x}{3(x^2+2)}-\frac{1}{3(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{2(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)$$

input
```
Int[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

output
```
1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 + Log[2 + x
^2]/3
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.112.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\ln(-1+x)}{3}+\frac{1}{2x^2+4}+\frac{\ln(x^2+2)}{3}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{6}$	37
risch	$\frac{\ln(-1+x)}{3}+\frac{1}{2x^2+4}+\frac{\ln(x^2+2)}{3}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{6}$	37

input
```
int((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(-1+x)+1/2/(x^2+2)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = -\frac{\sqrt{2}(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-2\left(x^2+2\right)\log\left(x^2+2\right)-2\left(x^2+2\right)\log\left(x-1\right)-3}{6\left(x^2+2\right)}$$

input
```
integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/6*(sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) - 2*(x^2 + 2)*log(x^2 + 2) -
 2*(x^2 + 2)*log(x - 1) - 3)/(x^2 + 2)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.29

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = \frac{\log\left(x-1\right)}{3}+\frac{1}{2x^2+4}$$

input
```
integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
```

output
```
log(x - 1)/3 + 1/(2*x**2 + 4)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)} + \frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="maxima")
```

output
```
-1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/
3*log(x - 1)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/
3*log(abs(x - 1))
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx = \frac{\ln(x-1)}{3}+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right)+\frac{1}{2\,\left(x^2+2\right)}$$

input
```
int((2*x^2 - x - x^3 + x^4 + 2)/((x^2 + 2)^2*(x - 1)),x)
```

output
```
log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/12 + 1/3) - log(x + 2^(1/
2)*1i)*((2^(1/2)*1i)/12 - 1/3) + 1/(2*(x^2 + 2))
```

3.113 $\int \frac{1}{\cos(x)+\sin(x)} \, dx$

3.113.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{\cos(x)+\sin(x)} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\cos(x)+\sin(x)} \, dx = (-1-i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

input
```
Integrate[(Cos[x] + Sin[x])^(-1),x]
```

output
```
(-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

3.113.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\cos(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin(x)+\cos(x)}dx$$
$$\Big\downarrow \text{ 3553}$$
$$-\int \frac{1}{2-(\cos(x)-\sin(x))^2}d(\cos(x)-\sin(x))$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{\text{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[(Cos[x] + Sin[x])^(-1),x]
```

output
```
-(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

3.113.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.113.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$	19
risch	$\frac{\sqrt{2}\,\ln\left(e^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}$	48

input
```
int(1/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)+1}\right)$$

input
```
integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*co
s(x)*sin(x) + 1))
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = \frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{2} - \frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{2}$$

input
```
integrate(1/(cos(x)+sin(x)),x)
```

output
```
sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/2 - sqrt(2)*log(tan(x/2) - sqrt(2) - 1
)/2
```

3.113.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

input
```
integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(c
os(x) + 1) - 1))
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

input
```
integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(
1/2*x) - 2))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}\tan\left(\frac{x}{2}\right)}{2}\right)$$

input
```
int(1/(cos(x) + sin(x)),x)
```

output
```
-2^(1/2)*atanh(2^(1/2)/2 - (2^(1/2)*tan(x/2))/2)
```

3.114 $\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx$

3.114.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(1+\sqrt{4-x^2}\right)$$

output
```
-ln(1+(-x^2+4)^(1/2))
```

3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(1+\sqrt{4-x^2}\right)$$

input
```
Integrate[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

output
```
-Log[1 + Sqrt[4 - x^2]]
```

3.114.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{-x^2+\sqrt{4-x^2}+4}dx$$

$$\Big\downarrow \text{2586}$$

$$\frac{1}{2}\int \frac{1}{-x^2+\sqrt{4-x^2}+4}dx^2$$

$$\Big\downarrow \text{7267}$$

$$-\int \frac{1}{\sqrt{4-x^2}+1}d\sqrt{4-x^2}$$

$$\Big\downarrow \text{16}$$

$$-\log\left(\sqrt{4-x^2}+1\right)$$

input
```
Int[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

output
```
-Log[1 + Sqrt[4 - x^2]]
```

3.114.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.114.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
trager	$-\ln\left(-1-\sqrt{-x^{2}+4}\right)$
default	$-\frac{\ln\left(x^{2}-3\right)}{2}+\frac{\sqrt{-\left(-2+x\right)^{2}-4x+8}-2\arcsin\left(\frac{x}{2}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}+\frac{\sqrt{-\left(2+x\right)^{2}+4x+8}+2\arcsin\left(\frac{x}{2}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}-\frac{\sqrt{-\left(x-\sqrt{3}\right)^{2}-2\sqrt{3}\left(x-\sqrt{3}\right)+1}-\sqrt{3}\,\ldots}{2\left(2\ldots\right.}$

input
```
int(x/(4-x^2+(-x^2+4)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1-(-x^2+4)^(1/2))
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

$$\int \frac{x}{4-x^{2}+\sqrt{4-x^{2}}}\,dx = -\frac{1}{2}\log\left(x^{2}-3\right)+\frac{1}{2}\log\left(-\frac{x^{2}+3\sqrt{-x^{2}+4}-6}{x^{2}}\right) - \frac{1}{2}\log\left(-\frac{x^{2}+\sqrt{-x^{2}+4}-2}{x^{2}}\right)$$

input
```
integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 - 3) + 1/2*log(-(x^2 + 3*sqrt(-x^2 + 4) - 6)/x^2) - 1/2*log(-
(x^2 + sqrt(-x^2 + 4) - 2)/x^2)
```

3.114.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(12) = 24.

Time = 1.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = \frac{\log\left(2\sqrt{4-x^2}\right)}{2} - \frac{\log\left(2\sqrt{4-x^2}+2\right)}{2} - \frac{\log\left(2x^2-2\sqrt{4-x^2}-8\right)}{2}$$

input
```
integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)
```

output
```
log(2*sqrt(4 - x**2))/2 - log(2*sqrt(4 - x**2) + 2)/2 - log(2*x**2 - 2*sqr
t(4 - x**2) - 8)/2
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(\sqrt{-x^2+4}+1\right)$$

input
```
integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="maxima")
```

output
```
-log(sqrt(-x^2 + 4) + 1)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\log\left(\sqrt{-x^2+4}+1\right)$$

input
```
integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="giac")
```

output
```
-log(sqrt(-x^2 + 4) + 1)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx = -\frac{\ln\left(x-\sqrt{3}\right)}{2}-\frac{\ln\left(\frac{\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x+\sqrt{3}}\right)}{2} - \frac{\ln\left(x+\sqrt{3}\right)}{2}-\frac{\ln\left(\frac{-\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x-\sqrt{3}}\right)}{2}$$

input
```
int(x/((4 - x^2)^(1/2) - x^2 + 4),x)
```

output
```
- log(x - 3^(1/2))/2 - log((3^(1/2)*x*1i + (4 - x^2)^(1/2)*1i + 4i)/(x + 3
^(1/2)))/2 - log(x + 3^(1/2))/2 - log(((4 - x^2)^(1/2)*1i - 3^(1/2)*x*1i +
 4i)/(x - 3^(1/2)))/2
```

3.115 $\int \frac{3+2x}{(-2+x)(5+x)} \, dx$

3.115.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{3+2x}{(-2+x)(5+x)} \, dx = \log(2-x) + \log(5+x)$$

output
```
ln(2-x)+ln(5+x)
```

3.115.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)} \, dx = \log(-2+x) + \log(5+x)$$

input
```
Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

output
```
Log[-2 + x] + Log[5 + x]
```

3.115.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(x-2)(x+5)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{1}{x+5}+\frac{1}{x-2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(2-x)+\log(x+5)$$

input
```
Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

output
```
Log[2 - x] + Log[5 + x]
```

3.115.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.115.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\ln\left(\left(-2+x\right)\left(5+x\right)\right)$	9
norman	$\ln\left(-2+x\right)+\ln\left(5+x\right)$	10
risch	$\ln\left(x^{2}+3x-10\right)$	10
parallelrisch	$\ln\left(-2+x\right)+\ln\left(5+x\right)$	10

input
```
int((3+2*x)/(-2+x)/(5+x),x,method=_RETURNVERBOSE)
```

output
```
ln((-2+x)*(5+x))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^2+3\,x-10\right)$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
```

output
```
log(x^2 + 3*x - 10)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^2+3x-10\right)$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x)
```

output
```
log(x**2 + 3*x - 10)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log{(x+5)} + \log{(x-2)}$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
```

output
```
log(x + 5) + log(x - 2)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log{(|x+5|)} + \log{(|x-2|)}$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

output
```
log(abs(x + 5)) + log(abs(x - 2))
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \ln\left(x^2+3\,x-10\right)$$

input
```
int((2*x + 3)/((x - 2)*(x + 5)),x)
```

output
```
log(3*x + x^2 - 10)
```

3.116 $\int \frac{x}{(1+x)(2+x)(3+x)}\,dx$

3.116.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

output
```
-1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

3.116.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

input
```
Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

output
```
-1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

3.116.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)(x+2)(x+3)} \, dx$$
$$\Big\downarrow \text{165}$$
$$\int \left(\frac{2}{x+2} - \frac{3}{2(x+3)} - \frac{1}{2(x+1)}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{2}\log(x+1) + 2\log(x+2) - \frac{3}{2}\log(x+3)$$

input
```
Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

output
```
-1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

3.116.3.1 Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.116.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
norman	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
risch	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
parallelrisch	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20

input
```
int(x/(1+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
```

output
```
-3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x)
```

output
```
-log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(x+3) + 2\log(x+2) - \frac{1}{2}\log(x+1)$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
```

output
```
-3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(|x+3|) + 2\log(|x+2|) - \frac{1}{2}\log(|x+1|)$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
```

output
```
-3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = 2\ln(x+2) - \frac{\ln(x+1)}{2} - \frac{3\ln(x+3)}{2}$$

input
```
int(x/((x + 1)*(x + 2)*(x + 3)),x)
```

output
```
2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

3.117 $\int \frac{x}{2-3x+x^3}\,dx$

3.117.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{x}{2-3x+x^3}\,dx=\frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(2+x)$$

output
```
1/3/(1-x)+2/9*ln(1-x)-2/9*ln(2+x)
```

3.117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{x}{2-3x+x^3}\,dx=-\frac{1}{3(-1+x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(2+x)$$

input
```
Integrate[x/(2 - 3*x + x^3),x]
```

output
```
-1/3*1/(-1 + x) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

3.117.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^3-3x+2}dx$$

$$\Big\downarrow \; 2462$$

$$\int \left(-\frac{2}{9(x+2)}+\frac{2}{9(x-1)}+\frac{1}{3(x-1)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

input
```
Int[x/(2 - 3*x + x^3),x]
```

output
```
1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.117.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
norman	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
risch	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
parallelrisch	$\frac{2\ln(-1+x)x-2\ln(2+x)x-3-2\ln(-1+x)+2\ln(2+x)}{-9+9x}$	36

input
```
int(x/(x^3-3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(-1+x)+2/9*ln(-1+x)-2/9*ln(2+x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{2\,(x-1)\log\left(x+2\right)-2\,(x-1)\log\left(x-1\right)+3}{9\,(x-1)}$$

input
```
integrate(x/(x^3-3*x+2),x, algorithm="fricas")
```

output
```
-1/9*(2*(x - 1)*log(x + 2) - 2*(x - 1)*log(x - 1) + 3)/(x - 1)
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-3x+x^3}\,dx = \frac{2\log\left(x-1\right)}{9}-\frac{2\log\left(x+2\right)}{9}-\frac{1}{3x-3}$$

input
```
integrate(x/(x**3-3*x+2),x)
```

output
```
2*log(x - 1)/9 - 2*log(x + 2)/9 - 1/(3*x - 3)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{1}{3\,(x-1)} - \frac{2}{9}\,\log\left(x+2\right) + \frac{2}{9}\,\log\left(x-1\right)$$

input
```
integrate(x/(x^3-3*x+2),x, algorithm="maxima")
```

output
```
-1/3/(x - 1) - 2/9*log(x + 2) + 2/9*log(x - 1)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{1}{3\,(x-1)} - \frac{2}{9}\,\log\left(|x+2|\right) + \frac{2}{9}\,\log\left(|x-1|\right)$$

input
```
integrate(x/(x^3-3*x+2),x, algorithm="giac")
```

output
```
-1/3/(x - 1) - 2/9*log(abs(x + 2)) + 2/9*log(abs(x - 1))
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{2-3x+x^3}\,dx = -\frac{4\,\mathrm{atanh}\left(\frac{2\,x}{3}+\frac{1}{3}\right)}{9} - \frac{1}{3\,\left(x-1\right)}$$

input
```
int(x/(x^3 - 3*x + 2),x)
```

output
```
- (4*atanh((2*x)/3 + 1/3))/9 - 1/(3*(x - 1))
```

3.118 $\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx = -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)$$

output
```
-x+1/2*x^2-ln(1-x)+3*ln(x)+ln(2+x)
```

3.118.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx = -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)$$

input
```
Integrate[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

output
```
-x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

3.118.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x-6}{x^3+x^2-2x}\,dx$$

$$\Big\downarrow \ 2026$$

$$\int \frac{x^4+2x-6}{x\left(x^2+x-2\right)}dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(x+\frac{1}{1-x}+\frac{1}{x+2}+\frac{3}{x}-1\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

input
```
Int[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

output
```
-x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.118.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-x+\frac{x^{2}}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24
norman	$-x+\frac{x^{2}}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24
risch	$-x+\frac{x^{2}}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24
parallelrisch	$-x+\frac{x^{2}}{2}-\ln\left(-1+x\right)+\ln\left(2+x\right)+3\ln\left(x\right)$	24

input
```
int((x^4+2*x-6)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2-ln(-1+x)+ln(2+x)+3*ln(x)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int\frac{-6+2x+x^{4}}{-2x+x^{2}+x^{3}}\,dx=\frac{1}{2}\,x^{2}-x+\log\left(x+2\right)-\log\left(x-1\right)+3\,\log\left(x\right)$$

input
```
integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="fricas")
```

output
```
1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int\frac{-6+2x+x^{4}}{-2x+x^{2}+x^{3}}\,dx=\frac{x^{2}}{2}-x+3\log\left(x\right)-\log\left(x-1\right)+\log\left(x+2\right)$$

input
```
integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)
```

output
```
x**2/2 - x + 3*log(x) - log(x - 1) + log(x + 2)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - x + \log\left(x+2\right) - \log\left(x-1\right) + 3\,\log\left(x\right)$$

input
```
integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="maxima")
```

output
```
1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - x + \log\left(|x+2|\right) - \log\left(|x-1|\right) + 3\,\log\left(|x|\right)$$

input
```
integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="giac")
```

output
```
1/2*x^2 - x + log(abs(x + 2)) - log(abs(x - 1)) + 3*log(abs(x))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx = 3\,\ln\left(x\right) - x + \frac{x^2}{2} + \operatorname{atan}\left(\frac{192\mathrm{i}}{7\,\left(28\,x-40\right)} + \frac{9}{7}\mathrm{i}\right)\,2\mathrm{i}$$

input
```
int((2*x + x^4 - 6)/(x^2 - 2*x + x^3),x)
```

output
```
atan(192i/(7*(28*x - 40)) + 9i/7)*2i - x + 3*log(x) + x^2/2
```

3.119 $\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx$

3.119.1 Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = -\frac{3}{(1+2x)^2} + \frac{3}{1+2x} + \log(1+x)$$

output
```
-3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

3.119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = \frac{6x+(1+2x)^2\log(1+x)}{(1+2x)^2}$$

input
```
Integrate[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

output
```
(6*x + (1 + 2*x)^2*Log[1 + x])/(1 + 2*x)^2
```

3.119.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3+7}{(x+1)(2x+1)^3}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(-\frac{6}{(2x+1)^2}+\frac{12}{(2x+1)^3}+\frac{1}{x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

input
```
Int[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

output
```
-3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.119.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
norman	$\frac{6x}{(1+2x)^2}+\ln(1+x)$	16
risch	$\frac{6x}{(1+2x)^2}+\ln(1+x)$	16
default	$-\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\ln(1+x)$	24
parallelrisch	$\frac{4\ln(1+x)x^2+4\ln(1+x)x+\ln(1+x)+6x}{(1+2x)^2}$	33

input
```
int((8*x^3+7)/(1+x)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
6*x/(1+2*x)^2+ln(1+x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx = \frac{(4\,x^2+4\,x+1)\log(x+1)+6\,x}{4\,x^2+4\,x+1}$$

input
```
integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
((4*x^2 + 4*x + 1)*log(x + 1) + 6*x)/(4*x^2 + 4*x + 1)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx = \frac{6x}{4x^2+4x+1}+\log(x+1)$$

input
```
integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)
```

output
```
6*x/(4*x**2 + 4*x + 1) + log(x + 1)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = \frac{6\,x}{4\,x^2+4\,x+1} + \log\left(x+1\right)$$

input
```
integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
6*x/(4*x^2 + 4*x + 1) + log(x + 1)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = \frac{6\,x}{(2\,x+1)^2} + \log\left(|x+1|\right)$$

input
```
integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="giac")
```

output
```
6*x/(2*x + 1)^2 + log(abs(x + 1))
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{7+8x^3}{(1+x)(1+2x)^3} \, dx = \ln\left(x+1\right) + \frac{6\,x}{(2\,x+1)^2}$$

input
```
int((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x)
```

output
```
log(x + 1) + (6*x)/(2*x + 1)^2
```

3.120 $\int \frac{1+x+4x^2}{-1+x^3} \, dx$

3.120.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1+x+4x^2}{-1+x^3} \, dx = 2\log(1-x) + \log\left(1+x+x^2\right)$$

output
```
2*ln(1-x)+ln(x^2+x+1)
```

3.120.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{-1+x^3} \, dx = 2\log(1-x) + \log\left(1+x+x^2\right)$$

input
```
Integrate[(1 + x + 4*x^2)/(-1 + x^3),x]
```

output
```
2*Log[1 - x] + Log[1 + x + x^2]
```

3.120.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2414, 16, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+x+1}{x^3-1}dx$$

$$\Big\downarrow \ 2414$$

$$-\frac{1}{3}\int -\frac{3(2x+1)}{x^2+x+1}dx-2\int \frac{1}{1-x}dx$$

$$\Big\downarrow \ 16$$

$$2\log(1-x)-\frac{1}{3}\int -\frac{3(2x+1)}{x^2+x+1}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{2x+1}{x^2+x+1}dx+2\log(1-x)$$

$$\Big\downarrow \ 1103$$

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

input
```
Int[(1 + x + 4*x^2)/(-1 + x^3),x]
```

output
```
2*Log[1 - x] + Log[1 + x + x^2]
```

3.120.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

3.120.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
norman	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
risch	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
parallelrisch	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
meijerg	$\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\frac{4\ln\left(-x^{3}+1\right)}{3}+\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)\right.}{}\right.}{}$

input
```
int((4*x^2+x+1)/(x^3-1),x,method=_RETURNVERBOSE)
```

output
```
2*ln(-1+x)+ln(x^2+x+1)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

input
```
integrate((4*x^2+x+1)/(x^3-1),x, algorithm="fricas")
```

output
```
log(x^2 + x + 1) + 2*log(x - 1)
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = 2\log\left(x-1\right)+\log\left(x^2+x+1\right)$$

input
```
integrate((4*x**2+x+1)/(x**3-1),x)
```

output
```
2*log(x - 1) + log(x**2 + x + 1)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

input
```
integrate((4*x^2+x+1)/(x^3-1),x, algorithm="maxima")
```

output
```
log(x^2 + x + 1) + 2*log(x - 1)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \log\left(x^2+x+1\right) + 2\,\log\left(|x-1|\right)$$

input
```
integrate((4*x^2+x+1)/(x^3-1),x, algorithm="giac")
```

output
```
log(x^2 + x + 1) + 2*log(abs(x - 1))
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x+4x^2}{-1+x^3}\,dx = \ln\left(x^2+x+1\right) + 2\,\ln\left(x-1\right)$$

input
```
int((x + 4*x^2 + 1)/(x^3 - 1),x)
```

output
```
log(x + x^2 + 1) + 2*log(x - 1)
```

3.121 $\int \frac{x^4}{4+5x^2+x^4} dx$

3.121.1 Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{x^4}{4+5x^2+x^4} dx = x - \frac{8}{3}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3}$$

output
```
x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

3.121.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{4+5x^2+x^4} dx = x + \frac{8}{3}\arctan\left(\frac{2}{x}\right) + \frac{\arctan(x)}{3}$$

input
```
Integrate[x^4/(4 + 5*x^2 + x^4),x]
```

output
```
x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

3.121.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1442, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^4+5x^2+4}\,dx$$

$$\Big\downarrow \text{ 1442}$$

$$x-\int \frac{5x^2+4}{x^4+5x^2+4}dx$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{3}\int \frac{1}{x^2+1}dx-\frac{16}{3}\int \frac{1}{x^2+4}dx+x$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}+x$$

input
```
Int[x^4/(4 + 5*x^2 + x^4),x]
```

output
```
x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

3.121.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.121.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
default	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan\left(x\right)}{3}$	13
risch	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan\left(x\right)}{3}$	13
parallelrisch	$x+\frac{i\ln\left(x+i\right)}{6}-\frac{i\ln\left(x-i\right)}{6}+\frac{4i\ln\left(x-2i\right)}{3}-\frac{4i\ln\left(x+2i\right)}{3}$	35

input
```
int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx=x-\frac{8}{3}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{3}\arctan\left(x\right)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}\,(x)}{3}$$

input
```
integrate(x**4/(x**4+5*x**2+4),x)
```

output
```
x - 8*atan(x/2)/3 + atan(x)/3
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\,(x)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\,(x)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

input
```
int(x^4/(5*x^2 + x^4 + 4),x)
```

output
```
x - (8*atan(x/2))/3 + atan(x)/3
```

3.122 $\int \frac{2+x}{x+x^2}\,dx$

3.122.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{2+x}{x+x^2}\,dx = 2\log(x)-\log(1+x)$$

output
```
2*ln(x)-ln(1+x)
```

3.122.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2}\,dx = 2\log(x)-\log(1+x)$$

input
```
Integrate[(2 + x)/(x + x^2),x]
```

output
```
2*Log[x] - Log[1 + x]
```

3.122.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^2+x}\,dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(\frac{2}{x}+\frac{1}{-x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(x)-\log(x+1)$$

input
```
Int[(2 + x)/(x + x^2),x]
```

output
```
2*Log[x] - Log[1 + x]
```

3.122.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.122.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$2\ln(x)-\ln(1+x)$	12
norman	$2\ln(x)-\ln(1+x)$	12
meijerg	$2\ln(x)-\ln(1+x)$	12
risch	$2\ln(x)-\ln(1+x)$	12
parallelrisch	$2\ln(x)-\ln(1+x)$	12

input
```
int((2+x)/(x^2+x),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)-ln(1+x)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2}\,dx = -\log(x+1)+2\,\log(x)$$

input
```
integrate((2+x)/(x^2+x),x, algorithm="fricas")
```

output
```
-log(x + 1) + 2*log(x)
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{x+x^2}\,dx = 2\log(x)-\log(x+1)$$

input
```
integrate((2+x)/(x**2+x),x)
```

output
```
2*log(x) - log(x + 1)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2}\,dx = -\log(x+1) + 2\log(x)$$

input
```
integrate((2+x)/(x^2+x),x, algorithm="maxima")
```

output
```
-log(x + 1) + 2*log(x)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{2+x}{x+x^2}\,dx = -\log(|x+1|) + 2\log(|x|)$$

input
```
integrate((2+x)/(x^2+x),x, algorithm="giac")
```

output
```
-log(abs(x + 1)) + 2*log(abs(x))
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{x+x^2}\,dx = 2\ln(x) - \ln(x+1)$$

input
```
int((x + 2)/(x + x^2),x)
```

output
```
2*log(x) - log(x + 1)
```

3.123 $\int \frac{1}{x(1+x^2)^2} \, dx$

3.123.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x(1+x^2)^2} \, dx = \frac{1}{2(1+x^2)} + \log(x) - \frac{1}{2}\log(1+x^2)$$

output
```
1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

3.123.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x^2)^2} \, dx = \frac{1}{2(1+x^2)} + \log(x) - \frac{1}{2}\log(1+x^2)$$

input
```
Integrate[1/(x*(1 + x^2)^2),x]
```

output
```
1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

3.123.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^2+1)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(x^2+1)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{1}{x^2}-\frac{1}{(x^2+1)^2}+\frac{1}{-x^2-1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{x^2+1}+\log\left(x^2\right)-\log\left(x^2+1\right)\right)$$

input
```
Int[1/(x*(1 + x^2)^2),x]
```

output
```
((1 + x^2)^(-1) + Log[x^2] - Log[1 + x^2])/2
```

3.123.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.123.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{1}{2x^2+2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	21
norman	$\frac{1}{2x^2+2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	21
risch	$\frac{1}{2x^2+2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	21
meijerg	$\frac{1}{2}+\ln(x)-\frac{x^2}{2x^2+2}-\frac{\ln(x^2+1)}{2}$	27
parallelrisch	$\frac{2x^2\ln(x)-\ln(x^2+1)x^2+1+2\ln(x)-\ln(x^2+1)}{2x^2+2}$	42

input
```
int(1/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(1+x^2)^2}\,dx = -\frac{(x^2+1)\log(x^2+1)-2(x^2+1)\log(x)-1}{2(x^2+1)}$$

input
```
integrate(1/x/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) - 1)/(x^2 + 1)
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(1+x^2)^2}\,dx = \log(x)-\frac{\log(x^2+1)}{2}+\frac{1}{2x^2+2}$$

input
```
integrate(1/x/(x**2+1)**2,x)
```

output
```
log(x) - log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \frac{1}{2\left(x^2+1\right)} - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \frac{x^2+2}{2\left(x^2+1\right)} - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*(x^2 + 2)/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1+x^2\right)^2}\,dx = \ln(x) - \frac{\ln\left(x^2+1\right)}{2} + \frac{1}{2\,\left(x^2+1\right)}$$

input
```
int(1/(x*(x^2 + 1)^2),x)
```

output
```
log(x) - log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

3.124 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

3.124.1 Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x) + 2\log(2+x)-\frac{17}{8}\log(3+x)$$

output
```
1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{8}\left(\frac{8}{2+x}+\frac{2}{(3+x)^2}+\frac{10}{3+x}+\log(-1-x)+16\log(2+x) - 17\log(3+x)\right)$$

input
```
Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

output
```
(8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*L
og[3 + x])/8
```

3.124.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)(x+2)^2(x+3)^3} \, dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{2}{x+2}-\frac{17}{8(x+3)}-\frac{1}{(x+2)^2}-\frac{5}{4(x+3)^2}-\frac{1}{2(x+3)^3}+\frac{1}{8(x+1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

input
```
Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

output
```
(2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 +
x] - (17*Log[3 + x])/8
```

3.124.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.124.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
default	$\frac{1}{2+x}+\frac{1}{4(3+x)^{2}}+\frac{5}{4(3+x)}+\frac{\ln(1+x)}{8}+2\ln\left(2+x\right)-\frac{17\ln(3+x)}{8}$
norman	$\frac{\frac{9}{4}x^{2}+\frac{25}{2}x+17}{(2+x)(3+x)^{2}}+\frac{\ln(1+x)}{8}+2\ln\left(2+x\right)-\frac{17\ln(3+x)}{8}$
risch	$\frac{\frac{9}{4}x^{2}+\frac{25}{2}x+17}{(2+x)(3+x)^{2}}+\frac{\ln(1+x)}{8}+2\ln\left(2+x\right)-\frac{17\ln(3+x)}{8}$
parallelrisch	$\frac{\ln(1+x)x^{3}+16\ln(2+x)x^{3}-17\ln(3+x)x^{3}+136+8\ln(1+x)x^{2}+128\ln(2+x)x^{2}-136\ln(3+x)x^{2}+21\ln(1+x)x+336\ln(2+x)x-3}{8(2+x)(3+x)^{2}}$

input
```
int(1/(1+x)/(2+x)^2/(3+x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(38) = 76.

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$$
$$=\frac{18\,x^2-17\,(x^3+8\,x^2+21\,x+18)\log\left(x+3\right)+16\,(x^3+8\,x^2+21\,x+18)\log\left(x+2\right)+(x^3+8\,x^2+21\,x}{8\,(x^3+8\,x^2+21\,x+18)}$$

input
```
integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

output
```
1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 +
21*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136
)/(x^3 + 8*x^2 + 21*x + 18)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9x^2+50x+68}{4x^3+32x^2+84x+72} + \frac{\log(x+1)}{8} + 2\log(x+2) - \frac{17\log(x+3)}{8}$$

input
```
integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

output
```
(9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log
(x + 2) - 17*log(x + 3)/8
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9\,x^2+50\,x+68}{4\,(x^3+8\,x^2+21\,x+18)} - \frac{17}{8}\,\log(x+3) + 2\,\log(x+2) + \frac{1}{8}\,\log(x+1)$$

input
```
integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

output
```
1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*lo
g(x + 2) + 1/8*log(x + 1)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{x+2} - \frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2} + \frac{1}{8}\,\log\left(\left|-\frac{1}{x+2}+1\right|\right) - \frac{17}{8}\,\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

input
```
integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

output
```
1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
 + 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{\ln{(x+1)}}{8} + 2\,\ln{(x+2)} - \frac{17\,\ln{(x+3)}}{8} + \frac{\frac{9\,x^2}{4} + \frac{25\,x}{2} + 17}{x^3 + 8\,x^2 + 21\,x + 18}$$

input
```
int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

output
```
log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 +
17)/(21*x + 8*x^2 + x^3 + 18)
```

3.125 $\int \frac{x}{(1+x)^2} \, dx$

3.125.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{1+x} + \log(1+x)$$

output
```
1/(1+x)+ln(1+x)
```

3.125.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{1+x} + \log(1+x)$$

input
```
Integrate[x/(1 + x)^2,x]
```

output
```
(1 + x)^(-1) + Log[1 + x]
```

3.125.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^2} dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{x+1}+\log(x+1)$$

input
```
Int[x/(1 + x)^2,x]
```

output
```
(1 + x)^(-1) + Log[1 + x]
```

3.125.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.125.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
norman	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
risch	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
meijerg	$-\frac{x}{1+x}+\ln\left(1+x\right)$	14
parallelrisch	$\frac{\ln(1+x)x+1+\ln(1+x)}{1+x}$	19

input
```
int(x/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(1+x)+ln(1+x)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x}{(1+x)^2}\,dx=\frac{(x+1)\log\left(x+1\right)+1}{x+1}$$

input
```
integrate(x/(1+x)^2,x, algorithm="fricas")
```

output
```
((x + 1)*log(x + 1) + 1)/(x + 1)
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)^2}\,dx=\log\left(x+1\right)+\frac{1}{x+1}$$

input
```
integrate(x/(1+x)**2,x)
```

output
```
log(x + 1) + 1/(x + 1)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{x+1} + \log{(x+1)}$$

input
```
integrate(x/(1+x)^2,x, algorithm="maxima")
```

output
```
1/(x + 1) + log(x + 1)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1+x)^2} \, dx = \frac{1}{x+1} + \log{(|x+1|)}$$

input
```
integrate(x/(1+x)^2,x, algorithm="giac")
```

output
```
1/(x + 1) + log(abs(x + 1))
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} \, dx = \ln{(x+1)} + \frac{1}{x+1}$$

input
```
int(x/(x + 1)^2,x)
```

output
```
log(x + 1) + 1/(x + 1)
```

3.126 $\int \frac{1}{-x+x^3} dx$

3.126.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{-x+x^3} dx = -\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

output
```
-ln(x)+1/2*ln(-x^2+1)
```

3.126.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3} dx = -\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[(-x + x^3)^(-1),x]
```

output
```
-Log[x] + Log[1 - x^2]/2
```

3.126.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2026, 243, 25, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3-x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(x^2-1)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int -\frac{1}{x^2(1-x^2)}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{x^2(1-x^2)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(-\int \frac{1}{x^2}dx^2-\int \frac{1}{1-x^2}dx^2\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(-\int \frac{1}{1-x^2}dx^2-\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(1-x^2\right)-\log\left(x^2\right)\right)$$

input
```
Int[(-x + x^3)^(-1),x]
```

output
```
(-Log[x^2] + Log[1 - x^2])/2
```

3.126.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.126.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
risch	$-\ln\left(x\right)+\frac{\ln\left(x^{2}-1\right)}{2}$	14
default	$\frac{\ln\left(-1+x\right)}{2}-\ln\left(x\right)+\frac{\ln\left(1+x\right)}{2}$	18
norman	$\frac{\ln\left(-1+x\right)}{2}-\ln\left(x\right)+\frac{\ln\left(1+x\right)}{2}$	18
parallelrisch	$\frac{\ln\left(-1+x\right)}{2}-\ln\left(x\right)+\frac{\ln\left(1+x\right)}{2}$	18
meijerg	$-\ln\left(x\right)-\frac{i\pi}{2}+\frac{\ln\left(-x^{2}+1\right)}{2}$	20

input
```
int(1/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+1/2*ln(x^2-1)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x+x^3}\,dx = \frac{1}{2}\log\left(x^2-1\right) - \log\left(x\right)$$

input
```
integrate(1/(x^3-x),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - 1) - log(x)
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{-x+x^3}\,dx = -\log\left(x\right) + \frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(1/(x**3-x),x)
```

output
```
-log(x) + log(x**2 - 1)/2
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3}\,dx = \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right) - \log\left(x\right)$$

input
```
integrate(1/(x^3-x),x, algorithm="maxima")
```

output
```
1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{-x+x^3}\,dx = -\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/(x^3-x),x, algorithm="giac")
```

output
```
-1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

3.126.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x+x^3}\,dx = \frac{\ln\left(x^2-1\right)}{2}-\ln\left(x\right)$$

input
```
int(-1/(x - x^3),x)
```

output
```
log(x^2 - 1)/2 - log(x)
```

3.127 $\int \frac{x^2}{-6+x+x^2}\,dx$

3.127.1 Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{x^2}{-6+x+x^2}\,dx = x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(3+x)$$

output
```
x+4/5*ln(2-x)-9/5*ln(3+x)
```

3.127.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-6+x+x^2}\,dx = x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(3+x)$$

input
```
Integrate[x^2/(-6 + x + x^2),x]
```

output
```
x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

3.127.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+x-6} \, dx$$
$$\Big\downarrow \; 1141$$
$$\int \left(-\frac{9}{5(x+3)}-\frac{4}{5(2-x)}+1\right)dx$$
$$\Big\downarrow \; 2009$$
$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

input
```
Int[x^2/(-6 + x + x^2),x]
```

output
```
x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

3.127.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.127.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15
norman	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15
risch	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15
parallelrisch	$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$	15

input
```
int(x^2/(x^2+x-6),x,method=_RETURNVERBOSE)
```

output
```
x-9/5*ln(3+x)+4/5*ln(-2+x)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x-\frac{9}{5}\,\log\left(x+3\right)+\frac{4}{5}\,\log\left(x-2\right)$$

input
```
integrate(x^2/(x^2+x-6),x, algorithm="fricas")
```

output
```
x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{-6+x+x^2}\,dx = x+\frac{4\log\left(x-2\right)}{5}-\frac{9\log\left(x+3\right)}{5}$$

input
```
integrate(x**2/(x**2+x-6),x)
```

output
```
x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\log(x+3) + \frac{4}{5}\log(x-2)$$

input
```
integrate(x^2/(x^2+x-6),x, algorithm="maxima")
```

output
```
x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{-6+x+x^2}\,dx = x - \frac{9}{5}\log(|x+3|) + \frac{4}{5}\log(|x-2|)$$

input
```
integrate(x^2/(x^2+x-6),x, algorithm="giac")
```

output
```
x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{-6+x+x^2}\,dx = x + \frac{4\ln(x-2)}{5} - \frac{9\ln(x+3)}{5}$$

input
```
int(x^2/(x + x^2 - 6),x)
```

output
```
x + (4*log(x - 2))/5 - (9*log(x + 3))/5
```

3.128 $\int \frac{2+x}{4-4x+x^2} \, dx$

3.128.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{2+x}{4-4x+x^2} \, dx = \frac{4}{2-x} + \log(2-x)$$

output
```
4/(2-x)+ln(2-x)
```

3.128.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{4-4x+x^2} \, dx = -\frac{4}{-2+x} + \log(-2+x)$$

input
```
Integrate[(2 + x)/(4 - 4*x + x^2),x]
```

output
```
-4/(-2 + x) + Log[-2 + x]
```

3.128.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^2-4x+4} \, dx$$
$$\Big\downarrow \text{ 1098}$$
$$\int \frac{x+2}{(2-x)^2}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{x-2}+\frac{4}{(x-2)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{4}{2-x}+\log(2-x)$$

input
```
Int[(2 + x)/(4 - 4*x + x^2),x]
```

output
```
4/(2 - x) + Log[2 - x]
```

3.128.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.128.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{4}{-2+x}+\ln\left(-2+x\right)$	13
norman	$-\frac{4}{-2+x}+\ln\left(-2+x\right)$	13
risch	$-\frac{4}{-2+x}+\ln\left(-2+x\right)$	13
meijerg	$\frac{x}{1-\frac{x}{2}}+\ln\left(1-\frac{x}{2}\right)$	17
parallelrisch	$\frac{\ln(-2+x)x-4-2\ln(-2+x)}{-2+x}$	21

input
```
int((2+x)/(x^2-4*x+4),x,method=_RETURNVERBOSE)
```

output
```
-4/(-2+x)+ln(-2+x)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{4-4x+x^2}\,dx=\frac{(x-2)\log\left(x-2\right)-4}{x-2}$$

input
```
integrate((2+x)/(x^2-4*x+4),x, algorithm="fricas")
```

output
```
((x - 2)*log(x - 2) - 4)/(x - 2)
```

3.128.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{2+x}{4-4x+x^2}\,dx=\log\left(x-2\right)-\frac{4}{x-2}$$

input
```
integrate((2+x)/(x**2-4*x+4),x)
```

output
```
log(x - 2) - 4/(x - 2)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{4-4x+x^2}\,dx = -\frac{4}{x-2} + \log(x-2)$$

input
```
integrate((2+x)/(x^2-4*x+4),x, algorithm="maxima")
```

output
```
-4/(x - 2) + log(x - 2)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{2+x}{4-4x+x^2}\,dx = -\frac{4}{x-2} + \log(|x-2|)$$

input
```
integrate((2+x)/(x^2-4*x+4),x, algorithm="giac")
```

output
```
-4/(x - 2) + log(abs(x - 2))
```

3.128.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{4-4x+x^2}\,dx = \ln(x-2) - \frac{4}{x-2}$$

input
```
int((x + 2)/(x^2 - 4*x + 4),x)
```

output
```
log(x - 2) - 4/(x - 2)
```

3.129 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \frac{1}{2-x} + \arctan(2-x)$$

output
```
1/(2-x)-arctan(-2+x)
```

3.129.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{-2+x} + \arctan(2-x)$$

input
```
Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

output
```
-(-2 + x)^(-1) + ArcTan[2 - x]
```

3.129.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1294, 1117, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-4x+4)(x^2-4x+5)}dx$$
$$\Big\downarrow \text{ 1294}$$
$$\int \frac{1}{(2-x)^2(x^2-4x+5)}dx$$
$$\Big\downarrow \text{ 1117}$$
$$\frac{1}{2-x}-\int \frac{1}{x^2-4x+5}dx$$
$$\Big\downarrow \text{ 1083}$$
$$2\int \frac{1}{-(2x-4)^2-4}d(2x-4)+\frac{1}{2-x}$$
$$\Big\downarrow \text{ 217}$$
$$\frac{1}{2-x}-\arctan\left(\frac{1}{2}(2x-4)\right)$$

input
```
Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

output
```
(2 - x)^(-1) - ArcTan[(-4 + 2*x)/2]
```

3.129.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1117
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Simp[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[m, -1] &
& (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || IntegerQ[(m + 2*p + 3)
/2])
```

rule 1294
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_.), x_Symbol] :> Simp[1/c^p   Int[(b/2 + c*x)^(2*p)*(d + e*x + f*x
^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && EqQ[b^2 - 4*a*c, 0] &&
IntegerQ[p]
```

3.129.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{1}{-2+x}-\arctan\left(-2+x\right)$	15
risch	$-\frac{1}{-2+x}-\arctan\left(-2+x\right)$	15
parallelrisch	$\frac{i\ln(x-2-i)x-i\ln(x-2+i)x-2i\ln(x-2-i)+2i\ln(x-2+i)-x}{-4+2x}$	50

input
```
int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
```

output
```
-1/(-2+x)-arctan(-2+x)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{(x-2)\arctan\left(x-2\right)+1}{x-2}$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

output
```
-((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

3.129.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\operatorname{atan}(x-2) - \frac{1}{x-2}$$

input
```
integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

output
```
-atan(x - 2) - 1/(x - 2)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

output
```
-1/(x - 2) - arctan(x - 2)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

output
```
-1/(x - 2) - arctan(x - 2)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\mathrm{atan}(x-2) - \frac{1}{x-2}$$

input
```
int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
```

output
```
- atan(x - 2) - 1/(x - 2)
```

3.130 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx=-\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

output
```
-3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

3.130.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx=-\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

input
```
Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

output
```
(-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

3.130.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1979, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-3}{x^3+3x^2+2x}dx$$

$$\Big\downarrow \text{1979}$$

$$\int \frac{x-3}{x(x^2+3x+2)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{4}{x+1}-\frac{5}{2(x+2)}-\frac{3}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

input
```
Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

output
```
(-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

3.130.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.130.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$	18
norman	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$	18
risch	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$	18
parallelrisch	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$	18

input
```
int((-3+x)/(x^3+3*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
-3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx=-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

input
```
integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
```

output
```
-5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

3.130.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5\log(x+2)}{2}$$

input
```
integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

output
```
-3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\log(x+2) + 4\log(x+1) - \frac{3}{2}\log(x)$$

input
```
integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
```

output
```
-5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\log(|x+2|) + 4\log(|x+1|) - \frac{3}{2}\log(|x|)$$

input
```
integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
```

output
```
-5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = 4\ln(x+1) - \frac{5\ln(x+2)}{2} - \frac{3\ln(x)}{2}$$

input
```
int((x - 3)/(2*x + 3*x^2 + x^3),x)
```

output
```
4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

3.131 $\int \frac{1}{(-1+x^2)^2}\,dx$

3.131.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{x}{2(1-x^2)} + \frac{\text{arctanh}(x)}{2}$$

output
```
1/2*x/(-x^2+1)+1/2*arctanh(x)
```

3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{1}{4}\left(-\frac{2x}{-1+x^2} - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[(-1 + x^2)^(-2),x]
```

output
```
((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

3.131.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {215, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-1)^2}dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{x}{2(1-x^2)}-\frac{1}{2}\int \frac{1}{x^2-1}dx$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\text{arctanh}(x)}{2}+\frac{x}{2(1-x^2)}$$

input
```
Int[(-1 + x^2)^(-2),x]
```

output
```
x/(2*(1 - x^2)) + ArcTanh[x]/2
```

3.131.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.131.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\,\mathrm{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
risch	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{4}-\frac{1}{4(1+x)}+\frac{\ln(1+x)}{4}$	28
parallelrisch	$-\frac{\ln(-1+x)x^2-\ln(1+x)x^2-\ln(-1+x)+\ln(1+x)+2x}{4(x^2-1)}$	41

input
```
int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(2*I*x/(-2*x^2+2)+I*arctanh(x))
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\,x}{4\,(x^2-1)}$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

input
```
integrate(1/(x**2-1)**2,x)
```

output
```
-x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2(x^2-1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2(x^2-1)} + \frac{1}{4}\log(|x+1|) - \frac{1}{4}\log(|x-1|)$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(-1+x^2\right)^2} \, dx = \frac{\operatorname{atanh}(x)}{2} - \frac{x}{2\,(x^2-1)}$$

input
```
int(1/(x^2 - 1)^2,x)
```

output
```
atanh(x)/2 - x/(2*(x^2 - 1))
```

3.132 $\int \frac{1+x}{-1+x^3} \, dx$

3.132.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1+x}{-1+x^3} \, dx = \frac{2}{3} \log(1-x) - \frac{1}{3} \log\left(1+x+x^2\right)$$

output
```
2/3*ln(1-x)-1/3*ln(x^2+x+1)
```

3.132.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{-1+x^3} \, dx = \frac{2}{3} \log(1-x) - \frac{1}{3} \log\left(1+x+x^2\right)$$

input
```
Integrate[(1 + x)/(-1 + x^3),x]
```

output
```
(2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

3.132.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2400, 16, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^3-1} \, dx$$

$$\Big\downarrow \text{ 2400}$$

$$\frac{1}{3} \int -\frac{2x+1}{x^2+x+1} dx - \frac{2}{3} \int \frac{1}{1-x} dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3} \int -\frac{2x+1}{x^2+x+1} dx + \frac{2}{3} \log(1-x)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{3} \log(1-x) - \frac{1}{3} \int \frac{2x+1}{x^2+x+1} dx$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{2}{3} \log(1-x) - \frac{1}{3} \log\left(x^2+x+1\right)$$

input
```
Int[(1 + x)/(-1 + x^3),x]
```

output
```
(2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

3.132.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

3.132.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
norman	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
risch	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
parallelrisch	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
meijerg	$\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,a\right.}{3(x^3)^{\frac{2}{3}}}$

input
```
int((1+x)/(x^3-1),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(-1+x)-1/3*ln(x^2+x+1)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{-1+x^3}\,dx = -\frac{1}{3}\,\log\left(x^2+x+1\right) + \frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((1+x)/(x^3-1),x, algorithm="fricas")
```

output
```
-1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{-1+x^3}\,dx = \frac{2\log\left(x-1\right)}{3} - \frac{\log\left(x^2+x+1\right)}{3}$$

input
```
integrate((1+x)/(x**3-1),x)
```

output
```
2*log(x - 1)/3 - log(x**2 + x + 1)/3
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{-1+x^3}\,dx = -\frac{1}{3}\,\log\left(x^2+x+1\right) + \frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((1+x)/(x^3-1),x, algorithm="maxima")
```

output
```
-1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{-1+x^3}\,dx = -\frac{1}{3}\,\log\left(x^2+x+1\right)+\frac{2}{3}\,\log\left(|x-1|\right)$$

input
```
integrate((1+x)/(x^3-1),x, algorithm="giac")
```

output
```
-1/3*log(x^2 + x + 1) + 2/3*log(abs(x - 1))
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{-1+x^3}\,dx = \frac{2\,\ln\left(x-1\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

input
```
int((x + 1)/(x^3 - 1),x)
```

output
```
(2*log(x - 1))/3 - log(x + x^2 + 1)/3
```

3.133 $\int \frac{1+x^4}{x(1+x^2)^2} dx$

3.133.1 Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1+x^4}{x(1+x^2)^2} dx = \frac{1}{1+x^2} + \log(x)$$

output
```
1/(x^2+1)+ln(x)
```

3.133.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x(1+x^2)^2} dx = \frac{1}{1+x^2} + \log(x)$$

input
```
Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
```

output
```
(1 + x^2)^(-1) + Log[x]
```

3.133.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1579, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^4+1}{x^2\left(x^2+1\right)^2}dx^2$$

$$\Big\downarrow \text{522}$$

$$\frac{1}{2}\int \left(\frac{1}{x^2}-\frac{2}{\left(x^2+1\right)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{x^2+1}+\log\left(x^2\right)\right)$$

input
```
Int[(1 + x^4)/(x*(1 + x^2)^2),x]
```

output
```
(2/(1 + x^2) + Log[x^2])/2
```

3.133.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.133.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
norman	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
risch	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
parallelrisch	$\frac{x^2\ln(x)+1+\ln(x)}{x^2+1}$	19
meijerg	$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}$	31

input
```
int((x^4+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+1)+ln(x)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\frac{(x^2+1)\log\left(x\right)+1}{x^2+1}$$

input
```
integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

output
```
((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\log\left(x\right)+\frac{1}{x^2+1}$$

input
```
integrate((x**4+1)/x/(x**2+1)**2,x)
```

output
```
log(x) + 1/(x**2 + 1)
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/(x^2 + 1) + 1/2*log(x^2)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/(x^2 + 1) + 1/2*log(x^2)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx=\ln\left(x\right)+\frac{1}{x^2+1}$$

input
```
int((x^4 + 1)/(x*(x^2 + 1)^2),x)
```

output
```
log(x) + 1/(x^2 + 1)
```

3.134 $\int \frac{1}{-2x^3+x^4}\,dx$

3.134.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{-2x^3+x^4}\,dx = \frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

output
```
1/4/x^2+1/4/x+1/8*ln(2-x)-1/8*ln(x)
```

3.134.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{-2x^3+x^4}\,dx = \frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

input
```
Integrate[(-2*x^3 + x^4)^(-1),x]
```

output
```
1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

3.134.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-2x^3}dx$$

$$\Big\downarrow \ 2026$$

$$\int \frac{1}{(x-2)x^3}dx$$

$$\Big\downarrow \ 54$$

$$\int \left(-\frac{1}{2x^3}-\frac{1}{4x^2}-\frac{1}{8x}+\frac{1}{8(x-2)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

input
```
Int[(-2*x^3 + x^4)^(-1),x]
```

output
```
1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

3.134.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.134.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
norman	$\frac{\frac{1}{4}+\frac{x}{4}}{x^2}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$	21
risch	$\frac{\frac{1}{4}+\frac{x}{4}}{x^2}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$	21
default	$\frac{1}{4x^2}+\frac{1}{4x}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$	22
parallelrisch	$-\frac{x^2\ln(x)-\ln(-2+x)x^2-2-2x}{8x^2}$	26
meijerg	$\frac{1}{4x^2}+\frac{1}{4x}-\frac{\ln(x)}{8}+\frac{\ln(2)}{8}-\frac{i\pi}{8}+\frac{\ln(1-\frac{x}{2})}{8}$	32

input
```
int(1/(x^4-2*x^3),x,method=_RETURNVERBOSE)
```

output
```
(1/4+1/4*x)/x^2-1/8*ln(x)+1/8*ln(-2+x)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-2x^3+x^4}\,dx=\frac{x^2\log(x-2)-x^2\log(x)+2\,x+2}{8\,x^2}$$

input
```
integrate(1/(x^4-2*x^3),x, algorithm="fricas")
```

output
```
1/8*(x^2*log(x - 2) - x^2*log(x) + 2*x + 2)/x^2
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{-2x^3+x^4}\,dx=-\frac{\log(x)}{8}+\frac{\log(x-2)}{8}+\frac{x+1}{4x^2}$$

input
```
integrate(1/(x**4-2*x**3),x)
```

output
```
-log(x)/8 + log(x - 2)/8 + (x + 1)/(4*x**2)
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{-2x^3+x^4}\,dx = \frac{x+1}{4\,x^2}+\frac{1}{8}\,\log\left(x-2\right)-\frac{1}{8}\,\log\left(x\right)$$

input
```
integrate(1/(x^4-2*x^3),x, algorithm="maxima")
```

output
```
1/4*(x + 1)/x^2 + 1/8*log(x - 2) - 1/8*log(x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{-2x^3+x^4}\,dx = \frac{x+1}{4\,x^2}+\frac{1}{8}\,\log\left(|x-2|\right)-\frac{1}{8}\,\log\left(|x|\right)$$

input
```
integrate(1/(x^4-2*x^3),x, algorithm="giac")
```

output
```
1/4*(x + 1)/x^2 + 1/8*log(abs(x - 2)) - 1/8*log(abs(x))
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{1}{-2x^3+x^4}\,dx = \frac{\frac{x}{4}+\frac{1}{4}}{x^2}-\frac{\mathrm{atanh}(x-1)}{4}$$

input
```
int(-1/(2*x^3 - x^4),x)
```

output
```
(x/4 + 1/4)/x^2 - atanh(x - 1)/4
```

3.135 $\int \frac{1-x^3}{x(1+x^2)}\,dx$

3.135.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.135.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 - x^3)/(x*(1 + x^2)),x]
```

output
```
-x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

3.135.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x\left(x^2+1\right)}dx$$
$$\Big\downarrow \ 2333$$
$$\int \left(\frac{1-x}{x^2+1}+\frac{1}{x}-1\right)dx$$
$$\Big\downarrow \ 2009$$
$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)$$

input
```
Int[(1 - x^3)/(x*(1 + x^2)),x]
```

output
```
-x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.135.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-x+\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	17
meijerg	$-x+\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	17
risch	$-x+\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	17
parallelrisch	$-x+\ln\left(x\right)-\frac{\ln(x-i)}{2}-\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}+\frac{i\ln(x+i)}{2}$	37

input
```
int((-x^3+1)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1-x^3}{x\left(1+x^2\right)}\,dx=-x+\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x\right)$$

input
```
integrate((-x^3+1)/x/(x^2+1),x, algorithm="fricas")
```

output
```
-x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1-x^3}{x\left(1+x^2\right)}\,dx=-x+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}\left(x\right)$$

input
```
integrate((-x**3+1)/x/(x**2+1),x)
```

output
```
-x + log(x) - log(x**2 + 1)/2 + atan(x)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

input
```
integrate((-x^3+1)/x/(x^2+1),x, algorithm="maxima")
```

output
```
-x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = -x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(|x|)$$

input
```
integrate((-x^3+1)/x/(x^2+1),x, algorithm="giac")
```

output
```
-x + arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = \ln(x)-x+\ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(-(x^3 - 1)/(x*(x^2 + 1)),x)
```

output
```
log(x) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) - x
```

3.136 $\int \frac{1}{-1+x^4} \, dx$

3.136.1 Optimal result

Integrand size = 7, antiderivative size = 13

$$\int \frac{1}{-1+x^4} \, dx = -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2}$$

output
```
-1/2*arctan(x)-1/2*arctanh(x)
```

3.136.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{-1+x^4} \, dx = -\frac{\arctan(x)}{2} + \frac{1}{4}\log(1-x) - \frac{1}{4}\log(1+x)$$

input
```
Integrate[(-1 + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

3.136.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-1}dx$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(-1 + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x] - ArcTanh[x]/2
```

3.136.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$-\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	18
parallelrisch	$\frac{i\ln(x-i)}{4}-\frac{i\ln(x+i)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	30
meijerg	$\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

input
```
int(1/(x^4-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)-1/2*arctanh(x)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

input
```
integrate(1/(x^4-1),x, algorithm="fricas")
```

output
```
-1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = \frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(x**4-1),x)
```

output
```
log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

input
```
integrate(1/(x^4-1),x, algorithm="maxima")
```

output
```
-1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log(|x+1|) + \frac{1}{4}\log(|x-1|)$$

input
```
integrate(1/(x^4-1),x, algorithm="giac")
```

output
```
-1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.136.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{-1+x^4}\,dx = -\frac{\operatorname{atan}(x)}{2} - \frac{\operatorname{atanh}(x)}{2}$$

input
```
int(1/(x^4 - 1),x)
```

output
```
- atan(x)/2 - atanh(x)/2
```

3.137 $\int \frac{1}{1+x^4} dx$

3.137.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \frac{1}{1+x^4} dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+
x^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^4} dx = \frac{-2\arctan\left(1-\sqrt{2}x\right)+2\arctan\left(1+\sqrt{2}x\right)-\log\left(1-\sqrt{2}x+x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

input
```
Integrate[(1 + x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x +
x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

3.137.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+1}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[(1 + x^4)^(-1),x]
```

output
```
(-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1
/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2])
)/2
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.137.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}$

input
```
int(1/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.137.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{1}{1+x^4}\,dx = \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right)-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right)+\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right)-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)$$

input
```
integrate(1/(x^4+1),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*l
og(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2
)) - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

input
```
integrate(1/(x**4+1),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(1/(x^4+1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(1/(x^4+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(
2)*log(x^2 - sqrt(2)*x + 1)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{1}{1+x^4}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x^4 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x
*(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.137. $\int \frac{1}{1+x^4} \, dx$

3.138 $\int \frac{x^2}{(2+2x+x^2)^2} dx$

3.138.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^2}{(2+2x+x^2)^2} dx = -\frac{x(2+x)}{2(2+2x+x^2)} + \arctan(1+x)$$

output
```
-1/2*x*(2+x)/(x^2+2*x+2)+arctan(1+x)
```

3.138.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2} dx = \frac{1}{2+2x+x^2} + \arctan(1+x)$$

input
```
Integrate[x^2/(2 + 2*x + x^2)^2,x]
```

output
```
(2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

3.138.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1153, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+2x+2)^2}dx$$

$$\Big\downarrow \text{1153}$$

$$\int \frac{1}{x^2+2x+2}dx - \frac{x(x+2)}{2(x^2+2x+2)}$$

$$\Big\downarrow \text{1082}$$

$$-\int \frac{1}{-(x+1)^2-1}d(x+1) - \frac{x(x+2)}{2(x^2+2x+2)}$$

$$\Big\downarrow \text{217}$$

$$\arctan(x+1) - \frac{x(x+2)}{2(x^2+2x+2)}$$

input
```
Int[x^2/(2 + 2*x + x^2)^2,x]
```

output
```
-1/2*(x*(2 + x))/(2 + 2*x + x^2) + ArcTan[1 + x]
```

3.138.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

3.138.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{1}{x^2+2x+2}+\arctan\left(1+x\right)$	16
risch	$\frac{1}{x^2+2x+2}+\arctan\left(1+x\right)$	16
parallelrisch	$-\frac{i\ln(x+1-i)x^2-i\ln(x+1+i)x^2+2i\ln(x+1-i)x-2i\ln(x+1+i)x-2+2i\ln(x+1-i)-2i\ln(x+1+i)}{2(x^2+2x+2)}$	77

input
```
int(x^2/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+2*x+2)+arctan(1+x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx=\frac{(x^2+2\,x+2)\arctan\left(x+1\right)+1}{x^2+2\,x+2}$$

input
```
integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
```

output
```
((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \operatorname{atan}(x+1) + \frac{1}{x^2+2x+2}$$

input
```
integrate(x**2/(x**2+2*x+2)**2,x)
```

output
```
atan(x + 1) + 1/(x**2 + 2*x + 2)
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \frac{1}{x^2+2\,x+2} + \arctan(x+1)$$

input
```
integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
```

output
```
1/(x^2 + 2*x + 2) + arctan(x + 1)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \frac{1}{x^2+2\,x+2} + \arctan(x+1)$$

input
```
integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
```

output
```
1/(x^2 + 2*x + 2) + arctan(x + 1)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(2+2x+x^2)^2}\,dx = \operatorname{atan}(x+1) + \frac{1}{x^2+2\,x+2}$$

input
```
int(x^2/(2*x + x^2 + 2)^2,x)
```

output
```
atan(x + 1) + 1/(2*x + x^2 + 2)
```

3.139 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

3.139.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

output
```
-x/(x^5+x+1)
```

3.139.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

input
```
Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

output
```
-(x/(1 + x + x^5))
```

3.139.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^5-1}{(x^5+x+1)^2}\,dx$$

$$\Big\downarrow \; 2021$$

$$-\frac{x}{x^5+x+1}$$

input
```
Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

output
```
-(x/(1 + x + x^5))
```

3.139.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.139.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
parallelrisch	$-\frac{x}{x^5+x+1}$	12
default	$-\frac{-3x^2+5x-1}{7(x^3-x^2+1)}+\frac{-3x-1}{7x^2+7x+7}$	41

input
```
int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(x^5+x+1)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
```

output
```
-x/(x^5 + x + 1)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

output
```
-x/(x**5 + x + 1)
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

output
```
-x/(x^5 + x + 1)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

output
```
-x/(x^5 + x + 1)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{x^5+x+1}$$

input
```
int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

output
```
-x/(x + x^5 + 1)
```

3.140 $\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx$

3.140.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx=\frac{x}{2\sqrt{5}}+\frac{\arctan\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}$$

output
```
1/10*x*5^(1/2)+1/5*arctan((2*cos(x)+sin(x))/(5-cos(x)+2*sin(x)+2*5^(1/2)))
*5^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx=\frac{\arctan\left(\frac{1+3\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Integrate[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

output
```
ArcTan[(1 + 3*Tan[x/2])/Sqrt[5]]/Sqrt[5]
```

3.140.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sin(x)-\cos(x)+5}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{2\sin(x)-\cos(x)+5}dx$$
$$\Big\downarrow \text{ 3603}$$
$$2\int \frac{1}{6\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+4}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{ 1083}$$
$$-4\int \frac{1}{-\left(12\tan\left(\frac{x}{2}\right)+4\right)^2-80}d\left(12\tan\left(\frac{x}{2}\right)+4\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{\arctan\left(\frac{12\tan\left(\frac{x}{2}\right)+4}{4\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Int[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

output
```
ArcTan[(4 + 12*Tan[x/2])/(4*Sqrt[5])]/Sqrt[5]
```

3.140.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.140.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\sqrt{5}\, \arctan\left(\frac{\left(6 \tan\left(\frac{x}{2}\right)+2\right)\sqrt{5}}{10}\right)}{5}$	20
risch	$\frac{i\sqrt{5}\, \ln\left(\mathrm{e}^{ix}-1+2i+\frac{4i\sqrt{5}}{5}-\frac{2\sqrt{5}}{5}\right)}{10}-\frac{i\sqrt{5}\, \ln\left(\mathrm{e}^{ix}-1+2i-\frac{4i\sqrt{5}}{5}+\frac{2\sqrt{5}}{5}\right)}{10}$	56

input
```
int(1/(5-cos(x)+2*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*5^(1/2)*arctan(1/10*(6*tan(1/2*x)+2)*5^(1/2))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\, dx = \frac{1}{10}\sqrt{5}\arctan\left(-\frac{\sqrt{5}\cos\left(x\right)-2\sqrt{5}\sin\left(x\right)-\sqrt{5}}{2\left(2\,\cos\left(x\right)+\sin\left(x\right)\right)}\right)$$

input
```
integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*arctan(-1/2*(sqrt(5)*cos(x) - 2*sqrt(5)*sin(x) - sqrt(5))/(2*
cos(x) + sin(x)))
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{3\sqrt{5}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{5}}{5}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{5}$$

input
```
integrate(1/(5-cos(x)+2*sin(x)),x)
```

output
```
sqrt(5)*(atan(3*sqrt(5)*tan(x/2)/5 + sqrt(5)/5) + pi*floor((x/2 - pi/2)/pi
))/5
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)\right)$$

input
```
integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*arctan(1/5*sqrt(5)*(3*sin(x)/(cos(x) + 1) + 1))
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{1}{10}\sqrt{5}\left(x+2\arctan\left(-\frac{\sqrt{5}\sin(x)-\cos(x)-3\sin(x)-1}{\sqrt{5}\cos(x)+\sqrt{5}-3\cos(x)+\sin(x)+3}\right)\right)$$

input
```
integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="giac")
```

output
```
1/10*sqrt(5)*(x + 2*arctan(-(sqrt(5)*sin(x) - cos(x) - 3*sin(x) - 1)/(sqrt
(5)*cos(x) + sqrt(5) - 3*cos(x) + sin(x) + 3)))
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

$$\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx = \frac{\sqrt{20}\,\mathrm{atan}\left(\frac{3\sqrt{20}\tan\left(\frac{x}{2}\right)}{10}+\frac{\sqrt{20}}{10}\right)}{10}$$

input
```
int(1/(2*sin(x) - cos(x) + 5),x)
```

output
```
(20^(1/2)*atan((3*20^(1/2)*tan(x/2))/10 + 20^(1/2)/10))/10
```

3.141 $\int \frac{1}{1+a\cos(x)}\,dx$

3.141.1 Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{1}{1+a\cos(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}$$

output
```
2*arctan((1-a)^(1/2)*tan(1/2*x)/(1+a)^(1/2))/(-a^2+1)^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+a\cos(x)}\,dx = \frac{2\text{arctanh}\left(\frac{(-1+a)\tan\left(\frac{x}{2}\right)}{\sqrt{-1+a^2}}\right)}{\sqrt{-1+a^2}}$$

input
```
Integrate[(1 + a*Cos[x])^(-1),x]
```

output
```
(2*ArcTanh[((-1 + a)*Tan[x/2])/Sqrt[-1 + a^2]])/Sqrt[-1 + a^2]
```

3.141.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(x)+1}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{a\sin\left(x+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \ 3138$$
$$2\int \frac{1}{(1-a)\tan^2\left(\frac{x}{2}\right)+a+1}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \ 218$$
$$\frac{2\arctan\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

input
```
Int[(1 + a*Cos[x])^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

3.141.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.141.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{(1+a)(a-1)}}\right)}{\sqrt{(1+a)(a-1)}}$	30
risch	$\frac{\ln\left(\mathrm{e}^{ix}+\frac{i a^2+\sqrt{a^2-1}-i}{a\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{-i a^2+\sqrt{a^2-1}+i}{a\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}$	87

input
```
int(1/(1+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((1+a)*(a-1))^(1/2)*arctanh((a-1)*tan(1/2*x)/((1+a)*(a-1))^(1/2))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.00

$$\int \frac{1}{1+a\cos(x)}\,dx = \left[\frac{\log\left(-\frac{(a^2-2)\cos(x)^2-2\sqrt{a^2-1}(a+\cos(x))\sin(x)-2a^2-2a\cos(x)+1}{a^2\cos(x)^2+2a\cos(x)+1}\right)}{2\sqrt{a^2-1}},\ -\frac{\sqrt{-a^2+1}\arctan\left(\frac{\sqrt{-a^2+1}(a+\cos(x))}{(a^2-1)\sin(x)}\right)}{a^2-1}\right]$$

input
```
integrate(1/(1+a*cos(x)),x, algorithm="fricas")
```

output
```
[1/2*log(-((a^2 - 2)*cos(x)^2 - 2*sqrt(a^2 - 1)*(a + cos(x))*sin(x) - 2*a^
2 - 2*a*cos(x) + 1)/(a^2*cos(x)^2 + 2*a*cos(x) + 1))/sqrt(a^2 - 1), -sqrt(
-a^2 + 1)*arctan(sqrt(-a^2 + 1)*(a + cos(x))/((a^2 - 1)*sin(x)))/(a^2 - 1)
]
```

3.141.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(29) = 58$.

Time = 1.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.97

$$\int \frac{1}{1+a\cos(x)}\,dx = \begin{cases} \tan\left(\frac{x}{2}\right) & \text{for } a = 1 \\ -\frac{1}{\tan\left(\frac{x}{2}\right)} & \text{for } a = -1 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} + \frac{\log\left(\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+a*cos(x)),x)
```

output
```
Piecewise((tan(x/2), Eq(a, 1)), (-1/tan(x/2), Eq(a, -1)), (-log(-sqrt(a/(a
 - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a - 1)) - sqrt(a/(a
- 1) + 1/(a - 1))) + log(sqrt(a/(a - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a
/(a - 1) + 1/(a - 1)) - sqrt(a/(a - 1) + 1/(a - 1))), True))
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{1+a\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(1+a*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a^2-1.0>0)', see `assume?` for m
ore detail
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+a\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)-\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+1}}\right)\right)}{\sqrt{-a^2+1}}$$

input
```
integrate(1/(1+a*cos(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) - tan(1/2
*x))/sqrt(-a^2 + 1)))/sqrt(-a^2 + 1)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+a\cos(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{a-1}}{\sqrt{a+1}}\right)}{\sqrt{a-1}\,\sqrt{a+1}}$$

input
```
int(1/(a*cos(x) + 1),x)
```

output
```
(2*atanh((tan(x/2)*(a - 1)^(1/2))/(a + 1)^(1/2)))/((a - 1)^(1/2)*(a + 1)^(
1/2))
```

3.142 $\int \frac{1}{1+2\cos(x)}\,dx$

3.142.1 Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

output
```
-1/3*ln(-sin(1/2*x)+cos(1/2*x)*3^(1/2))*3^(1/2)+1/3*ln(sin(1/2*x)+cos(1/2*
x)*3^(1/2))*3^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.36

$$\int \frac{1}{1+2\cos(x)}\,dx = \frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + 2*Cos[x])^(-1),x]
```

output
```
(2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

3.142.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\cos(x)+1}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{2\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \; 3138$$

$$2\int \frac{1}{3-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + 2*Cos[x])^(-1),x]
```

output
```
(2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

3.142.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.142.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{\sqrt{3}\,\ln\left(e^{ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{3}-\frac{\sqrt{3}\,\ln\left(e^{ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{3}$	40

input
```
int(1/(1+2*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctanh(1/3*tan(1/2*x)*3^(1/2))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+2\cos(x)}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{2\cos(x)^2-2\left(\sqrt{3}\cos(x)+2\sqrt{3}\right)\sin(x)-4\cos(x)-7}{4\cos(x)^2+4\cos(x)+1}\right)$$

input
```
integrate(1/(1+2*cos(x)),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(2*cos(x)^2 - 2*(sqrt(3)*cos(x) + 2*sqrt(3))*sin(x) - 4*c
os(x) - 7)/(4*cos(x)^2 + 4*cos(x) + 1))
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3} + \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

input
```
integrate(1/(1+2*cos(x)),x)
```

output
```
-sqrt(3)*log(tan(x/2) - sqrt(3))/3 + sqrt(3)*log(tan(x/2) + sqrt(3))/3
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{3}+\frac{\sin(x)}{\cos(x)+1}}\right)$$

input
```
integrate(1/(1+2*cos(x)),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*log(-(sqrt(3) - sin(x)/(cos(x) + 1))/(sqrt(3) + sin(x)/(cos(x
) + 1)))
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+2\cos(x)}\,dx = -\frac{1}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}\right)$$

input
```
integrate(1/(1+2*cos(x)),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x))/abs(2*sqrt(3) + 2*tan(1/2*
x)))
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+2\cos(x)}\,dx = \frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

input
```
int(1/(2*cos(x) + 1),x)
```

output
```
(2*3^(1/2)*atanh((3^(1/2)*tan(x/2))/3))/3
```

3.143 $\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx$

3.143.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{2x}{\sqrt{3}} - \frac{4\arctan\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

output
```
2/3*x*3^(1/2)-4/3*arctan(sin(x)/(2+cos(x)+3^(1/2)))*3^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + Cos[x]/2)^(-1),x]
```

output
```
(4*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

3.143.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{\cos(x)}{2}+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\frac{1}{2}\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{2x}{\sqrt{3}}-\frac{4\arctan\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

input
```
Int[(1 + Cos[x]/2)^(-1),x]
```

output
```
(2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

3.143.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

3.143.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{4\sqrt{3}\,\arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{ix}+\sqrt{3}+2\right)}{3}-\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{ix}-\sqrt{3}+2\right)}{3}$	38

input
```
int(1/(1+1/2*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
4/3*3^(1/2)*arctan(1/3*tan(1/2*x)*3^(1/2))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(x)+\sqrt{3}}{3\sin(x)}\right)$$

input
```
integrate(1/(1+1/2*cos(x)),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

input
```
integrate(1/(1+1/2*cos(x)),x)
```

output
```
4*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(x)}{3(\cos(x)+1)}\right)$$

input
```
integrate(1/(1+1/2*cos(x)),x, algorithm="maxima")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{2}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\sin(x)}{\sqrt{3}\cos(x)+\sqrt{3}-\cos(x)+1}\right)\right)$$

input
```
integrate(1/(1+1/2*cos(x)),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqr
t(3) - cos(x) + 1)))
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx = \frac{4\sqrt{3}\left(\frac{x}{2}-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)\right)\right)}{3}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

input
```
int(1/(cos(x)/2 + 1),x)
```

output
```
(4*3^(1/2)*(x/2 - atan(tan(x/2))))/3 + (4*3^(1/2)*atan((3^(1/2)*tan(x/2))/
3))/3
```

3.144 $\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx$

3.144.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = x-\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}$$

output
```
x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = x-\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

input
```
Integrate[Sin[x]^2/(1 + Sin[x]^2),x]
```

output
```
x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

3.144.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3650, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{\sin^2(x)+1} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^2}{\sin(x)^2+1}dx$$

$$\Big\downarrow \text{ 3650}$$

$$x-\int \frac{1}{\sin^2(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$x-\int \frac{1}{\sin(x)^2+1}dx$$

$$\Big\downarrow \text{ 3660}$$

$$x-\int \frac{1}{2\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 216}$$

$$x-\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

input
```
Int[Sin[x]^2/(1 + Sin[x]^2),x]
```

output
```
x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

3.144.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.144.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{2}+\arctan\left(\tan\left(x\right)\right)$	17
risch	$x-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)}{4}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)}{4}$	41

input
```
int(sin(x)^2/(1+sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctan(tan(x)*2^(1/2))+arctan(tan(x))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos{(x)}^2-2\,\sqrt{2}}{4\,\cos{(x)}\sin{(x)}}\right)+x$$

input
```
integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) +
 x
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(36) = 72.

Time = 24.44 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.89

$$\begin{aligned}\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx &= \frac{31988856\sqrt{2}x}{31988856\sqrt{2}+45239074}+\frac{45239074x}{31988856\sqrt{2}+45239074}\\ &\quad-\frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}\\ &\quad-\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}\\ &\quad-\frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}\\ &\quad-\frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}\end{aligned}$$

input
```
integrate(sin(x)**2/(1+sin(x)**2),x)
```

output
```
31988856*sqrt(2)*x/(31988856*sqrt(2) + 45239074) + 45239074*x/(31988856*sq
rt(2) + 45239074) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2
*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 54
608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + p
i*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 13250218*sqrt(2*
sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/p
i))/(31988856*sqrt(2) + 45239074) - 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(a
tan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*s
qrt(2) + 45239074)
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+x$$

input
```
integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)+x$$

input
```
integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x
) + sqrt(2) - 2*cos(2*x) + 2))) + x
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx = x - \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)}{2}$$

input
```
int(sin(x)^2/(sin(x)^2 + 1),x)
```

output
```
x - (2^(1/2)*(x - atan(tan(x))))/2 - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

3.145 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

3.145.1 Optimal result

Integrand size = 19, antiderivative size = 15

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

output
```
arctan(a*tan(x)/b)/a/b
```

3.145.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(a*Tan[x])/b]/(a*b)
```

3.145.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4889, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2\sin^2(x)+b^2\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a^2\sin(x)^2+b^2\cos(x)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{a^2\tan^2(x)+b^2}d\tan(x)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(a*Tan[x])/b]/(a*b)
```

3.145.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.145.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$	16
parallelrisch	$\frac{i\left(\ln\left(\frac{ia\sin(x)-b\cos(x)}{\cos(x)+1}\right)-\ln\left(\frac{-b\cos(x)-ia\sin(x)}{\cos(x)+1}\right)\right)}{2ab}$	53
risch	$\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a+b}{a-b}\right)}{2ab}-\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a-b}{a+b}\right)}{2ab}$	58

input
```
int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*tan(x)/b)/a/b
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

input
```
integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

3.145.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71839 vs. $2(10) = 20$.

Time = 18.58 (sec) , antiderivative size = 71839, normalized size of antiderivative = 4789.27

$$\int \frac{1}{b^2\cos^2(x) + a^2\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2
)/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1
)), Eq(a, 0)), (x/(b**2*sin(x)**2 + b**2*cos(x)**2), Eq(a, b) | Eq(a, -b))
, (8192*a**15*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b
**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 4556
8*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1...
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x) + a^2\sin^2(x)}\,dx = \frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x)/b)/(a*b)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{b^2\cos^2(x) + a^2\sin^2(x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi} + \frac{1}{2}\right\rfloor + \arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x) + a^2\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{a\tan(x)}{b}\right)}{a\,b}$$

input
```
int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
```

output
```
atan((a*tan(x))/b)/(a*b)
```

3.146 $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

3.146.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = \frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

output
```
sin(x)/b/(b*cos(x)+a*sin(x))
```

3.146.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = \frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

input
```
Integrate[(b*Cos[x] + a*Sin[x])^(-2),x]
```

output
```
Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

3.146.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(x)+b\cos(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(x)+b\cos(x))^2}dx$$

$$\Big\downarrow \text{3554}$$

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

input
```
Int[(b*Cos[x] + a*Sin[x])^(-2),x]
```

output
```
Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

3.146.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.146.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{a(a\tan(x)+b)}$	14
parallelrisch	$\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$	18
norman	$\frac{-\frac{1}{a}+\frac{\tan^2\left(\frac{x}{2}\right)}{a}}{-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b}$	38
risch	$-\frac{2i}{(a\,e^{2ix}+ib\,e^{2ix}-a+ib)(ib+a)}$	38

input
```
int(1/(b*cos(x)+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/(a*tan(x)+b)
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = -\frac{a\cos(x)-b\sin(x)}{(a^2b+b^3)\cos(x)+(a^3+ab^2)\sin(x)}$$

input
```
integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="fricas")
```

output
```
-(a*cos(x) - b*sin(x))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)*sin(x))
```

3.146.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(14) = 28.

Time = 134.57 (sec) , antiderivative size = 602, normalized size of antiderivative = 35.41

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} \\ \frac{x\tan^4\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)} + \frac{}{2b} \\ \frac{\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tan\left(\frac{x}{2}\right)}}{a^2} \\ \frac{2\tan\left(\frac{x}{2}\right)}{2ab\tan\left(\frac{x}{2}\right)-b^2\tan^2\left(\frac{x}{2}\right)+b^2} \end{cases}$$

input
```
integrate(1/(b*cos(x)+a*sin(x))**2,x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (x*tan(x/
2)**4/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**
2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(
x/2) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2*x*tan(x/2)**2/(2*b**2*sin(x)**2*t
an(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(
x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*t
an(x/2)**2) + x/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)*
*2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*c
os(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2*tan(x/2)**3/(2*b**2*sin
(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b
**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos
(x)**2*tan(x/2)**2) - 2*tan(x/2)/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*si
n(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3
- 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2), Eq(a, b*(
tan(x/2) - 1)*(tan(x/2) + 1)/(2*tan(x/2)))), ((tan(x/2)/2 - 1/(2*tan(x/2))
)/a**2, Eq(b, 0)), (2*tan(x/2)/(2*a*b*tan(x/2) - b**2*tan(x/2)**2 + b**2),
 True))
```

3.146. $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

3.146.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = -\frac{1}{a^2\tan{(x)}+ab}$$

input
```
integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="maxima")
```

output
```
-1/(a^2*tan(x) + a*b)
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = -\frac{1}{(a\tan{(x)}+b)a}$$

input
```
integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="giac")
```

output
```
-1/((a*tan(x) + b)*a)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{b\left(-b\tan\left(\frac{x}{2}\right)^2+2\,a\tan\left(\frac{x}{2}\right)+b\right)}$$

input
```
int(1/(b*cos(x) + a*sin(x))^2,x)
```

output
```
(2*tan(x/2))/(b*(b + 2*a*tan(x/2) - b*tan(x/2)^2))
```

3.147 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

3.147.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \frac{1}{2}\log(1+\cos(x)+\sin(x)) - \frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

output
```
1/2*x-1/2*ln(1+cos(x)+sin(x))-1/2*ln(1+tan(1/2*x))
```

3.147.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[Cos[x/2] + Sin[x/2]]
```

3.147.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3616, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)+1} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)+1}dx$$

$$\Big\downarrow \text{ 3616}$$

$$-\frac{1}{2}\int \frac{1}{\cos(x)+\sin(x)+1}dx+\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}\int \frac{1}{\cos(x)+\sin(x)+1}dx+\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

$$\Big\downarrow \text{ 3603}$$

$$-\int \frac{1}{2\tan\left(\frac{x}{2}\right)+2}d\tan\left(\frac{x}{2}\right)+\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

input
```
Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

3.147.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.147.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\ln\left(i+{\mathrm e}^{ix}\right)$	20
default	$\frac{\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	27
parallelrisch	$-\ln\left(-\frac{\left(\cot\left(x\right)-1-\csc\left(x\right)\right)\sqrt{2}}{2}\right)+\ln\left(\sqrt{\frac{1}{\cos\left(x\right)+1}}\right)+\frac{x}{2}$	30
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)}{2}$	46

input
```
int(sin(x)/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*I*x-ln(I+exp(I*x))
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.37

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
1/2*x - 1/2*log(sin(x) + 1)
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right)+1\right) + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2}$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

output
```
x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) - \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right) + \frac{1}{2}\,\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x
)^2/(cos(x) + 1)^2 + 1)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)+1\right) + \ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(sin(x)/(cos(x) + sin(x) + 1),x)
```

output
```
log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(
1/2 + 1i/2)
```

3.148 $\int \sqrt{3-x^2}\,dx$

3.148.1 Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\arcsin\left(\frac{x}{\sqrt{3}}\right)$$

output
```
3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}x\sqrt{3-x^2}+3\arctan\left(\frac{-\sqrt{3}+x}{\sqrt{3-x^2}}\right)$$

input
```
Integrate[Sqrt[3 - x^2],x]
```

output
```
(x*Sqrt[3 - x^2])/2 + 3*ArcTan[(-Sqrt[3] + x)/Sqrt[3 - x^2]]
```

3.148.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-x^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{2}\int \frac{1}{\sqrt{3-x^2}}dx+\frac{1}{2}\sqrt{3-x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}\arcsin\left(\frac{x}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3-x^2}x$$

input
```
Int[Sqrt[3 - x^2],x]
```

output
```
(x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

3.148.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.148.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3 \arcsin\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{-x^{2}+3}}{2}$	23
risch	$-\frac{x\left(x^{2}-3\right)}{2\sqrt{-x^{2}+3}}+\frac{3 \arcsin\left(\frac{x\sqrt{3}}{3}\right)}{2}$	28
pseudoelliptic	$\frac{x\sqrt{-x^{2}+3}}{2}-\frac{3 \arctan\left(\frac{\sqrt{-x^{2}+3}}{x}\right)}{2}$	30
meijerg	$\frac{3i\left(-\frac{2i\sqrt{\pi}\, x\sqrt{3}\, \sqrt{-\frac{x^{2}}{3}+1}}{3}-2i\sqrt{\pi}\, \arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{4\sqrt{\pi}}$	40
trager	$\frac{x\sqrt{-x^{2}+3}}{2}+\frac{3 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+3}+x\right)}{2}$	41

input
```
int((-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+3}x - \frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)$$

input
```
integrate((-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 3)*x - 3/2*arctan(sqrt(-x^2 + 3)/x)
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{3-x^2}\,dx = \frac{x\sqrt{3-x^2}}{2} + \frac{3\,\mathrm{asin}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

input
```
integrate((-x**2+3)**(1/2),x)
```

output
```
x*sqrt(3 - x**2)/2 + 3*asin(sqrt(3)*x/3)/2
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+3}x + \frac{3}{2}\,\arcsin\left(\frac{1}{3}\,\sqrt{3}x\right)$$

input
```
integrate((-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \sqrt{3-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+3}x + \frac{3}{2}\,\arcsin\left(\frac{1}{3}\,\sqrt{3}x\right)$$

input
```
integrate((-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \sqrt{3-x^2}\,dx=\frac{3\,\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}+\frac{x\,\sqrt{3-x^2}}{2}$$

input
```
int((3 - x^2)^(1/2),x)
```

output
```
(3*asin((3^(1/2)*x)/3))/2 + (x*(3 - x^2)^(1/2))/2
```

3.149 $\int \frac{x}{\sqrt{3-x^2}}\,dx$

3.149.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

output
```
-(-x^2+3)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

input
```
Integrate[x/Sqrt[3 - x^2],x]
```

output
```
-Sqrt[3 - x^2]
```

3.149.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3-x^2}}\,dx$$

$$\Big\downarrow \; 241$$

$$-\sqrt{3-x^2}$$

input
```
Int[x/Sqrt[3 - x^2],x]
```

output
```
-Sqrt[3 - x^2]
```

3.149.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.149.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\sqrt{-x^2+3}$	12
derivativedivides	$-\sqrt{-x^2+3}$	12
default	$-\sqrt{-x^2+3}$	12
trager	$-\sqrt{-x^2+3}$	12
pseudoelliptic	$-\sqrt{-x^2+3}$	12
risch	$\frac{x^2-3}{\sqrt{-x^2+3}}$	16
meijerg	$-\frac{\sqrt{3}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-\frac{x^2}{3}+1}\right)}{2\sqrt{\pi}}$	29

input
```
int(x/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+3)^(1/2)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{-x^2+3}$$

input
```
integrate(x/(-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 3)
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

input
```
integrate(x/(-x**2+3)**(1/2),x)
```

output
```
-sqrt(3 - x**2)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{-x^2+3}$$

input
```
integrate(x/(-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 3)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{-x^2+3}$$

input
```
integrate(x/(-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 3)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

input
```
int(x/(3 - x^2)^(1/2),x)
```

output
```
-(3 - x^2)^(1/2)
```

3.150 $\int \frac{\sqrt{3-x^2}}{x} dx$

3.150.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sqrt{3-x^2}}{x} dx = \sqrt{3-x^2} - \sqrt{3}\text{arctanh}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

output
```
-arctanh(1/3*(-x^2+3)^(1/2)*3^(1/2))*3^(1/2)+(-x^2+3)^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{3-x^2}}{x} dx = \sqrt{3-x^2} - \sqrt{3}\text{arctanh}\left(\sqrt{1-\frac{x^2}{3}}\right)$$

input
```
Integrate[Sqrt[3 - x^2]/x,x]
```

output
```
Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[1 - x^2/3]]
```

3.150.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3-x^2}}{x} \, dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int \frac{\sqrt{3-x^2}}{x^2} dx^2$$
$$\Big\downarrow \text{ 60}$$
$$\frac{1}{2} \left(3 \int \frac{1}{x^2\sqrt{3-x^2}} dx^2 + 2\sqrt{3-x^2}\right)$$
$$\Big\downarrow \text{ 73}$$
$$\frac{1}{2} \left(2\sqrt{3-x^2} - 6 \int \frac{1}{3-x^4} d\sqrt{3-x^2}\right)$$
$$\Big\downarrow \text{ 219}$$
$$\frac{1}{2} \left(2\sqrt{3-x^2} - 2\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)\right)$$

input
```
Int[Sqrt[3 - x^2]/x,x]
```

output
```
(2*Sqrt[3 - x^2] - 2*Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]])/2
```

3.150.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.150.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
default	$\sqrt{-x^{2}+3}-\sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{-x^{2}+3}}\right)$	30
pseudoelliptic	$-\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}+3}\, \sqrt{3}}{3}\right) \sqrt{3}+\sqrt{-x^{2}+3}$	31
trager	$\sqrt{-x^{2}+3}+\operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(\frac{\sqrt{-x^{2}+3}-\operatorname{RootOf}\left(\_Z^{2}-3\right)}{x}\right)$	41
meijerg	$-\frac{\sqrt{3}\, \left(-2\left(2-2\ln(2)+2\ln(x)-\ln(3)+i\pi\right)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\, \sqrt{-\frac{x^{2}}{3}+1}+4\sqrt{\pi}\, \ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^{2}}{3}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	71

input
```
int((-x^2+3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-x^2+3)^(1/2)-3^(1/2)*arctanh(3^(1/2)/(-x^2+3)^(1/2))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \frac{1}{2}\sqrt{3}\log\left(-\frac{x^2+2\sqrt{3}\sqrt{-x^2+3}-6}{x^2}\right)+\sqrt{-x^2+3}$$

input
```
integrate((-x^2+3)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*log(-(x^2 + 2*sqrt(3)*sqrt(-x^2 + 3) - 6)/x^2) + sqrt(-x^2 + 3
)
```

3.150.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \begin{cases} i\sqrt{x^2-3}-\sqrt{3}\log(x)+\frac{\sqrt{3}\log(x^2)}{2}+\sqrt{3}i\operatorname{asin}\left(\frac{\sqrt{3}}{x}\right) & \text{for } |x^2|>3 \\ \sqrt{3-x^2}+\frac{\sqrt{3}\log(x^2)}{2}-\sqrt{3}\log\left(\sqrt{1-\frac{x^2}{3}}+1\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-x**2+3)**(1/2)/x,x)
```

output
```
Piecewise((I*sqrt(x**2 - 3) - sqrt(3)*log(x) + sqrt(3)*log(x**2)/2 + sqrt(
3)*I*asin(sqrt(3)/x), Abs(x**2) > 3), (sqrt(3 - x**2) + sqrt(3)*log(x**2)/
2 - sqrt(3)*log(sqrt(1 - x**2/3) + 1), True))
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = -\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+3}}{|x|}+\frac{6}{|x|}\right)+\sqrt{-x^2+3}$$

input
```
integrate((-x^2+3)^(1/2)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 3)/abs(x) + 6/abs(x)) + sqrt(-x^2 + 3)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{3}-\sqrt{-x^2+3}}{\sqrt{3}+\sqrt{-x^2+3}}\right)+\sqrt{-x^2+3}$$

input
```
integrate((-x^2+3)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(3)*log((sqrt(3) - sqrt(-x^2 + 3))/(sqrt(3) + sqrt(-x^2 + 3))) + s
qrt(-x^2 + 3)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{3-x^2}}{x}\,dx = \sqrt{3}\,\ln\left(\sqrt{\frac{3}{x^2}-1}-\sqrt{3}\,\sqrt{\frac{1}{x^2}}\right)+\sqrt{3-x^2}$$

input
```
int((3 - x^2)^(1/2)/x,x)
```

output
```
3^(1/2)*log((3/x^2 - 1)^(1/2) - 3^(1/2)*(1/x^2)^(1/2)) + (3 - x^2)^(1/2)
```

3.151 $\int \frac{\sqrt{x+x^2}}{x} \, dx$

3.151.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{x+x^2}}{x} \, dx = \sqrt{x+x^2} + \operatorname{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

output
```
arctanh(x/(x^2+x)^(1/2))+(x^2+x)^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{x+x^2}}{x} \, dx = \sqrt{x(1+x)}\left(1 - \frac{\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}\sqrt{1+x}}\right)$$

input
```
Integrate[Sqrt[x + x^2]/x,x]
```

output
```
Sqrt[x*(1 + x)]*(1 - Log[-Sqrt[x] + Sqrt[1 + x]]/(Sqrt[x]*Sqrt[1 + x]))
```

3.151.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+x}}{x}\,dx$$
$$\Big\downarrow \text{ 1131}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+x}}dx+\sqrt{x^2+x}$$
$$\Big\downarrow \text{ 1091}$$
$$\int \frac{1}{1-\frac{x^2}{x^2+x}}d\frac{x}{\sqrt{x^2+x}}+\sqrt{x^2+x}$$
$$\Big\downarrow \text{ 219}$$
$$\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)+\sqrt{x^2+x}$$

input
```
Int[Sqrt[x + x^2]/x,x]
```

output
```
Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

3.151.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

3.151.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
default	$\sqrt{x^2+x}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2}$	22
trager	$\sqrt{x^2+x}+\frac{\ln\left(1+2x+2\sqrt{x^2+x}\right)}{2}$	26
risch	$\frac{x(1+x)}{\sqrt{x(1+x)}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2}$	27
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{x}\,\sqrt{1+x}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{2\sqrt{\pi}}$	29
pseudoelliptic	$\frac{\ln\left(\frac{\sqrt{x(1+x)}+x}{x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x(1+x)}-x}{x}\right)}{2}+\sqrt{x\,(1+x)}$	43

input
```
int((x^2+x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(x^2+x)^(1/2)+1/2*ln(x+1/2+(x^2+x)^(1/2))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\sqrt{x^2+x}-\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

input
```
integrate((x^2+x)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(x^2 + x) - 1/2*log(-2*x + 2*sqrt(x^2 + x) - 1)
```

3.151.6 Sympy [F]

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\int \frac{\sqrt{x\,(x+1)}}{x}\,dx$$

input
```
integrate((x**2+x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(x + 1))/x, x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\sqrt{x^2+x}+\frac{1}{2}\log\left(2\,x+2\,\sqrt{x^2+x}+1\right)$$

input
```
integrate((x^2+x)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(x^2 + x) + 1/2*log(2*x + 2*sqrt(x^2 + x) + 1)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x+x^2}}{x}\,dx=\sqrt{x^2+x}-\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

input
```
integrate((x^2+x)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(x^2 + x) - 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{x+x^2}}{x}\,dx = \frac{\ln\left(x+\sqrt{x\,(x+1)}+\frac{1}{2}\right)}{2}+\sqrt{x^2+x}$$

input
```
int((x + x^2)^(1/2)/x,x)
```

output
```
log(x + (x*(x + 1))^(1/2) + 1/2)/2 + (x + x^2)^(1/2)
```

3.152 $\int \sqrt{5+x^2}\,dx$

3.152.1 Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \sqrt{5+x^2}\,dx=\frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\text{arcsinh}\left(\frac{x}{\sqrt{5}}\right)$$

output
```
5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \sqrt{5+x^2}\,dx=\frac{1}{2}x\sqrt{5+x^2}-\frac{5}{2}\log\left(-x+\sqrt{5+x^2}\right)$$

input
```
Integrate[Sqrt[5 + x^2],x]
```

output
```
(x*Sqrt[5 + x^2])/2 - (5*Log[-x + Sqrt[5 + x^2]])/2
```

3.152.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+5}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{5}{2}\int \frac{1}{\sqrt{x^2+5}}dx+\frac{1}{2}\sqrt{x^2+5}x$$

$$\Big\downarrow \text{ 222}$$

$$\frac{5}{2}\text{arcsinh}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^2+5}x$$

input
```
Int[Sqrt[5 + x^2],x]
```

output
```
(x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

3.152.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.152.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{5\,\mathrm{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)}{2}+\frac{x\sqrt{x^2+5}}{2}$	21
risch	$\frac{5\,\mathrm{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)}{2}+\frac{x\sqrt{x^2+5}}{2}$	21
trager	$\frac{x\sqrt{x^2+5}}{2}-\frac{5\ln\left(x-\sqrt{x^2+5}\right)}{2}$	26
meijerg	$-\frac{5\left(-\frac{2\sqrt{\pi}\,x\sqrt{5}\,\sqrt{1+\frac{x^2}{5}}}{5}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)\right)}{4\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{x^2+5}}{2}+\frac{5\ln\left(\frac{\sqrt{x^2+5}+x}{x}\right)}{4}-\frac{5\ln\left(\frac{\sqrt{x^2+5}-x}{x}\right)}{4}$	46

input
```
int((x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{5+x^2}\,dx=\frac{1}{2}\,\sqrt{x^2+5}x-\frac{5}{2}\,\log\left(-x+\sqrt{x^2+5}\right)$$

input
```
integrate((x^2+5)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sqrt{5+x^2}\,dx = \frac{x\sqrt{x^2+5}}{2} + \frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

input
```
integrate((x**2+5)**(1/2),x)
```

output
```
x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{5+x^2}\,dx = \frac{1}{2}\sqrt{x^2+5}x + \frac{5}{2}\operatorname{arsinh}\left(\frac{1}{5}\sqrt{5}x\right)$$

input
```
integrate((x^2+5)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 5)*x + 5/2*arcsinh(1/5*sqrt(5)*x)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{5+x^2}\,dx = \frac{1}{2}\sqrt{x^2+5}x - \frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

input
```
integrate((x^2+5)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{5+x^2}\,dx=\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}+\frac{x\,\sqrt{x^2+5}}{2}$$

input
```
int((x^2 + 5)^(1/2),x)
```

output
```
(5*asinh((5^(1/2)*x)/5))/2 + (x*(x^2 + 5)^(1/2))/2
```

3.153 $\int \frac{x}{\sqrt{1+x+x^2}} dx$

3.153.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{x}{\sqrt{1+x+x^2}} dx = \sqrt{1+x+x^2} - \frac{1}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-1/2*arcsinh(1/3*(1+2*x)*3^(1/2))+(x^2+x+1)^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{1+x+x^2}} dx = \sqrt{1+x+x^2} + \frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x/Sqrt[1 + x + x^2],x]
```

output
```
Sqrt[1 + x + x^2] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

3.153.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$\sqrt{x^2+x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$\sqrt{x^2+x+1}-\frac{\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 222}$$

$$\sqrt{x^2+x+1}-\frac{1}{2}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)$$

input
```
Int[x/Sqrt[1 + x + x^2],x]
```

output
```
Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

3.153.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.153.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\sqrt{x^2+x+1}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	21
risch	$\sqrt{x^2+x+1}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	21
trager	$\sqrt{x^2+x+1}-\frac{\ln\left(2x+1+2\sqrt{x^2+x+1}\right)}{2}$	28

input
```
int(x/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+x+1)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1}+\frac{1}{2}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} - \frac{\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$$

input
```
integrate(x/(x**2+x+1)**(1/2),x)
```

output
```
sqrt(x**2 + x + 1) - asinh(2*sqrt(3)*(x + 1/2)/3)/2
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} - \frac{1}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate(x/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + x + 1) - 1/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} + \frac{1}{2}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1} - \frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{2}$$

input
```
int(x/(x + x^2 + 1)^(1/2),x)
```

output
```
(x + x^2 + 1)^(1/2) - log(x + (x + x^2 + 1)^(1/2) + 1/2)/2
```

3.154 $\int \frac{1}{\sqrt{x+x^2}} dx$

3.154.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{x+x^2}} dx = 2\text{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

output
```
2*arctanh(x/(x^2+x)^(1/2))
```

3.154.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x+x^2}} dx = -\frac{2\sqrt{x}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x(1+x)}}$$

input
```
Integrate[1/Sqrt[x + x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/Sqrt[x*(1 + x)]
```

3.154.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+x}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{x^2}{x^2+x}}d\frac{x}{\sqrt{x^2+x}}$$

$$\Big\downarrow \text{ 219}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

input
```
Int[1/Sqrt[x + x^2],x]
```

output
```
2*ArcTanh[x/Sqrt[x + x^2]]
```

3.154.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.154.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\sqrt{x}\right)$	7
default	$\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)$	12
pseudoelliptic	$2 \operatorname{arctanh}\left(\frac{\sqrt{x(1+x)}}{x}\right)$	15
trager	$\ln\left(1+2x+2\sqrt{x^2+x}\right)$	16

input
```
int(1/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(x^(1/2))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

input
```
integrate(1/(x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*x + 2*sqrt(x^2 + x) - 1)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2+x}+1\right)$$

input
```
integrate(1/(x**2+x)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 + x) + 1)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \log\left(2\,x + 2\,\sqrt{x^2+x} + 1\right)$$

input
```
integrate(1/(x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 + x) + 1)
```

3.154.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \frac{1}{4}\,\sqrt{x^2+x}(2\,x+1) + \frac{1}{8}\,\log\left(\left|-2\,x + 2\,\sqrt{x^2+x} - 1\right|\right)$$

input
```
integrate(1/(x^2+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 + x)*(2*x + 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x+x^2}}\,dx = \ln\left(x + \sqrt{x\,(x+1)} + \frac{1}{2}\right)$$

input
```
int(1/(x + x^2)^(1/2),x)
```

output
```
log(x + (x*(x + 1))^(1/2) + 1/2)
```

3.155 $\int \frac{\sqrt{2-x-x^2}}{x^2} dx$

3.155.1 Optimal result

Integrand size = 18, antiderivative size = 68

$$\int \frac{\sqrt{2-x-x^2}}{x^2} dx = -\frac{\sqrt{2-x-x^2}}{x} + \arcsin\left(\frac{1}{3}(-1-2x)\right) + \frac{\operatorname{arctanh}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}$$

output
```
-arcsin(1/3+2/3*x)+1/4*arctanh(1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)
-(-x^2-x+2)^(1/2)/x
```

3.155.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{2-x-x^2}}{x^2} dx = -\frac{\sqrt{2-x-x^2}}{x} + 2\arctan\left(\frac{\sqrt{2-x-x^2}}{2+x}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-x-x^2}}{\sqrt{2}(-1+x)}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

output
```
-(Sqrt[2 - x - x^2]/x) + 2*ArcTan[Sqrt[2 - x - x^2]/(2 + x)] - ArcTanh[Sqr
t[2 - x - x^2]/(Sqrt[2]*(-1 + x))]/Sqrt[2]
```

3.155.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1161, 25, 1269, 1090, 223, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^2-x+2}}{x^2}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{2}\int -\frac{2x+1}{x\sqrt{-x^2-x+2}}dx-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{2x+1}{x\sqrt{-x^2-x+2}}dx-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(-2\int \frac{1}{\sqrt{-x^2-x+2}}dx-\int \frac{1}{x\sqrt{-x^2-x+2}}dx\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{9}(-2x-1)^2}}d(-2x-1)-\int \frac{1}{x\sqrt{-x^2-x+2}}dx\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(2\arcsin\left(\frac{1}{3}(-2x-1)\right)-\int \frac{1}{x\sqrt{-x^2-x+2}}dx\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(2\int \frac{1}{8-\frac{(4-x)^2}{-x^2-x+2}}d\frac{4-x}{\sqrt{-x^2-x+2}}+2\arcsin\left(\frac{1}{3}(-2x-1)\right)\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\arcsin\left(\frac{1}{3}(-2x-1)\right)+\frac{\operatorname{arctanh}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{\sqrt{2}}\right)-\frac{\sqrt{-x^2-x+2}}{x}$$

input
```
Int[Sqrt[2 - x - x^2]/x^2,x]
```

output
```
-(Sqrt[2 - x - x^2]/x) + (2*ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt
[2]*Sqrt[2 - x - x^2])]/Sqrt[2])/2
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

3.155.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x^2+x-2}{x\sqrt{-x^2-x+2}}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)+\frac{\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)\sqrt{2}}{4}$
default	$-\frac{(-x^2-x+2)^{\frac{3}{2}}}{2x}-\frac{\sqrt{-x^2-x+2}}{4}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)+\frac{\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)\sqrt{2}}{4}+\frac{(-2x-1)\sqrt{-x^2-x+2}}{4}$
trager	$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2-x+2}+4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{4}-\operatorname{RootOf}\left(\_Z^2+1\right)$

input
```
int((-x^2-x+2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(x^2+x-2)/x/(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/4*arctanh(1/4*(4-x)*2^(1/
2)/(-x^2-x+2)^(1/2))*2^(1/2)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx$$
$$=\frac{\sqrt{2}x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}(x-4)+7x^2+16x-32}{x^2}\right)+8x\arctan\left(\frac{\sqrt{-x^2-x+2}(2x+1)}{2(x^2+x-2)}\right)-8\sqrt{-x^2-x+2}}{8x}$$

input
```
integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x -
 32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8
*sqrt(-x^2 - x + 2))/x
```

3.155.6 Sympy [F]

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \int \frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

input
```
integrate((-x**2-x+2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right) - \frac{\sqrt{-x^2-x+2}}{x} + \arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

input
```
integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt
(-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

3.155.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right) + \frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1} - \arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

input
```
integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) +
 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2
*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x
 + 1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3
)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{2}{x}+\frac{\sqrt{2}\,\sqrt{-x^2-x+2}}{x}-\frac{1}{2}\right)}{4}-\frac{\sqrt{-x^2-x+2}}{x}+\ln\left(x\,1\mathrm{i}+\sqrt{-x^2-x+2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((2 - x^2 - x)^(1/2)/x^2,x)
```

output
```
log(x*1i + (2 - x^2 - x)^(1/2) + 1i/2)*1i - (2 - x^2 - x)^(1/2)/x + (2^(1/
2)*log(2/x + (2^(1/2)*(2 - x^2 - x)^(1/2))/x - 1/2))/4
```

3.156 $\int \frac{\log(t)}{1+t}\,dt$

3.156.1 Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \frac{\log(t)}{1+t}\,dt = \log(t)\log(1+t) + \text{PolyLog}(2,-t)$$

output
```
ln(t)*ln(1+t)+polylog(2,-t)
```

3.156.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(t)}{1+t}\,dt = \log(t)\log(1+t) + \text{PolyLog}(2,-t)$$

input
```
Integrate[Log[t]/(1 + t),t]
```

output
```
Log[t]*Log[1 + t] + PolyLog[2, -t]
```

3.156.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(t)}{t+1}\,dt$$

$$\Big\downarrow \text{ 2754}$$

$$\log(t)\log(t+1)-\int \frac{\log(t+1)}{t}dt$$

$$\Big\downarrow \text{ 2838}$$

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

input
```
Int[Log[t]/(1 + t),t]
```

output
```
Log[t]*Log[1 + t] + PolyLog[2, -t]
```

3.156.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.156.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
default	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13
risch	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13
parts	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13

input
```
int(ln(t)/(1+t),t,method=_RETURNVERBOSE)
```

output
```
dilog(1+t)+ln(t)*ln(1+t)
```

3.156.5 Fricas [F]

$$\int \frac{\log(t)}{1+t}\,dt = \int \frac{\log(t)}{t+1}\,dt$$

input
```
integrate(log(t)/(1+t),t, algorithm="fricas")
```

output
```
integral(log(t)/(t + 1), t)
```

3.156.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.62

$$\int \frac{\log(t)}{1+t}\,dt = \begin{cases} -\operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|}<1 \wedge |t+1|<1 \\ i\pi\log(t+1)-\operatorname{Li}_2(t+1) & \text{for } |t+1|<1 \\ -i\pi\log\left(\frac{1}{t+1}\right)-\operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|}<1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| t+1\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| t+1\right)-\operatorname{Li}_2(t+1) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(t)/(1+t),t)
```

output
```
Piecewise((-polylog(2, t + 1), (Abs(t + 1) < 1) & (1/Abs(t + 1) < 1)), (I*
pi*log(t + 1) - polylog(2, t + 1), Abs(t + 1) < 1), (-I*pi*log(1/(t + 1))
- polylog(2, t + 1), 1/Abs(t + 1) < 1), (-I*pi*meijerg(((), (1, 1)), ((0,
0), ()), t + 1) + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), t + 1) - polylo
g(2, t + 1), True))
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\log(t)}{1+t}\,dt = \log(t+1)\log(t) + \mathrm{Li}_2(-t)$$

input
```
integrate(log(t)/(1+t),t, algorithm="maxima")
```

output
```
log(t + 1)*log(t) + dilog(-t)
```

3.156.8 Giac [F]

$$\int \frac{\log(t)}{1+t}\,dt = \int \frac{\log(t)}{t+1}\,dt$$

input
```
integrate(log(t)/(1+t),t, algorithm="giac")
```

output
```
integrate(log(t)/(t + 1), t)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(t)}{1+t}\, dt = \mathrm{polylog}(2,-t) + \ln(t+1)\, \ln(t)$$

input

```
int(log(t)/(t + 1),t)
```

output

```
polylog(2, -t) + log(t + 1)*log(t)
```

3.157 $\int \log\left(e^{\cos(x)}\right)\,dx$

3.157.1 Optimal result

Integrand size = 5, antiderivative size = 15

$$\int \log\left(e^{\cos(x)}\right)\,dx = -x\cos(x) + x\log\left(e^{\cos(x)}\right) + \sin(x)$$

output
```
-x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log\left(e^{\cos(x)}\right)\,dx = x\left(-\cos(x) + \log\left(e^{\cos(x)}\right)\right) + \sin(x)$$

input
```
Integrate[Log[E^Cos[x]],x]
```

output
```
x*(-Cos[x] + Log[E^Cos[x]]) + Sin[x]
```

3.157.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3028, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(e^{\cos (x)}\right) \, dx$$

$$\Big\downarrow \text{ 3028}$$

$$x \log \left(e^{\cos (x)}\right)-\int -x \sin (x)dx$$

$$\Big\downarrow \text{ 25}$$

$$\int x \sin (x)dx+x \log \left(e^{\cos (x)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int x \sin (x)dx+x \log \left(e^{\cos (x)}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\int \cos (x)dx-x \cos (x)+x \log \left(e^{\cos (x)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin \left(x+\frac{\pi }{2}\right) \, dx-x \cos (x)+x \log \left(e^{\cos (x)}\right)$$

$$\Big\downarrow \text{ 3117}$$

$$\sin (x)-x \cos (x)+x \log \left(e^{\cos (x)}\right)$$

input
```
Int[Log[E^Cos[x]],x]
```

output
```
-(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3028
```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.157.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

method	result	size
default	$-x\cos\left(x\right)+x\ln\left({\mathrm e}^{\cos\left(x\right)}\right)+\sin\left(x\right)$	15
risch	$-x\cos\left(x\right)+x\ln\left({\mathrm e}^{\cos\left(x\right)}\right)+\sin\left(x\right)$	15
parallelrisch	$-x\cos\left(x\right)+x\ln\left({\mathrm e}^{\cos\left(x\right)}\right)+\sin\left(x\right)$	15
parts	$-x\cos\left(x\right)+x\ln\left({\mathrm e}^{\cos\left(x\right)}\right)+\sin\left(x\right)$	15

input
```
int(ln(exp(cos(x))),x,method=_RETURNVERBOSE)
```

output
```
-x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log\left(e^{\cos(x)}\right)\,dx = \sin(x)$$

input
```
integrate(log(exp(cos(x))),x, algorithm="fricas")
```

output
```
sin(x)
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log\left(e^{\cos(x)}\right)\,dx = x\log\left(e^{\cos(x)}\right) - x\cos(x) + \sin(x)$$

input
```
integrate(ln(exp(cos(x))),x)
```

output
```
x*log(exp(cos(x))) - x*cos(x) + sin(x)
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log\left(e^{\cos(x)}\right)\,dx = \sin(x)$$

input
```
integrate(log(exp(cos(x))),x, algorithm="maxima")
```

output
```
sin(x)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log \left(e^{\cos (x)}\right) \, dx = \sin (x)$$

input
```
integrate(log(exp(cos(x))),x, algorithm="giac")
```

output
```
sin(x)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.13

$$\int \log \left(e^{\cos (x)}\right) \, dx = \sin (x)$$

input
```
int(log(exp(cos(x))),x)
```

output
```
sin(x)
```

3.158 $\int \frac{e^t}{t}\,dt$

3.158.1 Optimal result

Integrand size = 7, antiderivative size = 2

$$\int \frac{e^t}{t}\,dt = \text{ExpIntegralEi}(t)$$

output
```
Ei(t)
```

3.158.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t}\,dt = \text{ExpIntegralEi}(t)$$

input
```
Integrate[E^t/t,t]
```

output
```
ExpIntegralEi[t]
```

3.158.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^t}{t} \, dt$$
$$\Big\downarrow \text{2609}$$
$$\text{ExpIntegralEi}(t)$$

input
```
Int[E^t/t,t]
```

output
```
ExpIntegralEi[t]
```

3.158.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 4.00

method	result	size
default	$-\operatorname{Ei}_1(-t)$	8
risch	$-\operatorname{Ei}_1(-t)$	8
meijerg	$\ln(t) + i\pi - \ln(-t) - \operatorname{Ei}_1(-t)$	21

input
```
int(exp(t)/t,t,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-t)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

input
```
integrate(exp(t)/t,t, algorithm="fricas")
```

output
```
Ei(t)
```

3.158.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}\,(t)$$

input
```
integrate(exp(t)/t,t)
```

output
```
Ei(t)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

input
```
integrate(exp(t)/t,t, algorithm="maxima")
```

output
```
Ei(t)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

input
```
integrate(exp(t)/t,t, algorithm="giac")
```

output
```
Ei(t)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t} \, dt = \text{ei}(t)$$

input
```
int(exp(t)/t,t)
```

output
```
ei(t)
```

3.159 $\int \frac{e^{at}}{t} \, dt$

3.159.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int \frac{e^{at}}{t} \, dt = \text{ExpIntegralEi}(at)$$

output
```
Ei(a*t)
```

3.159.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t} \, dt = \text{ExpIntegralEi}(at)$$

input
```
Integrate[E^(a*t)/t,t]
```

output
```
ExpIntegralEi[a*t]
```

3.159.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{at}}{t} \, dt$$

$$\Big\downarrow \text{ 2609}$$

$$\text{ExpIntegralEi}(at)$$

input
```
Int[E^(a*t)/t,t]
```

output
```
ExpIntegralEi[a*t]
```

3.159.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.159.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 2.25

method	result	size
derivativedivides	$-\mathrm{Ei}_{1}\left(-at\right)$	9
default	$-\mathrm{Ei}_{1}\left(-at\right)$	9
risch	$-\mathrm{Ei}_{1}\left(-at\right)$	9
meijerg	$\ln\left(t\right)+\ln\left(-a\right)-\ln\left(-at\right)-\mathrm{Ei}_{1}\left(-at\right)$	23

input
```
int(exp(a*t)/t,t,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-a*t)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}(at)$$

input
```
integrate(exp(a*t)/t,t, algorithm="fricas")
```

output
```
Ei(a*t)
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}\,(at)$$

input
```
integrate(exp(a*t)/t,t)
```

output
```
Ei(a*t)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}(at)$$

input
```
integrate(exp(a*t)/t,t, algorithm="maxima")
```

output
```
Ei(a*t)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \mathrm{Ei}(at)$$

input
```
integrate(exp(a*t)/t,t, algorithm="giac")
```

output
```
Ei(a*t)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{at}}{t}\,dt = \mathrm{ei}(a\,t)$$

input
```
int(exp(a*t)/t,t)
```

output
```
ei(a*t)
```

3.160 $\int \frac{e^t}{t^2} \, dt$

3.160.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{e^t}{t^2} \, dt = -\frac{e^t}{t} + \text{ExpIntegralEi}(t)$$

output
```
-exp(t)/t+Ei(t)
```

3.160.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{t^2} \, dt = -\frac{e^t}{t} + \text{ExpIntegralEi}(t)$$

input
```
Integrate[E^t/t^2,t]
```

output
```
-(E^t/t) + ExpIntegralEi[t]
```

3.160.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^t}{t^2} \, dt$$
$$\Big\downarrow \; 2608$$
$$\int \frac{e^t}{t}dt - \frac{e^t}{t}$$
$$\Big\downarrow \; 2609$$
$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

input
```
Int[E^t/t^2,t]
```

output
```
-(E^t/t) + ExpIntegralEi[t]
```

3.160.3.1 Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.160.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{e^t}{t} - \mathrm{Ei}_1(-t)$	16
risch	$-\frac{e^t}{t} - \mathrm{Ei}_1(-t)$	16
meijerg	$-\frac{1}{t} - 1 + \ln(t) + i\pi + \frac{2+2t}{2t} - \frac{e^t}{t} - \ln(-t) - \mathrm{Ei}_1(-t)$	44

input
```
int(exp(t)/t^2,t,method=_RETURNVERBOSE)
```

output
```
-exp(t)/t-Ei(1,-t)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{e^t}{t^2}\,dt = \frac{t\mathrm{Ei}(t) - e^t}{t}$$

input
```
integrate(exp(t)/t^2,t, algorithm="fricas")
```

output
```
(t*Ei(t) - e^t)/t
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{e^t}{t^2}\,dt = \mathrm{Ei}(t) - \frac{e^t}{t}$$

input
```
integrate(exp(t)/t**2,t)
```

output
```
Ei(t) - exp(t)/t
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{e^t}{t^2}\,dt = \Gamma(-1, -t)$$

input
```
integrate(exp(t)/t^2,t, algorithm="maxima")
```

output
```
gamma(-1, -t)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{e^t}{t^2}\,dt = \frac{t\mathrm{Ei}(t) - e^t}{t}$$

input
```
integrate(exp(t)/t^2,t, algorithm="giac")
```

output
```
(t*Ei(t) - e^t)/t
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{e^t}{t^2}\,dt = -\frac{\mathrm{e}^t}{t} - \mathrm{expint}(-t)$$

input
```
int(exp(t)/t^2,t)
```

output
```
- exp(t)/t - expint(-t)
```

3.161 $\int e^{\frac{1}{t}}\,dt$

3.161.1 Optimal result

Integrand size = 5, antiderivative size = 14

$$\int e^{\frac{1}{t}}\,dt = e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

output
```
exp(1/t)*t-Ei(1/t)
```

3.161.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{\frac{1}{t}}\,dt = e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

input
```
Integrate[E^t^(-1),t]
```

output
```
E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

3.161.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{t}}dt$$

$$\Big\downarrow \text{ 2635}$$

$$\int \frac{e^{\frac{1}{t}}}{t}dt+e^{\frac{1}{t}}t$$

$$\Big\downarrow \text{ 2639}$$

$$e^{\frac{1}{t}}t-\text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

input
```
Int[E^t^(-1),t]
```

output
```
E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

3.161.3.1 Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

3.161.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$e^{\frac{1}{t}} t+\mathrm{Ei}_{1}\left(-\frac{1}{t}\right)$	15
default	$e^{\frac{1}{t}} t+\mathrm{Ei}_{1}\left(-\frac{1}{t}\right)$	15
risch	$e^{\frac{1}{t}} t+\mathrm{Ei}_{1}\left(-\frac{1}{t}\right)$	15
meijerg	$t+1+\ln\left(t\right)-i\pi-\frac{t\left(2+\frac{2}{t}\right)}{2}+e^{\frac{1}{t}} t+\ln\left(-\frac{1}{t}\right)+\mathrm{Ei}_{1}\left(-\frac{1}{t}\right)$	39

input
```
int(exp(1/t),t,method=_RETURNVERBOSE)
```

output
```
exp(1/t)*t+Ei(1,-1/t)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{\frac{1}{t}}\,dt = te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

input
```
integrate(exp(1/t),t, algorithm="fricas")
```

output
```
t*e^(1/t) - Ei(1/t)
```

3.161.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int e^{\frac{1}{t}}\,dt = te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

input
```
integrate(exp(1/t),t)
```

output
```
t*exp(1/t) - Ei(1/t)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{\frac{1}{t}}\,dt = -\Gamma\left(-1,-\frac{1}{t}\right)$$

input
```
integrate(exp(1/t),t, algorithm="maxima")
```

output
```
-gamma(-1, -1/t)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int e^{\frac{1}{t}}\,dt = -t\left(\frac{\mathrm{Ei}\left(\frac{1}{t}\right)}{t} - e^{\frac{1}{t}}\right)$$

input
```
integrate(exp(1/t),t, algorithm="giac")
```

output
```
-t*(Ei(1/t)/t - e^(1/t))
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{\frac{1}{t}}\,dt = t\,\mathrm{expint}\left(2,-\frac{1}{t}\right)$$

input
```
int(exp(1/t),t)
```

output
```
t*expint(2, -1/t)
```

3.162 $\int \frac{e^{-t}}{-1-a+t}\,dt$

3.162.1 Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{e^{-t}}{-1-a+t}\,dt = e^{-1-a}\,\text{ExpIntegralEi}(1+a-t)$$

output
```
exp(-1-a)*Ei(1+a-t)
```

3.162.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{-t}}{-1-a+t}\,dt = e^{-1-a}\,\text{ExpIntegralEi}(1+a-t)$$

input
```
Integrate[1/(E^t*(-1 - a + t)),t]
```

output
```
E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

3.162.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-t}}{-a+t-1} \, dt$$

$$\Big\downarrow \text{2609}$$

$$e^{-a-1} \text{ExpIntegralEi}(a-t+1)$$

input
```
Int[1/(E^t*(-1 - a + t)),t]
```

output
```
E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

3.162.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.162.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
default	$-\mathrm{e}^{-1-a} \operatorname{Ei}_1(-1-a+t)$	17
risch	$-\mathrm{e}^{-1-a} \operatorname{Ei}_1(-1-a+t)$	17

input
```
int(1/exp(t)/(-1-a+t),t,method=_RETURNVERBOSE)
```

output
```
-exp(-1-a)*Ei(1,-1-a+t)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \operatorname{Ei}(a-t+1)\,e^{(-a-1)}$$

input
```
integrate(1/exp(t)/(-1-a+t),t, algorithm="fricas")
```

output
```
Ei(a - t + 1)*e^(-a - 1)
```

3.162.6 Sympy [F]

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \int \frac{e^{-t}}{-a+t-1}\,dt$$

input
```
integrate(1/exp(t)/(-1-a+t),t)
```

output
```
Integral(exp(-t)/(-a + t - 1), t)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{e^{-t}}{-1-a+t}\,dt = -e^{(-a-1)}E_1(-a+t-1)$$

input
```
integrate(1/exp(t)/(-1-a+t),t, algorithm="maxima")
```

output
```
-e^(-a - 1)*exp_integral_e(1, -a + t - 1)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \operatorname{Ei}(a-t+1)\,e^{(-a-1)}$$

input
```
integrate(1/exp(t)/(-1-a+t),t, algorithm="giac")
```

output
```
Ei(a - t + 1)*e^(-a - 1)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{e^{-t}}{-1-a+t}\,dt = \mathrm{e}^{-a-1}\,\mathrm{ei}(a-t+1)$$

input
```
int(-exp(-t)/(a - t + 1),t)
```

output
```
exp(- a - 1)*ei(a - t + 1)
```

3.163 $\int \frac{e^{t^2} t}{1+t^2} \, dt$

3.163.1 Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^{t^2} t}{1+t^2} \, dt = \frac{\text{ExpIntegralEi}(1+t^2)}{2e}$$

output
```
1/2*Ei(t^2+1)/exp(1)
```

3.163.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{t^2} t}{1+t^2} \, dt = \frac{\text{ExpIntegralEi}(1+t^2)}{2e}$$

input
```
Integrate[(E^t^2*t)/(1 + t^2),t]
```

output
```
ExpIntegralEi[1 + t^2]/(2*E)
```

3.163.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7266, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{t^2} t}{t^2+1} \, dt$$

$$\Big\downarrow \ 7266$$

$$\frac{1}{2} \int \frac{e^{t^2}}{t^2+1} dt^2$$

$$\Big\downarrow \ 2609$$

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

input
```
Int[(E^t^2*t)/(1 + t^2),t]
```

output
```
ExpIntegralEi[1 + t^2]/(2*E)
```

3.163.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.163.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{e^{-1} \operatorname{Ei}_1(-t^2-1)}{2}$	14
default	$-\frac{e^{-1} \operatorname{Ei}_1(-t^2-1)}{2}$	14
risch	$-\frac{e^{-1} \operatorname{Ei}_1(-t^2-1)}{2}$	14

input
```
int(exp(t^2)*t/(t^2+1),t,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-1)*Ei(1,-t^2-1)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{t^2} t}{1+t^2} \, dt = \frac{1}{2} \operatorname{Ei}\left(t^2+1\right) e^{(-1)}$$

input
```
integrate(exp(t^2)*t/(t^2+1),t, algorithm="fricas")
```

output
```
1/2*Ei(t^2 + 1)*e^(-1)
```

3.163.6 Sympy [F]

$$\int \frac{e^{t^2} t}{1+t^2} \, dt = \int \frac{t e^{t^2}}{t^2+1} \, dt$$

input
```
integrate(exp(t**2)*t/(t**2+1),t)
```

output
```
Integral(t*exp(t**2)/(t**2 + 1), t)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = -\frac{1}{2}\,e^{(-1)}E_1\left(-t^2-1\right)$$

input
```
integrate(exp(t^2)*t/(t^2+1),t, algorithm="maxima")
```

output
```
-1/2*e^(-1)*exp_integral_e(1, -t^2 - 1)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

input
```
integrate(exp(t^2)*t/(t^2+1),t, algorithm="giac")
```

output
```
1/2*Ei(t^2 + 1)*e^(-1)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{t^2}t}{1+t^2}\,dt = \frac{\mathrm{e}^{-1}\,\mathrm{ei}\left(t^2+1\right)}{2}$$

input
```
int((t*exp(t^2))/(t^2 + 1),t)
```

output
```
(exp(-1)*ei(t^2 + 1))/2
```

3.164 $\int \frac{e^t}{(1+t)^2} \, dt$

3.164.1 Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{e^t}{(1+t)^2} \, dt = -\frac{e^t}{1+t} + \frac{\text{ExpIntegralEi}(1+t)}{e}$$

output
```
-exp(t)/(1+t)+Ei(1+t)/exp(1)
```

3.164.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^t}{(1+t)^2} \, dt = -\frac{e^t}{1+t} + \frac{\text{ExpIntegralEi}(1+t)}{e}$$

input
```
Integrate[E^t/(1 + t)^2,t]
```

output
```
-(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

3.164.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^t}{(t+1)^2}dt$$
$$\Big\downarrow \ 2608$$
$$\int \frac{e^t}{t+1}dt-\frac{e^t}{t+1}$$
$$\Big\downarrow \ 2609$$
$$\frac{\text{ExpIntegralEi}(t+1)}{e}-\frac{e^t}{t+1}$$

input
```
Int[E^t/(1 + t)^2,t]
```

output
```
-(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

3.164.3.1 Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.164.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{e^{t}}{1+t}-e^{-1} \operatorname{Ei}_{1}\left(-1-t\right)$	22
risch	$-\frac{e^{t}}{1+t}-e^{-1} \operatorname{Ei}_{1}\left(-1-t\right)$	22

input
```
int(exp(t)/(1+t)^2,t,method=_RETURNVERBOSE)
```

output
```
-exp(t)/(1+t)-exp(-1)*Ei(1,-1-t)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{e^t}{(1+t)^2}\,dt = \frac{\left((t+1)\mathrm{Ei}(t+1) - e^{(t+1)}\right)e^{(-1)}}{t+1}$$

input
```
integrate(exp(t)/(1+t)^2,t, algorithm="fricas")
```

output
```
((t + 1)*Ei(t + 1) - e^(t + 1))*e^(-1)/(t + 1)
```

3.164.6 Sympy [F]

$$\int \frac{e^t}{(1+t)^2}\,dt = \int \frac{e^t}{(t+1)^2}\,dt$$

input
```
integrate(exp(t)/(1+t)**2,t)
```

output
```
Integral(exp(t)/(t + 1)**2, t)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{e^t}{(1+t)^2}\,dt = -\frac{e^{(-1)}E_2(-t-1)}{t+1}$$

input
```
integrate(exp(t)/(1+t)^2,t, algorithm="maxima")
```

output
```
-e^(-1)*exp_integral_e(2, -t - 1)/(t + 1)
```

3.164.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.21

$$\int \frac{e^t}{(1+t)^2}\,dt = \frac{(t+1)\left(\frac{1}{t+1}-1\right)\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)-\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)+e^{\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)}}{(t+1)\left(\frac{1}{t+1}-1\right)e-e}$$

input
```
integrate(exp(t)/(1+t)^2,t, algorithm="giac")
```

output
```
((t + 1)*(1/(t + 1) - 1)*Ei(-(t + 1)*(1/(t + 1) - 1) + 1) - Ei(-(t + 1)*(1
/(t + 1) - 1) + 1) + e^(-(t + 1)*(1/(t + 1) - 1) + 1))/((t + 1)*(1/(t + 1)
 - 1)*e - e)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{e^t}{(1+t)^2}\,dt = \mathrm{ei}(t+1)\ \mathrm{e}^{-1} - \frac{\mathrm{e}^t}{t+1}$$

input
```
int(exp(t)/(t + 1)^2,t)
```

output
```
ei(t + 1)*exp(-1) - exp(t)/(t + 1)
```

3.165 $\int e^t \log(1+t)\,dt$

3.165.1 Optimal result

Integrand size = 8, antiderivative size = 18

$$\int e^t \log(1+t)\,dt = -\frac{\text{ExpIntegralEi}(1+t)}{e} + e^t \log(1+t)$$

output
```
-Ei(1+t)/exp(1)+exp(t)*ln(1+t)
```

3.165.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^t \log(1+t)\,dt = -\frac{\text{ExpIntegralEi}(1+t)}{e} + e^t \log(1+t)$$

input
```
Integrate[E^t*Log[1 + t],t]
```

output
```
-(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

3.165.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3034, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^t \log(t+1)\,dt$$

$$\Big\downarrow \text{3034}$$

$$e^t \log(t+1) - \int \frac{e^t}{t+1}dt$$

$$\Big\downarrow \text{2609}$$

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

input
```
Int[E^t*Log[1 + t],t]
```

output
```
-(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

3.165.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

3.165.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
risch	$e^t \ln(1+t) + e^{-1} \operatorname{Ei}_1(-1-t)$	19

input
```
int(exp(t)*ln(1+t),t,method=_RETURNVERBOSE)
```

output
```
exp(t)*ln(1+t)+exp(-1)*Ei(1,-1-t)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^t \log(1+t)\,dt = \left(e^{(t+1)} \log(t+1) - \operatorname{Ei}(t+1)\right)e^{(-1)}$$

input
```
integrate(exp(t)*log(1+t),t, algorithm="fricas")
```

output
```
(e^(t + 1)*log(t + 1) - Ei(t + 1))*e^(-1)
```

3.165.6 Sympy [F]

$$\int e^t \log(1+t)\,dt = \int e^t \log(t+1)\,dt$$

input
```
integrate(exp(t)*ln(1+t),t)
```

output
```
Integral(exp(t)*log(t + 1), t)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^t \log(1+t)\,dt = e^{(-1)}E_1(-t-1) + e^t \log(t+1)$$

input
```
integrate(exp(t)*log(1+t),t, algorithm="maxima")
```

output
```
e^(-1)*exp_integral_e(1, -t - 1) + e^t*log(t + 1)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int e^t \log(1+t)\,dt = -\mathrm{Ei}(t+1)\,e^{(-1)} + e^t \log(t+1)$$

input
```
integrate(exp(t)*log(1+t),t, algorithm="giac")
```

output
```
-Ei(t + 1)*e^(-1) + e^t*log(t + 1)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int e^t \log(1+t)\,dt = \int \ln(t+1)\,\mathrm{e}^t\,dt$$

input
```
int(log(t + 1)*exp(t),t)
```

output
```
int(log(t + 1)*exp(t), t)
```

3.166 $\int e^{-t} t \, dt$

3.166.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int e^{-t} t \, dt = -e^{-t} - e^{-t} t$$

output
```
-1/exp(t)-t/exp(t)
```

3.166.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int e^{-t} t \, dt = e^{-t}(-1 - t)$$

input
```
Integrate[t/E^t,t]
```

output
```
(-1 - t)/E^t
```

3.166.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-t}t\,dt$$
$$\Big\downarrow \text{2607}$$
$$\int e^{-t}dt - e^{-t}t$$
$$\Big\downarrow \text{2624}$$
$$-e^{-t}t - e^{-t}$$

input
```
Int[t/E^t,t]
```

output
```
-E^(-t) - t/E^t
```

3.166.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.166.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
gosper	$-(1+t)\,\mathrm{e}^{-t}$	10
norman	$(-1-t)\,\mathrm{e}^{-t}$	11
risch	$(-1-t)\,\mathrm{e}^{-t}$	11
parallelrisch	$(-1-t)\,\mathrm{e}^{-t}$	11
meijerg	$1-\frac{(2+2t)\mathrm{e}^{-t}}{2}$	14
default	$-\mathrm{e}^{-t}-t\,\mathrm{e}^{-t}$	15

input
```
int(t/exp(t),t,method=_RETURNVERBOSE)
```

output
```
-(1+t)/exp(t)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

input
```
integrate(t/exp(t),t, algorithm="fricas")
```

output
```
-(t + 1)*e^(-t)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int e^{-t}t\,dt = (-t-1)\,e^{-t}$$

input
```
integrate(t/exp(t),t)
```

output
```
(-t - 1)*exp(-t)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

input
```
integrate(t/exp(t),t, algorithm="maxima")
```

output
```
-(t + 1)*e^(-t)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -(t+1)e^{(-t)}$$

input
```
integrate(t/exp(t),t, algorithm="giac")
```

output
```
-(t + 1)*e^(-t)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int e^{-t}t\,dt = -\mathrm{e}^{-t}\,(t+1)$$

input
```
int(t*exp(-t),t)
```

output
```
-exp(-t)*(t + 1)
```

3.167 $\int e^{-t}t^2\,dt$

3.167.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int e^{-t}t^2\,dt = -2e^{-t} - 2e^{-t}t - e^{-t}t^2$$

output
```
-2/exp(t)-2*t/exp(t)-t^2/exp(t)
```

3.167.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-t}t^2\,dt = e^{-t}(-2-2t-t^2)$$

input
```
Integrate[t^2/E^t,t]
```

output
```
(-2 - 2*t - t^2)/E^t
```

3.167.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-t}t^2\,dt$$

$$\Big\downarrow \text{2607}$$

$$2\int e^{-t}tdt-e^{-t}t^2$$

$$\Big\downarrow \text{2607}$$

$$2\left(\int e^{-t}dt-e^{-t}t\right)-e^{-t}t^2$$

$$\Big\downarrow \text{2624}$$

$$2\left(-e^{-t}t-e^{-t}\right)-e^{-t}t^2$$

input
```
Int[t^2/E^t,t]
```

output
```
-(t^2/E^t) + 2*(-E^(-t) - t/E^t)
```

3.167.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.167.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

method	result	size
gosper	$-(t^2+2t+2)\,\mathrm{e}^{-t}$	15
norman	$(-t^2-2t-2)\,\mathrm{e}^{-t}$	16
risch	$(-t^2-2t-2)\,\mathrm{e}^{-t}$	16
parallelrisch	$(-t^2-2t-2)\,\mathrm{e}^{-t}$	16
meijerg	$2-\frac{(3t^2+6t+6)\,\mathrm{e}^{-t}}{3}$	19
default	$-2\,\mathrm{e}^{-t}-2t\,\mathrm{e}^{-t}-t^2\mathrm{e}^{-t}$	24

input
```
int(t^2/exp(t),t,method=_RETURNVERBOSE)
```

output
```
-(t^2+2*t+2)/exp(t)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -\left(t^2+2\,t+2\right)e^{(-t)}$$

input
```
integrate(t^2/exp(t),t, algorithm="fricas")
```

output
```
-(t^2 + 2*t + 2)*e^(-t)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-t}t^2\,dt = \left(-t^2-2t-2\right)e^{-t}$$

input
```
integrate(t**2/exp(t),t)
```

output
```
(-t**2 - 2*t - 2)*exp(-t)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -(t^2 + 2\,t + 2)e^{(-t)}$$

input
```
integrate(t^2/exp(t),t, algorithm="maxima")
```

output
```
-(t^2 + 2*t + 2)*e^(-t)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -(t^2 + 2\,t + 2)e^{(-t)}$$

input
```
integrate(t^2/exp(t),t, algorithm="giac")
```

output
```
-(t^2 + 2*t + 2)*e^(-t)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int e^{-t}t^2\,dt = -\mathrm{e}^{-t}\,(t^2 + 2\,t + 2)$$

input
```
int(t^2*exp(-t),t)
```

output
```
-exp(-t)*(2*t + t^2 + 2)
```

3.168 $\int e^{-t}t^3\,dt$

3.168.1 Optimal result

Integrand size = 9, antiderivative size = 36

$$\int e^{-t}t^3\,dt = -6e^{-t} - 6e^{-t}t - 3e^{-t}t^2 - e^{-t}t^3$$

output
```
-6/exp(t)-6*t/exp(t)-3*t^2/exp(t)-t^3/exp(t)
```

3.168.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{-t}t^3\,dt = e^{-t}\left(-6 - 6t - 3t^2 - t^3\right)$$

input
```
Integrate[t^3/E^t,t]
```

output
```
(-6 - 6*t - 3*t^2 - t^3)/E^t
```

3.168.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-t}t^3\,dt$$
$$\Big\downarrow \text{2607}$$
$$3\int e^{-t}t^2dt - e^{-t}t^3$$
$$\Big\downarrow \text{2607}$$
$$3\left(2\int e^{-t}tdt - e^{-t}t^2\right) - e^{-t}t^3$$
$$\Big\downarrow \text{2607}$$
$$3\left(2\left(\int e^{-t}dt - e^{-t}t\right) - e^{-t}t^2\right) - e^{-t}t^3$$
$$\Big\downarrow \text{2624}$$
$$3(2(-e^{-t}t - e^{-t}) - e^{-t}t^2) - e^{-t}t^3$$

input
```
Int[t^3/E^t,t]
```

output
```
-(t^3/E^t) + 3*(-(t^2/E^t) + 2*(-E^(-t) - t/E^t))
```

3.168.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.168.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
gosper	$-(t^3+3t^2+6t+6)\,e^{-t}$	20
norman	$(-t^3-3t^2-6t-6)\,e^{-t}$	21
risch	$(-t^3-3t^2-6t-6)\,e^{-t}$	21
parallelrisch	$(-t^3-3t^2-6t-6)\,e^{-t}$	21
meijerg	$6-\frac{(4t^3+12t^2+24t+24)e^{-t}}{4}$	24
default	$-6\,e^{-t}-6t\,e^{-t}-3t^2e^{-t}-t^3e^{-t}$	33

input
```
int(t^3/exp(t),t,method=_RETURNVERBOSE)
```

output
```
-(t^3+3*t^2+6*t+6)/exp(t)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\left(t^3+3\,t^2+6\,t+6\right)e^{(-t)}$$

input
```
integrate(t^3/exp(t),t, algorithm="fricas")
```

output
```
-(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

$$\int e^{-t}t^3\,dt = \left(-t^3-3t^2-6t-6\right)e^{-t}$$

input
```
integrate(t**3/exp(t),t)
```

output
```
(-t**3 - 3*t**2 - 6*t - 6)*exp(-t)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\left(t^3+3\,t^2+6\,t+6\right)e^{(-t)}$$

input
```
integrate(t^3/exp(t),t, algorithm="maxima")
```

output
```
-(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\left(t^3+3\,t^2+6\,t+6\right)e^{(-t)}$$

input
```
integrate(t^3/exp(t),t, algorithm="giac")
```

output
```
-(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int e^{-t}t^3\,dt = -\mathrm{e}^{-t}\left(t^3+3\,t^2+6\,t+6\right)$$

input
```
int(t^3*exp(-t),t)
```

output
```
-exp(-t)*(6*t + 3*t^2 + t^3 + 6)
```

3.169 $\int \frac{\mathbf{b1}\cos(x)+\mathbf{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx$

3.169.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\mathrm{a1}+b\mathrm{b1})x}{a^2+b^2}-\frac{(\mathrm{a1}b-a\mathrm{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

output
```
(a*a1+b*b1)*x/(a^2+b^2)-(-a*b1+a1*b)*ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\mathrm{a1}+b\mathrm{b1})x+(-\mathrm{a1}b+a\mathrm{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

input
```
Integrate[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

output
```
((a*a1 + b*b1)*x + (-(a1*b) + a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

3.169.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{a1}\sin(x)+\text{b1}\cos(x)}{a\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\text{a1}\sin(x)+\text{b1}\cos(x)}{a\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \ 3612$$

$$\frac{x(a\text{a1}+b\text{b1})}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

input
```
Int[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

output
```
((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*Sin[x]])/(
a^2 + b^2)
```

3.169.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.169.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

method	result	si
default	$\frac{(a\,\mathrm{b1}-\mathrm{a1}\,b)\ln(a\tan(x)+b)}{a^2+b^2}+\frac{\frac{(-a\,\mathrm{b1}+\mathrm{a1}\,b)\ln\left(1+\tan^2(x)\right)}{2}+(a\,\mathrm{a1}+b\,\mathrm{b1})\arctan(\tan(x))}{a^2+b^2}$	6
parallelrisch	$\frac{-a\,\mathrm{b1}\ln\left(\frac{1}{\cos(x)+1}\right)+\mathrm{a1}\,b\ln\left(\frac{1}{\cos(x)+1}\right)+a\,\mathrm{b1}\ln\left(\frac{-b\cos(x)-a\sin(x)}{\cos(x)+1}\right)-\mathrm{a1}\,b\ln\left(\frac{-b\cos(x)-a\sin(x)}{\cos(x)+1}\right)+xa\,\mathrm{a1}+xb\,\mathrm{b1}}{a^2+b^2}$	8
norman	$\frac{\frac{(a\,\mathrm{a1}+b\,\mathrm{b1})x}{a^2+b^2}+\frac{(a\,\mathrm{a1}+b\,\mathrm{b1})x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{(a\,\mathrm{b1}-\mathrm{a1}\,b)\ln\left(-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b\right)}{a^2+b^2}-\frac{(a\,\mathrm{b1}-\mathrm{a1}\,b)\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}$	1
risch	$\frac{ix\,\mathrm{b1}}{ib+a}+\frac{x\,\mathrm{a1}}{ib+a}-\frac{2ixa\,\mathrm{b1}}{a^2+b^2}+\frac{2ix\,\mathrm{a1}\,b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{ib-a}{ib+a}\right)a\,\mathrm{b1}}{a^2+b^2}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{ib-a}{ib+a}\right)\mathrm{a1}\,b}{a^2+b^2}$	1

input
```
int((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
(a*b1-a1*b)/(a^2+b^2)*ln(a*tan(x)+b)+1/(a^2+b^2)*(1/2*(-a*b1+a1*b)*ln(1+ta
n(x)^2)+(a*a1+b*b1)*arctan(tan(x)))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\mathrm{b1}\cos(x)+\mathrm{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{2\,(aa_1+bb_1)x-(a_1b-ab_1)\log\left(2\,ab\cos{(x)}\sin{(x)}-(a^2-b^2)\cos{(x)}^2+a^2\right)}{2\,(a^2+b^2)}$$

input
```
integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(2*(a*a1 + b*b1)*x - (a1*b - a*b1)*log(2*a*b*cos(x)*sin(x) - (a^2 - b^
2)*cos(x)^2 + a^2))/(a^2 + b^2)
```

3.169.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 360, normalized size of antiderivative = 7.50

$$\int \frac{\mathrm{b1}\cos(x) + \mathrm{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx$$

$$= \begin{cases} \tilde{\infty}(-a_1 \log(\cos(x)) + b_1 x) \\ \frac{a_1 x + b_1 \log(\sin(x))}{a} \\ \frac{a_1 x \sin(x)}{-2ib\sin(x)+2b\cos(x)} + \frac{ia_1 x\cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{a_1\cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{ib_1 x\sin(x)}{-2ib\sin(x)+2b\cos(x)} + \frac{b_1 x\cos(x)}{-2ib\sin(x)+2b\cos(x)} - \\ \frac{a_1 x \sin(x)}{2ib\sin(x)+2b\cos(x)} - \frac{ia_1 x\cos(x)}{2ib\sin(x)+2b\cos(x)} - \frac{a_1\cos(x)}{2ib\sin(x)+2b\cos(x)} + \frac{ib_1 x\sin(x)}{2ib\sin(x)+2b\cos(x)} + \frac{b_1 x\cos(x)}{2ib\sin(x)+2b\cos(x)} + \\ \frac{aa_1 x}{a^2+b^2} + \frac{ab_1\log\left(\frac{a\sin(x)}{b}+\cos(x)\right)}{a^2+b^2} - \frac{a_1 b\log\left(\frac{a\sin(x)}{b}+\cos(x)\right)}{a^2+b^2} + \frac{bb_1 x}{a^2+b^2} \end{cases}$$

input
```
integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

output
```
Piecewise((zoo*(-a1*log(cos(x)) + b1*x), Eq(a, 0) & Eq(b, 0)), ((a1*x + b1
*log(sin(x)))/a, Eq(b, 0)), (a1*x*sin(x)/(-2*I*b*sin(x) + 2*b*cos(x)) + I*
a1*x*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)) - a1*cos(x)/(-2*I*b*sin(x) + 2*b*
cos(x)) - I*b1*x*sin(x)/(-2*I*b*sin(x) + 2*b*cos(x)) + b1*x*cos(x)/(-2*I*b
*sin(x) + 2*b*cos(x)) - I*b1*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)), Eq(a, -I
*b)), (a1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) - I*a1*x*cos(x)/(2*I*b*sin(
x) + 2*b*cos(x)) - a1*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) + I*b1*x*sin(x)/(
2*I*b*sin(x) + 2*b*cos(x)) + b1*x*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) + I*b
1*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)), Eq(a, I*b)), (a*a1*x/(a**2 + b**2) +
 a*b1*log(a*sin(x)/b + cos(x))/(a**2 + b**2) - a1*b*log(a*sin(x)/b + cos(x
))/(a**2 + b**2) + b*b1*x/(a**2 + b**2), True))
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(48) = 96$.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.77

$$\int \frac{\text{b1}\cos(x) + \text{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx$$
$$= a_1\left(\frac{2\,a\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2} - \frac{b\log\left(-b - \frac{2\,a\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2} + \frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$
$$+ b_1\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2} + \frac{a\log\left(-b - \frac{2\,a\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2} - \frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$

input
```
integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

output
```
a1*(2*a*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - b*log(-b - 2*a*sin(x)/(c
os(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + b*log(sin(x)^2/(cos(
x) + 1)^2 + 1)/(a^2 + b^2)) + b1*(2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b
^2) + a*log(-b - 2*a*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2
 + b^2) - a*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^2 + b^2))
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{\text{b1}\cos(x) + \text{a1}\sin(x)}{b\cos(x) + a\sin(x)}\,dx = \frac{(aa_1 + bb_1)x}{a^2+b^2} + \frac{(a_1b - ab_1)\log\left(\tan(x)^2 + 1\right)}{2(a^2+b^2)} - \frac{(aa_1b - a^2b_1)\log\left(|a\tan(x) + b|\right)}{a^3 + ab^2}$$

input
```
integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

output
```
(a*a1 + b*b1)*x/(a^2 + b^2) + 1/2*(a1*b - a*b1)*log(tan(x)^2 + 1)/(a^2 + b
^2) - (a*a1*b - a^2*b1)*log(abs(a*tan(x) + b))/(a^3 + a*b^2)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 2034, normalized size of antiderivative = 42.38

$$\int \frac{\text{b1}\cos(x)+\text{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x)
```

output
```
(2*atan((tan(x/2)*(((((a*a1 + b*b1)^3*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)
^3 + ((((a*a1 + b*b1)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a
^2*b^3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 +
 b^2) - ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2
+ b^2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2)) - ((a*a1 + b*b1)*(32*b^3*b1^
2 - ((2*a*b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*
a^2*b^3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a
^2 + b^2)) + 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(a^2 + b^2)
)*(a^4*a1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^2 + 13*a^
2*b^2*b1^2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((a^2 + b^2)^2*(a^2*a1^2 +
4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2) - ((((((a*a1 + b*b1)*
(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2
)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^2) - ((a*a1 + b*b1)
*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)^2))*(a*a1 + b*b
1))/(a^2 + b^2) - 32*a1*b^2*b1^2 - 64*a1^3*b^2 + ((2*a*b1 - 2*a1*b)*(32*b^
3*b1^2 - ((2*a*b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b
+ 96*a^2*b^3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/
(2*(a^2 + b^2)) + 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(2*(a^
2 + b^2)) + 32*a*b*b1^3 - ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a^4*b + 9
6*a^2*b^3))/(2*(a^2 + b^2)^3) + 64*a*a1^2*b*b1)*(12*a*a1^2*b^3 - 6*a^3*...
```

3.170 $\int \frac{1}{\log(t)} \, dt$

3.170.1 Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \frac{1}{\log(t)} \, dt = \text{LogIntegral}(t)$$

output
```
Li(t)
```

3.170.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)} \, dt = \text{LogIntegral}(t)$$

input
```
Integrate[Log[t]^(-1),t]
```

output
```
LogIntegral[t]
```

3.170.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log(t)}\,dt$$
$$\Big\downarrow \; 2735$$
$$\text{LogIntegral}(t)$$

input
```
Int[Log[t]^(-1),t]
```

output
```
LogIntegral[t]
```

3.170.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

method	result	size
default	$-\text{Ei}_1\left(-\ln\left(t\right)\right)$	9
risch	$-\text{Ei}_1\left(-\ln\left(t\right)\right)$	9

input
```
int(1/ln(t),t,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-ln(t))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\,dt = \text{log\_integral}\,(t)$$

input
```
integrate(1/log(t),t, algorithm="fricas")
```

output
```
log_integral(t)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\,dt = \text{li}\,(t)$$

input
```
integrate(1/ln(t),t)
```

output
```
li(t)
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(t)}\,dt = \text{Ei}(\log\,(t))$$

input
```
integrate(1/log(t),t, algorithm="maxima")
```

output
```
Ei(log(t))
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(t)}\,dt = \mathrm{Ei}(\log{(t)})$$

input
```
integrate(1/log(t),t, algorithm="giac")
```

output
```
Ei(log(t))
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(t)}\,dt = \mathrm{logint}(t)$$

input
```
int(1/log(t),t)
```

output
```
logint(t)
```

3.171 $\int \frac{1}{\log^2(t)}\,dt$

3.171.1 Optimal result

Integrand size = 4, antiderivative size = 10

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log(t)} + \text{LogIntegral}(t)$$

output
```
Li(t)-t/ln(t)
```

3.171.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log(t)} + \text{LogIntegral}(t)$$

input
```
Integrate[Log[t]^(-2),t]
```

output
```
-(t/Log[t]) + LogIntegral[t]
```

3.171.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2734, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^2(t)}dt$$
$$\Big\downarrow \text{2734}$$
$$\int \frac{1}{\log(t)}dt - \frac{t}{\log(t)}$$
$$\Big\downarrow \text{2735}$$
$$\text{LogIntegral}(t) - \frac{t}{\log(t)}$$

input
```
Int[Log[t]^(-2),t]
```

output
```
-(t/Log[t]) + LogIntegral[t]
```

3.171.3.1 Defintions of rubi rules used

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

3.171.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

method	result	size
default	$-\frac{t}{\ln(t)} - \mathrm{Ei}_1(-\ln(t))$	17
risch	$-\frac{t}{\ln(t)} - \mathrm{Ei}_1(-\ln(t))$	17

input
```
int(1/ln(t)^2,t,method=_RETURNVERBOSE)
```

output
```
-t/ln(t)-Ei(1,-ln(t))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{\log^2(t)}\, dt = \frac{\log(t)\,\mathrm{log\_integral}(t) - t}{\log(t)}$$

input
```
integrate(1/log(t)^2,t, algorithm="fricas")
```

output
```
(log(t)*log_integral(t) - t)/log(t)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\log^2(t)}\, dt = -\frac{t}{\log(t)} + \mathrm{li}(t)$$

input
```
integrate(1/ln(t)**2,t)
```

output
```
-t/log(t) + li(t)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\log^2(t)}\,dt = \Gamma(-1, -\log(t))$$

input
```
integrate(1/log(t)^2,t, algorithm="maxima")
```

output
```
gamma(-1, -log(t))
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\log^2(t)}\,dt = -\frac{t}{\log(t)} + \mathrm{Ei}(\log(t))$$

input
```
integrate(1/log(t)^2,t, algorithm="giac")
```

output
```
-t/log(t) + Ei(log(t))
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(t)}\,dt = \mathrm{logint}(t) - \frac{t}{\ln(t)}$$

input
```
int(1/log(t)^2,t)
```

output
```
logint(t) - t/log(t)
```

3.172 $\int \log^{-1-n}(t)\,dt$

3.172.1 Optimal result

Integrand size = 8, antiderivative size = 22

$$\int \log^{-1-n}(t)\,dt = -\Gamma(-n,-\log(t))(-\log(t))^n\log^{-n}(t)$$

output
```
-GAMMA(-n,-ln(t))*(-ln(t))^n/(ln(t)^n)
```

3.172.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log^{-1-n}(t)\,dt = -\Gamma(-n,-\log(t))(-\log(t))^n\log^{-n}(t)$$

input
```
Integrate[Log[t]^(-1 - n),t]
```

output
```
-((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

3.172.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^{-n-1}(t)\,dt$$
$$\Big\downarrow \text{ 2736}$$
$$\int t\log^{-n-1}(t)d\log(t)$$
$$\Big\downarrow \text{ 2612}$$
$$(-\log(t))^n\log^{-n}(t)(-\Gamma(-n,-\log(t)))$$

input
```
Int[Log[t]^(-1 - n),t]
```

output
```
-((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

3.172.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

3.172.4 Maple [F]

$$\int \ln(t)^{-1-n} dt$$

input
```
int(ln(t)^(-1-n),t)
```

output
```
int(ln(t)^(-1-n),t)
```

3.172.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \log^{-1-n}(t)\, dt = e^{(i\pi+i\pi n)}\Gamma(-n, -\log(t))$$

input
```
integrate(log(t)^(-1-n),t, algorithm="fricas")
```

output
```
e^(I*pi + I*pi*n)*gamma(-n, -log(t))
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \log^{-1-n}(t)\, dt = (-\log(t))^{n+1}\log(t)^{-n-1}\Gamma(-n, -\log(t))$$

input
```
integrate(ln(t)**(-1-n),t)
```

output
```
(-log(t))**(n + 1)*log(t)**(-n - 1)*uppergamma(-n, -log(t))
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log^{-1-n}(t)\,dt = -(-\log(t))^n \log(t)^{-n}\,\Gamma(-n, -\log(t))$$

input
```
integrate(log(t)^(-1-n),t, algorithm="maxima")
```

output
```
-(-log(t))^n*log(t)^(-n)*gamma(-n, -log(t))
```

3.172.8 Giac [F]

$$\int \log^{-1-n}(t)\,dt = \int \log(t)^{-n-1}\,dt$$

input
```
integrate(log(t)^(-1-n),t, algorithm="giac")
```

output
```
integrate(log(t)^(-n - 1), t)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log^{-1-n}(t)\,dt = -\frac{(-\ln(t))^n\,\Gamma(-n, -\ln(t))}{\ln(t)^n}$$

input
```
int(1/log(t)^(n + 1),t)
```

output
```
-((-log(t))^n*igamma(-n, -log(t)))/log(t)^n
```

3.173 $\int \frac{e^{2t}}{-1+t}\,dt$

3.173.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\,\text{ExpIntegralEi}(-2(1-t))$$

output
```
exp(2)*Ei(-2+2*t)
```

3.173.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\,\text{ExpIntegralEi}(2(-1+t))$$

input
```
Integrate[E^(2*t)/(-1 + t),t]
```

output
```
E^2*ExpIntegralEi[2*(-1 + t)]
```

3.173.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2t}}{t-1}\,dt$$

$$\Big\downarrow \text{ 2609}$$

$$e^2 \,\text{ExpIntegralEi}(-2(1-t))$$

input
```
Int[E^(2*t)/(-1 + t),t]
```

output
```
E^2*ExpIntegralEi[-2*(1 - t)]
```

3.173.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

3.173.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\mathrm{e}^2 \,\mathrm{Ei}_1(-2t+2)$	12
default	$-\mathrm{e}^2 \,\mathrm{Ei}_1(-2t+2)$	12
risch	$-\mathrm{e}^2 \,\mathrm{Ei}_1(-2t+2)$	12

input
```
int(exp(2*t)/(-1+t),t,method=_RETURNVERBOSE)
```

output
```
-exp(2)*Ei(1,-2*t+2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^{2t}}{-1+t}\,dt = \mathrm{Ei}(2\,t-2)\,e^2$$

input
```
integrate(exp(2*t)/(-1+t),t, algorithm="fricas")
```

output
```
Ei(2*t - 2)*e^2
```

3.173.6 Sympy [F]

$$\int \frac{e^{2t}}{-1+t}\,dt = \int \frac{e^{2t}}{t-1}\,dt$$

input
```
integrate(exp(2*t)/(-1+t),t)
```

output
```
Integral(exp(2*t)/(t - 1), t)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^{2t}}{-1+t}\,dt = -e^2E_1(-2\,t+2)$$

input
```
integrate(exp(2*t)/(-1+t),t, algorithm="maxima")
```

output
```
-e^2*exp_integral_e(1, -2*t + 2)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^{2t}}{-1+t}\,dt = \mathrm{Ei}(2\,t-2)\,e^{2}$$

input
```
integrate(exp(2*t)/(-1+t),t, algorithm="giac")
```

output
```
Ei(2*t - 2)*e^2
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^{2t}}{-1+t}\,dt = \mathrm{e}^{2}\,\mathrm{ei}(2\,t-2)$$

input
```
int(exp(2*t)/(t - 1),t)
```

output
```
exp(2)*ei(2*t - 2)
```

3.174 $\int \frac{e^{2x}}{2-3x+x^2} \, dx$

3.174.1 Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{e^{2x}}{2-3x+x^2} \, dx = e^4 \operatorname{ExpIntegralEi}(-4+2x) - e^2 \operatorname{ExpIntegralEi}(-2+2x)$$

output
```
exp(4)*Ei(-4+2*x)-exp(2)*Ei(-2+2*x)
```

3.174.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{2-3x+x^2} \, dx = e^4 \operatorname{ExpIntegralEi}(-4+2x) - e^2 \operatorname{ExpIntegralEi}(-2+2x)$$

input
```
Integrate[E^(2*x)/(2 - 3*x + x^2),x]
```

output
```
E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

3.174.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2698, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{x^2-3x+2}\,dx$$

$$\Big\downarrow \text{2698}$$

$$\int \left(-\frac{2e^{2x}}{2x-2}-\frac{2e^{2x}}{4-2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^4\,\text{ExpIntegralEi}(2x-4)-e^2\,\text{ExpIntegralEi}(2x-2)$$

input
```
Int[E^(2*x)/(2 - 3*x + x^2),x]
```

output
```
E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

3.174.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2698
```
Int[(F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^
2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]
```

3.174.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\mathrm{e}^{4} \mathrm{Ei}_{1}\left(-2 x+4\right)+\mathrm{e}^{2} \mathrm{Ei}_{1}\left(-2 x+2\right)$	23
default	$-\mathrm{e}^{4} \mathrm{Ei}_{1}\left(-2 x+4\right)+\mathrm{e}^{2} \mathrm{Ei}_{1}\left(-2 x+2\right)$	23
risch	$-\mathrm{e}^{4} \mathrm{Ei}_{1}\left(-2 x+4\right)+\mathrm{e}^{2} \mathrm{Ei}_{1}\left(-2 x+2\right)$	23

input
```
int(exp(2*x)/(x^2-3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-exp(4)*Ei(1,-2*x+4)+exp(2)*Ei(1,-2*x+2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \mathrm{Ei}(2\,x-4)\,e^{4} - \mathrm{Ei}(2\,x-2)\,e^{2}$$

input
```
integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="fricas")
```

output
```
Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

3.174.6 Sympy [F]

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \int \frac{e^{2x}}{(x-2)\,(x-1)}\,dx$$

input
```
integrate(exp(2*x)/(x**2-3*x+2),x)
```

output
```
Integral(exp(2*x)/((x - 2)*(x - 1)), x)
```

3.174.7 Maxima [F]

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \int \frac{e^{(2\,x)}}{x^2-3\,x+2}\,dx$$

input
```
integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="maxima")
```

output
```
integrate(e^(2*x)/(x^2 - 3*x + 2), x)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \mathrm{Ei}(2\,x-4)\,e^4 - \mathrm{Ei}(2\,x-2)\,e^2$$

input
```
integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="giac")
```

output
```
Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2x}}{2-3x+x^2}\,dx = \int \frac{\mathrm{e}^{2\,x}}{x^2-3\,x+2}\,dx$$

input
```
int(exp(2*x)/(x^2 - 3*x + 2),x)
```

output
```
int(exp(2*x)/(x^2 - 3*x + 2), x)
```

3.175 $\int \frac{1}{\sqrt{1+t^3}}\,dt$

3.175.1 Optimal result

Integrand size = 9, antiderivative size = 103

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \frac{2\sqrt{2+\sqrt{3}}(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\sqrt{1+t^3}}$$

output
```
2/3*(1+t)*EllipticF((1+t-3^(1/2))/(1+t+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((t^2-t+1)/(1+t+3^(1/2))^2)^(1/2)*3^(3/4)/(t^3+1)^(1/2)/((1
+t)/(1+t+3^(1/2))^2)^(1/2)
```

3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = t\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-t^3\right)$$

input
```
Integrate[1/Sqrt[1 + t^3],t]
```

output
```
t*Hypergeometric2F1[1/3, 1/2, 4/3, -t^3]
```

3.175.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{t^3+1}}\,dt$$

$$\Big\downarrow \ 759$$

$$\frac{2\sqrt{2+\sqrt{3}}(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

input
```
Int[1/Sqrt[1 + t^3],t]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4*Sqrt[3]])/(3^(1/4
)*Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^3])
```

3.175.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.175.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.14

method	result	size
meijerg	$t\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-t^3\right)$	14
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$	116
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$	116

input
```
int(1/(t^3+1)^(1/2),t,method=_RETURNVERBOSE)
```

output
```
t*hypergeom([1/3,1/2],[4/3],-t^3)
```

3.175.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = 2\,\text{weierstrassPInverse}(0,-4,t)$$

input
```
integrate(1/(t^3+1)^(1/2),t, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4, t)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \frac{t\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| t^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(t**3+1)**(1/2),t)
```

output
```
t*gamma(1/3)*hyper((1/3, 1/2), (4/3,), t**3*exp_polar(I*pi))/(3*gamma(4/3)
)
```

3.175.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \int \frac{1}{\sqrt{t^3+1}}\,dt$$

input
```
integrate(1/(t^3+1)^(1/2),t, algorithm="maxima")
```

output
```
integrate(1/sqrt(t^3 + 1), t)
```

3.175.8 Giac [F]

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \int \frac{1}{\sqrt{t^3+1}}\,dt$$

input
```
integrate(1/(t^3+1)^(1/2),t, algorithm="giac")
```

output
```
integrate(1/sqrt(t^3 + 1), t)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{t-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-t+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{t^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)t-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(t^3 + 1)^(1/2),t)
```

output
```
((3^(1/2)*1i + 3)*((t + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - t + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(t^3 - t*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2))^(1/2)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
              finalresult={"A"," "}
            ,(*ELSE*)
              finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
             ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```